Title: PSI 2018/2019 - Statistical Mechanics - Lecture 7

Date: Oct 19, 2018 10:45 AM

URL: http://pirsa.org/18100029

Abstract:

$$Z = \int \prod d\vec{x}_{\vec{n}} \; \exp\left[- S_{LATICE}[\vec{x}_{\vec{n}}] \right] \quad , \quad S_{LATICE}$$

transforms in vector representation of O(N)

LARGE DISTANCES, $x_{\vec{n}} \to x(\vec{n})$ smooth

field

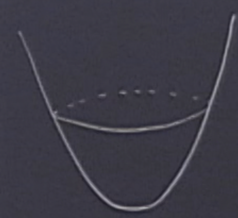
$$Z = \int \mathcal{D}\vec{x}(\vec{n}) \; \exp\left[- S_{FT}[\vec{x}(\vec{n})] \right] \quad , \quad S_{FT} = \int d^D \vec{x}$$

smooth space coordinate

$\vec n]]$, $S_{LATICE} = \sum_{\vec n} \frac{1}{4} \sum_{\vec l} \left(\vec x_{\vec n} - \vec x_{\vec n + \vec l}\right)^2 + \frac{m^2}{2} \vec x_{\vec n}^2 + \mu \left(\vec x_n^2\right)^2$

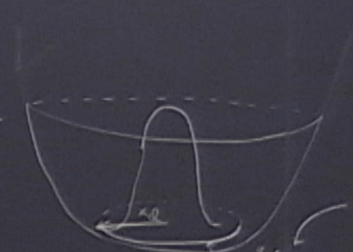
transforms in vector representation of O(N)

$V(\vec x) =$



$m^2 > 0$

or



$m^2 < 0$

N-1 dimensional

line of minima

$S_{FT} = \int d^D \vec x \left(\frac{1}{2} (\partial_\mu \vec x)^2 + \overbrace{\frac{m^2}{2} \vec x^2 + \mu \vec x^4}^{V(\vec x)} \right)$

ite

$$X = X_{cl} + \delta x$$

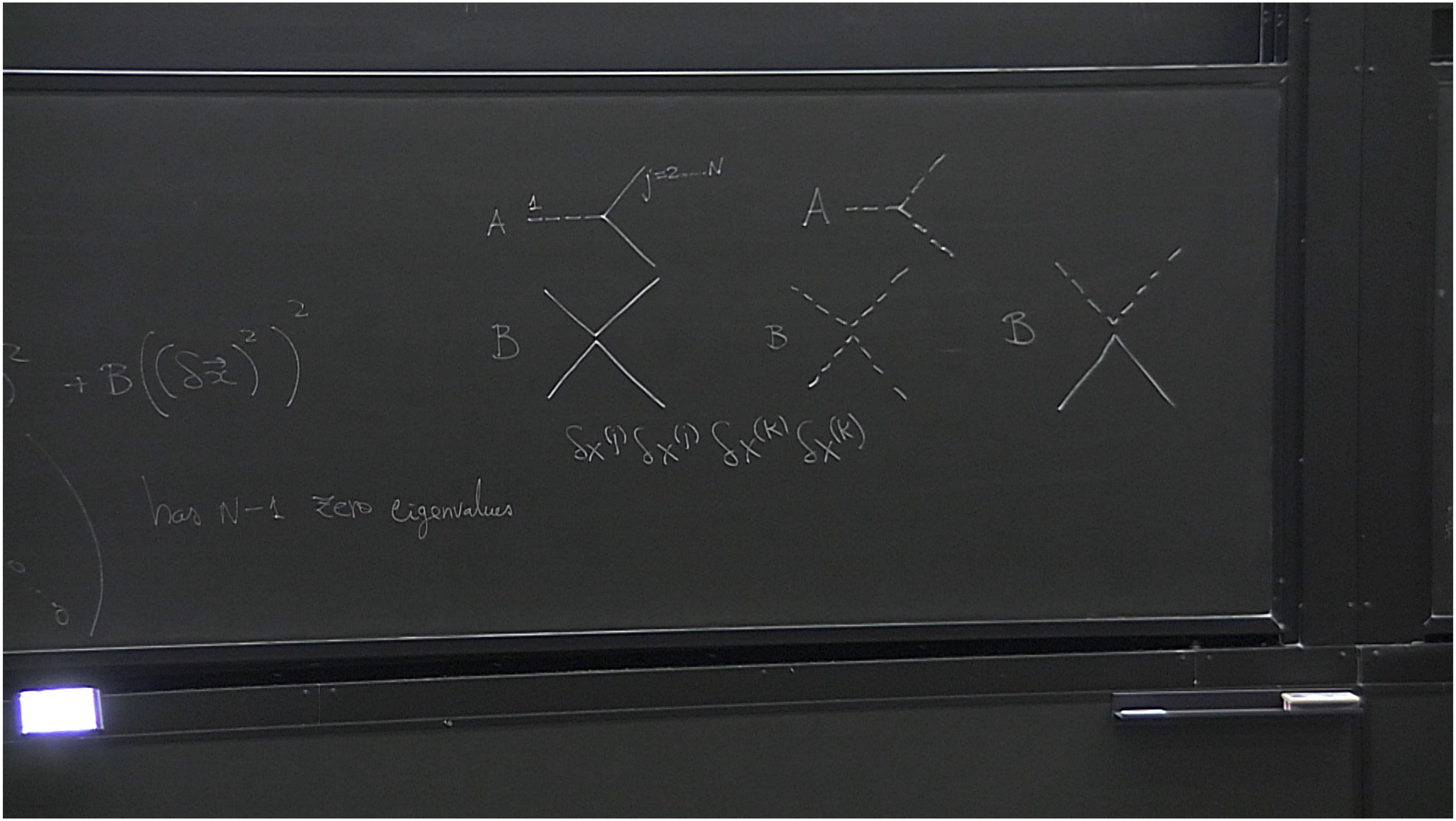
X_{cl}: const $\propto (1, 0, 0, \dots, 0)$

$N-1$ massless modes $\delta x^{(j)}$, $j = 2, \dots, N$

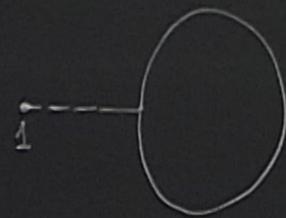
$$S = \int \frac{1}{2}\left(\partial_\mu \delta \vec{x}\right)^2 + \underbrace{0 + \frac{\tilde{m}^2}{2}\left[\delta x^{(1)}\right]^2}_{\frac{1}{2}\delta x^{(j)}\left[m^2_{jk}\right]\delta x^{(k)}} + A\,\delta x^{(1)}\left(\delta \vec{x}\right)^2 + B\left(\left(\delta \vec{x}\right)^2\right)^2$$

$$\begin{pmatrix} \tilde{m}^2 & & & & \\ & 0 & & & \\ & & 0 & & \\ & & & 0 & \\ & & & & \ddots \; 0 \end{pmatrix}$$

has $N-1$ zero


A
1
j=2....N
A
B
B
B
$+B((\delta\vec{x})^2)^2$
$\delta x^{(i)}\delta x^{(i)}\delta x^{(k)}\delta x^{(k)}$
has N−1 zero eigenvalues

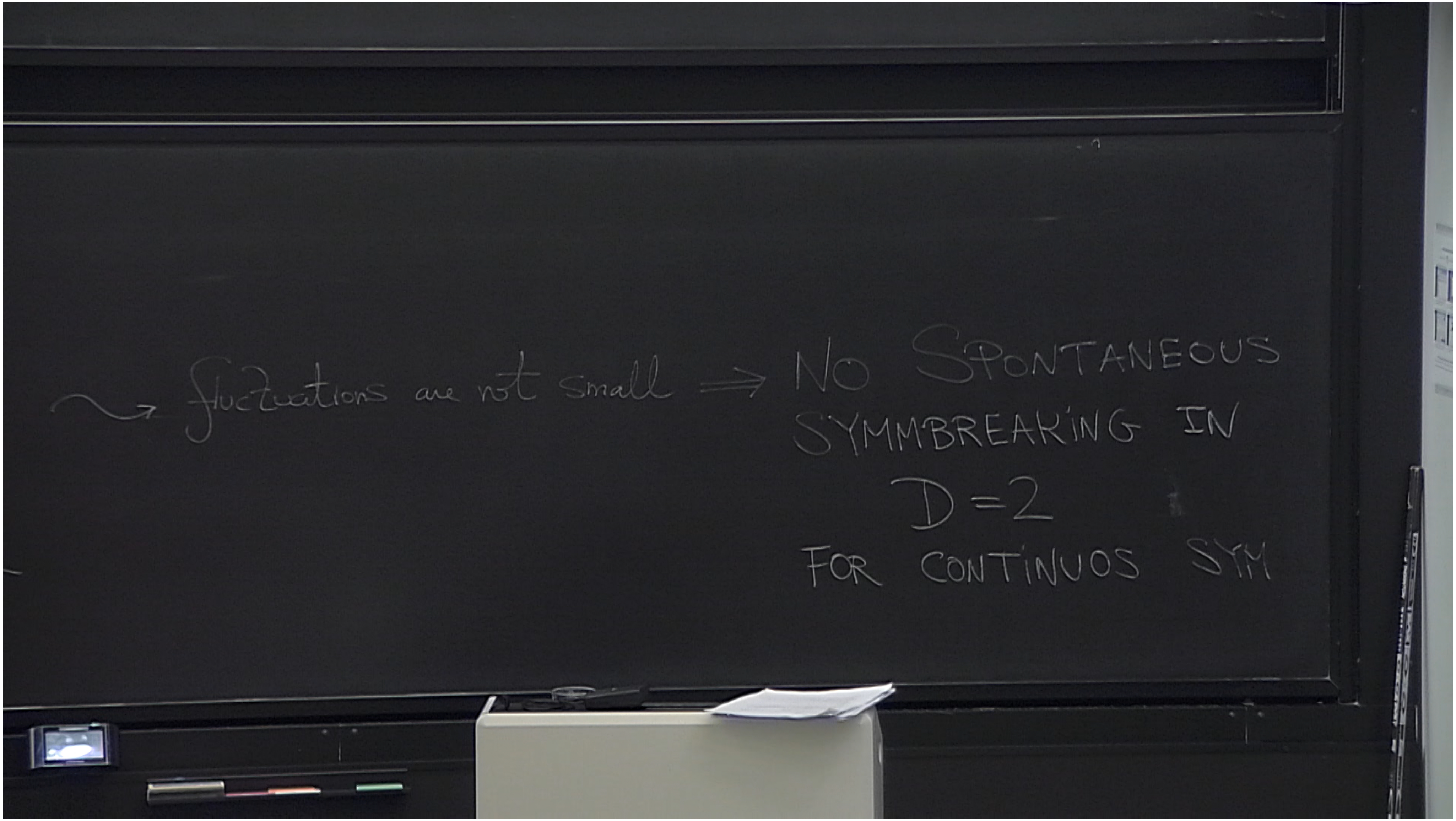
$$\langle \delta x^{(1)} \rangle =$$



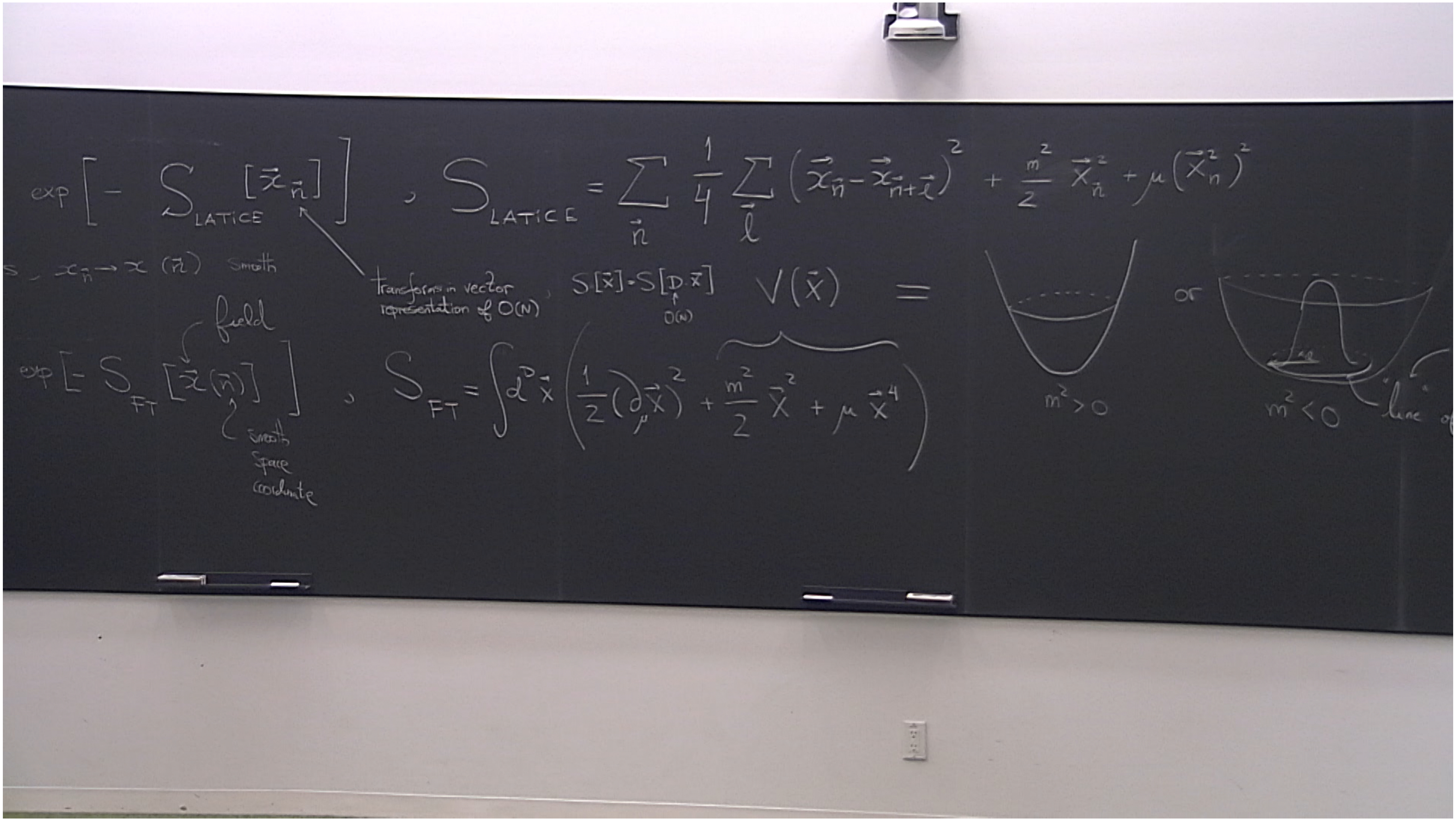
$$+ \cdots$$

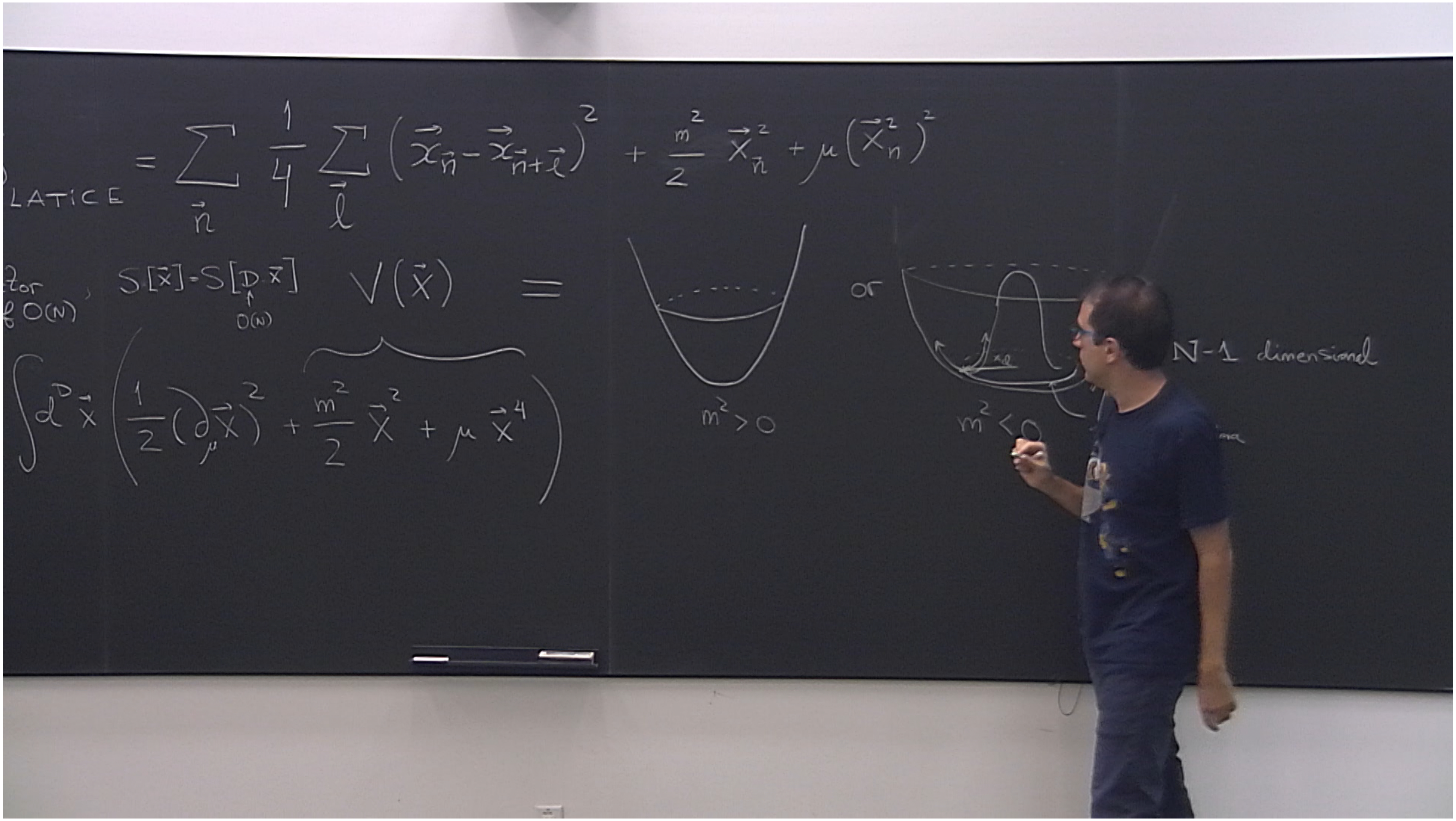
$$\int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2} = \begin{cases} \text{finite} & D > 2 \\ \infty & D = 2 \end{cases}$$

$\rightsquigarrow$ fluc


fluctuations are not small ⟹ NO SPONTANEOUS
SYMMBREAKING IN
D=2
FOR CONTINUOS SYM

SPONT. ~~SYM~~	D=1	D=2	D>2
DISCRETE	×	✓	✓
CONTINUOUS	×	×	✓


$\exp\left[-S_{LATICE}[\vec{x}_{\vec{n}}]\right]$, $S_{LATICE} = \sum_{\vec{n}} \frac{1}{4} \sum_{\vec{l}} \left(\vec{x}_{\vec{n}} - \vec{x}_{\vec{n}+\vec{l}}\right)^2 + \frac{m^2}{2} \vec{x}_{\vec{n}}^2 + \mu\left(\vec{x}_n^2\right)^2$
s , $x_{\vec{n}} \to x(\vec{n})$ smooth
transforms in vector representation of O(N)
$S[\vec{x}] = S[D \cdot \vec{x}]$
O(N)
$V(\vec{x}) =$
or
$m^2 > 0$
$m^2 < 0$
line of
field
$\exp\left[-S_{FT}[\vec{x}(\vec{n})]\right]$, $S_{FT} = \int d^D \vec{x} \left(\frac{1}{2}(\partial_\mu \vec{x})^2 + \frac{m^2}{2}\vec{x}^2 + \mu \vec{x}^4\right)$
smooth space coordinate



$\mathcal{L}_{\text{LATICE}} = \sum_{\vec{n}} \frac{1}{4} \sum_{\vec{l}} \left(\vec{x}_{\vec{n}} - \vec{x}_{\vec{n}+\vec{l}}\right)^2 + \frac{m^2}{2} \vec{x}_{\vec{n}}^2 + \mu \left(\vec{x}_n^2\right)^2$
tor
f O(N)
$S[\vec{x}] = S[D \cdot \vec{x}]$
O(N)
$V(\vec{x}) =$
$\int d^D \vec{x} \left(\frac{1}{2} (\partial_\mu \vec{x})^2 + \frac{m^2}{2} \vec{x}^2 + \mu \vec{x}^4 \right)$
$m^2 > 0$
or
$m^2 < 0$
N-1 dimensional

$$Z = \int d\vec{x}_n \, \exp\left[-S[\vec{x}_n]\right] = \int d\,\delta\vec{x}_n \, \exp\left[-S[x_{cl} + \delta x_n]\right]$$

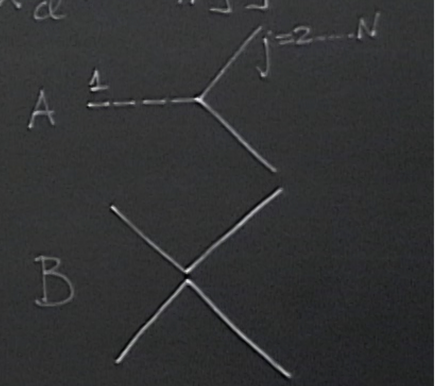
$(1, 0, 0, \ldots 0)$

N−1 massless modes $\delta x^{(j)}, \; j = 2, \ldots, N$

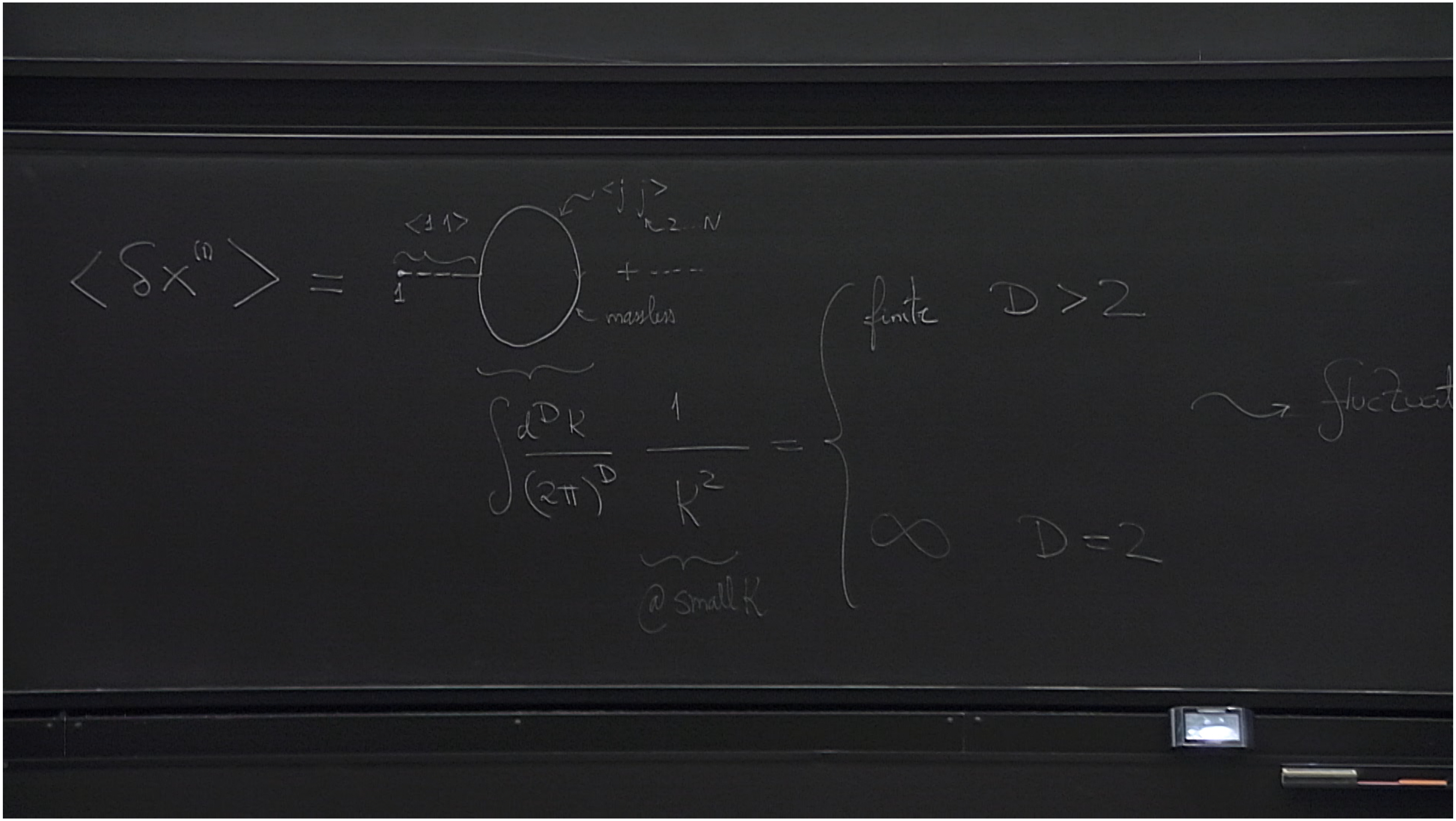
$$\ldots) + 0 + \underbrace{\frac{\tilde{m}^2}{2}\left[\delta x^{(1)}\right]^2}_{\frac{1}{2}\delta x^{(j)}\left[m^2_{jk}\right]\delta x^{(k)}} + A\,\delta x^{(1)}(\delta\vec{x})^2 + B\left((\delta\vec{x})^2\right)^2$$

