Title: PSI 2018/2019 - Quantum Field Theory I - Lecture 12

Date: Oct 29, 2018 09:00 AM

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Abstract:

Recap. Quantize Dirac.

$$\mathcal{L} = \bar{\psi}\left(i\gamma^\mu \partial_\mu - m\right)\psi$$

$$\left\{\psi^r_a(\vec{x}), \psi^{s\dagger}_b(\vec{y})\right\} = \delta(\vec{x}-\vec{y})\,\delta^{rs}\delta_{ab}$$

Recap: Quantize Dirac.

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi$$

$$\{\psi_a(\vec{x}), \psi_b^\dagger(\vec{y})\} = \delta(\vec{x}-\vec{y})\,\delta_{ab}$$

Causality $\mathcal{O} = \bar{\psi}\Gamma_A\psi$ $\quad [\mathcal{O}_1, \mathcal{O}_2] = 0$

$$\Rightarrow \{\quad\} = 0$$

$$\psi_{a+}(\vec{x}) = \int dV_p\, c^{s\dagger}_{\vec{p}}\, v^s_a(\vec{p})\, e^{+ipx}$$

$$\psi_{a-}(\vec{x}) = \int dV_p\, b^s_{\vec{p}}\, u^s_a(\vec{p})\, e^{-ipx}$$

δ_{ab}

$\{O_1, O_2\} = 0$

$\{ \quad \} = 0$

Recap: Quantize Dirac.

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$$

$$\{\psi_a(\vec{x}), \psi^\dagger_b(\vec{y})\} = \delta(\vec{x}-\vec{y})\,\delta_{ab}$$

Causality $\mathcal{O} = \bar{\psi}\Gamma\psi$ $[\mathcal{O}_1, \mathcal{O}_2] = 0$

$\Rightarrow \{\quad\}=0$

$$\psi_{a+}(x) = \int dV_p \, c^{s\dagger}_{\vec{p}} \, v^s_a(\vec{p}) \, e^{+ipx}$$

$$\psi_{b-}(x) = \int dV_p \, b^s_{\vec{p}} \, u^s_a(\vec{p}) \, e^{-ipx}$$

δ_{ab}

$$\bar{\psi}_{b+}(x) = \int dV_p \, b^{\dagger s}_{\vec{p}} \, \bar{u}^s_b(\vec{p}) \, e^{ipx}$$

$[O_1, O_2] = 0$

$\} = 0$

$$\bar{\psi}_{b-}(x) = \int dV_p \, c^s_{\vec{p}} \, \bar{v}^s_b(\vec{p}) \, e^{-ipx}$$

Interactions.

cross section

$\downarrow$

Matrix element square

$\downarrow$ some gamething

Matrix element

$\langle f | S | i \rangle$

$\downarrow$

LSZ

$$\langle f|S|i\rangle$$

$$S|i\rangle = a_1^\dagger(-\infty)\, a_2^\dagger(-\infty)\,|\Omega\rangle$$

$$|f\rangle = a_3^\dagger(+\infty)\, a_4^\dagger(+\infty)\,|\Omega\rangle$$

$$\langle f|S|i\rangle = \langle\Omega|\, T\, a_4(+\infty)\, a_3(+\infty)\, a_1^\dagger(-\infty)\, a_2^\dagger(-\infty)\,|\Omega\rangle$$

$$T^2 = T$$

$$a_4(+\infty) = a_4(+\infty) - a_4(-\infty) + a_4(-\infty)$$

$$= I_4 + a_4(-\infty)$$

an integral

$$I_4 = \int_{-\infty}^{+\infty} \partial_0 a_4(t)$$

$$a_1^\dagger(-\infty)\, a_2^\dagger(-\infty) |\Omega\rangle$$

$$\langle f|S|i\rangle = \langle \Omega | I_4 I_3 I_1^\dagger I_2^\dagger | \Omega \rangle$$

$$I_1^\dagger = -i \int d^4x \, e^{-ik_1\cdot x_1} (\partial^2 + m^2)\, \varphi(x_1)$$

something will give 4-momentum conservation

KG operator

$$I_1 = -i \int d^4x \, e \ldots$$

$$\Big[\partial_0 a_4(t)\Big]_{-\infty}^{+\infty}$$

$\langle S_2$

$$\langle f|S|i\rangle = \ldots \langle \Omega | \varphi_4 \varphi_3 \varphi_1 \varphi_2 | \Omega \rangle$$

