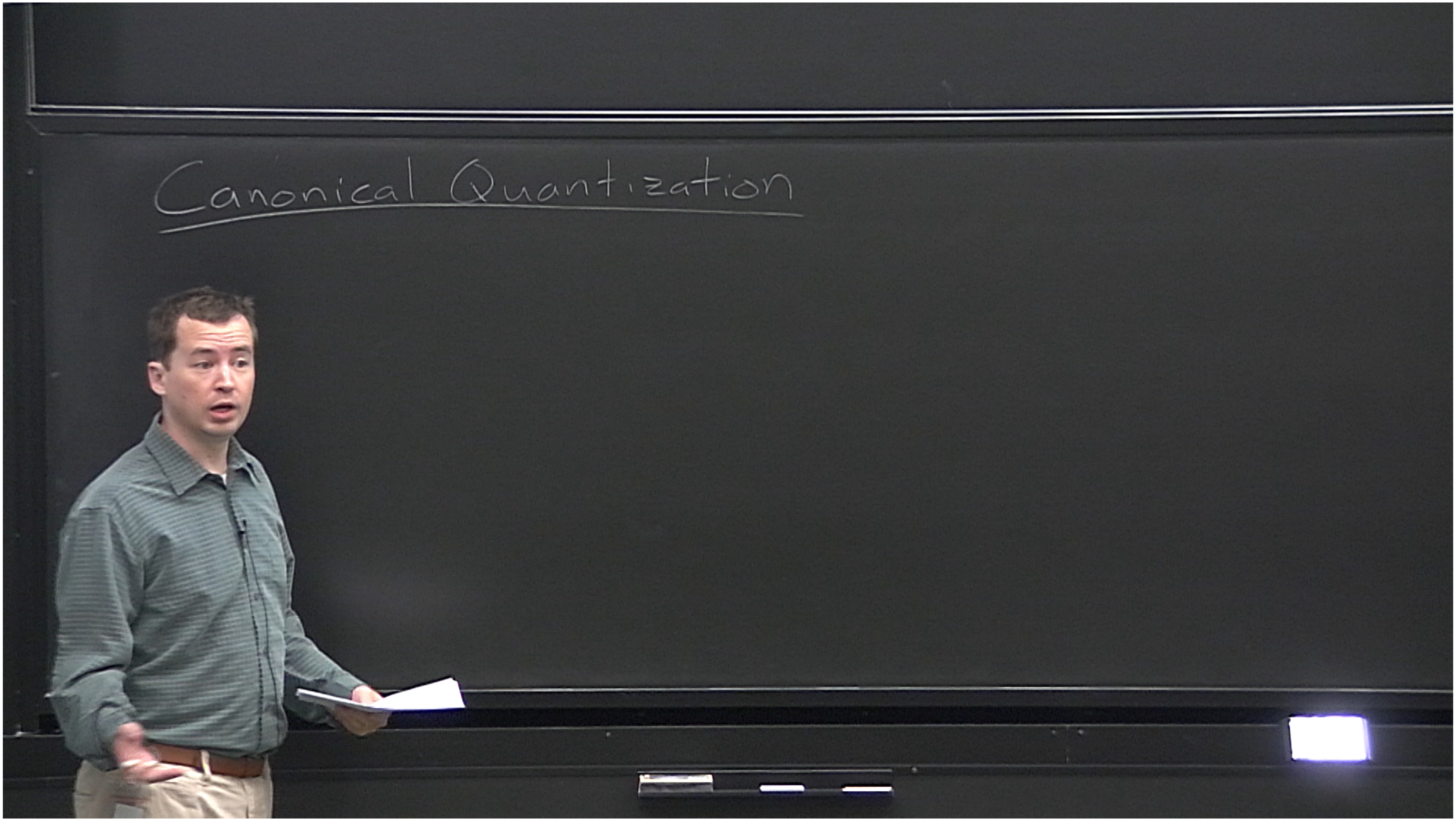
Title: PSI 2018/2019 - Quantum Field Theory I - Lecture 2

Date: Oct 10, 2018 09:00 AM

URL: http://pirsa.org/18100043

Abstract:

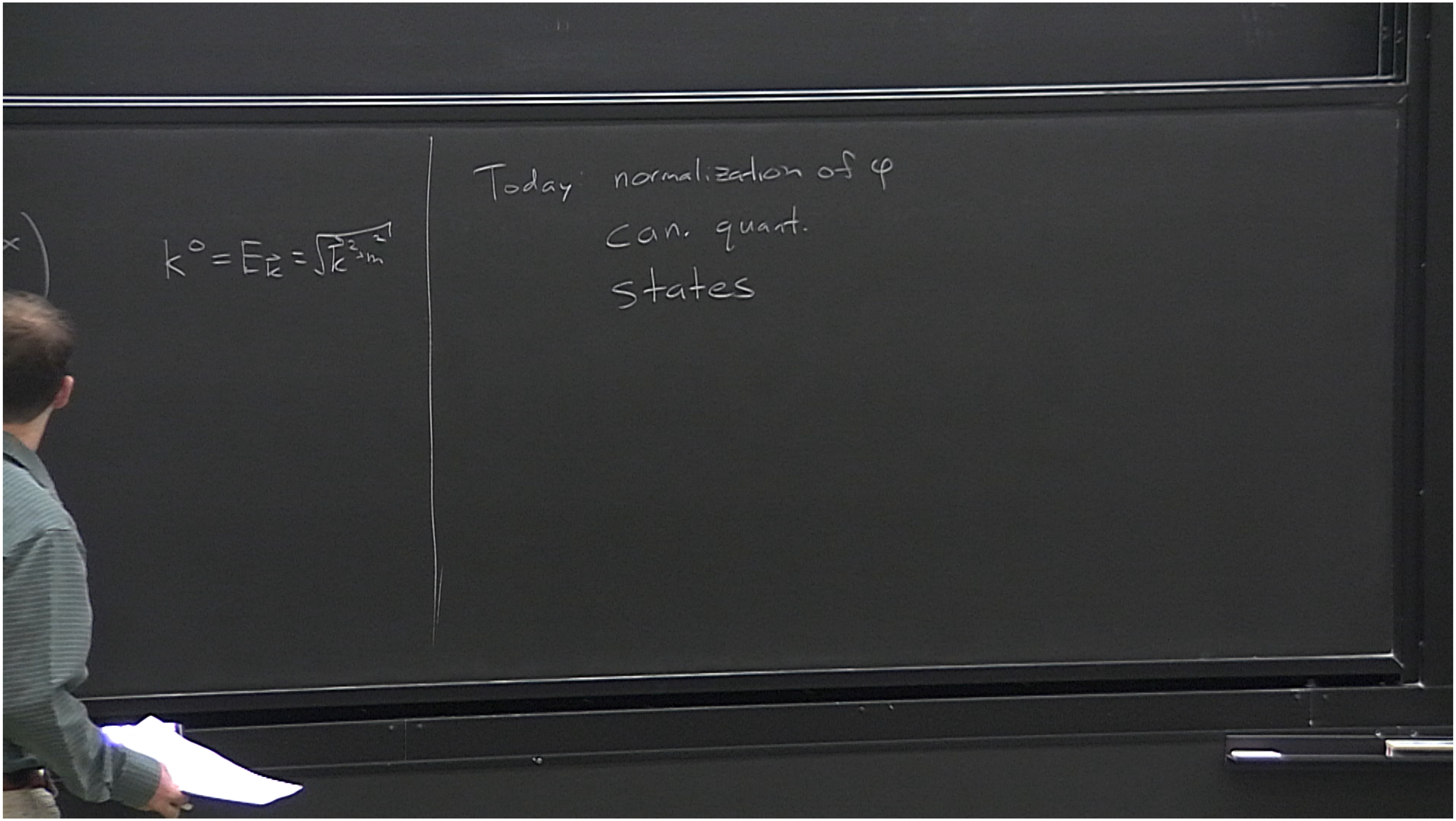

Canonical Quantization

Canonical Quantization

$$\varphi(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^3}\left(A(\vec{k})\, e^{-ik\cdot x} + A^*(\vec{k})\, e^{ik\cdot x} \right) \qquad k^0 = E_{\vec{k}} = \sqrt{\vec{k}^2 + m^2}$$

Today: normalization of φ

$$k^0 = E_{\vec{k}} = \sqrt{\vec{k}^2 + m^2}$$

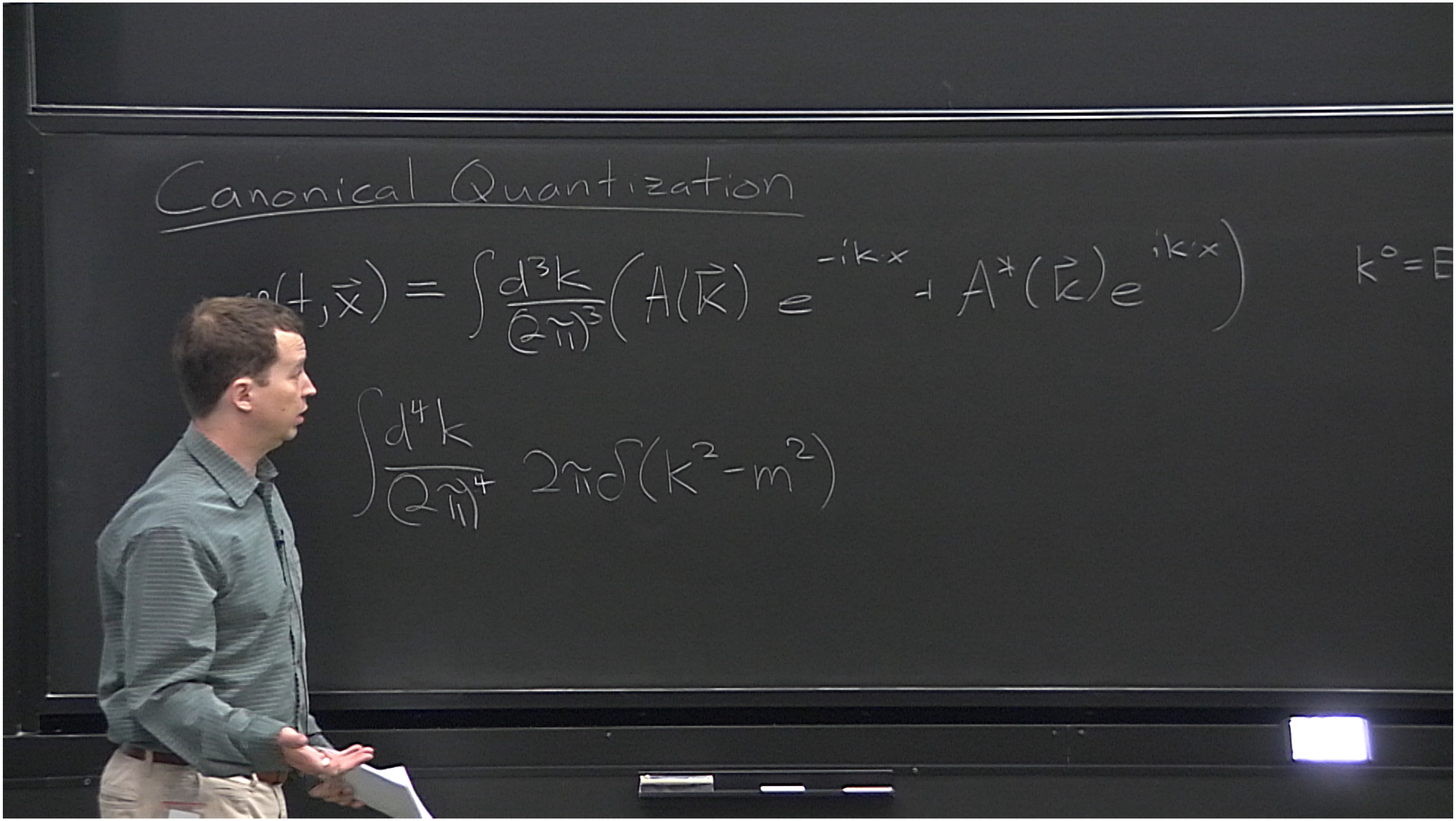

$k^0 = E_{\vec{k}} = \sqrt{\vec{k}^2 + m^2}$
Today: normalization of φ
can. quant.
states

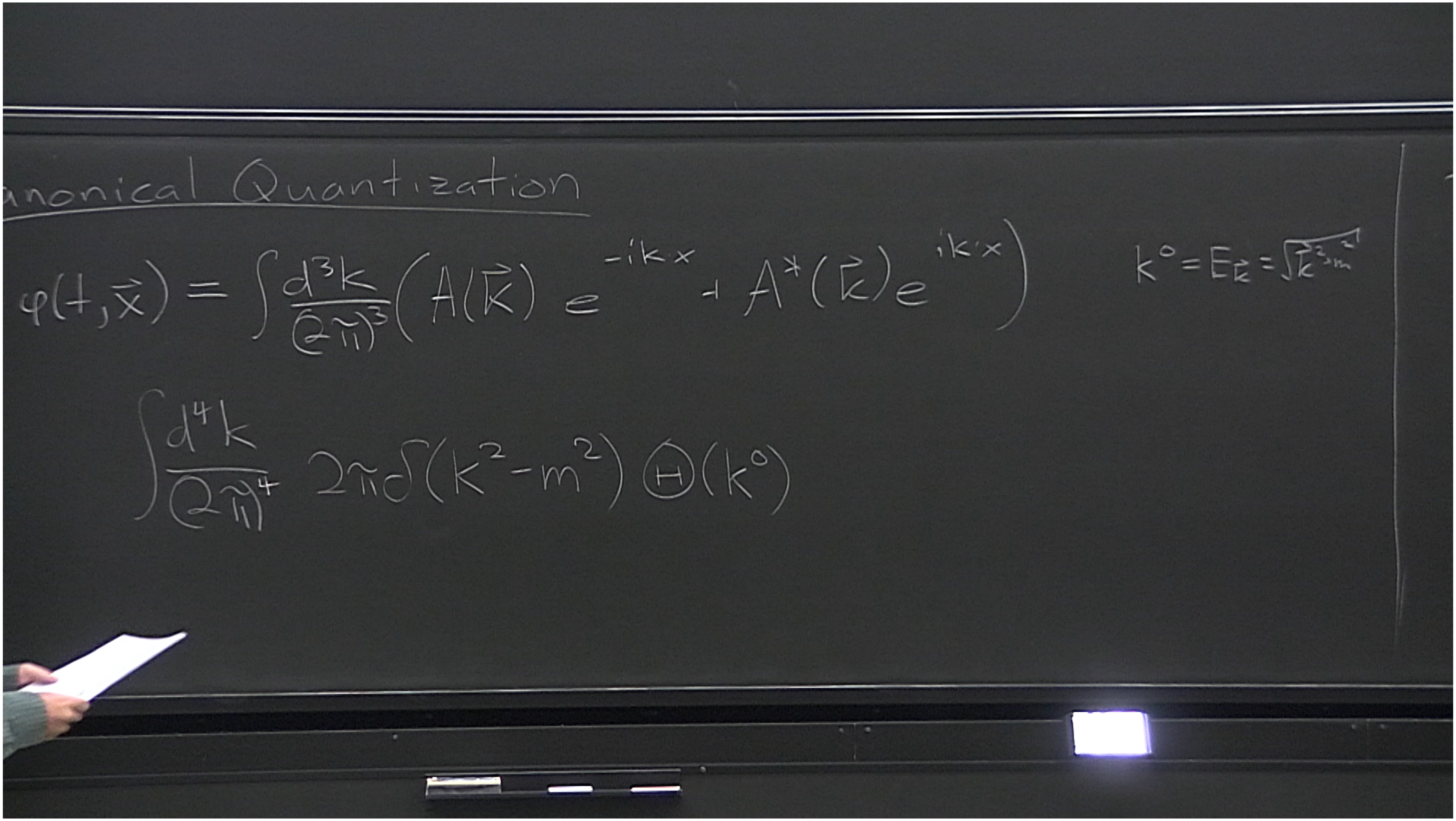
$$k^0 = E_{\vec{k}} = \sqrt{\vec{k}^2 + m^2}$$

Today: normalization of φ

can. quant.
states $\Big\}$ Schrödinger

Heisenberg


Canonical Quantization
(t, x⃗) = ∫ d³k/(2π)³ (A(k⃗) e^{-ik·x} + A*(k⃗) e^{ik·x})
k⁰=E
∫ d⁴k/(2π)⁴ 2πδ(k²-m²)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$k^0 = E_{\vec{k}} = \sqrt{\vec{k}^2 + m^2}$
$\int \frac{d^4k}{(2\pi)^4}\, 2\pi\delta(k^2 - m^2)\, \Theta(k^0)$

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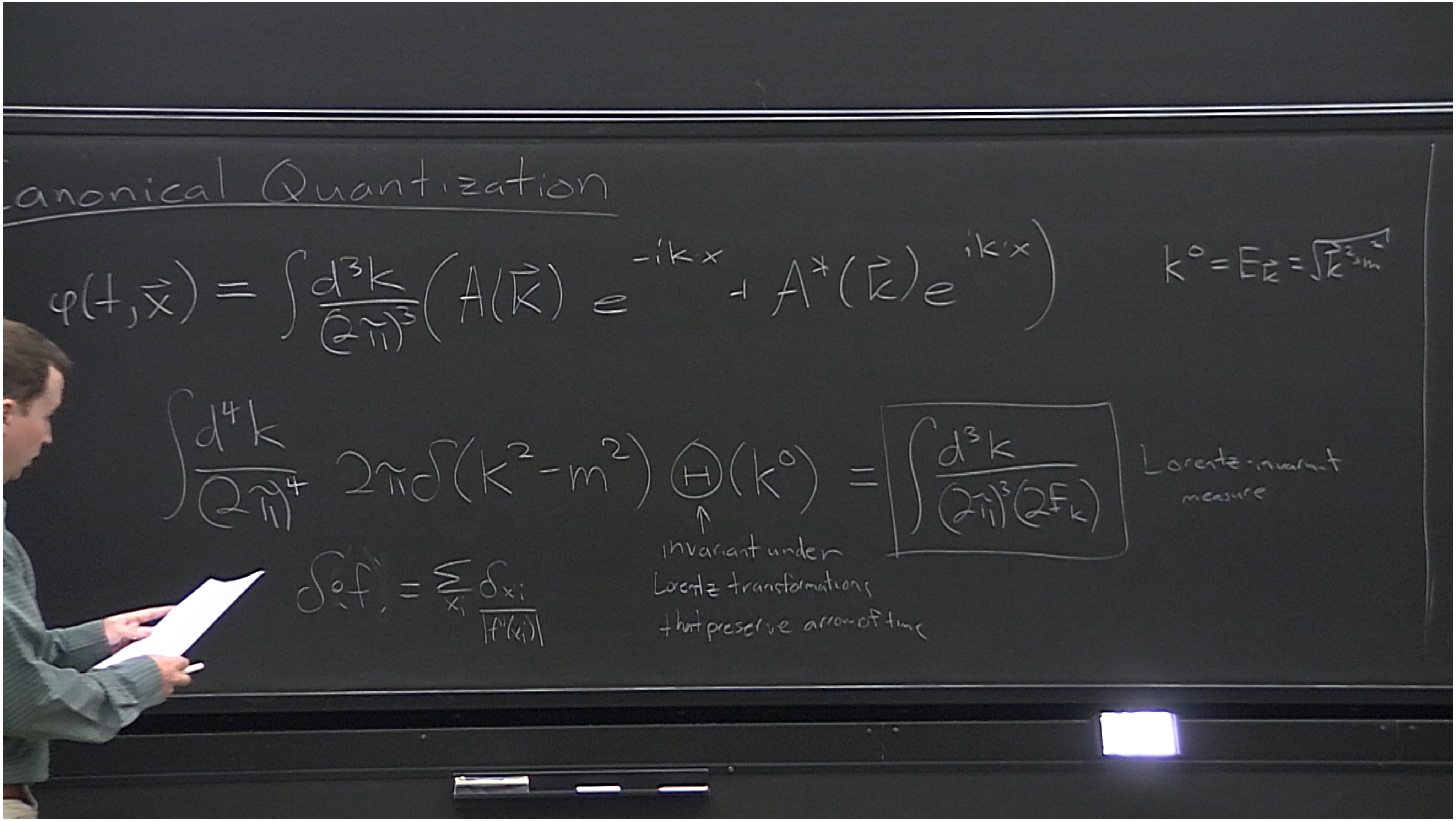
invariant under
Lorentz transformations
that preserve arrow of time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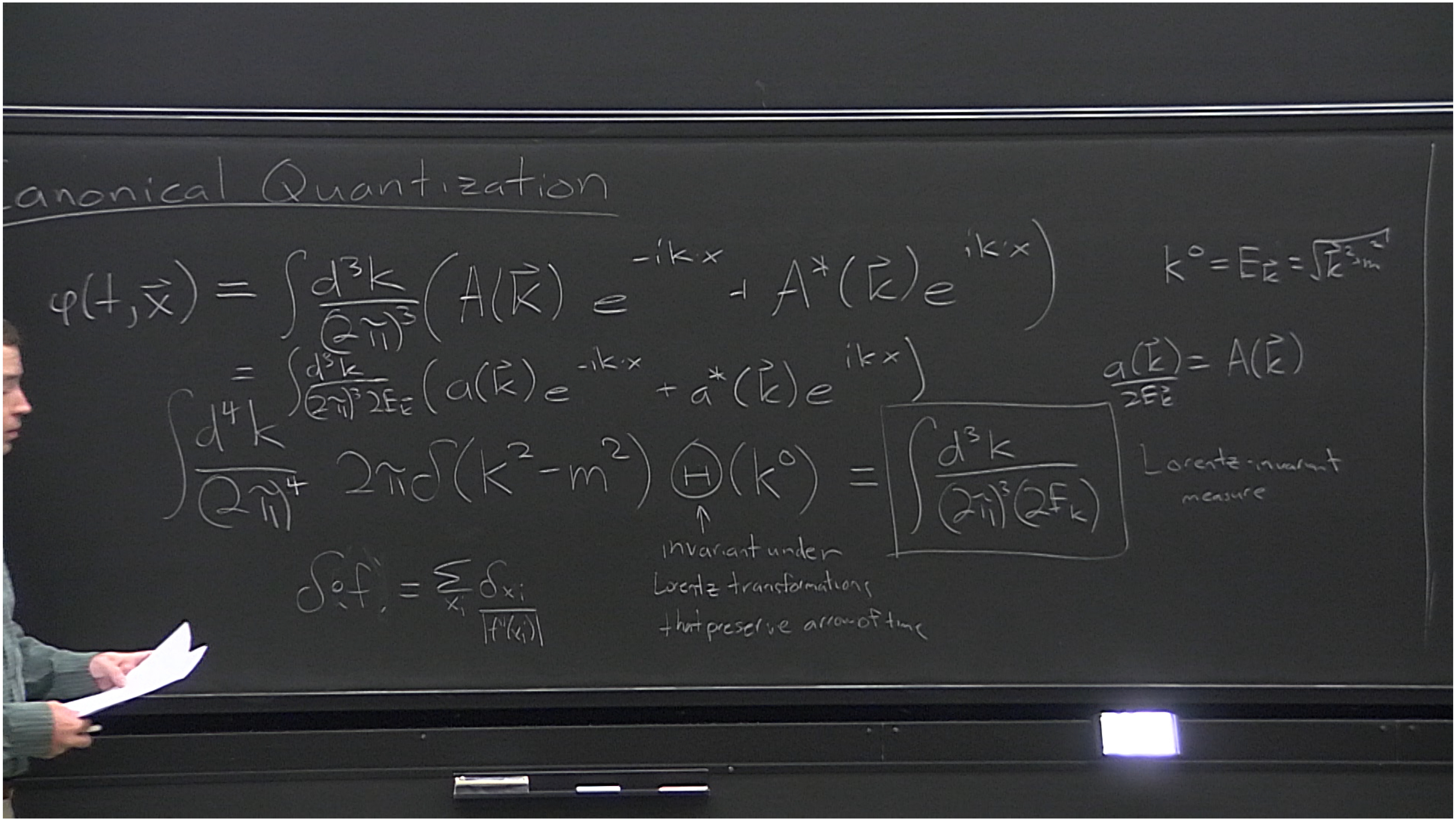
$k^0 = E_{\vec{k}} = \sqrt{\vec{k}^2+m^2}$
$\int \frac{d^4k}{(2\pi)^4}\, 2\pi\,\delta(k^2-m^2)\,\Theta(k^0) = \boxed{\int \frac{d^3k}{(2\pi)^3(2E_k)}}$
Lorentz-invariant measure
invariant under Lorentz transformations that preserve arrow of time
$\delta \circ f = \sum_{x_i} \frac{\delta_{x_i}}{|f'(x_i)|}$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$\varphi(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^3}\left(A(\vec{k})\, e^{-ik\cdot x} + A^*(\vec{k})\, e^{ik\cdot x}\right)$
$k^0 = E_{\vec{k}} = \sqrt{\vec{k}^2 + m^2}$
$= \int \frac{d^3k}{(2\pi)^3\, 2E_{\vec{k}}}\left(a(\vec{k})\, e^{-ik\cdot x} + a^*(\vec{k})\, e^{ik\cdot x}\right)$
$\frac{a(\vec{k})}{2E_{\vec{k}}} = A(\vec{k})$
$\int \frac{d^4k}{(2\pi)^4}\, 2\pi\delta(k^2 - m^2)\, \Theta(k^0) = \boxed{\int \frac{d^3k}{(2\pi)^3 (2E_k)}}$
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$$= \boxed{\int \frac{d^3k}{(2\pi)^3\, 2E_{\vec{k}}}\left(a(\vec{k})\, e^{-ik\cdot x} + a^*(\vec{k})\, e^{ik\cdot x}\right)}$$

$$k^0 = E_{\vec{k}} = \sqrt{\vec{k}^2 + m^2}$$

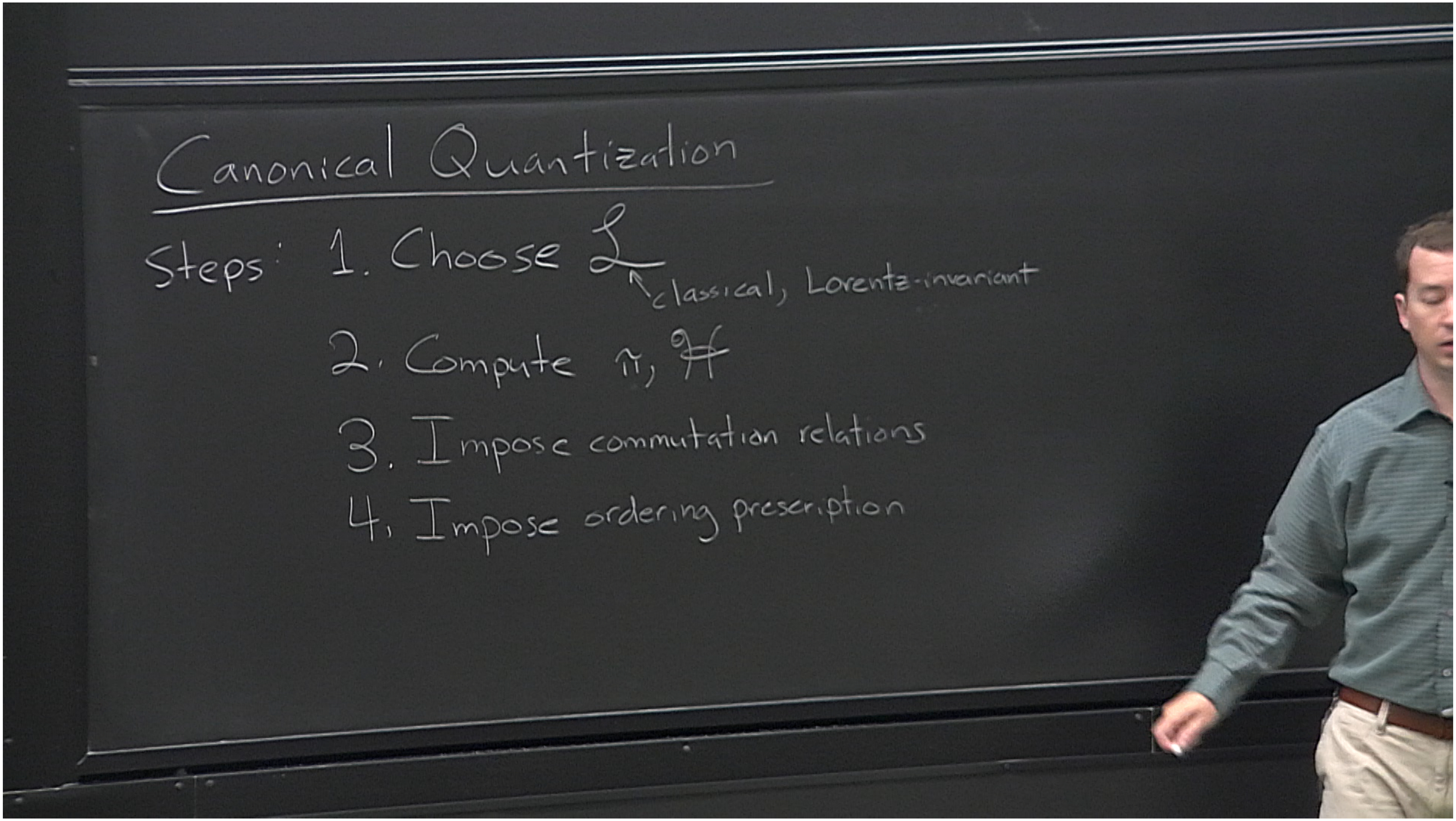
$$\frac{a(\vec{k})}{2E_{\vec{k}}} = A(\vec{k})$$

