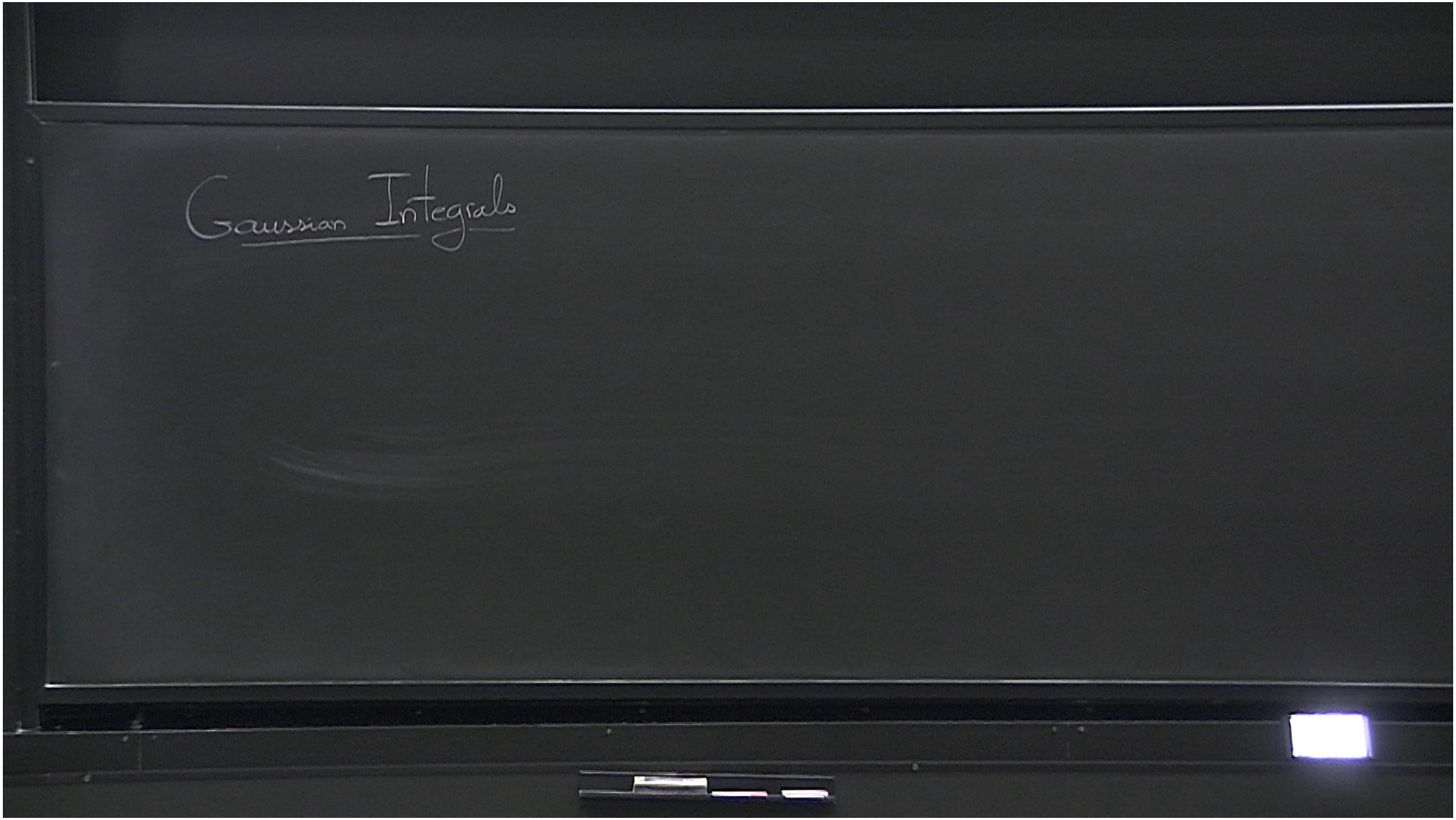
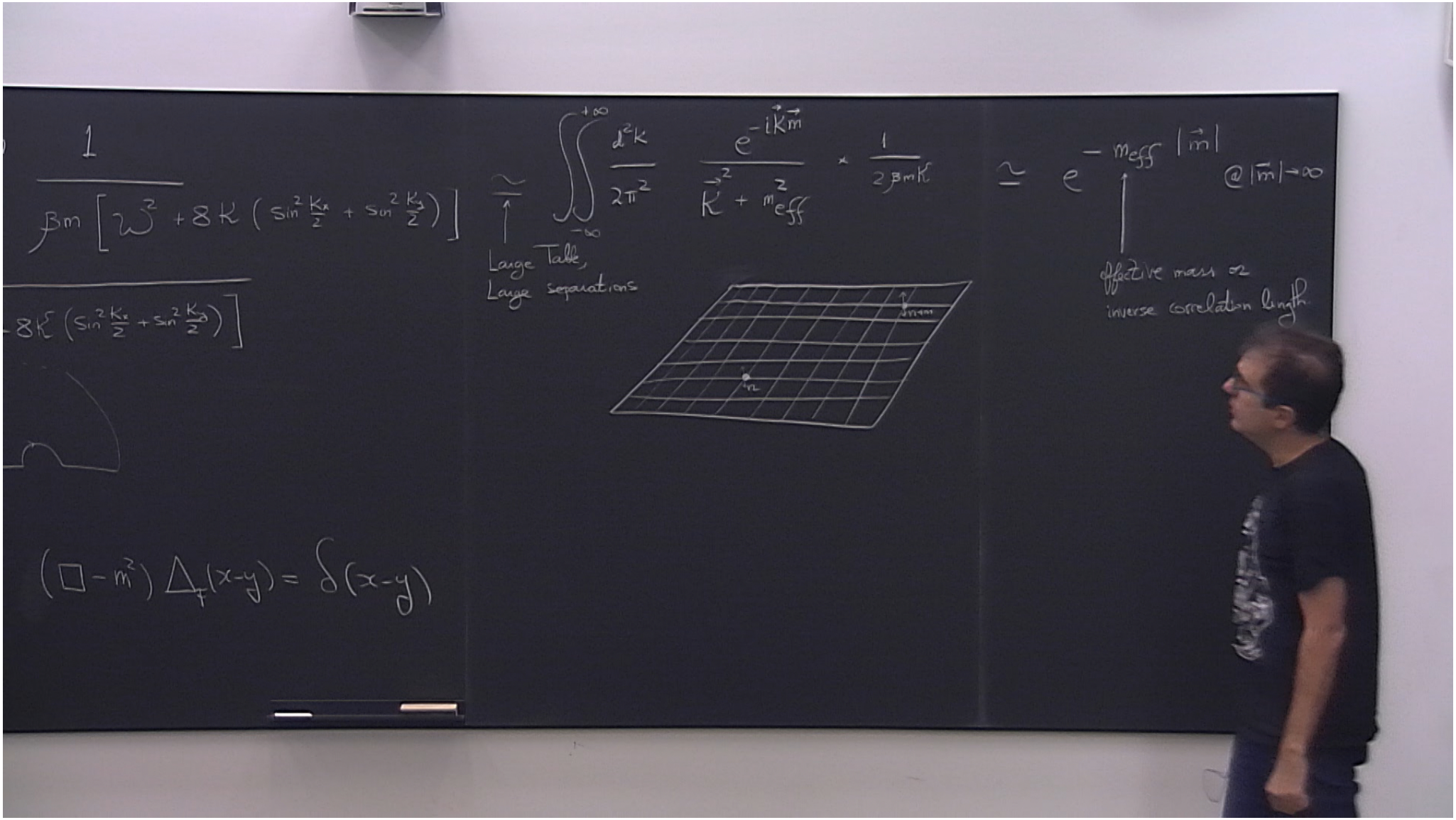
Title: PSI 2018/2019 - Statistical Mechanics - Lecture 5

Date: Oct 16, 2018 10:45 AM

URL: http://pirsa.org/18100026

Abstract:


Gaussian Integrals


$\frac{1}{\beta m\left[w^2 + 8k\left(\sin^2\frac{k_x}{2} + \sin^2\frac{k_y}{2}\right)\right]} \simeq \int_{-\infty}^{+\infty}\int \frac{d^2k}{2\pi^2} \frac{e^{-i\vec{k}\vec{m}}}{\vec{k}^2 + m_{eff}^2} \times \frac{1}{2\beta m k} \simeq e^{-m_{eff}|\vec{m}|} \quad @\,|\vec{m}|\to\infty$
Large Table, Large separations
effective mass or inverse correlation length
$(\Box - m^2)\,\Delta_f(x-y) = \delta(x-y)$

Our Table

$$\langle X_{\vec{n}} X_{\vec{n}+\vec{m}} \rangle = \frac{1}{L^2} \sum_{\vec{k}} e^{-ikm} \frac{1}{\beta m \left[\omega^2 + 8k \left(\sin^2 \frac{k_x}{2} + \sin^2 \frac{k_y}{2} \right) \right]}$$

$$\Omega_k^2$$

$$\langle X_{\vec{k}} X_{\vec{k}'} \rangle = \delta_{\vec{k}+\vec{k}'} \frac{1}{\beta m \left[\omega^2 + 8k^2 \left(\sin^2 \frac{k_x}{2} + \sin^2 \frac{k_y}{2} \right) \right]}$$

Large Tall
Large sep

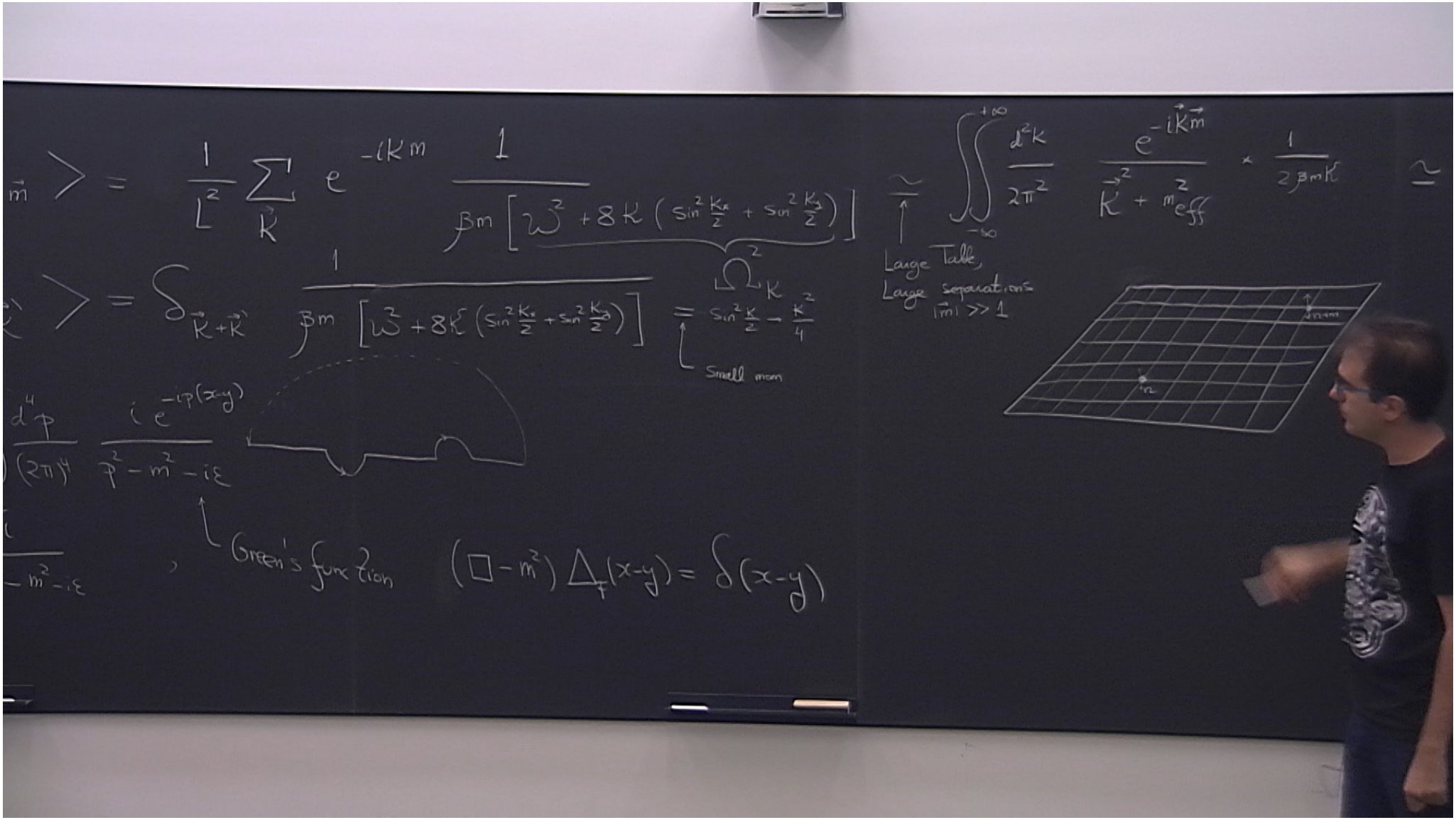
Dan's Lecture

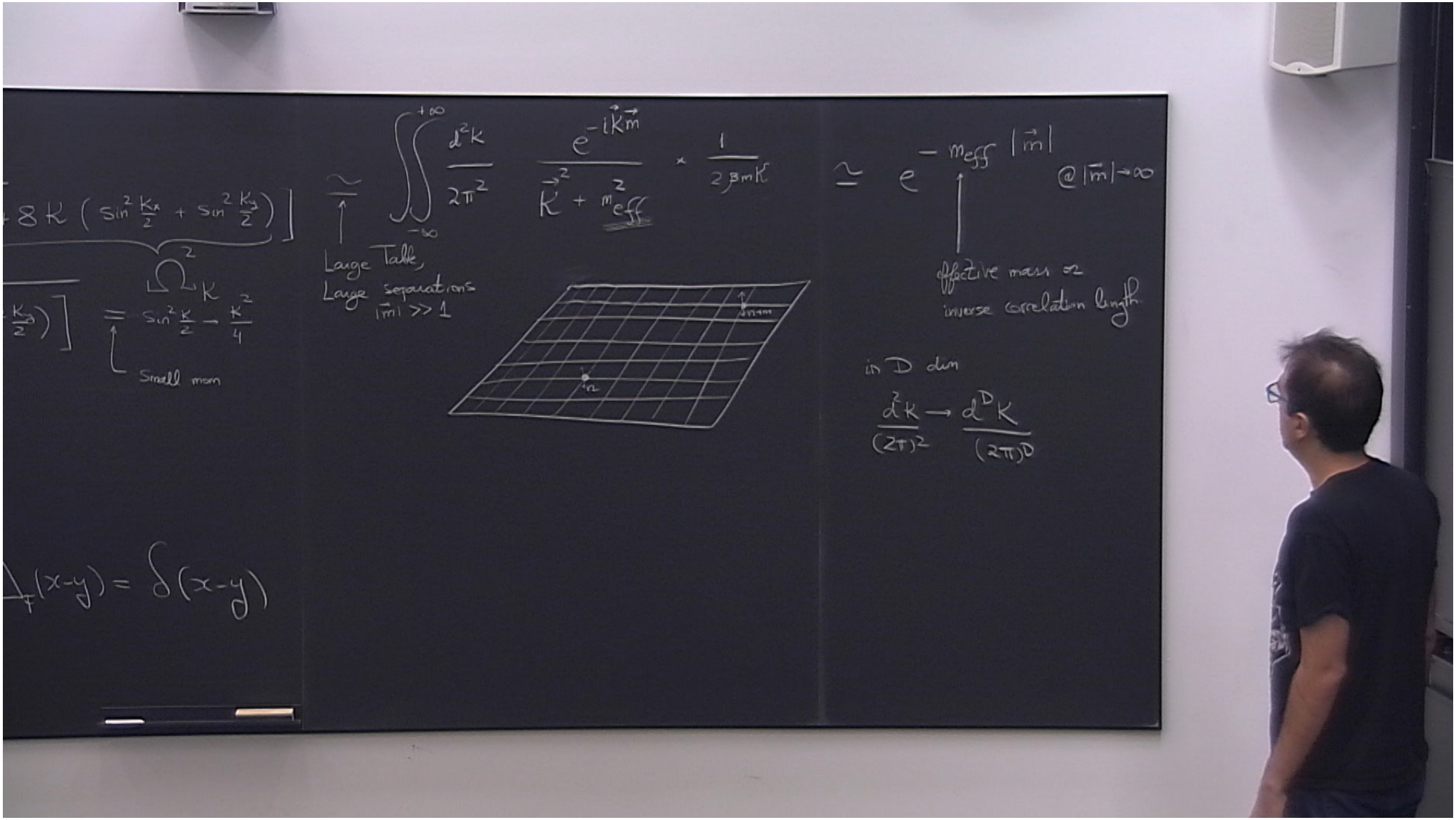
$$\Delta_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{i e^{-ip(x-y)}}{p^2 - m^2 - i\varepsilon}$$

$$\Delta_F(k) = \frac{i}{p^2 - m^2 - i\varepsilon}$$

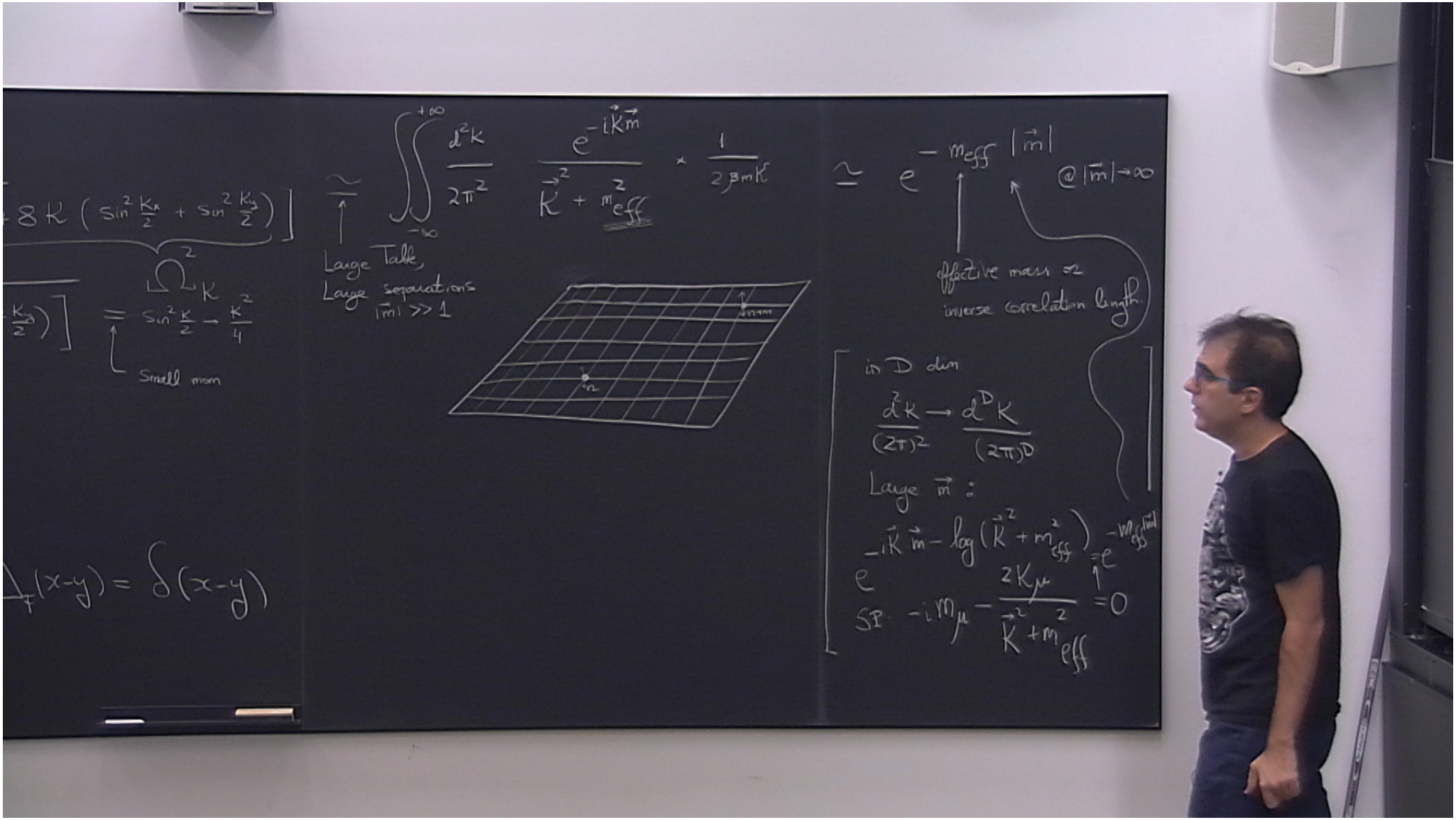
Green's function

$$(\Box - m^2) \Delta_F(x-y) = \delta(x-y)$$


Large Table,
Large separations
|m| >> 1
Small mom
Green's function
(□ − m²) Δ_F(x−y) = δ(x−y)



$+8K\left(\sin^2\frac{K_x}{2}+\sin^2\frac{K_y}{2}\right)\Big]$
Ω_K^2
$\frac{K_y}{2}\Big)\Big] = \sin^2\frac{K}{2} \to \frac{K^2}{4}$
Small mom
$\Delta_F(x-y) = \delta(x-y)$
$\simeq \iint_{-\infty}^{+\infty} \frac{d^2K}{2\pi^2} \frac{e^{-i\vec{K}\vec{m}}}{\vec{K}^2 + m_{eff}^2} \times \frac{1}{2\beta m K}$
Large Table,
Large separations
$|\vec{m}| \gg 1$
$\simeq e^{-m_{eff}|\vec{m}|}$ @ $|\vec{m}| \to \infty$
effective mass or
inverse correlation length
in D dim
$\frac{d^2K}{(2\pi)^2} \to \frac{d^DK}{(2\pi)^D}$


Large Table,
Large separations
|m| >> 1
Small mom
effective mass or
inverse correlation length
in D dim
Large m :
SP

Our Table

$$\langle X_{\vec{n}} X_{\vec{n}+\vec{m}} \rangle = \frac{1}{L^2} \sum_{\vec{K}} e^{-iKm} \frac{1}{\beta m \left[\omega^2 + 8k \left(\sin^2 \frac{K_x}{2} + \sin^2 \frac{K_y}{2} \right) \right]}$$

(Gaussian)

$$\langle X_{\vec{K}} X_{\vec{K}'} \rangle = \delta_{\vec{K}+\vec{K}'} \frac{1}{\beta m \left[\omega^2 + 8k \left(\sin^2 \frac{K_x}{2} + \sin^2 \frac{K_y}{2} \right) \right]}$$

$$\Omega_K^2 = \sin^2 \frac{K}{2} \to \frac{K^2}{4}$$

(Small momenta)

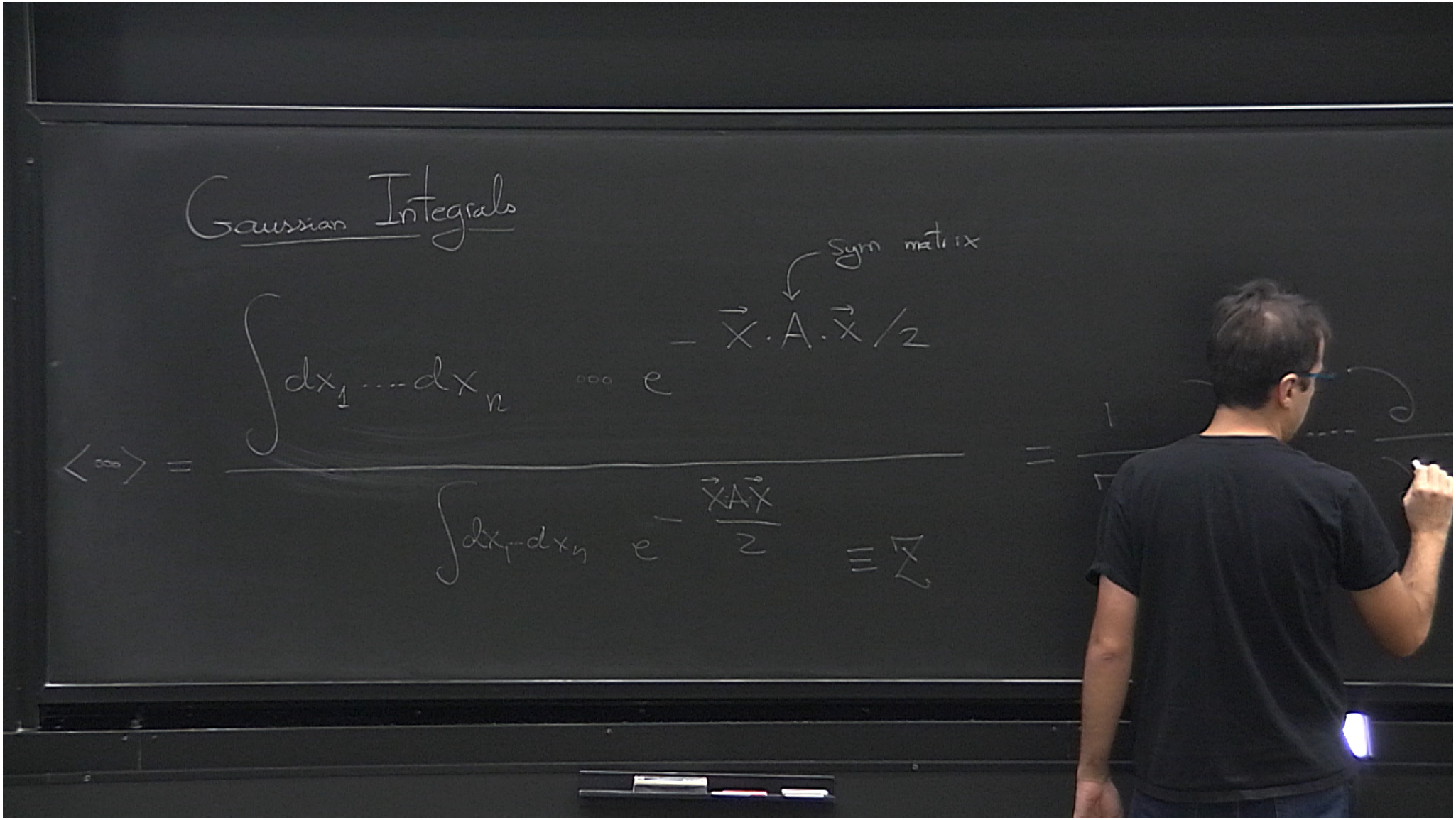
Dan's Lecture

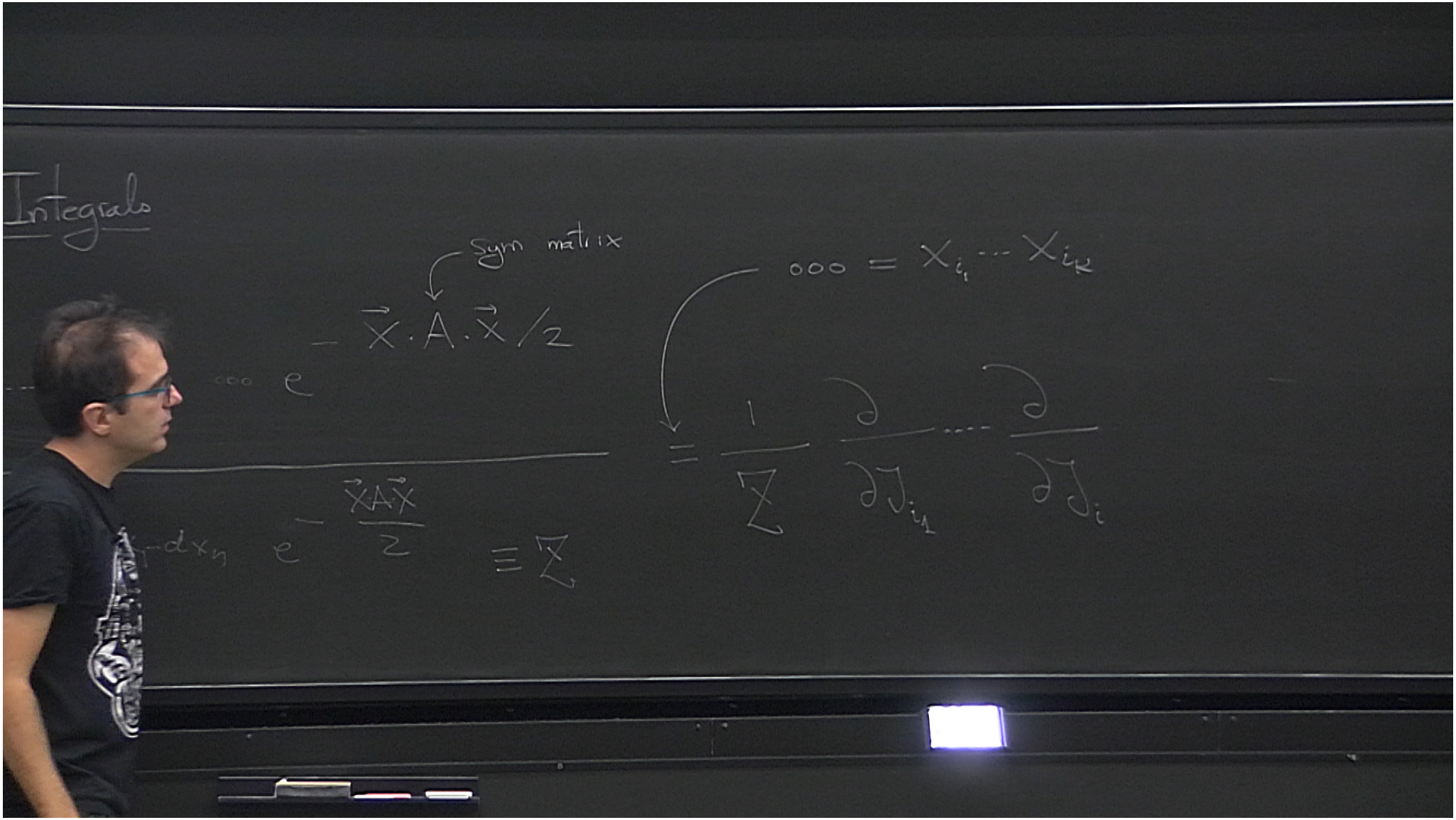
$$\Delta_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{i e^{-ip(x-y)}}{p^2 - m^2 - i\varepsilon}$$

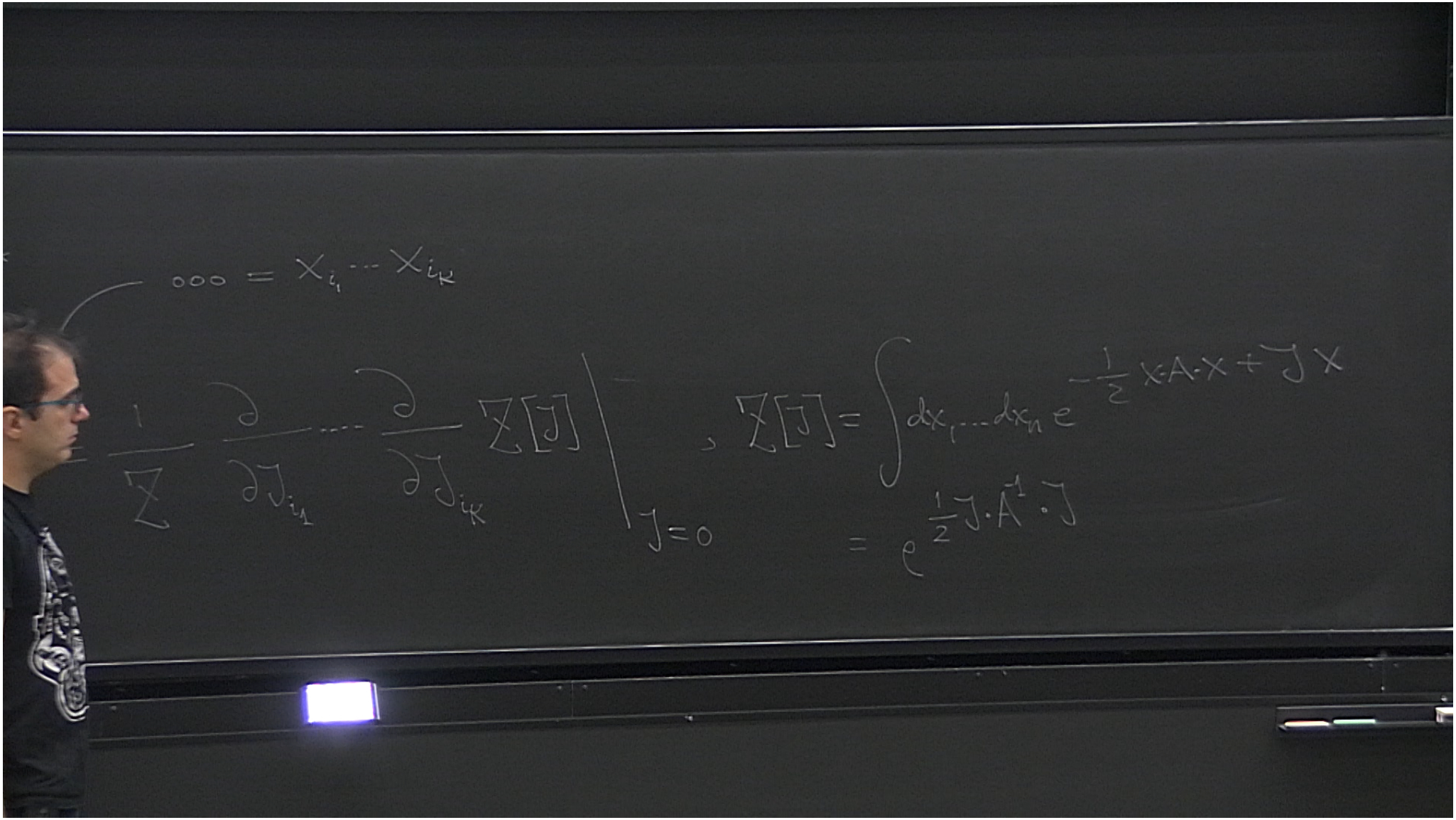
$$\Delta_F(K) = \frac{i}{p^2 - m^2 - i\varepsilon}$$

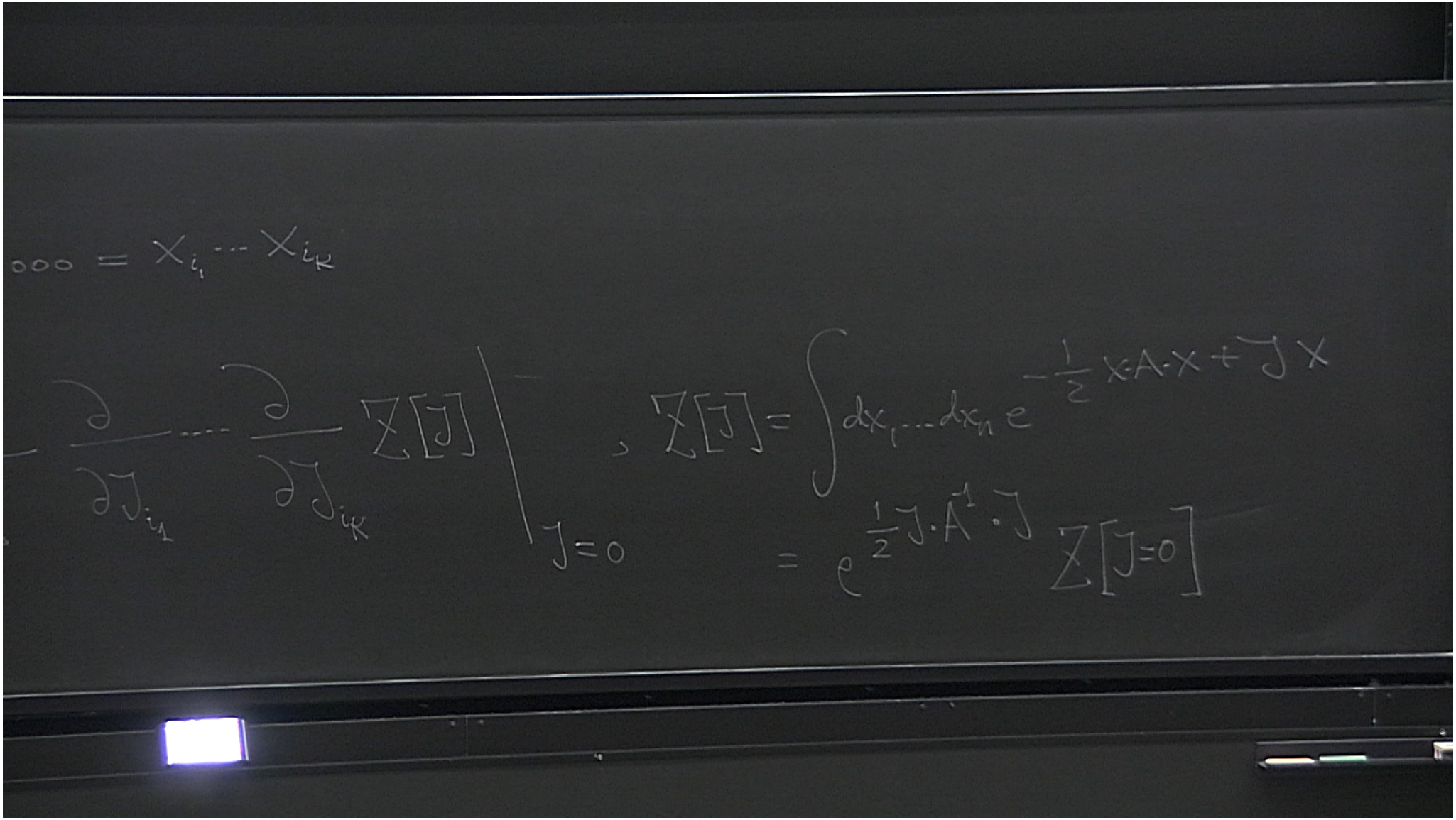
(Green's function)

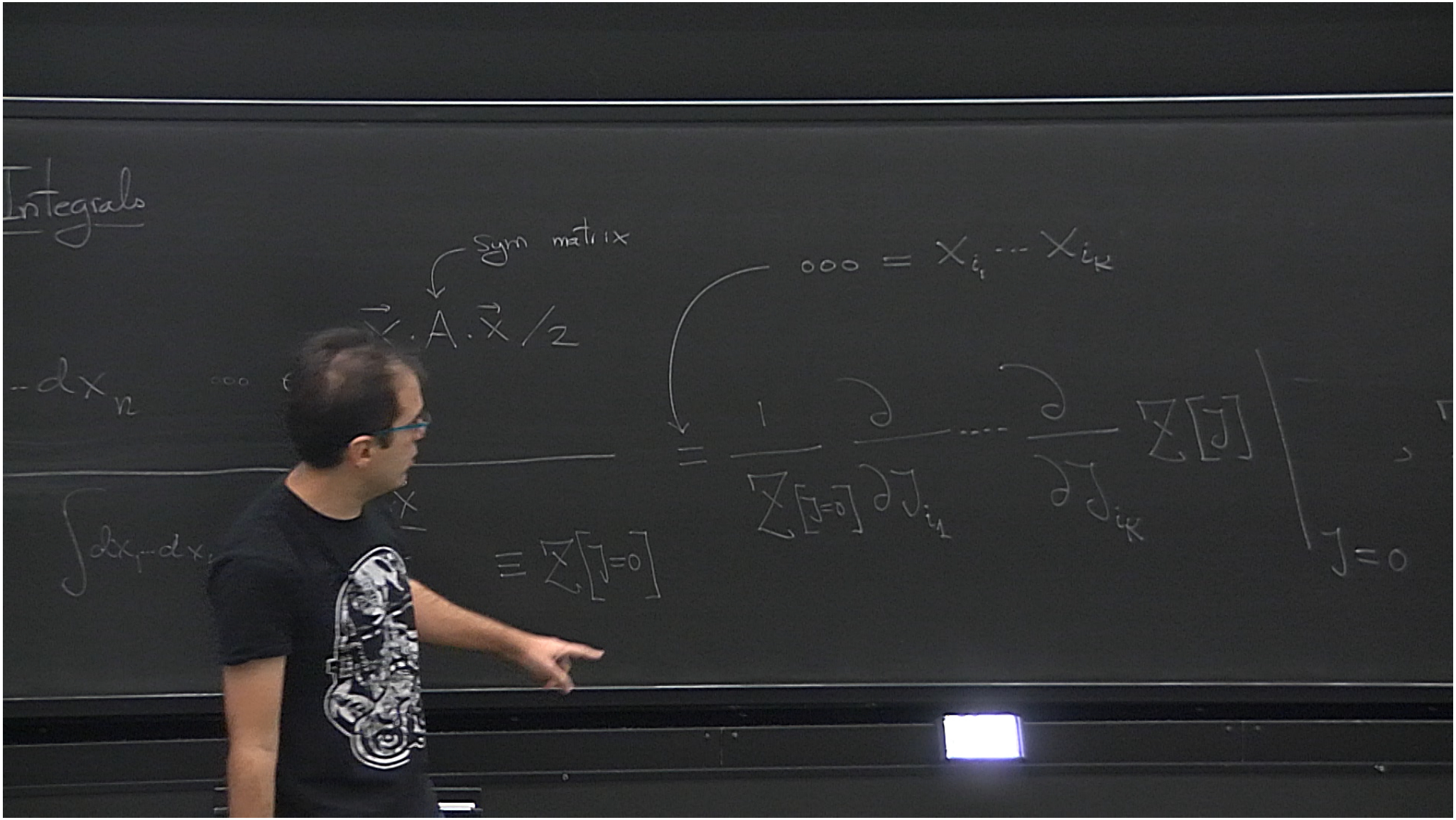
$$(\Box - m^2) \Delta_F(x-y) = \delta(x-y)$$

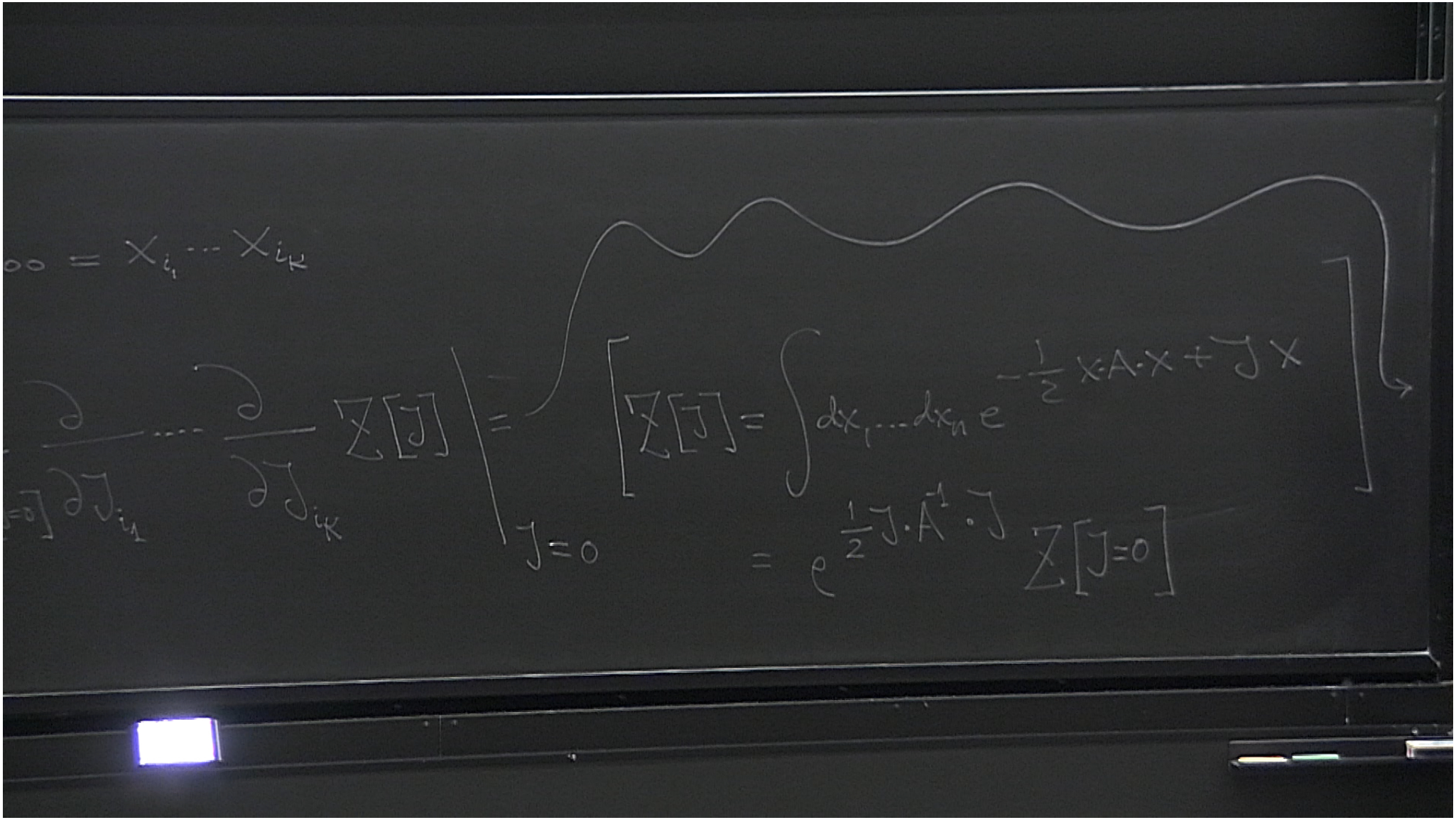

Gaussian Integrals
Sym matrix
$\langle \cdots \rangle = \frac{\int dx_1 \cdots dx_n \cdots e^{-\vec{x}\cdot A\cdot\vec{x}/2}}{\int dx_1 \cdots dx_n \, e^{-\frac{\vec{x}\cdot A\cdot\vec{x}}{2}}} \equiv \Sigma$


Integrals
sym matrix
$\vec{x}\cdot A\cdot\vec{x}/2$
$\circ\circ\circ = x_{i_1}\cdots x_{i_k}$
$\equiv Z$
$= \frac{1}{Z}\frac{\partial}{\partial J_{i_1}}\cdots\frac{\partial}{\partial J_i}$


$\langle \cdots \rangle = \frac{1}{Z} \frac{\partial}{\partial J_{i_1}} \cdots \frac{\partial}{\partial J_{i_K}} Z[J] \Big|_{J=0}$
$Z[J] = \int dx_1 \cdots dx_n \, e^{-\frac{1}{2} x \cdot A \cdot x + J x}$
$= e^{\frac{1}{2} J \cdot A^{-1} \cdot J}$


$\langle x_{i_1} \cdots x_{i_K} \rangle = \frac{\partial}{\partial J_{i_1}} \cdots \frac{\partial}{\partial J_{i_K}} Z[J] \Big|_{J=0}$
$Z[J] = \int dx_1 \ldots dx_n \, e^{-\frac{1}{2} x \cdot A \cdot x + J x}$
$= e^{\frac{1}{2} J \cdot A^{-1} \cdot J} \, Z[J=0]$

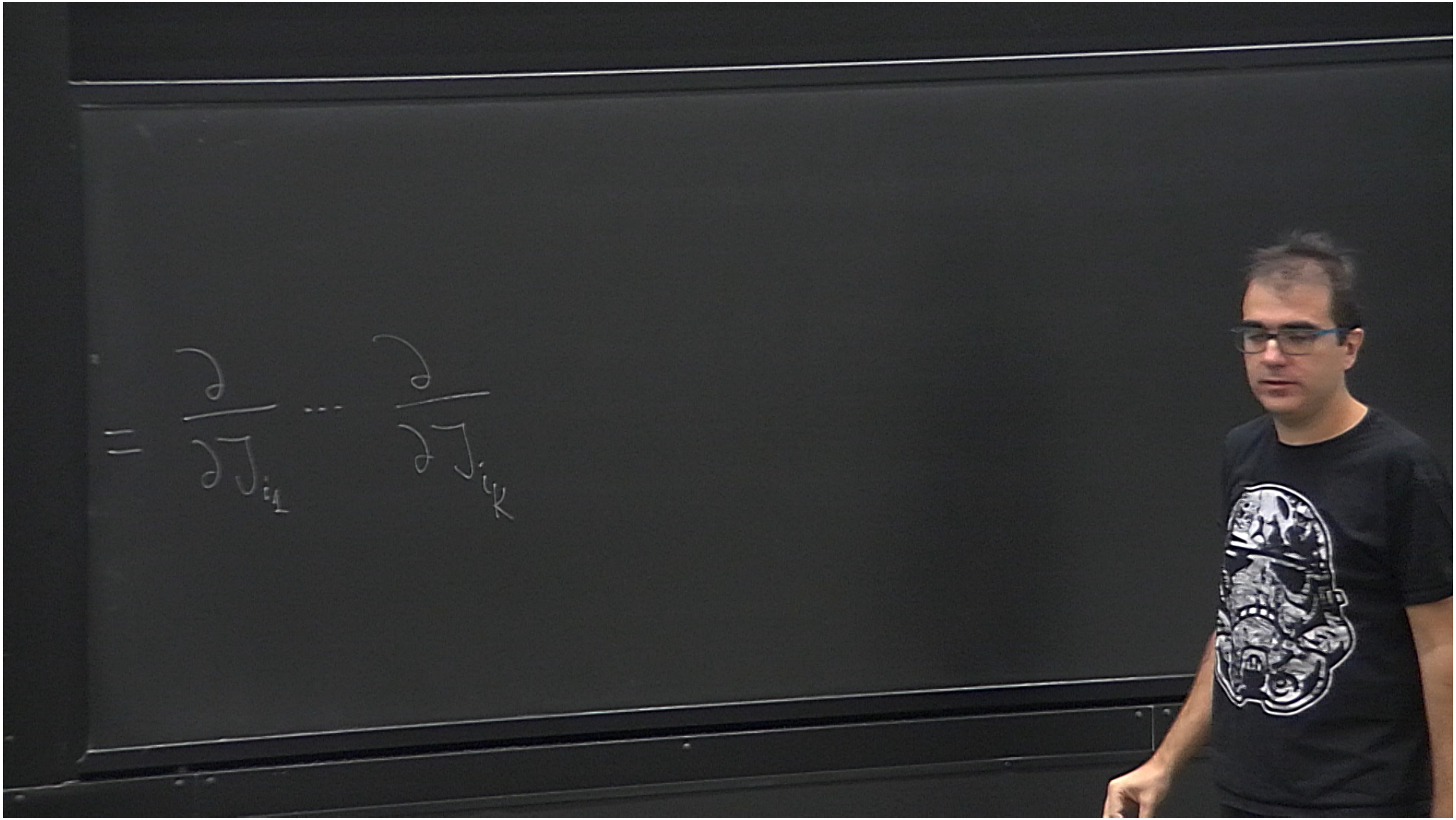

Integrals
Sym matrix
$\vec{x} \cdot A \cdot \vec{x}/2$
$\equiv Z[J=0]$
$= \frac{1}{Z[J=0]} \frac{\partial}{\partial J_{i_1}} \cdots \frac{\partial}{\partial J_{i_K}} Z[J] \Big|_{J=0}$

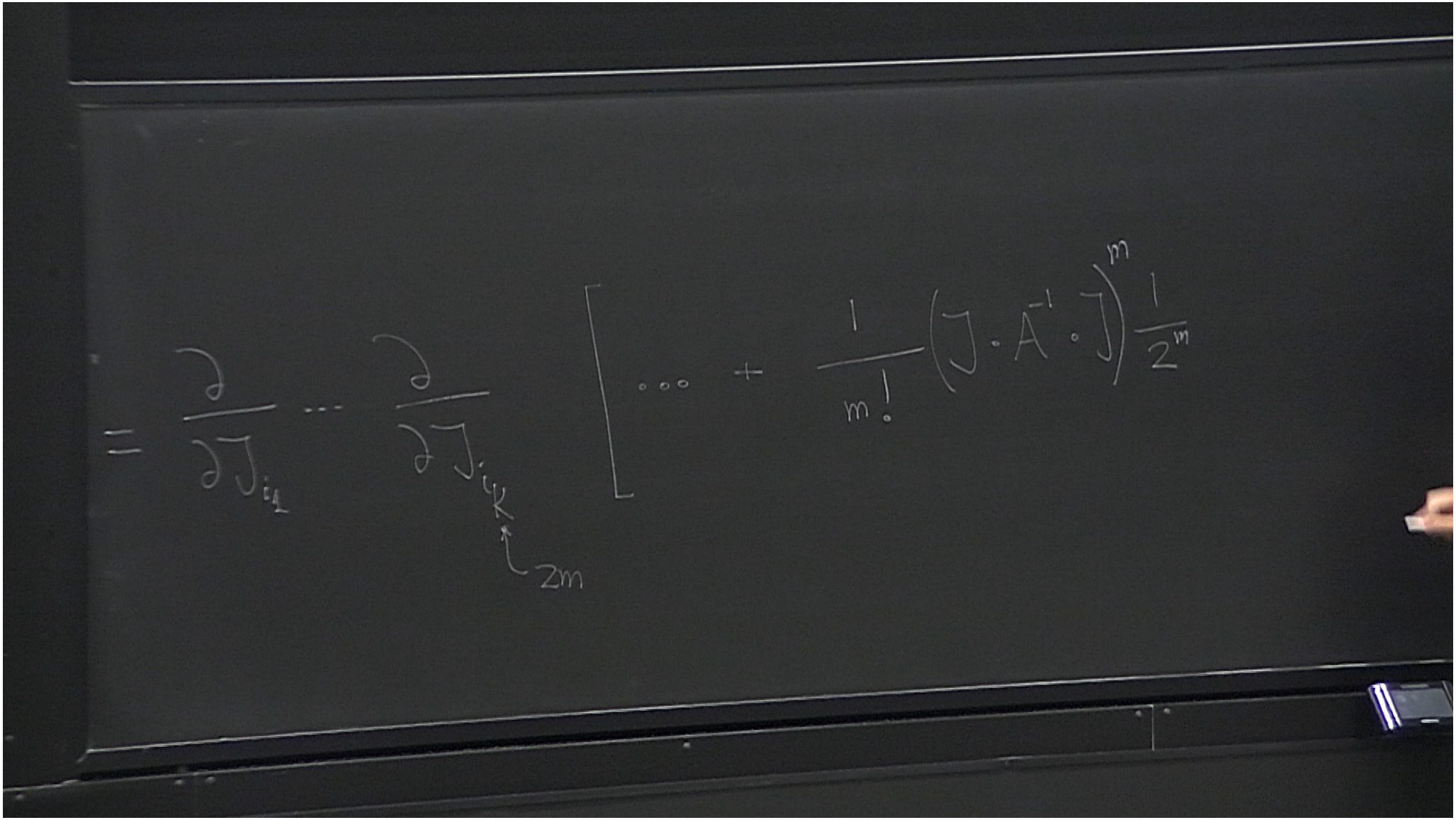


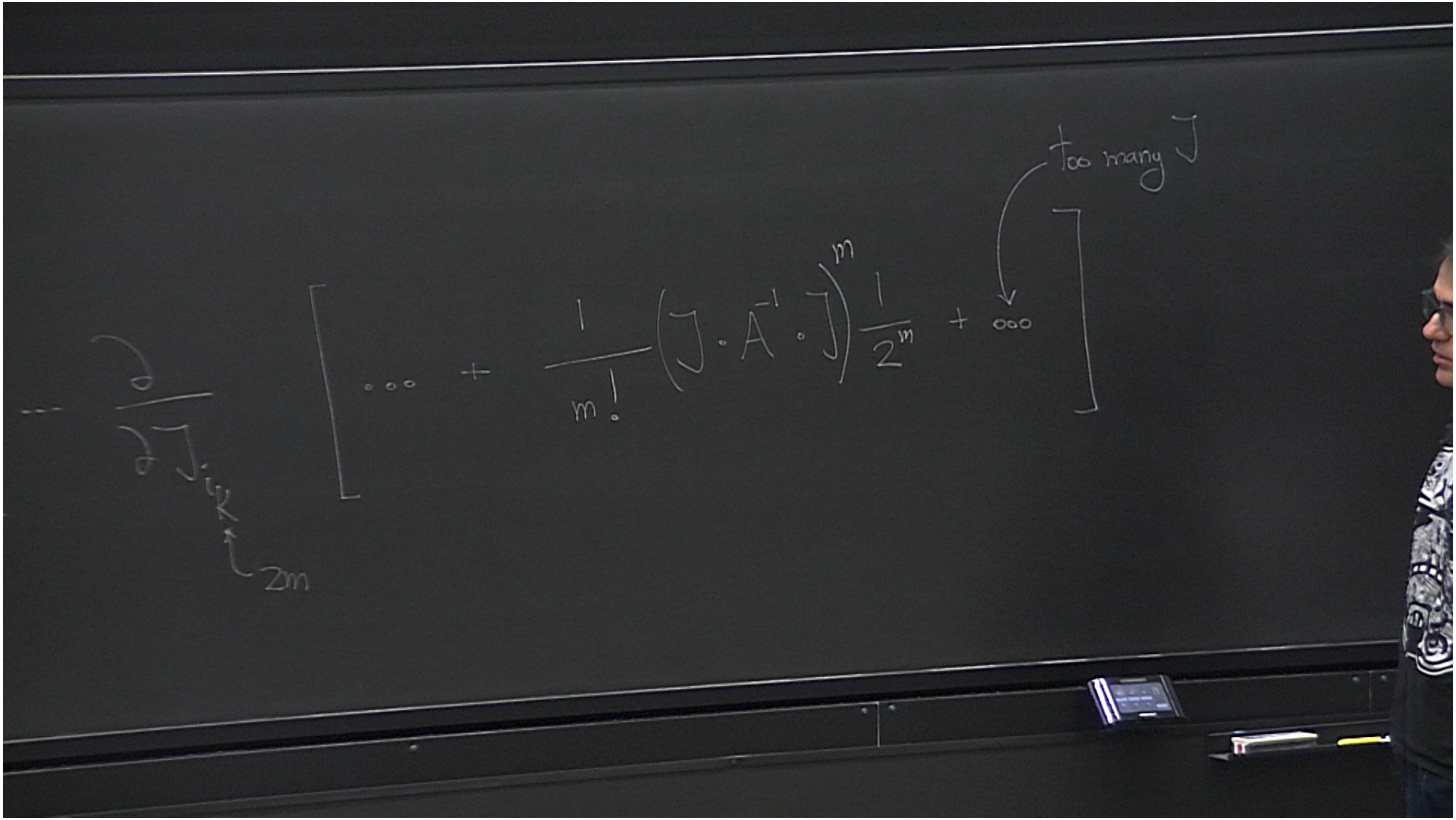
$$\ldots\ldots = x_{i_1} \cdots x_{i_k}$$

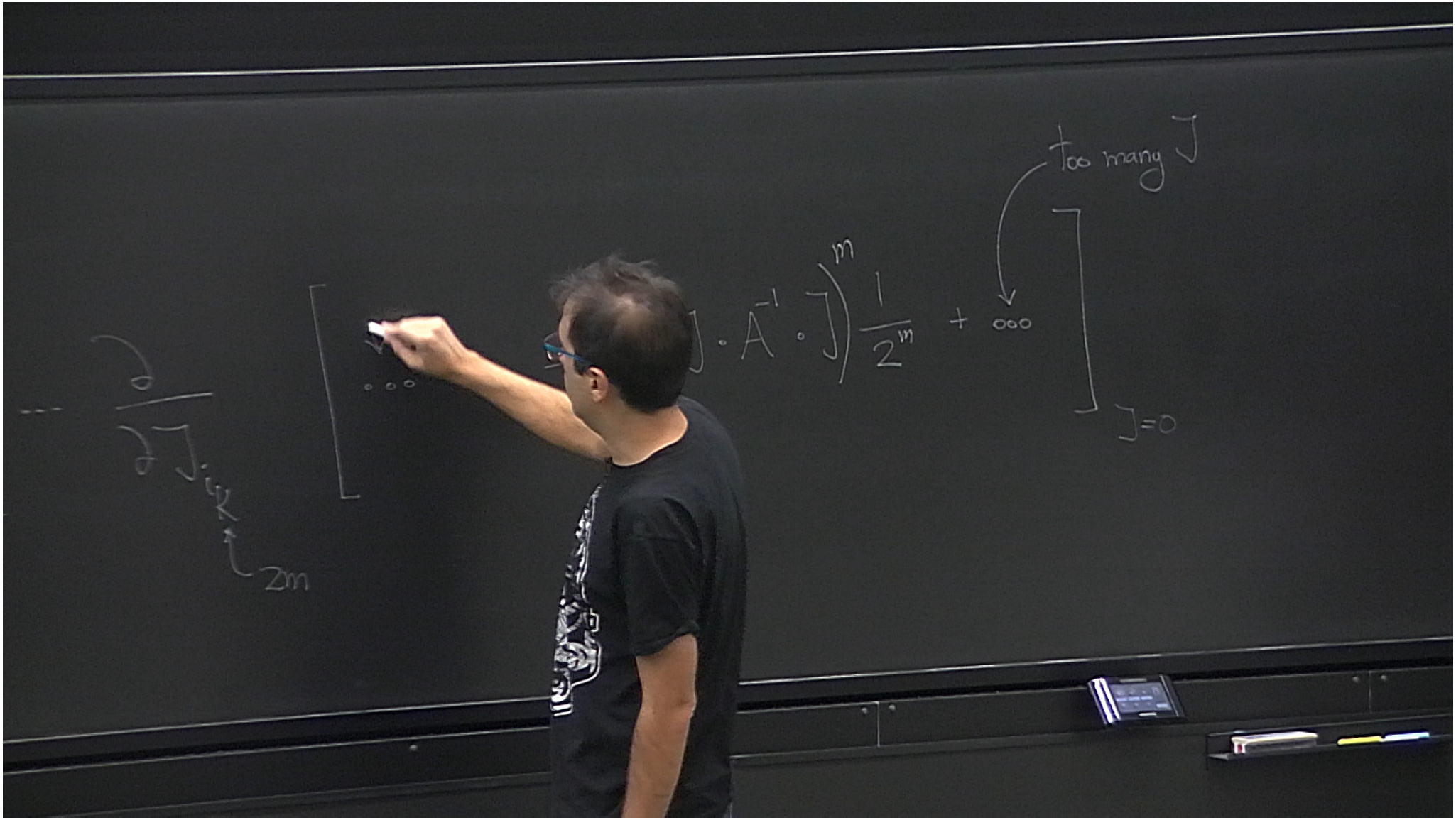
$$\ldots \frac{\partial}{\partial J_{i_1}} \cdots \frac{\partial}{\partial J_{i_k}} Z[J] \Big|_{J=0} = \left[Z[J] = \int dx_1 \ldots dx_n \, e^{-\frac{1}{2} x \cdot A \cdot x + J x} \right]$$

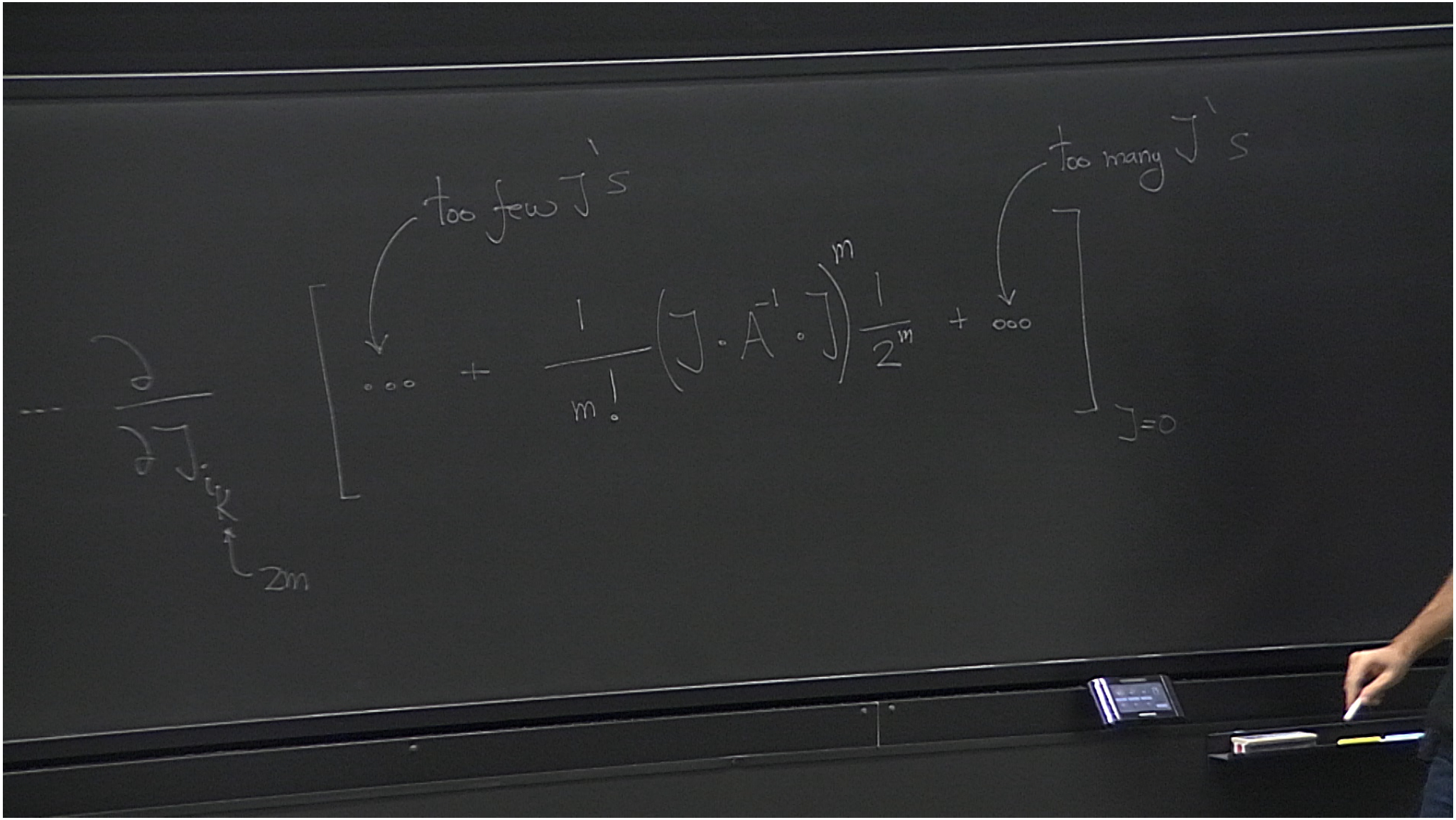
$$= e^{\frac{1}{2} J \cdot A^{-1} \cdot J} \, Z[J=0]$$

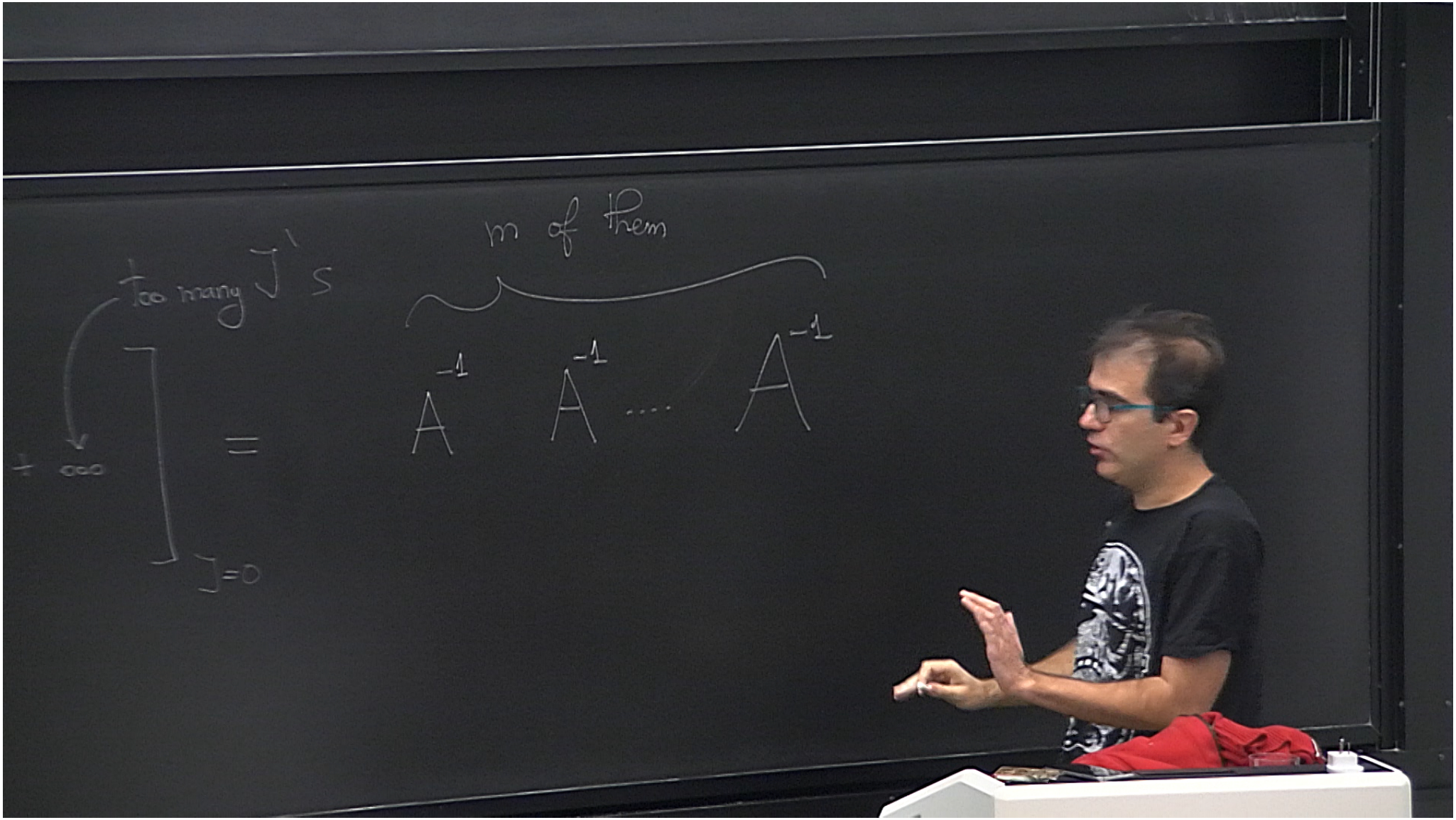

$= \frac{\partial}{\partial J_{i_1}} \cdots \frac{\partial}{\partial J_{i_K}}$

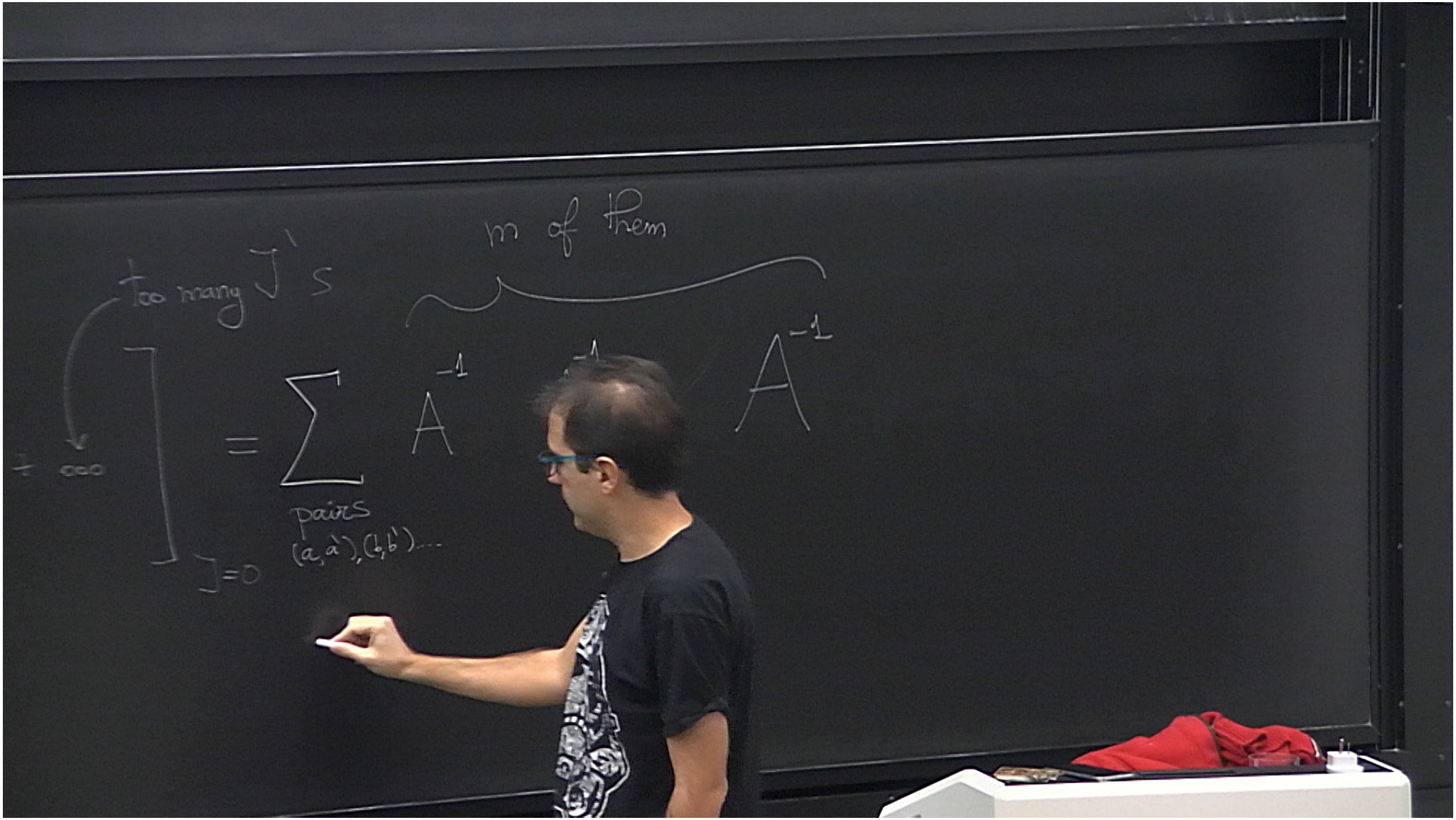

$= \frac{\partial}{\partial J_{i_1}} \cdots \frac{\partial}{\partial J_{i_{2m}}} \left[\cdots + \frac{1}{m!} \left(J \cdot A^{-1} \cdot J \right)^m \frac{1}{2^m} \right.$
2m