KG operator
plane waves

i
something will give 4-momentum conservation

KG operator

$$\langle \Omega | T\, \varphi_4 \varphi_3 \varphi_1 \varphi_2 | \Omega \rangle$$

$$\overset{\text{Dyson}}{=} \frac{\langle 0 | T\, \varphi_4 \varphi_3 \varphi_1 \varphi_2\, e^{i\int \mathcal{L}_{int}} | 0 \rangle}{\langle 0 | e^{i\int \mathcal{L}_{int}} | 0 \rangle}$$

$\downarrow$ perturbation.

$$\langle 0 | T\, \varphi_1 \varphi_2 \cdots | 0 \rangle$$

Wick contraction: $T - :\;:$

$$\varphi_1 \varphi_2 | \Omega \rangle$$

$$\varphi_1 \varphi_2 \, e^{i \int \mathcal{L}_{int}} | 0 \rangle$$

$$e^{i \int \mathcal{L}_{int}} | 0 \rangle$$

tion.

$$\langle 0 | T \varphi_1 \varphi_2 \cdots | 0 \rangle$$

Wick contraction: $T - : \; :$

$$T \varphi_1 \cdots = : \quad : + \text{all possible contractions}$$

$$\underbrace{\varphi_1 \varphi_2} = \Delta_F (x_1 - x_2)$$

Fermionic variations: $b_1(t) \equiv b^{s_1}_{\vec{k}_1}(t)$

$$S|i\rangle = b_1^{\dagger}(-\infty)\, b_2^{\dagger}(-\infty)|\Omega\rangle$$

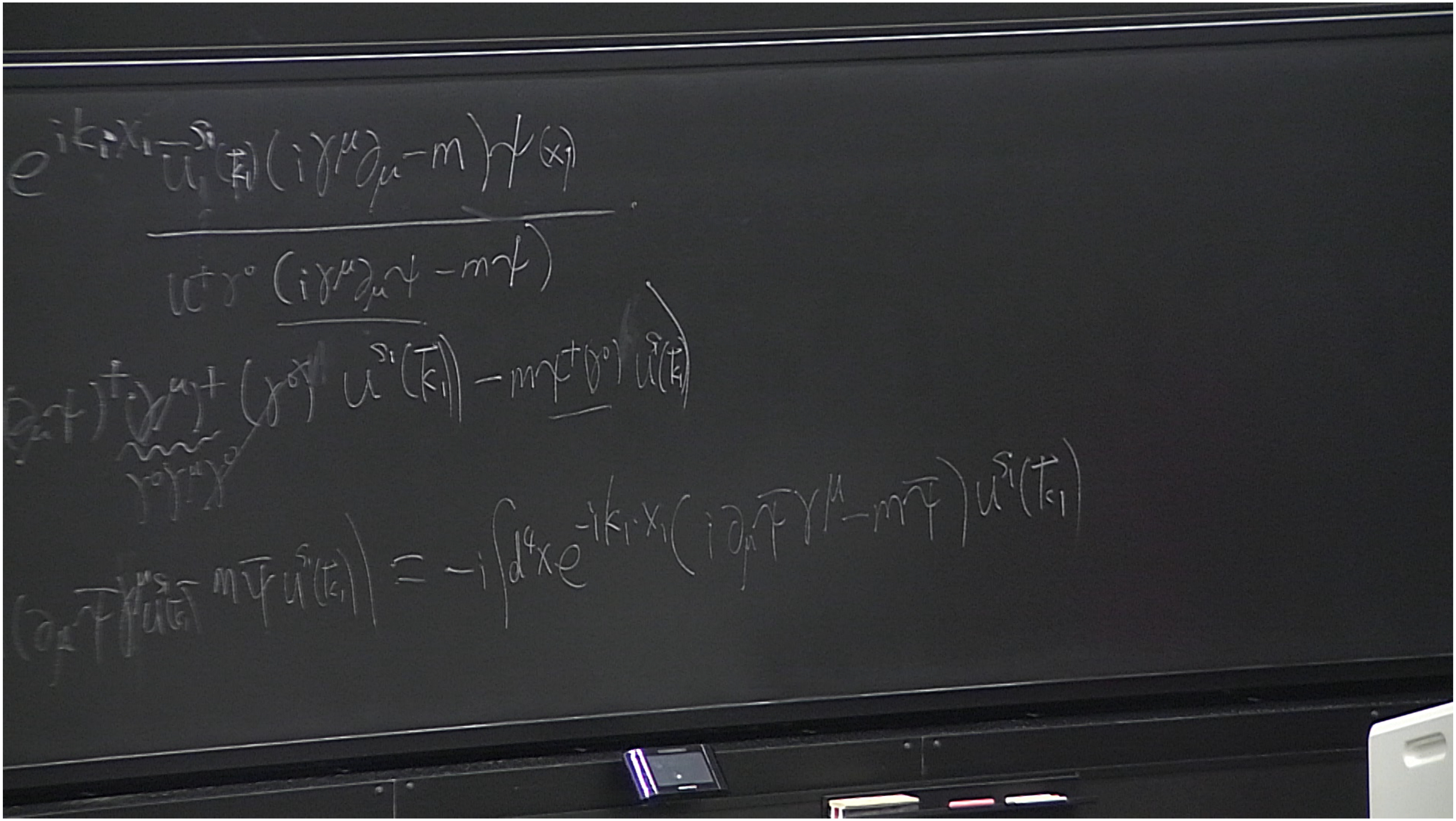
↳ aside. minus sign if we swap

$$\langle f|S|i\rangle = \langle\Omega|\, I_4 I_3 I_1^{\dagger} I_2^{\dagger}\,|\Omega\rangle$$