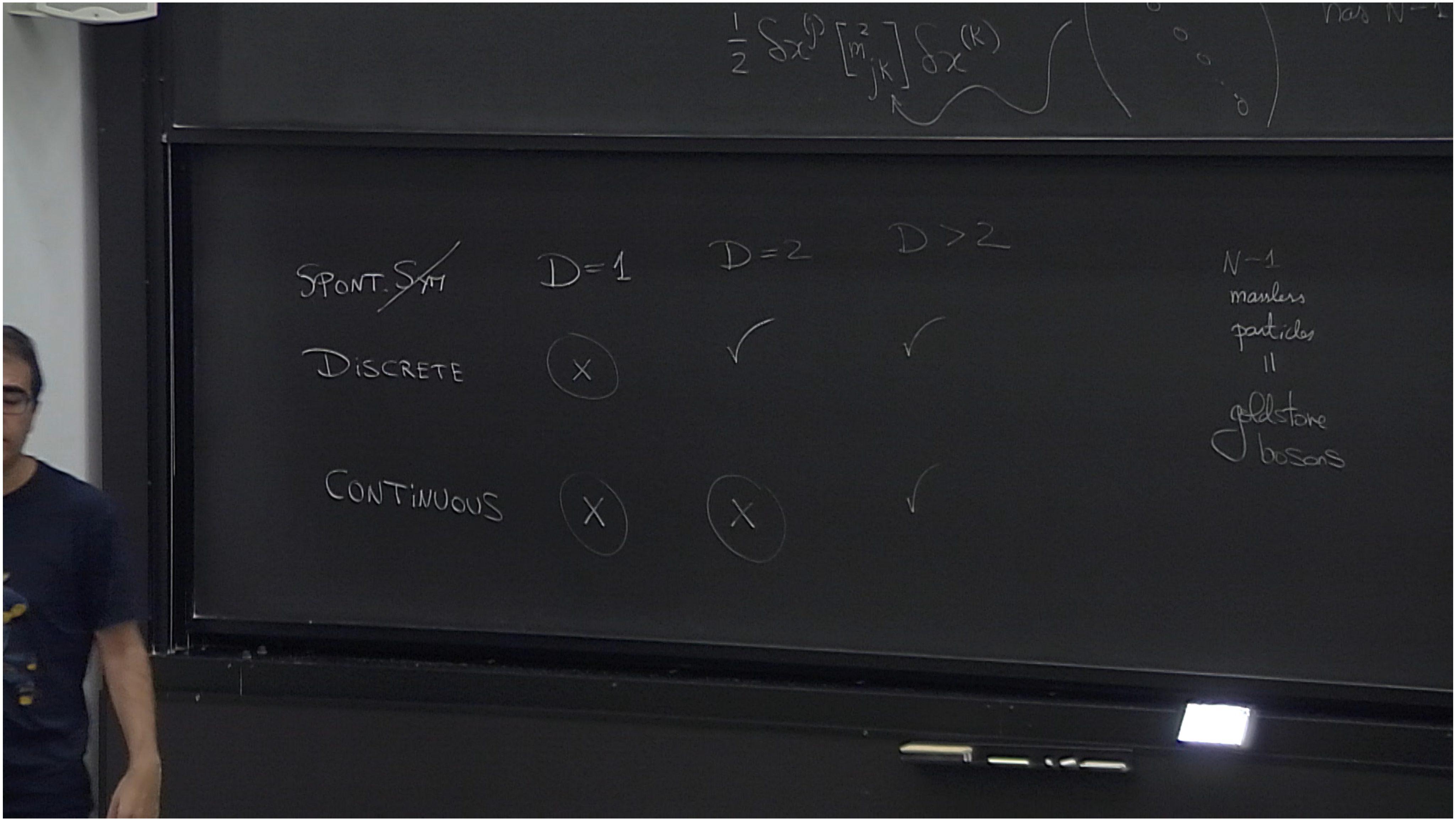
$$\begin{pmatrix} \tilde{m}^2 & & & & \\ & 0 & & & \\ & & 0 & & \\ & & & 0 & \\ & & & & \ddots \; 0 \end{pmatrix}$$

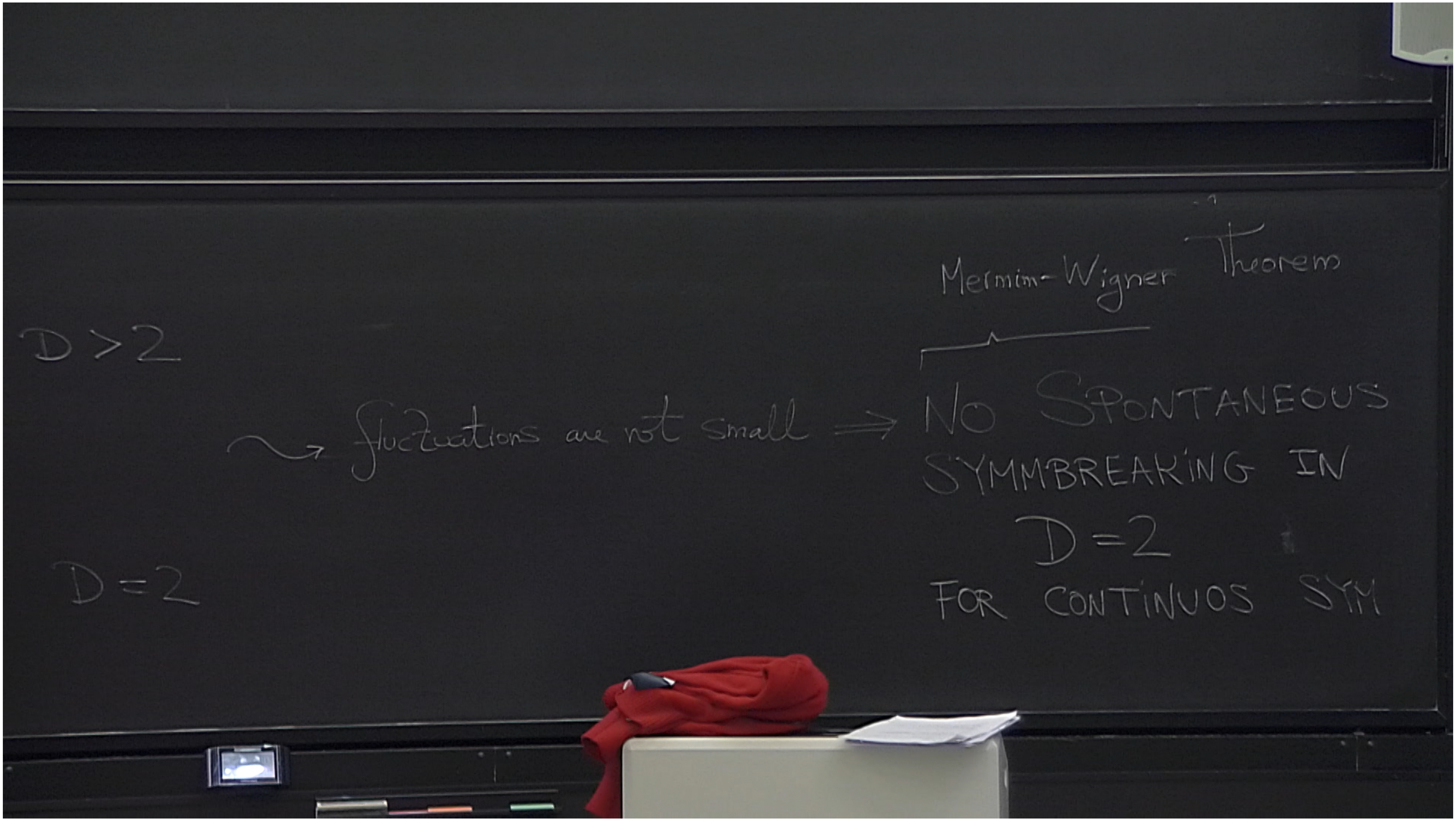
has N−1 zero eigenvalues



$\delta x^{(j)} \delta x^{(j)} \delta x$


$\langle \delta x^{(1)} \rangle =$
$\langle 1\,1 \rangle$
$\langle j\,j \rangle$
$2 \ldots N$
$+ \ldots$
1
massless
$\int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2} = \begin{cases} \text{finite} & D > 2 \\ \infty & D = 2 \end{cases}$
@ small k
fluctuat


$\frac{1}{2}\delta x^{(j)} [m^2_{jk}] \delta x^{(k)}$
SPONT. SYM
D=1
D=2
D>2
DISCRETE
CONTINUOUS
N-1
massless
particles
goldstone
bosons



Mermim-Wigner Theorem
D > 2
fluctuations are not small ⟹ NO SPONTANEOUS SYMMBREAKING IN D = 2 FOR CONTINUOS SYM
D = 2

$$\langle \delta x^{(i)} \rangle = \underbrace{\langle 1\,1\rangle}_{1} \text{ (loop: } \langle j\,j\rangle_{j=2\ldots N}\text{, massless)} + \cdots$$

$$0 \leftarrow \frac{1}{|m|^{D-2}} \text{ if } D>2$$

$$\log|m| \quad D=2 \rightarrow \infty$$

$$\sim \langle X_n X_{n+m} \rangle \longleftrightarrow \int \frac{d^D k}{(2\pi)^D} \underbrace{\frac{1}{k^2}}_{@\text{ small } k} = \begin{cases} \text{finite} & D>2 \\ \infty & D=2 \end{cases} \sim\!\!\rightarrow$$

Goldstone modes in D=2 would be path. hence they don't exist.

has N−1 zero eigenvalues

N−1
massless
particles
||
goldstone
bosons

If sym ⟶ broken ⟹ ∃ massless modes

not

GOLDSTONE
THEOREM

i.e

m^2_{ij} has zero eigenvalues

in D=2

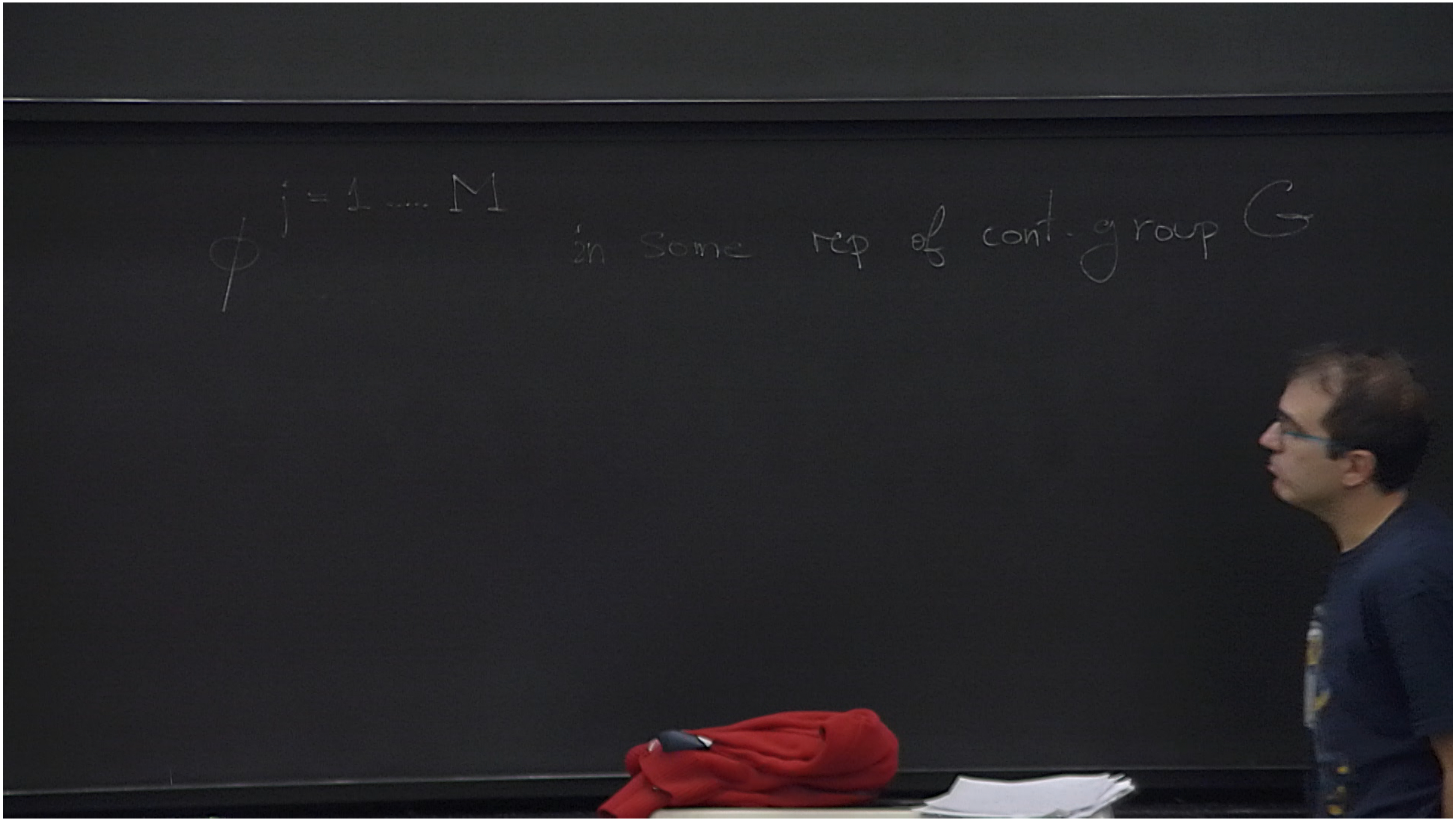
~~sym~~ ⟶ goldstone, but they are not good ⟹

no ~~sym~~

MERMIN-WIGNER

hence they don't exist.

$\vec{X}$ in vector rep of $O(N)$


$\phi^{j=1 \ldots M}$ in some rep of cont. group G

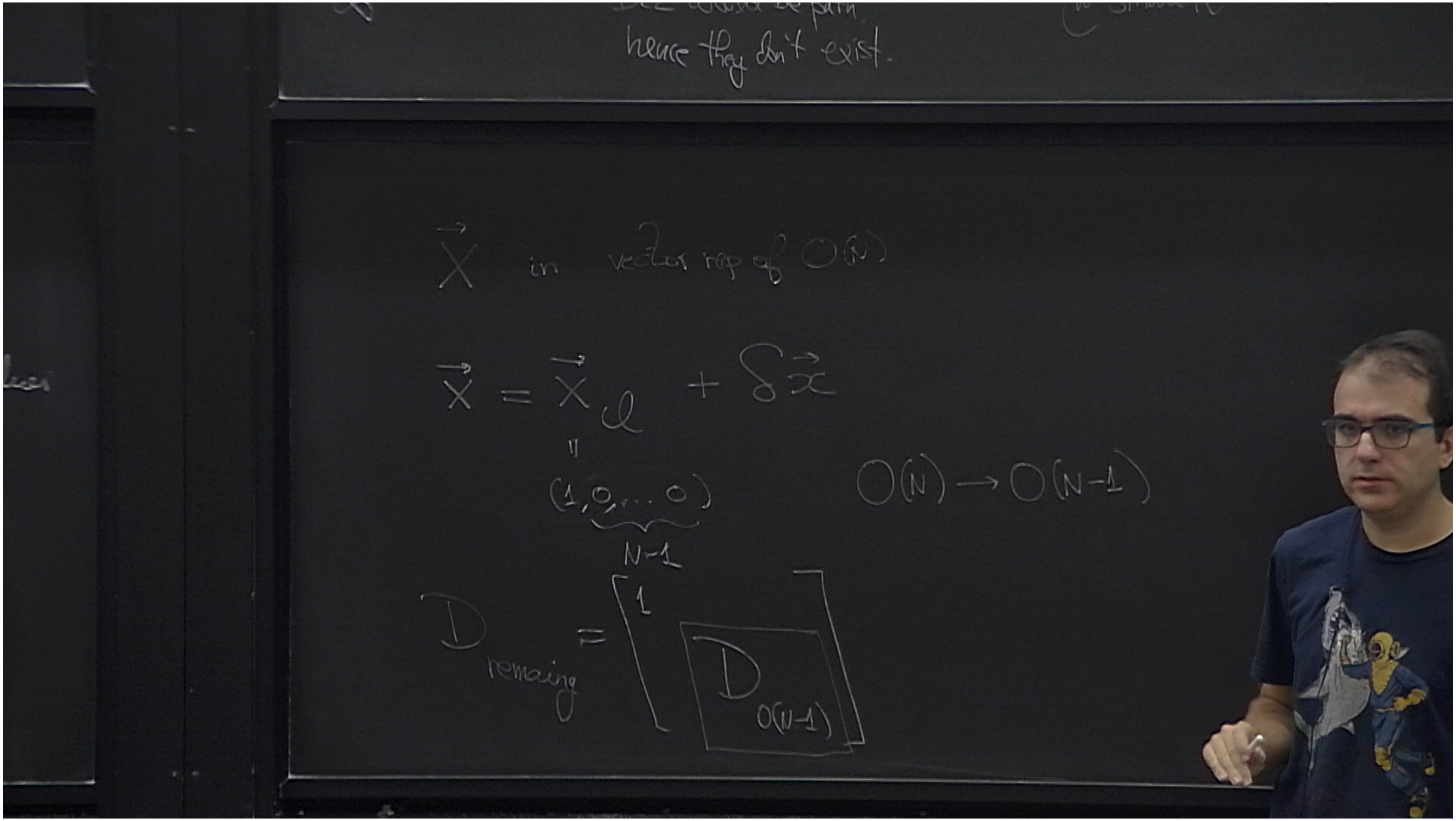
hence they don't exist.

$\vec{X}$ in vector rep of $O(N)$

$$\vec{X} = \vec{X}_{cl} + \delta\vec{x}$$

$$\vec{X}_{cl} = (1, \underbrace{0, \dots 0}_{N-1})$$

$$D_{\text{remaining}} = \begin{bmatrix} 1 & \\ & D_{O(N-1)} \end{bmatrix}$$


hence they don't exist.
X in vector rep of O(N)
X = X_cl + δx
(1, 0, ... 0)
N-1
O(N) → O(N-1)
D_remaing =
1
D_O(N-1)

hence they don't exist.

$\vec{X}$ in vector rep of $O(N)$

$$\vec{X} = \vec{X}_{cl} + \delta\vec{x}$$

$$\vec{X}_{cl} = (1, \underbrace{0, \ldots 0}_{N-1})$$

$$O(N) \rightarrow O(N-1)$$

$\frac{N(N-1)}{2}$ symmetries

$$D_{\text{remaing}} = \begin{bmatrix} 1 & \\ & D_{O(N-1)} \end{bmatrix}$$

hence they don't exist.

$\vec{X}$ in vector rep of $O(N)$

$$\vec{X} = \vec{X}_{cl} + \delta\vec{x}$$

$$\vec{X}_{cl} = (1, \underbrace{0, \ldots 0}_{N-1})$$

$$O(N) \to O(N-1)$$

$\frac{N(N-1)}{2}$ symmetries

$\frac{(N-1)(N-2)}{2}$ sym

$$D_{\text{remaing}} = \begin{bmatrix} 1 & \\ & \boxed{D_{O(N-1)}} \end{bmatrix}$$

hence they don't exist.

$\vec{X}$ in vector rep of $O(N)$

$$\vec{X} = \vec{X}_{cl} + \delta\vec{x}$$

$$\vec{X}_{cl} = (\underbrace{1,0,\ldots 0}_{N-1})$$

$$O(N) \to O(N-1)$$

$\frac{N(N-1)}{2}$ symmetries

$\frac{(N-1)(N-2)}{2}$ sym

DIFFERENCE $= N-1$

$$D_{remaing} = \begin{bmatrix} 1 & \\ & D_{O(N-1)} \end{bmatrix}$$

hence they don't exist.

$\vec{X}$ in vector rep of $O(N)$

$$\vec{X} = \vec{X}_{cl} + \delta\vec{x}$$

$$\vec{X}_{cl} = (1, \underbrace{0, \ldots 0}_{N-1})$$

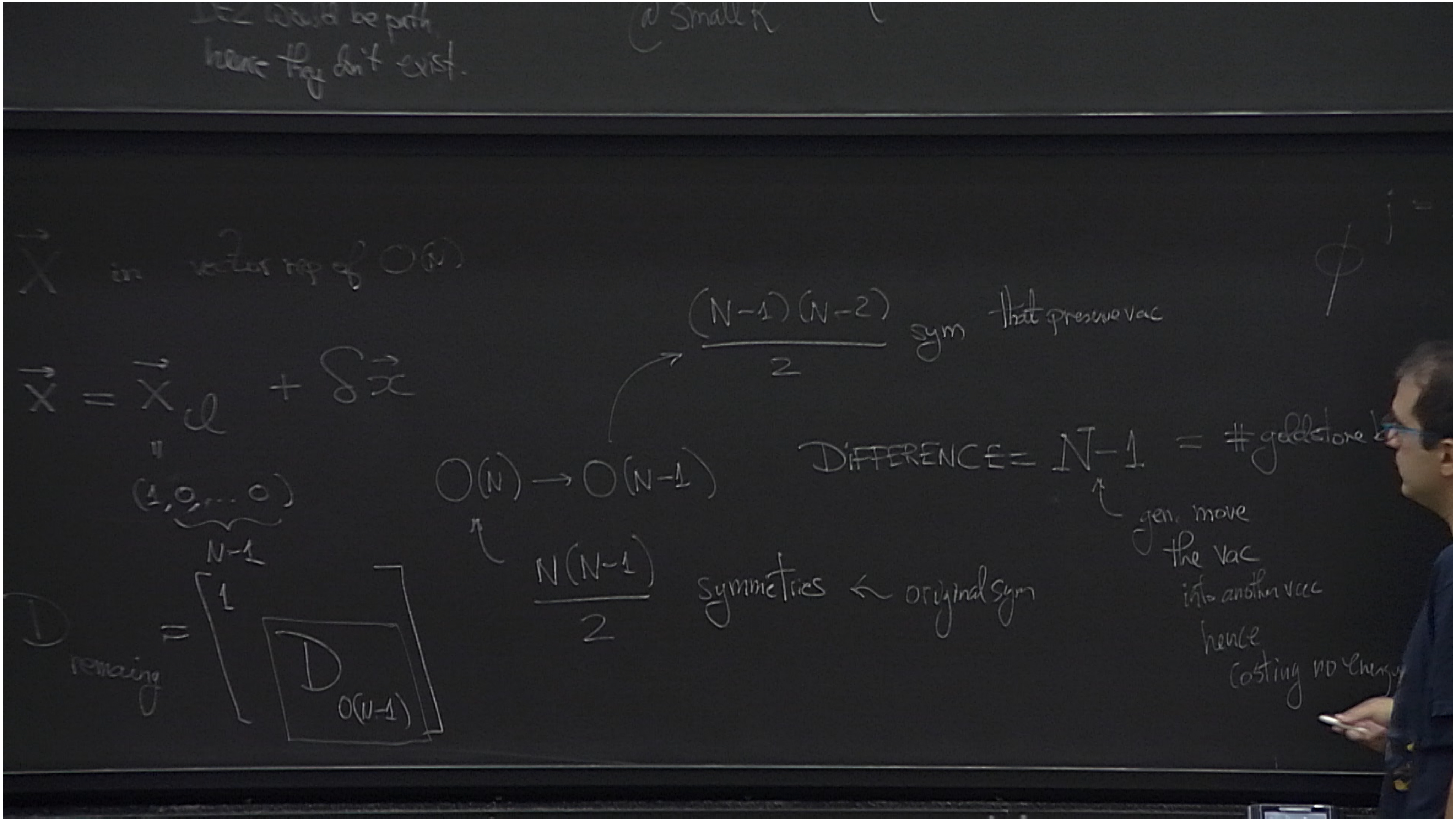
$$D_{\text{remaing}} = \begin{bmatrix} 1 & \\ & \boxed{D_{O(N-1)}} \end{bmatrix}$$

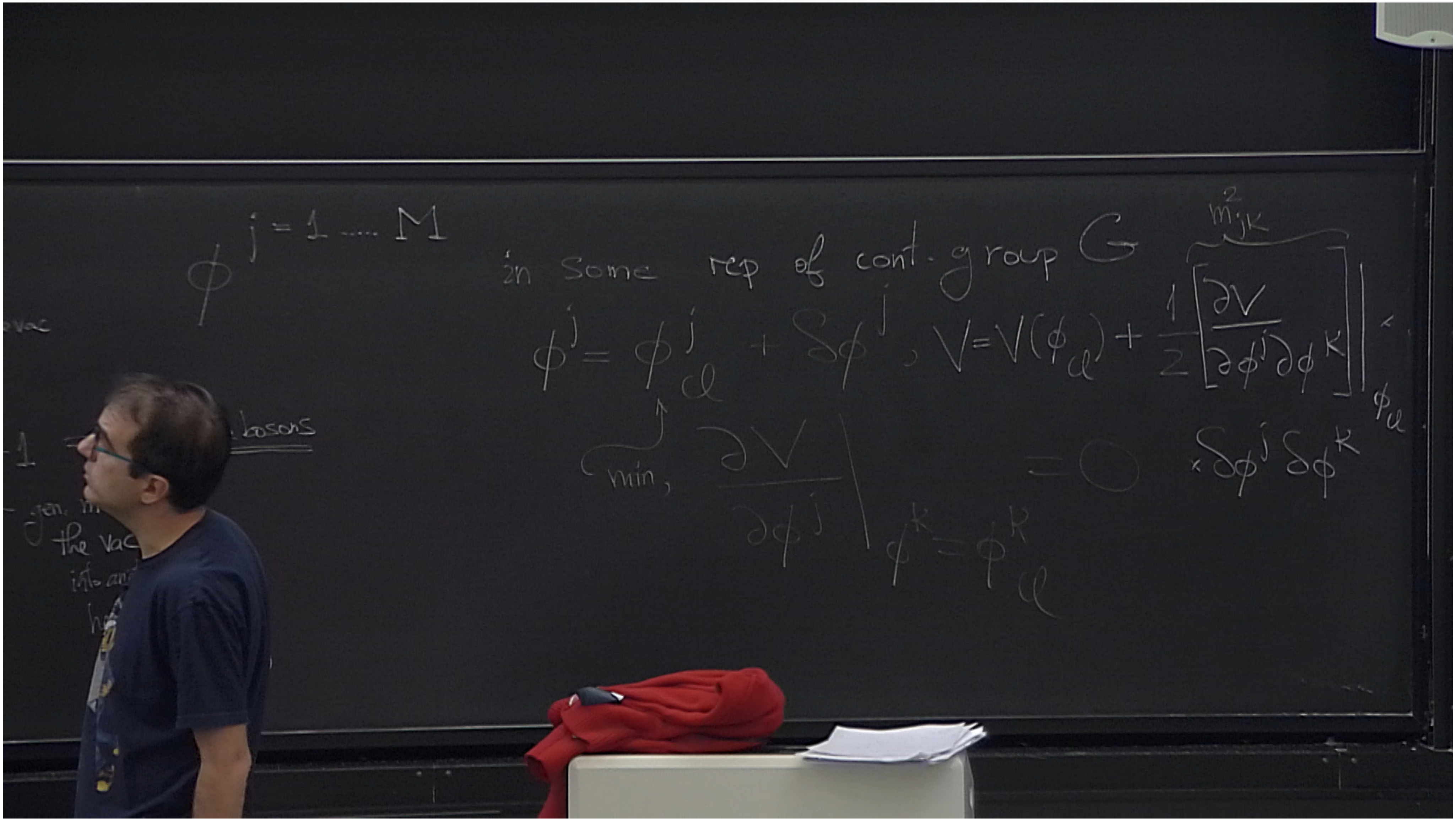
$$O(N) \longrightarrow O(N-1)$$

$\frac{N(N-1)}{2}$ symmetries ← original sym

$\frac{(N-1)(N-2)}{2}$ sym that preserve vac

DIFFERENCE $= N-1$ ← gen. move the vac


would be path.
hence they don't exist.
@ small R
X in vector rep of O(N)
(N-1)(N-2)/2 sym that preserve vac
X = X_cl + δx
(1, 0, ... 0)
N-1
O(N) → O(N-1)
N(N-1)/2 symmetries ← original sym
DIFFERENCE = N-1 = # goldstone b
gen. move the vac into another vac hence costing no energy
D remaining = 1 D O(N-1)