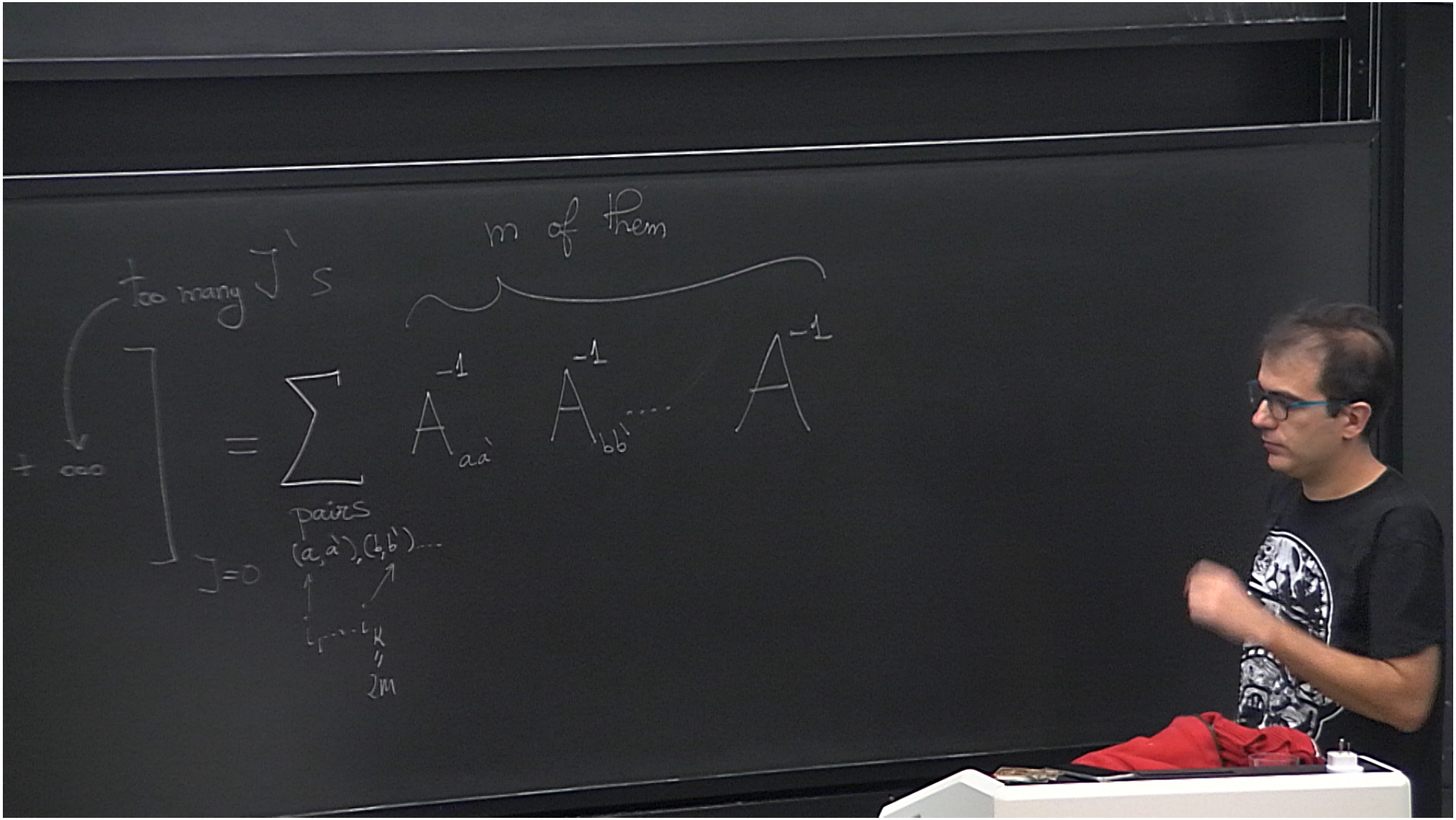

too many J
2m

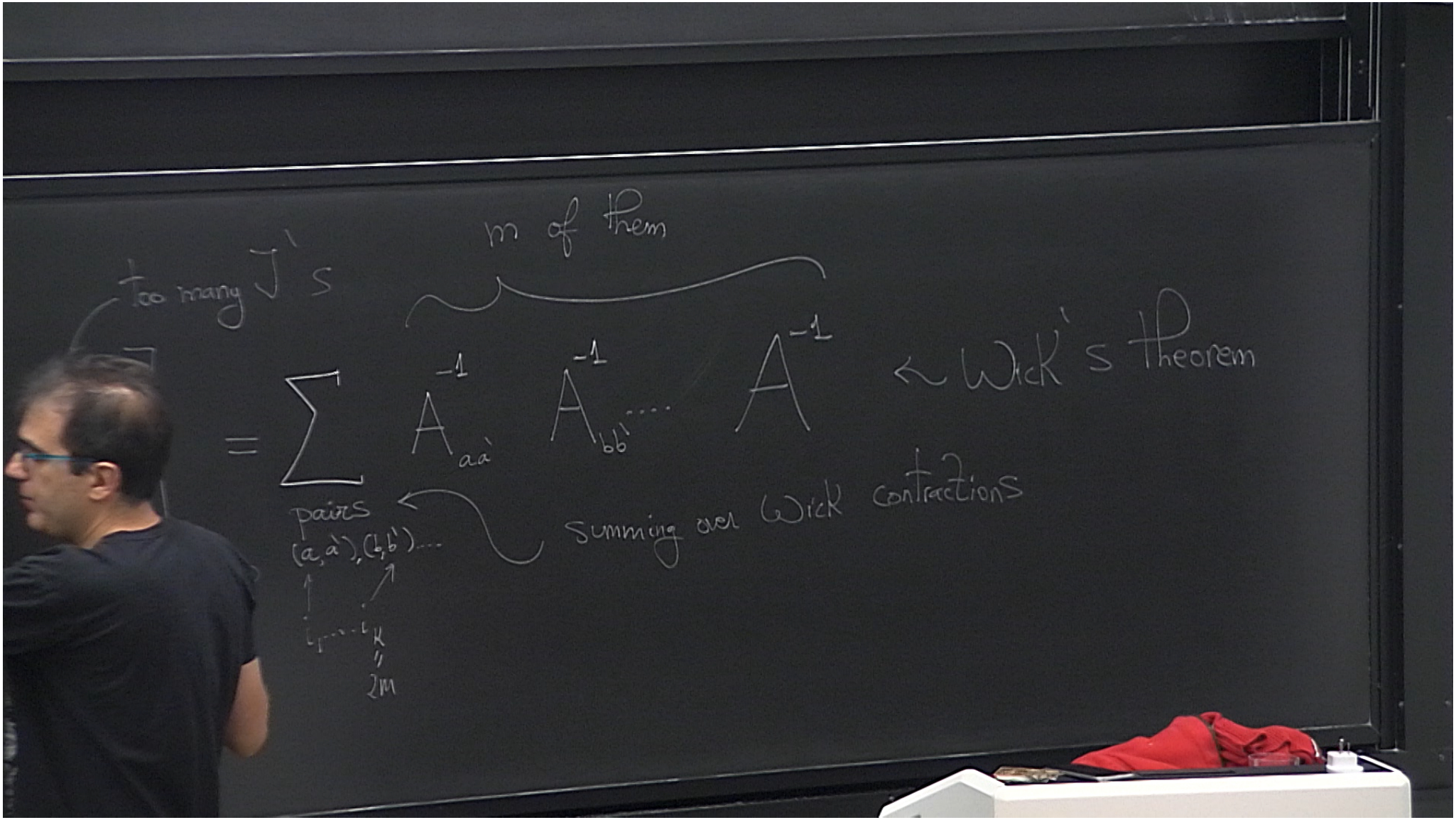

too many J
J=0

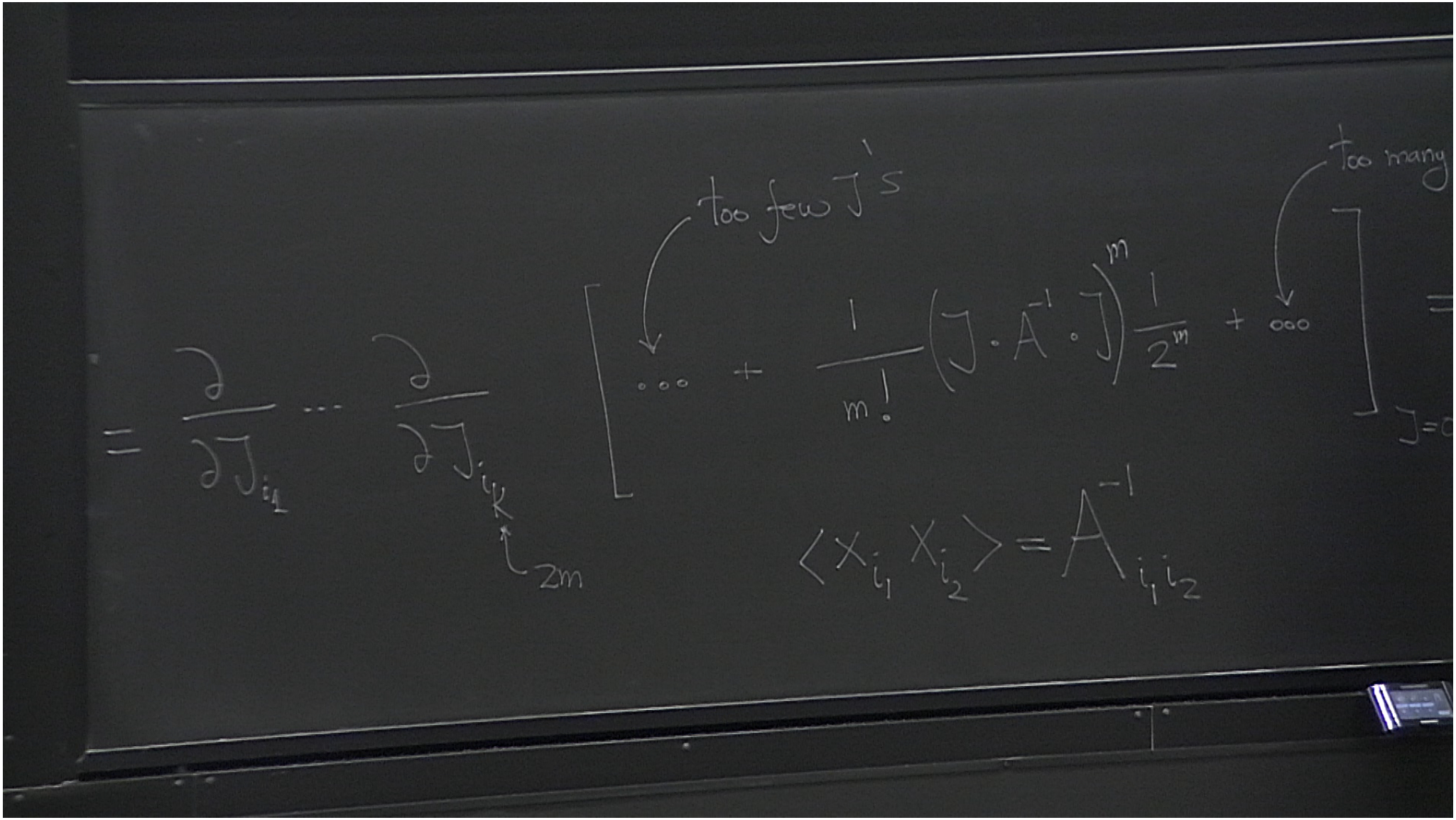

too few J's
too many J's
J=0

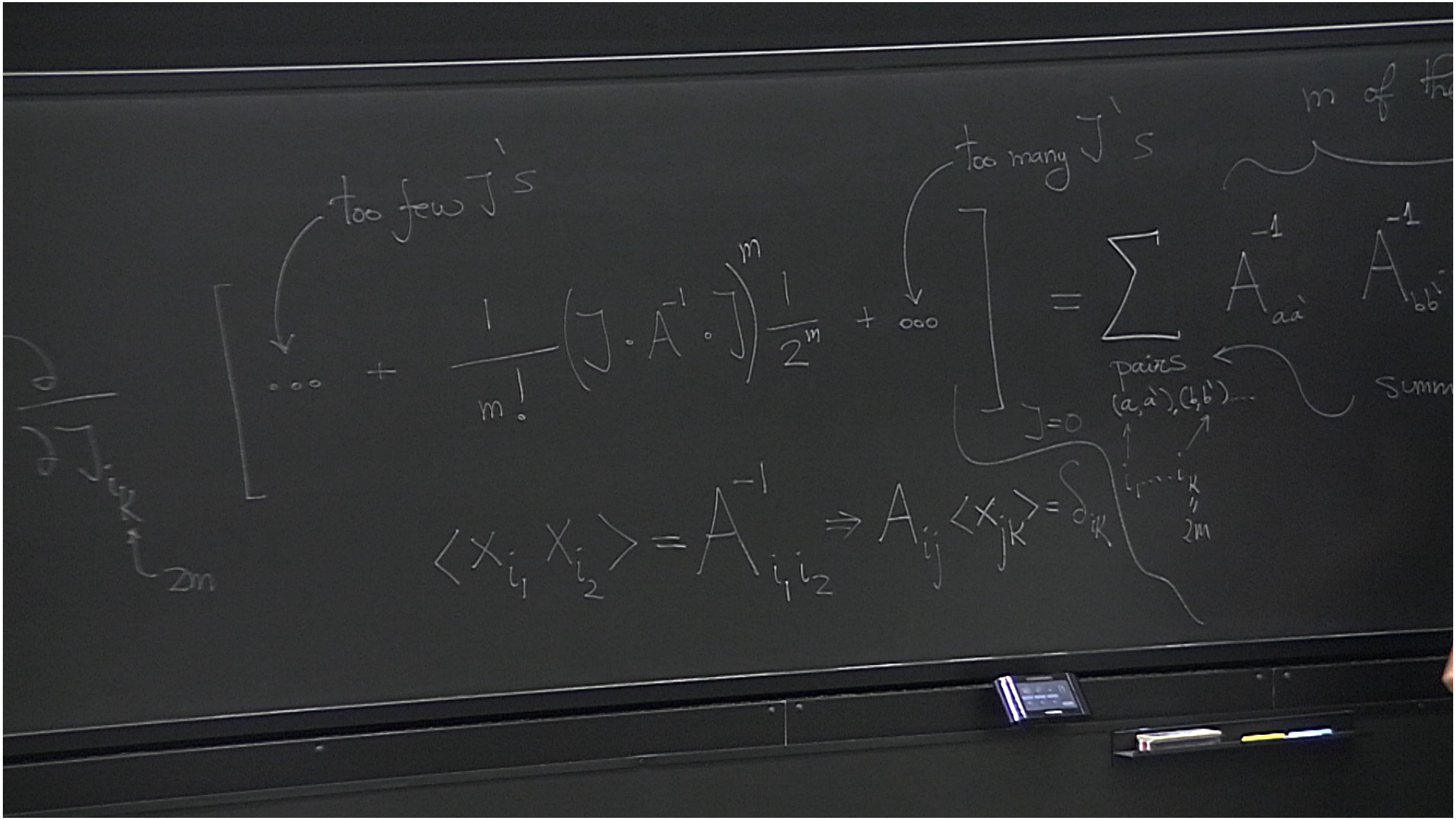

too many j's
m of them
A^-1 A^-1 .... A^-1
t → ∞
j=0

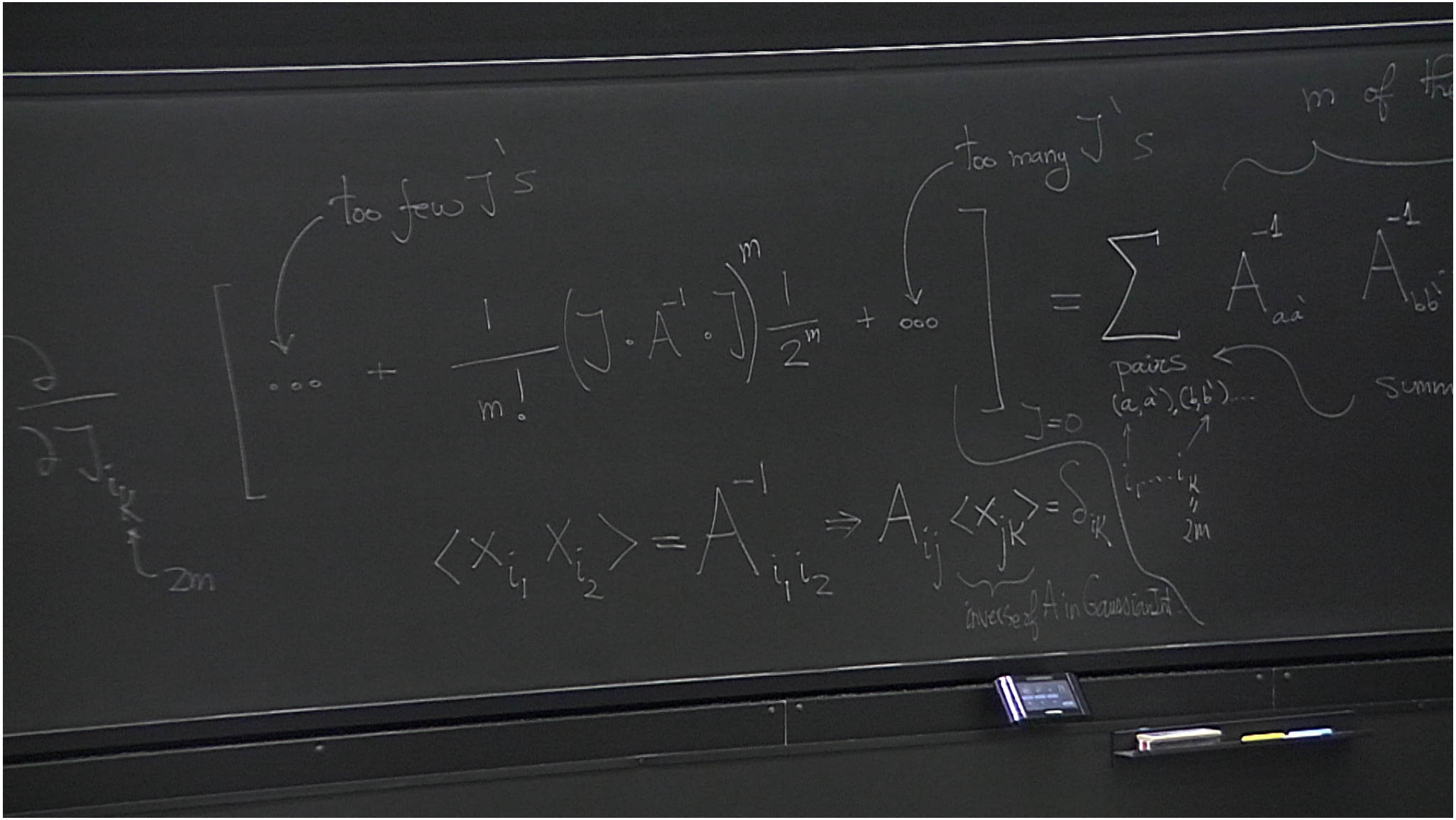

m of them
too many J's
+ ...
= Σ A^-1 A^-1
pairs
(a,a'),(b,b')....
J=0

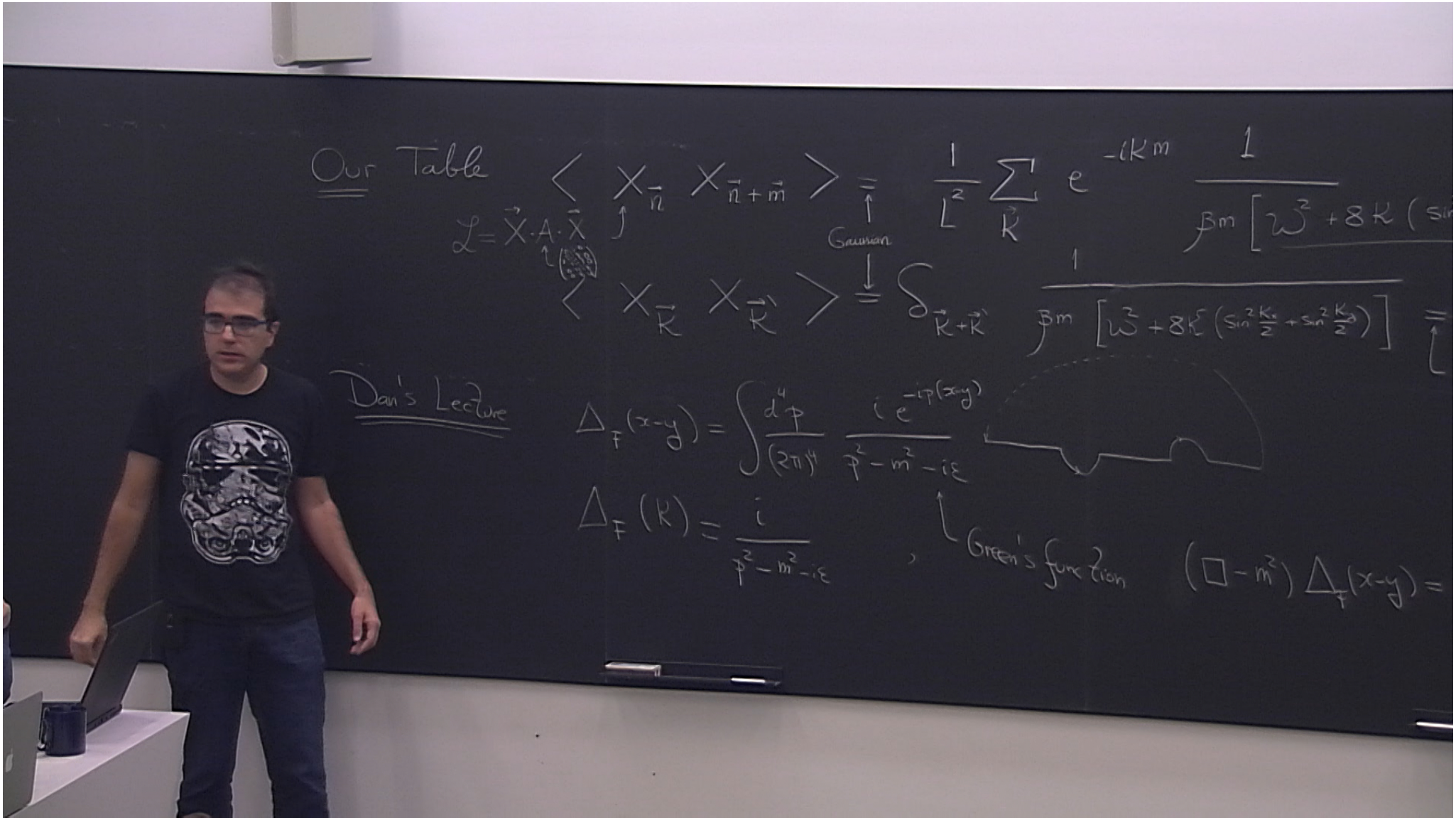

m of them
too many J's
+ ...
= Σ A^-1_aa' A^-1_bb' .... A^-1
pairs
(a,a'),(b,b')....
J=0
i_1 ... i_k
2m

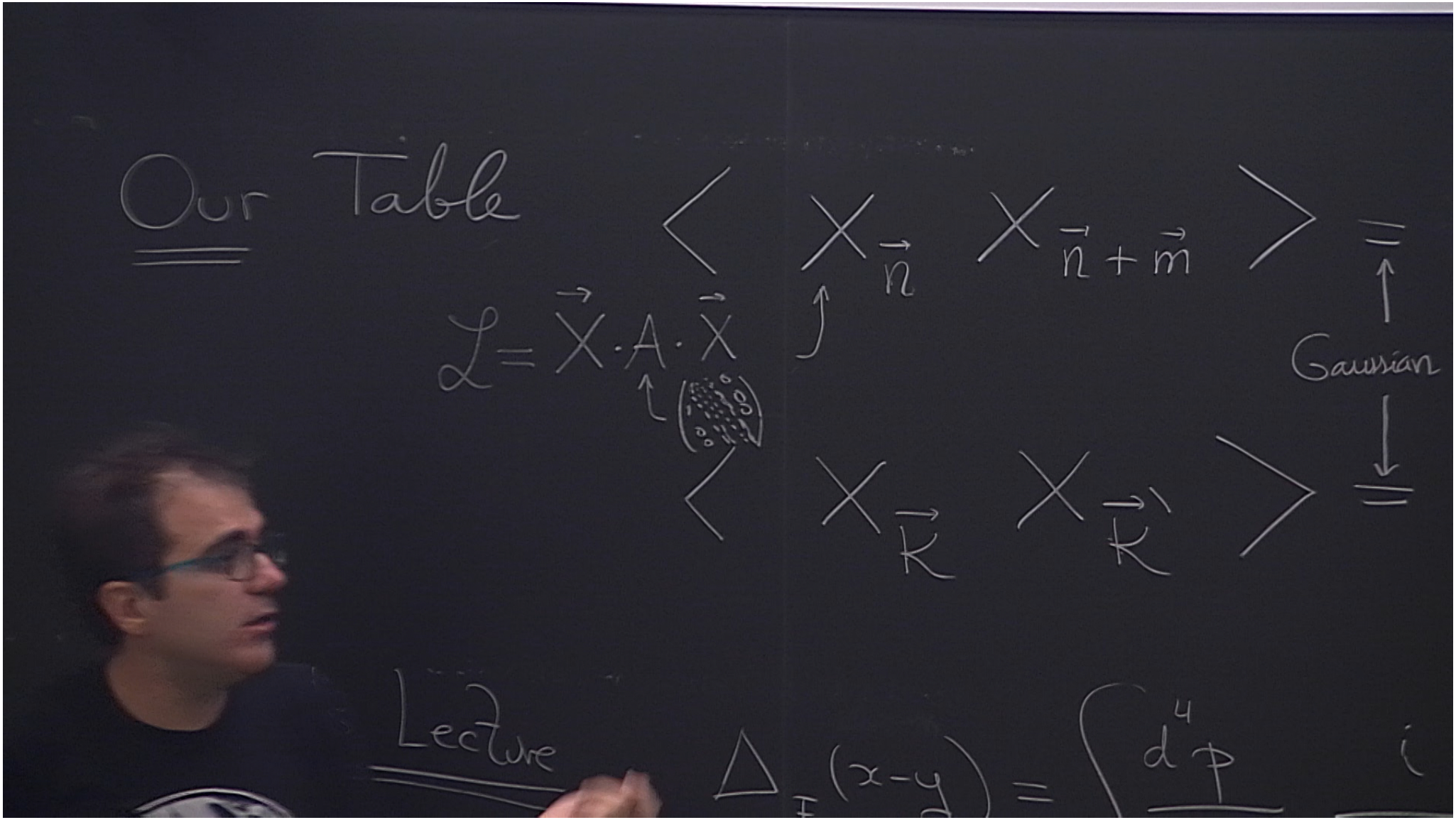

too many J's
m of them
$= \sum A^{-1}_{aa'} A^{-1}_{bb'} \cdots A^{-1}$
← Wick's theorem
pairs
(a,a'),(b,b')....
summing over Wick contractions
$i_1 \cdots i_K$
$K = 2m$

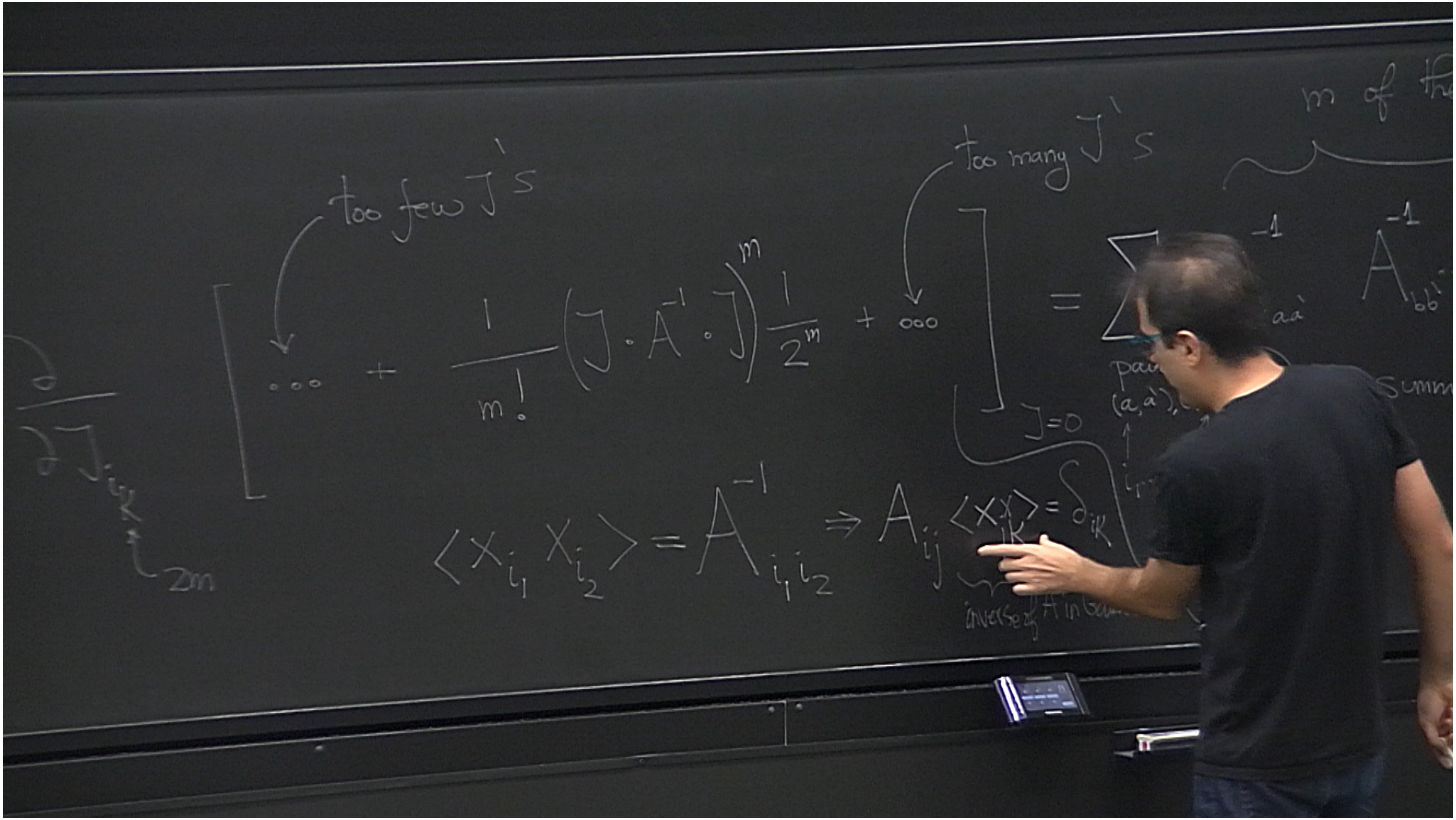

too few J's
too many
$= \frac{\partial}{\partial J_{i_1}} \cdots \frac{\partial}{\partial J_{i_{2m}}} \left[\cdots + \frac{1}{m!} \left(J \cdot A^{-1} \cdot J \right)^m \frac{1}{2^m} + \cdots \right]_{J=0} =$
$\langle X_{i_1} X_{i_2} \rangle = A^{-1}_{i_1 i_2}$

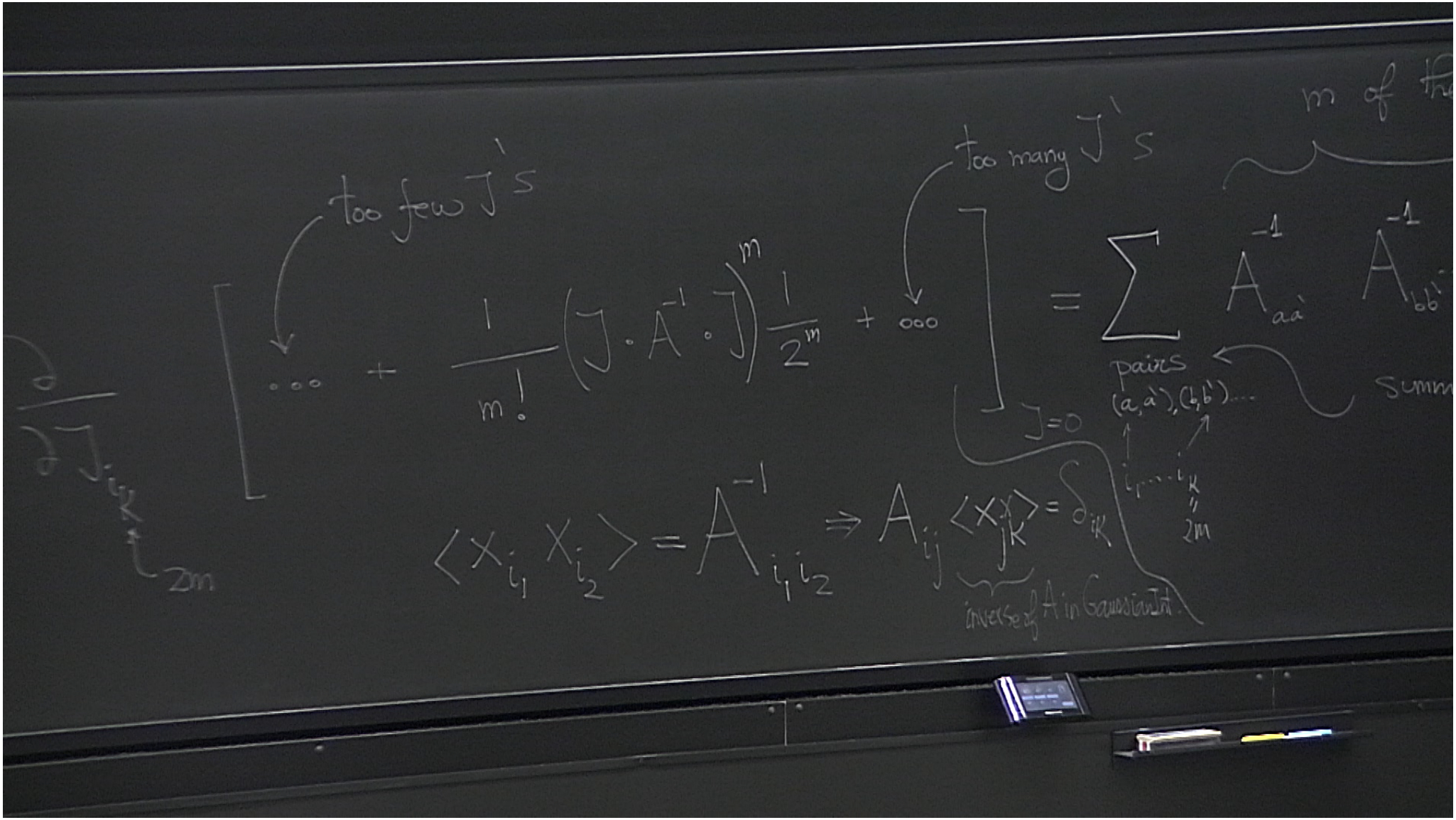

too few J's
too many J's
m of the
pairs
(a,a'),(b,b')....
summ
J=0
2m
2m

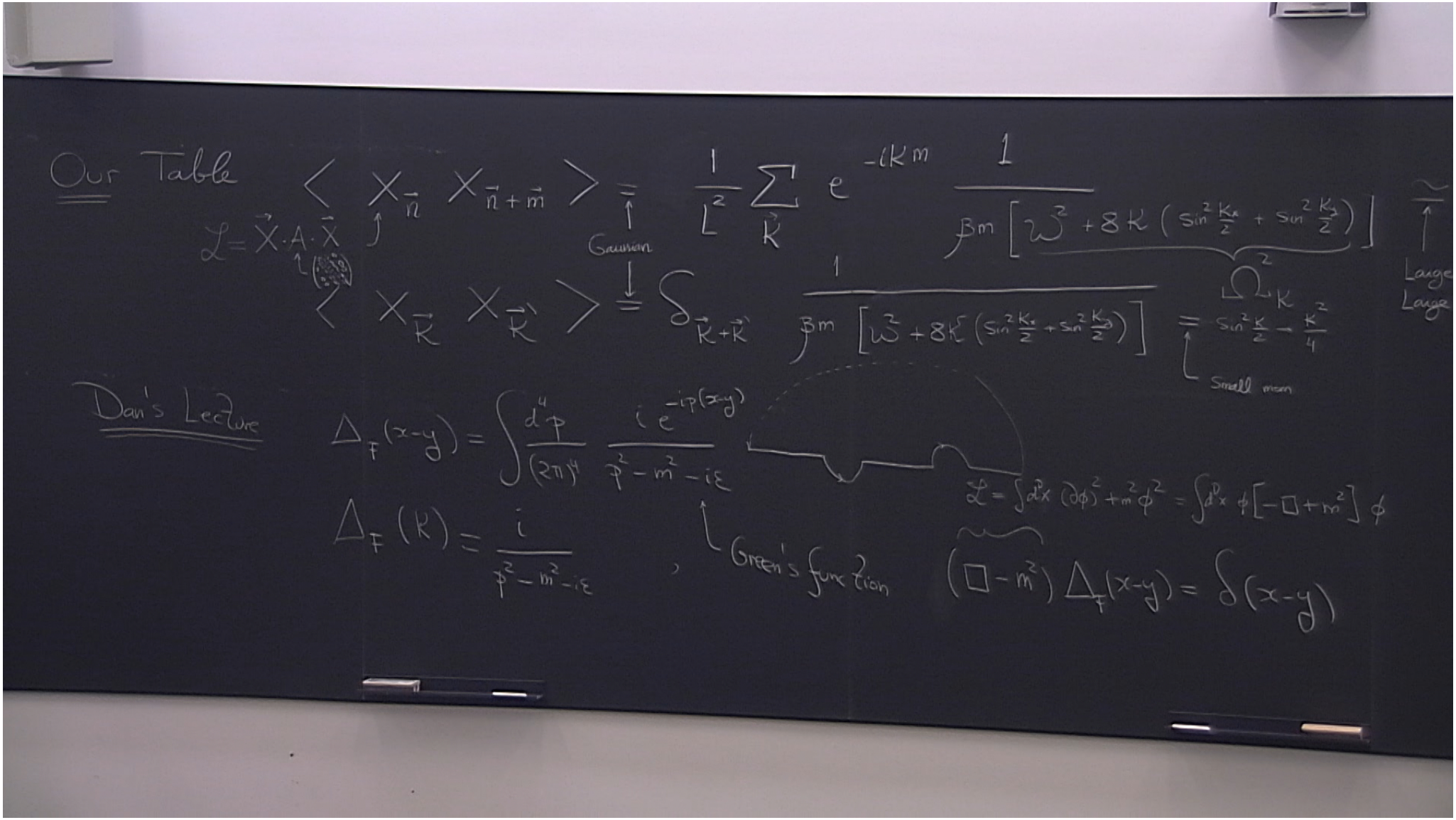

too few J's
too many J's
m of th
$\frac{\partial}{\partial J_{i_k}}$
2m
$\left[\cdots + \frac{1}{m!}\left(J \cdot A^{-1} \cdot J\right)^m \frac{1}{2^m} + \cdots \right]_{J=0} = \sum_{\text{pairs}} A^{-1}_{aa'} A^{-1}_{bb'}$
$(a,a'),(b,b')\ldots$
$i_1 \ldots i_k$
$= 2m$
summ
$\langle x_{i_1} x_{i_2} \rangle = A^{-1}_{i_1 i_2} \Rightarrow A_{ij} \langle x_{jk} \rangle = \delta_{ik}$
inverse of A in Gaussian Int.

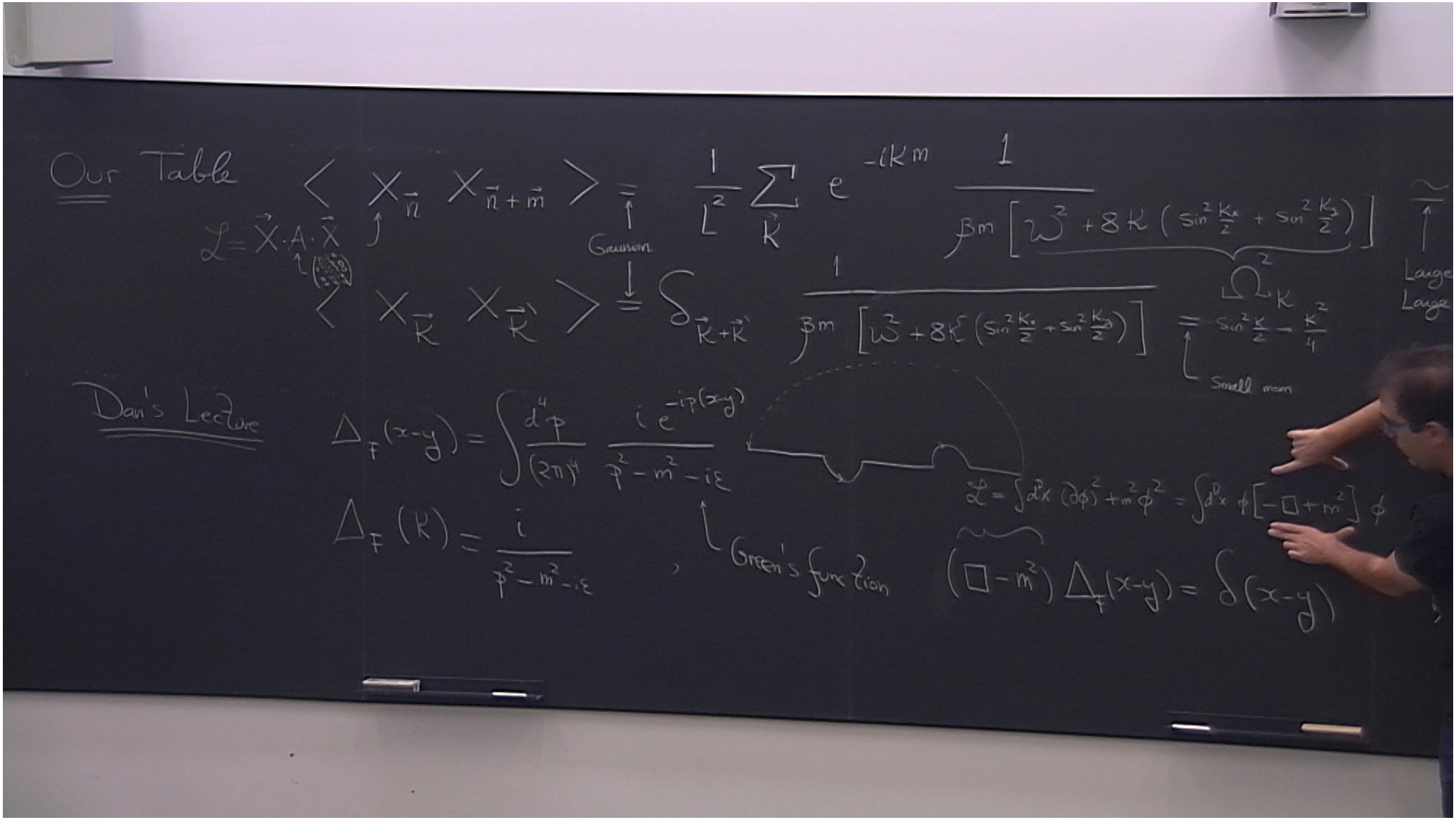

Our Table
Gaussian
Dan's Lecture
Green's function


Our Table
$\mathcal{L} = \vec{X} \cdot A \cdot \vec{X}$
$\langle X_{\vec{n}} X_{\vec{n}+\vec{m}} \rangle$
Gaussian
$\langle X_{\vec{k}} X_{\vec{k}'} \rangle$
Lecture
$\Delta_F(x-y) = \int d^4 p \; i$


too few J's
too many J's
m of the
$\frac{\partial}{\partial J_{i_1} \cdots J_{i_{2m}}}$
$\left[\cdots + \frac{1}{m!} \left(J \cdot A^{-1} \cdot J \right)^m \frac{1}{2^m} + \cdots \right]_{J=0} = \sum_{pairings} \prod_{(a,a')} A^{-1}_{bb'}$
$\langle X_{i_1} X_{i_2} \rangle = A^{-1}_{i_1 i_2} \Rightarrow A_{ij} \langle X_j X_k \rangle = \delta_{ik}$
inverse of A in Gaussian
Summ


too few J's
too many J's
m of th
pairs
(a,a'),(b,b')....
Summ
J=0
inverse of A in GaussianInt.
2m


Our Table
$\mathcal{L} = \vec{X} \cdot A \cdot \vec{X}$
$\langle X_{\vec{n}} X_{\vec{n}+\vec{m}} \rangle = \frac{1}{L^2} \sum_{\vec{K}} e^{-iKm} \frac{1}{\beta m \left[\omega^2 + 8k\left(\sin^2\frac{K_x}{2} + \sin^2\frac{K_y}{2}\right)\right]}$
Gaussian
$\langle X_{\vec{K}} X_{\vec{K}'} \rangle = \delta_{\vec{K}+\vec{K}'} \frac{1}{\beta m \left[\omega^2 + 8k\left(\sin^2\frac{K_x}{2} + \sin^2\frac{K_y}{2}\right)\right]}$
Ω_K^2
$\sin^2\frac{K}{2} \to \frac{K^2}{4}$
Small mom
Large
Large
Dan's Lecture
$\Delta_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{i e^{-ip(x-y)}}{p^2 - m^2 - i\varepsilon}$
$\Delta_F(K) = \frac{i}{p^2 - m^2 - i\varepsilon}$
Green's function
$\mathcal{L} = \int d^3x \, (\partial\phi)^2 + m^2\phi^2 = \int d^Dx \, \phi\left[-\Box + m^2\right]\phi$
$(\Box - m^2)\Delta_F(x-y) = \delta(x-y)$


Our Table
$\mathcal{L} = \vec{X} \cdot A \cdot \vec{X}$
$\langle X_{\vec{n}} X_{\vec{n}+\vec{m}} \rangle = \frac{1}{L^2} \sum_{\vec{K}} e^{-iKm} \frac{1}{\beta m \left[\omega^2 + 8k \left(\sin^2 \frac{K_x}{2} + \sin^2 \frac{K_y}{2} \right) \right]}$
Gaussian
$\langle X_{\vec{K}} X_{\vec{K}'} \rangle = \delta_{\vec{K}+\vec{K}'} \frac{1}{\beta m \left[\omega^2 + 8k \left(\sin^2 \frac{K_x}{2} + \sin^2 \frac{K_y}{2} \right) \right]}$
Ω_K^2
$\sin^2 \frac{K}{2} \to \frac{K^2}{4}$
Small mom
Large
Large
Dan's Lecture
$\Delta_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{i e^{-ip(x-y)}}{p^2 - m^2 - i\varepsilon}$
$\Delta_F(K) = \frac{i}{p^2 - m^2 - i\varepsilon}$
Green's function
$\mathcal{L} = \int d^3x \, (\partial\phi)^2 + m^2\phi^2 = \int d^Dx \, \phi \left[-\Box + m^2 \right] \phi$
$(\Box - m^2) \Delta_F(x-y) = \delta(x-y)$

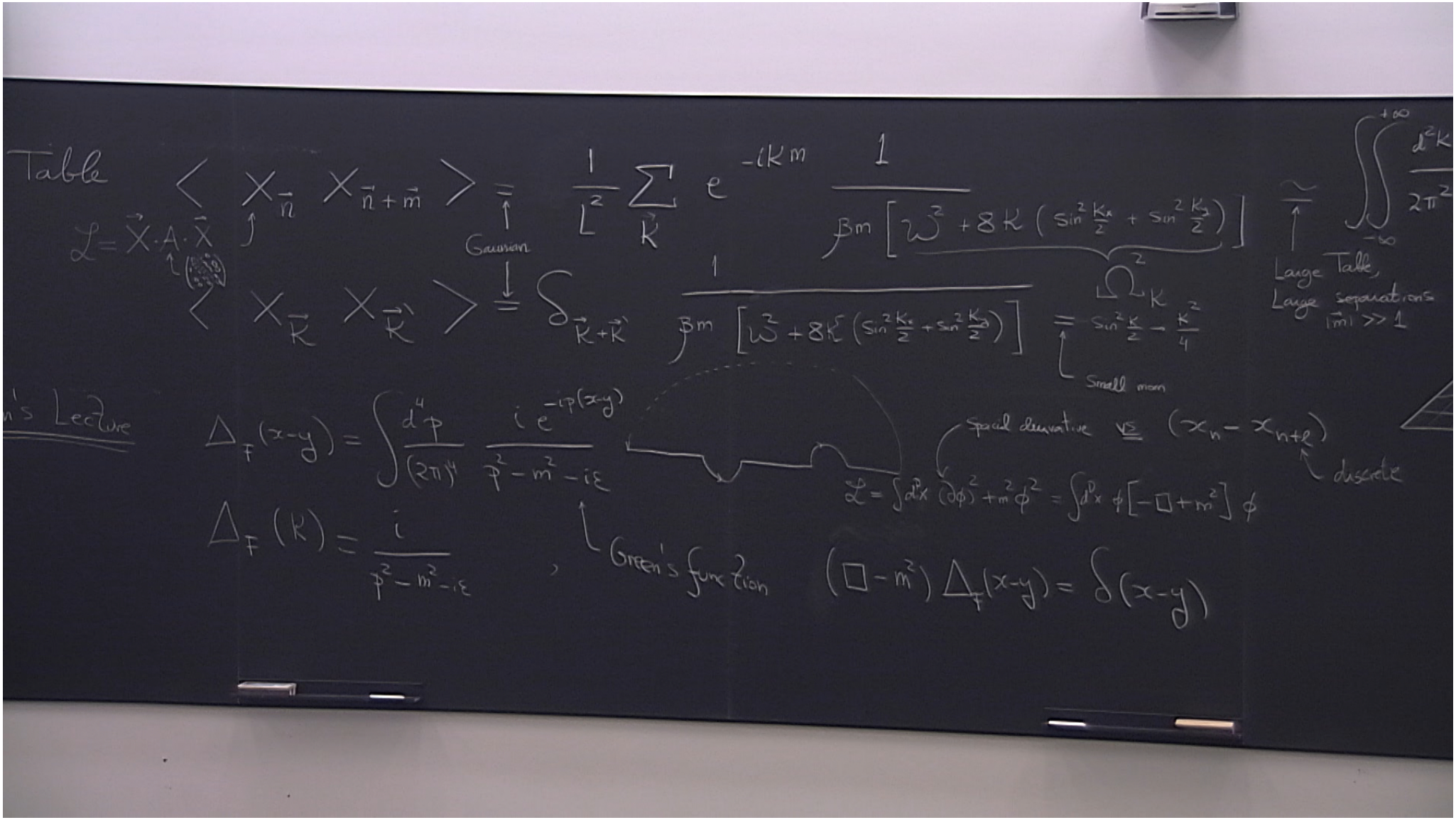
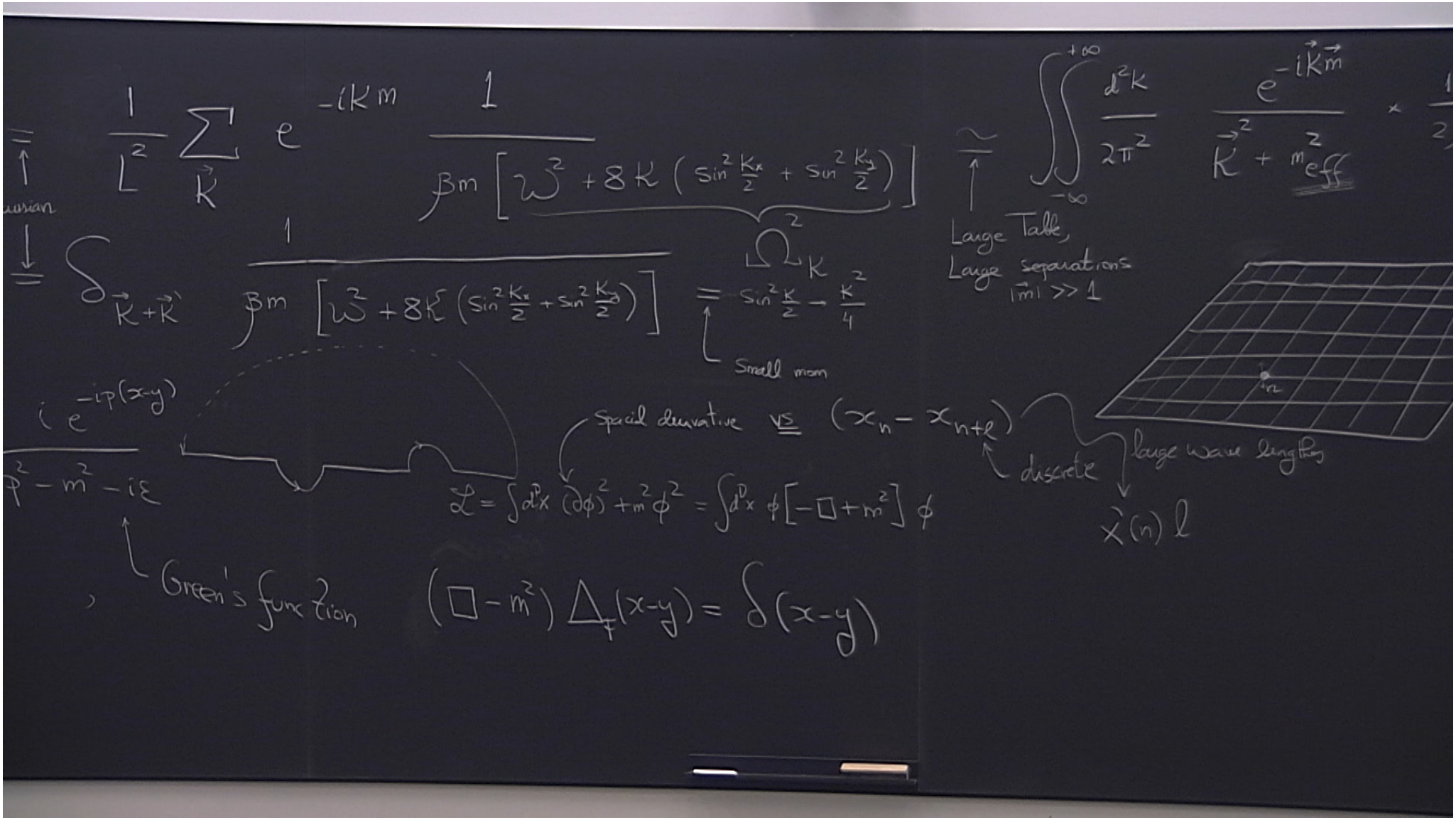


Table
Gaussian
Large Table, Large separations
Small momn
Special derivative
discrete
Green's function


Large Table,
Large separations
$|\vec{m}| \gg 1$
Small mom
Spacial derivative vs $(x_n - x_{n+\ell})$
discrete
large wave lengths
$\mathcal{L} = \int d^D x \, (\partial\phi)^2 + m^2\phi^2 = \int d^D x \, \phi\left[-\Box + m^2\right]\phi$
Green's function
$(\Box - m^2)\,\Delta_F(x-y) = \delta(x-y)$

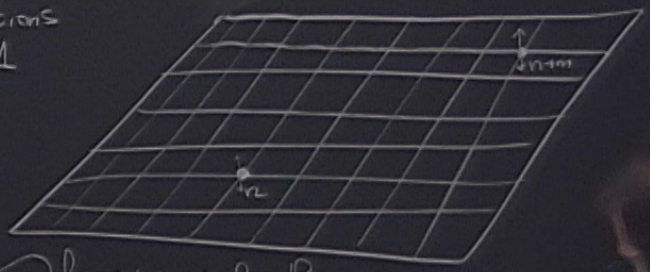
$$\sum_{\vec{k}} e^{-ikm} \frac{1}{\beta m\left[\omega^2 + 8k\left(\sin^2\frac{k_x}{2} + \sin^2\frac{k_y}{2}\right)\right]} \simeq \int_{-\infty}^{+\infty} \frac{d^2k}{2\pi^2} \frac{e^{-i\vec{k}\vec{m}}}{\vec{k}^2 + m_{eff}^2} \times \frac{1}{2\beta m k}$$

$$\frac{1}{\beta m\left[\omega^2 + 8k\left(\sin^2\frac{k_x}{2} + \sin^2\frac{k_y}{2}\right)\right]} = \frac{1}{\beta m\left[\omega^2 + 8k\,\Omega_k^2\right]}$$

$$\sin^2\frac{k}{2} \to \frac{k^2}{4}$$

Small mom

Large Talk, Large separations $|\vec{m}| \gg 1$



Special derivative $\equiv (x_n - x_{n+\ell})$

discrete

large wave lengths

$$\mathcal{L} = \int d^D x\,(\partial\phi)^2 + m^2\phi^2 = \int d^D x\,\phi\left[-\Box + m^2\right]\phi$$

$x(n)\,\ell$, our action $\sum (x_n - x_{n+\ell})^2$

$$\int dn\,(\partial\phi)^2$$

Green's function $\left(\Box - m^2\right)\Delta_F(x-y) = \delta(x-y)$

$$\sum_{\vec{k}} e^{-ik\,m} \frac{1}{\beta m\left[\omega^2 + 8k\left(\sin^2\frac{k_x}{2} + \sin^2\frac{k_y}{2}\right)\right]} \simeq \int_{-\infty}^{+\infty} \frac{d^2k}{2\pi^2} \frac{e^{-i\vec{k}\vec{m}}}{\vec{k}^2 + m_{eff}^2} \times \frac{1}{2\beta m \Omega_k}$$

$$\frac{1}{\beta m\left[\omega^2 + 8k\left(\sin^2\frac{k_x}{2} + \sin^2\frac{k_y}{2}\right)\right]} = \Omega_k^2$$

$$\sin^2\frac{k}{2} \to \frac{k^2}{4}$$

Small mom

Large Table, Large separations $|\vec{m}| \gg 1$

Spacial derivative $\underline{vs}$ $(x_n - x_{n+\ell})$

discrete

large wave lengths

$$\mathcal{L} = \int d^D x\, (\partial\phi)^2 + m^2\phi^2 = \int d^D x\, \phi\left[-\Box + m^2\right]\phi$$

$x(n)\,\ell$, our action $\sum (x_n - x_{n+\ell})^2$

$$\int dn\, (\partial\phi)^2$$

x

Green's function $\left(\Box - m^2\right)\Delta_F(x-y) = \delta(x-y)$

Our Table

$$\mathcal{L} = \vec{X} \cdot A \cdot \vec{X}$$

$$\langle X_{\vec{n}} X_{\vec{n}+\vec{m}} \rangle = \frac{1}{L^2} \sum_{\vec{k}} e^{-ikm} \frac{1}{\beta m \left[\omega^2 + 8k \right]}$$

Gaussian

$$\langle X_{\vec{k}} X_{\vec{k}'} \rangle = \delta_{\vec{k}+\vec{k}'} \frac{1}{\beta m \left[\omega^2 + 8k \left(\sin^2 \frac{k_x}{2} + \sin^2 \frac{k_y}{2} \right) \right]}$$

Dan's Lecture

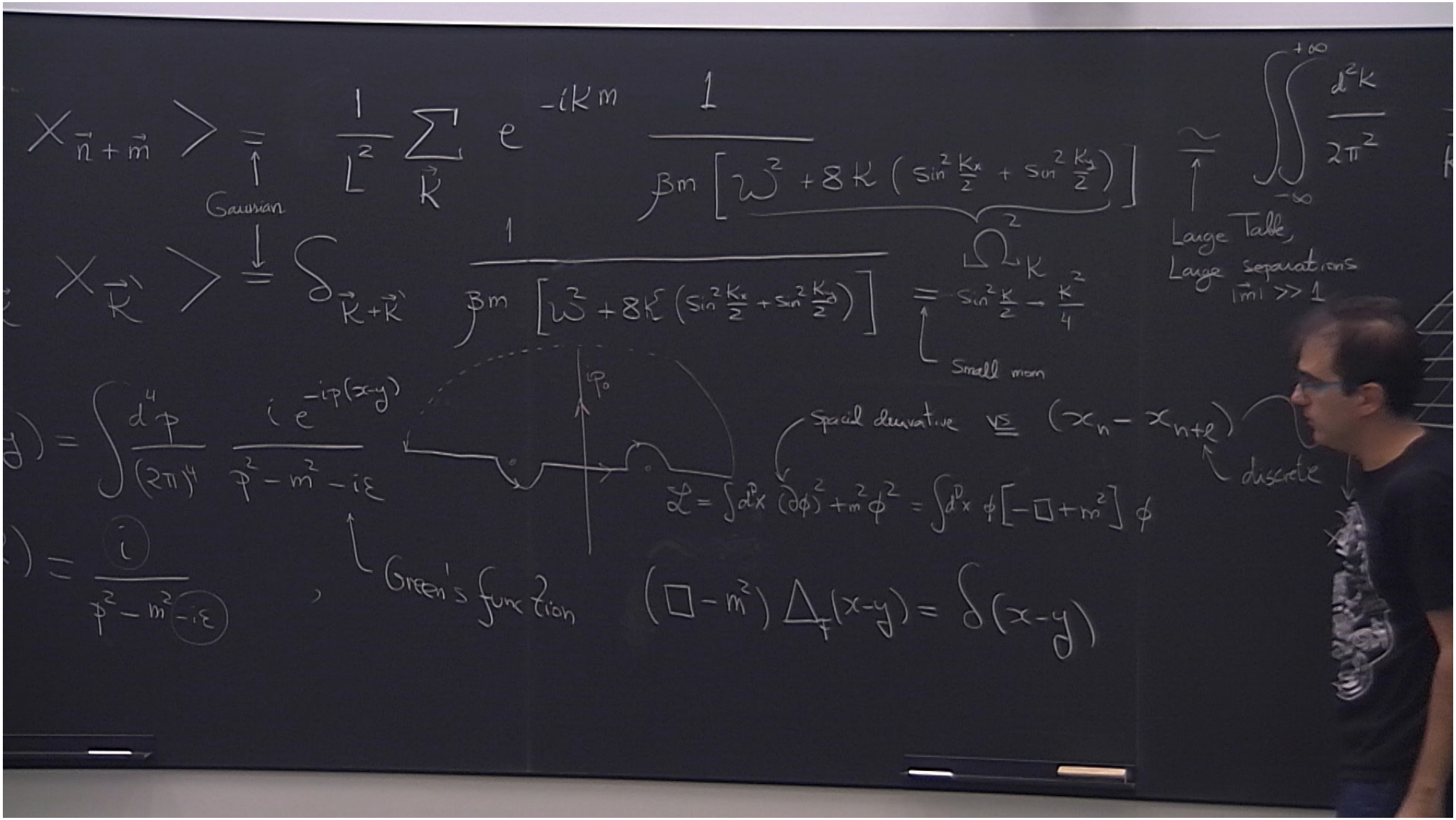
$$\Delta_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{i e^{-ip(x-y)}}{p^2 - m^2 - i\varepsilon}$$

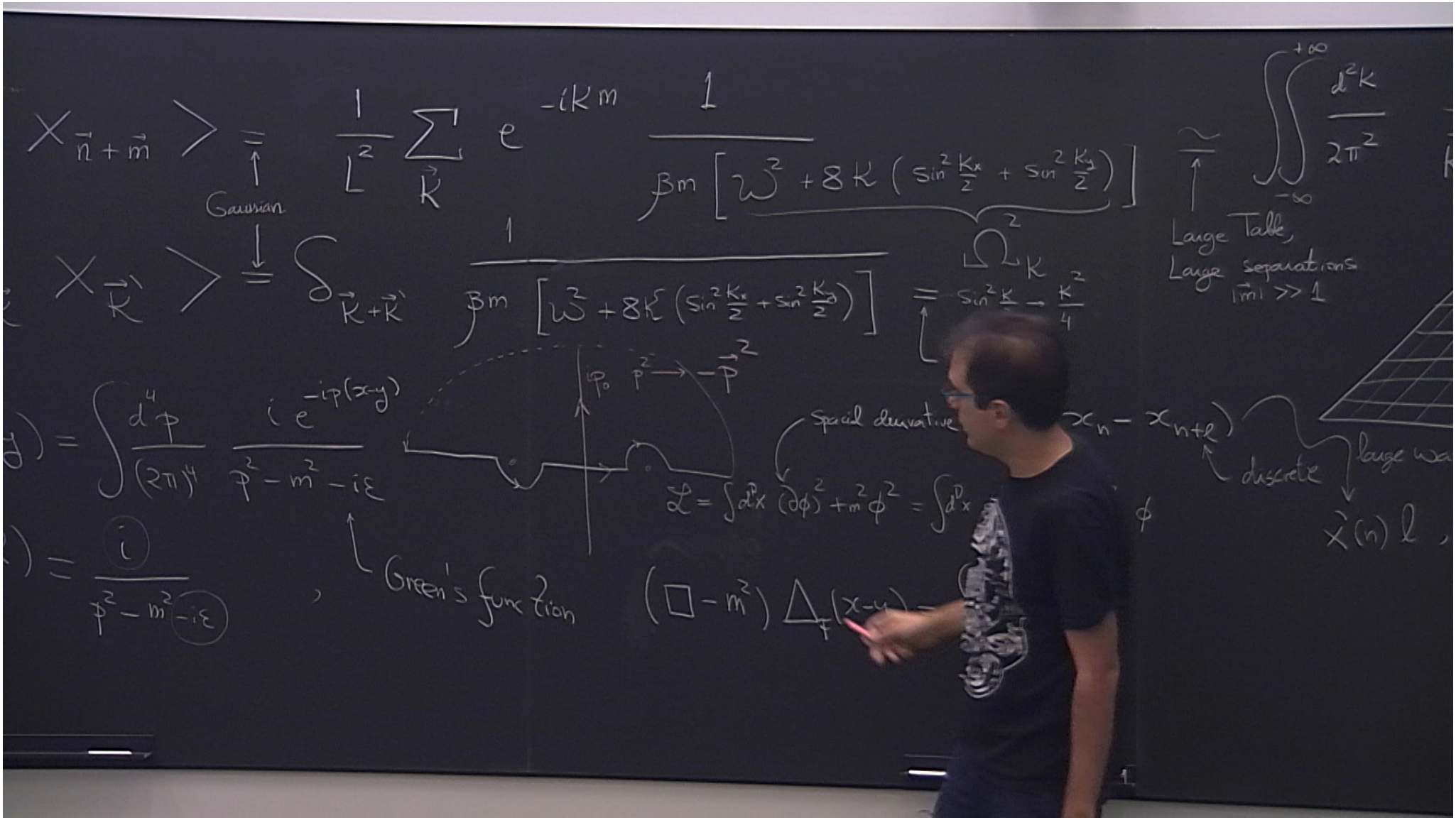
$$\Delta_F(k) = \frac{i}{p^2 - m^2 - i\varepsilon}$$

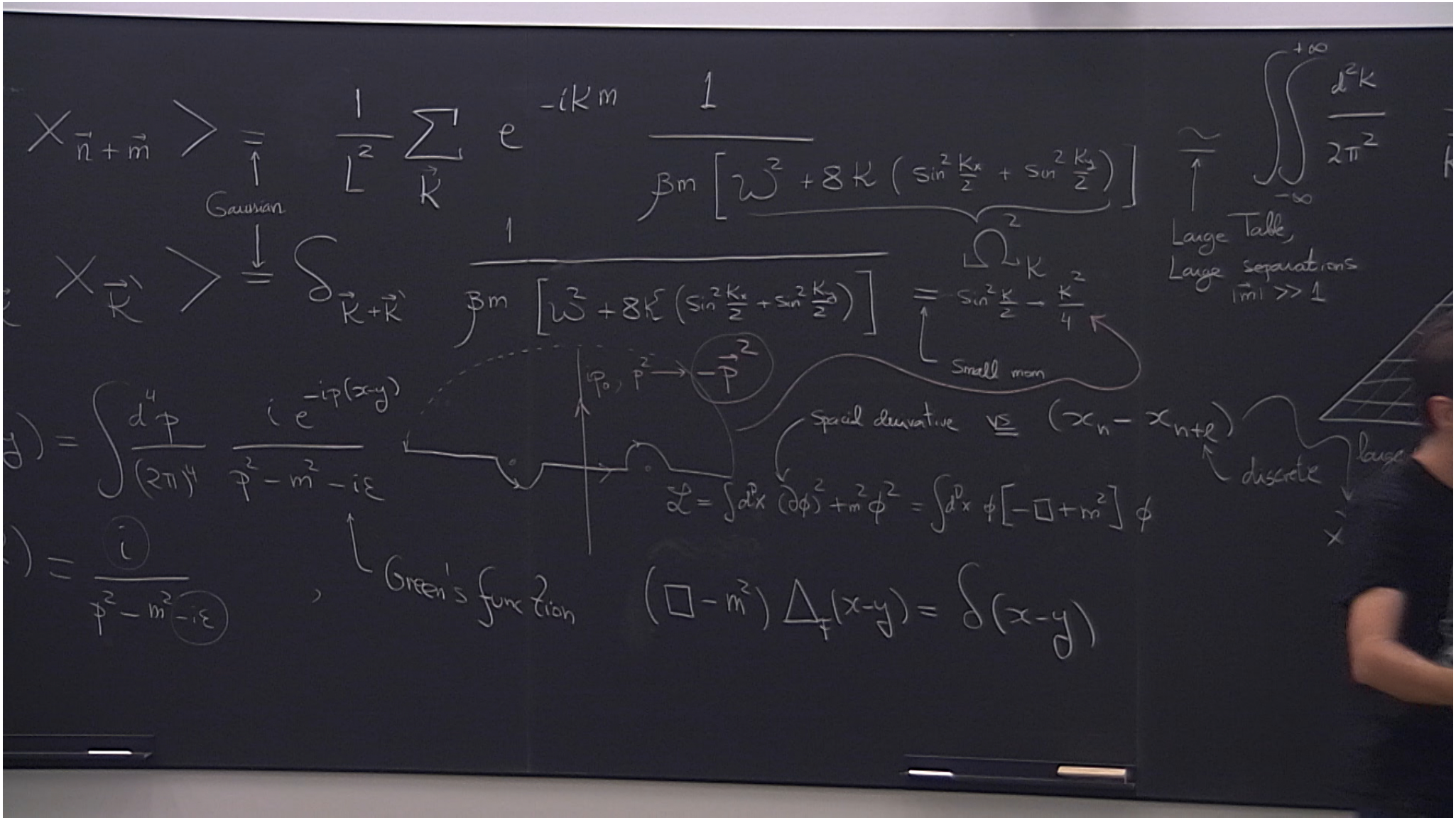
Green's function

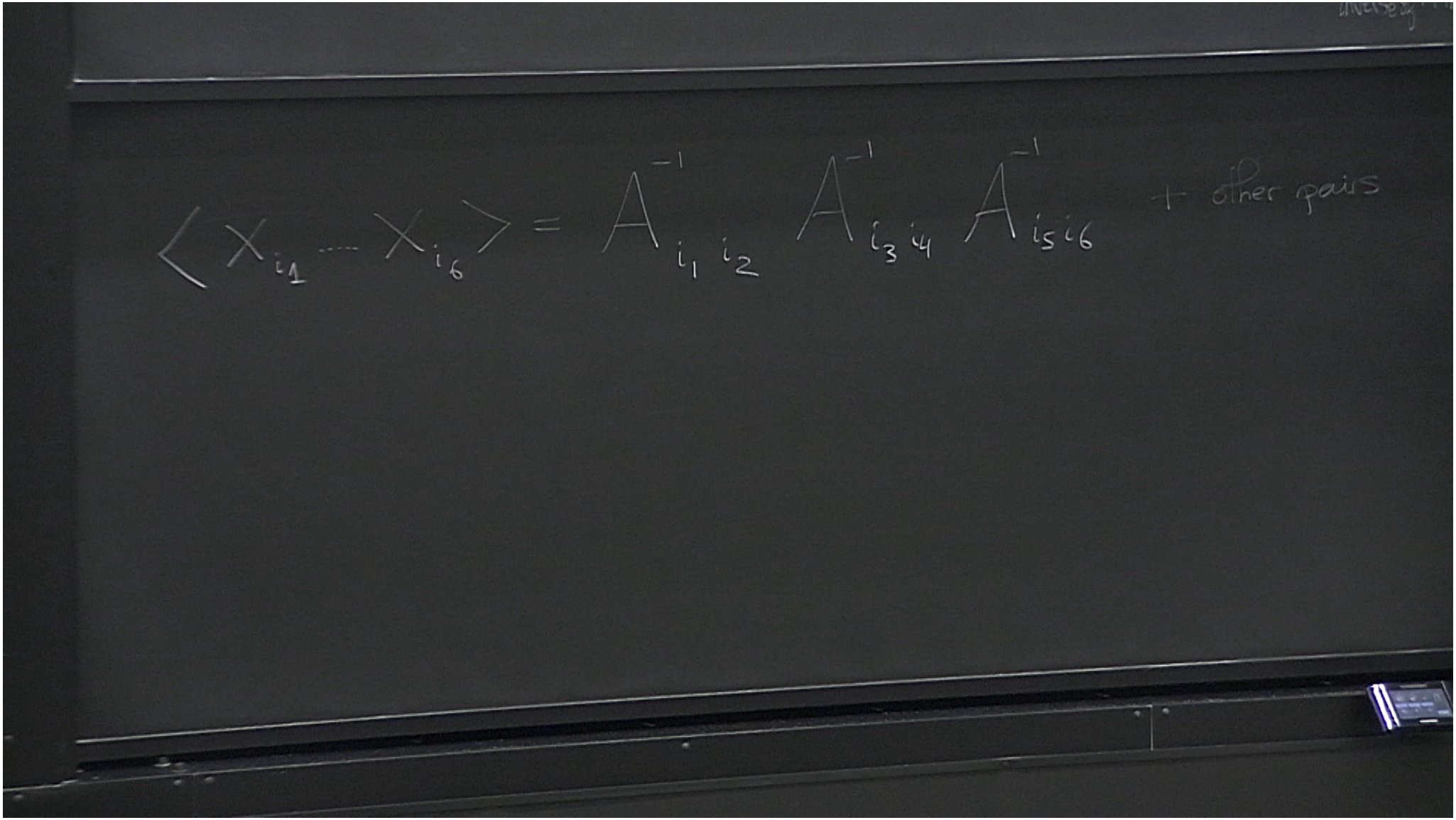
$$\mathcal{L} = \int d^3x \, (\partial\phi)^2 + m^2$$