$$I_1 = b_1(+\infty) - b_1(-\infty) = -i\int d^4x\, e^{ik_1\cdot x_1}\, \underbrace{\bar{u}^{s_1}(\vec{k}_1)\left(i\gamma^\mu\partial_\mu - m\right)\psi(x_1)}_{u^\dagger\gamma^0\left(i\gamma^\mu\partial_\mu\psi - m\psi\right)}$$

$$I_1^\dagger = i\int d^4x\, e^{-ik_1\cdot x_1}\left(-i(\partial_\mu\psi)^\dagger \underbrace{\gamma^{\mu\dagger}}_{\gamma^0\gamma^\mu\gamma^0}(\gamma^0)^\dagger u^{s_1}(\vec{k}_1) - m\psi^\dagger(\gamma^0)^\dagger u^{s_1}(\vec{k}_1)\right)$$

$$= i\int d^4x\, e^{-ik_1\cdot x_1}\left(-i(\partial_\mu\bar{\psi})u^{s_1}(\vec{k}_1) - m\bar{\psi}u^{s_1}(\vec{k}_1)\right) =$$


$e^{ik_1 \cdot x_1} \bar{u}_1^{s_1}(\vec{k}_1)(i\gamma^\mu \partial_\mu - m)\psi(x_1)$
$u^\dagger \gamma^0 (i\gamma^\mu \partial_\mu \psi - m\psi)$
$(\partial_\mu \psi)^\dagger \gamma^{\mu\dagger} (\gamma^0)^\dagger u^{s_1}(\vec{k}_1) - m\psi^\dagger (\gamma^0)^\dagger u^{s_1}(\vec{k}_1)$
$\gamma^0 \gamma^\mu \gamma^0$
$(\partial_\mu \bar{\psi} \gamma^\mu u^{s_1}(\vec{k}_1) - m\bar{\psi} u^{s_1}(\vec{k}_1)) = -i \int d^4x\, e^{-ik_1 \cdot x_1} (i\partial_\mu \bar{\psi} \gamma^\mu - m\bar{\psi}) u^{s_1}(\vec{k}_1)$

$$= -i\int d^4x\, e^{-ik_1\cdot x_1}\,\bar\Psi\,(i\overleftarrow{\partial}_\mu\gamma^\mu + m)\,u^{s_1}(\vec k_1)$$