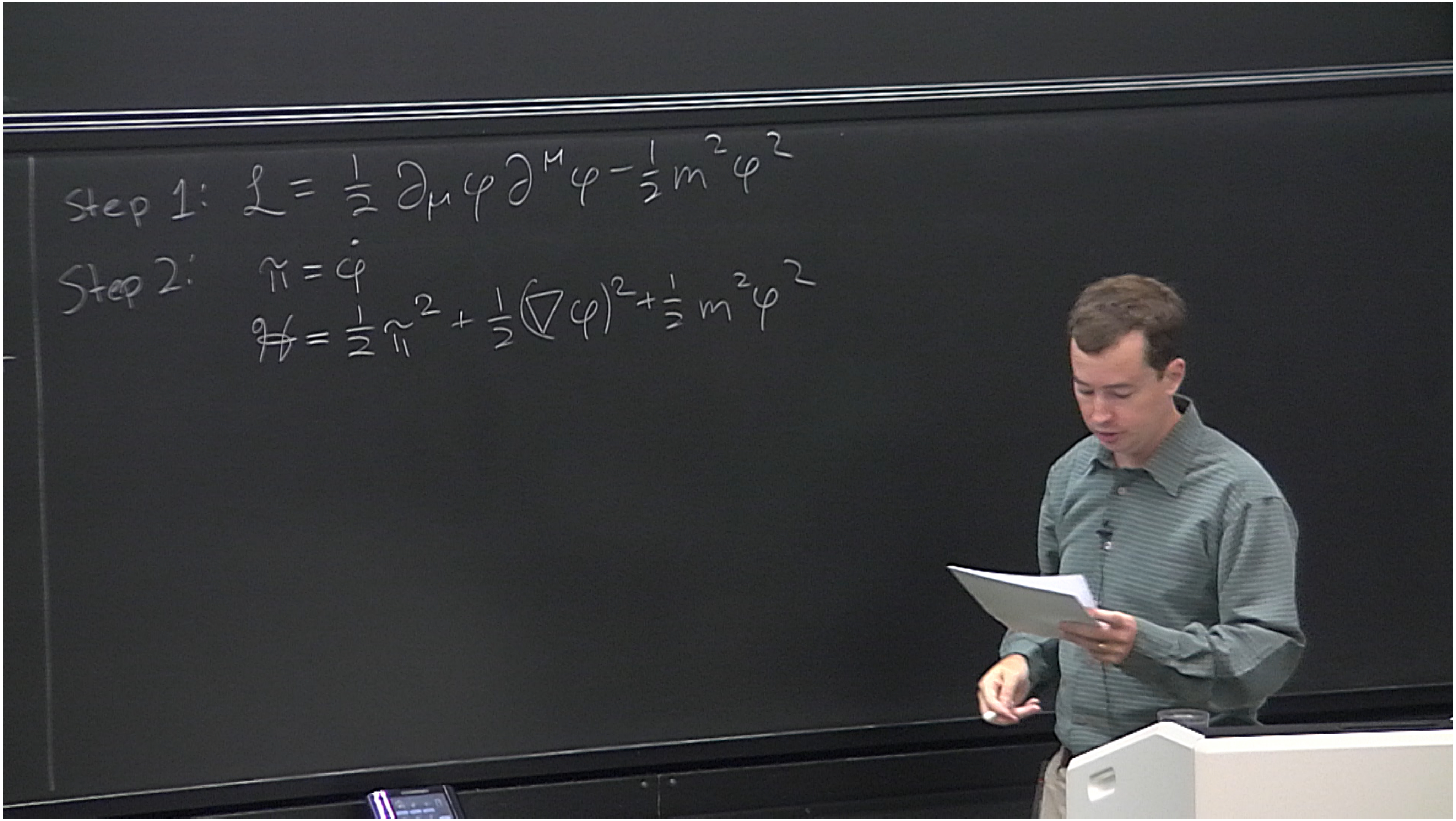
$$\int \frac{d^4k}{(2\pi)^4}\, 2\pi\,\delta(k^2 - m^2)\, \Theta(k^0) = \boxed{\int \frac{d^3k}{(2\pi)^3 (2E_k)}}$$

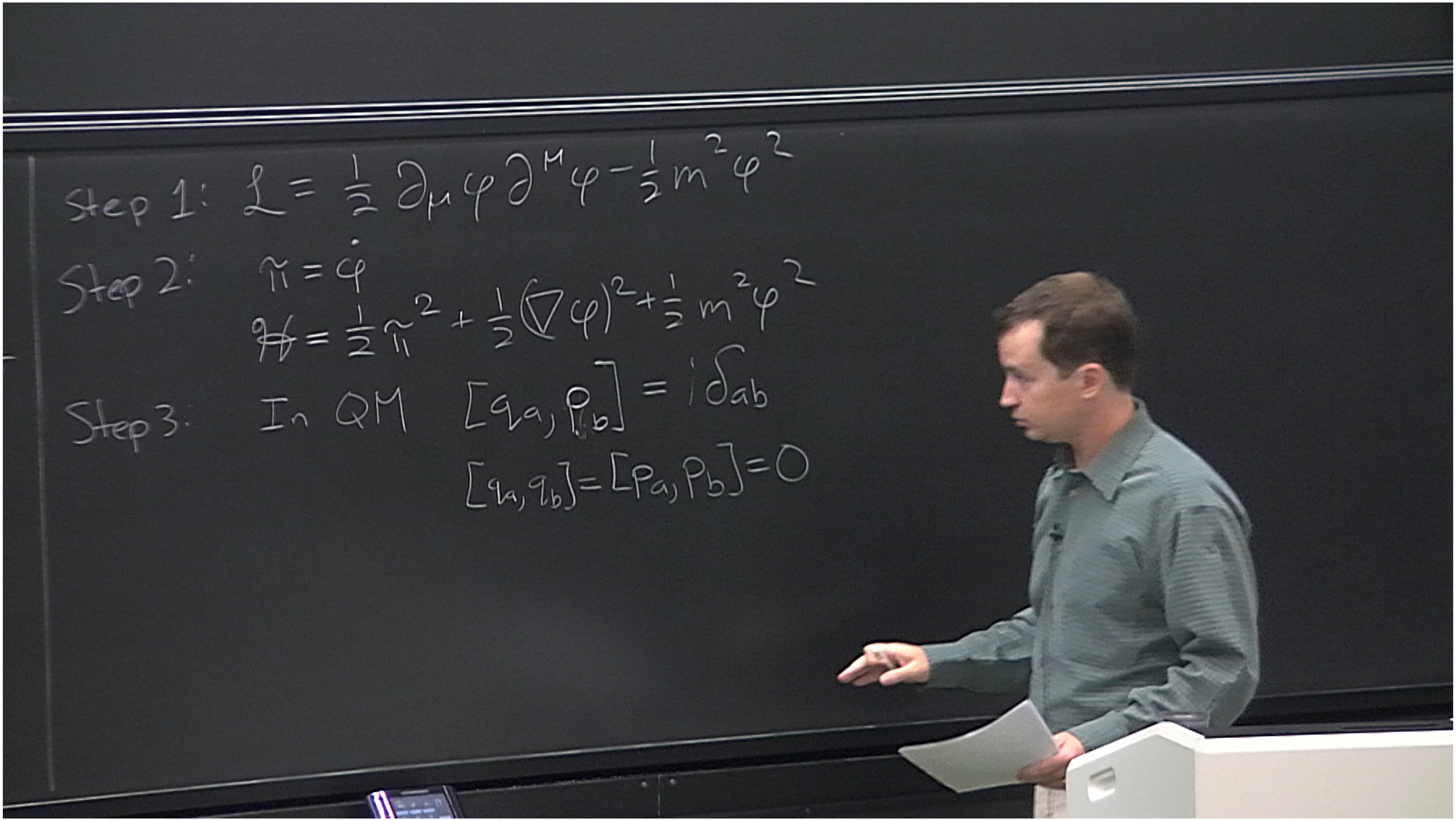
Lorentz-invariant measure

invariant under Lorentz transformations that preserve arrow of time

$$\delta \circ f = \sum_{x_i} \frac{\delta_{x_i}}{|f'(x_i)|}$$


Canonical Quantization
Steps: 1. Choose $\mathcal{L}$
classical, Lorentz-invariant
2. Compute π, $\mathcal{H}$
3. Impose commutation relations
4. Impose ordering prescription


Step 1: $\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi - \frac{1}{2}m^2\varphi^2$
Step 2: $\pi = \dot{\varphi}$
$\mathcal{H} = \frac{1}{2}\pi^2 + \frac{1}{2}(\nabla\varphi)^2 + \frac{1}{2}m^2\varphi^2$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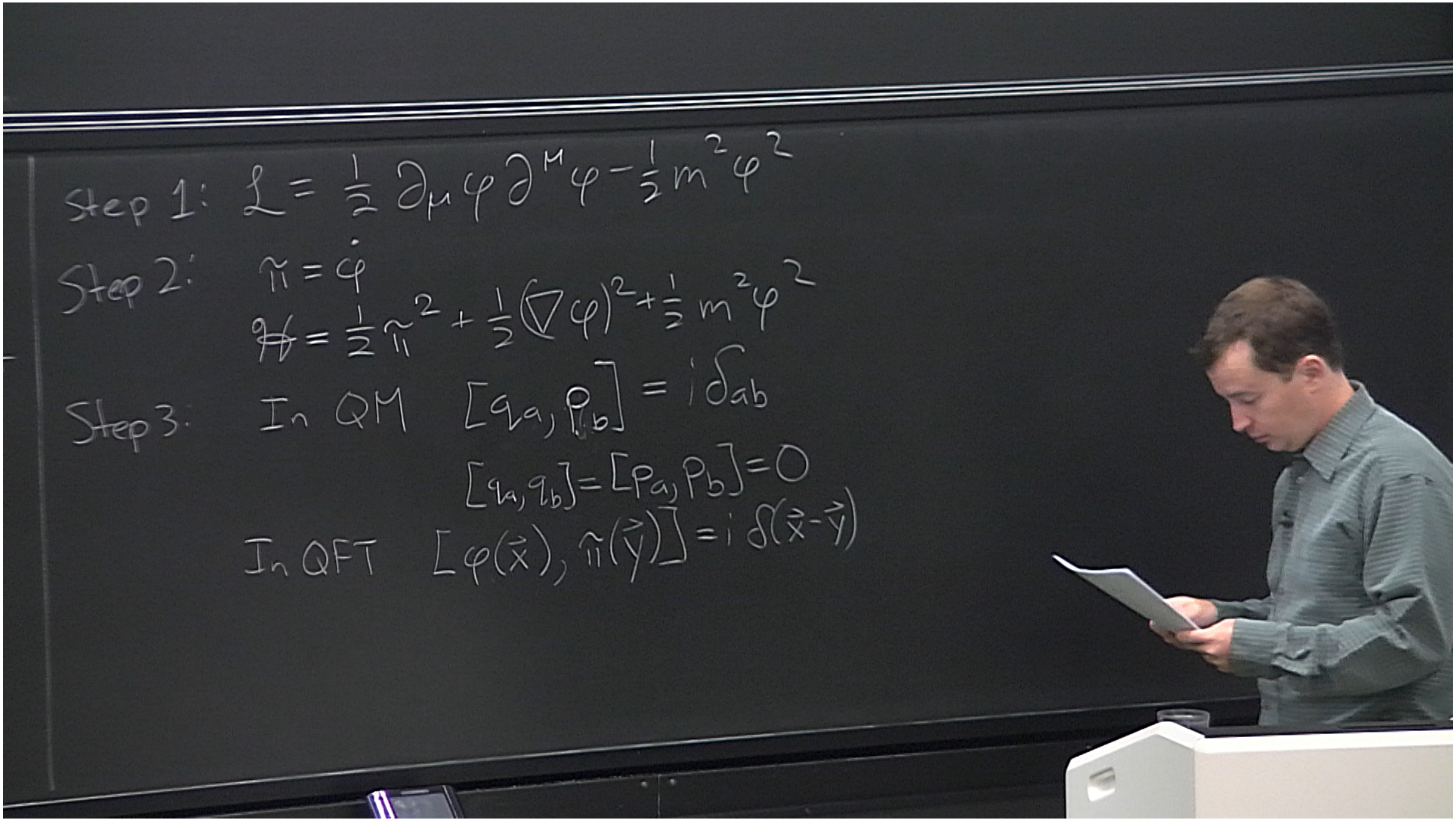
Step 3: In QM $[q_a, p_b] = i\delta_{ab}$
$[q_a, q_b] = [p_a, p_b] = 0$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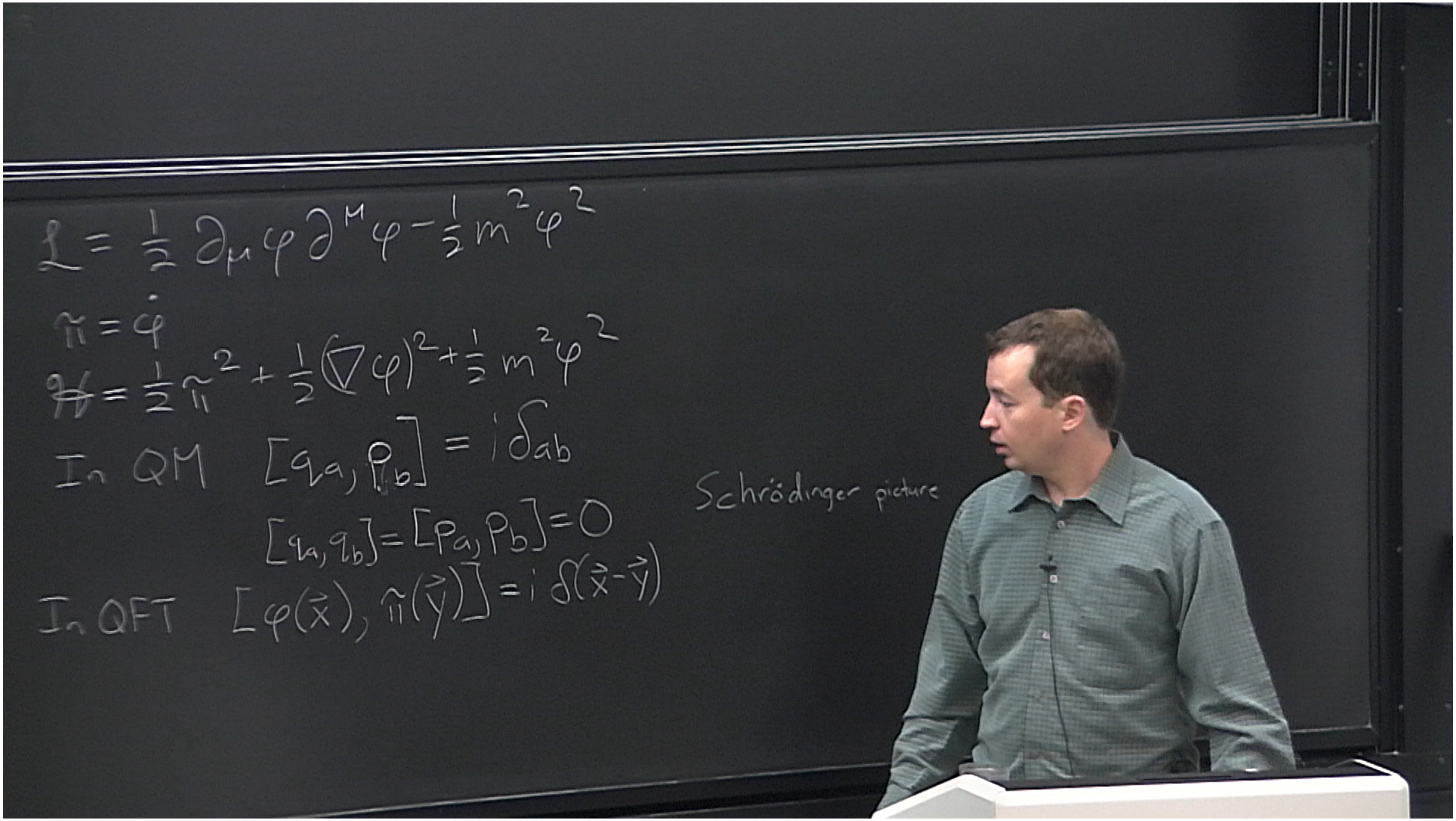
In QFT $[\varphi(\vec{x}), \pi(\vec{y})] = i\delta(\vec{x} - \vec{y})$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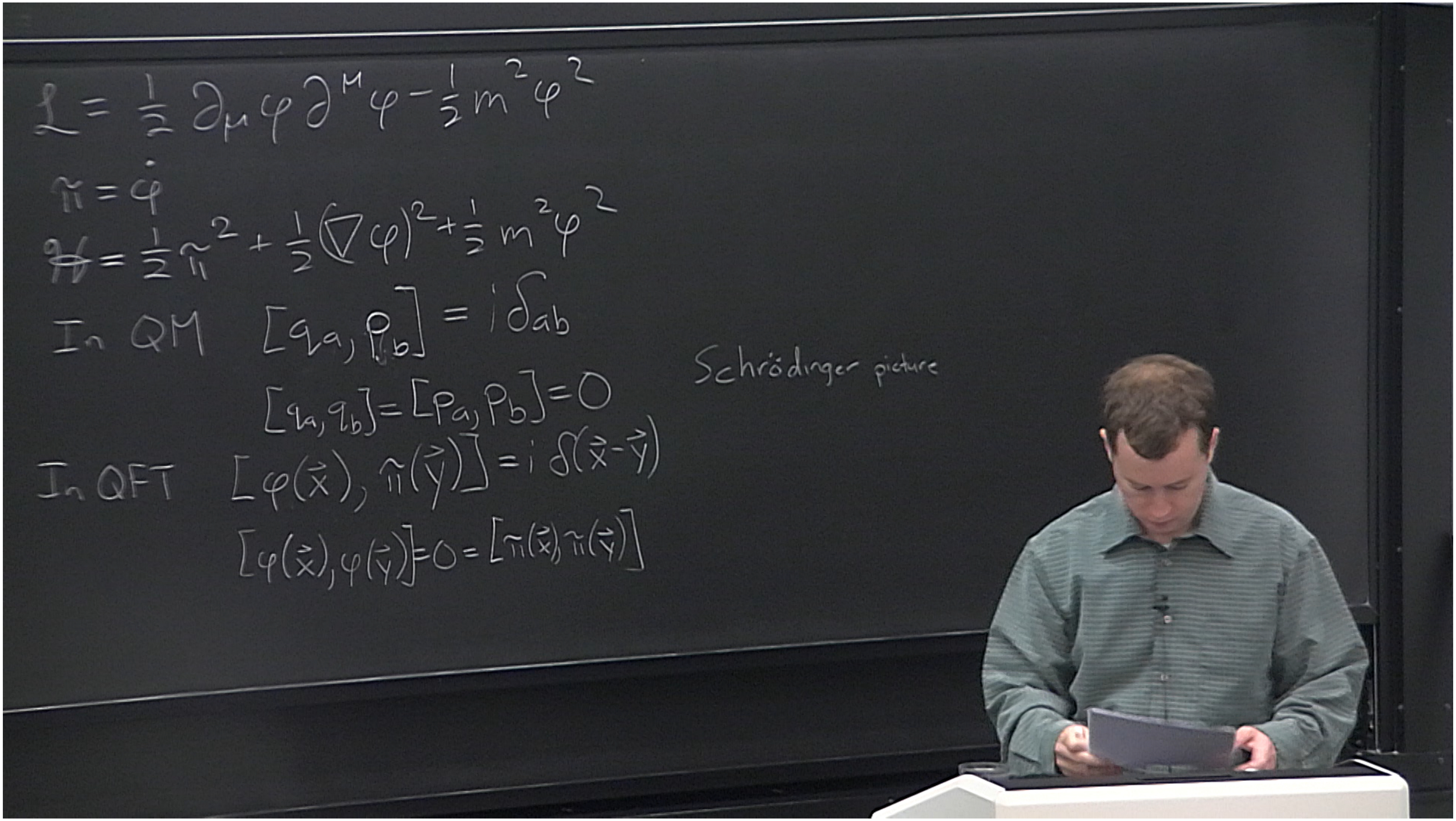
Schrödinger picture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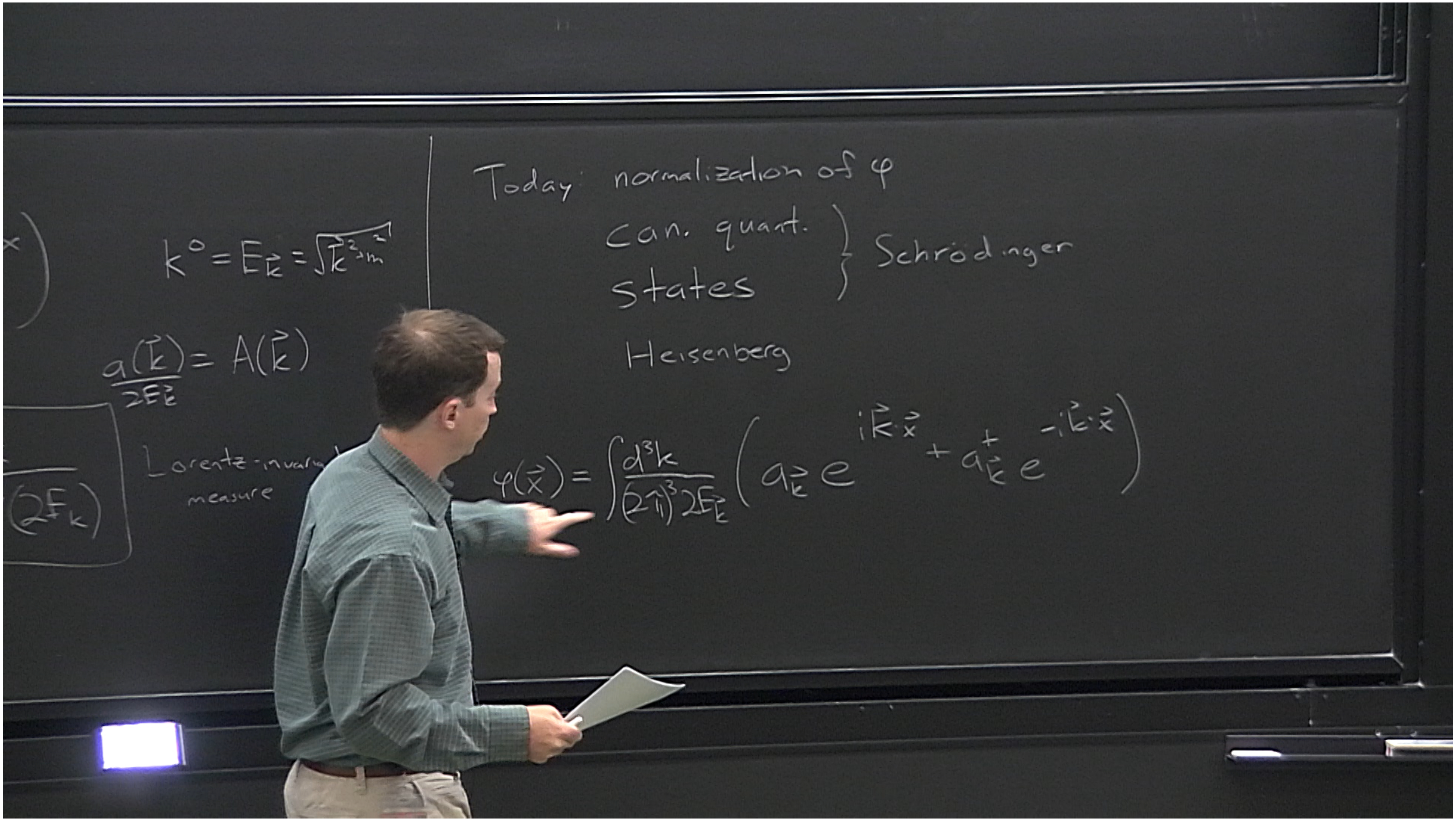
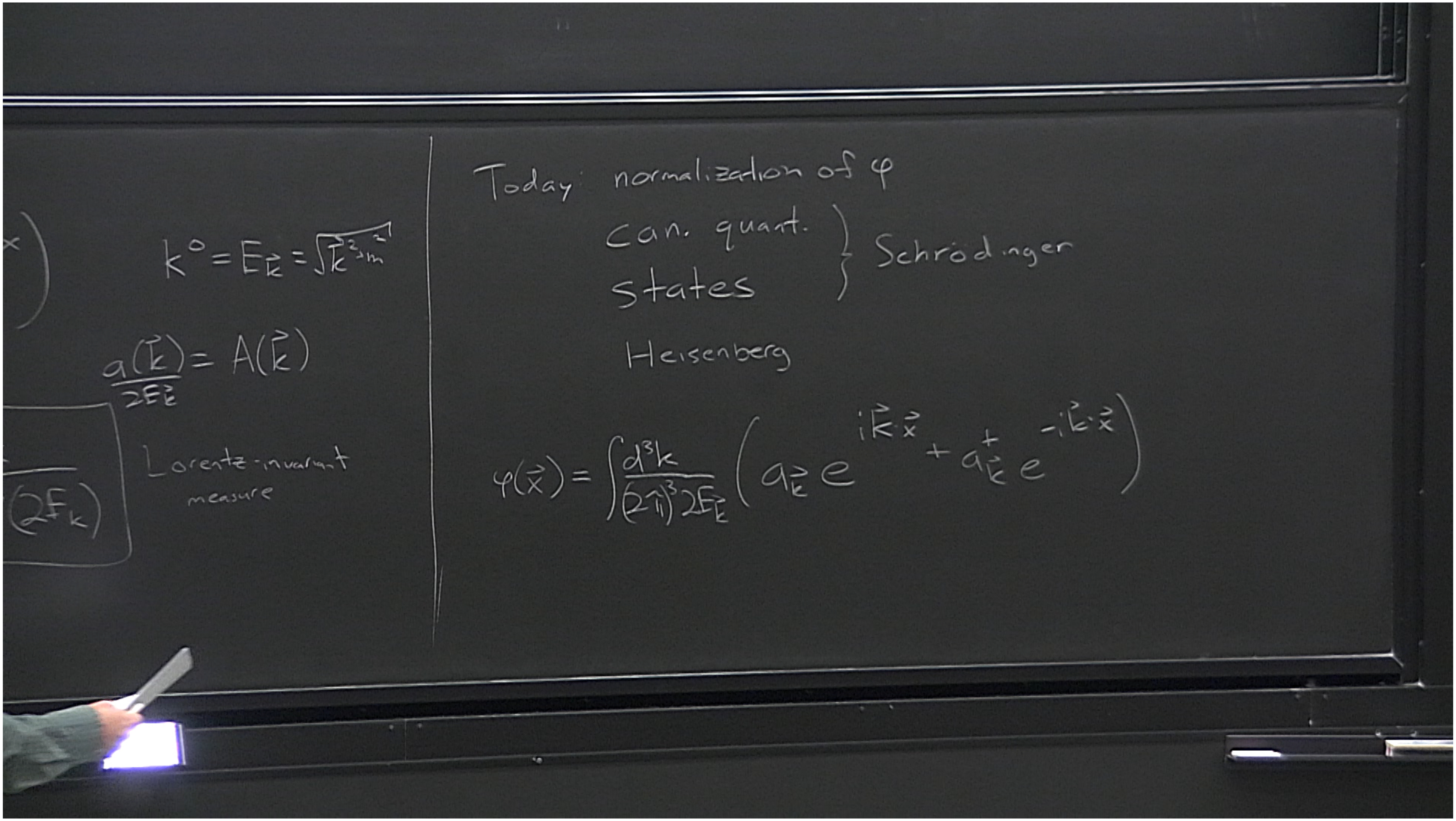
Schrödinger picture
In QFT $[\varphi(\vec{x}), \pi(\vec{y})] = i\delta(\vec{x}-\vec{y})$
$[\varphi(\vec{x}), \varphi(\vec{y})] = 0 = [\pi(\vec{x}), \pi(\vec{y})]$