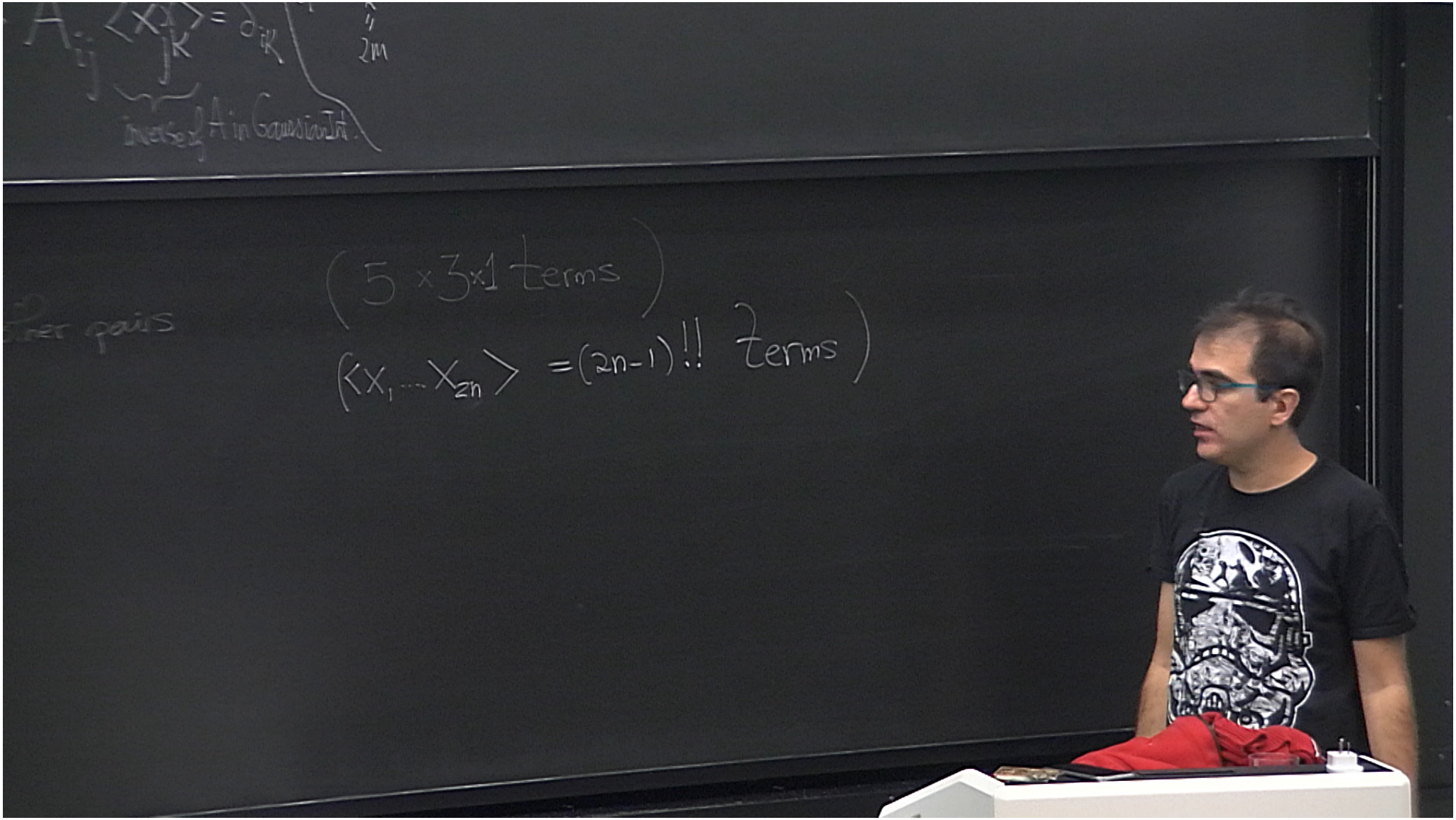
$$(\Box - m^2) \Delta$$

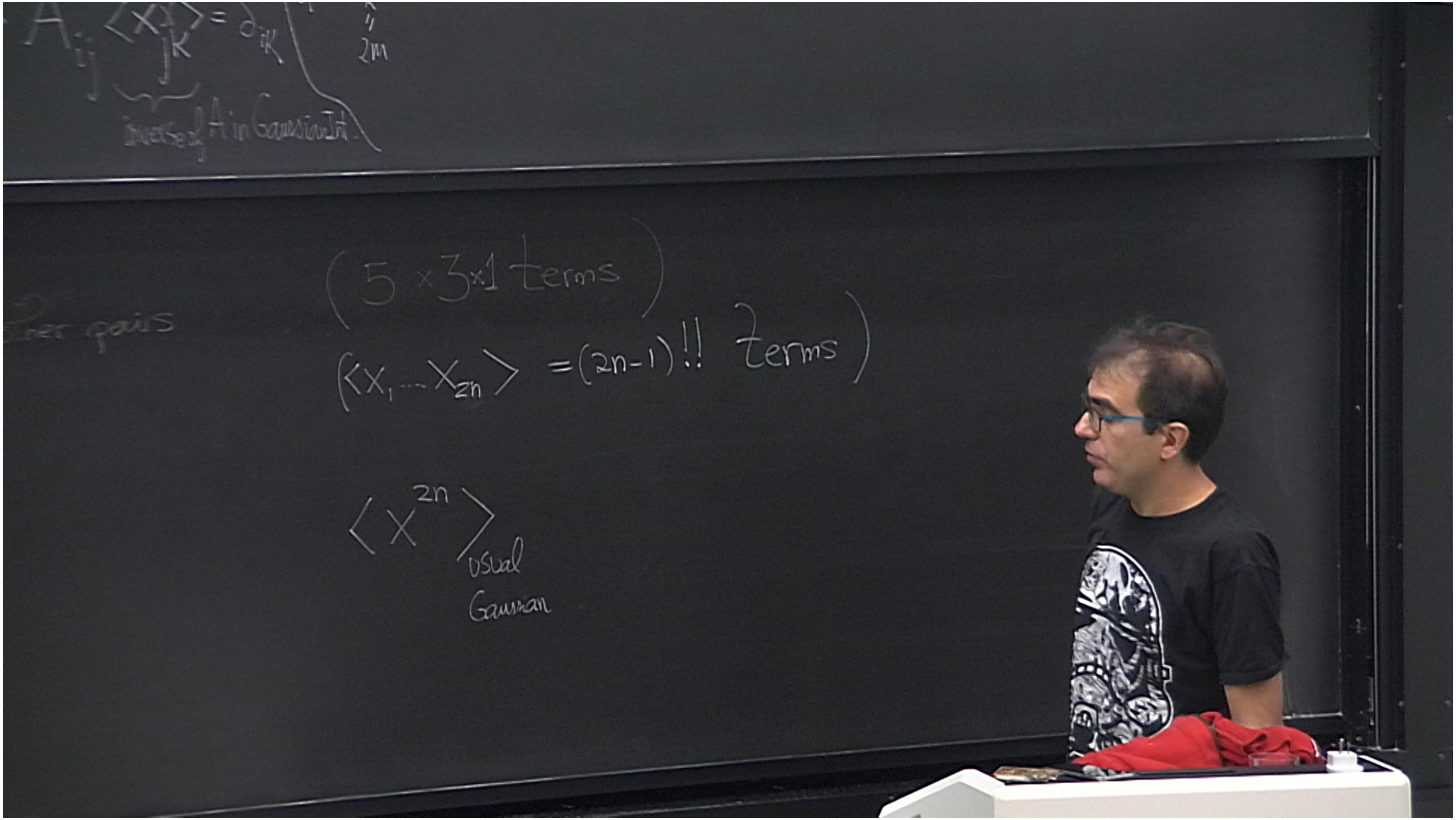

Gaussian
Large Table,
Large separations
|m| >> 1
Small mom
Spacial derivative vs (x_n − x_{n+ℓ})
discrete
Green's function
(□ − m²) Δ_F(x−y) = δ(x−y)

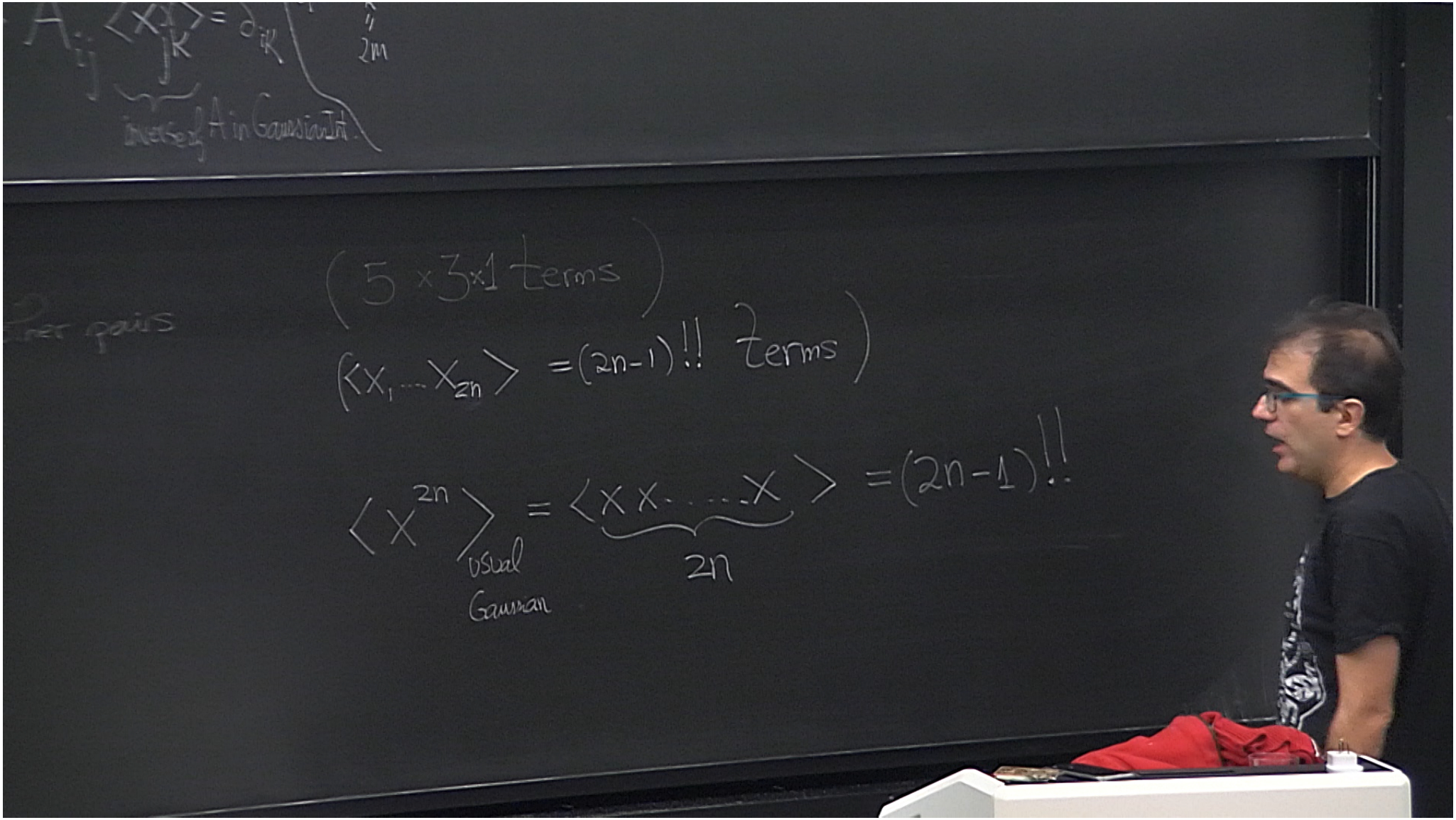

Gaussian
Large Table,
Large separations
spacial derivative
discrete
Green's function

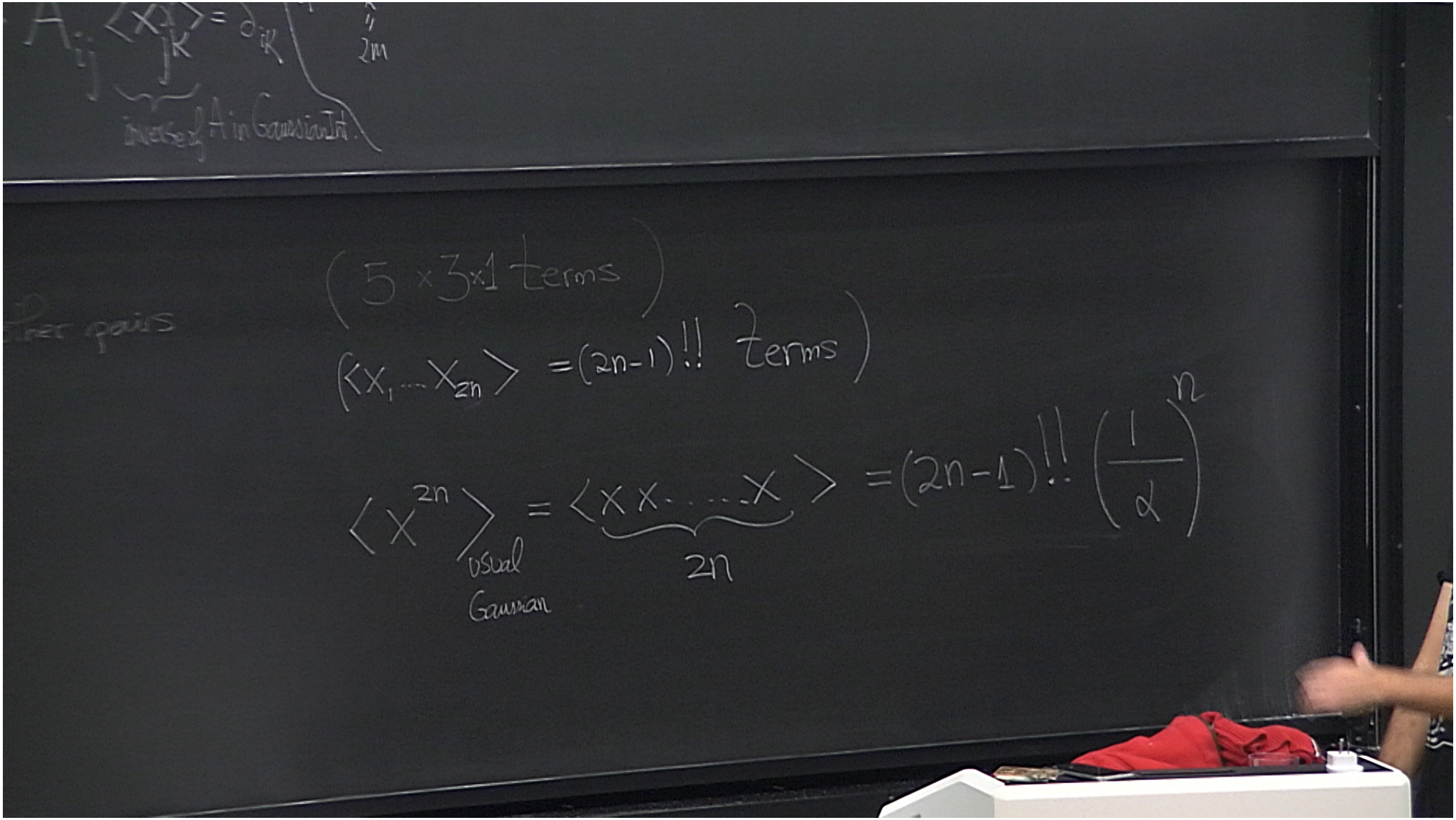

Gaussian
Large Table,
Large separations
$|\vec{m}| >> 1$
Small mom
Spacial derivative
discrete
Green's function

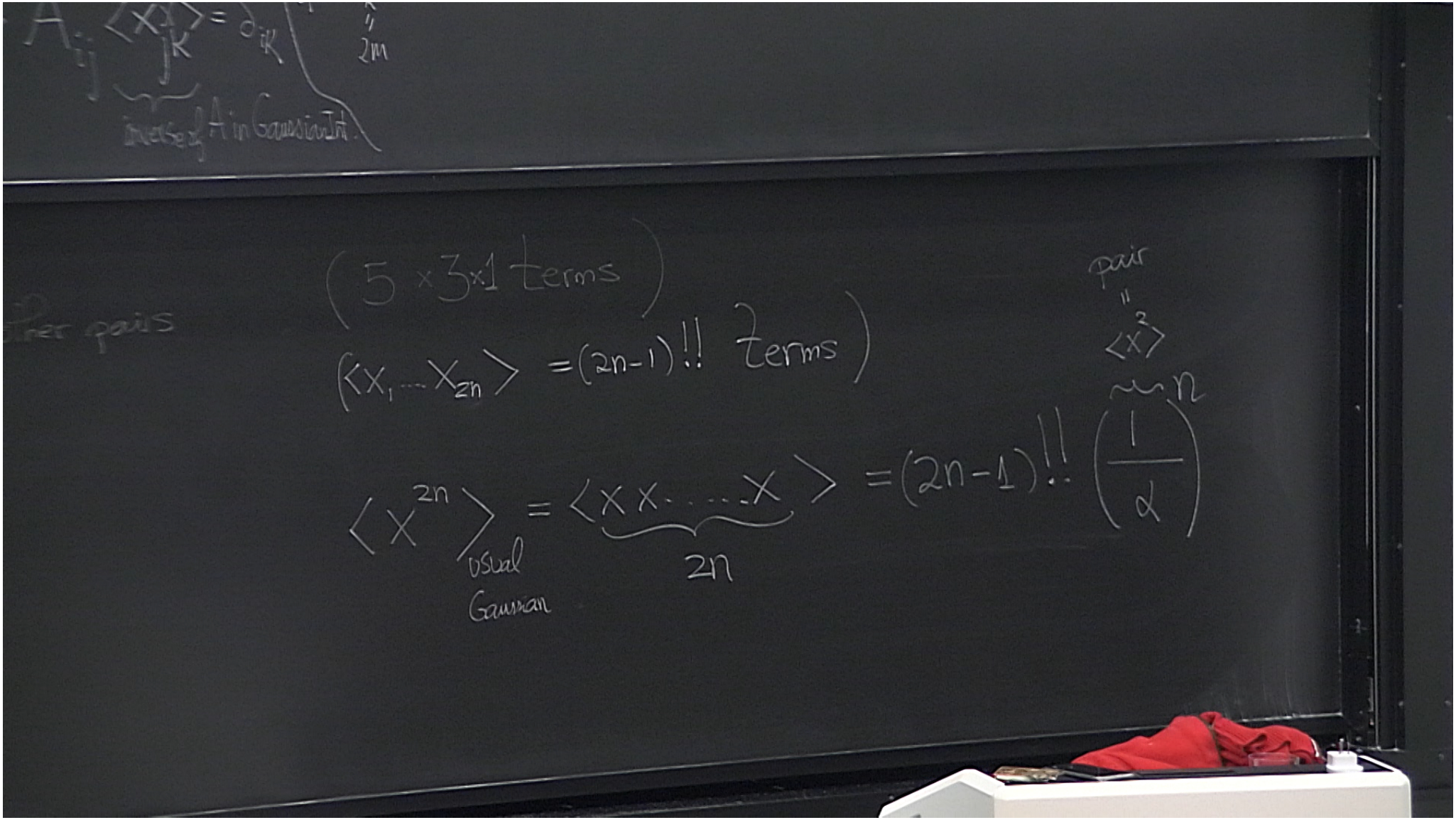

$\langle X_{i_1} \cdots X_{i_6} \rangle = A^{-1}_{i_1 i_2} A^{-1}_{i_3 i_4} A^{-1}_{i_5 i_6}$ + other pairs

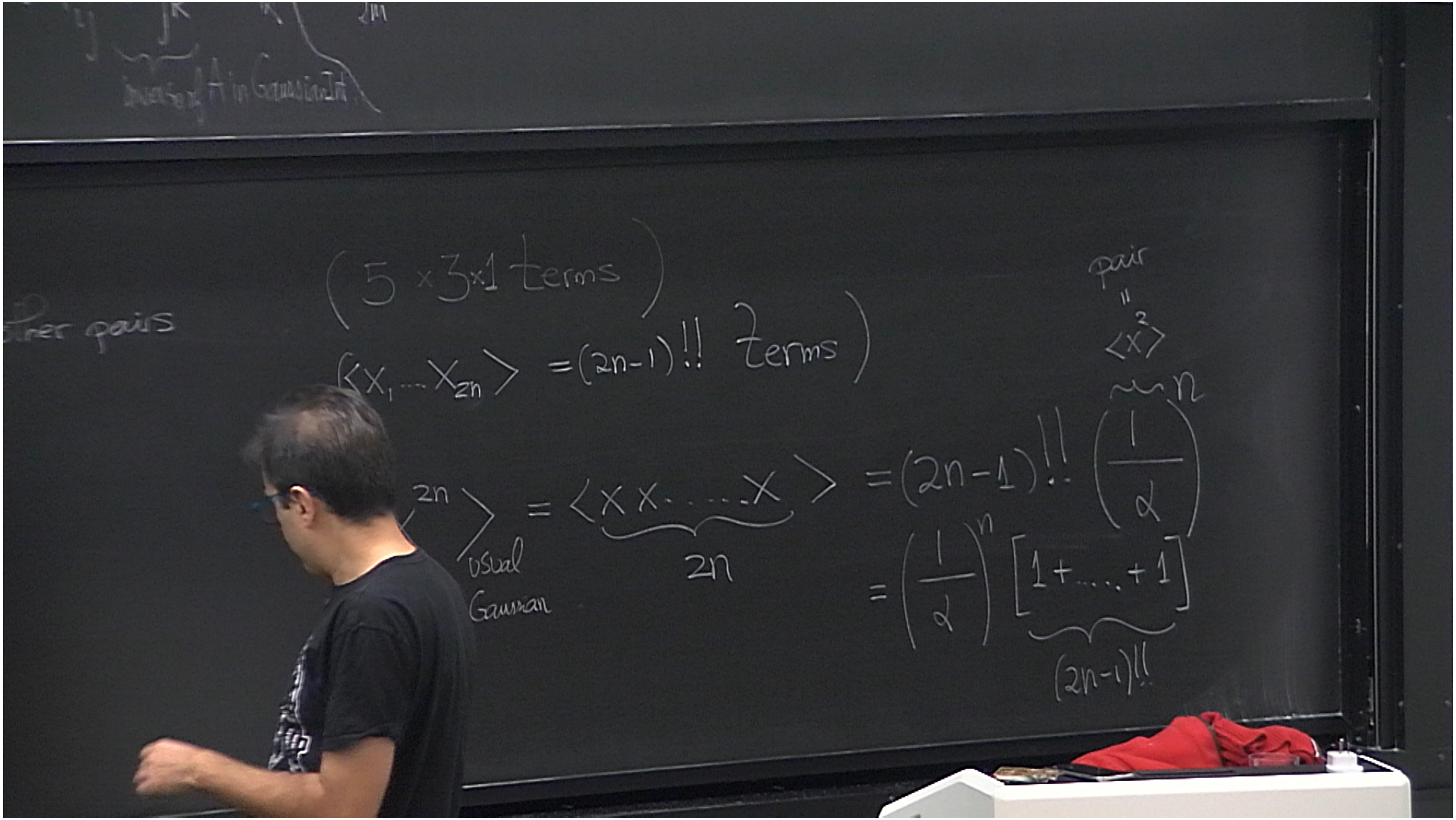

inverse of A in Gaussian Int.
2m
(5 × 3×1 terms)
(⟨X1 ... X2n⟩ = (2n-1)!! terms)

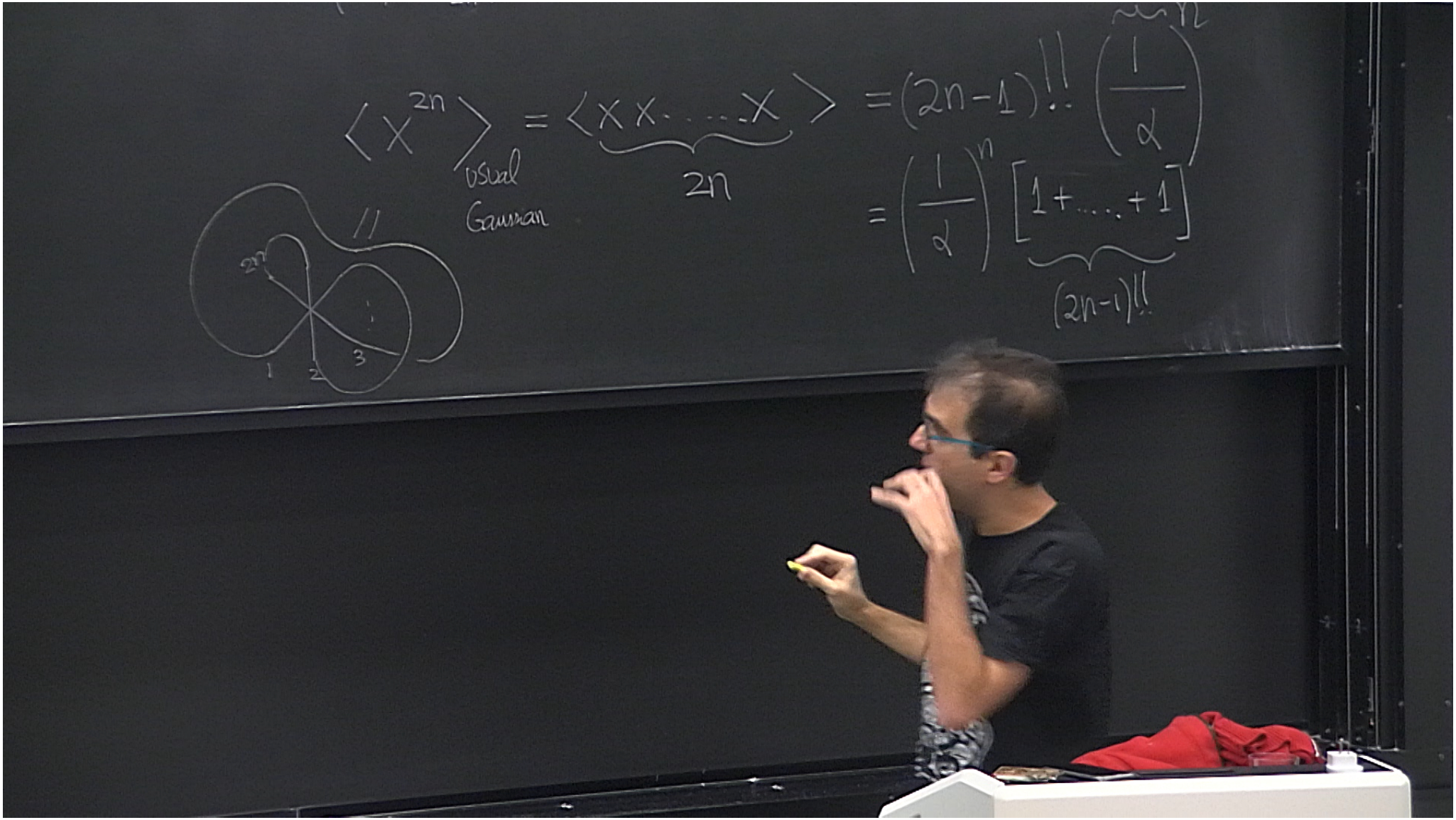

inverse of A in Gaussian Int.
2m
(5 × 3×1 terms)
(⟨x1 ... x2n⟩ = (2n-1)!! terms)
⟨x^2n⟩
usual
Gaussian

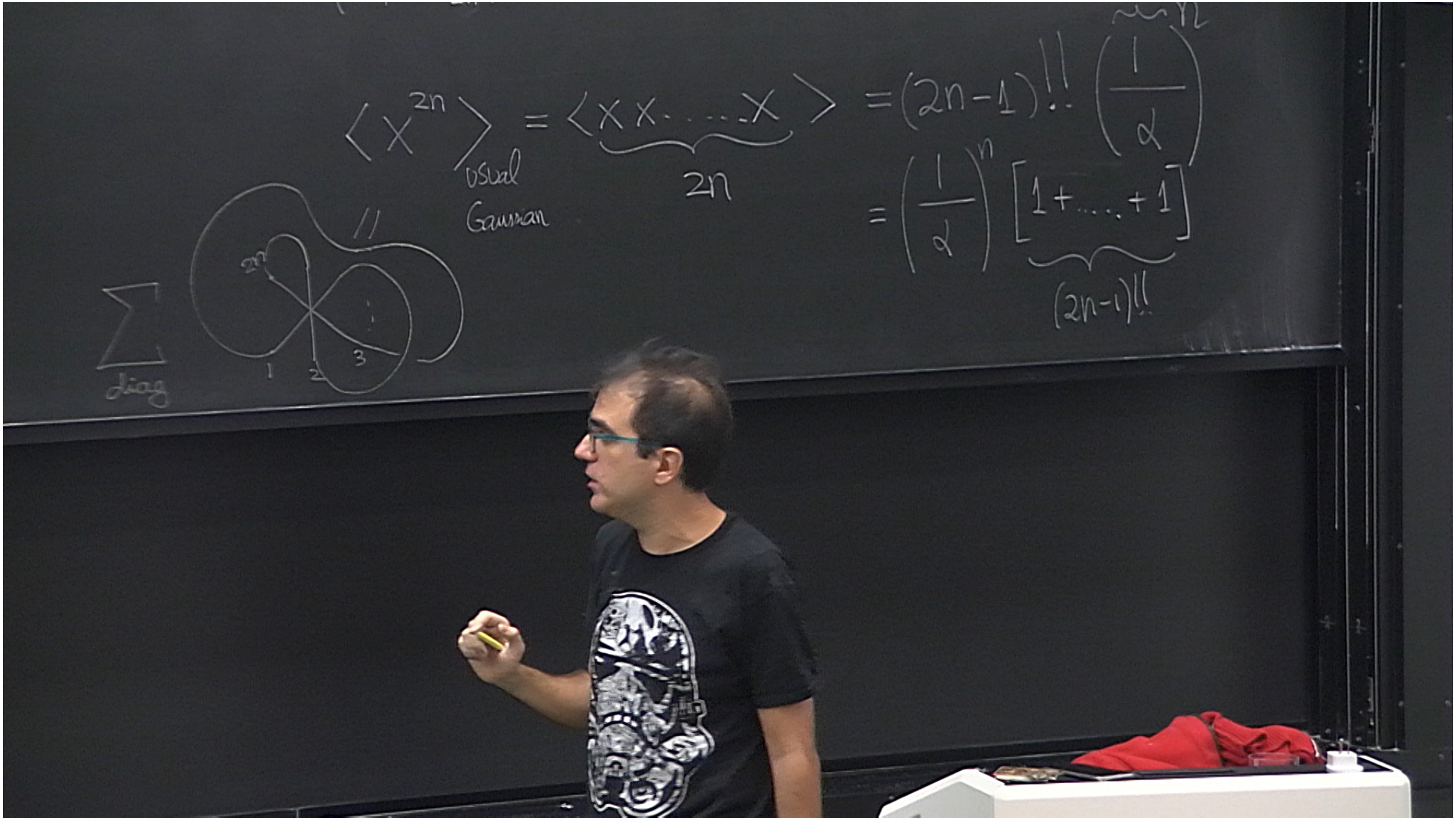

$A_{ij}\langle x_j x_k\rangle = \delta_{ik}$
inverse of A in Gaussian Int.
2m
(5 × 3×1 terms)
$(\langle x_1 \dots x_{2n}\rangle = (2n-1)!!$ terms)
$\langle x^{2n}\rangle_{\text{usual Gaussian}} = \langle \underbrace{x\,x \dots x}_{2n}\rangle = (2n-1)!!$

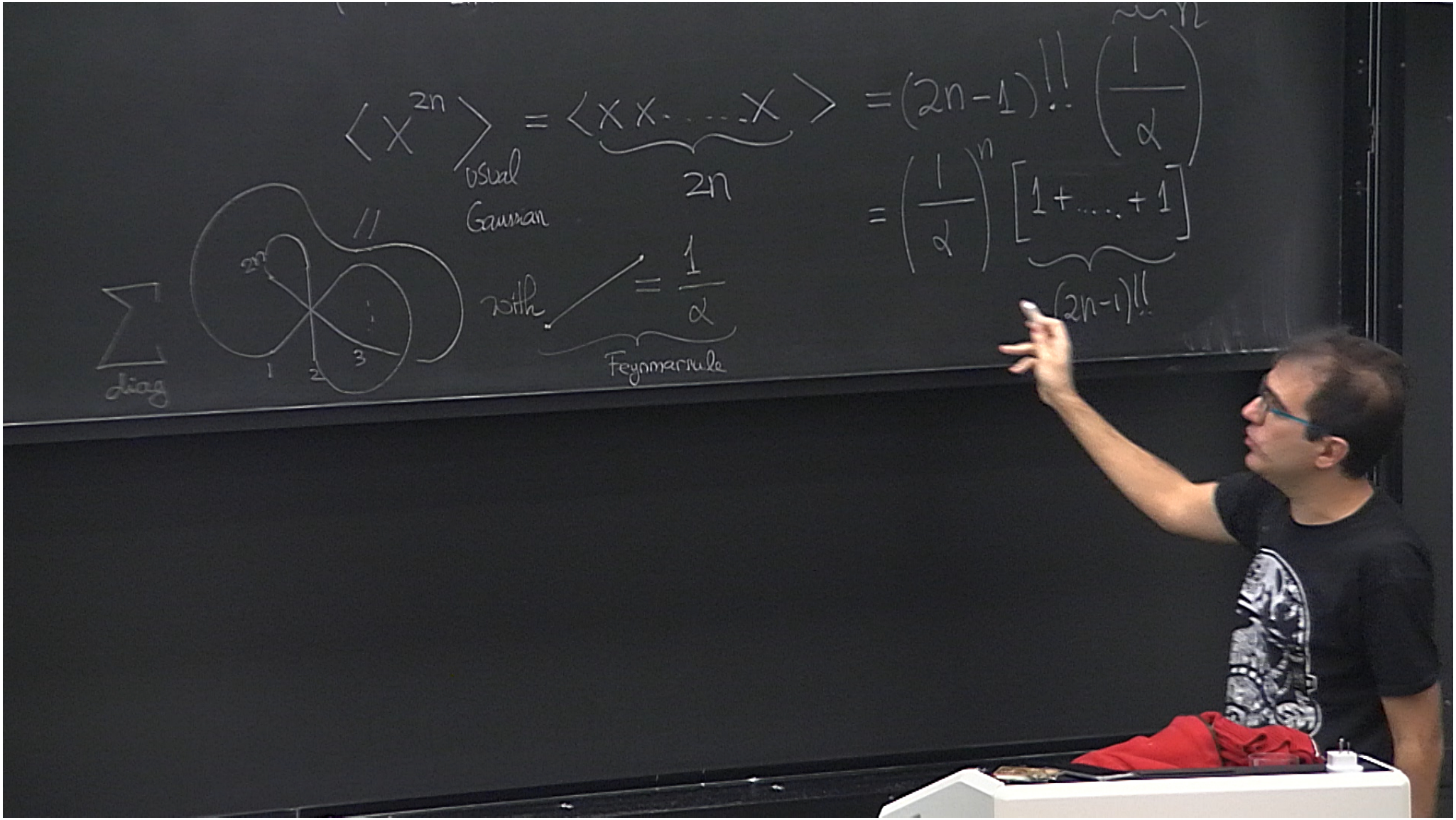

$A_{ij}\langle x_j x_k \rangle = \delta_{ik}$
$2m$
inverse of A in Gaussian Int.
$(5 \times 3 \times 1$ terms$)$
$\langle x_1 \cdots x_{2n} \rangle = (2n-1)!!$ terms
$\langle x^{2n} \rangle = \langle x x \cdots x \rangle = (2n-1)!! \left(\frac{1}{\alpha}\right)^n$
$2n$
usual Gaussian


$A_{ij}\langle x_j x_k\rangle = \delta_{ik}$
inverse of A in Gaussian Int.
2m
other pairs
(5 × 3×1 terms)
$(\langle x_1 \cdots x_{2n}\rangle = (2n-1)!!$ terms)
pair
$\langle x^2\rangle$
$\langle x^{2n}\rangle_{\text{usual Gaussian}} = \langle \underbrace{x\,x \cdots x}_{2n}\rangle = (2n-1)!!\left(\frac{1}{\alpha}\right)^n$


inverse of A in Gaussian Int.
other pairs
(5 × 3 × 1 terms)
(⟨x1 ... x2n⟩ = (2n-1)!! terms)
pair
= ⟨x²⟩
usual
Gaussian
= ⟨x x . . . . x⟩ = (2n-1)!! (1/α)^n
2n
= (1/α)^n [1 + . . . + 1]
(2n-1)!!

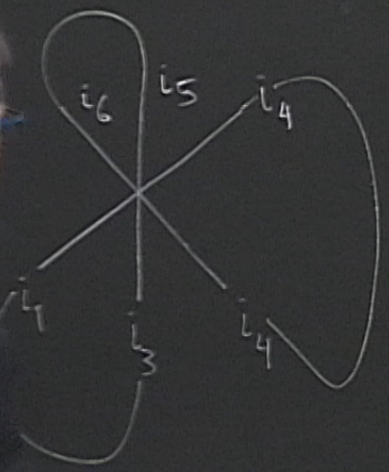

$\langle X^{2n} \rangle = \langle X X \ldots X \rangle = (2n-1)!! \left(\frac{1}{\alpha}\right)^n$
$= \left(\frac{1}{\alpha}\right)^n [1 + \ldots + 1]$
$2n$
$(2n-1)!!$
usual
Gaussian
$2n$
1
2
3


$\langle X^{2n} \rangle$
usual
Gaussian
$= \langle X X \ldots X \rangle$
$2n$
$= (2n-1)!!$
$\left(\frac{1}{\alpha}\right)^n$
$= \left(\frac{1}{\alpha}\right)^n [1 + \ldots + 1]$
$(2n-1)!!$
Σ
diag
$2n$
1
2
3



$\langle x^{2n} \rangle_{\text{usual Gaussian}} = \langle \underbrace{x\,x \ldots x}_{2n} \rangle = (2n-1)!! \left(\frac{1}{\alpha}\right)^n$
$= \left(\frac{1}{\alpha}\right)^n \underbrace{[1 + \ldots + 1]}_{(2n-1)!!}$
$\sum_{\text{diag}}$
2n
1
2
3
with
$= \frac{1}{\alpha}$
Feynmarsule

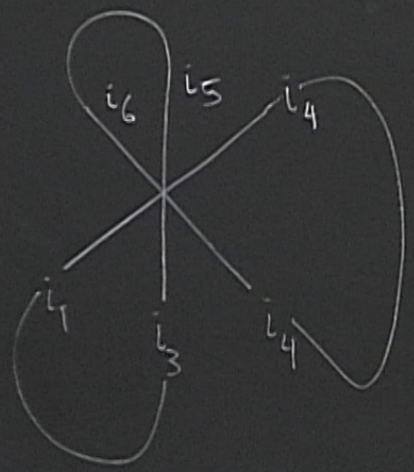
$$\langle X_{i_1} \cdots X_{i_6} \rangle = A^{-1}_{i_1 i_2} A^{-1}_{i_3 i_4} A^{-1}_{i_5 i_6} + \text{other pairs}$$



$\sum$ diag

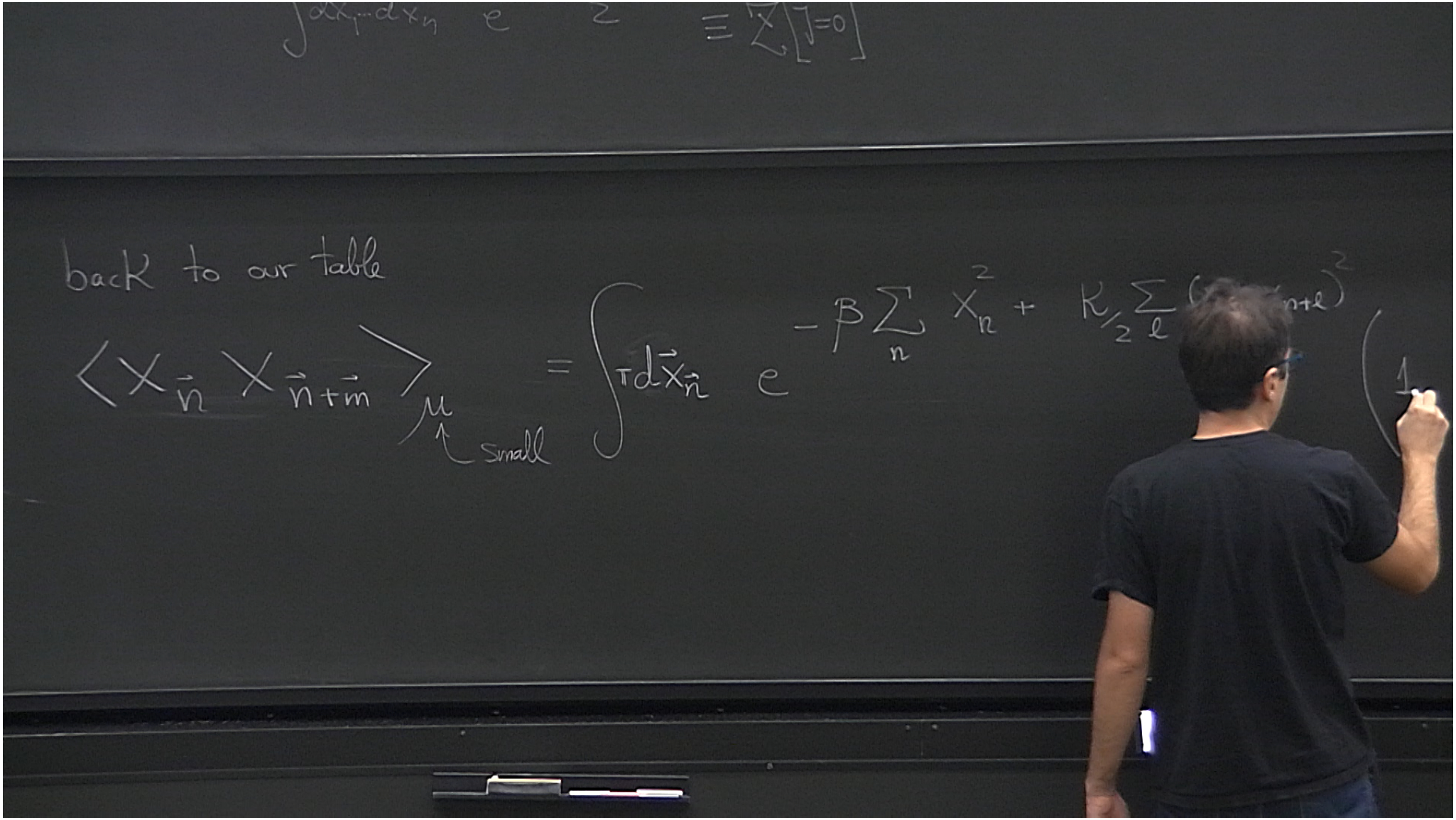
$$\langle X_{i_1} \cdots X_{i_6} \rangle = A^{-1}_{i_1 i_2} A^{-1}_{i_3 i_4} A^{-1}_{i_5 i_6} + \text{other pairs}$$

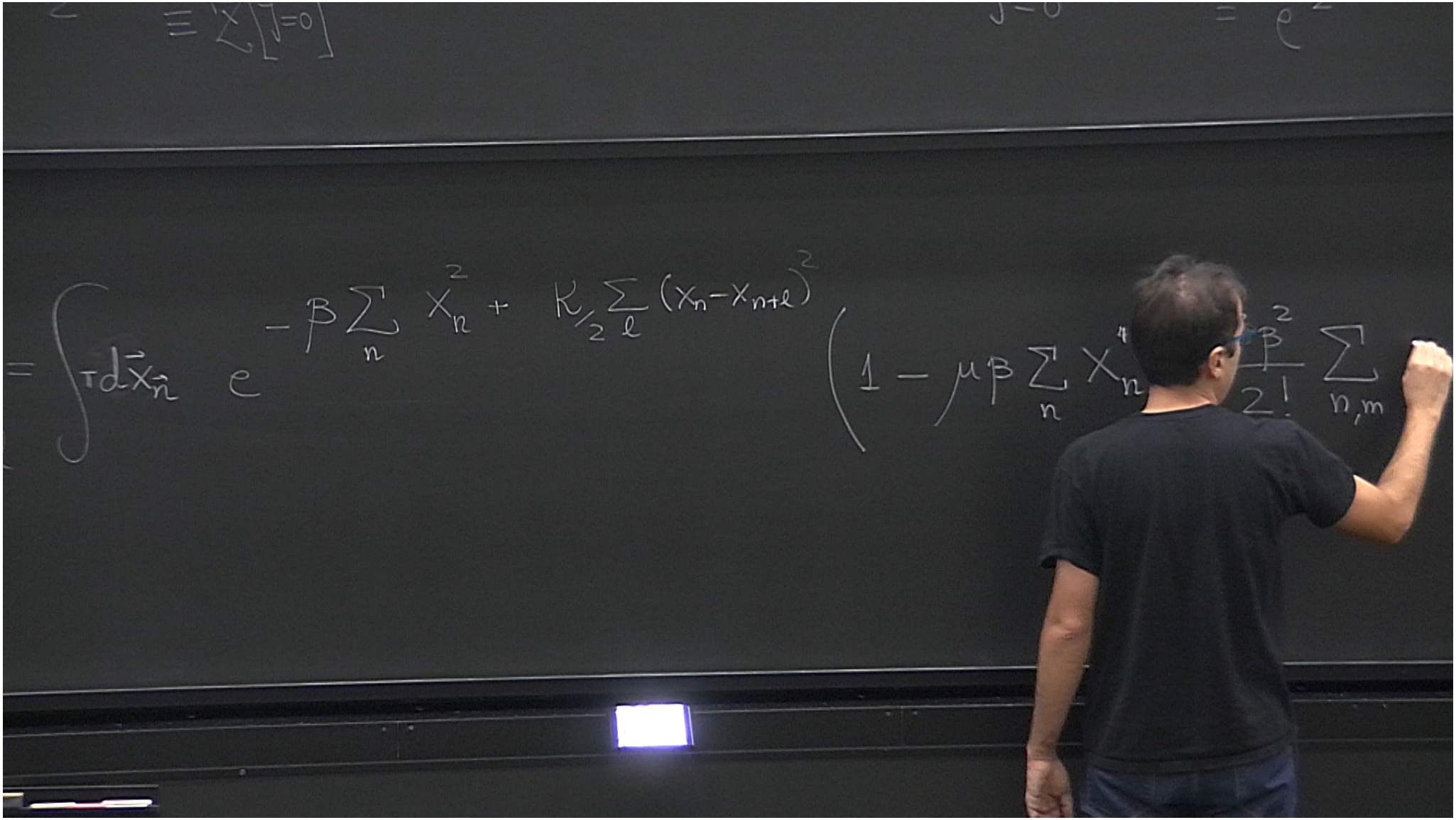
$\sum_{\text{diags}}$

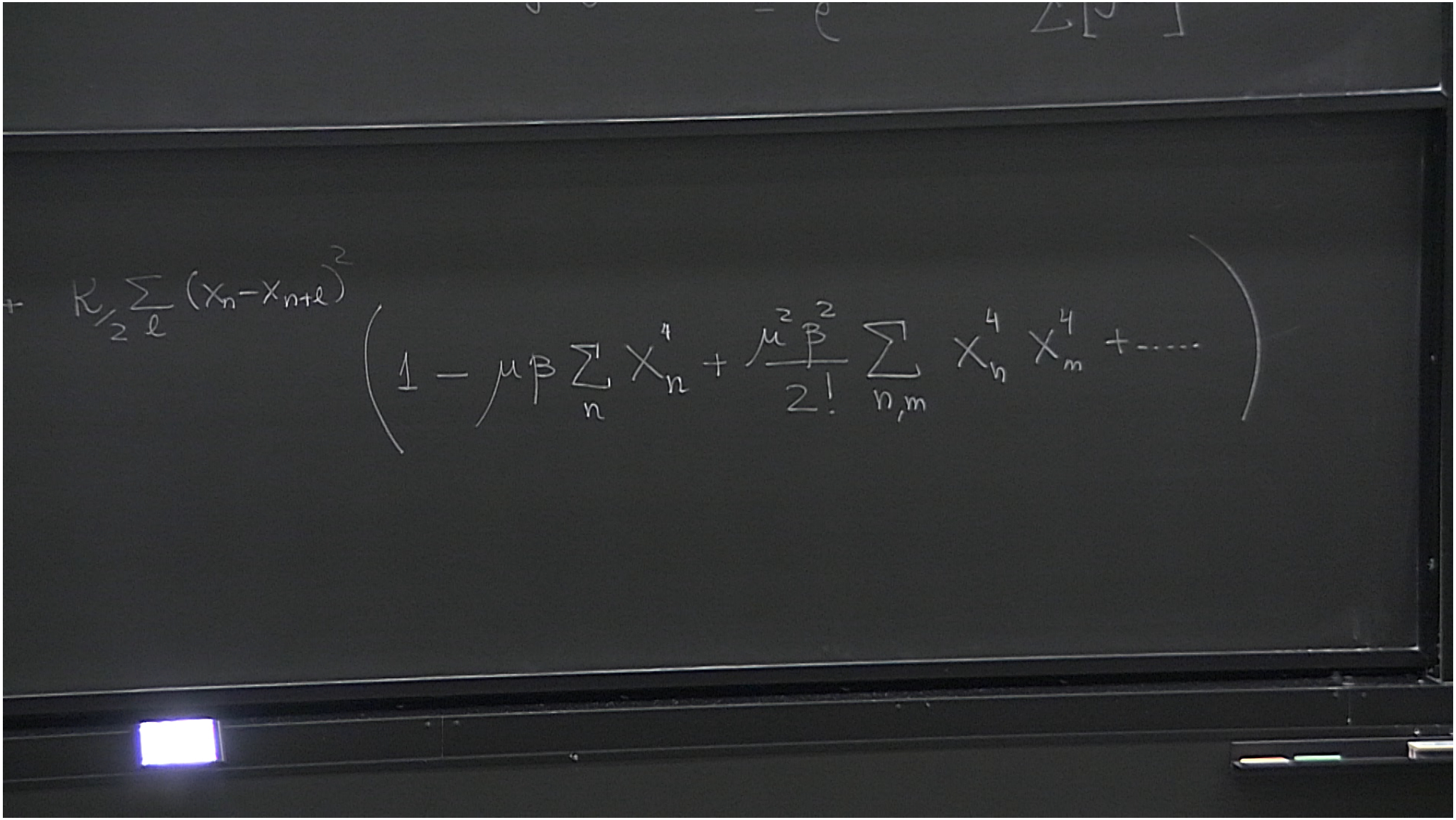


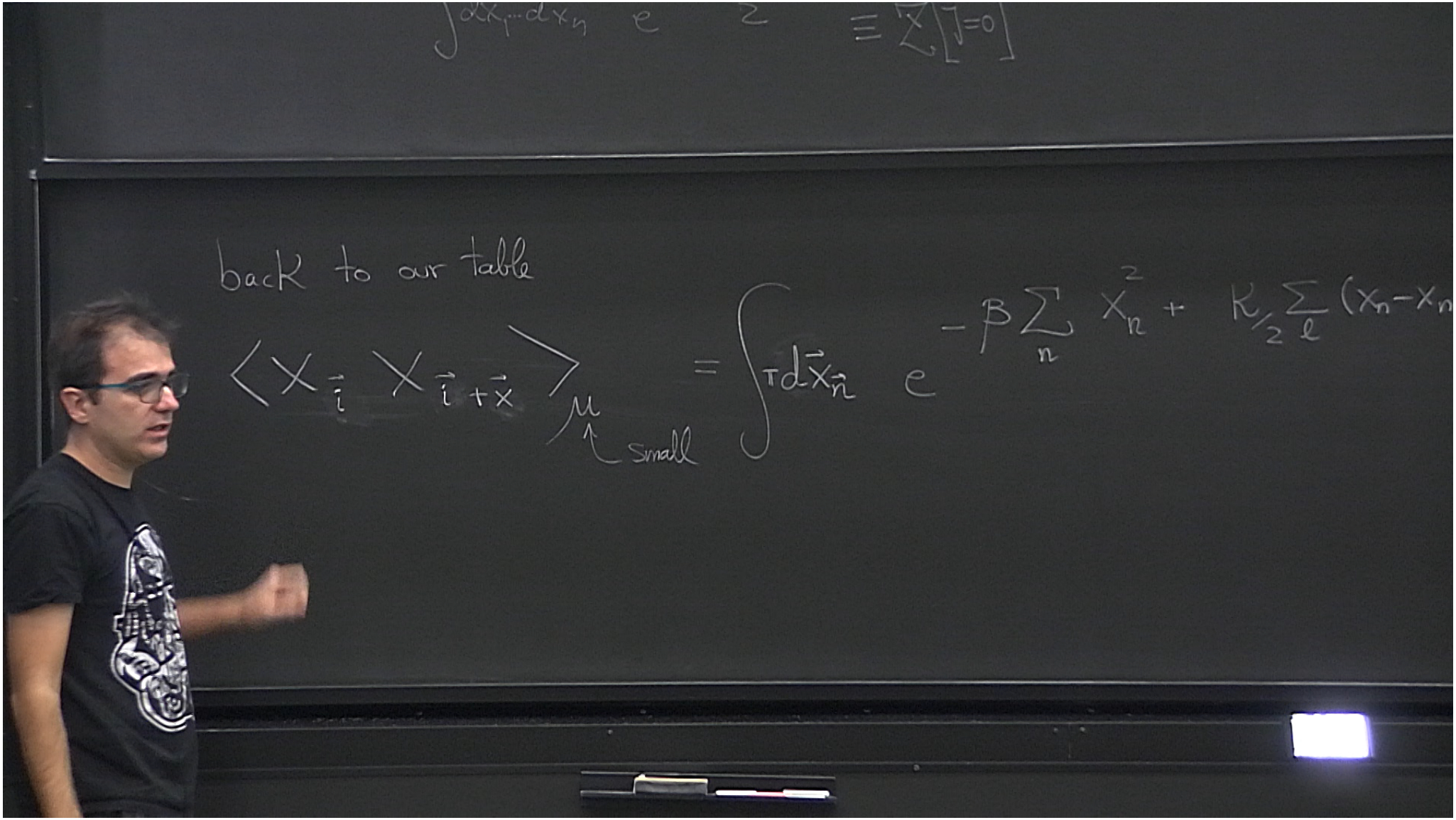
with i — j $= A^{-1}_{ij}$

$\sum_{\text{diag}}$


back to our table
small


$= \int \prod d\vec{x}_{\vec{n}} \; e^{-\beta \sum_n x_n^2 + \frac{K}{2} \sum_\ell (x_n - x_{n+\ell})^2}$
$\left(1 - \mu\beta \sum_n x_n^4 \ldots \frac{\beta^2}{2!} \sum_{n,m} \ldots\right.$


$+ \frac{K}{2} \sum_{\ell} (x_n - x_{n+\ell})^2$
$\left(1 - \mu\beta \sum_n x_n^4 + \frac{\mu^2\beta^2}{2!} \sum_{n,m} x_n^4 x_m^4 + \cdots \right)$


back to our table
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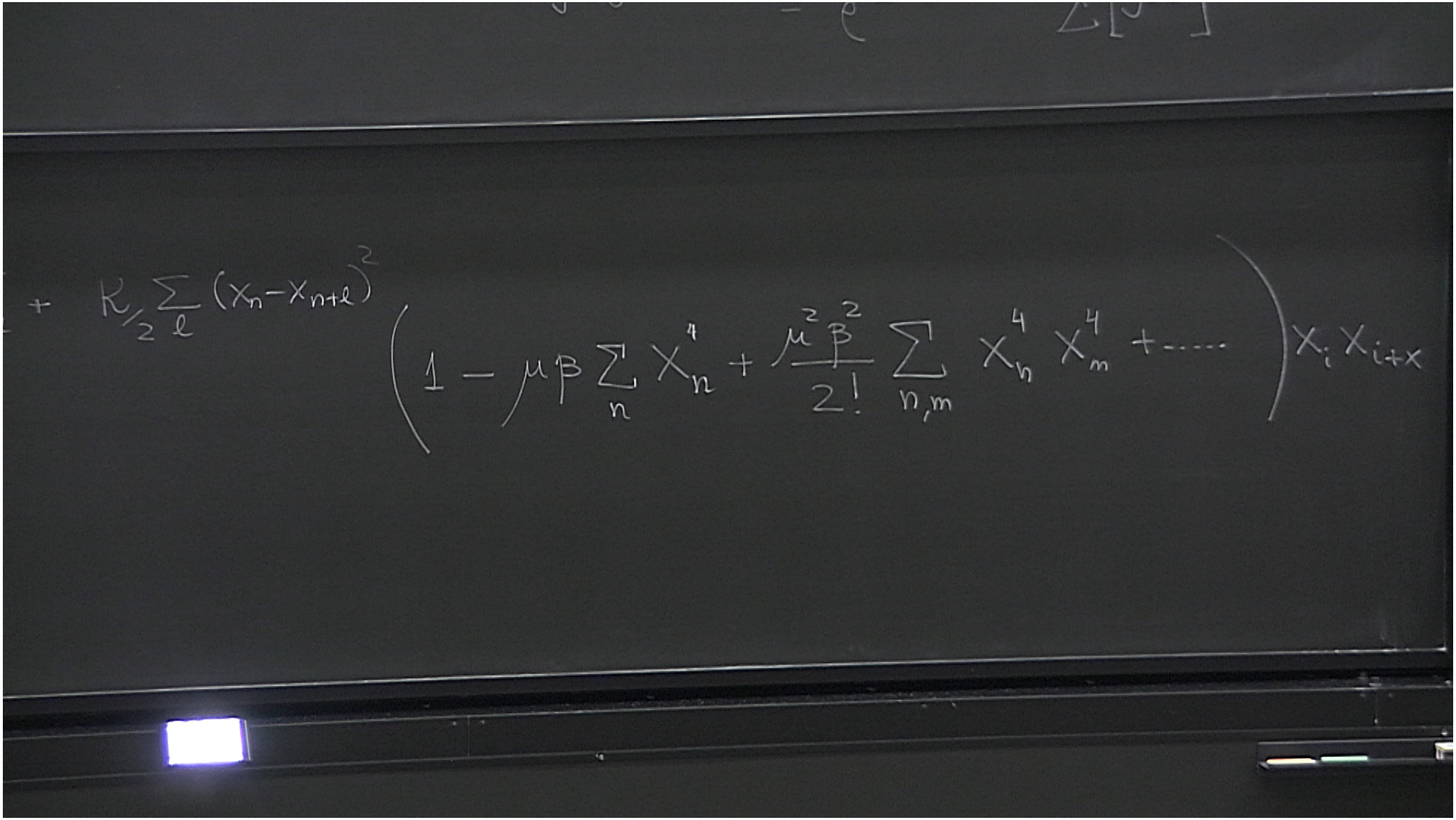
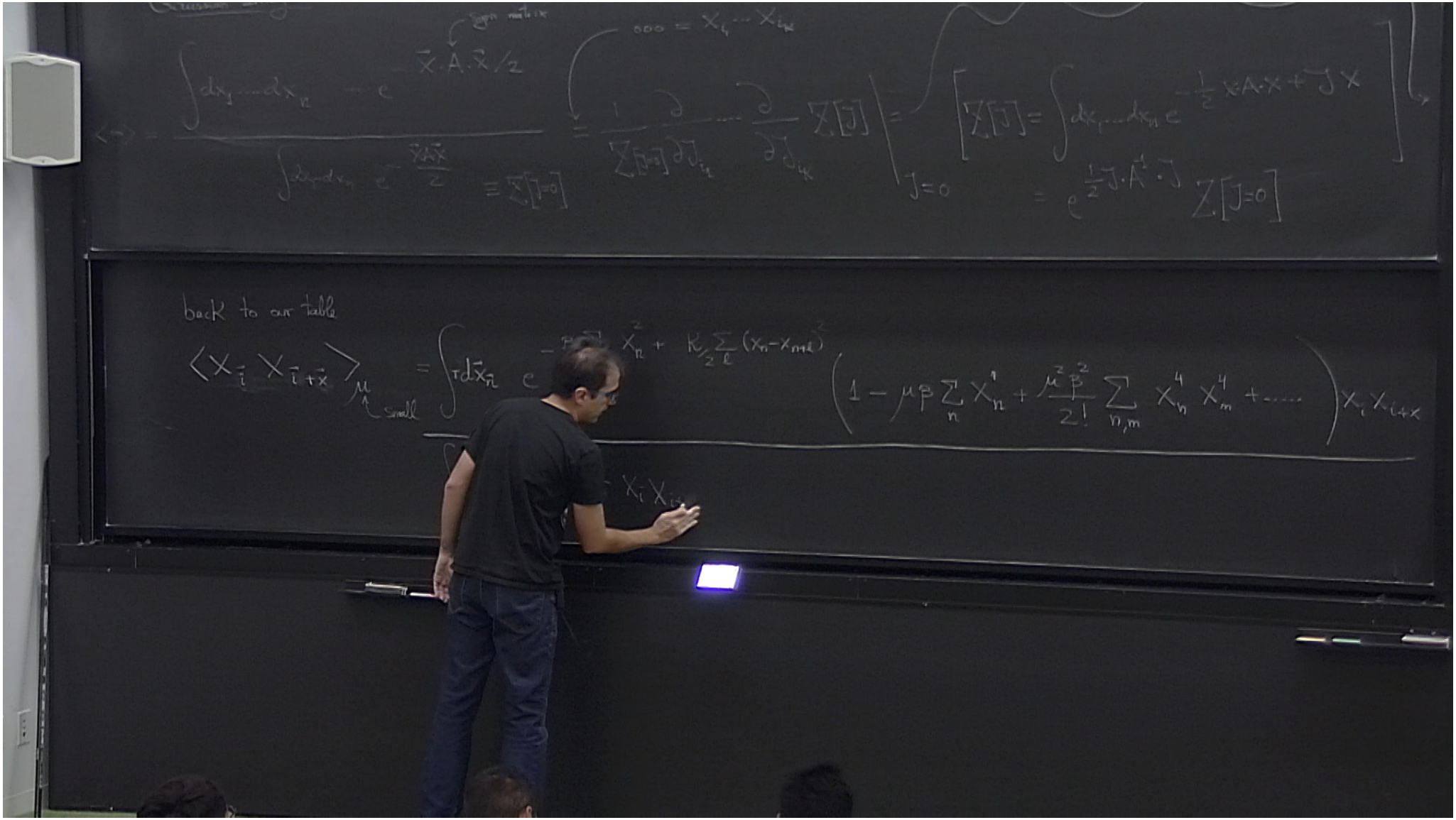
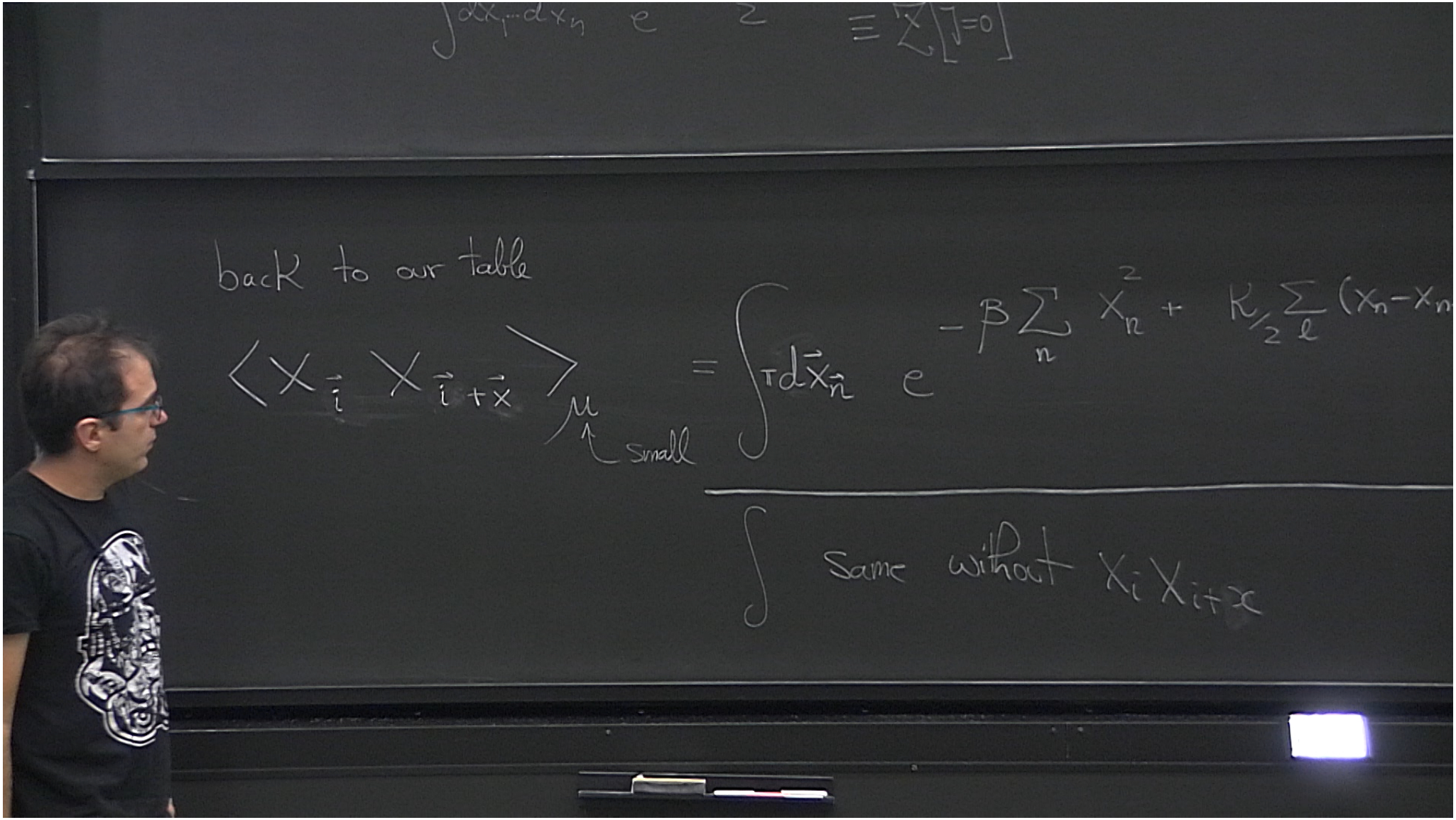
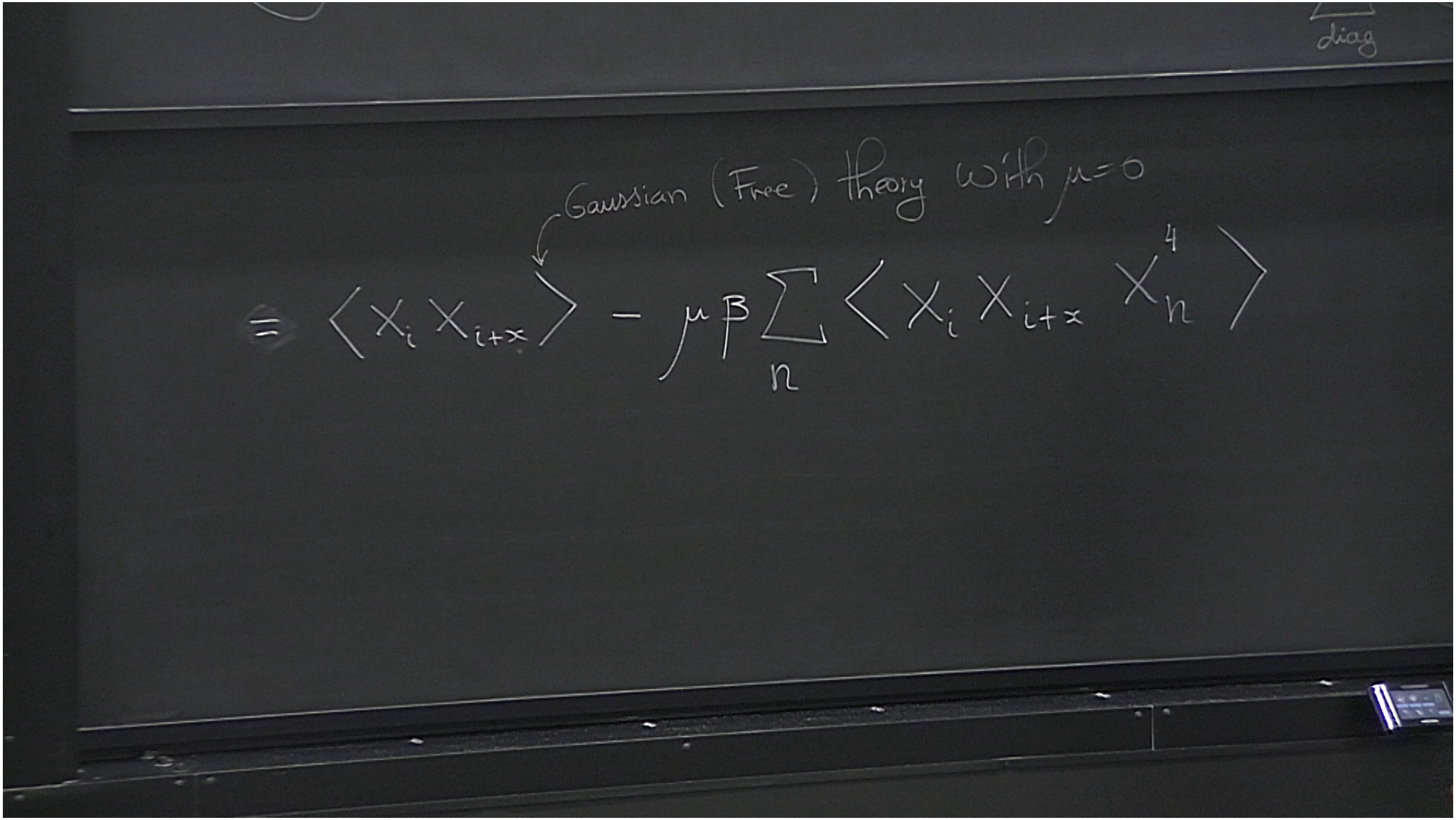
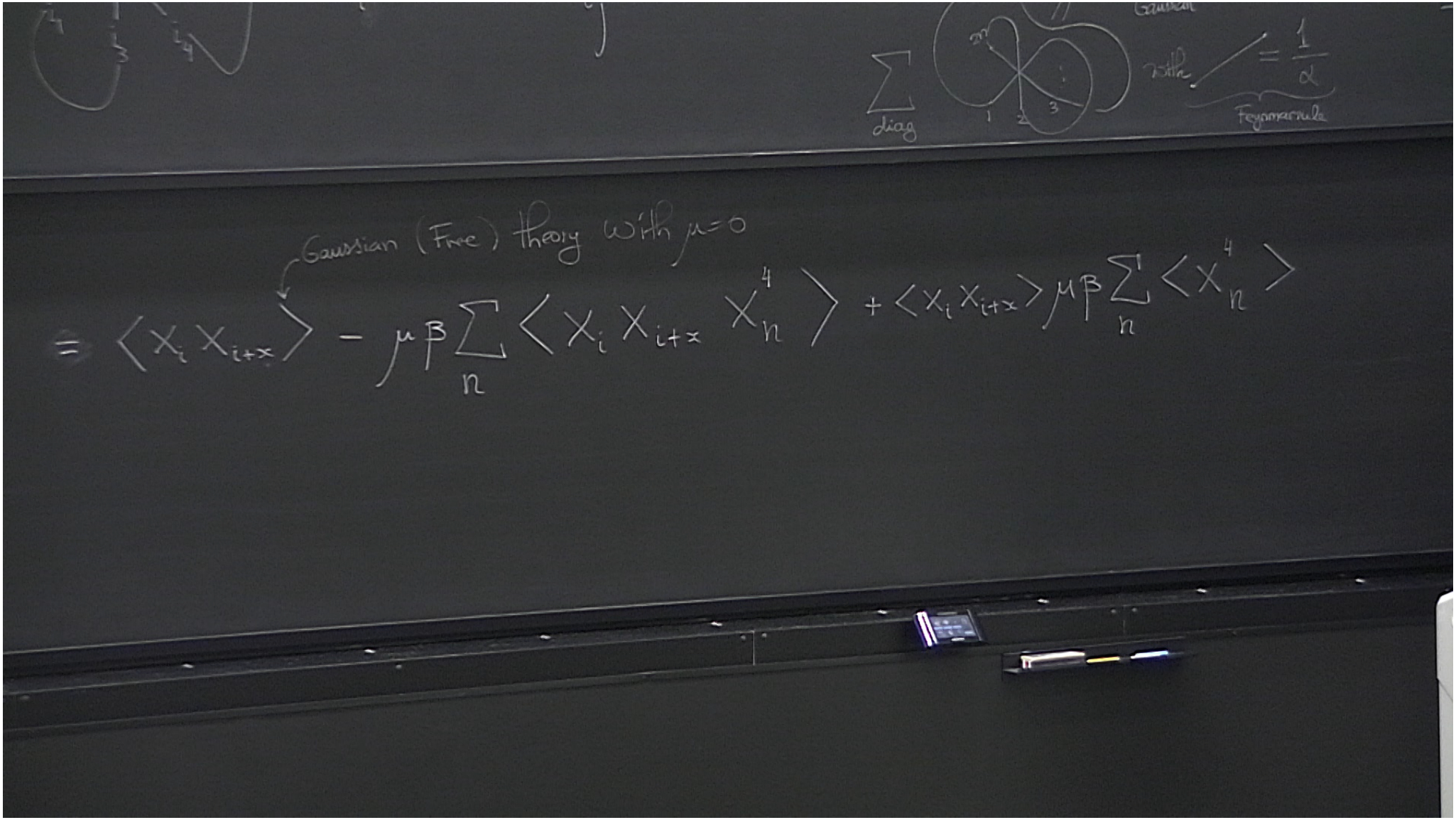
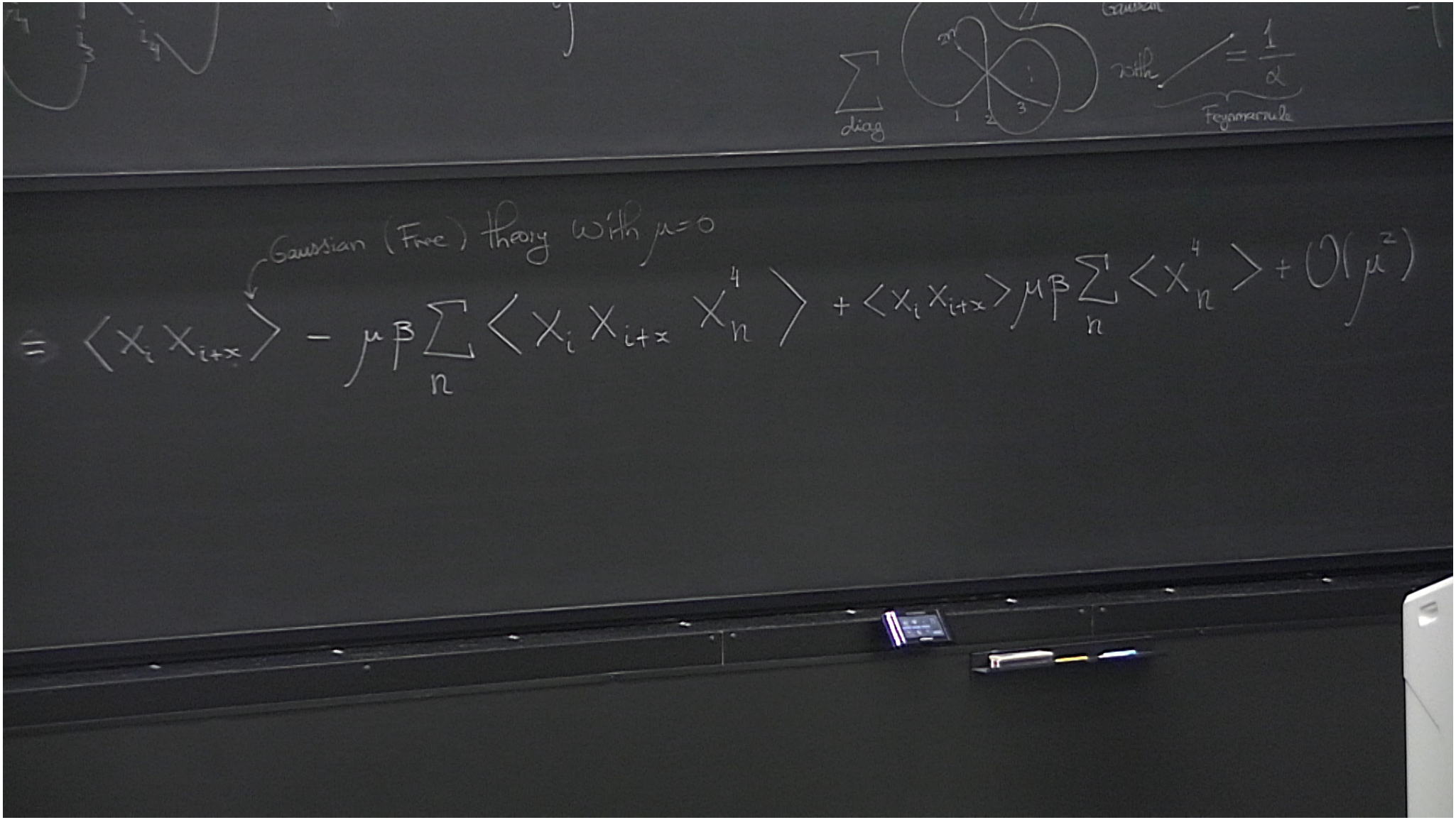

