Title: PSI 2018/2019 - Statistical Mechanics - Lecture 11

Date: Oct 26, 2018 10:45 AM

URL: http://pirsa.org/18100034

Abstract:

$$Z = \int \prod_{\vec{n}} d x_{\vec{n}}^{(j)} \quad \exp\left[-\beta \sum_{\langle ij \rangle} \left(\vec{x}_i - \vec{x}_j \right)^2 + m^2 \sum \vec{X}^2 + \mu^2 \sum \left(\vec{X}^2 \right)^2 \right]$$

$\equiv \vec{X}_{\vec{n}}$ with N components

large distances

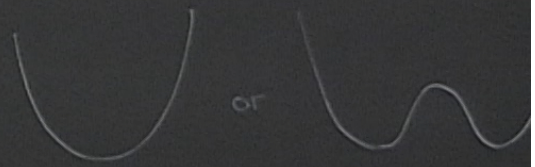
$$\int d^D \vec{n} \, (\nabla \vec{X})^2 + m_{\text{eff}}^2 \vec{X}^2$$

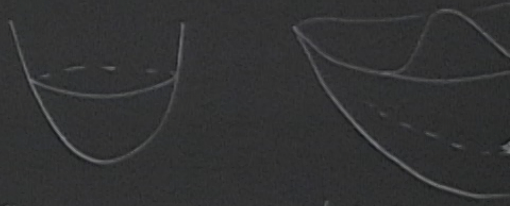
INTERESTING AT LARGE DISTANCES ⇒ Power law decay if $m_{\text{eff}} = 0$ (requires tuning)

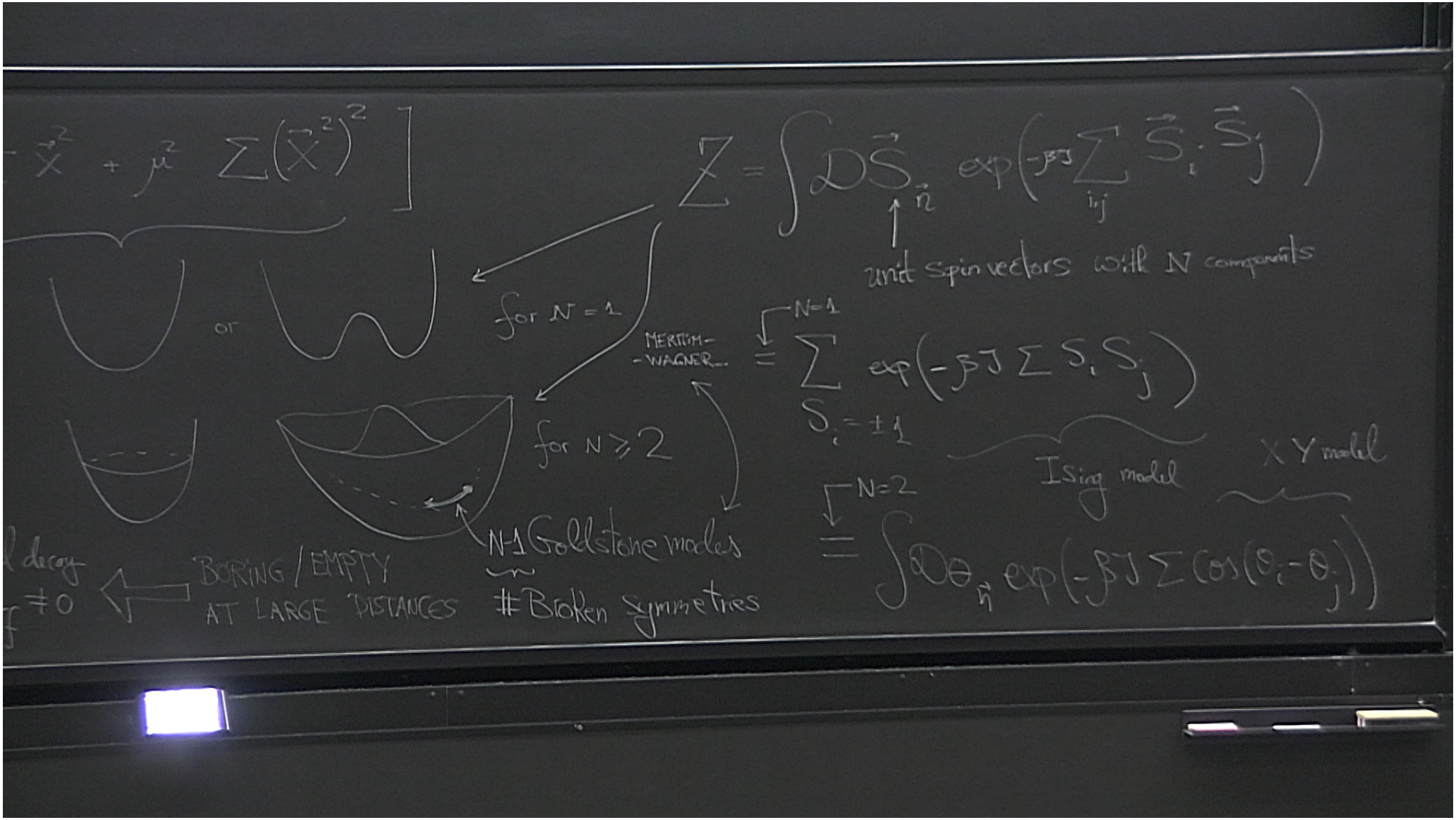
exponential decay if $m_{\text{eff}} \neq 0$

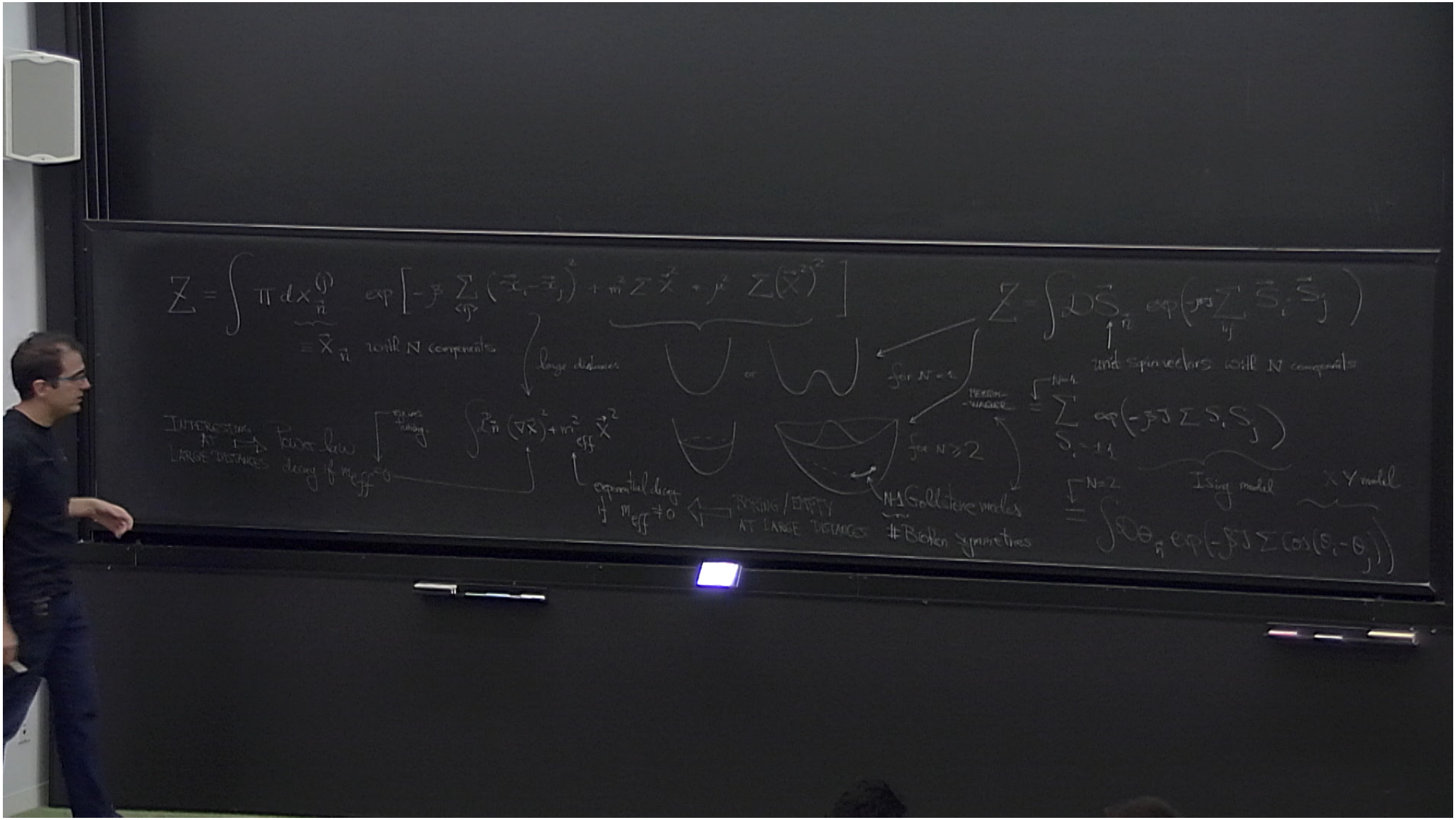
⇐ BORING / EMPTY AT LARGE DISTANCES

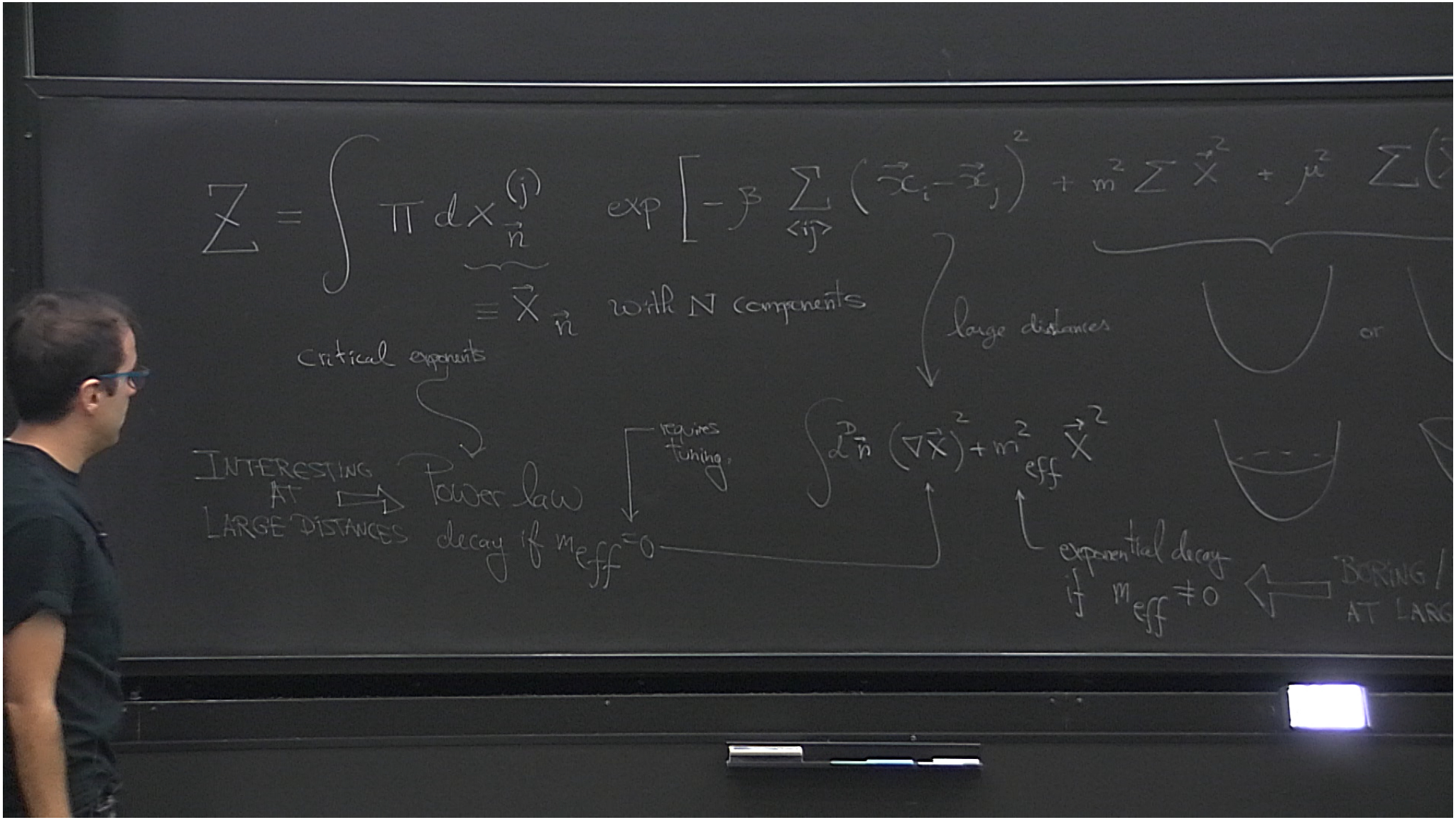
or

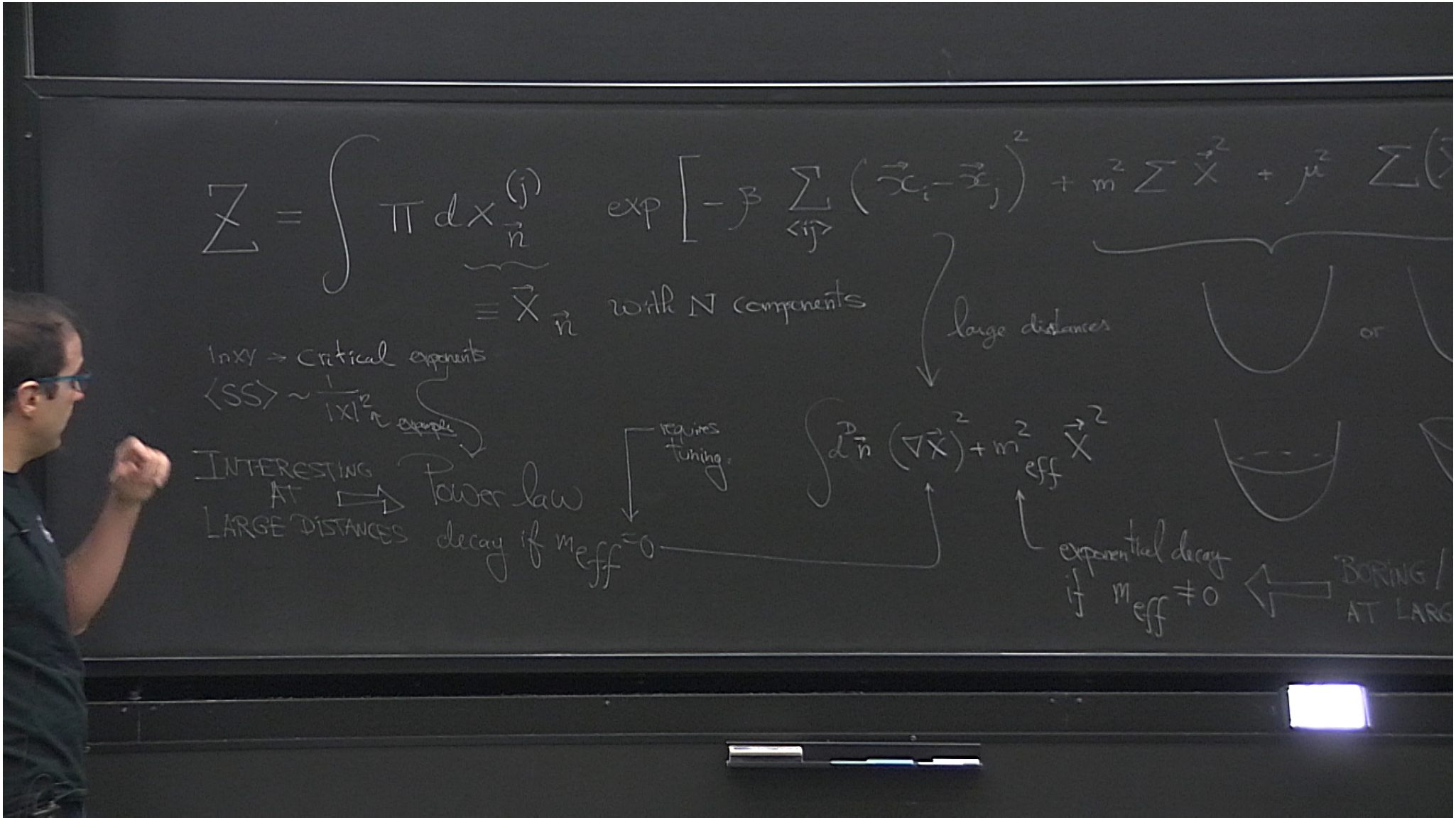


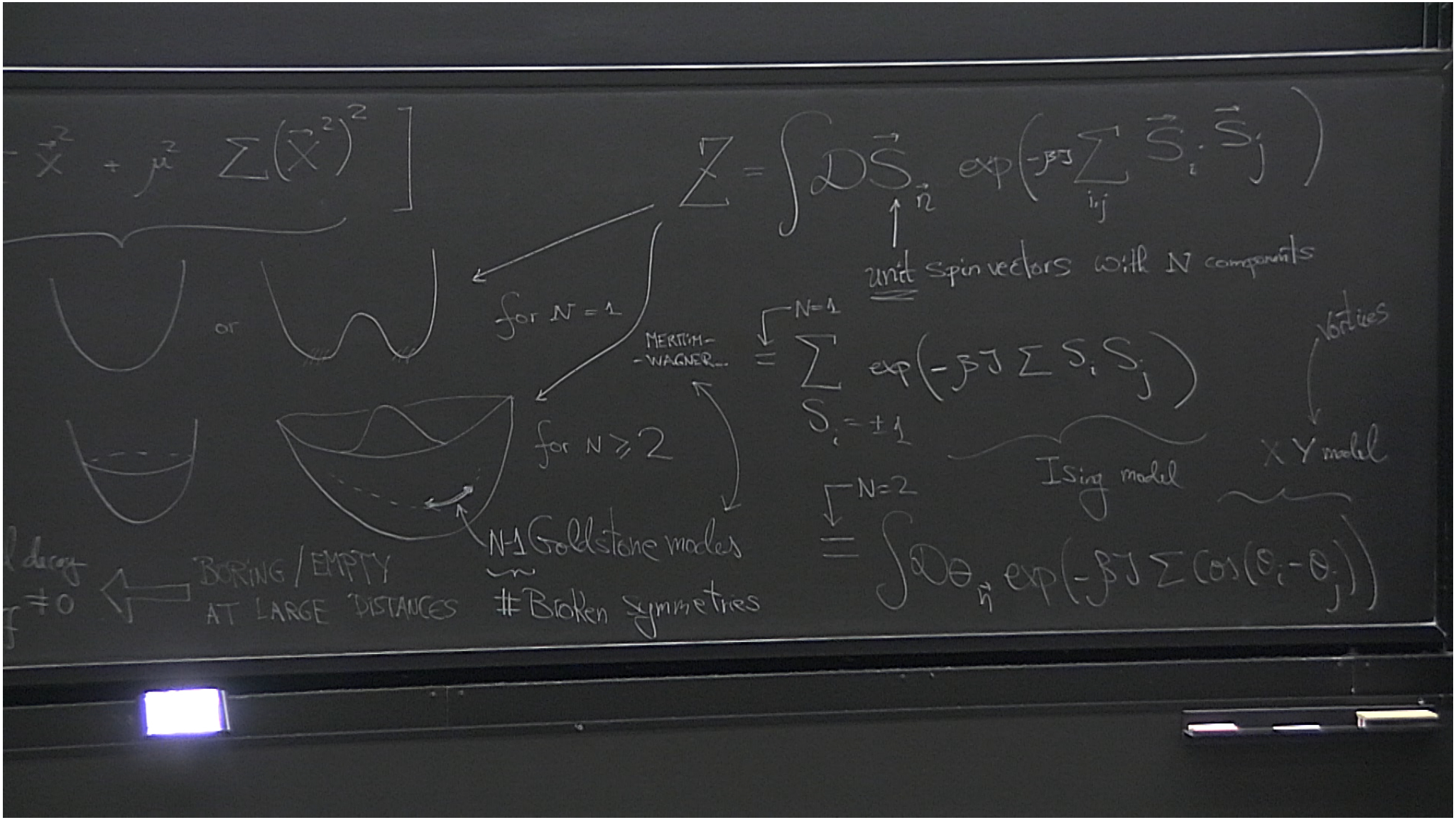


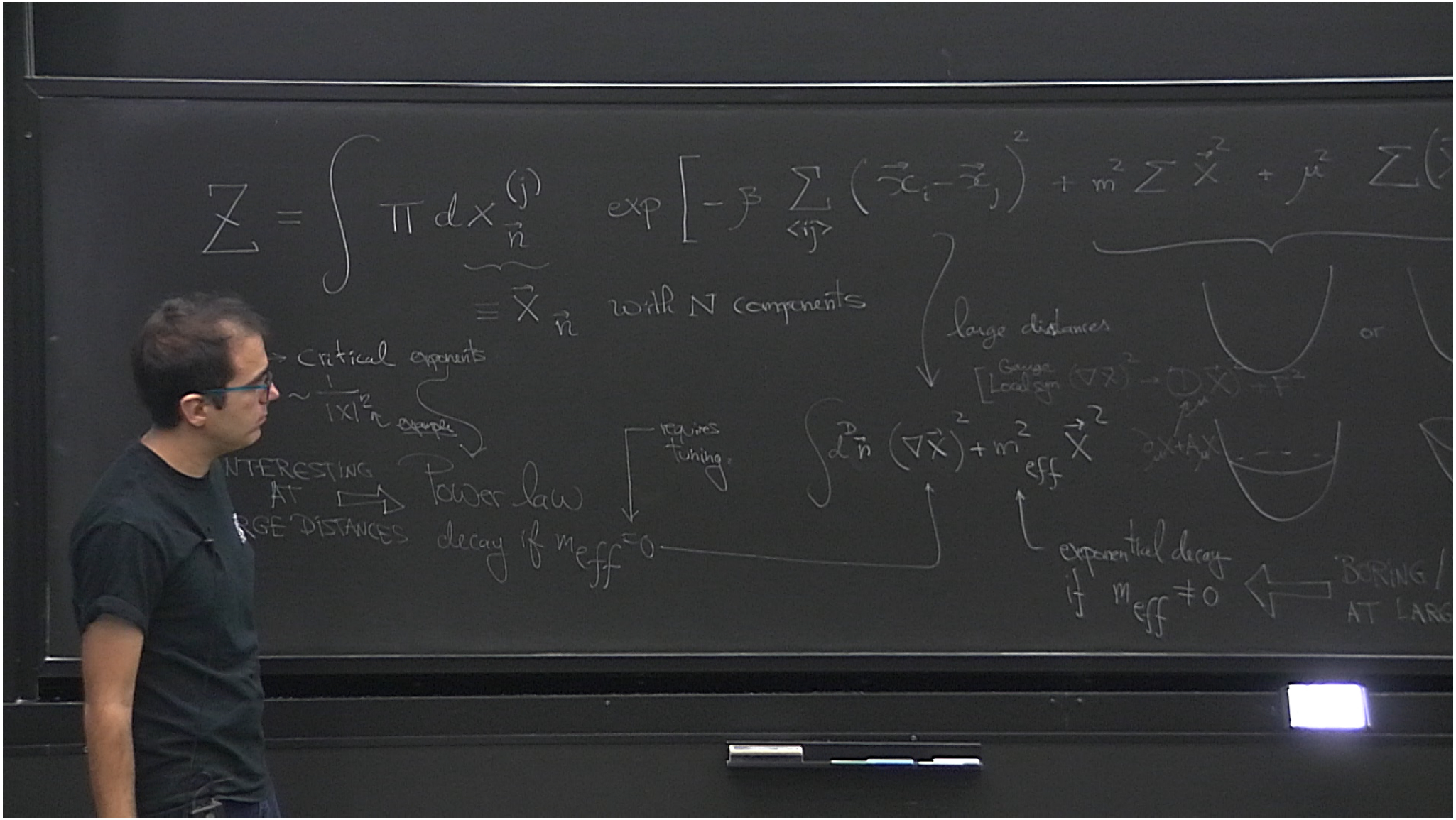

$\vec{x}^2 + \mu^2 \sum (\vec{x}^2)^2$
$Z = \int \mathcal{D}\vec{S}_{\vec{n}} \exp\left(-\beta J \sum_{i,j} \vec{S}_i \cdot \vec{S}_j\right)$
unit spin vectors with N components
or
for $N = 1$
MERMIN-WAGNER...
$= \sum_{S_i = \pm 1}^{N=1} \exp\left(-\beta J \sum S_i S_j\right)$
Ising model
XY model
for $N \geq 2$
$= \int \mathcal{D}\theta_{\vec{n}} \exp\left(-\beta J \sum \cos(\theta_i - \theta_j)\right)$
$N-1$ Goldstone modes
Broken symmetries
BORING / EMPTY AT LARGE DISTANCES

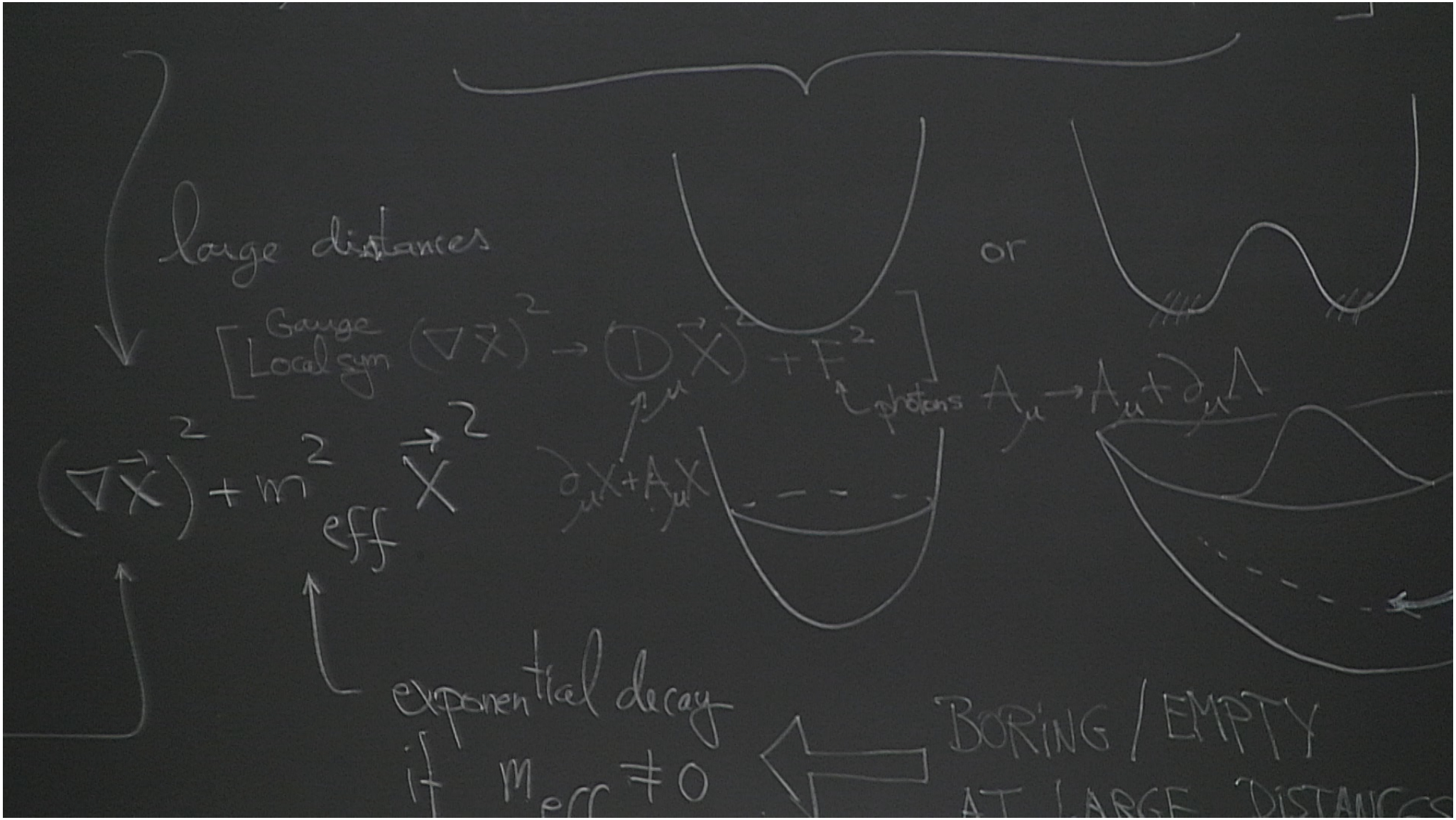

with N components
large distances
for N = 1
for N ≥ 2
Goldstone modes
Broken symmetries
Ising model
XY model
power law
decay if m_eff = 0
exponential decay if m_eff ≠ 0
AT LARGE DISTANCES

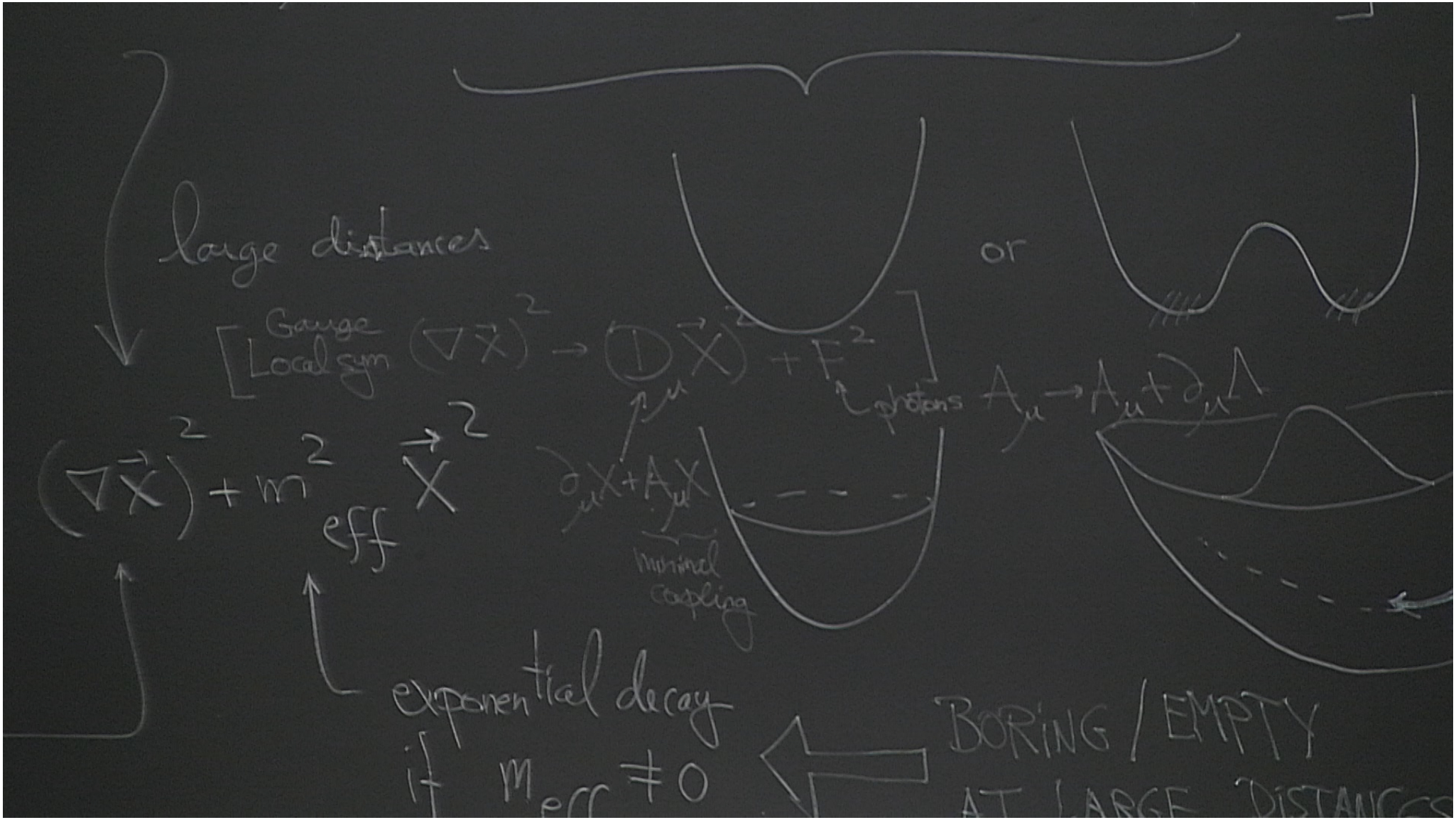

$Z = \int \prod d x_{\vec{n}}^{(j)} \exp\left[-\beta \sum_{\langle ij \rangle} \left(\vec{x}_i - \vec{x}_j\right)^2 + m^2 \sum \vec{x}^2 + \mu^2 \sum (\vec{x}^2)^2\right]$
$\equiv \vec{X}_{\vec{n}}$ with N components
Critical exponents
large distances
or
INTERESTING AT LARGE DISTANCES
Power law decay if $m_{eff} = 0$
requires tuning
$\int d^D \vec{n} \, (\nabla \vec{X})^2 + m_{eff}^2 \vec{X}^2$
exponential decay if $m_{eff} \neq 0$
BORING / AT LARG

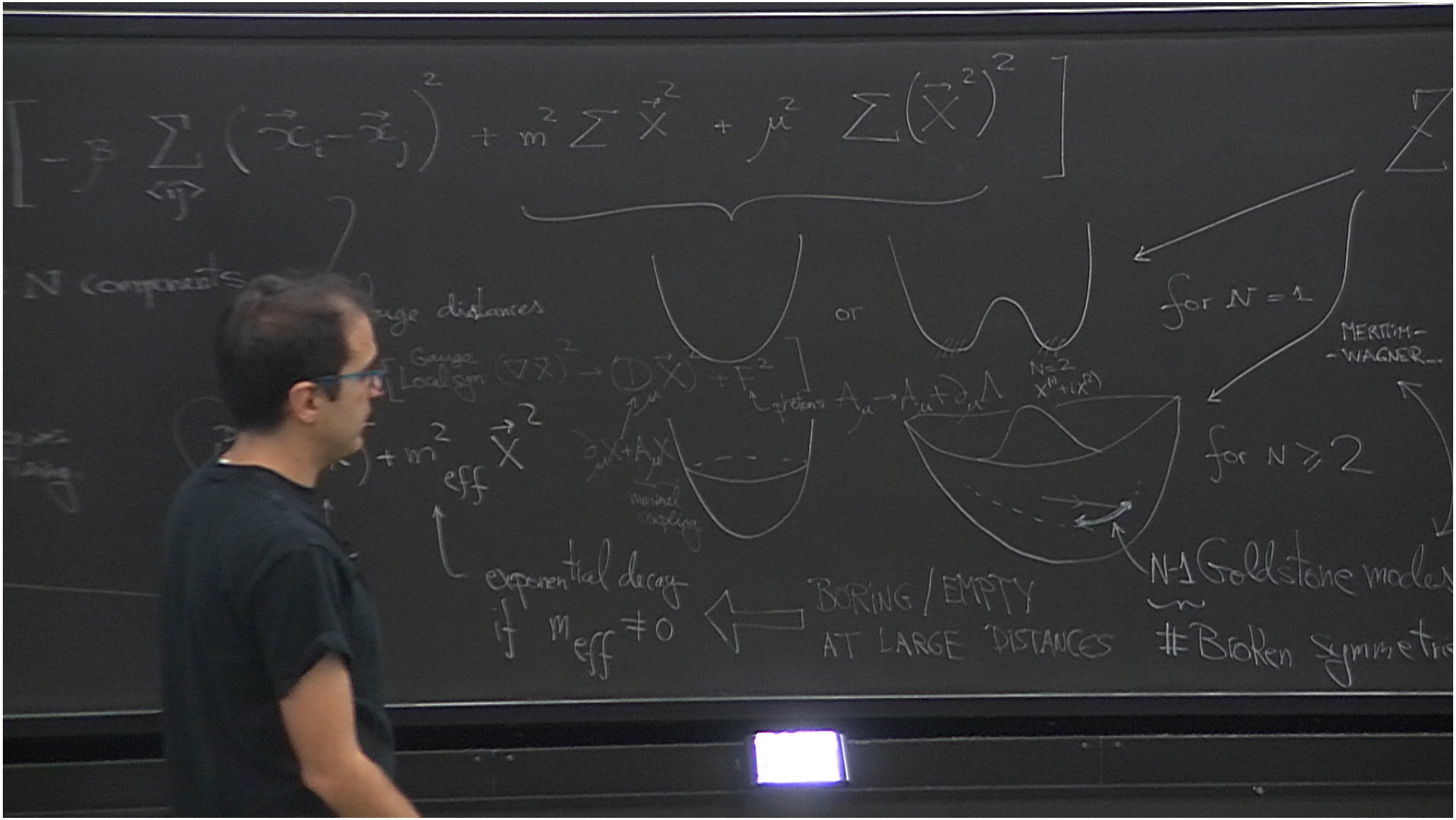

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$\equiv \vec{X}_{\vec{n}}$ with N components
large distances
or
$\int d^D \vec{n} \, (\nabla \vec{X})^2 + m_{eff}^2 \vec{X}^2$
$\ln xy \Rightarrow$ Critical exponents
$\langle SS \rangle \sim \frac{1}{|x|^2}$
example
INTERESTING AT LARGE DISTANCES
Power law decay if $m_{eff} = 0$
requires tuning
exponential decay if $m_{eff} \neq 0$
BORING / AT LARG

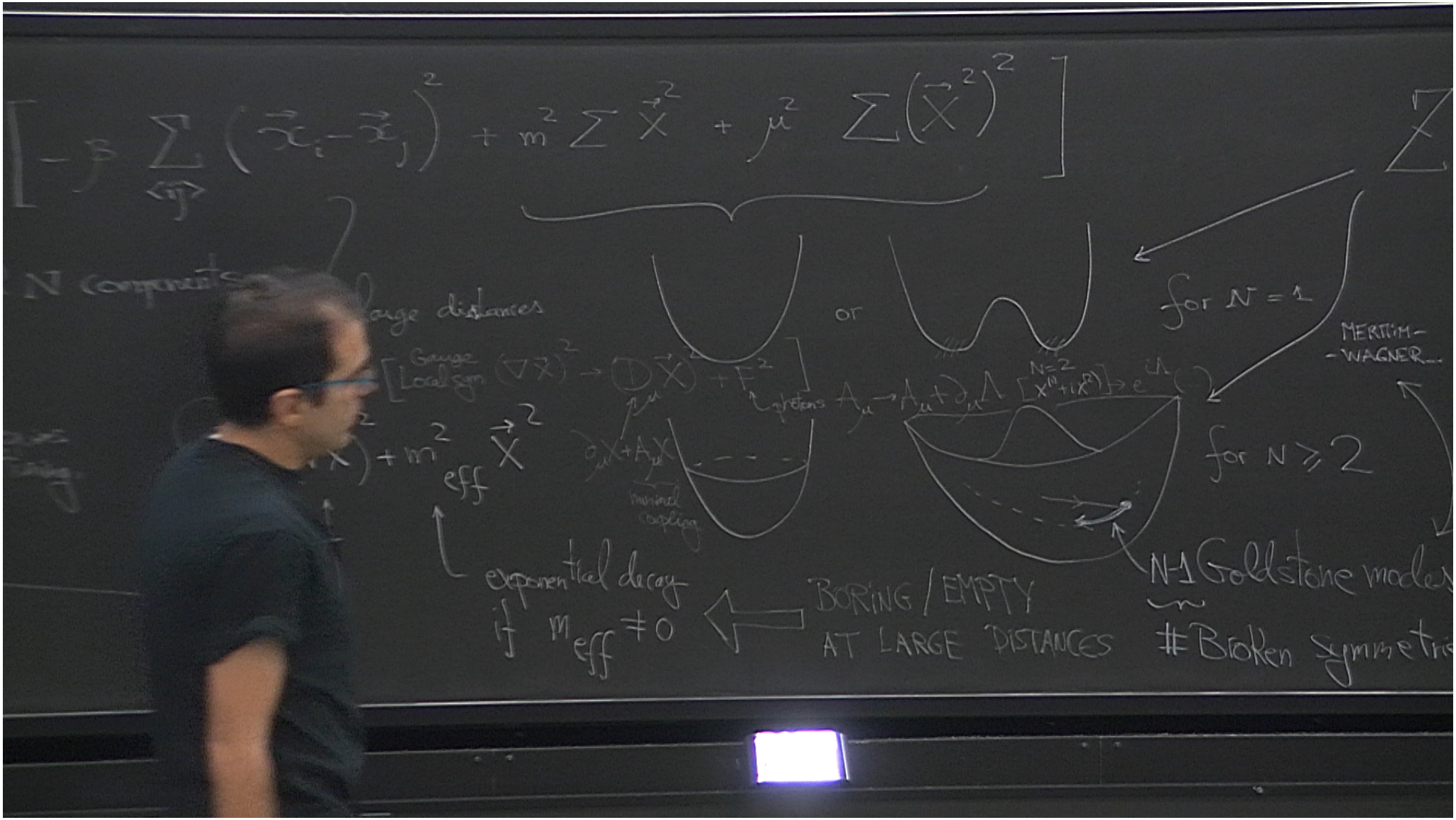

$\vec{x}^2 + \mu^2 \sum (\vec{x}^2)^2$
$Z = \int \mathcal{D}\vec{S}_{\vec{n}} \exp\left(-\beta J \sum_{i,j} \vec{S}_i \cdot \vec{S}_j\right)$
unit spin vectors with N components
or
for $N = 1$
for $N \geq 2$
MERMIN-WAGNER...
$= \sum_{S_i = \pm 1} \exp\left(-\beta J \sum S_i S_j\right)$
$N=1$
Ising model
vortices
XY model
$= \int \mathcal{D}\theta_{\vec{n}} \exp\left(-\beta J \sum \cos(\theta_i - \theta_j)\right)$
$N=2$
N-1 Goldstone modes
Broken symmetries
BORING / EMPTY AT LARGE DISTANCES

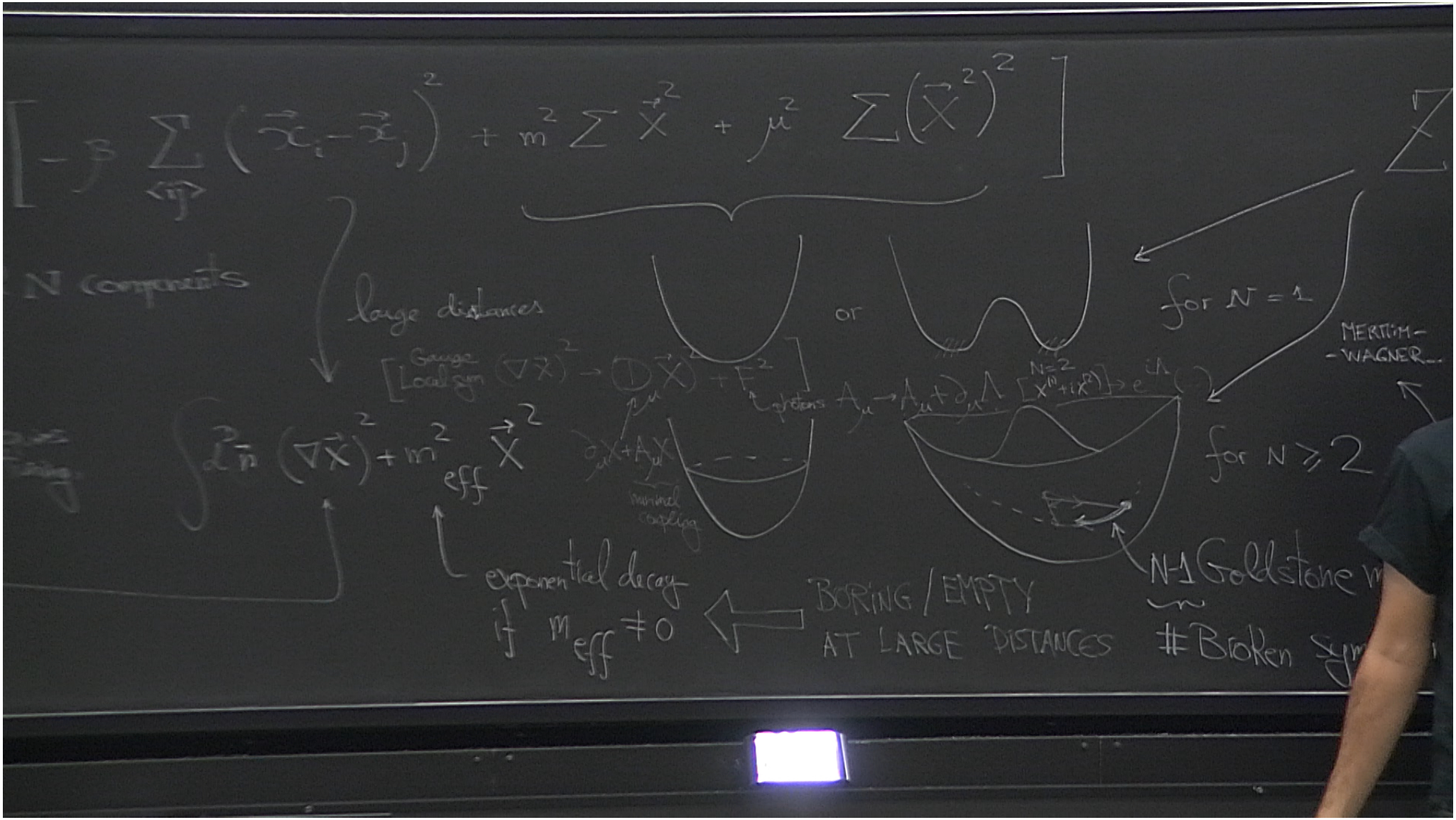

$Z = \int \prod d x_{\vec{n}}^{(j)} \exp\left[-\beta \sum_{\langle ij \rangle} \left(\vec{x}_i - \vec{x}_j\right)^2 + m^2 \sum \vec{x}^2 + \mu^2 \sum (\vec{x}^2)^2\right]$
$\equiv \vec{X}_{\vec{n}}$ with N components
large distances
or
Gauge
Local sym
$(\nabla \chi)^2 \to$
$\partial_\mu \chi + A_\mu$
$\int d^D \vec{n} \, (\nabla \vec{X})^2 + m_{eff}^2 \vec{X}^2$
requires tuning
Critical exponents
$\sim \frac{1}{|x|^2}$
INTERESTING AT LARGE DISTANCES
Power law decay if $m_{eff} = 0$
exponential decay if $m_{eff} \neq 0$
BORING AT LARG

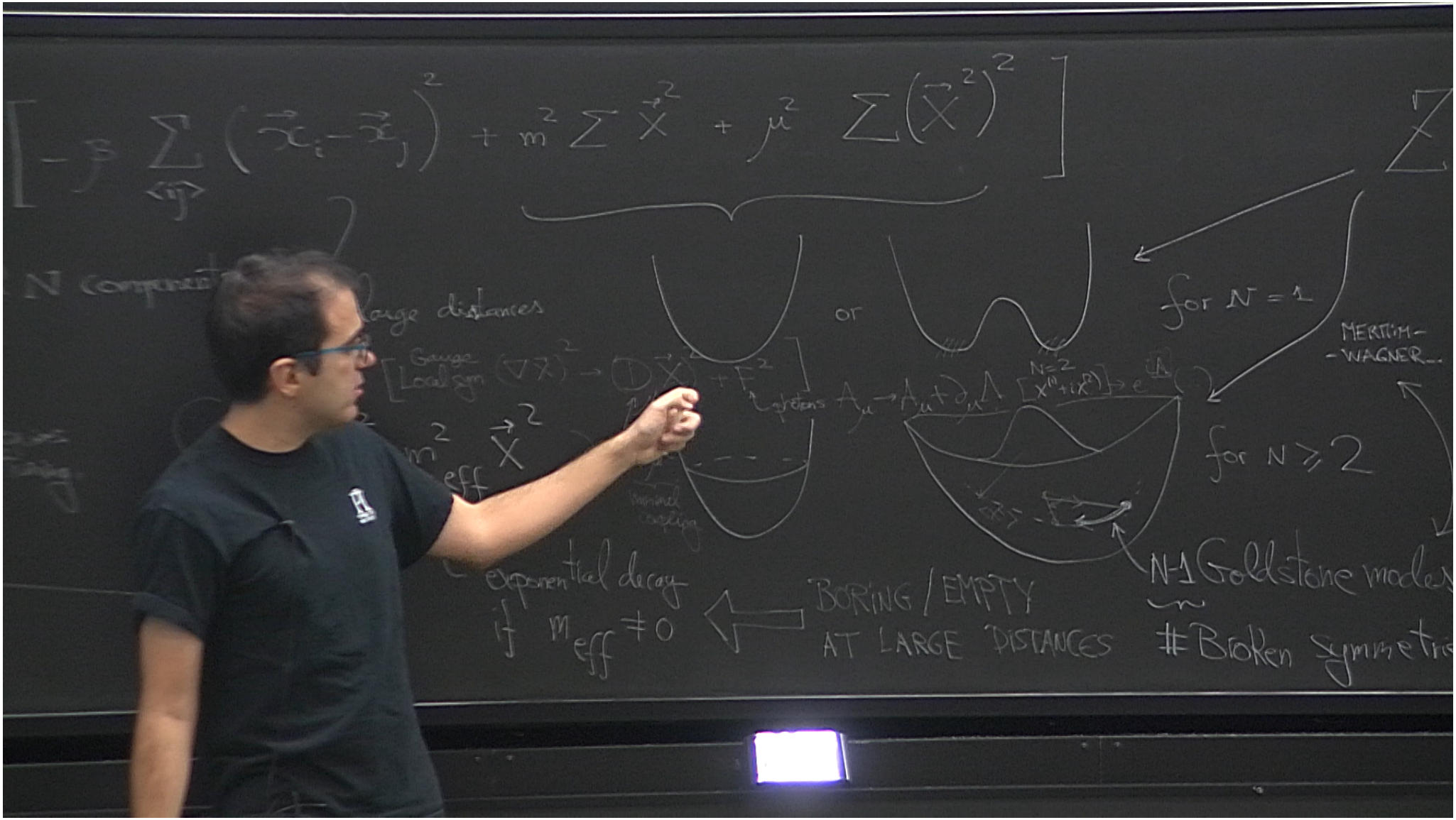

large distances
or
Gauge
Local sym
$(\nabla\vec{X})^2 \to (D\vec{X})^2 + F^2$
$\partial_\mu X + A_\mu X$
photons
$A_\mu \to A_\mu + \partial_\mu \Lambda$
$(\nabla\vec{X})^2 + m_{eff}^2 \vec{X}^2$
exponential decay
if $m_{eff} \neq 0$
BORING / EMPTY
AT LARGE DISTANCES

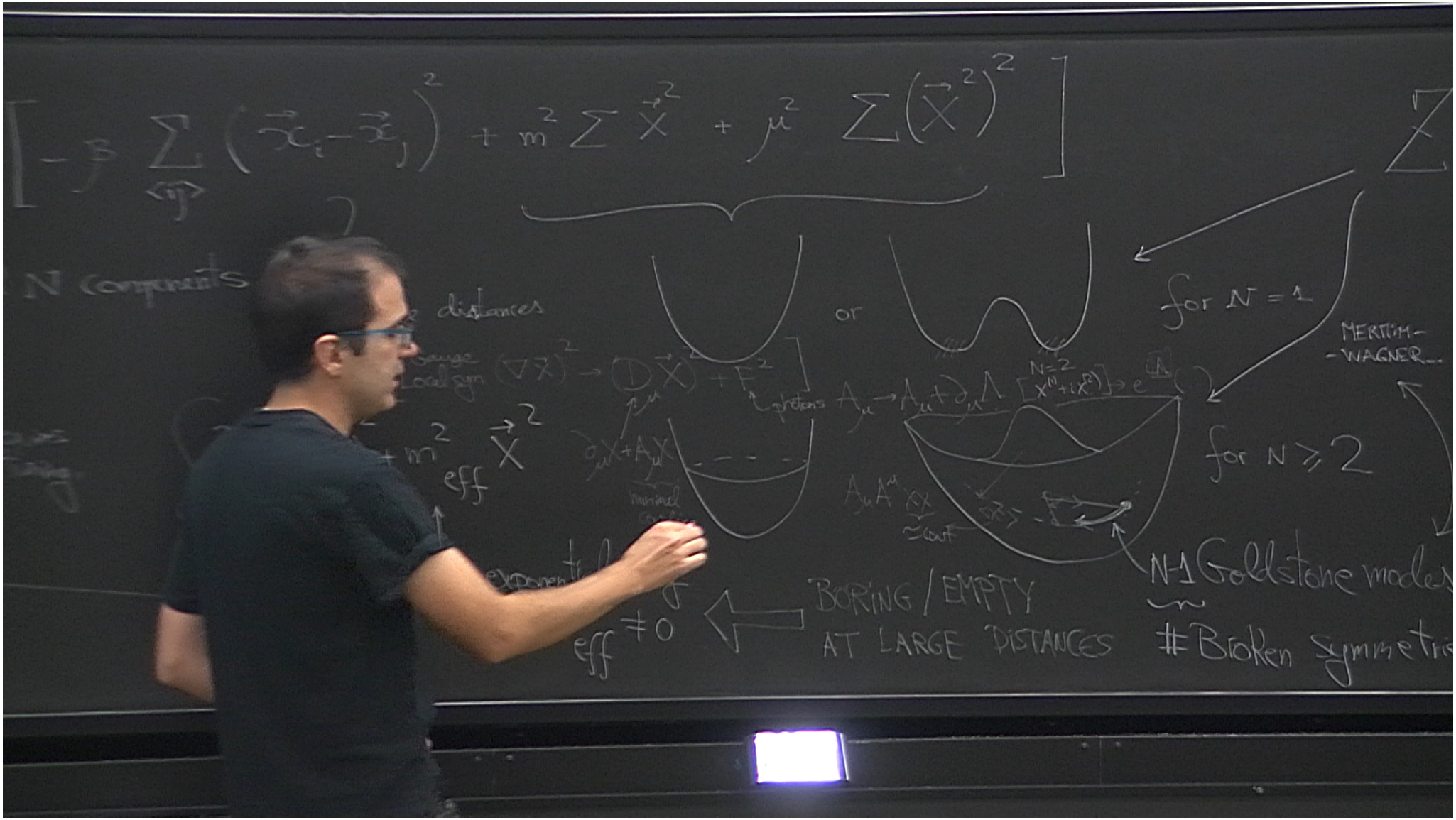

large distances
or
Gauge
Local sym
$(\nabla\vec{X})^2 \to (D\vec{X})^2 + F^2$
photons $A_\mu \to A_\mu + \partial_\mu \Lambda$
$\partial_\mu X + A_\mu X$
minimal coupling
$(\nabla\vec{X})^2 + m_{eff}^2 \vec{X}^2$
exponential decay
if $m_{eff} \neq 0$
BORING / EMPTY
AT LARGE DISTANCES