$\phi^{j=1 \ldots\ldots M}$
in some rep of cont. group G
$\phi^j = \phi^j_{cl} + \delta\phi^j$, $V = V(\phi_{cl}) + \frac{1}{2}\overbrace{\left[\frac{\partial V}{\partial\phi^j\partial\phi^k}\right]}^{m^2_{jk}}\Big|_{\phi_{cl}} \delta\phi^j\delta\phi^k$
min,
$\frac{\partial V}{\partial\phi^j}\Big|_{\phi^k=\phi^k_{cl}} = 0$
bosons
the vac

$\phi^{j=1 \ldots M}$ in some rep of cont. group G

vac

$$\phi^j = \phi^j_{cl} + \delta\phi^j, \quad V = V(\phi_{cl}) + \frac{1}{2}\overbrace{\left[\frac{\partial V}{\partial \phi^j \partial \phi^k}\right]}^{m^2_{jk}} \times \delta\phi^j \delta\phi^k + O(\delta\phi^3)$$

min, $\left.\frac{\partial V}{\partial \phi^j}\right|_{\phi^k = \phi^k_{cl}} = 0$

1 = # goldstone bosons

gen. move the vac into another vac hence costing no energy

$$D_{\text{remaining}} = \left[\boxed{D_{O(N-1)}} \right] \qquad 2$$

hence costing no energy

infinitesimal sym transf of G

$$\phi^j \longrightarrow \phi^j + \underbrace{\alpha \Delta^j[\vec{\phi}]}_{(\text{e.g. } \alpha\omega^{jk}\phi_k)}$$

which are not sym of ϕ_{cl}

hence
costing no energy

sign of ϕ_{cl} , for $\underline{\text{const.}}$ config , $V[\phi^j + \alpha \Delta^j(\phi)] = V[\phi^j]$

Kin can be ignored

$$\Downarrow$$

$$\left[\frac{\partial}{\partial \phi^k}\left[\Delta^j(\phi) \frac{\partial V}{\partial \phi^j} = 0\right]\right]$$ and set $\phi = \phi_{cl}$

$\boxed{D_{O(N-1)}}$ 2

hence
costing no energy

...of G

$$\phi^j + \underbrace{\alpha \Delta^j[\vec{\phi}]}_{(\text{e.g. } \alpha\omega^{jk}\phi_k)}$$

which are not sym of ϕ_{cl}, for const. config

Kin can be ignored

$$\frac{\partial \Delta^j}{\partial \phi^k}\bigg|_{\phi_{cl}} \frac{\partial V}{\partial \phi^j}\bigg|_{\phi_{cl}} + \Delta^j(\phi_{cl}) \frac{\partial^2 V}{\partial \phi^j \partial \phi^k}\bigg|_{\phi_{cl}} = 0$$

$$\frac{\partial}{\partial \phi^k}\Big[\Delta^j \ldots$$

$D_{O(N-1)}$

hence costing no energy

...nsf of G

eigenvector of m with eigenvalue

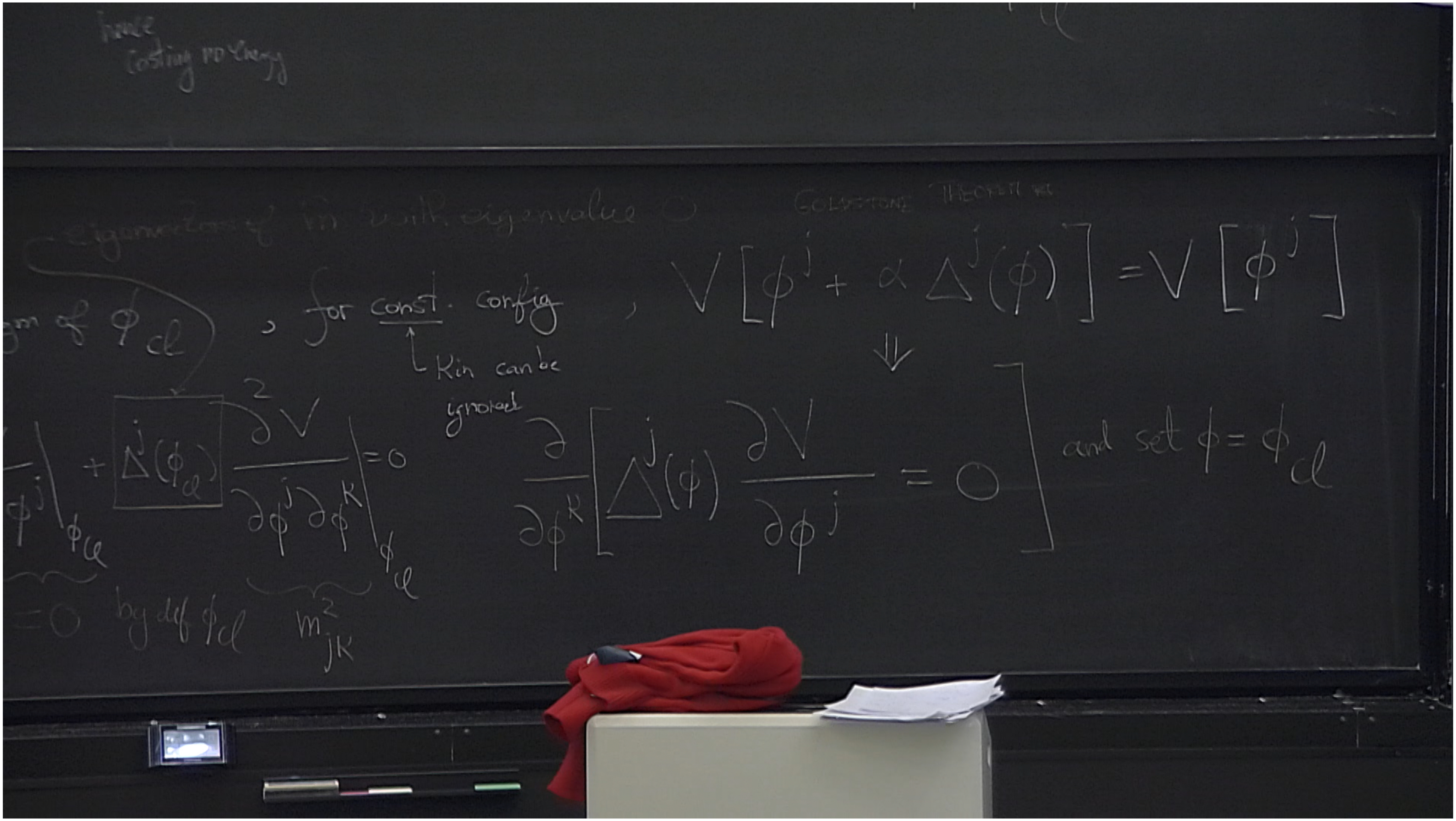
$$\phi^j + \alpha \underbrace{\Delta^j[\vec{\phi}]}_{(\text{e.g. } \alpha \omega^{jk}\phi_k)}$$

which are not sym of ϕ_{cl}, for const. config

↳ Kin can be ignored

$$\left.\frac{\partial \Delta^j}{\partial \phi^k}\right|_{\phi_{cl}} \underbrace{\left.\frac{\partial V}{\partial \phi^j}\right|_{\phi_{cl}}}_{=0 \text{ by def } \phi_{cl}} + \boxed{\Delta^j(\phi_{cl})} \underbrace{\left.\frac{\partial^2 V}{\partial \phi^j \partial \phi^k}\right|_{\phi_{cl}}}_{m^2_{jk}} = 0$$

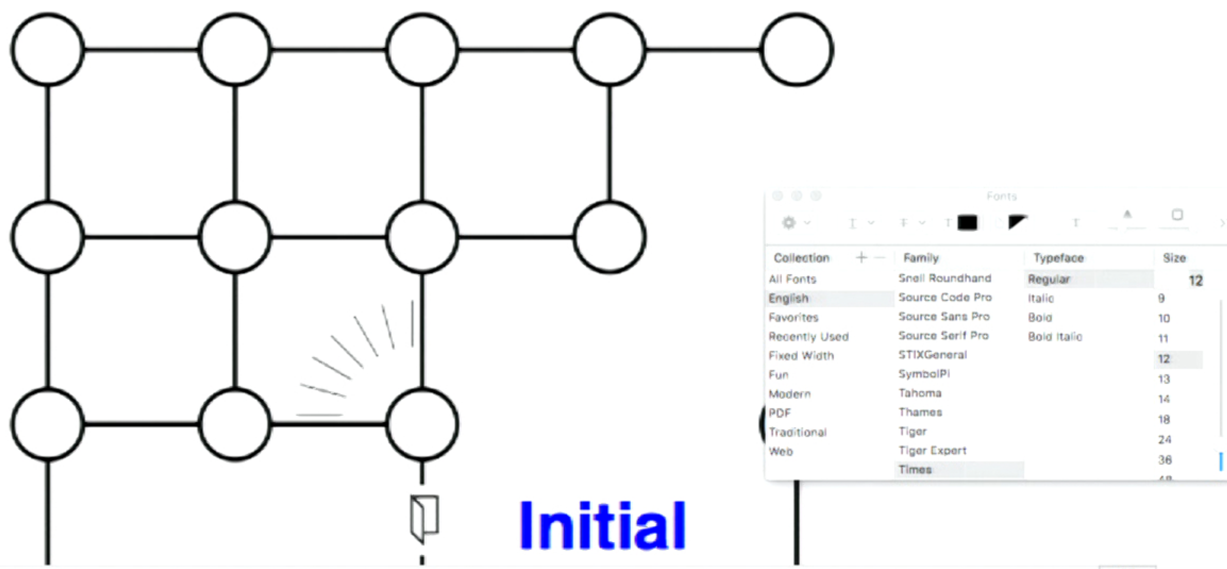
$$\frac{\partial}{\partial \phi^k}\left[\Delta^j \ldots\right.$$


for const. config
Kin can be ignored
$V[\phi^j + \alpha \Delta^j(\phi)] = V[\phi^j]$
$\Downarrow$
$\frac{\partial}{\partial \phi^k}\left[\Delta^j(\phi) \frac{\partial V}{\partial \phi^j} = 0\right]$ and set $\phi = \phi_{cl}$
$+ \boxed{\Delta^j(\phi_{cl})} \frac{\partial^2 V}{\partial \phi^j \partial \phi^k}\Big|_{\phi_{cl}} = 0$
$= 0$ by def ϕ_{cl}
m^2_{jk}

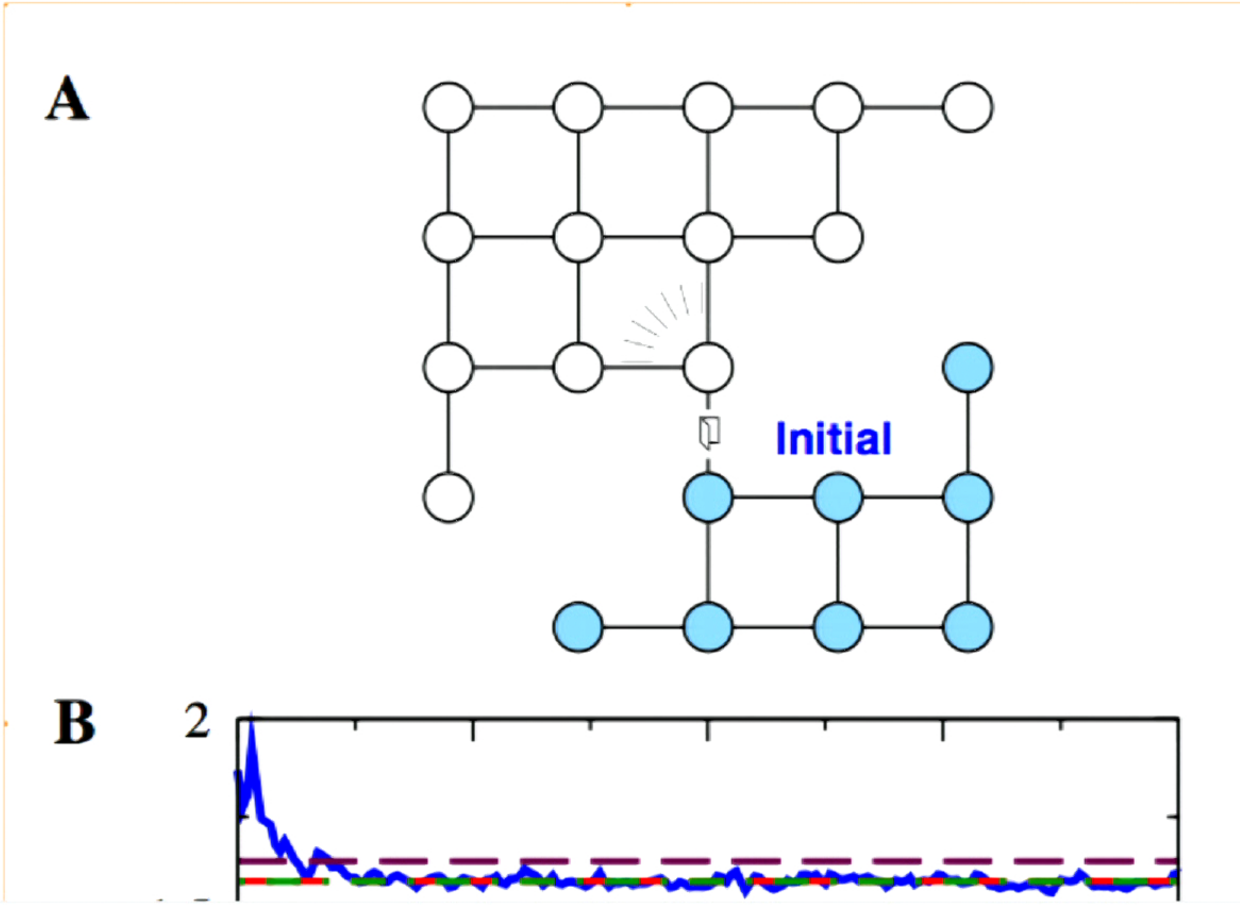
Goal

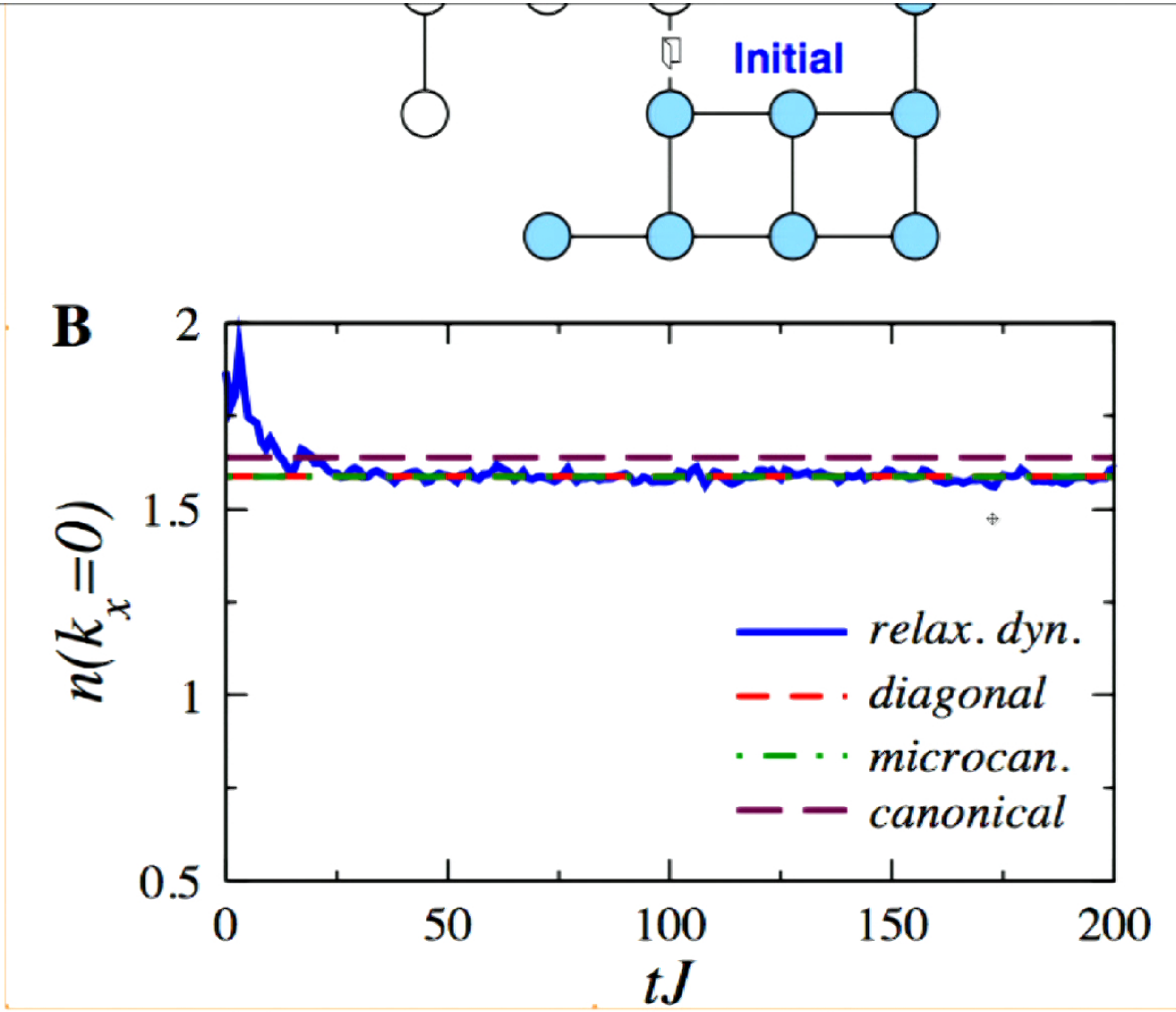
$$\widehat{H} = -J \sum_{\langle i,j \rangle} \left(\hat{b}_i^\dagger \hat{b}_j + \mathrm{h.c.} \right) + U \sum_{\langle i,j \rangle} \hat{n}_i \hat{n}_j$$

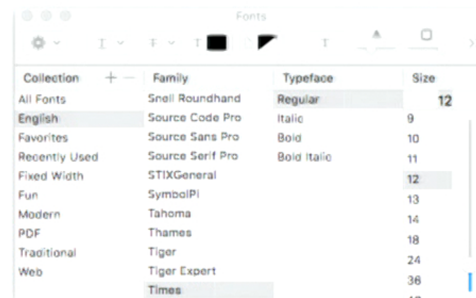
A



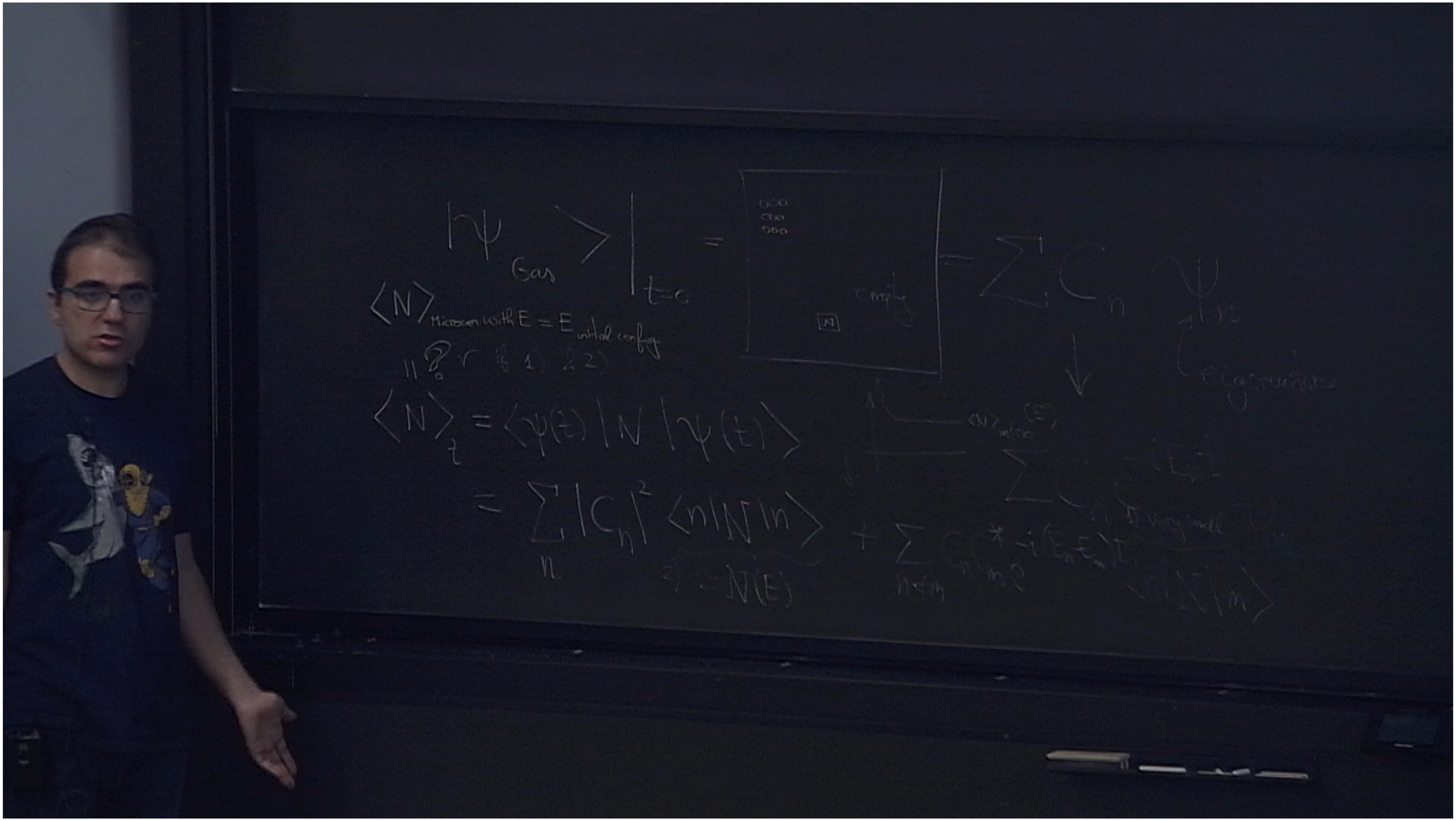
$$\widehat{H} = -J \sum_{\langle i,j \rangle} \left(\hat{b}_i^\dagger \hat{b}_j + \text{h.c.} \right) + U \sum_{\langle i,j \rangle} \hat{n}_i \hat{n}_j$$

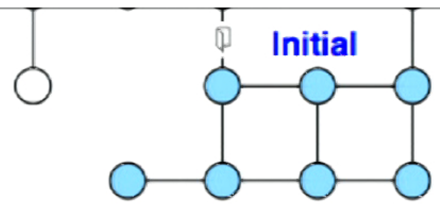


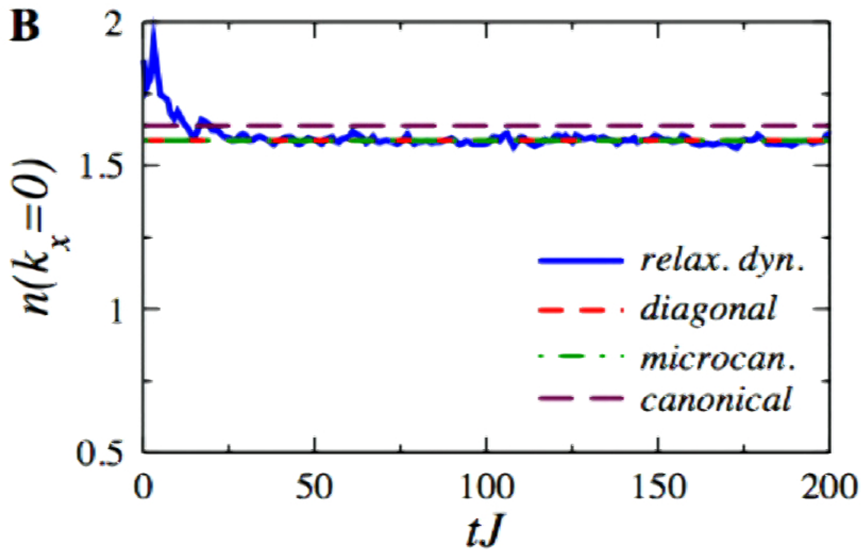

Initial
B
2
1.5
1
0.5
0
50
100
150
200
$n(k_x=0)$
tJ
relax. dyn.
diagonal
microcan.
canonical


Collection
Family
Typeface
Size

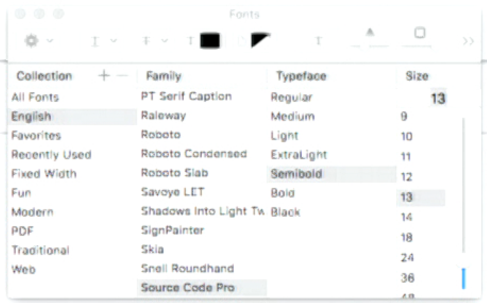
```
a == 1/(b + a) // Solve[#, a] &
```


Gas
t=0
empty
Microcan with E = E initial config



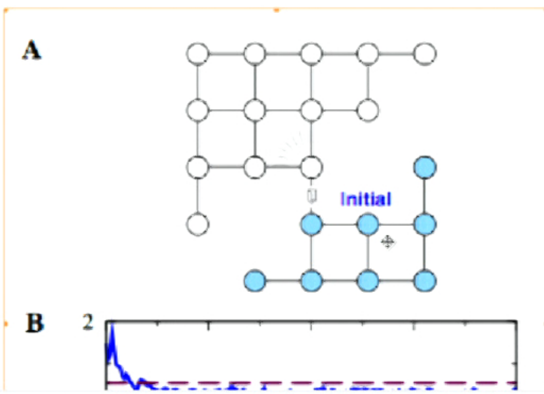


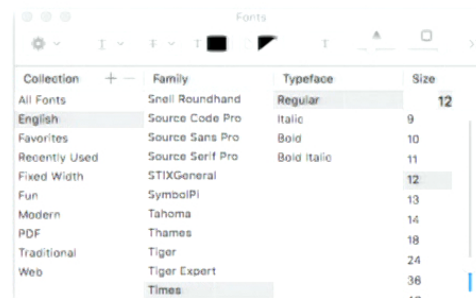
Lattice and H

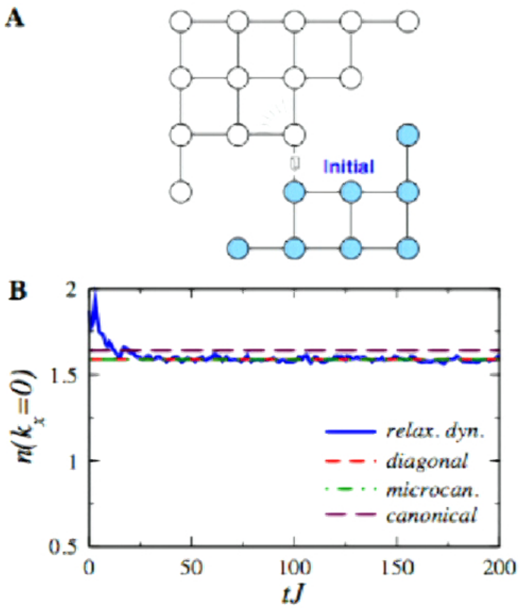


Goal

$$\hat{H} = -J \sum_{\langle i,j \rangle} \left(\hat{b}_i^\dagger \hat{b}_j + \text{h.c.} \right) + U \sum_{\langle i,j \rangle} \hat{n}_i \hat{n}_j$$