+ $K/2 \sum_l (x_n - x_{n+l})^2$
$\left(1 - \mu\beta \sum_n x_n^4 + \frac{\mu^2\beta^2}{2!} \sum_{n,m} x_n^4 x_m^4 + \cdots\right) x_i x_{i+x}$

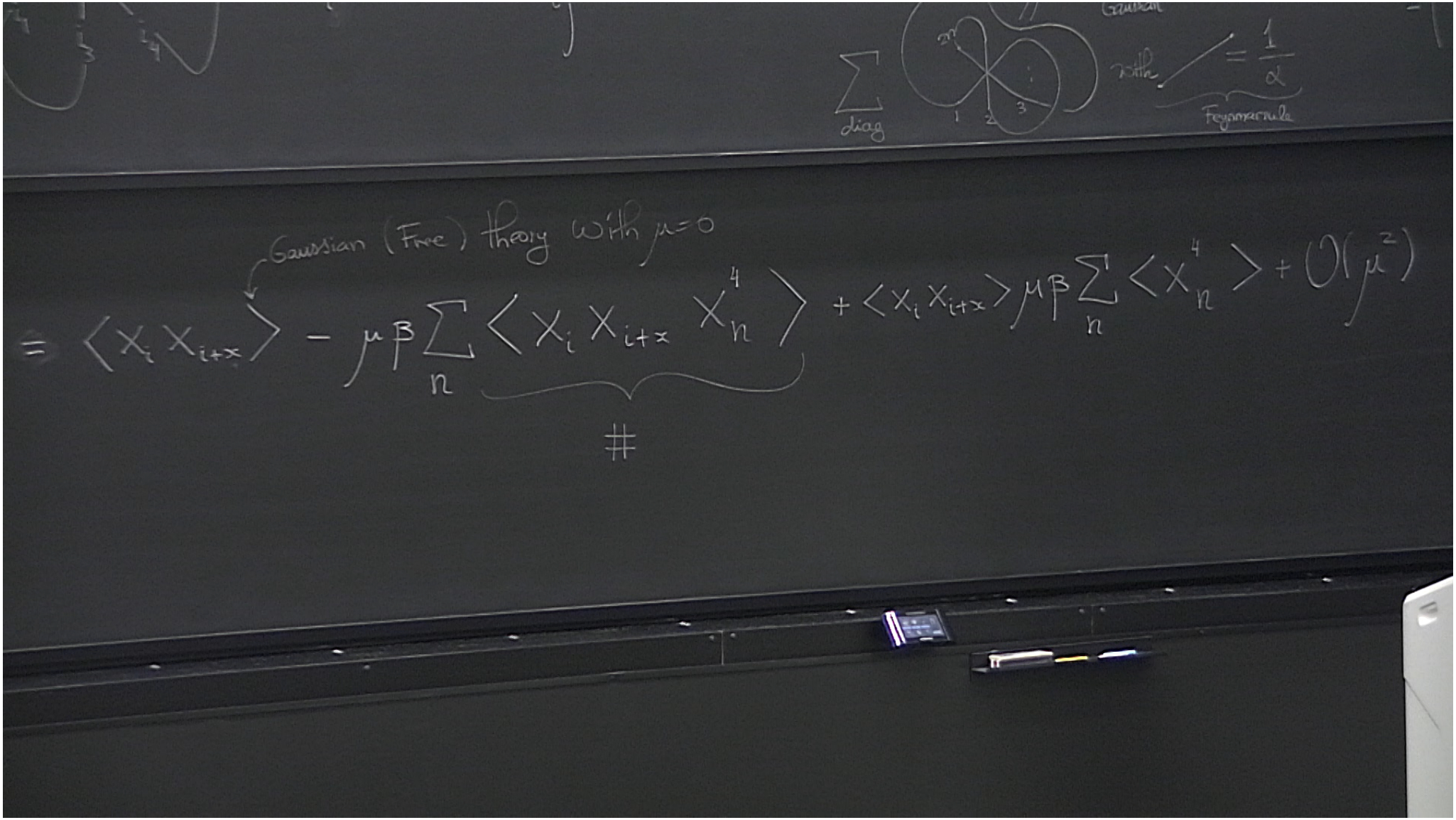

back to our table
small

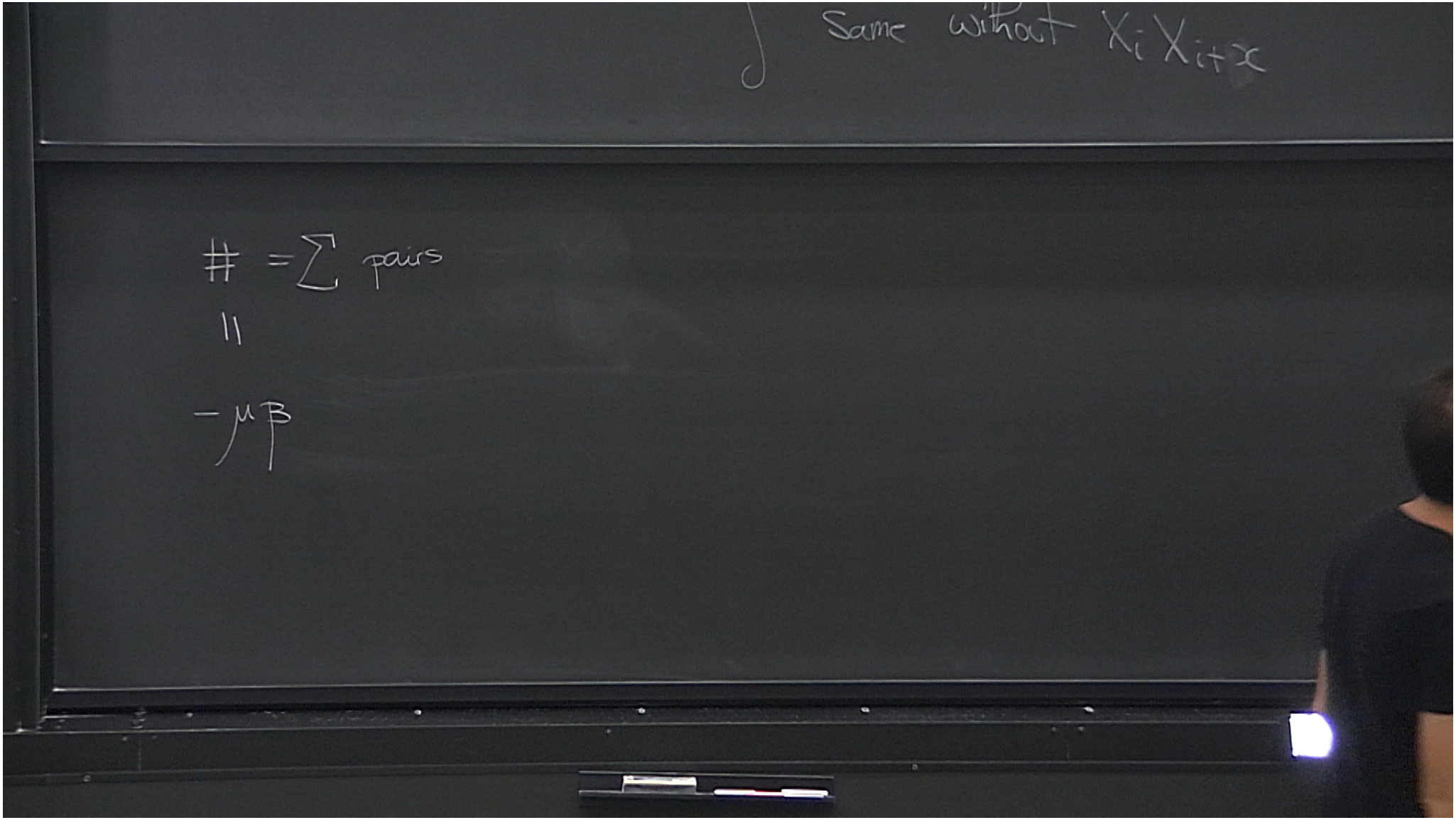

back to our table
small
Same without $X_i X_{i+x}$

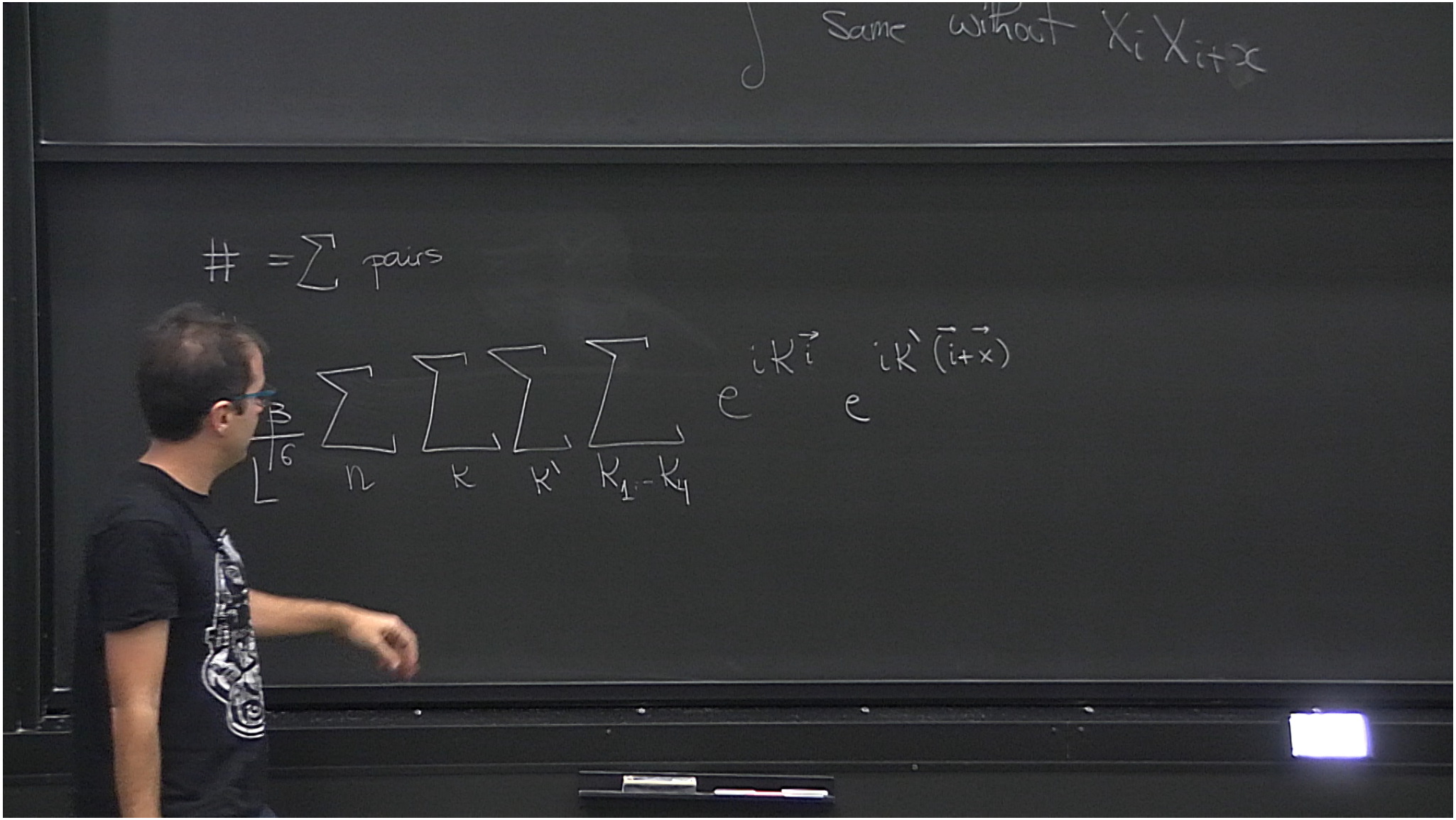

diag
Gaussian (Free) theory with $\mu=0$
$= \langle X_i X_{i+x} \rangle - \mu\beta \sum_n \langle X_i X_{i+x} X_n^4 \rangle$

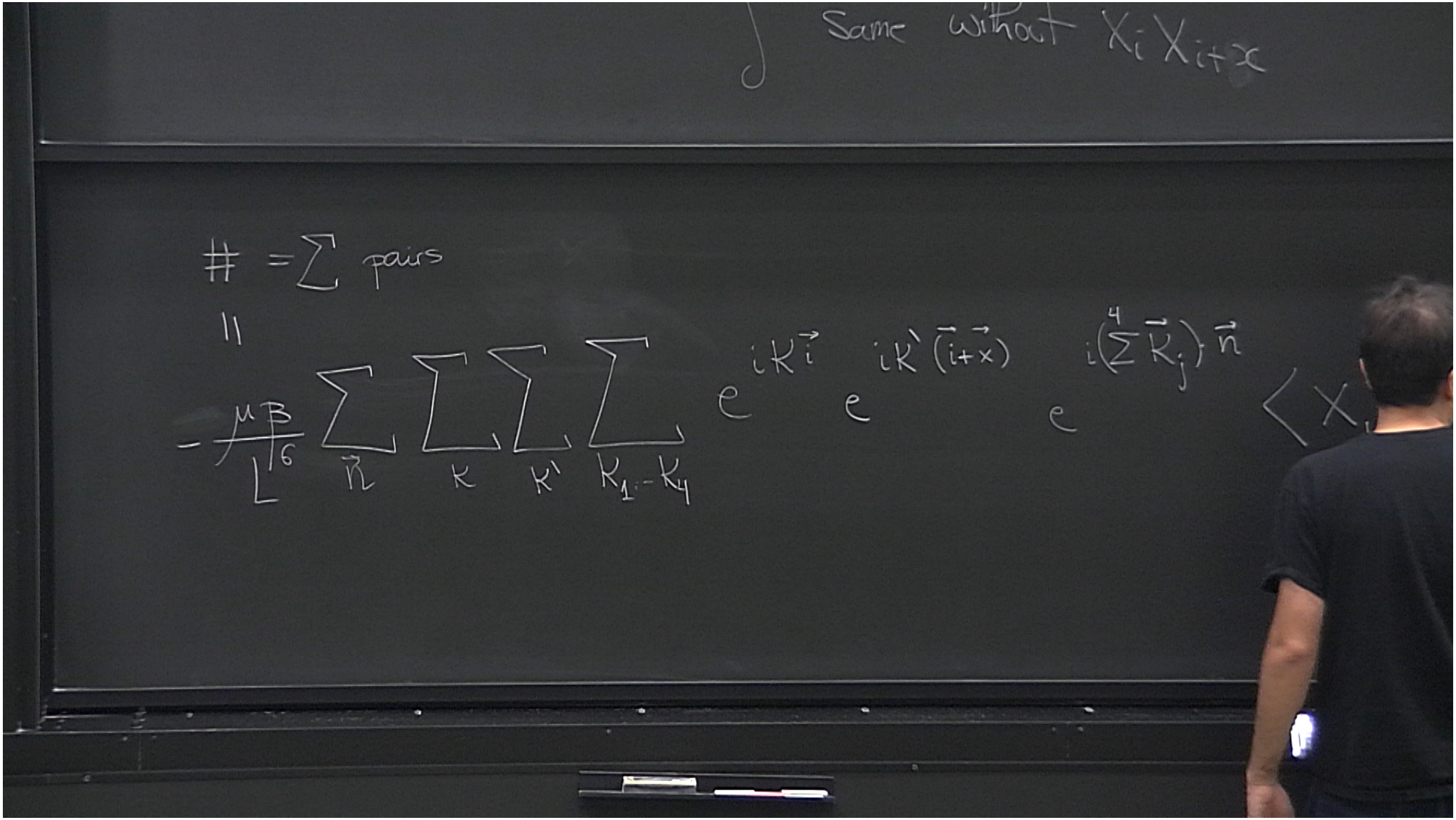

Gaussian (Free) theory with $\mu=0$
$= \langle X_i X_{i+x} \rangle - \mu\beta \sum_n \langle X_i X_{i+x} X_n^4 \rangle + \langle X_i X_{i+x} \rangle \mu\beta \sum_n \langle X_n^4 \rangle$
$\sum_{diag}$
1
2
3
with
$= \frac{1}{\alpha}$
Feynman rule

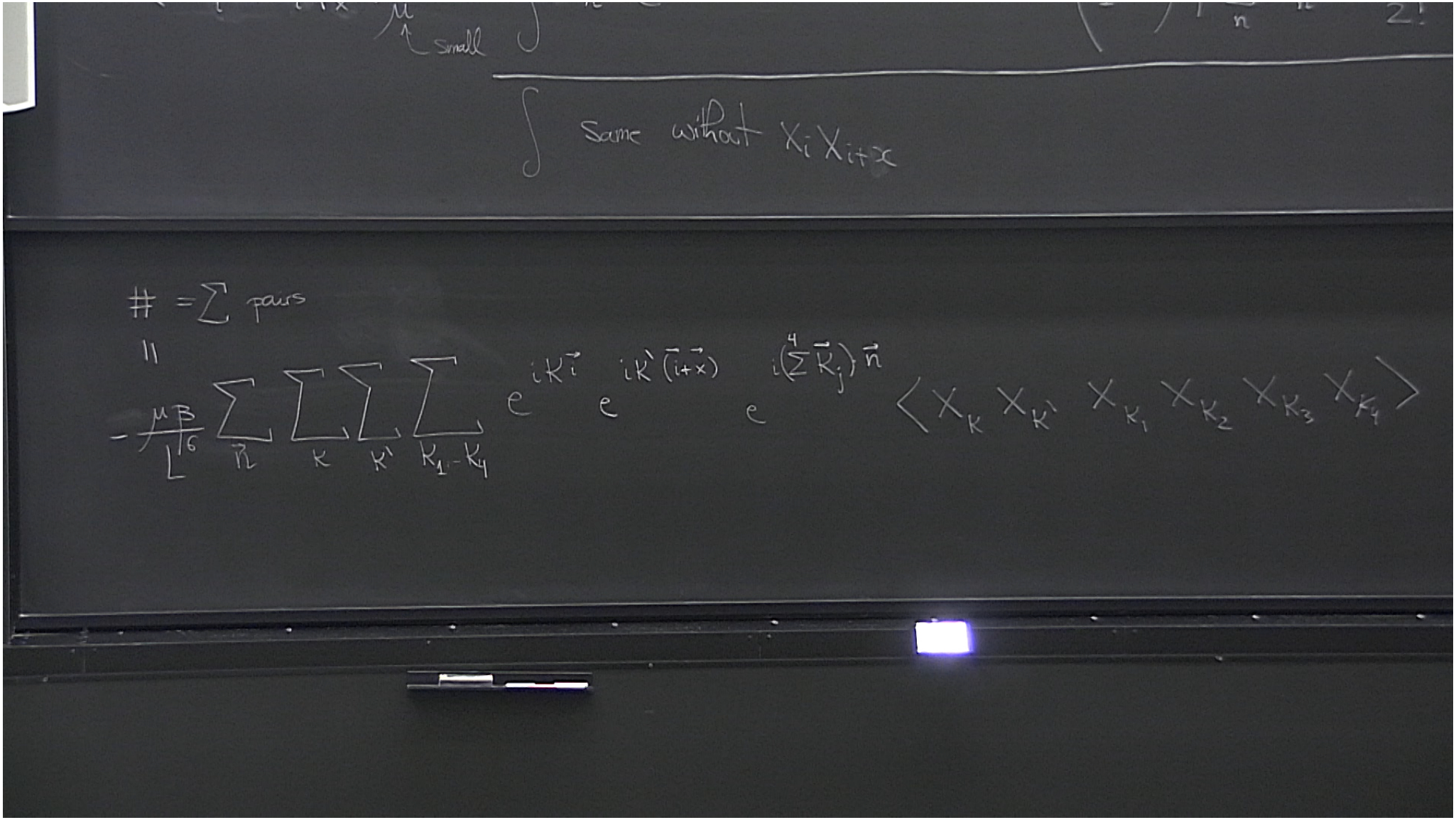

Gaussian (Free) theory with $\mu=0$
$= \langle X_i X_{i+x} \rangle - \mu\beta \sum_n \langle X_i X_{i+x} X_n^4 \rangle + \langle X_i X_{i+x} \rangle \mu\beta \sum_n \langle X_n^4 \rangle + O(\mu^2)$
$\sum_{\text{diag}}$
1
2
3
with
$= \frac{1}{\alpha}$
Feynman rule

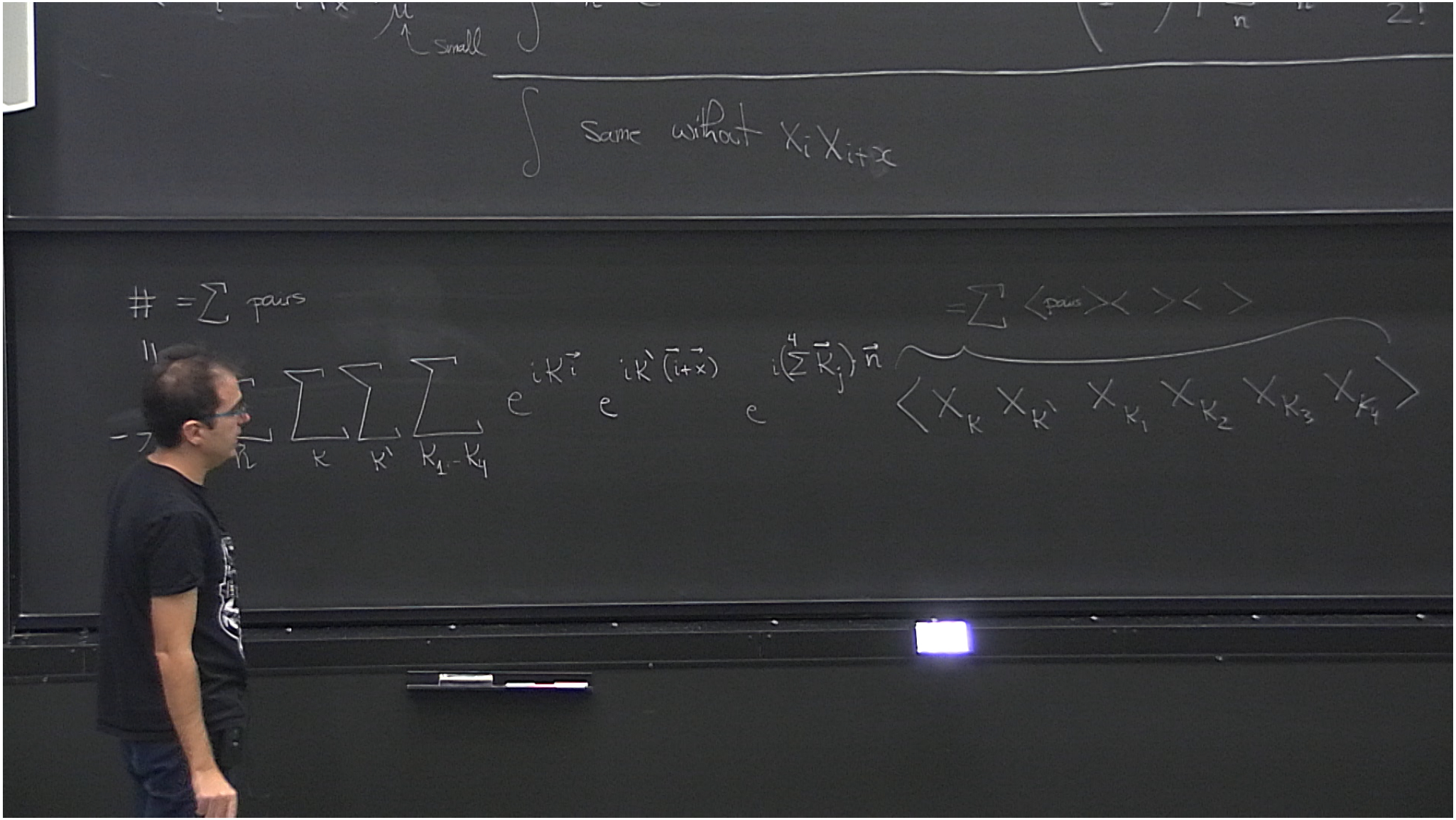

Gaussian (Free) theory with $\mu=0$
$= \langle X_i X_{i+x} \rangle - \mu\beta \sum_n \underbrace{\langle X_i X_{i+x} X_n^4 \rangle}_{\#} + \langle X_i X_{i+x} \rangle \mu\beta \sum_n \langle X_n^4 \rangle + O(\mu^2)$
$\sum_{\text{diag}}$
1
2
3
with
$= \frac{1}{\alpha}$
Feynman rule

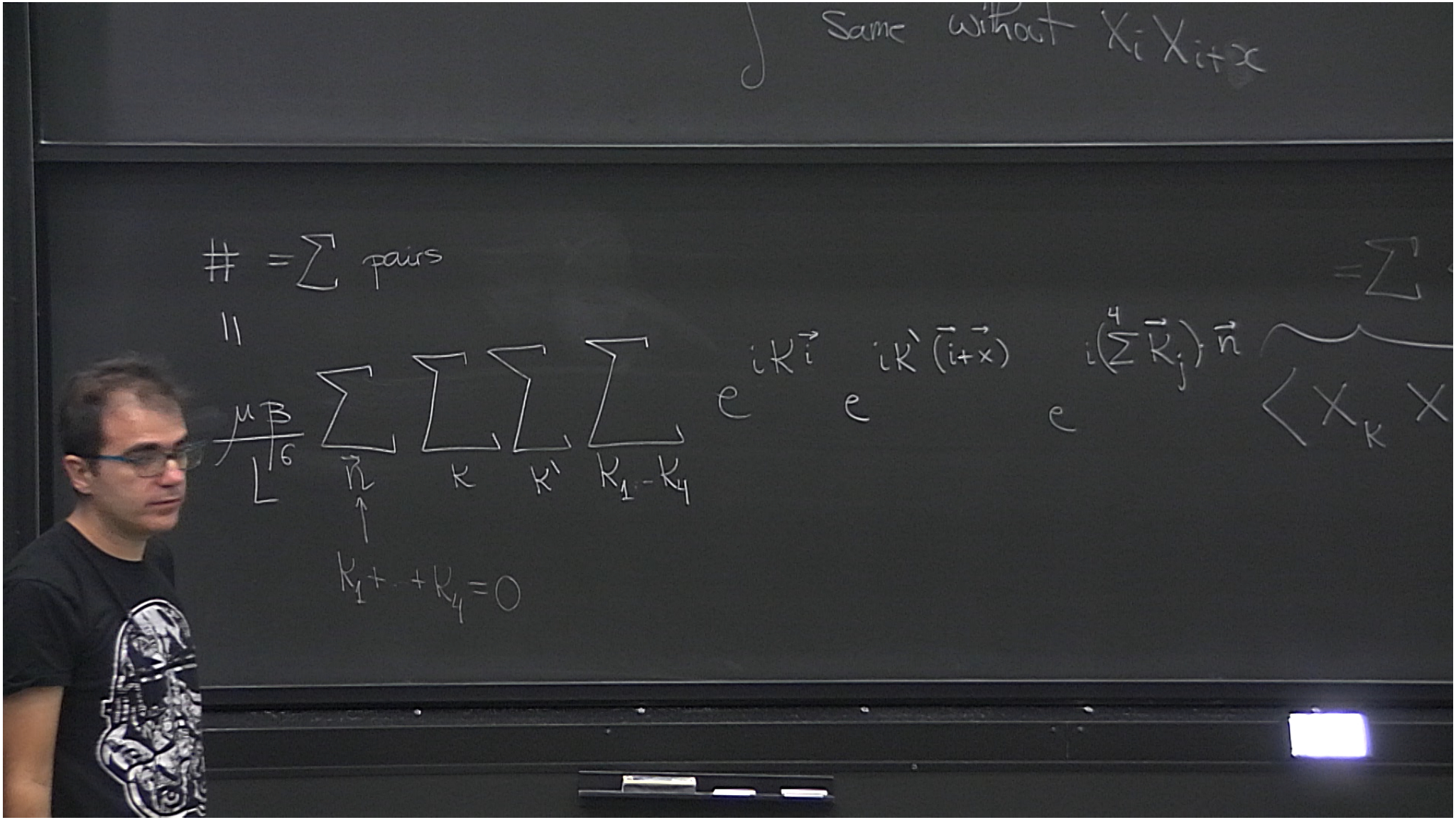

Same without $X_i X_{i+x}$
= ∑ pairs
=
$-\mu B$

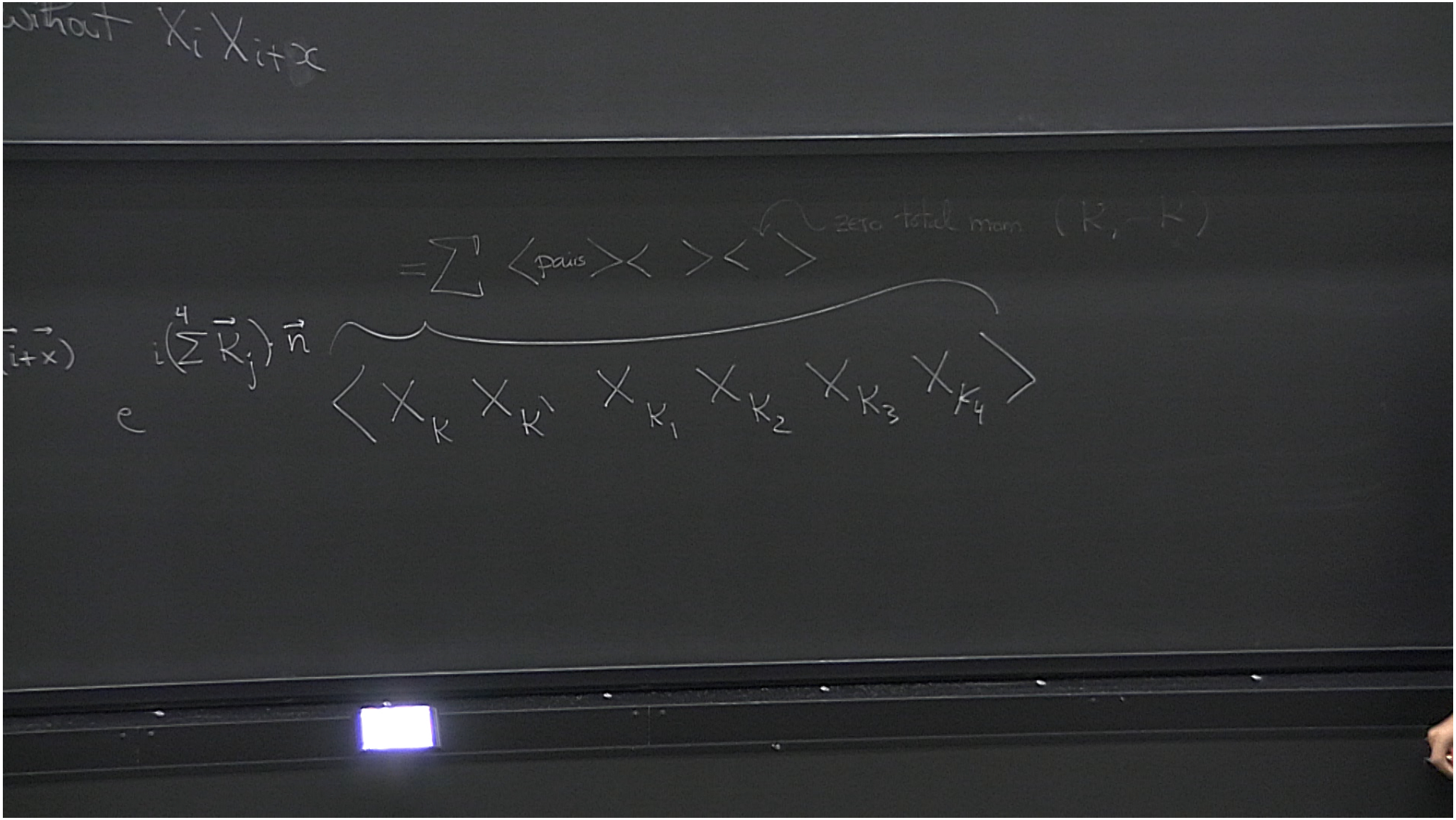

Same without $X_i X_{i+x}$
= ∑ pairs
$\frac{B}{L^6} \sum_{n} \sum_{k} \sum_{k'} \sum_{k_1 \ldots k_4} e^{iK\vec{i}} \; e^{iK'(\vec{i}+\vec{x})}$

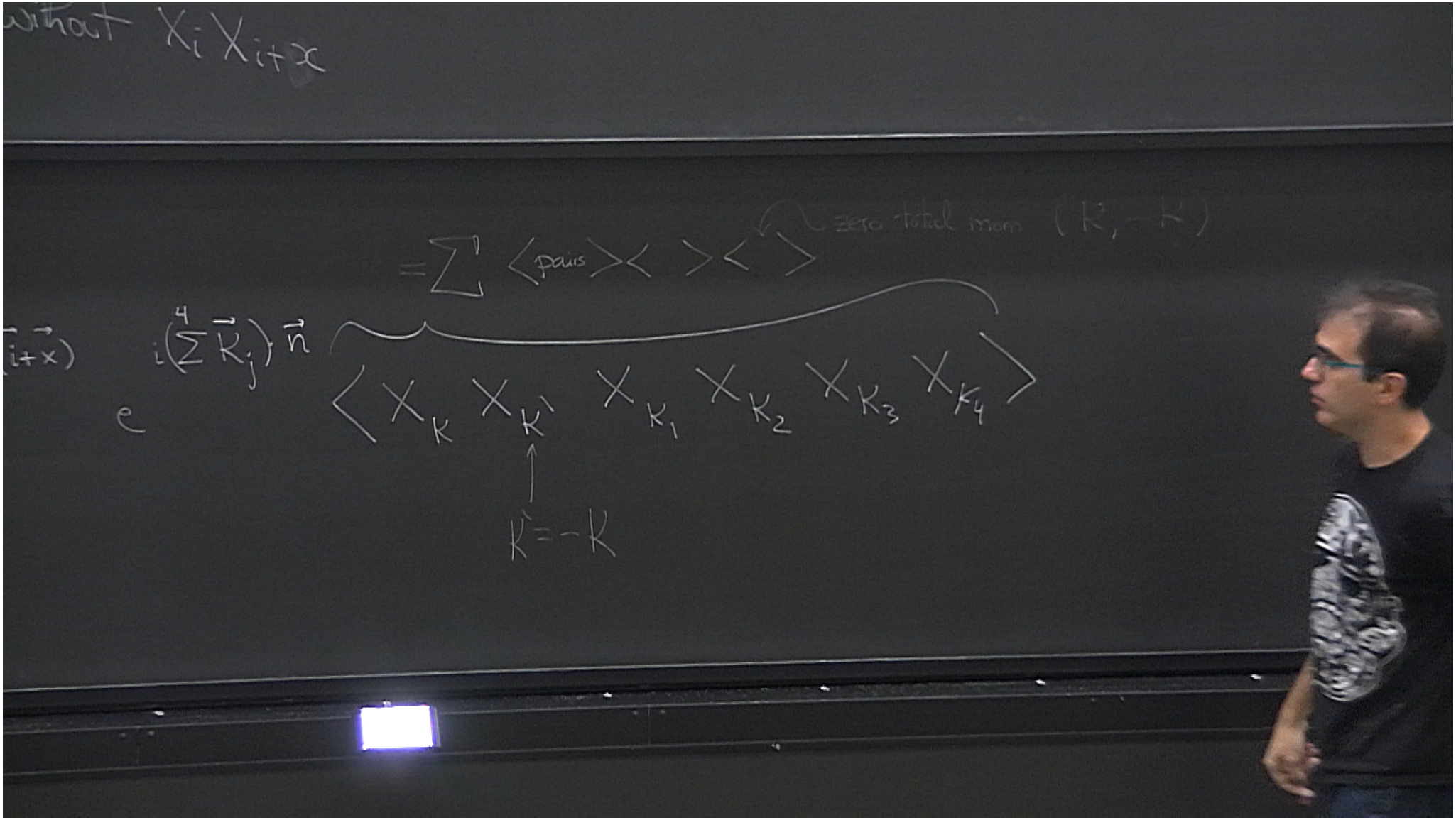

Same without $X_i X_{i+x}$
$\# = \sum$ pairs
$=$
$-\frac{\mu_B}{L^6} \sum_{\vec{n}} \sum_{K} \sum_{K'} \sum_{K_1 \ldots K_4} e^{iK\vec{i}} e^{iK'(\vec{i}+\vec{x})} e^{i(\sum^4 \vec{K}_j)\cdot\vec{n}} \langle X$

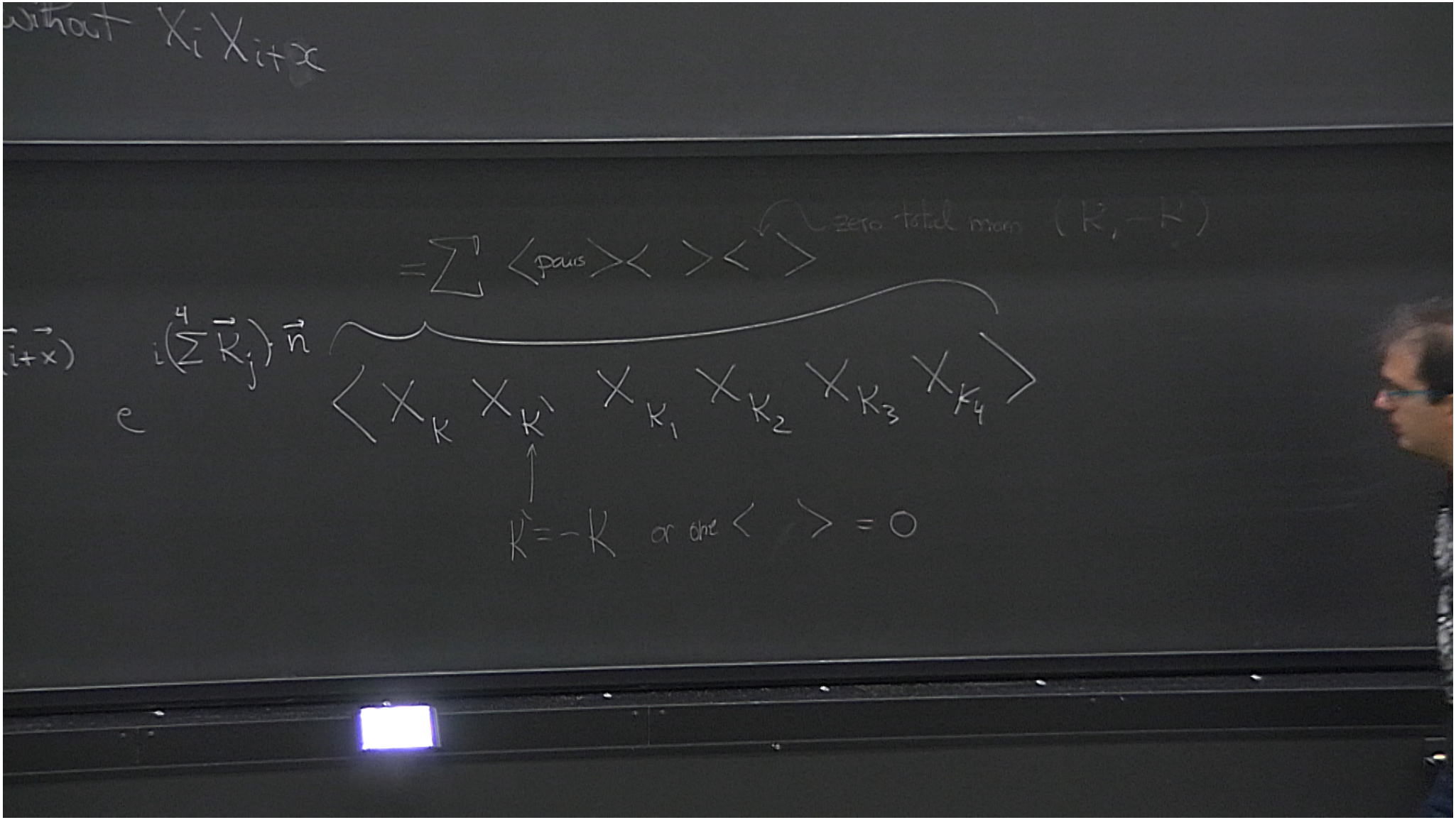

small
Same without $X_i X_{i+x}$
= ∑ pairs
$-\frac{\mu B}{L^6} \sum_{\vec{n}} \sum_{K} \sum_{K'} \sum_{K_1 \cdots K_4} e^{iK\vec{i}}\, e^{iK'(\vec{i}+\vec{x})}\, e^{i(\sum^4 \vec{K}_j)\cdot\vec{n}} \langle X_K X_{K'} X_{K_1} X_{K_2} X_{K_3} X_{K_4} \rangle$

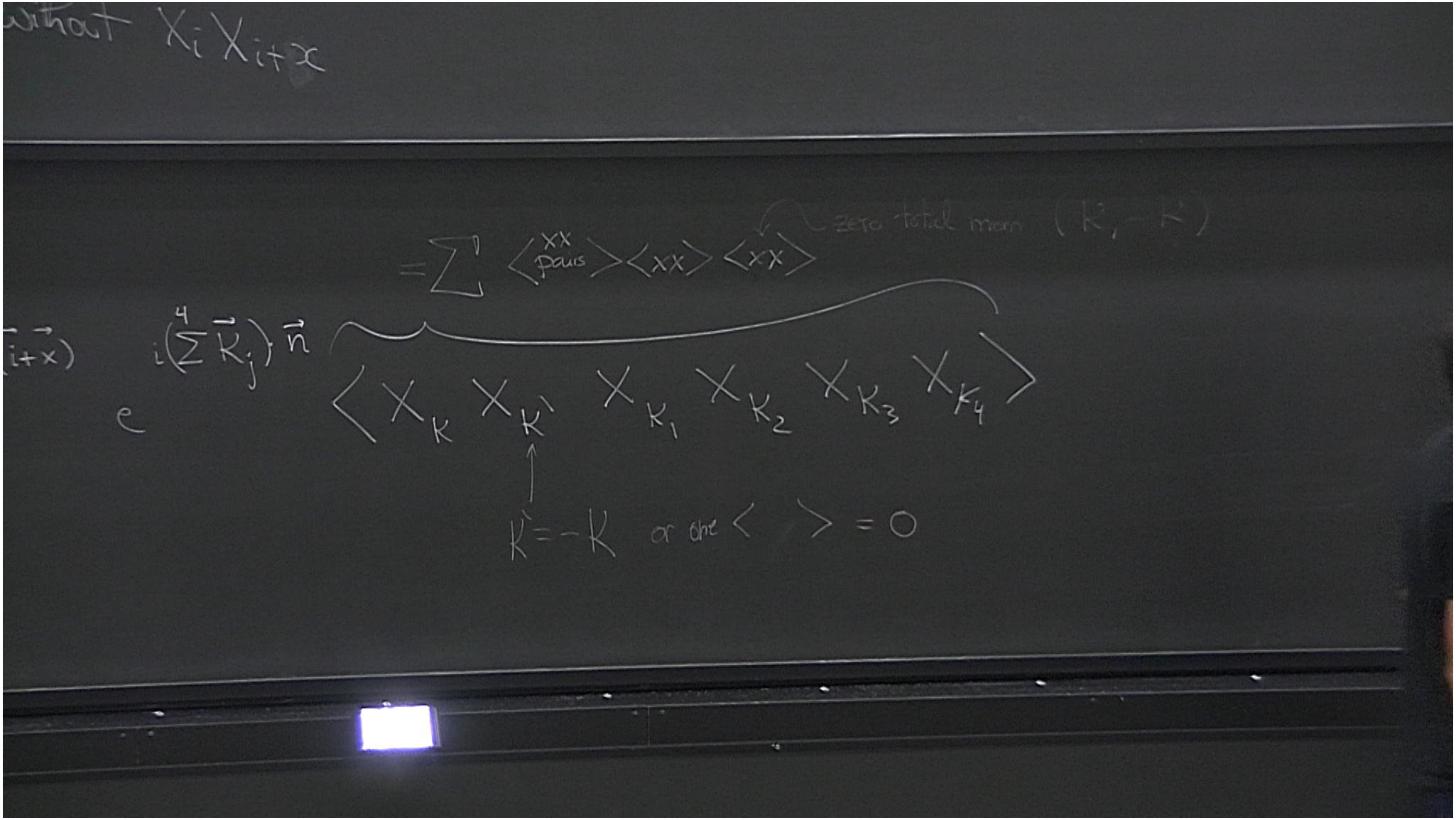

small
Same without $X_i X_{i+x}$
$\# = \sum$ pairs
$e^{iK\vec{i}}$
$e^{iK'(\vec{i}+\vec{x})}$
$e^{i(\sum_{j=1}^{4} \vec{K}_j)\vec{n}}$
$K_1 \cdots K_4$
$= \sum$ pairs $\langle X_K X_{K'} X_{K_1} X_{K_2} X_{K_3} X_{K_4} \rangle$

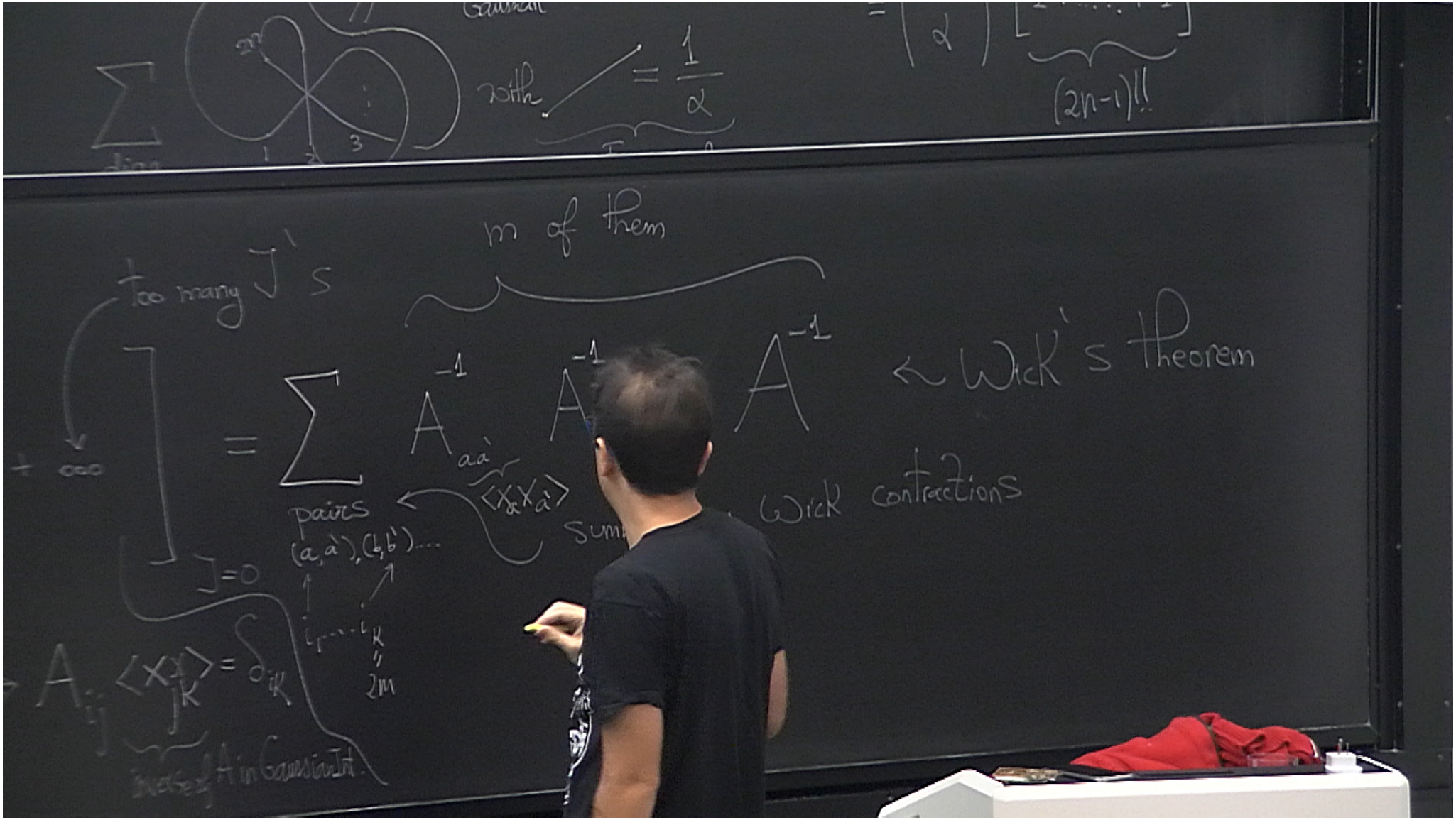

Same without $X_i X_{i+x}$
$\# = \sum$ pairs
$=$
$\frac{\mu_B}{L^6} \sum_{\vec{n}} \sum_{K} \sum_{K'} \sum_{K_1 \ldots K_4} e^{iK\vec{i}} e^{iK'(\vec{i}+\vec{x})} e^{i(\sum_j^4 \vec{K}_j)\cdot\vec{n}}$
$K_1 + \ldots + K_4 = 0$
$= \sum$
$\langle X_K X$

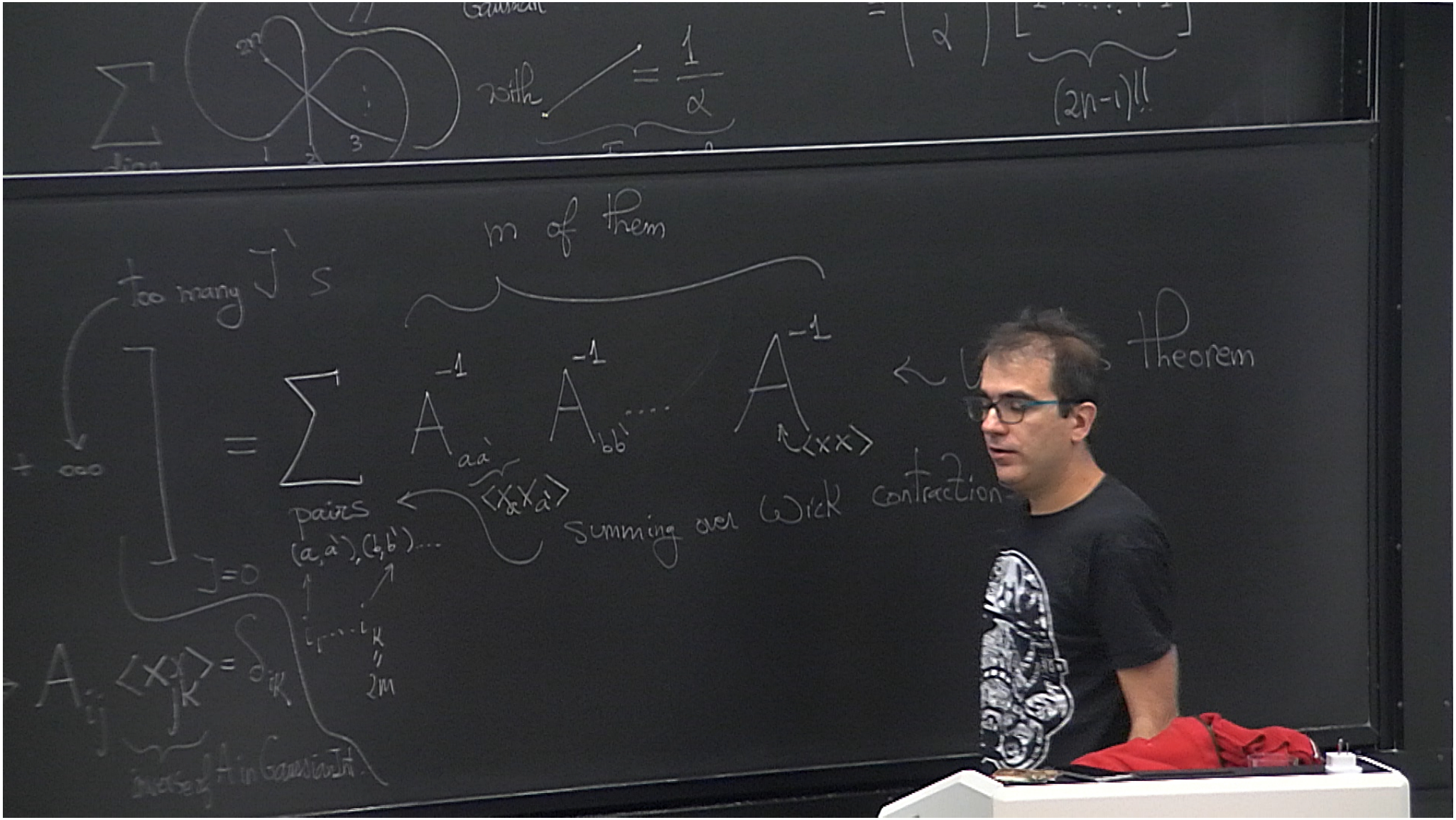

without $X_i X_{i+x}$
$= \sum'$ ⟨pairs⟩⟨ ⟩⟨ ⟩
zero total mom $(K_1 - K)$
$e^{i(\sum_j^4 \vec{K}_j)\cdot\vec{n}}$
$\langle X_K X_{K'} X_{K_1} X_{K_2} X_{K_3} X_{K_4} \rangle$

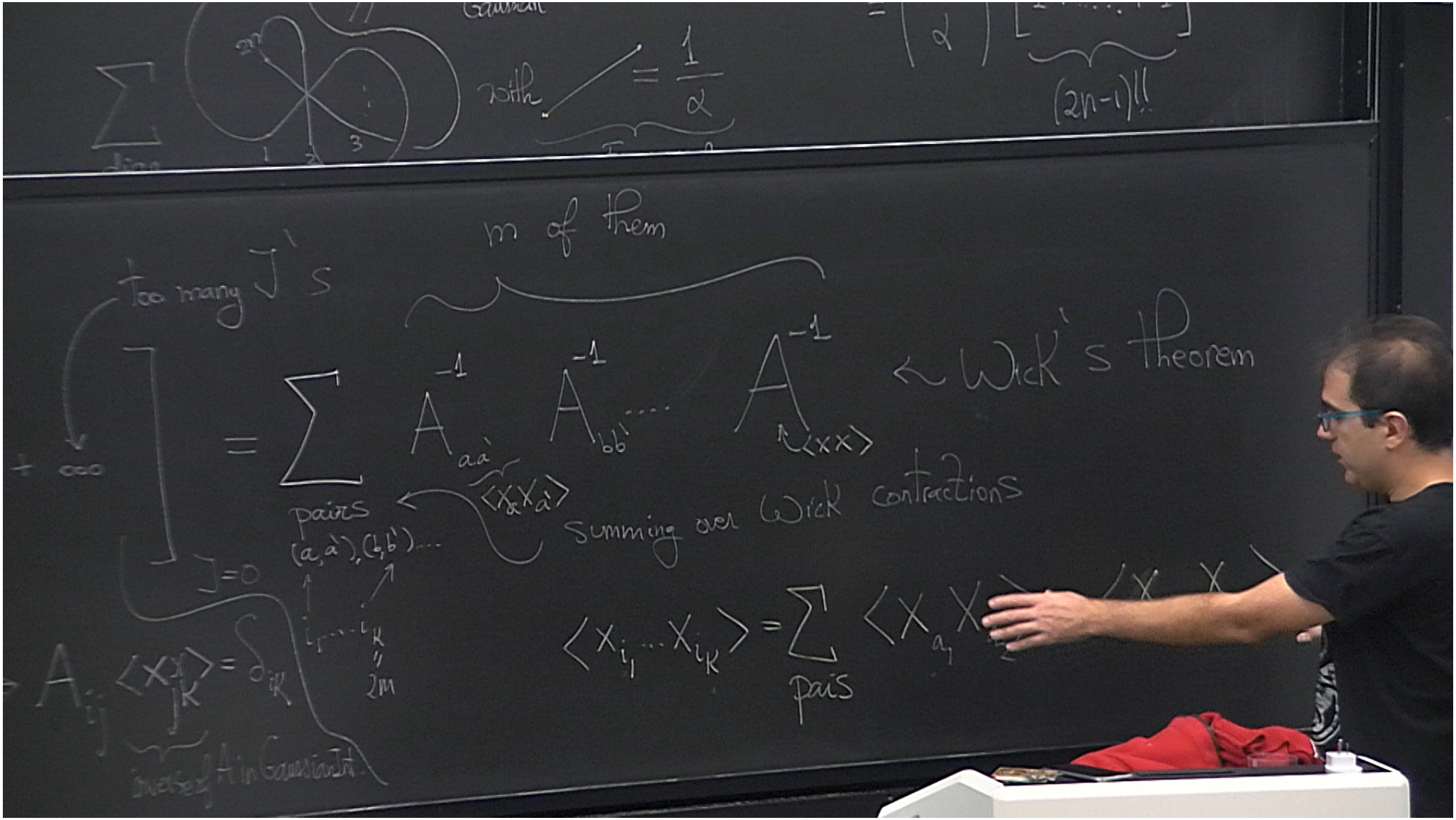

without $X_i X_{i+x}$
$= \sum'$ ⟨pairs⟩⟨ ⟩⟨ ⟩
zero total mom $(K, -K)$
$e^{i(\sum_j^4 \vec{K}_j)\cdot\vec{n}}$
$\langle X_K X_{K'} X_{K_1} X_{K_2} X_{K_3} X_{K_4} \rangle$
$K' = -K$

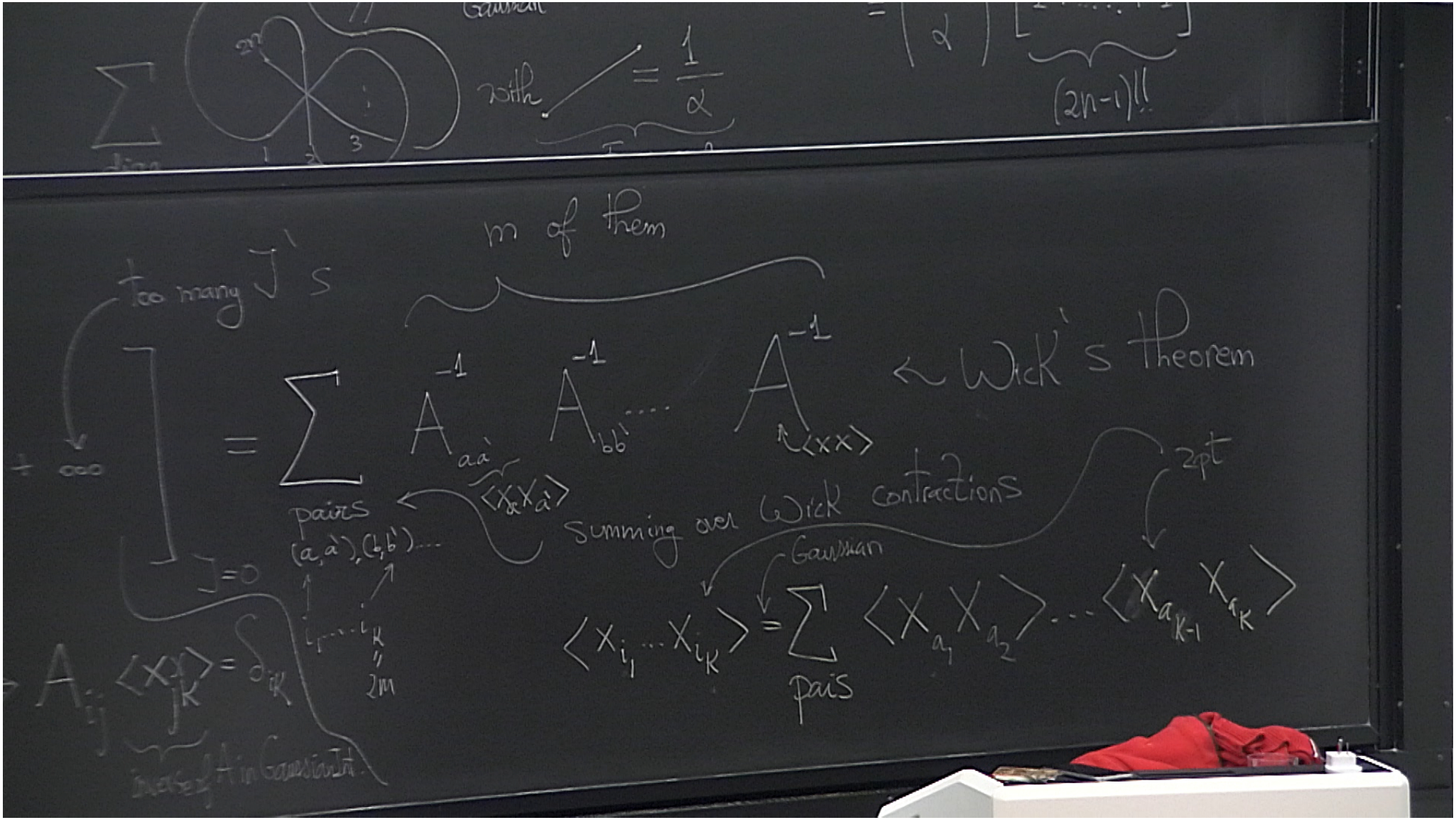

without $X_i X_{i+x}$
$= \sum$ ⟨pairs⟩⟨ ⟩⟨ ⟩
zero total mom $(K, -K)$
$e^{i(\sum_j^4 \vec{K}_j)\cdot\vec{n}}$
$\langle X_K X_{K'} X_{K_1} X_{K_2} X_{K_3} X_{K_4} \rangle$
$K' = -K$ or else $\langle\ \rangle = 0$

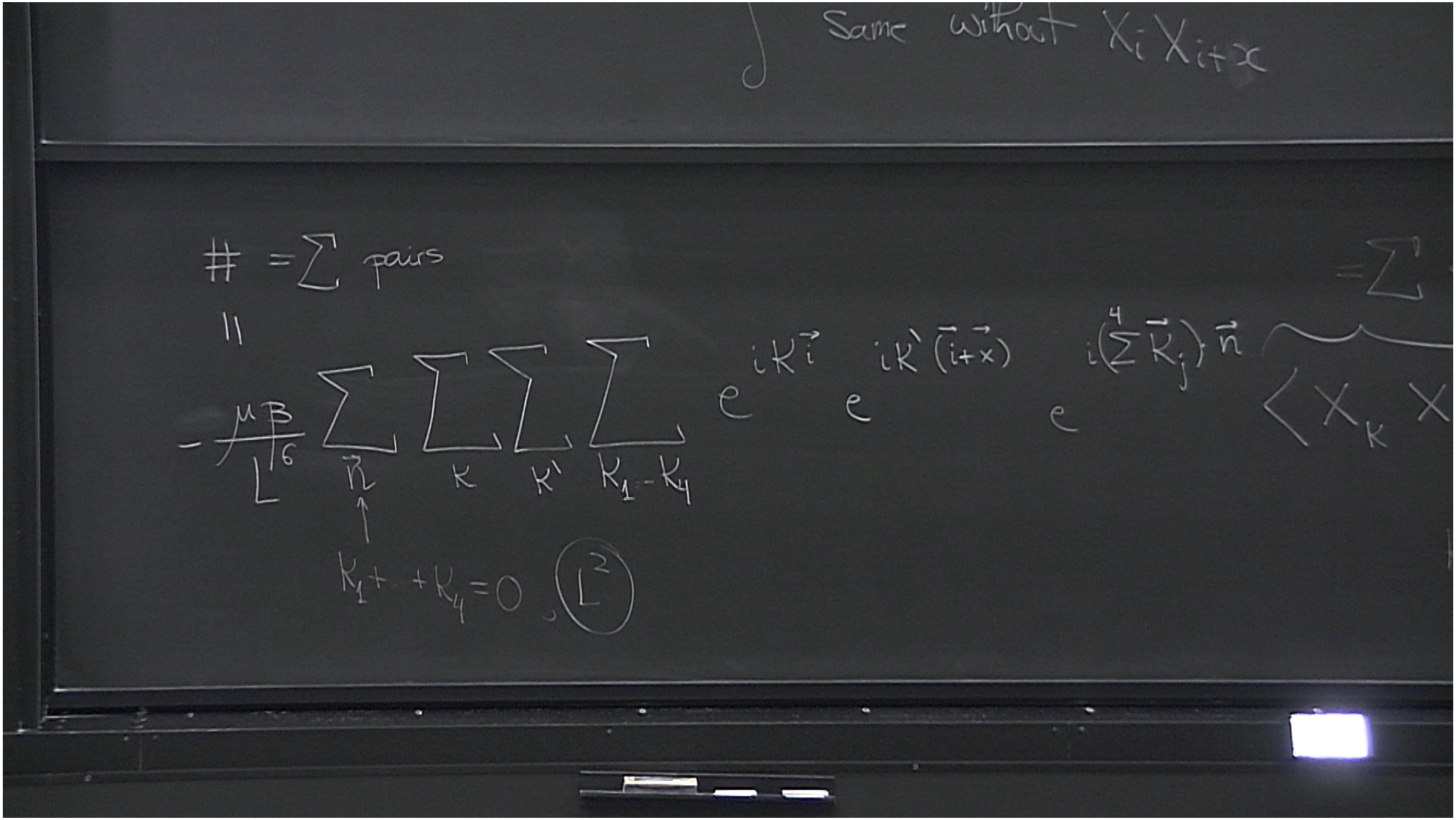

without $X_i X_{i+x}$
$= \sum'$ (xx pairs) $\langle xx \rangle \langle xx \rangle$
zero total mom $(K', -K)$
$e^{i(\sum_j^4 \vec{K}_j)\cdot\vec{n}}$
$\langle X_K X_{K'} X_{K_1} X_{K_2} X_{K_3} X_{K_4} \rangle$
$K' = -K$ or else $\langle \; \rangle = 0$

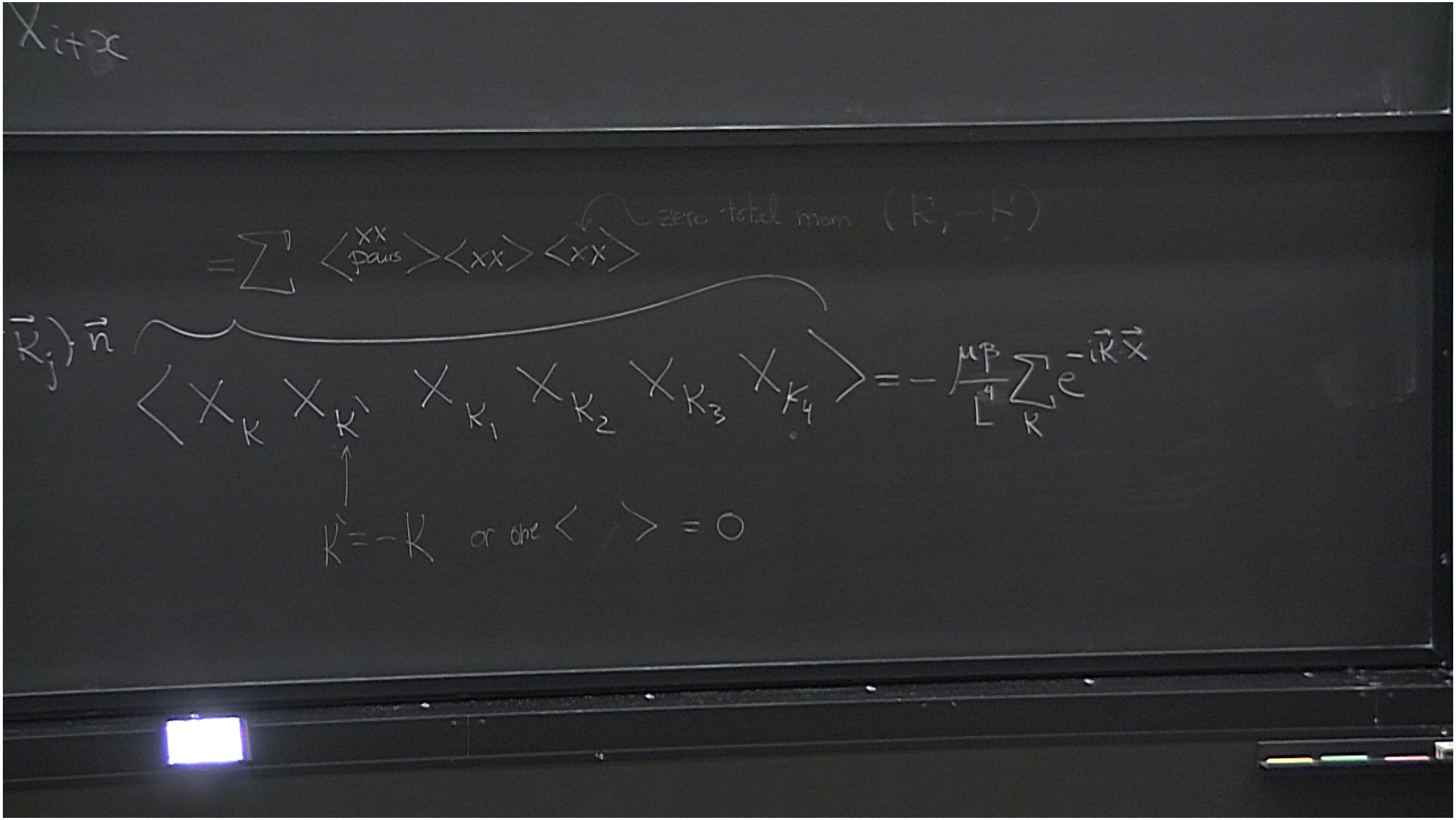

with
= 1/α
(2n-1)!!
m of them
too many J's
= Σ A^-1 A^-1 A^-1
← Wick's theorem
pairs
(a,a'),(b,b')...
Wick contractions
J=0
2m
inverse of A in Gaussian Int.


with
$= \frac{1}{\alpha}$
$(2n-1)!!$
m of them
too many J's
$= \sum$
pairs
$(a,a'),(b,b')\ldots$
$A^{-1}_{aa'}$
$A^{-1}_{bb'}$
$\langle x_a x_{a'} \rangle$
summing over Wick contraction
theorem
$J=0$
$\langle x_i x_k \rangle = \delta_{ik}$
$i_1 \ldots i_k = 2m$


with = 1/α
(2n-1)!!
m of them
too many J's
= Σ A^-1_aa' A^-1_bb' .... A^-1
← Wick's theorem
pairs
(a,a'),(b,b')....
⟨X_a X_a'⟩
summing over Wick contractions
J=0
i_1 ... i_k
2m
⟨X_i1 ... X_ik⟩ = Σ ⟨X_a1 X
pairs
A_ij ⟨X_j X_k⟩ = δ_ik
inverse of A in Gaussian Int.


with = 1/α
(2n-1)!!
m of them
too many J's
Wick's theorem
pairs
summing over Wick contractions
2pt
Gaussian
pairs


Same without $X_i X_{i+x}$
= ∑ pairs
=
$-\frac{\mu_B}{L^6} \sum_{\vec{R}} \sum_{K} \sum_{K'} \sum_{K_1 \ldots K_4} e^{iK\vec{i}} e^{iK'(\vec{i}+\vec{x})} e^{i(\sum_j^4 \vec{K}_j)\cdot\vec{n}}$
$K_1 + \ldots + K_4 = 0$
L^3
$= \sum'$
$\langle X_K X$