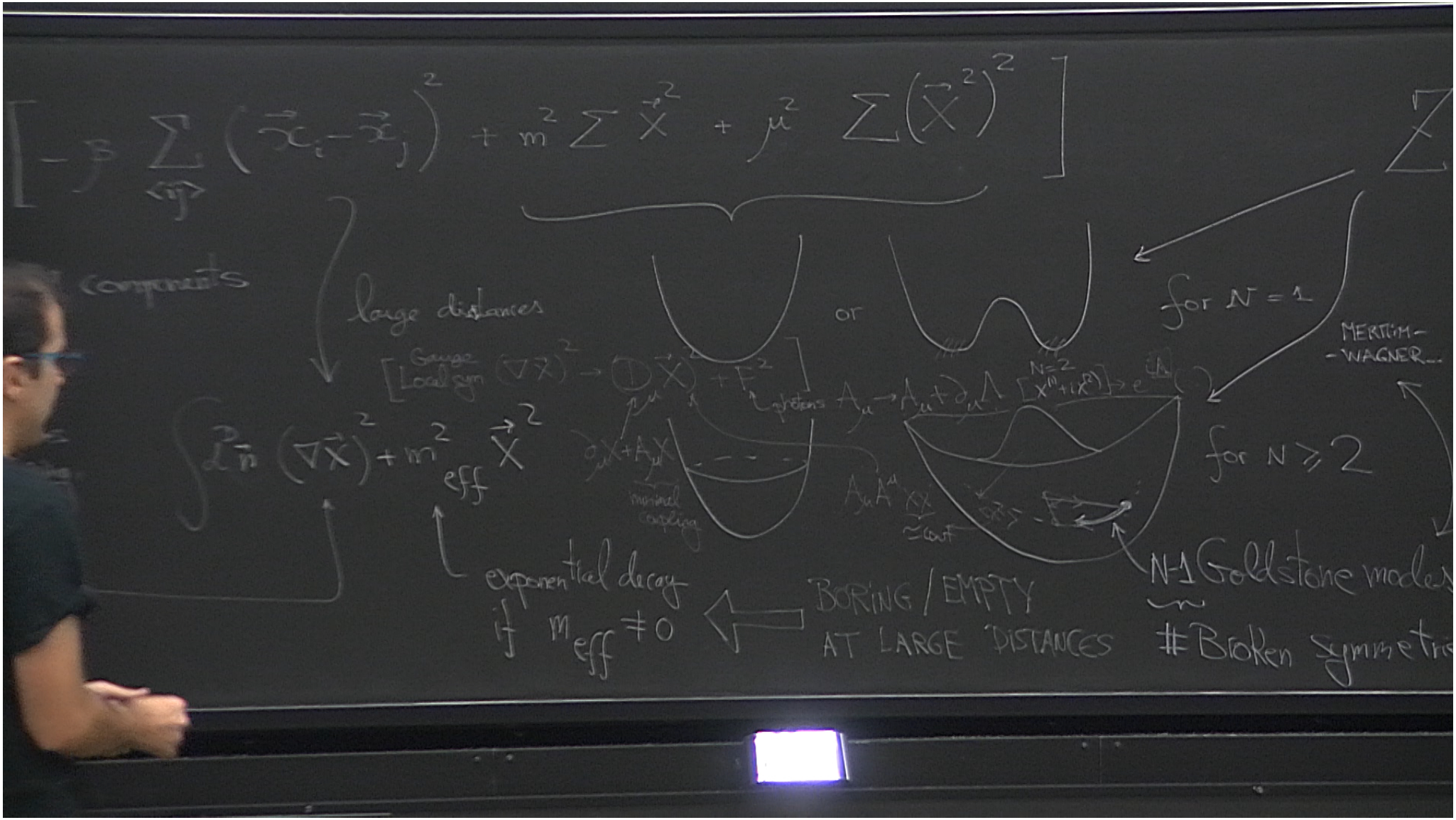

N (components
large distances
Gauge
Local sym
or
for N = 1
MERMIN-
-WAGNER...
N=2
for N ≥ 2
eff
exponential decay
if m_eff ≠ 0
BORING / EMPTY
AT LARGE DISTANCES
N-1 Goldstone modes
Broken symmetries

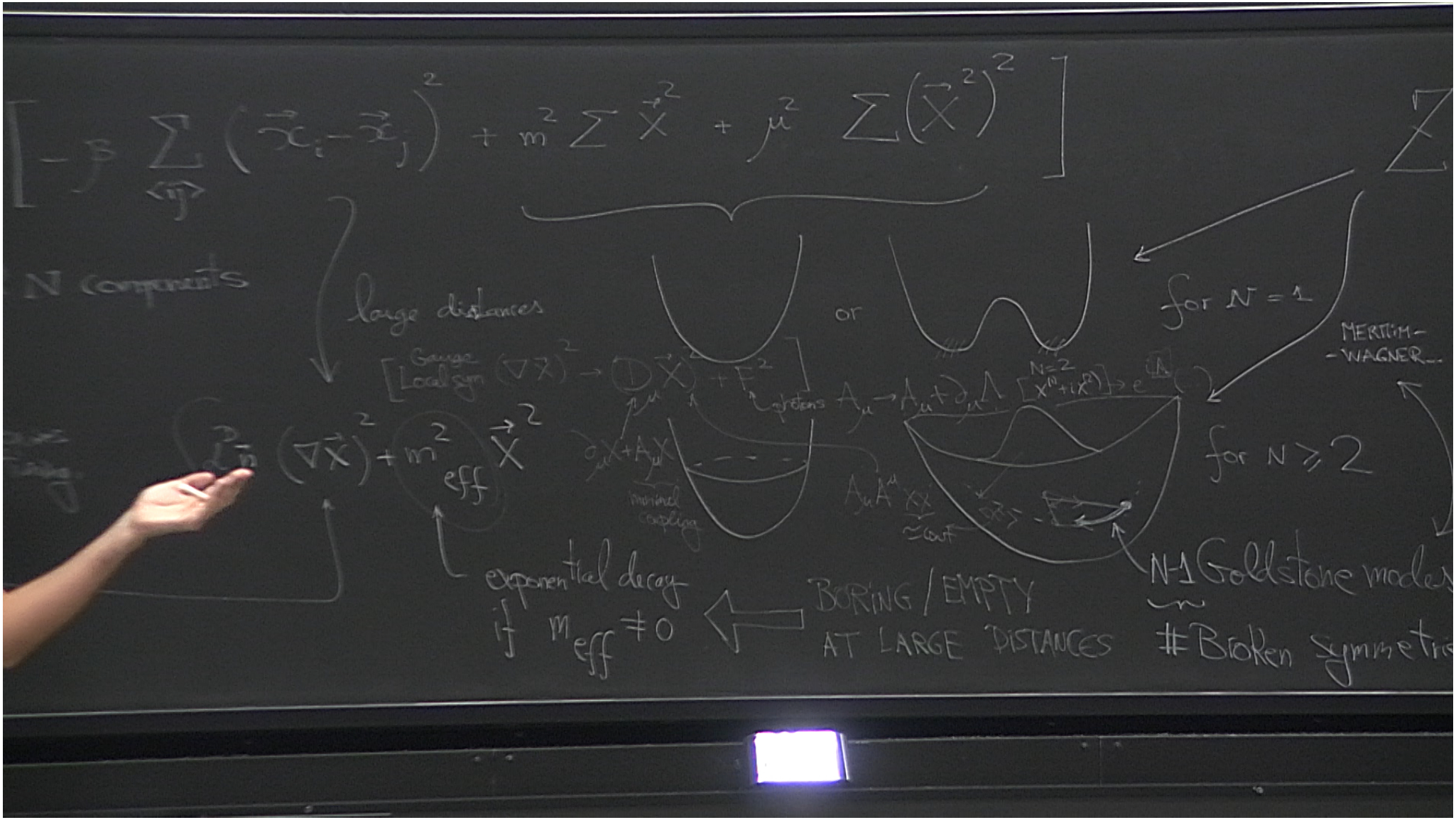

N (components)
large distances
Gauge
Local sym
or
for N = 1
for N ≥ 2
MERMIN-
-WAGNER...
N=2
eff
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if m_eff ≠ 0
BORING / EMPTY
AT LARGE DISTANCES
N-1 Goldstone modes
Broken symmetries

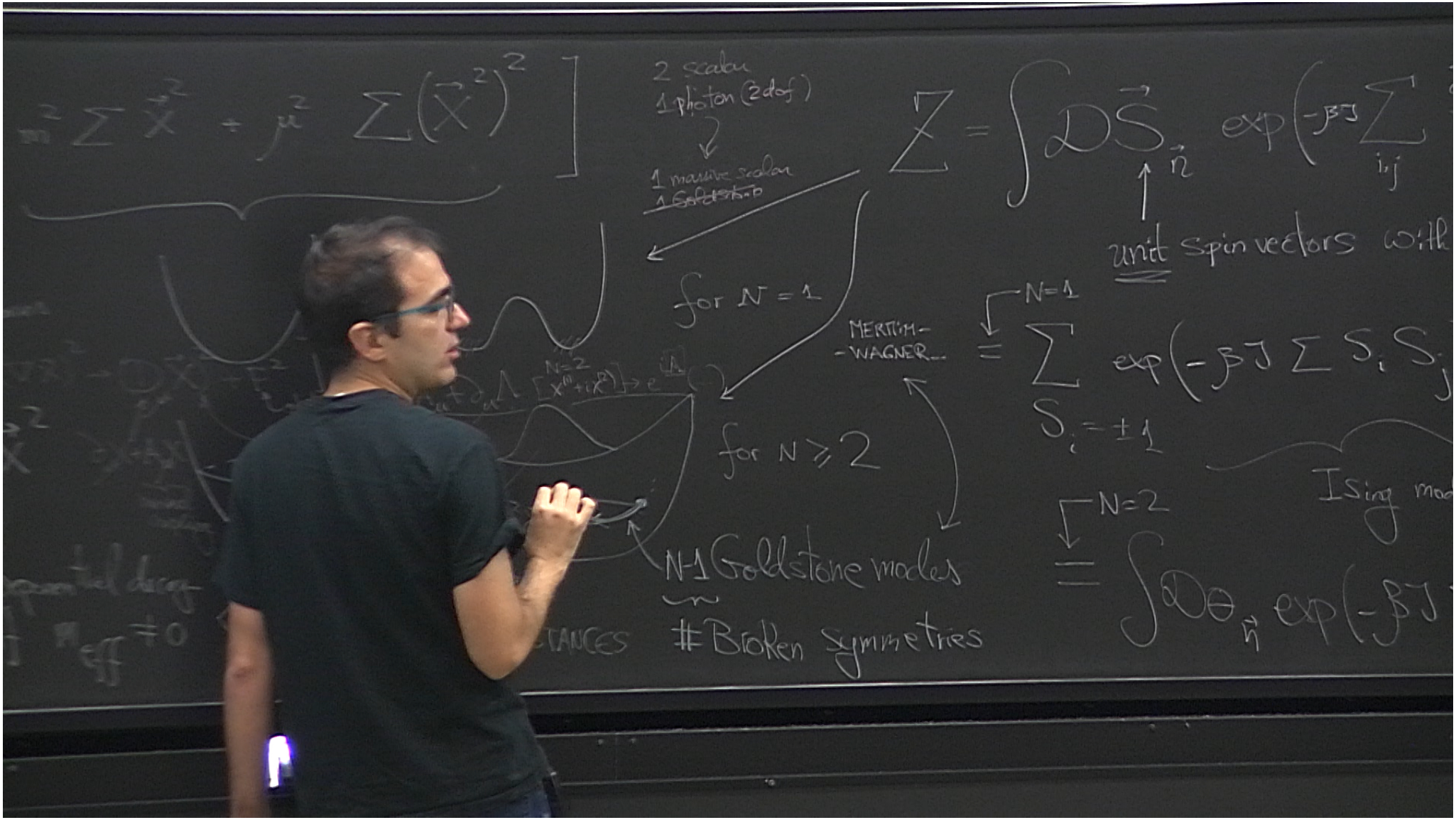

N components
large distances
or
for N = 1
for N ≥ 2
MERMIN-WAGNER...
exponential decay
if $m_{eff} \neq 0$
BORING / EMPTY
AT LARGE DISTANCES
N-1 Goldstone
#Broken

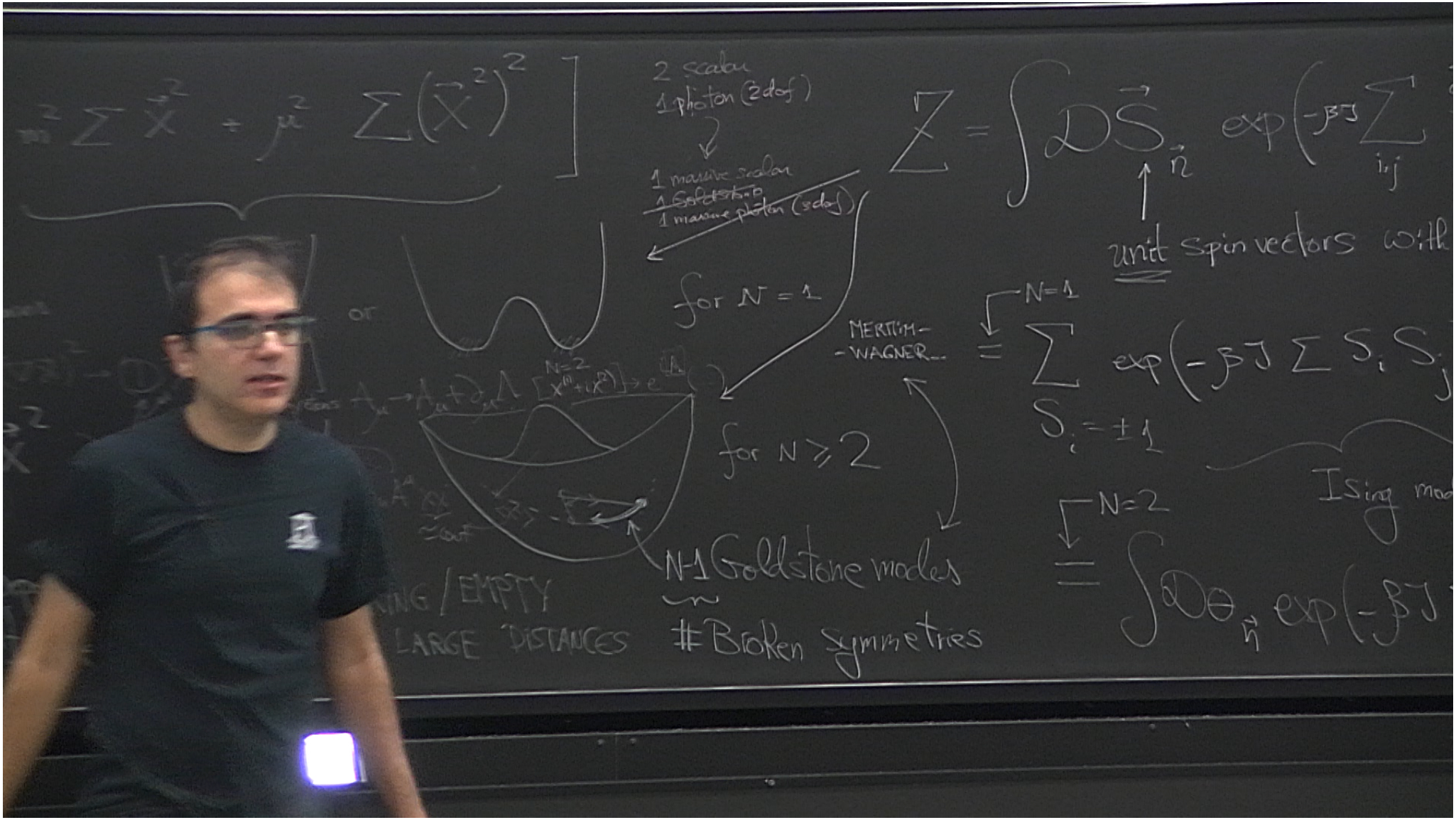

for N = 1
or
for N ≥ 2
MERMIN-WAGNER...
BORING / EMPTY AT LARGE DISTANCES
N-1 Goldstone modes
Broken symmetries

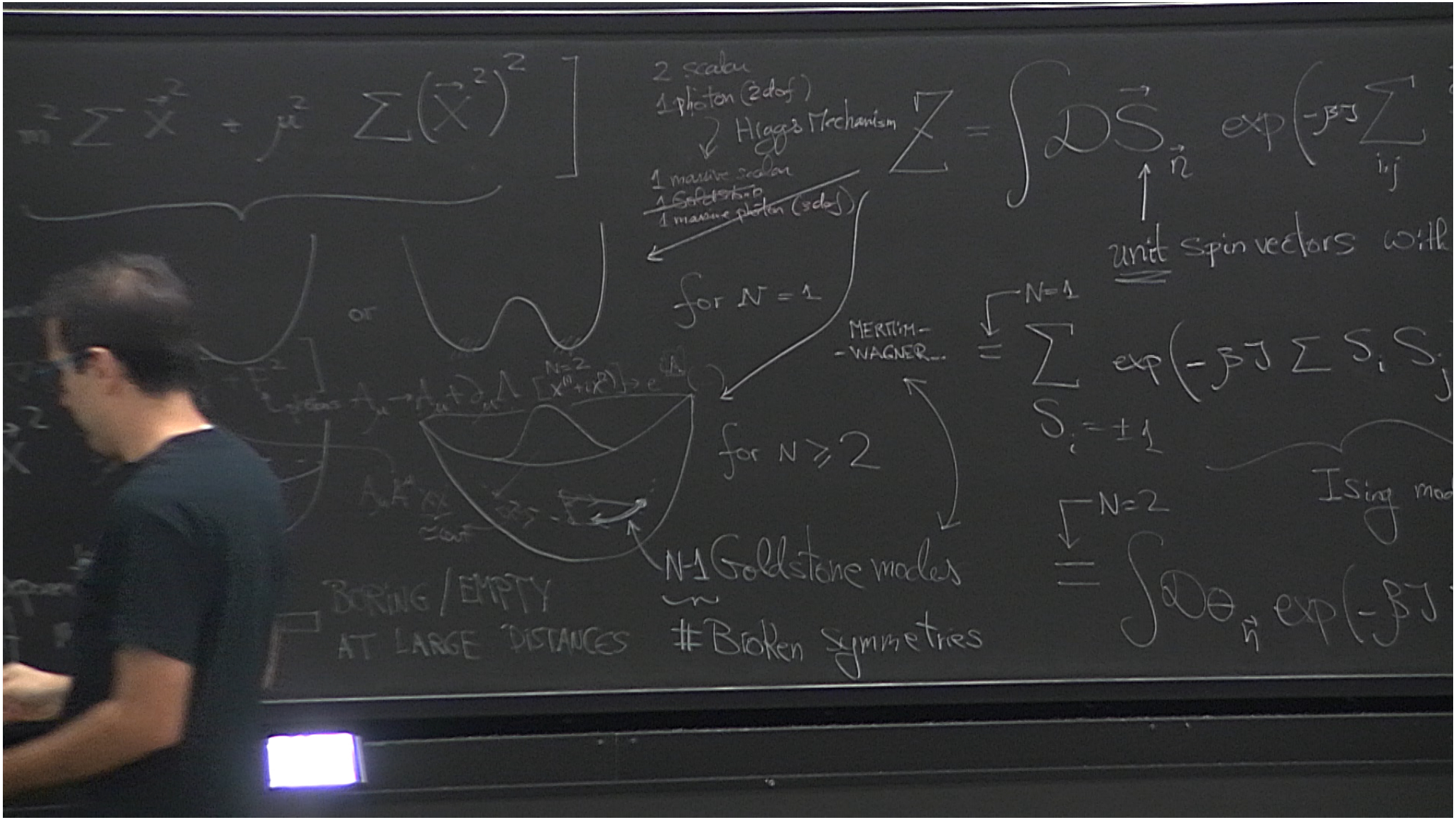

N (contents
distances
or
for N = 1
for N ≥ 2
MERTIM-
-WAGNER...
BORING / EMPTY
AT LARGE DISTANCES
N-1 Goldstone mode
#Broken symmetri

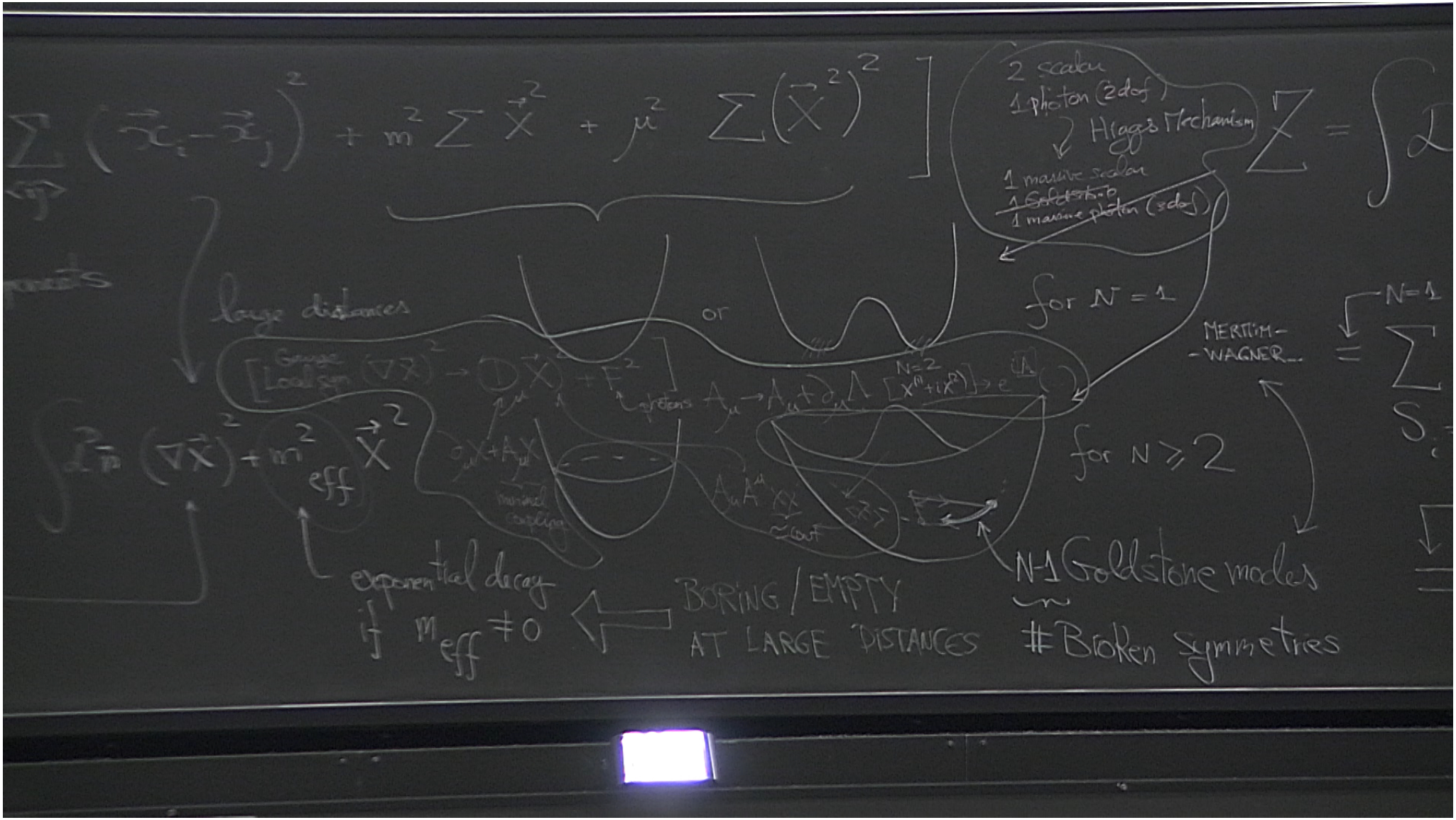

components
large distances
or
for N = 1
for N ≥ 2
MERMIN-WAGNER...
exponential decay
if $m_{eff} \neq 0$
BORING / EMPTY
AT LARGE DISTANCES
N-1 Goldstone modes
Broken symmetries

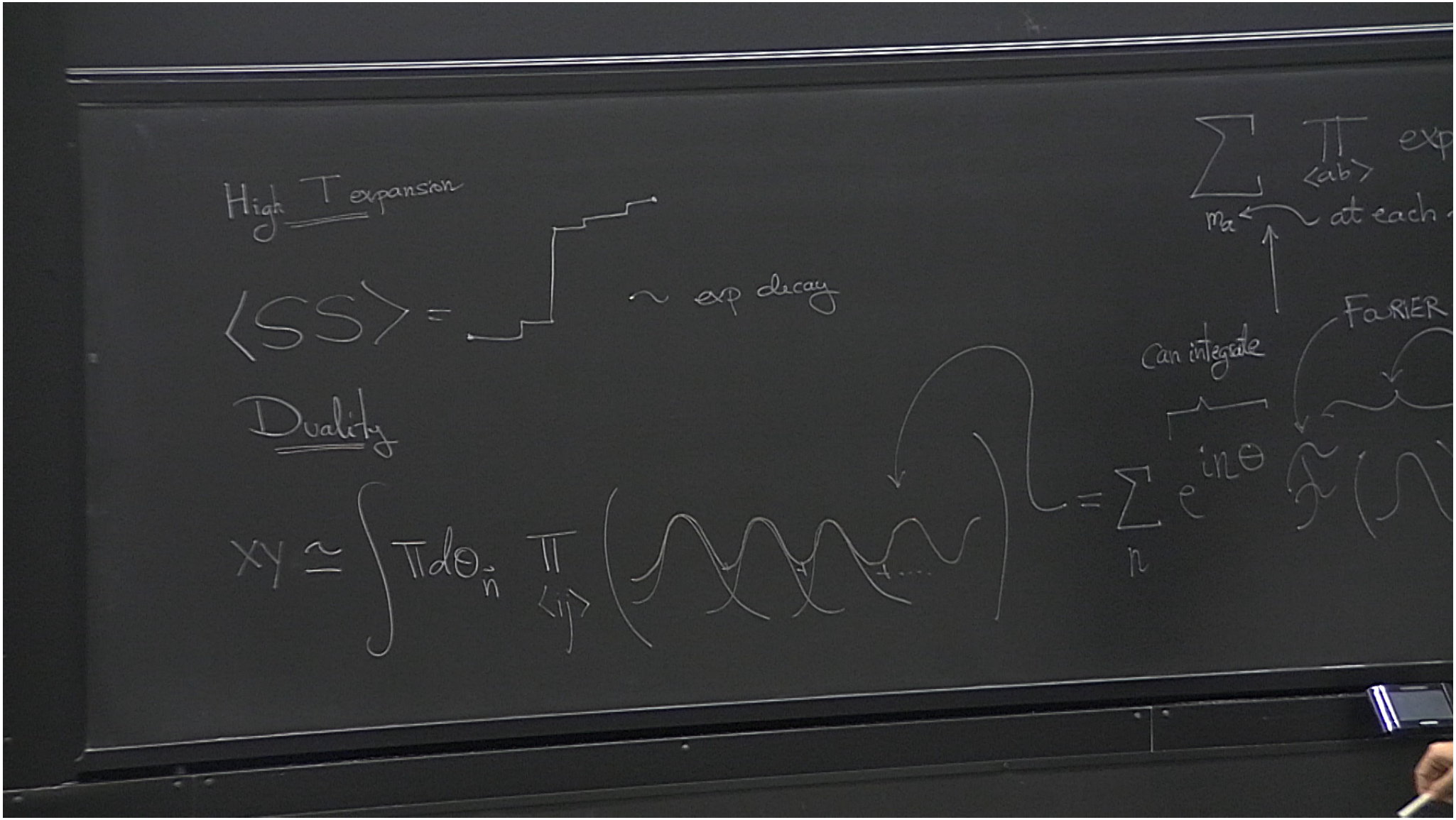

N components
large distances
or
for N = 1
for N ≥ 2
MERTIM-WAGNER...
exponential decay
if $m_{eff} \neq 0$
BORING / EMPTY
AT LARGE DISTANCES
N-1 Goldstone modes
#Broken symmetries

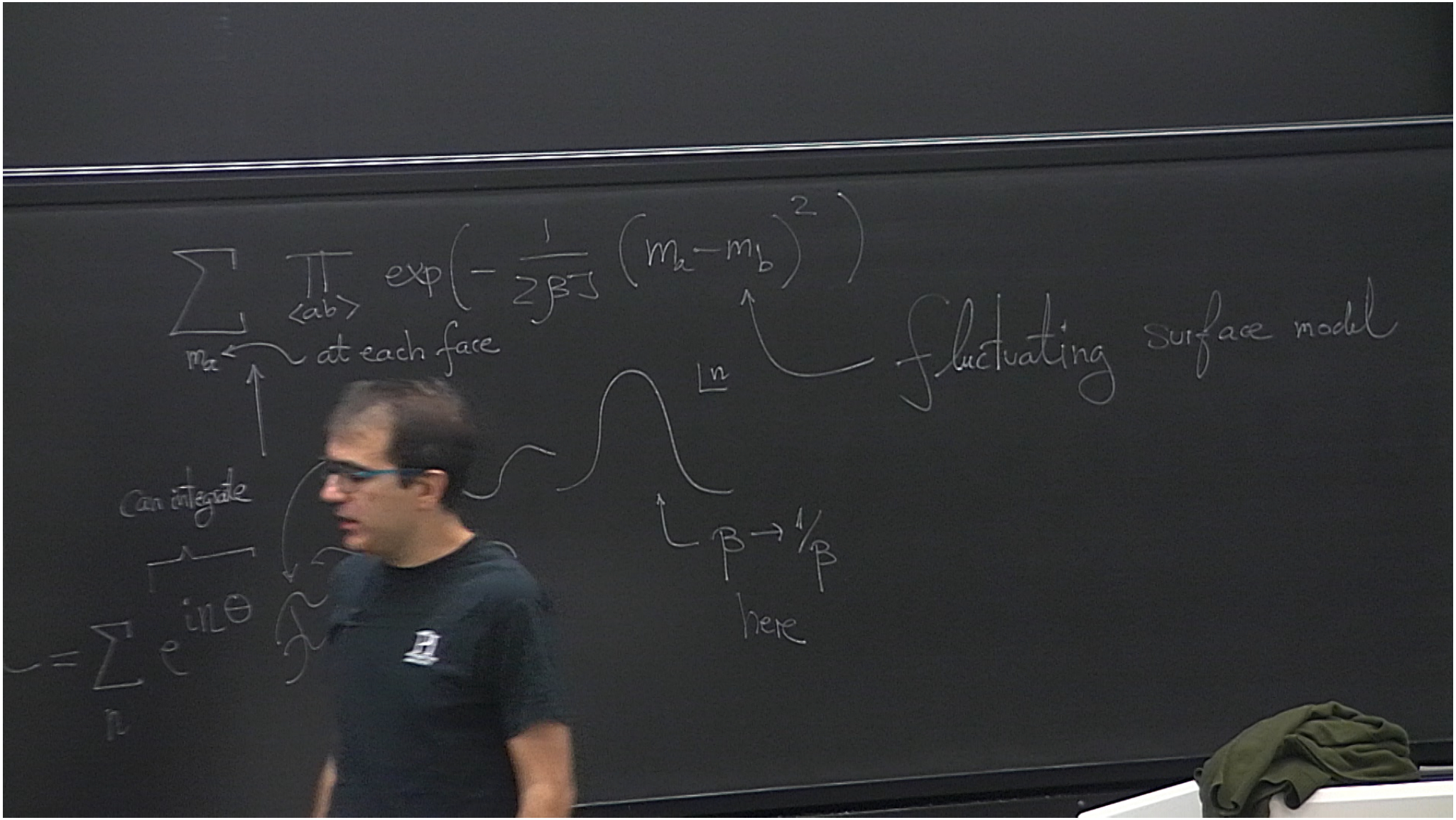

2 scalar
1 photon (2dof)
1 massive scalar
for N = 1
for N ≥ 2
MERMIN-WAGNER...
N=1
N=2
unit spin vectors
N-1 Goldstone modes
Broken symmetries
Ising

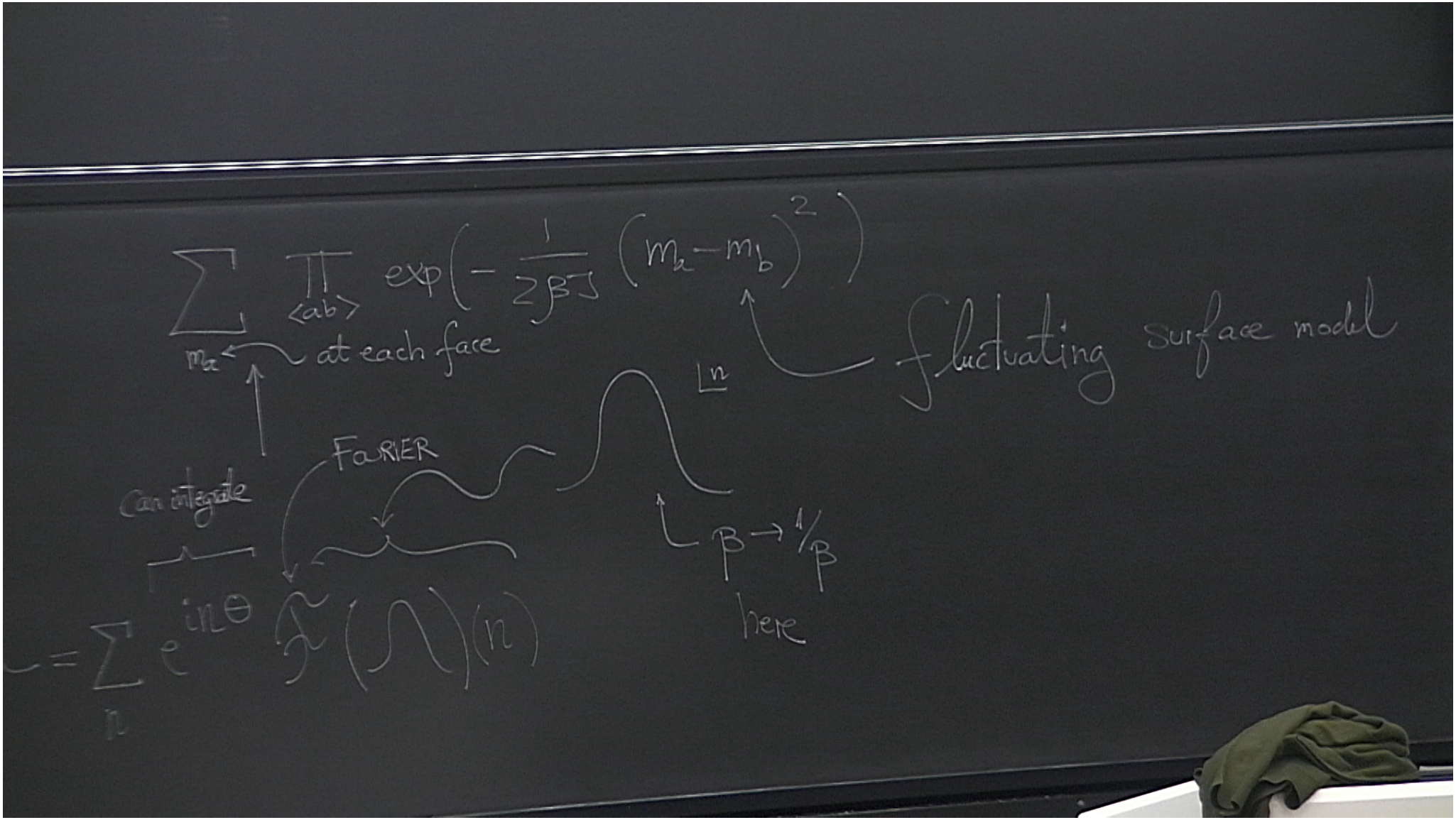

2 scalar
1 photon (2dof)
1 massive scalar
1 massive photon (3dof)
for N = 1
for N ≥ 2
MERMIN-WAGNER...
N=1
N=2
unit spin vectors with
Ising
N-1 Goldstone modes
Broken symmetries
LARGE DISTANCES
or

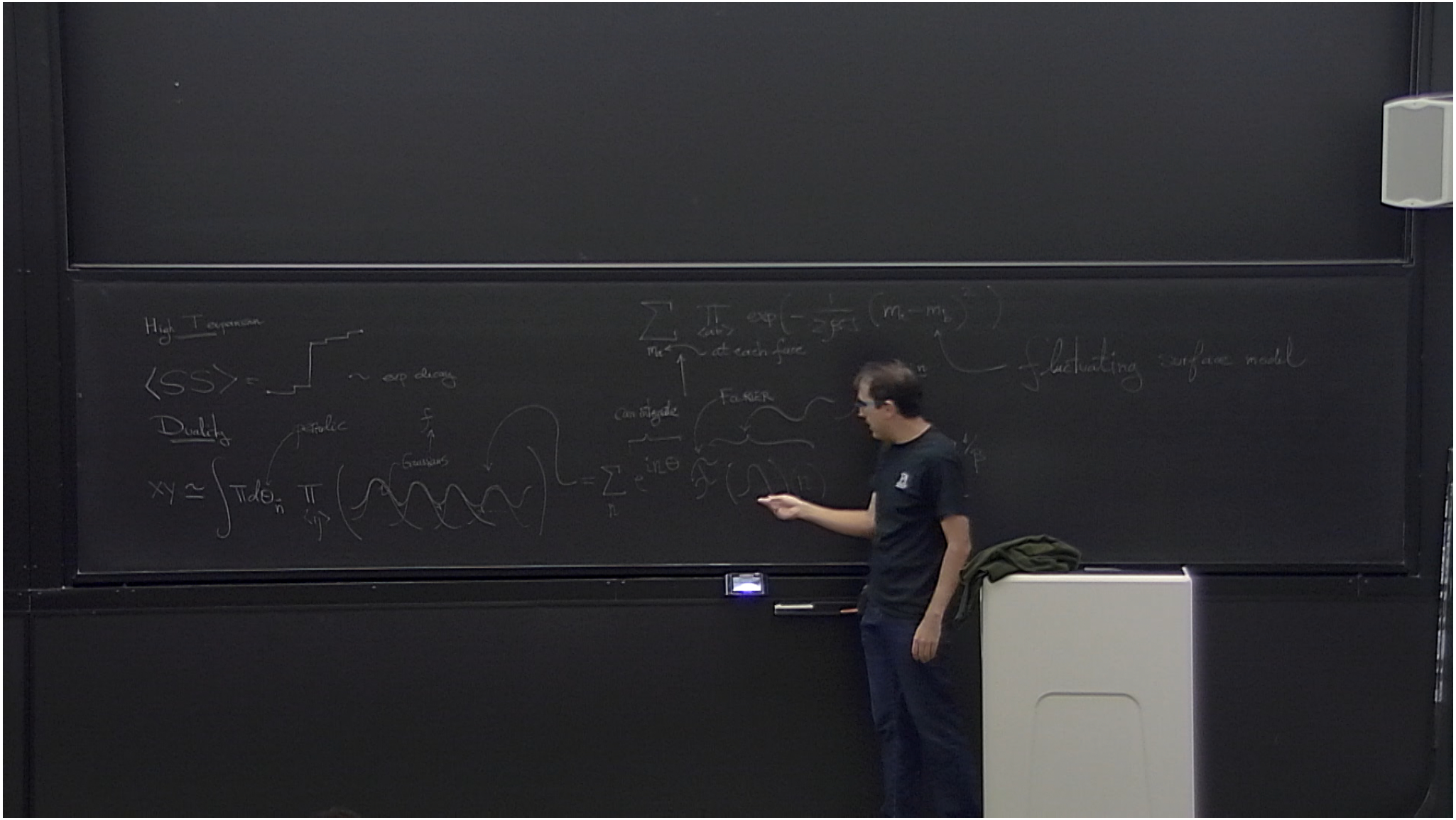

2 scalar
1 photon (2dof)
Higgs Mechanism
1 massive scalar
1 massive photon (3dof)
for N = 1
for N ≥ 2
or
MERMIN-WAGNER...
unit spin vectors with
Ising mod
BORING / EMPTY
AT LARGE DISTANCES
N-1 Goldstone modes
Broken symmetries

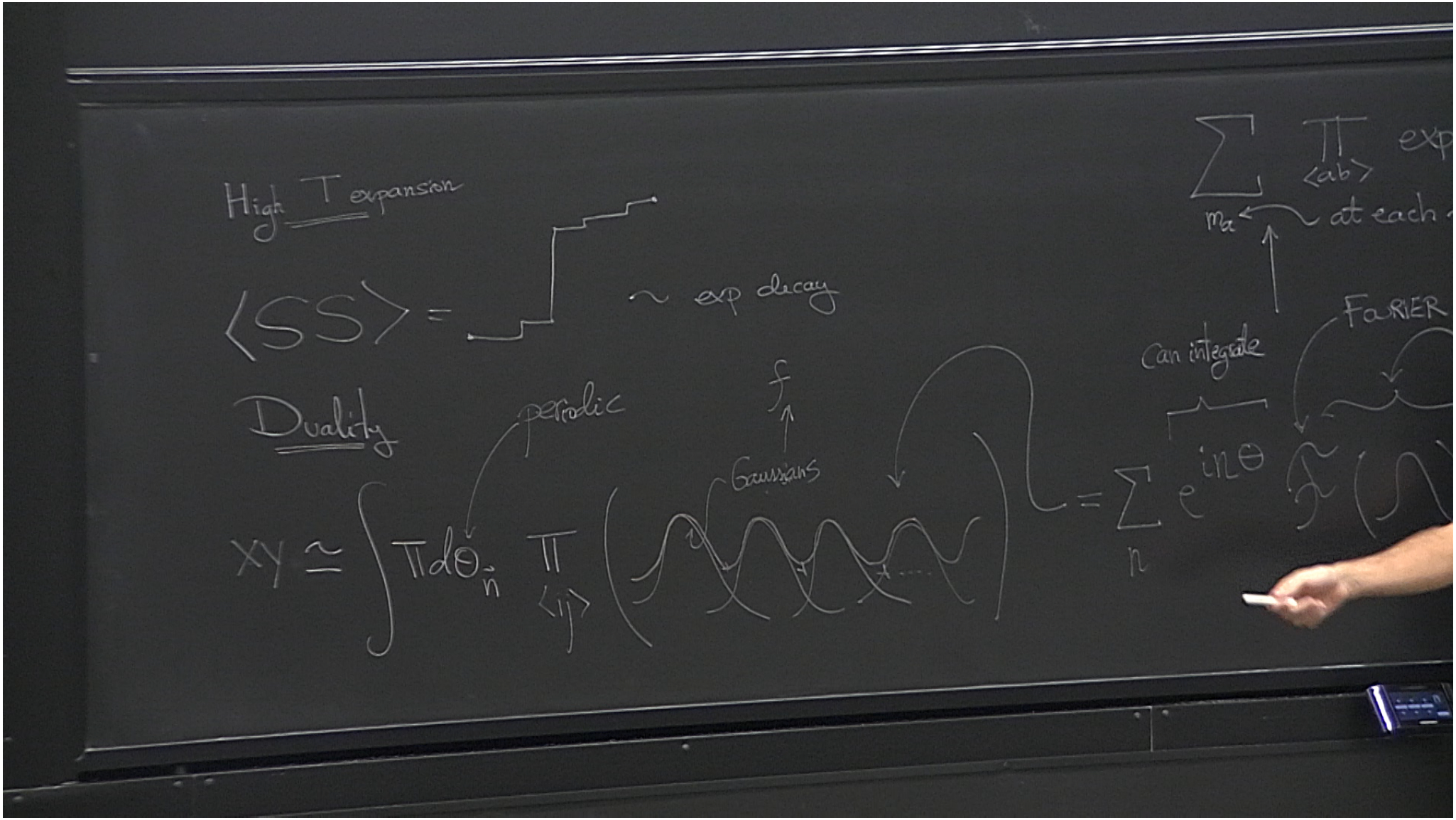

2 scalars
1 photon (2 dof)
Higgs Mechanism
1 massive scalar
1 Goldstone
1 massive photon (3 dof)
for N = 1
large distances
or
for N ≥ 2
MERTIM-
-WAGNER...
N=1
exponential decay
if m_eff ≠ 0
BORING / EMPTY
AT LARGE DISTANCES
N-1 Goldstone modes
Broken symmetries

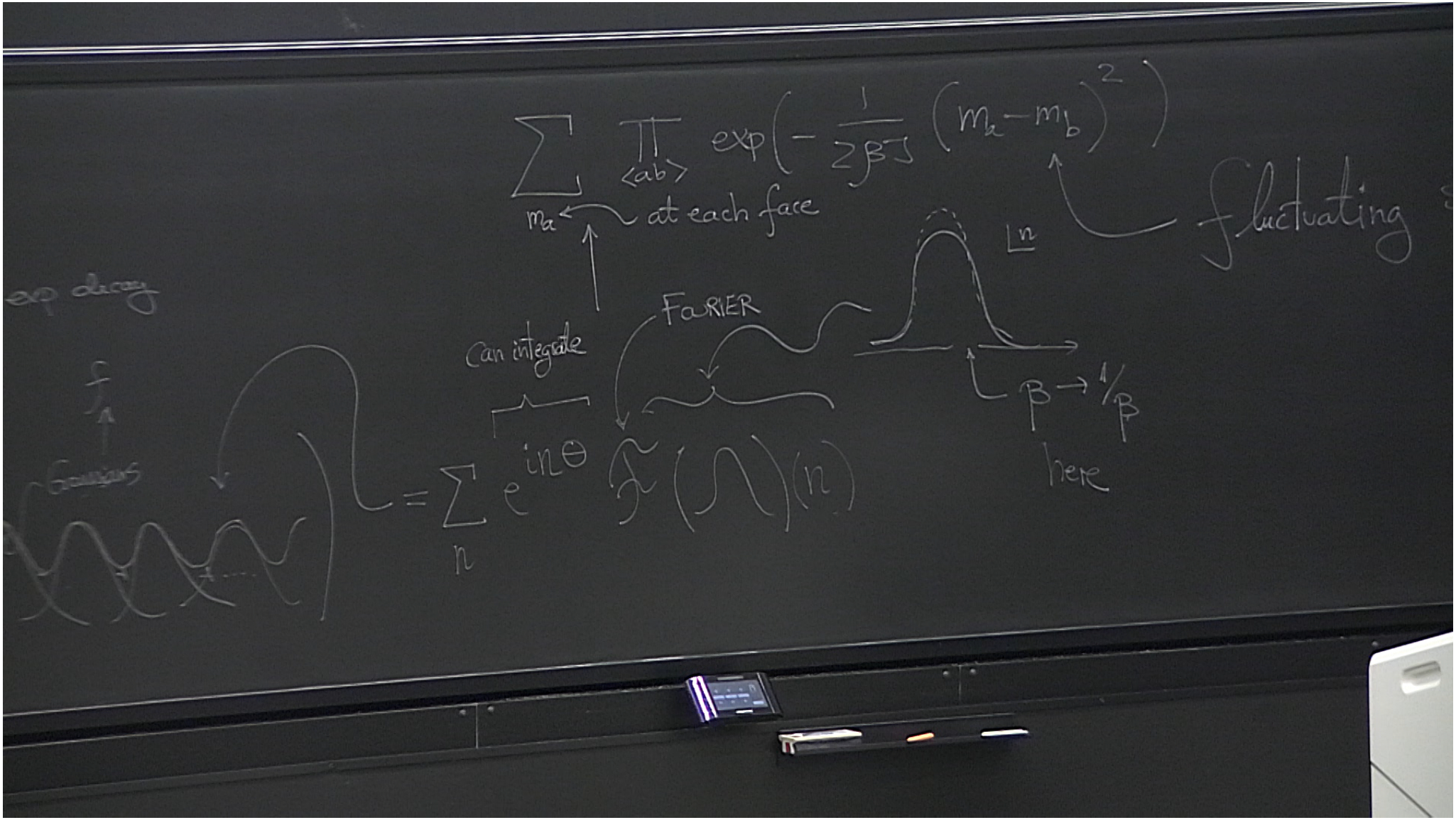

High T expansion
⟨SS⟩ =
~ exp decay
Duality
XY ≃ ∫ Π dθ_n Π_⟨ij⟩
= Σ_n e^{inθ}
Can integrate
FOURIER
Σ_{m_a}
Π_⟨ab⟩
at each

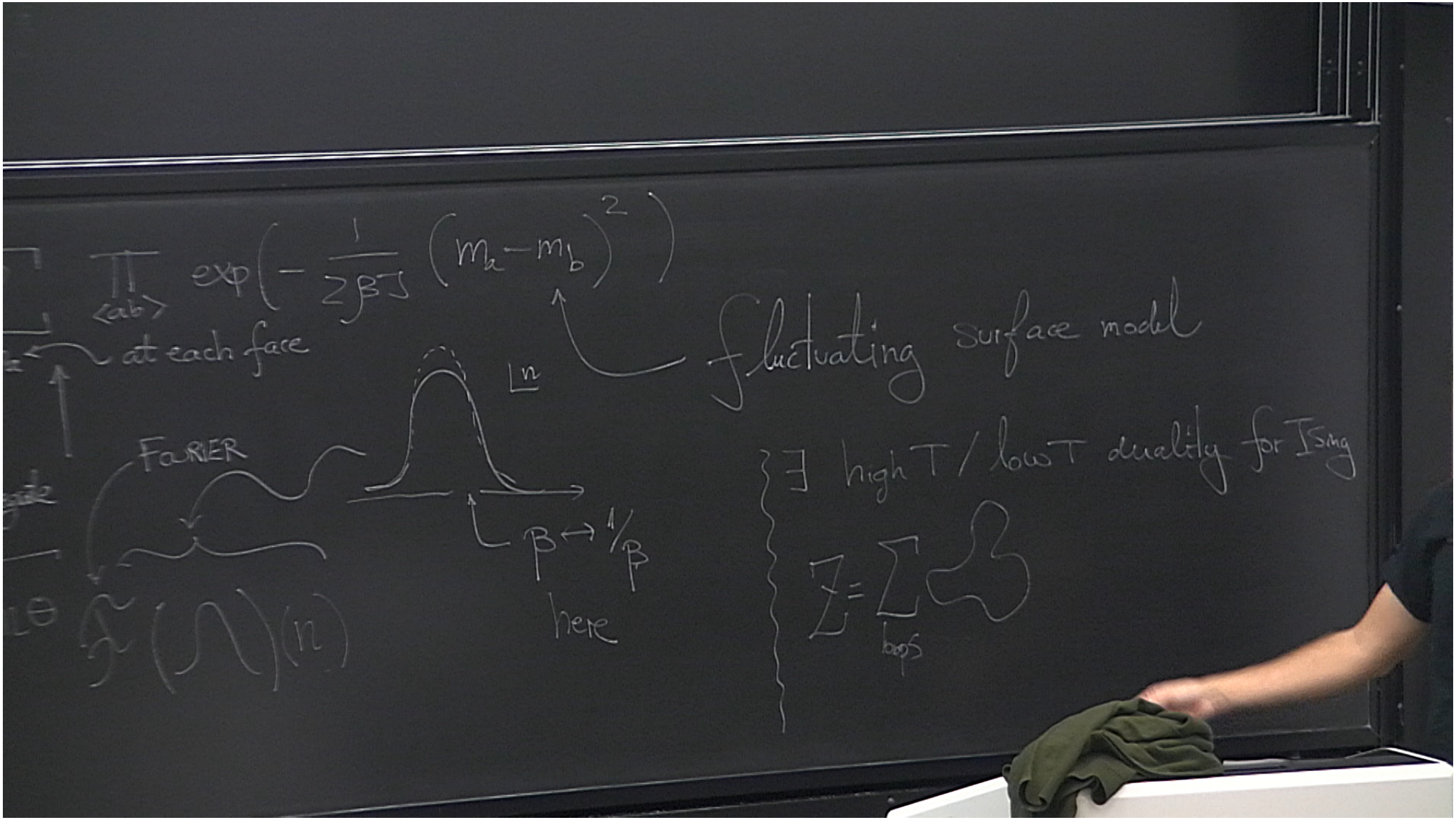

$\sum_{m_a}$ $\prod_{\langle ab \rangle}$ $\exp\left(-\frac{1}{2\beta J}(m_a - m_b)^2\right)$
at each face
fluctuating surface model
Can integrate
$\beta \to 1/\beta$
here

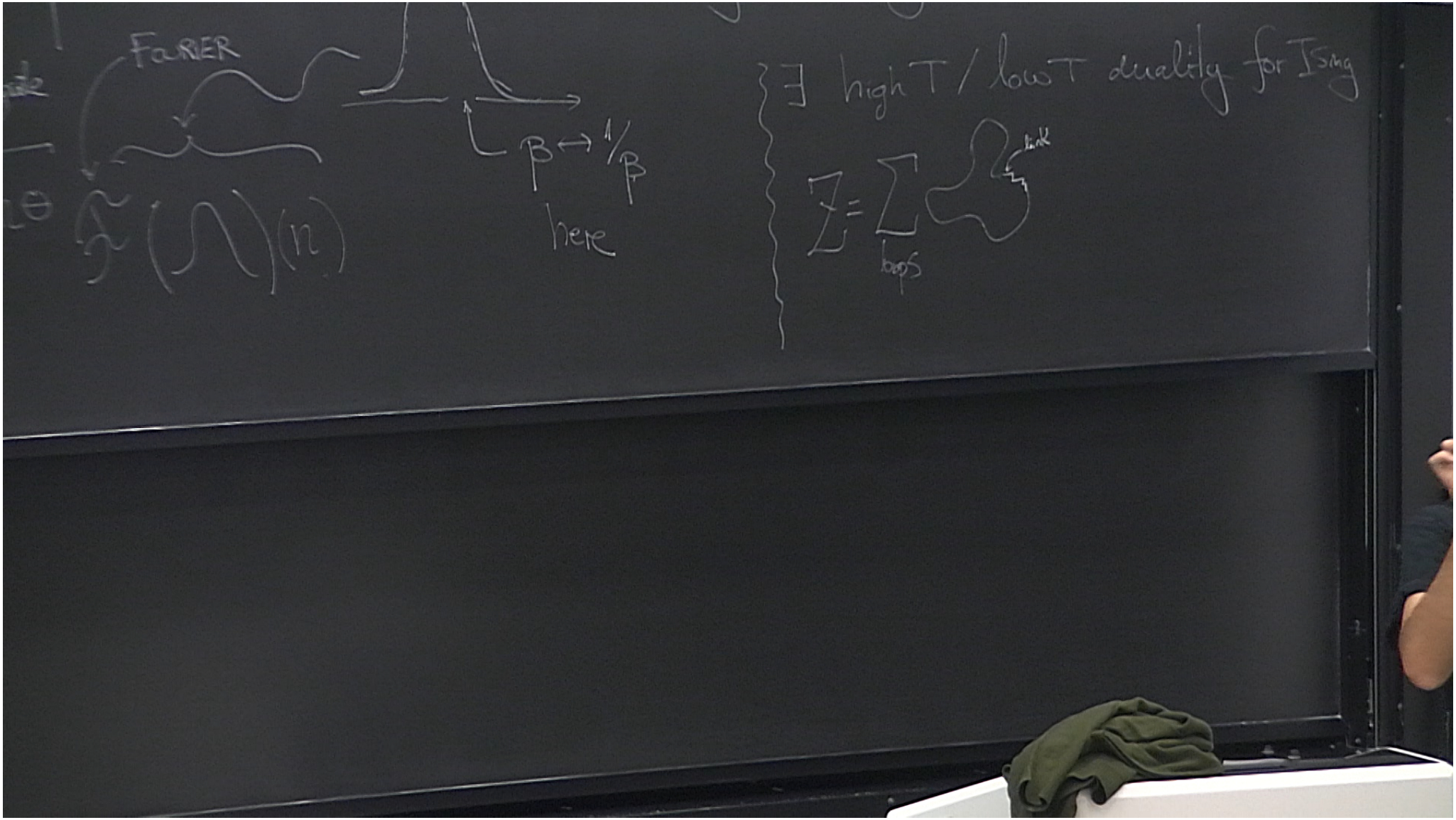

$\sum_{m_a}$ $\prod_{\langle ab \rangle}$ $\exp\left(-\frac{1}{2\beta J}(m_a - m_b)^2\right)$
at each face
fluctuating surface model
L^n
FOURIER
Can integrate
$\beta \to 1/\beta$
here
$= \sum_n e^{in\theta} \tilde{f}(n)$