Lattice and H

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Table[{|
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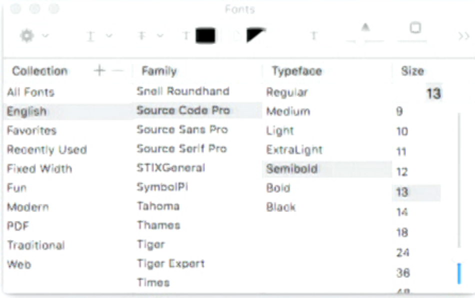
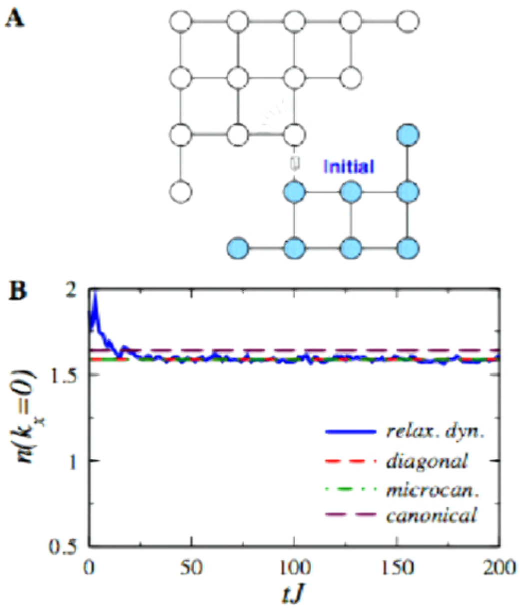


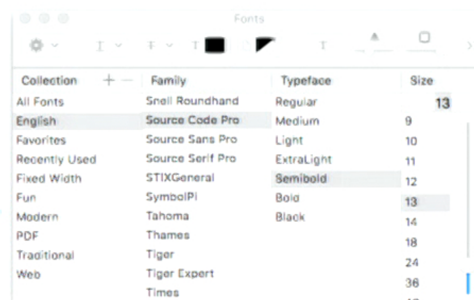


Lattice and H

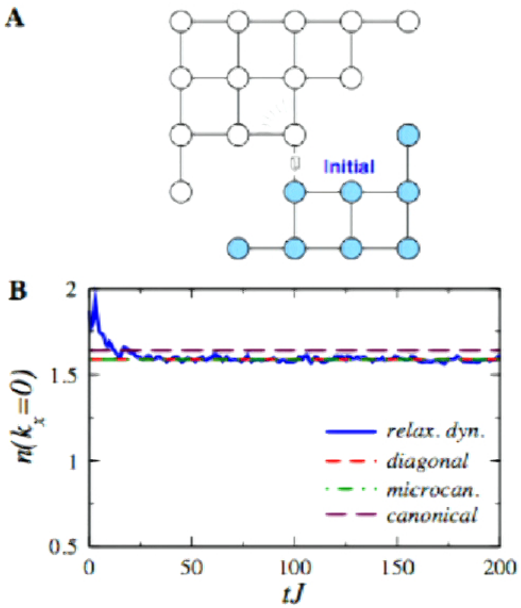
```
In[78]:= five = 5;

In[79]:= Table[{i, j}, {i, 5}, {j, five}]

Out[79]= {{{1, 1}, {1, 2}, {1, 3}, {1, 4}, {1, 5}}, {{2, 1}, {2, 2}, {2, 3}, {2, 4}, {2, 5}},
  {{3, 1}, {3, 2}, {3, 3}, {3, 4}, {3, 5}}, {{4, 1}, {4, 2}, {4, 3}, {4, 4}, {4, 5}}, {{5, 1}, {5, 2}, {5, 3}, {5, 4},
```



150%

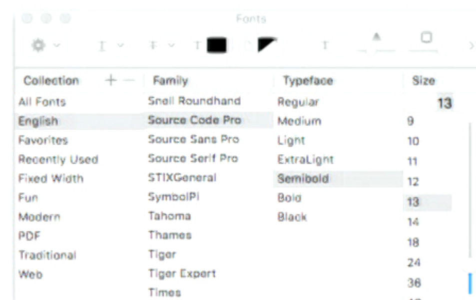


Lattice and H

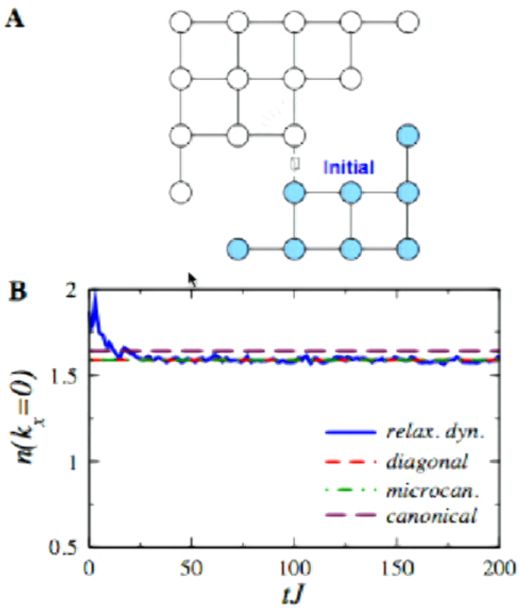
```
In[78]:= five = 5;

In[80]:= Table[{i, j}, {i, 5}, {j, five}] // Flatten[#, 1] &

Out[80]= {{1, 1}, {1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}, {2, 2}, {2, 3}, {2, 4}, {2, 5}, {3, 1},
  {3, 2}, {3, 3}, {3, 4}, {3, 5}, {4, 1}, {4, 2}, {4, 3}, {4, 4}, {4, 5}, {5, 1}, {5, 2}, {5, 3}, {5, 4}, {5, 5}}
```



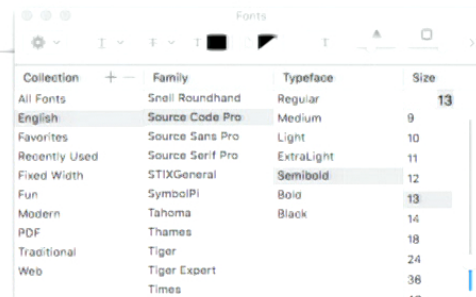
$\langle i,j \rangle$ $\langle i,j \rangle$



Lattice and H

In[78]:=
```
five = 5;
```

In[81]:=
```
(start = Table[{i, j}, {i, 5}, {j, five}] // Flatten[#, 1] &) // ListPlot
```

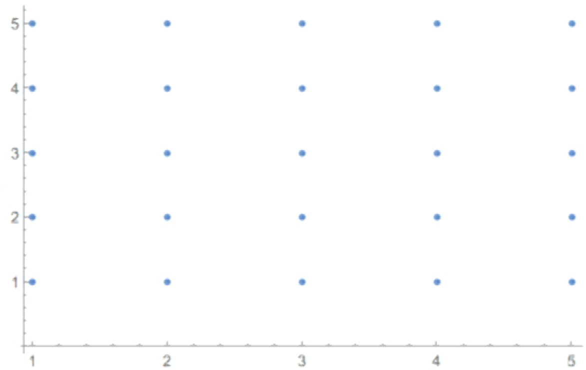


Lattice and H

```
In[78]:= five = 5;

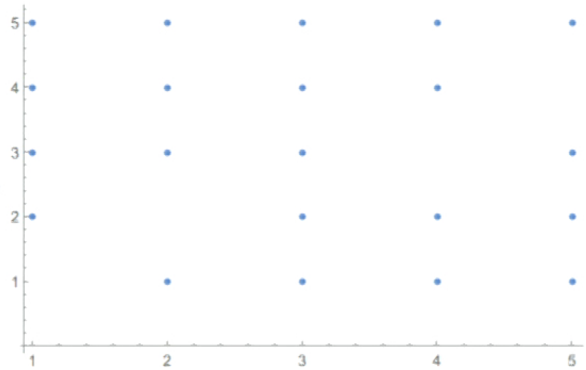
In[81]:= (start = Table[{i, j}, {i, 5}, {j, five}] // Flatten[#, 1] &) // ListPlot
```

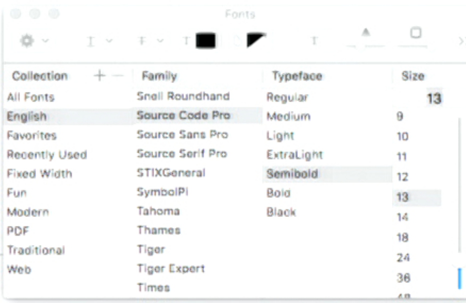
Out[81]=




```
In[82]:= (erased = start /. {{1, 1} → Sequence[], {2, 2} -> Sequence[], {4, 3} → Sequence[], {5, 4} → Sequence[]}) // ListPlot
```

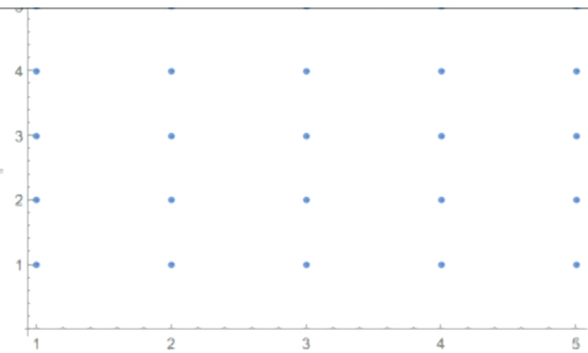
Out[82]=





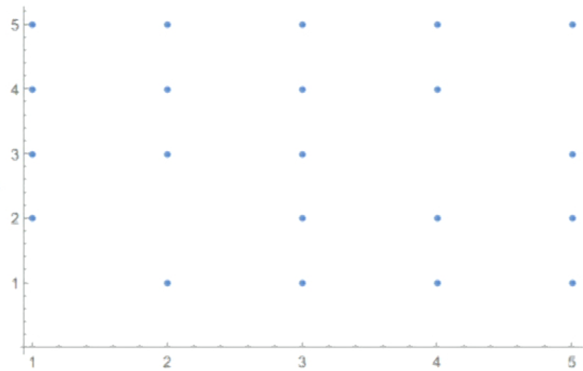
150%

Out[81]=



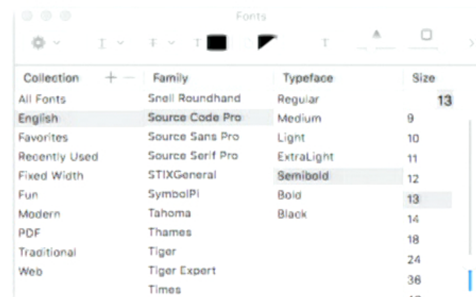
In[82]:= (erased = start /. {{1, 1} → Sequence[], {2, 2} -> Sequence[], {4, 3} → Sequence[], {5, 4} → Sequence[]}) // ListPlot

Out[82]=



In[83]:= erased

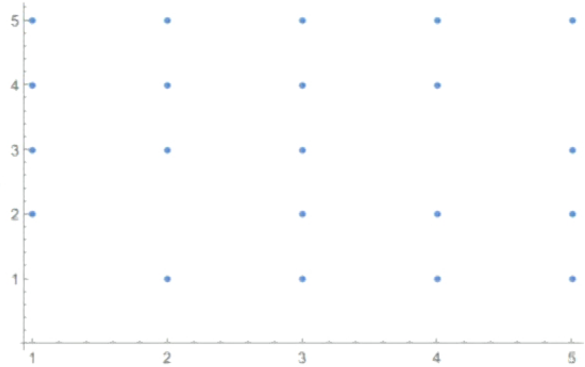
Out[83]= {{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}, {2, 3}, {2, 4}, {2, 5}, {3, 1}, {3, 2}, {3, 3}, {3, 4}, {3, 5}, {4, 1}, {4, 2}, {4, 4}, {4, 5}, {5, 1}, {5, 2}, {5, 3}, {5, 5}}



150%

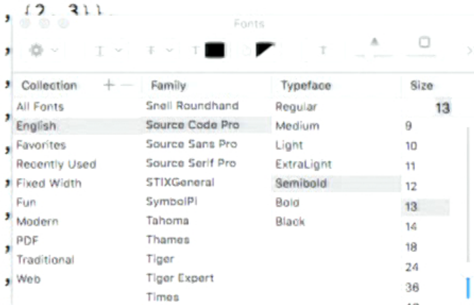
In[82]:=
```
(erased = start /. {{1, 1} → Sequence[], {2, 2} -> Sequence[], {4, 3} → Sequence[], {5, 4} → Sequence[]}) // ListPlot
```

Out[82]=



In[84]:=
```
Subsets[erased, {2}]
```

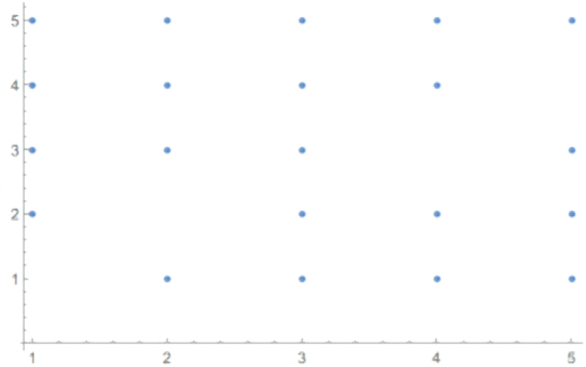
Out[84]=
```
{{{1, 2}, {1, 3}}, {{1, 2}, {1, 4}}, {{1, 2}, {1, 5}}, {{1, 2}, {2, 1}}, {{1, 2}, {2, 3}}, {{1, 2}, {2, 4}}, {{1, 2}, {2, 5}},
 {{1, 2}, {3, 1}}, {{1, 2}, {3, 2}}, {{1, 2}, {3, 3}}, {{1, 2}, {3, 4}}, {{1, 2}, {3, 5}}, {{1, 2}, {4, 1}}, {{1, 2}, {4, 2}},
 {{1, 2}, {4, 4}}, {{1, 2}, {4, 5}}, {{1, 2}, {5, 1}}, {{1, 2}, {5, 2}}, {{1, 2}, {5, 3}}, {{1, 2}, {5, 5}}, {{1, 3}, {1, 4}},
 {{1, 3}, {1, 5}}, {{1, 3}, {2, 1}}, {{1, 3}, {2, 3}}, {{1, 3}, {2, 4}}, {{1, 3}, {2, 5}}, {{1, 3}, {3, 1}}, {{1, 3}, {3, 2}},
 {{1, 3}, {3, 3}}, {{1, 3}, {3, 4}}, {{1, 3}, {3, 5}}, {{1, 3}, {4, 1}}, {{1, 3}, {4, 2}}, {{1, 3}, {4, 4}}, {{1, 3}, {4, 5}},
 {{1, 3}, {5, 1}}, {{1, 3}, {5, 2}}, {{1, 3}, {5, 3}}, {{1, 3}, {5, 5}}, {{1, 4}, {1, 5}}, {{1, 4}, {2, 1}}, {{1, 4}, {2, 3}},
 {{1, 4}, {2, 4}}, {{1, 4}, {2, 5}}, {{1, 4}, {3, 1}}, {{1, 4}, {3, 2}}, {{1, 4}, {3, 3}}, {{1, 4}, {3, 4}}, {{1, 4},
 {{1, 4}, {4, 2}}, {{1, 4}, {4, 4}}, {{1, 4}, {4, 5}}, {{1, 4}, {5, 1}}, {{1, 4}, {5, 2}}, {{1, 4}, {5, 3}}, {{1, 4},
 {{1, 5}, {2, 3}}, {{1, 5}, {2, 4}}, {{1, 5}, {2, 5}}, {{1, 5}, {3, 1}}, {{1, 5}, {3, 2}}, {{1, 5}, {3, 3}}, {{1, 5},
 {{1, 5}, {4, 1}}, {{1, 5}, {4, 2}}, {{1, 5}, {4, 4}}, {{1, 5}, {4, 5}}, {{1, 5}, {5, 1}}, {{1, 5}, {5, 2}}, {{1, 5},
 {{2, 1}, {2, 3}}, {{2, 1}, {2, 4}}, {{2, 1}, {2, 5}}, {{2, 1}, {3, 1}}, {{2, 1}, {3, 2}}, {{2, 1}, {3, 3}}, {{2, 1},
 {{2, 1}, {4, 1}}, {{2, 1}, {4, 2}}, {{2, 1}, {4, 4}}, {{2, 1}, {4, 5}}, {{2, 1}, {5, 1}}, {{2, 1}, {5, 2}}, {{2, 1},
 {{2, 3}, {2, 4}}, {{2, 3}, {2, 5}}, {{2, 3}, {3, 1}}, {{2, 3}, {3, 2}}, {{2, 3}, {3, 3}}, {{2, 3}, {3, 4}}, {{2, 3},
 {{2, 3}, {4, 2}}, {{2, 3}, {4, 4}}, {{2, 3}, {4, 5}}, {{2, 3}, {5, 1}}, {{2, 3}, {5, 2}}, {{2, 3}, {5, 3}}, {{2, 3},
 {{2, 4}, {3, 1}}, {{2, 4}, {3, 2}}, {{2, 4}, {3, 3}}, {{2, 4}, {3, 4}}, {{2, 4}, {3, 5}}, {{2, 4}, {4, 1}}, {{2, 4}, {4, 2}}, {{2, 4}, {4, 4}},
 {{2, 4}, {4, 5}}, {{2, 4}, {5, 1}}, {{2, 4}, {5, 2}}, {{2, 4}, {5, 3}}, {{2, 4}, {5, 5}}, {{2, 5}, {3, 1}}, {{2, 5}, {3, 2}}, {{2, 5}, {3, 3}},
 {{2, 5}, {3, 4}}, {{2, 5}, {3, 5}}, {{2, 5}, {4, 1}}, {{2, 5}, {4, 2}}, {{2, 5}, {4, 4}}, {{2, 5}, {4, 5}}, {{2, 5}, {5, 1}}, {{2, 5}, {5, 2}},
```



150%

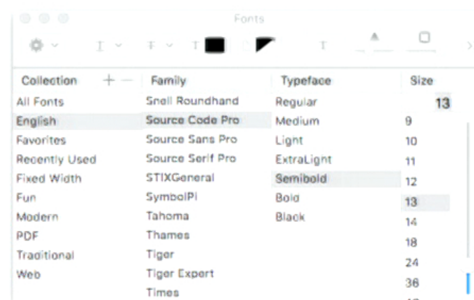
```
In[82]:= (erased = start /. {{1, 1} → Sequence[], {2, 2} -> Sequence[], {4, 3} → Sequence[], {5, 4} → Sequence[]}) // ListPlot
```

Out[82]=



```
In[86]:= f @@@ Subsets[erased, {2}] /. f[a_, b_] :→ Sequence[] /; Norm[a - b] ≠ 1
```

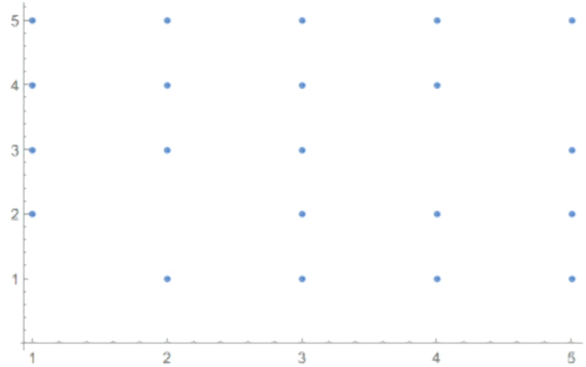
```
Out[86]= {f[{1, 2}, {1, 3}], f[{1, 3}, {1, 4}], f[{1, 3}, {2, 3}], f[{1, 4}, {1, 5}], f[{1, 4}, {2, 4}], f[{1, 5}, {2, 5}],
  f[{2, 1}, {3, 1}], f[{2, 3}, {2, 4}], f[{2, 3}, {3, 3}], f[{2, 4}, {2, 5}], f[{2, 4}, {3, 4}], f[{2, 5}, {3, 5}], f[{3, 1}, {3, 2}],
  f[{3, 1}, {4, 1}], f[{3, 2}, {3, 3}], f[{3, 2}, {4, 2}], f[{3, 3}, {3, 4}], f[{3, 4}, {3, 5}], f[{3, 4}, {4, 4}], f[{3, 5}, {4, 5}],
  f[{4, 1}, {4, 2}], f[{4, 1}, {5, 1}], f[{4, 2}, {5, 2}], f[{4, 4}, {4, 5}], f[{4, 5}, {5, 5}], f[{5, 1}, {5, 2}], f[{5, 2}, {5, 3}]}
```



150%

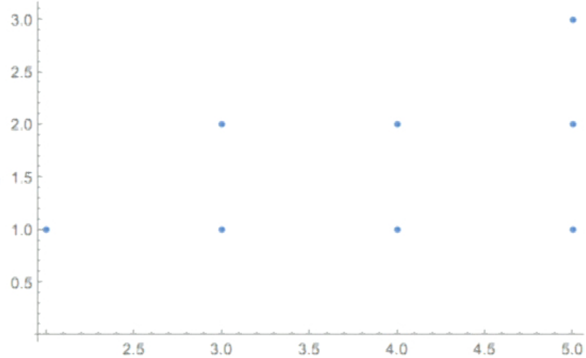
In[82]:= `(erased = start /. {{1, 1} → Sequence[], {2, 2} -> Sequence[], {4, 3} → Sequence[], {5, 4} → Sequence[]}) // ListPlot`

Out[82]=



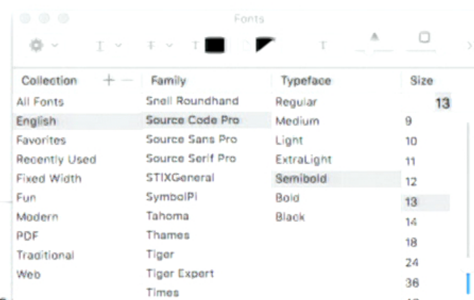
In[90]:= `(bottom = erased /. {i_, j_} :→ Sequence[] /; i < j + 1) // ListPlot`

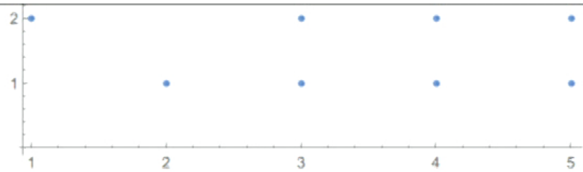
Out[90]=



In[88]:= `links = f @@@ Subsets[erased, {2}] /. f[a_, b_] :→ Sequence[] /; Norm[a - b] ≠ 1`

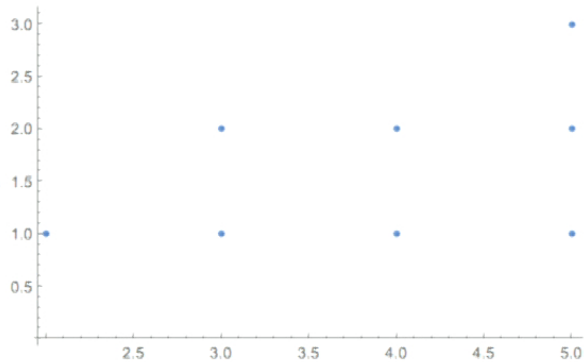
Out[88]=
```
{f[{1, 2}, {1, 3}], f[{1, 3}, {1, 4}], f[{1, 3}, {2, 3}], f[{1, 4}, {1, 5}], f[{1, 4}, {2, 4}], f[{1, 5}, {2, 5}],
 f[{2, 1}, {3, 1}], f[{2, 3}, {2, 4}], f[{2, 3}, {3, 3}], f[{2, 4}, {2, 5}], f[{2, 4}, {3, 4}], f[{2, 5}, {3, 5}], f[{3, 1}, {3, 2}],
 f[{3, 1}, {4, 1}], f[{3, 2}, {3, 3}], f[{3, 2}, {4, 2}], f[{3, 3}, {3, 4}], f[{3, 4}, {3, 5}], f[{3, 4}, {4, 4}], f[{3, 5}, {4, 5}],
 f[{4, 1}, {4, 2}], f[{4, 1}, {5, 1}], f[{4, 2}, {5, 2}], f[{4, 4}, {4, 5}], f[{4, 5}, {5, 5}], f[{5, 1}, {5, 2}], f[{5, 2}, {5, 3}]}
```






```
In[90]:= (bottom = erased /. {i_, j_} :> Sequence[] /; i < j + 1) // ListPlot
```

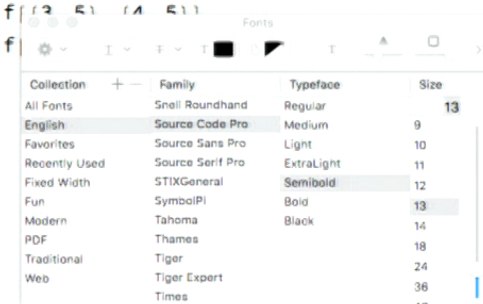
Out[90]=




```
In[88]:= links = f @@@ Subsets[erased, {2}] /. f[a_, b_] :> Sequence[] /; Norm[a - b] ≠ 1

Out[88]= {f[{1, 2}, {1, 3}], f[{1, 3}, {1, 4}], f[{1, 3}, {2, 3}], f[{1, 4}, {1, 5}], f[{1, 4}, {2, 4}], f[{1, 5}, {2, 5}],
  f[{2, 1}, {3, 1}], f[{2, 3}, {2, 4}], f[{2, 3}, {3, 3}], f[{2, 4}, {2, 5}], f[{2, 4}, {3, 4}], f[{2, 5}, {3, 5}], f[{3, 1}, {3, 2}],
  f[{3, 1}, {4, 1}], f[{3, 2}, {3, 3}], f[{3, 2}, {4, 2}], f[{3, 3}, {3, 4}], f[{3, 4}, {3, 5}], f[{3, 4}, {4, 4}], f[
  f[{4, 1}, {4, 2}], f[{4, 1}, {5, 1}], f[{4, 2}, {5, 2}], f[{4, 4}, {4, 5}], f[{4, 5}, {5, 5}], f[{5, 1}, {5, 2}], f[

linksBottom = links = f @@@ Subsets[erased, {2}] /. f[a_, b_] :> Sequence[] /; Norm[a - b] ≠ 1
```