X_{i+x}

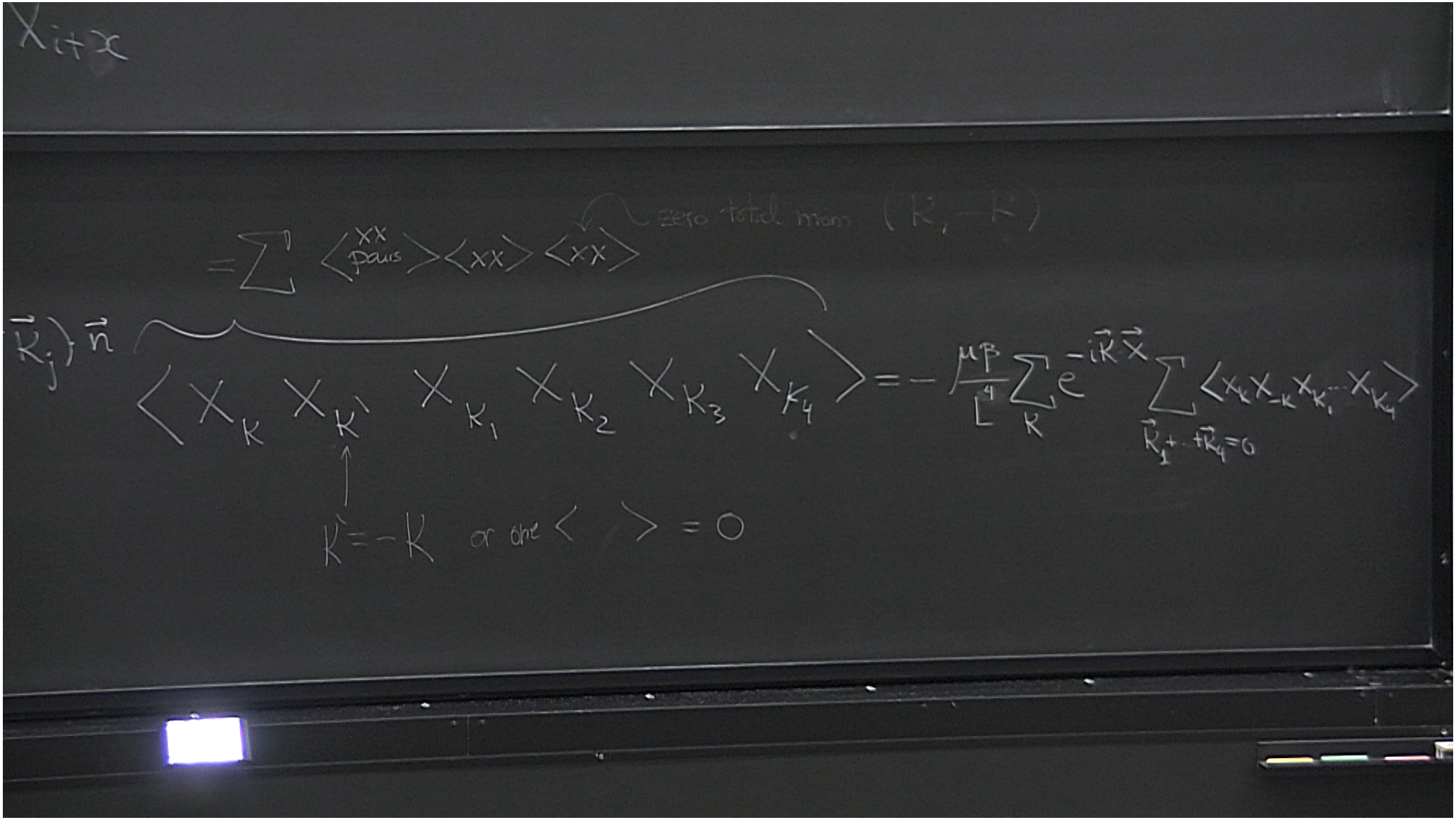
zero total mom $(K, -K)$

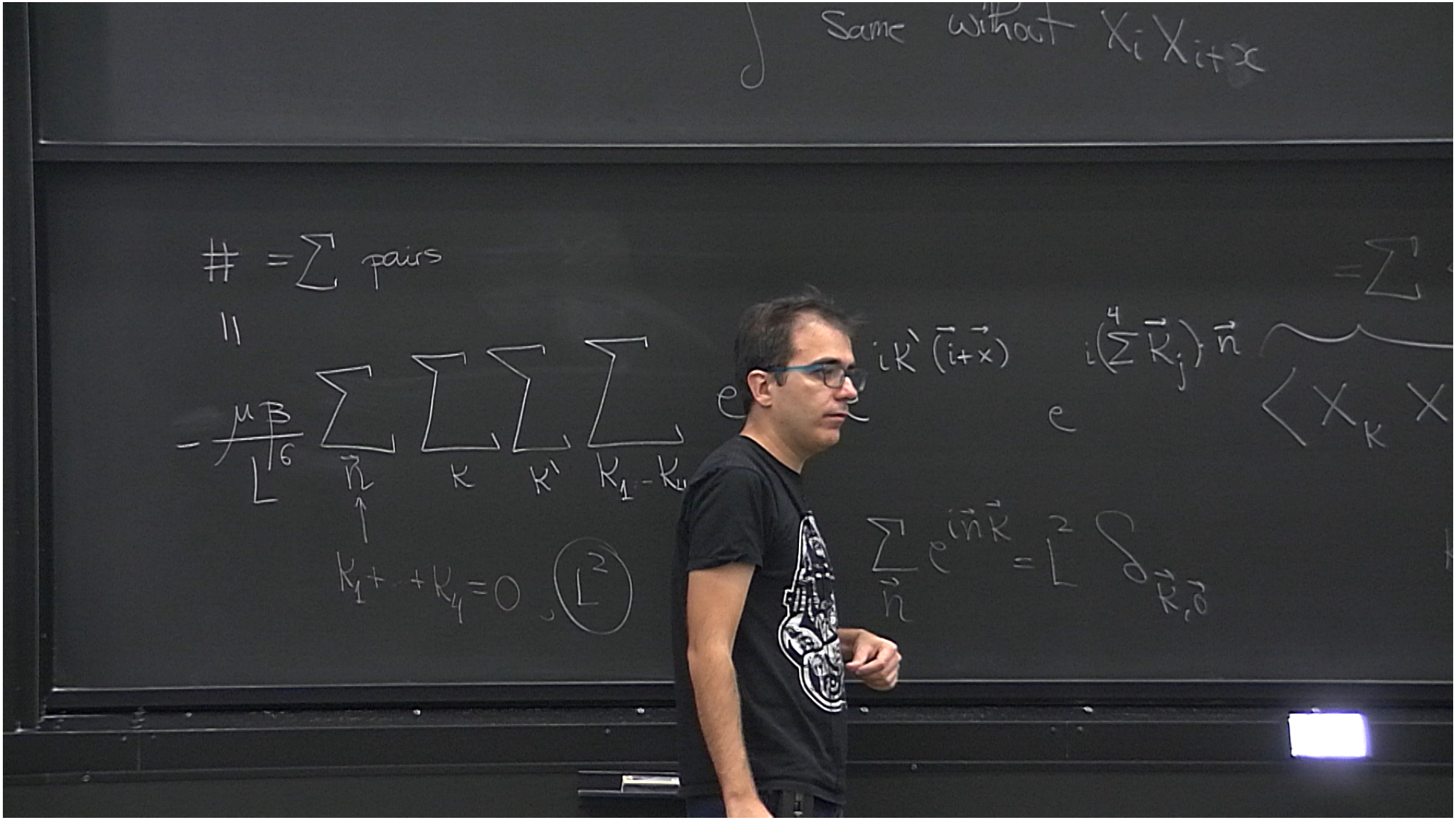
$$= \sum \langle \overset{xx}{\text{pairs}} \rangle \langle xx \rangle \langle xx \rangle$$

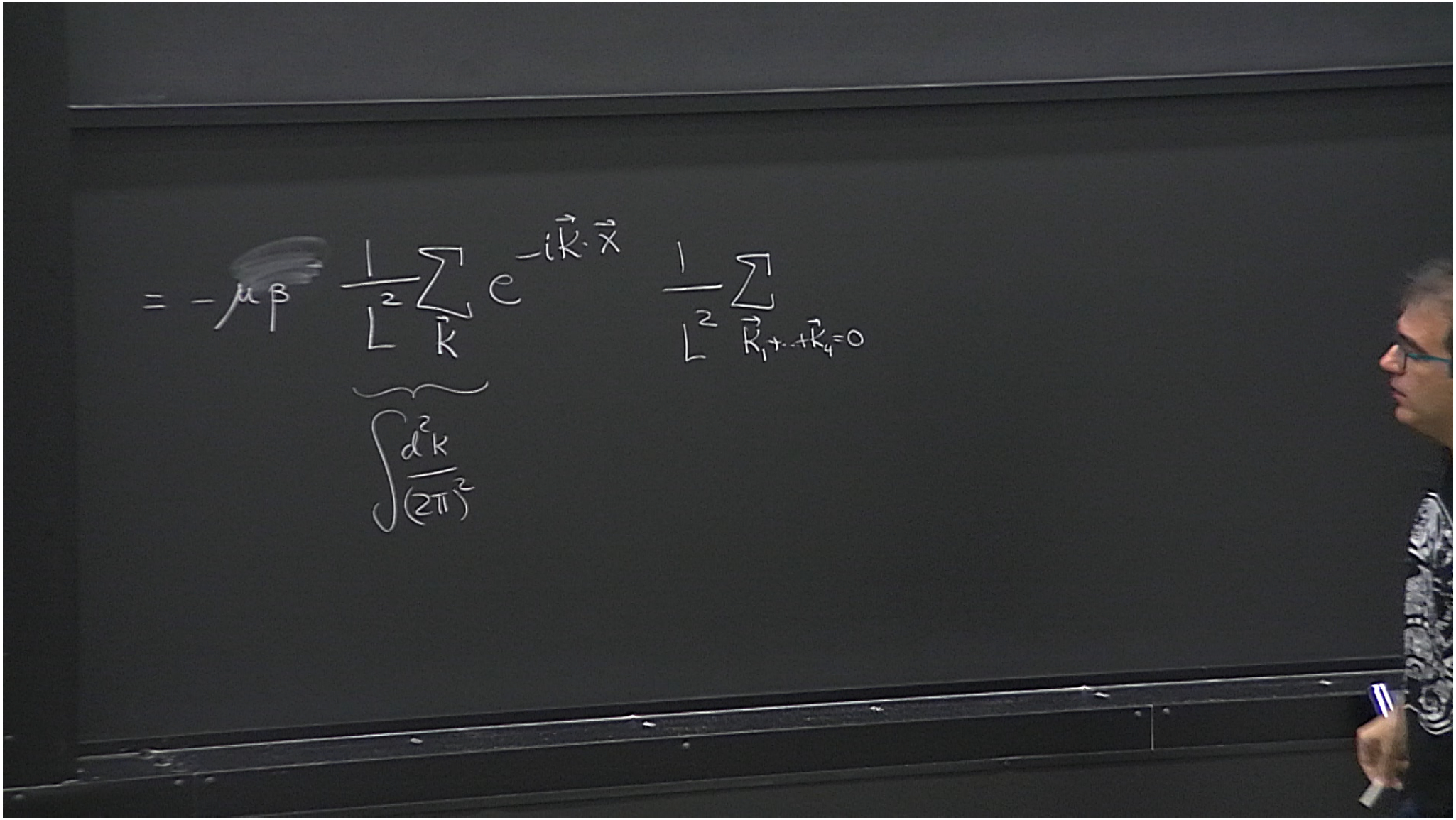
$\vec{K}_j) \cdot \vec{n}$

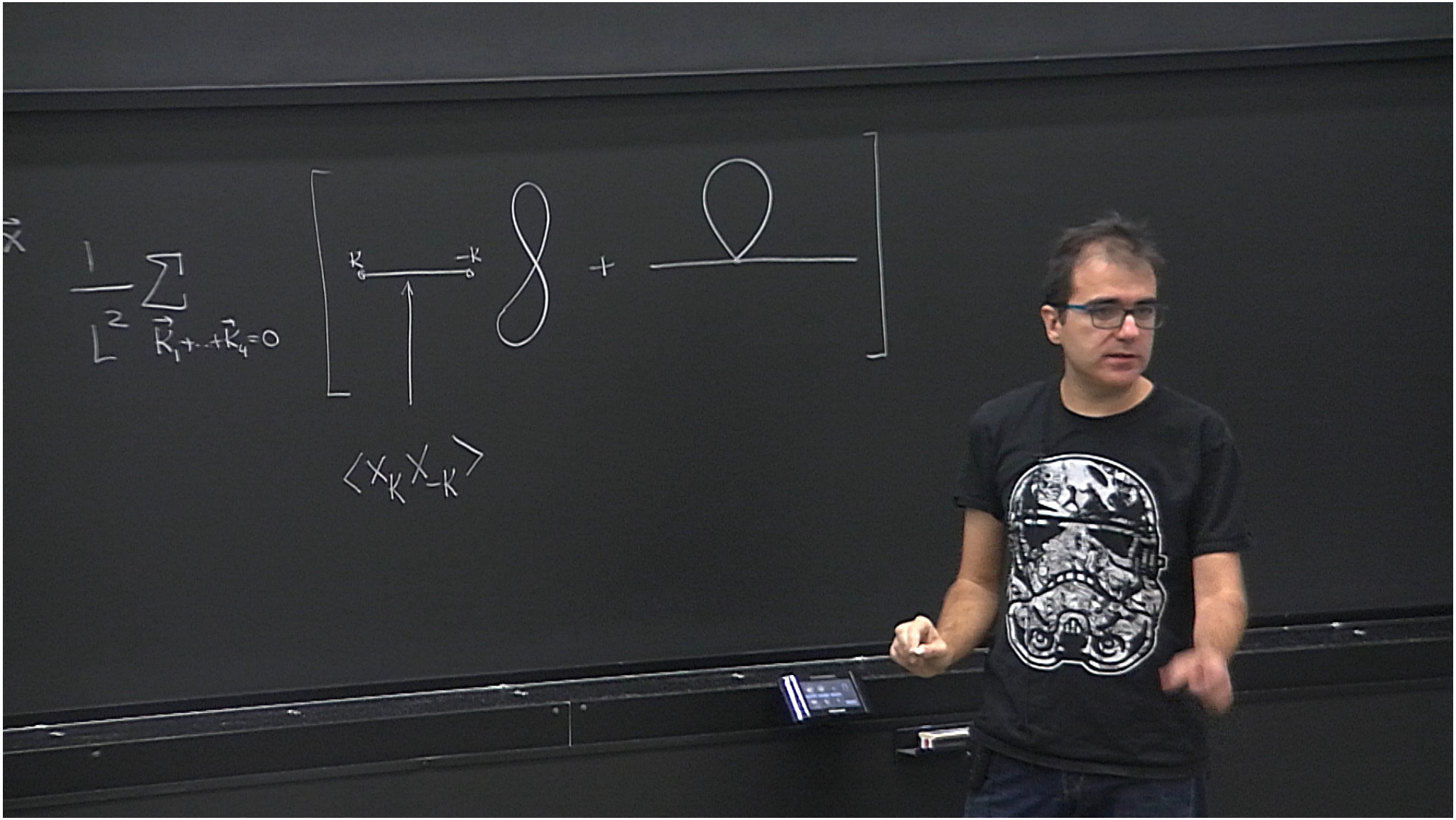
$$\langle X_K X_{K'} X_{K_1} X_{K_2} X_{K_3} X_{K_4} \rangle = -\frac{\mu\beta}{L^4} \sum_K e^{-i\vec{K}\cdot\vec{x}}$$

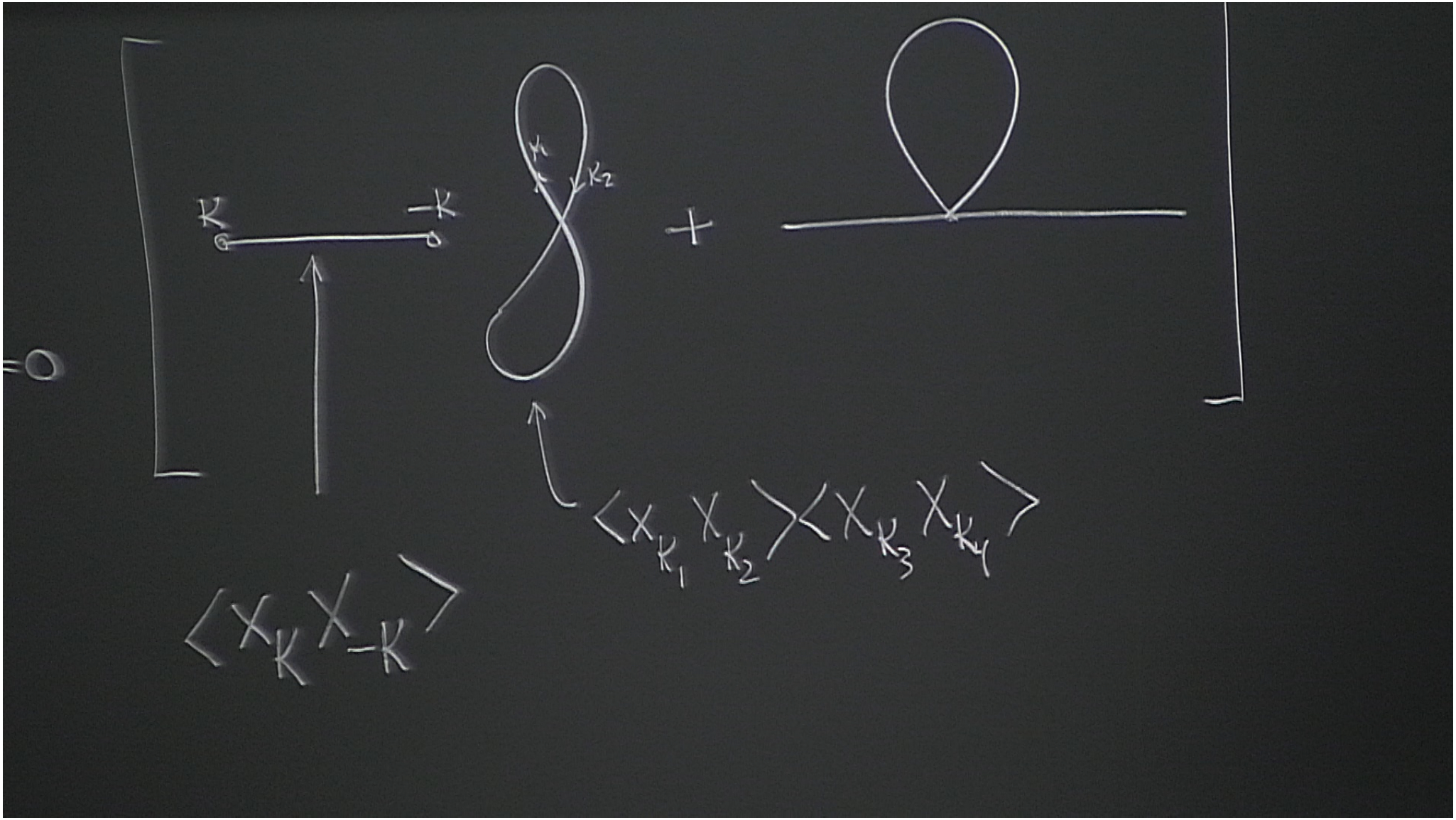
$K' = -K$ or one $\langle \quad \rangle = 0$

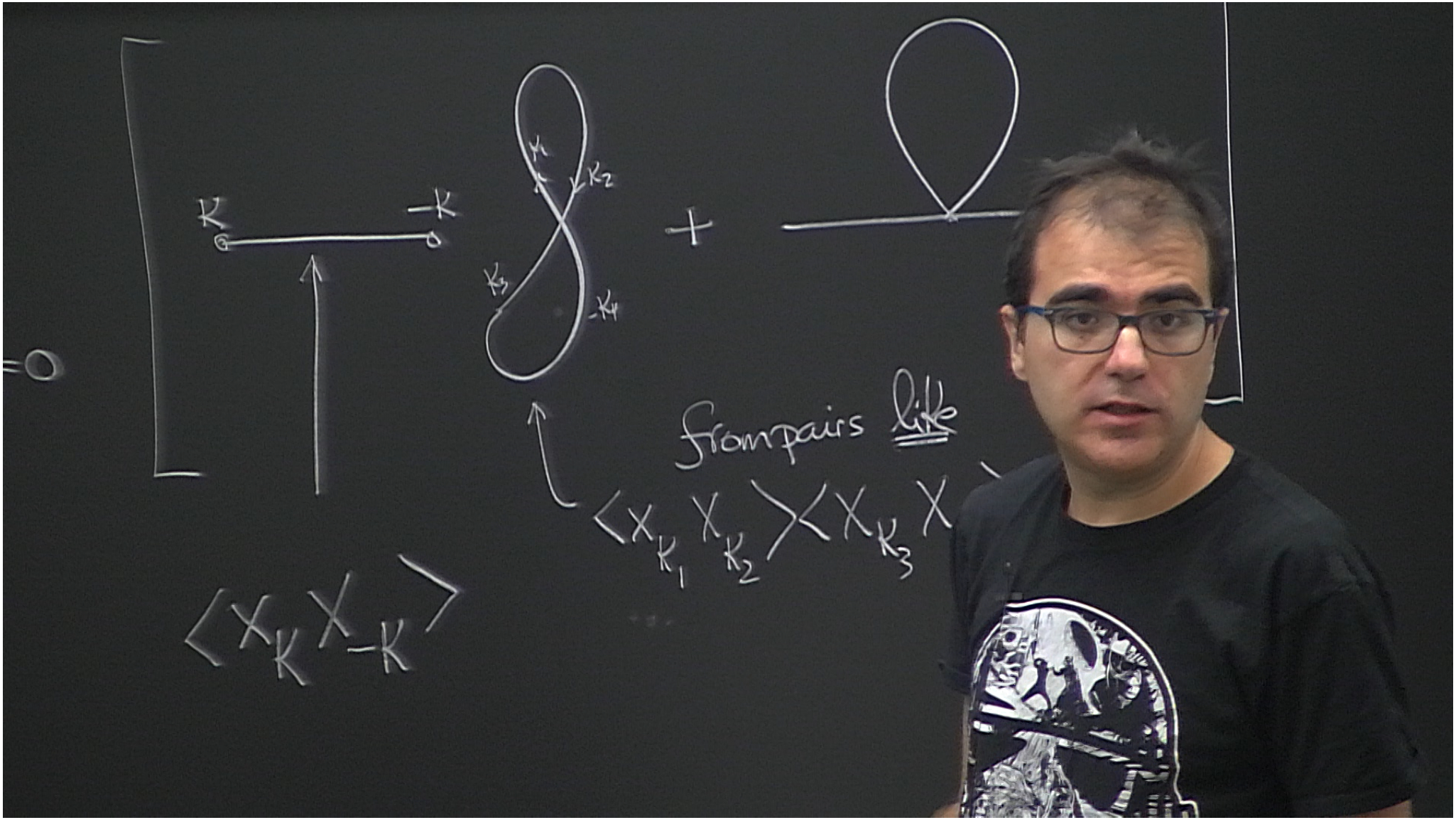

$\chi_{i+\chi}$
zero total mom $(\vec{K}, -\vec{K})$
$= \sum' \langle \chi\chi_{\text{pairs}} \rangle \langle \chi\chi \rangle \langle \chi\chi \rangle$
$\vec{K}_j) \cdot \vec{n}$
$\langle \chi_K \chi_{K'} \chi_{K_1} \chi_{K_2} \chi_{K_3} \chi_{K_4} \rangle = -\frac{\mu_B}{L^4} \sum_K e^{-i\vec{K}\vec{x}} \sum_{\vec{K}_1 + \cdots + \vec{K}_4 = 0} \langle \chi_K \chi_{-K} \chi_{K_1} \cdots \chi_{K_4} \rangle$
$K' = -K$ or else $\langle \; \rangle = 0$

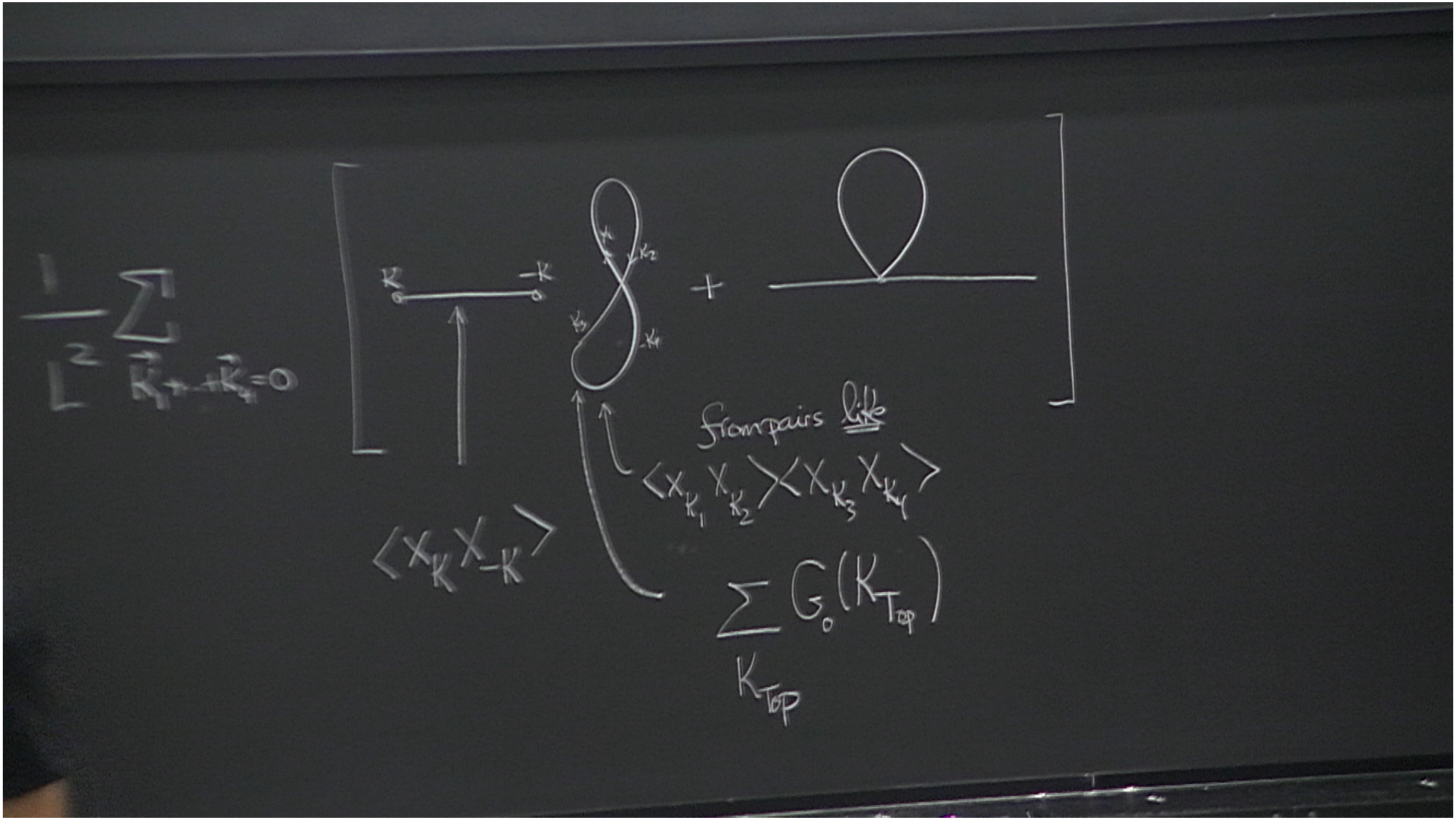

Same without $X_i X_{i+x}$
= ∑ pairs
=
$-\frac{\mu_B}{L^6}$
$\vec{K}_1$
K
K'
$K_1 = -K_1$
$K_1 + \cdots + K_4 = 0$
L^3
$e^{iK'(\vec{i}+\vec{x})}$
$e^{i(\sum_j^4 \vec{K}_j)\cdot\vec{n}}$
$\sum_{\vec{n}} e^{i\vec{n}\vec{K}} = L^2 \delta_{\vec{K},\vec{0}}$
$\langle X_k X$

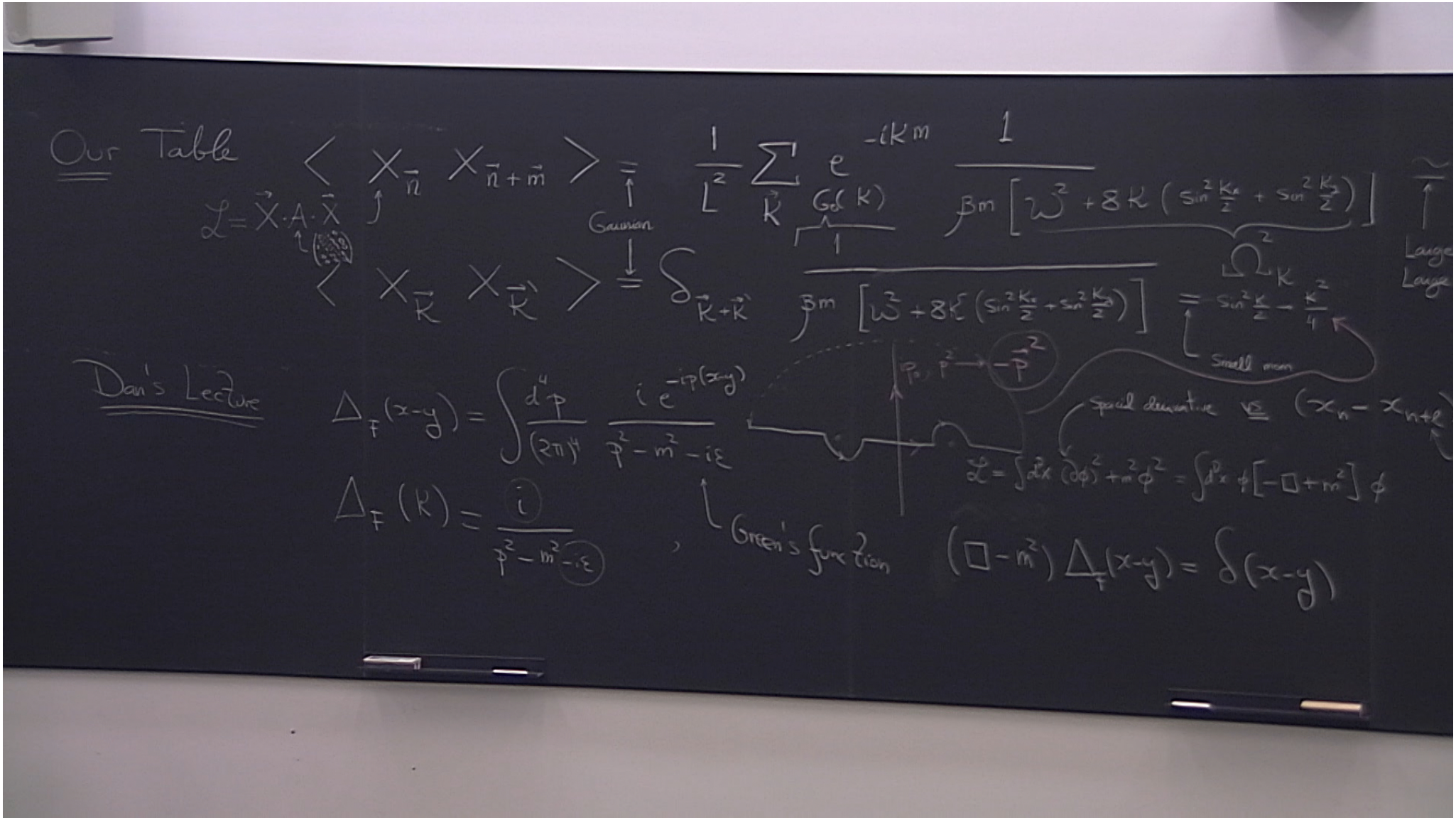

$= -\mu\beta \frac{1}{L^2}\sum_{\vec{K}} e^{-i\vec{K}\cdot\vec{x}} \frac{1}{L^2}\sum_{\vec{K}_1+\cdots+\vec{K}_4=0}$
$\int \frac{d^2k}{(2\pi)^2}$

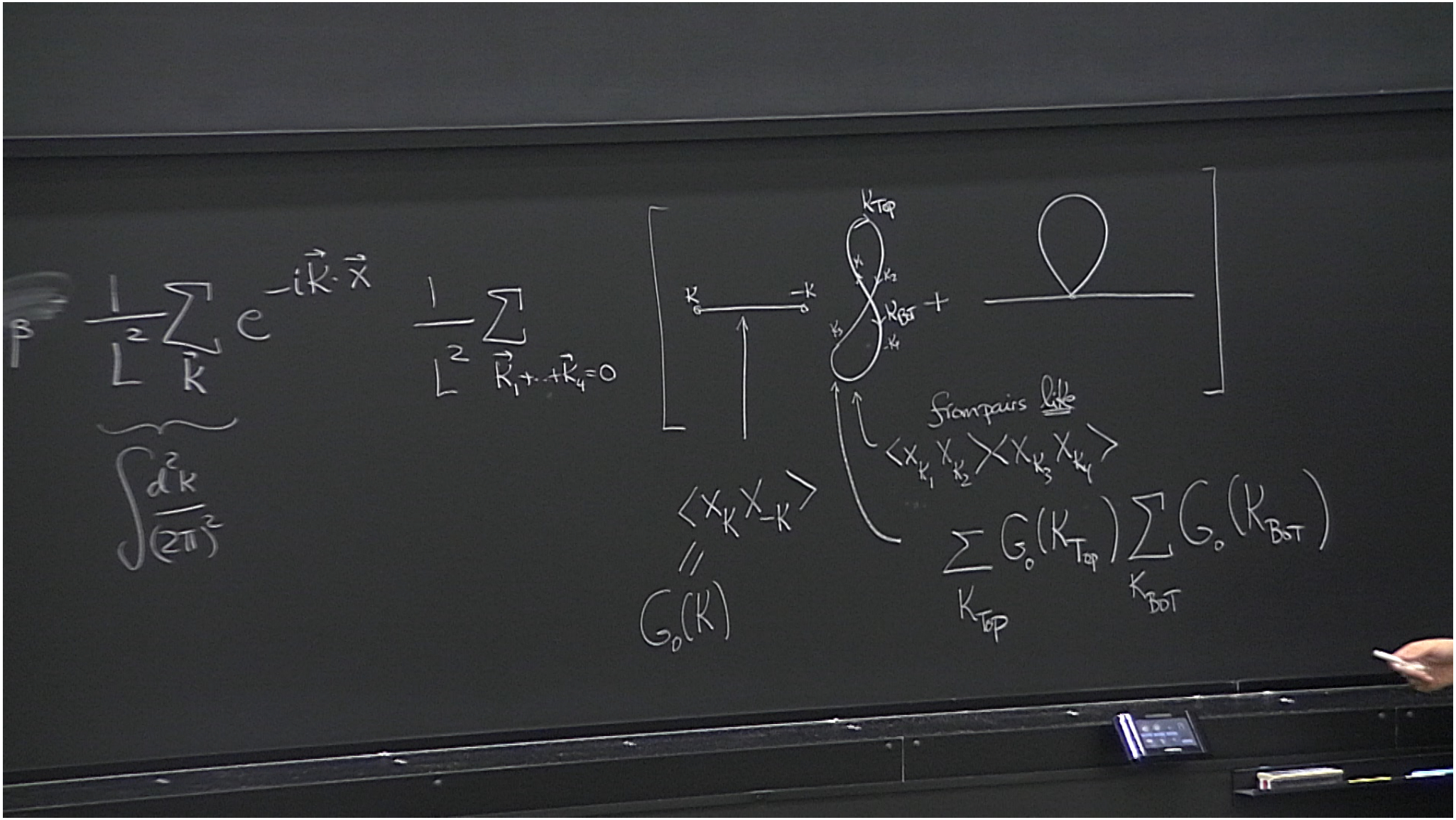

$\frac{1}{L^2}\sum_{\vec{k}_1+\cdots+\vec{k}_4=0}$
k
$-k$
$+$
$\langle X_k X_{-k} \rangle$

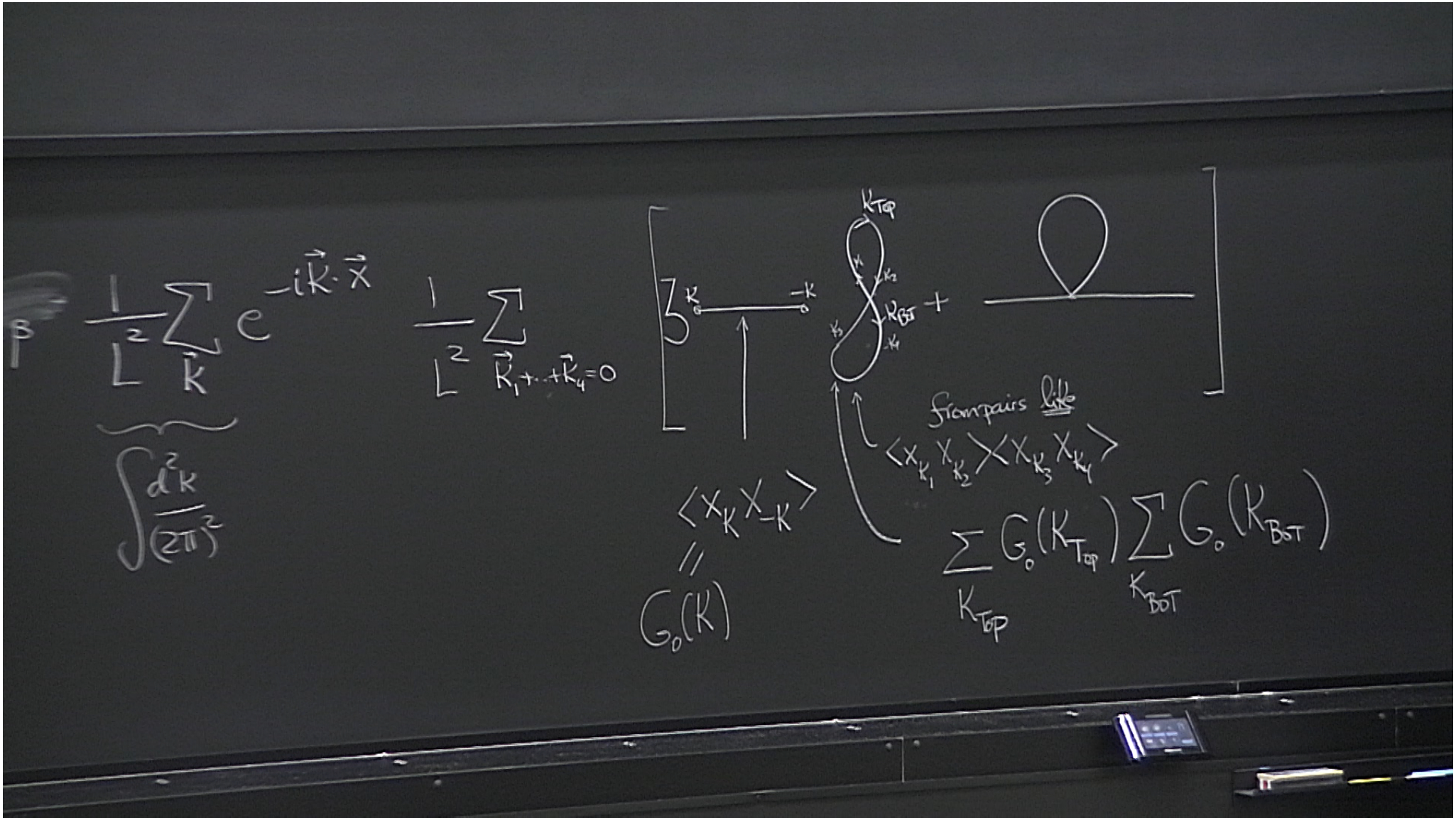

=0
K
-K
k1
k2
+
⟨X_K X_-K⟩
⟨X_k1 X_k2⟩⟨X_k3 X_k4⟩

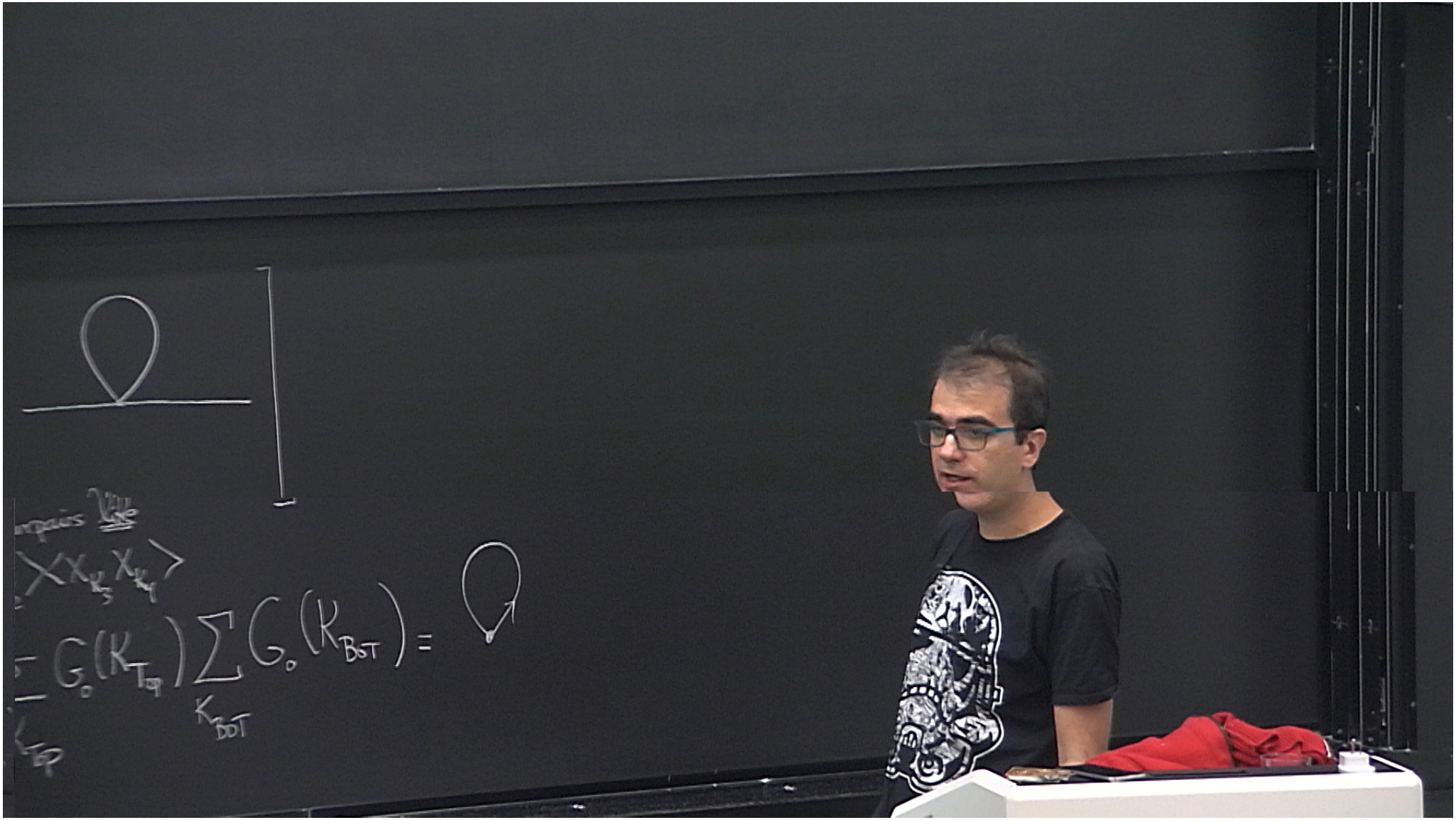

K
-K
K1
K2
K3
-K4
+
=0
frompairs like
⟨X_K X_-K⟩
⟨X_K1 X_K2⟩⟨X_K3 X

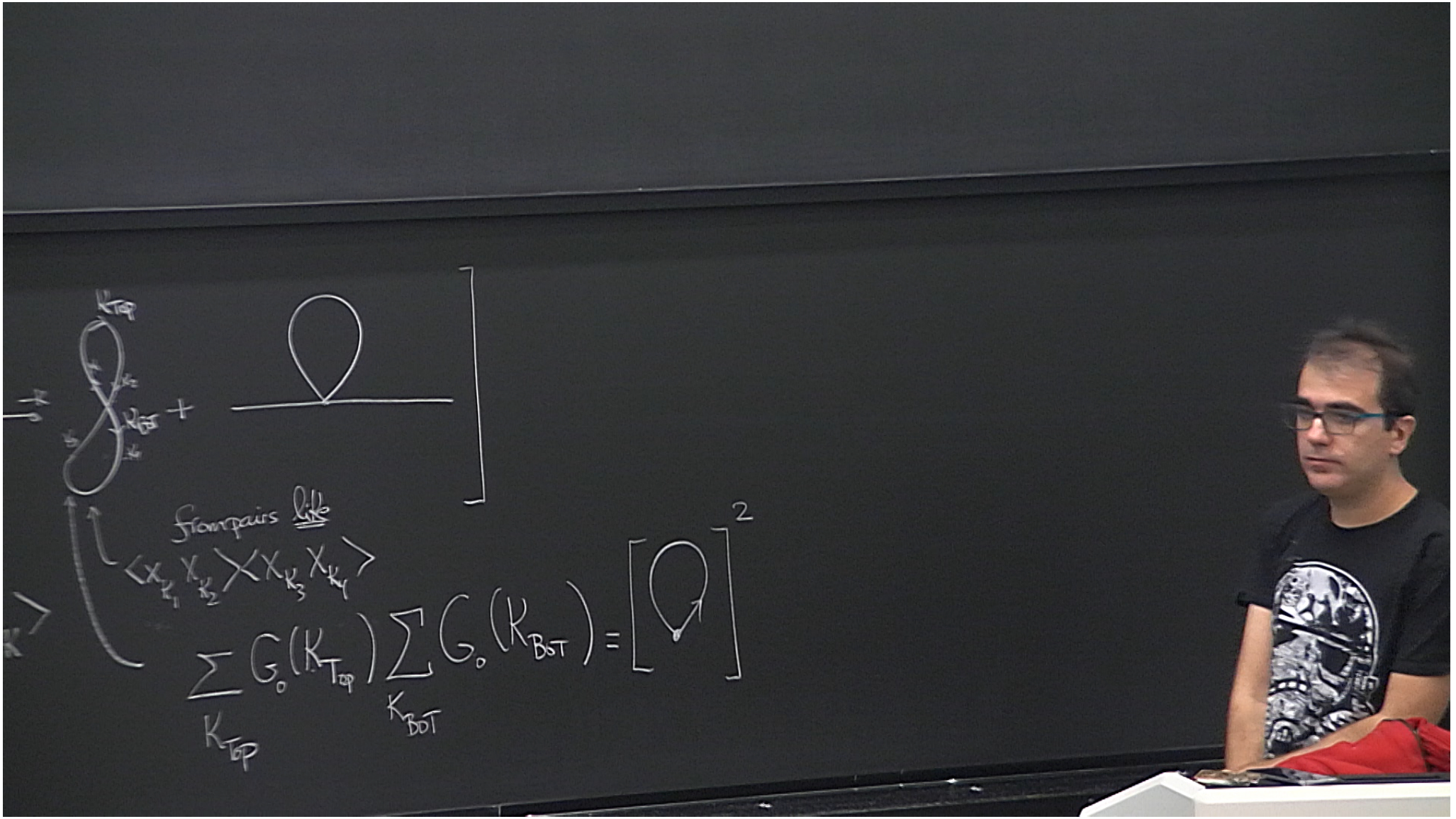

from pairs like
$\langle X_{K_1} X_{K_2} \rangle \langle X_{K_3} X_{K_4} \rangle$
$\langle X_K X_{-K} \rangle$
$\sum_{K_{Top}} G_0(K_{Top})$

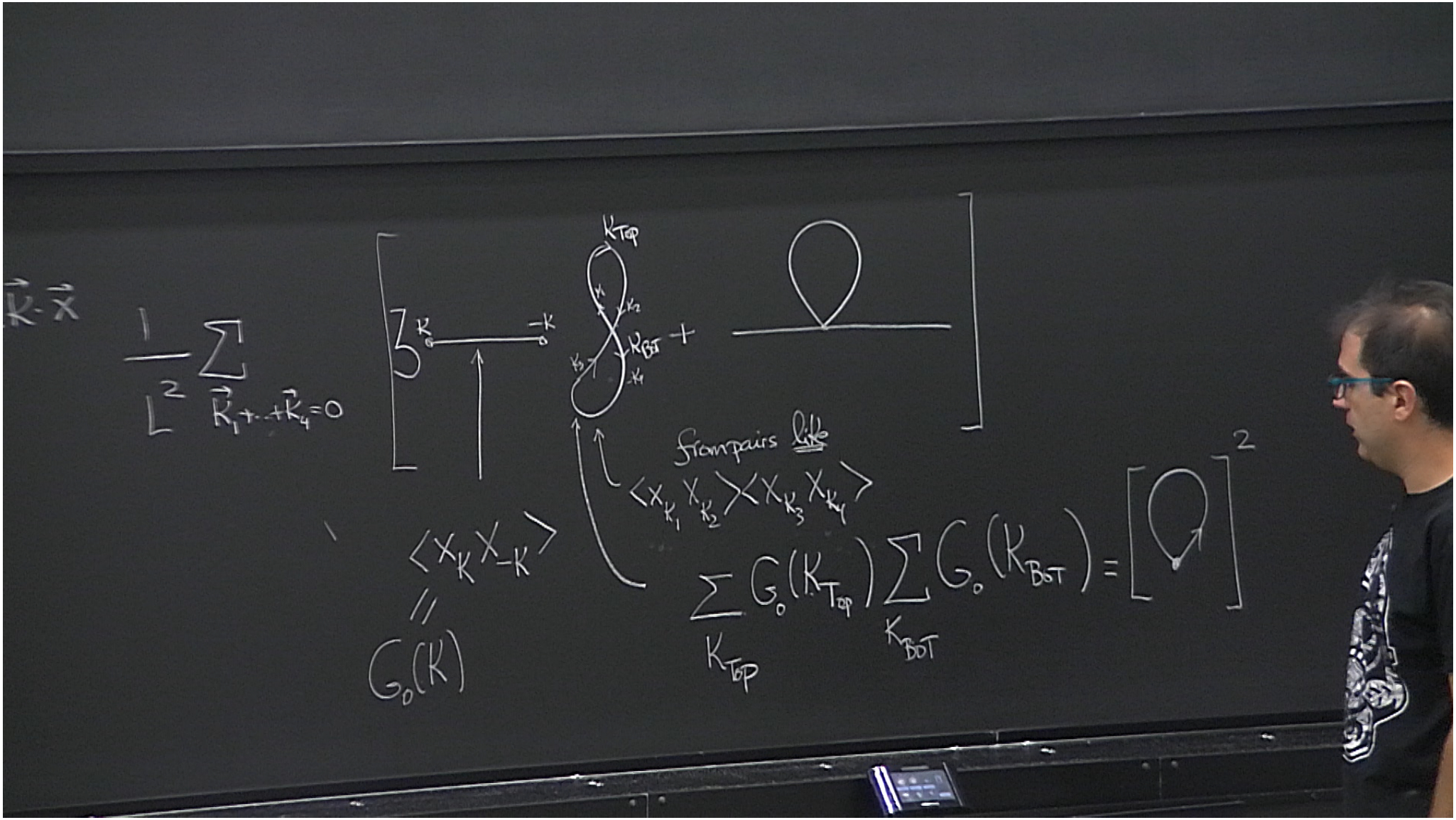

Our Table
$\langle X_{\vec{n}} X_{\vec{n}+\vec{m}} \rangle = \frac{1}{L^2} \sum_{\vec{k}} e^{-ikm} \frac{1}{\beta m \left[\omega^2 + 8k \left(\sin^2 \frac{k_x}{2} + \sin^2 \frac{k_y}{2} \right) \right]}$
$\mathcal{L} = \vec{X} \cdot A \cdot \vec{X}$
Gaussian
$G_0(k)$
Ω_k^2
Large
Large
$\langle X_{\vec{k}} X_{\vec{k}'} \rangle = \delta_{\vec{k}+\vec{k}'} \frac{1}{\beta m \left[\omega^2 + 8k \left(\sin^2 \frac{k_x}{2} + \sin^2 \frac{k_y}{2} \right) \right]}$
$= \sin^2 \frac{k}{2} \to \frac{k^2}{4}$
Small mom
Dan's Lecture
$\Delta_F(x-y) = \int \frac{d^4p}{(2\pi)^4} \frac{i e^{-ip(x-y)}}{p^2 - m^2 - i\varepsilon}$
$p_0, p^2 \to -\vec{p}^2$
Spacial derivative vs $(x_n - x_{n+\ell})$
$\mathcal{L} = \int d^Dx \, (\partial\phi)^2 + m^2\phi^2 = \int d^Dx \, \phi \left[-\Box + m^2 \right] \phi$
$\Delta_F(k) = \frac{i}{p^2 - m^2 - i\varepsilon}$
Green's function
$(\Box - m^2) \Delta_F(x-y) = \delta(x-y)$

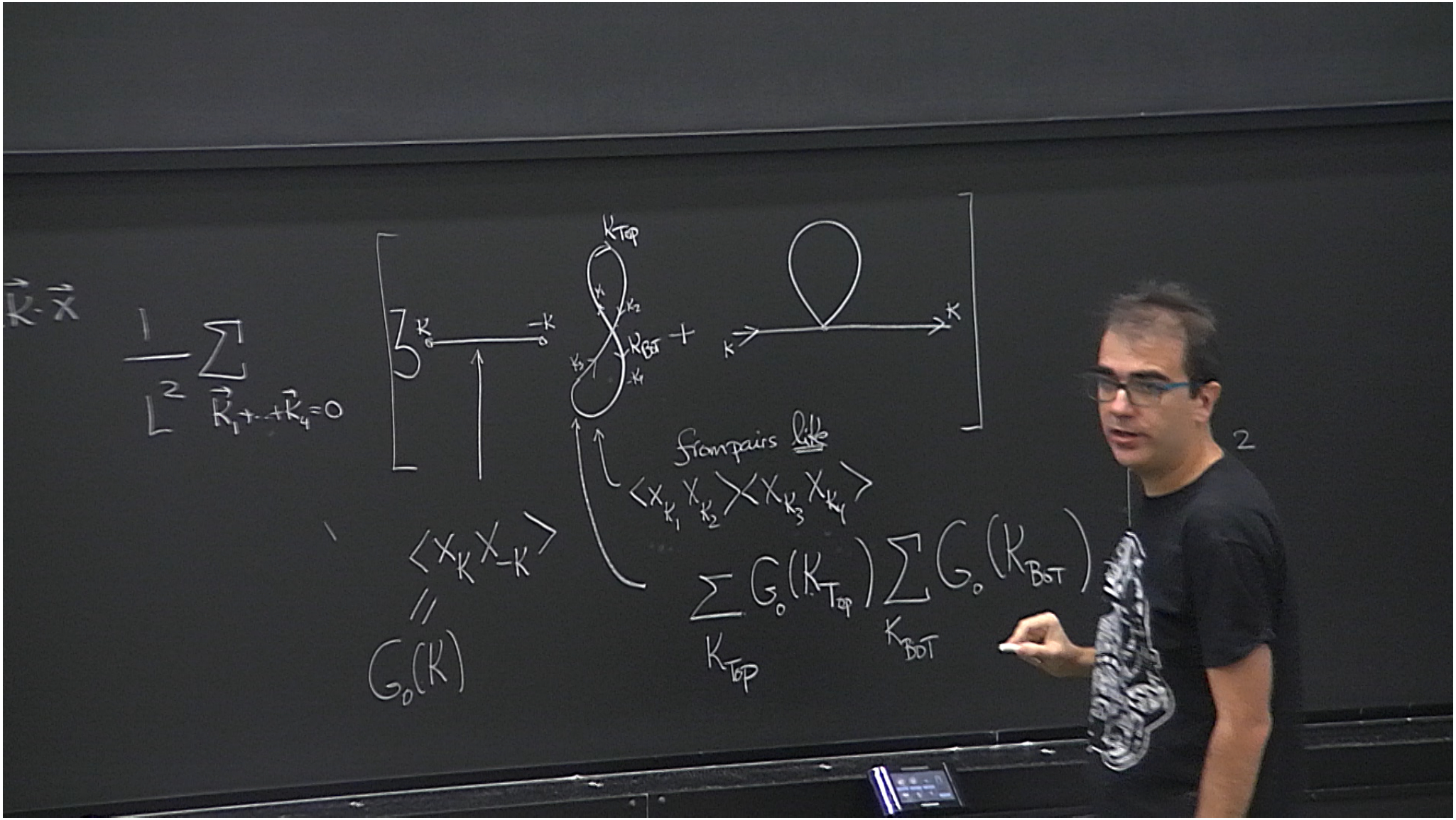

$\frac{1}{L^2}\sum_{\vec{k}} e^{-i\vec{k}\cdot\vec{x}}$
$\int \frac{d^2k}{(2\pi)^2}$
$\frac{1}{L^2}\sum_{\vec{k}_1+\dots+\vec{k}_4=0}$
k
$-k$
k_{Top}
k_{BOT}
$\langle x_k x_{-k}\rangle$
$G_0(k)$
from pairs like
$\langle x_{k_1} x_{k_2}\rangle\langle x_{k_3} x_{k_4}\rangle$
$\sum_{k_{Top}} G_0(k_{Top}) \sum_{k_{BOT}} G_0(k_{BOT})$

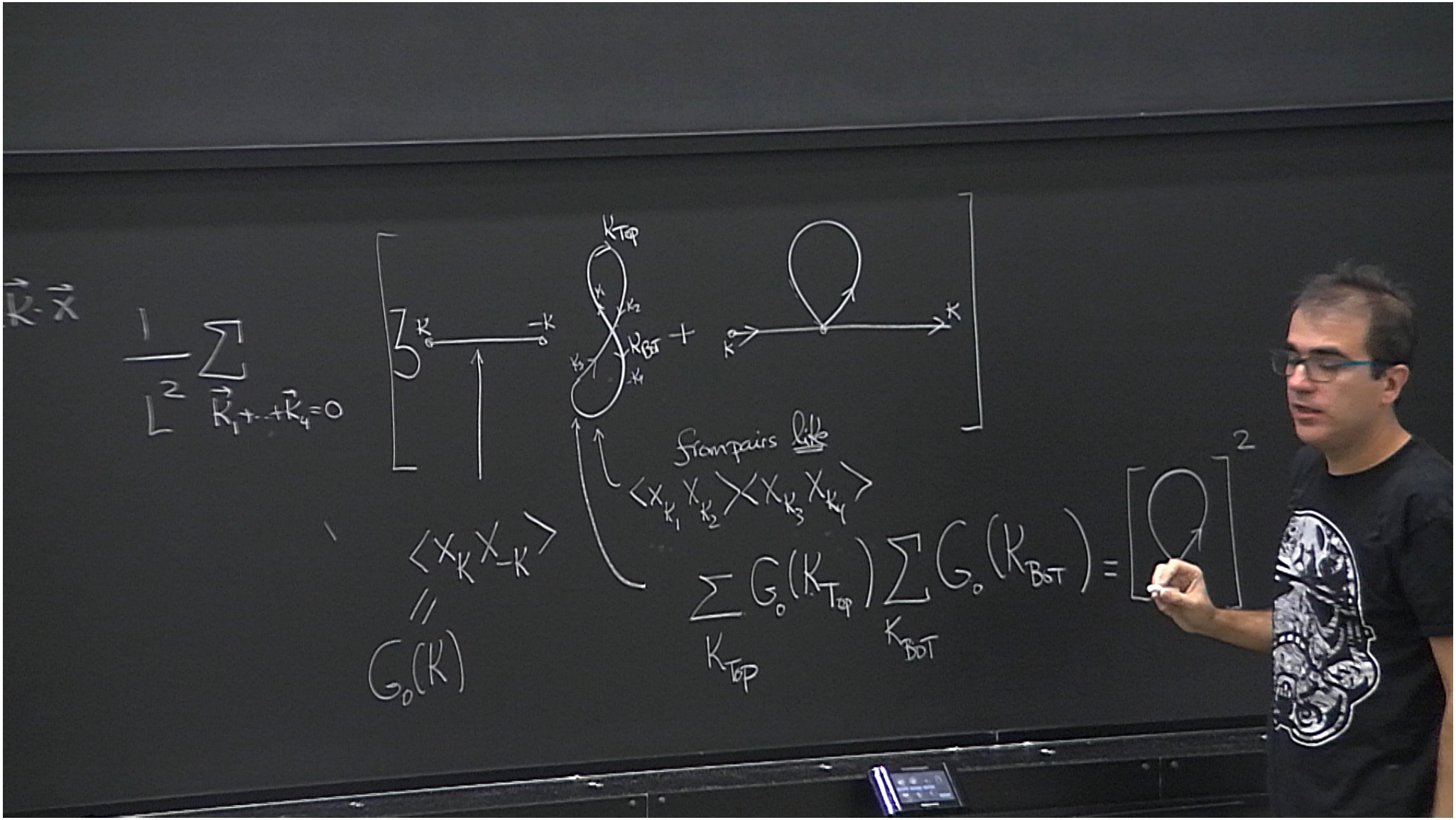

$\frac{1}{L^2}\sum_{\vec{k}} e^{-i\vec{k}\cdot\vec{x}}$
$\int \frac{d^2k}{(2\pi)^2}$
$\frac{1}{L^2}\sum_{\vec{k}_1+\dots+\vec{k}_4=0}$
k
$-k$
k_{Top}
k_{BOT}
$+$
from pairs like
$\langle x_{k_1} x_{k_2}\rangle\langle x_{k_3} x_{k_4}\rangle$
$\langle x_k x_{-k}\rangle$
$\parallel$
$G_0(k)$
$\sum_{k_{Top}} G_0(k_{Top}) \sum_{k_{BOT}} G_0(k_{BOT})$

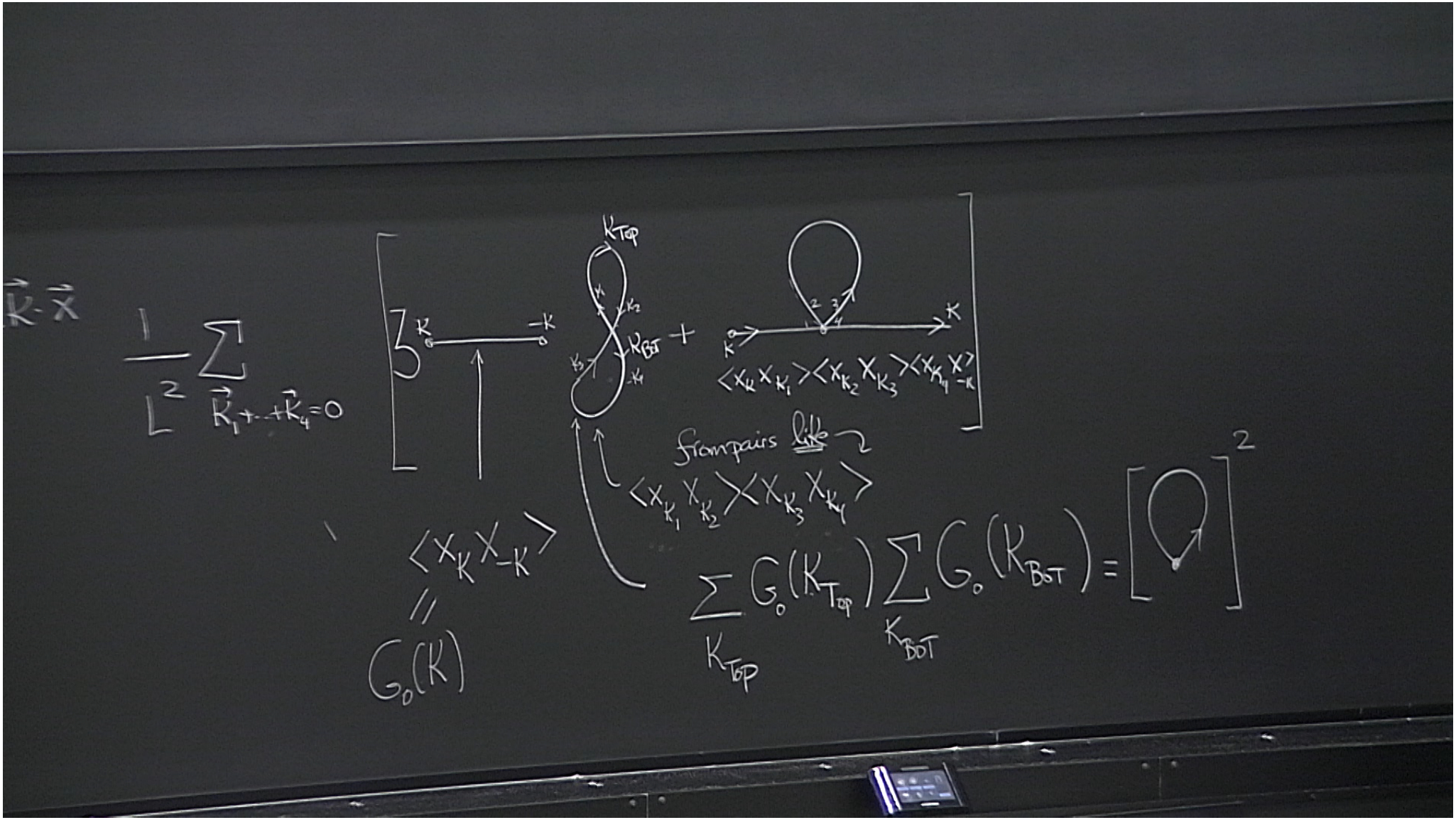

$G_0(K_{T_{op}}) \sum_{K_{BDT}} G_0(K_{BDT}) =$

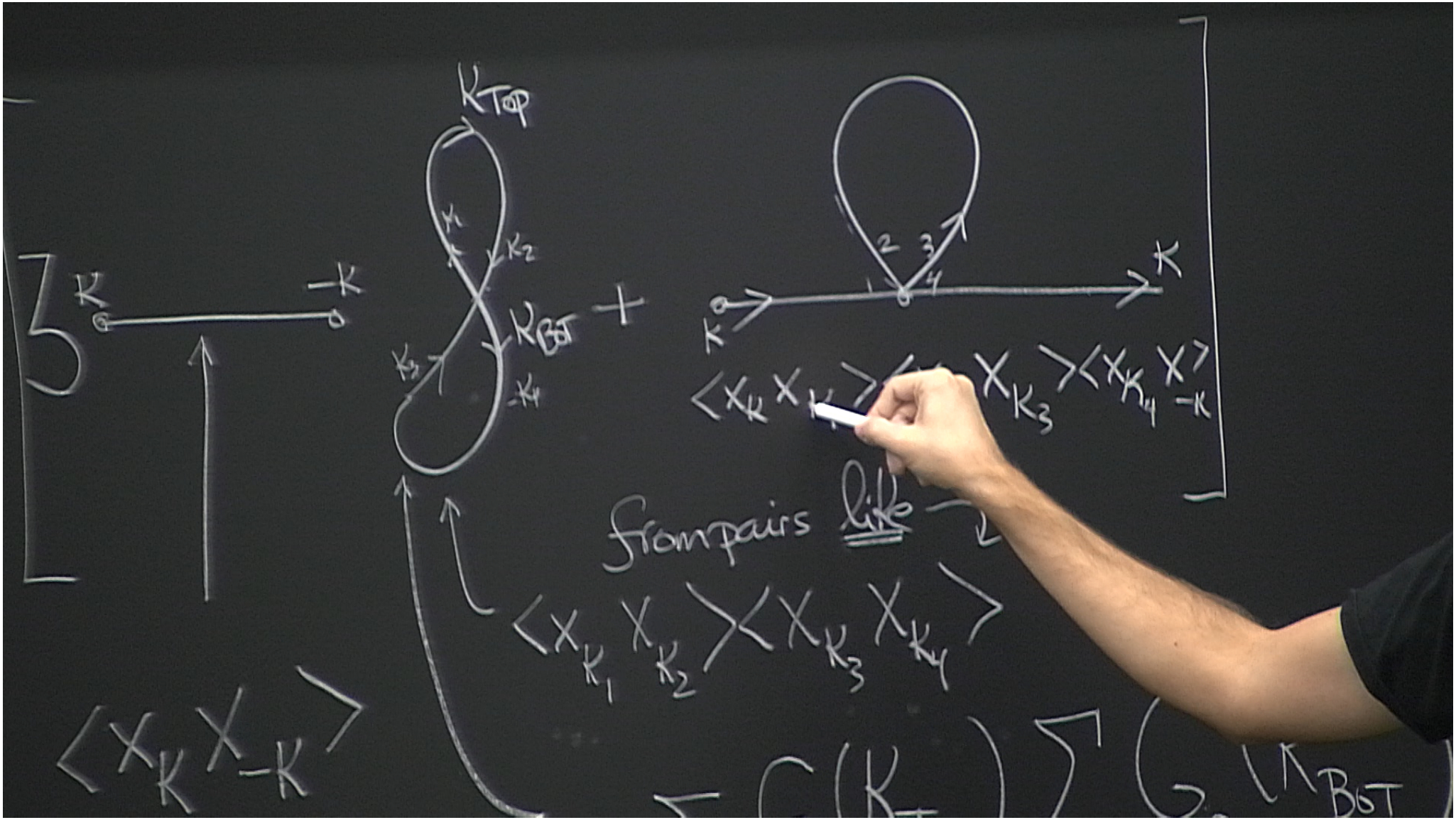

from pairs like
$\sum_{K_{Top}} G_0(K_{Top}) \sum_{K_{BOT}} G_0(K_{BOT}) = [\;\;]^2$

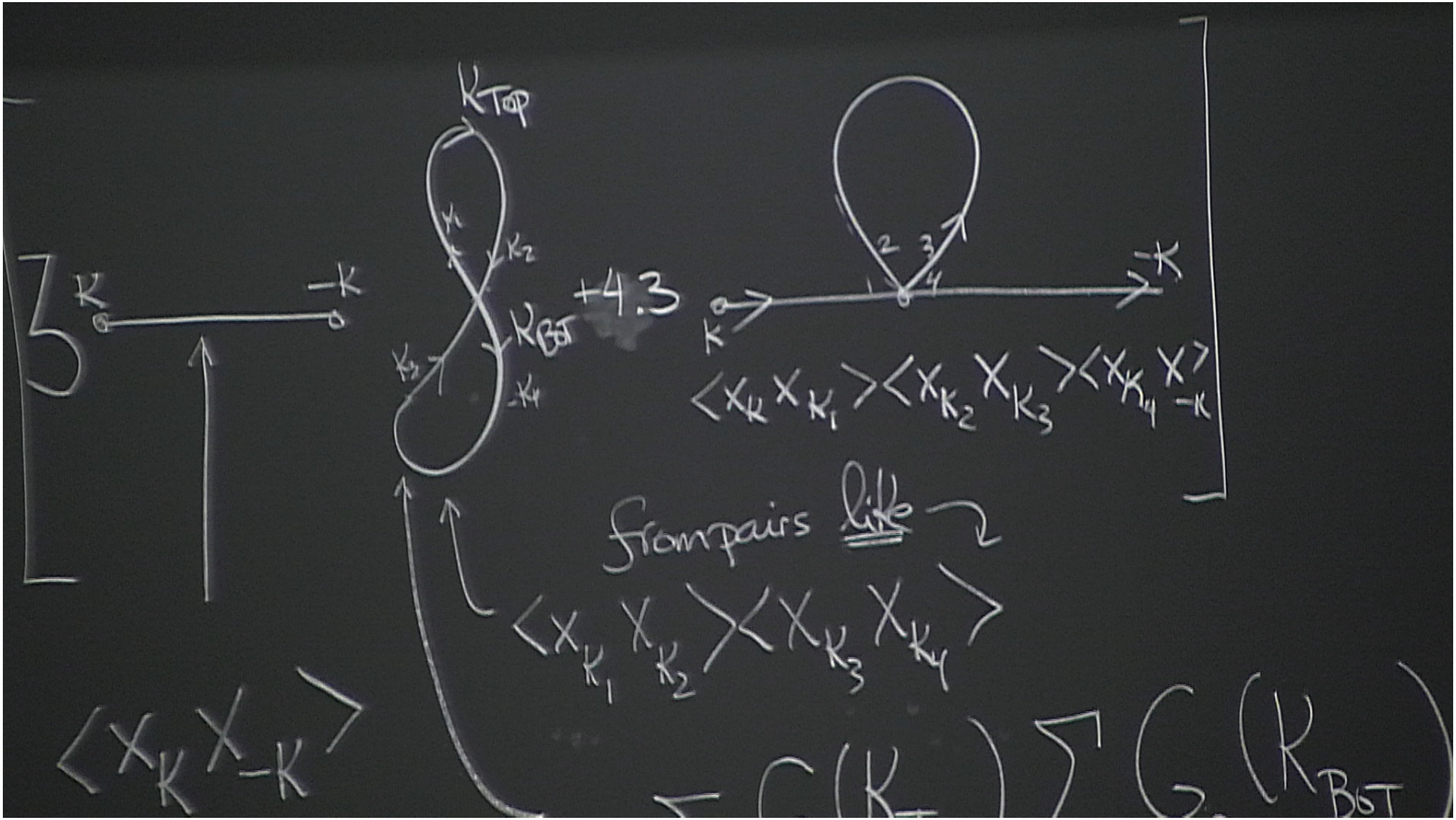

$\vec{k}\cdot\vec{x}$
$\frac{1}{L^2}\sum_{\vec{k}_1+\cdots+\vec{k}_4=0}$
k
$-k$
k_{Top}
k_1
k_2
k_{BOT}
k_3
$-k_4$
+
from pairs
$\langle x_{k_1} x_{k_2}\rangle\langle x_{k_3} x_{k_4}\rangle$
$\langle x_k x_{-k}\rangle$
$=$
$G_0(k)$
$\sum_{k_{Top}} G_0(k_{Top}) \sum_{k_{BOT}} G_0(k_{BOT}) =$
^{2}

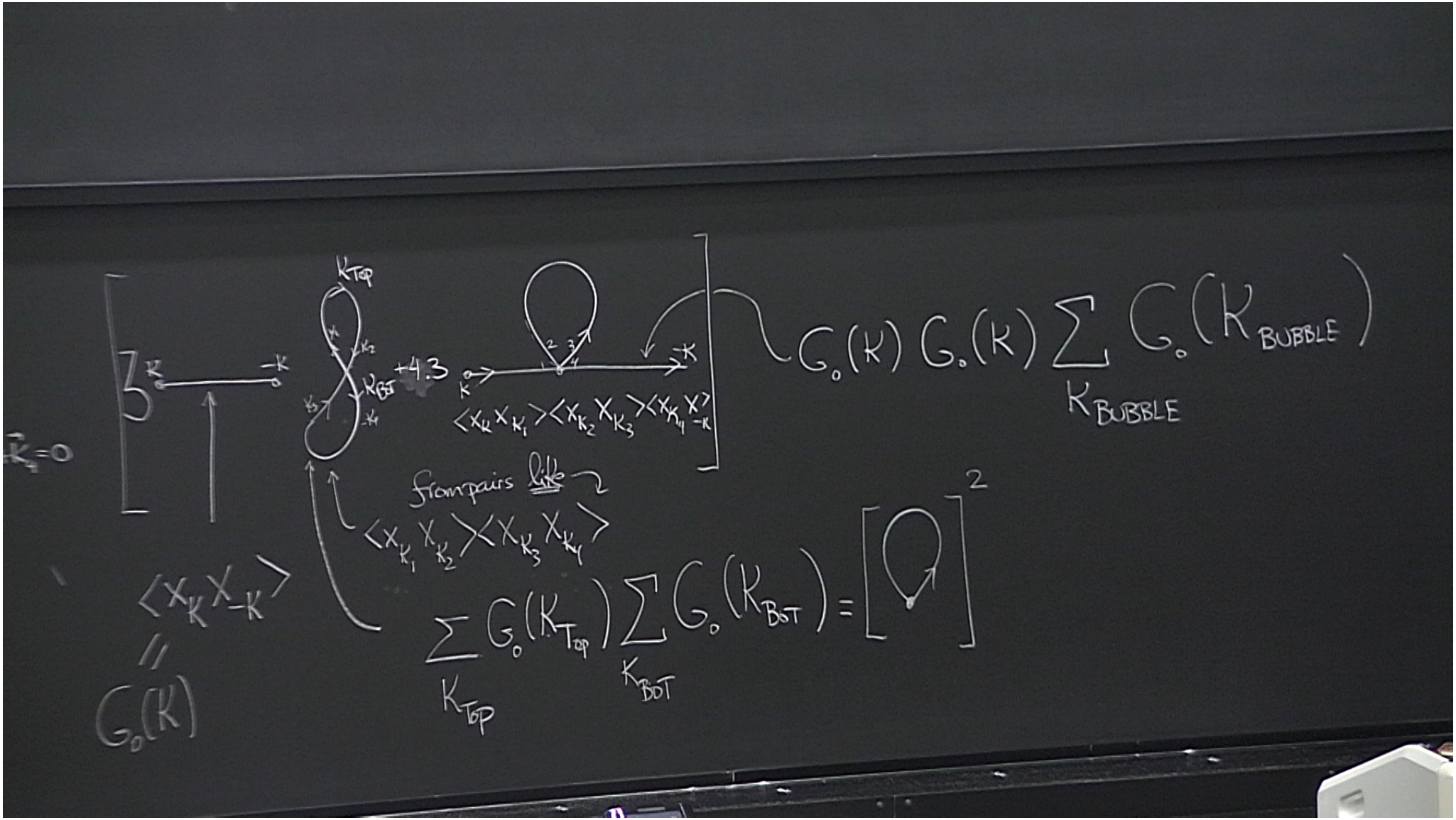

$\vec{k}\cdot\vec{x}$
$\frac{1}{L^2}\sum_{\vec{k}_1+\cdots+\vec{k}_4=0}$
$\langle X_K X_{-K}\rangle$
$=$
$G_0(K)$
K_{Top}
K_{BOT}
$+$
from pairs like
$\langle X_{K_1} X_{K_2}\rangle\langle X_{K_3} X_{K_4}\rangle$
$\sum_{K_{Top}} G_0(K_{Top}) \sum_{K_{BOT}} G_0(K_{BOT})$
2