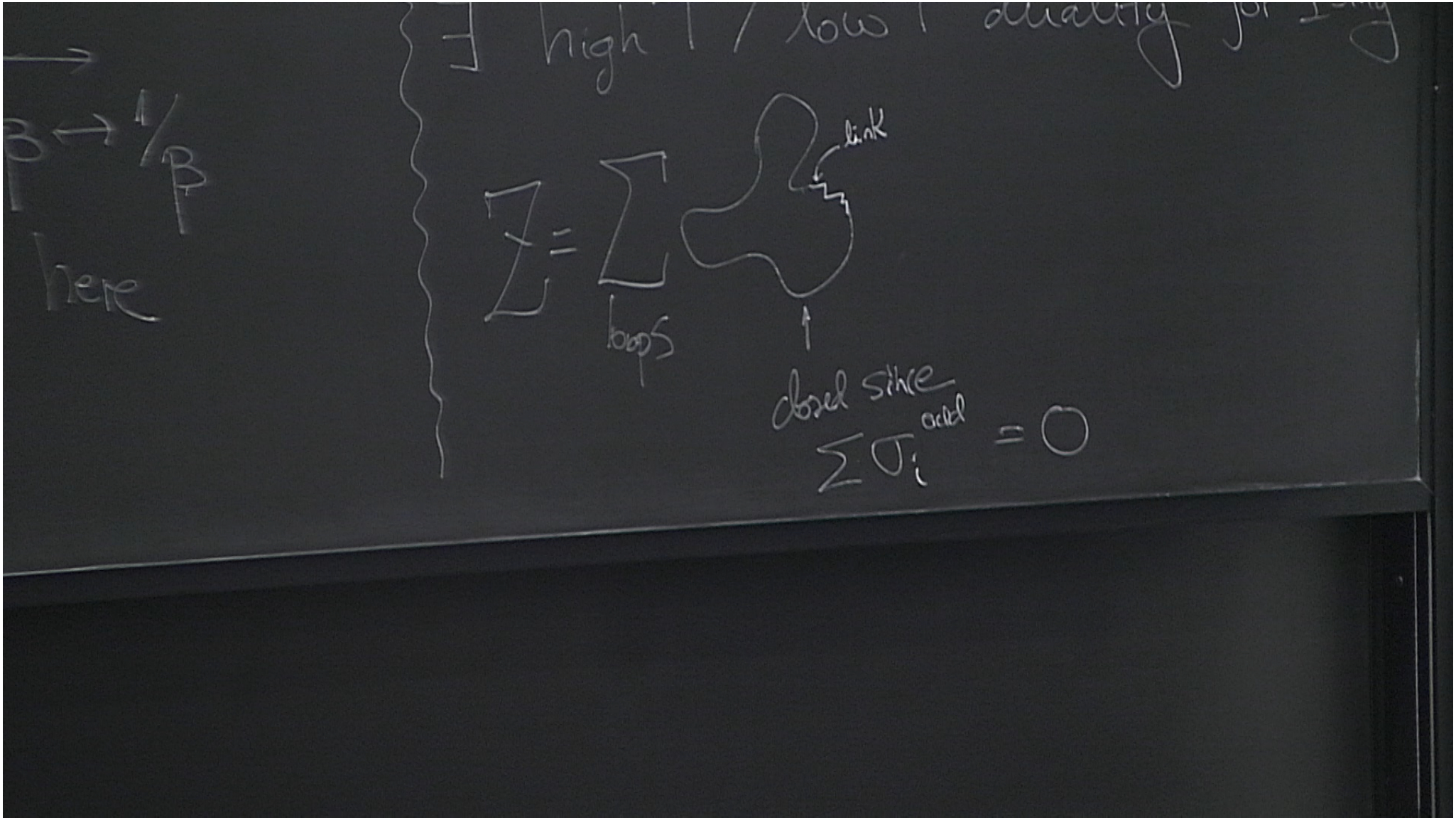

High T expansion
exp decay
Duality
periodic
Gaussians
can integrate
Fourier
at each face
fluctuating surface model

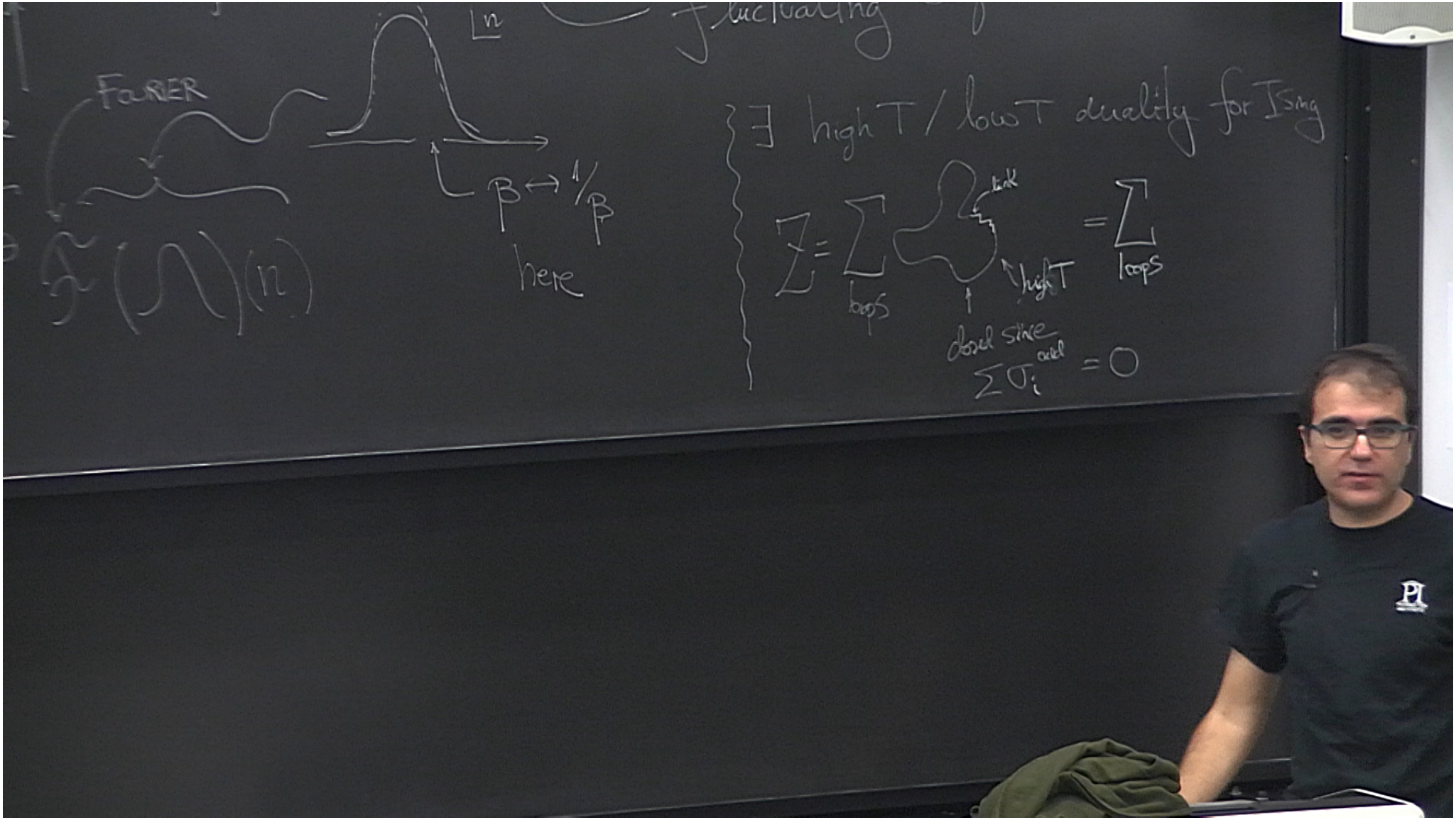

High T expansion
⟨SS⟩ =
~ exp decay
Duality
periodic
f
Gaussians
Can integrate
FOURIER
XY ≃ ∫ Π dθ_n Π_⟨ij⟩ (...) = Σ_n e^{inθ}
Σ_m Π_⟨ab⟩ exp
m_a
at each

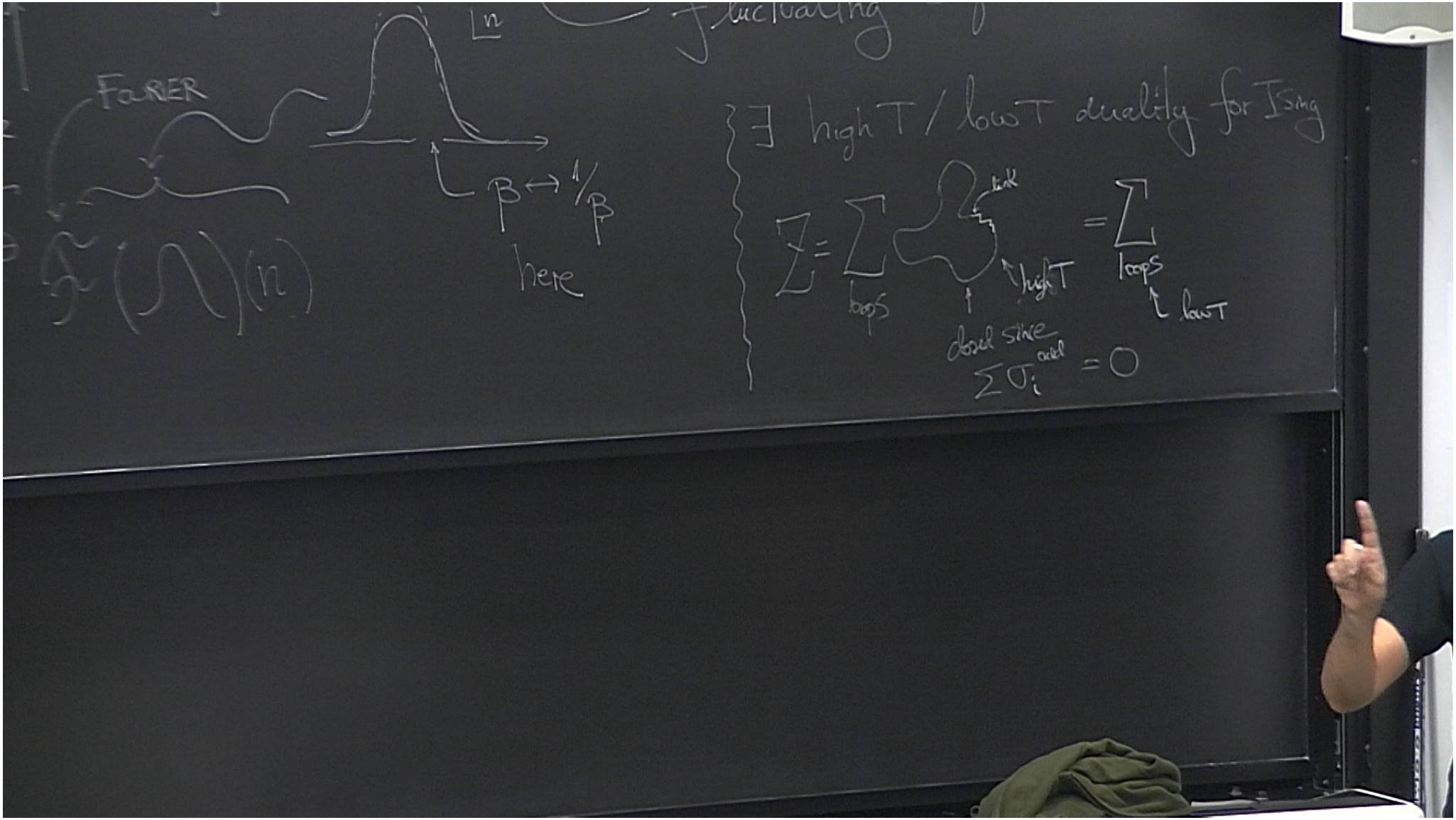

exp decay
f
fourier
Can integrate
FOURIER
at each face
fluctuating
here

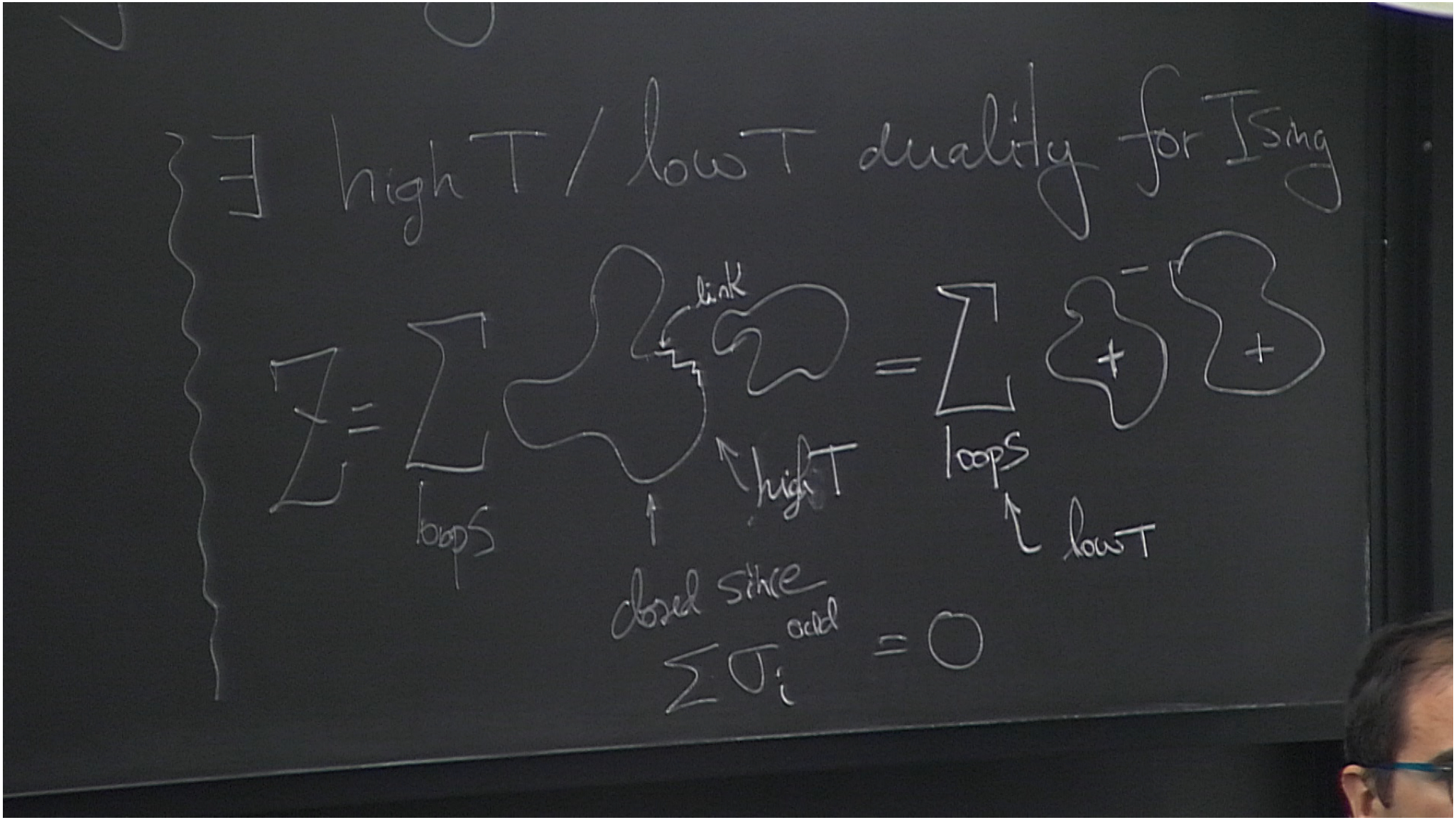

$\prod_{\langle ab \rangle} \exp\left(-\frac{1}{2\beta J}(m_a - m_b)^2\right)$
at each face
fluctuating surface model
Ln
FOURIER
$\beta \leftrightarrow 1/\beta$
here
$\exists$ high T / low T duality for Ising
$Z = \sum$
loops

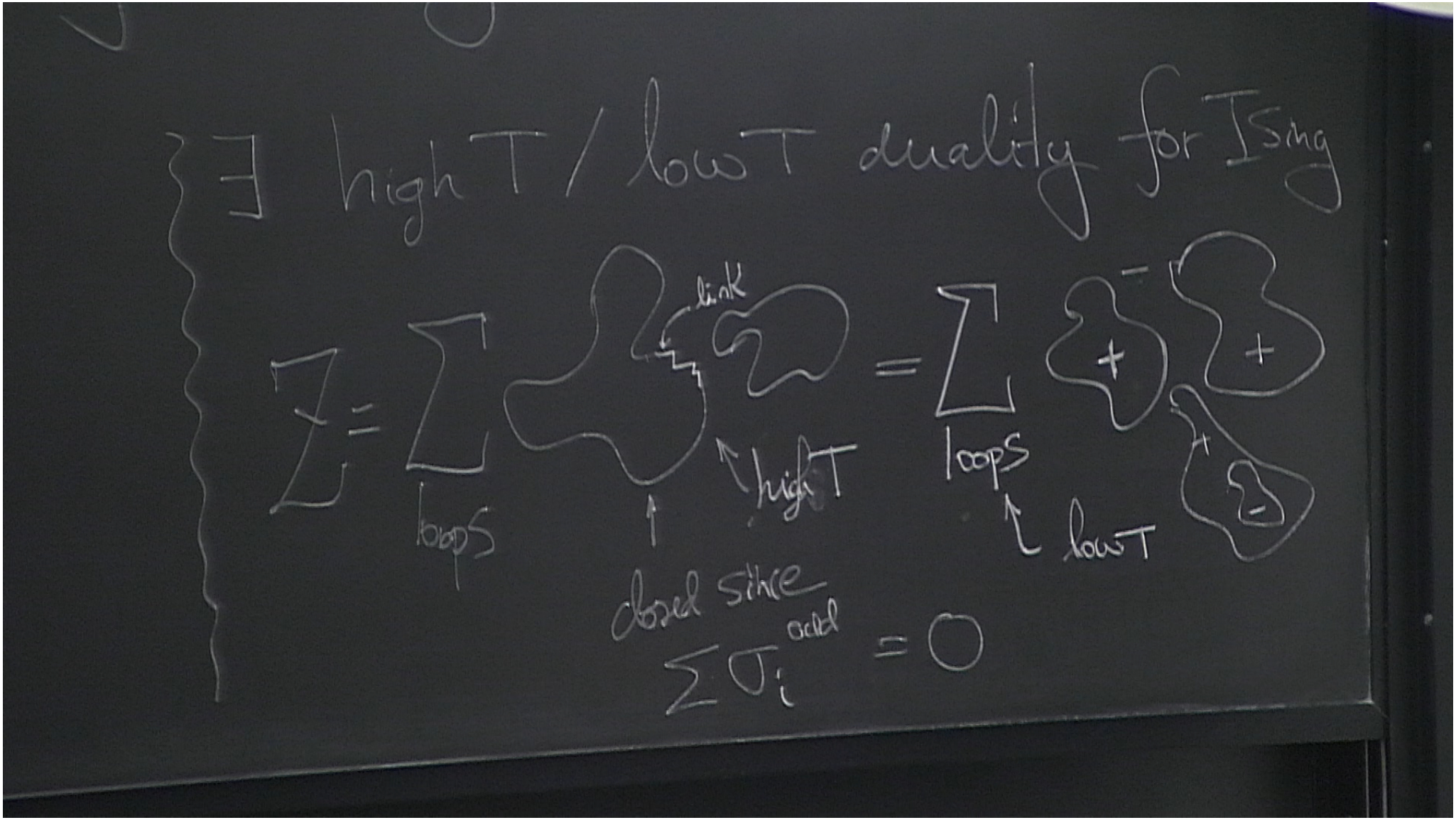

FOURIER
$\beta \mapsto 1/\beta$
here
$\exists$ high T / low T duality for Ising
$Z = \sum_{\text{loops}}$


high
low
link
loops
closed surface
$\beta \to 1/\beta$
here
$Z = \sum$
$\sum \sigma_i^{\text{odd}} = 0$

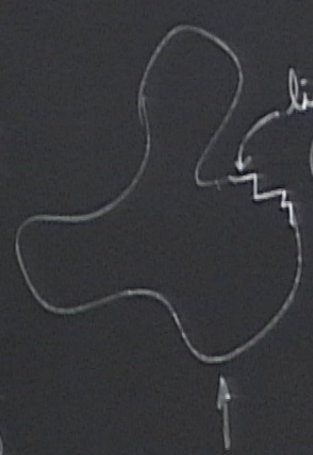

FOURIER
$\beta \leftrightarrow 1/\beta$
here
$\exists$ high T / low T duality for Ising
$Z = \sum_{\text{loops}}$
link
high T
$= \sum_{\text{loops}}$
closed since
$\sum \sigma_i^{\text{odd}} = 0$

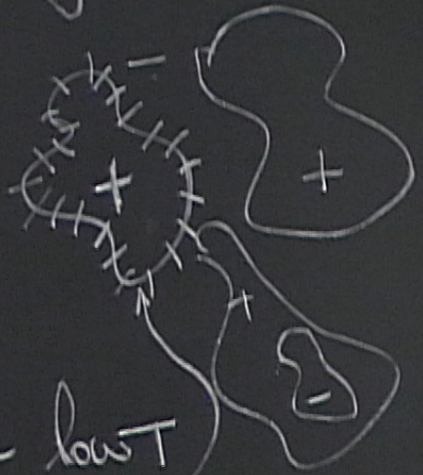

FOURIER
here
high T / low T duality for Ising
loops
high T
low T
closed surface


∃ high T / low T duality for Ising
Z = Σ
link
loops
high T
= Σ
loops
low T
closed since
Σ σ_i^odd = 0


∃ high T / low T duality for Ising
Z = Σ
loops
link
high T
= Σ
loops
low T
+
−
+
−
closed since
Σ σ_i = 0
odd

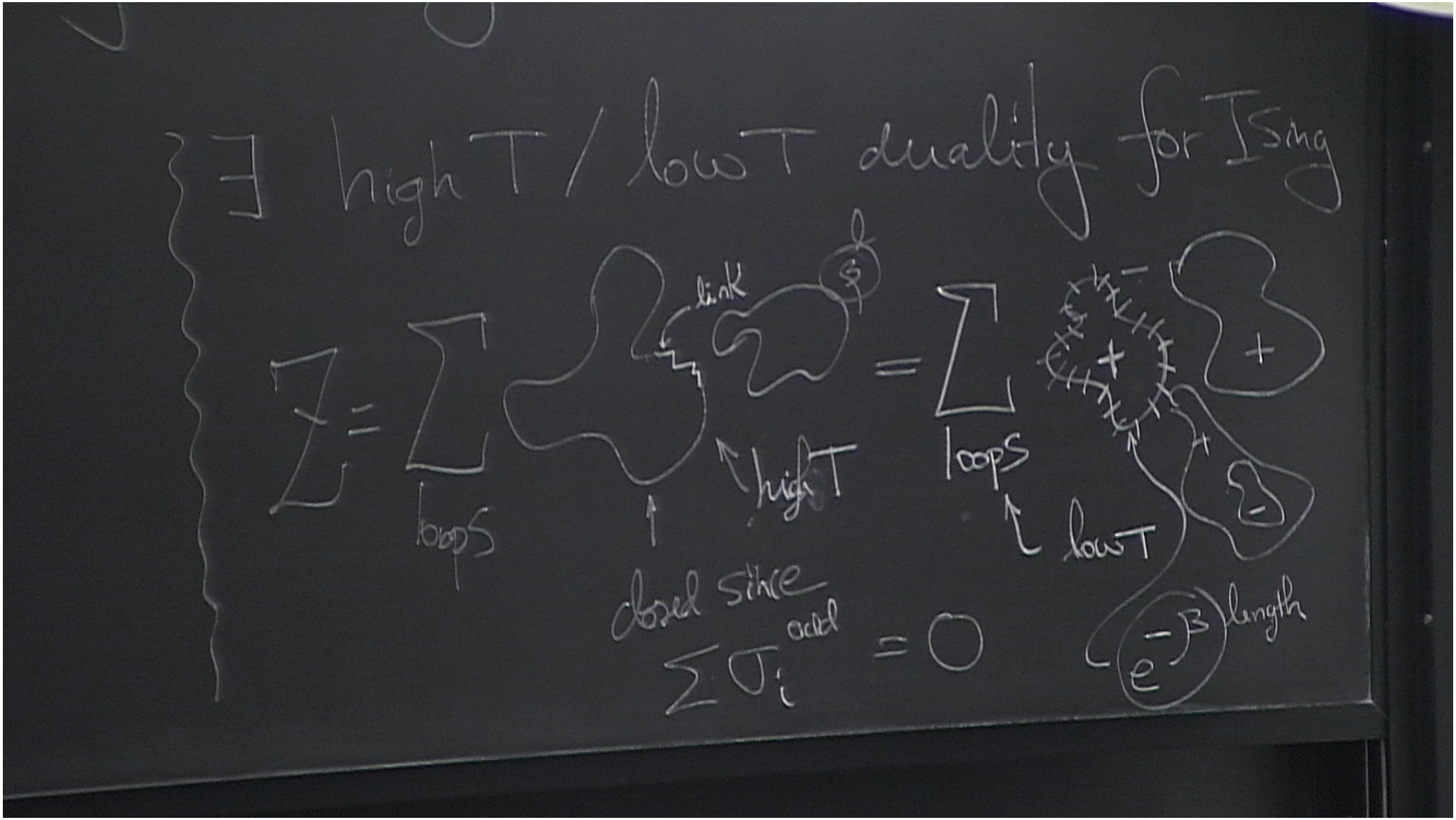
∃ high T / low T duality for Ising

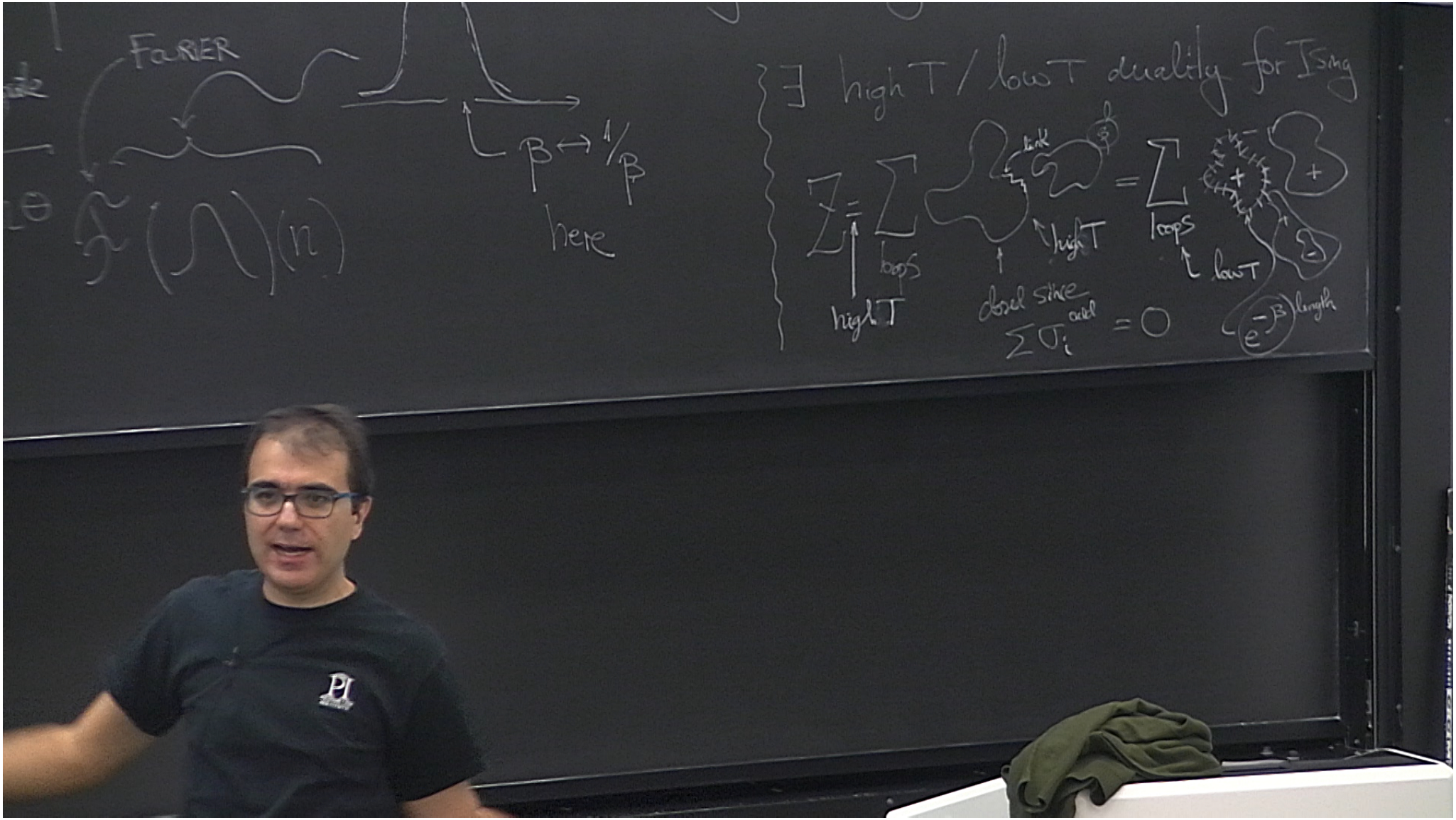
$Z = \sum_{\text{loops}}$ 

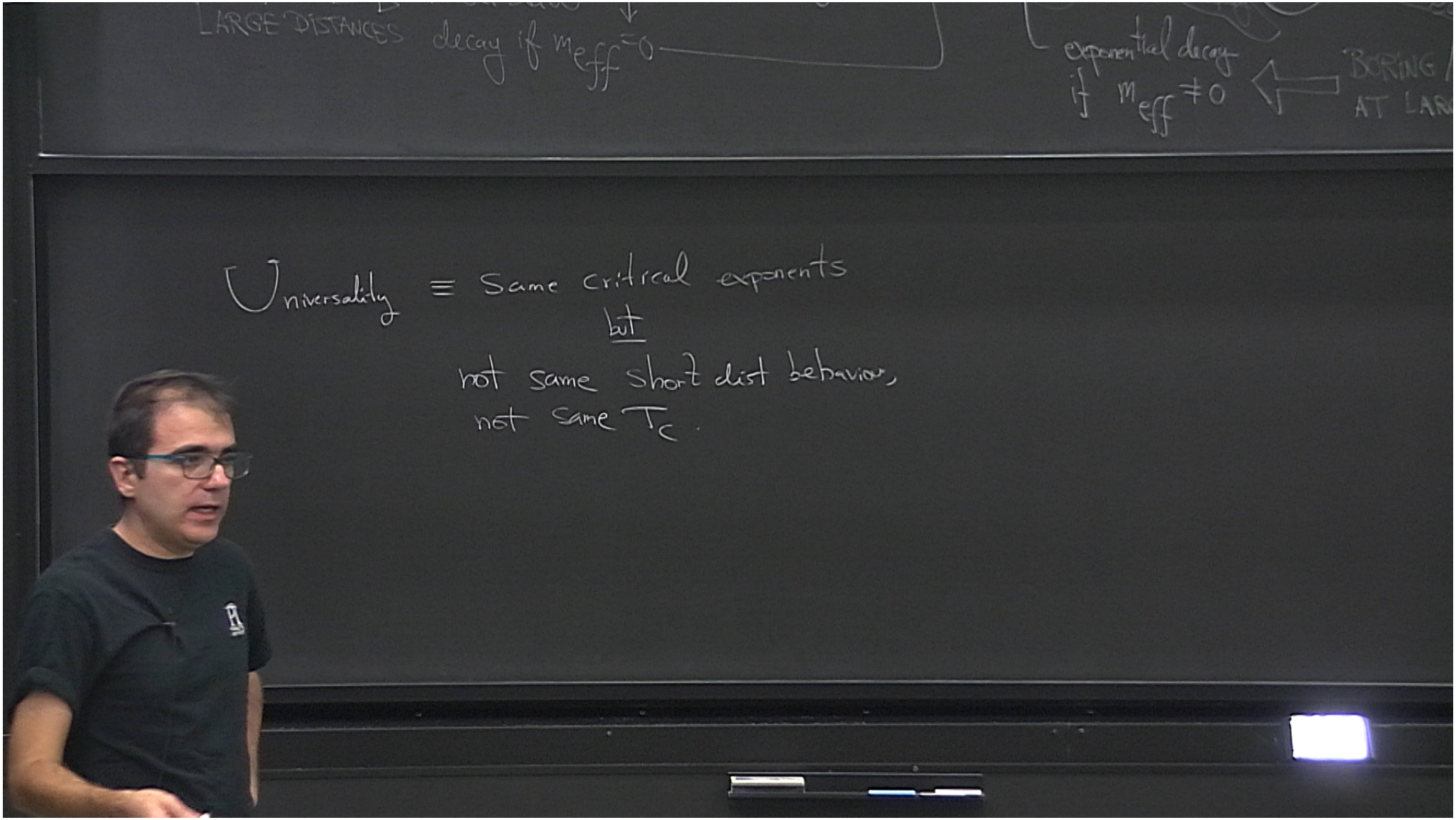
$= \sum_{\text{loops}}$ 

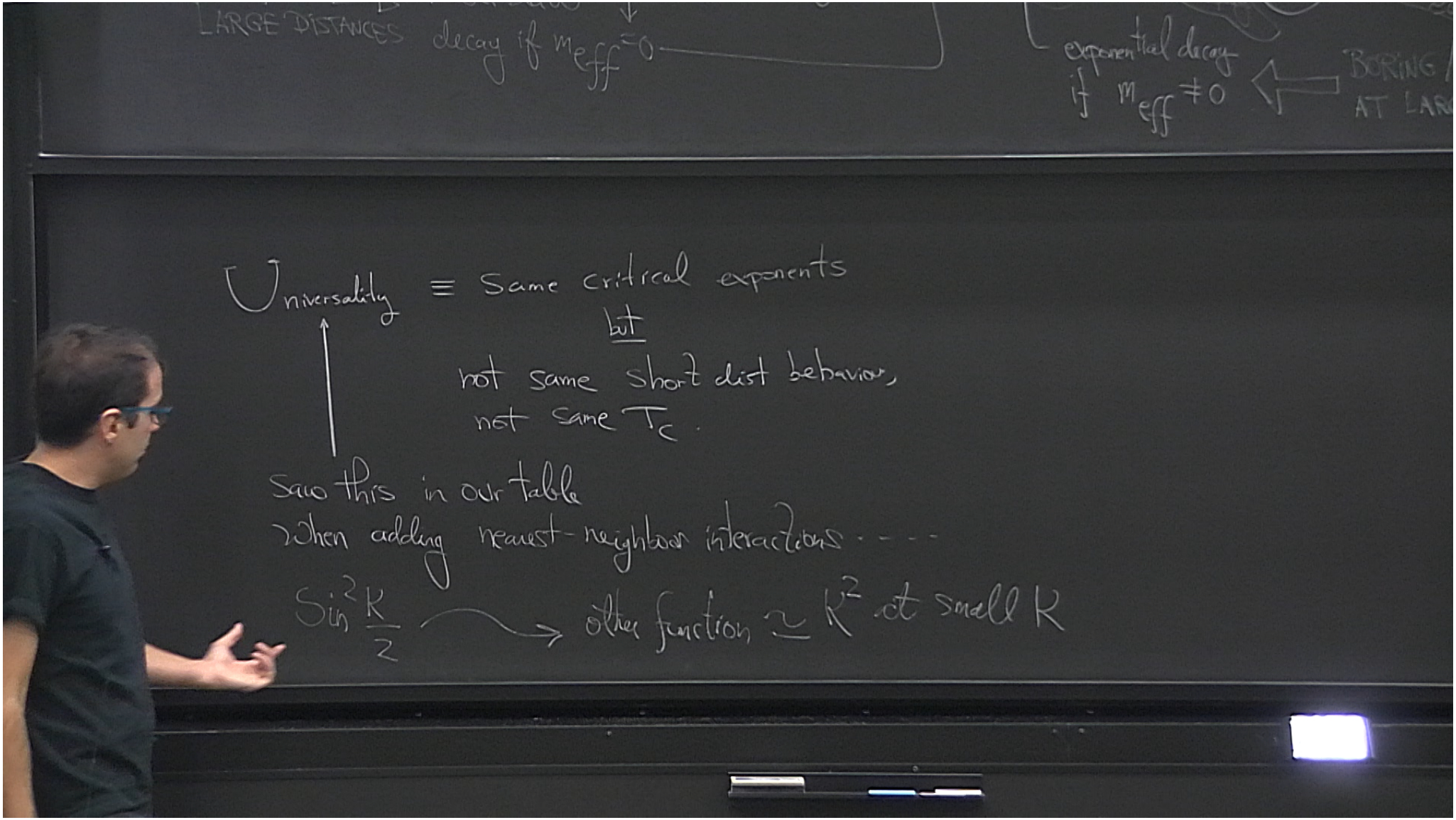
closed since $\sum_{\text{odd}} \sigma_i = 0$

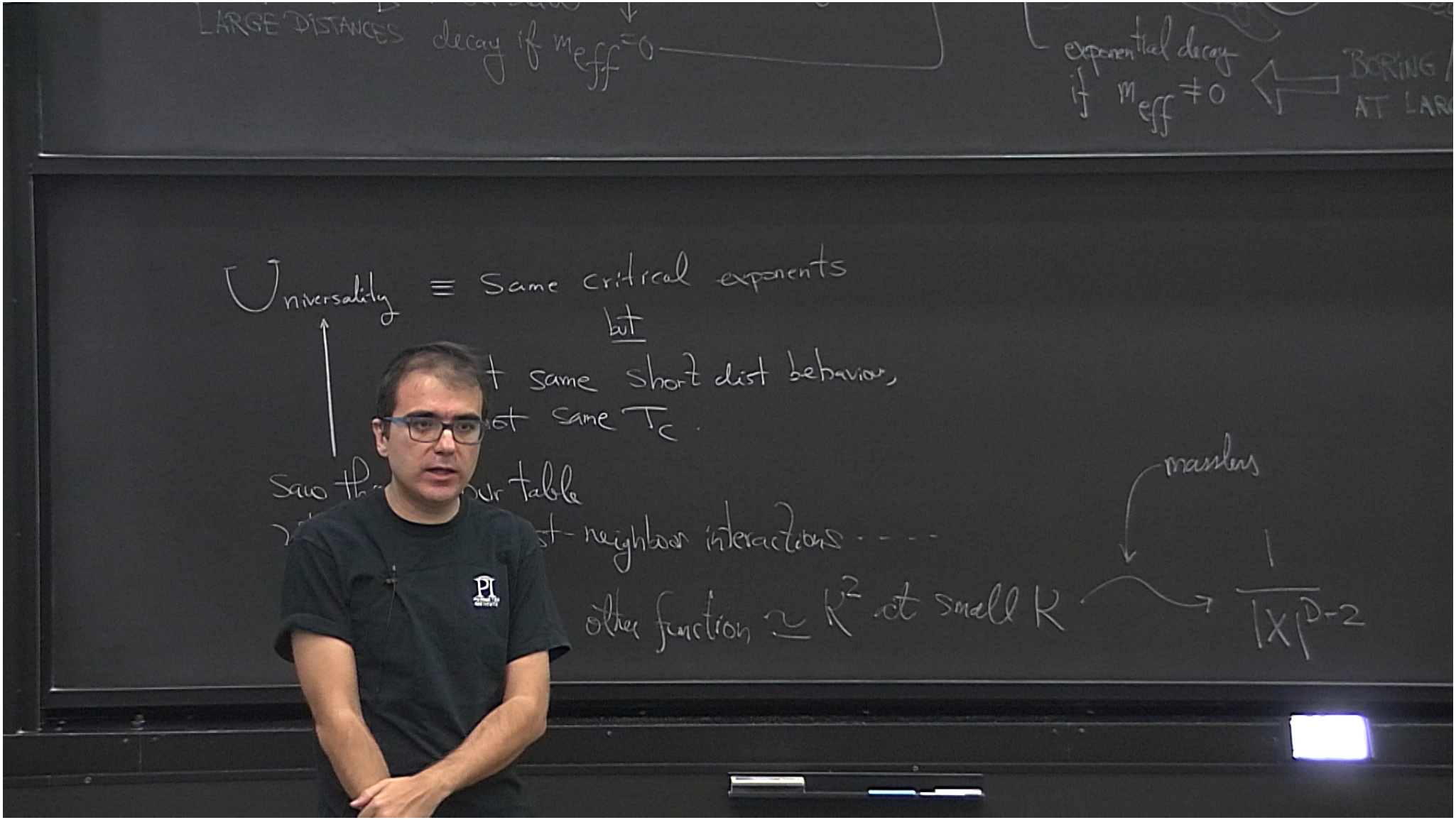
$e^{-\beta \text{ length}}$

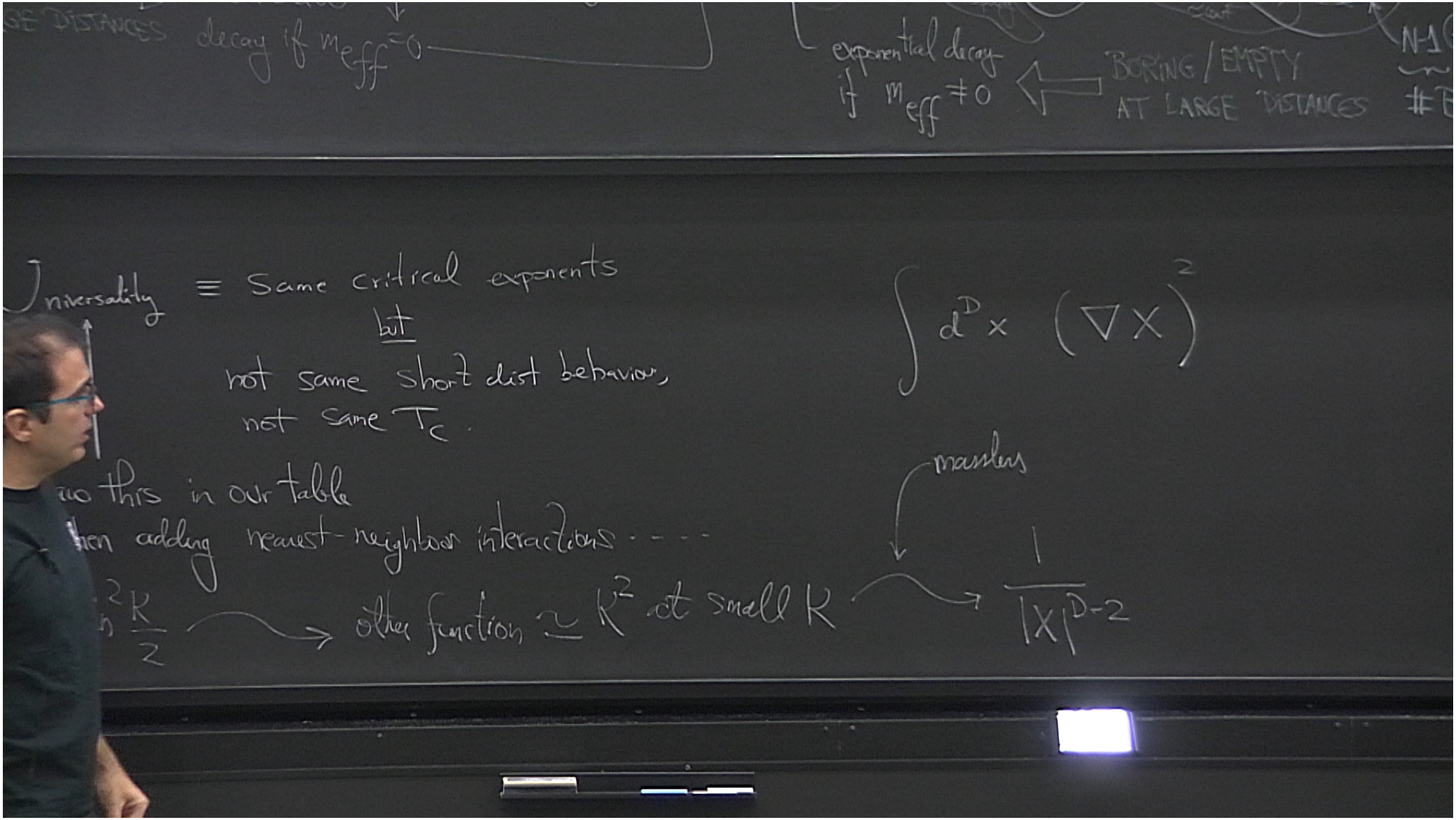

∃ high T / low T duality for Ising
Z = Σ
loops
link
high T
= Σ
loops
low T
closed since
Σ σ_i^odd = 0
e^{-βl} length

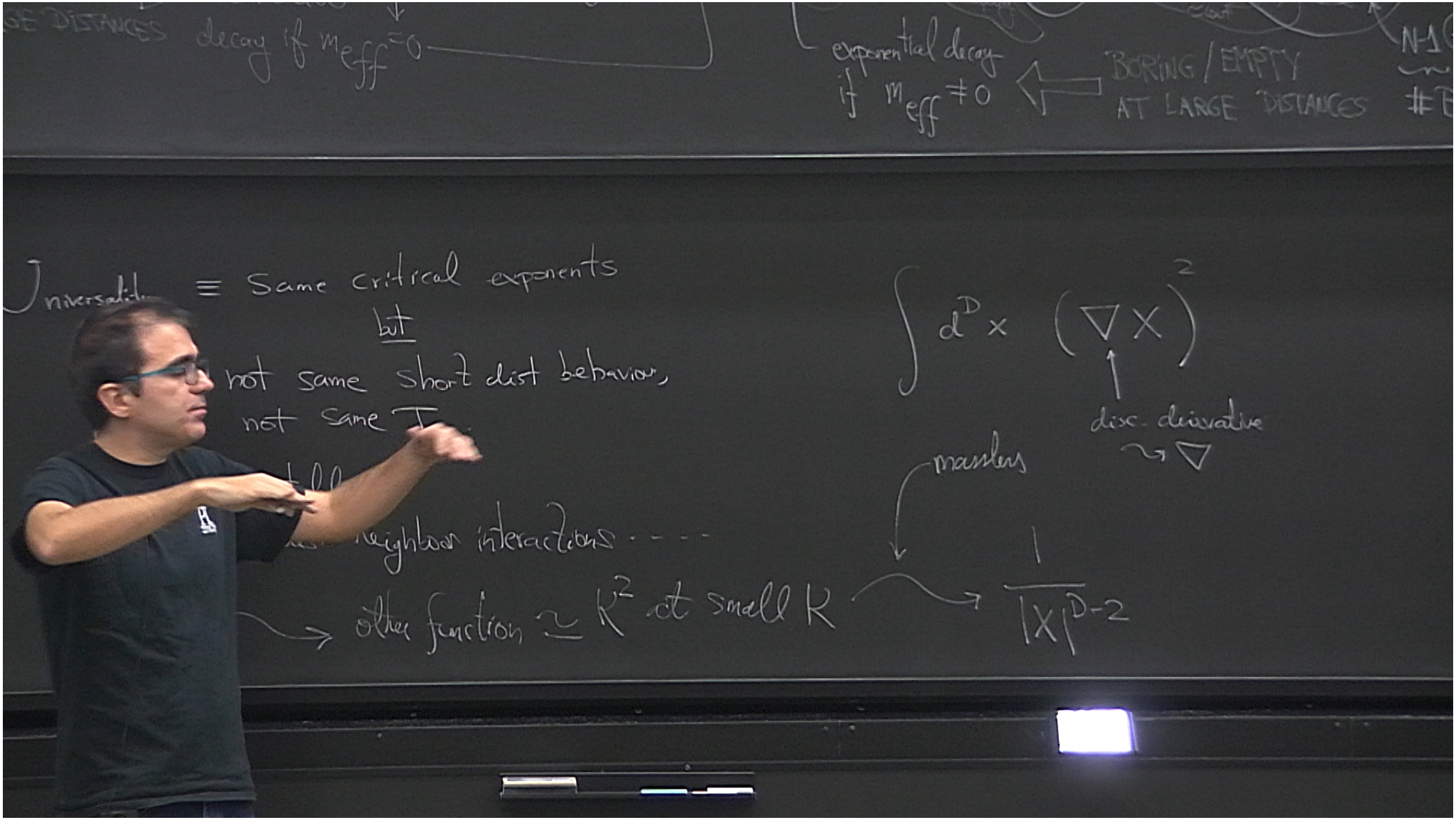

FOURIER
here
high T / low T duality for Ising
high T
loops
low T
closed slice
length

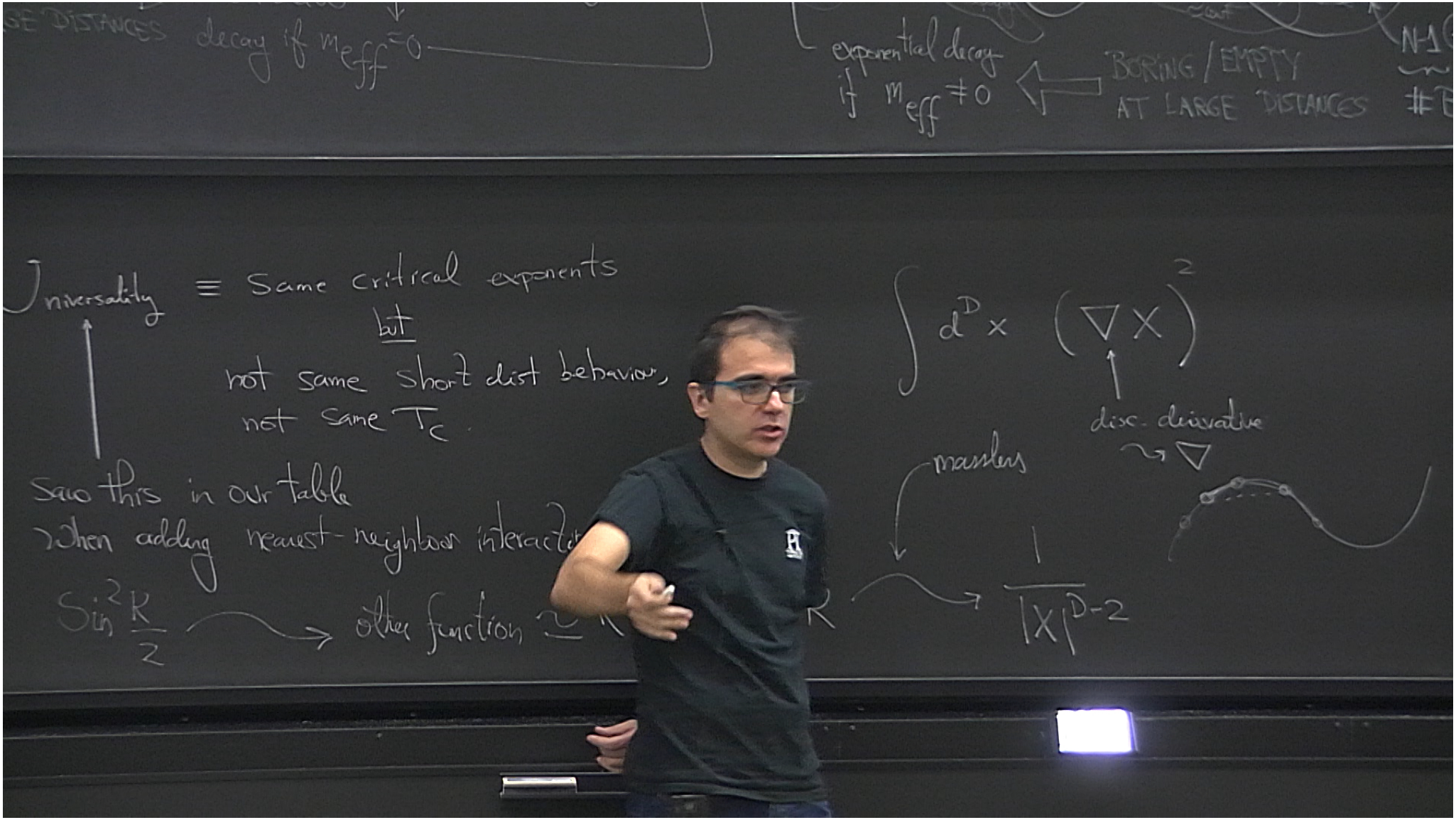

LARGE DISTANCES
decay if $m_{eff} = 0$
exponential decay if $m_{eff} \neq 0$
BORING
AT LAR
Universality ≡ Same critical exponents
but
not same short dist behavior,
not same T_c.