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```
Out[88]= {f[{1, 2}, {1, 3}], f[{1, 3}, {1, 4}], f[{1, 3}, {2, 3}], f[{1, 4}, {1, 5}], f[{1, 4}, {2, 4}], f[{1, 5}, {2, 5}],
  f[{2, 1}, {3, 1}], f[{2, 3}, {2, 4}], f[{2, 3}, {3, 3}], f[{2, 4}, {2, 5}], f[{2, 4}, {3, 4}], f[{2, 5}, {3, 5}], f[{3, 1}, {3, 2}],
  f[{3, 1}, {4, 1}], f[{3, 2}, {3, 3}], f[{3, 2}, {4, 2}], f[{3, 3}, {3, 4}], f[{3, 4}, {3, 5}], f[{3, 4}, {4, 4}], f[{3, 5}, {4, 5}],
  f[{4, 1}, {4, 2}], f[{4, 1}, {5, 1}], f[{4, 2}, {5, 2}], f[{4, 4}, {4, 5}], f[{4, 5}, {5, 5}], f[{5, 1}, {5, 2}], f[{5, 2}, {5, 3}]}

In[91]:= linksBottom = f @@@ Subsets[bottom, {2}] /. f[a_, b_] :> Sequence[] /; Norm[a - b] ≠ 1

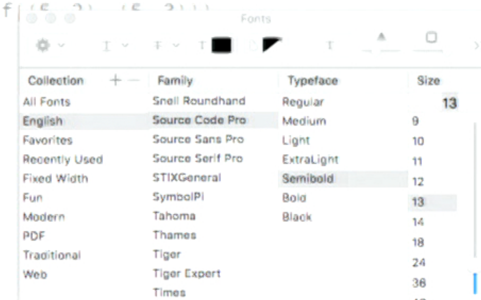
Out[91]= {f[{2, 1}, {3, 1}], f[{3, 1}, {3, 2}], f[{3, 1}, {4, 1}], f[{3, 2}, {4, 2}],
  f[{4, 1}, {4, 2}], f[{4, 1}, {5, 1}], f[{4, 2}, {5, 2}], f[{5, 1}, {5, 2}], f[{5, 2}, {5, 3}]}
```

$$\hat{H} = -J\sum_{\langle i,j\rangle}\left(\hat{b}_i^{\dagger}\hat{b}_j + \mathrm{h.c.}\right) + U\sum_{\langle i,j\rangle}\hat{n}_i\hat{n}_j$$

```
J

links /. f[i_, j_] → -J bd[i].b[j].state - J bd[j].b[i].state + U bd[i].b[i].bd[j].b[j].state

Out[92]= {f[{1, 2}, {1, 3}], f[{1, 3}, {1, 4}], f[{1, 3}, {2, 3}], f[{1, 4}, {1, 5}], f[{1, 4}, {2, 4}], f[{1, 5}, {2, 5}],
  f[{2, 1}, {3, 1}], f[{2, 3}, {2, 4}], f[{2, 3}, {3, 3}], f[{2, 4}, {2, 5}], f[{2, 4}, {3, 4}], f[{2, 5}, {3, 5}], f[{3, 1}, {3, 2}],
  f[{3, 1}, {4, 1}], f[{3, 2}, {3, 3}], f[{3, 2}, {4, 2}], f[{3, 3}, {3, 4}], f[{3, 4}, {3, 5}], f[{3, 4}, {4, 4}], f[{3, 5}, {4, 5}],
  f[{4, 1}, {4, 2}], f[{4, 1}, {5, 1}], f[{4, 2}, {5, 2}], f[{4, 4}, {4, 5}], f[{4, 5}, {5, 5}], f[{5, 1}, {5, 2}], f[
```



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```
f[{4, 1}, {4, 2}], f[{4, 1}, {5, 1}], f[{4, 2}, {5, 2}], f[{5, 1}, {5, 2}], f[{5, 2}, {5, 3}]}
```

$$\hat{H} = -J \sum_{\langle i,j \rangle} \left(\hat{b}_i^\dagger \hat{b}_j + \text{h.c.} \right) + U \sum_{\langle i,j \rangle} \hat{n}_i \hat{n}_j$$

```
In[94]:= J = 1; (* measure time in 1/J units *)
         U = 1 / 10;

In[98]:= h = links /. f[i_, j_] → -J bd[i].b[j].state - J bd[j].b[i].state + U bd[i].b[i].bd[j].b[j].state // Total;

In[100]:= hBottom = linksBottom /. f[i_, j_] → -J bd[i].b[j].state - J bd[j].b[i].state + U bd[i].b[i].bd[j].b[j].state // Total

Out[100]= -bd[{2, 1}].b[{3, 1}].state - bd[{3, 1}].b[{2, 1}].state - bd[{3, 1}].b[{3, 2}].state - bd[{3, 1}].b[{4, 1}].state - bd[{3, 2}].b[{3, 1}].state -
  bd[{3, 2}].b[{4, 2}].state - bd[{4, 1}].b[{3, 1}].state - bd[{4, 1}].b[{4, 2}].state - bd[{4, 1}].b[{5, 1}].state - bd[{4, 2}].b[{3, 2}].state -
  bd[{4, 2}].b[{4, 1}].state - bd[{4, 2}].b[{5, 2}].state - bd[{5, 1}].b[{4, 1}].state - bd[{5, 1}].b[{5, 2}].state - bd[{5, 2}].b[{4, 2}].state -
  bd[{5, 2}].b[{5, 1}].state - bd[{5, 2}].b[{5, 3}].state - bd[{5, 3}].b[{5, 2}].state + 1/10 bd[{2, 1}].b[{2, 1}].bd[{3, 1}].b[{3, 1}].state +
 1/10 bd[{3, 1}].b[{3, 1}].bd[{3, 2}].b[{3, 2}].state + 1/10 bd[{3, 1}].b[{3, 1}].bd[{4, 1}].b[{4, 1}].state +
 1/10 bd[{3, 2}].b[{3, 2}].bd[{4, 2}].b[{4, 2}].state + 1/10 bd[{4, 1}].b[{4, 1}].bd[{4, 2}].b[{4, 2}].state +
 1/10 bd[{4, 1}].b[{4, 1}].bd[{5, 1}].b[{5, 1}].state + 1/10 bd[{4, 2}].b[{4, 2}].bd[{5, 2}].b[{5, 2}].state +
 1/10 bd[{5, 1}].b[{5, 1}].bd[{5, 2}].b[{5, 2}].state + 1/10 bd[{5, 2}].b[{5, 2}].bd[{5, 3}].b[{5, 3}].state
```

Fonts

Collection	Family	Typeface	Size
All Fonts	Snell Roundhand	Regular	13
English	Source Code Pro	Medium	9
Favorites	Source Sans Pro	Light	10
Recently Used	Source Serif Pro	ExtraLight	11
Fixed Width	STIXGeneral	Semibold	12
Fun	SymbolPi	Bold	13
Modern	Tahoma	Black	14
PDF	Thames		18
Traditional	Tiger		24
Web	Tiger Expert		36
	Times		

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Matrix Form Problem

5 bosons that move around.

In[102]:=
```
basis = Ket @@@ Subsets[erased, {5}]
```

Out[102]=
```
{|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩, |{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 3}⟩,
 |{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 4}⟩, |{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 5}⟩, |{1, 2}, {1, 3}, {1, 4}, {1, 5}, {3, 1}⟩,
 ··· 20 339 ···, |{4, 4}, {4, 5}, {5, 1}, {5, 2}, {5, 5}⟩, |{4, 4}, {4, 5}, {5, 1}, {5, 3}, {5, 5}⟩,
 |{4, 4}, {4, 5}, {5, 2}, {5, 3}, {5, 5}⟩, |{4, 4}, {5, 1}, {5, 2}, {5, 3}, {5, 5}⟩, |{4, 5}, {5, 1}, {5, 2}, {5, 3}, {5, 5}⟩}
```

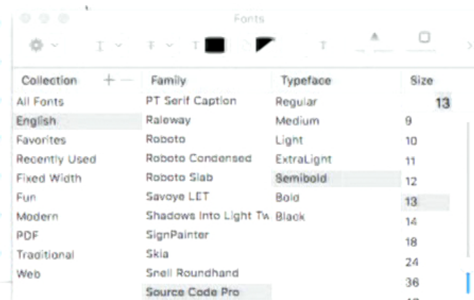
large output | show less | show more | show all | set size limit...

```
      {|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩, |{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 3}⟩,
       |{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 4}⟩, |{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 5}⟩, |{1, 2}, {1, 3}, {1, 4}, {1, 5}, {3, 1}⟩,
Out[102]=  ... 20339 ... , |{4, 4}, {4, 5}, {5, 1}, {5, 2}, {5, 5}⟩, |{4, 4}, {4, 5}, {5, 1}, {5, 3}, {5, 5}⟩,
       |{4, 4}, {4, 5}, {5, 2}, {5, 3}, {5, 5}⟩, |{4, 4}, {5, 1}, {5, 2}, {5, 3}, {5, 5}⟩, |{4, 5}, {5, 1}, {5, 2}, {5, 3}, {5, 5}⟩}

  large output    show less    show more    show all    set size limit...

In[103]:= basisBottom = Ket @@@ Subsets[bottom, {5}]

Out[103]= {|{2, 1}, {3, 1}, {3, 2}, {4, 1}, {4, 2}⟩, |{2, 1}, {3, 1}, {3, 2}, {4, 1}, {5, 1}⟩,
  |{2, 1}, {3, 1}, {3, 2}, {4, 1}, {5, 2}⟩, |{2, 1}, {3, 1}, {3, 2}, {4, 1}, {5, 3}⟩, |{2, 1}, {3, 1}, {3, 2}, {4, 2}, {5, 1}⟩,
  |{2, 1}, {3, 1}, {3, 2}, {4, 2}, {5, 2}⟩, |{2, 1}, {3, 1}, {3, 2}, {4, 2}, {5, 3}⟩, |{2, 1}, {3, 1}, {3, 2}, {5, 1}, {5, 2}⟩,
  |{2, 1}, {3, 1}, {3, 2}, {5, 1}, {5, 3}⟩, |{2, 1}, {3, 1}, {3, 2}, {5, 2}, {5, 3}⟩, |{2, 1}, {3, 1}, {4, 1}, {4, 2}, {5, 1}⟩,
  |{2, 1}, {3, 1}, {4, 1}, {4, 2}, {5, 2}⟩, |{2, 1}, {3, 1}, {4, 1}, {4, 2}, {5, 3}⟩, |{2, 1}, {3, 1}, {4, 1}, {5, 1}, {5, 2}⟩,
  |{2, 1}, {3, 1}, {4, 1}, {5, 1}, {5, 3}⟩, |{2, 1}, {3, 1}, {4, 1}, {5, 2}, {5, 3}⟩, |{2, 1}, {3, 1}, {4, 2}, {5, 1}, {5, 2}⟩,
  |{2, 1}, {3, 1}, {4, 2}, {5, 1}, {5, 3}⟩, |{2, 1}, {3, 1}, {4, 2}, {5, 2}, {5, 3}⟩, |{2, 1}, {3, 1}, {5, 1}, {5, 2}, {5, 3}⟩,
  |{2, 1}, {3, 2}, {4, 1}, {4, 2}, {5, 1}⟩, |{2, 1}, {3, 2}, {4, 1}, {4, 2}, {5, 2}⟩, |{2, 1}, {3, 2}, {4, 1}, {4, 2}, {5, 3}⟩,
  |{2, 1}, {3, 2}, {4, 1}, {5, 1}, {5, 2}⟩, |{2, 1}, {3, 2}, {4, 1}, {5, 1}, {5, 3}⟩, |{2, 1}, {3, 2}, {4, 1}, {5, 2}, {5, 3}⟩,
  |{2, 1}, {3, 2}, {4, 2}, {5, 1}, {5, 2}⟩, |{2, 1}, {3, 2}, {4, 2}, {5, 1}, {5, 3}⟩, |{2, 1}, {3, 2}, {4, 2}, {5, 2}, {5, 3}⟩,
  |{2, 1}, {3, 2}, {5, 1}, {5, 2}, {5, 3}⟩, |{2, 1}, {4, 1}, {4, 2}, {5, 1}, {5, 2}⟩, |{2, 1}, {4, 1}, {4, 2}, {5, 1}, {5, 3}⟩,
  |{2, 1}, {4, 1}, {4, 2}, {5, 2}, {5, 3}⟩, |{2, 1}, {4, 1}, {5, 1}, {5, 2}, {5, 3}⟩, |{2, 1}, {4, 2}, {5, 1}, {5, 2}, {5, 3}⟩,
  |{3, 1}, {3, 2}, {4, 1}, {4, 2}, {5, 1}⟩, |{3, 1}, {3, 2}, {4, 1}, {4, 2}, {5, 2}⟩, |{3, 1}, {3, 2}, {4, 1}, {4, 2},
  |{3, 1}, {3, 2}, {4, 1}, {5, 1}, {5, 2}⟩, |{3, 1}, {3, 2}, {4, 1}, {5, 1}, {5, 3}⟩, |{3, 1}, {3, 2}, {4, 1}, {5, 2},
  |{3, 1}, {3, 2}, {4, 2}, {5, 1}, {5, 2}⟩, |{3, 1}, {3, 2}, {4, 2}, {5, 1}, {5, 3}⟩, |{3, 1}, {3, 2}, {4, 2}, {5, 2},
  |{3, 1}, {3, 2}, {5, 1}, {5, 2}, {5, 3}⟩, |{3, 1}, {4, 1}, {4, 2}, {5, 1}, {5, 2}⟩, |{3, 1}, {4, 1}, {4, 2}, {5, 1},
  |{3, 1}, {4, 1}, {4, 2}, {5, 2}, {5, 3}⟩, |{3, 1}, {4, 1}, {5, 1}, {5, 2}, {5, 3}⟩, |{3, 1}, {4, 2}, {5, 1}, {5, 2},
  |{3, 2}, {4, 1}, {4, 2}, {5, 1}, {5, 2}⟩, |{3, 2}, {4, 1}, {4, 2}, {5, 1}, {5, 3}⟩, |{3, 2}, {4, 1}, {4, 2}, {5, 2},
  |{3, 2}, {4, 1}, {5, 1}, {5, 2}, {5, 3}⟩, |{3, 2}, {4, 2}, {5, 1}, {5, 2}, {5, 3}⟩, |{4, 1}, {4, 2}, {5, 1}, {5, 2},
```



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Matrix Form Problem

5 bosons that move around.

```
In[102]:= basis = Ket @@@ Subsets[erased, {5}]

Out[102]= {|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩, |{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 3}⟩,
 |{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 4}⟩, |{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 5}⟩, |{1, 2}, {1, 3}, {1, 4}, {1, 5}, {3, 1}⟩,
 ··· 20 339 ···, |{4, 4}, {4, 5}, {5, 1}, {5, 2}, {5, 5}⟩, |{4, 4}, {4, 5}, {5, 1}, {5, 3}, {5, 5}⟩,
 |{4, 4}, {4, 5}, {5, 2}, {5, 3}, {5, 5}⟩, |{4, 4}, {5, 1}, {5, 2}, {5, 3}, {5, 5}⟩, |{4, 5}, {5, 1}, {5, 2}, {5, 3}, {5, 5}⟩}

large output   show less   show more   show all   set size limit...

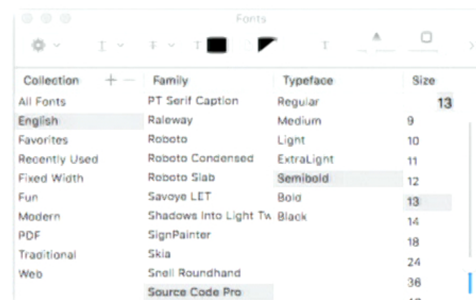
In[103]:= basisBottom = Ket @@@ Subsets[bottom, {5}]

In[104]:= Length[basisBottom]

Out[104]= 56

In[105]:= Length[basis]

Out[105]= 20 349
```



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```
Out[102]= {|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩, |{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 3}⟩,
  |{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 4}⟩, |{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 5}⟩, |{1, 2}, {1, 3}, {1, 4}, {1, 5}, {3, 1}⟩,
  … 20 339 …, |{4, 4}, {4, 5}, {5, 1}, {5, 2}, {5, 5}⟩, |{4, 4}, {4, 5}, {5, 1}, {5, 3}, {5, 5}⟩,
  |{4, 4}, {4, 5}, {5, 2}, {5, 3}, {5, 5}⟩, |{4, 4}, {5, 1}, {5, 2}, {5, 3}, {5, 5}⟩, |{4, 5}, {5, 1}, {5, 2}, {5, 3}, {5, 5}⟩}

  large output   show less   show more   show all   set size limit...

In[103]:= basisBottom = Ket @@@ Subsets[bottom, {5}]

In[104]:= Length[basisBottom]

Out[104]= 56

In[105]:= Length[basis]

Out[105]= 20 349

In[109]:= h /. state → basis[[1]]

Out[109]= -bd[{1, 2}].b[{1, 3}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{1, 3}].b[{1, 2}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ -
  bd[{1, 3}].b[{1, 4}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{1, 3}].b[{2, 3}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ -
  bd[{1, 4}].b[{1, 3}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{1, 4}].b[{1, 5}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ -
  bd[{1, 4}].b[{2, 4}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{1, 5}].b[{1, 4}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ -
  bd[{1, 5}].b[{2, 5}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{2, 1}].b[{3, 1}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ -
  bd[{2, 3}].b[{1, 3}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{2, 3}].b[{2, 4}].|{1, 2}, {1, 3}, {1, 4}, {1, 5
  bd[{2, 3}].b[{3, 3}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{2, 4}].b[{1, 4}].|{1, 2}, {1, 3}, {1, 4}, {1, 5
  bd[{2, 4}].b[{2, 3}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{2, 4}].b[{2, 5}].|{1, 2}, {1, 3}, {1, 4}, {1, 5
  bd[{2, 4}].b[{3, 4}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{2, 5}].b[{1, 5}].|{1, 2}, {1, 3}, {1, 4}, {1, 5
  bd[{2, 5}].b[{2, 4}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{2, 5}].b[{3, 5}].|{1, 2}, {1, 3}, {1, 4}, {1, 5
  bd[{3, 1}].b[{2, 1}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{3, 1}].b[{3, 2}].|{1, 2}, {1, 3}, {1, 4}, {1, 5
  bd[{3, 1}].b[{4, 1}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{3, 2}].b[{3, 1}].|{1, 2}, {1, 3}, {1, 4}, {1, 5
  bd[{3, 2}].b[{3, 3}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{3, 2}].b[{4, 2}].|{1, 2}, {1, 3}, {1, 4}, {1, 5
  bd[{3, 3}].b[{2, 3}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{3, 3}].b[{3, 2}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ -
  bd[{3, 3}].b[{3, 4}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{3, 4}].b[{2, 4}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ -
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Out[102]= ··· 20 339 ··· , |{4, 4}, {4, 5}, {5, 1}, {5, 2}, {5, 5}⟩, |{4, 4}, {4, 5}, {5, 1}, {5, 3}, {5, 5}⟩,
  |{4, 4}, {4, 5}, {5, 2}, {5, 3}, {5, 5}⟩, |{4, 4}, {5, 1}, {5, 2}, {5, 3}, {5, 5}⟩, |{4, 5}, {5, 1}, {5, 2}, {5, 3}, {5, 5}⟩}
```

large output | show less | show more | show all | set size limit...

```
In[103]:= basisBottom = Ket @@@ Subsets[bottom, {5}]

In[104]:= Length[basisBottom]

Out[104]= 56

In[105]:= Length[basis]

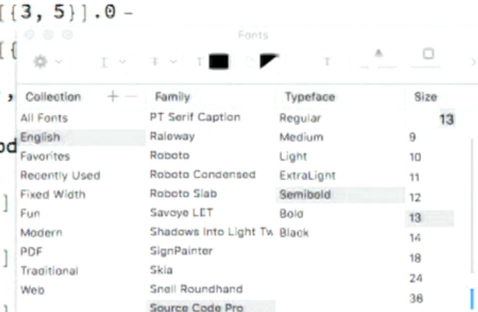
Out[105]= 20 349

In[110]:= rule = {b[a_].c_Ket :> If[FreeQ[c, a], 0, Sort[c /. a → Sequence[]]]}

Out[110]= {b[a_].c_Ket :> If[FreeQ[c, a], 0, Sort[c /. a → Sequence[]]]}

In[111]:= h /. state → basis[[1]] /. rule

Out[111]= -bd[{1, 2}].|{1, 2}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{1, 3}].0 - bd[{1, 3}].|{1, 2}, {1, 3}, {1, 5}, {2, 1}⟩ - bd[{1, 3}].|{1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ -
  bd[{1, 4}].0 - bd[{1, 4}].|{1, 2}, {1, 3}, {1, 4}, {2, 1}⟩ - bd[{1, 4}].|{1, 2}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{1, 5}].0 -
  bd[{1, 5}].|{1, 2}, {1, 3}, {1, 5}, {2, 1}⟩ - bd[{2, 1}].0 - 2 bd[{2, 3}].0 - bd[{2, 3}].|{1, 2}, {1, 4}, {1, 5}, {2, 1}⟩ -
  3 bd[{2, 4}].0 - bd[{2, 4}].|{1, 2}, {1, 3}, {1, 5}, {2, 1}⟩ - 2 bd[{2, 5}].0 - bd[{2, 5}].|{1, 2}, {1, 3}, {1, 4}, {2, 1}⟩ -
  2 bd[{3, 1}].0 - bd[{3, 1}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}⟩ - 3 bd[{3, 2}].0 - 3 bd[{3, 3}].0 - 4 bd[{3, 4}].0 - 3 bd[{3, 5}].0 -
  3 bd[{4, 1}].0 - 3 bd[{4, 2}].0 - 2 bd[{4, 4}].0 - 3 bd[{4, 5}].0 - 2 bd[{5, 1}].0 - 3 bd[{5, 2}].0 - bd[{5, 3}].0 - bd[{
  1/10 bd[{1, 2}].b[{1, 2}].bd[{1, 3}].|{1, 2}, {1, 4}, {1, 5}, {2, 1}⟩ + 1/10 bd[{1, 3}].b[{1, 3}].bd[{1, 4}].|{1, 2},
  1/10 bd[{1, 3}].b[{1, 3}].bd[{2, 3}].0 + 1/10 bd[{1, 4}].b[{1, 4}].bd[{1, 5}].|{1, 2}, {1, 3}, {1, 4}, {2, 1}⟩ + 1/10 bd
  1/10 bd[{1, 5}].b[{1, 5}].bd[{2, 5}].0 + 1/10 bd[{2, 1}].b[{2, 1}].bd[{3, 1}].0 + 1/10 bd[{2, 3}].b[{2, 3}].bd[{2, 4}]   0 +
  1/10 bd[{2, 4}].b[{2, 4}].bd[{2, 5}].0 + 1/10 bd[{2, 4}].b[{2, 4}].bd[{3, 4}].0 + 1/10 bd[{2, 5}].b[{2, 5}].bd[{3, 5}]   0 +
  1/10 bd[{3, 1}].b[{3, 1}].bd[{4, 1}].0 + 1/10 bd[{3, 2}].b[{3, 2}].bd[{3, 3}].0 + 1/10 bd[{3, 2}].b[{3, 2}].bd[{4, 2}]   0 +
  1/10 bd[{3, 4}].b[{3, 4}].bd[{3, 5}].0 + 1/10 bd[{3, 4}].b[{3, 4}].bd[{4, 4}].0 + 1/10 bd[{3, 5}].b[{3, 5}].bd[{4, 5}].0 +
```



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Out[102]= ⋯ 20 339 ⋯ , |{4, 4}, {4, 5}, {5, 1}, {5, 2}, {5, 5}⟩, |{4, 4}, {4, 5}, {5, 1}, {5, 3}, {5, 5}⟩,
 |{4, 4}, {4, 5}, {5, 2}, {5, 3}, {5, 5}⟩, |{4, 4}, {5, 1}, {5, 2}, {5, 3}, {5, 5}⟩, |{4, 5}, {5, 1}, {5, 2}, {5, 3}, {5, 5}⟩}

large output   show less   show more   show all   set size limit...

In[103]:= basisBottom = Ket @@@ Subsets[bottom, {5}]

In[104]:= Length[basisBottom]

Out[104]= 56

In[105]:= Length[basis]

Out[105]= 20 349

In[110]:= rule = {b[a_].c_Ket :→ If[FreeQ[c, a], 0, Sort[c /. a → Sequence[]]]}

Out[110]= {b[a_].c_Ket :→ If[FreeQ[c, a], 0, Sort[c /. a → Sequence[]]]}

In[113]:= bd[{1, 2}].b[{1, 3}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ /. rule

Out[113]= bd[{1, 2}].|{1, 2}, {1, 4}, {1, 5}, {2, 1}⟩

In[112]:= h /. state → basis[[1]]

Out[112]= -bd[{1, 2}].b[{1, 3}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{1, 3}].b[{1, 2}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ -
 bd[{1, 3}].b[{1, 4}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{1, 3}].b[{2, 3}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ -
 bd[{1, 4}].b[{1, 3}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{1, 4}].b[{1, 5}].|{1, 2}, {1, 3}, {1, 4}, {1, 5
 bd[{1, 4}].b[{2, 4}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{1, 5}].b[{1, 4}].|{1, 2}, {1, 3}, {1, 4}, {1, 5
 bd[{1, 5}].b[{2, 5}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{2, 1}].b[{3, 1}].|{1, 2}, {1, 3}, {1, 4}, {1, 5
 bd[{2, 3}].b[{1, 3}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{2, 3}].b[{2, 4}].|{1, 2}, {1, 3}, {1, 4}, {1, 5
 bd[{2, 3}].b[{3, 3}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{2, 4}].b[{1, 4}].|{1, 2}, {1, 3}, {1, 4}, {1, 5
 bd[{2, 4}].b[{2, 3}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{2, 4}].b[{2, 5}].|{1, 2}, {1, 3}, {1, 4}, {1, 5
 bd[{2, 4}].b[{3, 4}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{2, 5}].b[{1, 5}].|{1, 2}, {1, 3}, {1, 4}, {1, 5
 bd[{2, 5}].b[{2, 4}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{2, 5}].b[{3, 5}].|{1, 2}, {1, 3}, {1, 4}, {1, 5, ...
 bd[{3, 1}].b[{2, 1}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{3, 1}].b[{3, 2}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ -
 bd[{3, 1}].b[{4, 1}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{3, 2}].b[{3, 1}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ -
```

```
Out[102]= … 20 339 … , |{4, 4}, {4, 5}, {5, 1}, {5, 2}, {5, 5}⟩, |{4, 4}, {4, 5}, {5, 1}, {5, 3}, {5, 5}⟩,
  |{4, 4}, {4, 5}, {5, 2}, {5, 3}, {5, 5}⟩, |{4, 4}, {5, 1}, {5, 2}, {5, 3}, {5, 5}⟩, |{4, 5}, {5, 1}, {5, 2}, {5, 3}, {5, 5}⟩}

  large output   show less   show more   show all   set size limit...

In[103]:= basisBottom = Ket @@@ Subsets[bottom, {5}]

In[104]:= Length[basisBottom]

Out[104]= 56

In[105]:= Length[basis]

Out[105]= 20 349

rule = {
   b[a_].c_Ket :→ If[FreeQ[c, a], 0, Sort[c /. a → Sequence[]]],
   b[a_].c_Ket :→ If[FreeQ[c, a]|, , 0],
   Dot[a_, 0] → 0}

Out[115]= {b[a_].c_Ket :→ If[FreeQ[c, a], 0, Sort[c /. a → Sequence[]]], (a_).0 → 0}

In[113]:= bd[{1, 2}].b[{1, 3}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ /. rule

Out[113]= bd[{1, 2}].|{1, 2}, {1, 4}, {1, 5}, {2, 1}⟩

In[116]:= bd[{1, 2}].b[{1, 3}].|{1, 2}, {2, 3}, {1, 4}, {1, 5}, {2, 1}⟩ //. rule

Out[116]= 0

In[112]:= h /. state → basis[[1]]

Out[112]= -bd[{1, 2}].b[{1, 3}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{1, 3}].b[{1, 2}].|{1, 2}, {1, 3}, {1, 4}, {1, 5
  bd[{1, 3}].b[{1, 4}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{1, 3}].b[{2, 3}].|{1, 2}, {1, 3}, {1, 4}, {1, 5
  bd[{1, 4}].b[{1, 3}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{1, 4}].b[{1, 5}].|{1, 2}, {1, 3}, {1, 4}, {1, 5
  bd[{1, 4}].b[{2, 4}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{1, 5}].b[{1, 4}].|{1, 2}, {1, 3}, {1, 4}, {1, 5
  bd[{1, 5}].b[{2, 5}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{2, 1}].b[{3, 1}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ -
  bd[{2, 3}].b[{1, 3}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{2, 3}].b[{2, 4}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ -
```

```
Out[102]=  … 20 339 … , |{4, 4}, {4, 5}, {5, 1}, {5, 2}, {5, 5}⟩, |{4, 4}, {4, 5}, {5, 1}, {5, 3}, {5, 5}⟩,
  |{4, 4}, {4, 5}, {5, 2}, {5, 3}, {5, 5}⟩, |{4, 4}, {5, 1}, {5, 2}, {5, 3}, {5, 5}⟩, |{4, 5}, {5, 1}, {5, 2}, {5, 3}, {5, 5}⟩}
```