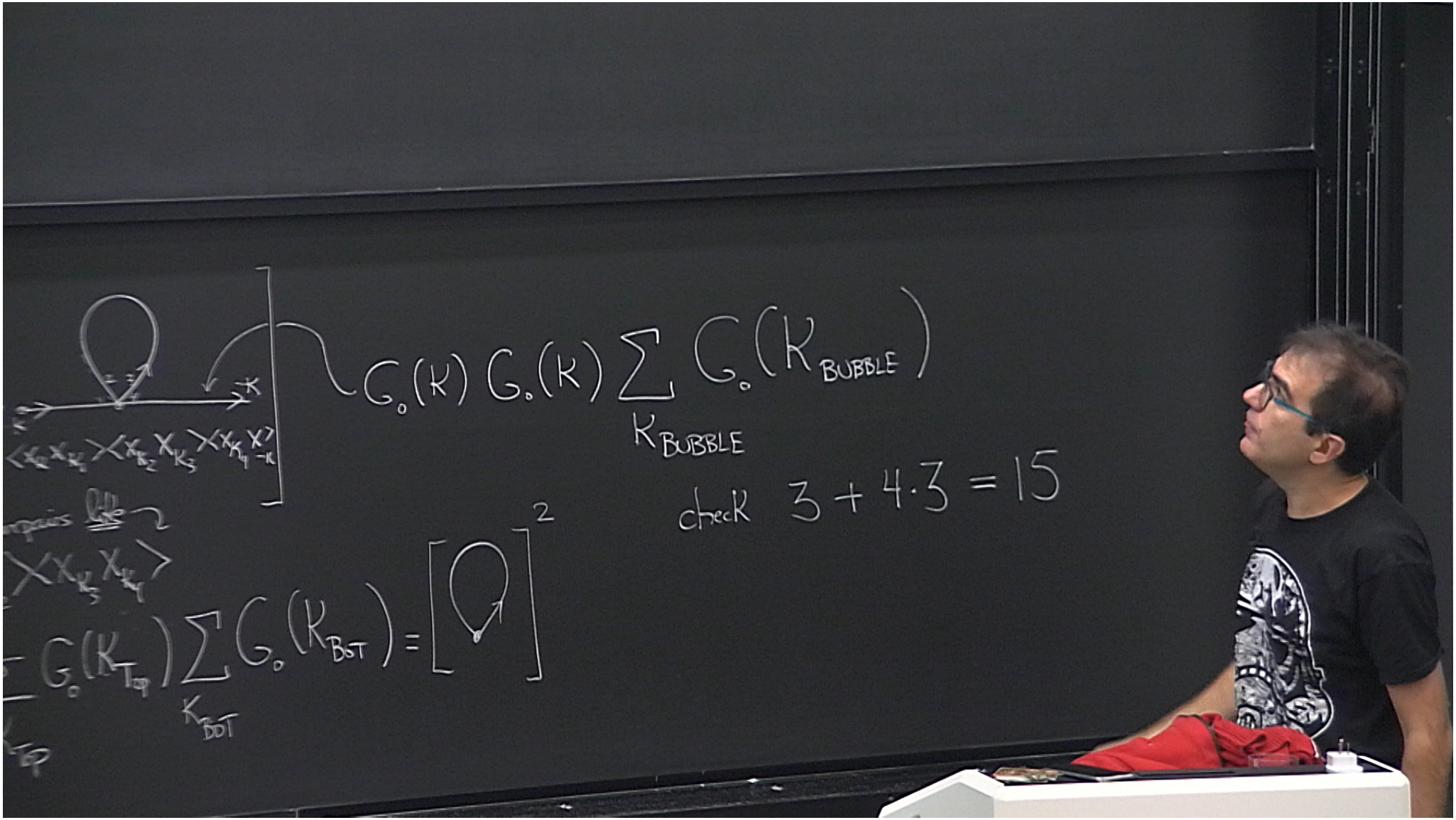

$\vec{k}\cdot\vec{x}$
$\frac{1}{L^2}\sum_{\vec{k}_1+\cdots+\vec{k}_4=0}$
k_{Top}
k_{BOT}
from pairs like
$\langle x_{k_1} x_{k_2}\rangle\langle x_{k_3} x_{k_4}\rangle$
$\langle x_k x_{-k}\rangle$
$= G_0(k)$
$\sum_{k_{Top}} G_0(k_{Top}) \sum_{k_{BOT}} G_0(k_{BOT}) =$
2

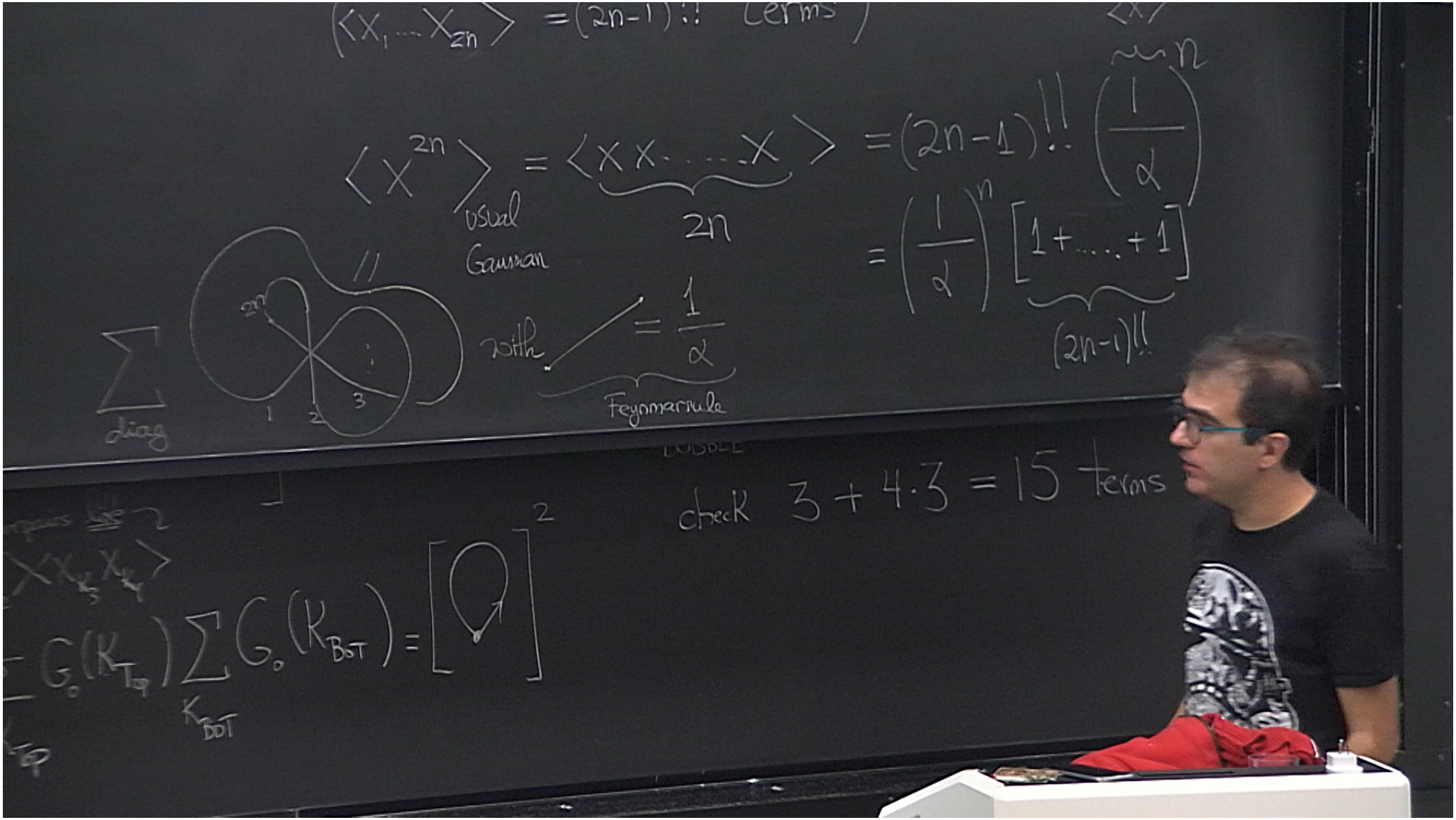

$\vec{K}\cdot\vec{x}$
$\frac{1}{L^2}\sum_{\vec{K}_1+\cdots+\vec{K}_4=0}$
K_{Top}
K_{BOT}
$\langle X_K X_{-K}\rangle$
$= G_0(K)$
from pairs like
$\langle X_{K_1} X_{K_2}\rangle\langle X_{K_3} X_{K_4}\rangle$
$\sum_{K_{Top}} G_0(K_{Top}) \sum_{K_{BOT}} G_0(K_{BOT}) = \left[\ \right]^2$


K_Top
K_Bot
frompairs like
⟨X_K X_-K⟩
⟨X_K1 X_K2⟩⟨X_K3 X_K4⟩

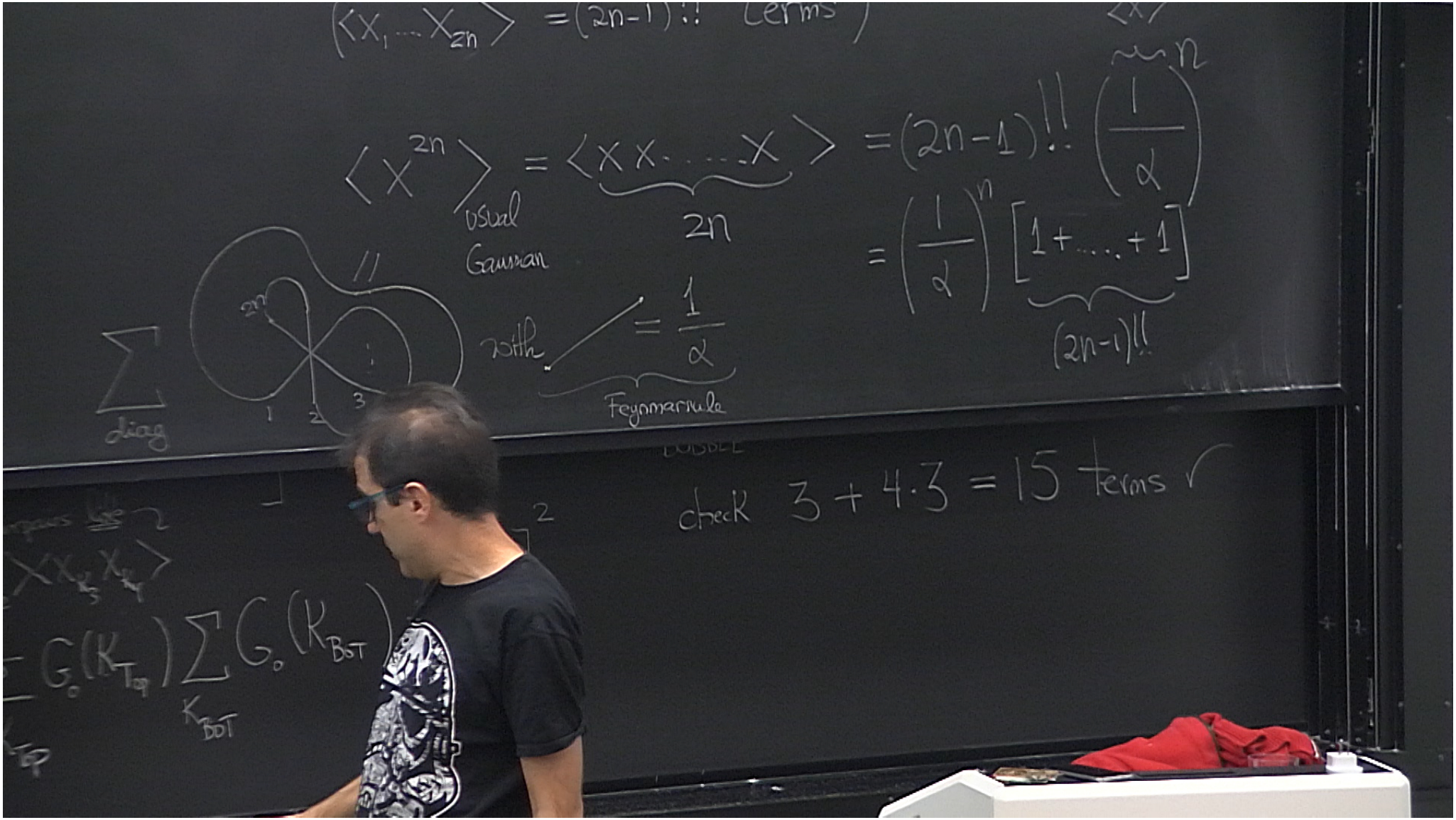

K_{Top}
K_{Bot}
$+4.3$
from pairs like
$\langle X_K X_{-K} \rangle$
$\sum C(K_{Top}) \sum C(K_{Bot})$


K_{TOP}
K_{BOT}
$+4.3$
$\langle X_K X_{K_1} \rangle \langle X_{K_2} X_{K_3} \rangle \langle X_{K_4} X_{-K} \rangle$
$G_0(K)\, G_0(K) \sum_{K_{BUBBLE}} G_0(K_{BUBBLE})$
$K_1 = 0$
from pairs like
$\langle X_{K_1} X_{K_2} \rangle \langle X_{K_3} X_{K_4} \rangle$
$\langle X_K X_{-K} \rangle = G_0(K)$
$\sum_{K_{TOP}} G_0(K_{TOP}) \sum_{K_{BOT}} G_0(K_{BOT}) = [\,\,]^2$


$G_0(K)\,G_0(K)\sum_{K_{BUBBLE}} G_0(K_{BUBBLE})$
check $3+4\cdot 3=15$
$G_0(K_{TOP})\sum_{K_{BOT}} G_0(K_{BOT}) = \left[\;\right]^2$



$\langle X_1 \dots X_{2n} \rangle = (2n-1)!!$ terms
$\langle X^{2n} \rangle_{\text{usual Gaussian}} = \langle \underbrace{X X \dots X}_{2n} \rangle = (2n-1)!! \left(\frac{1}{\alpha}\right)^n$
$= \left(\frac{1}{\alpha}\right)^n \underbrace{[1 + \dots + 1]}_{(2n-1)!!}$
with $= \frac{1}{\alpha}$
Feynmanrule
diag
check $3 + 4 \cdot 3 = 15$ terms
$\sum_{K_{BOT}} G_0(K_{BOT}) =$



$\langle X_1 \dots X_{2n} \rangle = (2n-1)!!$ terms
$\langle X^{2n} \rangle_{\text{usual Gaussian}} = \langle \underbrace{X X \dots X}_{2n} \rangle = (2n-1)!! \left(\frac{1}{\alpha}\right)^n$
$= \left(\frac{1}{\alpha}\right)^n \underbrace{[1 + \dots + 1]}_{(2n-1)!!}$
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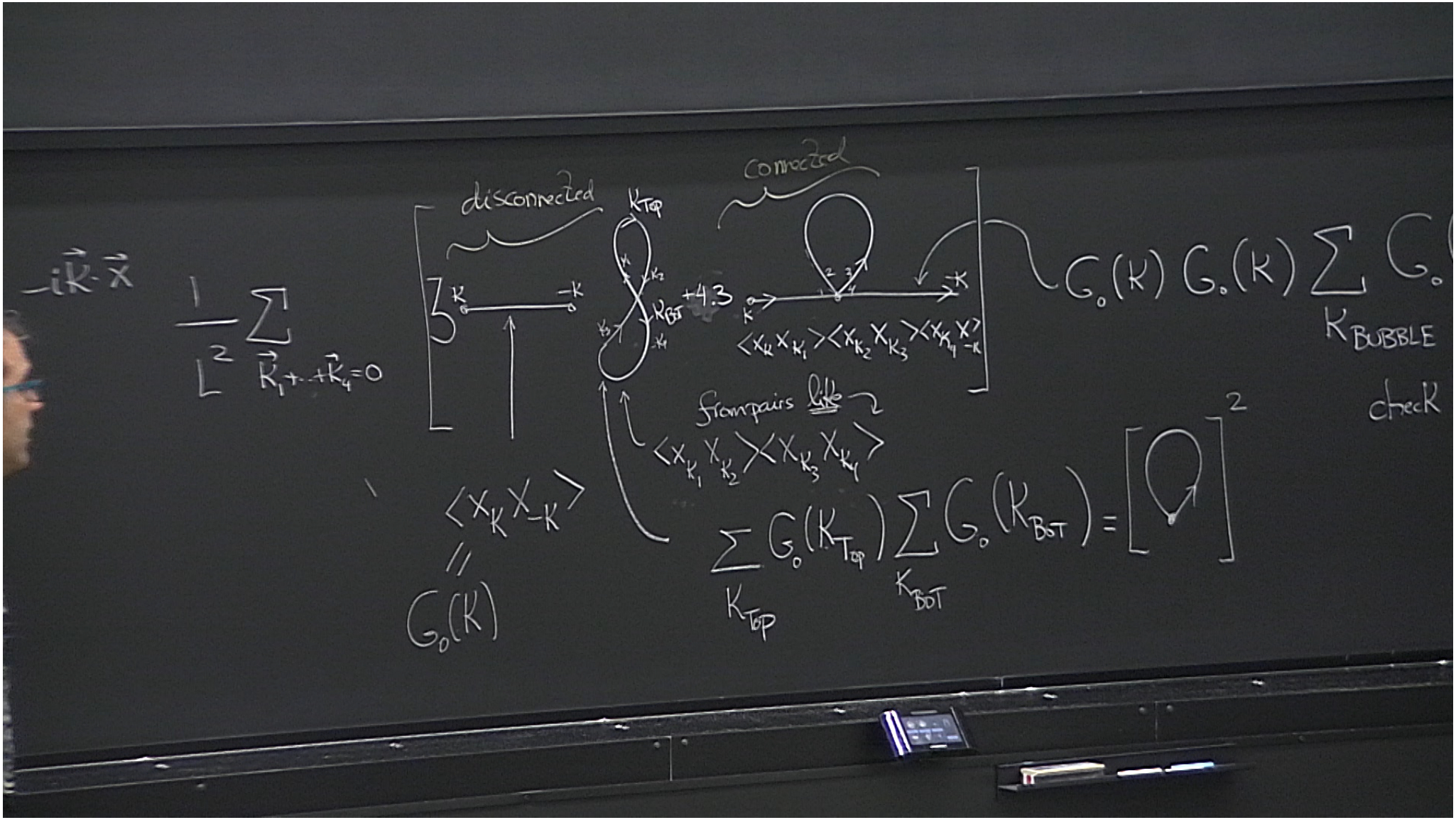
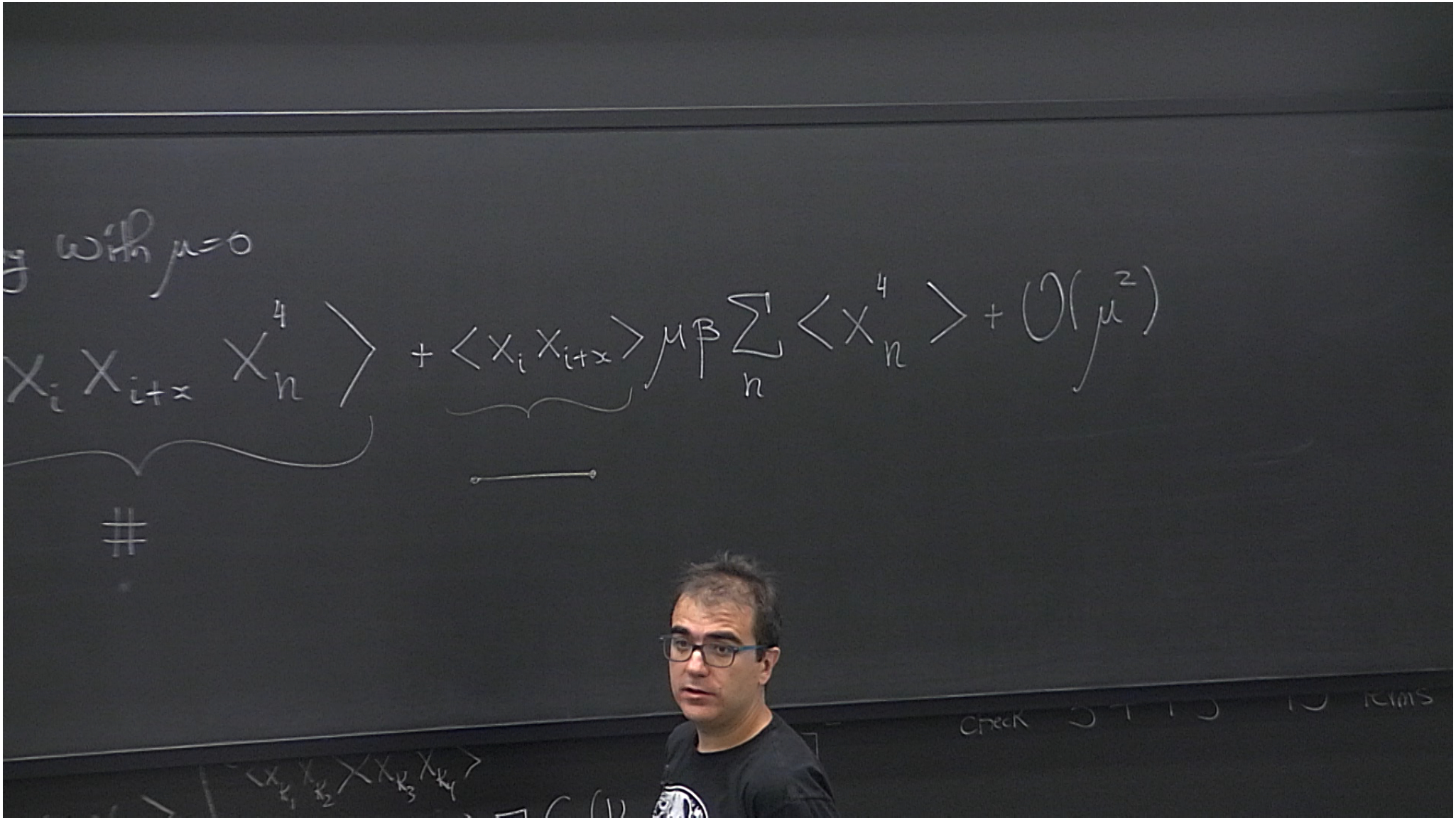
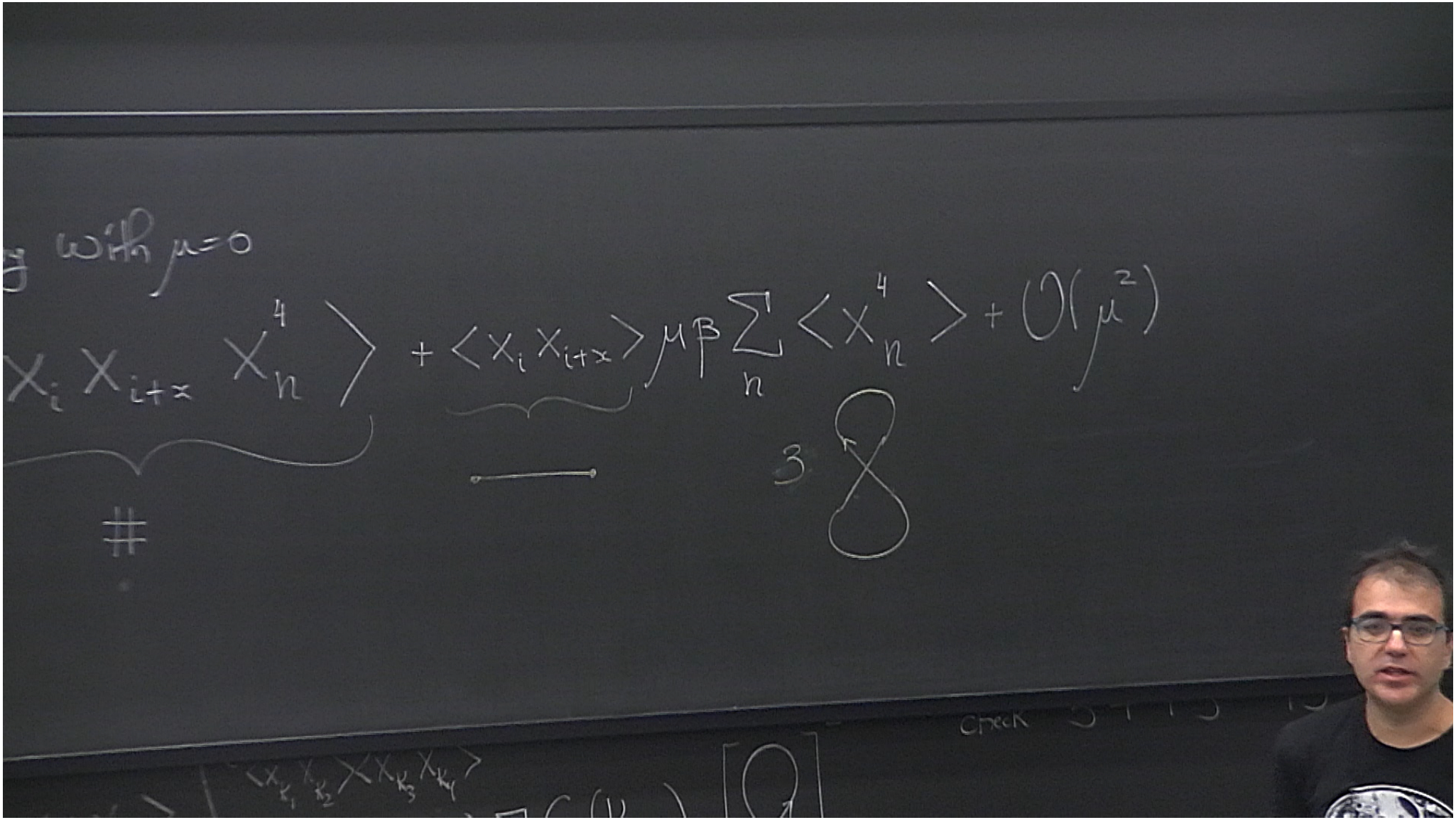
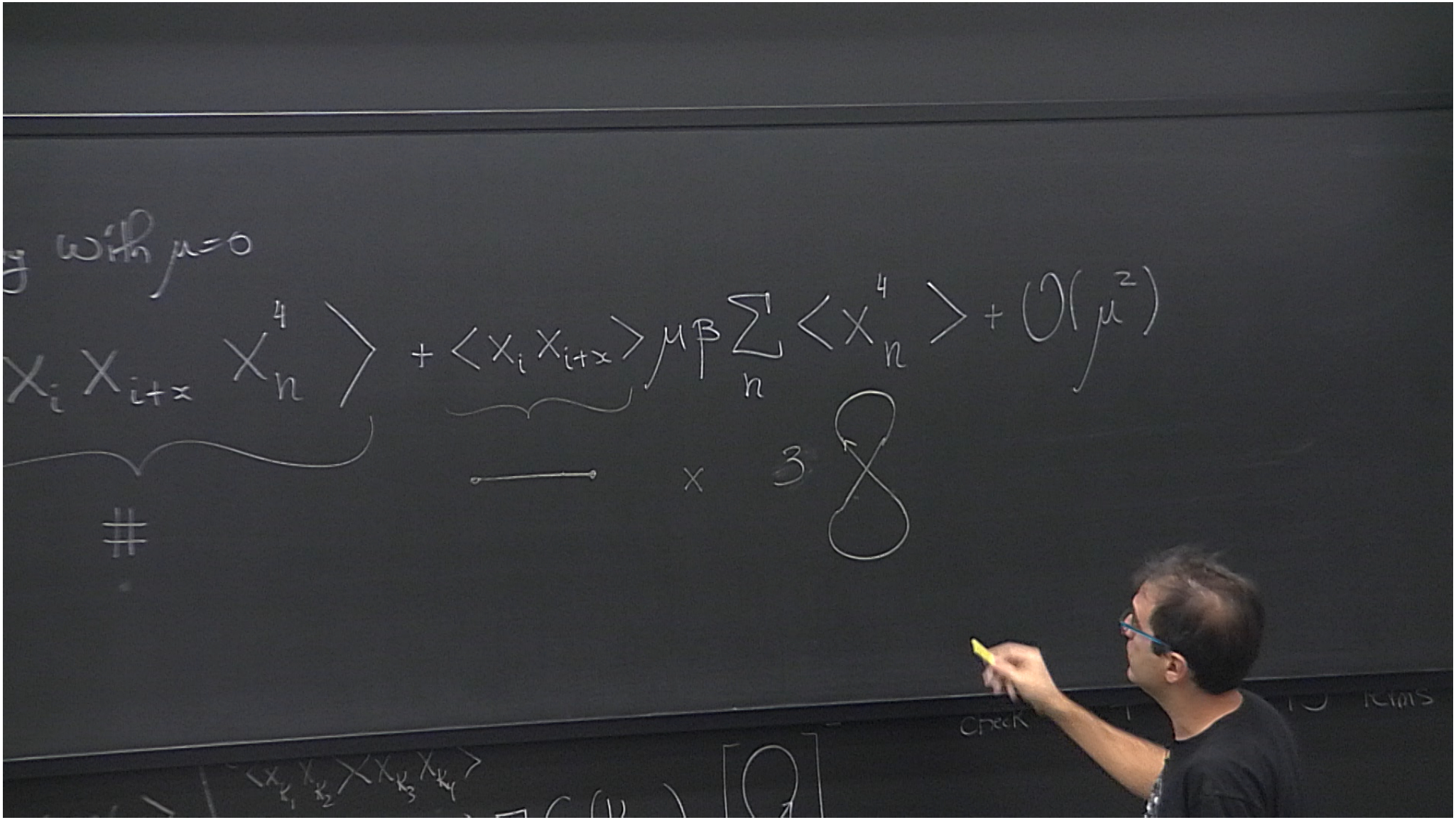
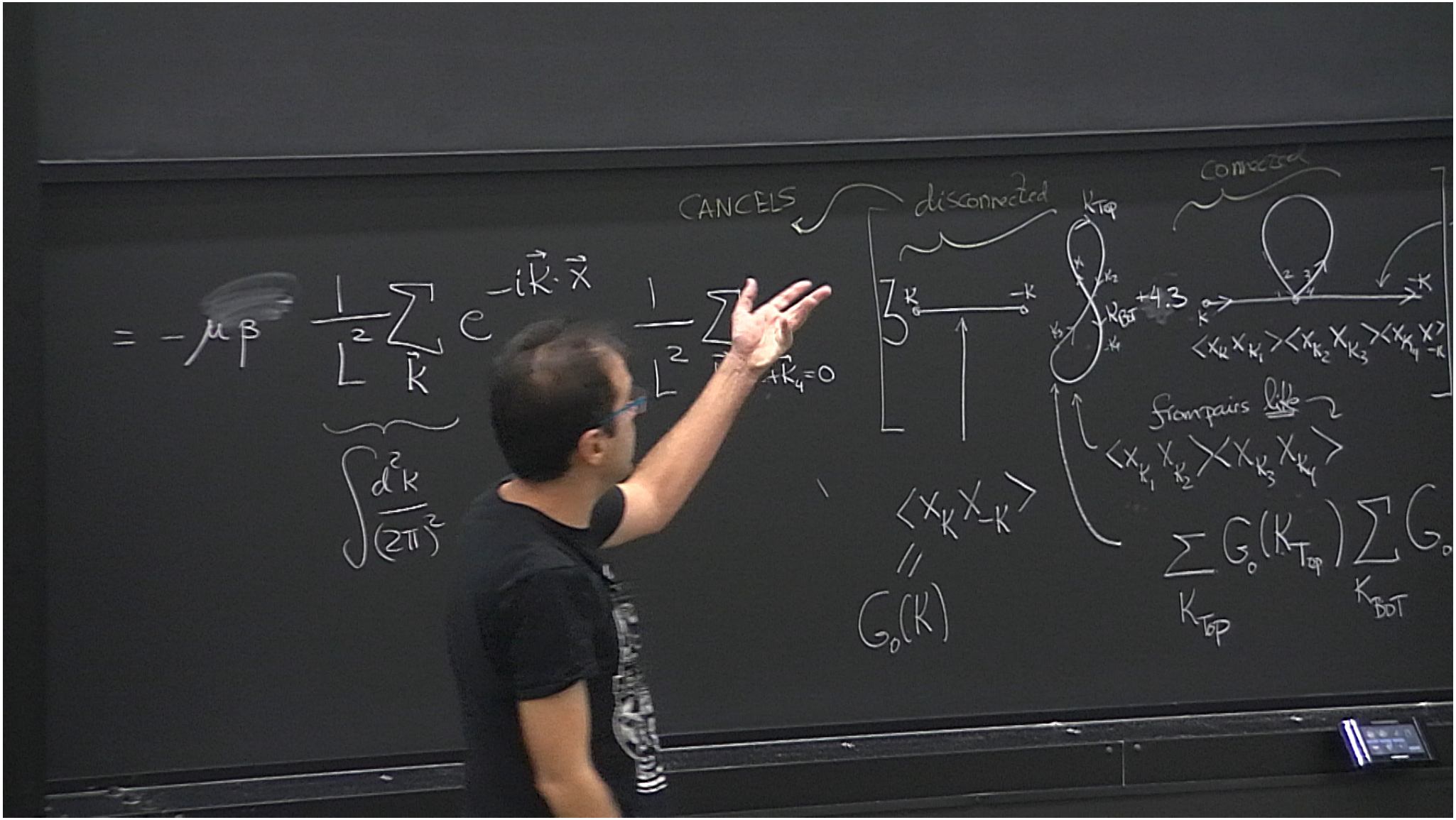
Feynmanrule
diag
check $3 + 4 \cdot 3 = 15$ terms

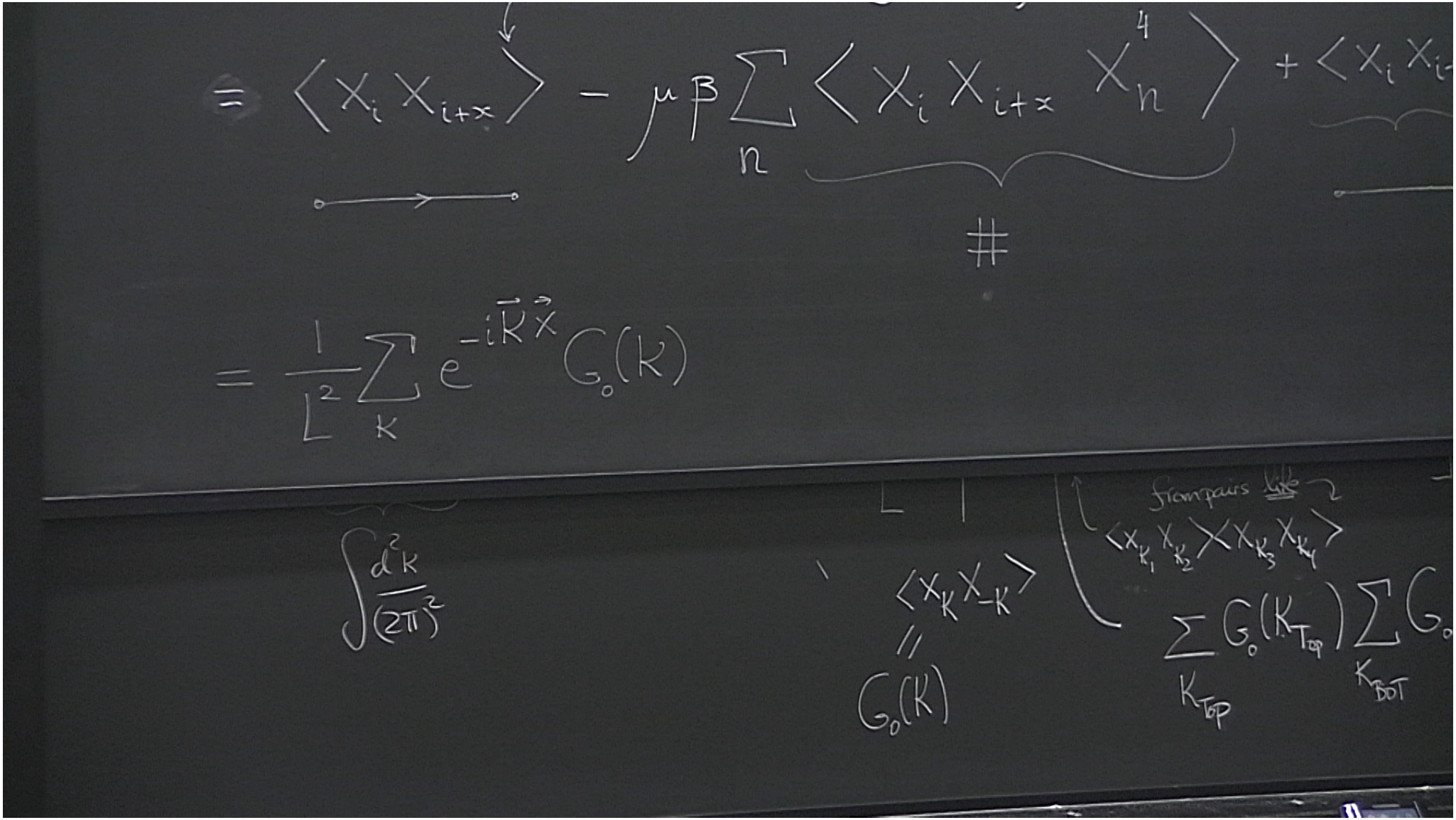

disconnected
connected
K_{Top}
K_{BOT}
+4.3
$-i\vec{k}\cdot\vec{x}$
$\frac{1}{L^2}\sum_{\vec{k}_1+\dots+\vec{k}_4=0}$
$\langle x_k x_{k_1}\rangle\langle x_{k_2} x_{k_3}\rangle\langle x_{k_4} x_{-k}\rangle$
from pairs like
$\langle x_{k_1} x_{k_2}\rangle\langle x_{k_3} x_{k_4}\rangle$
$\langle x_k x_{-k}\rangle$
$G_0(k)$
$\sum_{K_{Top}} G_0(K_{Top}) \sum_{K_{BOT}} G_0(K_{BOT}) =$
$G_0(k)\, G_0(k) \sum_{K_{BUBBLE}} G_0$
check

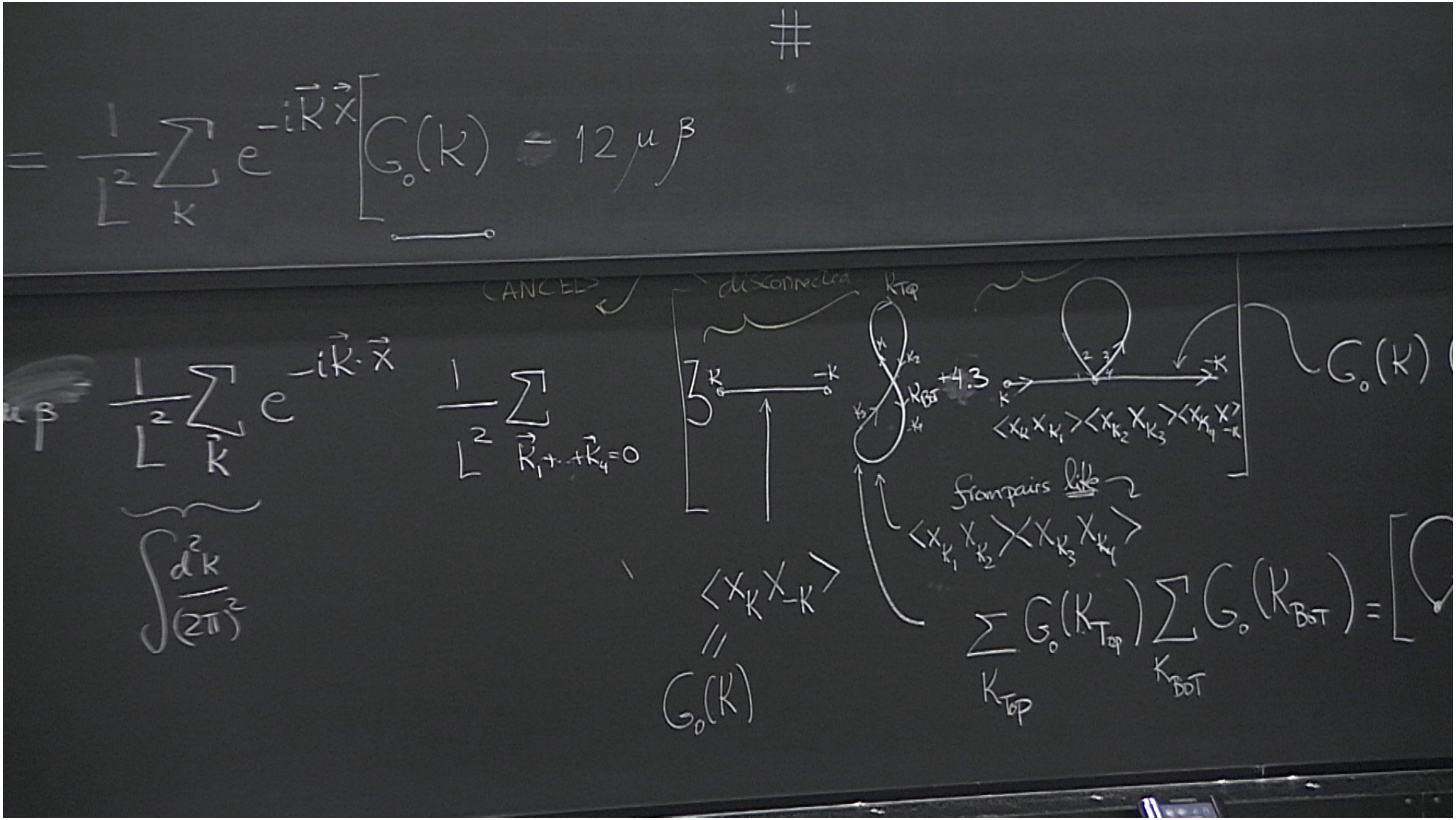

with $\mu=0$
$\langle X_i X_{i+x} X_n^4 \rangle + \langle X_i X_{i+x} \rangle \mu\beta \sum_n \langle X_n^4 \rangle + O(\mu^2)$
#
check


with $\mu=0$
$\langle X_i X_{i+x} X_n^4 \rangle + \langle X_i X_{i+x} \rangle \mu\beta \sum_n \langle X_n^4 \rangle + O(\mu^2)$
3
#

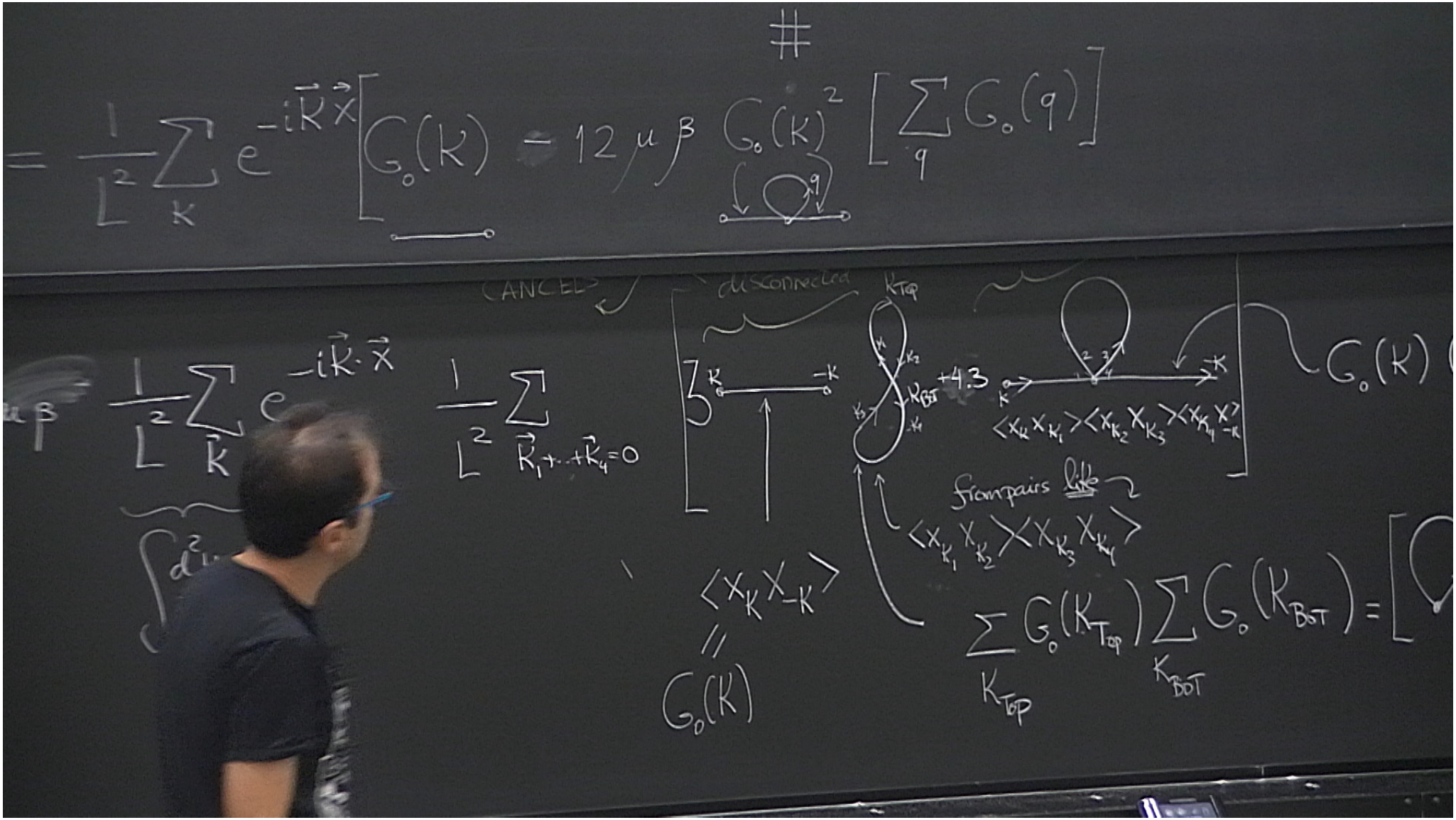

with $\mu=0$
$\langle X_i X_{i+x} X_n^4 \rangle + \langle X_i X_{i+x} \rangle \mu\beta \sum_n' \langle X_n^4 \rangle + O(\mu^2)$
$\#$
$\times 3$
check

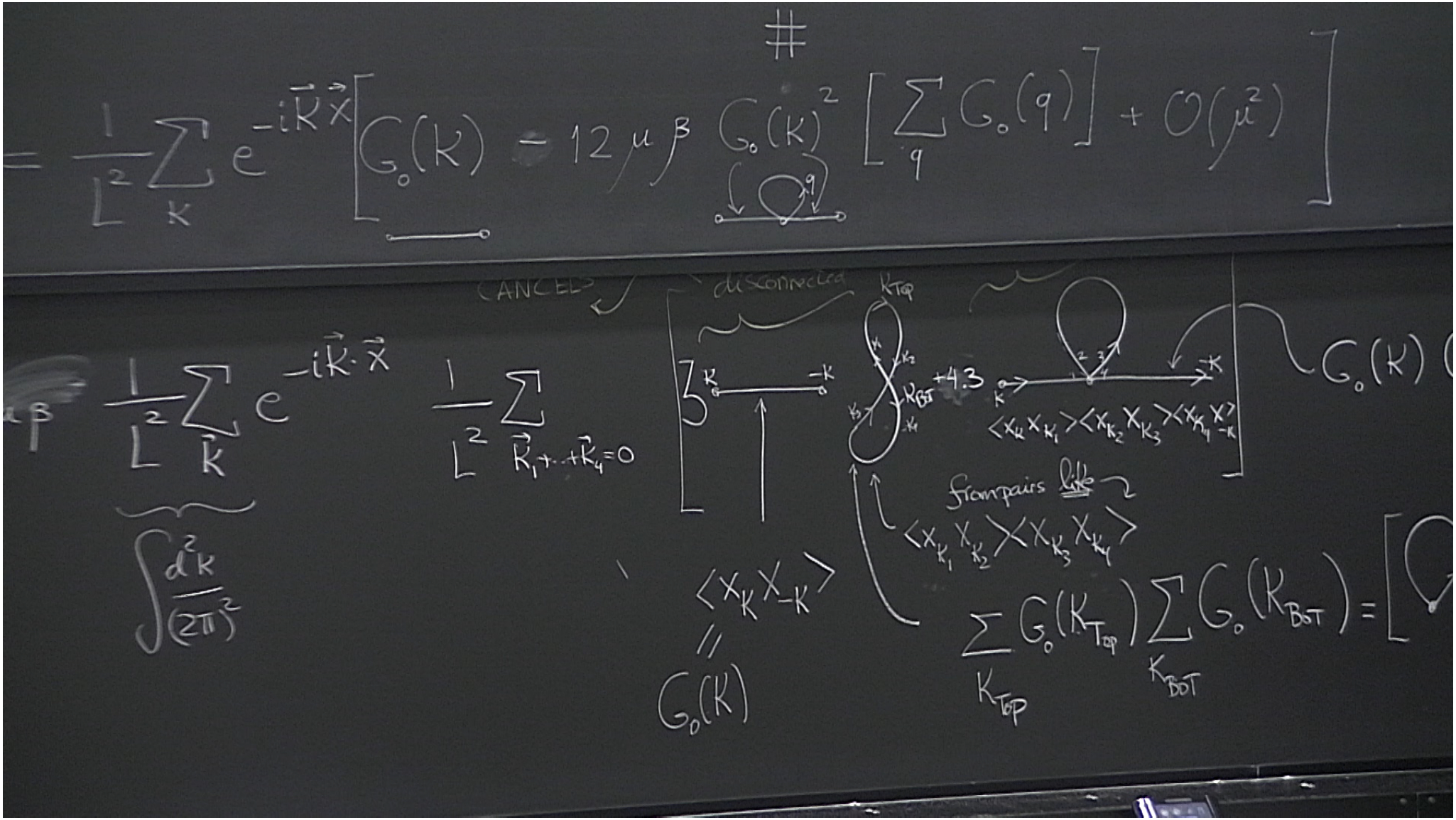

CANCELS
disconnected
connected
$= -\mu\beta \frac{1}{L^2}\sum_{\vec{K}} e^{-i\vec{K}\cdot\vec{x}} \int \frac{d^2k}{(2\pi)^2}$
$\frac{1}{L^2}\sum$
$+\vec{K}_4 = 0$
$\langle X_K X_{-K} \rangle$
$= G_0(K)$
K_{Top}
K_{BOT}
$+4.3$
$\langle X_{K_1} X_{K_2} \rangle \langle X_{K_3} X_{K_4} \rangle$
from pairs like
$\sum_{K_{Top}} G_0(K_{Top}) \sum_{K_{BOT}} G_0$

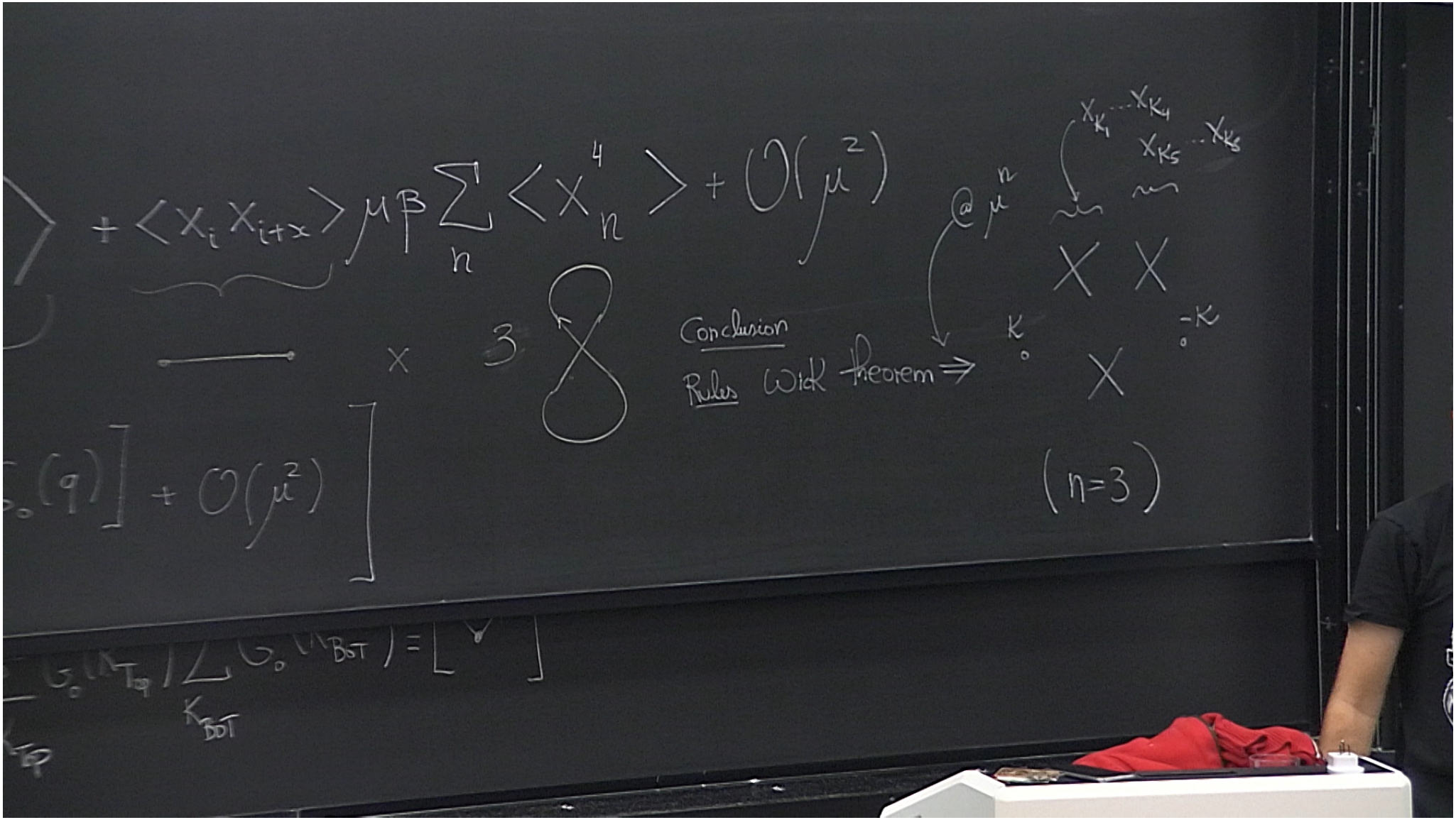

$= \langle X_i X_{i+x} \rangle - \mu\beta \sum_n \langle X_i X_{i+x} X_n^4 \rangle + \langle X_i X_{i+x} \rangle$
#
$= \frac{1}{L^2} \sum_k e^{-i\vec{k}\vec{x}} G_0(k)$
$\int \frac{d^2k}{(2\pi)^2}$
$\langle X_k X_{-k} \rangle$
$= G_0(k)$
from pairs like 2
$\langle X_{k_1} X_{k_2} \rangle \langle X_{k_3} X_{k_4} \rangle$
$\sum_{k_{TOP}} G_0(k_{T_{OP}}) \sum_{k_{BOT}} G_0$

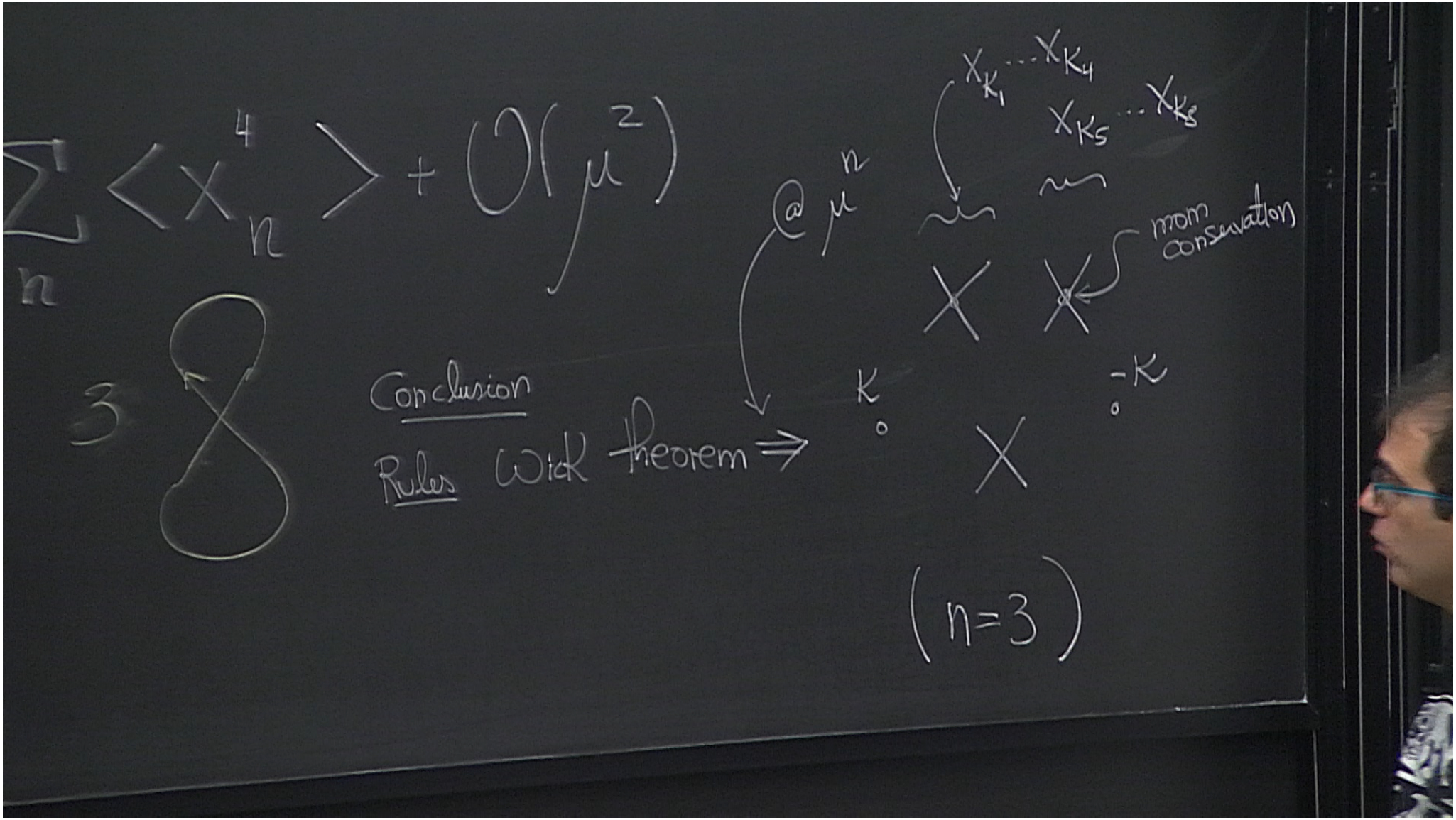


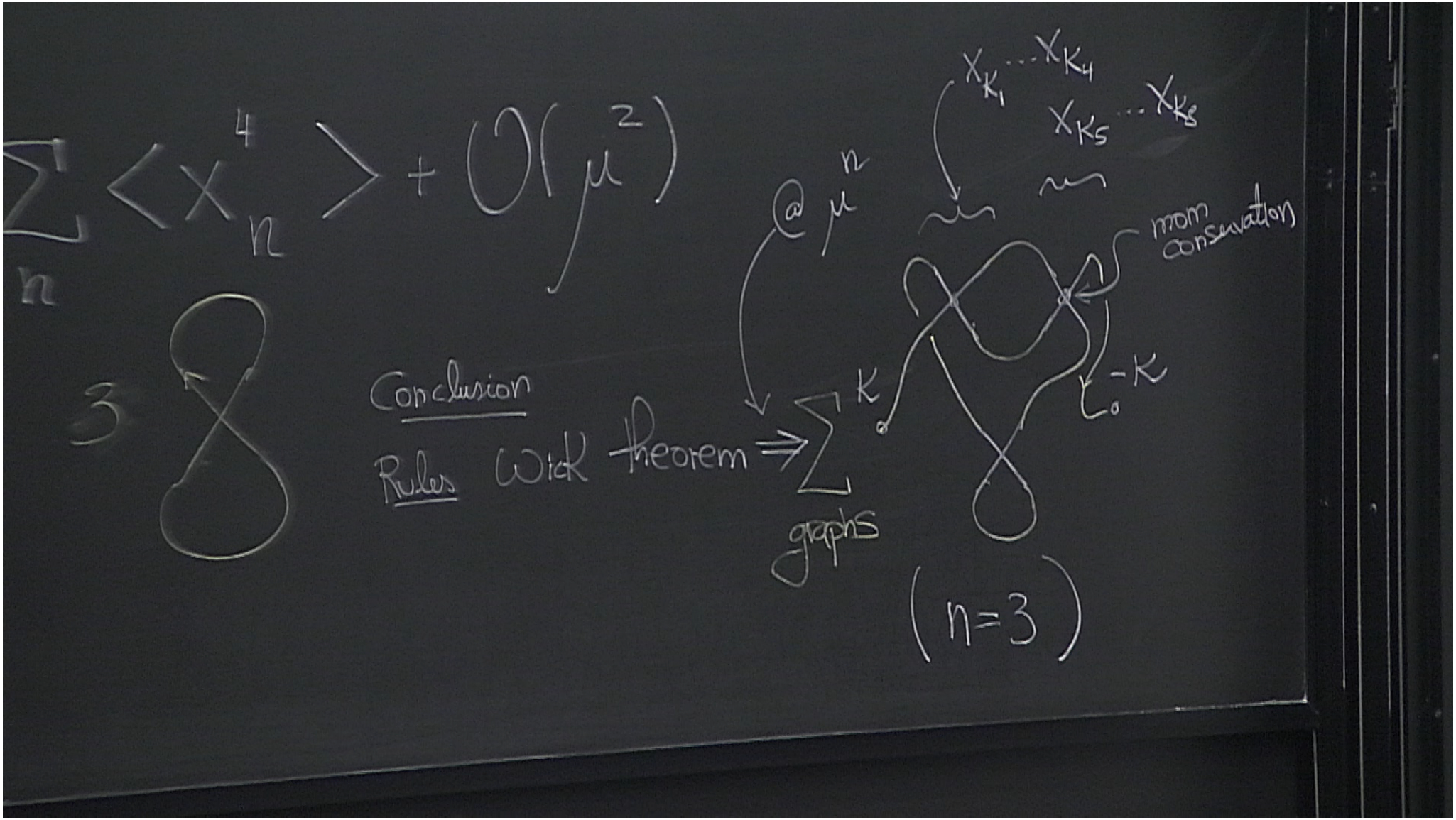
$= \frac{1}{L^2}\sum_k e^{-i\vec{k}\vec{x}}\left[G_0(k) - 12\mu\beta\right.$
CANCEL
disconnected
k_{Top}
k_{BOT}
+4.3
$\langle x_k x_{k_1}\rangle\langle x_{k_2} x_{k_3}\rangle\langle x_{k_4} x_{-k}\rangle$
$G_0(k)$
$\mu\beta = \frac{1}{L^2}\sum_{\vec{k}} e^{-i\vec{k}\cdot\vec{x}}$
$\frac{1}{L^2}\sum_{\vec{k}_1+\dots+\vec{k}_4=0}$
$\int \frac{d^2k}{(2\pi)^2}$
from pairs like
$\langle x_{k_1} x_{k_2}\rangle\langle x_{k_3} x_{k_4}\rangle$
$\langle x_k x_{-k}\rangle$
$G_0(k)$
$\sum_{k_{Top}} G_0(k_{Top}) \sum_{k_{BOT}} G_0(k_{BOT}) =$

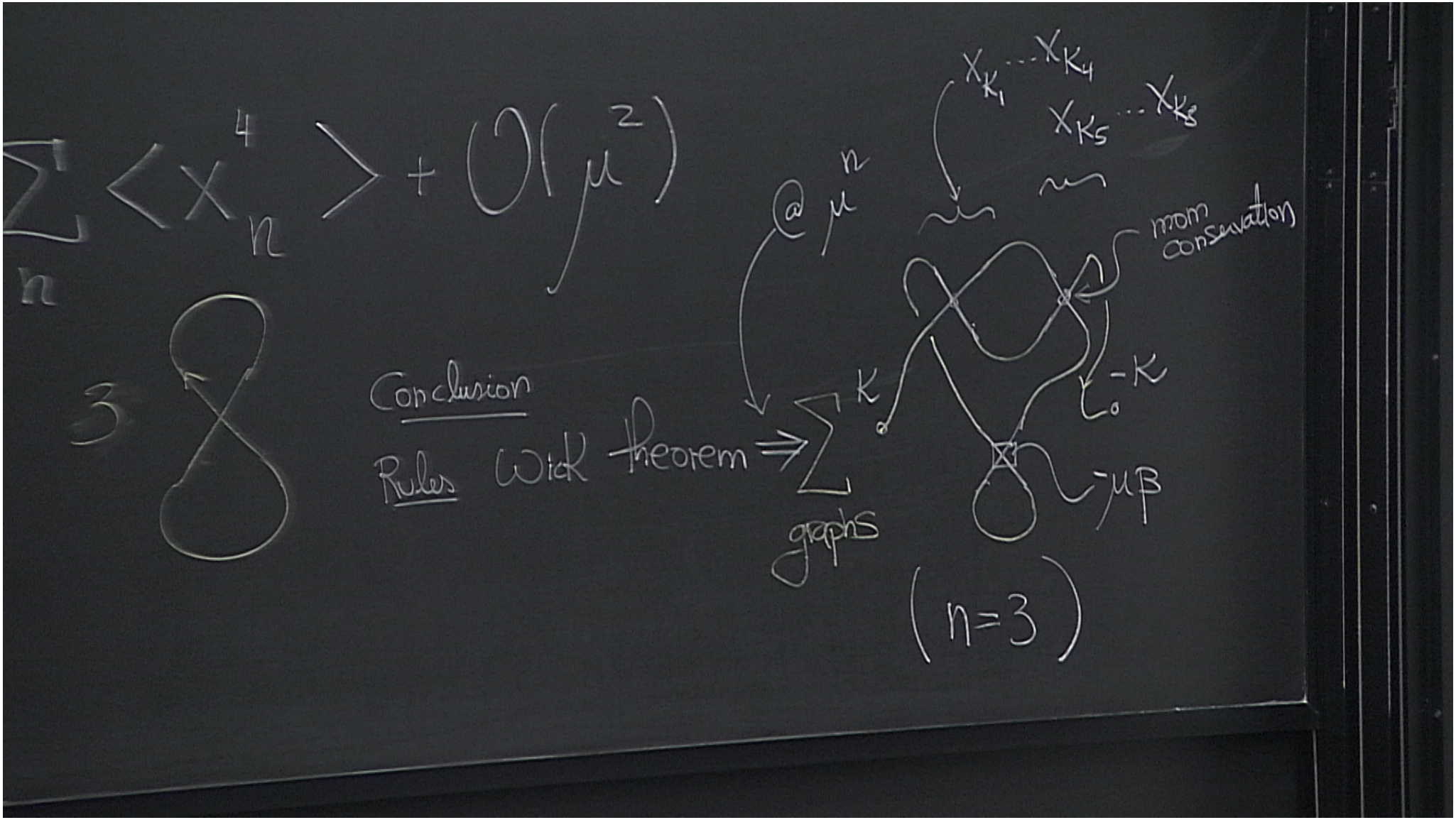

CANCEL
disconnected
from pairs like

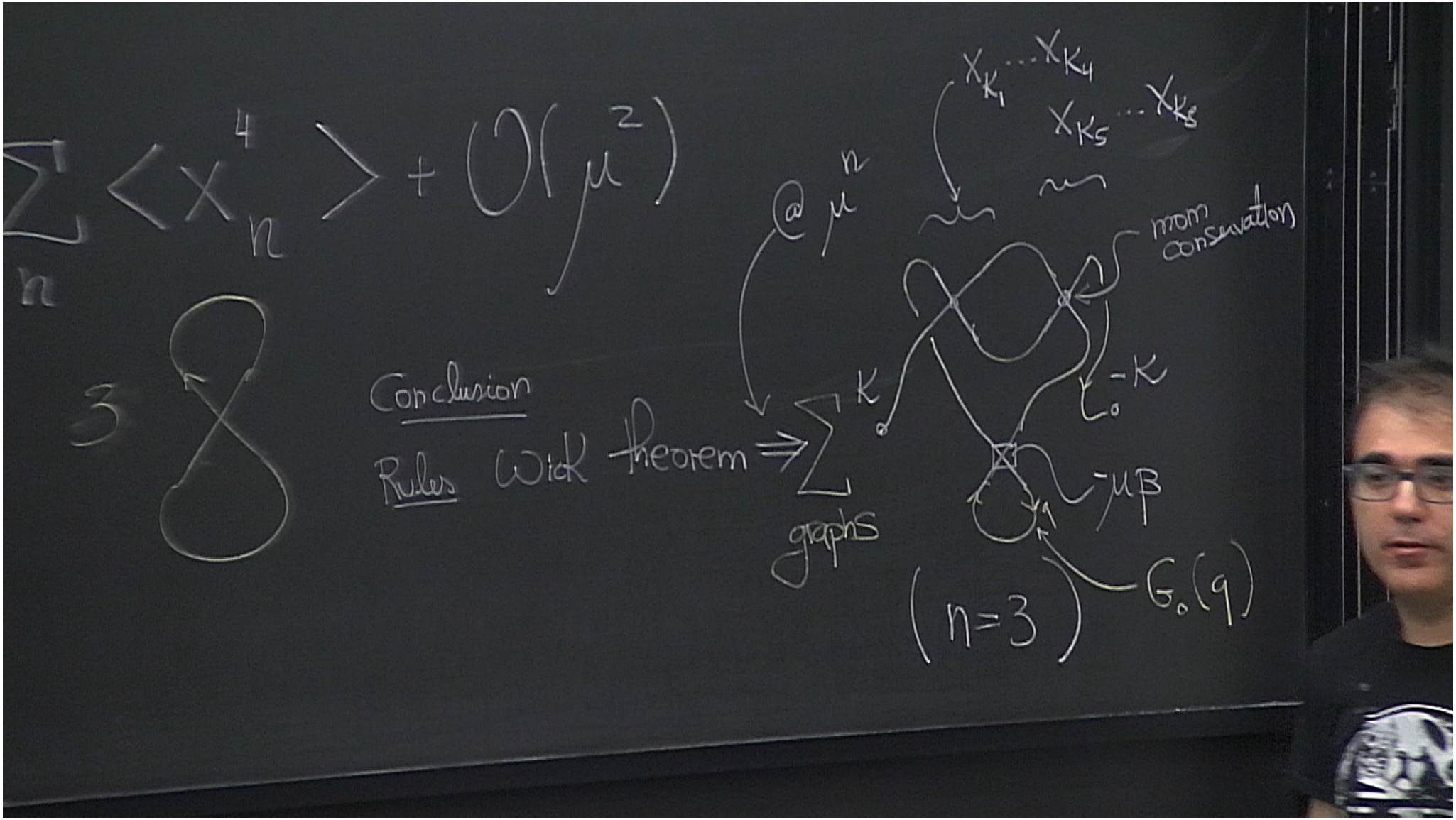

CANCELS
disconnected
+4.3
from pairs like

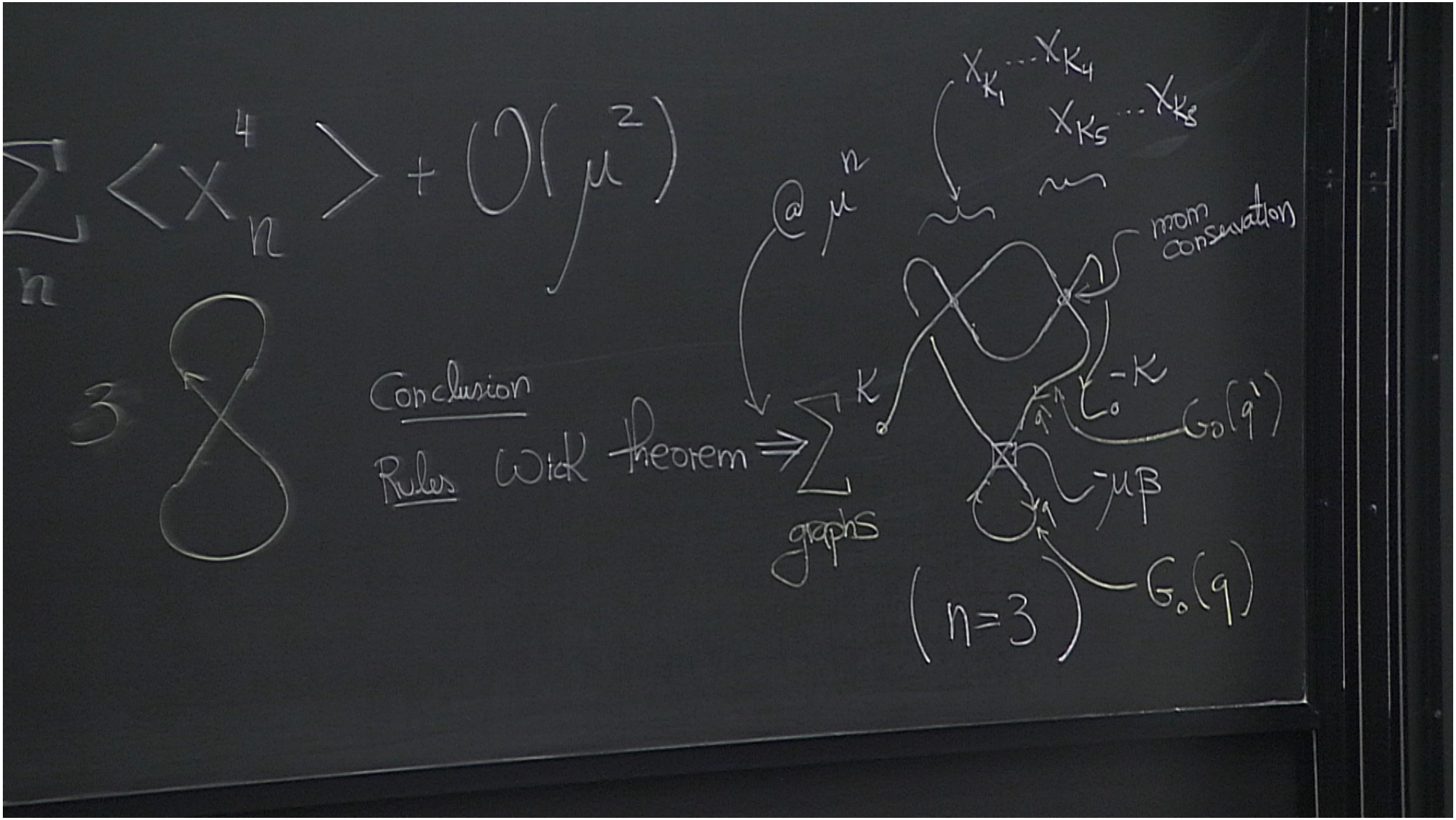

Conclusion
Rules Wick theorem ⇒
(n=3)