LARGE DISTANCES
decay if $m_{eff} \neq 0$
exponential decay if $m_{eff} \neq 0$
BORING AT LARGE
Universality ≡ Same critical exponents
but
not same short dist behavior,
not same T_c.
Saw this in our table
when adding nearest-neighbor interactions....
$\sin^2 \frac{K}{2}$
other function $\simeq K^2$ at small K


LARGE DISTANCES
decay if $m_{eff} = 0$
exponential decay if $m_{eff} \neq 0$
BORING
AT LARG
Universality ≡ Same critical exponents
but
same short dist behavior,
same T_c.
Same th... our table
...st-neighbor interactions ....
massless
other function $\simeq k^2$ at small k
$\frac{1}{|x|^{D-2}}$


SE DISTANCES
decay if $m_{eff} = 0$
exponential decay if $m_{eff} \neq 0$
BORING / EMPTY AT LARGE DISTANCES
N-1
Universality ≡ Same critical exponents
but
not same short dist behavior,
not same T_c.
$\int d^D x \, (\nabla X)^2$
masless
$\frac{1}{|X|^{D-2}}$
$\frac{K}{2}$
other function $\simeq K^2$ at small K


decay if $m_{eff} \neq 0$
exponential decay if $m_{eff} \neq 0$
BORING / EMPTY AT LARGE DISTANCES
Universality ≡ Same critical exponents
but
not same short dist behavior,
not same
neighbor interactions . . . .
other function $\simeq k^2$ at small k
masless
$\frac{1}{|x|^{D-2}}$
$\int d^D x \left(\nabla X\right)^2$
disc. derivative
∇


DISTANCES
decay if $m_{eff} \neq 0$
exponential decay if $m_{eff} \neq 0$
BORING / EMPTY AT LARGE DISTANCES
Universality ≡ Same critical exponents
but
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$\int d^D x \left(\nabla X\right)^2$
disc. derivative
masses
Saw this in our table
When adding nearest-neighbor interac
$\sin^2 \frac{K}{2}$
other function
$\frac{1}{|X|^{D-2}}$

exponential decay if $m_{eff} \neq 0$ ⇐ BORING / EMPTY AT LARGE DISTANCES

N-1 Goldstone modes = # Broken symmetries

$\int \mathcal{D}\theta_{\vec{n}}$ e

exponents

list behavior,

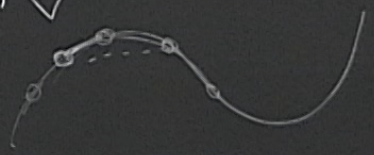
$$\int d^D x \left(\nabla X\right)^2 + b\left(\nabla^2 X\right)^2 + \cdots$$

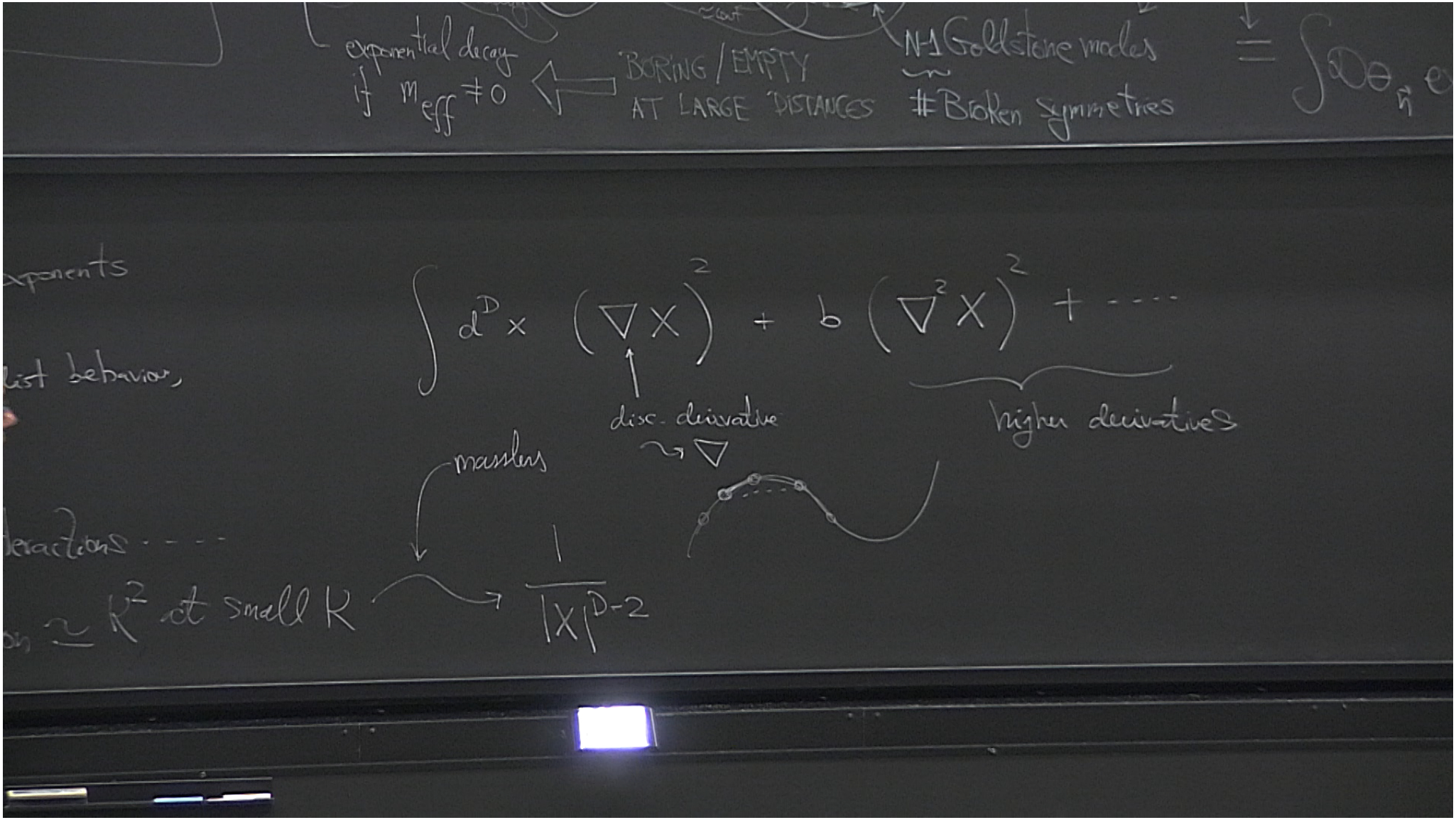
disc. derivative ⇝ ∇

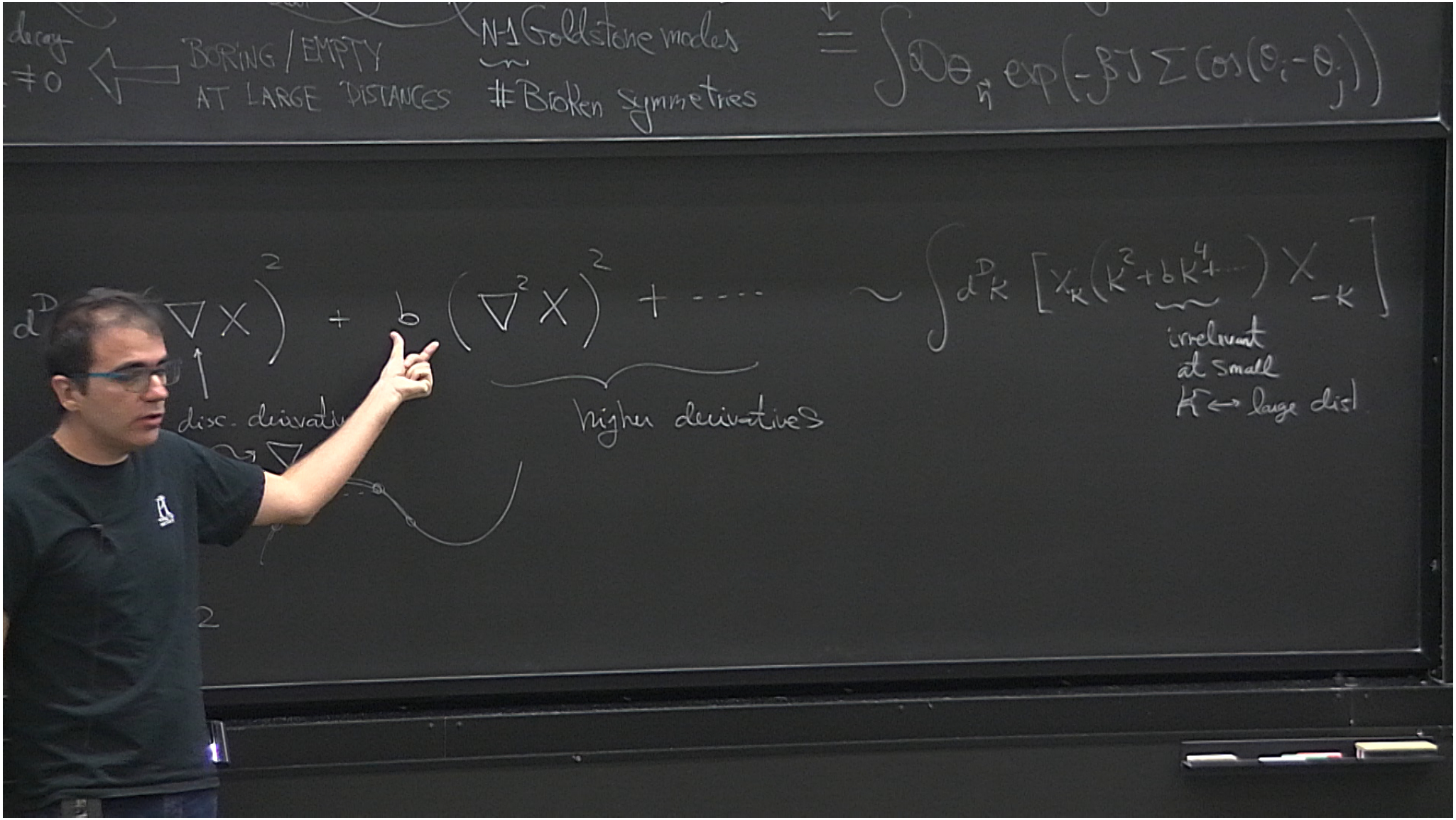
massless

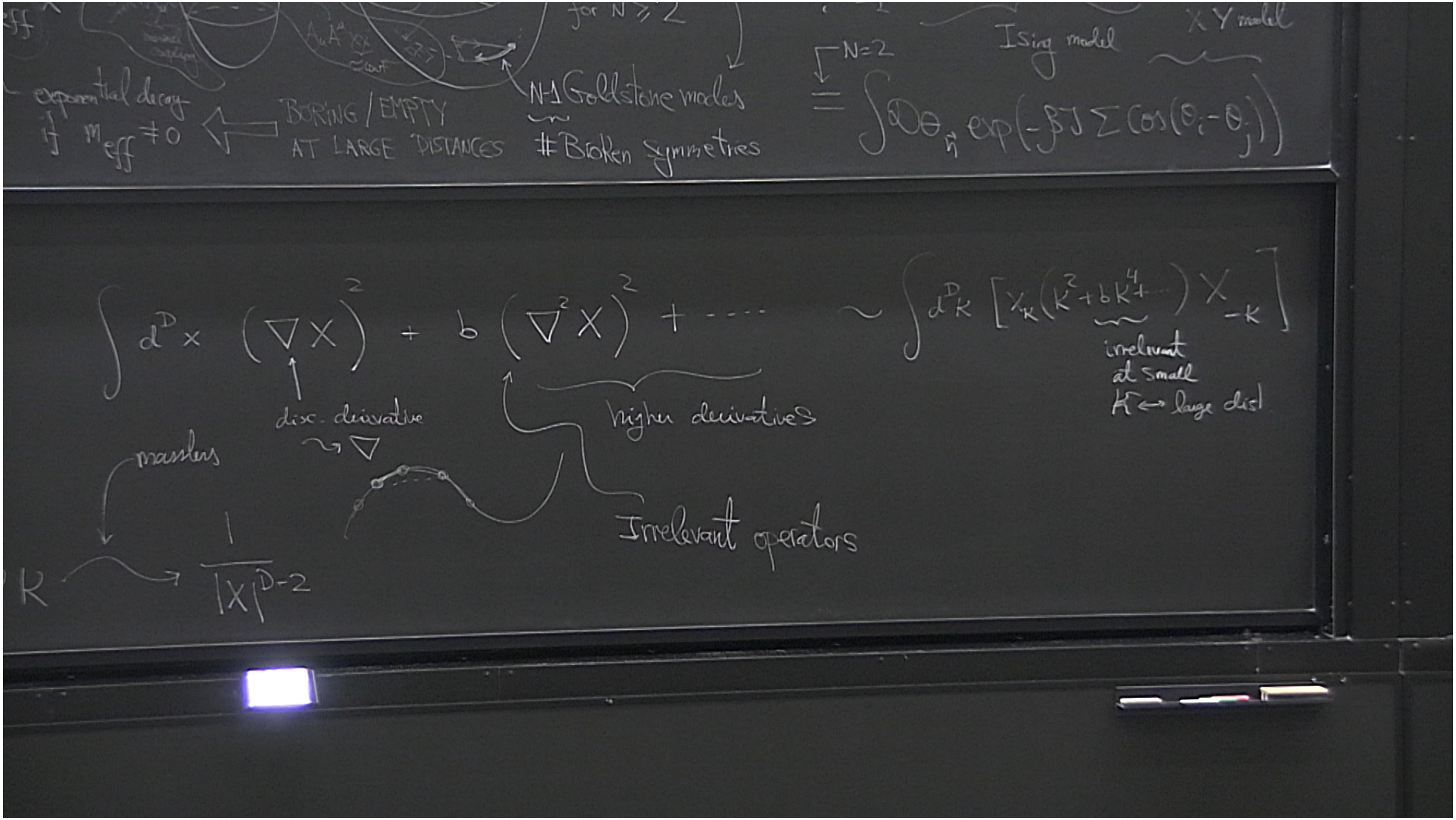
interactions ----

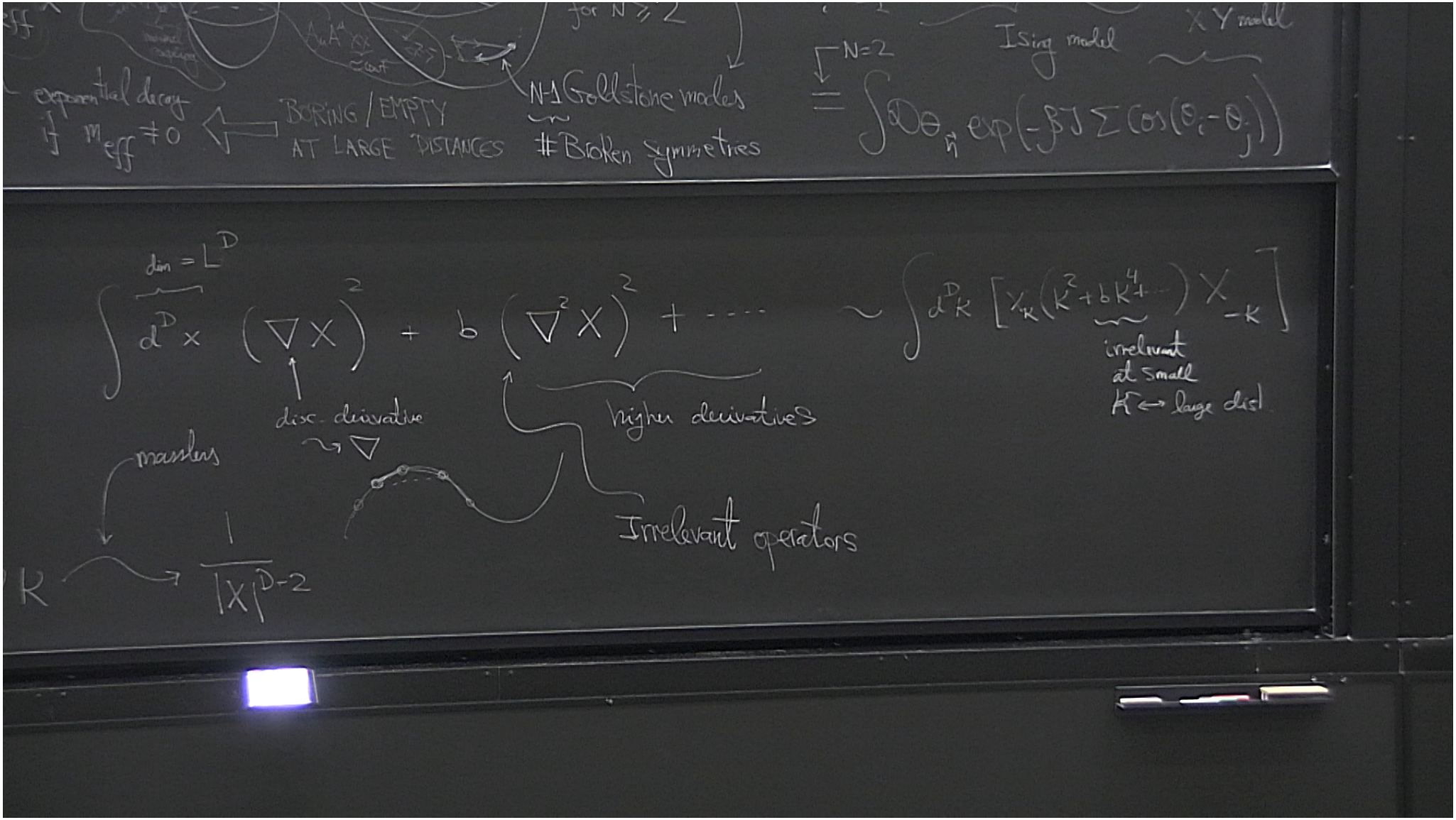
$\sim K^2$ at small K ⇝ $\frac{1}{|X|^{D-2}}$

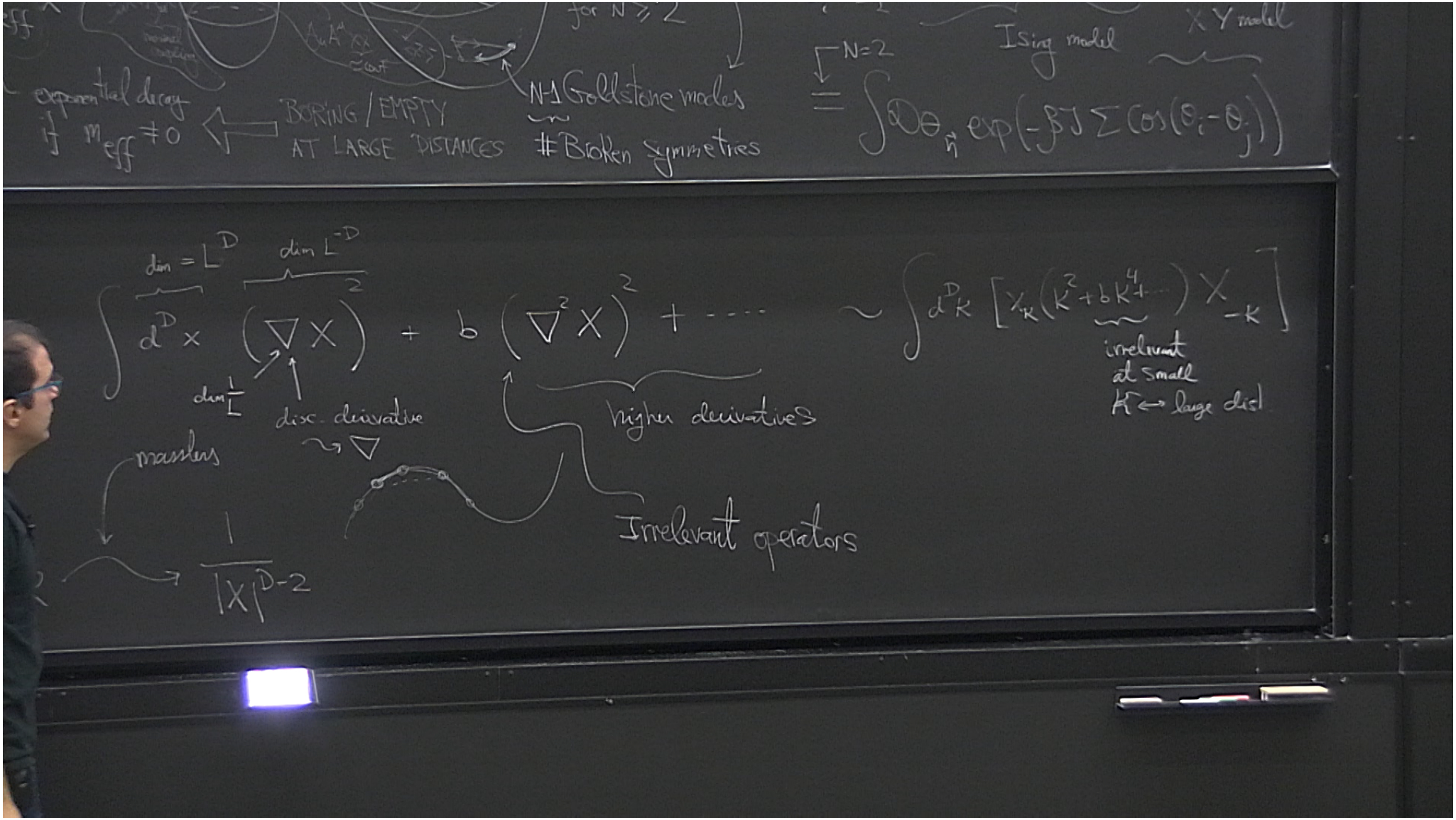


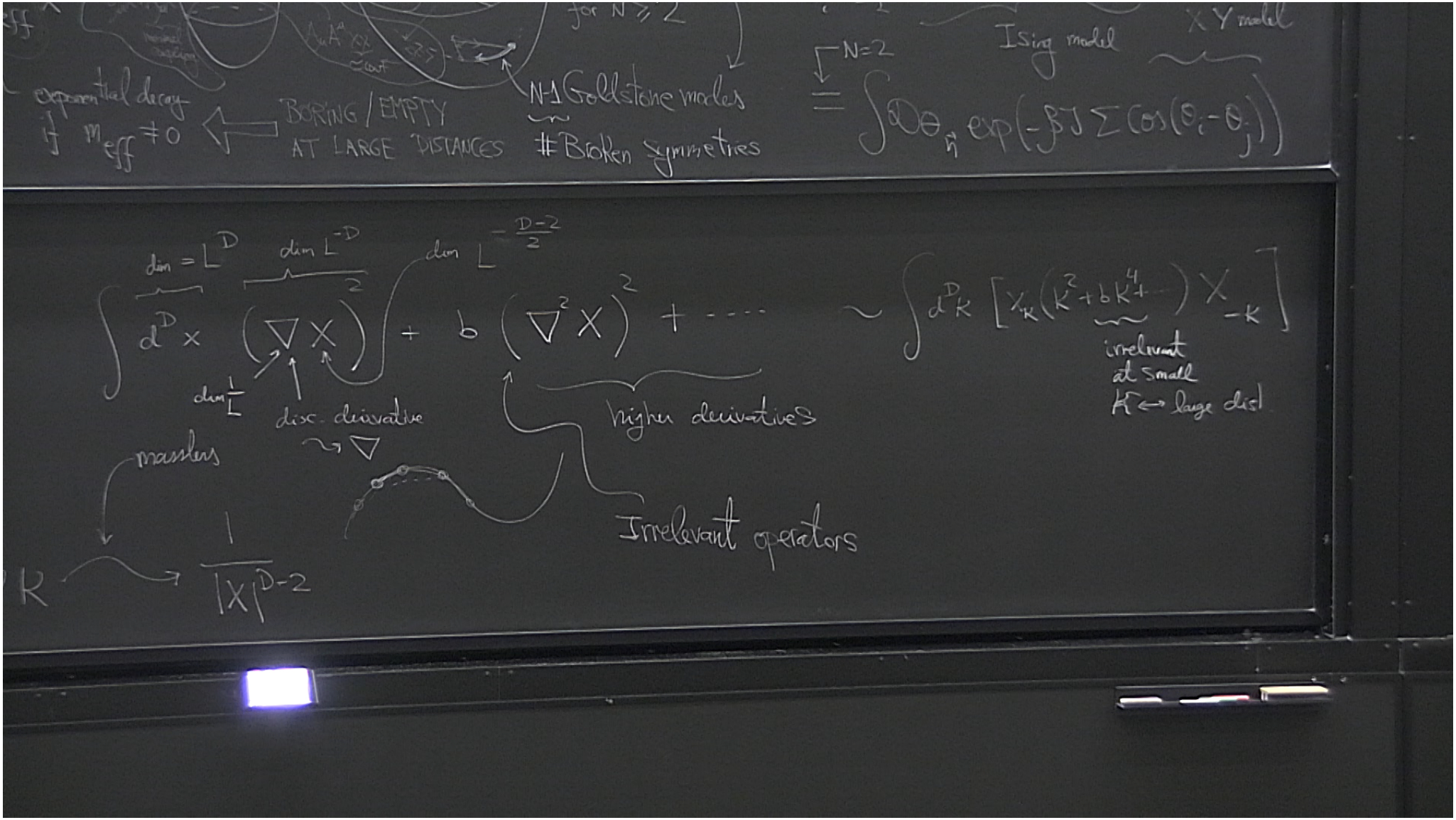

exponential decay
if $m_{eff} \neq 0$
BORING / EMPTY
AT LARGE DISTANCES
N-1 Goldstone modes
Broken symmetries
$\int d^D x \left(\nabla X\right)^2 + b\left(\nabla^2 X\right)^2 + \cdots$
disc. derivative
∇
higher derivatives
massless
$\frac{1}{|x|^{D-2}}$

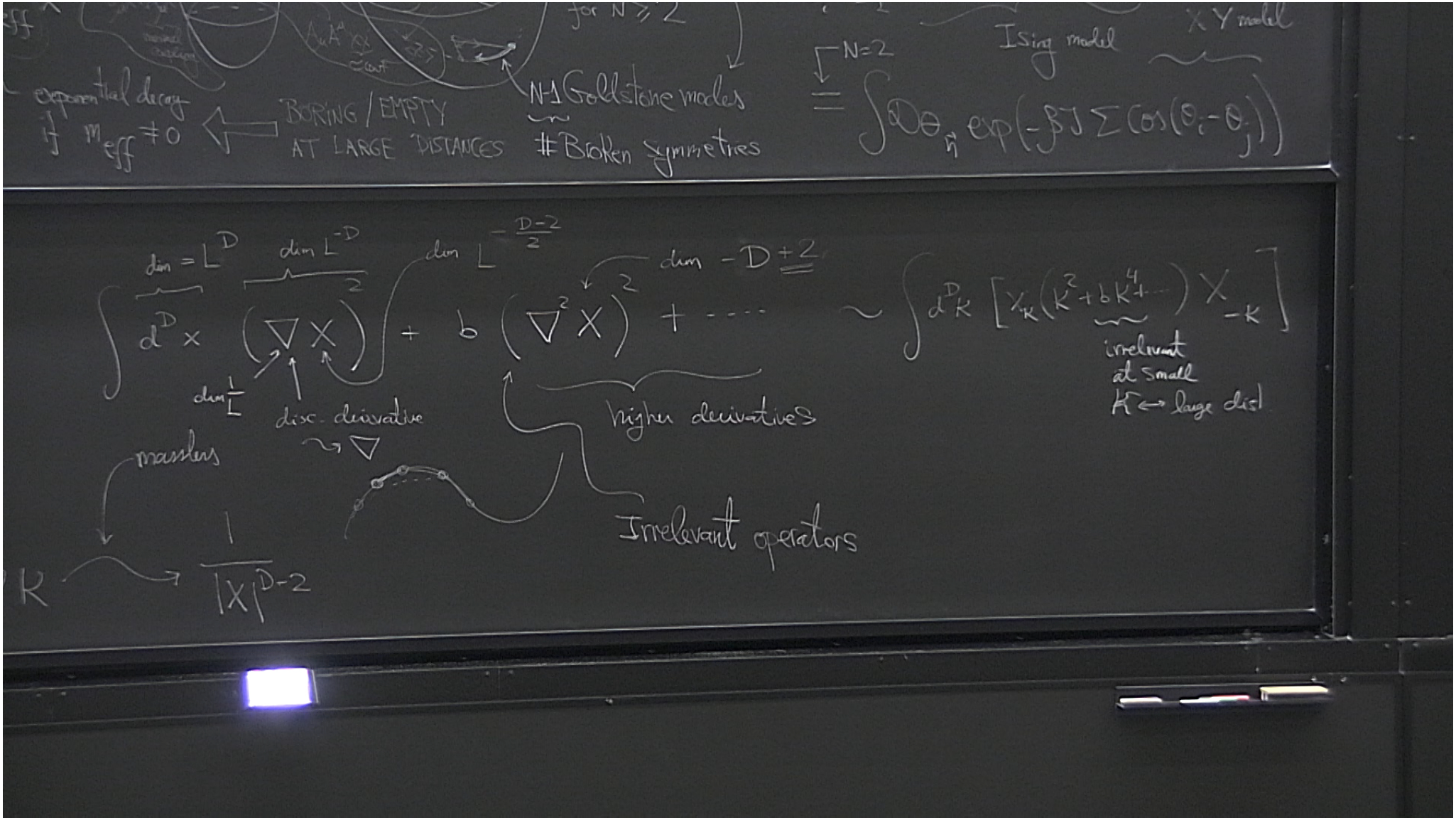

decay
≠0
BORING / EMPTY
AT LARGE DISTANCES
N-1 Goldstone modes
Broken symmetries
$= \int \prod_i d\theta_i \exp\left(-\beta J \sum \cos(\theta_i - \theta_j)\right)$
d^D
$\nabla X)^2 + b(\nabla^2 X)^2 + \cdots$
$\sim \int d^D k \left[X_k (k^2 + b k^4 + \cdots) X_{-k} \right]$
disc. derivati
higher derivatives
irrelevant
at small
$k \leftrightarrow$ large dist

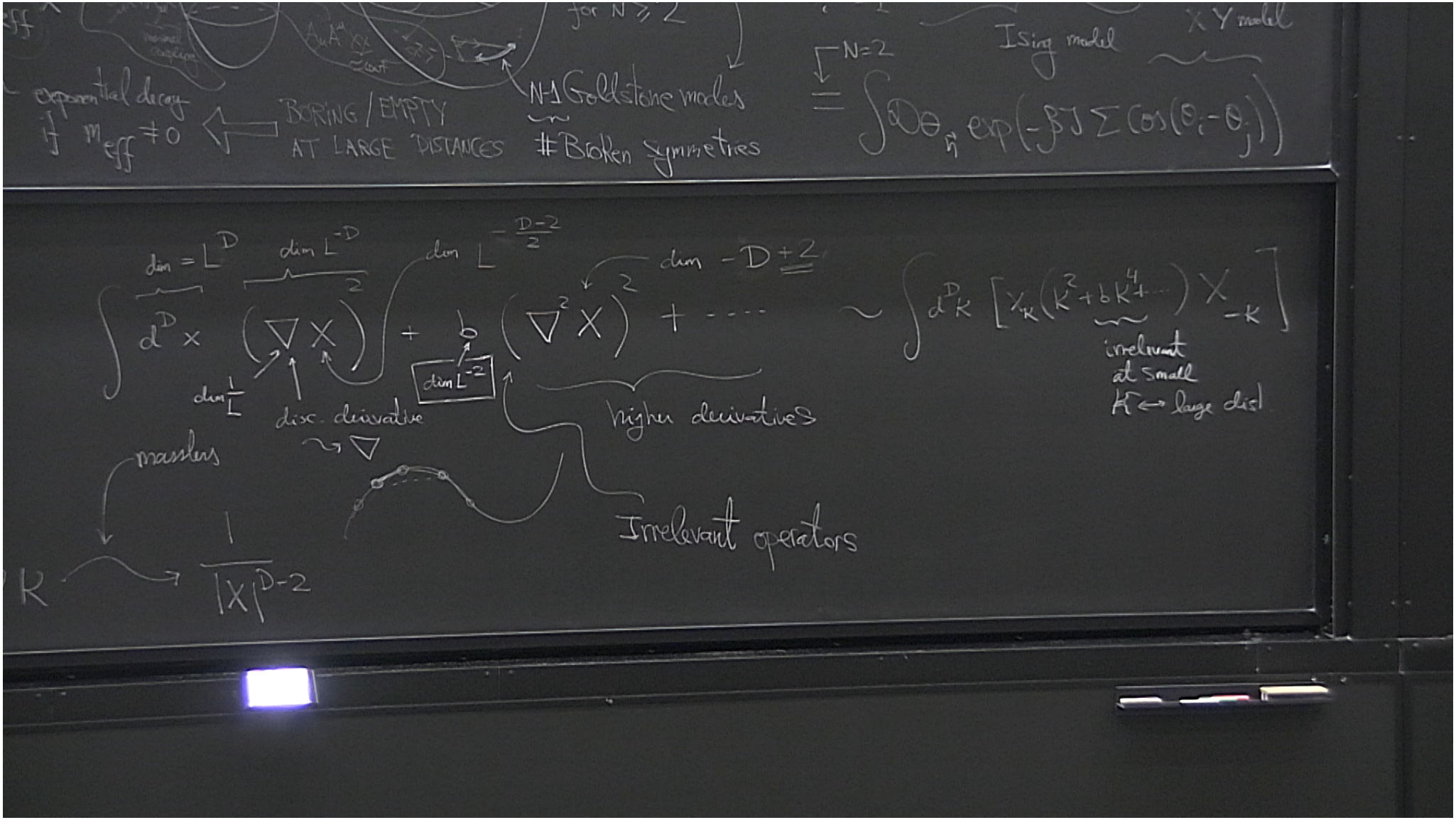

for N ≥ 2
N=2
Ising model
XY model
exponential decay
if $m_{eff} \neq 0$
BORING / EMPTY
AT LARGE DISTANCES
N-1 Goldstone modes
Broken symmetries
$= \int \mathcal{D}\theta_{\vec{n}} \exp\left(-\beta J \sum \cos(\theta_i - \theta_j)\right)$
$\int d^D x \left(\nabla X\right)^2 + b\left(\nabla^2 X\right)^2 + \cdots \sim \int d^D k \left[\chi_k \left(k^2 + b k^4 + \cdots\right) X_{-k}\right]$
disc. derivative
∇
higher derivatives
irrelevant at small k ↔ large dist
massless
$\frac{1}{|x|^{D-2}}$
Irrelevant operators


for N ≥ 2
N-1 Goldstone modes
Broken symmetries
BORING / EMPTY
AT LARGE DISTANCES
exponential decay
if $m_{eff} \neq 0$
Ising model
XY model
$\Gamma^{N=2}$
$= \int \mathcal{D}\theta_{\vec{n}} \exp\left(-\beta J \sum \cos(\theta_i - \theta_j)\right)$
dim $= L^D$
$\int d^D x \left(\nabla X\right)^2 + b\left(\nabla^2 X\right)^2 + \cdots$
$\sim \int d^D k \left[\chi_k \left(k^2 + b k^4 + \cdots\right) X_{-k}\right]$
irrelevant at small k ↔ large dist
disc. derivative
higher derivatives
masses
Irrelevant operators
$k \rightsquigarrow \frac{1}{|x|^{D-2}}$


exponential decay
if $m_{eff} \neq 0$
BORING / EMPTY
AT LARGE DISTANCES
N-1 Goldstone modes
Broken symmetries
for $N \geq 2$
$\Gamma^{N=2}$
Ising model
XY model
$= \int \mathcal{D}\theta_{\vec{n}} \exp\left(-\beta J \sum \cos(\theta_i - \theta_j)\right)$
$\int d^D x \left(\nabla X\right)^2 + b\left(\nabla^2 X\right)^2 + \cdots$
dim $= L^D$
dim L^{-D}
dim $\frac{1}{L}$
disc. derivative
∇
higher derivatives
$\sim \int d^D k \left[\chi_k \left(k^2 + b k^4 + \cdots\right) X_{-k}\right]$
irrelevant at small $k \leftrightarrow$ large dist
massless
$\frac{1}{|x|^{D-2}}$
Irrelevant operators


for N ≥ 2
N-1 Goldstone modes
Broken symmetries
BORING / EMPTY AT LARGE DISTANCES
exponential decay
$m_{eff} \neq 0$
$\Gamma^{N=2}$
Ising model
XY model
$= \int \mathcal{D}\theta_{\vec{n}} \exp\left(-\beta J \sum \cos(\theta_i - \theta_j)\right)$
$\int d^D x \left(\nabla X\right)^2 + b\left(\nabla^2 X\right)^2 + \cdots$
$\dim = L^D$
$\dim L^{-D}$
$\dim L^{-\frac{D-2}{2}}$
$\dim \frac{1}{L}$
disc. derivative
∇
higher derivatives
$\sim \int d^D k \left[\chi_k \left(k^2 + b k^4 + \cdots\right) X_{-k}\right]$
irrelevant at small k ↔ large dist
massless
$k \rightsquigarrow \frac{1}{|x|^{D-2}}$
Irrelevant operators


for N ≥ 2
N-1 Goldstone modes
Broken symmetries
Ising model
XY model
exponential decay
if $m_{eff} \neq 0$
BORING / EMPTY
AT LARGE DISTANCES
$\Gamma^{N=2} = \int \mathcal{D}\theta_{\vec{n}} \exp\left(-\beta J \sum \cos(\theta_i - \theta_j)\right)$
$\int d^D x \left(\nabla X\right)^2 + b\left(\nabla^2 X\right)^2 + \cdots$
dim = L^D
dim L^{-D}
dim $L^{-\frac{D-2}{2}}$
dim $-D+2$
dim $\frac{1}{L}$
disc. derivative
higher derivatives
$\sim \int d^D k \left[\chi_k\left(k^2 + b k^4 + \cdots\right) X_{-k}\right]$
irrelevant at small k ↔ large dist
massless
Irrelevant operators
$k \rightsquigarrow \frac{1}{|x|^{D-2}}$


for N ≥ 2
N=2
Ising model
XY model
exponential decay
if $m_{eff} \neq 0$
BORING / EMPTY
AT LARGE DISTANCES
N-1 Goldstone modes
Broken symmetries
$\int \mathcal{D}\theta_{\vec{n}} \exp(-\beta J \sum \cos(\theta_i - \theta_j))$
$\int d^D x \left(\nabla X\right)^2$
$\dim = L^D$
$\dim L^{-D}$
$\dim L^{-\frac{D-2}{2}}$
$\dim \frac{1}{L}$
$\dim L^{-2}$
$+ b \left(\nabla^2 X\right)^2 + \cdots$
$\dim -D+2$
disc. derivative
∇
higher derivatives
$\sim \int d^D k \left[\chi_k (k^2 + b k^4 + \cdots) X_{-k}\right]$
irrelevant at small k ↔ large dist
massless
$\frac{1}{|x|^{D-2}}$
Irrelevant operators

AT LARGE DISTANCES #Broken Symmetries

$$\int d^D x \left(\nabla X\right)^2 + b\left(\nabla^2 X\right)^2 + \cdots$$

dim $= L^D$; dim L^{-D} ; dim $L^{-\frac{D-2}{2}}$; dim $-D-2$

dim $\frac{1}{L}$; dim L^{+2}

disc. derivative $\rightsquigarrow \nabla$

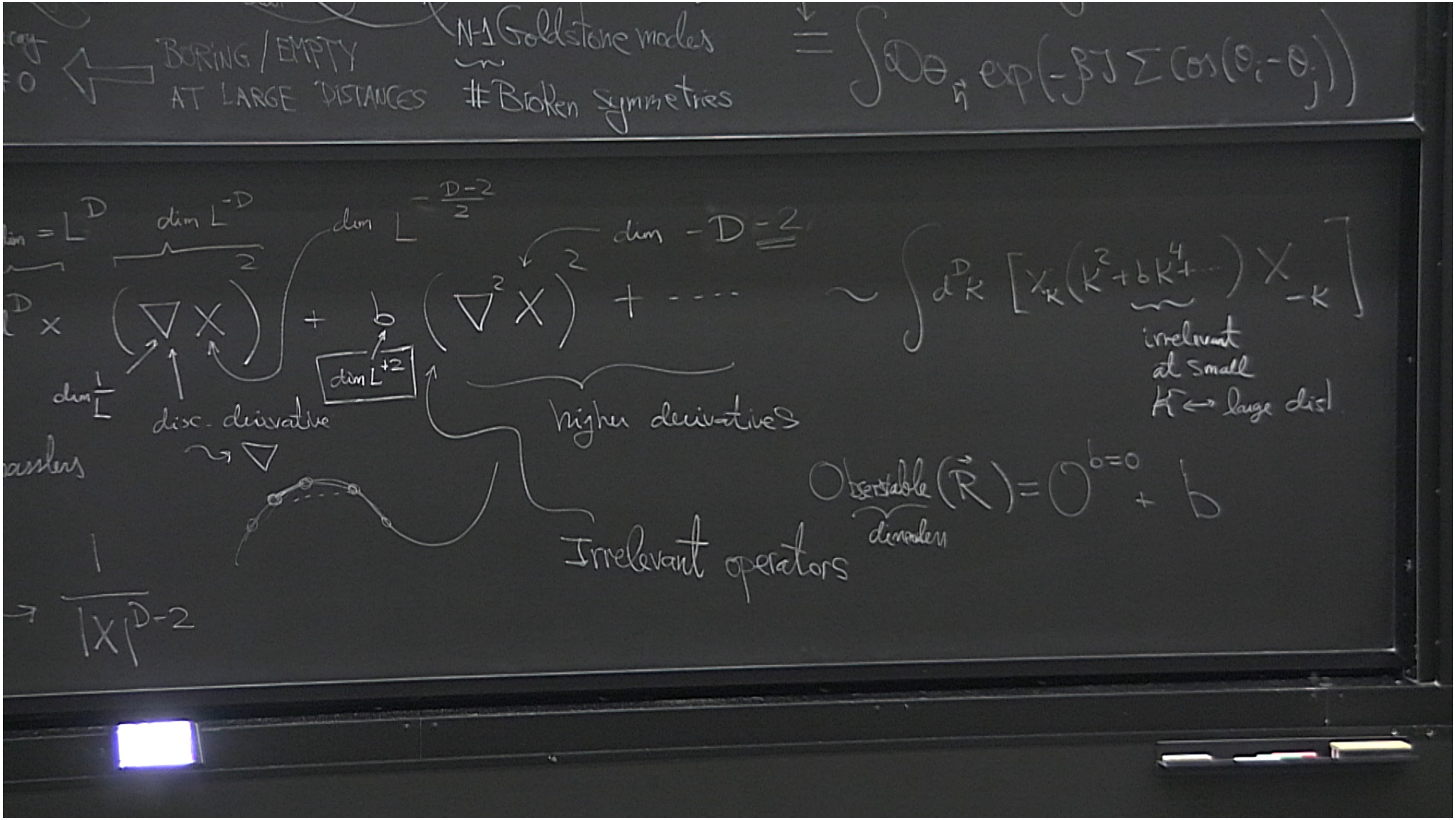
higher derivatives

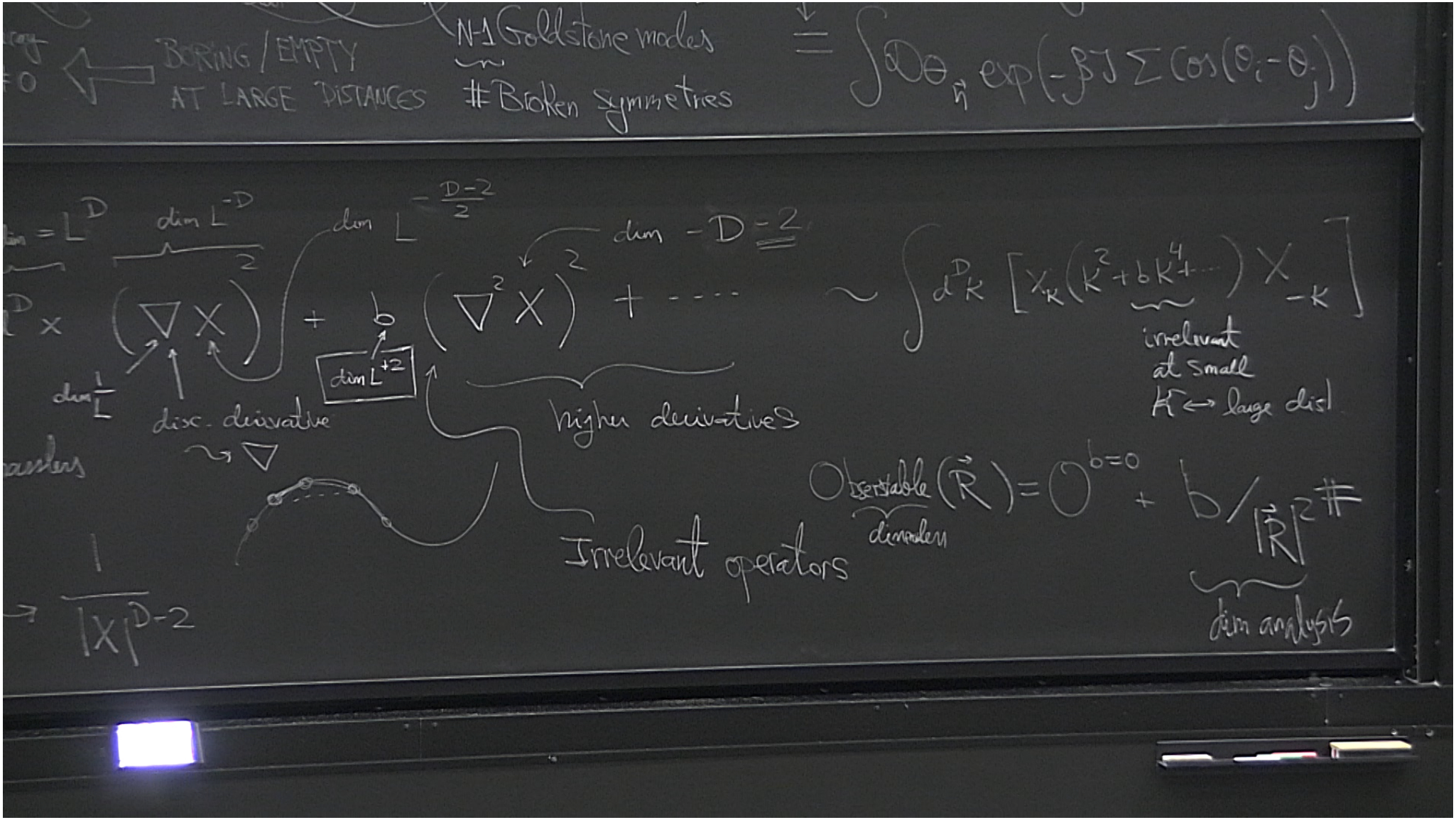
Irrelevant operators

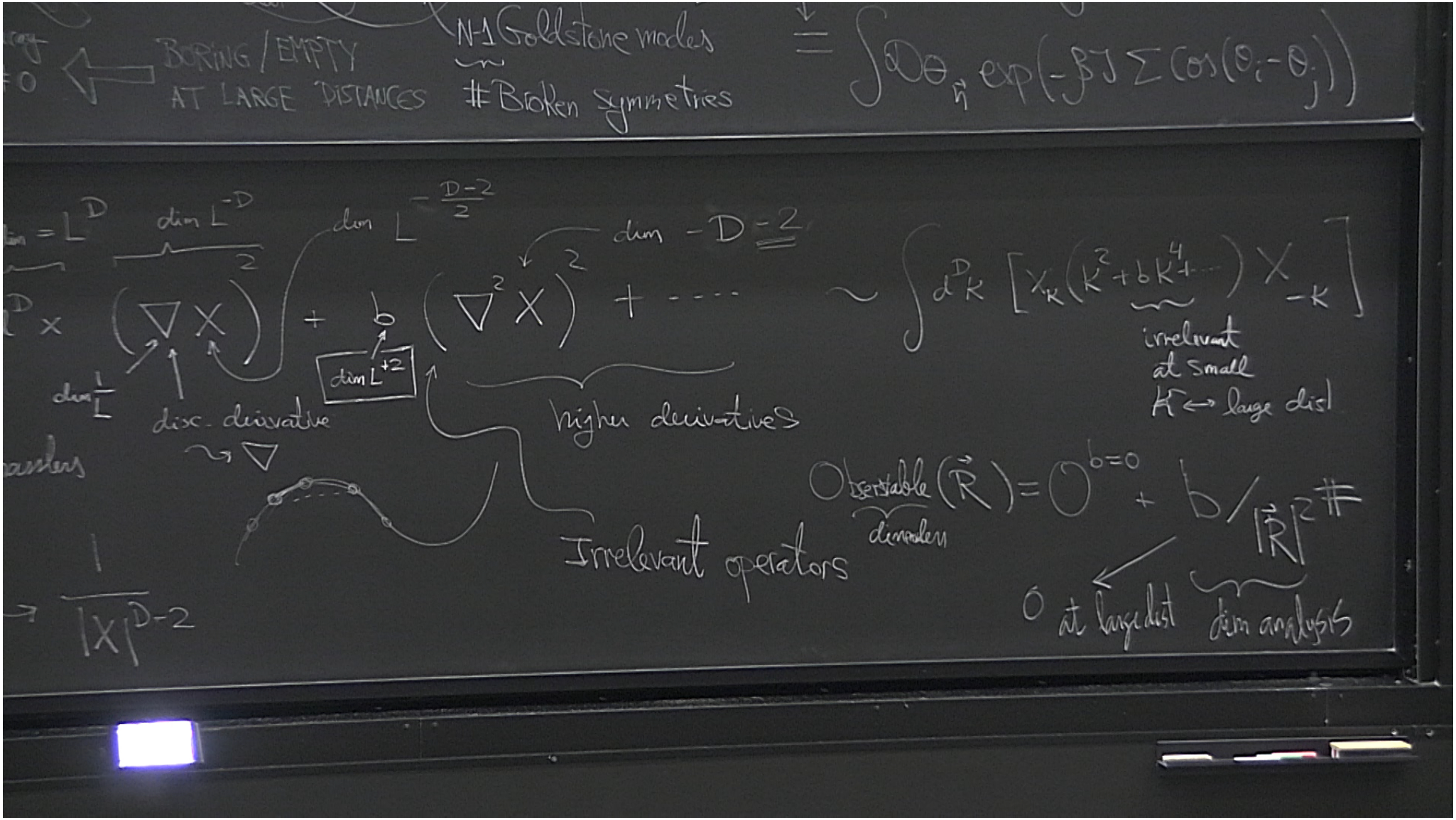
masses

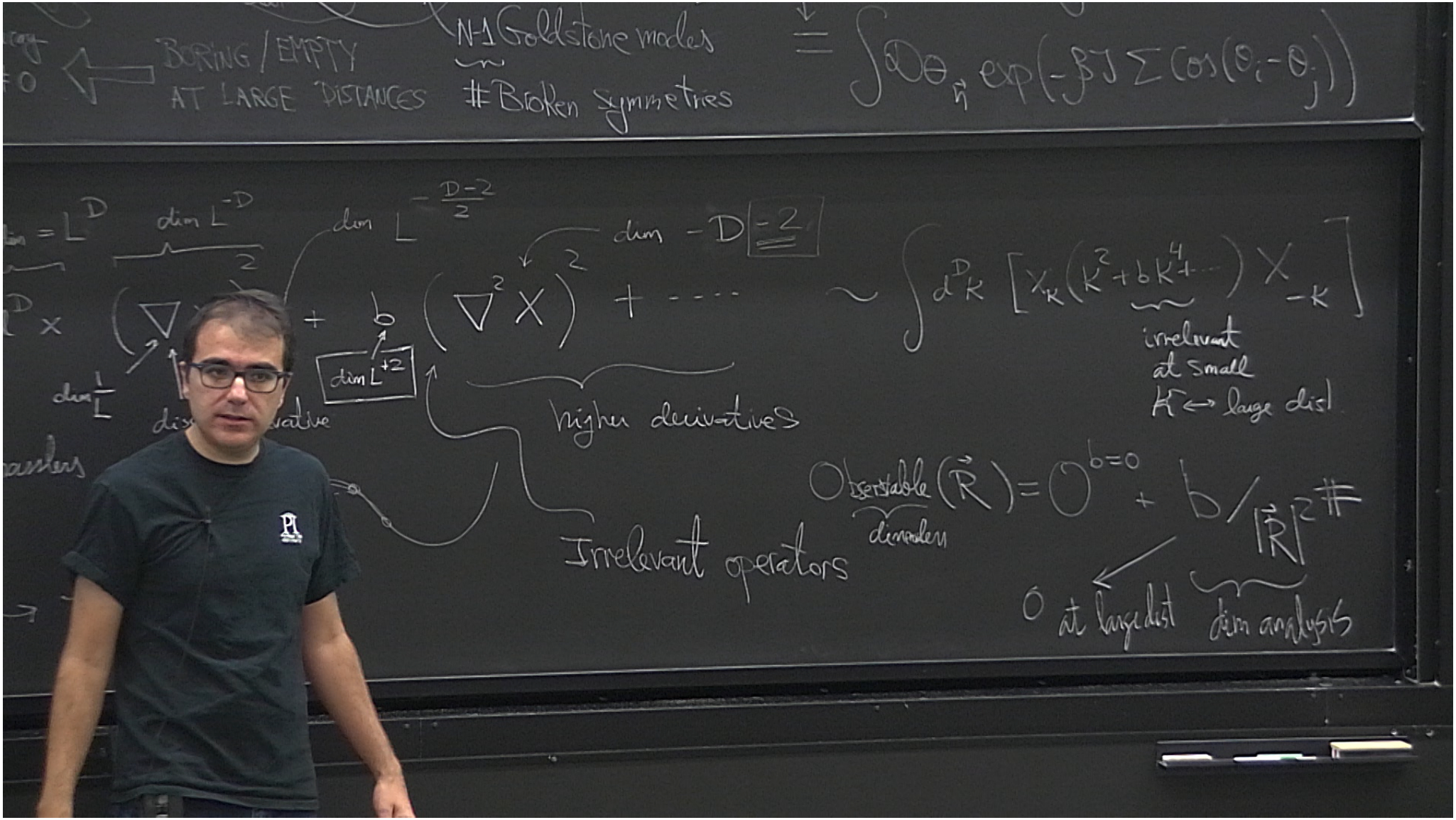
$$\sim \frac{1}{|x|^{D-2}}$$

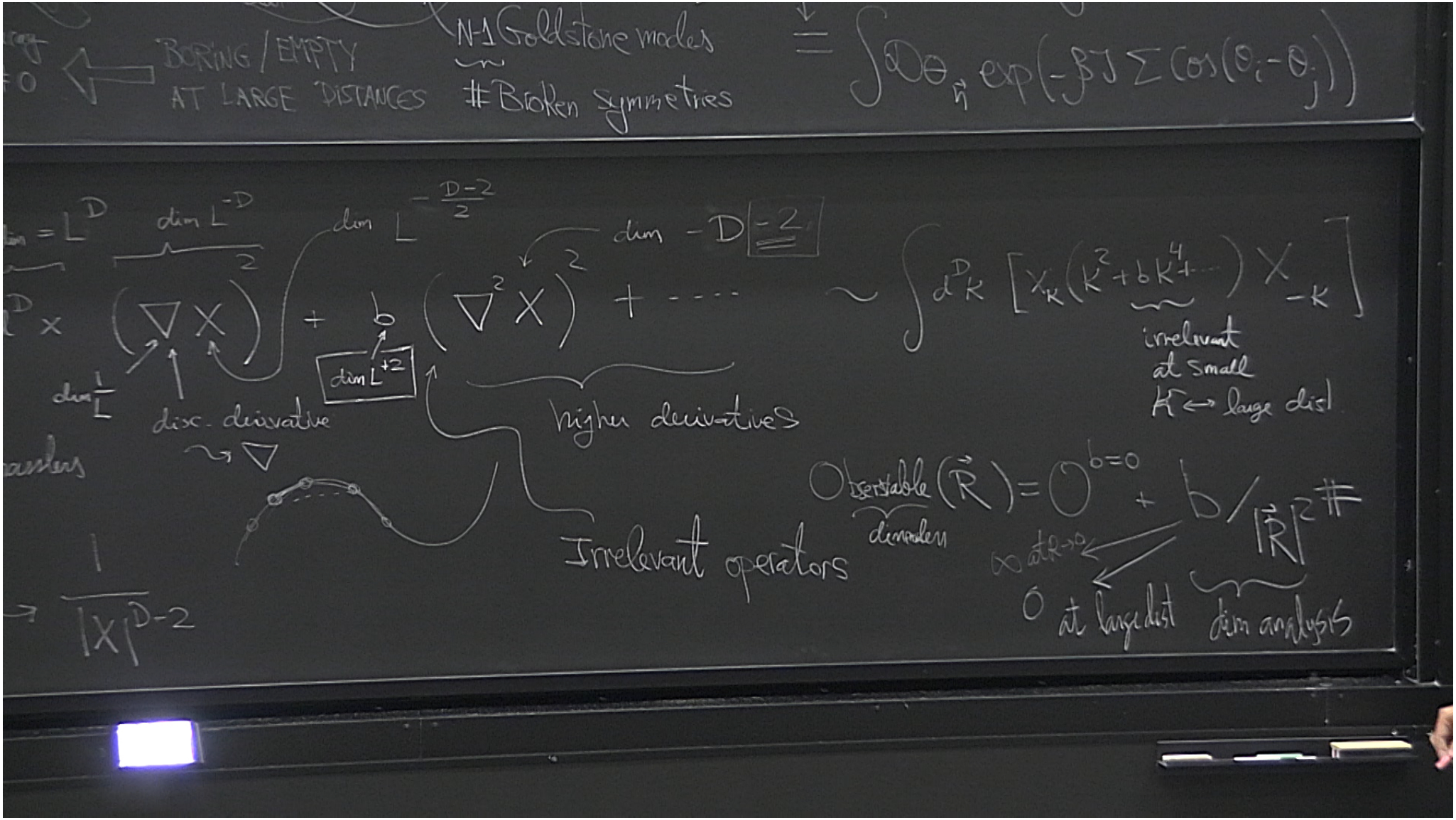
$$\sim \int d^D k \left[X_k (k^2 \right.$$

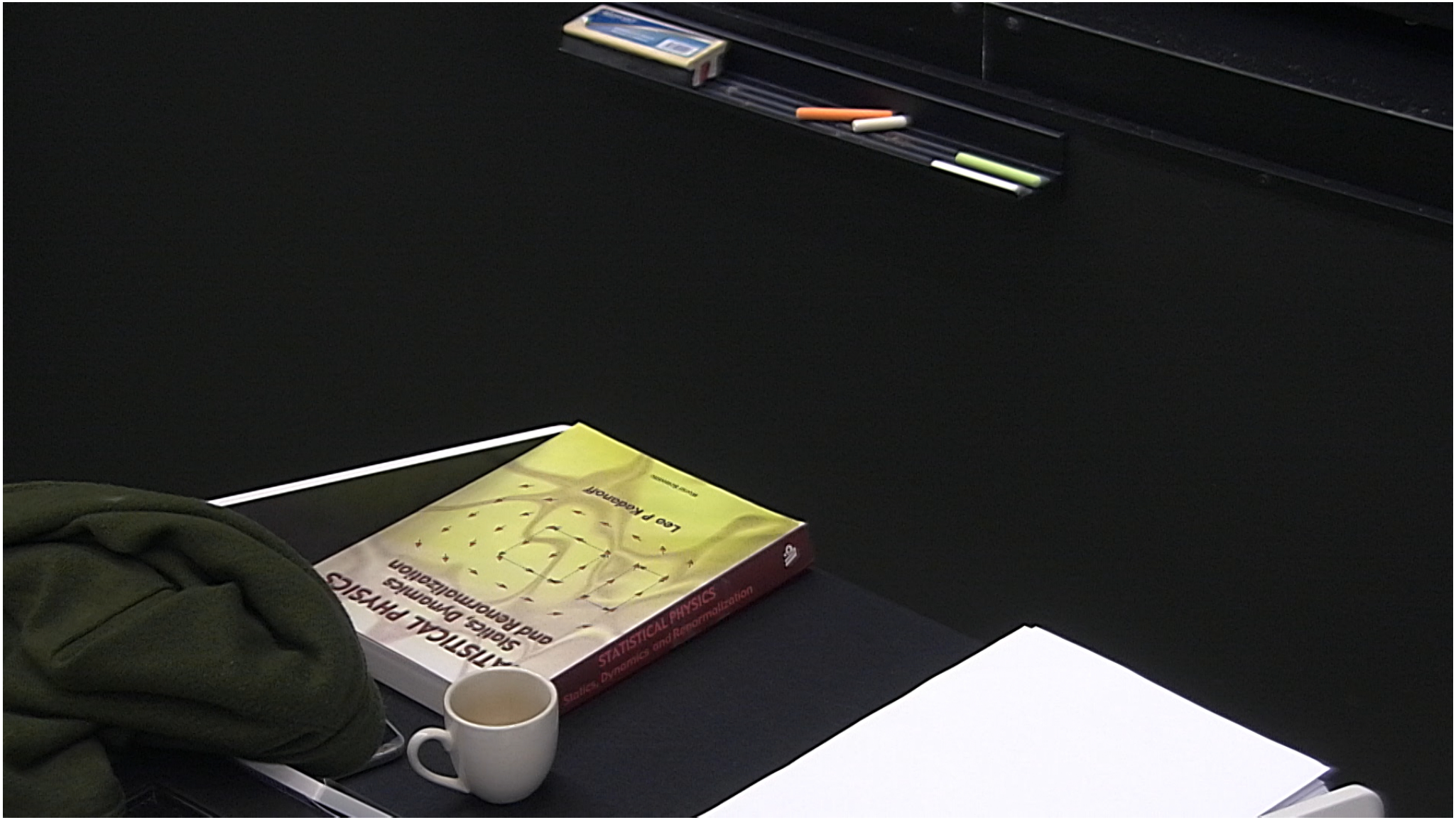

BORING / EMPTY
AT LARGE DISTANCES
N-1 Goldstone modes
Broken symmetries
dim L^{-D}
dim L^{-(D-2)/2}
dim -D-2
disc. derivative
higher derivatives
irrelevant at small K ↔ large dist
Irrelevant operators
dim L^{+2}
dim 1/L
dimension