Dirac operator

$$\langle f|S|i\rangle = \boxed{\bar u^{s_4}_4 u^{s_3}_3(\vec k_3)}\,\langle 0|\psi_4\psi_3\bar\psi_1\bar\psi_2|0\rangle\,\boxed{\text{Dirac operator}\; u^{s_1}(\vec k_1)\,u^{s_2}(\vec k_2)}$$

$$)\,\psi(x_1)$$

$$m\psi)$$

$$-m\Psi^\dagger\gamma^0\gamma\, u(\vec k_1)$$

$$-i\int d^4x\, e^{-ik_1\cdot x_1}\,(i\partial_\mu\bar\Psi\gamma^\mu + m\bar\Psi)\,u^{s_1}(\vec k_1)$$

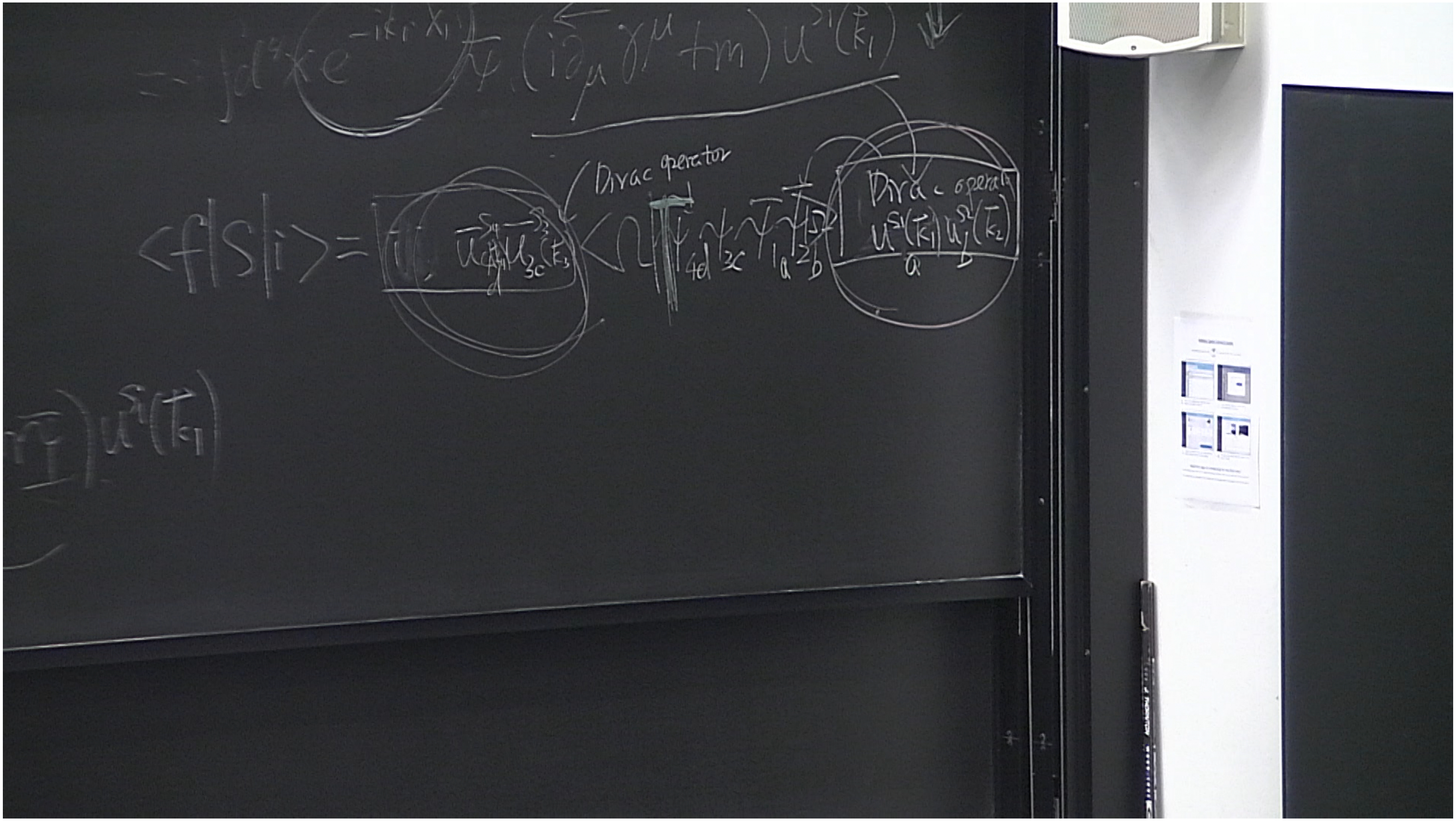
$$= -i\int d^4x\, e^{-ik_1\cdot x_1}\,\bar{\psi}\,(i\overleftarrow{\partial}_\mu \gamma^\mu + m)\,u^{s_1}(\vec{k}_1)$$