Today: normalization of φ
can. quant.
states
Schrödinger
Heisenberg
k⁰ = E_k = √(k² + m²)
a(k)/2E_k = A(k)
Lorentz-invariant
measure
φ(x) = ∫ d³k/((2π)³ 2E_k) (a_k e^{ik·x} + a_k^† e^{-ik·x})


$k^0 = E_{\vec{k}} = \sqrt{\vec{k}^2 + m^2}$
$\frac{a(\vec{k})}{2E_{\vec{k}}} = A(\vec{k})$
Lorentz-invariant measure
Today: normalization of φ
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$\varphi(\vec{x}) = \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} \left(a_{\vec{k}} e^{i\vec{k}\cdot\vec{x}} + a^+_{\vec{k}} e^{-i\vec{k}\cdot\vec{x}} \right)$

1: $\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi - \frac{1}{2}m^2\varphi^2$

$\pi = \dot{\varphi}$

$\mathcal{H} = \frac{1}{2}\pi^2 + \frac{1}{2}(\nabla\varphi)^2 + \frac{1}{2}m^2\varphi^2$

In QM $[q_a, p_b] = i\delta_{ab}$

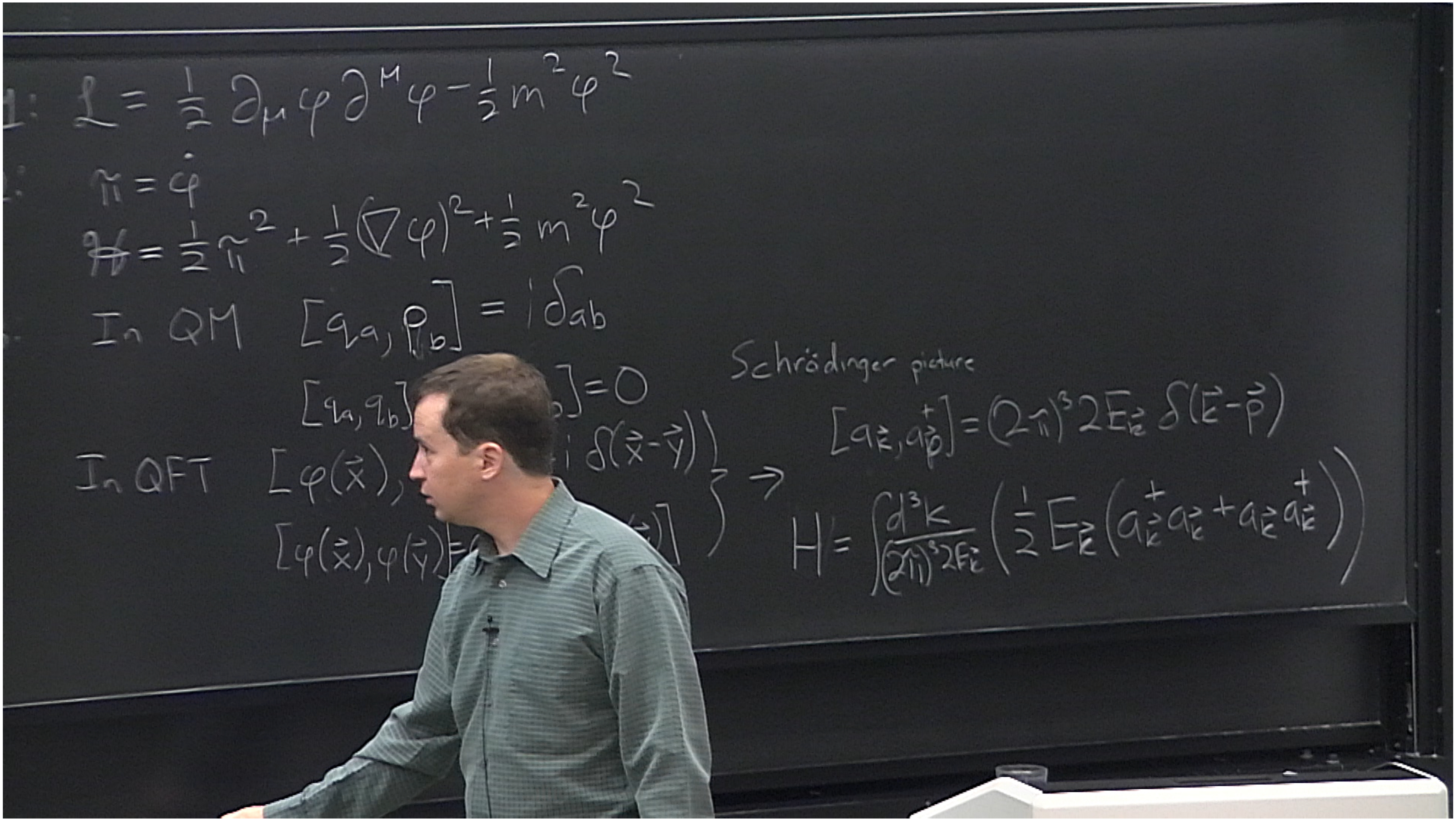
$[q_a, q_b] = [p_a, p_b] = 0$

Schrödinger picture

In QFT $[\varphi(\vec{x}), \pi(\vec{y})] = i\,\delta(\vec{x}-\vec{y})$

$[\varphi(\vec{x}), \varphi(\vec{y})] = 0 = [\pi(\vec{x}), \pi(\vec{y})]$

$\rightarrow \; [a_{\vec{k}}, a^{\dagger}_{\vec{p}}] = (2\pi)^3\, 2E_{\vec{k}}\,\delta(\vec{k}-\vec{p})$



$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi - \frac{1}{2}m^2\varphi^2$
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In QM $[q_a, p_b] = i\delta_{ab}$
$[q_a, q_b]$
$] = 0$
Schrödinger picture
$[a_{\vec k}, a^\dagger_{\vec p}] = (2\pi)^3\, 2E_{\vec k}\,\delta(\vec k - \vec p)$
In QFT $[\varphi(\vec x),$
$i\delta(\vec x - \vec y)]$
$[\varphi(\vec x), \varphi(\vec y)] =$
$H = \int \frac{d^3k}{(2\pi)^3 2E_{\vec k}}\left(\frac{1}{2}E_{\vec k}\left(a^\dagger_{\vec k}a_{\vec k} + a_{\vec k}a^\dagger_{\vec k}\right)\right)$

$$\mathcal{H} = \frac{1}{2}\pi^2 + \frac{1}{2}(\nabla\varphi)^2 + \frac{1}{2}m^2\varphi^2$$

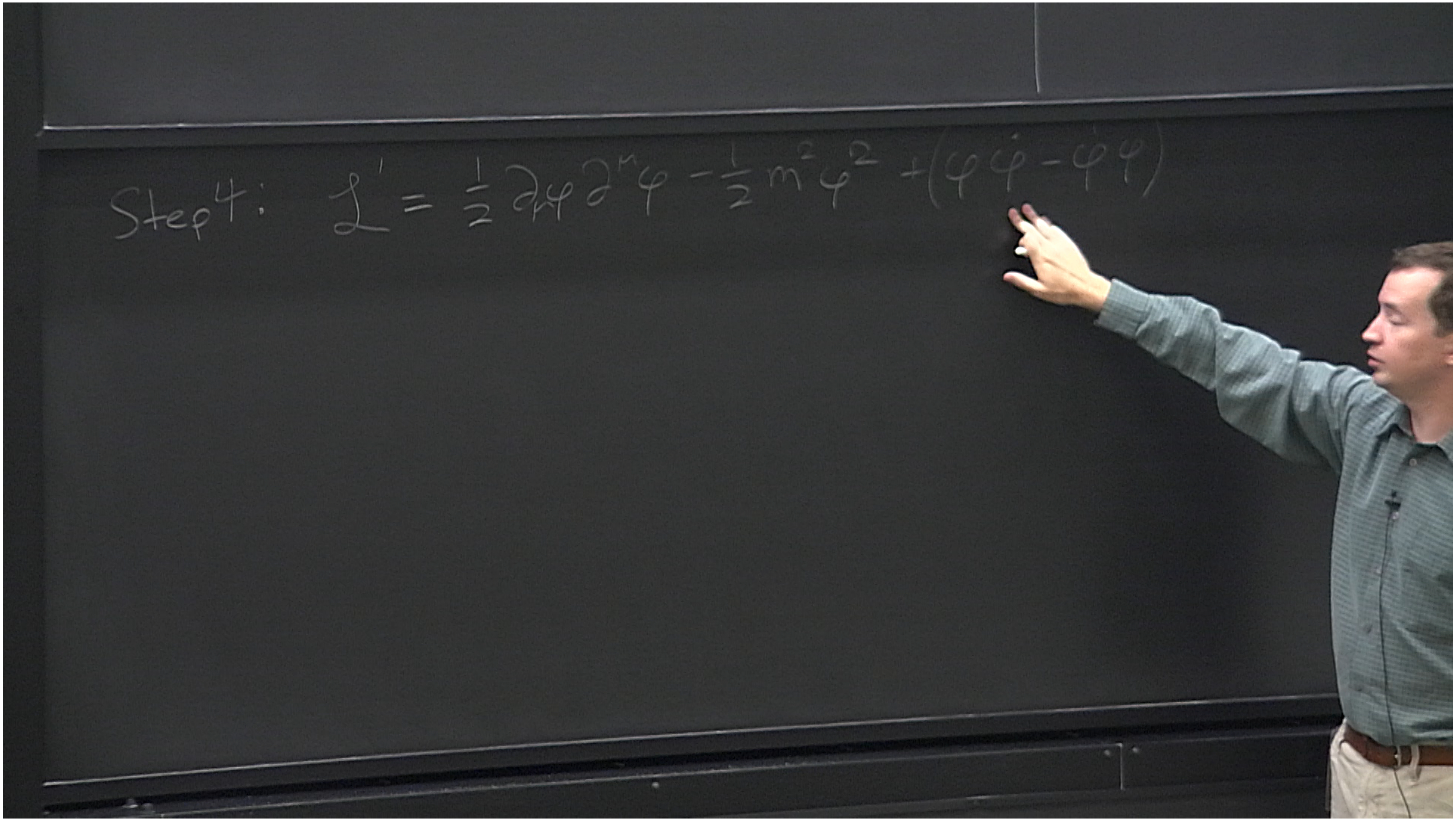
In QM $\quad [q_a, p_b] = i\delta_{ab}$

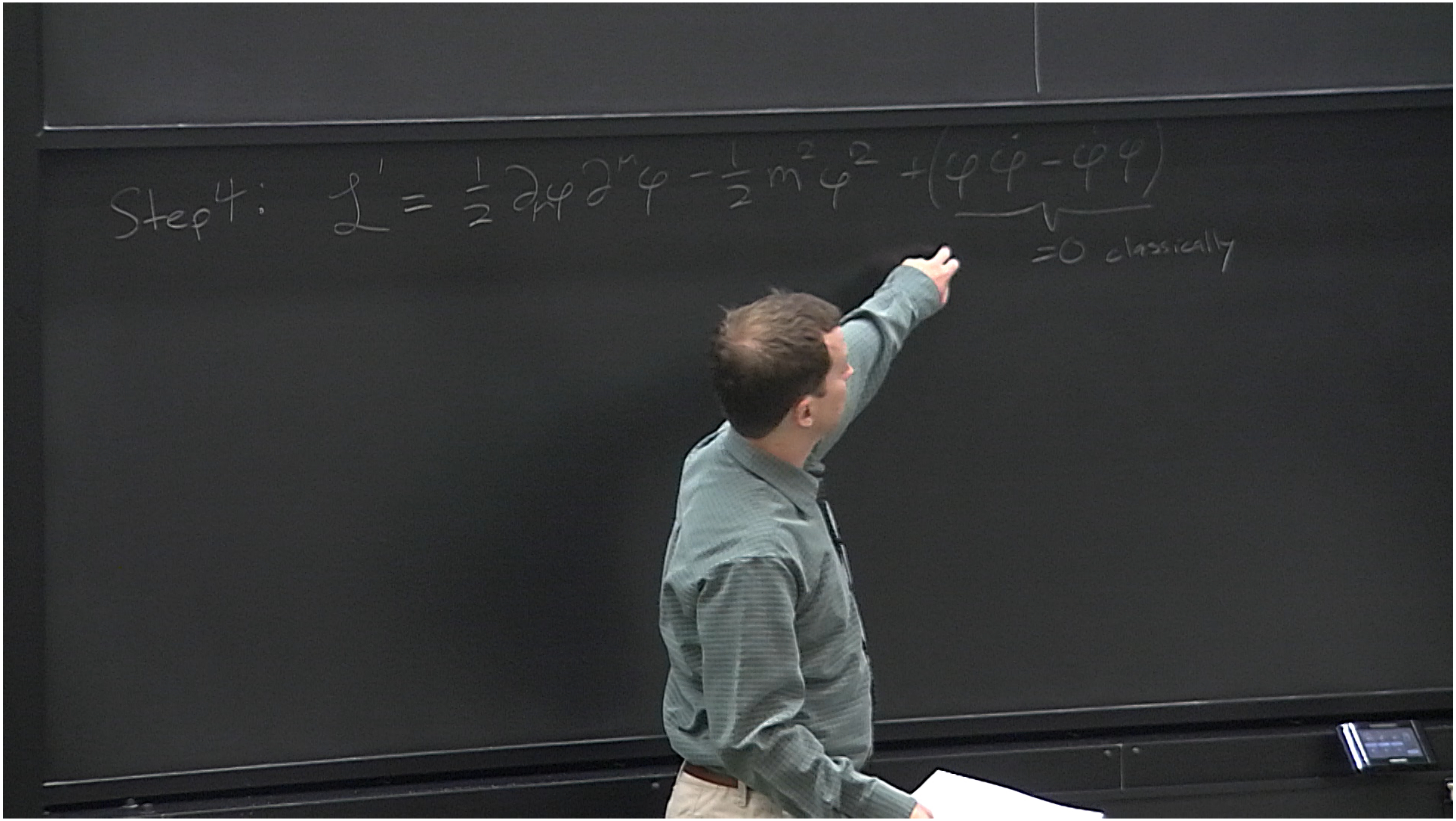
$$[q_a, q_b] = [p_a, p_b] = 0$$

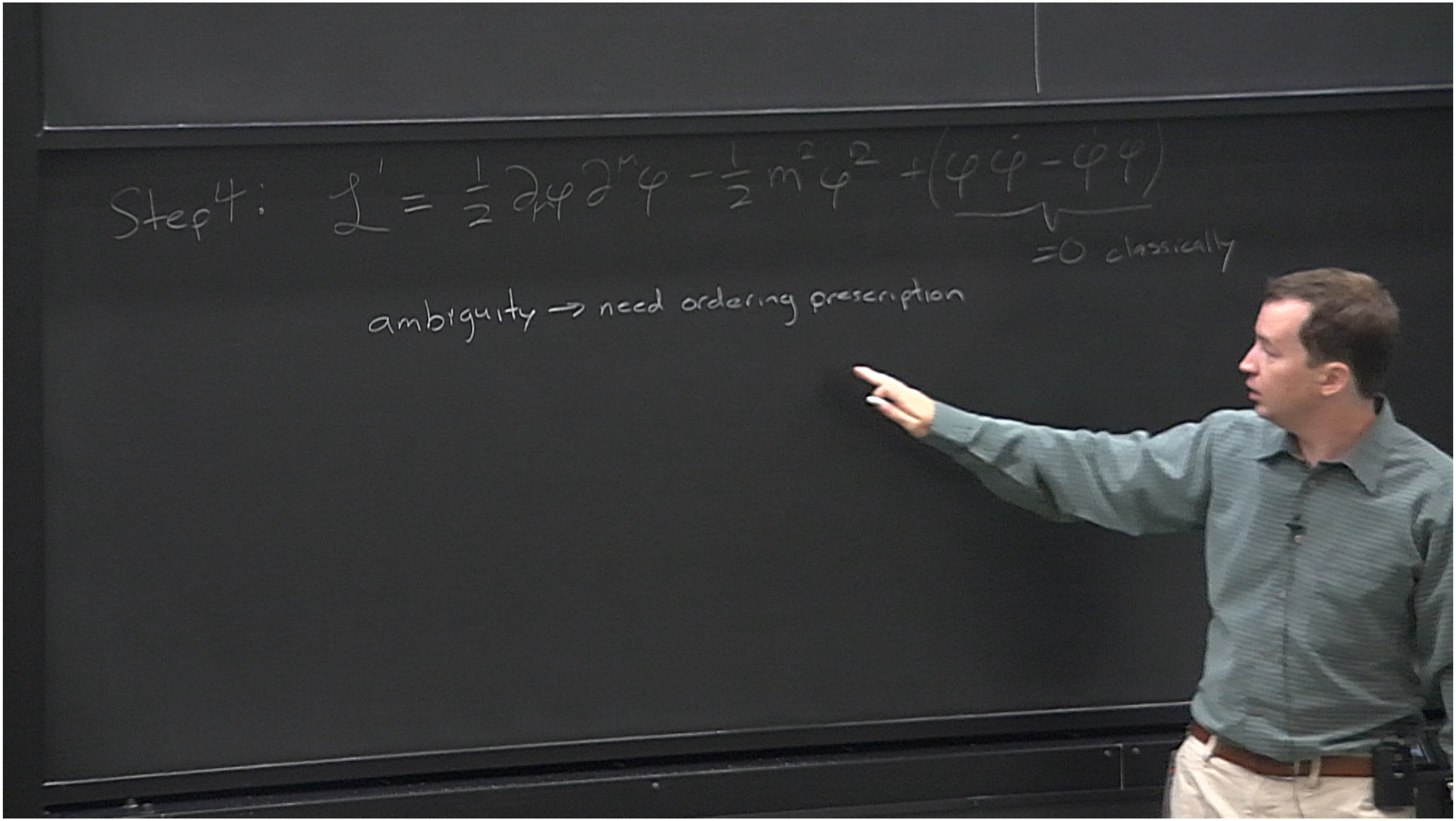
Schrödinger picture

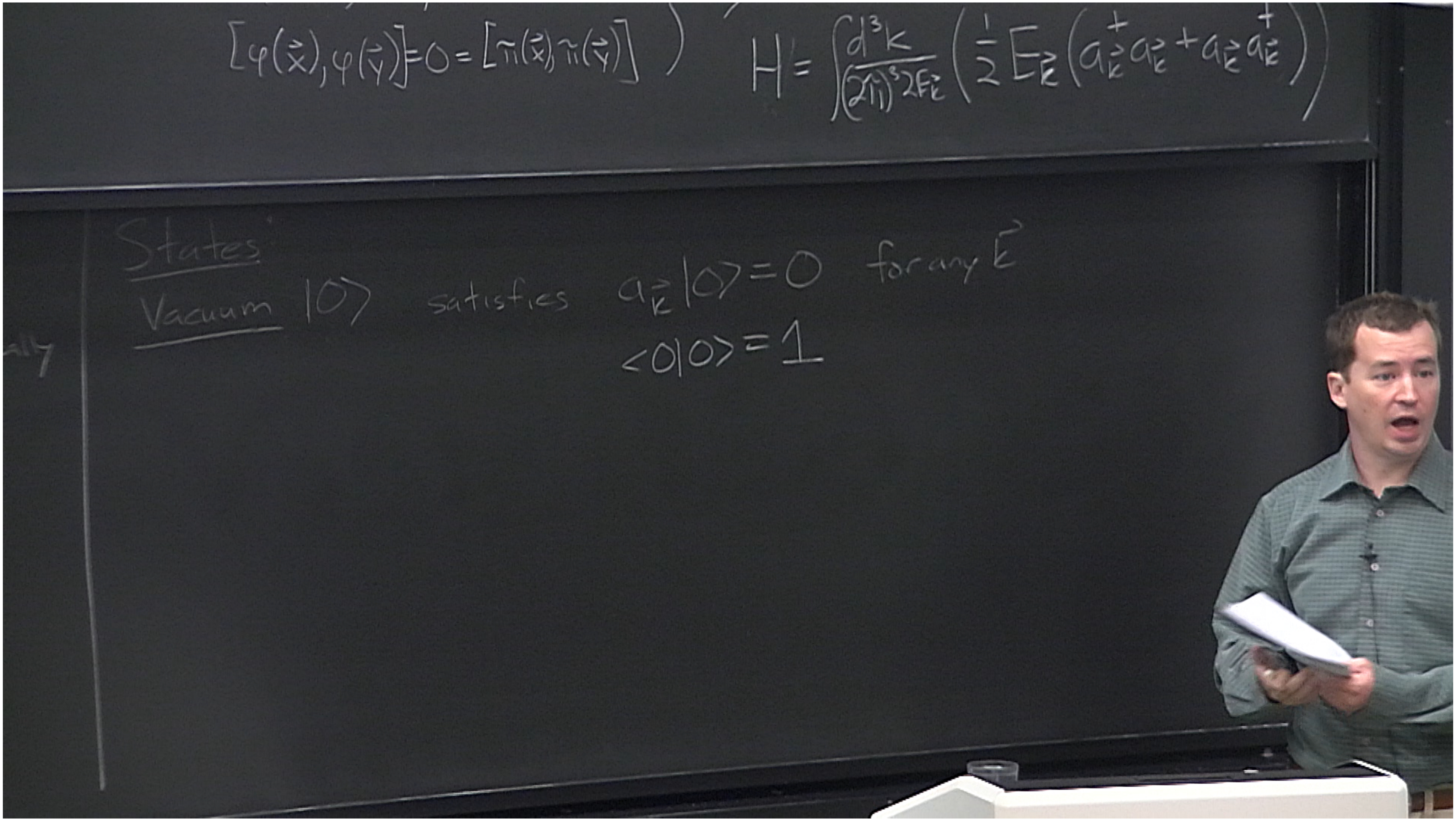
In QFT $\quad [\varphi(\vec{x}), \pi(\vec{y})] = i\,\delta(\vec{x}-\vec{y})$

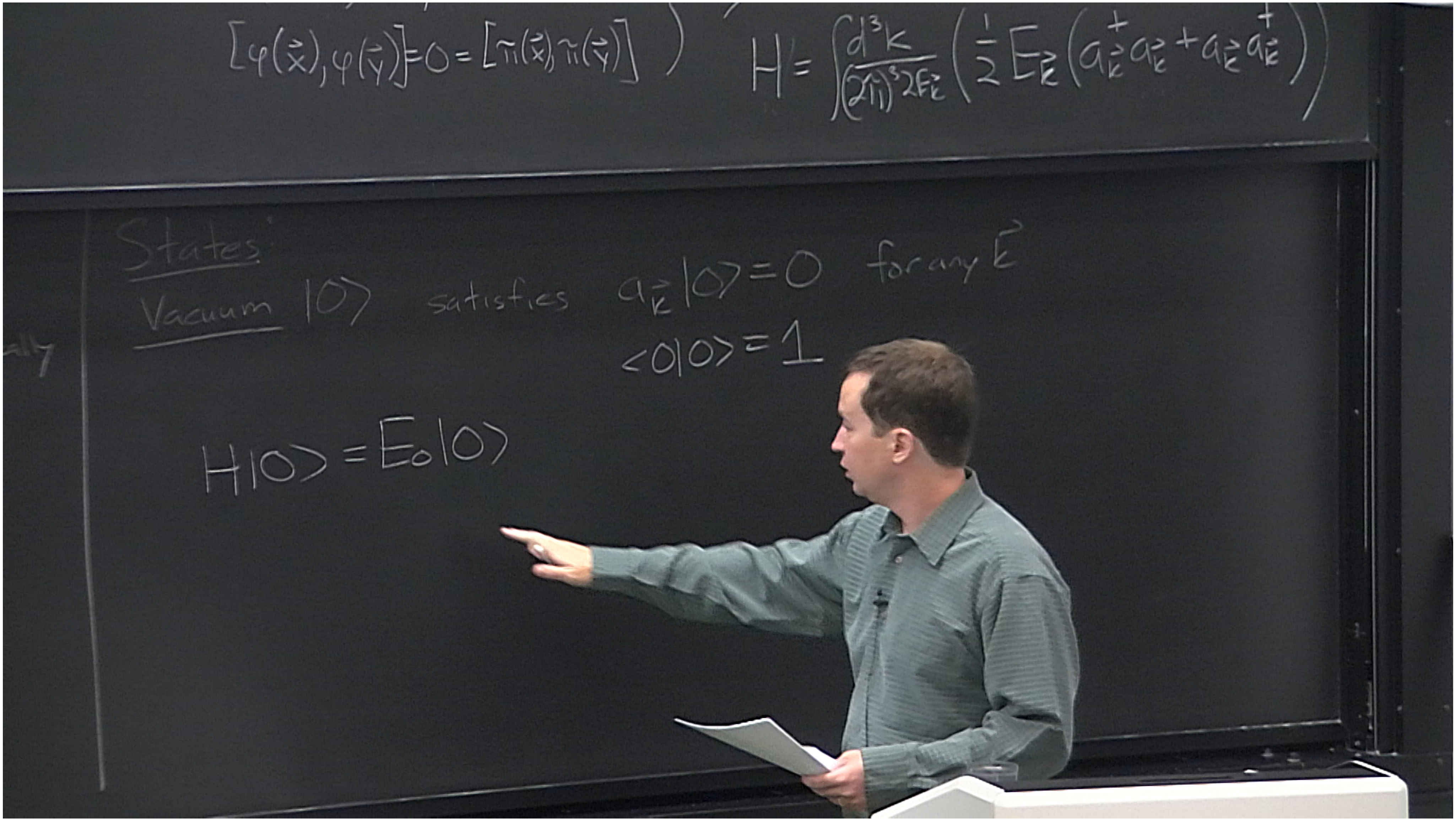
$$\left.\begin{aligned} &[\varphi(\vec{x}), \pi(\vec{y})] = i\,\delta(\vec{x}-\vec{y}) \\ &[\varphi(\vec{x}), \varphi(\vec{y})] = 0 = [\pi(\vec{x}), \pi(\vec{y})] \end{aligned}\right\} \rightarrow \quad \begin{aligned} &[a_{\vec{k}}, a^{+}_{\vec{p}}] = (2\pi)^3\, 2E_{\vec{k}}\, \delta(\vec{k}-\vec{p}) \\ &H = \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} \left(\frac{1}{2}E_{\vec{k}}\left(a^{+}_{\vec{k}} a_{\vec{k}} + a_{\vec{k}} a^{+}_{\vec{k}}\right)\right) \end{aligned}$$


Step 4: $\mathcal{L}' = \frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi - \frac{1}{2}m^2\varphi^2 + (\varphi\dot{\varphi} - \dot{\varphi}\varphi)$


Step 4: $\mathcal{L}' = \frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi - \frac{1}{2}m^2\varphi^2 + \underbrace{(\varphi\dot{\varphi} - \dot{\varphi}\varphi)}_{=0 \text{ classically}}$


Step 4: $\mathcal{L}' = \frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi - \frac{1}{2}m^2\varphi^2 + \underbrace{(\varphi\dot{\varphi} - \dot{\varphi}\varphi)}_{=0 \text{ classically}}$
ambiguity → need ordering prescription