BORING / EMPTY
AT LARGE DISTANCES
N-1 Goldstone modes
Broken symmetries
$= \int \mathcal{D}\theta_{\vec{n}} \exp\left(-\beta J \sum \cos(\theta_i - \theta_j)\right)$
$\dim = L^D$
$\dim L^{-D}$
$\dim L^{-\frac{D-2}{2}}$
$\dim -D-2$
$\left(\nabla X\right)^2$
$+ \; b \left(\nabla^2 X\right)^2 + \cdots$
$\dim L^{+2}$
$\dim \frac{1}{L}$
disc. derivative
∇
higher derivatives
$\sim \int d^D K \left[X_K \left(K^2 + b K^4 + \cdots\right) X_{-K} \right]$
irrelevant at small $K \leftrightarrow$ large dist
$\mathcal{O}_{\text{irrelevant}}(\vec{R}) = 0^{b=0} + b / |\vec{R}|^{\#}$
dimension
dim analysis
Irrelevant operators
$\frac{1}{|X|^{D-2}}$


BORING / EMPTY
AT LARGE DISTANCES
N-1 Goldstone modes
Broken symmetries
dim L^{-D}
dim L^{-(D-2)/2}
dim -D-2
dim 1/L
disc. derivative
dim L^{+2}
higher derivatives
irrelevant at small K ↔ large dist
Irrelevant operators
dimension
0 at large dist
dim analysis


BORING / EMPTY
AT LARGE DISTANCES
N-1 Goldstone modes
Broken symmetries
higher derivatives
irrelevant at small K ↔ large dist
Irrelevant operators
dim analysis
at large dist


BORING / EMPTY
AT LARGE DISTANCES
N-1 Goldstone modes
Broken symmetries
dim L^{-D}
dim L^{-(D-2)/2}
dim -D-2
dim 1/L
disc. derivative
dim L^{+2}
higher derivatives
irrelevant
at small
K ↔ large dist
Irrelevant operators
dimension
∞ at R→0
0 at large dist
dim analysis


STATISTICAL PHYSICS
Statics, Dynamics and Renormalization
Leo P Kadanoff

Real Space Renormalization for Vortex gas

$$Z = \sum_{n}$$

$n+1$ vortices
$n-1$ vortices

Real Space Renormalization for Vortex gas

$$Z = \sum_{n} \frac{K^{2n}}{(n!)^2} \int \prod_{j=1}^{2n} d\vec{r}_j \exp\Big(\beta J \sum_{i \neq j} n$$

$|\vec{r}_i - \vec{r}_j| > a$

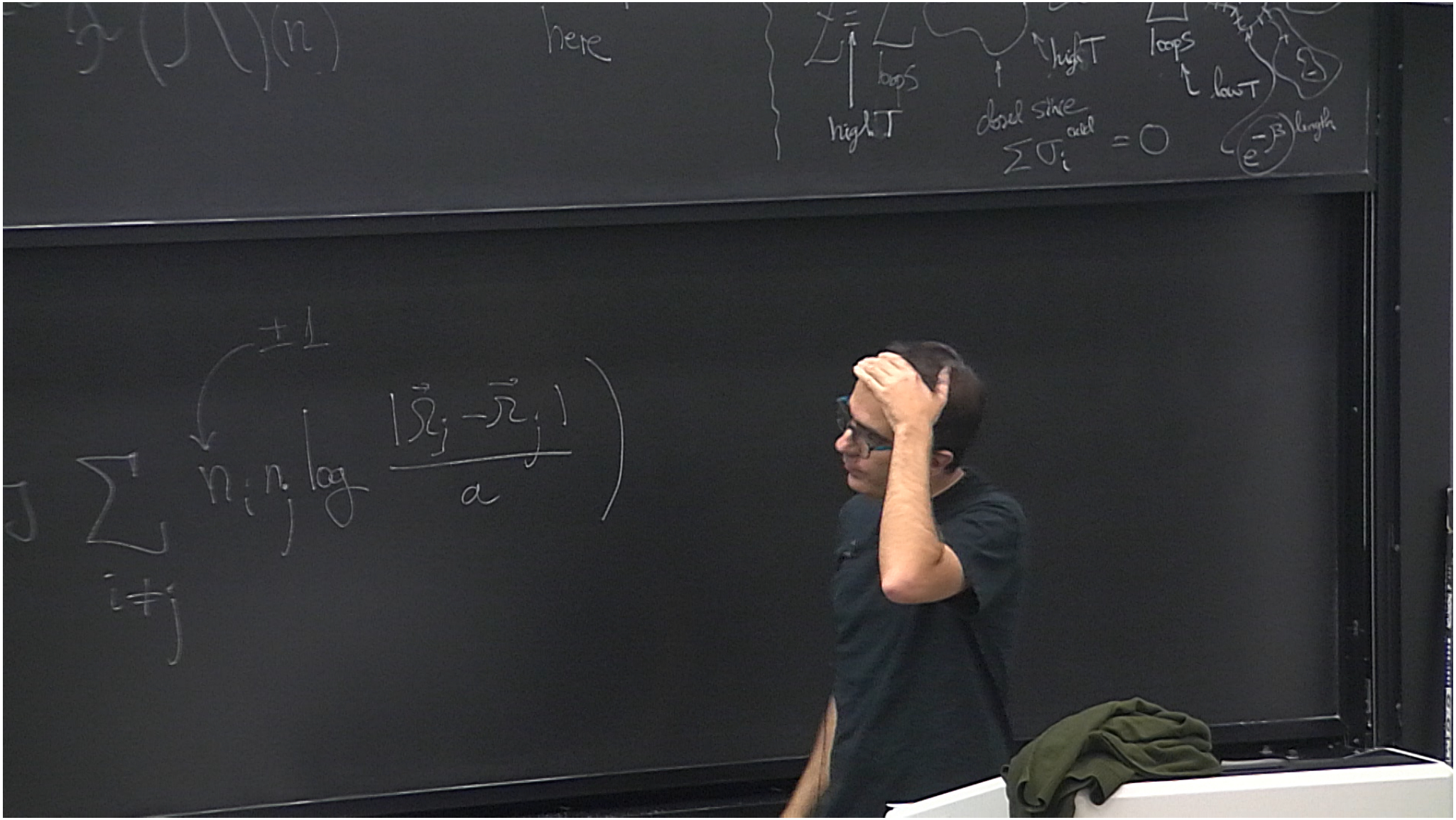
$n+1$ vortices
$n-1$ vortices

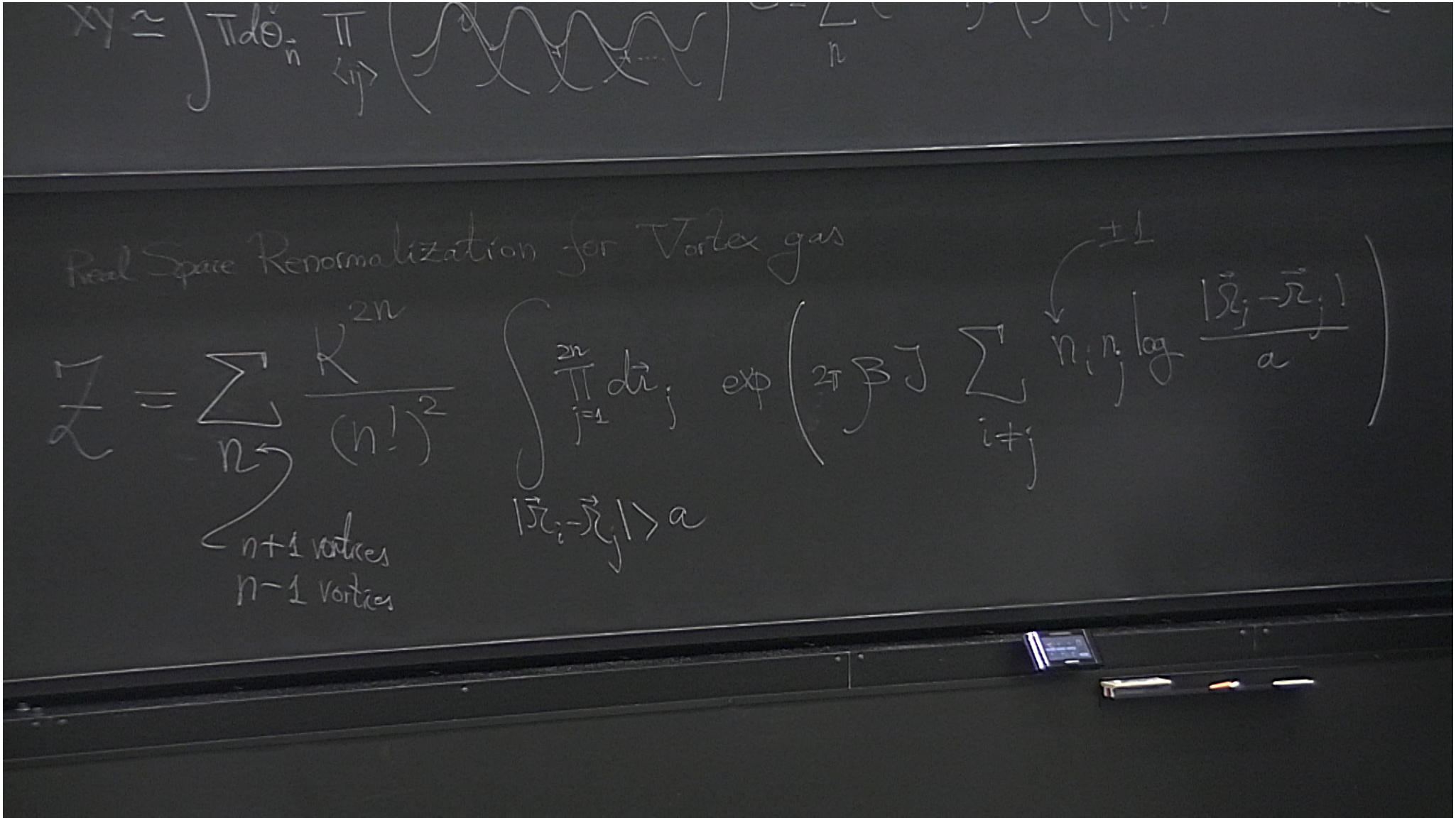
Real Space Renormalization for Vortex gas

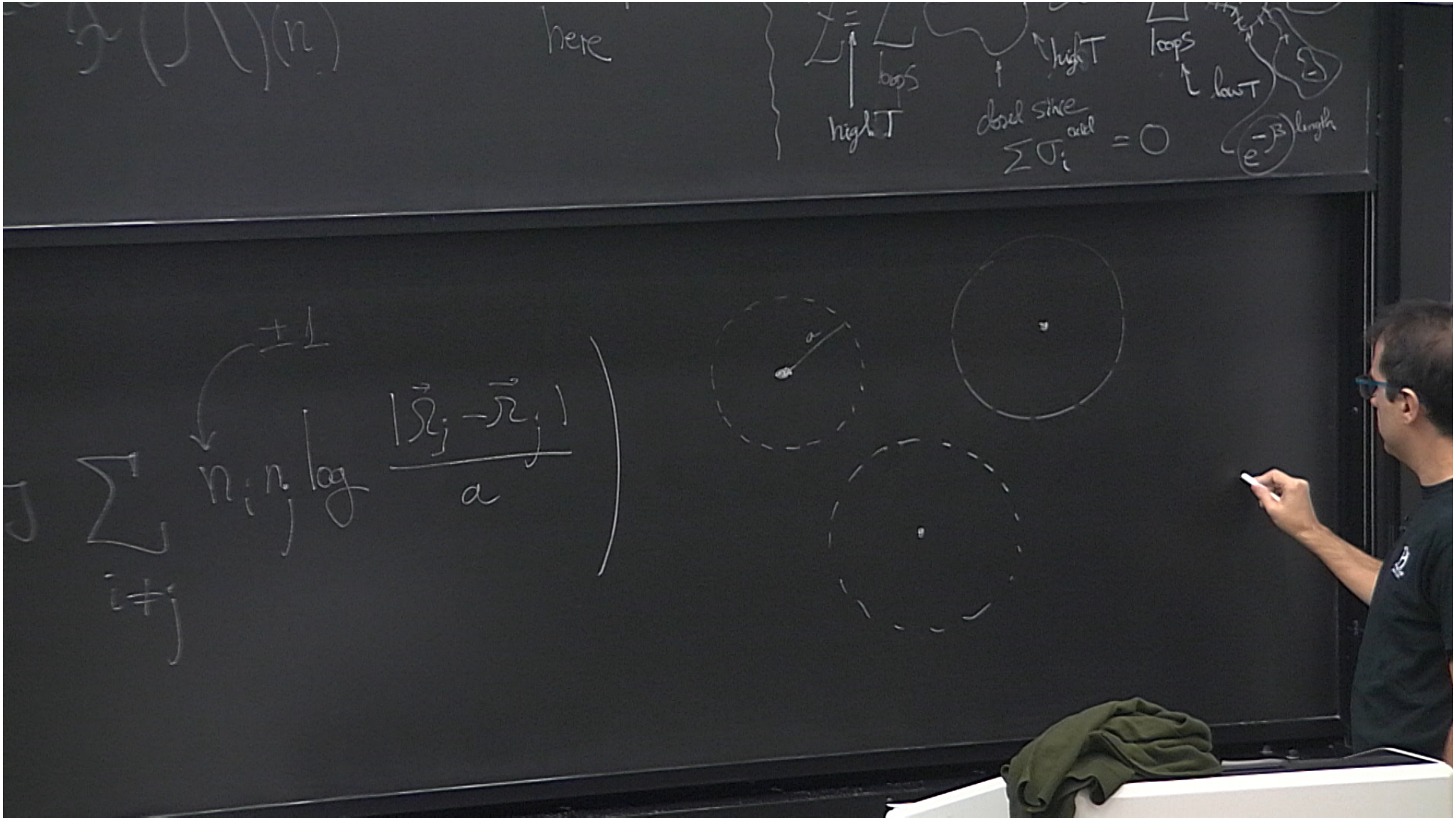
$$Z = \sum_{n} \frac{K^{2n}}{(n!)^2} \int \prod_{j=1}^{2n} d\vec{r}_j \exp\left(2\pi\beta J \sum_{i\neq j}' n_i n_j \right.$$

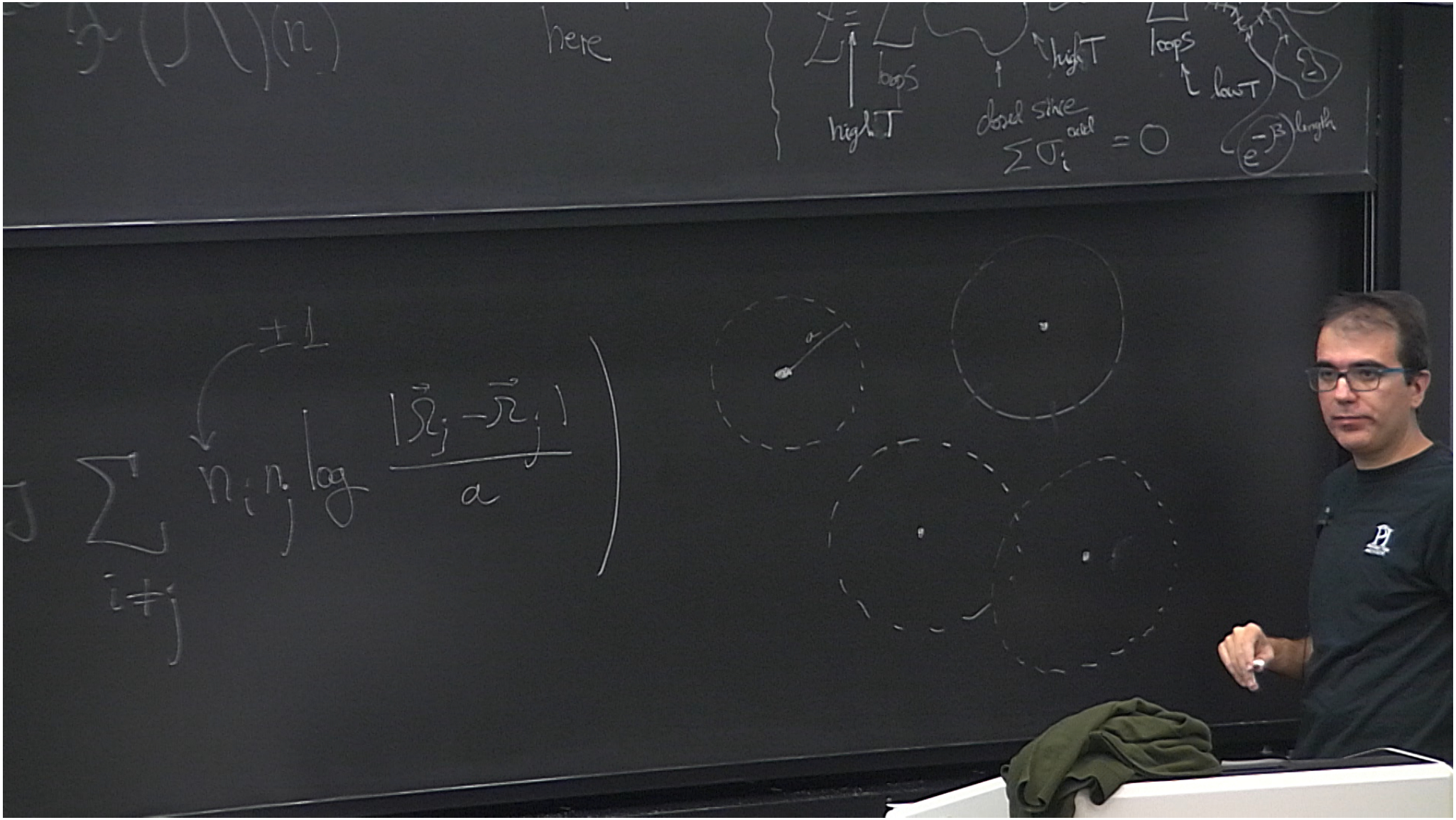
$|\vec{r}_i - \vec{r}_j| > a$

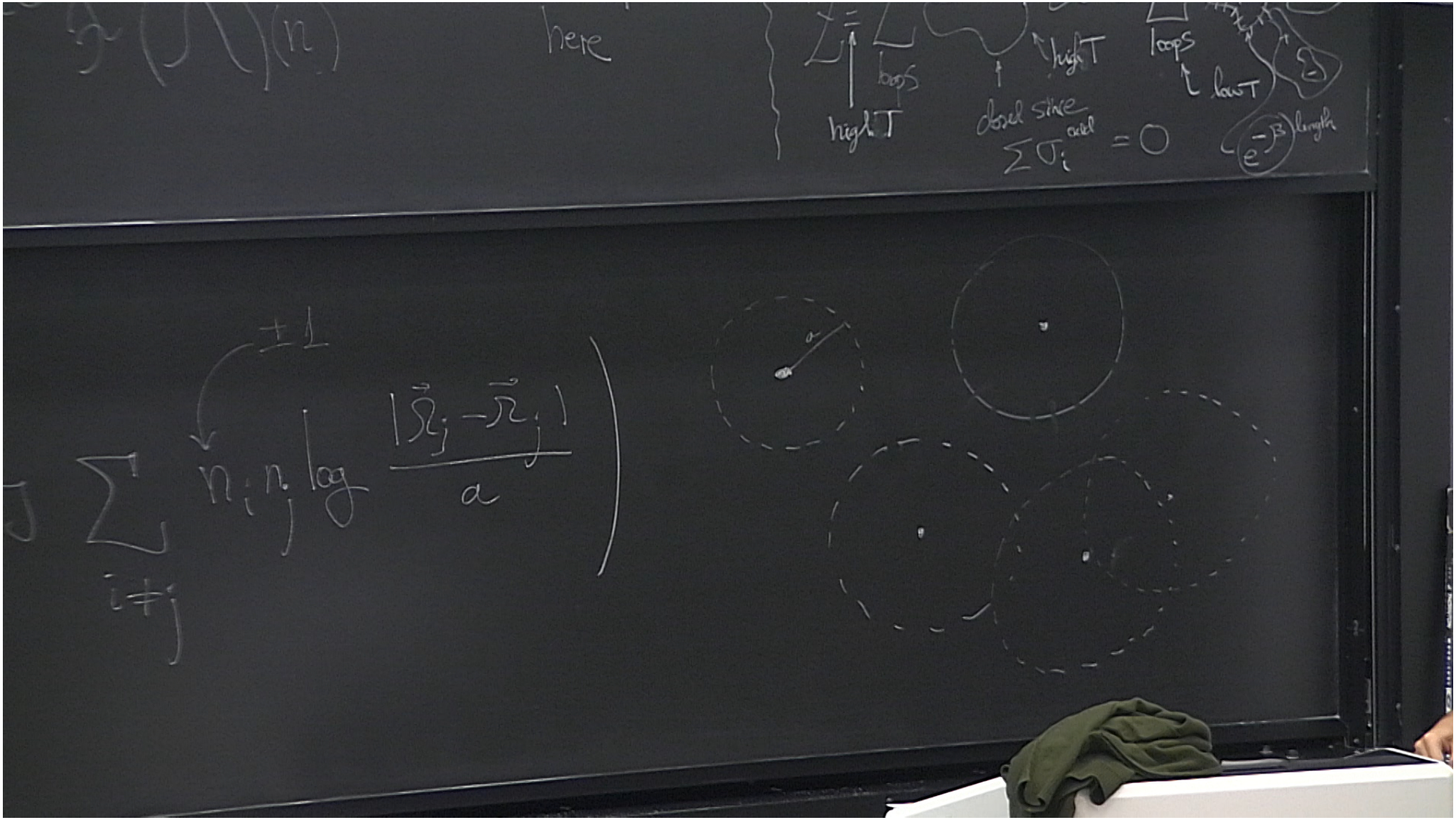
$n+1$ vortices
$n-1$ vortices

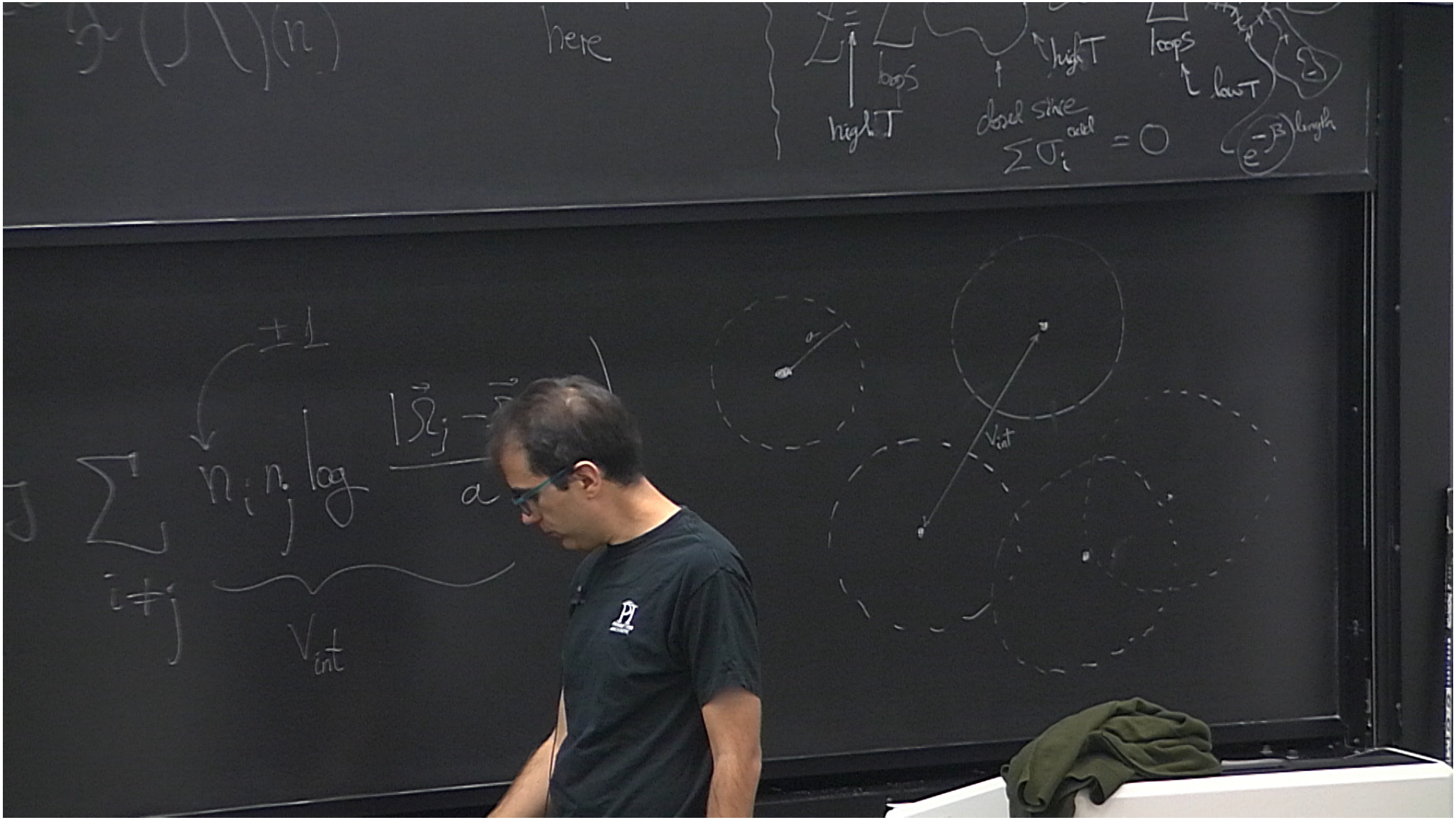

here
loops
high T
loops
high T
low T
closed stine
$\sum \sigma_i^{\text{axial}} = 0$
length
± 1
$\sum_{i \neq j} n_i n_j \log \frac{|\vec{x}_i - \vec{x}_j|}{a}$

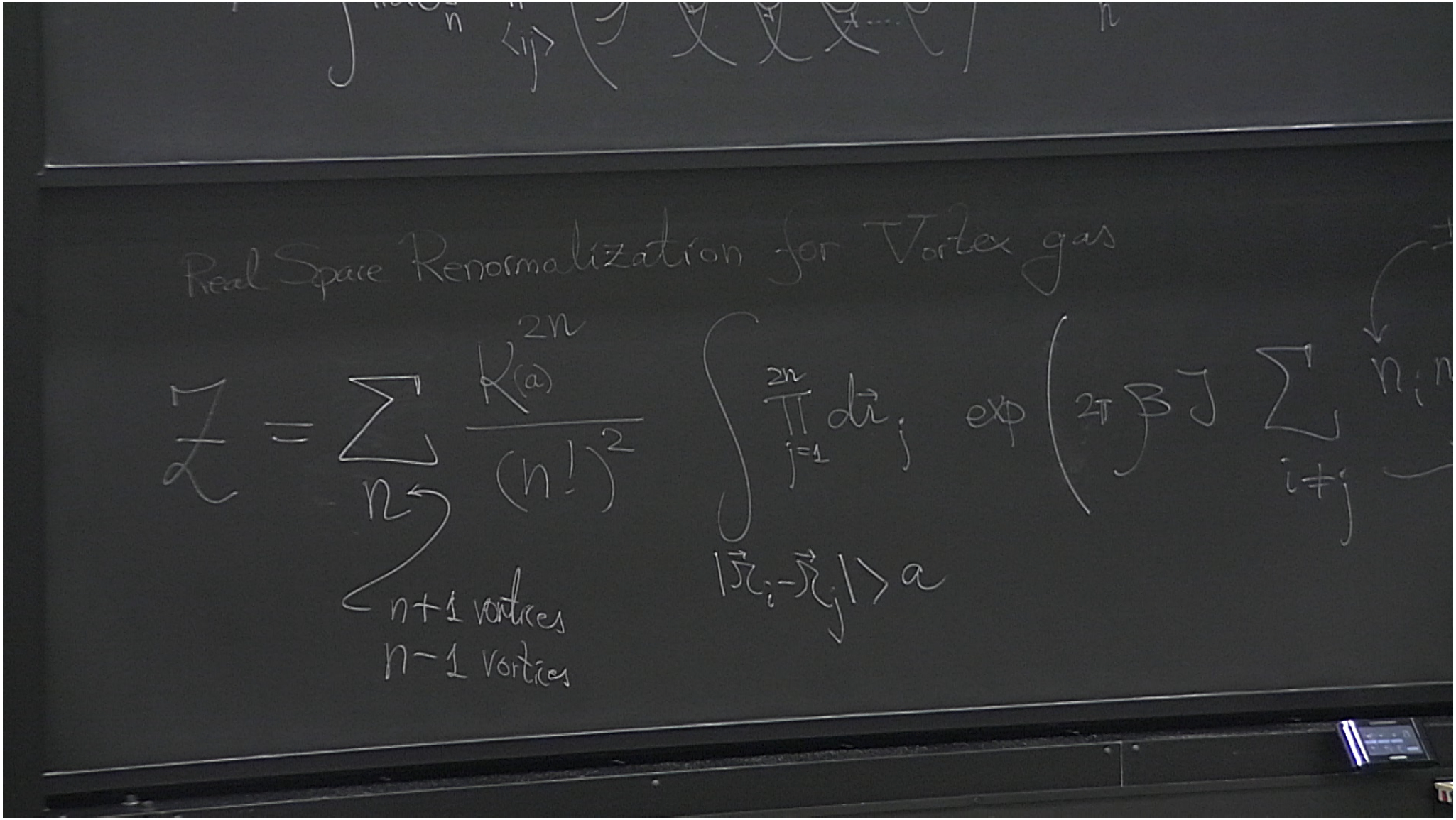

XY ≃ ∫ Π dθ_n Π_⟨ij⟩
Real Space Renormalization for Vortex gas
Z = Σ_n K^{2n} / (n!)^2 ∫ Π_{j=1}^{2n} dr_j exp(2π β J Σ_{i≠j} n_i n_j log |r_i − r_j| / a)
|r_i − r_j| > a
n_i = ±1
n+1 vortices
n−1 vortices

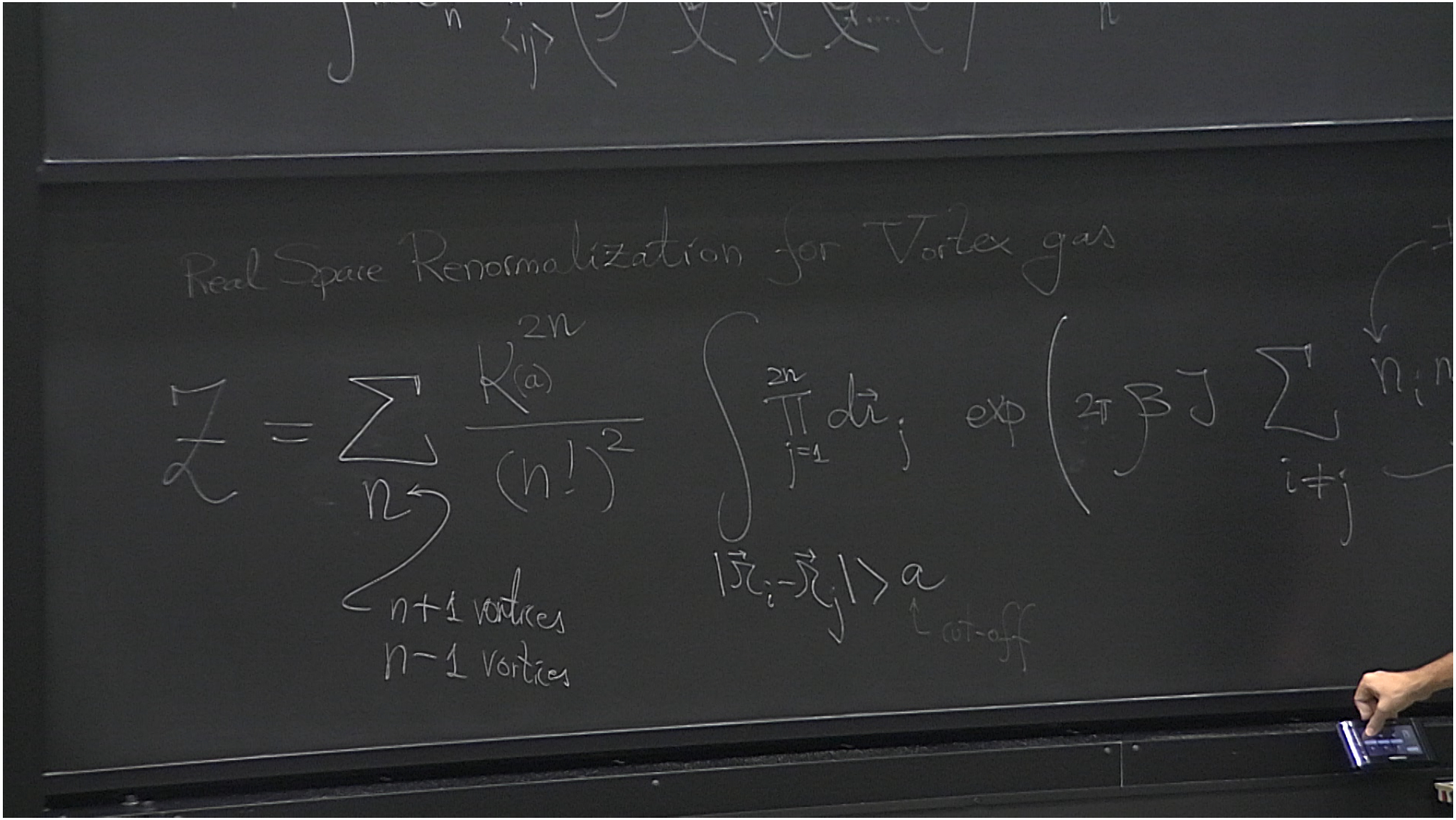

here
high T
loops
high T
loops
low T
high T
closed strike
$\sum \sigma_i^{axial} = 0$
$e^{-\beta}$ length
± 1
$\sum_{i \neq j} n_i n_j \log \frac{|\vec{R}_i - \vec{R}_j|}{a}$
a

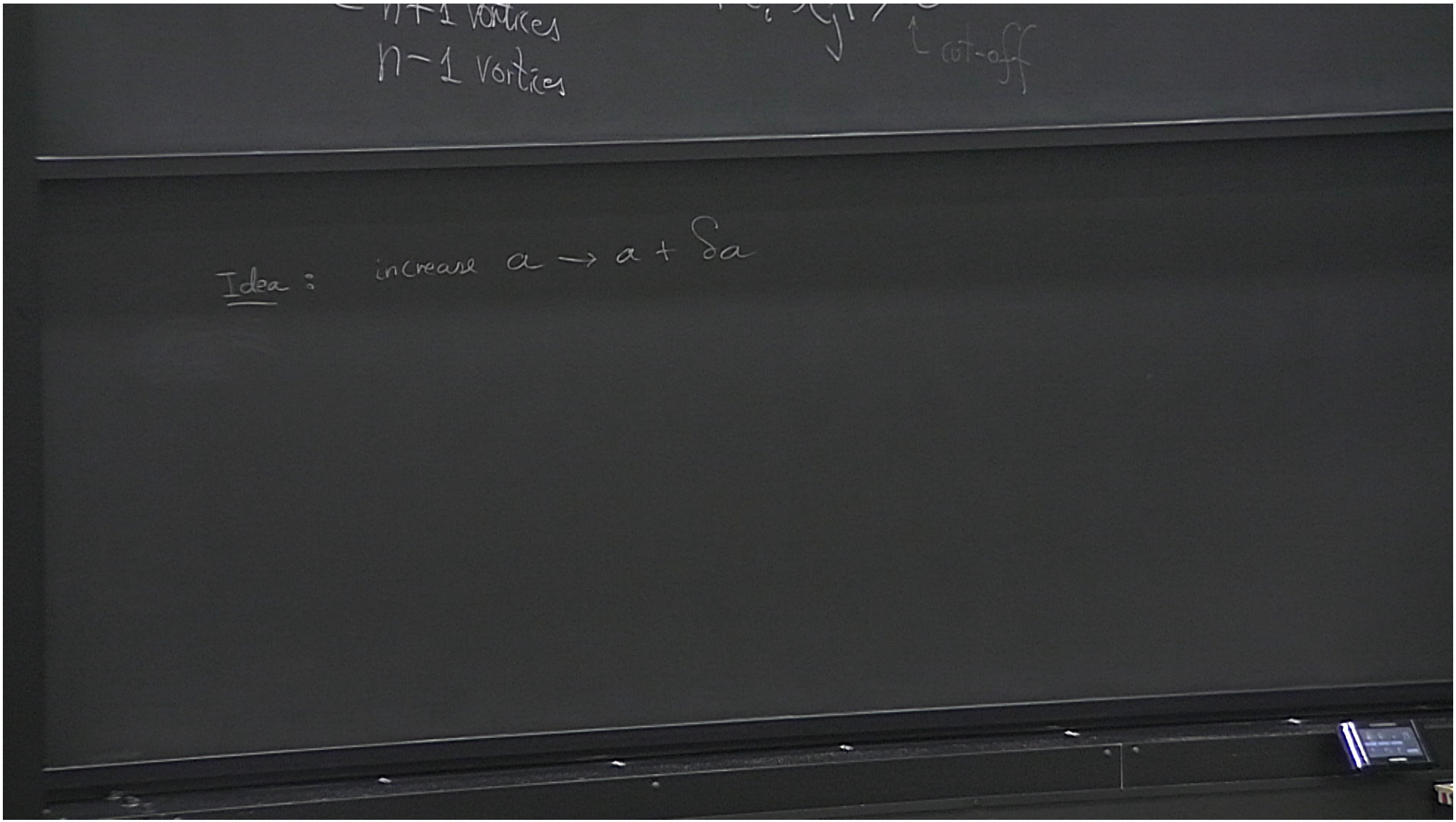

here
loops
high T
loops
high T
low T
closed strike
$\sum \sigma_i^{axial} = 0$
length
± 1
$\sum_{i \neq j} n_i n_j \log \frac{|\vec{R}_j - \vec{R}_i|}{a}$
a


here
loops
high T
loops
high T
low T
closed strike
$\sum \sigma_i^{axial} = 0$
$e^{-\beta}$ length
± 1
$\sum_{i \neq j} n_i n_j \log \frac{|\vec{x}_i - \vec{x}_j|}{a}$
a


here
high T
loops
high T
loops
low T
closed strike
$\sum \sigma_i^{axial} = 0$
$e^{-\beta}$ length
± 1
$n_i n_j \log$
a
V_{int}
$i \neq j$
V_{int}


Real Space Renormalization for Vortex gas
$Z = \sum_n \frac{K(a)^{2n}}{(n!)^2} \int \prod_{j=1}^{2n} d\vec{r}_j \exp\left(2\pi\beta J \sum_{i\neq j} n_i n_j \right.$
$|\vec{r}_i - \vec{r}_j| > a$
n+1 vortices
n-1 vortices


Real Space Renormalization for Vortex gas
$Z = \sum_n \frac{K(a)^{2n}}{(n!)^2} \int \prod_{j=1}^{2n} d\vec{r}_j \exp\left(2\pi\beta J \sum_{i\neq j}' n_i n_j \ln\right)$
$|\vec{r}_i - \vec{r}_j| > a$
cut-off
n+1 vortices
n-1 vortices

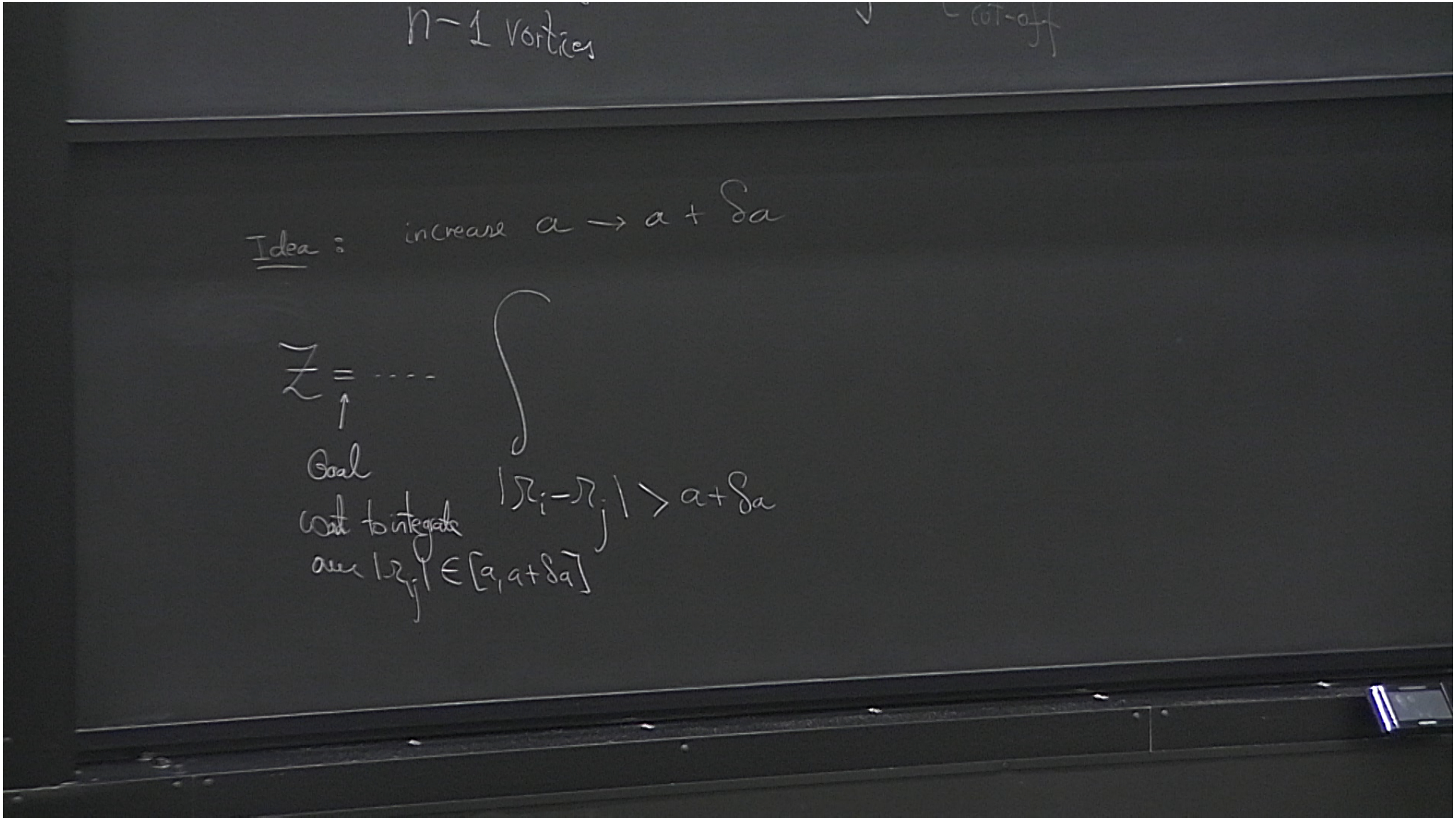

n+1 vortices
n-1 vortices
cut-off
Idea : increase $a \to a + \delta a$

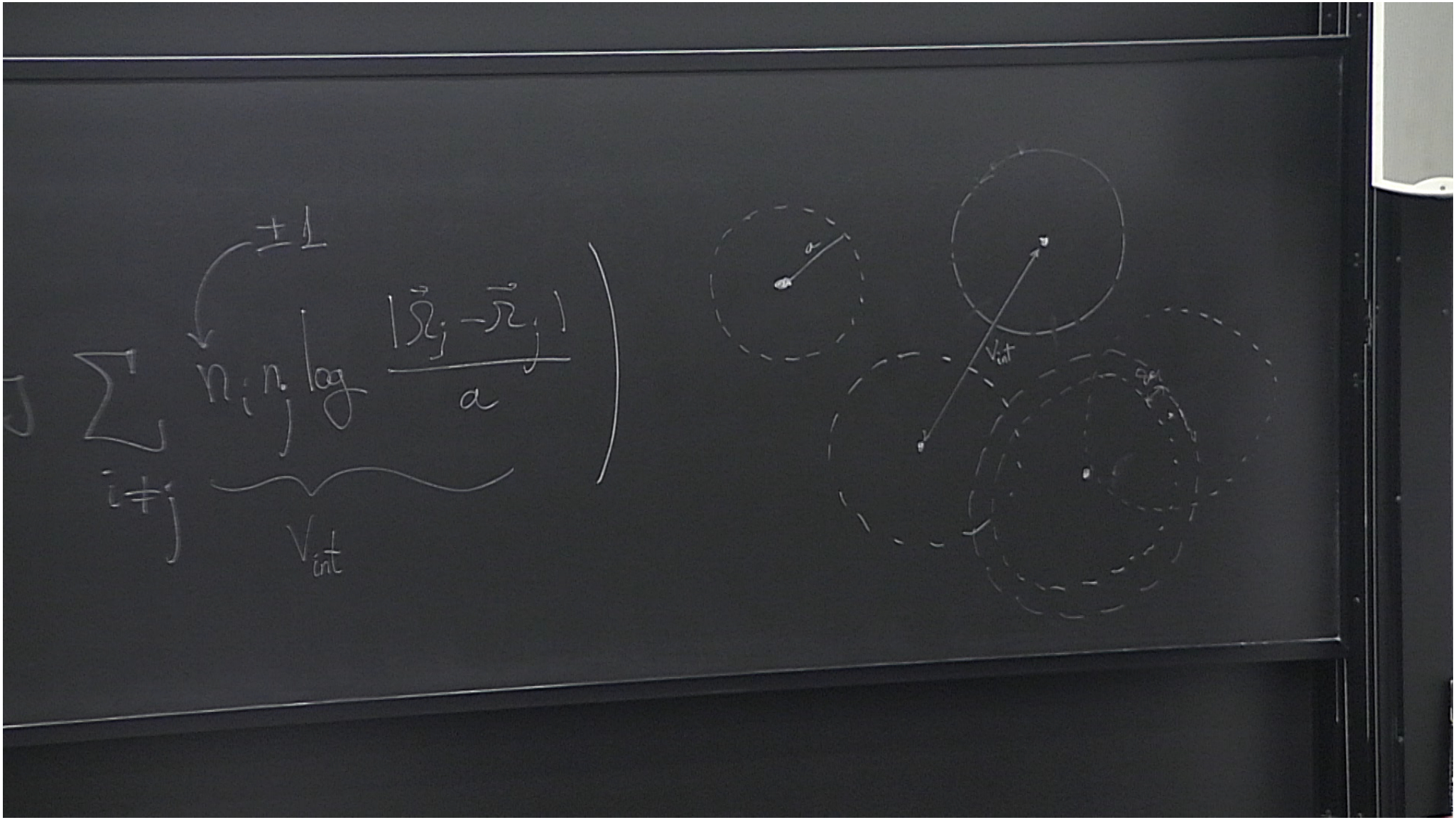
$n-1$ vortices

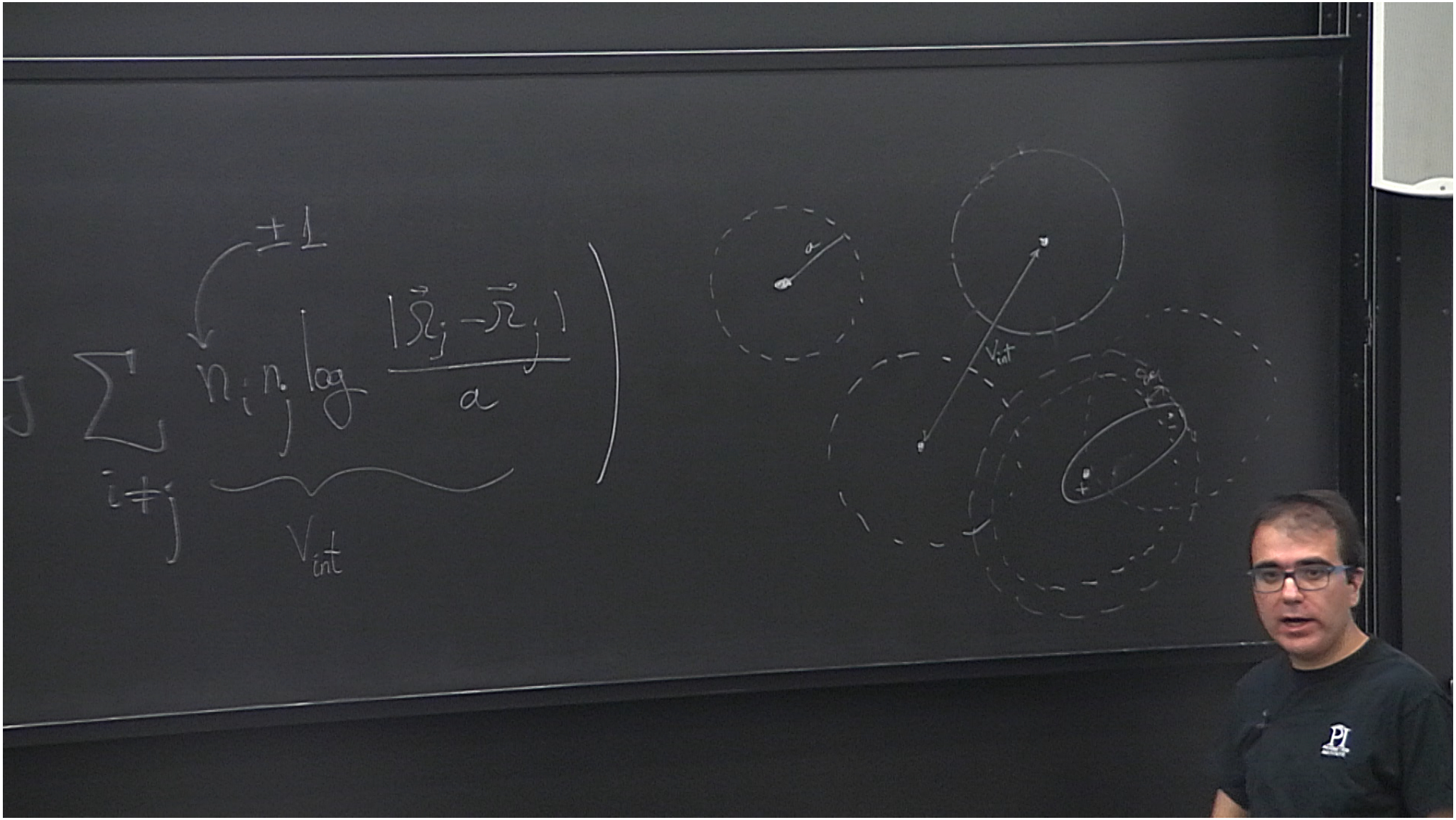
cut-off

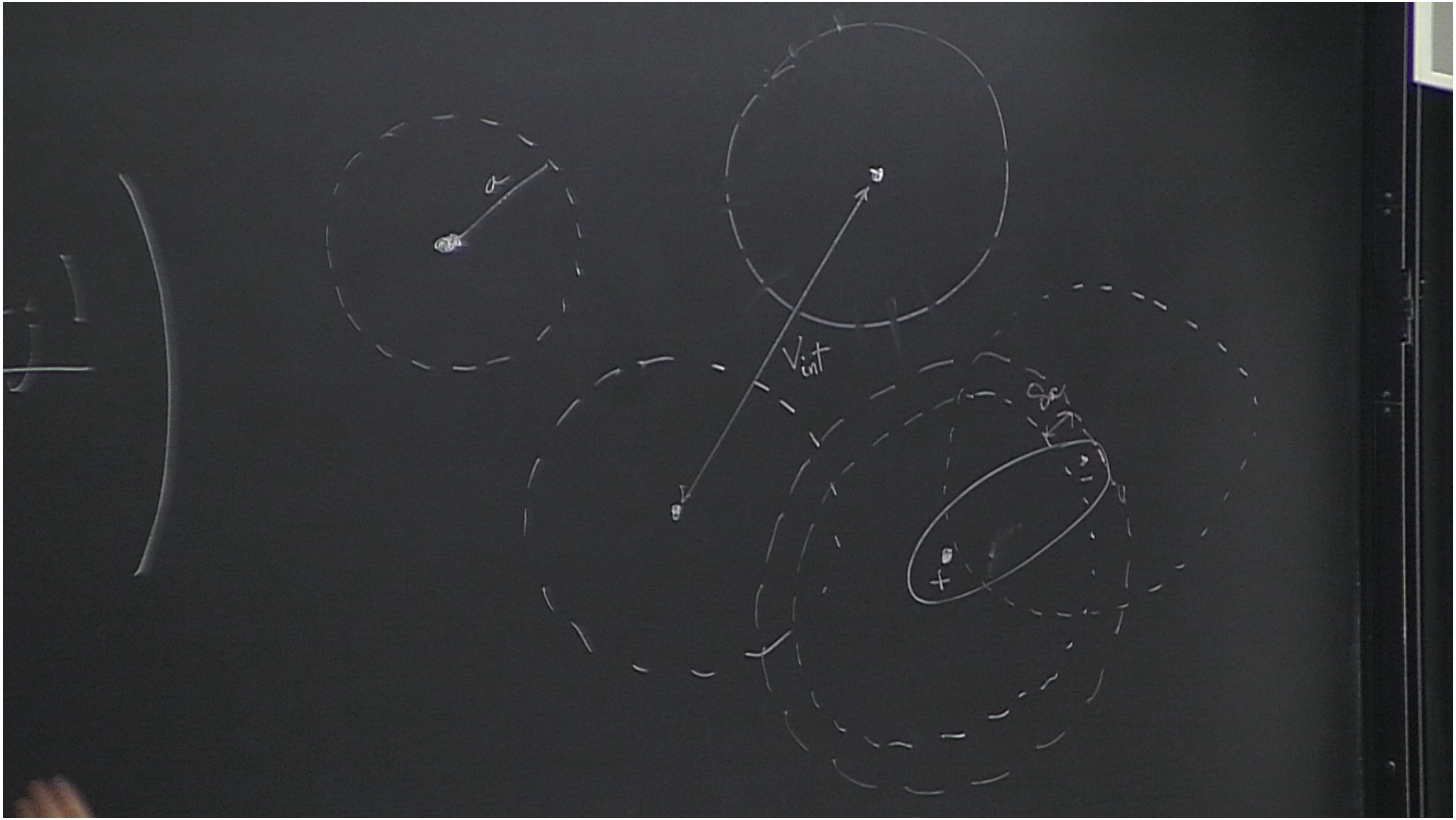
Idea: increase $a \to a + \delta a$

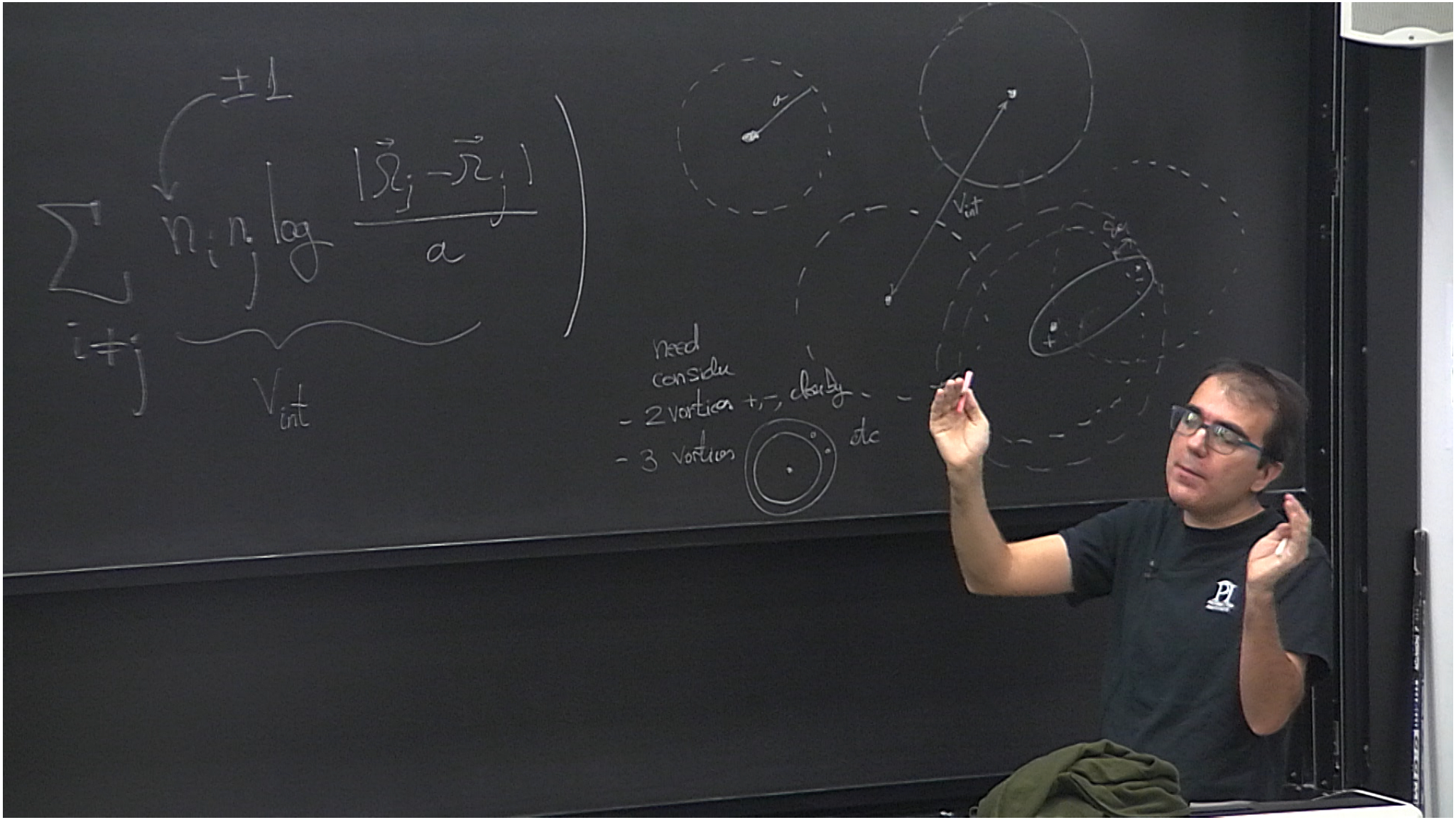
$$Z = \cdots \int_{|\vec{r}_i - \vec{r}_j| > a+\delta a}$$