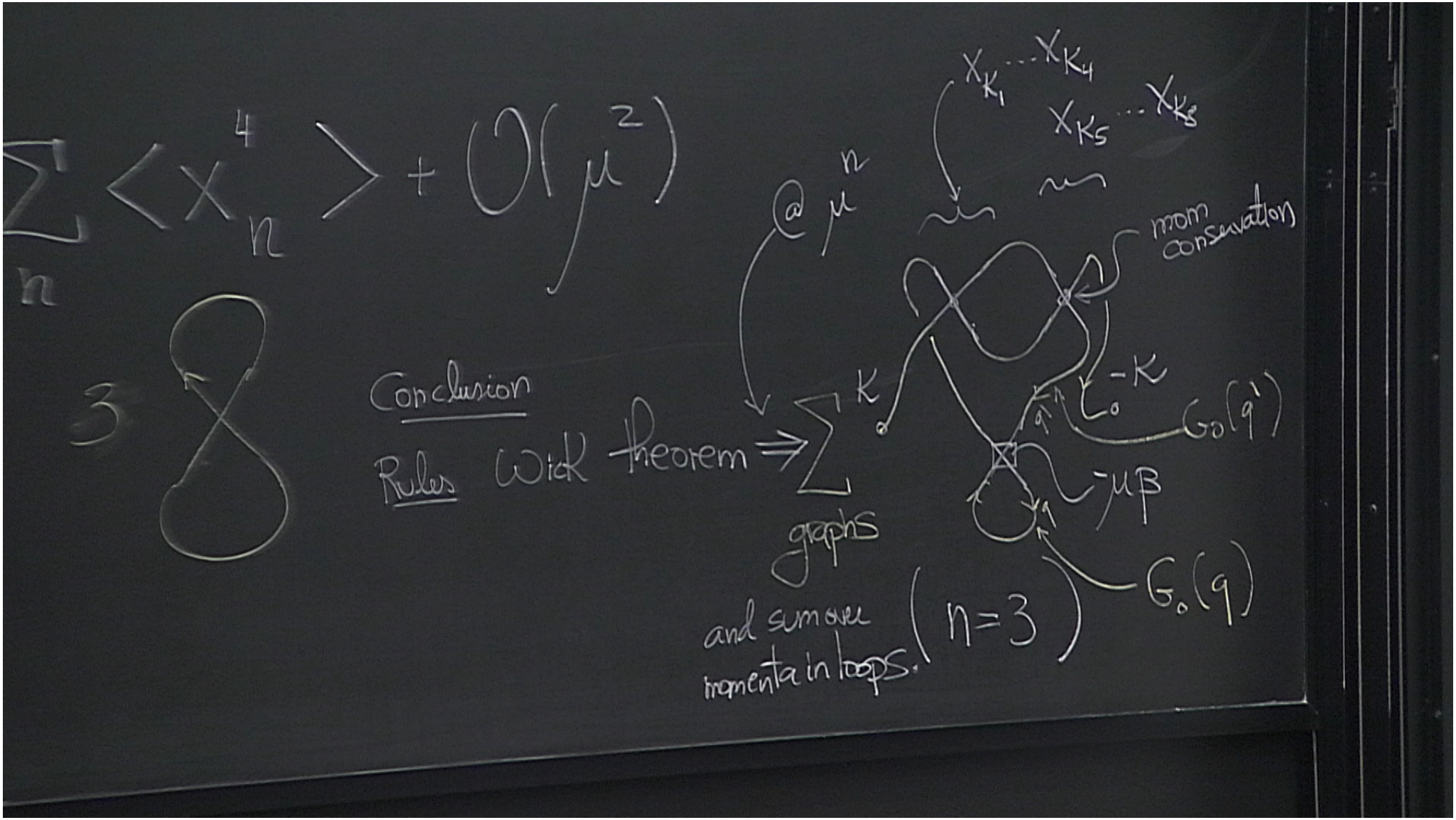

$\sum_n \langle x_n^4 \rangle + O(\mu^2)$
3
Conclusion
Rules Wick theorem ⇒
@ μ^n
$x_{k_1} \dots x_{k_4}$
$x_{k_5} \dots x_{k_8}$
mom conservation
k
$-k$
$(n=3)$

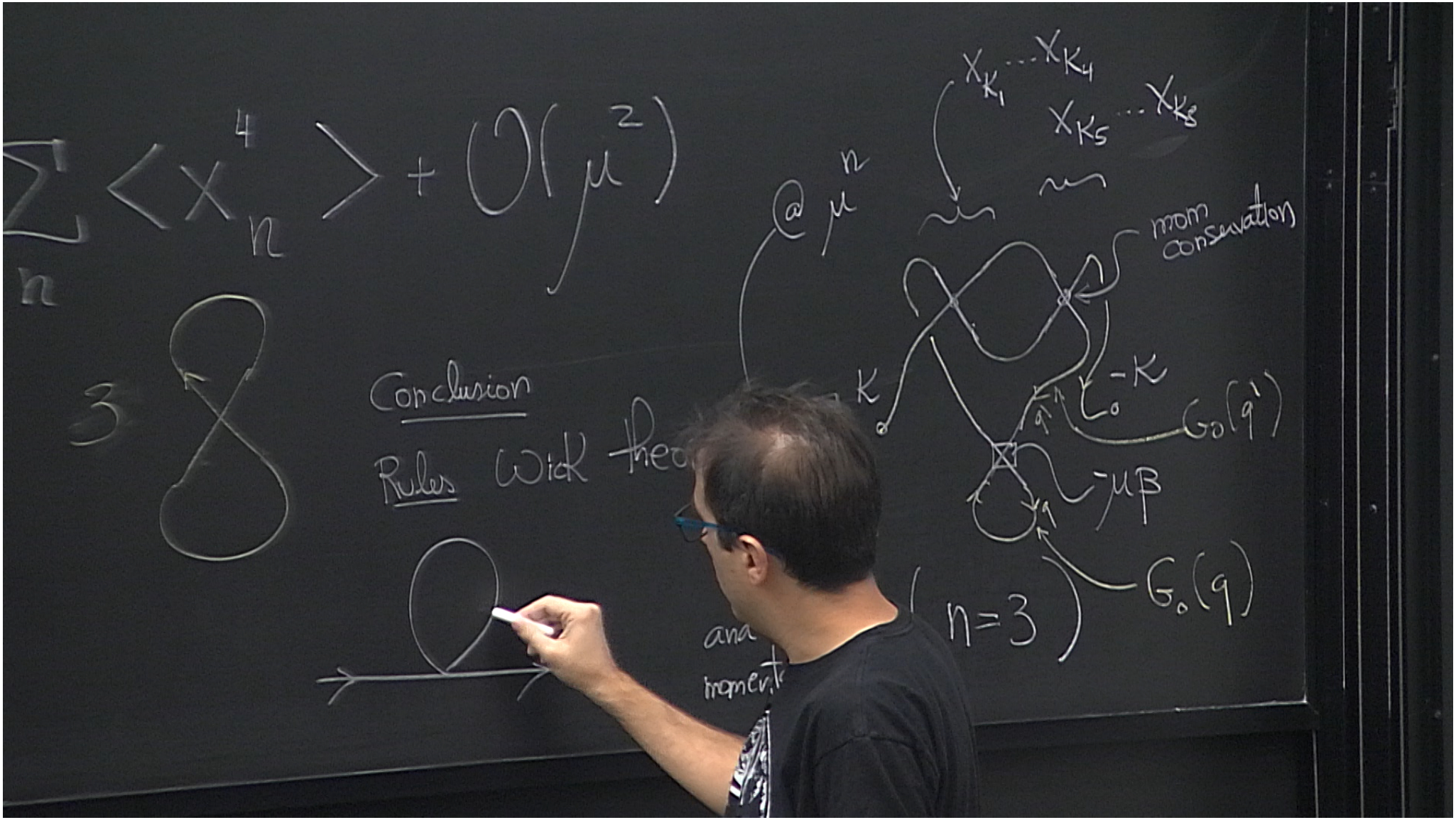

$\sum_n \langle x_n^4 \rangle + O(\mu^2)$
3
Conclusion
Rules Wick theorem $\Rightarrow \sum_{\text{graphs}}$
@ μ^n
$x_{k_1} \cdots x_{k_4}$
$x_{k_5} \cdots x_{k_8}$
mom conservation
k
$-k$
$(n=3)$


$\sum_n \langle x_n^4 \rangle + O(\mu^2)$
3
Conclusion
Rules Wick theorem $\Rightarrow \sum_{\text{graphs}}$
$@\mu^n$
$x_{k_1} \cdots x_{k_4}$
$x_{k_5} \cdots x_{k_8}$
mom conservation
$-\kappa$
$-\mu\beta$
$(n=3)$

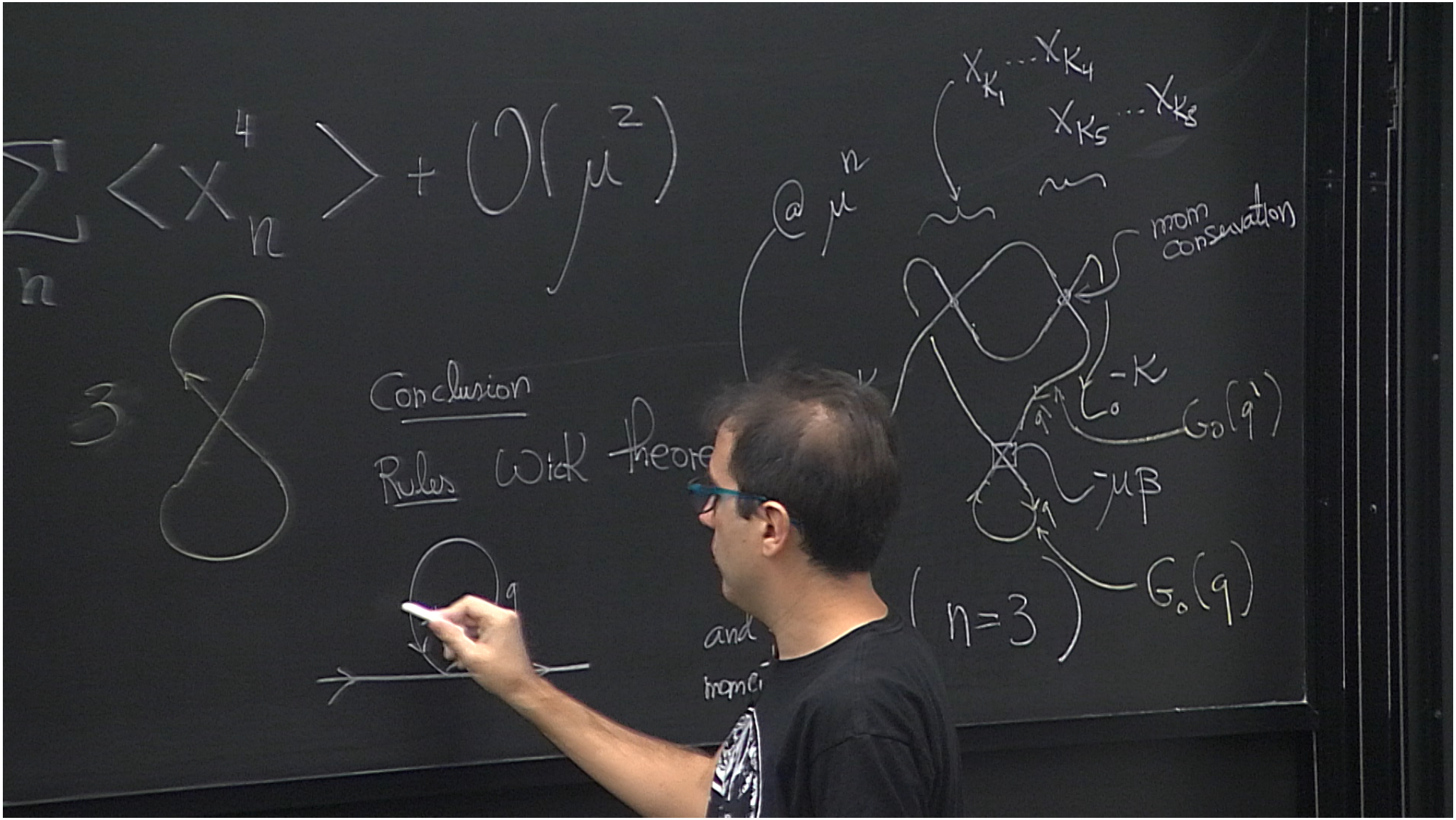

$\sum_n \langle x_n^4 \rangle + O(\mu^2)$
3
Conclusion
Rules Wick theorem $\Rightarrow \sum_{\text{graphs}}$
$@\mu^n$
$x_{k_1} \dots x_{k_4}$
$x_{k_5} \dots x_{k_8}$
mom conservation
$-K$
$-\mu B$
$G_0(q)$
$(n=3)$

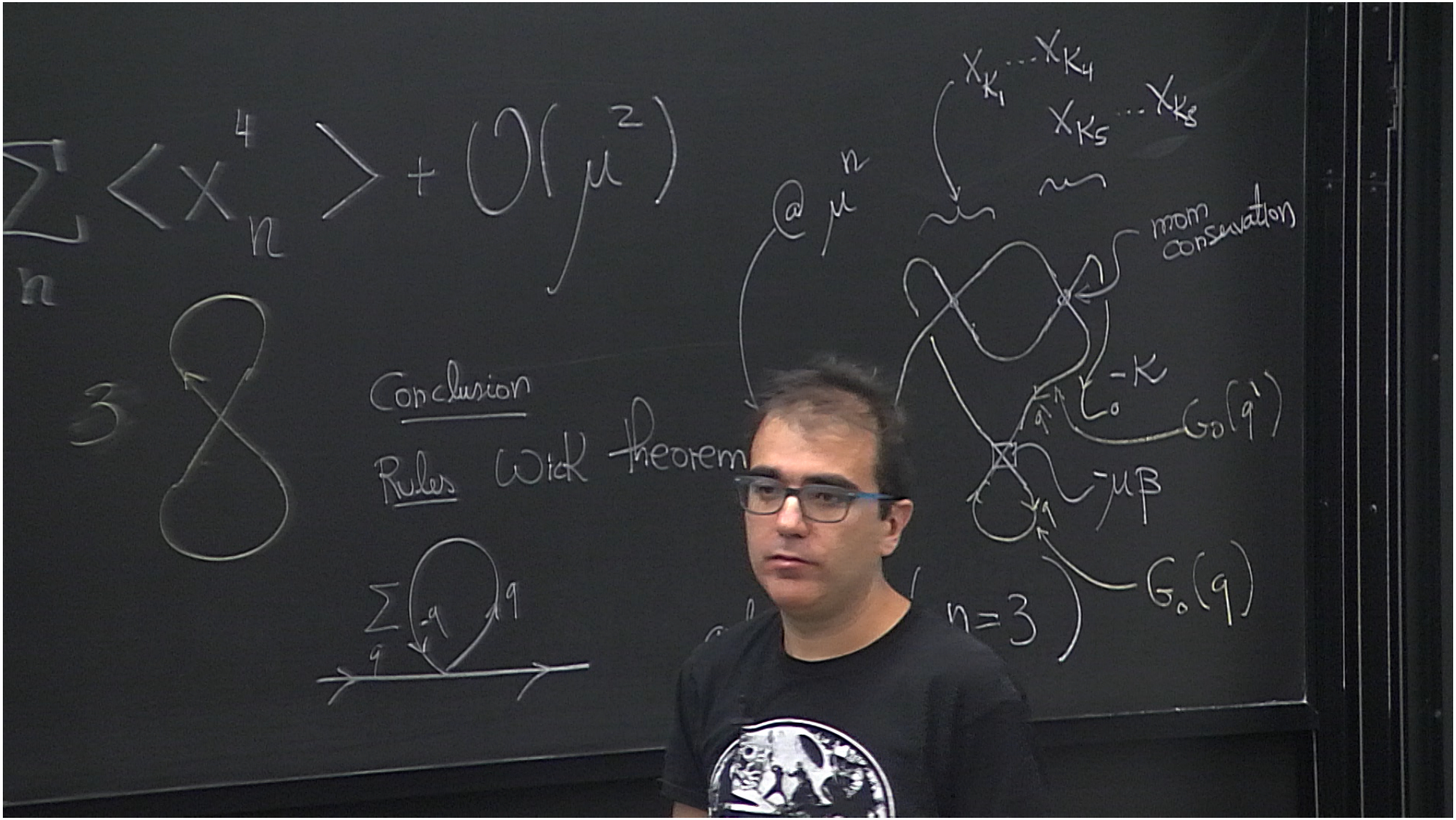

$\sum_n \langle x_n^4 \rangle + O(\mu^2)$
3
Conclusion
Rules Wick theorem $\Rightarrow \sum_{\text{graphs}}$
$@\ \mu^n$
$x_{k_1} \cdots x_{k_4}$
$x_{k_5} \cdots x_{k_8}$
mom conservation
$-k$
$G_0(q')$
$-\mu\beta$
$G_0(q)$
$(n=3)$

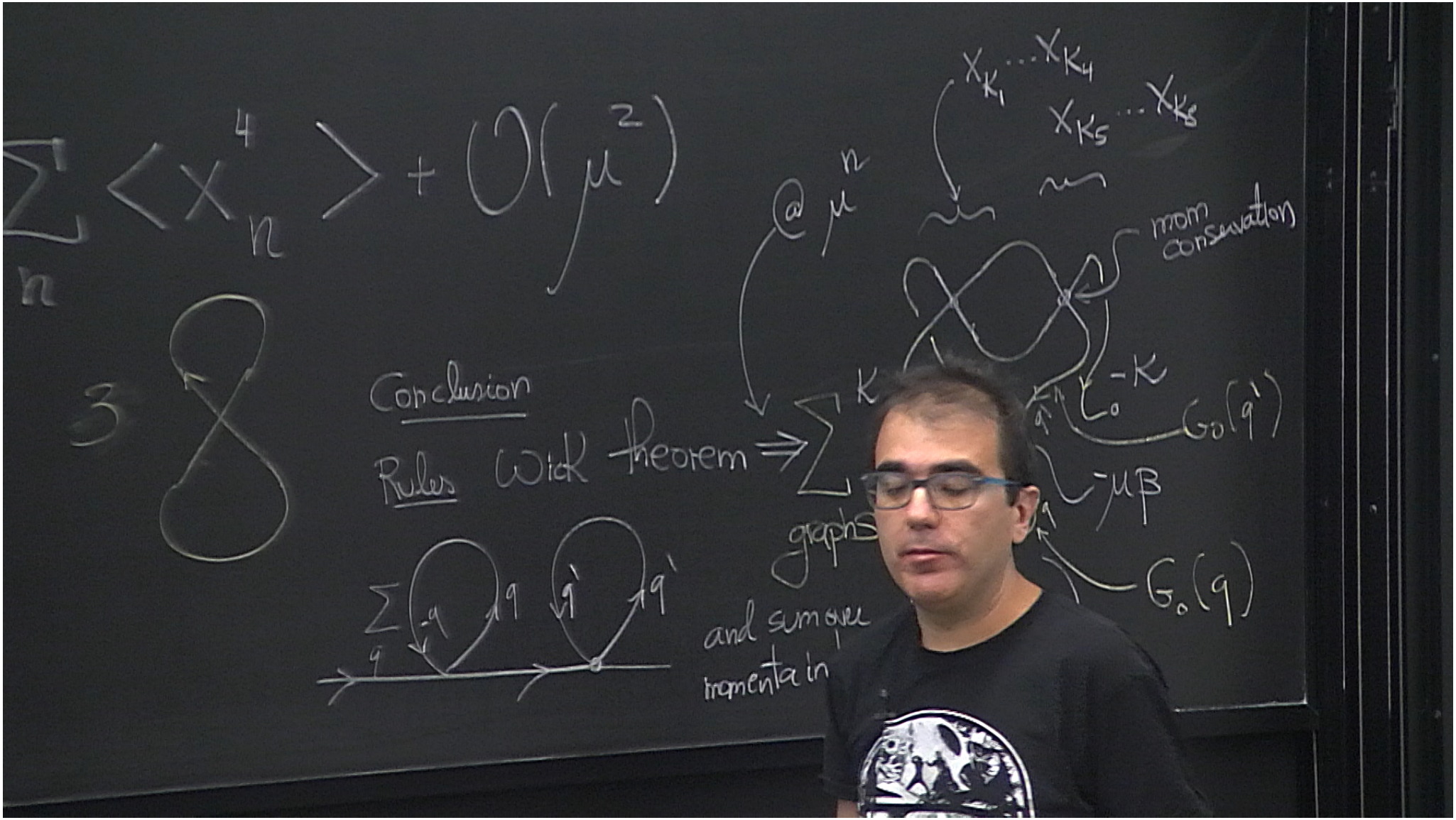

$\sum_n \langle x_n^4 \rangle + O(\mu^2)$
3
Conclusion
Rules Wick theorem $\Rightarrow \sum_{\text{graphs}}$
and sum over momenta in loops.
$@\mu^n$
$x_{k_1} \cdots x_{k_4}$
$x_{k_5} \cdots x_{k_8}$
mom conservation
$-K$
$G_0(q')$
$-\mu\beta$
$G_0(q)$
$(n=3)$

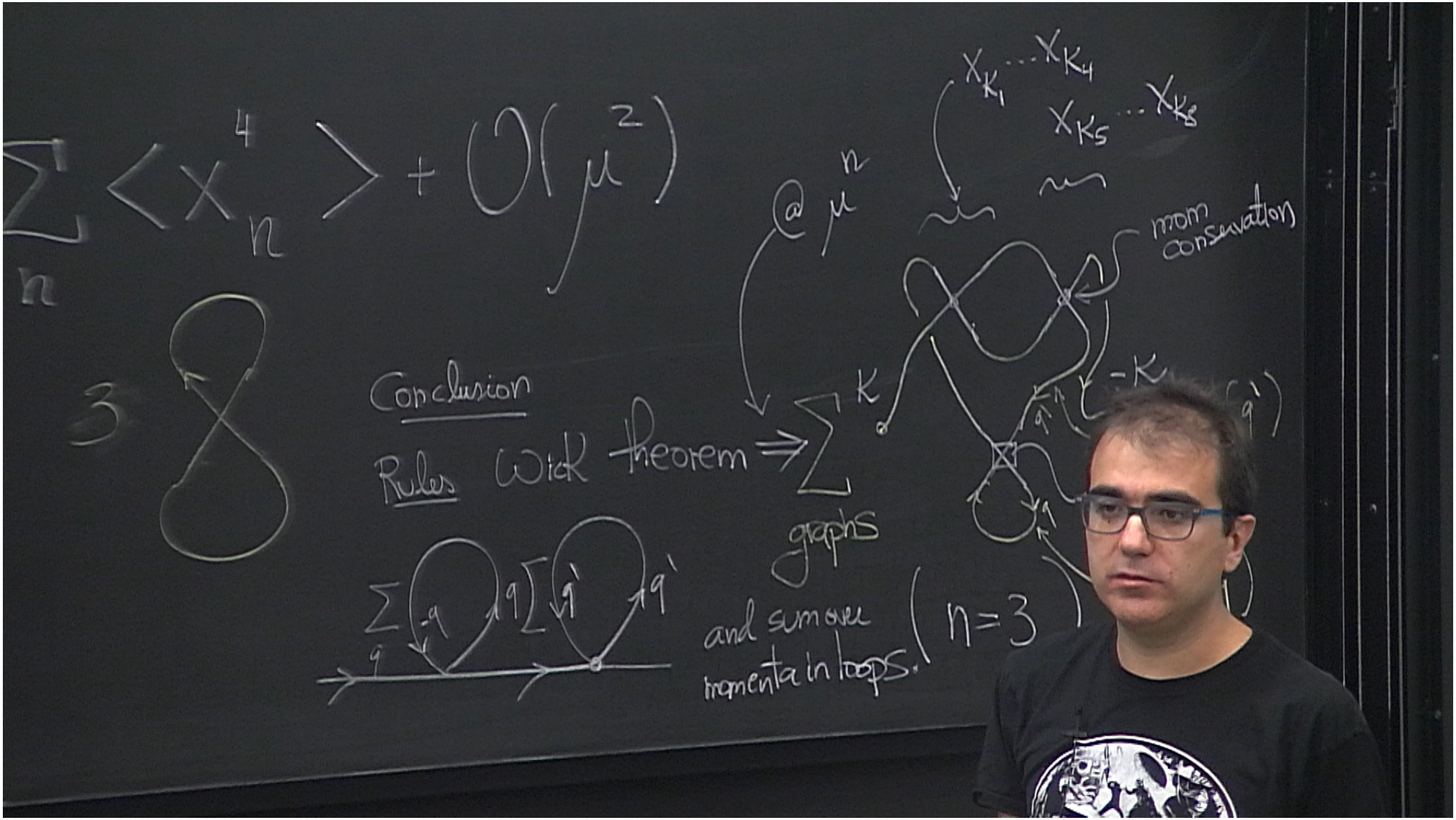


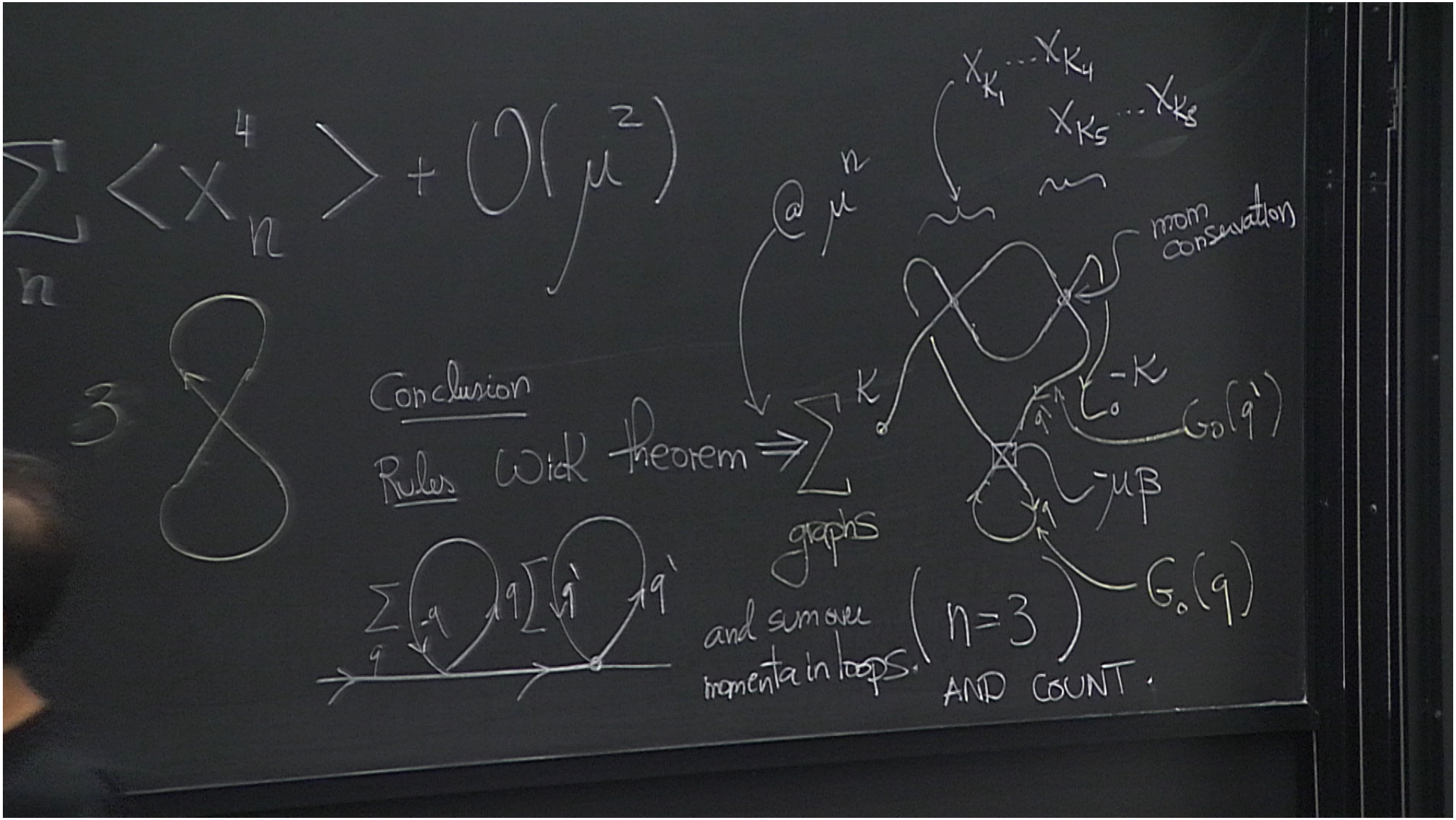
$\sum_n \langle x_n^4 \rangle + O(\mu^2)$
$@\ \mu^n$
$x_{k_1} \cdots x_{k_4}$
$x_{k_5} \cdots x_{k_8}$
mom conservation
$-k$
$G_0(q')$
$-\mu\beta$
$G_0(q)$
$n=3$
Conclusion
Rules Wick theo
ana
moment

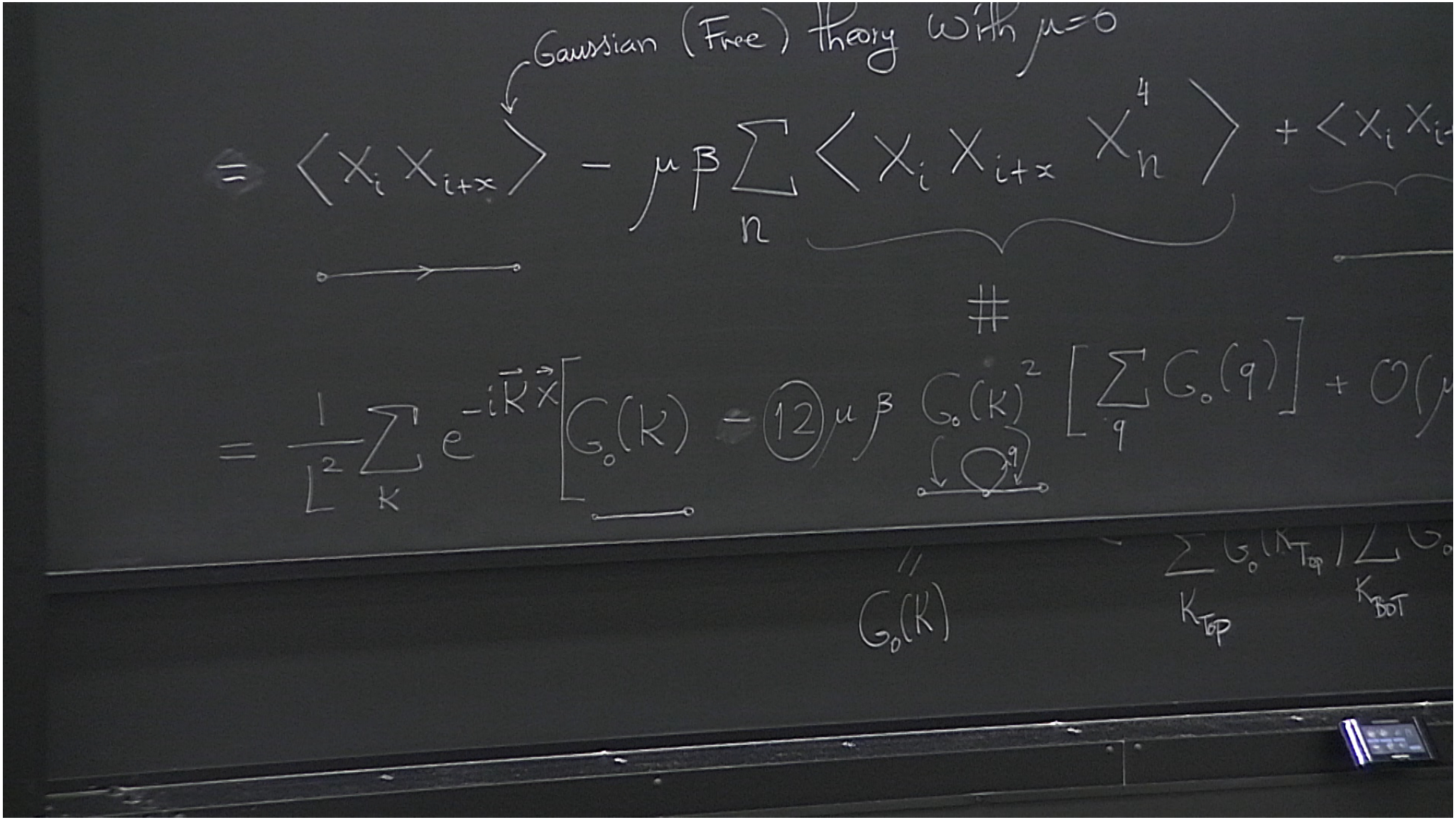

Conclusion
Rules Wick theore
mom conservation
$G_0(q')$
$-\mu\beta$
$G_0(q)$
$(n=3)$
and
$@\mu^n$
$X_{K_1} \cdots X_{K_4}$
$X_{K_5} \cdots X_{K_8}$

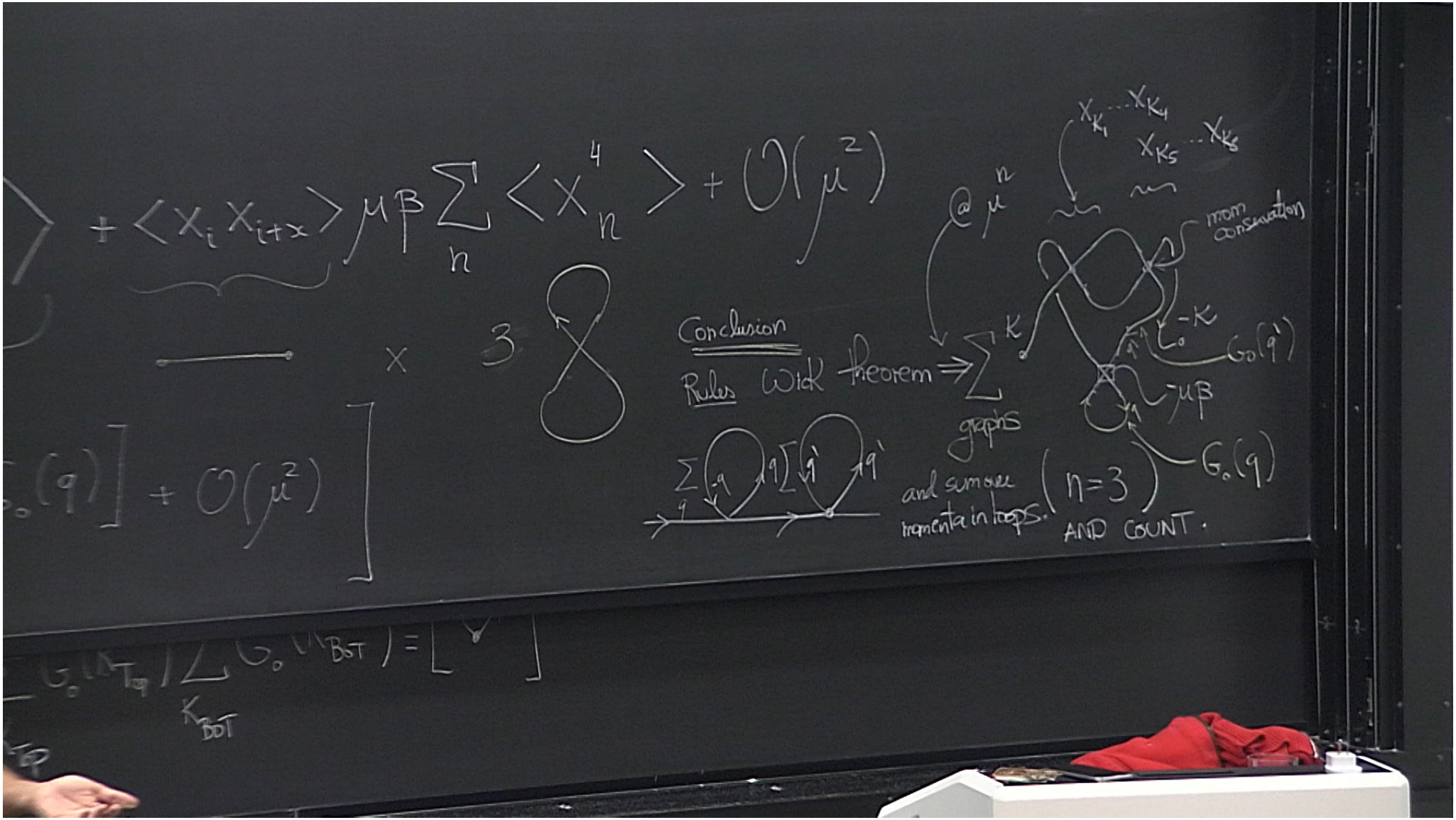

Conclusion
Rules Wick theorem
mom conservation
n=3

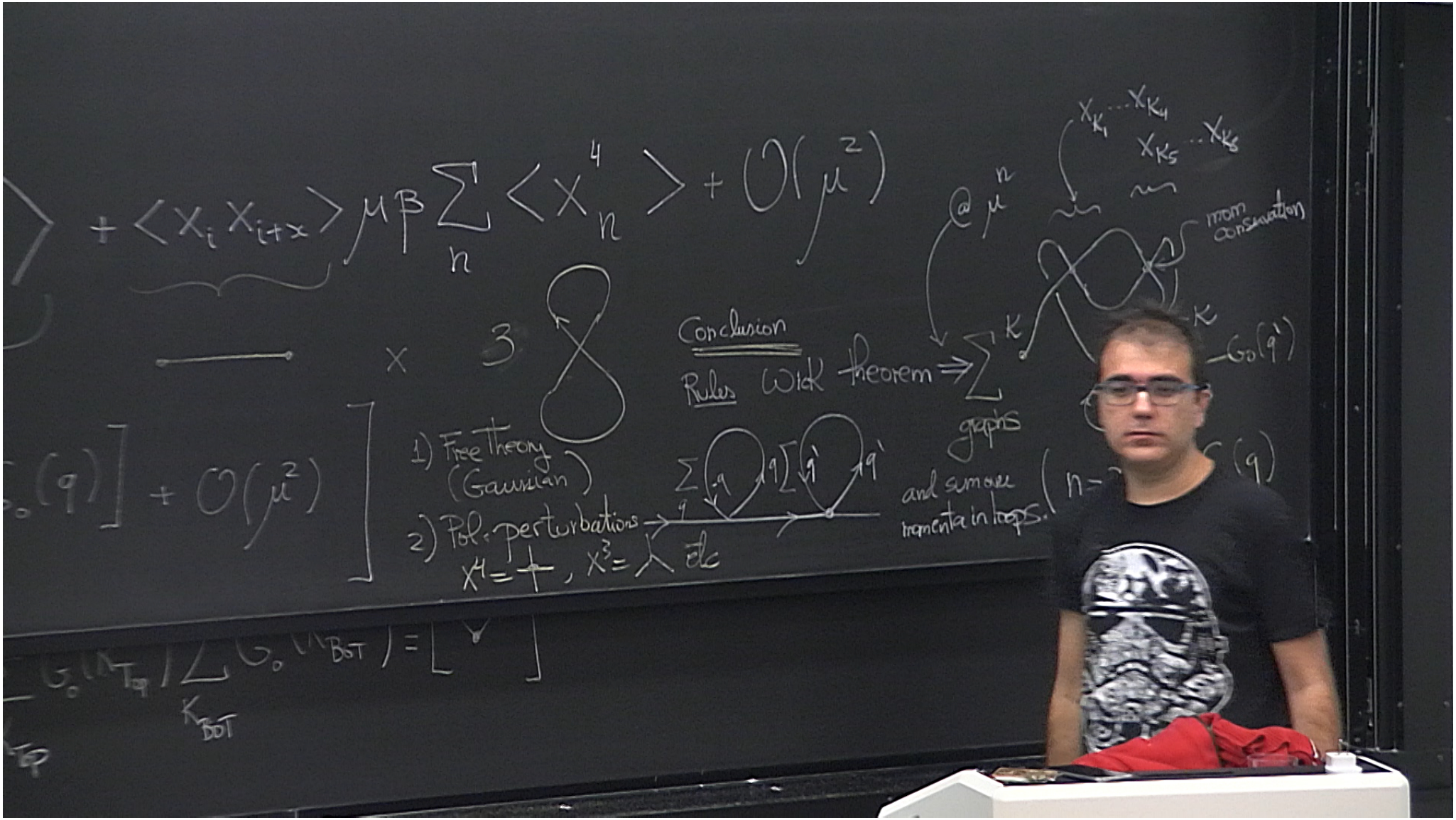

Conclusion
Rules Wick theorem
graphs
and sum over
momenta in
mom conservation
3

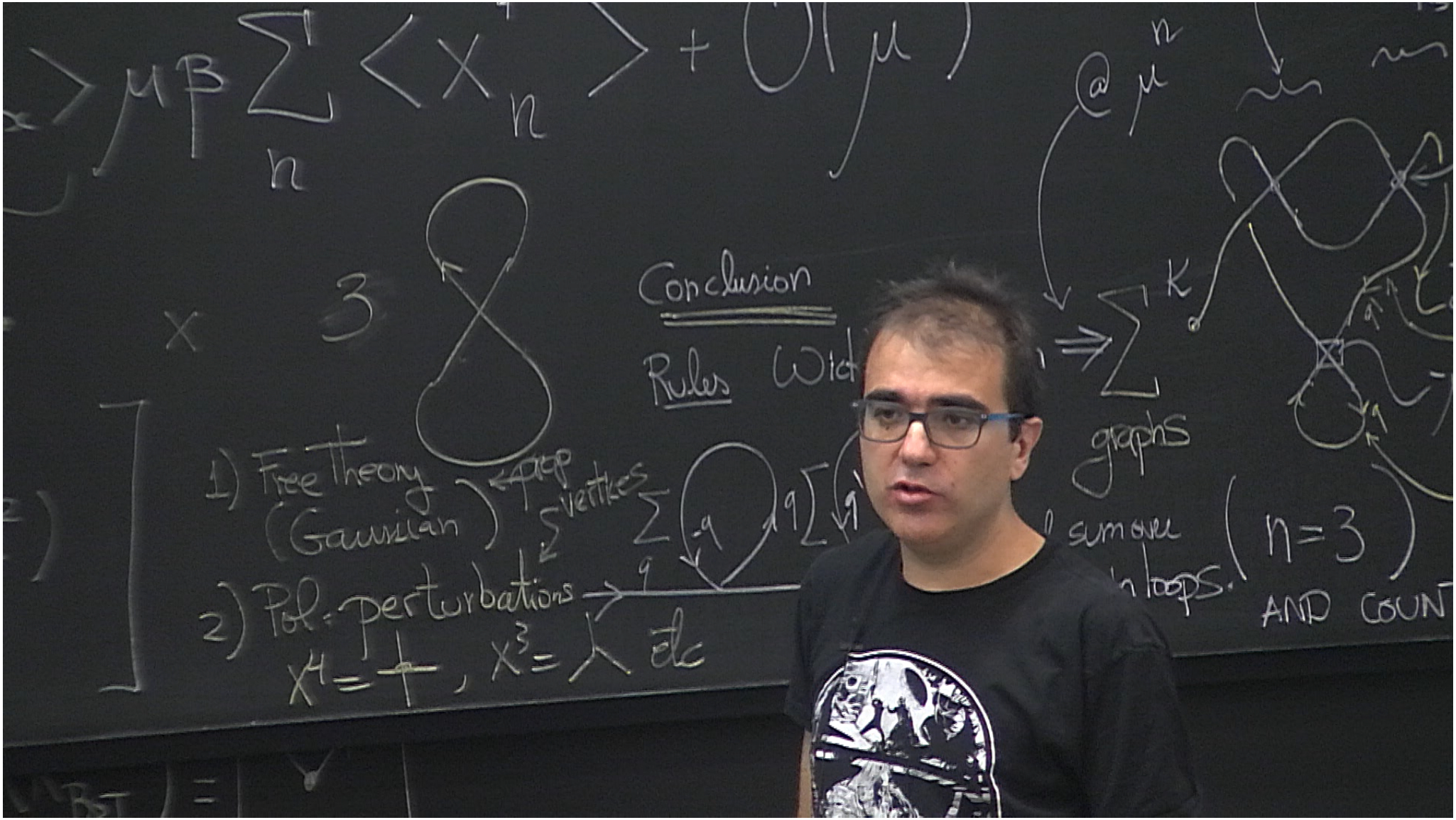

Conclusion
Rules Wick theorem
graphs
and sum over
momenta in loops.
n=3
mom
conservation


$\sum_n \langle x_n^4 \rangle + O(\mu^2)$
3
Conclusion
Rules Wick theorem ⇒ $\sum_{\text{graphs}}$
$\sum_q$ $-q$ q $\sum$ q' q'
and sum over momenta in loops.
$(n=3)$
AND COUNT.
$@\ \mu^n$
$x_{k_1} \cdots x_{k_4}$
$x_{k_5} \cdots x_{k_8}$
mom conservation
$-k$
$G_0(q')$
$-\mu\beta$
$G_0(q)$


Gaussian (Free) theory with $\mu=0$
$= \langle X_i X_{i+x} \rangle - \mu\beta \sum_n \langle X_i X_{i+x} X_n^4 \rangle + \langle X_i X_{i+$
#
$= \frac{1}{L^2} \sum_k e^{-i\vec{K}\vec{x}} [G_0(K) - 12\mu\beta\, G_0(K)^2 [\sum_q G_0(q)] + O(\mu$
$G_0(K)$
$\sum_{K_{TOP}} G_0(K_{TOP}) \angle_{K_{BOT}} G_0$

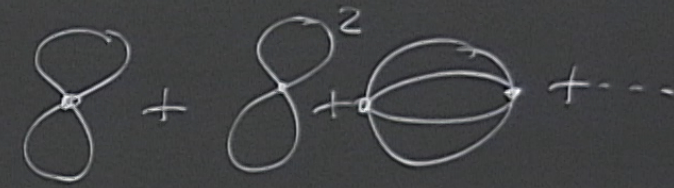

Conclusion
Rules Wick theorem
graphs
and sum over momenta in loops.
(n=3)
AND COUNT.
momentum conservation


Conclusion
Rules Wick theorem ⇒ Σ graphs
1) Free Theory (Gaussian)
2) Pol. perturbations
and sum over momenta in loops.
mom conservation


Conclusion
Rules
1) Free Theory
(Gaussian)
prop
vertices
2) Pol: perturbations
graphs
sum over
n loops.
n=3
AND COUN

$$\sum_n \quad n \qquad 2! \quad n,m$$

$$\ldots_{i+x} = Z = \sum \text{vacum diags} =$$



$$= \sum \langle \underset{\text{pairs}}{xx} \rangle \langle xx \rangle \langle xx \rangle$$

zero total mom $(k, -k)$

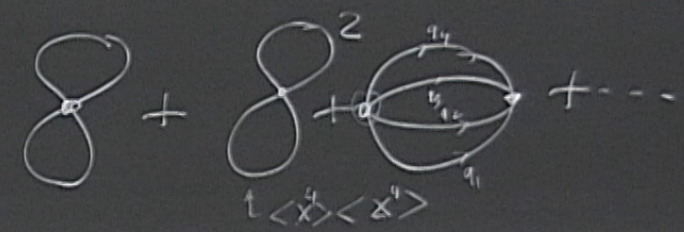
$$\ldots_j) \cdot \vec{n} \quad \langle X_k X_{k'} X_{k_1} X_{k_2} X_{k_3} X_{k_4} \rangle = -\frac{\mu\beta}{L^4} \sum_k e^{-i\vec{k}\cdot\vec{x}} \sum_{\vec{k}_1 + \ldots + \vec{k}_4 = 0} \langle x_k x_{-k} x_{k_1} \ldots x_{k_4} \rangle$$

$k' = -k$ or one $\langle \quad \rangle = 0$

$\vec{k}, \vec{0}$

$$\sum_n \quad n \qquad 2! \quad n,m$$

$$\ldots_{i+x} = Z = \sum \text{vacum diags} =$$



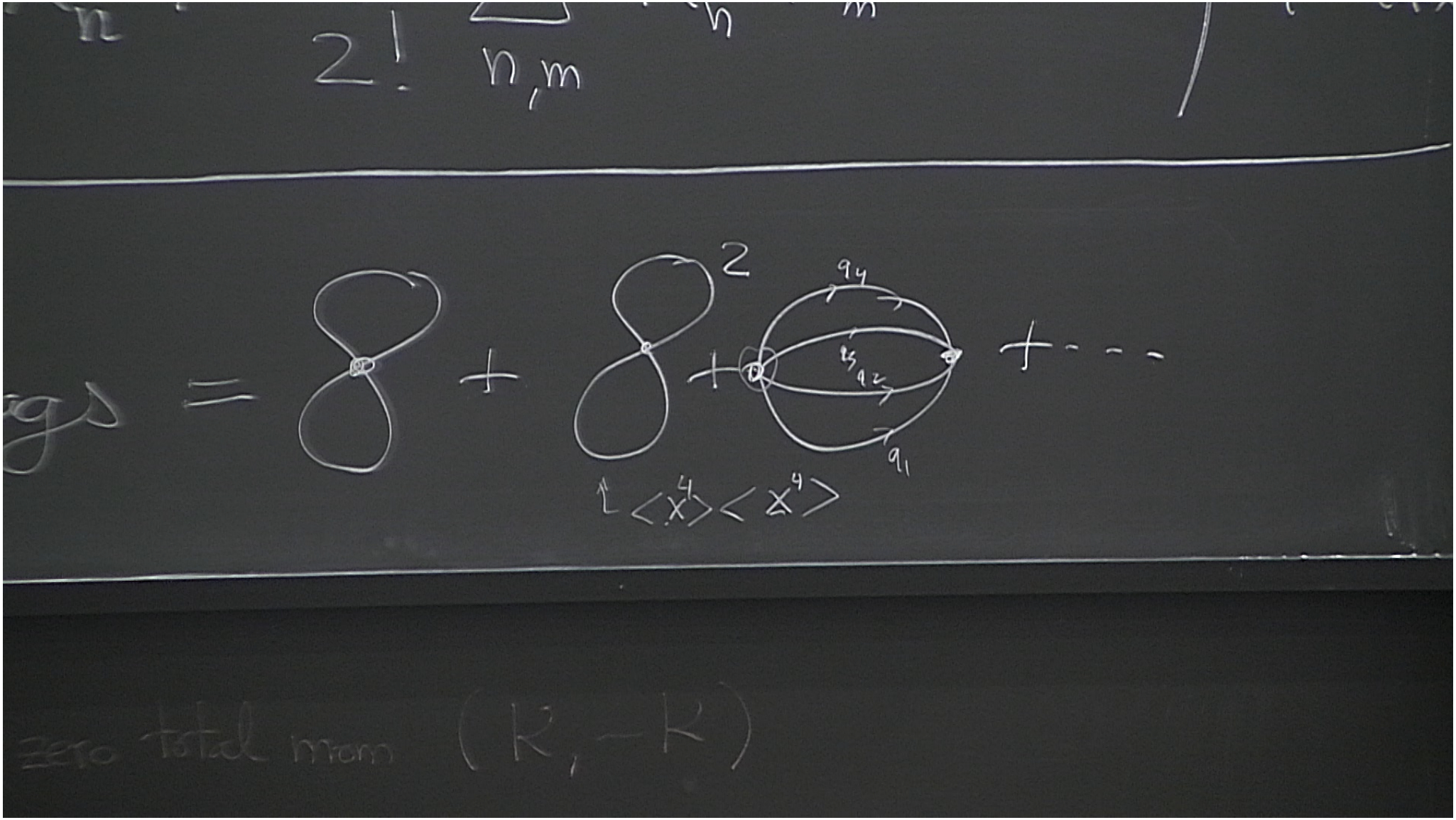
$$= \sum \langle \underset{\text{pairs}}{xx} \rangle \langle xx \rangle \langle xx \rangle$$

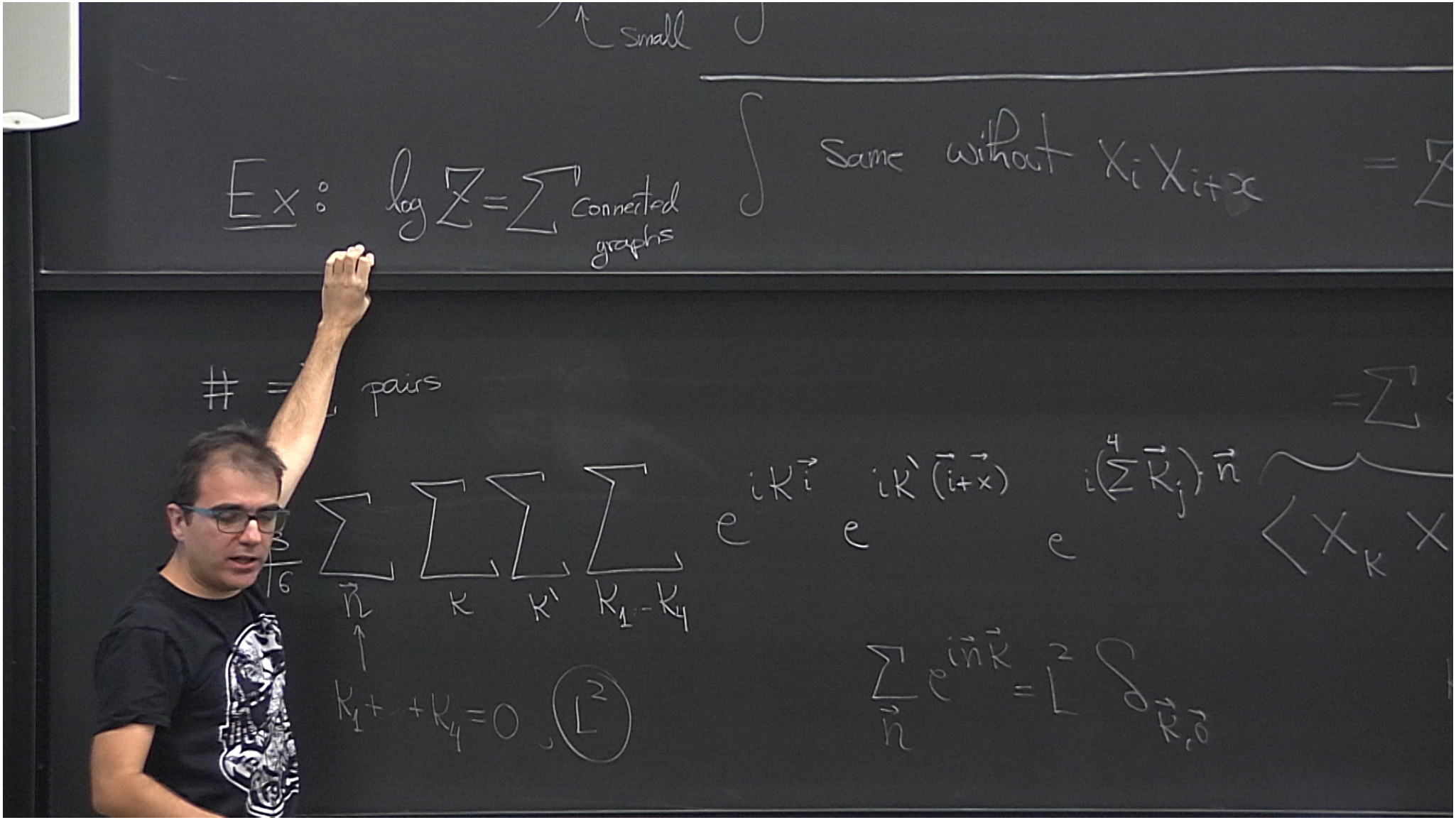
zero total mom $(K, -K)$

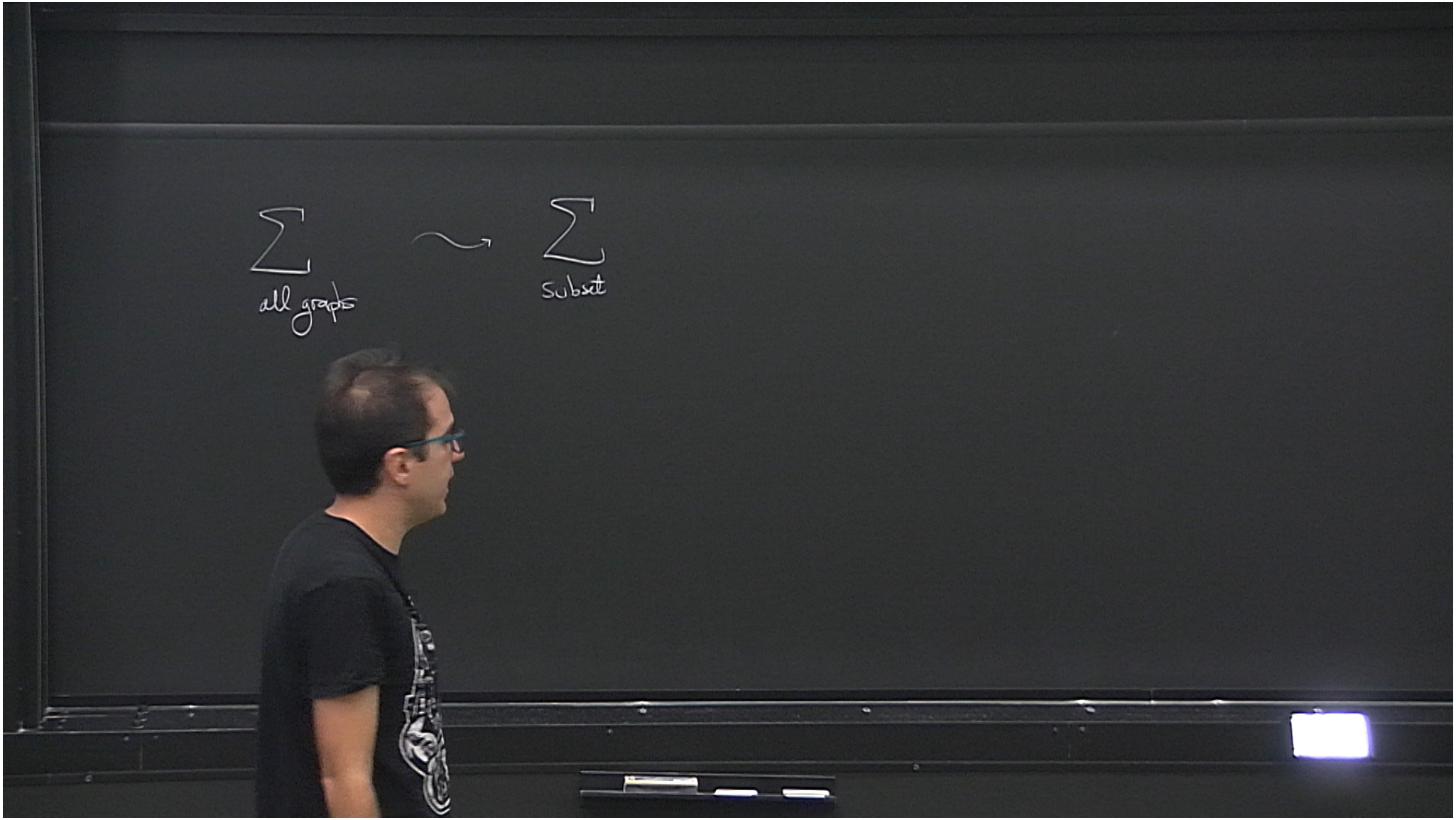
$$\ldots_j)\,\vec{n} \quad \langle X_K X_{K'} X_{K_1} X_{K_2} X_{K_3} X_{K_4} \rangle = -\frac{\mu\beta}{L^4} \sum_K e^{-i\vec{K}\cdot\vec{x}} \sum_{\vec{K}_1 + \cdots + \vec{K}_4 = 0} \langle x_K x_{-K} x_{K_1} \cdots x_{K_4} \rangle$$

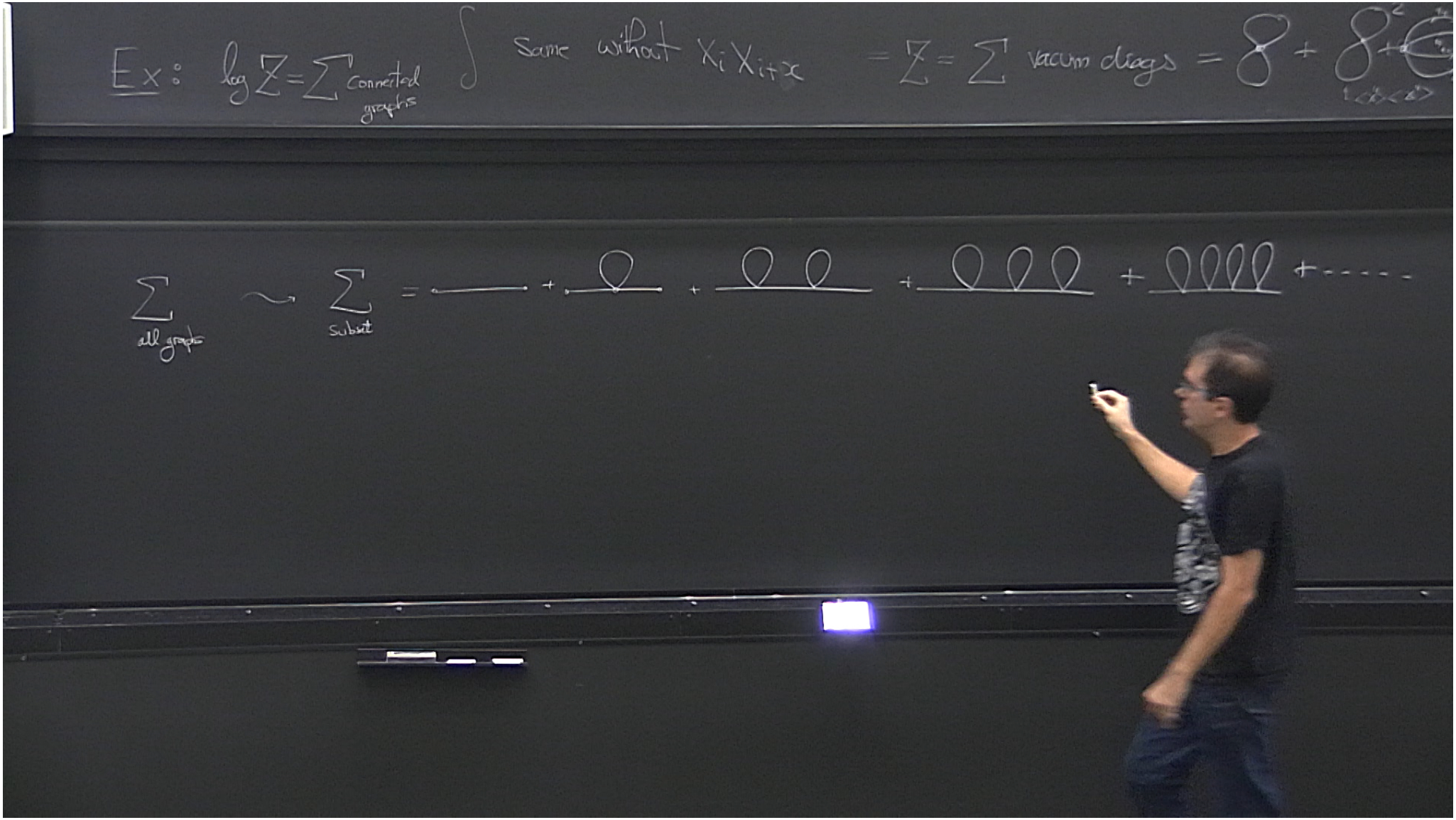
$K' = -K$ or one $\langle \quad \rangle = 0$

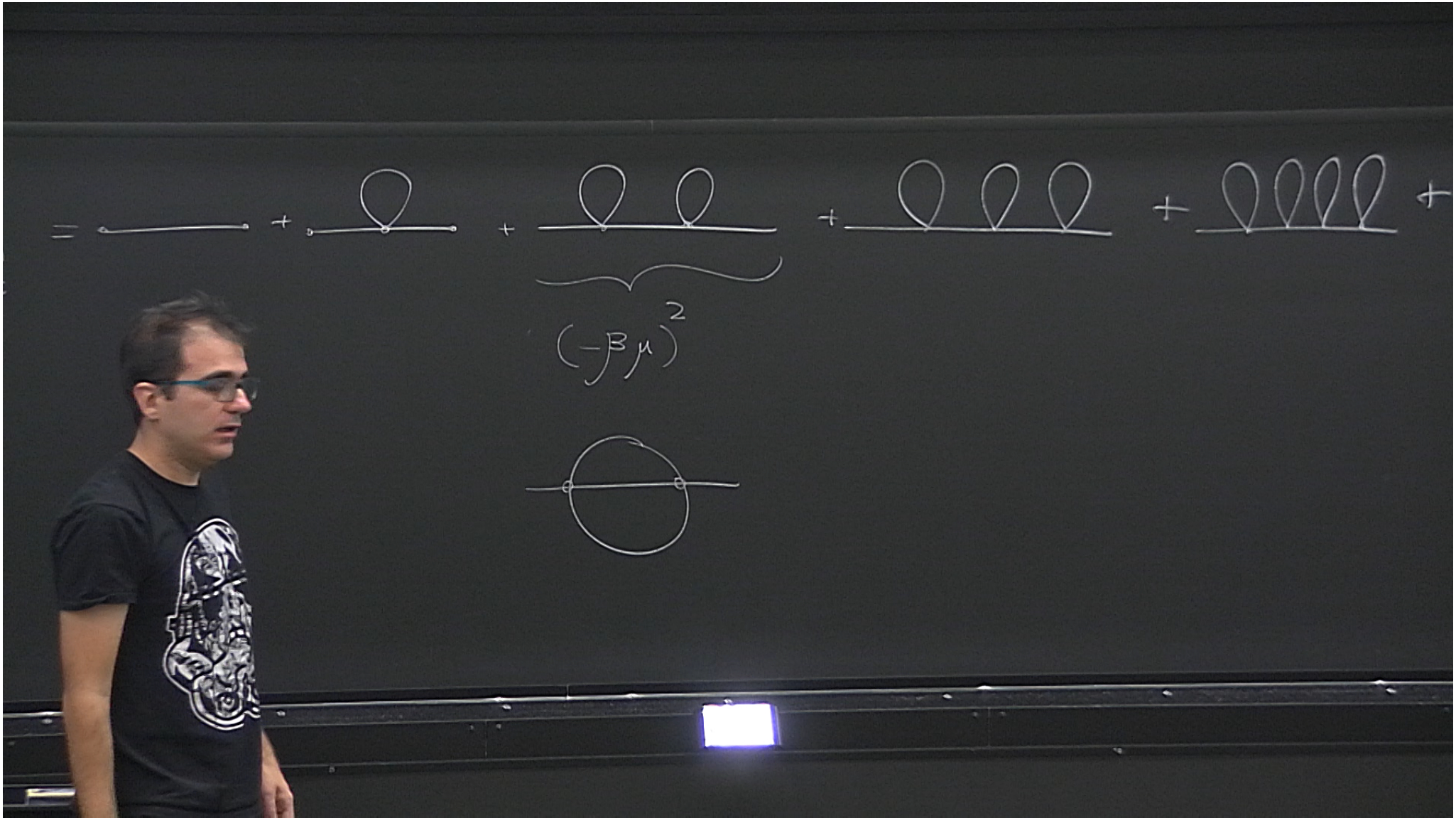
$\vec{K}, \vec{0}$

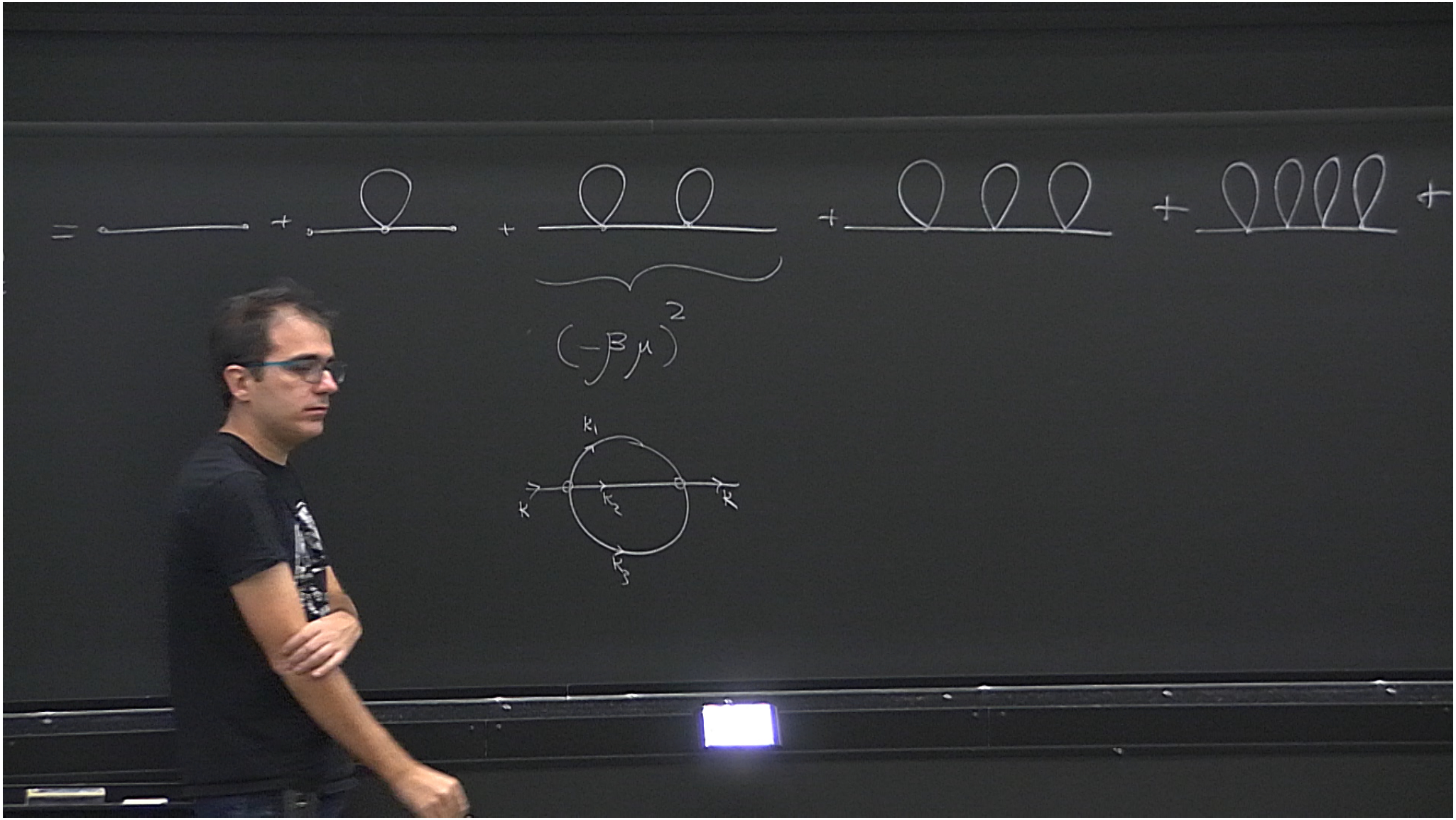

2!
n,m
2
q4
q3
q2
q1
+ - - -
<x^4><x^4>
zero total mom (K, -K)

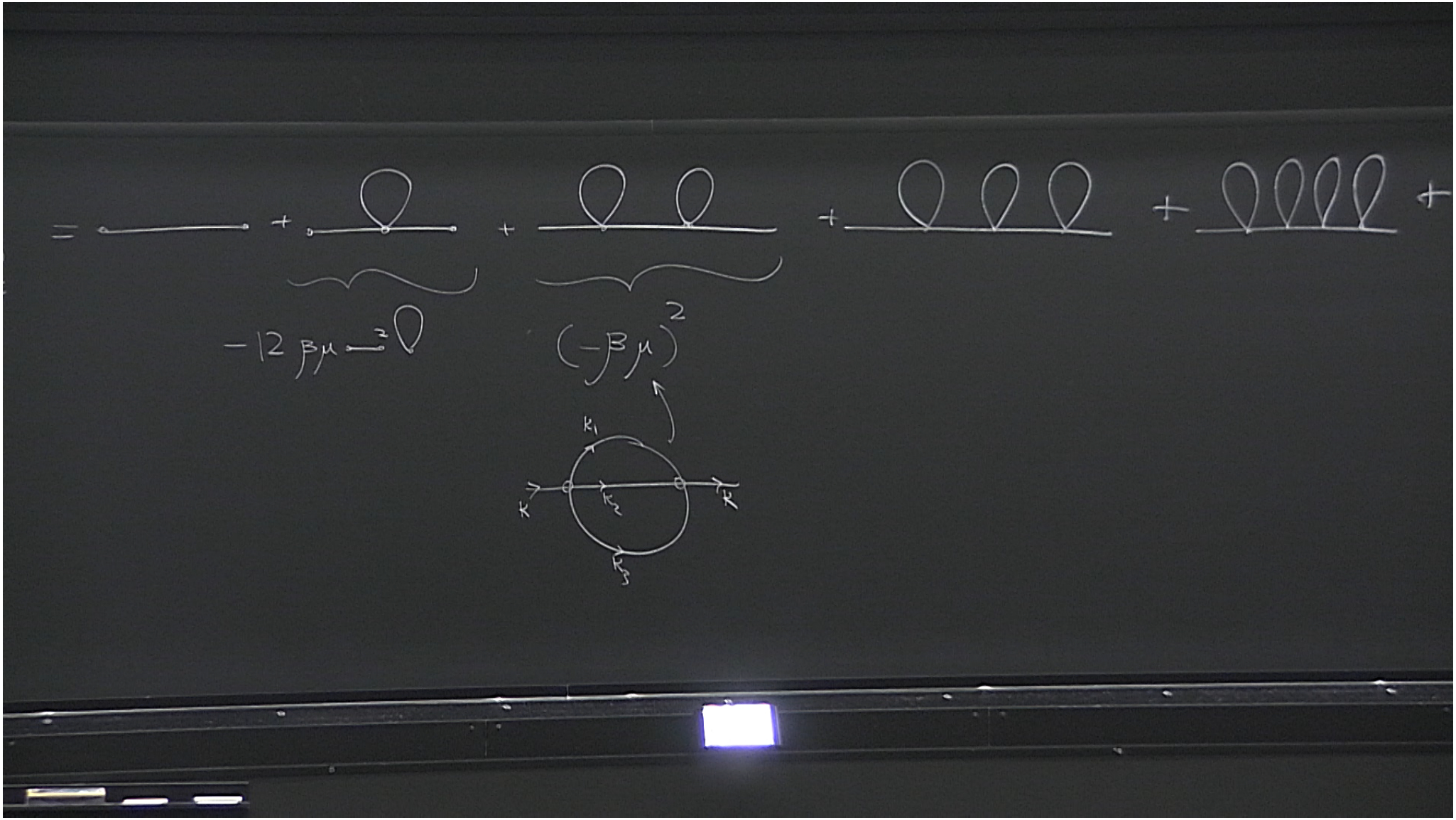

small
Ex: log Z = Σ connected graphs
Same without X_i X_{i+x}
= pairs
e^{iK i} e^{iK'(i+x)} e^{i(Σ K_j) n}
K_1 + ... + K_4 = 0 , L^2
Σ_n e^{in K} = L^2 δ_{K,0}