Dirac operator

Dirac operator

$$\langle f|S|i\rangle = \boxed{i\,\bar{u}_4^{s_4}(\vec{k}_4)\,u_3^{s_3}(\vec{k}_3)}\,\langle\Omega|\,\psi_4\,\psi_3\,\bar{\psi}_1\,\bar{\psi}_2\,|\Omega\rangle\,\boxed{u_1^{s_1}(\vec{k}_1)\,u^{s_2}(\vec{k}_2)}$$

$s_1(\vec{k}_1)$



Dirac operator

$$\langle 0 | T \psi_a(x) \bar{\psi}_b(y) | 0 \rangle$$

one issue boson $T \varphi(x) \varphi(y) = \varphi(x)\varphi(y) \quad x^0 > y^0$

$= -\varphi(y)\varphi(x) \quad y^0 > x^0$

$\psi \psi = 0$

$\bar{\psi} \bar{\psi} = 0$

Try 1:

$$T\psi_a(x)\bar{\psi}_b(y) = -T\bar{\psi}_b(y)\psi_a(x)$$

suppose $x^0 > y^0$ follow the same definition

$$T\psi_a(x)\bar{\psi}_b(y) = -\psi_a(x)\bar{\psi}_b(y)$$ ↙ bosonic def

fermion

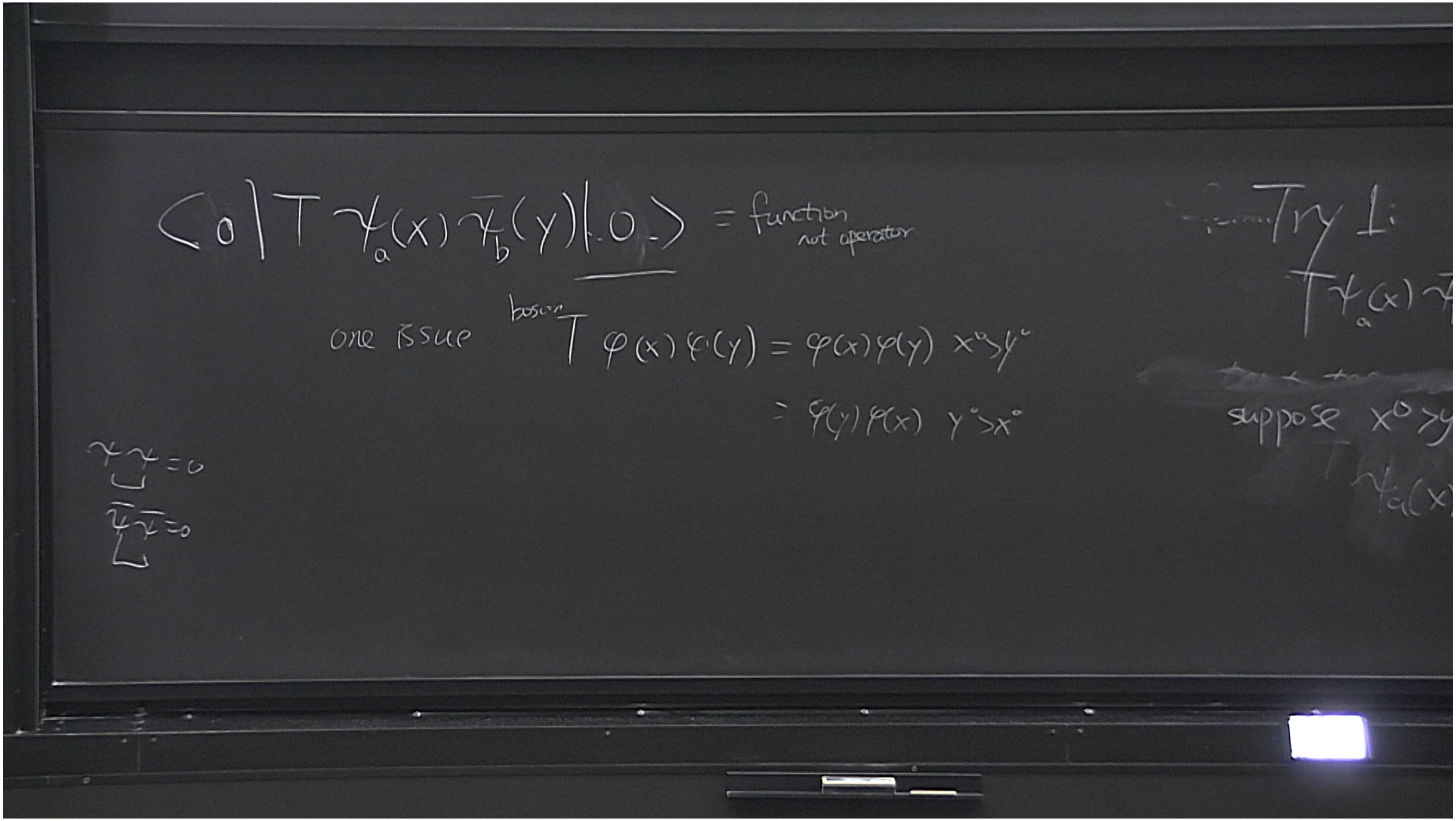
$x^0 > y^0$

$$T\psi_a(x)\bar{\psi}_b(y) = \psi_a(x)\bar{\psi}_b(y)$$

$$= -\bar{\psi}_b(y)\psi_a(x)$$

$y_0 > x^0$

$$T^2 = T$$



$\langle 0 | T \psi_a(x) \bar{\psi}_b(y) | 0 \rangle$ = function
not operator
one issue
boson
$T \varphi(x) \varphi(y) = \varphi(x)\varphi(y) \quad x^0 > y^0$
$= \varphi(y)\varphi(x) \quad y^0 > x^0$
$\psi \psi = 0$
$\bar{\psi} \bar{\psi} = 0$
Try 1:
$T \psi_a(x)$
suppose $x^0 >$