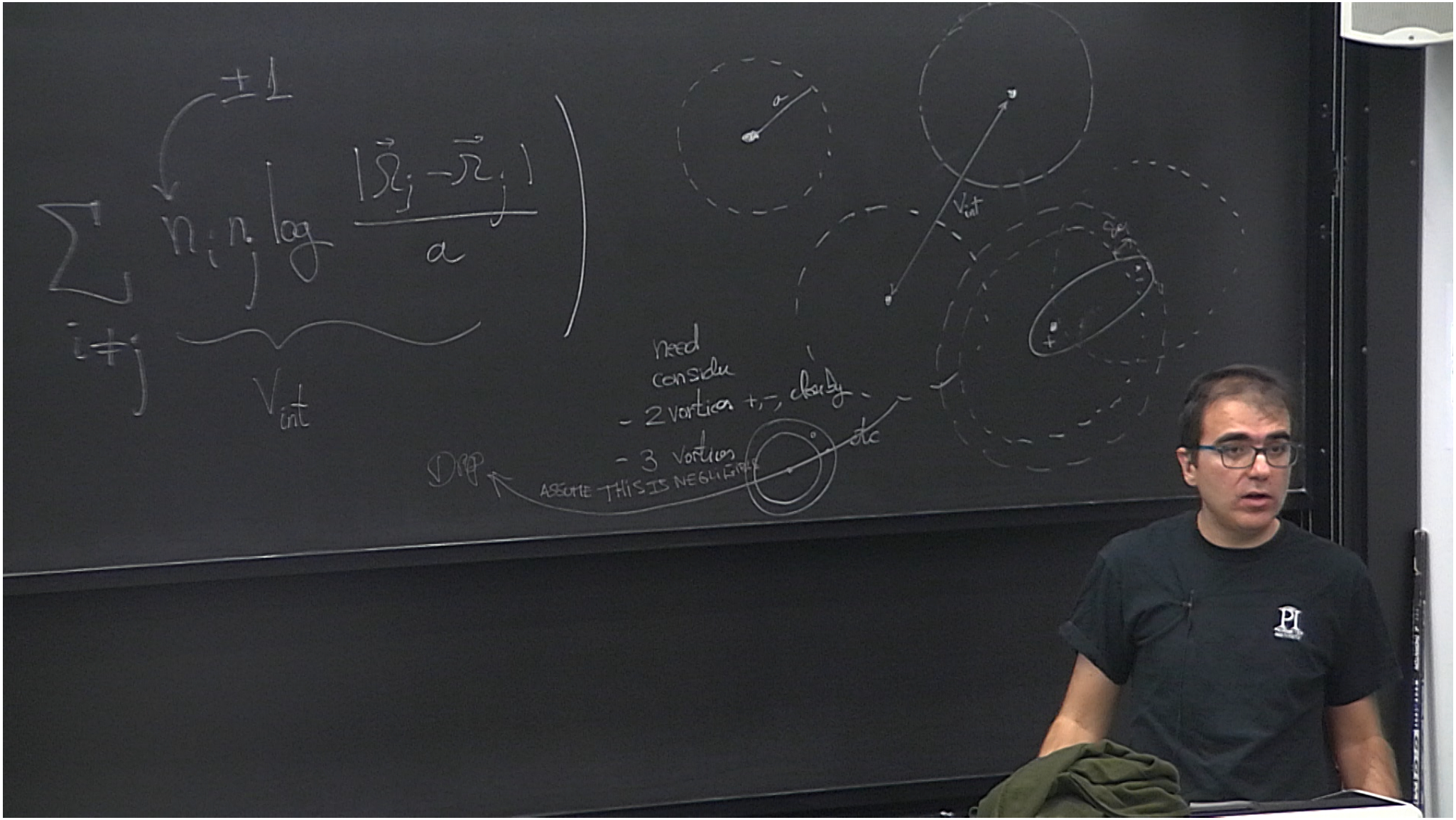

n−1 vertices
Idea : increase $a \to a + \delta a$
$Z = \cdots \int$
$|\vec{r}_i - \vec{r}_j| > a + \delta a$
Goal
Cost to integrate
over $|\vec{r}_{ij}| \in [a, a+\delta a]$


± 1
$\sum_{i \neq j} n_i n_j \log \frac{|\vec{R}_j - \vec{r}_j|}{a}$
V_{int}
a
V_{int}

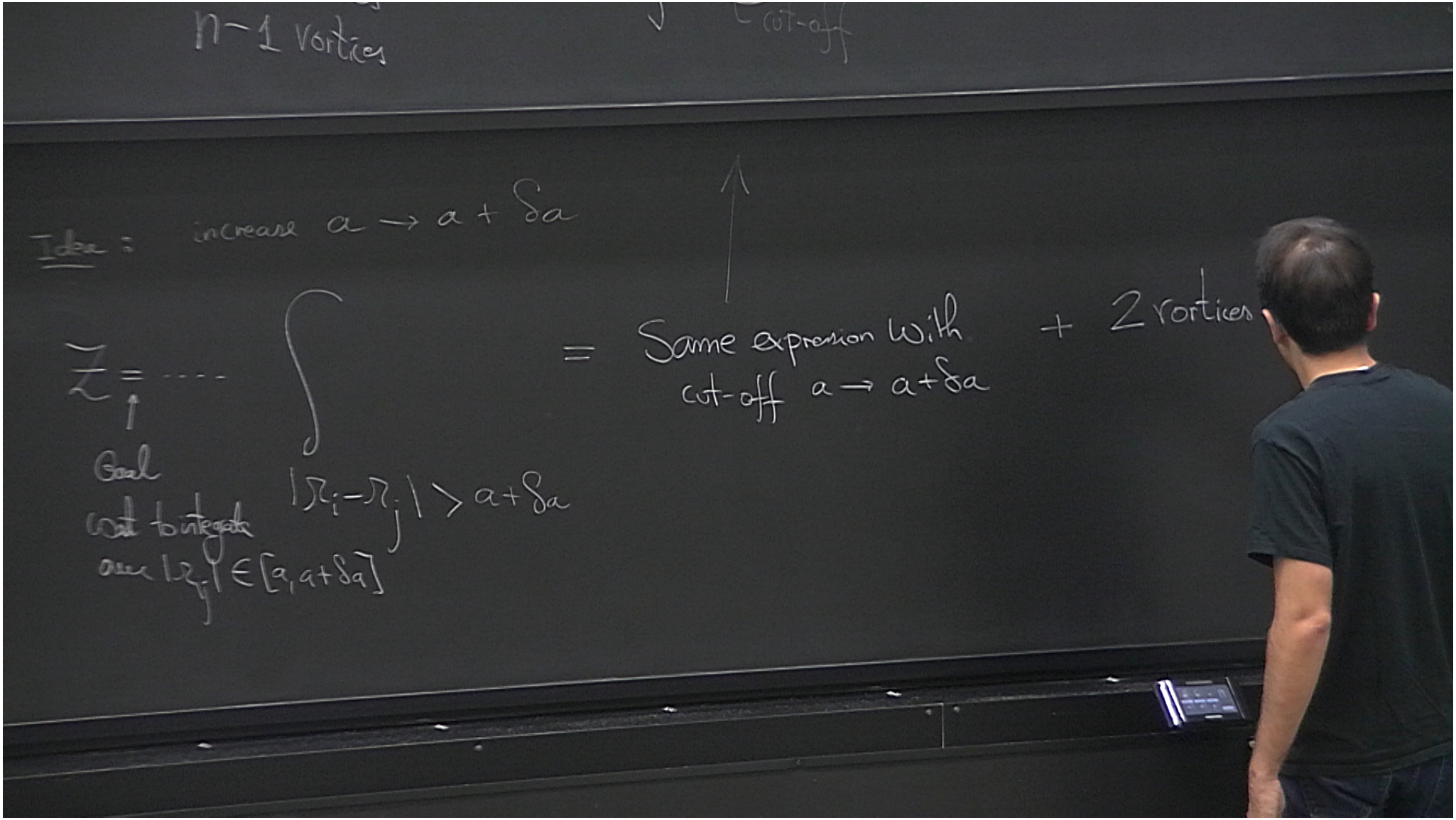

± 1
$\sum_{i \neq j} n_i n_j \log \frac{|\vec{x}_j - \vec{x}_i|}{a}$
V_{int}
a
V_{int}

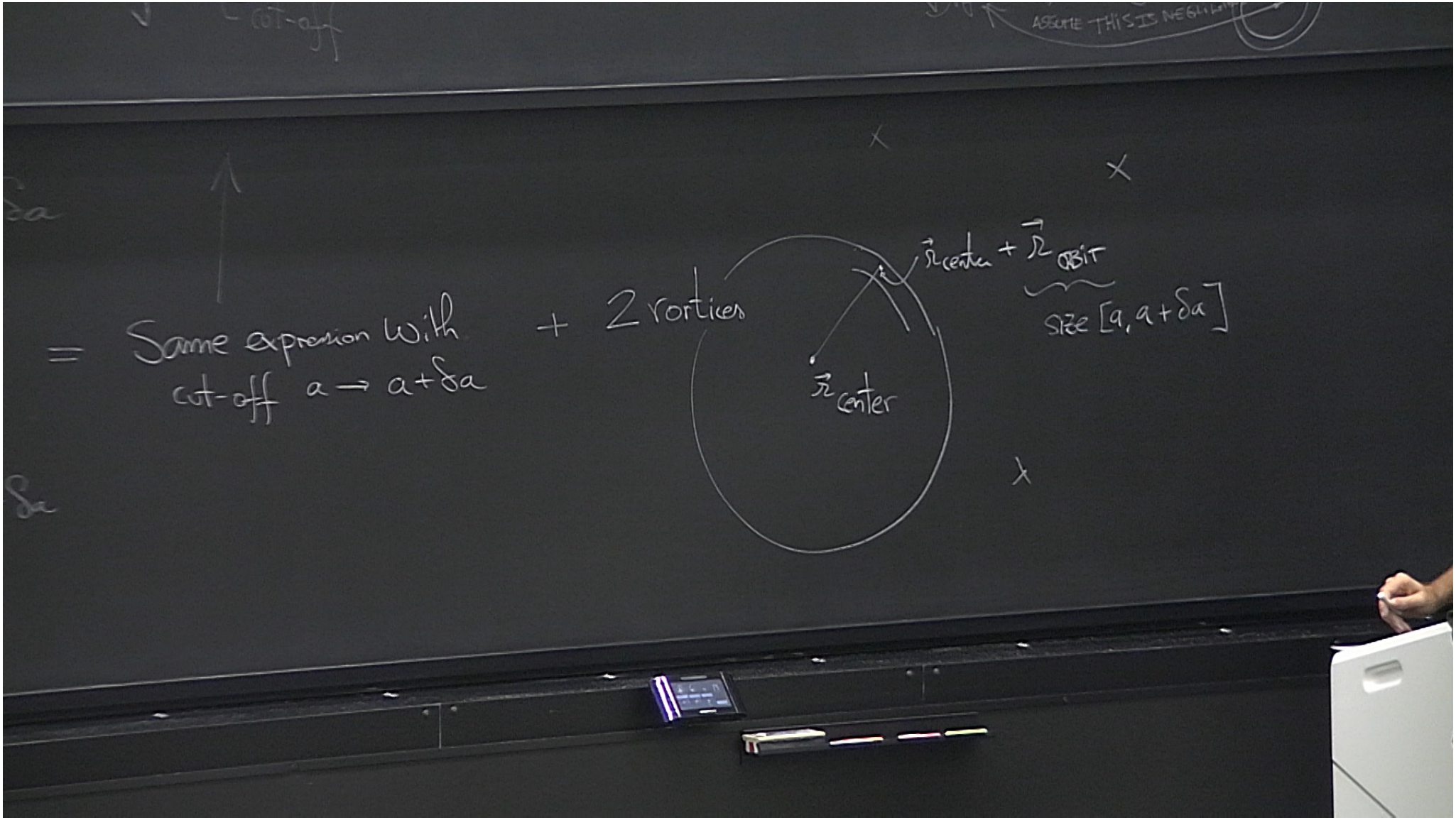

V_{int}

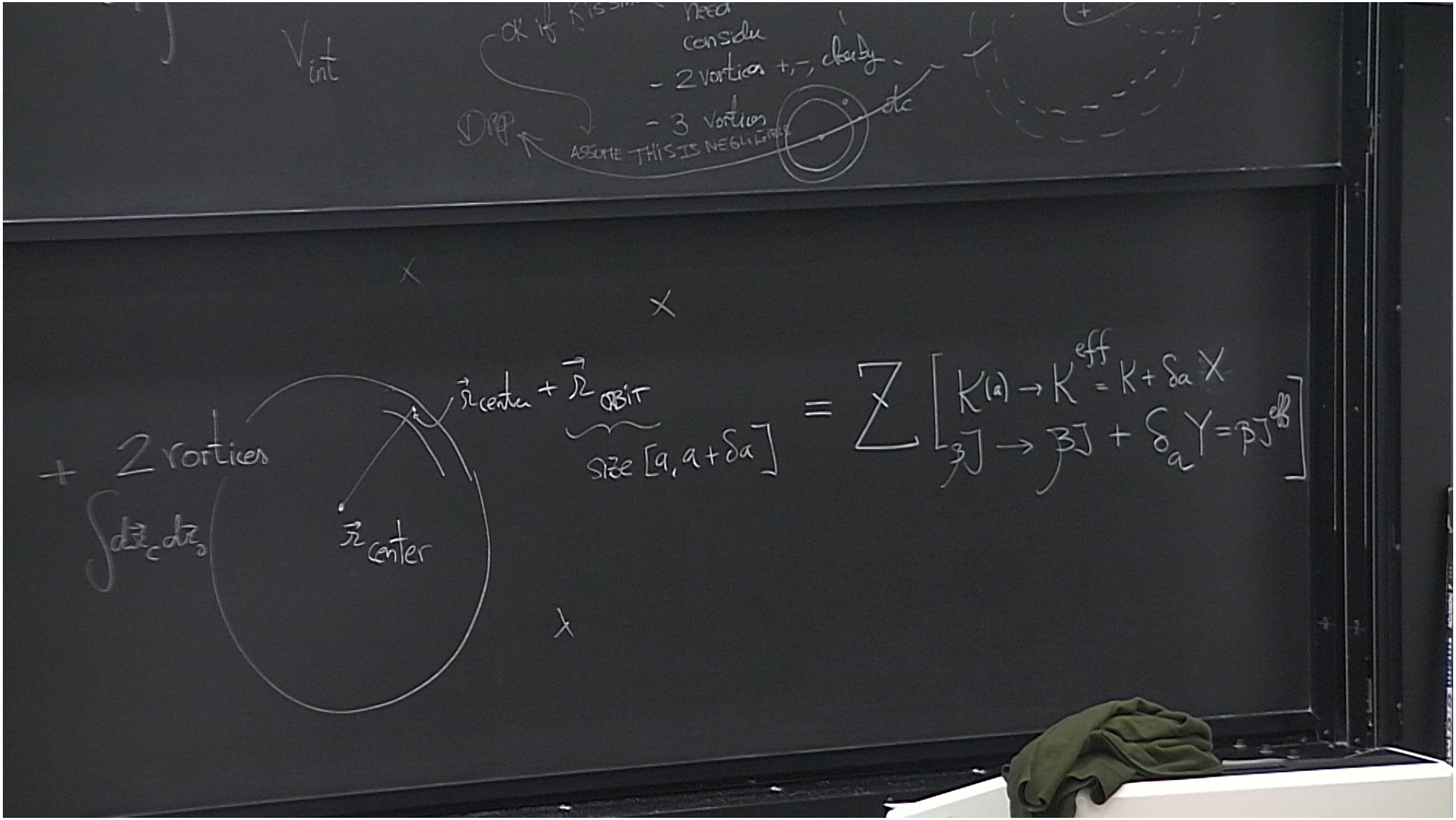

± 1
$\sum_{i \neq j} n_i n_j \log \frac{|\vec{z}_i - \vec{z}_j|}{a}$
V_{int}
need
consider
- 2 vortices +,−, clearly
- 3 vortices
etc
V_{int}

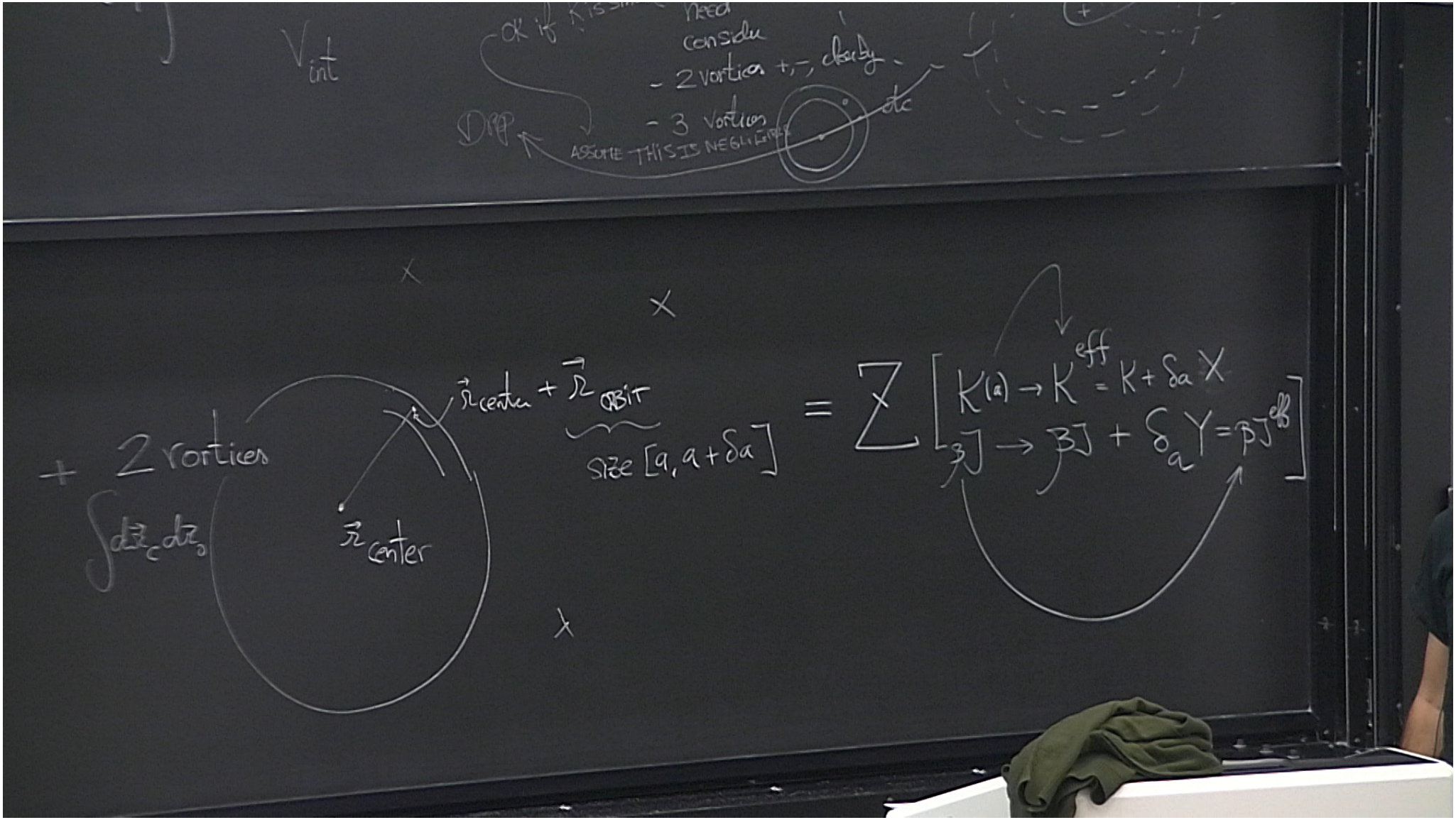

± 1
$\sum_{i \neq j} n_i n_j \log \frac{|\vec{z}_i - \vec{z}_j|}{a}$
V_{int}
a
V_{int}
need
consider
- 2 vortices +, −, clearly
- 3 vortices
ABSORB THIS IS NEGLIGIBLE
etc
DRP

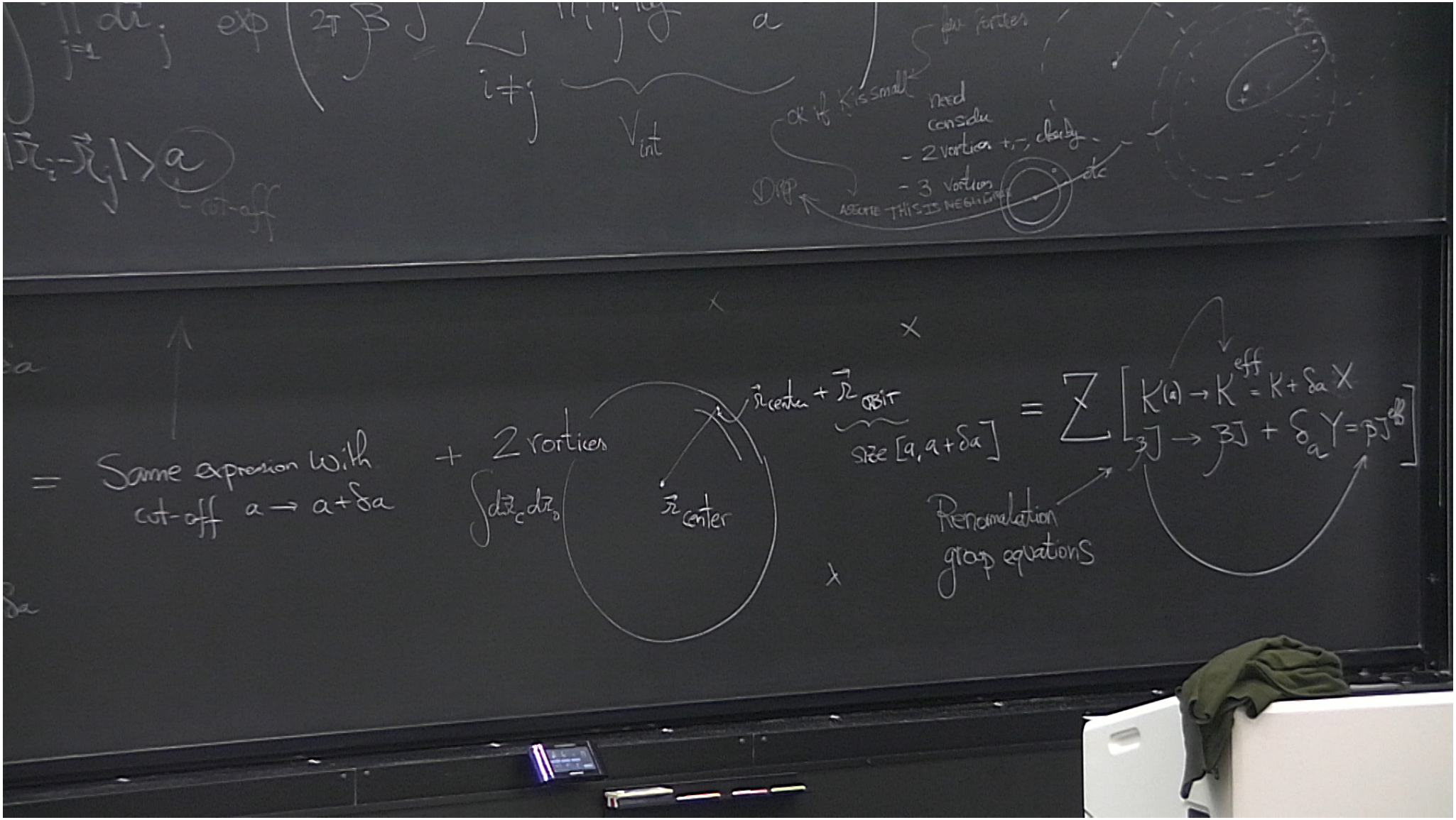

± 1
$\sum_{i \neq j} n_i n_j \log \frac{|\vec{x}_i - \vec{x}_j|}{a}$
V_{int}
ok if K is small
few particles
need consider
- 2 vortices +, −, clearly
- 3 vortices
etc
DIPP
ASSUME THIS IS NEGLIGIBLE
V_{int}

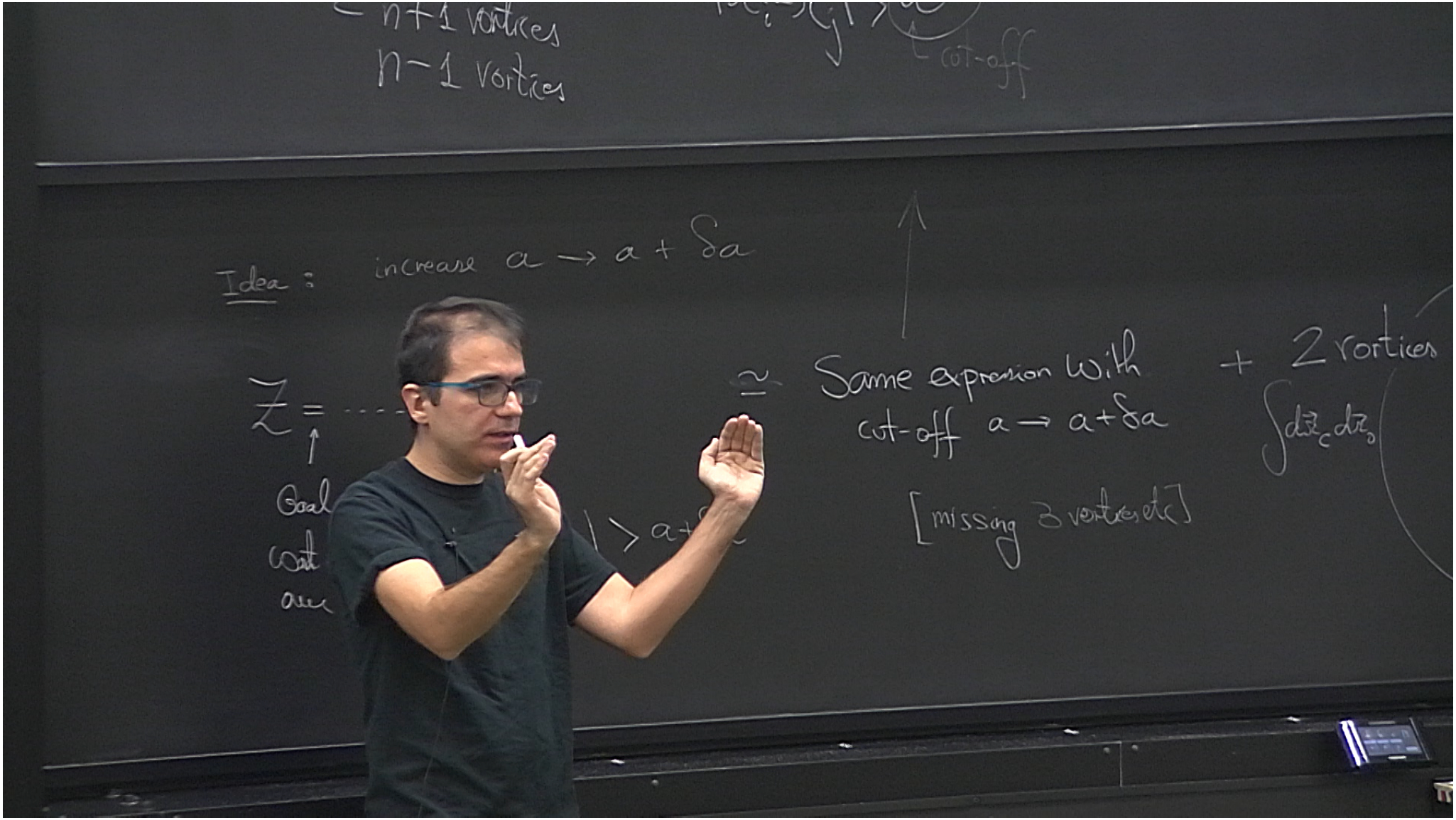

n-1 vortices
cut-off
Idea : increase $a \to a + \delta a$
$\tilde{Z} = \cdots$
Goal
cost to integrate over $|z_i| \in [a, a+\delta a]$
$\int_{|z_i - z_j| > a+\delta a}$
$=$ Same expression with cut-off $a \to a+\delta a$ $+$ 2 vortices


= Same expression with
cut-off $a \to a+\delta a$
+ 2 vortices
$\vec{r}_{center} + \vec{r}_{ORBIT}$
size $[a, a+\delta a]$
$\vec{r}_{center}$

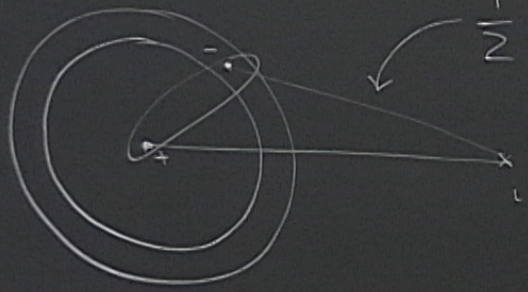

V_{int}
ok if
need
consider
- 2 vortices +,−, clearly
- 3 vortices
DPP
ASSUME THIS IS NEGLIGIBLE
etc
+ 2 vortices
$\int dz_c dz_5$
$\vec{r}_{center} + \vec{r}_{ORBIT}$
size $[a, a+\delta a]$
$\vec{r}_{center}$
$= \sum \left[\begin{matrix} K(a) \to K^{eff} = K + \delta_a X \\ \mathcal{J} \to \mathcal{J} + \delta_a Y = \mathcal{J}^{eff} \end{matrix} \right]$


V int
OK if
need
consider
- 2 vortices +,- clearly
- 3 vortices
etc
DIPP
ASSUME THIS IS NEGLIGIBLE
+ 2 vortices
$\int dz_c dz_2$
$\vec{r}_{center} + \vec{r}_{ORBIT}$
size $[a, a+\delta a]$
$\vec{r}_{center}$
$= \sum \left[\begin{matrix} K(a) \to K^{eff} = K + \delta_a X \\ J \to J + \delta_a Y = J^{eff} \end{matrix} \right]$


$\prod_{j=1} dz_j \exp\left(2\pi\beta J \sum_{i\neq j} \ln|z_i - z_j| > a\right)$
V_{int}
few vortices
ok if K is small
need consider
- 2 vortices +,−, charge
- 3 vortices
etc
DISP
ASSUME THIS IS NEGLIGIBLE
cut-off
$= \sum \left[K(a) \to K^{eff} = K + \delta a\, X \right.$
$\left. J \to J + \delta a\, Y = J^{eff} \right]$
Renormalization group equations
$= $ Same expression with cut-off $a \to a + \delta a$
$+$ 2 vortices
$\int dz_c dz_0$
z_{center}
$z_{center} + \vec{r}$ ORBIT
size $[a, a+\delta a]$


n+1 vortices
n-1 vortices
cut-off
Idea : increase $a \to a + \delta a$
$Z =$
Goal
$\simeq$ Same expression with cut-off $a \to a+\delta a$
+ 2 vortices
$\int dz_c dz_c$
[missing 3 vortices etc]

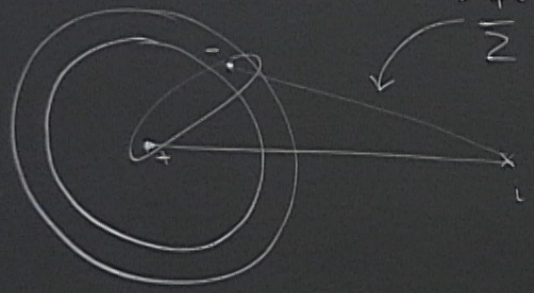
$\sin^2 \frac{k}{2}$ → other function $\simeq k^2$ at small k → $\frac{1}{|x|^{D-2}}$



$$\frac{1}{2} n_i \underbrace{\log \frac{|\vec{x}_i - \vec{x}_{center}|^2}{|\vec{x}_i - \vec{x}_{center} - \vec{x}_{ORBIT}|^2}}$$

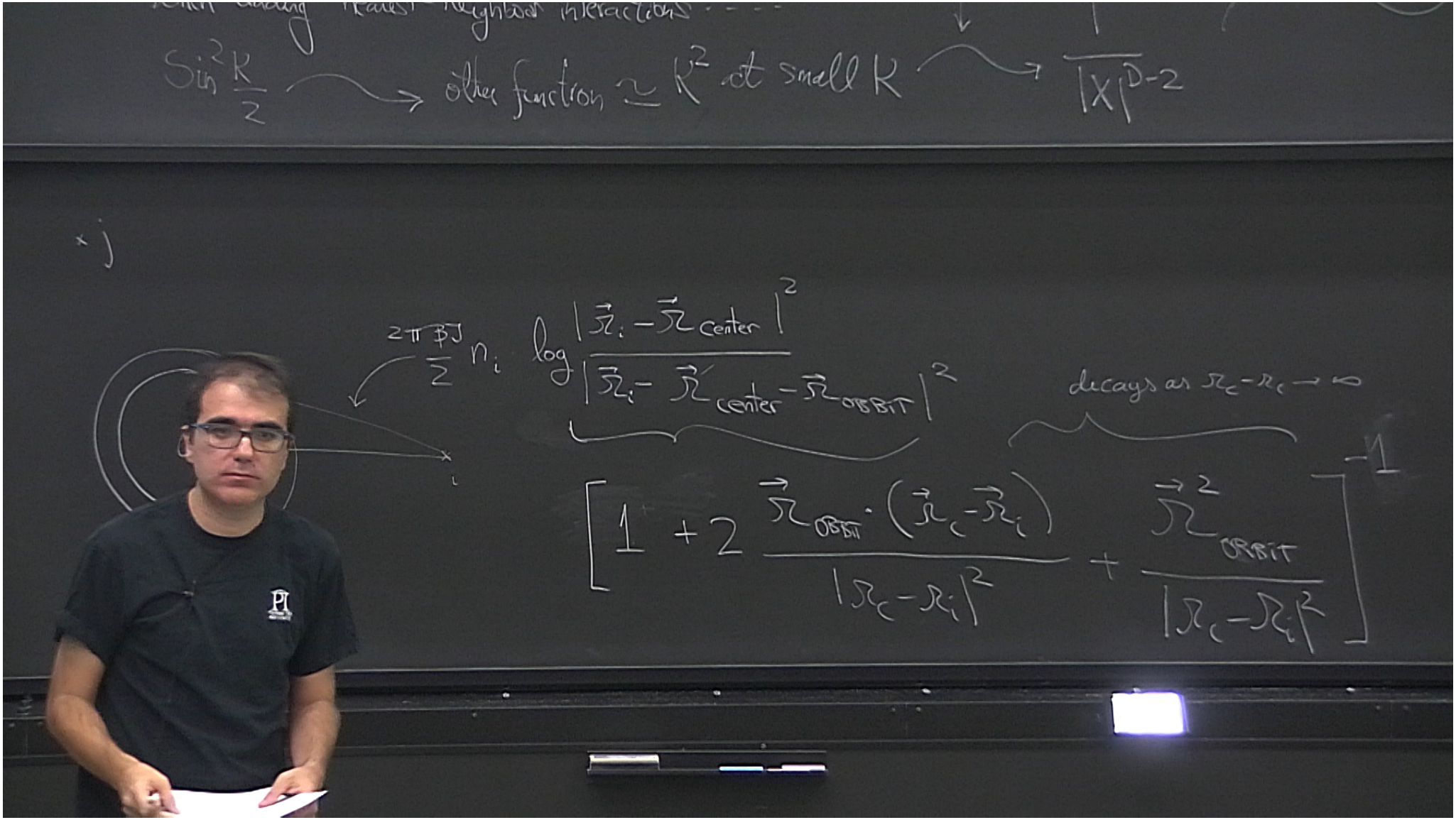
$$\left[1 + 2 \frac{\vec{x}_{ORBIT} \cdot (\vec{x}_c - \vec{x}_i)}{|x_c - x_i|^2} + \frac{\vec{x}^2_{ORBIT}}{|x_c - x_i|^2} \right]^{-1}$$

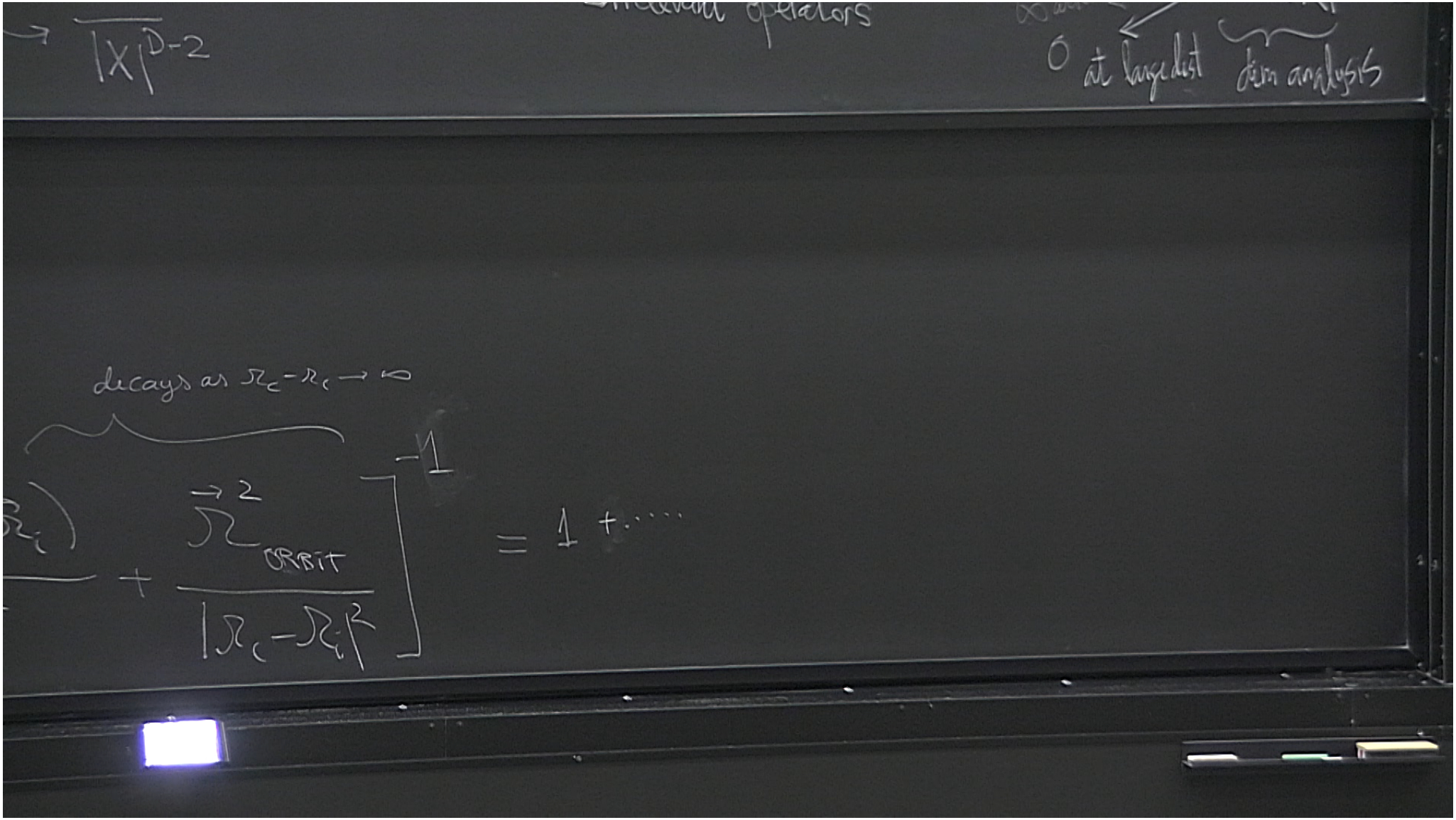
$\sin^2 \frac{K}{2}$ → other function $\simeq K^2$ at small K → $\frac{1}{|x|^{D-2}}$



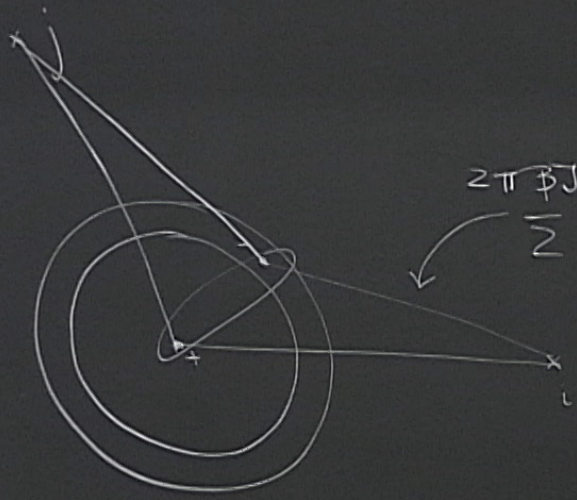
$$\frac{2\pi \beta J}{z} n_i \log \frac{|\vec{x}_i - \vec{x}_{center}|^2}{|\vec{x}_i - \vec{x}_{center} - \vec{x}_{ORBIT}|^2}$$

$$\left[1 + 2\frac{\vec{x}_{ORBIT} \cdot (\vec{x}_c - \vec{x}_i)}{|x_c - x_i|^2} + \frac{\vec{x}^2_{ORBIT}}{|x_c - x_i|^2}\right]^{-1}$$


$\sin^2 \frac{K}{2}$
other function $\simeq K^2$ at small K
$\frac{1}{|x|^{D-2}}$
$\frac{2\pi}{L} n_i \log \frac{|\vec{x}_i - \vec{x}_{center}|^2}{|\vec{x}_i - \vec{x}'_{center} - \vec{x}_{ORBIT}|^2}$
decays as $x_c - x_i \to \infty$
$\left[1 + 2 \frac{\vec{x}_{ORBIT} \cdot (\vec{x}_c - \vec{x}_i)}{|x_c - x_i|^2} + \frac{\vec{x}^2_{ORBIT}}{|x_c - x_i|^2}\right]^{-1}$


$\overline{|x|^{D-2}}$
irrelevant operators
0 at large dist
dim analysis
decays as $x_c - x_i \to \infty$
$\vec{x}_i$
$+ \frac{\vec{x}^2_{ORBIT}}{|x_c - x_i|^2}$
$^{-1}$
$= 1 + \cdots$

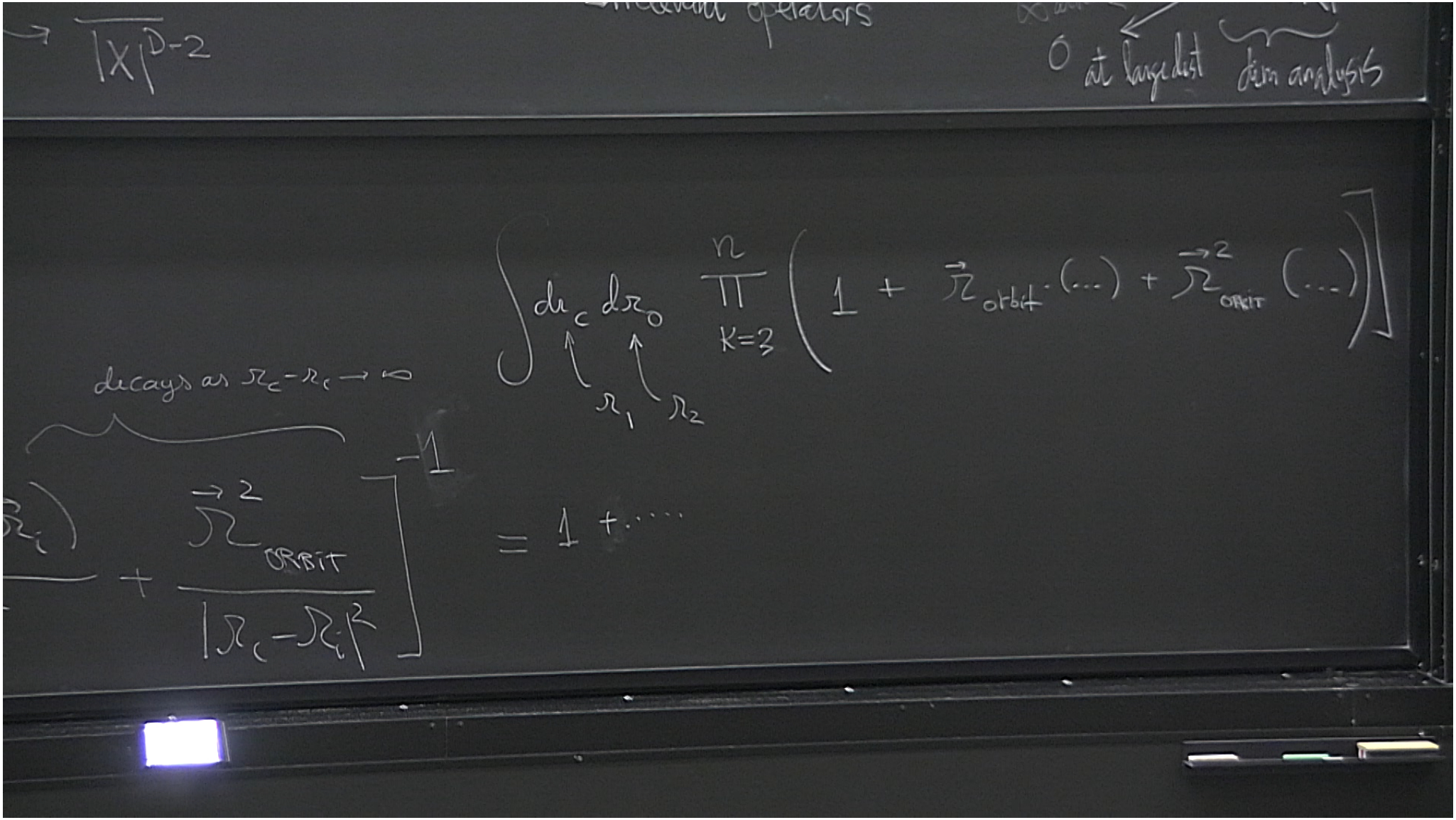
$\sin^2 \frac{K}{2}$ → other function $\simeq K^2$ at small K → $\frac{1}{|x|^{D-2}}$

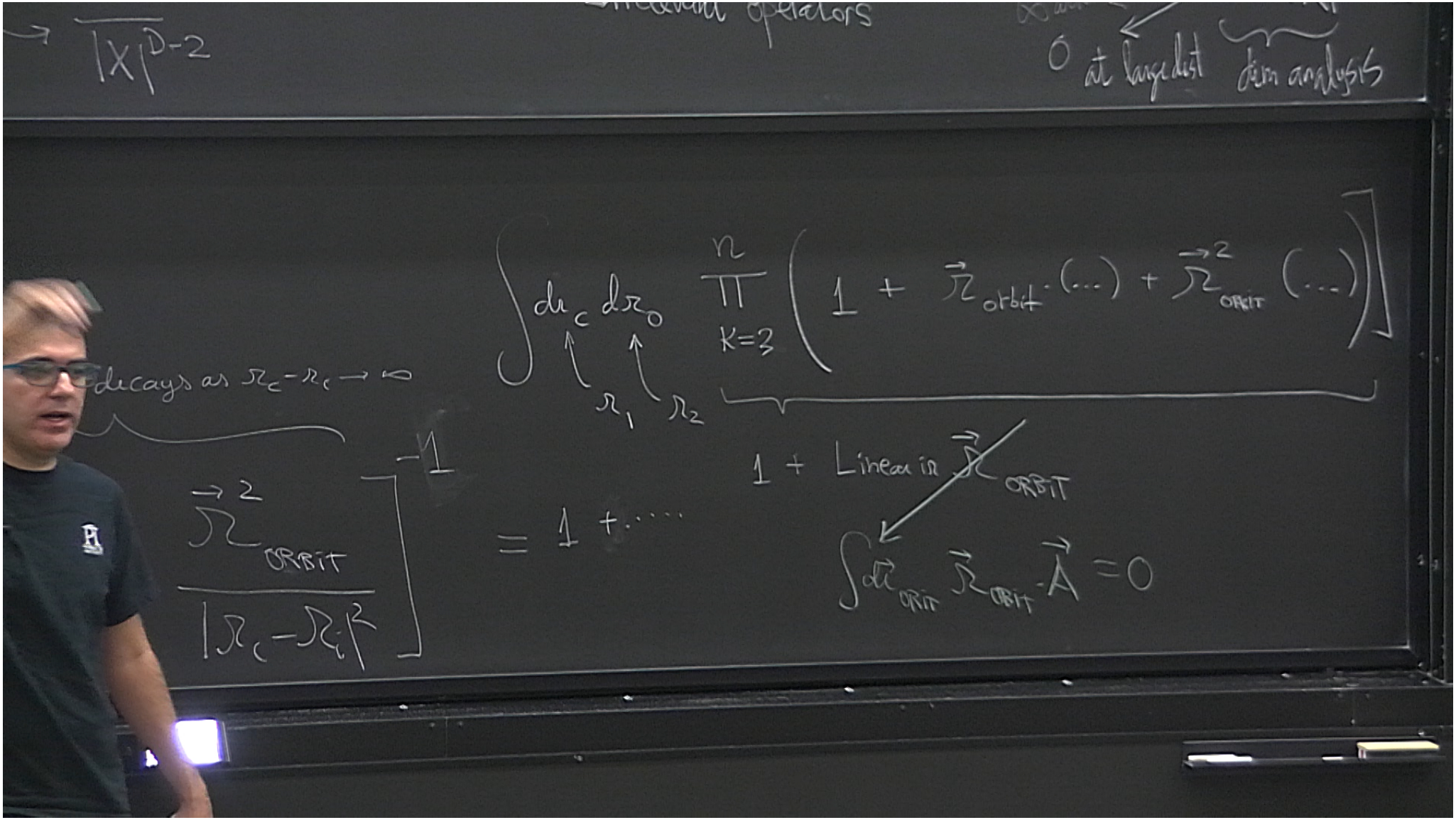


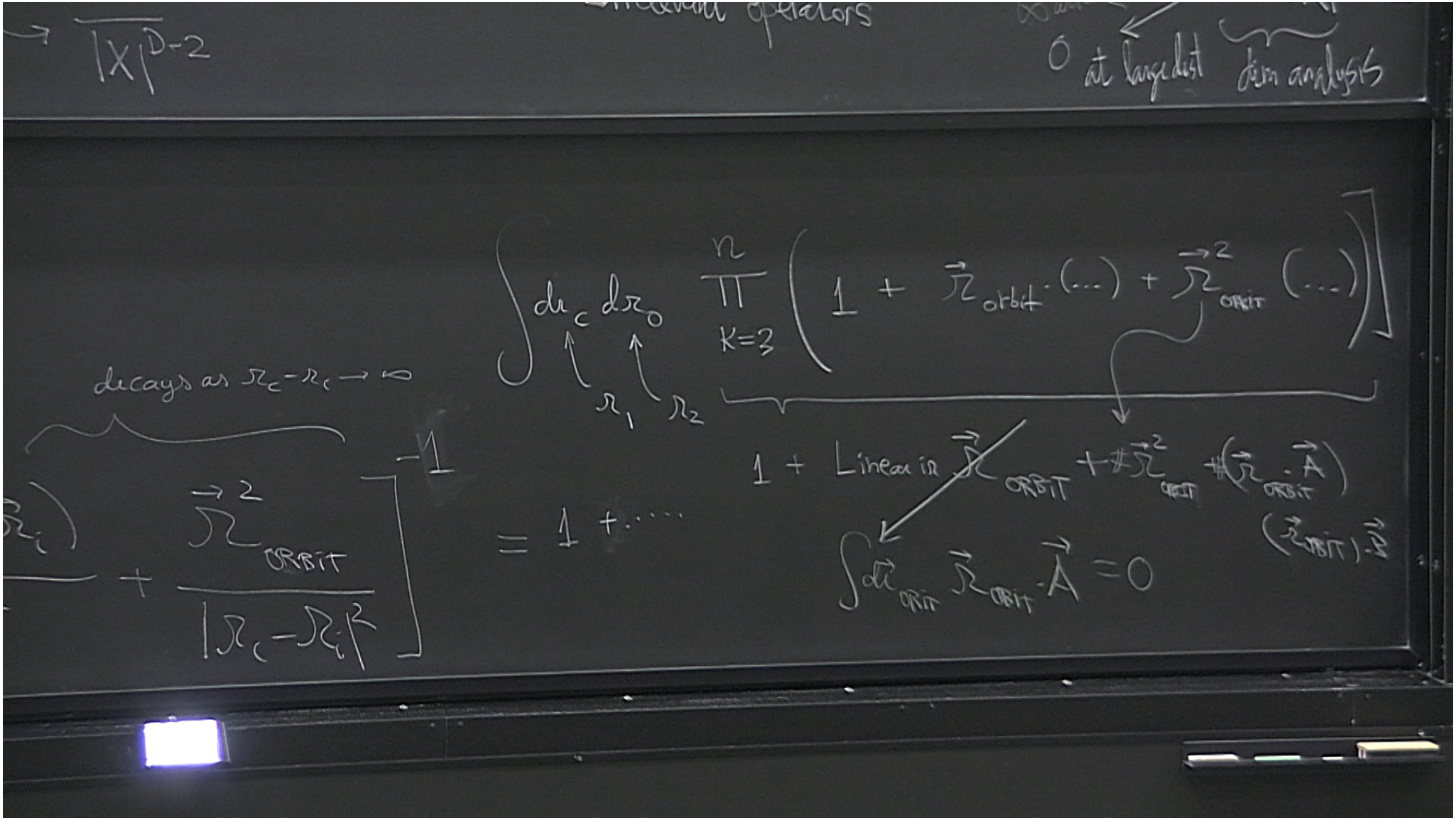
$$\frac{2\pi \beta J}{z} n_i \log \frac{|\vec{x}_i - \vec{x}_{center}|^2}{|\vec{x}_i - \vec{x}'_{center} - \vec{x}_{orbit}|^2}$$