$[\varphi(\vec{x}), \varphi(\vec{y})] = 0 = [\pi(\vec{x}), \pi(\vec{y})]$
$H = \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} \left(\frac{1}{2} E_{\vec{k}} \left(a^\dagger_{\vec{k}} a_{\vec{k}} + a_{\vec{k}} a^\dagger_{\vec{k}} \right) \right)$
States
Vacuum $|0\rangle$ satisfies $a_{\vec{k}}|0\rangle = 0$ for any $\vec{k}$
$\langle 0|0\rangle = 1$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States:
Vacuum $|0\rangle$ satisfies $a_{\vec{k}}|0\rangle = 0$ for any $\vec{k}$
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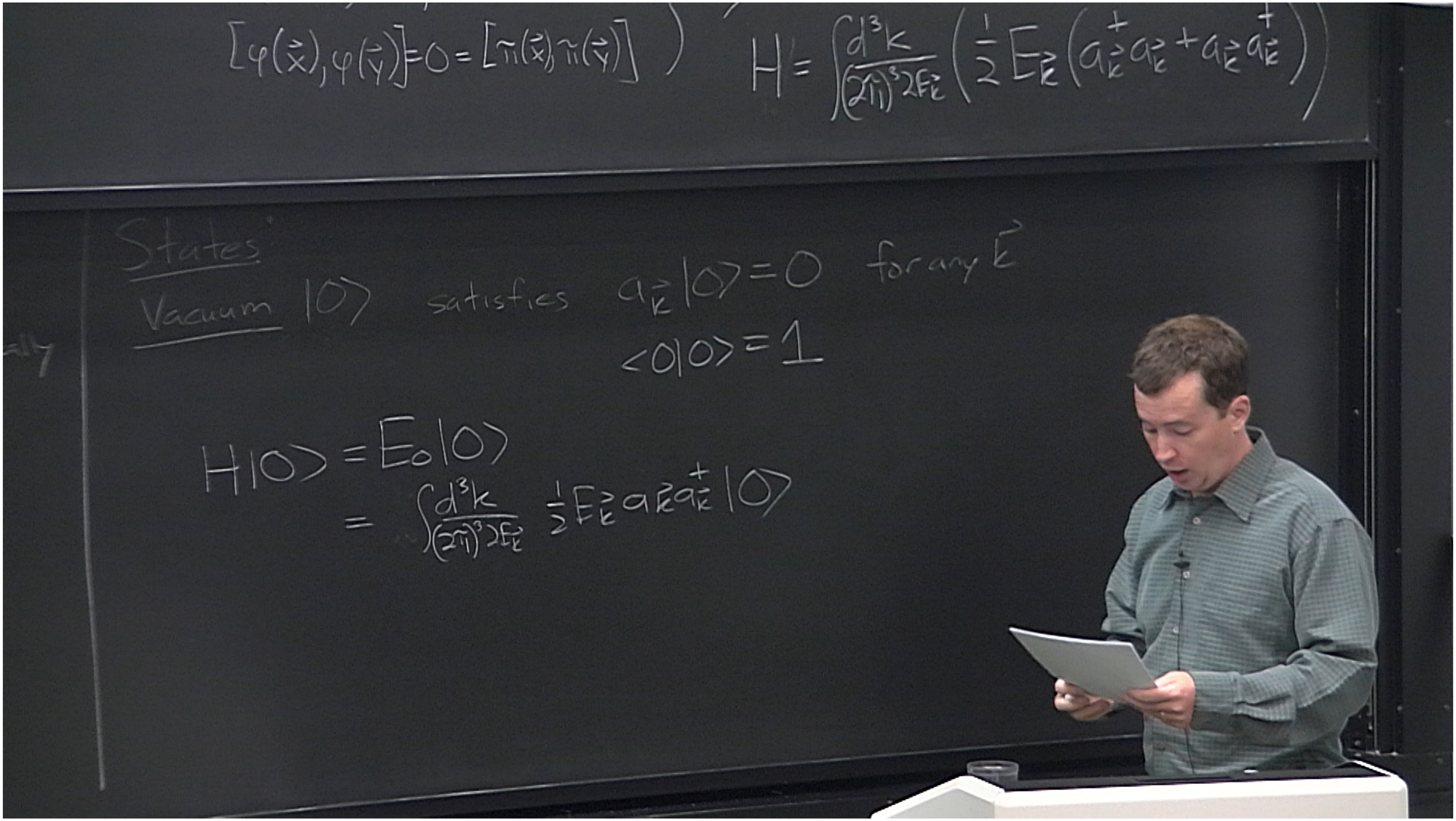
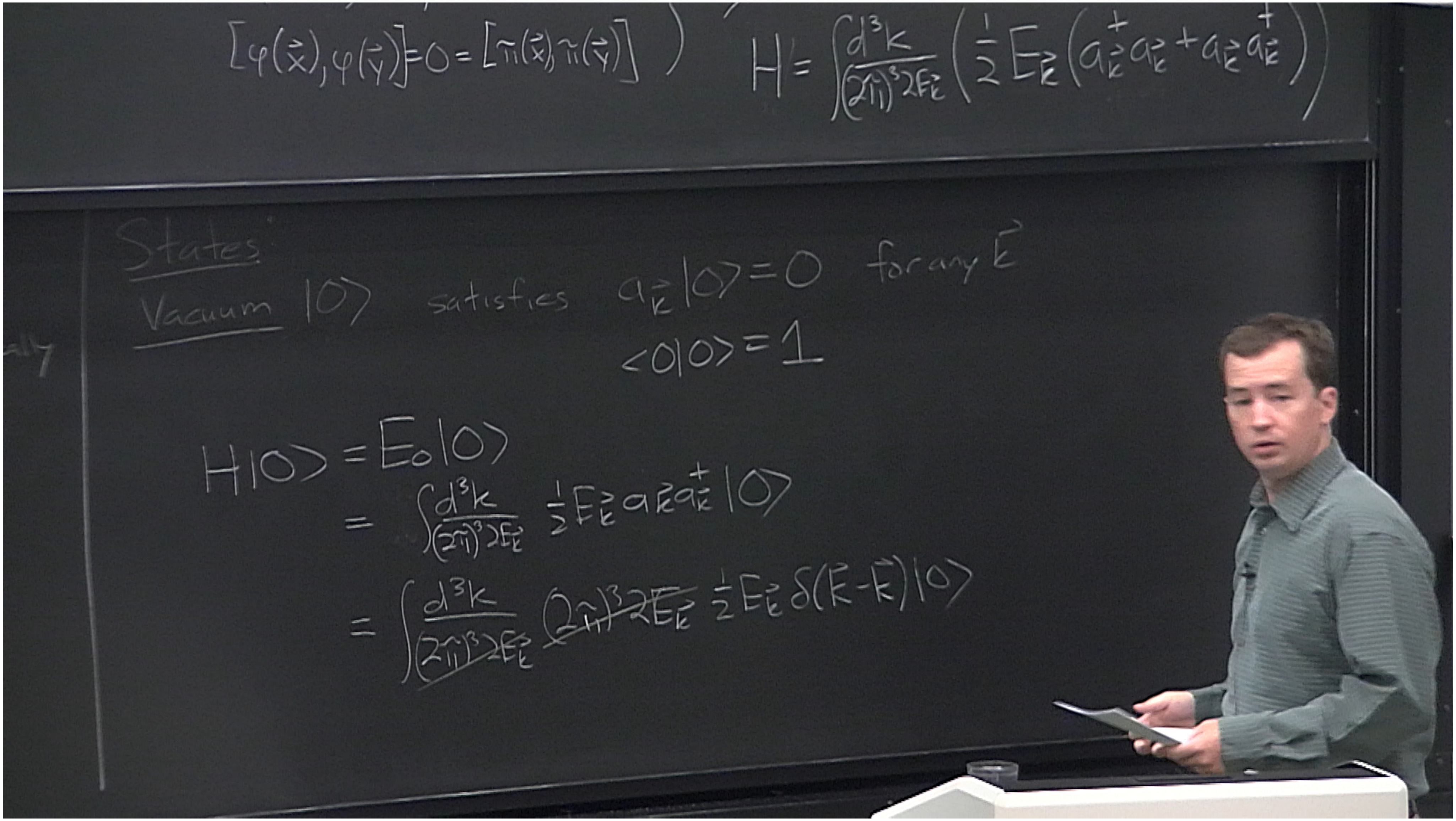
$H|0\rangle = E_0|0\rangle$



$[\varphi(\vec{x}),\varphi(\vec{y})]=0=[\pi(\vec{x}),\pi(\vec{y})]$
$H=\int\frac{d^3k}{(2\pi)^3 2E_{\vec{k}}}\left(\frac{1}{2}E_{\vec{k}}\left(a_{\vec{k}}^{\dagger}a_{\vec{k}}+a_{\vec{k}}a_{\vec{k}}^{\dagger}\right)\right)$
States:
Vacuum $|0\rangle$ satisfies $a_{\vec{k}}|0\rangle=0$ for any $\vec{k}$
$\langle 0|0\rangle=1$
$H|0\rangle=E_0|0\rangle$
$=\int\frac{d^3k}{(2\pi)^3 2E_{\vec{k}}}\frac{1}{2}E_{\vec{k}}a_{\vec{k}}a_{\vec{k}}^{\dagger}|0\rangle$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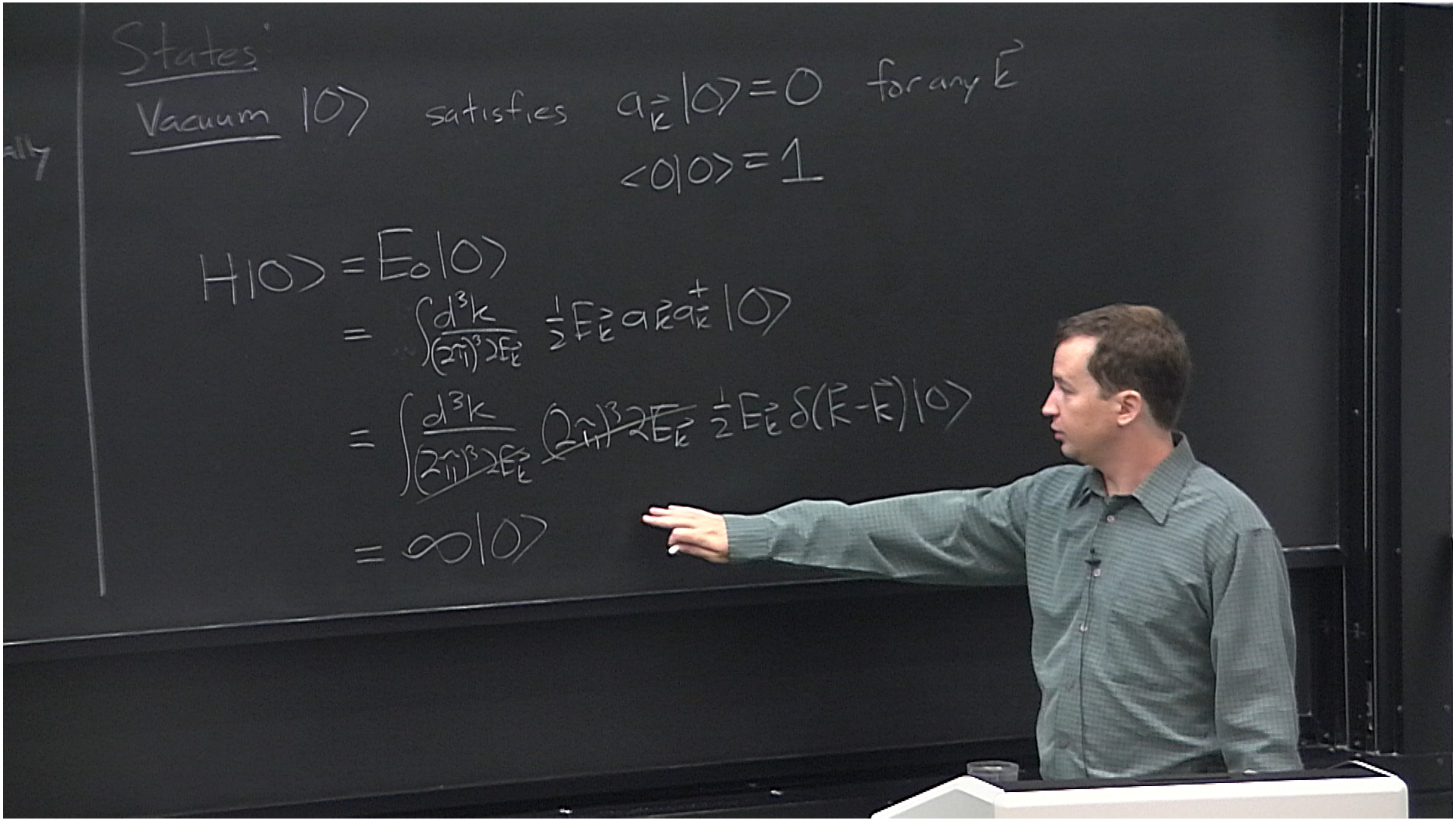
States
Vacuum $|0\rangle$ satisfies $a_{\vec{k}}|0\rangle = 0$ for any $\vec{k}$
$\langle 0|0\rangle = 1$
$H|0\rangle = E_0|0\rangle$
$= \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} \frac{1}{2} E_{\vec{k}} a_{\vec{k}} a^\dagger_{\vec{k}} |0\rangle$
$= \int \frac{d^3k}{\cancel{(2\pi)^3 2E_{\vec{k}}}} \cancel{(2\pi)^3 2E_{\vec{k}}} \frac{1}{2} E_{\vec{k}} \delta(\vec{k}-\vec{k})|0\rangle$


States:
Vacuum $|0\rangle$ satisfies $a_{\vec{k}}|0\rangle = 0$ for any $\vec{k}$
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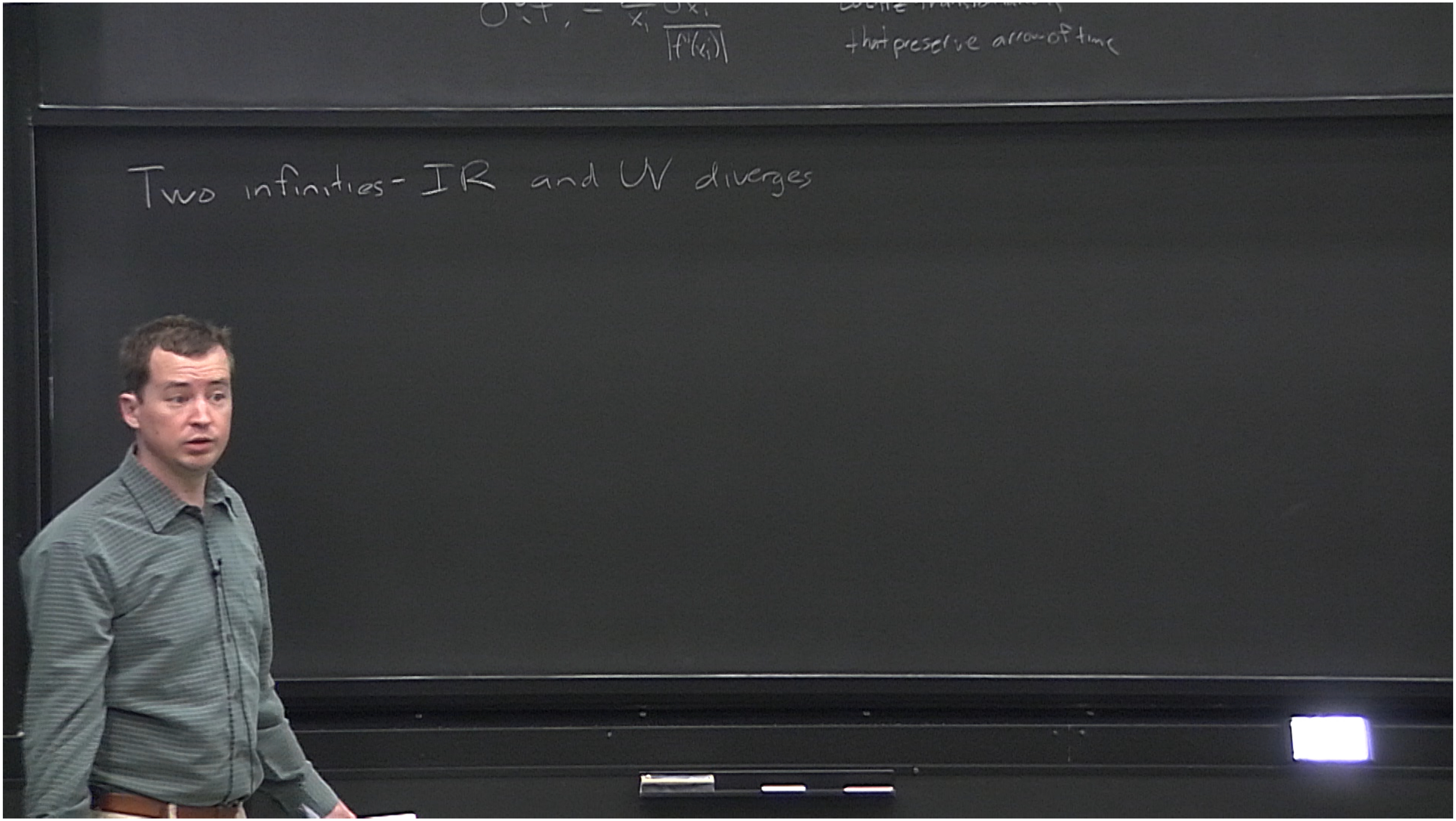
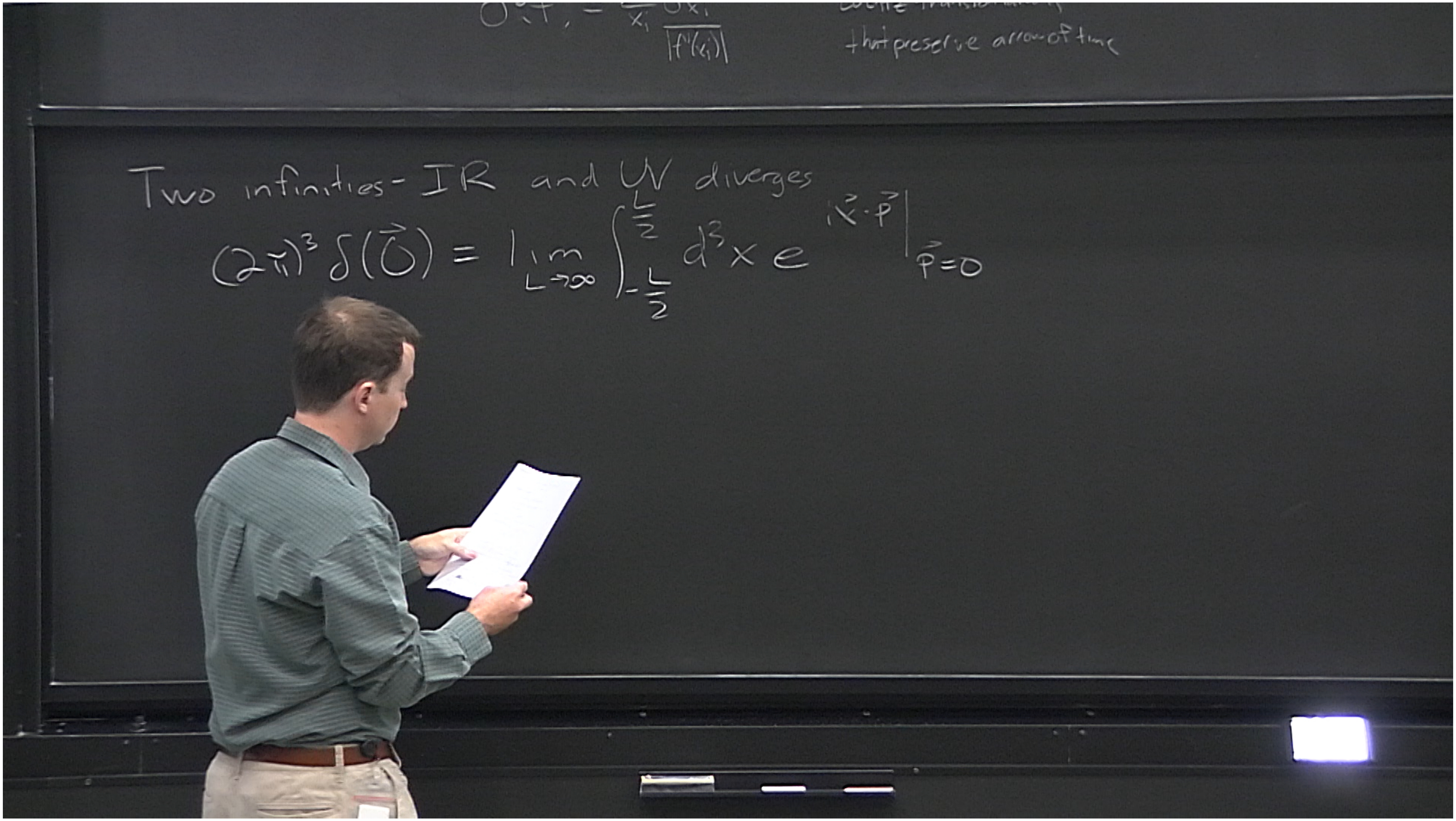
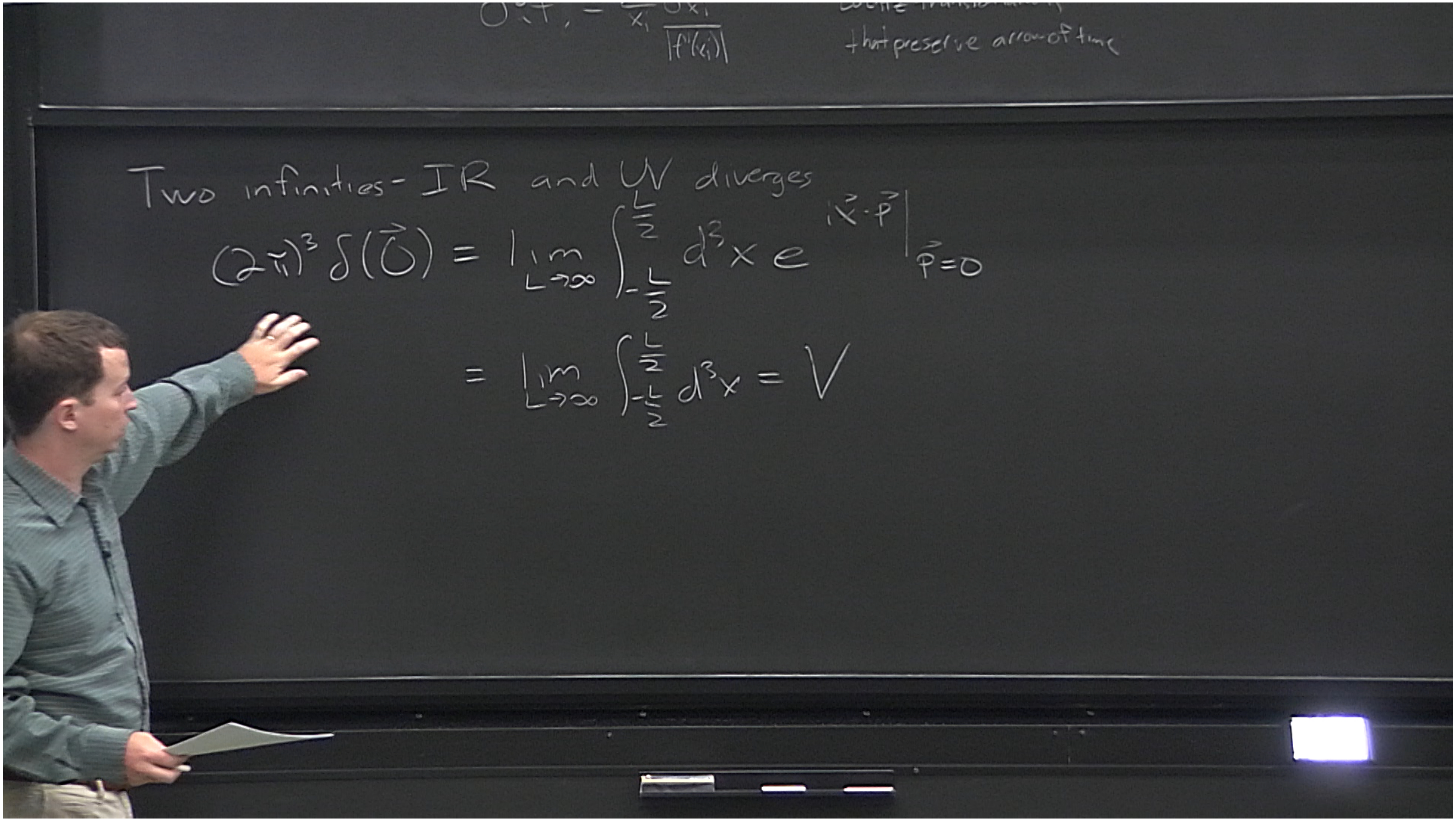
$= \int \frac{d^3k}{\cancel{(2\pi)^3 2E_{\vec{k}}}} \cancel{(2\pi)^3 2E_{\vec{k}}} \frac{1}{2} E_{\vec{k}} \delta(\vec{k}-\vec{k})|0\rangle$
$= \infty|0\rangle$


that preserve arrow of time
Two infinities - IR and UV diverges



that preserve arrow of time
Two infinities - IR and UV diverges
$(2\pi)^3\delta(\vec{0}) = \lim_{L\to\infty}\int_{-\frac{L}{2}}^{\frac{L}{2}} d^3x\, e^{i\vec{x}\cdot\vec{p}}\Big|_{\vec{p}=0}$