$\psi_a(x)$

$y^0 > x^0$: $T\psi_a(x)\bar{\psi}_b(y) = -\bar{\psi}_b(y)\psi_a(x)$

$= \text{good guys} - \bar{\psi}_{b-}(y)\psi_a(x)$

$= :\;: - \{\bar{\psi}_{b-}(y), \psi_{a+}(x)\}$

...ntize Dirac.

$(i\gamma^\mu \partial_\mu - m)\psi$

$\{\psi_a(\vec{x}), \psi^\dagger_b(\vec{y})\} = \delta(\vec{x}-\vec{y})\,\delta_{ab}$

$\mathcal{O} = \bar{\psi}\Gamma_i\psi \qquad [\mathcal{O}_1, \mathcal{O}_2] = 0$

$\Rightarrow \{\ \ \} = 0$

$$\psi_{a+}(x) = \int dV_p\, c^{s\dagger}_{\vec{p}}\, v^s_a(\vec{p})\, e^{+ipx}$$

$$\psi_{a-}(x) = \int dV_p\, b^s_{\vec{p}}\, u^s_a(\vec{p})\, e^{-ipx}$$

$$\bar{\psi}_{b+}(y) = \int dV_q\, b^{\dagger s}_{\vec{q}}\, \bar{u}^s_b(\vec{q})\, e^{iq\cdot y}$$

$$\bar{\psi}_{b-}(y) = \int dV_q\, c^s_{\vec{q}}\, \bar{v}^s_b(\vec{q})\, e^{-iq\cdot y}$$

Int...

cro...

$$\{\psi_{a-}(x), \bar{\psi}_{b+}(y)\} = \int dV_p \left(i\partial_x{}_\mu \gamma^\mu + m\right)_{ab} e^{-ip(x-y)}$$

$$-\{\bar{\psi}_{b-}(y), \psi_{a+}(x)\} = -\int dV_p \left(-i\partial_x{}_\mu \gamma^\mu - m\right)_{ab} e^{-ip(y-x)}$$

$p(x-y)$

$ip(y-x)$

$$\text{Wick} = \Theta(x^0-y^0) + \Theta(y^0-x^0))$$

$$= (i\partial_{\mu x}\gamma^\mu + m)\Delta_F(x-y)$$