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```
In[103]:= basisBottom = Ket @@@ Subsets[bottom, {5}]

In[104]:= Length[basisBottom]

Out[104]= 56

In[105]:= Length[basis]

Out[105]= 20 349

In[117]:= rule = {
    b[a_].c_Ket :→ If[FreeQ[c, a], 0, Sort[c /. a → Sequence[]]],
    bd[a_].c_Ket :→ If[FreeQ[c, a], Sort[Join[Ket[a], c]], 0],
    Dot[a_, 0] → 0}

Out[117]= {b[a_].c_Ket :→ If[FreeQ[c, a], 0, Sort[c /. a → Sequence[]]], bd[a_].c_Ket :→ If[FreeQ[c, a], Sort[Join[|a⟩, c]], 0], (a_).0 → 0}

In[119]:= bd[{1, 2}].b[{1, 3}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ /. rule
          bd[{1, 2}].b[{1, 3}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ /. rule /. rule

Out[119]= bd[{1, 2}].|{1, 2}, {1, 4}, {1, 5}, {2, 1}⟩

Out[120]= 0

In[116]:= bd[{1, 2}].b[{1, 3}].|{1, 2}, {2, 3}, {1, 4}, {1, 5}, {2, 1}⟩ //. rule

Out[116]= 0

In[112]:= h /. state → basis[[1]]

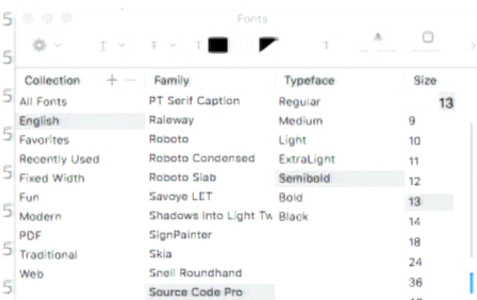
Out[112]= -bd[{1, 2}].b[{1, 3}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{1, 3}].b[{1, 2}].|{1, 2}, {1, 3}, {1, 4}, {1, 5
  bd[{1, 3}].b[{1, 4}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{1, 3}].b[{2, 3}].|{1, 2}, {1, 3}, {1, 4}, {1, 5
  bd[{1, 4}].b[{1, 3}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{1, 4}].b[{1, 5}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ -
  bd[{1, 4}].b[{2, 4}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{1, 5}].b[{1, 4}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ -
```

```
In[122]:= rule = {
    b[a_].c_Ket :> If[FreeQ[c, a], 0, Sort[c /. a → Sequence[]]],
    bd[a_].c_Ket :> If[FreeQ[c, a], Sort[Join[Ket[a], c]], 0],
    Dot[a_, 0] → 0};

  actionh[[1]]

In[123]:= actionh = Table[h /. state → X //. rule, {X, basis}];

Out[112]= -bd[{1, 2}].b[{1, 3}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{1, 3}].b[{1, 2}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ -
  bd[{1, 3}].b[{1, 4}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{1, 3}].b[{2, 3}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ -
  bd[{1, 4}].b[{1, 3}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{1, 4}].b[{1, 5}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ -
  bd[{1, 4}].b[{2, 4}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{1, 5}].b[{1, 4}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ -
  bd[{1, 5}].b[{2, 5}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{2, 1}].b[{3, 1}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ -
  bd[{2, 3}].b[{1, 3}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{2, 3}].b[{2, 4}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ -
  bd[{2, 3}].b[{3, 3}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{2, 4}].b[{1, 4}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ -
  bd[{2, 4}].b[{2, 3}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{2, 4}].b[{2, 5}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ -
  bd[{2, 4}].b[{3, 4}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{2, 5}].b[{1, 5}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ -
  bd[{2, 5}].b[{2, 4}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{2, 5}].b[{3, 5}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ -
  bd[{3, 1}].b[{2, 1}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{3, 1}].b[{3, 2}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ -
  bd[{3, 1}].b[{4, 1}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{3, 2}].b[{3, 1}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ -
  bd[{3, 2}].b[{3, 3}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{3, 2}].b[{4, 2}].|{1, 2}, {1, 3}, {1, 4}, {1, 5
  bd[{3, 3}].b[{2, 3}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{3, 3}].b[{3, 2}].|{1, 2}, {1, 3}, {1, 4}, {1, 5
  bd[{3, 3}].b[{3, 4}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{3, 4}].b[{2, 4}].|{1, 2}, {1, 3}, {1, 4}, {1, 5
  bd[{3, 4}].b[{3, 3}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{3, 4}].b[{3, 5}].|{1, 2}, {1, 3}, {1, 4}, {1, 5
  bd[{3, 4}].b[{4, 4}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{3, 5}].b[{2, 5}].|{1, 2}, {1, 3}, {1, 4}, {1, 5
  bd[{3, 5}].b[{3, 4}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{3, 5}].b[{4, 5}].|{1, 2}, {1, 3}, {1, 4}, {1, 5
  bd[{4, 1}].b[{3, 1}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{4, 1}].b[{4, 2}].|{1, 2}, {1, 3}, {1, 4}, {1, 5
  bd[{4, 1}].b[{5, 1}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{4, 2}].b[{3, 2}].|{1, 2}, {1, 3}, {1, 4}, {1, 5
  bd[{4, 2}].b[{4, 1}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{4, 2}].b[{5, 2}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ -
  bd[{4, 4}].b[{3, 4}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ - bd[{4, 4}].b[{4, 5}].|{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩ -
```



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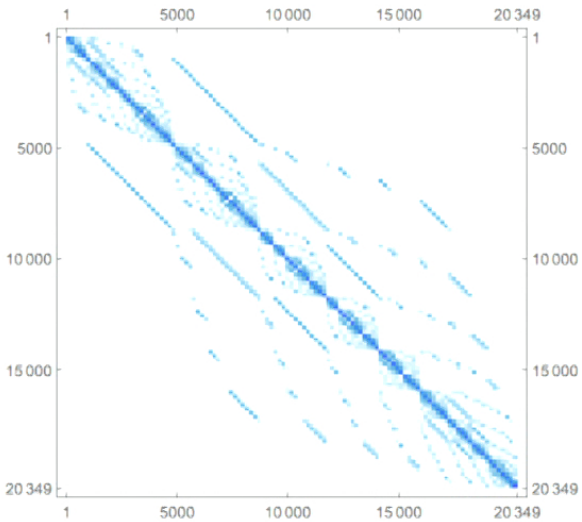
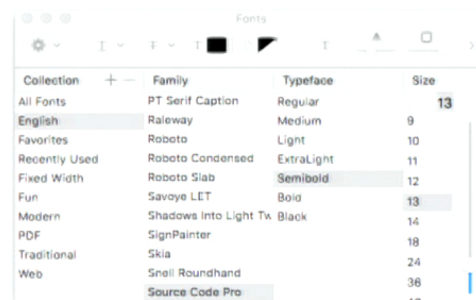
Out[127]= SparseArray[Specified elements: 225 202 Dimensions: {20 349, 20 349}]

In[128]:= MatrixPlot[hMatrix]

Out[128]=



hMatrix



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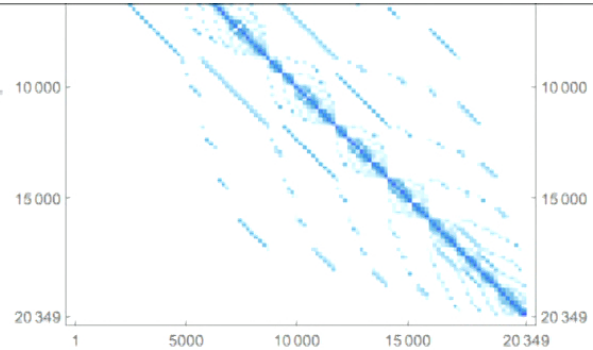
Out[128]=



In[132]:=

```
hMatrix.Table[If[i == 1, 1, 0], {i, 20349}]
```

Out[132]=

{$\frac{3}{10}$, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
··· 20 175 ···, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}

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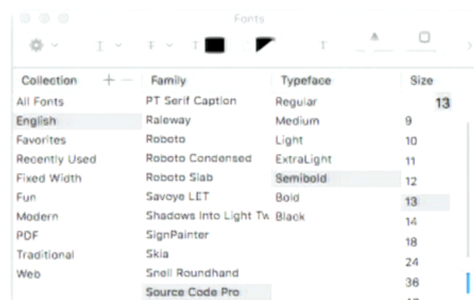

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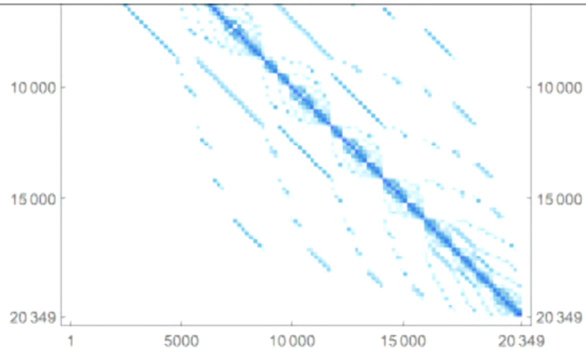
Out[128]=



In[134]:=
```
(hMatrix.Table[If[i == 1, 1, 0], {i, 20349}]).basis
```

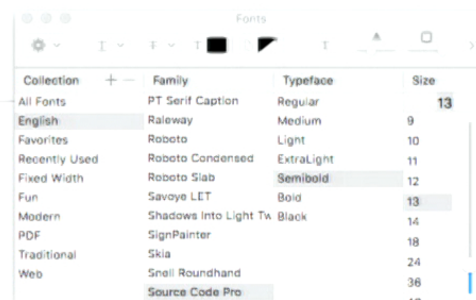
Out[134]=
$$\frac{3}{10}\left|\{1,2\},\{1,3\},\{1,4\},\{1,5\},\{2,1\}\right\rangle - \left|\{1,2\},\{1,3\},\{1,4\},\{1,5\},\{3,1\}\right\rangle - \left|\{1,2\},\{1,3\},\{1,4\},\{2,1\},\{2,5\}\right\rangle - \left|\{1,2\},\{1,3\},\{1,5\},\{2,1\},\{2,4\}\right\rangle - \left|\{1,2\},\{1,4\},\{1,5\},\{2,1\},\{2,3\}\right\rangle$$

In[135]:=
```
actionh[[1]]
```

Out[135]=
$$\frac{3}{10}\left|\{1,2\},\{1,3\},\{1,4\},\{1,5\},\{2,1\}\right\rangle - \left|\{1,2\},\{1,3\},\{1,4\},\{1,5\},\{3,1\}\right\rangle - \left|\{1,2\},\{1,3\},\{1,4\},\{2,1\},\{2,5\}\right\rangle - \left|\{1,2\},\{1,3\},\{1,5\},\{2,1\},\{2,4\}\right\rangle - \left|\{1,2\},\{1,4\},\{1,5\},\{2,1\},\{2,3\}\right\rangle$$

In[136]:=
```
basis[[10]]
```

Out[136]=
$$\left|\{1,2\},\{1,3\},\{1,4\},\{1,5\},\{4,1\}\right\rangle$$



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```
|{4, 4}, {4, 5}, {5, 2}, {5, 3}, {5, 5}⟩, |{4, 4}, {5, 1}, {5, 2}, {5, 3}, {5, 5}⟩, |{4, 5}, {5, 1}, {5, 2}, {5, 3}, {5, 5}⟩}
```

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```
In[103]:= basisBottom = Ket @@@ Subsets[bottom, {5}]

In[104]:= Length[basisBottom]

Out[104]= 56

In[105]:= Length[basis]

Out[105]= 20 349

In[122]:= rule = {
    b[a_].c_Ket :→ If[FreeQ[c, a], 0, Sort[c /. a → Sequence[]]],
    bd[a_].c_Ket :→ If[FreeQ[c, a], Sort[Join[Ket[a], c]], 0],
    Dot[a_, 0] → 0};

In[125]:= basis[[1]]

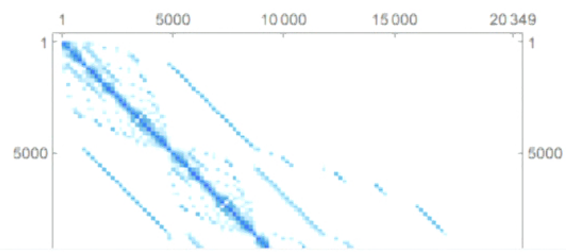
Out[125]= |{1, 2}, {1, 3}, {1, 4}, {1, 5}, {2, 1}⟩

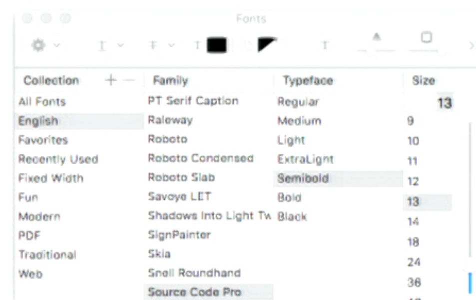
In[123]:= actionh = Table[h /. state → X //. rule, {X, basis}];

In[127]:= hMatrix = CoefficientArrays[actionh, basis][[2]]

Out[127]= SparseArray[ Specified elements: 225 202
                       Dimensions: {20 349, 20 349} ]

In[128]:= MatrixPlot[hMatrix]
```





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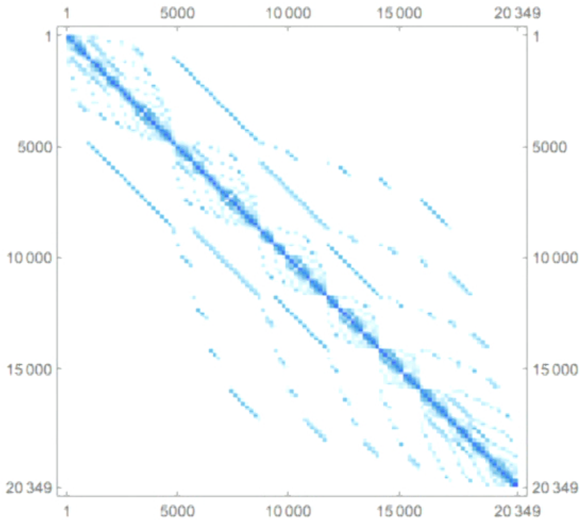
```
Out[127]= SparseArray[ Specified elements: 225 202
                       Dimensions: {20 349, 20 349} ]

In[128]:= MatrixPlot[hMatrix]
```

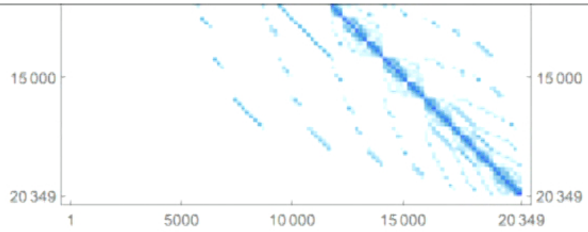
Out[128]=




```
In[138]:= actionhBottom = Table[hBottom /. state → X //. rule, {X, basisBottom}];

In[139]:= hMatrixBottom = CoefficientArrays[actionhBottom, basisBottom][[2]]

Out[139]= SparseArray[ Specified elements: 326
                       Dimensions: {56, 56} ]
```



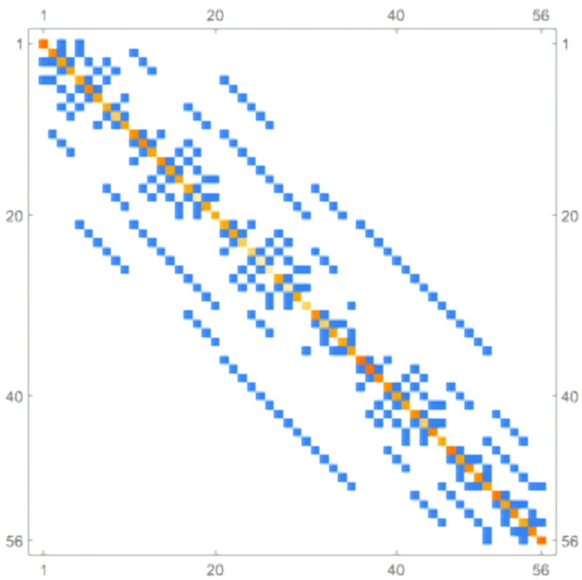

```
In[138]:= actionhBottom = Table[hBottom /. state → X //. rule, {X, basisBottom}];

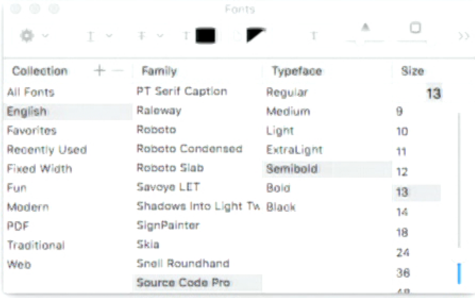
In[139]:= hMatrixBottom = CoefficientArrays[actionhBottom, basisBottom][[2]]

Out[139]= SparseArray[ Specified elements: 326
                       Dimensions: {56, 56} ]

In[140]:= hMatrixBottom // MatrixPlot
```

Out[140]=





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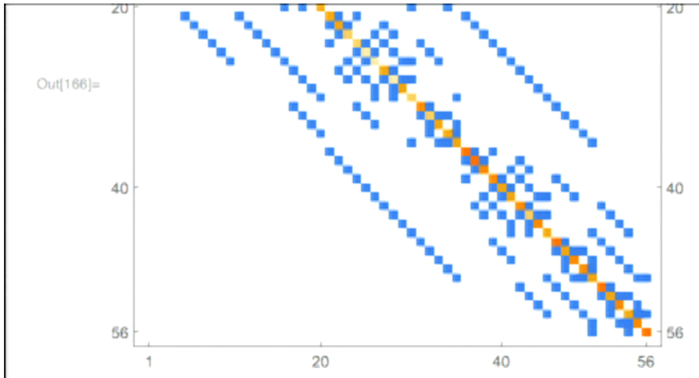
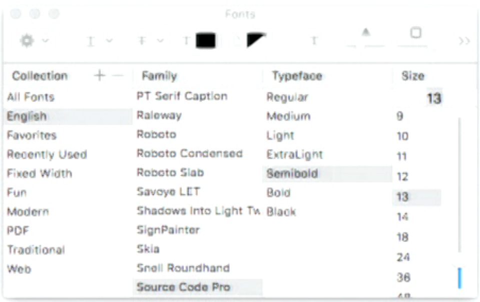


Diagonalization of Bottom Corner - Starting Point

```
esystemBottom = Normal[hMatrixBottom] // Eigensystem;
```

In[168]:=
```
esystemBottom[[1]]
```

Out[168]= {5.55218, -5.05949, 4.54145, 4.1225, -3.90234, 3.67066, 3.63159, -3.51263, -3.08507, 2.97131, -2.96561, 2.82078, 2.80671, 2.5243, 2.37145, -2.366, 2.32121, -2.26437, 2.25392, -2.14222, 2.01227, 1.95908, -1.89604, -1.76071, -1.6565, -1.61984, 1.60939, 1.59309, 1.52732, 1.52445, -1.42776, 1.33957, -1.29464, 1.24339, 1.19584, 1.08594, 0.964793, -0.941969, -0.923316, -0.865122, -0.824175, 0.804653, 0.77696, -0.676683, 0.645935, -0.632213, -0.612335, 0.481571, 0.45322, -0.451797, 0.341713, -0.335292, 0.223489, -0.0904178, -0.0682545, 0.00405907}



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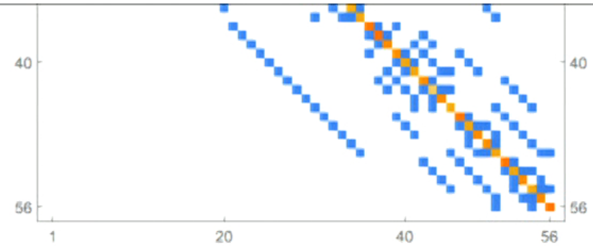


Diagonalization of Bottom Corner - Starting Point

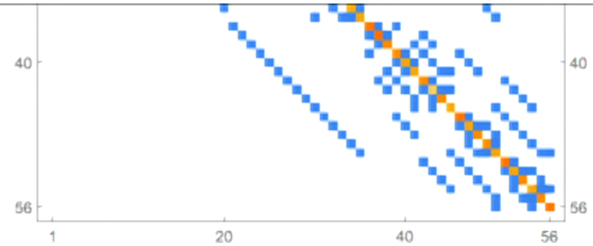
```
esystemBottom = Normal[hMatrixBottom] // Eigensystem;

In[171]:= E0 = esystemBottom[[1]] // Sort // First

Out[171]= -5.05949

esystemBottom[[2, 2]]

Out[174]= {{-0.0334995, -0.0471528, 0.0955061, -0.0934498, 0.0737396, -0.0774657, 0.0905912, -0.128263, 0.153721, -0.121645,
  0.0736942, -0.121785, 0.150997, -0.109835, 0.141503, -0.153981, 0.16798, -0.240609, 0.153981, 0.121645, -0.100913, 0.13928,
  -0.183241, 0.182713, -0.238776, 0.240609, -0.161286, 0.238776, -0.141503, -0.153721, -0.120487, 0.183241, -0.150997,
  -0.0905912, 0.0934498, 0.0605879, -0.0868352, 0.120487, -0.118353, 0.161286, -0.16798, 0.118353, -0.182713, 0.109835, 0.128263,
  0.0868352, -0.13928, 0.121785, 0.0774657, -0.0955061, -0.0605879, 0.100913, -0.0736942, -0.0737396, 0.0471528, 0.0334995},
 {-0.0266892, -0.0384228, -0.0843937, -0.0915022, -0.0639845, -0.066657, -0.088835, -0.121949, -0.165426, -0.126397, -0.0613908
  -0.107088, -0.15175, -0.100446, -0.147211, -0.158321, -0.160409, -0.26484, -0.158321, -0.126397, -0.0892065,
  -0.191555, -0.179371, -0.26537, -0.26484, -0.155404, -0.26537, -0.147211, -0.165426, -0.109987, -0.191555, -0.
  -0.0915022, -0.0466556, -0.0691392, -0.109987, -0.101036, -0.155404, -0.160409, -0.101036, -0.179371, -0.1004
  -0.0691392, -0.1267, -0.107088, -0.066657, -0.0843937, -0.0466556, -0.0892065, -0.0613908, -0.0639845, -0.0384
 {0.101802, 0.138757, -0.224309, 0.124505, -0.187117, 0.163319, -0.0842055, 0.209977, -0.105287, 0.0213025, -0.1
  -0.0609833, 0.012925, -0.199719, 0.0862138, 0.012925, 0.0213025, 0.179792, -0.180745, 0.0480269, -0.174174, 0.0
  -0.0609833, -0.105287, 0.0694102, 0.0480269, -0.114279, -0.0842055, 0.124505, -0.0573129, 0.0384071, 0.0694102
  -0.174174, 0.144419, 0.209977, 0.0384071, -0.180745, 0.213037, 0.163319, -0.224309, -0.0573129, 0.179792, -0.1
 {-0.0524226, -0.0775603, 0.0691928, 0.0692128, 0.120708, -0.0722741, -0.0641402, -0.0752194, -0.122608, 0.17270
  -0.0562255, -0.14701, -0.0501314, -0.132436, 0.242141, 0.0450195, 0.231288, -0.242141, -0.172708, -0.183929, 0.14328, 0.0755287,
  0.175214, 0.0829402, -0.231288, -0.135961, -0.0829402, 0.132436, 0.122608, -0.0858375, -0.0755287, 0.14701, 0.0641402,
```



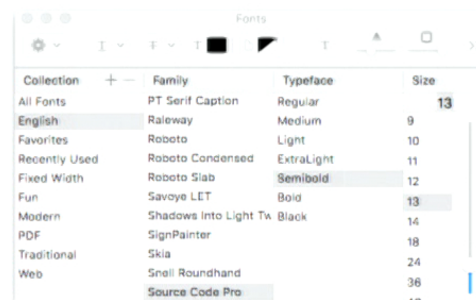
Diagonalization of Bottom Corner - Starting Point

```
esystemBottom = Normal[hMatrixBottom] // Eigensystem;