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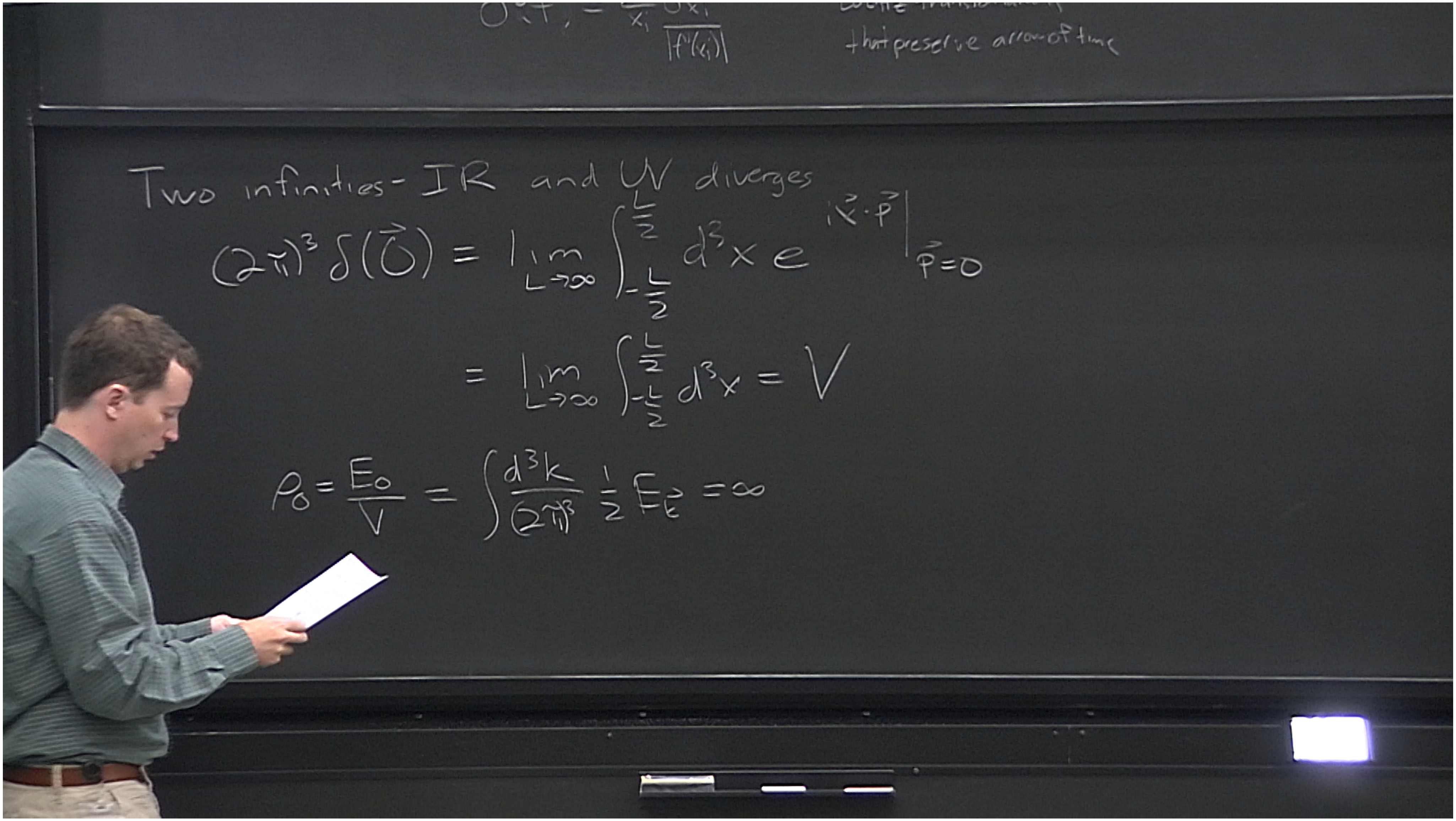
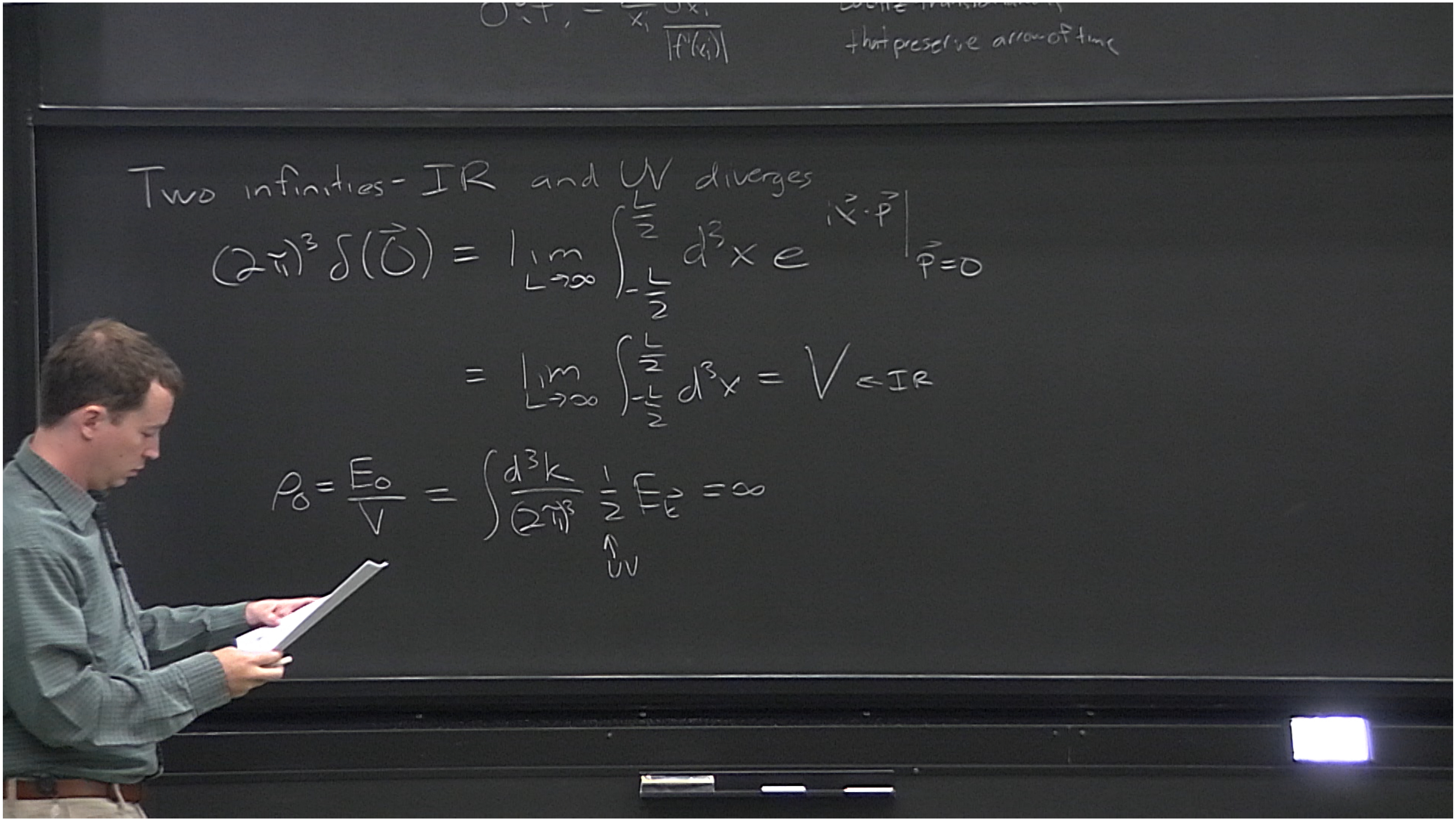
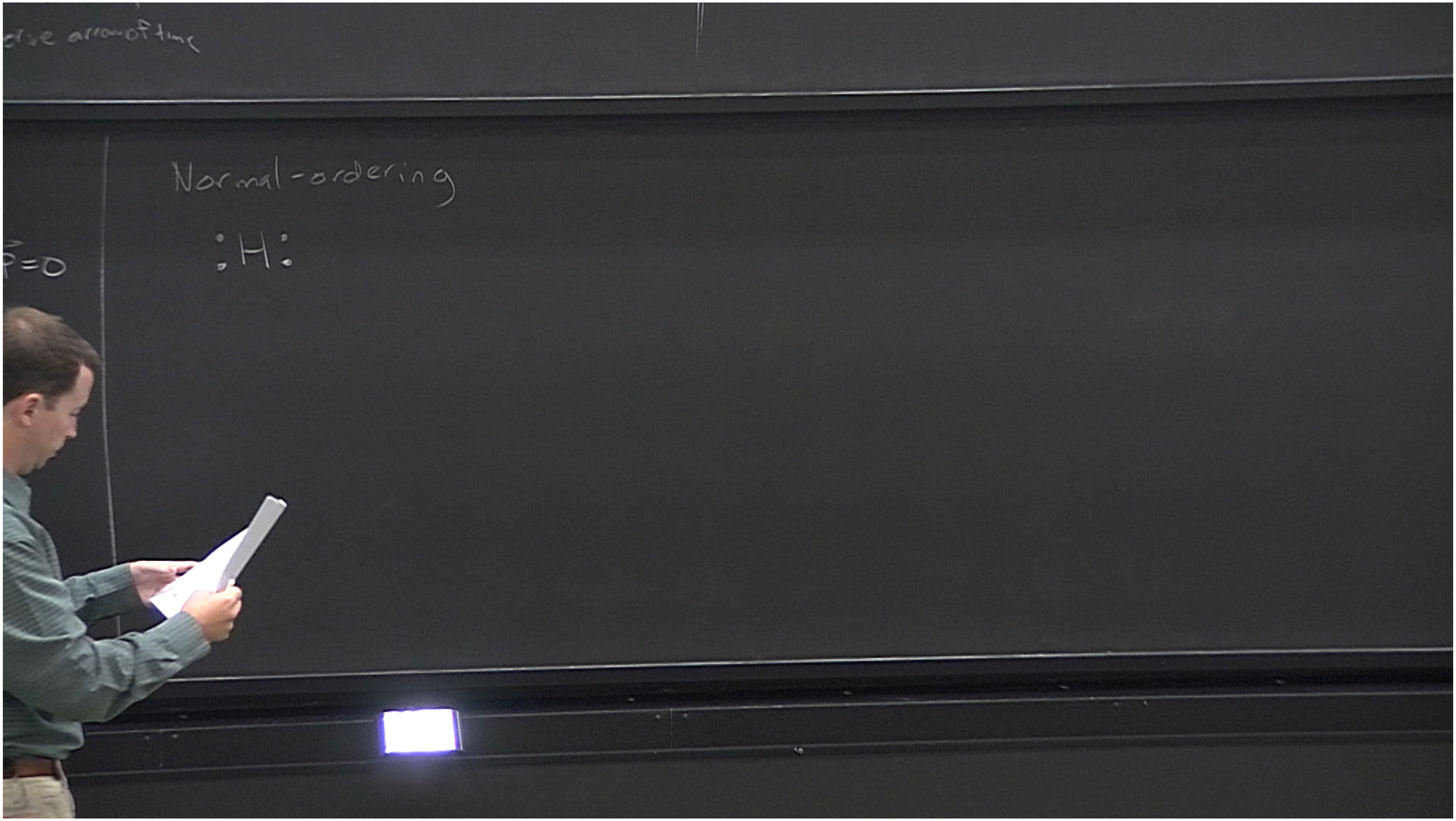
$= \lim_{L\to\infty}\int_{-\frac{L}{2}}^{\frac{L}{2}} d^3x = V$


that preserve arrow of time
Two infinities - IR and UV diverges
$(2\pi)^3 \delta(\vec{0}) = \lim_{L\to\infty} \int_{-\frac{L}{2}}^{\frac{L}{2}} d^3x \, e^{i\vec{x}\cdot\vec{p}} \Big|_{\vec{p}=0}$
$= \lim_{L\to\infty} \int_{-\frac{L}{2}}^{\frac{L}{2}} d^3x = V$
$\rho_0 = \frac{E_0}{V} = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2} E_{\vec{k}} = \infty$


$|f'(x_i)|$
that preserve arrow of time
Two infinities - IR and UV diverges
$(2\pi)^3 \delta(\vec{0}) = \lim_{L\to\infty} \int_{-\frac{L}{2}}^{\frac{L}{2}} d^3x \, e^{i\vec{x}\cdot\vec{p}} \Big|_{\vec{p}=0}$
$= \lim_{L\to\infty} \int_{-\frac{L}{2}}^{\frac{L}{2}} d^3x = V \leftarrow \text{IR}$
$\rho_0 = \frac{E_0}{V} = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2} E_{\vec{k}} = \infty$
UV


arrow of time
Normal-ordering
:H:
=0

Normal-ordering

$$:H: = \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} \frac{1}{2} E_{\vec{k}} \left(2 a^{\dagger}_{\vec{k}} a_{\vec{k}}\right)$$

moved creation ops to left

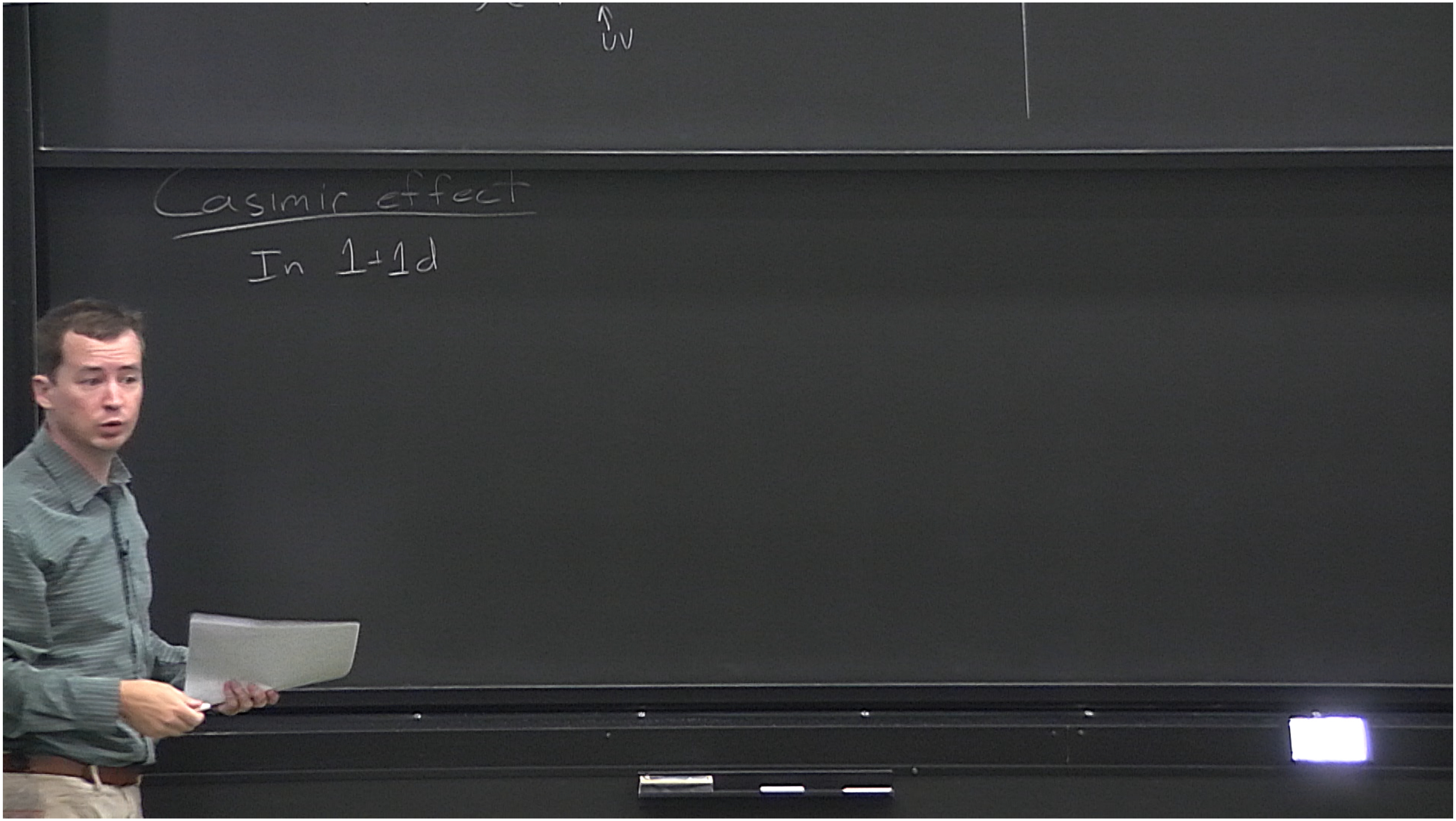
…ve arrow of time

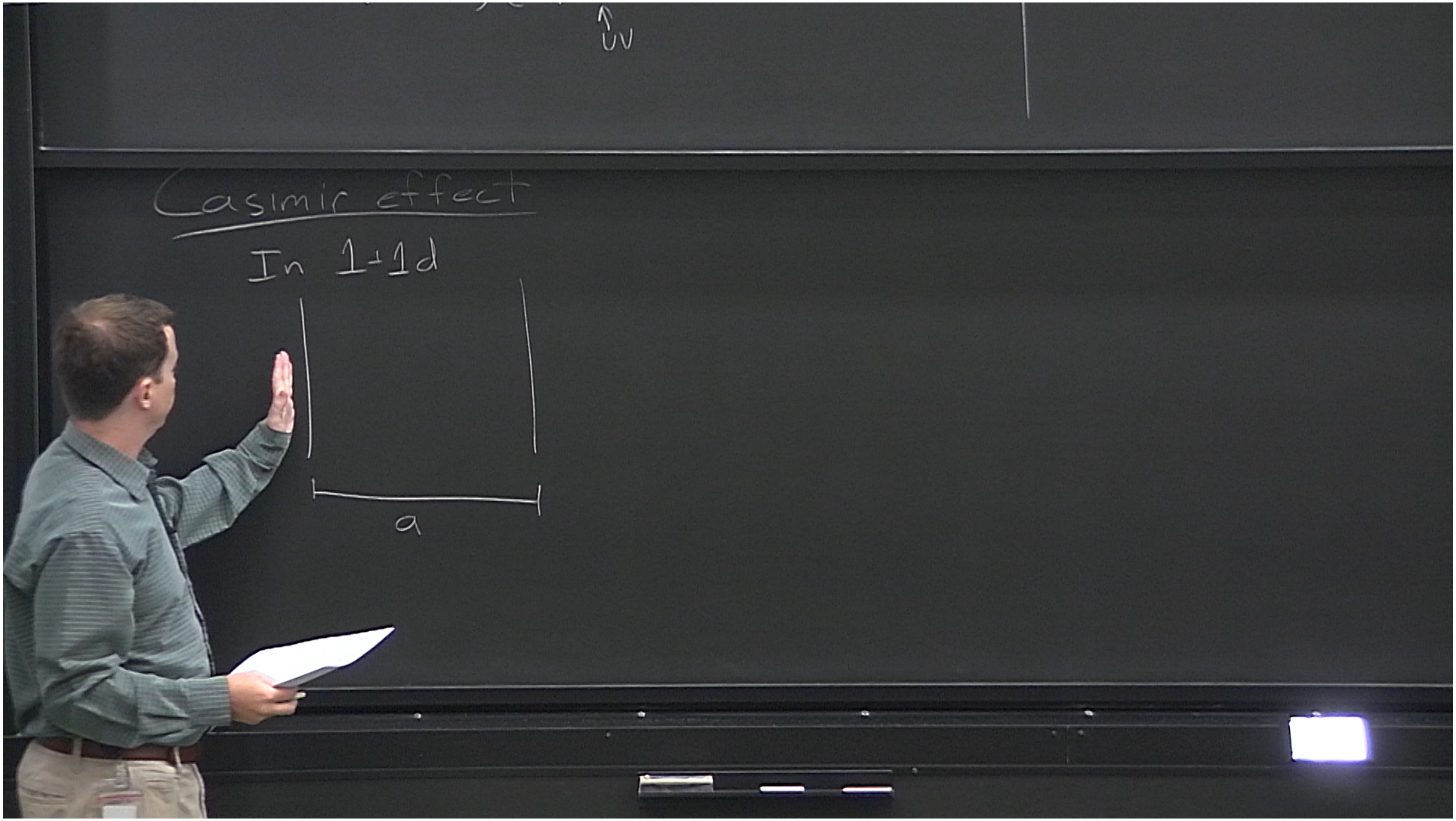
Normal-ordering

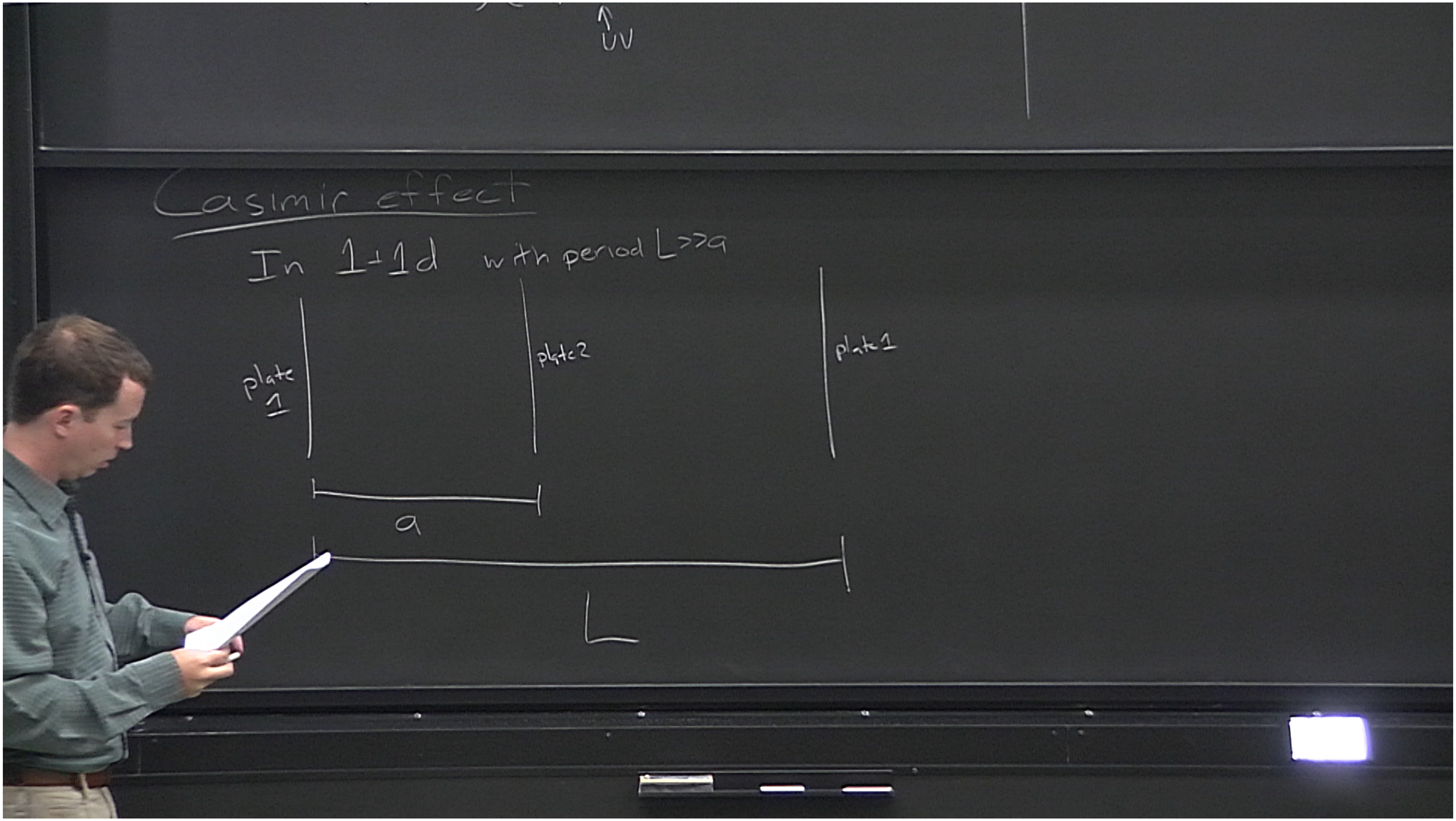
$$:H: = \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} \frac{1}{2} E_{\vec{k}} \left(2 a^{+}_{\vec{k}} a_{\vec{k}}\right)$$

moved creation ops to left

$$:a^{+}_{k} a_{k} a^{+}_{k}: = a^{+}_{k} a^{+}_{k} a_{k}$$


UV
Casimir effect
In 1+1d


UV
Casimir effect
In 1+1d
a


UV
Casimir effect
In 1+1d with period L>>a
plate 1
plate 2
plate 1
a
L

box size r $\quad \omega_n = \frac{n\pi}{r}$

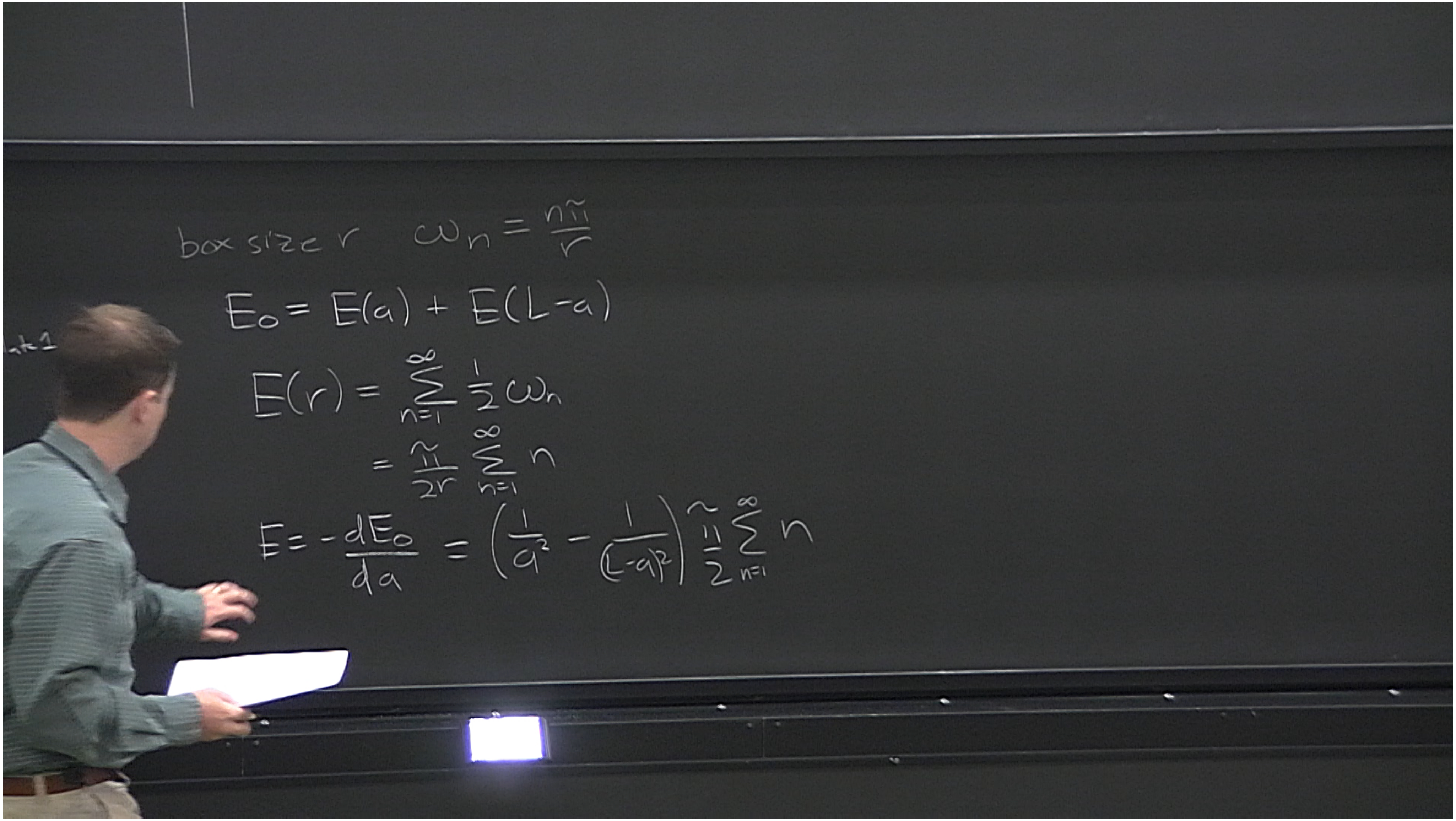
$$E_0 = E(a)$$

box size r $\quad \omega_n = \frac{n\pi}{r}$

$$E_0 = E(a) + E(L-a)$$

$$E(r) = \sum_{n=1}^{\infty} \frac{1}{2}\omega_n$$

$$= \frac{\pi}{2r} \sum_{n=1}^{\infty} n$$


box size r $\omega_n = \frac{n\pi}{r}$
$E_0 = E(a) + E(L-a)$
$E(r) = \sum_{n=1}^{\infty} \frac{1}{2}\omega_n$
$= \frac{\pi}{2r} \sum_{n=1}^{\infty} n$
$E = -\frac{dE_0}{da} = \left(\frac{1}{a^2} - \frac{1}{(L-a)^2}\right)\frac{\pi}{2}\sum_{n=1}^{\infty} n$