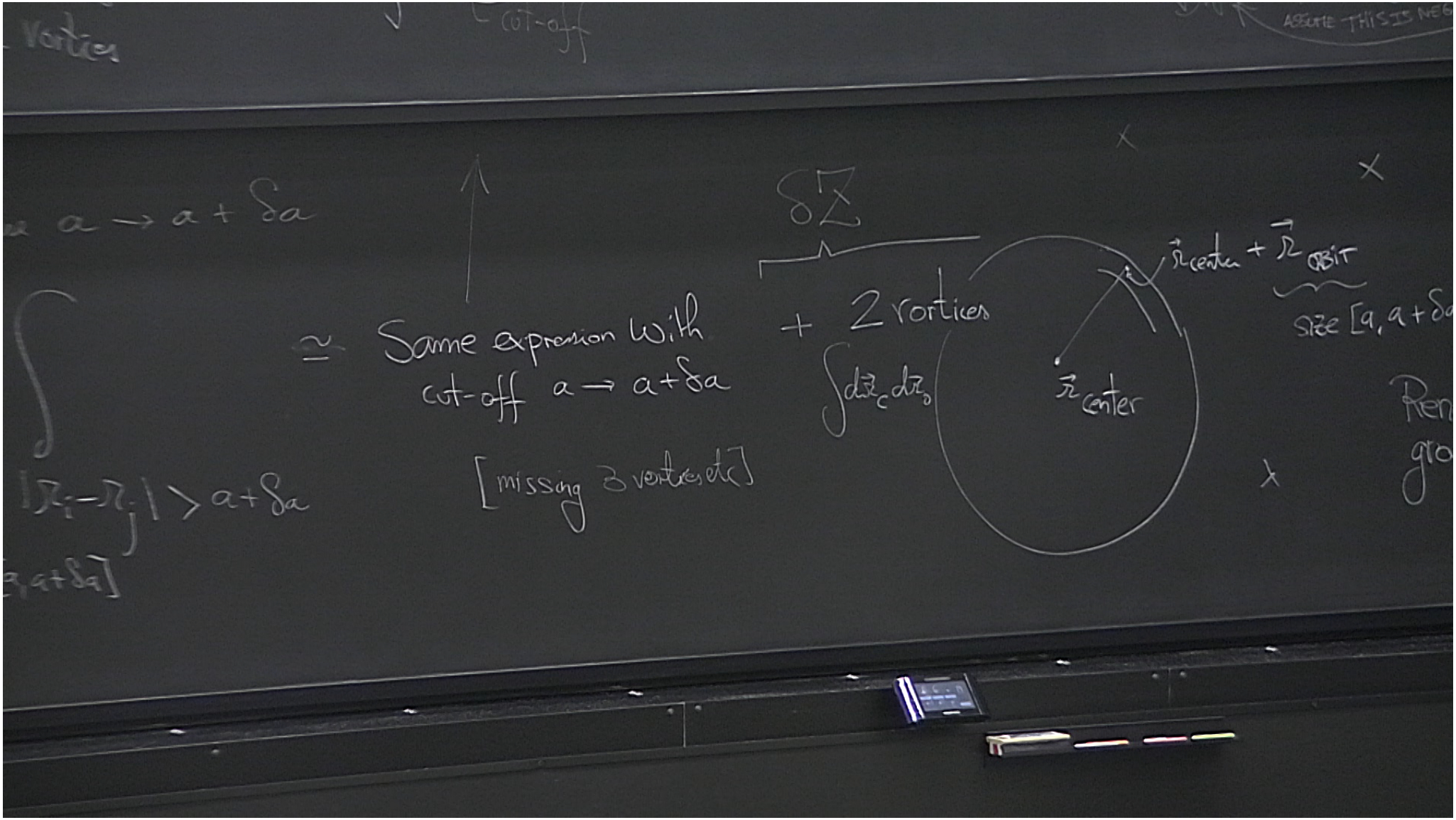
$$\left[1 + 2 \frac{\vec{x}_{orbit} \cdot (\vec{x}_c - \vec{x}$$

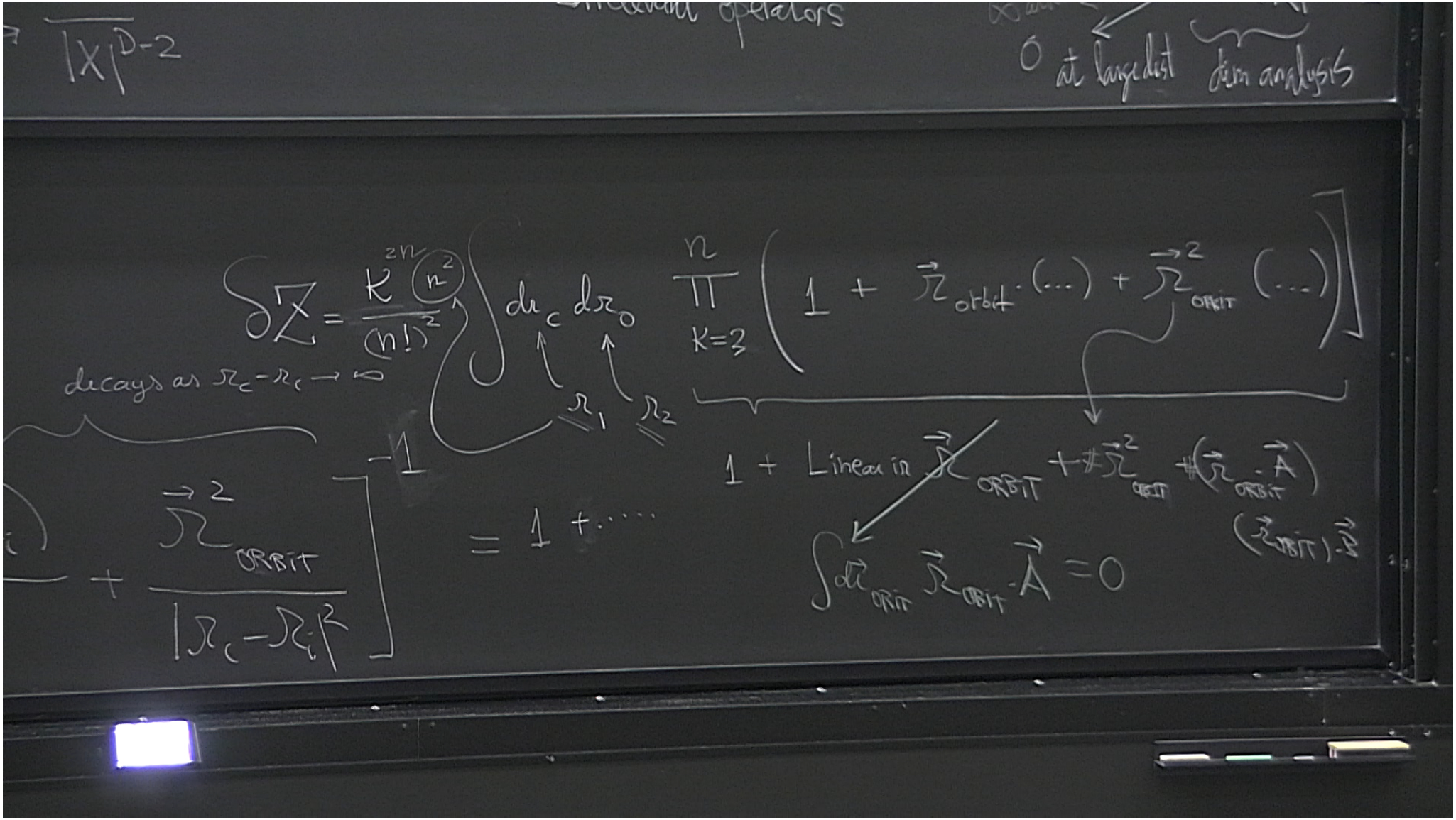
$$|x_c - x_i|^2$$

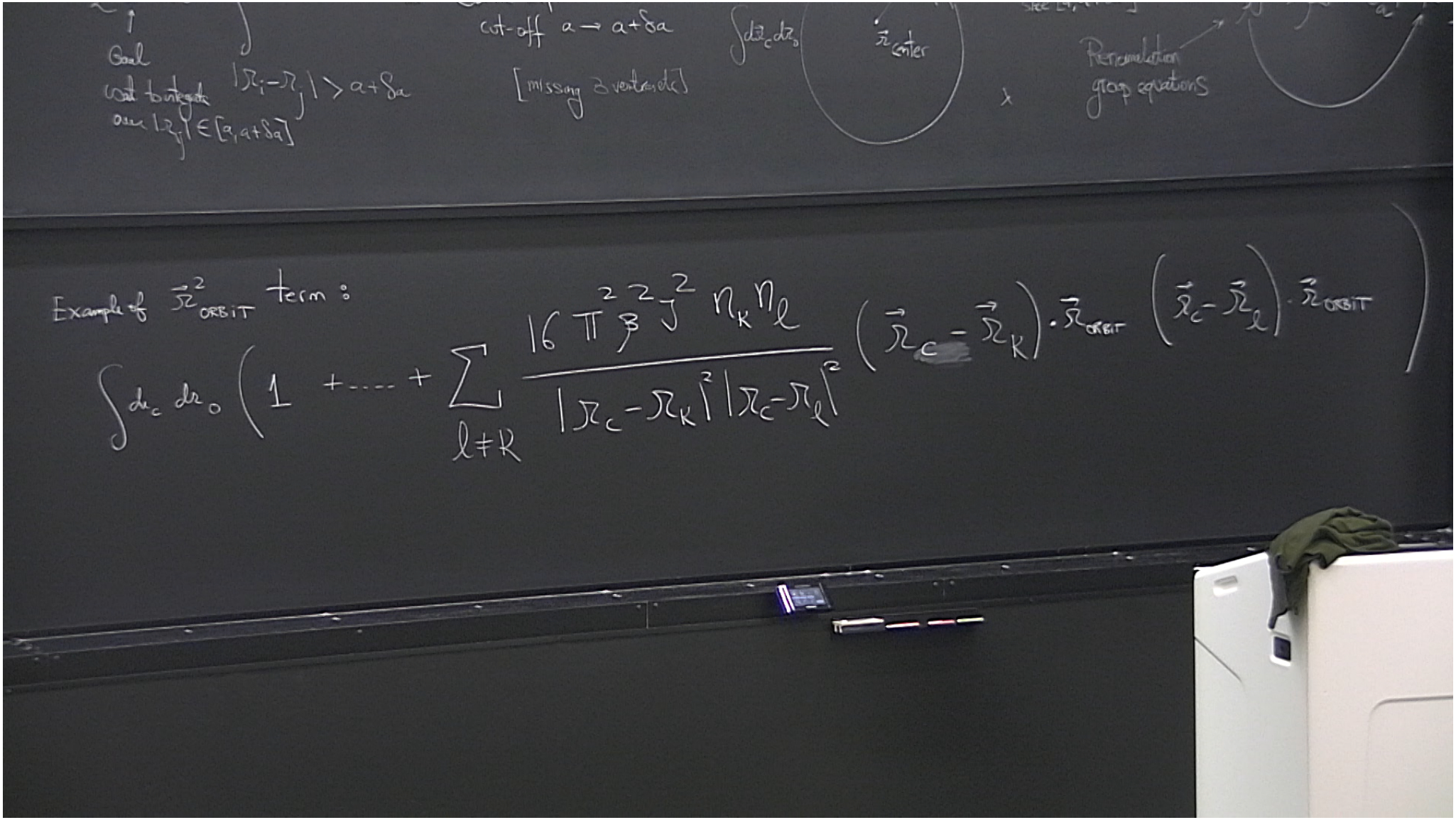

$\overline{|x|^{D-2}}$
relevant operators
0 at large dist
dim analysis
$\int d\tau_c\, d\tau_0 \prod_{k=3}^{n} \left(1 + \vec{\tau}_{orbit}\cdot(\ldots) + \vec{\tau}^{\,2}_{ORBIT}(\ldots)\right)$
τ_1 τ_2
decays as $\tau_c - \tau_i \to \infty$
$\left(\frac{\vec{\tau}^{\,2}_{ORBIT}}{|\tau_c - \tau_i|^2}\right)^{-1}$
$= 1 + \cdots$

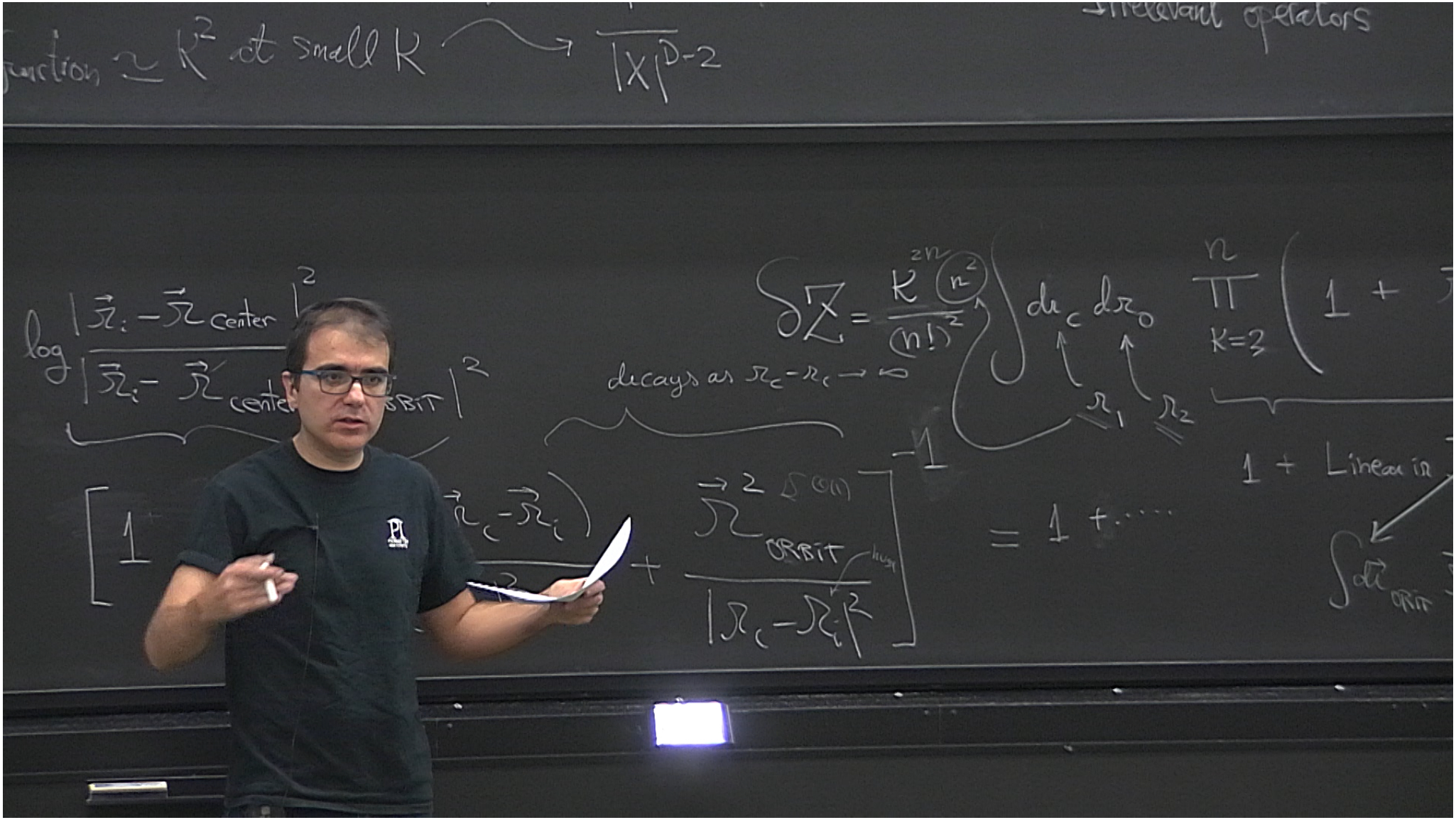

$\overline{|x|^{D-2}}$
relevant operators
0 at large dist
dim analysis
$\int d\tau_c\, d\tau_0 \prod_{k=3}^{n} \left(1 + \vec{\Omega}_{orbit}\cdot(\ldots) + \vec{\Omega}^2_{ORBIT}(\ldots)\right)$
τ_1 τ_2
decays as $\tau_c - \tau_i \to \infty$
$\left[1 - \frac{\vec{\Omega}^2_{ORBIT}}{|\tau_c - \tau_i|^2}\right]^{-1} = 1 + \cdots$
$1 +$ Linear in $\vec{\Omega}_{ORBIT}$
$\int d\vec{\tau}_{ORIT}\, \vec{\Omega}_{ORBIT}\cdot\vec{A} = 0$

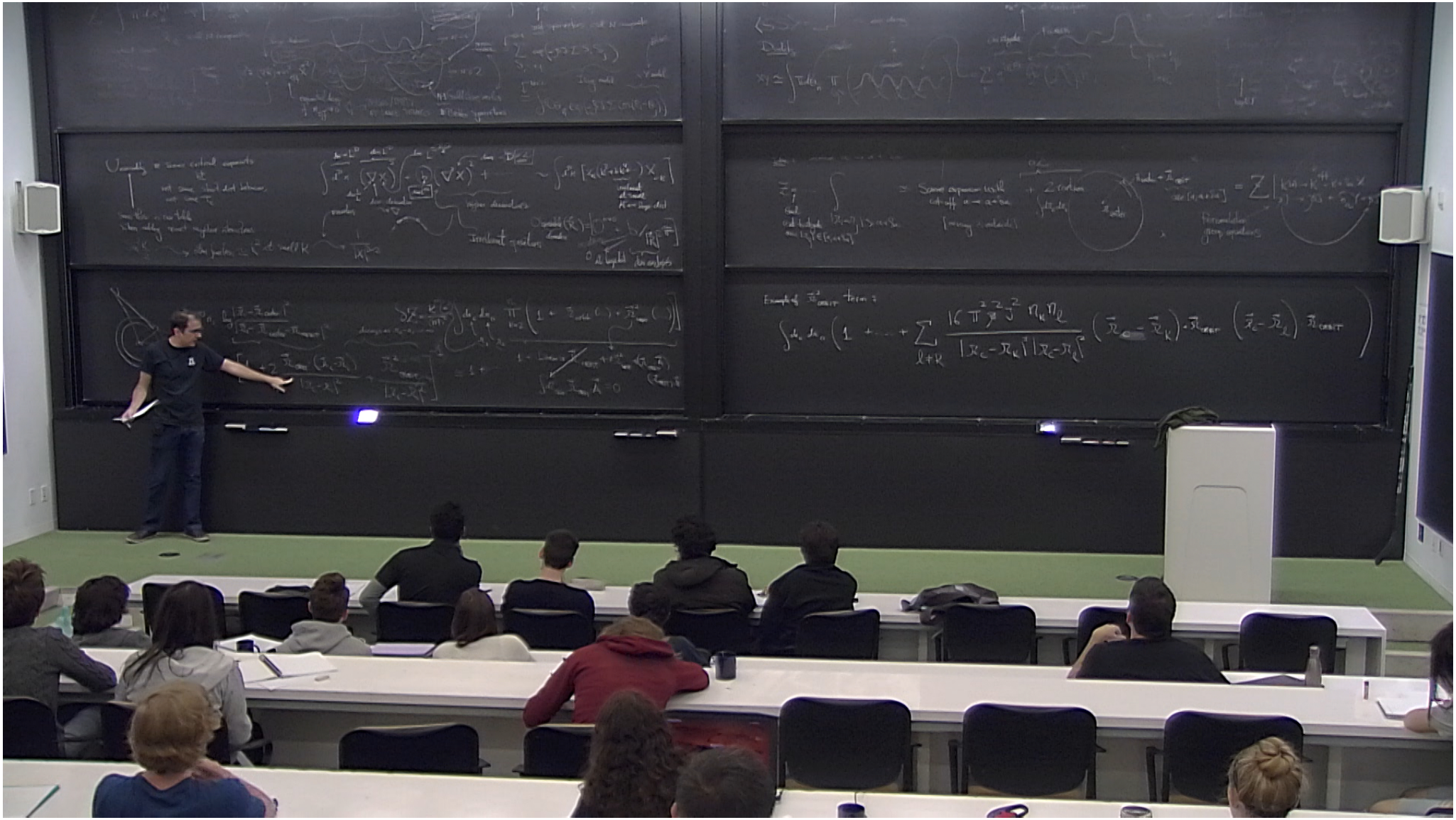

$|X|^{D-2}$
relevant operators
0 at large dist
dim analysis
$\int du_c\, d\tau_0 \prod_{k=3}^{n} \left(1 + \vec{\tau}_{orbit}\cdot(\ldots) + \vec{\tau}^2_{ORBIT}(\ldots)\right)$
τ_1 τ_2
decays as $\tau_c - \tau_i \to \infty$
$1 + \text{Linear in } \vec{\tau}_{ORBIT} + \#\vec{\tau}^2_{ORBIT} + \#(\vec{\tau}_{ORBIT}\cdot\vec{A})$
$\left(\vec{\tau}_{ORBIT}\right)\cdot\vec{A}$
$\int d\vec{\tau}_{ORIT}\, \vec{\tau}_{ORBIT}\cdot\vec{A} = 0$
$\left(\vec{\tau}_c\right)$
$+ \dfrac{\vec{\tau}^2_{ORBIT}}{|\tau_c - \tau_i|^2}\Big]^{-1}$
$= 1 + \cdots$

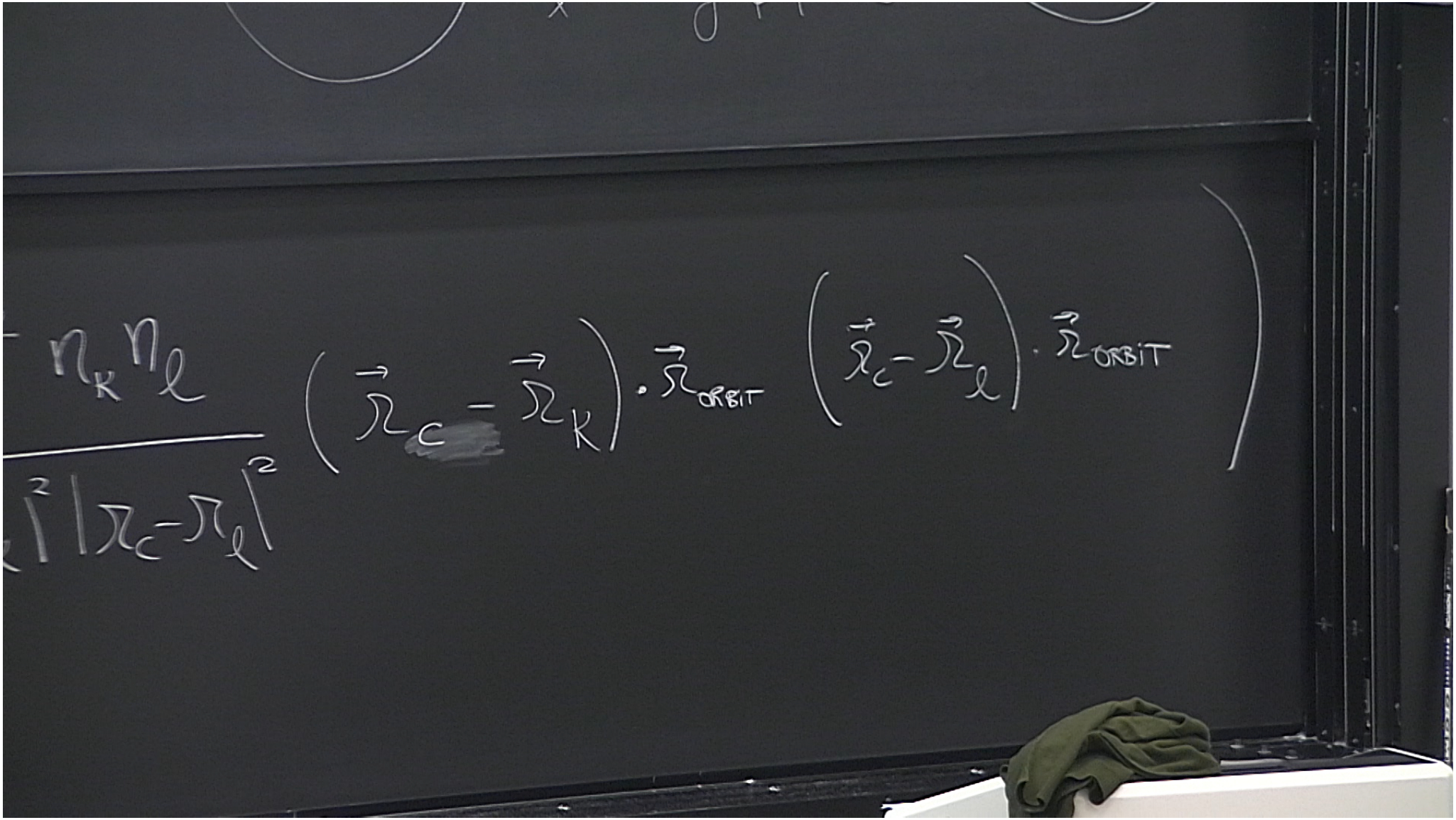

Vortices
cut-off
$a \to a + \delta a$
δZ
$\simeq$ Same expression with cut-off $a \to a + \delta a$
+ 2 vortices
$\int d\vec{x}_c dz_0$
$\vec{x}_{center} + \vec{x}_{ORBIT}$
$\vec{x}_{center}$
size $[a, a+\delta a]$
$|\vec{x}_i - \vec{x}_j| > a + \delta a$
[missing 3 vortices]


$|x|^{D-2}$
0 at large dist
dim analysis
$\delta Z = \frac{k^{2n}}{(n!)^2} n^2 \int d\tau_c\, d\tau_0 \prod_{k=3}^{n} \left(1 + \vec{r}_{orbit}\cdot(\ldots) + \vec{r}^{\,2}_{ORBIT}(\ldots)\right)$
decays as $\tau_c - \tau_i \to \infty$
τ_1 τ_2
$+ \frac{\vec{r}^{\,2}_{ORBIT}}{|\tau_c - \tau_i|^2}\Big]^{-1} = 1 + \ldots$
$1 + \text{Linear in } \vec{r}_{ORBIT} + \vec{r}^{\,2}_{ORBIT} + (\vec{r}_{ORBIT}\cdot\vec{A})$
$\int d\vec{r}_{ORBIT}\, \vec{r}_{ORBIT}\cdot\vec{A} = 0$


Goal
cut to integrate over $|x_i| \in [a, a+\delta a]$
$|x_i - x_j| > a + \delta a$
cut-off $a \to a + \delta a$
[missing 3 vertices etc.]
$\int dx_c dx_0$
$\vec{x}_{center}$
Renormalization group equations
Example of $\vec{x}^2_{ORBIT}$ term:
$\int dx_c dx_0 \left(1 + \cdots + \sum_{l \neq k} \frac{16 \pi^2 \beta^2 J^2 n_k n_l}{|x_c - x_k|^2 |x_c - x_l|^2} (\vec{x}_c - \vec{x}_k) \cdot \vec{x}_{ORBIT} (\vec{x}_c - \vec{x}_l) \cdot \vec{x}_{ORBIT} \right)$


irrelevant operators
decays as $x_c - x_i \to \infty$
1 + Linear in



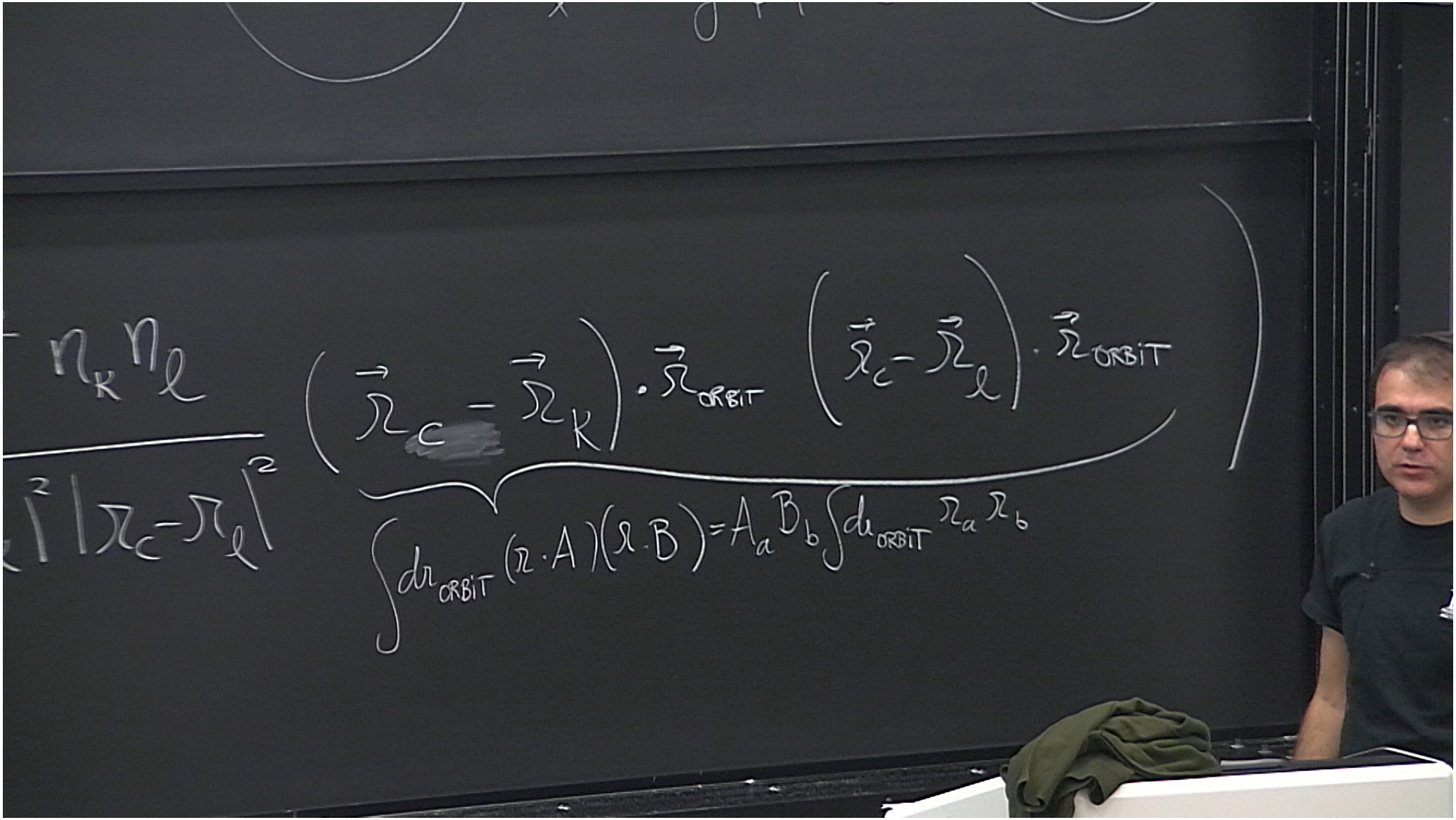

$\frac{n_k n_\ell}{|r_c - r_\ell|^2} \left(\vec{r}_c - \vec{r}_k \right) \cdot \vec{r}_{ORBIT} \left(\vec{r}_c - \vec{r}_\ell \right) \cdot \vec{r}_{ORBIT}$

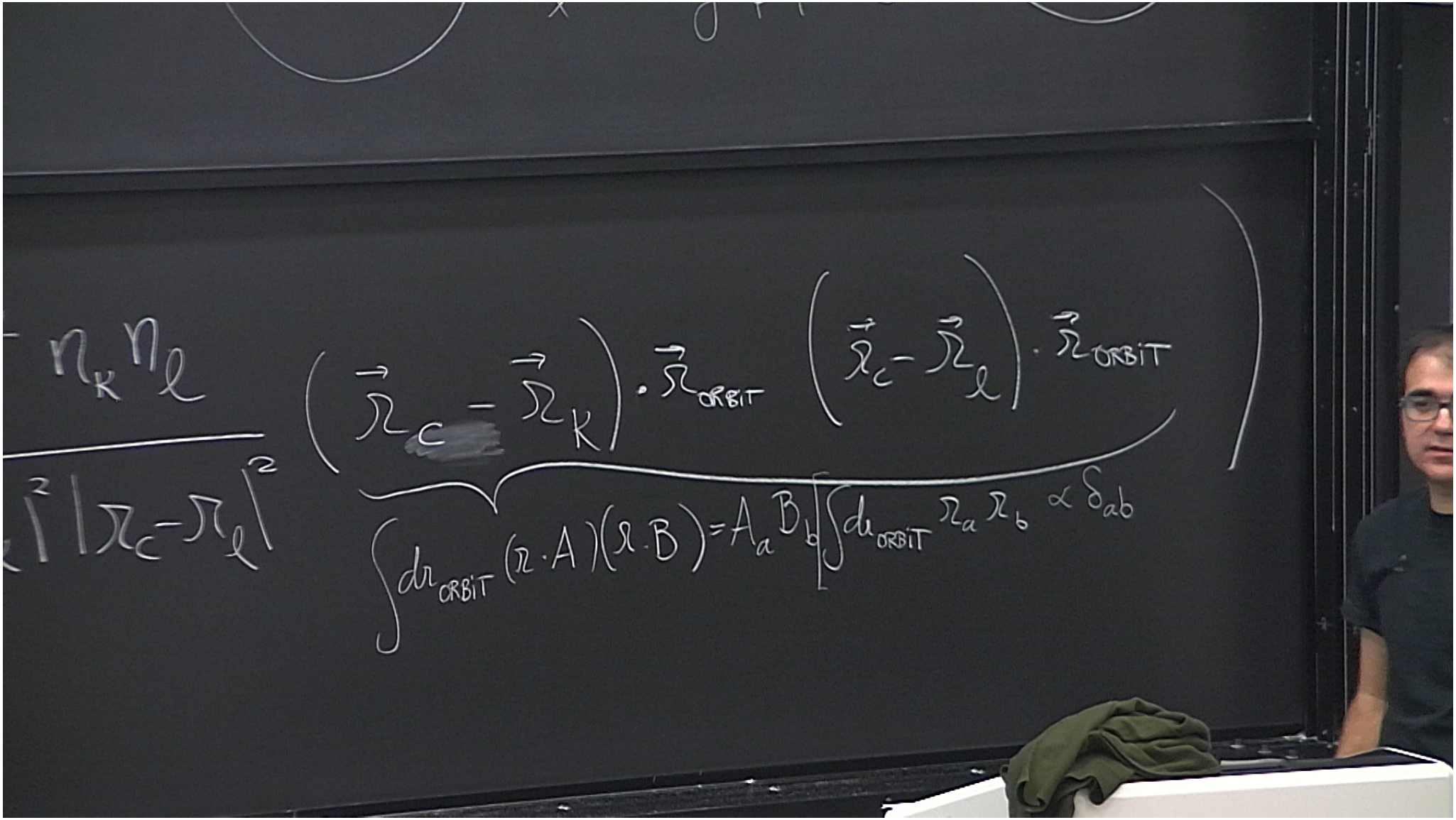
Example of $\vec{\Omega}^2_{\text{ORBIT}}$ term:

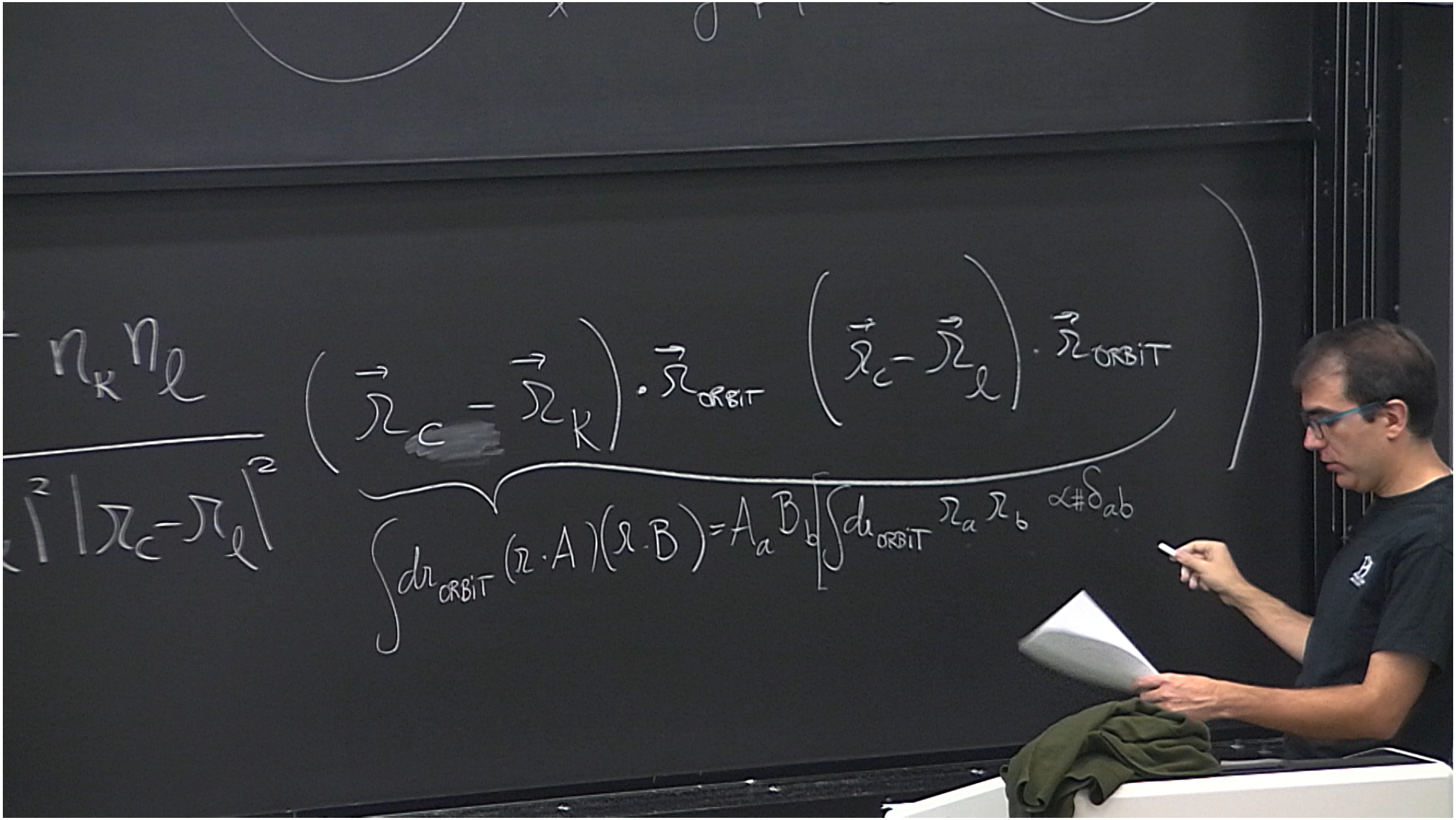
$$\int d\vec{r}_c \, d\vec{r}_0 \left(1 + \cdots + \sum_{l \neq K} \frac{\overbrace{16}^{4?} \pi^2 \beta^2 J^2 n_K n_l}{|\vec{r}_c - \vec{r}_K| \, |\vec{r}_c - \vec{r}_l|} \right.$$

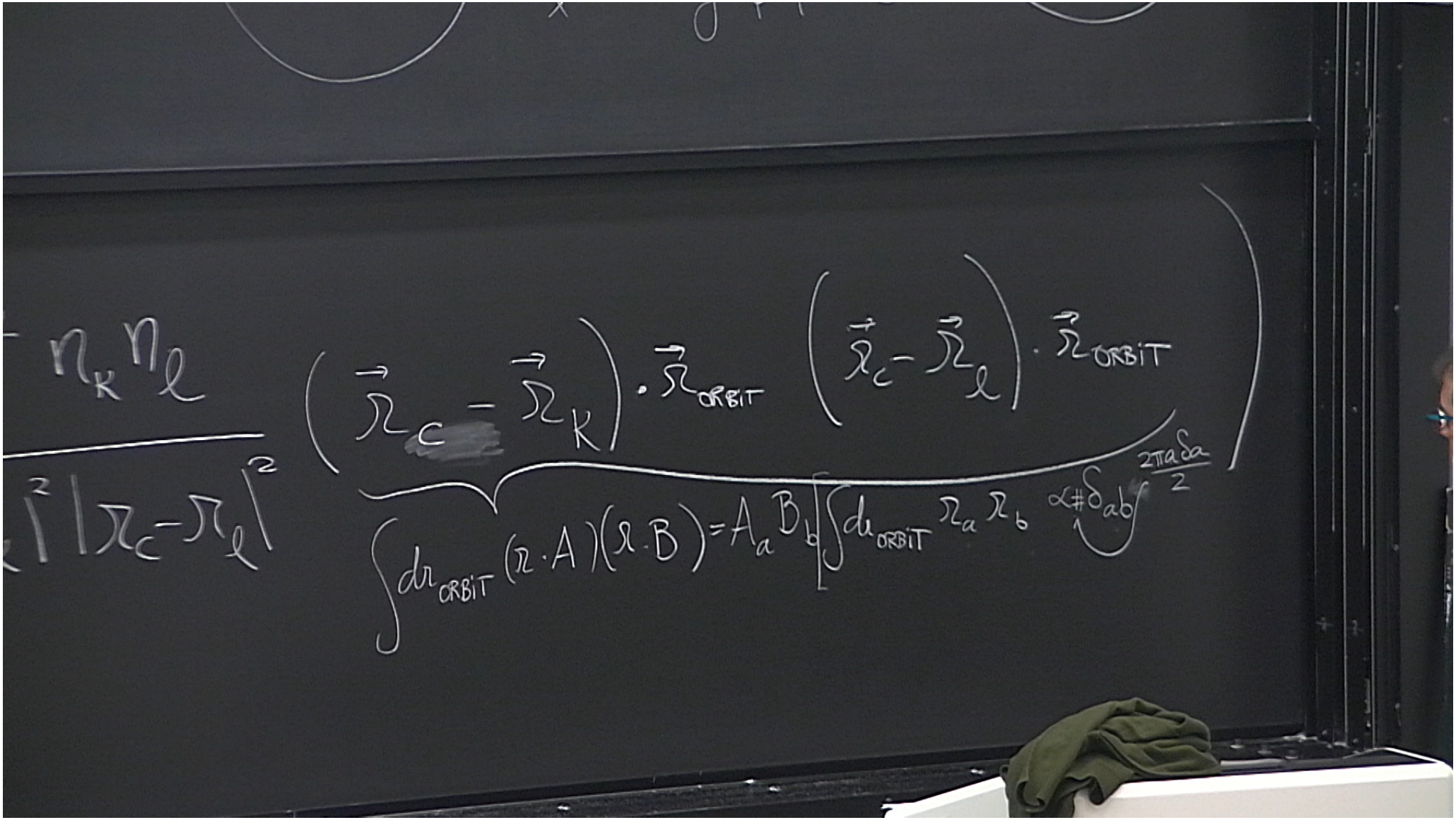
Example of $\vec{\Omega}^2_{\text{ORBIT}}$ term:

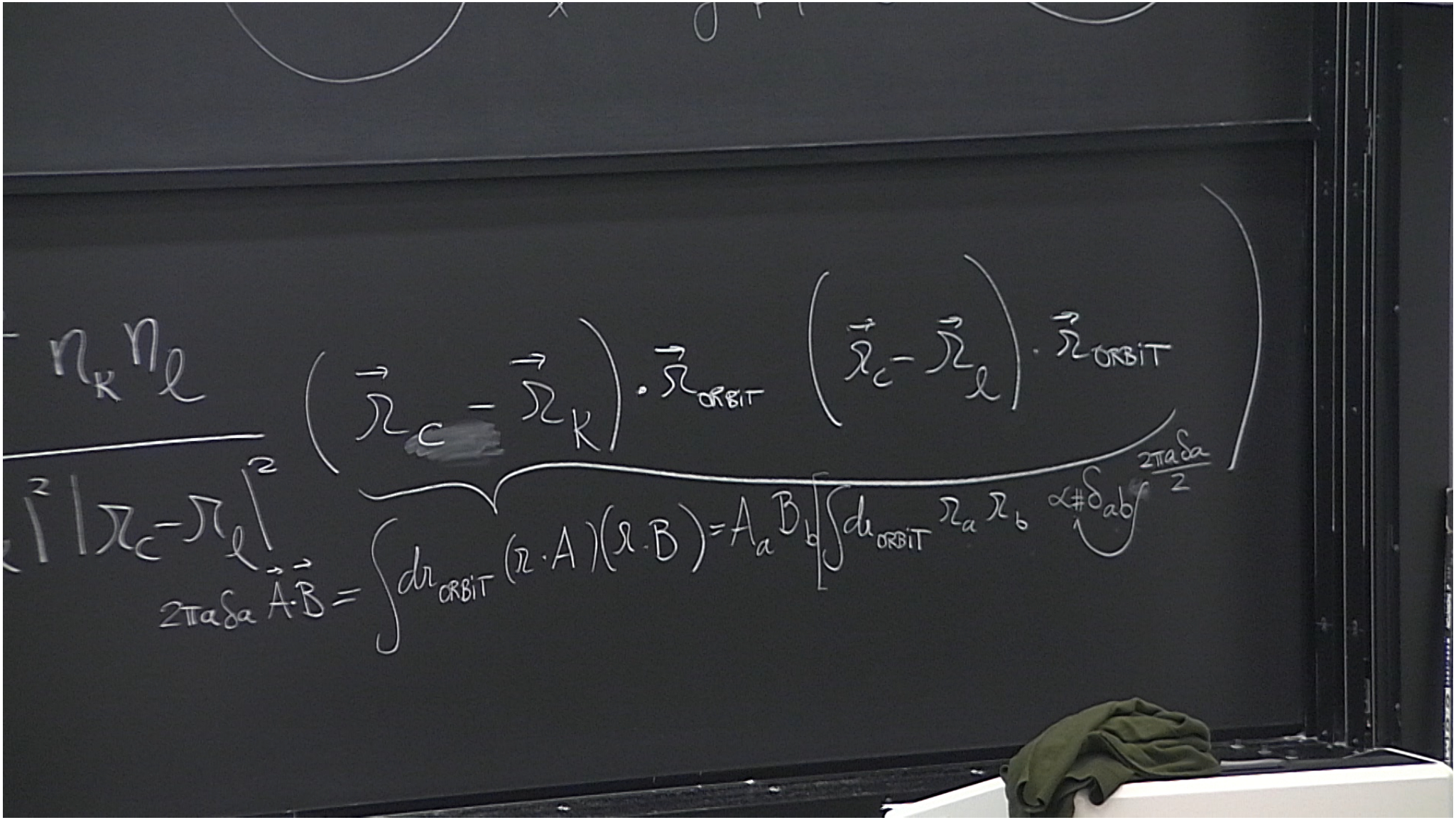
$$\int dt_c \, d\vec{x}_0 \left(1 + \cdots + \sum_{l \neq K} \frac{\overbrace{16}^{4?} \pi^2 \beta^2 J^2 n_K n_l}{|x_c - x_K|^2 |x_c - x_l|^2} \right.$$

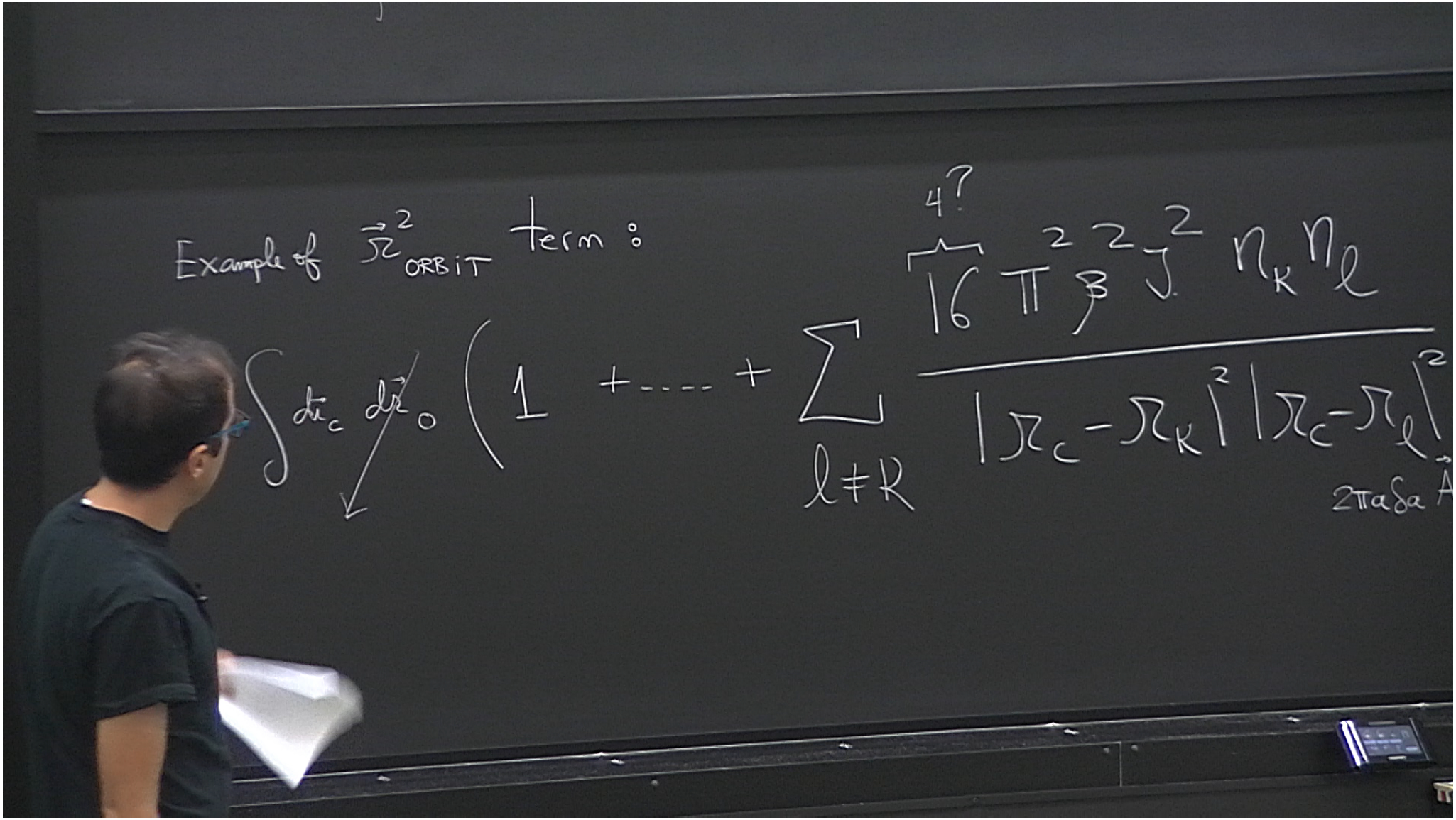

$\frac{n_k n_l}{|r_c - r_l|^2} \left((\vec{r}_c - \vec{r}_k) \cdot \vec{r}_{ORBIT} \right) \left((\vec{r}_c - \vec{r}_l) \cdot \vec{r}_{ORBIT} \right)$
$\int dr_{ORBIT} (r \cdot A)(r \cdot B) = A_a B_b \int dr_{ORBIT} r_a r_b$


$\frac{n_k n_\ell}{|r_c - r_\ell|^2} \left((\vec{r}_c - \vec{r}_k) \cdot \vec{r}_{ORBIT} \right) \left((\vec{r}_c - \vec{r}_\ell) \cdot \vec{r}_{ORBIT} \right)$
$\int dr_{ORBIT} (r \cdot A)(r \cdot B) = A_a B_b \int dr_{ORBIT} r_a r_b \propto \delta_{ab}$


$\frac{n_k n_\ell}{|\vec{x}_c - \vec{x}_\ell|^2}$
$\left(\vec{x}_c - \vec{x}_k\right) \cdot \vec{x}_{ORBIT}$
$\left(\vec{x}_c - \vec{x}_\ell\right) \cdot \vec{x}_{ORBIT}$
$\int dx_{ORBIT} (x \cdot A)(x \cdot B) = A_a B_b \int dx_{ORBIT} x_a x_b \propto \delta_{ab}$

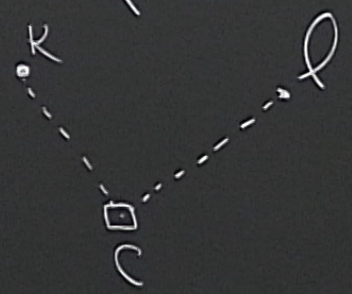

$\frac{n_k n_\ell}{|\vec{x}_c - \vec{x}_\ell|^2} \left((\vec{x}_c - \vec{x}_k) \cdot \vec{x}_{ORBIT} \right) \left((\vec{x}_c - \vec{x}_\ell) \cdot \vec{x}_{ORBIT} \right)$
$\int dt_{ORBIT} (x \cdot A)(x \cdot B) = A_a B_b \int dt_{ORBIT} x_a x_b \propto \delta_{ab} \frac{2\pi a \delta a}{2}$


$\frac{n_k n_\ell}{|\vec{r}_c - \vec{r}_\ell|^2} \left(\vec{r}_c - \vec{r}_k\right) \cdot \vec{r}_{ORBIT} \left(\vec{r}_c - \vec{r}_\ell\right) \cdot \vec{r}_{ORBIT}$
$2\pi a \delta_a \vec{A} \cdot \vec{B} = \int dr_{ORBIT} (r \cdot A)(r \cdot B) = A_a B_b \int dr_{ORBIT} r_a r_b \propto \delta_{ab} \int_0^{\frac{2\pi a \delta_a}{2}}$