In[171]:= E0 = esystemBottom[[1]] // Sort // First

Out[171]= -5.05949

GSBottom = esystemBottom[[2, 2]]; (* very ugly!! *)
```



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```
In[171]:= E0 = esystemBottom[[1]] // Sort // First

Out[171]= -5.05949

GSBottom = esystemBottom[[2, 2]]; (* very ugly!! *)

GSBottom.basisBottom, basis
                      basis
                      basisBottom

Out[177]= -0.0266892 |{2, 1}, {   2}, {4, 1}, {4, 2}⟩ - 0.0384228 |{2, 1}, {3, 1}, {3, 2}, {4, 1}, {5, 1}⟩ -
 0.0843937 |{2, 1}, {3, 1}, {3, 2}, {4, 1}, {5, 2}⟩ - 0.0915022 |{2, 1}, {3, 1}, {3, 2}, {4, 1}, {5, 3}⟩ - 0.0639845 |{2, 1}, {3, 1}, {3, 2}, {4, 2}, {5, 1}⟩ -
 0.066657 |{2, 1}, {3, 1}, {3, 2}, {4, 2}, {5, 2}⟩ - 0.088835 |{2, 1}, {3, 1}, {3, 2}, {4, 2}, {5, 3}⟩ - 0.121949 |{2, 1}, {3, 1}, {3, 2}, {5, 1}, {5, 2}⟩ -
 0.165426 |{2, 1}, {3, 1}, {3, 2}, {5, 1}, {5, 3}⟩ - 0.126397 |{2, 1}, {3, 1}, {3, 2}, {5, 2}, {5, 3}⟩ - 0.0613908 |{2, 1}, {3, 1}, {4, 1}, {4, 2}, {5, 1}⟩ -
 0.107088 |{2, 1}, {3, 1}, {4, 1}, {4, 2}, {5, 2}⟩ - 0.15175 |{2, 1}, {3, 1}, {4, 1}, {4, 2}, {5, 3}⟩ - 0.100446 |{2, 1}, {3, 1}, {4, 1}, {5, 1}, {5, 2}⟩ -
 0.147211 |{2, 1}, {3, 1}, {4, 1}, {5, 1}, {5, 3}⟩ - 0.158321 |{2, 1}, {3, 1}, {4, 1}, {5, 2}, {5, 3}⟩ - 0.160409 |{2, 1}, {3, 1}, {4, 2}, {5, 1}, {5, 2}⟩ -
 0.26484 |{2, 1}, {3, 1}, {4, 2}, {5, 1}, {5, 3}⟩ - 0.158321 |{2, 1}, {3, 1}, {4, 2}, {5, 2}, {5, 3}⟩ - 0.126397 |{2, 1}, {3, 1}, {5, 1}, {5, 2}, {5, 3}⟩ -
 0.0892065 |{2, 1}, {3, 2}, {4, 1}, {4, 2}, {5, 1}⟩ - 0.1267 |{2, 1}, {3, 2}, {4, 1}, {4, 2}, {5, 2}⟩ - 0.191555 |{2, 1}, {3, 2}, {4, 1}, {4, 2}, {5, 3}⟩ -
 0.179371 |{2, 1}, {3, 2}, {4, 1}, {5, 1}, {5, 2}⟩ - 0.26537 |{2, 1}, {3, 2}, {4, 1}, {5, 1}, {5, 3}⟩ - 0.26484 |{2, 1}, {3, 2}, {4, 1}, {5, 2}, {5, 3}⟩ -
 0.155404 |{2, 1}, {3, 2}, {4, 2}, {5, 1}, {5, 2}⟩ - 0.26537 |{2, 1}, {3, 2}, {4, 2}, {5, 1}, {5, 3}⟩ - 0.147211 |{2, 1}, {3, 2}, {4, 2}, {5, 2}, {5, 3}⟩ -
 0.165426 |{2, 1}, {3, 2}, {5, 1}, {5, 2}, {5, 3}⟩ - 0.109987 |{2, 1}, {4, 1}, {4, 2}, {5, 1}, {5, 2}⟩ - 0.191555 |{2, 1}, {4, 1}, {4, 2}, {5, 1}, {5, 3}⟩ -
 0.15175 |{2, 1}, {4, 1}, {4, 2}, {5, 2}, {5, 3}⟩ - 0.088835 |{2, 1}, {4, 1}, {5, 1}, {5, 2}, {5, 3}⟩ - 0.0915022 |{2, 1}, {4, 2}, {5, 1}, {5, 2}, {5, 3}⟩ -
 0.0466556 |{3, 1}, {3, 2}, {4, 1}, {4, 2}, {5, 1}⟩ - 0.0691392 |{3, 1}, {3, 2}, {4, 1}, {4, 2}, {5, 2}⟩ - 0.109987 |{3, 1}, {3, 2}, {4, 1}, {4, 2}, {5, 3}⟩ -
 0.101036 |{3, 1}, {3, 2}, {4, 1}, {5, 1}, {5, 2}⟩ - 0.155404 |{3, 1}, {3, 2}, {4, 1}, {5, 1}, {5, 3}⟩ - 0.160409 |{3, 1}, {3, 2}, {4, 1}, {5, 2}, {5, 3}⟩ -
 0.101036 |{3, 1}, {3, 2}, {4, 2}, {5, 1}, {5, 2}⟩ - 0.179371 |{3, 1}, {3, 2}, {4, 2}, {5, 1}, {5, 3}⟩ - 0.100446 |{3,
 0.121949 |{3, 1}, {3, 2}, {5, 1}, {5, 2}, {5, 3}⟩ - 0.0691392 |{3, 1}, {4, 1}, {4, 2}, {5, 1}, {5, 2}⟩ - 0.1267 |{3, 1
 0.107088 |{3, 1}, {4, 1}, {4, 2}, {5, 2}, {5, 3}⟩ - 0.066657 |{3, 1}, {4, 1}, {5, 1}, {5, 2}, {5, 3}⟩ - 0.0843937 |{3
 0.0466556 |{3, 2}, {4, 1}, {4, 2}, {5, 1}, {5, 2}⟩ - 0.0892065 |{3, 2}, {4, 1}, {4, 2}, {5, 1}, {5, 3}⟩ - 0.0613908 |
 0.0639845 |{3, 2}, {4, 1}, {5, 1}, {5, 2}, {5, 3}⟩ - 0.0384228 |{3, 2}, {4, 2}, {5, 1}, {5, 2}, {5, 3}⟩ - 0.0266892 |
```

Fonts

Collection	Family	Typeface	Size
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Favorites	Roboto	Light	10
Recently Used	Roboto Condensed	ExtraLight	11
Fixed Width	Roboto Slab	Semibold	12
Fun	Savoye LET	Bold	13
Modern	Shadows Into Light Tw	Black	14
PDF	SignPainter		18
Traditional	Skia		24
Web	Snell Roundhand		36
	Source Code Pro		

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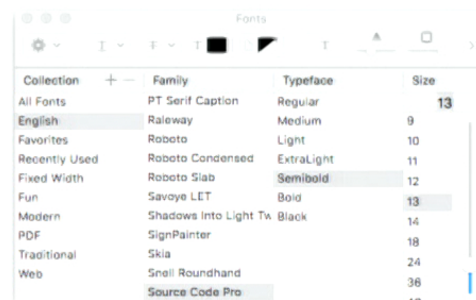
```
In[171]:= E0 = esystemBottom[[1]] // Sort // First
Out[171]= -5.05949

GSBottom = esystemBottom[[2, 2]]; (* very ugly!! *)

In[179]:= vectorStart = Table[Coefficient[GSBottom.basisBottom, basis[[j]]], {j, Length[basis]}]
Out[179]= {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, … 20 183 …,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 -0.0266892, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}

large output    show less    show more    show all    set size limit...
```

We now decompose vectorStart = sum c_n vector_n of the big system ->   sum c_n vector_n exp(-i E_n t)



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```
(esystem = hMatrix // Normal // Eigensystem;) // AbsoluteTiming (* 20 minutes for five = 5; 10 seconds for five = 4 *)

Out[209]= {9.99779, Null}

In[213]:= evectors = esystem[[2]];
          evalues = esystem[[1]];

In[215]:= evectors[[33]]

Out[215]= {0.00209011, 0.00211582, 0.00486943, 0.0074794, 0.00563282, 0.00441255, 0.0109527, 0.0113761, 0.00177388, 0.0104447, 0.0112135, 0.00451961, 0.000312637,
  0.00525285, 0.00474753, 0.00117081, 0.000851314, 0.00721606, 0.00734856, 0.000495861, 0.00581188, 0.00678116, 0.00295712, 0.00563427, 0.00546206,
  0.00171781, 0.00050337, 0.00773327, 0.00804637, 0.000285978, 0.00623781, 0.00737631, 0.00325187, 0.0105009, 0.0127992, 0.0118547, 0.0130843,
  0.0154807, 0.00543567, 0.0144569, 0.0156884, 0.00616134, 0.00985888, 0.0115311, 0.0161196, 0.0137163, 0.00591089, 0.0147469, 0.0161094,
  0.00689142, 0.00323975, 0.0182788, 0.0181935, 0.00209981, 0.015318, 0.0177886, 0.00785861, 0.0166387, 0.0175005, 0.000495268, 0.0137355,
  0.0164323, 0.00737417, 0.00743956, 0.00798245, 0.00502887, 0.005606, 0.00118646, 0.00883236, 0.00555407, 0.00472433, 0.00146784, 0.00688145,
  0.00856181, 0.00401354, 0.000776359, -0.000882318, -0.00060169, 0.00375613, 0.00664415, 0.00892633, 0.00549055, 0.00518694, 0.0149136,
  0.014565, 0.0016804, 0.0141479, 0.0143858, 0.00543458, 0.0080512, 0.0108023, 0.00838315, 0.00530843, 0.0181325, 0.0173728, 0.00192187,
  0.0178675, 0.018207, 0.0073156, 0.00842666, 0.00882116, 0.00561765, 0.00901786, 0.0117058, -0.000481103, 0.0105413, 0.0102656, 0.00215711,
  0.00607637, 0.00303725, 0.0204785, 0.0128409, -0.00430732, 0.0197622, 0.0136968, 0.00137064, 0.00267684, 0.0201282, 0.0151184, -0.00171095,
  0.0195347, 0.0160593, 0.00449189, 0.0131682, 0.00877923, -0.000678675, 0.0146903, 0.0126204, 0.00508662, 0.0147054, -0.0000990575, 0.00972086,
  0.0118495, 0.00133663, -0.00325715, 0.0159067, 0.00785394, -0.00117584, 0.00239856, 0.0011228, 0.00130242, 0.0077829, 0.00145769, -0.00119262,
  0.00962803, 0.0089706, 0.00218753, 0.00110306, 0.0134301, 0.0139332, 0.000738513, 0.0110443, 0.0130861, 0.00581737, 0.0110594, 0.0120905,
  0.0139046, 0.01565, 0.017031, 0.00563208, 0.0171097, 0.0169394, 0.00574151, 0.00922027, 0.0137272, 0.0165005, 0.0141059, 0.00663977, 0.0151461,
  0.0161098, 0.00633237, 0.0034289, 0.0173022, 0.0172555, 0.00214477, 0.0146056, 0.0169536, 0.00745623, 0.019831, 0.0212089, 0.000643621,
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  0.00553443, 0.0104093, 0.00480705, 0.00226146, -0.00174394, 0.0120437, 0.00324621, 0.00291982, -0.000477035, 0.
  -0.00257484, -0.00379113, -0.00232703, 0.0018083, -0.00234955, 0.0109646, 0.00432338, -0.00971283, 0.0117685, 0
  0.00658571, 0.0120258, 0.00568171, -0.00186666, 0.0136978, 0.00580127, -0.0022132, 0.00482966, -0.00211061, -0.
  0.00388379, 0.000940513, 0.00761772, -0.00683779, 0.00539711, 0.00278187, -0.00518584, -0.0129072, 0.0104608, -
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```

```
(esystem = hMatrix // Normal // Eigensystem;) // AbsoluteTiming (* 20 minutes for five = 5; 10 seconds for five = 4 *)

Out[209]= {9.99779, Null}

In[213]:= evectors = esystem[[2]];
          evalues = esystem[[1]];

          evectors[[33]].evectors[[34]]

Out[215]= {0.00209011, 0.00211582, 0.00486943, 0.0074794, 0.00563282, 0.00441255, 0.0109527, 0.0113761, 0.00177388, 0.0104447, 0.0112135, 0.00451961, 0.000312637,
 0.00525285, 0.00474753, 0.00117081, 0.000851314, 0.00721606, 0.00734856, 0.000495861, 0.00581188, 0.00678116, 0.00295712, 0.00563427, 0.00546206,
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```

Fonts

Collection	Family	Typeface	Size
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Favorites	Roboto	Light	10
Recently Used	Roboto Condensed	ExtraLight	11
Fixed Width	Roboto Slab	Semibold	12
Fun	Savoye LET	Bold	13
Modern	Shadows Into Light Tw	Black	14
PDF	SignPainter		18
Traditional	Skia		24
Web	Snell Roundhand		36
	Source Code Pro		

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```
(esystem = hMatrix // Normal // Eigensystem;) // AbsoluteTiming (* 20 minutes for five = 5; 10 seconds for five = 4 *)

Out[209]= {9.99779, Null}

In[213]:= evectors = esystem[[2]];
          evalues = esystem[[1]];

In[217]:= evectors[[33]].evectors[[33]]

Out[217]= 1.

In[218]:= vectorT0 = vectorStart.# & /@ evectors;

In[219]:= vectorT0

Out[219]= {1.22351 × 10^-6, 0.000808639, 3.05168 × 10^-6, 0.00228645, -4.57737 × 10^-6, 1.97687 × 10^-6, 0.00158044, -0.000626715, 0.0000116042, -0.0000123814, -0.0000128806,
 7.46937 × 10^-6, -0.0000107508, 0.015679, -0.000011629, 0.00135706, 0.0162849, -0.00738838, -0.00421072, -0.0277437, 0.0000103986, 3.60481 × 10^-6,
 -1.8985 × 10^-6, -0.0000158628, -0.0000168725, -7.58977 × 10^-6, -0.0000137308, 0.0000115493, -0.00205595, -0.0255597, 0.0000113955, 0.0235047,
 -0.00998903, -6.26787 × 10^-6, -0.00825421, 0.00002646, 0.0126613, 0.00628508, -0.0126494, -0.0000391654, -0.0000341391, -0.0334357, -0.0000107291,
 0.016291, 9.47019 × 10^-6, 0.0000102611, -0.024375, -3.70334 × 10^-6, -6.56301 × 10^-7, -0.0000120822, -0.00778558, -7.2141 × 10^-6, 6.35363 × 10^-6,
 -0.0267705, -5.27467 × 10^-6, 0.0181523, -0.00228021, 0.0129211, 0.00560511, -5.12681 × 10^-6, 6.76793 × 10^-6, -0.0000106693, 0.000034405, 0.00488053,
 0.0113162, 0.0000328236, 0.0124757, -0.0000126137, 0.0183649, 6.04191 × 10^-6, -0.0182795, 0.0000111907, 4.63027 × 10^-6, 1.85602 × 10^-6, 0.047319,
 0.0000147904, -0.000018911, -0.0227262, -0.0000283644, -0.0000192238, 0.0194931, 0.0266864, 0.000026383, 0.0000221424, -0.0000266188,
 0.0184025, 0.000025946, 0.024428, 0.0179084, -0.0000195442, -0.0000206883, -0.0135256, -3.61421 × 10^-6, -0.0000164147, -0.0167203, 8.97423 × 10^-7,
 -9.87486 × 10^-6, 0.0252038, -0.0324867, 0.0000247621, 5.38886 × 10^-6, 0.0207123, -0.00816966, -0.0000118087, 3.01535 × 10^-6, -0.0331444,
 -0.0111569, 8.49783 × 10^-7, 0.00175912, -0.0000443939, 0.0805992, -0.0640916, 7.31993 × 10^-6, -0.0000124754, -2.424
 -0.0101657, 0.0351028, -0.0000268282, 0.0269412, 7.22156 × 10^-6, 0.00894694, 0.0000178178, -0.0381731, -0.0300232
 -0.0000117888, 0.0111637, 0.0000268852, -0.0184493, 6.62221 × 10^-6, -0.00839702, -0.0000223771, -3.36271 × 10^-6, -(
 -8.32263 × 10^-6, -0.0769332, 0.0000148809, 0.00462588, 9.414 × 10^-6, 0.0000328976, -0.0742913, -0.0491696, -0.00001
 -3.19915 × 10^-6, 0.0193267, -0.0676435, 0.0000108915, 5.76956 × 10^-6, 0.000013614, 0.0145707, -9.2769 × 10^-6, -0.0489
 -1.78134 × 10^-6, 1.18145 × 10^-6, 0.0753724, -0.00855731, 0.0233574, 7.89819 × 10^-6, -0.00390303, 0.0000554332, -0.000
 -0.0930202, 0.0000232586, 0.00847781, 7.99738 × 10^-6, 7.81415 × 10^-6, 0.0171424, 0.0183478, 0.0000685035, -0.000057
 -0.0254996, 0.0553878, -0.0000320629, 0.0000263716, 0.0000266934, 0.030552, -0.0000171183, 0.0169145, -0.064715
 -0.0549408, -0.000010831, -0.0247845, 0.0000586193, 0.0000444343, 0.0000481379, -0.0000326603, 0.0136362, 0.085
 0.0174254, 0.0000135428, -6.13601 × 10^-6, 0.0000278409, -0.155823, -0.0000360533, 2.9709 × 10^-6, -0.00739347, -0.0727362, -0.0000302021,
 0.0171817, 2.96806 × 10^-6, 5.39711 × 10^-6, -2.73486 × 10^-6, 0.0567965, 0.0000189282, -0.0848258, 0.000024697, 0.000037463, -0.0467321, -0.0155766,
 -9.4075 × 10^-6. -0.0000323875. -0.0000410392. -4.2708 × 10^-6. -0.0137356. -0.042928. 0.0000474469. -5.76316 × 10^-6. 0.0000210136. 0.000052061.
```

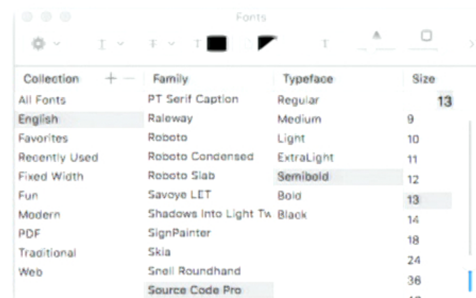
```
In[217]:= evectors[[33]].evectors[[33]]

Out[217]= 1.

In[218]:= vectorT0 = vectorStart.# & /@ evectors;

{c1

v[t_] := v[t] = (vectorT0 Exp[I t evalues]).evectors
```



150%

```
In[217]:= evectors[[33]].evectors[[33]]

Out[217]= 1.

In[218]:= vectorT0 = vectorStart.# & /@ evectors;

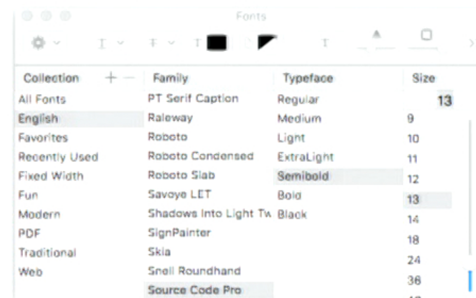
In[220]:= {c1, c2, c3} Exp[I {E1, E2, E3} t]
```

Out[220]= $\{c1\, e^{i\, E1\, t},\ c2\, e^{i\, E2\, t},\ c3\, e^{i\, E3\, t}\}$

```
In[221]:= {c1, c2, c3}.Exp[I {E1, E2, E3} t]
```

Out[221]= $c1\, e^{i\, E1\, t} + c2\, e^{i\, E2\, t} + c3\, e^{i\, E3\, t}$

```
v[t_] := v[t] = (vectorT0 Exp[I t evalues]).evectors
```



150%

In[217]:= `evectors[[33]].evectors[[33]]`

Out[217]= `1.`

In[218]:= `vectorT0 = vectorStart.# & /@ evectors;`

In[220]:= `{c1, c2, c3} Exp[I {E1, E2, E3} t]`

Out[220]= $\{c1\, e^{i\, E1\, t}, c2\, e^{i\, E2\, t}, c3\, e^{i\, E3\, t}\}$

In[222]:= `v[t_] := v[t] = (vectorT0 Exp[I t evalues]).evectors;`

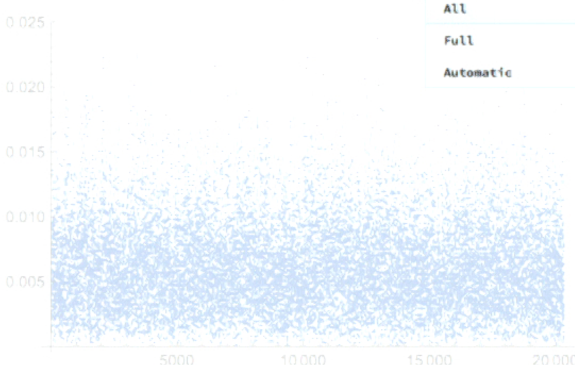
`v[40] // Abs // ListPlot[#, PlotRange ->`

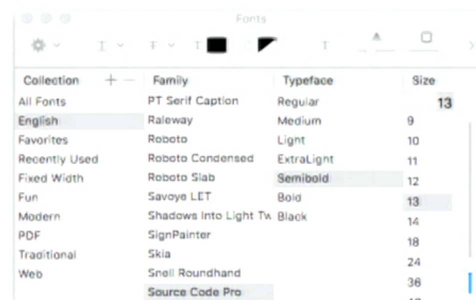
All

Full

Automatic

Out[226]=





150%

Diagonalization of Bottom Corner - Starting Point

```
In[205]:= esystemBottom = Normal[hMatrixBottom] // Eigensystem;

In[206]:= E0 = esystemBottom[[1]] // Sort // First

Out[206]= -5.05949

In[207]:= GSBottom = esystemBottom[[2, 2]]; (* very ugly!! *)

In[208]:= vectorStart = Table[Coefficient[GSBottom.basisBottom, basis[[j]]], {j, Length[basis]}]

Out[208]= {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
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  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
  0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
```

In[213]:=
```
evectors = esystem[[2]];
evalues = esystem[[1]];
```

In[217]:=
```
evectors[[33]].evectors[[33]]
```

Out[217]= 1.

In[218]:=
```
vectorT0 = vectorStart.# & /@ evectors;
```

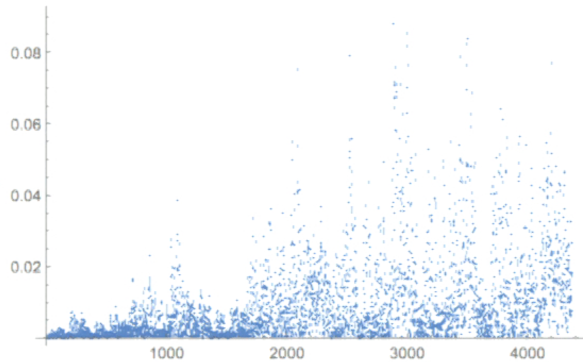
In[220]:=
```
{c1, c2, c3} Exp[I {E1, E2, E3} t]
```

Out[220]= $\{c1\, e^{i\, E1\, t},\ c2\, e^{i\, E2\, t},\ c3\, e^{i\, E3\, t}\}$

In[243]:=
```
w[t_] := w[t] = (vectorT0 Exp[I t evalues]).evectors;
```

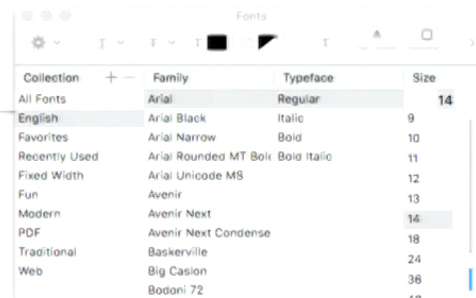
In[254]:=
```
w[3] // Abs // ListPlot[#, PlotRange → All] &
```

Out[254]=



Compute an observable by time evolution

$\frac{1}{Lx\,Ly}$ Sum[Exp[−*I* kx xn − *I* ky ny] bd[{nx, ny}].*b*[{nx, ny}] , {nx, ny}]



150%

1000 2000 3000 4000

Compute an observable by time evolution

```
In[255]:= goal = Sum[Sum[1/(5 * five) Exp[-2 π I {erased[[j]] - erased[[k]]}.{0/5, ny/five}] bd[erased[[j]]].b[erased[[k]]].state, {j, Length[erased]}, {k, Length[erased]}],
   {ny, 0, five - 1}]

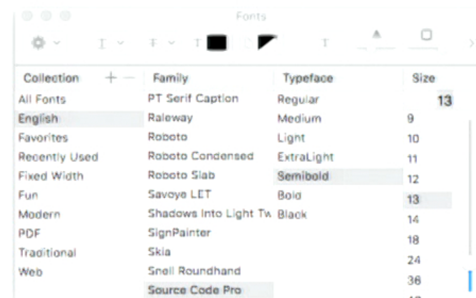
Out[255]= {1/5 bd[{1, 2}].b[{1, 2}].state + 1/5 bd[{1, 2}].b[{3, 2}].state + 1/5 bd[{1, 2}].b[{4, 2}].state + 1/5 bd[{1, 2}].b[{5, 2}].state + 1/5 bd[{1, 3}].b[{1, 3}].state +
   1/5 bd[{1, 3}].b[{2, 3}].state + 1/5 bd[{1, 3}].b[{3, 3}].state + 1/5 bd[{1, 3}].b[{5, 3}].state + 1/5 bd[{1, 4}].b[{1, 4}].state + 1/5 bd[{1, 4}].b[{2, 4}].state +
   1/5 bd[{1, 4}].b[{3, 4}].state + 1/5 bd[{1, 4}].b[{4, 4}].state + 1/5 bd[{2, 1}].b[{2, 1}].state + 1/5 bd[{2, 1}].b[{3, 1}].state + 1/5 bd[{2, 1}].b[{4, 1}].state +
   1/5 bd[{2, 1}].b[{5, 1}].state + 1/5 bd[{2, 3}].b[{1, 3}].state + 1/5 bd[{2, 3}].b[{2, 3}].state + 1/5 bd[{2, 3}].b[{3, 3}].state + 1/5 bd[{2, 3}].b[{5, 3}].state +
   1/5 bd[{2, 4}].b[{1, 4}].state + 1/5 bd[{2, 4}].b[{2, 4}].state + 1/5 bd[{2, 4}].b[{3, 4}].state + 1/5 bd[{2, 4}].b[{4, 4}].state + 1/5 bd[{3, 1}].b[{2, 1}].state +
   1/5 bd[{3, 1}].b[{3, 1}].state + 1/5 bd[{3, 1}].b[{4, 1}].state + 1/5 bd[{3, 1}].b[{5, 1}].state + 1/5 bd[{3, 2}].b[{1, 2}].state + 1/5 bd[{3, 2}].b[{3, 2}].state +
   1/5 bd[{3, 2}].b[{4, 2}].state + 1/5 bd[{3, 2}].b[{5, 2}].state + 1/5 bd[{3, 3}].b[{1, 3}].state + 1/5 bd[{3, 3}].b[{2, 3}].state + 1/5 bd[{3, 3}].b[{3, 3}].state +
   1/5 bd[{3, 3}].b[{5, 3}].state + 1/5 bd[{3, 4}].b[{1, 4}].state + 1/5 bd[{3, 4}].b[{2, 4}].state + 1/5 bd[{3, 4}].b[{3, 4}].state +
   1/5 bd[{3, 4}].b[{4, 4}].state + 1/5 bd[{4, 1}].b[{2, 1}].state + 1/5 bd[{4, 1}].b[{3, 1}].state + 1/5 bd[{4, 1}].b[{4, 1}].state +
   1/5 bd[{4, 1}].b[{5, 1}].state + 1/5 bd[{4, 2}].b[{1, 2}].state + 1/5 bd[{4, 2}].b[{3, 2}].state + 1/5 bd[{4, 2}].b[{4,
   1/5 bd[{4, 2}].b[{5, 2}].state + 1/5 bd[{4, 4}].b[{1, 4}].state + 1/5 bd[{4, 4}].b[{2, 4}].state + 1/5 bd[{4, 4}].b[{3,
   1/5 bd[{4, 4}].b[{4, 4}].state + 1/5 bd[{5, 1}].b[{2, 1}].state + 1/5 bd[{5, 1}].b[{3, 1}].state + 1/5 bd[{5, 1}].b[{4,
   1/5 bd[{5, 1}].b[{5, 1}].state + 1/5 bd[{5, 2}].b[{1, 2}].state + 1/5 bd[{5, 2}].b[{3, 2}].state + 1/5 bd[{5, 2}].b[{4,
   1/5 bd[{5, 2}].b[{5, 2}].state + 1/5 bd[{5, 3}].b[{1, 3}].state + 1/5 bd[{5, 3}].b[{2, 3}].state + 1/5 bd[{5, 3}].b[{3,   }
```

Compute an observable by time evolution

In[257]:=
```
goal = Sum[Sum[1/(5 * five) Exp[-2 π I {erased[[j]] - erased[[k]]}.{0/5, ny/five}] bd[erased[[j]]].b[erased[[k]]].state, {j, Length[erased]}, {k, Length[erased]}],
    {ny, 0, five - 1}][[1]];
```

In[258]:=
```
actionGoal = Table[goal /. state → X //. rule, {X, basis}];

GoalMatrix = CoefficientArrays[actionGoal, basis]
```



150%

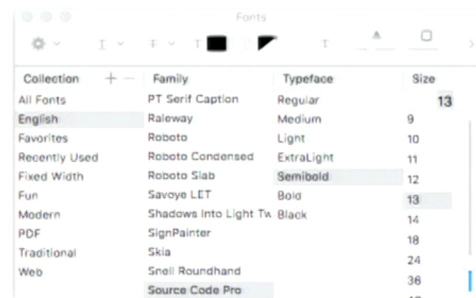
Compute an observable by time evolution

```
In[257]:= goal = Sum[Sum[1/(5*five) Exp[-2 π I {erased[[j]] - erased[[k]]}.{0/5, ny/five}] bd[erased[[j]]].b[erased[[k]]].state, {j, Length[erased]}, {k, Length[erased]}],
        {ny, 0, five - 1}][[1]];

In[258]:= actionGoal = Table[goal /. state → X //. rule, {X, basis}];

In[260]:= GoalMatrix = CoefficientArrays[actionGoal, basis][[2]];

          Conjugate[v[t]].GoalMatrix
```



150%

Compute an observable by time evolution

```
In[257]:= goal = Sum[Sum[1/(5 * five) Exp[-2 π I (erased[[j]] - erased[[k]]).{0/5, ny/five}] bd[erased[[j]]].b[erased[[k]]].state, {j, Length[erased]}, {k, Length[erased]}],
      {ny, 0, five - 1}][[1]];

In[258]:= actionGoal = Table[goal /. state → X //. rule, {X, basis}];

In[260]:= GoalMatrix = CoefficientArrays[actionGoal, basis][[2]];

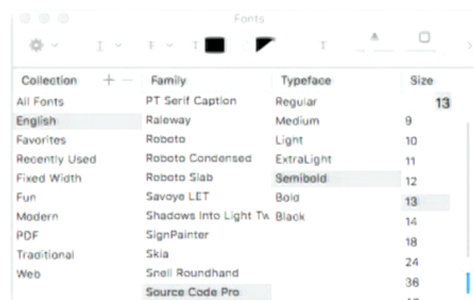
table = Monitor[Table[{t, Conjugate[w[t]].GoalMatrix.v[t]}, {t, 0, 100, 1/2}], t]
```

··· Dot: Tensors {-2.80214 × 10^-15, 2.98718 × 10^-16, 1.84041 × 10^-15, 3.13168 × 10^-15, -1.9096 × 10^-15, 2.06503 × 10^-16, -8.68922 × 10^-16, 7.60241 × 10^-16, ≪36≫, 3.48309 × 10^-15, 3.33572 × 10^-16, -2.32922 × 10^-16, 4.2162 × 10^-16, 1.08852 × 10^-15, 2.10267 × 10^-15, ≪20299≫} and SparseArray[Automatic, ≪3≫] have incompatible shapes.