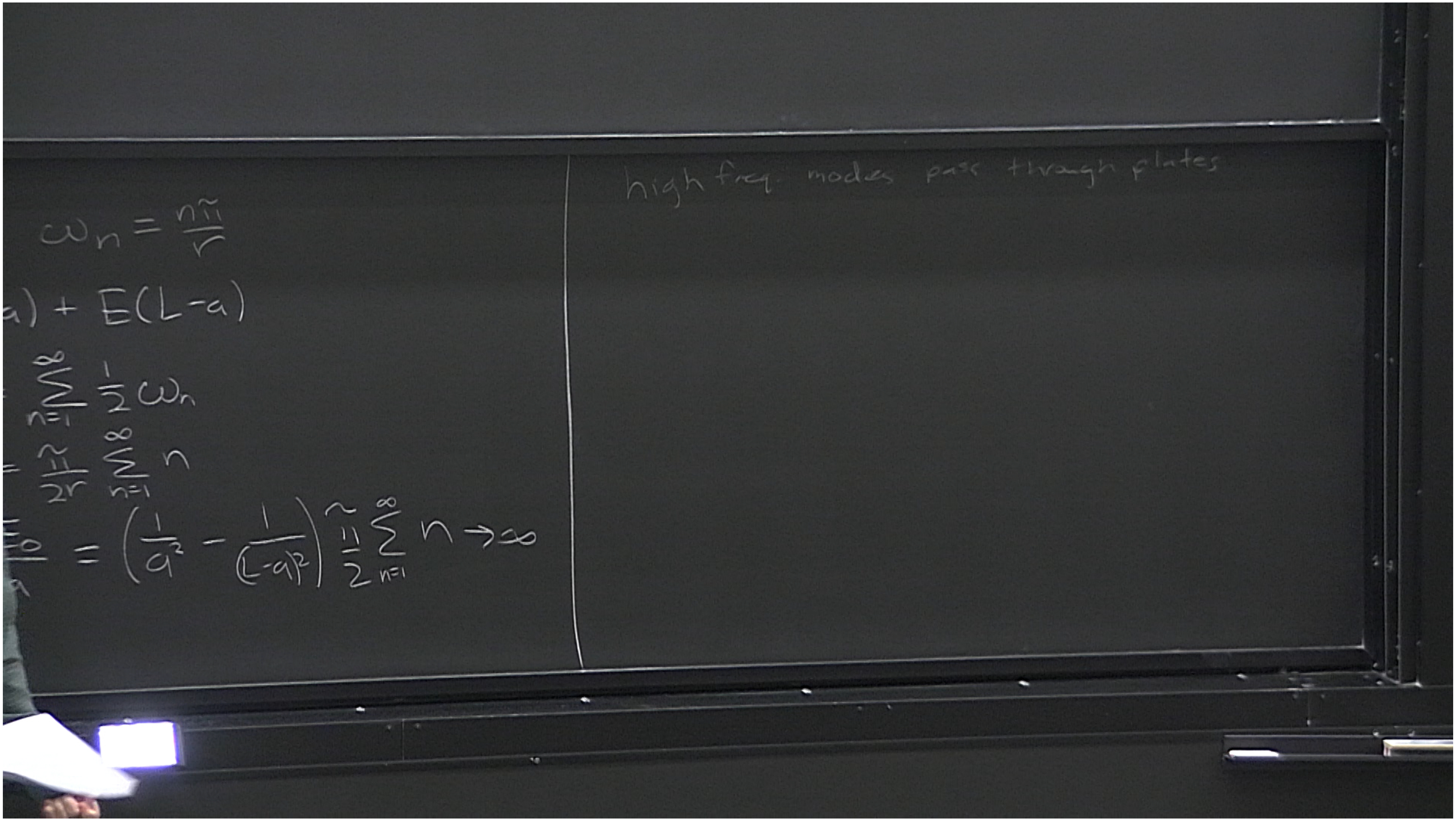
box size r $\quad \omega_n = \frac{n\pi}{r}$

$$E_0 = E(a) + E(L-a)$$

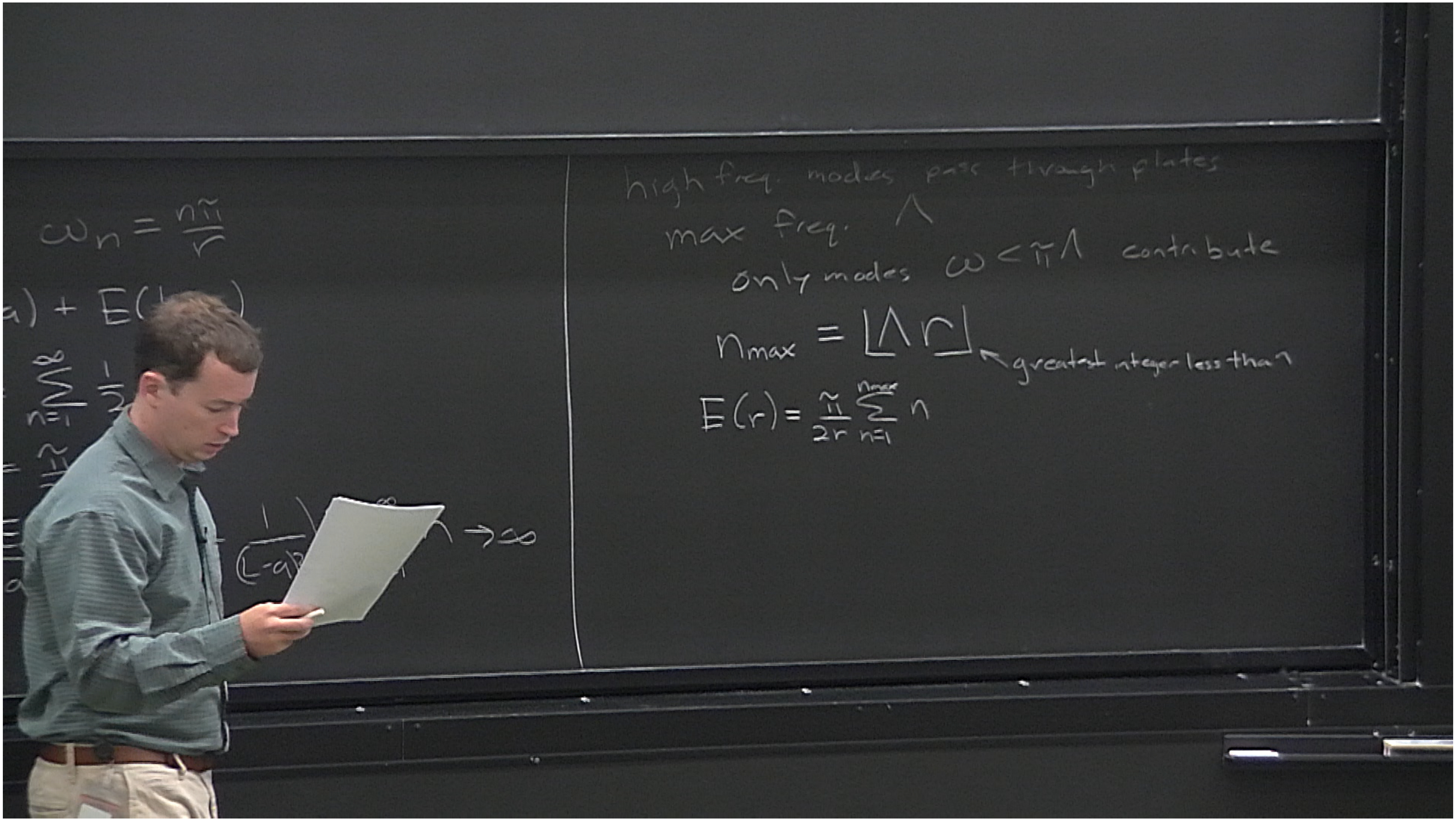
$$E(r) = \sum_{n=1}^{\infty} \frac{1}{2}\omega_n$$

$$= \frac{\pi}{2r} \sum_{n=1}^{\infty} n$$

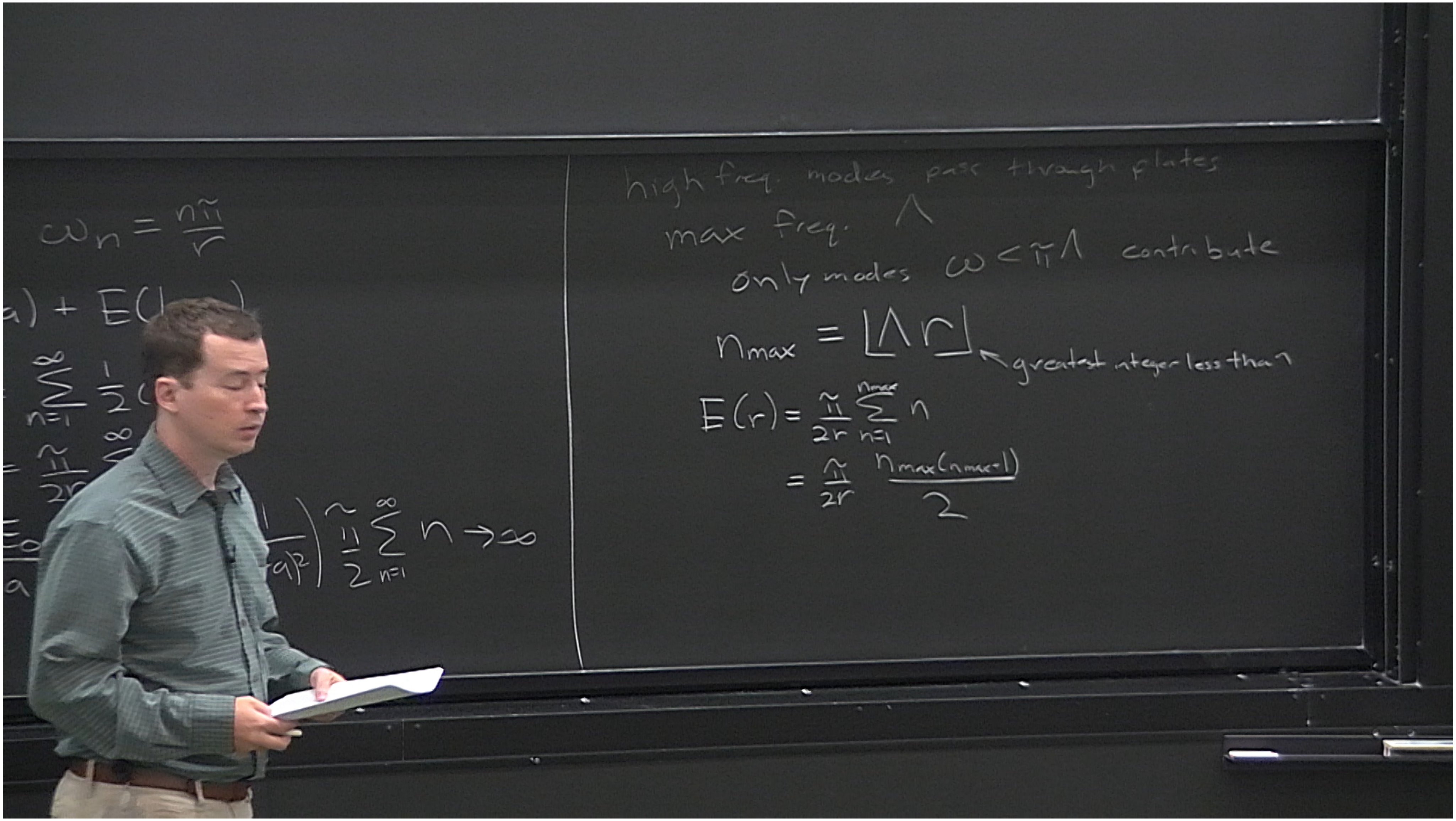
$$F = -\frac{dE_0}{da} = \left(\frac{1}{a^2} - \frac{1}{(L-a)^2}\right)\frac{\pi}{2}\sum_{n=1}^{\infty} n \to \infty$$

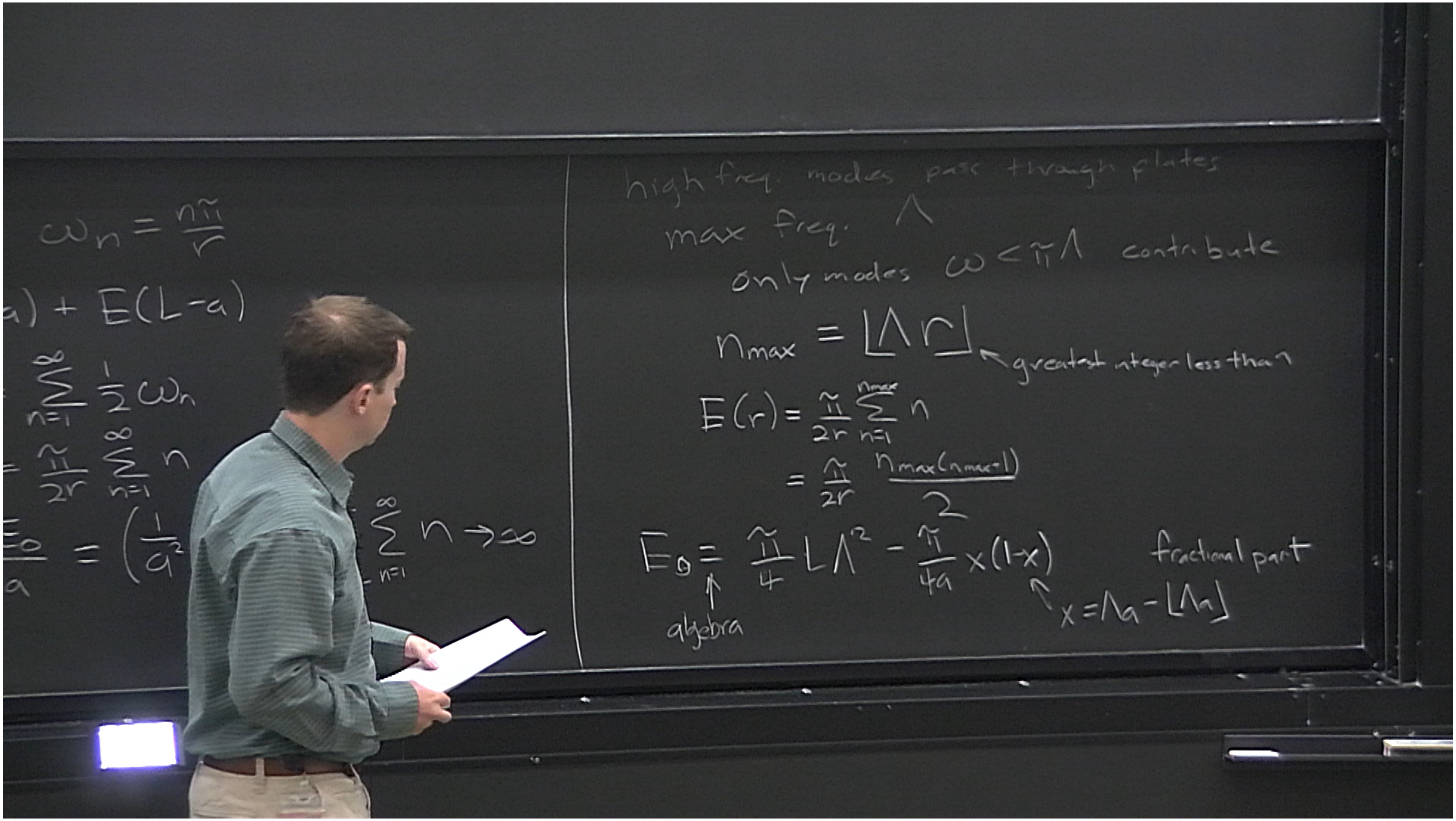


$\omega_n = \frac{n\pi}{r}$
$a) + E(L-a)$
$\sum_{n=1}^{\infty} \frac{1}{2}\omega_n$
$= \frac{\pi}{2r} \sum_{n=1}^{\infty} n$
$= \left(\frac{1}{a^2} - \frac{1}{(L-a)^2}\right) \frac{\pi}{2} \sum_{n=1}^{\infty} n \to \infty$
high freq. modes pass through plates

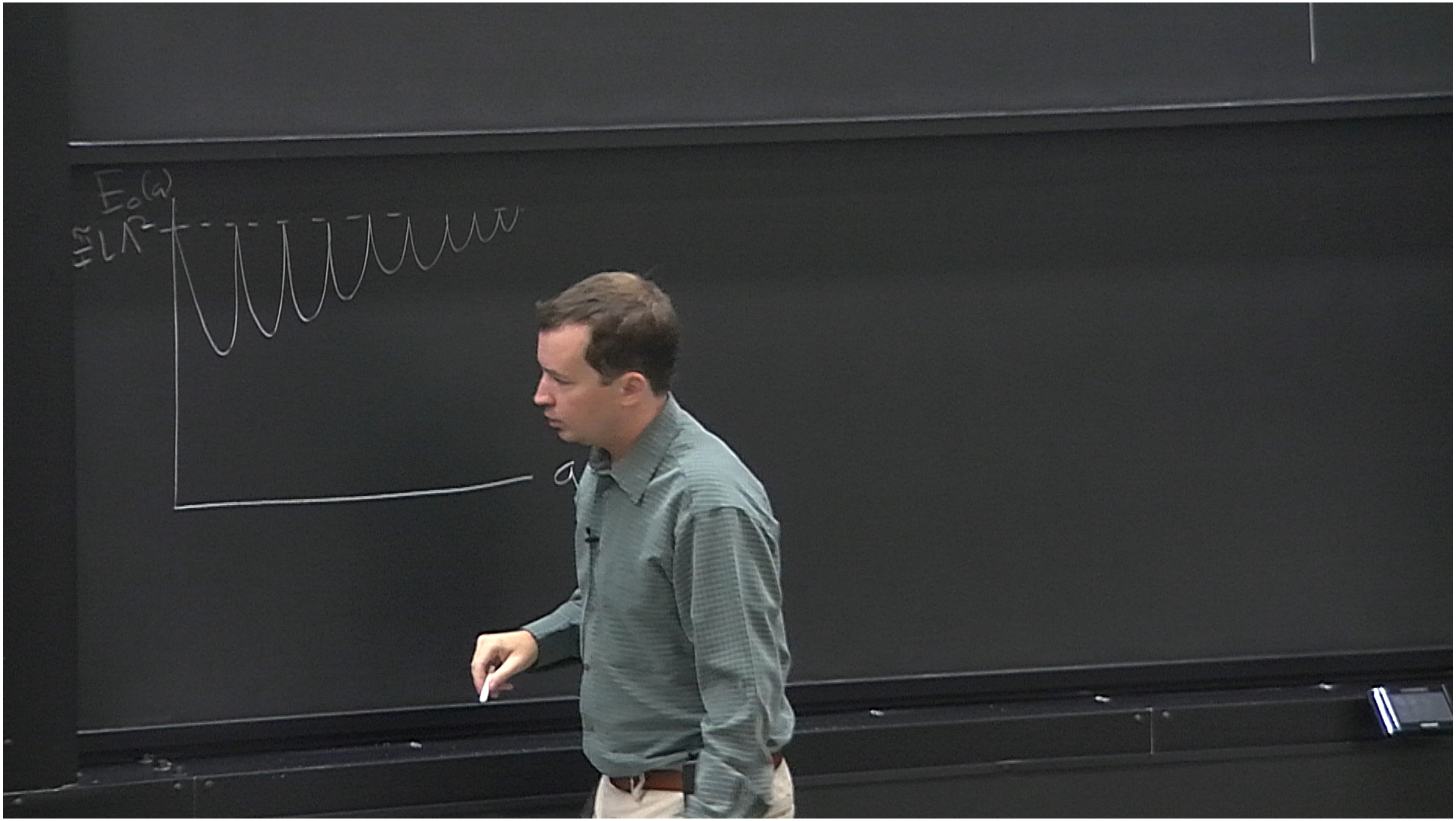


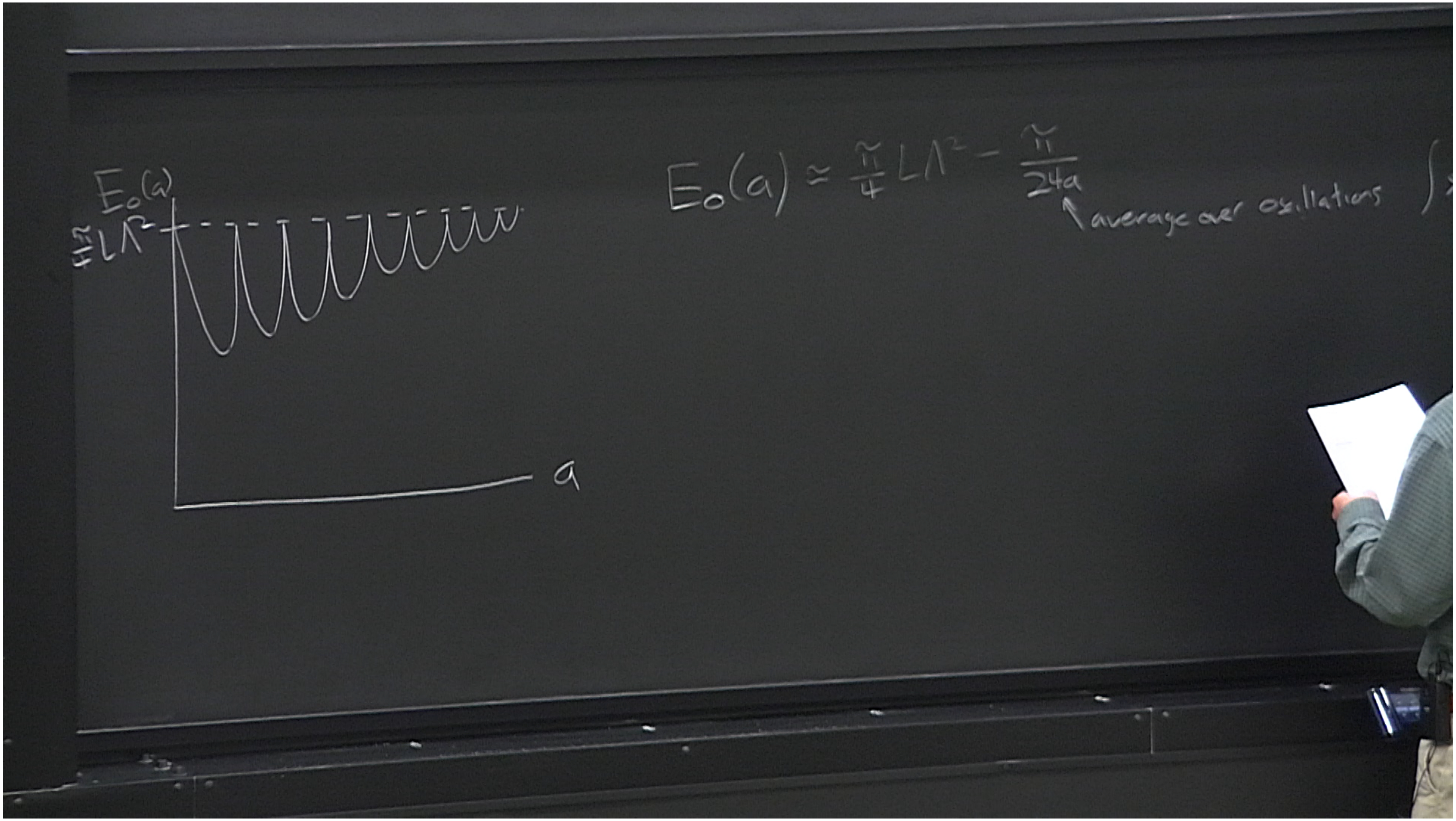
$\omega_n = \frac{n\tilde{\pi}}{r}$
high freq. modes pass through plates
max freq. Λ
only modes $\omega < \tilde{\pi}\Lambda$ contribute
$n_{max} = \lfloor \Lambda r \rfloor$
greatest integer less than
$E(r) = \frac{\tilde{\pi}}{2r} \sum_{n=1}^{n_{max}} n$

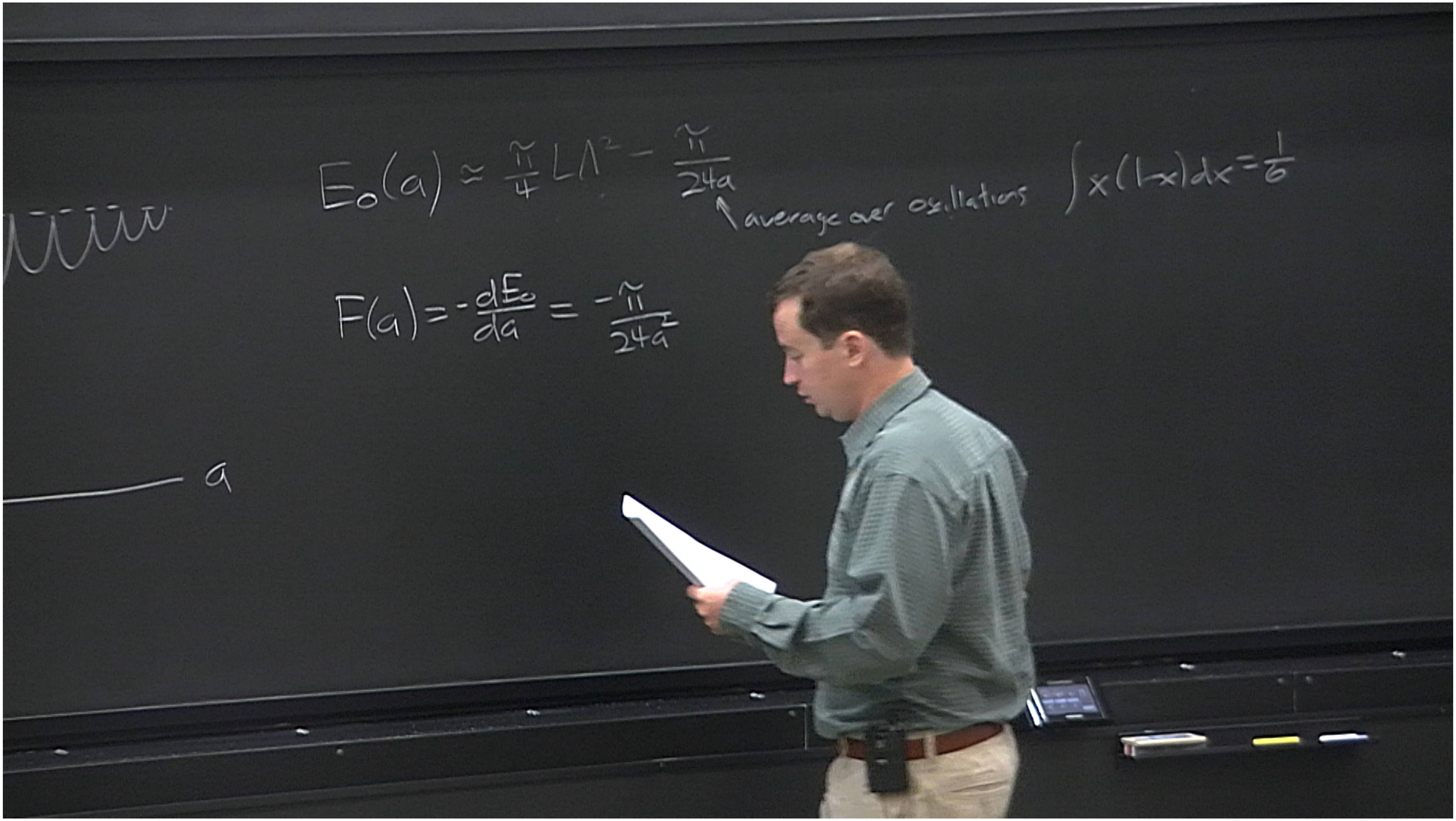

high freq. modes pass through plates
max freq. Λ
only modes ω < π̃Λ contribute
nmax = ⌊Λr⌋
greatest integer less than
E(r) = π̃/2r Σ_{n=1}^{nmax} n
= π̃/2r · nmax(nmax+1)/2
ωn = nπ̃/r



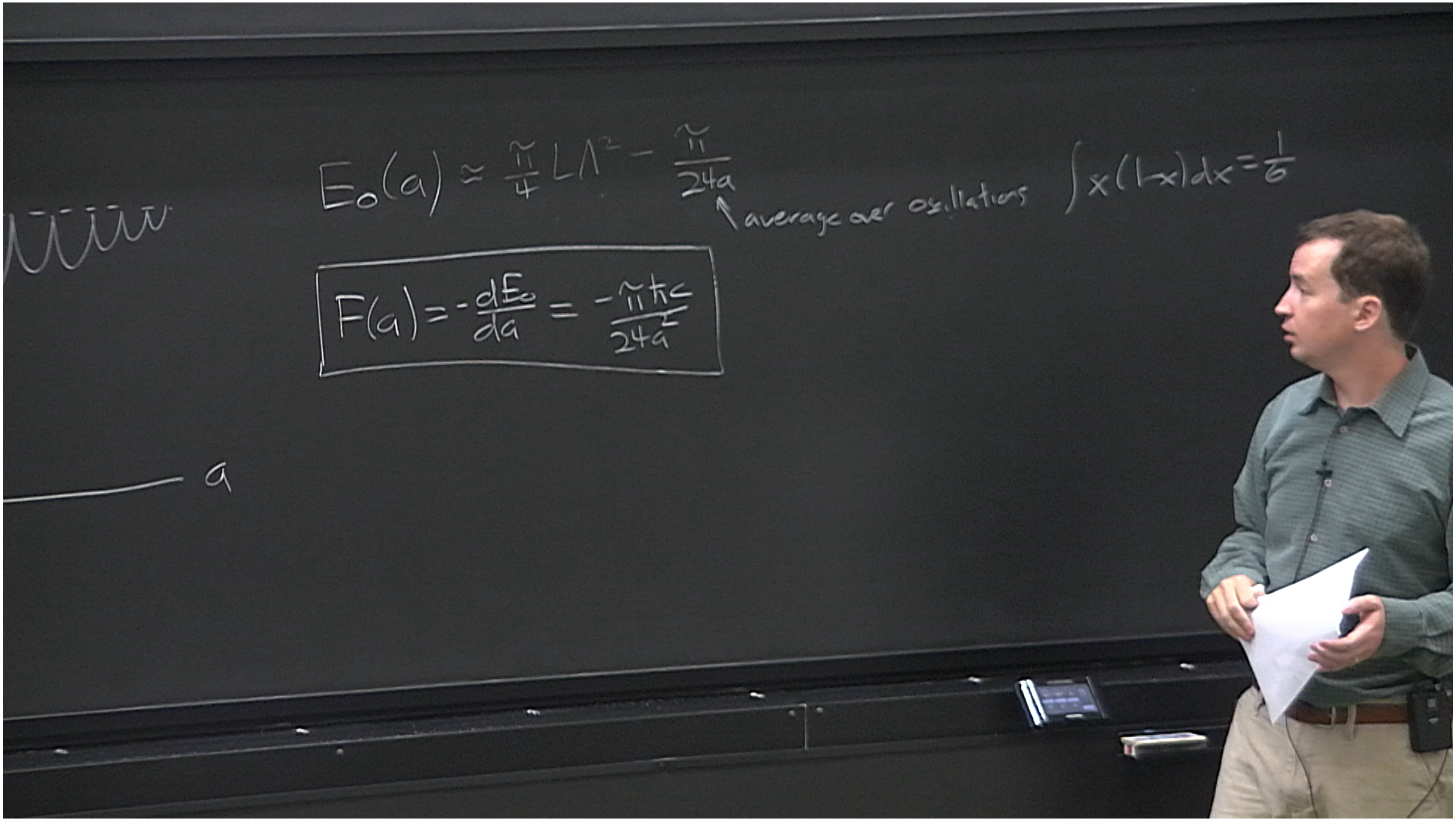
high freq. modes pass through plates
max freq. Λ
only modes ω < π Λ contribute
n_max = ⌊Λ r⌋
greatest integer less than
E(r) = π/2r Σ_{n=1}^{n_max} n
= π/2r n_max(n_max+1)/2
E_0 = π/4 L Λ^2 − π/4a x(1−x)
fractional part
algebra
x = Λa − ⌊Λa⌋
ω_n = nπ/r
a) + E(L−a)
Σ_{n=1}^{∞} 1/2 ω_n
= π/2r Σ_{n=1}^{∞} n
Σ_{n=1}^{∞} n → ∞