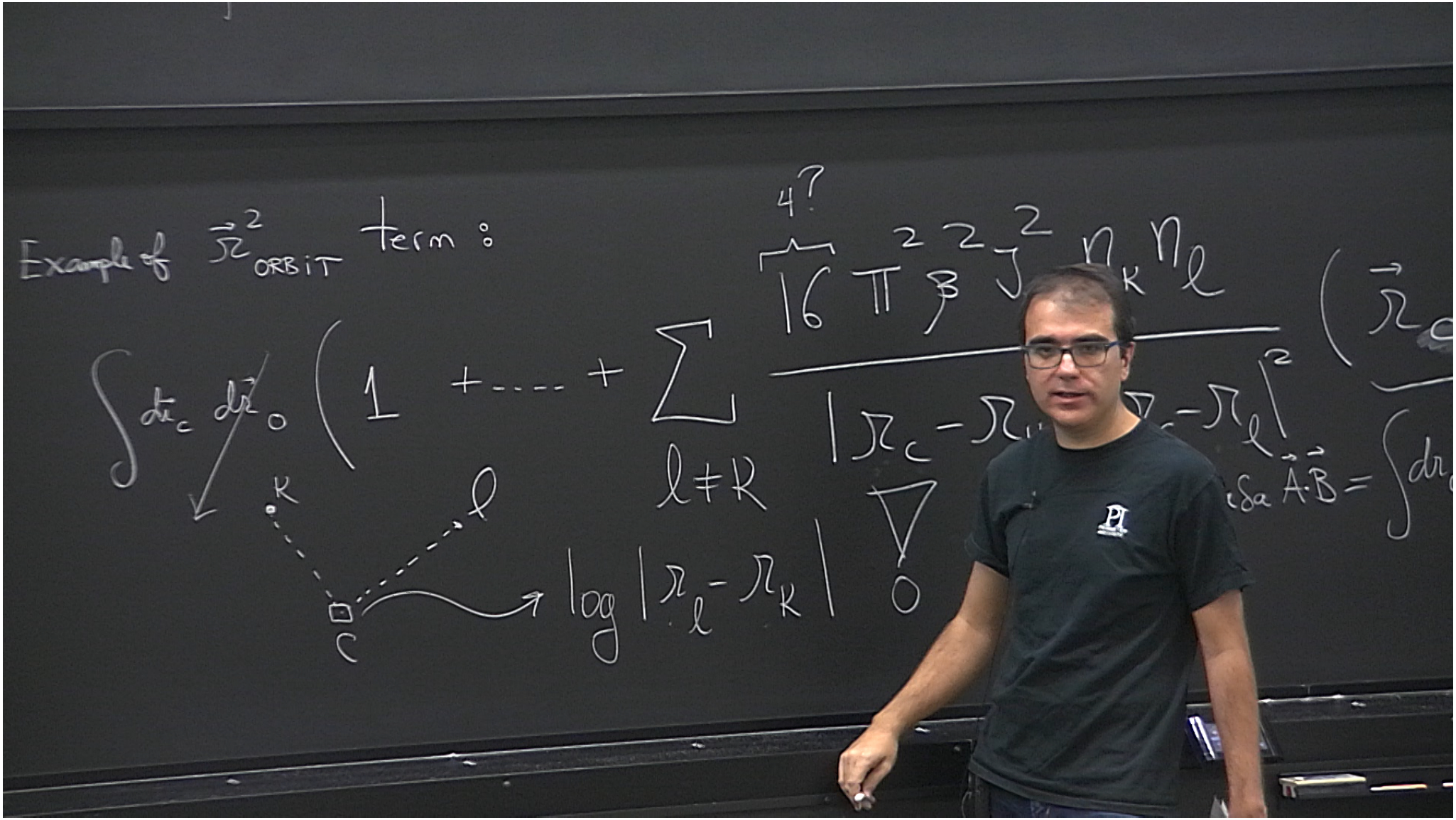

Example of $\vec{\Omega}^2_{ORBIT}$ term:
$\int dt_c \, d\vec{\Omega}_0 \left(1 + \cdots + \sum_{l \neq K} \frac{16\pi^2 \beta^2 J^2 n_K n_l}{|r_c - r_K|^2 |r_c - r_l|^2} \right)$
4?
$2\pi a \delta a$

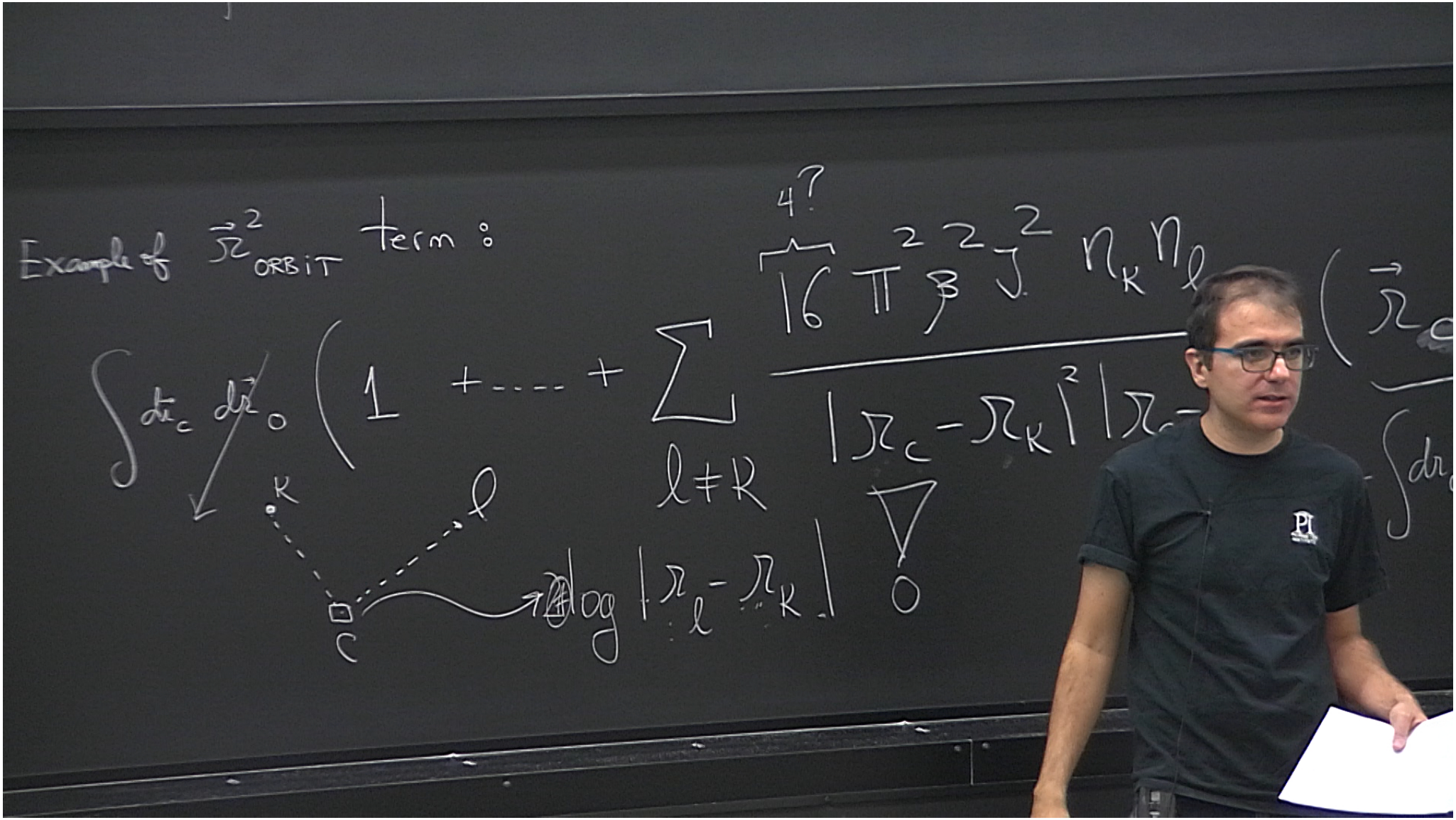
Example of $\vec{\Omega}^2_{ORBIT}$ term:

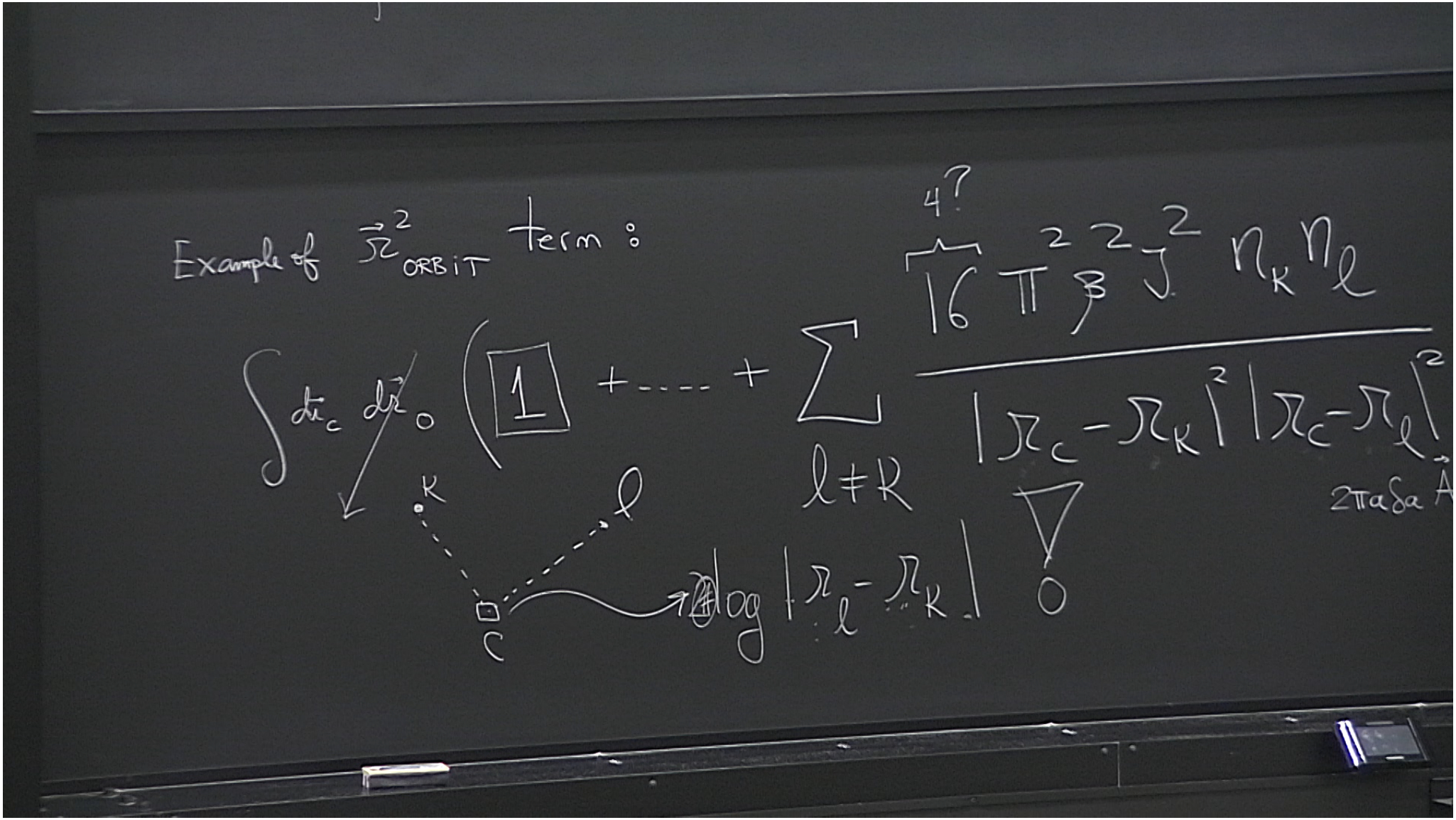
$$\int dt_c \, d\vec{x}_0 \left(1 + \cdots + \sum_{l \neq K} \frac{\overbrace{16}^{4?} \pi^2 \beta^2 J^2 n_K n_l}{|x_c - x_K|^2 |x_c - x_l|^2} \right)$$

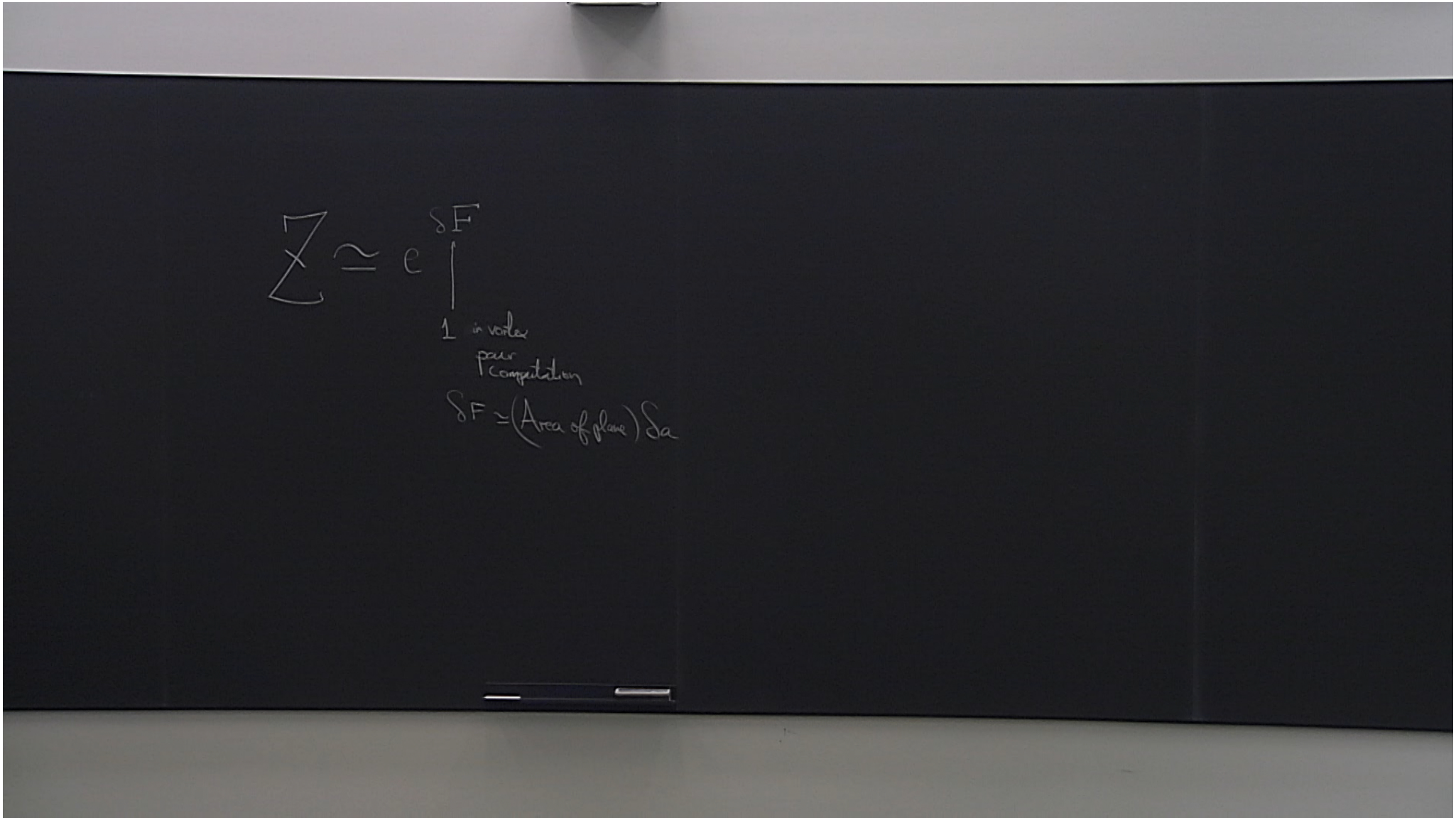


$2\pi a \delta a$ A


Example of $\vec{x}^2_{ORBIT}$ term:
4?
$\int dx_c\, dx_0 \left(1 + \cdots + \sum_{l \neq K} \frac{16\,\pi^2 \beta^2 J^2 n_K n_l}{|x_c - x_l|^2 |x_c - x_l|^2} \left(\vec{x}_c \right.\right.$
K
l
C
$\log |x_l - x_K|$
$\vec{A}\cdot\vec{B} = \int dx$


Example of $\vec{\Omega}^2_{ORBIT}$ term :
$\int dx_c\, dx_0 \left(1 + \cdots + \sum_{l \neq K} \frac{16\,\pi^2 \beta^2 J^2\, n_K n_l}{|x_c - x_K|^2 |x_c -} \left(\frac{\vec{x}_c}{\int dx_c}\right.\right.$
4?
K
l
C
$\log |x_l - x_K|$
0


Example of $\vec{\Omega}^2_{ORBIT}$ term:
$\int dx_c\, dx_0 \left(\boxed{1} + \cdots + \sum_{l \neq K} \right.$
$\frac{4?\ 16\,\pi^2 \beta^2 J^2\, n_K n_l}{|x_c - x_K|^2 |x_c - x_l|^2}$
$\left. \right)$
$2\pi a_0 a$
K
l
C
$\log |x_l - x_K|$
0

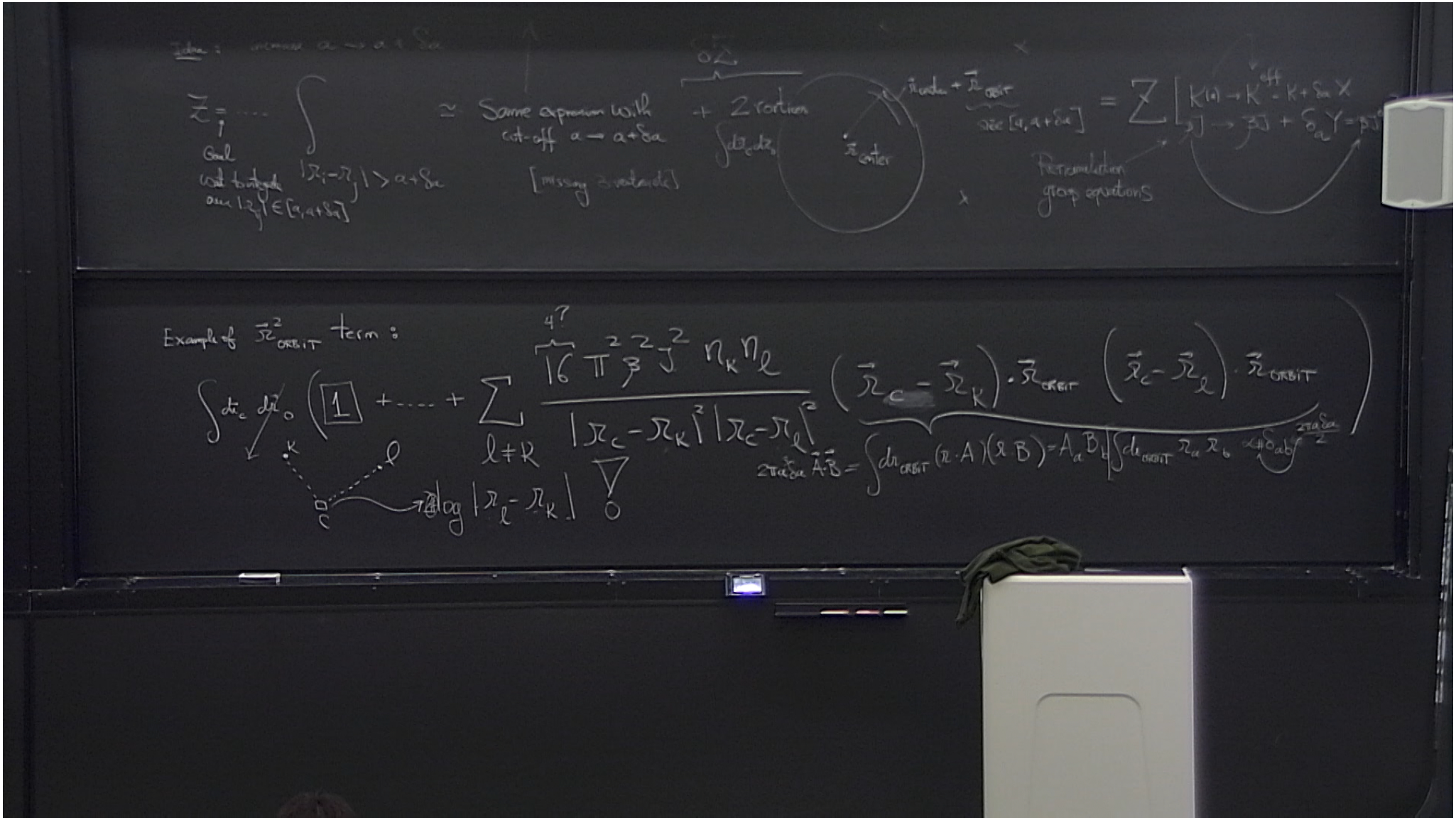

$Z \simeq \int^{\delta F} 1$
in vortex
pour
computation
$\delta F \simeq (\text{Area of plane})\,\delta a$

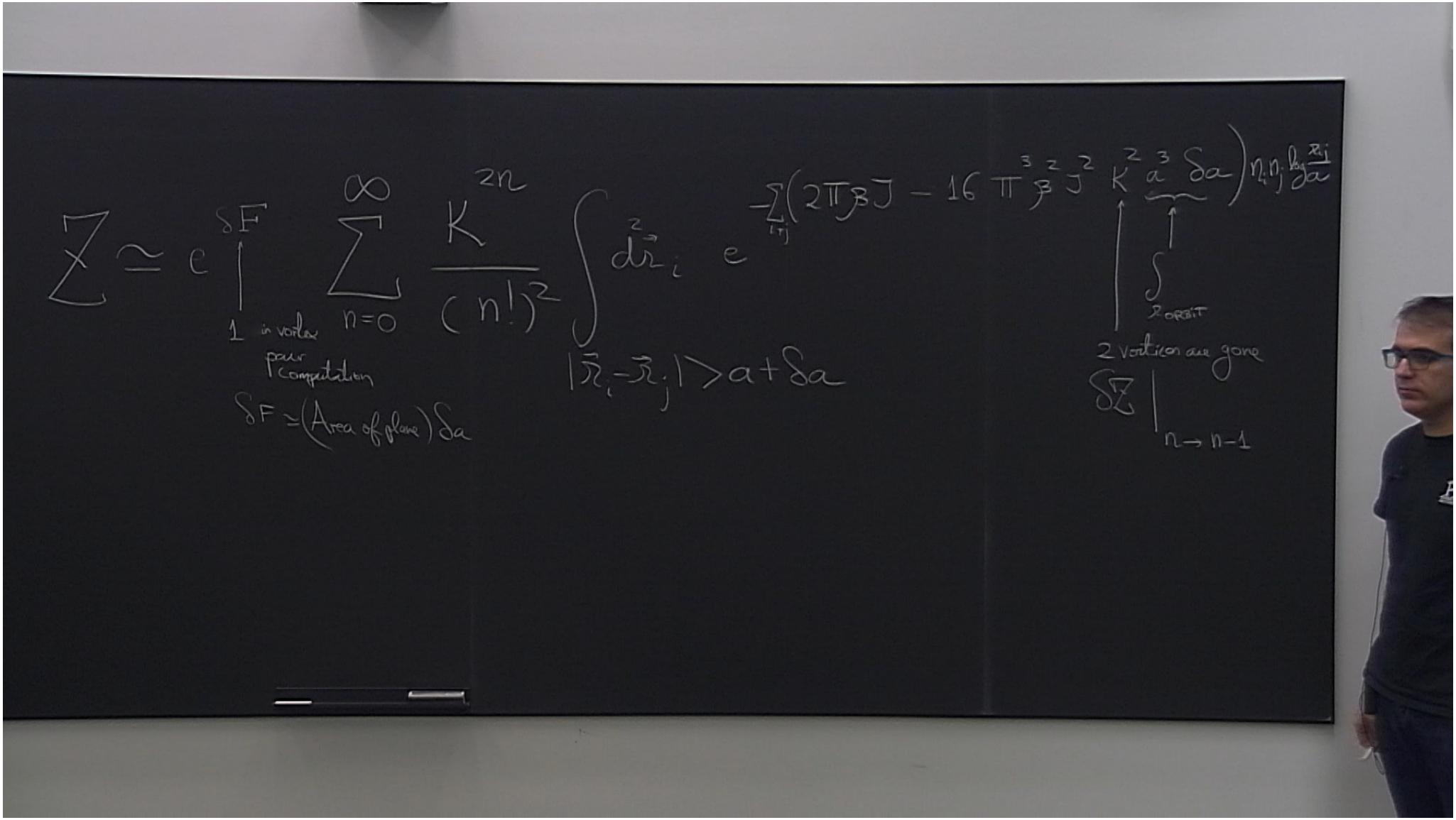
$$Z \simeq c \int_1^{\delta F} \sum_{n=0}^{\infty} \frac{K^{2n}}{(n!)^2} \int d^2\vec{r}_i \; e^{-\sum_{i,j}\left(2\pi\beta J - 16\pi^3\beta^2 J^2 K^2 a^3 \delta a\right) n_i n_j \frac{r_{ij}}{a}}$$

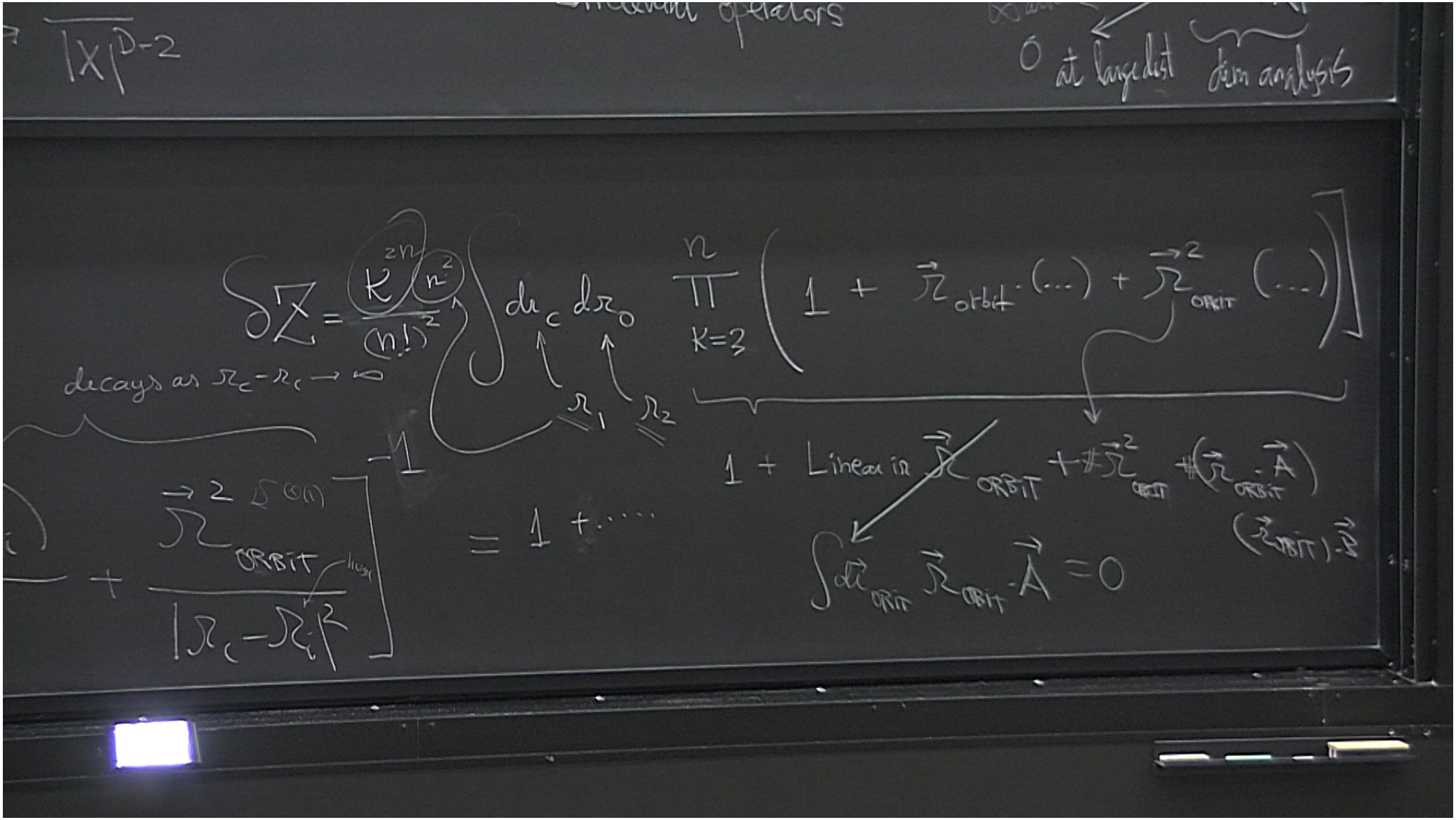
$|\vec{r}_i - \vec{r}_j| > a + \delta a$

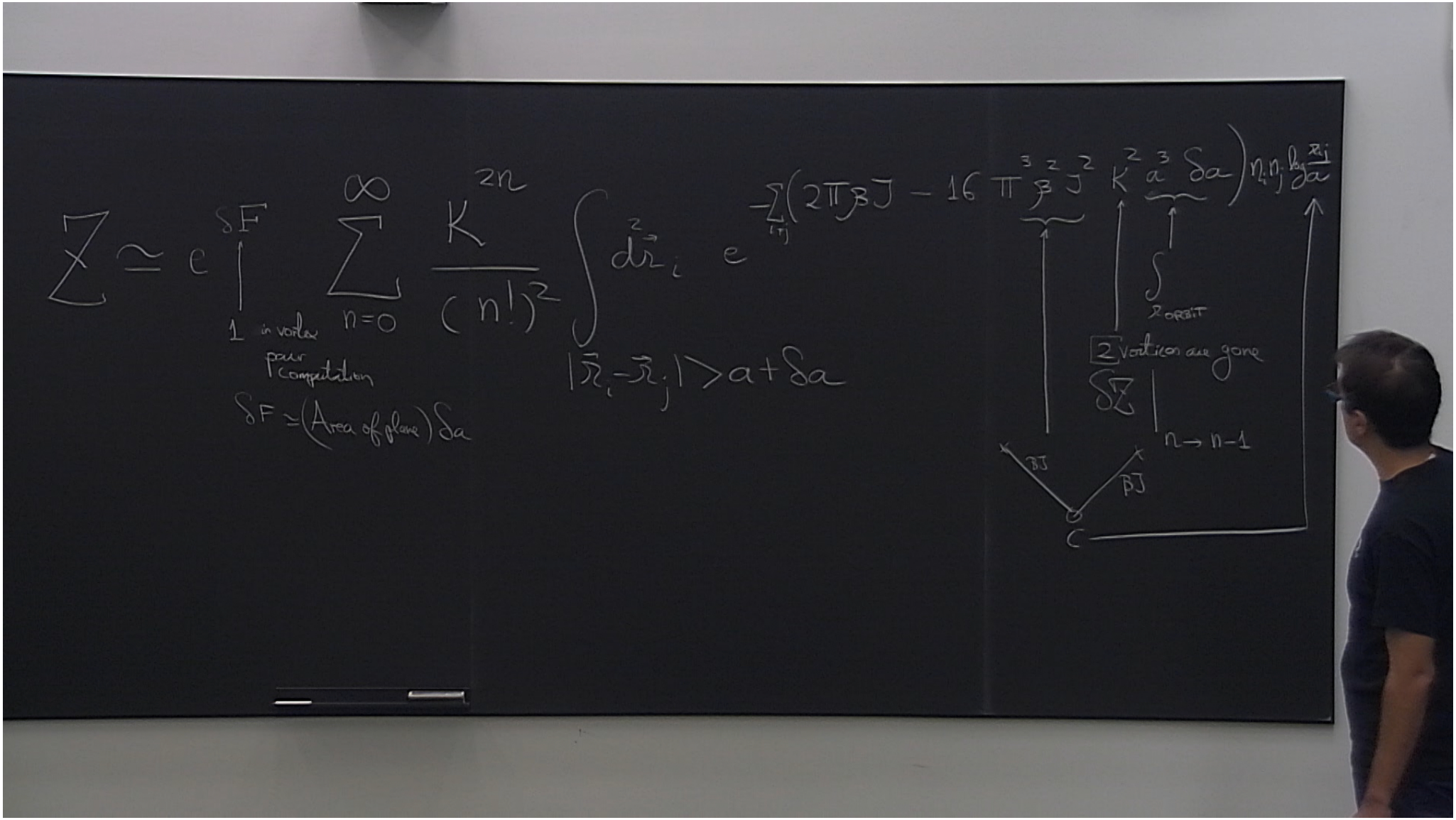
1 in voiler pour computation

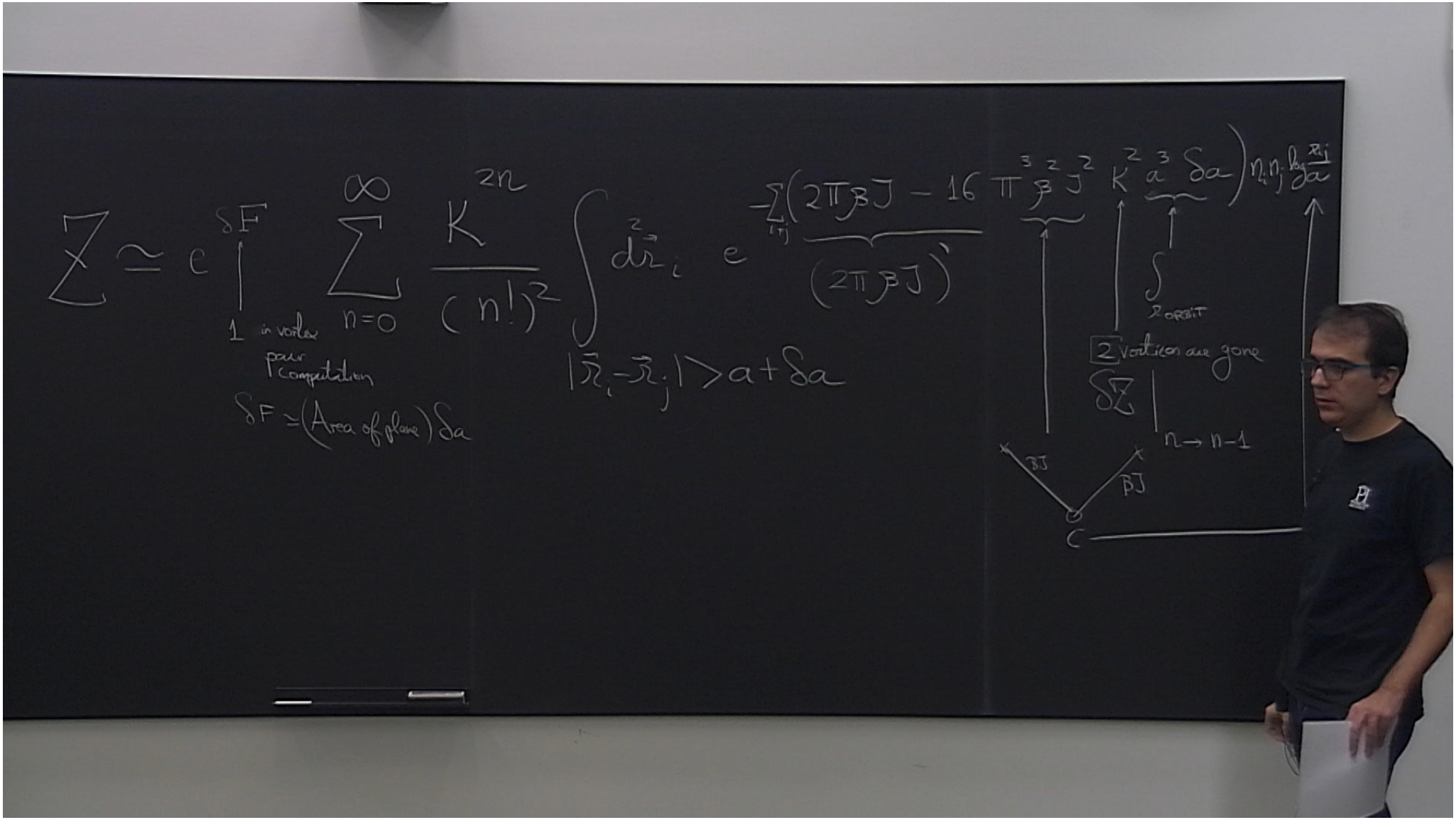
$\delta F \simeq (\text{Area of plane})\,\delta a$

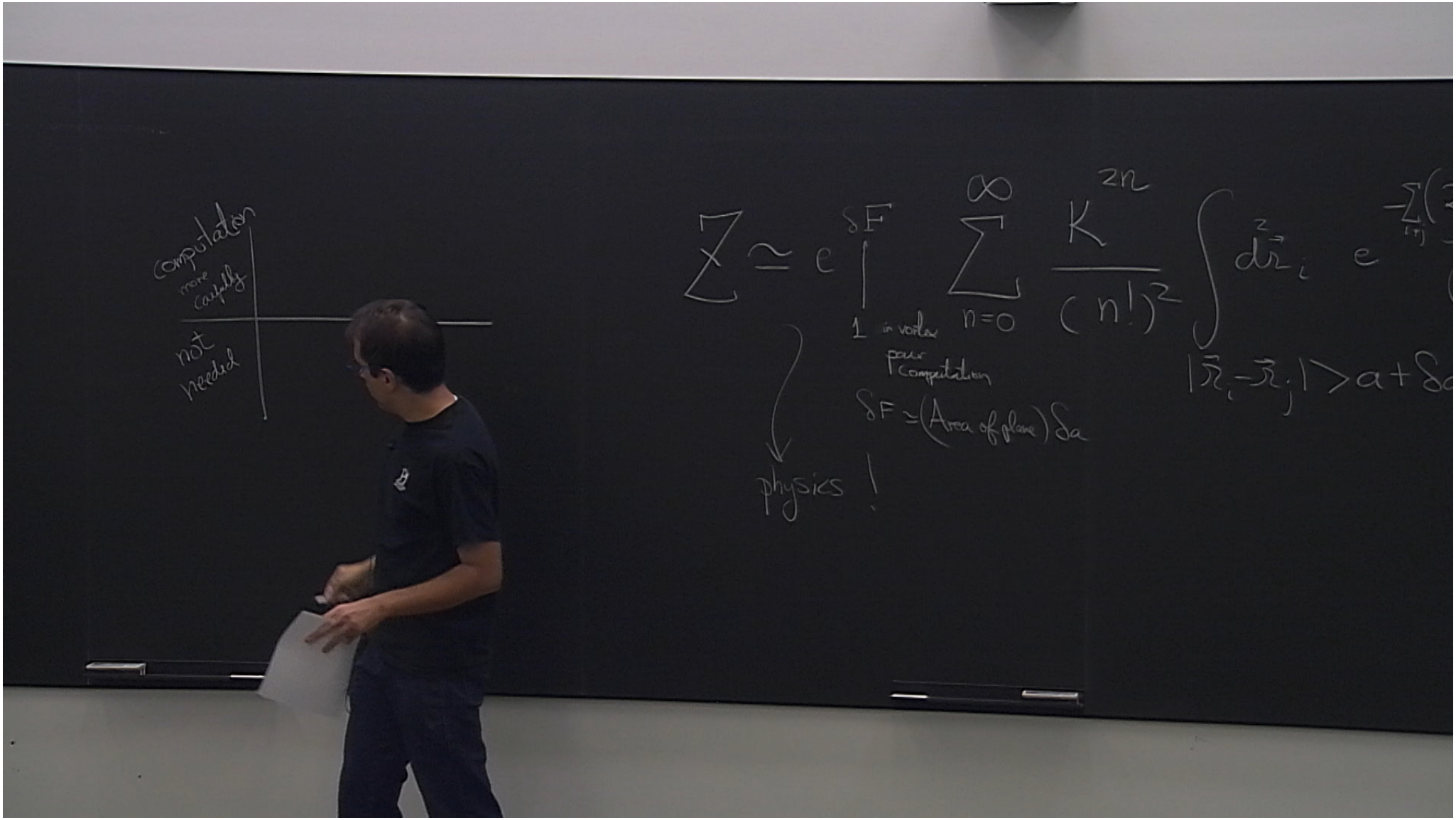

Idea : increase $a \to a + \delta a$
$\tilde{Z} = \int_{|x_i - x_j| > a + \delta a}$
Goal
cut-off integrals
over $|x_i - x_j| \in [a, a+\delta a]$
$\simeq$ Same expression with cut-off $a \to a + \delta a$
[missing 2-vertices]
$+$ 2 vertices
$\int dx_1 dx_2$
$\vec{x}_{center}$
$\vec{x}_{center} + \vec{x}_{orbit}$
size $[a, a+\delta a]$
$= Z \left[K(a) \to K^{eff} = K + \delta_a X \right.$
$\left. J \to J + \delta_a Y = \beta J \right]$
Renormalization group equations
Example of $\vec{x}^2_{ORBIT}$ term :
$\int dx_c \, dx_0 \left(1 + \dots + \sum_{l \neq K} \frac{16 \pi^2 \beta^2 J^2 \eta_K \eta_l}{|x_c - x_K|^2 |x_c - x_l|^2} \right.$
$\left. (\vec{x}_c - \vec{x}_K) \cdot \vec{x}_{ORBIT} \; (\vec{x}_c - \vec{x}_l) \cdot \vec{x}_{ORBIT} \right)$
K
l
c
$\to \log |x_l - x_K|$
0
$2\pi a \delta a \, \vec{A} \cdot \vec{B} = \int dx_{ORBIT} \, (x \cdot A)(x \cdot B) = A_a B_b \int dx_{ORBIT} \, x_a x_b$
$= \delta_{ab} \frac{2\pi a \delta a}{2}$

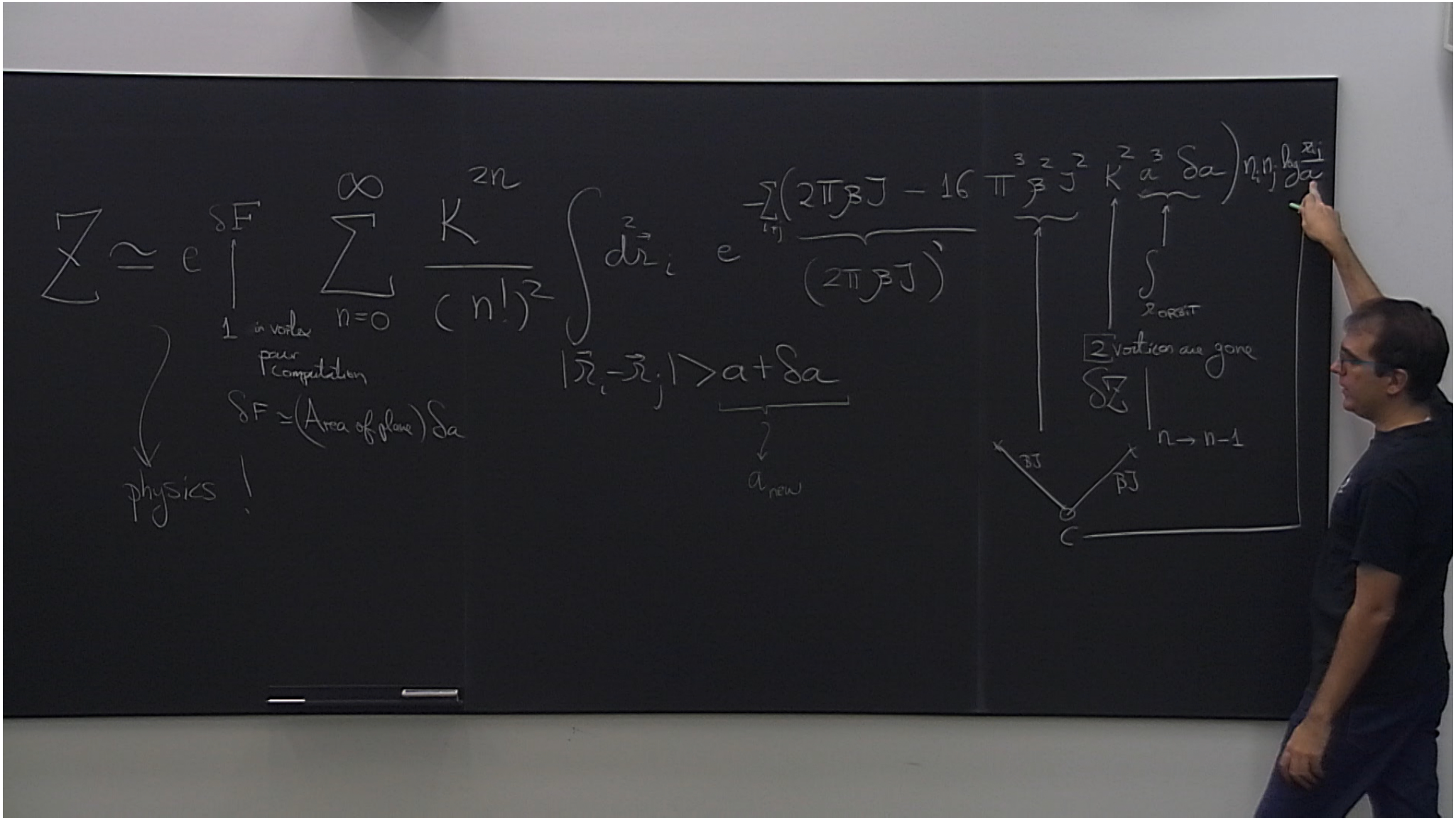

$Z \simeq c \int^{\delta F} \sum_{n=0}^{\infty} \frac{K^{2n}}{(n!)^2} \int d\vec{r}_i^{\,2}\, e^{-\sum_{i<j} \left(2\pi\beta J - 16\pi^3\beta^2 J^2 K^2 a^3 \delta a\right) n_i n_j \log \frac{r_{ij}}{a}}$
1 in voiles
pour computation
$\delta F \simeq (\text{Area of plane})\,\delta a$
$|\vec{r}_i - \vec{r}_j| > a + \delta a$
$\int$ ORBIT
2 vortices are gone
$\delta Z \Big|_{n \to n-1}$

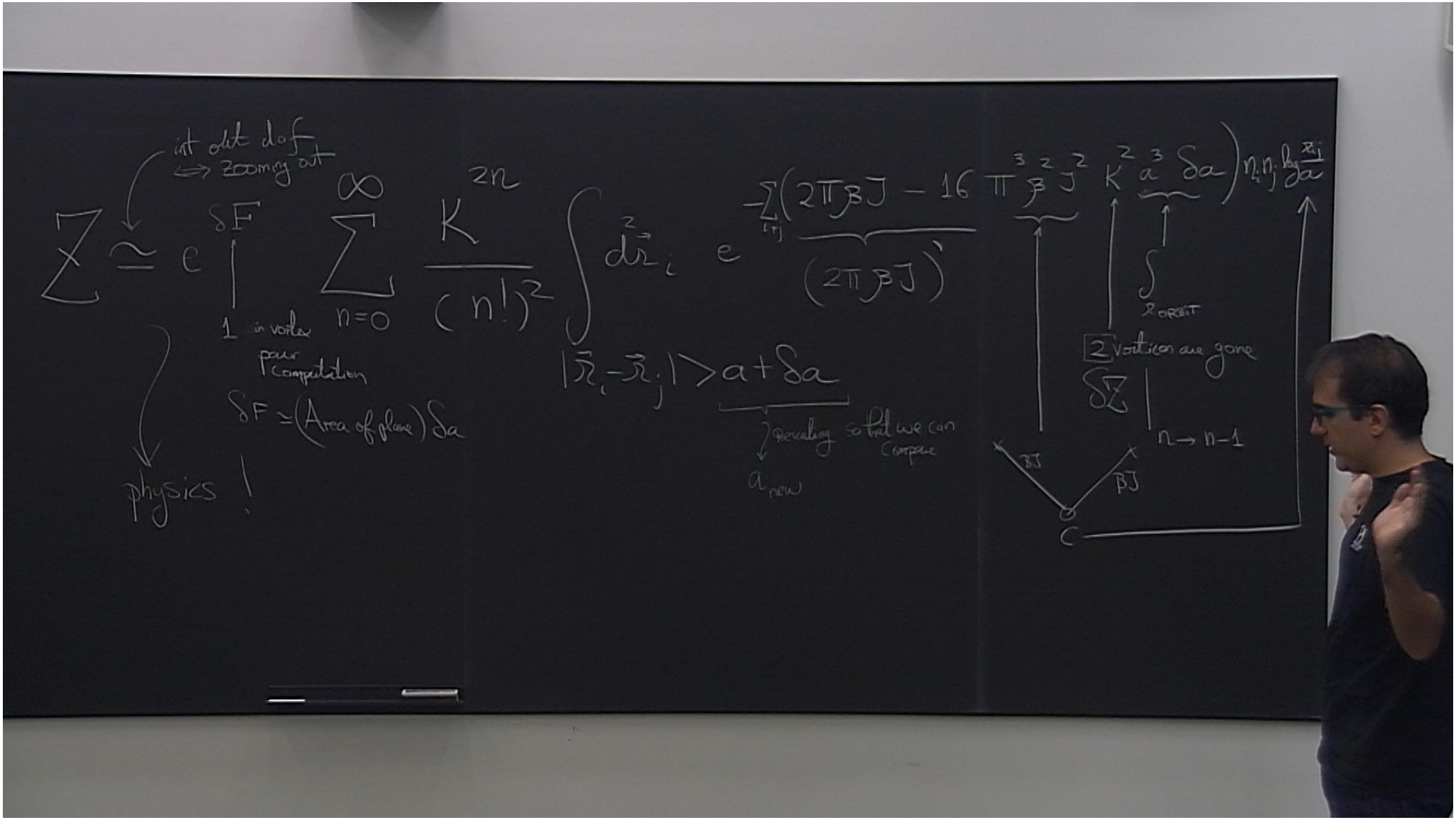

$|X|^{D-2}$
irrelevant operators
0 at large dist
dim analysis
$\delta Z = \frac{K^{2n}}{(n!)^2} n^2 \int d\tau_c \, d\tau_0 \prod_{k=3}^{n} \left(1 + \vec{r}_{orbit} \cdot (\dots) + \vec{r}^{\,2}_{ORBIT} (\dots)\right)$
decays as $\tau_c - \tau_i \to \infty$
τ_1 τ_2
-1
$= 1 + \dots$
$1 +$ Linear in $\vec{r}_{ORBIT} + \vec{r}^{\,2}_{ORBIT} + (\vec{r}_{ORBIT} \cdot \vec{A})$
$(\vec{r}_{ORBIT}) \cdot \vec{A}$
$\int d\vec{r}_{ORBIT} \, \vec{r}_{ORBIT} \cdot \vec{A} = 0$
$+ \frac{\vec{r}^{\,2}_{ORBIT}}{|\tau_c - \tau_i|^2}$
linear

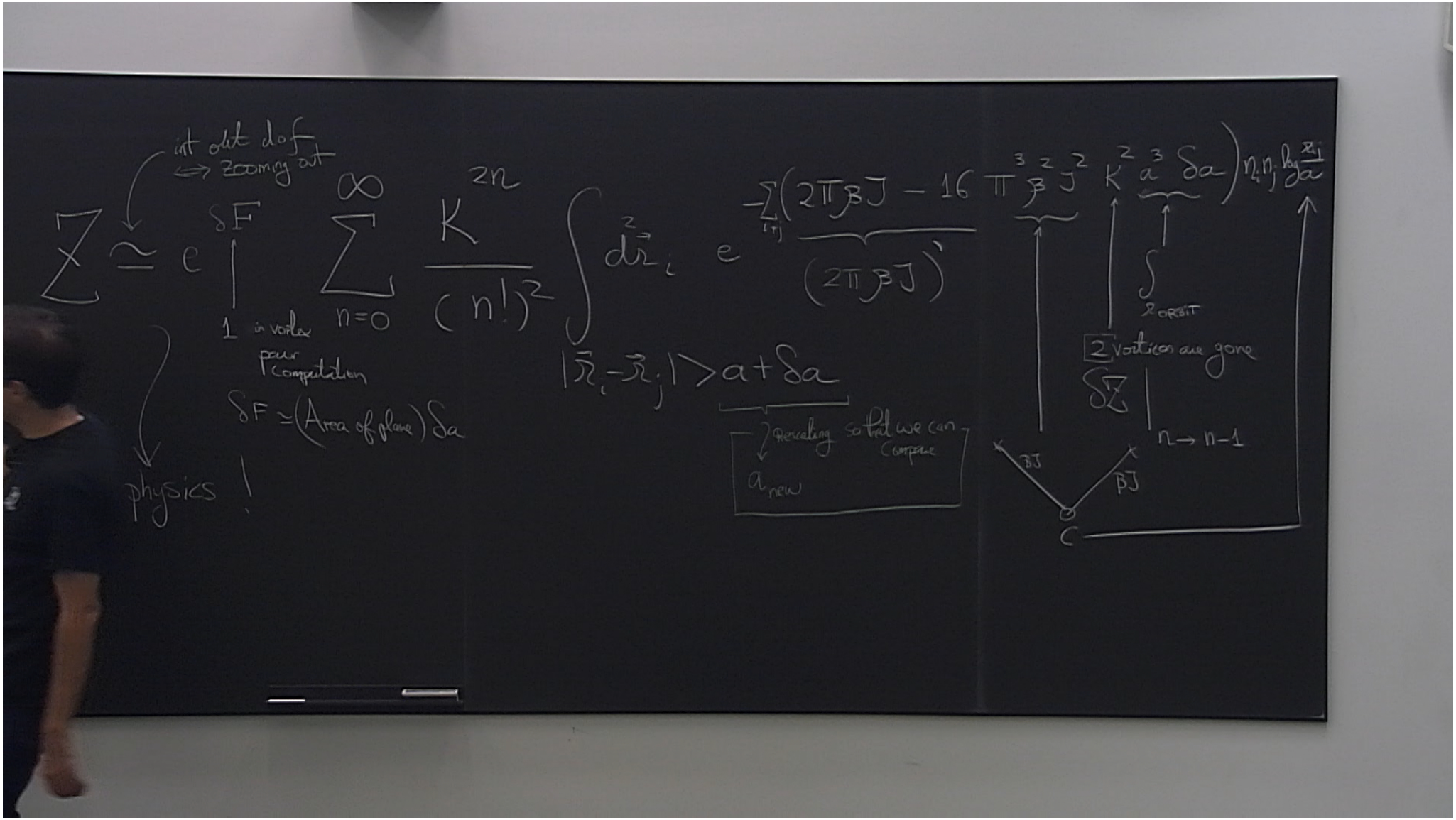

$Z \simeq c \int_1^{\delta F} \sum_{n=0}^{\infty} \frac{K^{2n}}{(n!)^2} \int d\vec{r}_i \, e^{-\sum_{ij}(2\pi\beta J - 16\pi^3\beta^2 J^2 K^2 a^3 \delta a)} n_i n_j \log \frac{r_{ij}}{\delta a}$
1 in volar
pour computation
$\delta F \simeq$ (Area of plane) δa
$|\vec{r}_i - \vec{r}_j| > a + \delta a$
$\int$ ORBIT
2 vortices are gone
δZ
$n \to n-1$
βJ
βJ
C


$Z \simeq c \int_1^{\delta F} \sum_{n=0}^{\infty} \frac{K^{2n}}{(n!)^2} \int d\vec{r}_i^{\,2} \, e^{-\sum_{ij} \frac{(2\pi\beta J - 16\pi^3\beta^2 J^2 K^2 a^3 \delta a)}{(2\pi\beta J)}} n_i n_j \frac{\vec{r}_{ij}}{\delta a}$
1 in volume
pour computation
$\delta F \simeq$ (Area of plane) δa
$|\vec{r}_i - \vec{r}_j| > a + \delta a$
$\int$ ORBIT
2 vortices are gone
δZ
$n \to n-1$
βJ
βJ
C


computation
more
costly
not
needed
$Z \simeq e^{\delta F} \sum_{n=0}^{\infty} \frac{K^{2n}}{(n!)^2} \int d\vec{r}_i \, e^{-\sum_{(ij)}}$
1 in vortex
pour
Computation
$\delta F \simeq (\text{Area of plane})\,\delta a$
physics !
$|\vec{r}_i - \vec{r}_j| > a + \delta a$


physics !
1 in vortex pour l'computation
$\delta F \simeq (\text{Area of plane})\,\delta a$
$|\vec{r}_i - \vec{r}_j| > a + \delta a$
a_{new}
γ_{ORBIT}
2 vortices are gone
$n \to n-1$
βJ
βJ
C


int dof ⟺ zooming out
physics !
1 in vortex pour Computation
δF ≃ (Area of plane) δa
|r_i − r_j| > a + δa
a_new
iterating so that we can compare
2 vortices are gone
n → n−1
βJ
βJ
C


int dof ⟺ zooming out
physics !
1 in vortex pour Computation
δF ≃ (Area of plane) δa
|r_i − r_j| > a + δa
Recalling so that we can Compare
a_new
2 Vortices are gone
n → n−1
βJ
βJ
C
r_ORBIT