··· Dot: Tensors SparseArray[Automatic, ≪3≫] and {-2.80214 × 10^-15, 2.98718 × 10^-16, 1.84041 × 10^-15, 3.13168 × 10^-15, -1.9096 × 10^-15, 2.06503 × 10^-16, -8.68922 × 10^-16, 7.60241 × 10^-16, ≪36≫, 3.48309 × 10^-15, 3.33572 × 10^-16, -2.32922 × 10^-16, 4.2162 × 10^-16, 1.08852 × 10^-15, 2.10267 × 10^-15, ≪20299≫} have incompatible shapes.

··· Dot: Tensors {6.37685 × 10^-16 - 2.17878 × 10^-15 i, 2.95487 × 10^-16 + 7.5629 × 10^-16 i, 1.75193 × 10^-16 + 2.10209 × 10^-15 i, ≪45≫, 1.48409 × 10^-15 + 9.06765 × 10^-16 i, -1.86475 × 10^-15 + 2.40137 × 10^-15 i, ≪20299≫} and SparseArray[Automatic, ≪3≫] have incompatible shapes.

··· General: Further output of Dot::dotsh will be suppressed during this calculation.



150%

$\{\frac{171}{2}, 1.61709 + 2.08167\times10^{-17}\, i\}, \{86, 1.61343 + 8.67362\times10^{-18}\, i\}, \{\frac{173}{2}, 1.61912 + 4.33681\times10^{-18}\, i\}, \{87, 1.63769 + 1.12757\times10^{-17}\, i\},$

$\{\frac{175}{2}, 1.65351 + 6.93889\times10^{-18}\, i\}, \{88, 1.65331 - 1.73472\times10^{-18}\, i\}, \{\frac{177}{2}, 1.63747 - 1.38778\times10^{-17}\, i\}, \{89, 1.61934 - 1.73472\times10^{-18}\, i\},$

$\{\frac{179}{2}, 1.612 - 8.67362\times10^{-18}\, i\}, \{90, 1.61942 - 2.60209\times10^{-17}\, i\}, \{\frac{181}{2}, 1.62442 + 6.93889\times10^{-18}\, i\}, \{91, 1.61654 - 1.99493\times10^{-17}\, i\},$

$\{\frac{183}{2}, 1.61085 - 4.33681\times10^{-18}\, i\}, \{92, 1.61945 + 3.46945\times10^{-18}\, i\}, \{\frac{185}{2}, 1.63243 + 1.73472\times10^{-17}\, i\}, \{93, 1.64953 + 2.60209\times10^{-18}\, i\},$

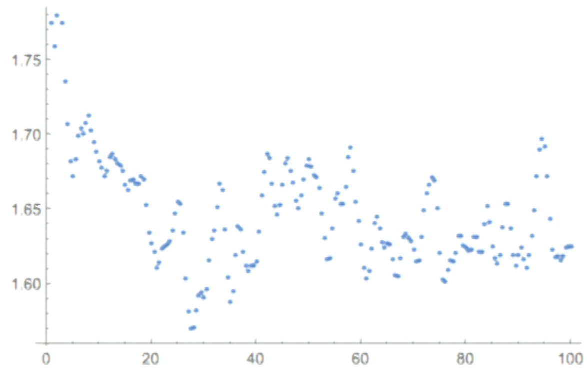
$\{\frac{187}{2}, 1.67217 + 0.\, i\}, \{94, 1.69 - 5.20417\times10^{-18}\, i\}, \{\frac{189}{2}, 1.6972 + 1.73472\times10^{-18}\, i\}, \{95, 1.69173 + 0.\, i\}, \{\frac{191}{2}, 1.67197 - 1.73472\times10^{-18}\, i\},$

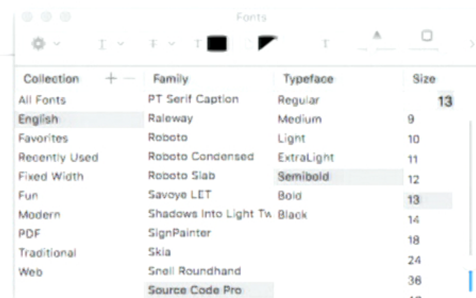
$\{96, 1.64391 + 0.\, i\}, \{\frac{193}{2}, 1.6228 + 9.54098\times10^{-18}\, i\}, \{97, 1.6181 - 2.25514\times10^{-17}\, i\}, \{\frac{195}{2}, 1.61873 + 2.08167\times10^{-17}\, i\}, \{98, 1.61578 - 8.67362\times10^{-18}\, i\},$

$\{\frac{197}{2}, 1.6188 + 1.47451\times10^{-17}\, i\}, \{99, 1.62462 - 3.46945\times10^{-17}\, i\}, \{\frac{199}{2}, 1.62497 - 1.30104\times10^{-17}\, i\}, \{100, 1.62517 + 0.\, i\}\}$

In[263]:= `experiment = Chop[table] // ListPlot`

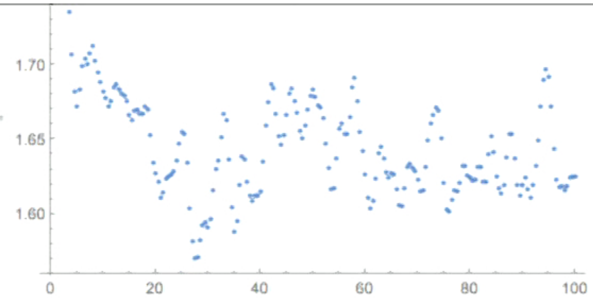
Out[263]=





150%

Out[264]=



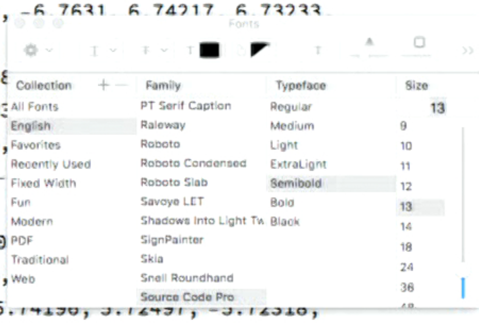
Computation by micro

```
In[265]:= E0

Out[265]= -5.05949

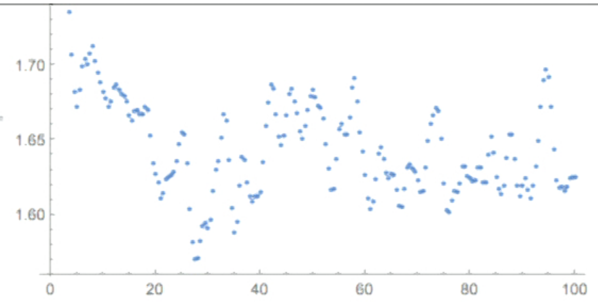
In[266]:= evalues

Out[266]= {10.0942, -9.85756, 9.63633, -9.40325, 9.00448, 8.97207, -8.78115, -8.75409, 8.73469, 8.55699, 8.52569, 8.4965, 8.4942, -8.47602, 8.36745, -8.32552,
  -8.28106, -8.27546, -8.26613, -8.0971, 8.07275, 8.0641, 7.98195, 7.95713, 7.92173, 7.90367, 7.87816, 7.86077, -7.83733, -7.82441, 7.77216,
  -7.74127, -7.7176, 7.70766, -7.69422, 7.66977, -7.65466, -7.64933, -7.62934, 7.58836, 7.54987, -7.51692, 7.47078, -7.45321, 7.44695, 7.42132,
  -7.40833, 7.38638, 7.33857, 7.32963, -7.32407, 7.30756, 7.30375, -7.29568, 7.28335, -7.22802, -7.2093, -7.19754, -7.16503, 7.16496, 7.15587,
  7.10457, 7.10002, -7.09859, -7.08775, 7.06059, -7.05089, 7.0415, -7.03398, 7.01803, -7.00652, 6.97083, 6.95127, 6.92974, -6.91755, 6.90663,
  6.90086, -6.89006, 6.88359, 6.86073, -6.85691, -6.84707, 6.82988, 6.81063, 6.80456, -6.78454, 6.78088, -6.77588, -6.7631, 6.74217, 6.73233,
  -6.71691, 6.71258, 6.71132, -6.70318, 6.69365, 6.6867, -6.66971, -6.66755, 6.66506, 6.64952, -6.64792, -6.6336,
  -6.57738, 6.55636, -6.54929, 6.54659, -6.54149, -6.52471, 6.51425, 6.50892, 6.50627, 6.50366, -6.48721, -6.48198
  -6.46134, 6.45559, -6.44153, -6.42518, 6.41689, 6.40716, 6.39732, -6.39383, 6.38549, -6.37064, 6.36297, -6.35883
  6.33305, 6.31446, -6.29074, 6.28935, -6.28532, 6.26349, 6.25771, -6.25043, -6.24634, 6.24523, 6.23503, -6.22605,
  6.20638, 6.19867, 6.19436, -6.18603, 6.17826, -6.1767, 6.16326, -6.15199, 6.1491, 6.14492, -6.14244, -6.14182, -
  6.11278, -6.10745, 6.09962, -6.09242, 6.08018, -6.07816, 6.0768, 6.06571, -6.06414, -6.03614, 6.02589, 6.02031,
  -5.98196, 5.97526, 5.96915, 5.95654, -5.95606, 5.94653, -5.94234, -5.942, 5.93, 5.92794, -5.92548, 5.9252, -5.90
  5.89117, -5.89112, -5.89105, 5.86915, -5.86436, 5.86112, 5.85533, 5.85198, -5.84776, 5.83999, 5.83964, -5.83725,
  5.8202, 5.81343, 5.80977, -5.80239, 5.78809, -5.78735, 5.78533, 5.77612, -5.76264, -5.75995, 5.75849, 5.74433, 5.74196, 5.72497, -5.72318,
  -5.71123, 5.70573, 5.69738, 5.69324, 5.69205, -5.68606, -5.6739, 5.67359, -5.66943, 5.66936, 5.6665, 5.66132, 5.65885, -5.65751, -5.64898,
  -5.64626, 5.63492, 5.62675, -5.62611, 5.61911, 5.61888, -5.61837, 5.61104, -5.60829, 5.59949, 5.59507, -5.59442, -5.5874, 5.58062, 5.57653,
```



150%

Out[264]=



Computation by micro

```
In[267]:= E0
          δ = 1 / 10;

Out[267]= -5.05949

In[271]:= positions = Position[evalues, _? (E0 - δ < # < E0 + δ &)] // Flatten

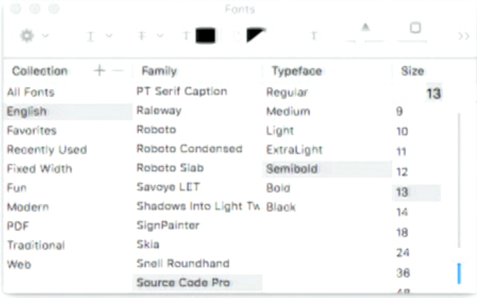
Out[271]= {358, 360, 362, 364, 369, 370, 371, 373, 375, 378, 381, 382, 384, 385, 387, 389, 392, 393, 396, 397, 401, 402, 404, 408, 409, 410}

In[272]:= VectorMicrocanonicalEnsemble = Table[evectors[[j]], {j, positions}];

In[273]:= VectorMicrocanonicalEnsemble // Dimensions

Out[273]= {26, 4368}
```

$$\text{micro} = \frac{1}{\text{Length[positions]}} \text{Sum[Conjugate[}\psi\text{].GoalMatrix.}\psi\text{, \{}\psi\text{, VectorMicrocanonicalEnsemble\}]}$$



150%

In[267]:= E0

δ = 1 / 10;

Out[267]= -5.05949

In[271]:= positions = Position[evalues, _? (E0 - δ < # < E0 + δ &)] // Flatten

Out[271]= {358, 360, 362, 364, 369, 370, 371, 373, 375, 378, 381, 382, 384, 385, 387, 389, 392, 393, 396, 397, 401, 402, 404, 408, 409, 410}

In[272]:= VectorMicrocanonicalEnsemble = Table[evectors[[j]], {j, positions}];

In[273]:= VectorMicrocanonicalEnsemble // Dimensions

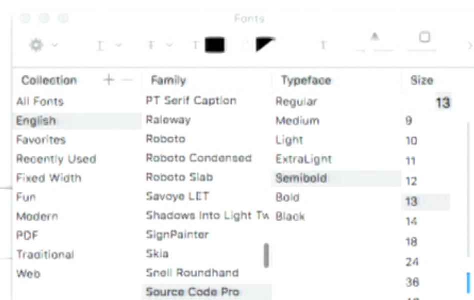
Out[273]= {26, 4368}

$$\text{micro} = \frac{1}{\text{Length[positions]}}\ \text{Sum[Conjugate[}\psi\text{].GoalMatrix.}\psi\text{, \{}\psi\text{, VectorMicrocanonicalEnsemble\}];}$$

theory = Plot[micro, {t, 0, 100}];

Out[275]=

Comparison



150%

```
theory = Plot[micro, {t, 0, 100}];
```

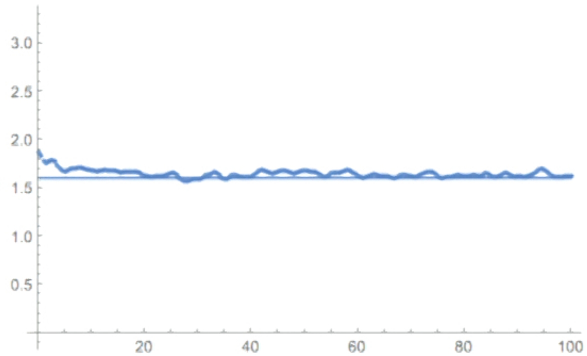
Out[275]=

Comparison

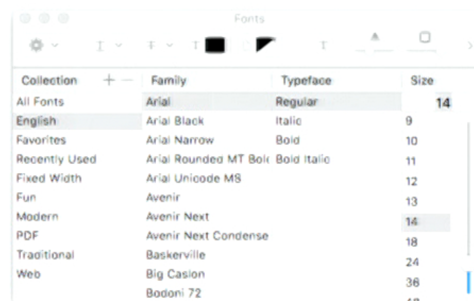
In[276]:=

```
Show[theory, experiment]
```

Out[276]=



ETH working :-)



150%

ETH. Quantum Hardcore Bosons Thermalization

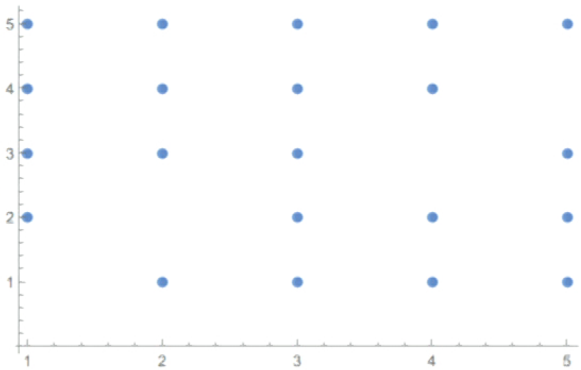
Lattice and H

```
In[2]:= five = 4;
        five = 5;

In[4]:= (start = Table[{i, j}, {i, 5}, {j, five}] // Flatten[#, 1] &) // ListPlot[#, PlotStyle → PointSize[0.02]] &;

In[5]:= (erased = start /. {{1, 1} → Sequence[], {2, 2} → Sequence[], {4, 3} → Sequence[], {5, 4} → Sequence[]}) // ListPlot[#, PlotStyle → PointSize[0.02]] &
```

Out[5]=



```
In[6]:= (bottom = erased /. {i_, j_} :> Sequence[] /; i < j + 1) // ListPlot[#, PlotStyle → PointSize[0.02]] &;

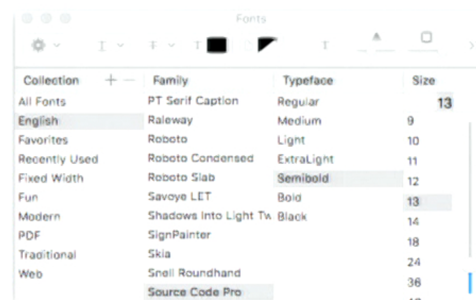
In[7]:= links = f @@@ Subsets[erased, {2}] /. f[a_, b_] :> Sequence[] /; Norm[a - b] ≠ 1;

In[8]:= linksBottom = f @@@ Subsets[bottom, {2}] /. f[a_, b_] :> Sequence[] /; Norm[a - b] ≠ 1;

In[9]:= J = 1; U = 0.1;

In[10]:= h = links /. f[i_, j_] :> -J bd[i].b[j].state - J bd[j].b[i].state + U bd[i].b[i].bd[j].b[j].state // Total;

In[11]:= hBottom = linksBottom /. f[i_, j_] :> -J bd[i].b[j].state - J bd[j].b[i].state + U bd[i].b[i].bd[j].b[j].state // Total;
```



```
δ4 = 1 / 40;
```

```
In[46]:= pos1 = Position[evalues, _?(E0 - δ1 < # < E0 + δ1 &)] // Flatten
         pos2 = Position[evalues, _?(E0 - δ2 < # < E0 + δ2 &)] // Flatten
         pos3 = Position[evalues, _?(E0 - δ3 < # < E0 + δ3 &)] // Flatten
         pos4 = Position[evalues, _?(E0 - δ4 < # < E0 + δ4 &)] // Flatten
```

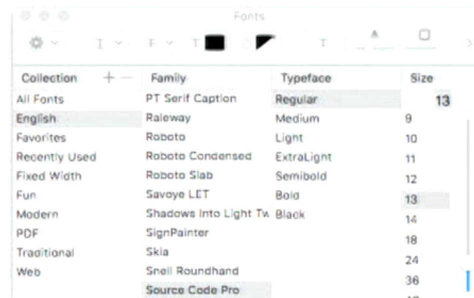
```
Out[46]= {2227, 2230, 2231, 2234, 2238, 2240, 2241, 2243, 2244, 2245, 2248, 2251, 2253, 2256, 2258, 2262, 2263, 2265, 2266, 2268, 2270, 2271, 2274, 2276, 2277,
 2280, 2282, 2283, 2286, 2287, 2289, 2291, 2294, 2295, 2297, 2300, 2301, 2305, 2306, 2309, 2313, 2315, 2316, 2318, 2320, 2323, 2324, 2326, 2328,
 2330, 2333, 2335, 2337, 2338, 2340, 2342, 2345, 2348, 2350, 2351, 2353, 2355, 2356, 2358, 2359, 2362, 2365, 2367, 2370, 2371, 2373, 2375, 2377,
 2379, 2382, 2385, 2386, 2388, 2391, 2394, 2396, 2397, 2399, 2401, 2403, 2405, 2407, 2411, 2413, 2414, 2415, 2418, 2421, 2423, 2425, 2427,
 2428, 2429, 2432, 2435, 2438, 2439, 2441, 2442, 2444, 2447, 2448, 2450, 2453, 2455, 2457, 2458, 2461, 2463, 2466, 2469, 2471, 2473, 2476,
 2479, 2481, 2483, 2485, 2486, 2489, 2490, 2492, 2493, 2496, 2497, 2500, 2501, 2505, 2506, 2507, 2510, 2512, 2515, 2517, 2519, 2520, 2523}

Out[47]= {2297, 2300, 2301, 2305, 2306, 2309, 2313, 2315, 2316, 2318, 2320, 2323, 2324, 2326, 2328, 2330, 2333, 2335, 2337, 2338, 2340, 2342, 2345, 2348, 2350,
 2351, 2353, 2355, 2356, 2358, 2359, 2362, 2365, 2367, 2370, 2371, 2373, 2375, 2377, 2379, 2382, 2385, 2386, 2388, 2391, 2394, 2396, 2397,
 2399, 2401, 2403, 2405, 2407, 2411, 2413, 2414, 2415, 2418, 2421, 2423, 2425, 2427, 2428, 2429, 2432, 2435, 2438, 2439, 2441, 2442, 2444}

Out[48]= {2323, 2324, 2326, 2328, 2330, 2333, 2335, 2337, 2338, 2340, 2342, 2345, 2348, 2350, 2351, 2353, 2355, 2356, 2358, 2359, 2362, 2365, 2367, 2370, 2371,
 2373, 2375, 2377, 2379, 2382, 2385, 2386, 2388, 2391, 2394, 2396, 2397, 2399, 2401, 2403, 2405, 2407, 2411, 2413, 2414, 2415, 2418, 2421}

Out[49]= {2335, 2337, 2338, 2340, 2342, 2345, 2348, 2350, 2351, 2353, 2355, 2356, 2358, 2359, 2362, 2365, 2367,
 2370, 2371, 2373, 2375, 2377, 2379, 2382, 2385, 2386, 2388, 2391, 2394, 2396, 2397, 2399, 2401, 2403, 2405, 2407}
```

```
In[50]:= VectorsEnsemble1 = Table[evectors[[j]], {j, pos1}];
         VectorsEnsemble2 = Table[evectors[[j]], {j, pos2}];
         VectorsEnsemble3 = Table[evectors[[j]], {j, pos3}];
         VectorsEnsemble4 = Table[evectors[[j]], {j, pos4}];
```



In[54]:=

$$\text{micro1} = \frac{1}{\text{Length[pos1]}}\ \text{Sum[Conjugate[X].GoalMatrix.X, \{X, VectorsEnsemble1\}]};$$

$$\text{micro2} = \frac{1}{\text{Length[pos2]}}\ \text{Sum[Conjugate[X].GoalMatrix.X, \{X, VectorsEnsemble2\}]};$$

$$\text{micro3} = \frac{1}{\text{Length[pos3]}}\ \text{Sum[Conjugate[X].GoalMatrix.X, \{X, VectorsEnsemble3\}]};$$

$$\text{micro4} = \frac{1}{\text{Length[pos4]}}\ \text{Sum[Conjugate[X].GoalMatrix.X, \{X, VectorsEnsemble4\}]};$$

150%

$$= \sum_{\vec n} \frac{1}{4} \sum_{\vec l} \left(\vec x_{\vec n} - \vec x_{\vec n+\vec l}\right)^2 + \frac{m^2}{2} \vec x_{\vec n}^2 + \mu \left(\vec x_n^2\right)^2$$

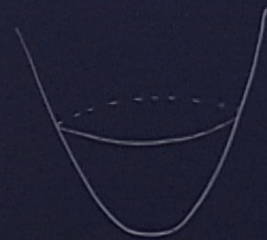
Q1) $N \to \infty$

can sum all graphs!

Q2) Q Th

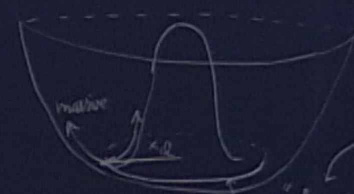
$$S[\vec x] = S[\underset{\substack{\uparrow \\ O(N)}}{D} \cdot \vec x]$$

$$V(\vec x) =$$



$m^2 > 0$

or



$m^2 < 0$

$N-1$ dimensional

"line" of minima

$$\overbrace{\frac{1}{2}(\partial_\mu \vec x)^2 + \frac{m^2}{2} \vec x^2 + \mu \vec x^4}$$