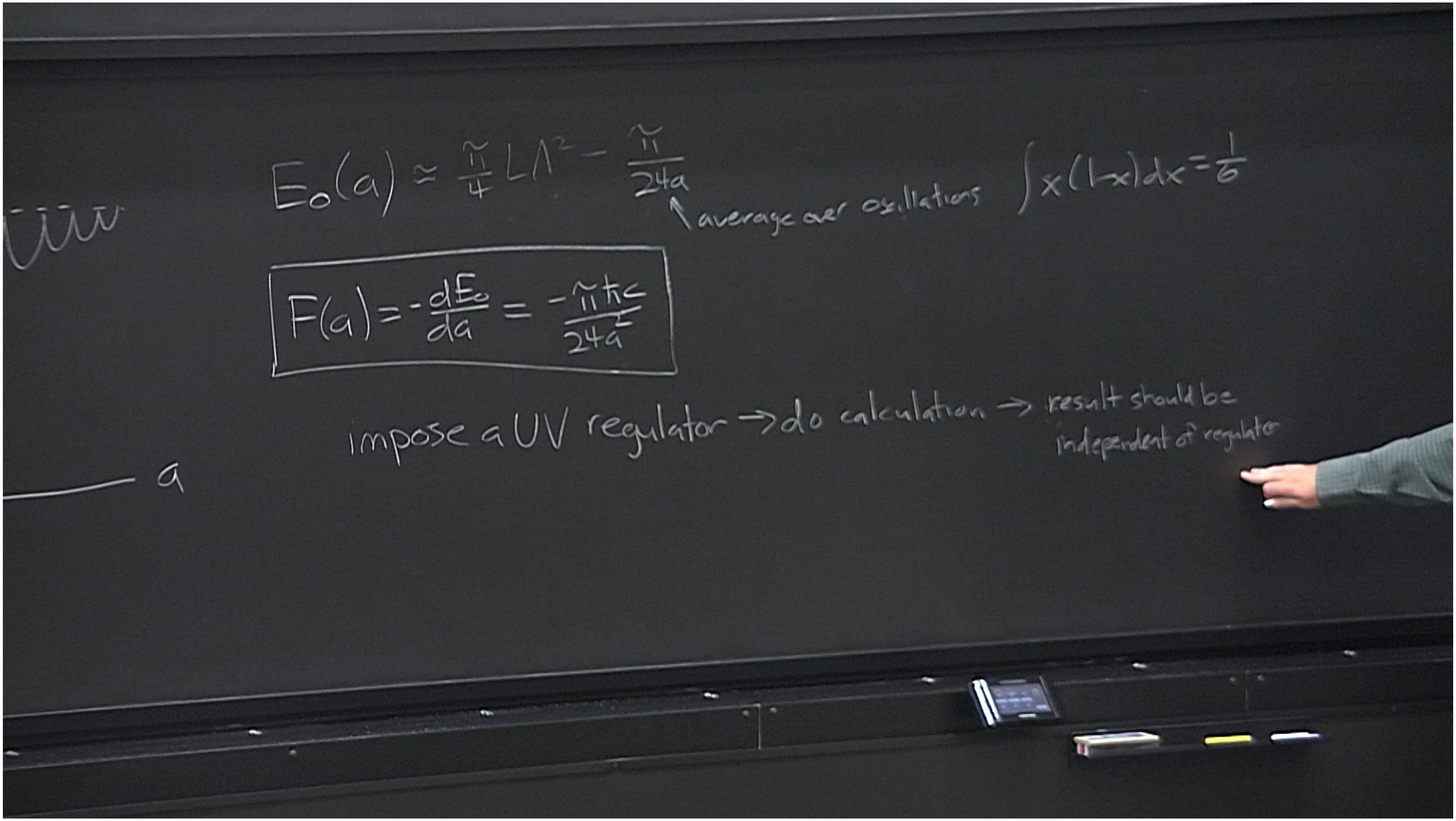

$E_0(a)$
$\approx L\Lambda^2$
a

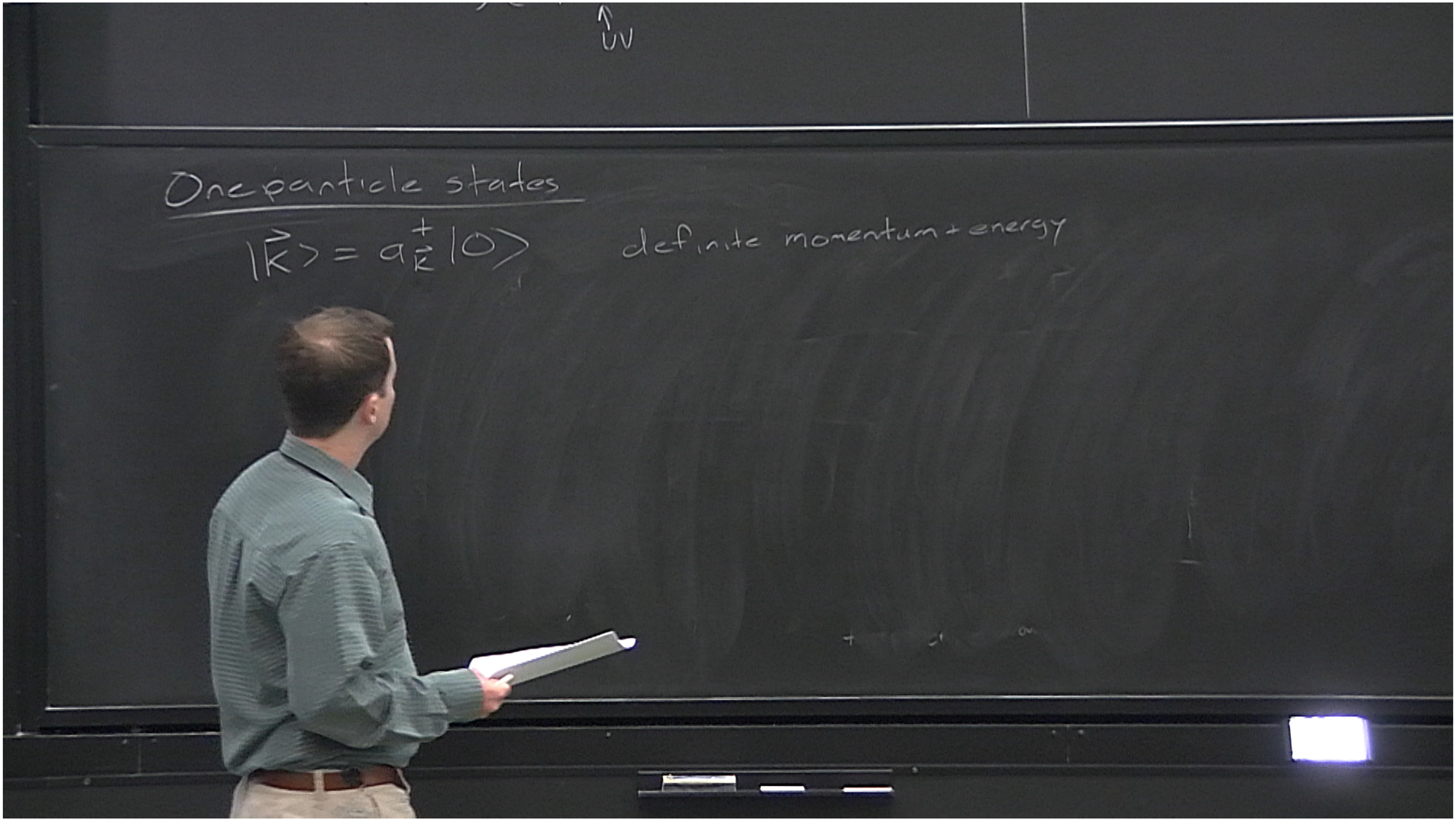

$E_0(a)$
$\frac{\pi}{4} L \Lambda^2$
a
$E_0(a) \simeq \frac{\pi}{4} L \Lambda^2 - \frac{\pi}{24a}$
average over oscillations

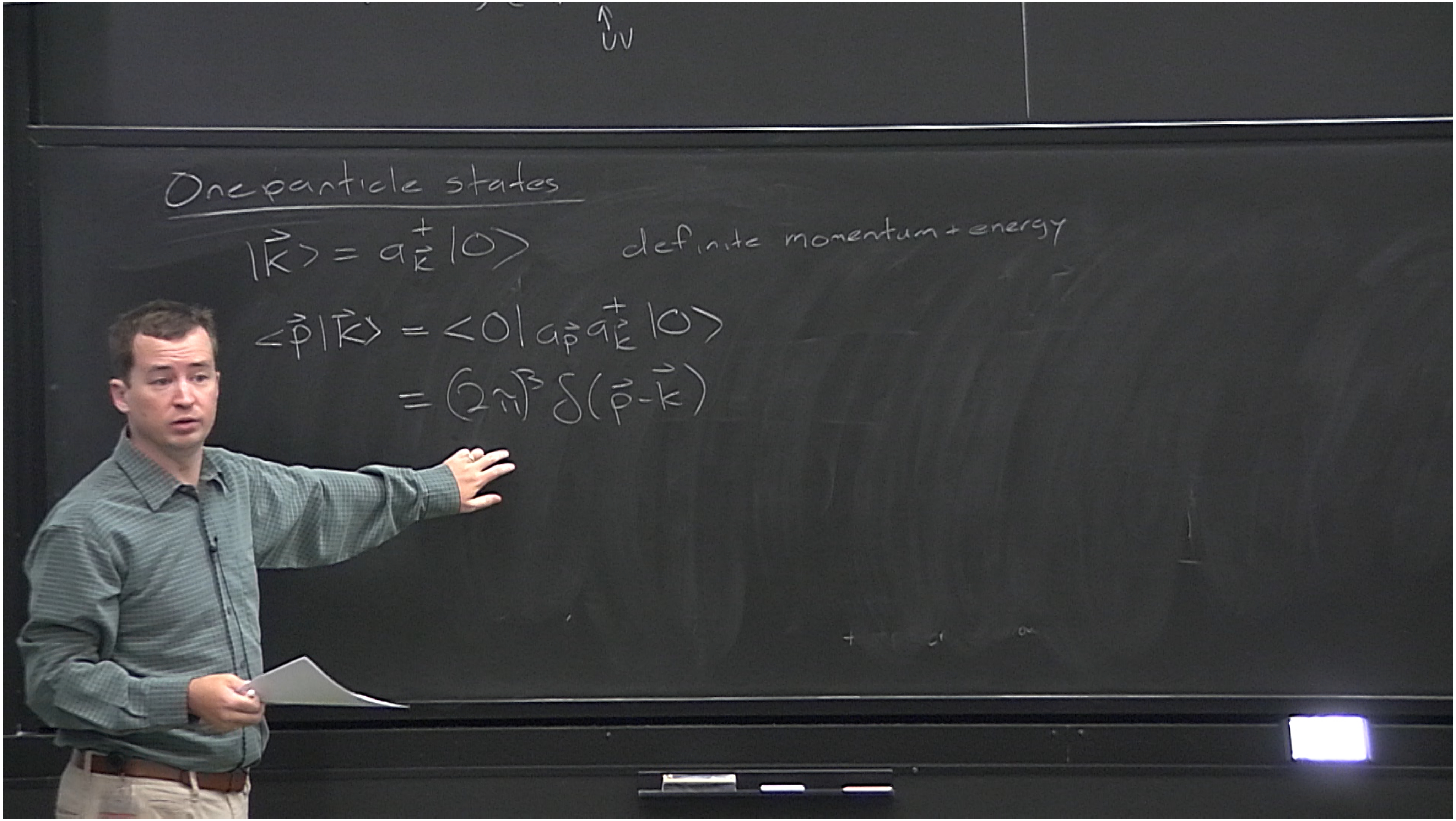


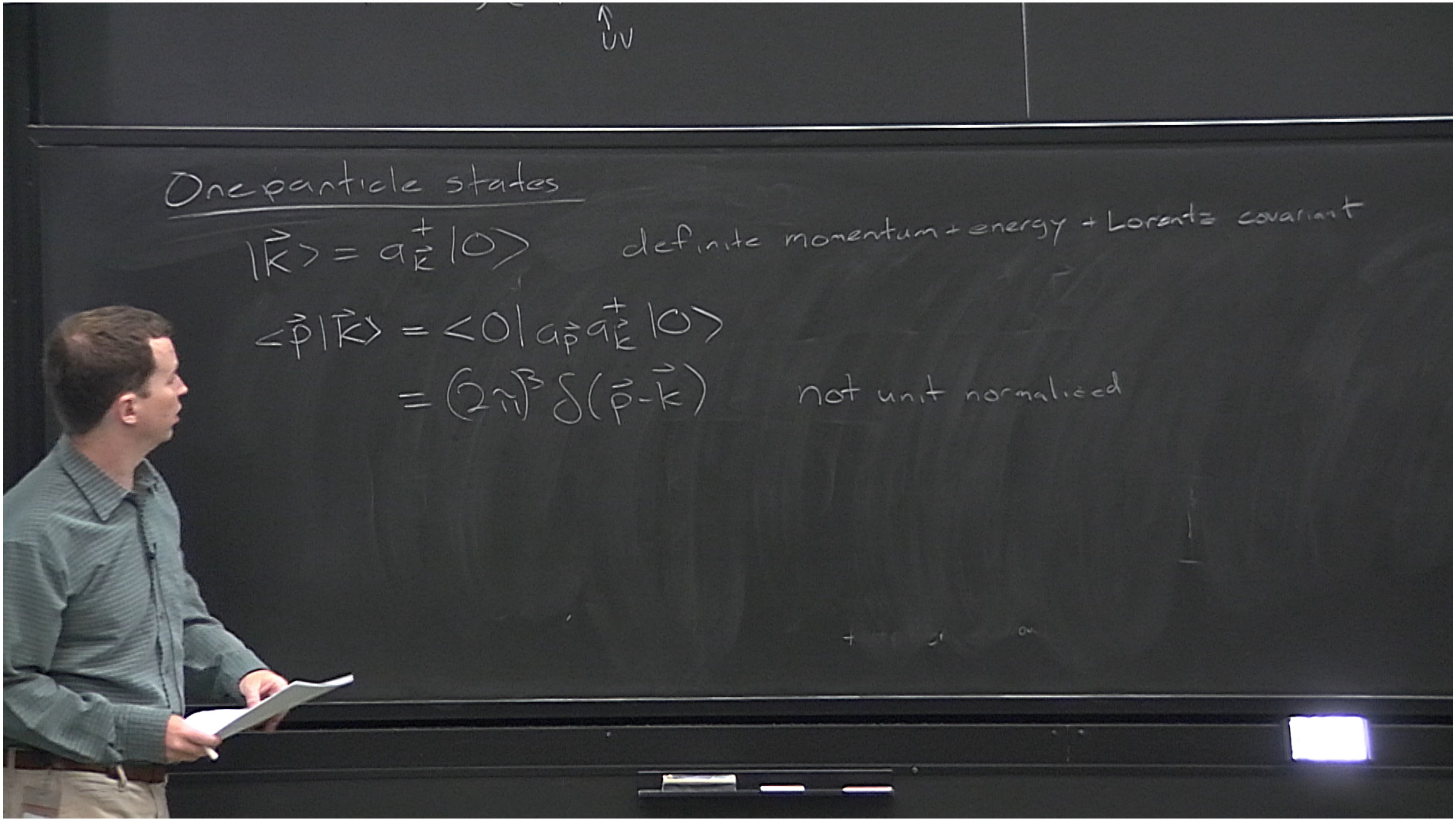
$E_0(a) \simeq \frac{\pi}{4} L \Lambda^2 - \frac{\pi}{24a}$
average over oscillations
$\int x(1-x)\,dx = \frac{1}{6}$
$F(a) = -\frac{dE_0}{da} = -\frac{\pi}{24a^2}$
a

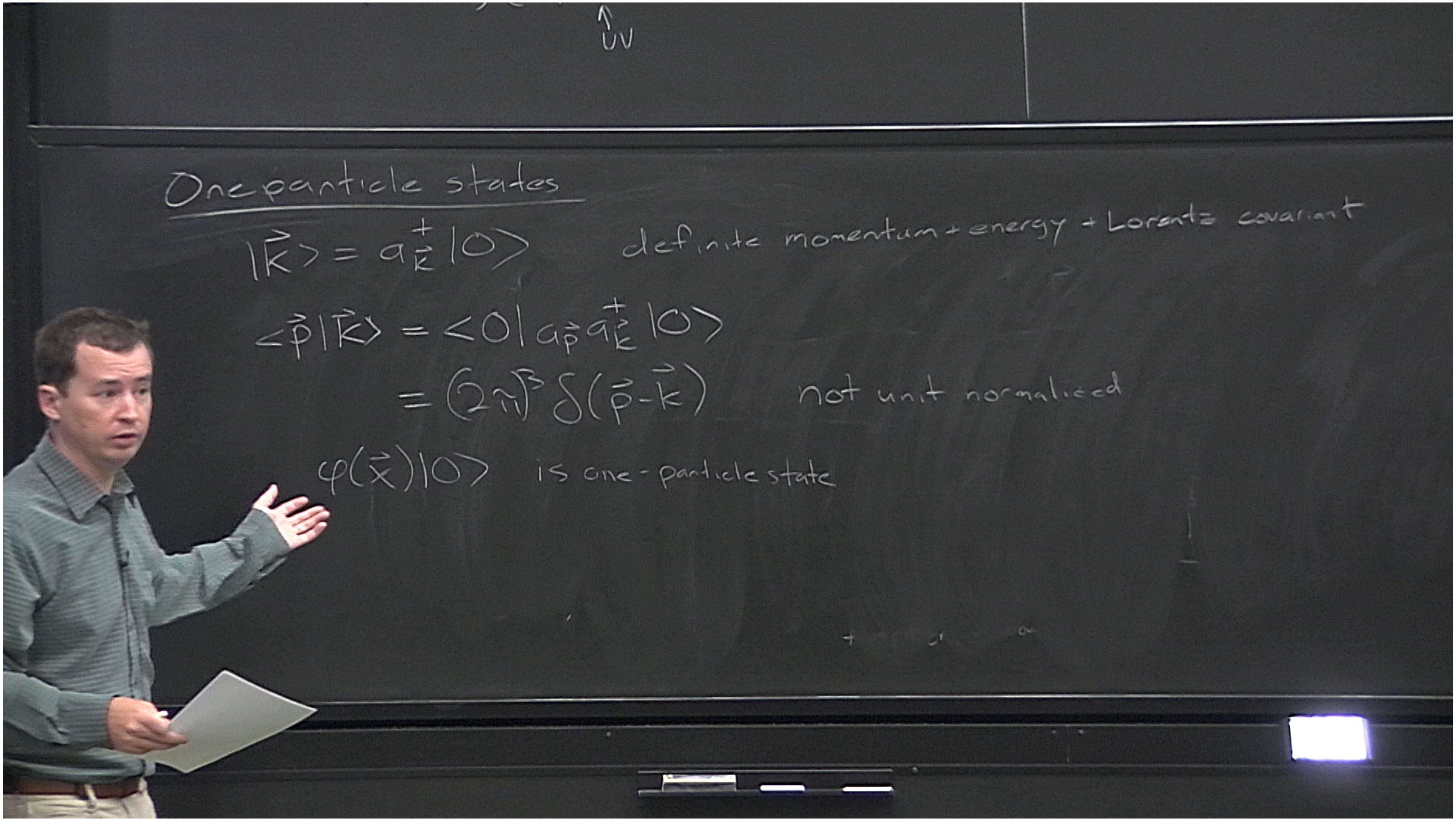

$E_0(a) \simeq \frac{\pi}{4} L \Lambda^2 - \frac{\pi}{24a}$
average over oscillations
$\int x(1-x)\,dx = \frac{1}{6}$
$F(a) = -\frac{dE_0}{da} = -\frac{\pi \hbar c}{24 a^2}$
a

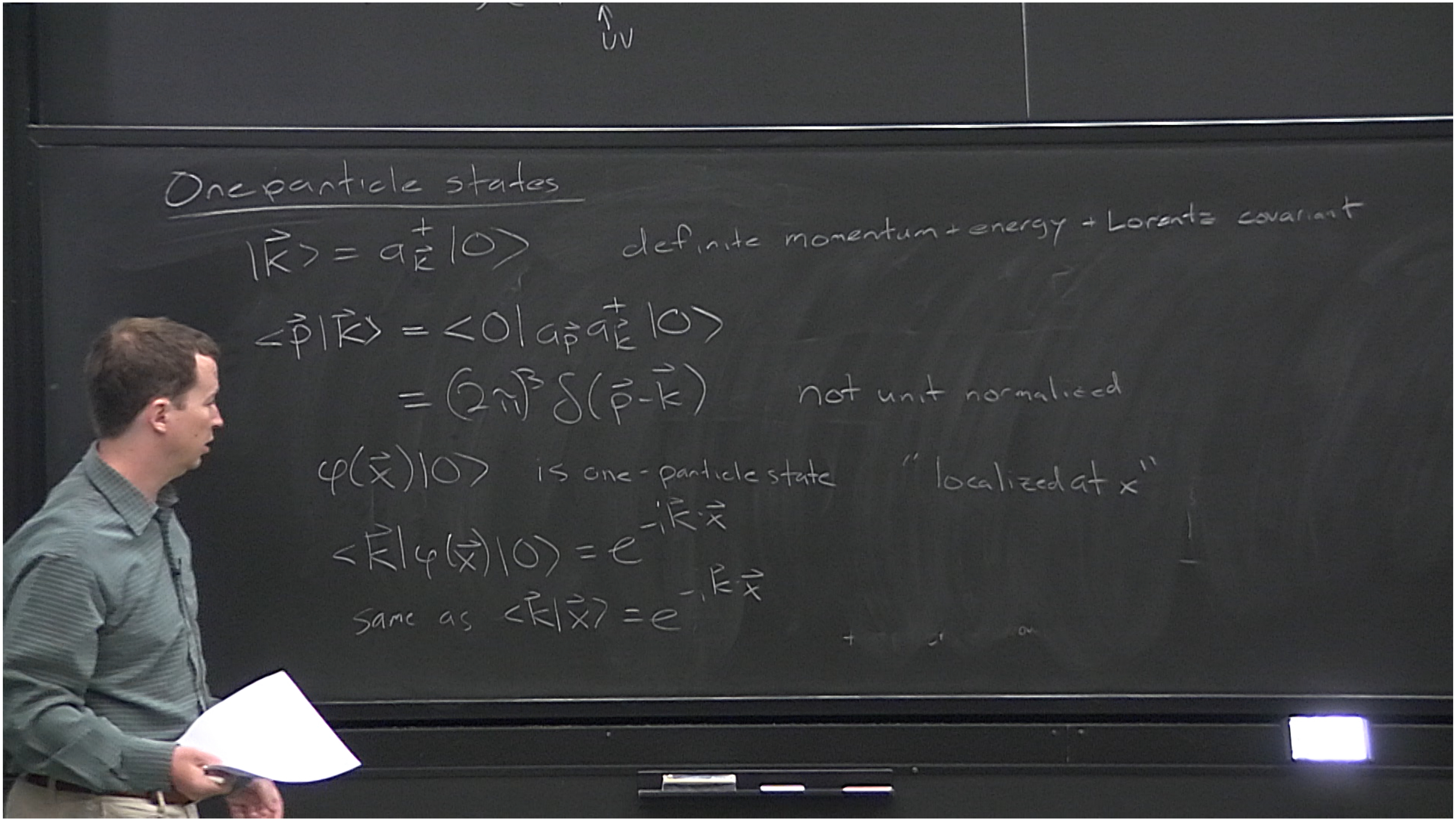

$E_0(a) \simeq \frac{\pi}{4} L \Lambda^2 - \frac{\pi}{24a}$
average over oscillations
$\int x(1-x)\,dx = \frac{1}{6}$
$\boxed{F(a) = -\frac{dE_0}{da} = -\frac{\pi \hbar c}{24 a^2}}$
impose a UV regulator → do calculation → result should be independent of regulator
a

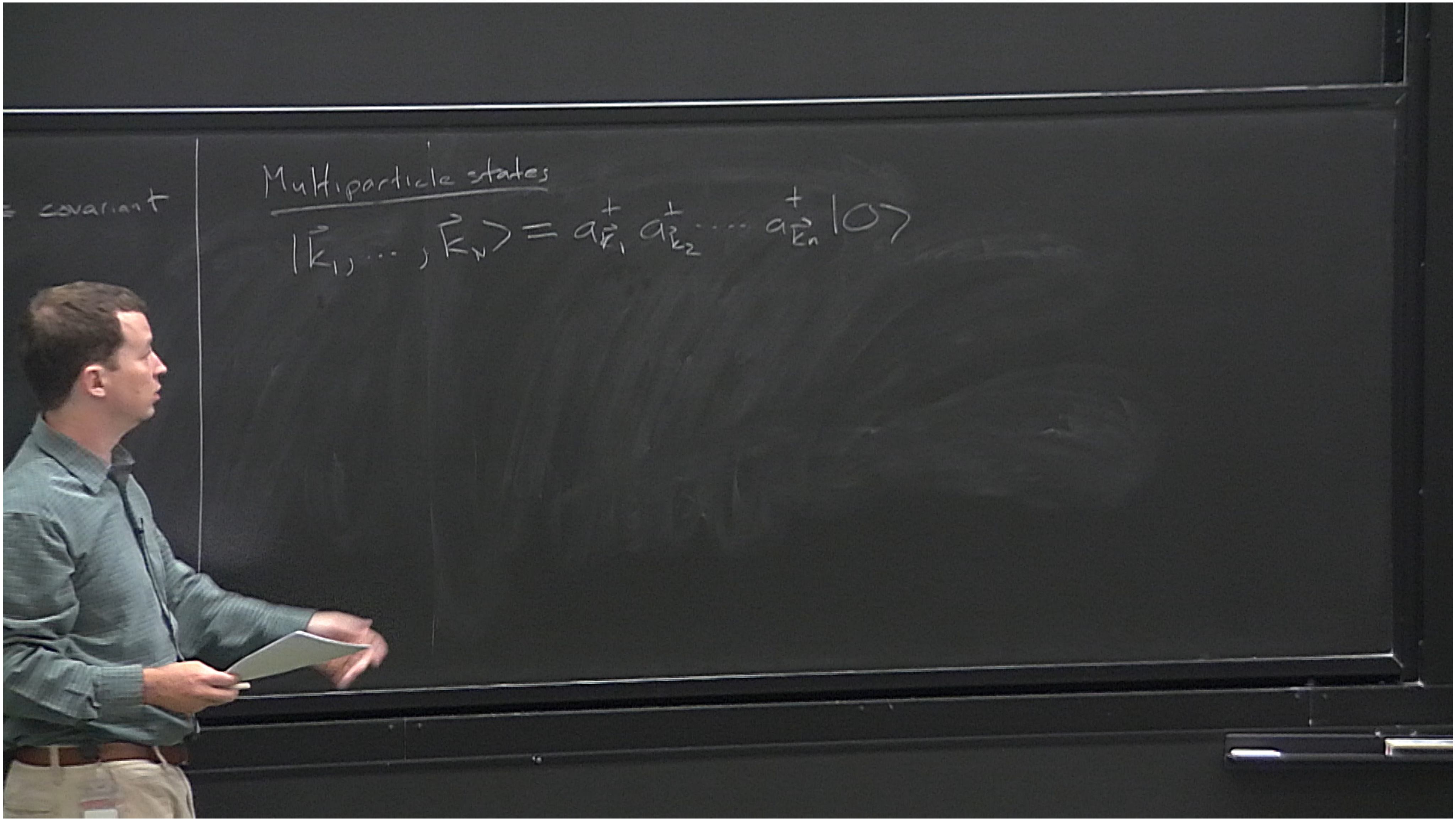

UV
One particle states
$|\vec{k}\rangle = a^\dagger_{\vec{k}}|0\rangle$
definite momentum + energy

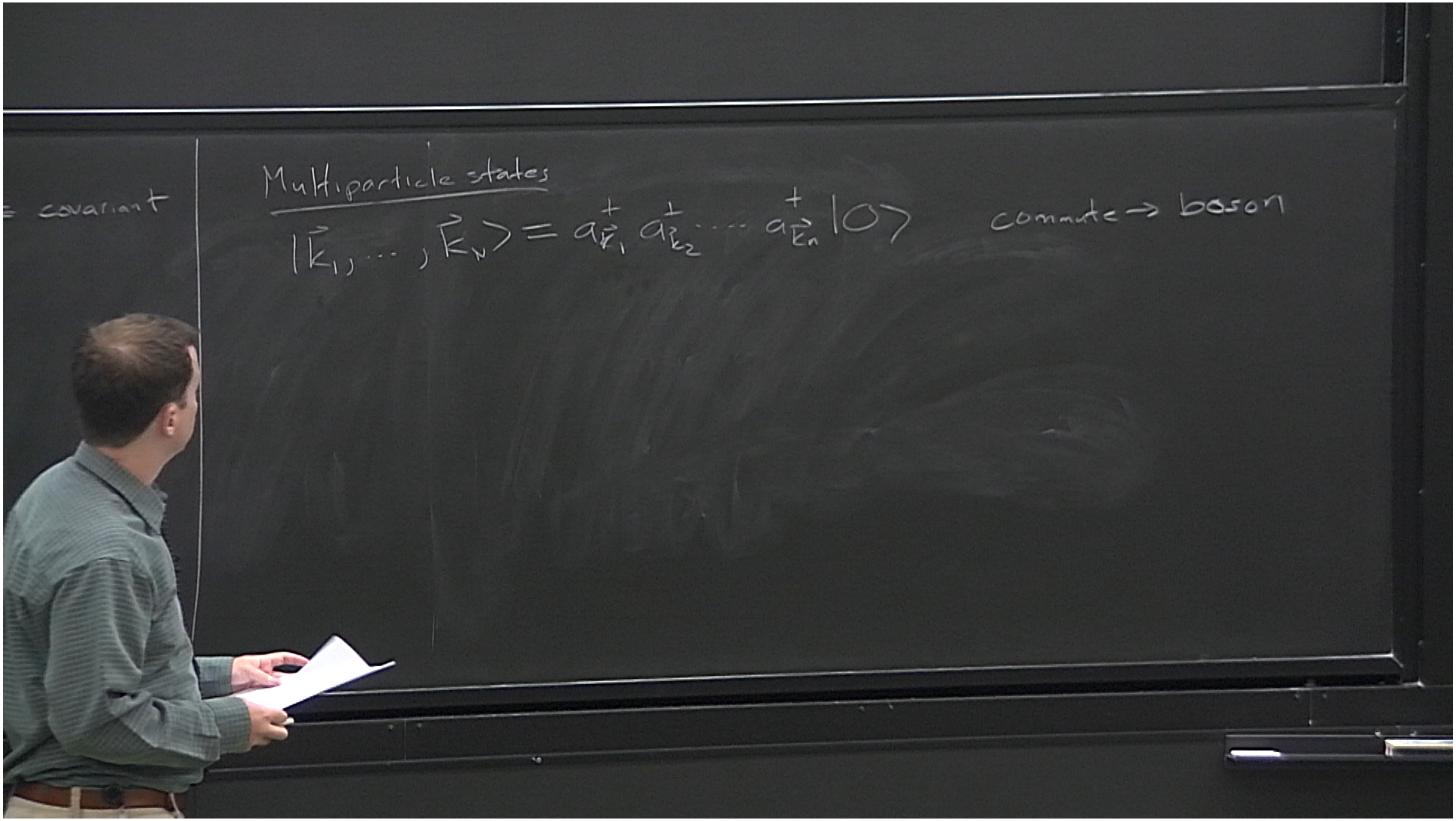

UV
One particle states
$|\vec{k}\rangle = a_{\vec{k}}^{\dagger}|0\rangle$
definite momentum + energy
$\langle\vec{p}|\vec{k}\rangle = \langle 0|a_{\vec{p}}a_{\vec{k}}^{\dagger}|0\rangle$
$= (2\pi)^3\delta(\vec{p}-\vec{k})$

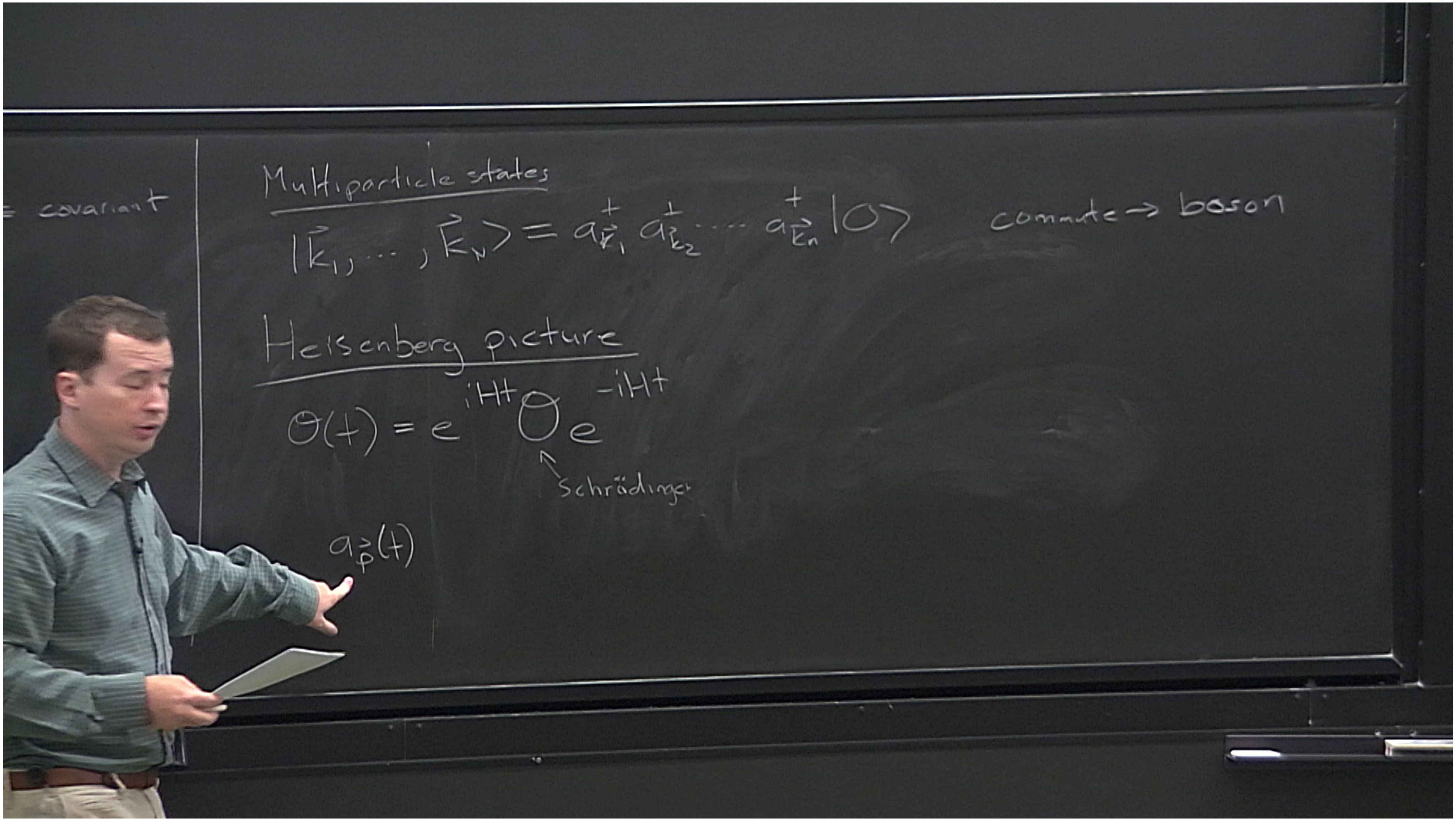

UV
One particle states
$|\vec{k}\rangle = a^{\dagger}_{\vec{k}}|0\rangle$
definite momentum + energy + Lorentz covariant
$\langle\vec{p}|\vec{k}\rangle = \langle 0|a_{\vec{p}}a^{\dagger}_{\vec{k}}|0\rangle$
$= (2\pi)^3\delta(\vec{p}-\vec{k})$
not unit normalized

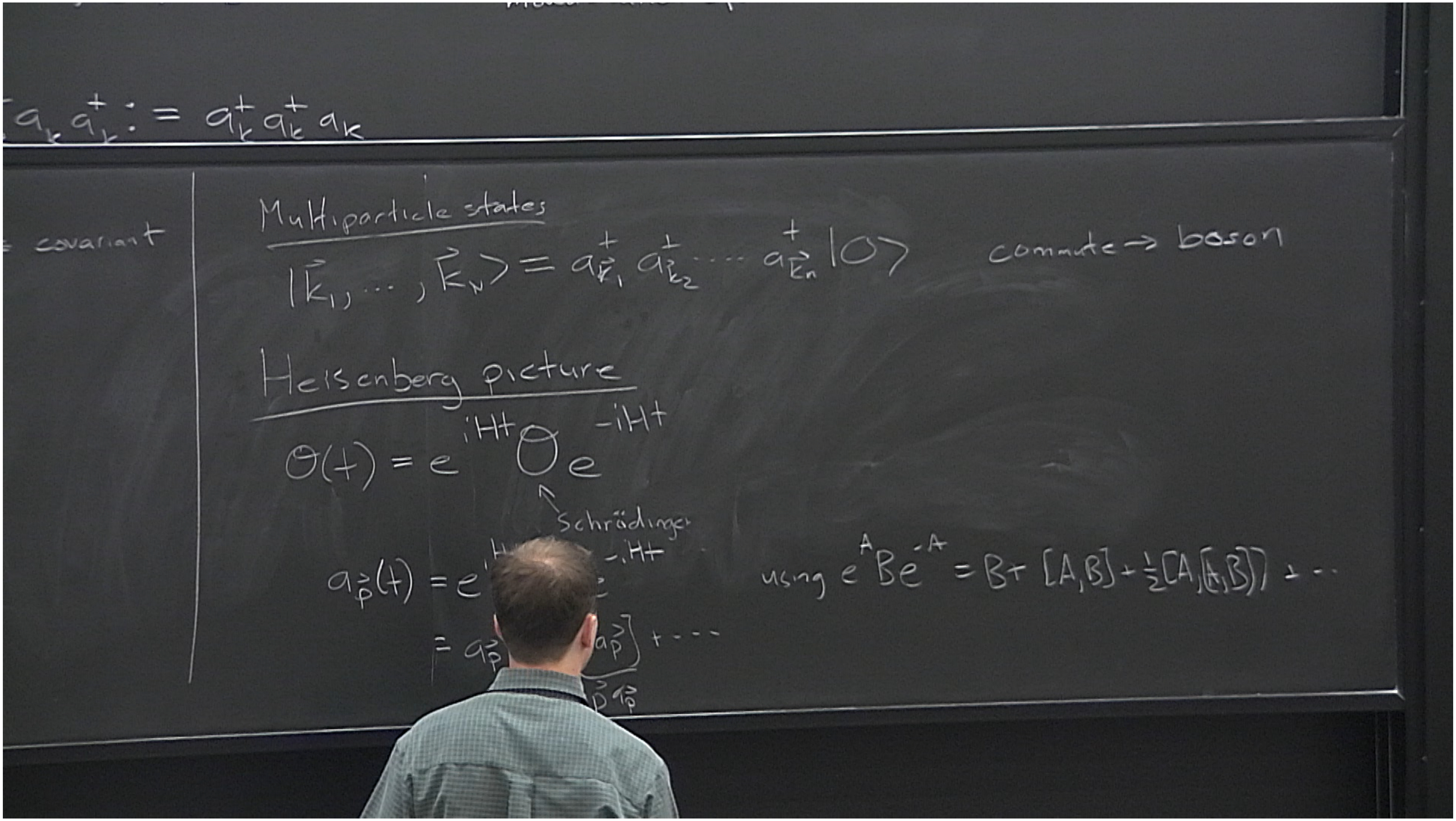

UV
One particle states
$|\vec{k}\rangle = a^\dagger_{\vec{k}}|0\rangle$
definite momentum + energy + Lorentz covariant
$\langle\vec{p}|\vec{k}\rangle = \langle 0|a_{\vec{p}}a^\dagger_{\vec{k}}|0\rangle$
$= (2\pi)^3\delta(\vec{p}-\vec{k})$
not unit normalized
$\varphi(\vec{x})|0\rangle$
is one-particle state

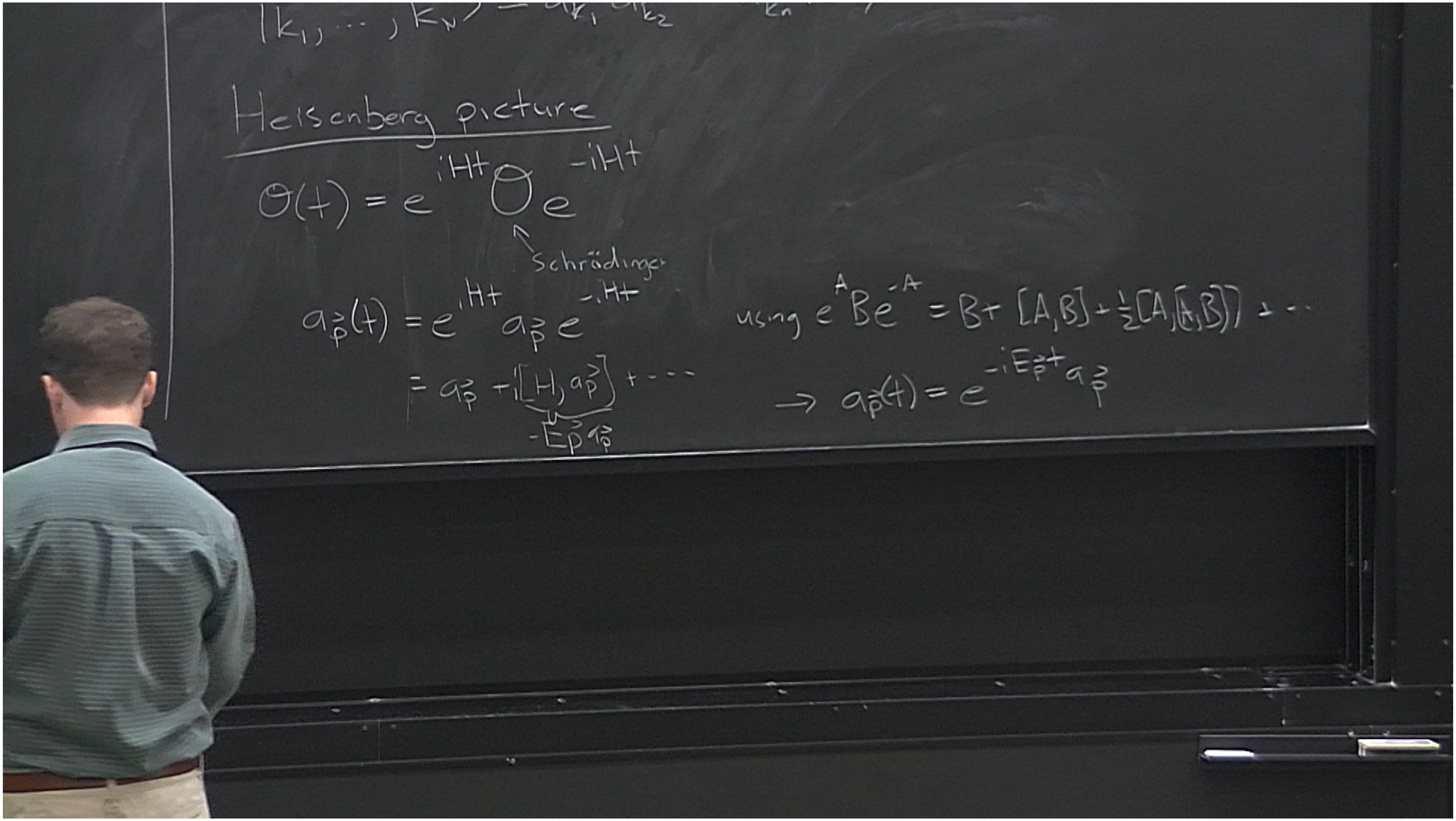

UV
One particle states
$|\vec{k}\rangle = a^{\dagger}_{\vec{k}}|0\rangle$
definite momentum + energy + Lorentz covariant
$\langle\vec{p}|\vec{k}\rangle = \langle 0|a_{\vec{p}}a^{\dagger}_{\vec{k}}|0\rangle$
$= (2\pi)^3\delta(\vec{p}-\vec{k})$
not unit normalized
$\varphi(\vec{x})|0\rangle$
is one-particle state
"localized at x"
$\langle\vec{k}|\varphi(\vec{x})|0\rangle = e^{-i\vec{k}\cdot\vec{x}}$
same as $\langle\vec{k}|\vec{x}\rangle = e^{-i\vec{k}\cdot\vec{x}}$

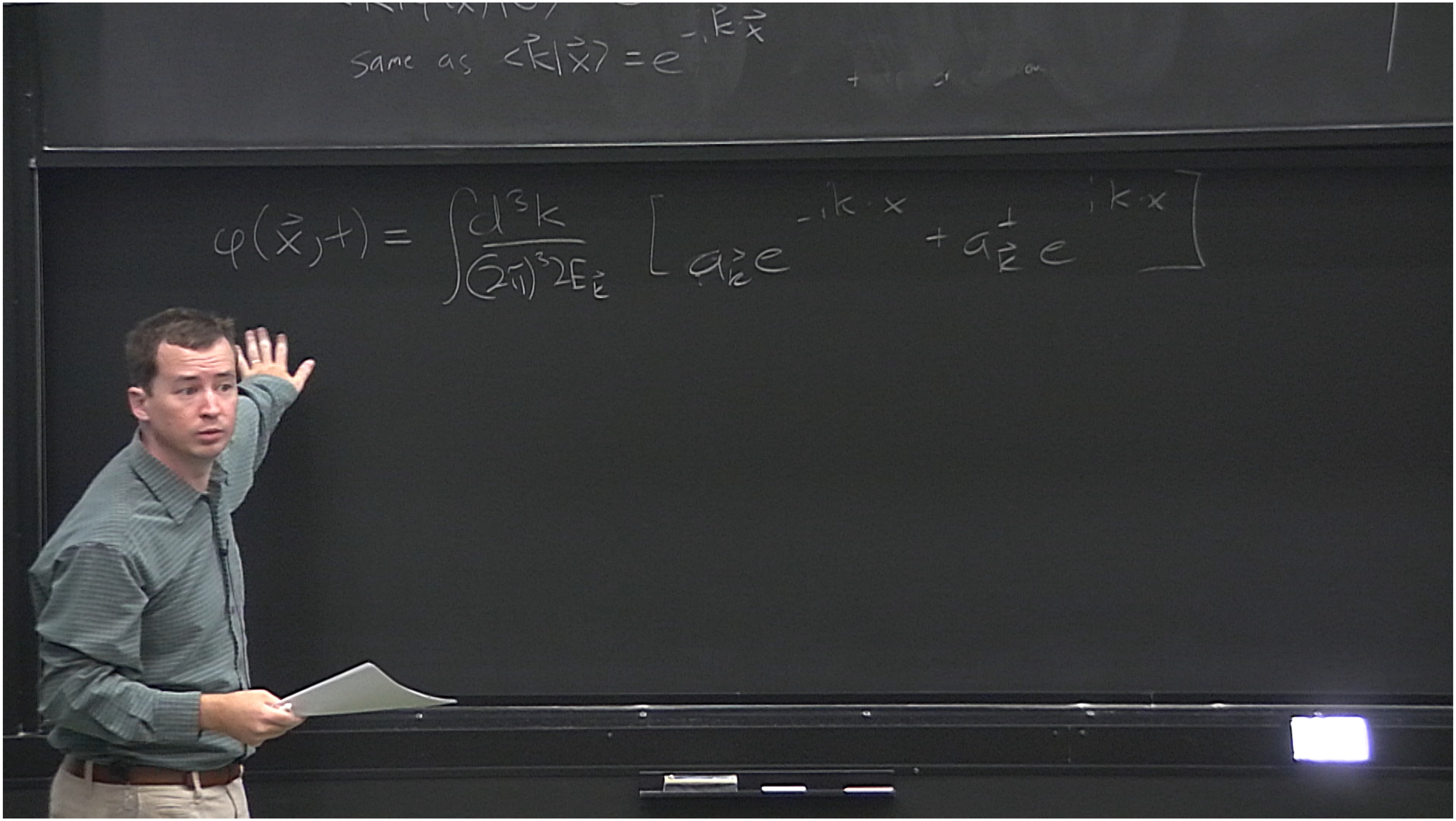

covariant
Multiparticle states
$|\vec{k}_1, \dots, \vec{k}_N\rangle = a^\dagger_{\vec{k}_1} a^\dagger_{\vec{k}_2} \cdots a^\dagger_{\vec{k}_n} |0\rangle$

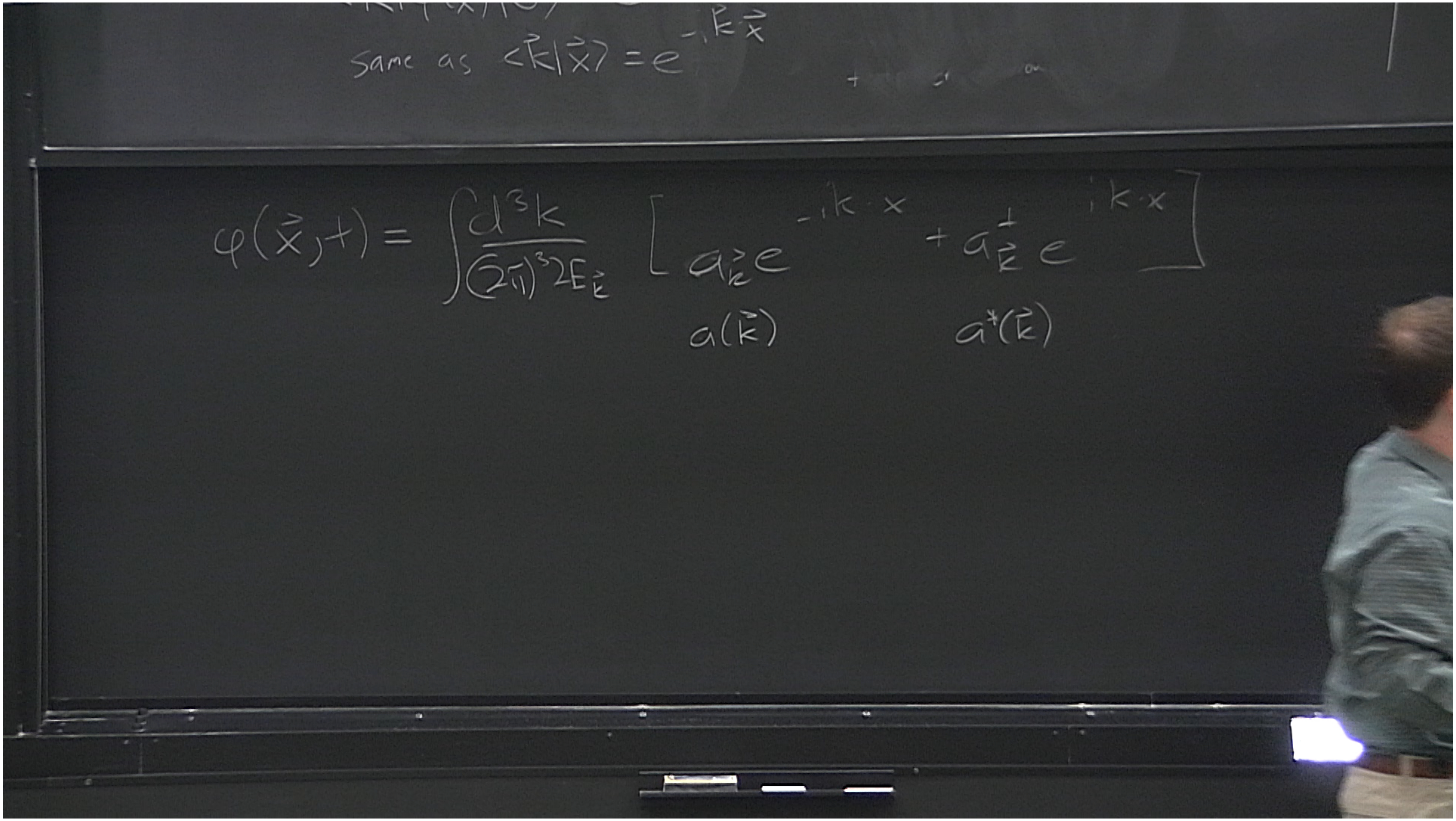

covariant
Multiparticle states
$|\vec{k}_1, \ldots, \vec{k}_N\rangle = a^\dagger_{\vec{k}_1} a^\dagger_{\vec{k}_2} \cdots a^\dagger_{\vec{k}_n} |0\rangle$
commute → boson


covariant
Multiparticle states
$|\vec{k}_1, \dots, \vec{k}_N\rangle = a^\dagger_{\vec{k}_1} a^\dagger_{\vec{k}_2} \cdots a^\dagger_{\vec{k}_n} |0\rangle$
commute → boson
Heisenberg picture
$\mathcal{O}(t) = e^{iHt} \mathcal{O} e^{-iHt}$
Schrödinger
$a_{\vec{p}}(t)$


:a_k a_k^+: = a_k^+ a_k^+ a_k
covariant
Multiparticle states
|k_1, ..., k_N> = a^+_{k_1} a^+_{k_2} ... a^+_{k_n} |0>
commute → boson
Heisenberg picture
O(t) = e^{iHt} O e^{-iHt}
Schrödinger
using e^A B e^{-A} = B + [A,B] + 1/2 [A,[A,B]] + ...


Heisenberg picture
$O(t) = e^{iHt} O e^{-iHt}$
Schrödinger
$a_{\vec{p}}(t) = e^{iHt} a_{\vec{p}} e^{-iHt}$
$= a_{\vec{p}} + i\underbrace{[H, a_{\vec{p}}]}_{-E_{\vec{p}} a_{\vec{p}}} + \cdots$
using $e^{A} B e^{-A} = B + [A,B] + \frac{1}{2}[A,[A,B]] + \cdots$
$\rightarrow a_{\vec{p}}(t) = e^{-iE_{\vec{p}} t} a_{\vec{p}}$


same as $\langle \vec{k} | \vec{x} \rangle = e^{-i\vec{k}\cdot\vec{x}}$
$\varphi(\vec{x},t) = \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} \left[a_{\vec{k}} e^{-ik\cdot x} + a^{\dagger}_{\vec{k}} e^{ik\cdot x} \right]$



Same as $\langle \vec{k}|\vec{x}\rangle = e^{-i\vec{k}\cdot\vec{x}}$
$\varphi(\vec{x},t) = \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} \left[a_{\vec{k}} e^{-ik\cdot x} + a^\dagger_{\vec{k}} e^{ik\cdot x} \right]$
$a(\vec{k})$
$a^\dagger(\vec{k})$

Same as $\langle \vec{k} | \vec{x} \rangle = e^{-i\vec{k}\cdot\vec{x}}$

$$\varphi(\vec{x},t) = \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} \left[a_{\vec{k}} e^{-ik\cdot x} + a^\dagger_{\vec{k}} e^{ik\cdot x} \right]$$

$a(\vec{k})$ $\quad$ $a^\dagger(\vec{k})$

interacting field

$$\Phi(\vec{x},t) = \int \frac{d^3k}{(2\pi)^3 2E_k} \left[b_{\vec{k}}(t) e^{-ik\cdot x} + b^\dagger_k(t) e^{ik\cdot x} \right]$$