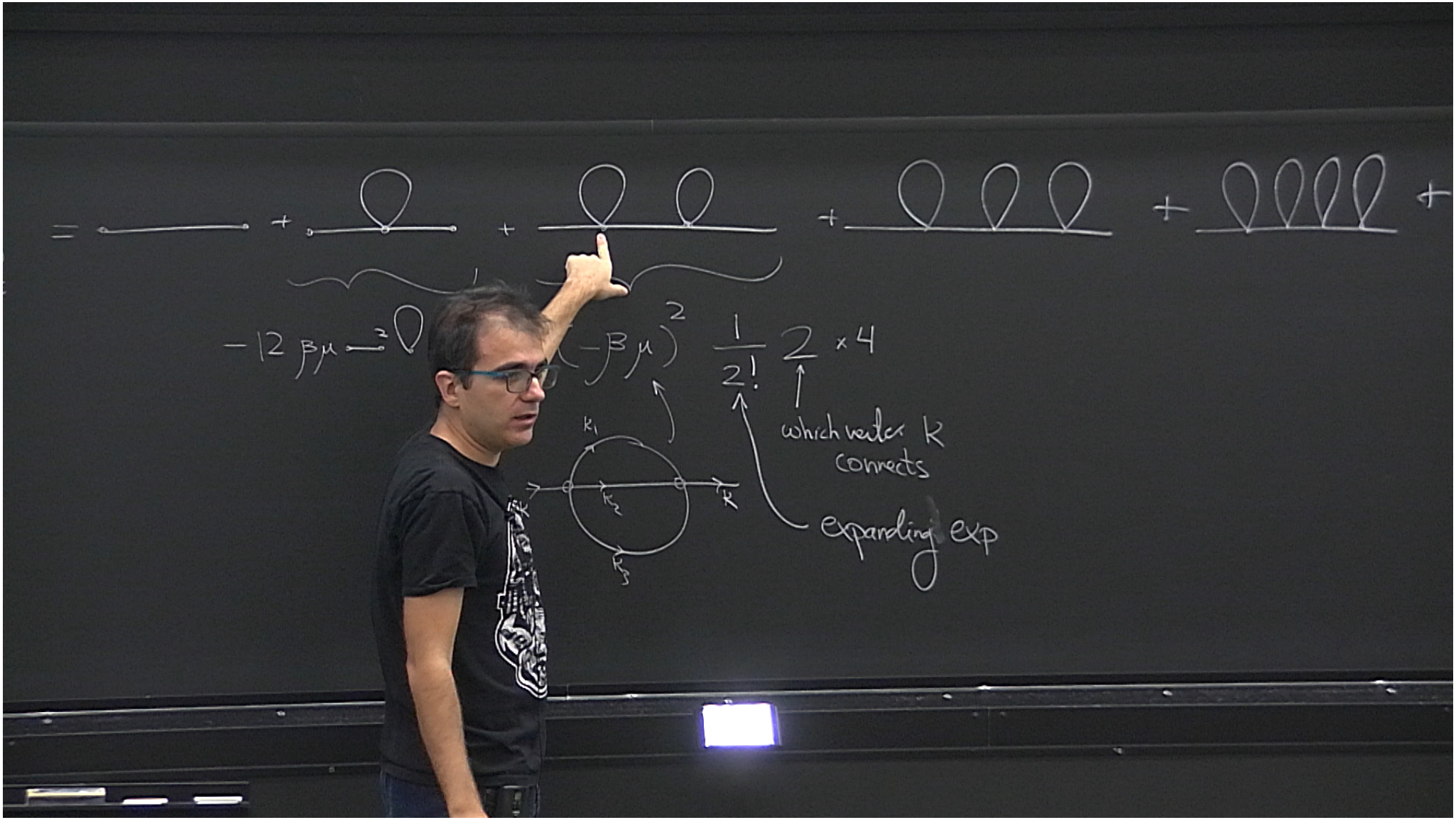

all graphs
Subset

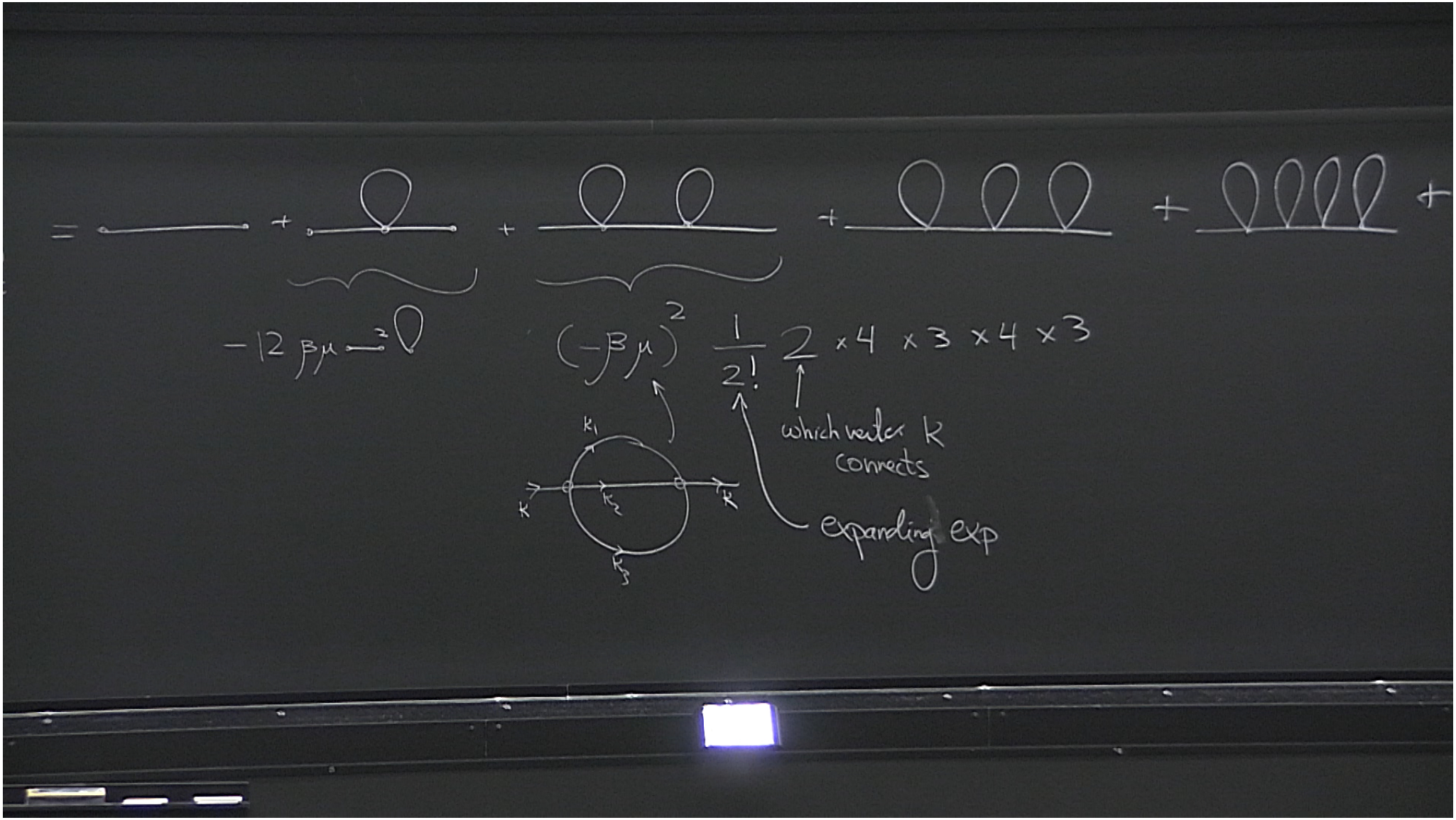

Ex: log Z = Σ connected graphs
∫ same without X_i X_{i+x}
= Z = Σ vacuum diags =
Σ all graphs ~ Σ subset =

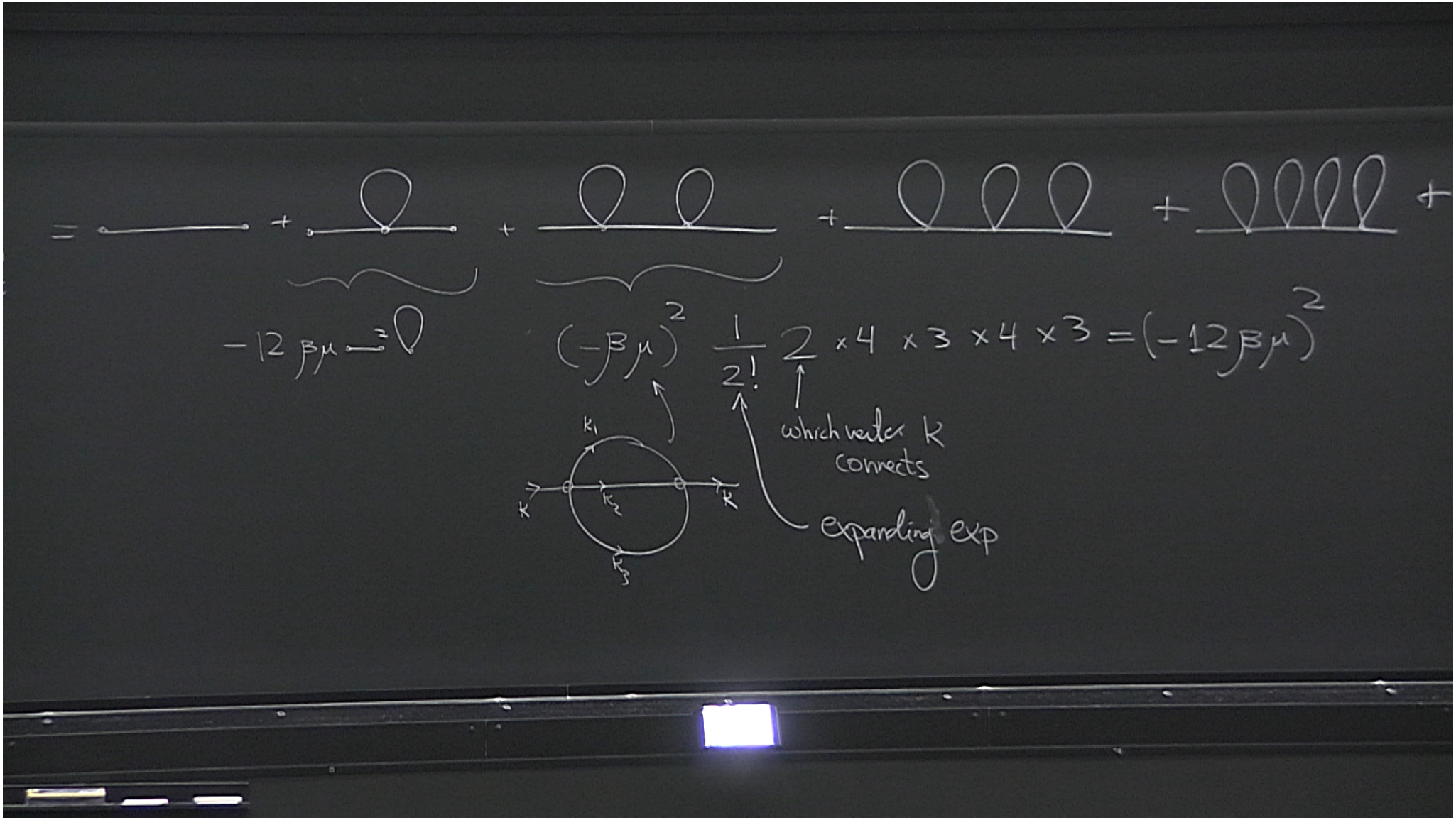

$(-\beta\mu)^2$

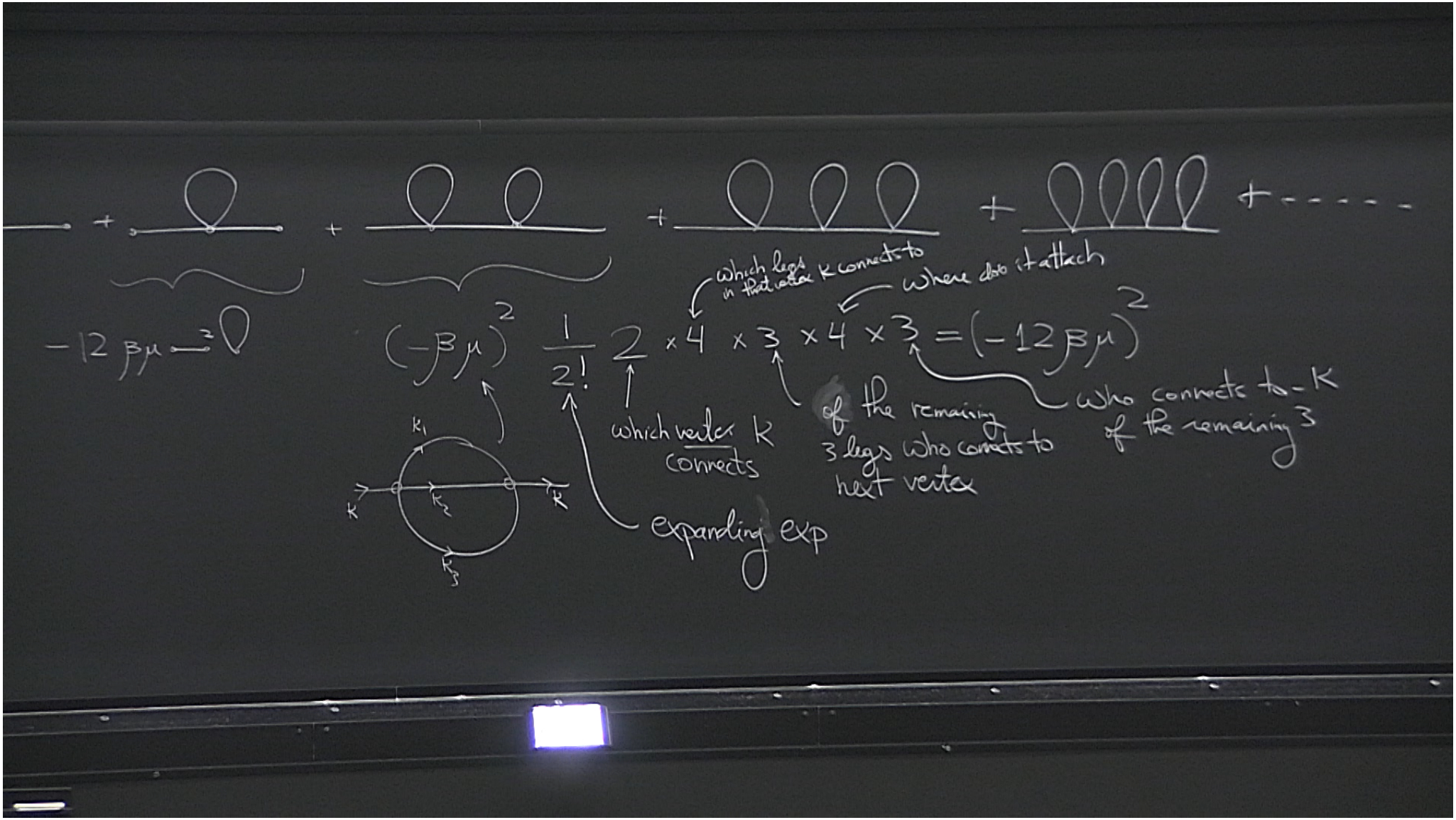

$(-\beta\mu)^2$
k_1
k
k_2
k
k_3

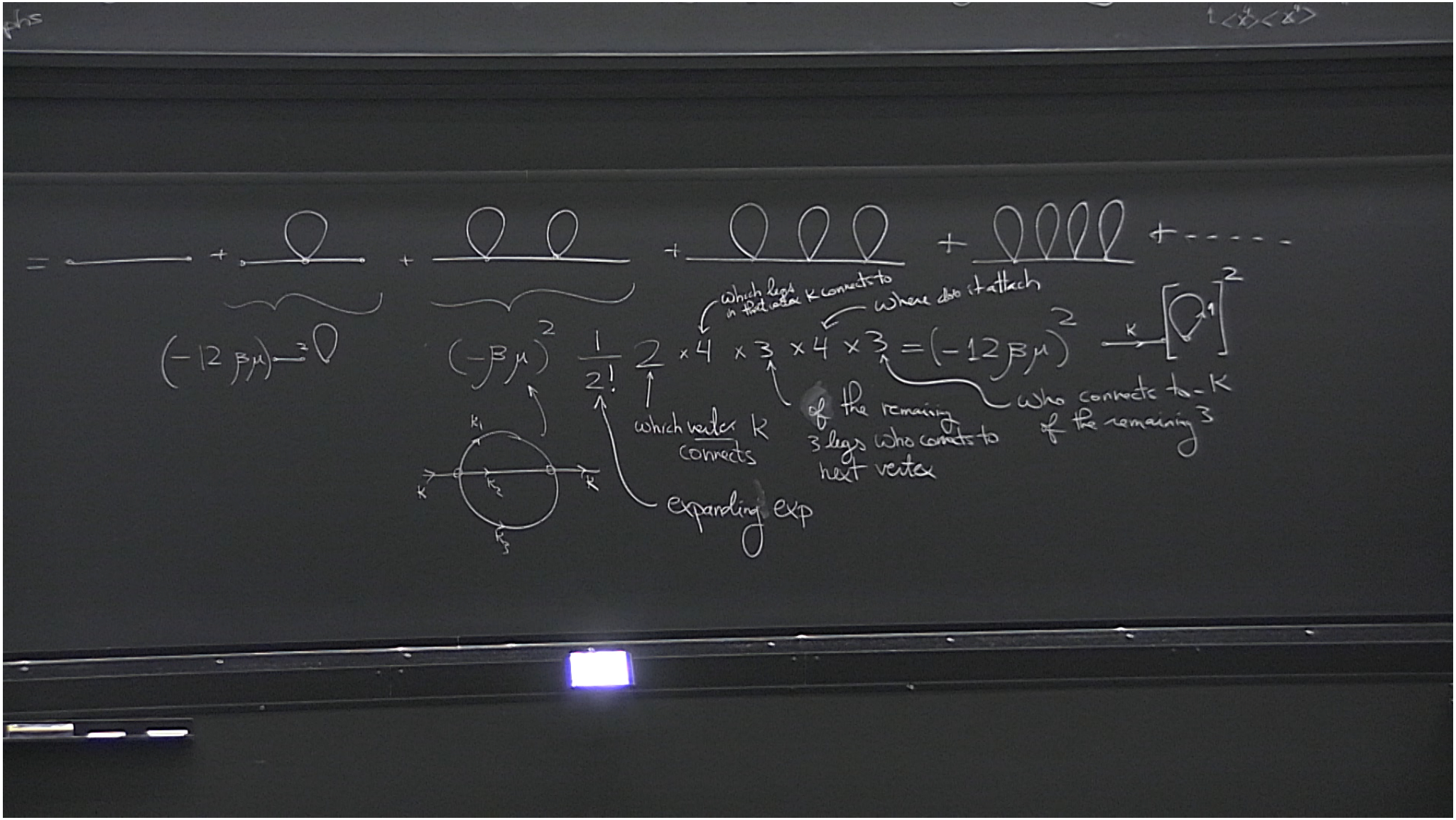

$-12\,\beta\mu^2$
$(-\beta\mu)^2$
k_1
k_2
k_3
k
k

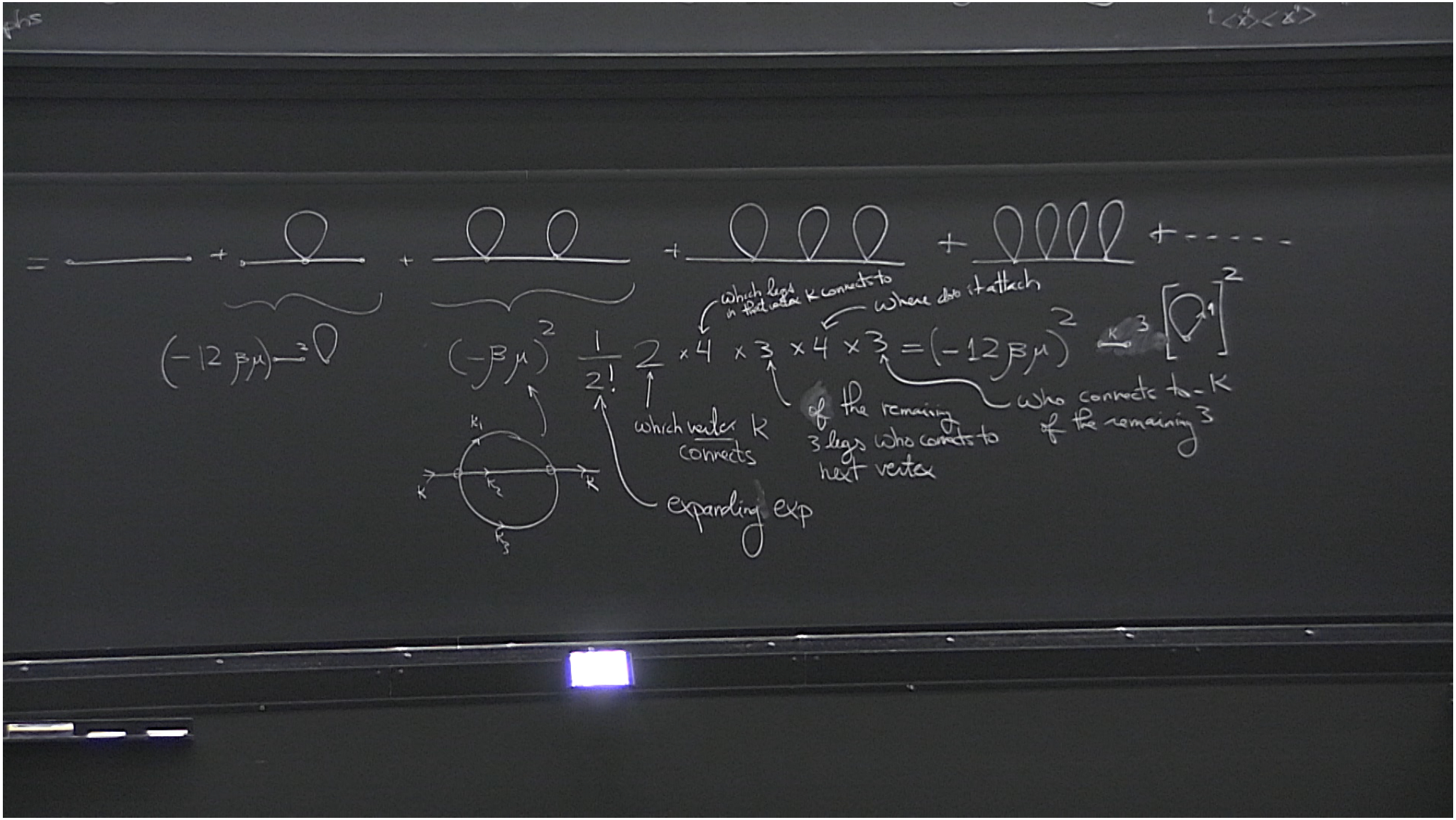

$-12\,\beta\mu$
$(-\beta\mu)^2$
$\frac{1}{2!}$
2
×4
which vertex K connects
k_1
k_2
k_3
K
K
expanding exp


$-12\,\beta\mu$
$(-\beta\mu)^2$
$\frac{1}{2!}$
2
$\times 4 \times 3 \times 4 \times 3$
which vertex k connects
expanding exp
k_1
k_2
k_3
k
k

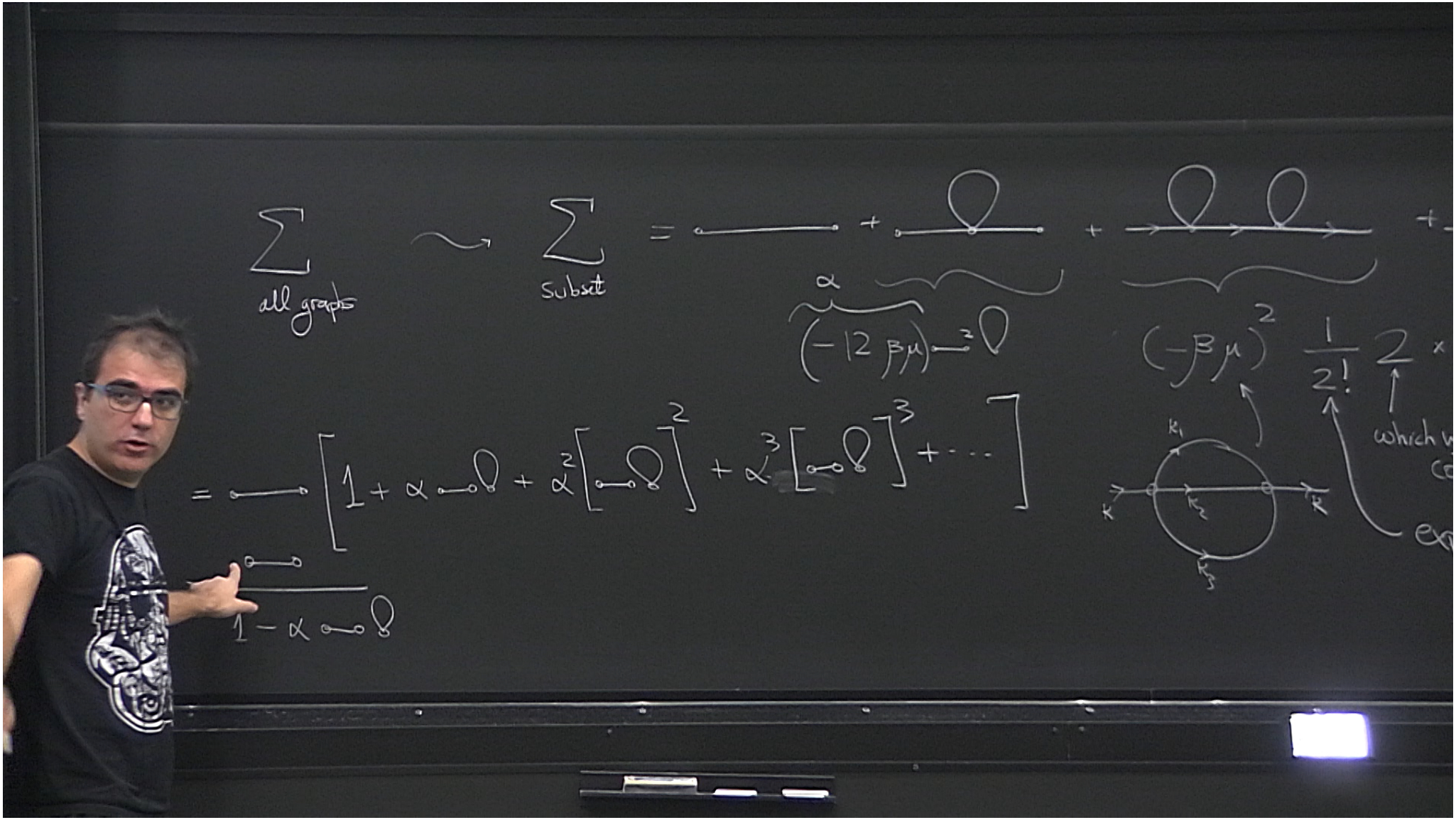

$-12\beta\mu$
$(-\beta\mu)^2 \frac{1}{2!} 2 \times 4 \times 3 \times 4 \times 3 = (-12\beta\mu)^2$
which vertex k connects
expanding exp
k_1
k_2
k_3
k
k

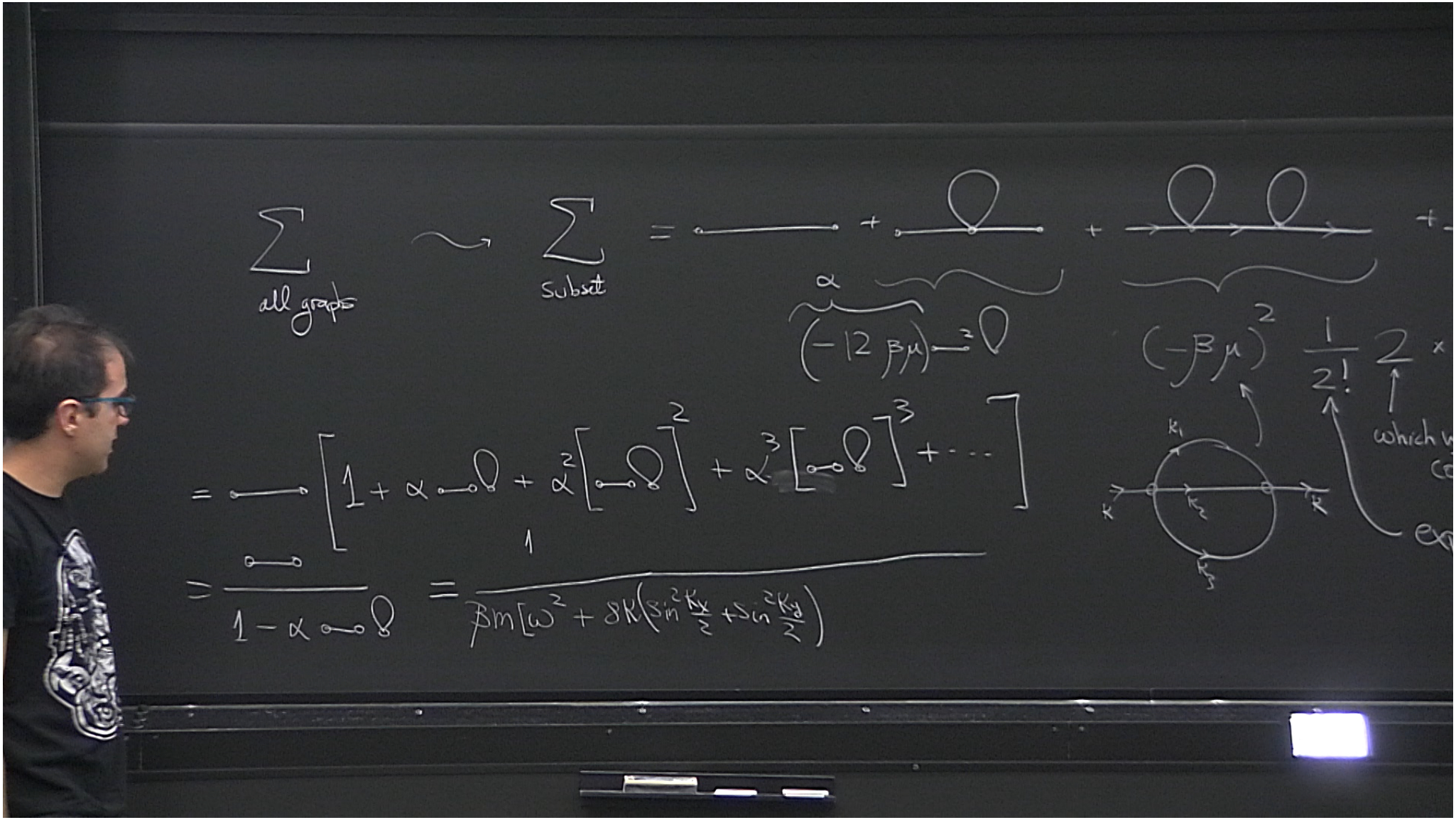

$-12\beta\mu^3$
$(-\beta\mu)^2 \frac{1}{2!} 2 \times 4 \times 3 \times 4 \times 3 = (-12\beta\mu)^2$
which legs in that vertex K connects to
where does it attach
which vertex K connects
of the remaining 3 legs who connects to next vertex
who connects to - K of the remaining 3
expanding exp
k_1
k_2
k_3
k
k

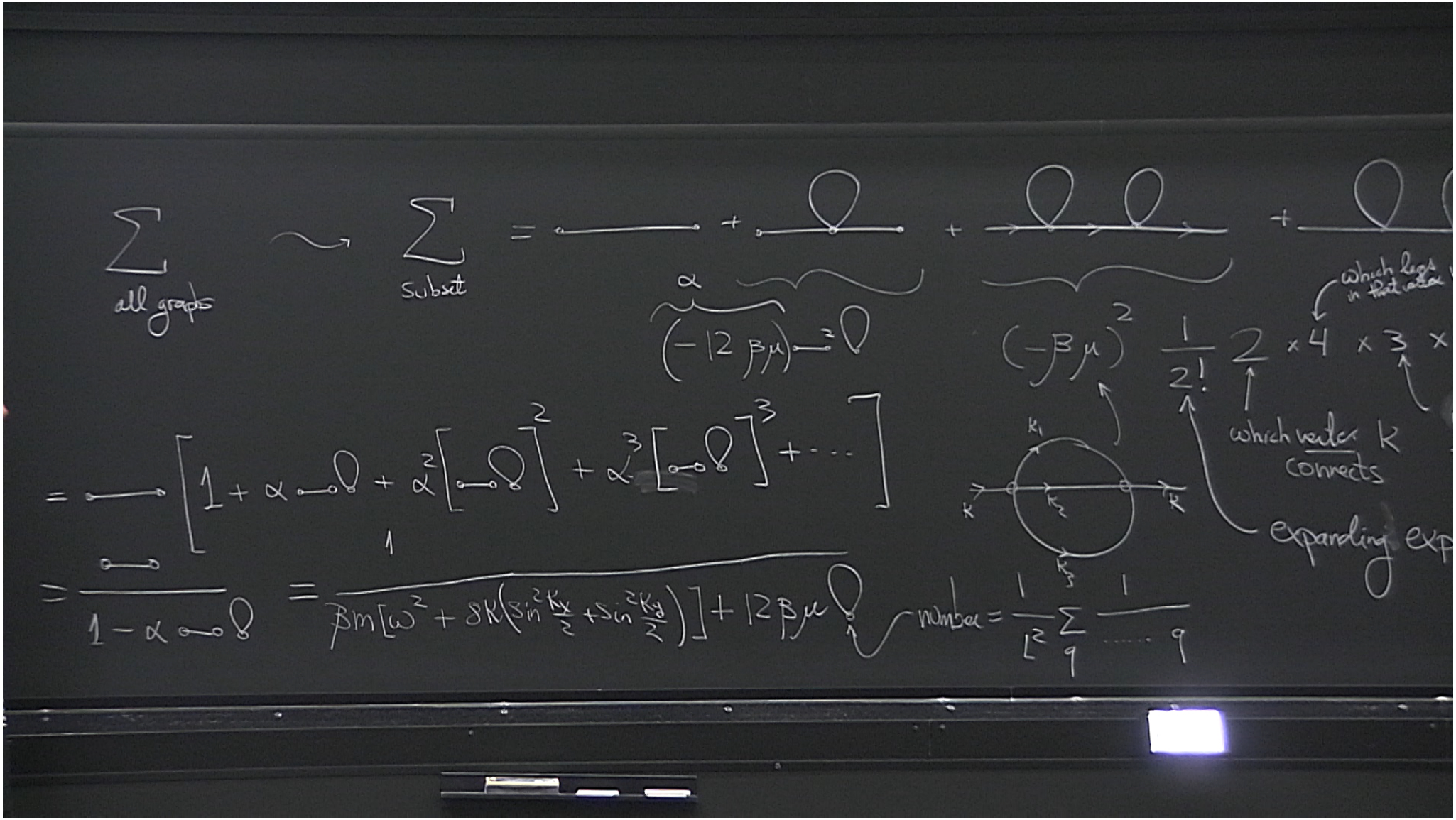

$(-12\beta\mu)^3$
$(-\beta\mu)^2$
$\frac{1}{2!}$
2
$\times 4 \times 3 \times 4 \times 3 = (-12\beta\mu)^2$
which legs in that vertex K connects to
where does it attach
which vertex K connects
of the remaining 3 legs who connects to next vertex
who connects to -K of the remaining 3
expanding exp
K
k_1
k_2
k_3

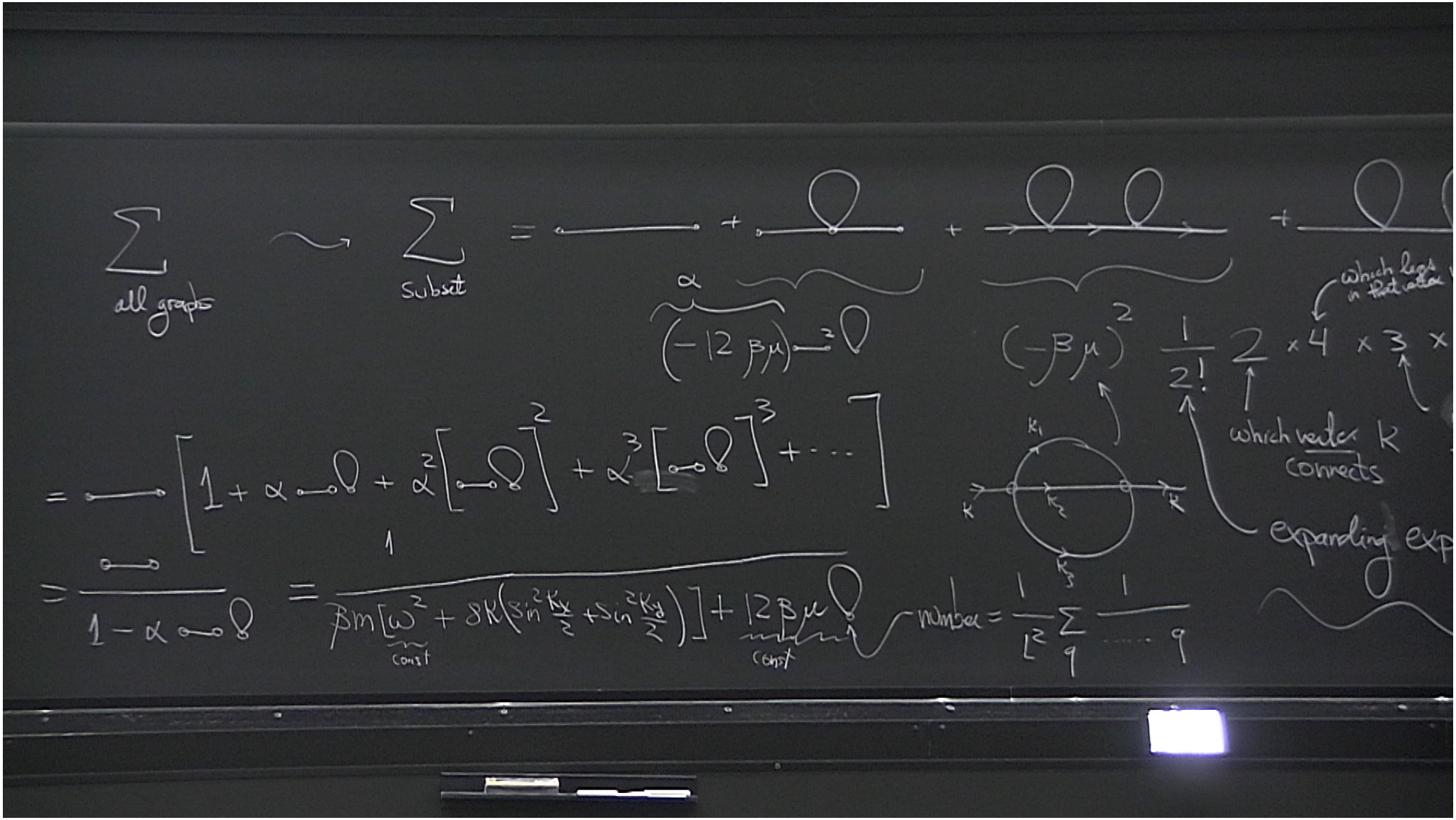


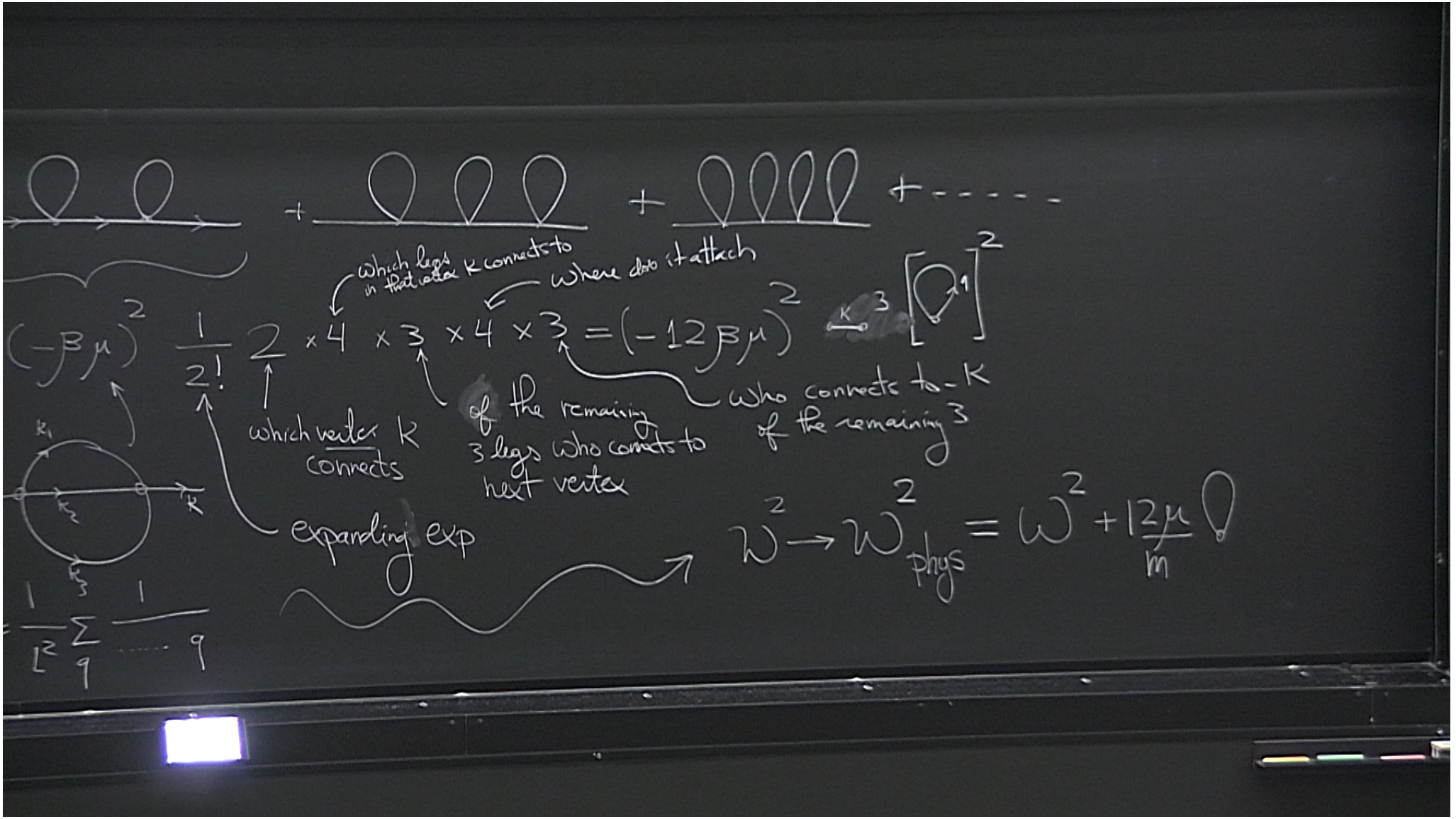
which legs in that vertex K connects to
where does it attach
which vertex K connects
of the remaining 3 legs who connects to next vertex
who connects to -K of the remaining 3
expanding exp

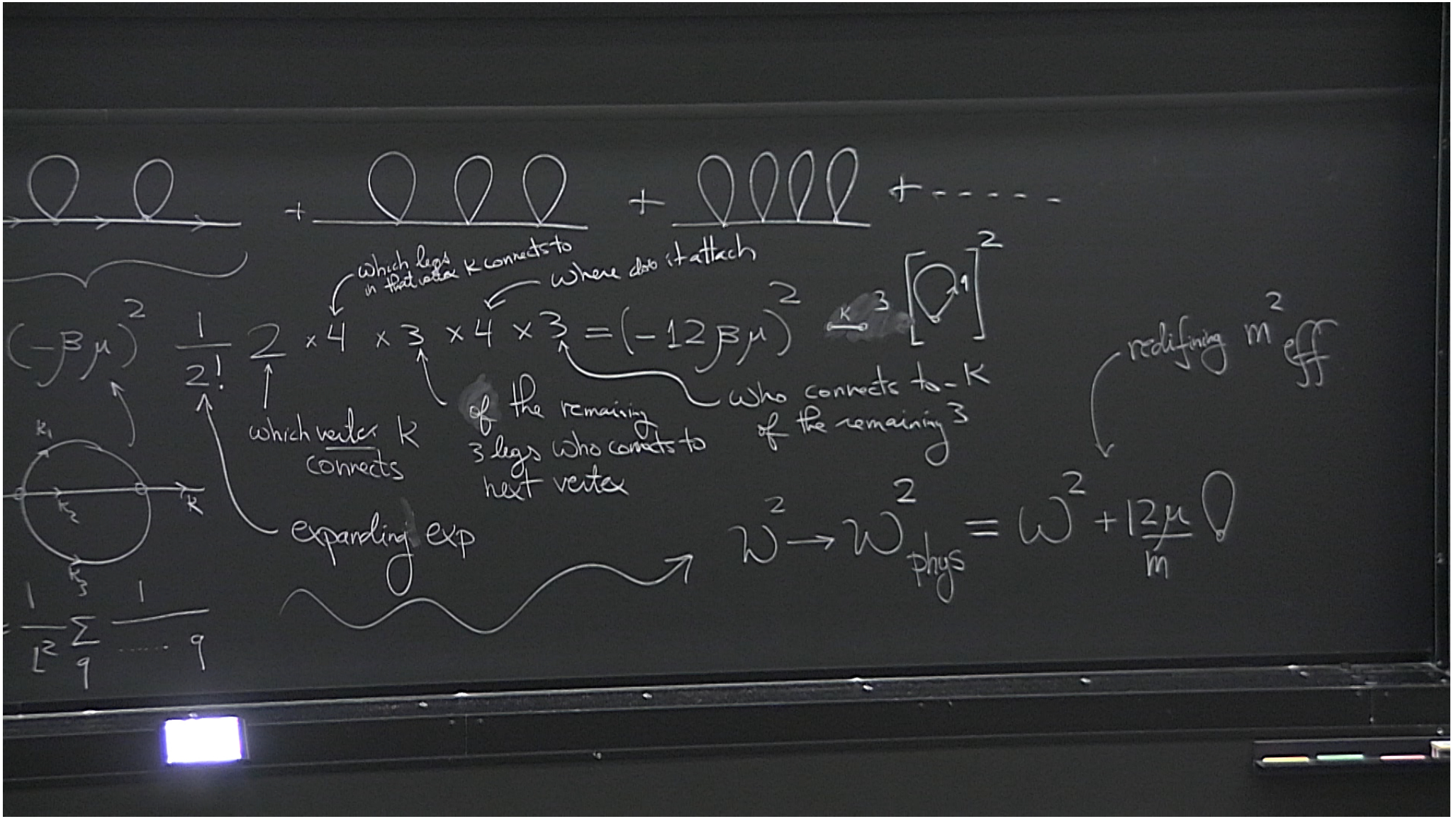

all graphs
subset
α
$(-12\beta\mu)^{-3}$
$(-\beta\mu)^2$
$\frac{1}{2!}$
2
which
k_1
k_2
k_3
k
k
$= \left[1 + \alpha + \alpha^2\left[\right]^2 + \alpha^3\left[\right]^3 + \cdots\right]$
$1 - \alpha$

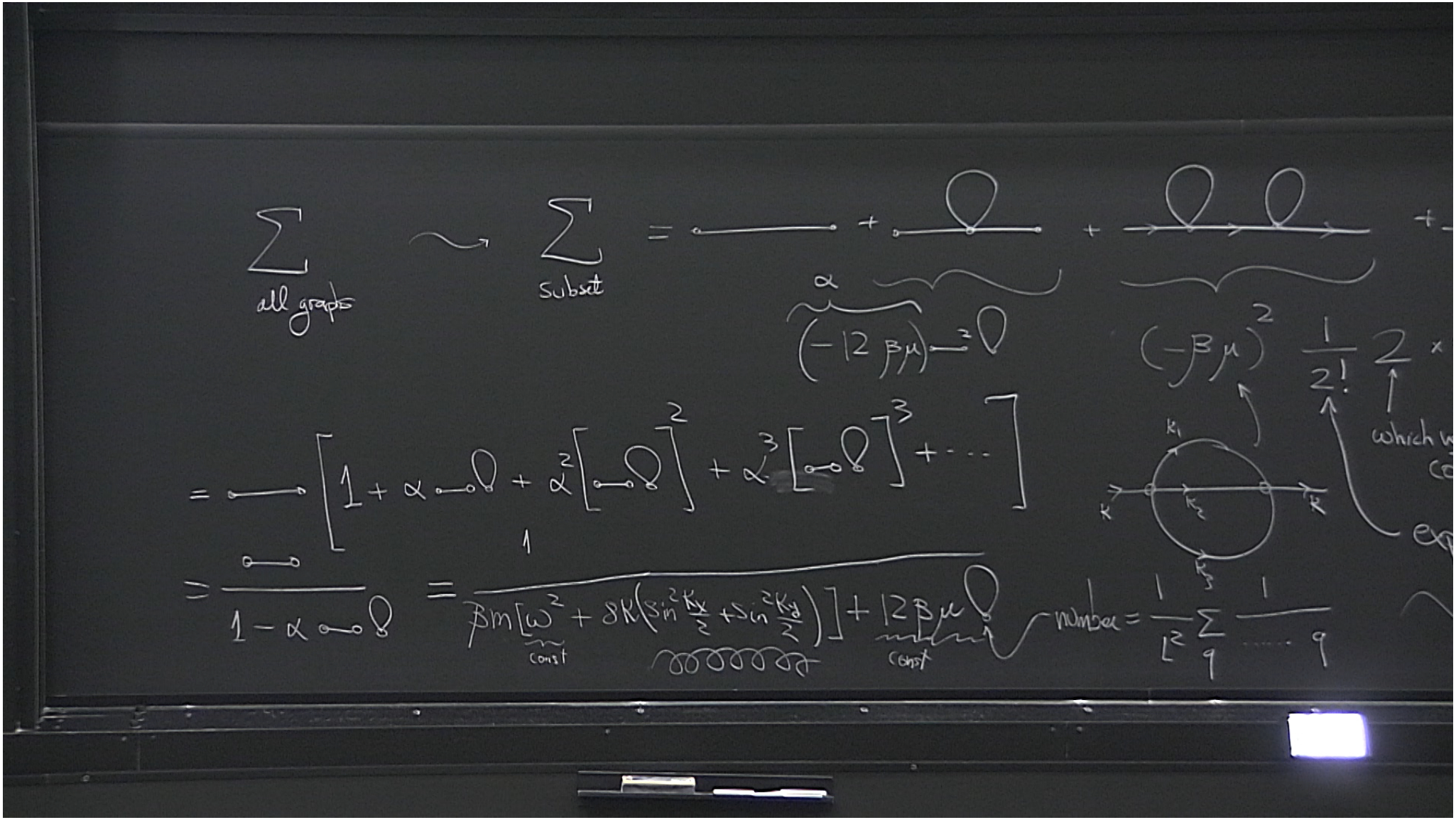

all graphs
Subset
α
$(-12\beta\mu)$
$(-\beta\mu)^2$
$\frac{1}{2!}$
2
which w
k_1
k_2
k_3
k
k
$= \frac{1}{1-\alpha}$
$= \frac{1}{\beta m[\omega^2 + 8K(\sin^2\frac{k_x}{2} + \sin^2\frac{k_y}{2})]}$

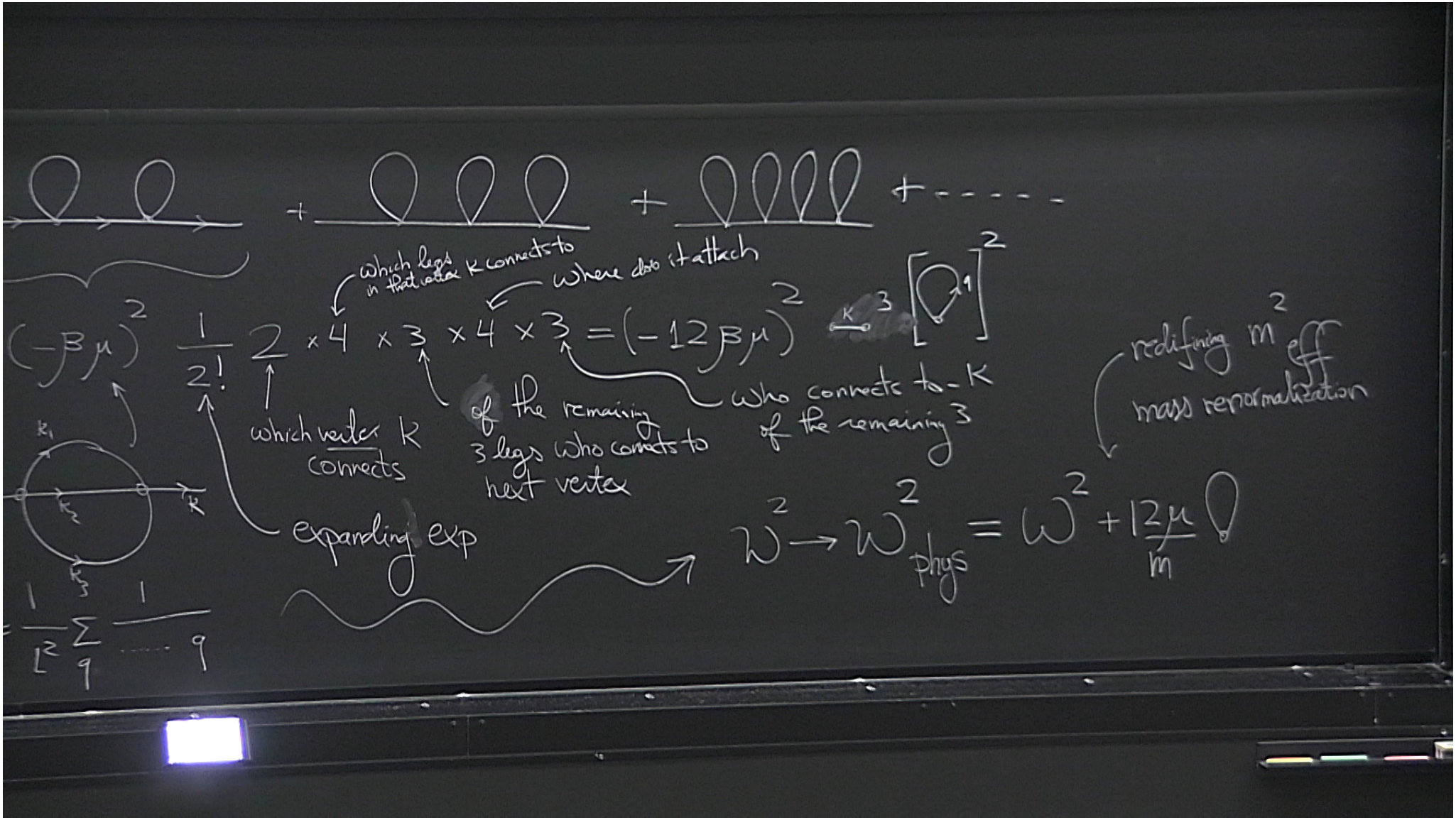

all graphs
subset
which legs in that vertex
which vertex K connects
expanding exp

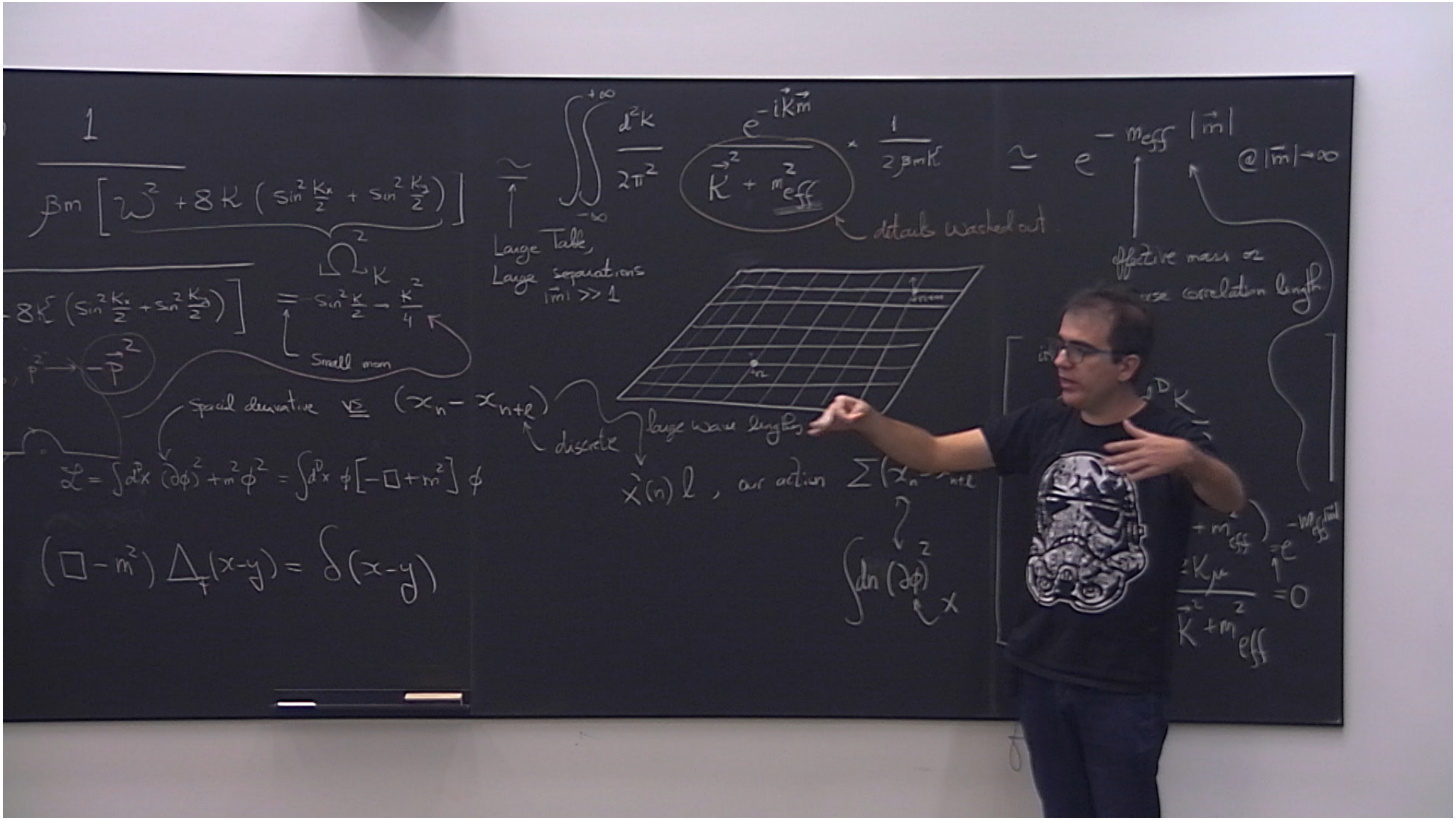

all graphs
subset
which legs in that vertex
which vertex k connects
expanding exp
number
const
const

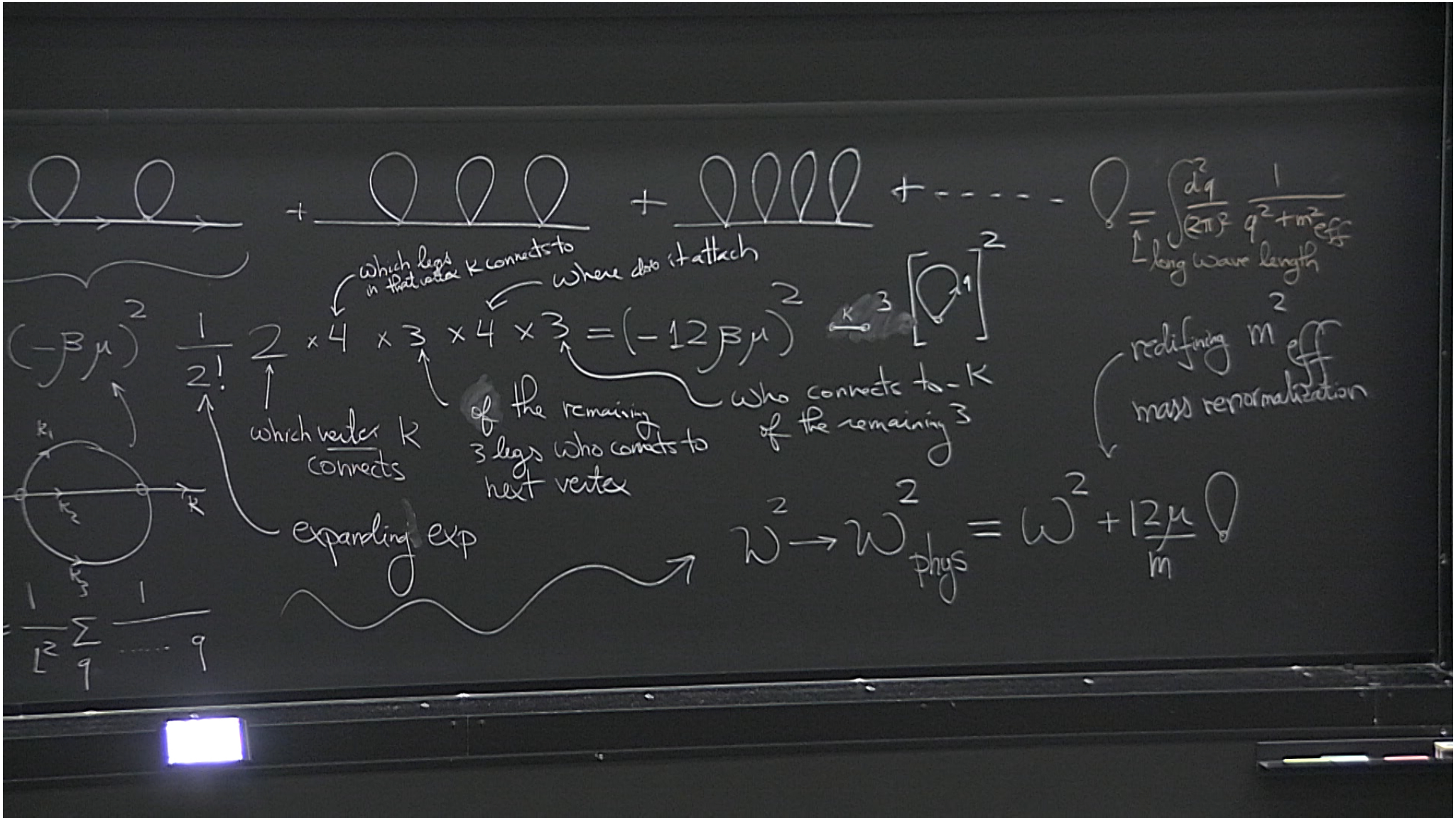

which legs in that vertex K connects to
where does it attach
$(-\beta\mu)^2 \frac{1}{2!} 2 \times 4 \times 3 \times 4 \times 3 = (-12\beta\mu)^2$
which vertex K connects
of the remaining 3 legs who connects to next vertex
who connects to -K of the remaining 3
expanding exp
$\omega^2 \to \omega^2_{phys} = \omega^2 + \frac{12\mu}{m}$
$\frac{1}{L^2}\sum_q \frac{1}{\cdots q}$

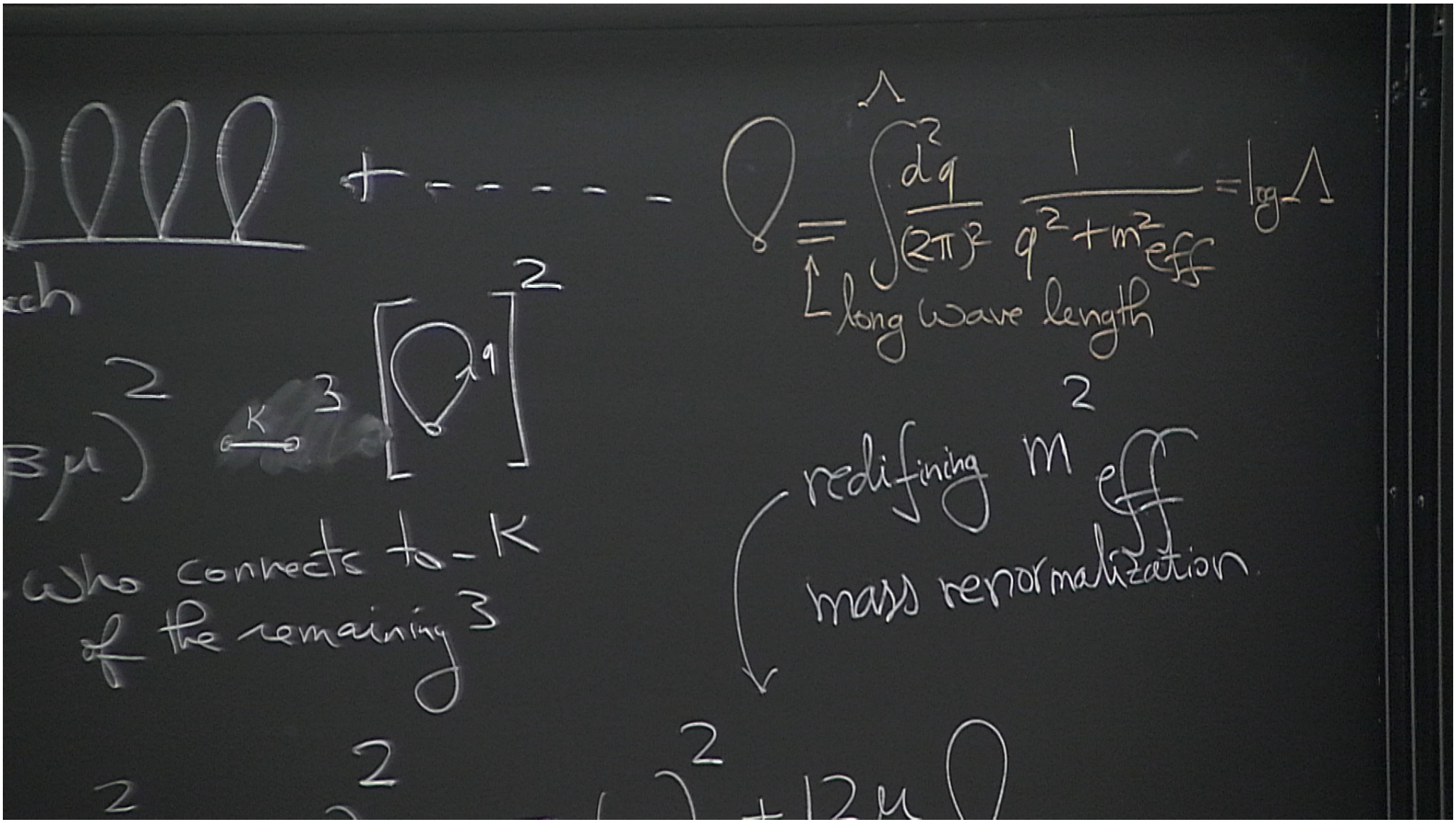

+
+
+ ------
which legs in that vertex K connects to
where does it attach
$(-\beta\mu)^2 \frac{1}{2!} 2 \times 4 \times 3 \times 4 \times 3 = (-12\beta\mu)^2 \sum_K^3 \left[\frac{1}{L^2}\right]^2$
which vertex K connects
of the remaining 3 legs who connects to next vertex
who connects to -K of the remaining 3
redifining m^2_{eff}
expanding exp
$\omega^2 \to \omega^2_{phys} = \omega^2 + 12\frac{\mu}{m}$
k_1
k_2
k_3
K
$\frac{1}{L^2} \sum_q \frac{1}{\cdots q}$

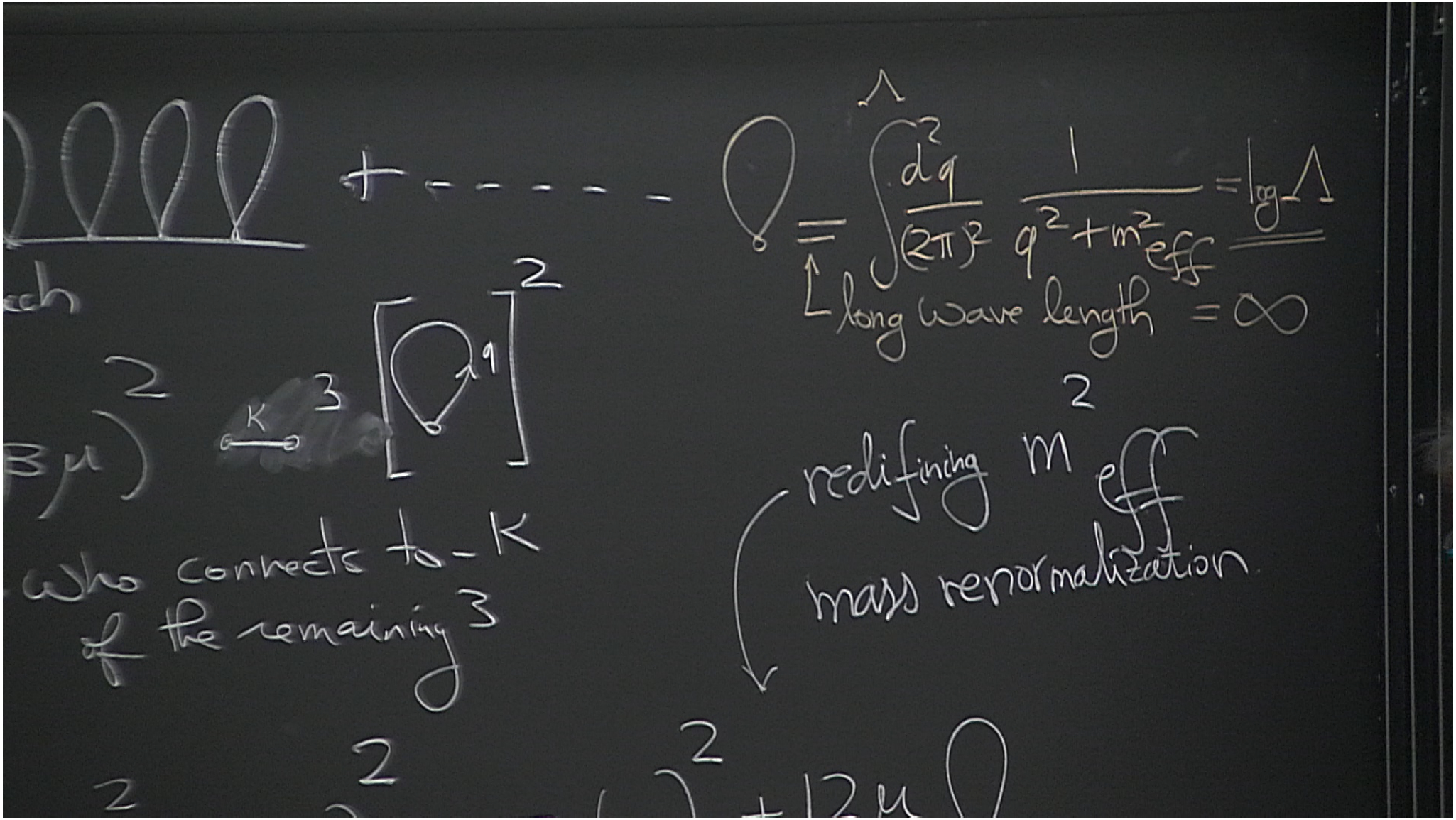

all graphs
subset
α
(−12 βμ)
(−βμ)²
1/2!
2
which
1 + α + α² + α³ + ...
1 − α
const
const
number = 1/L² Σ q

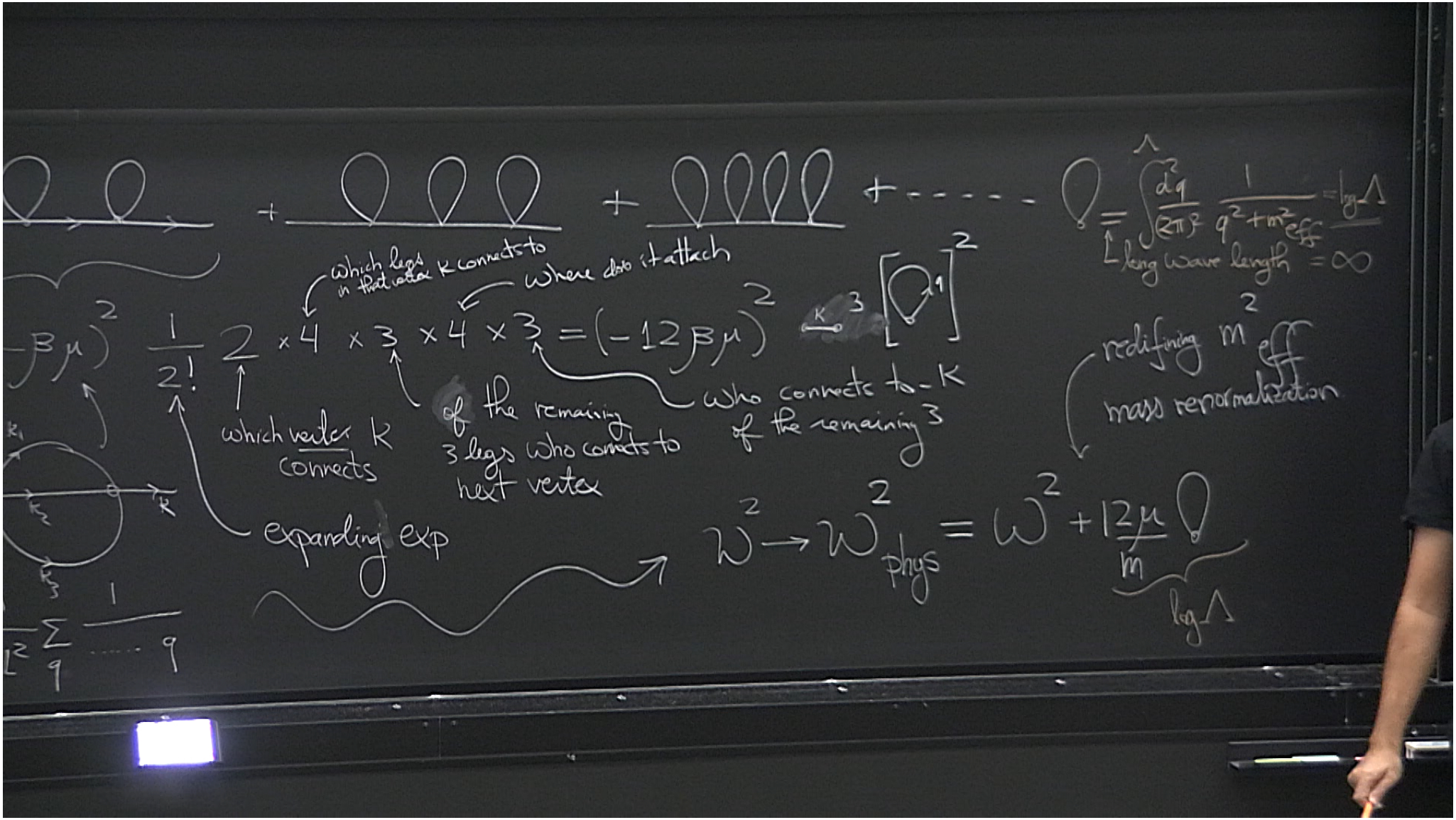

+ + + ⋯⋯
which legs in that vertex K connects to
where does it attach
$(-B\mu)^2 \frac{1}{2!} 2 \times 4 \times 3 \times 4 \times 3 = (-12B\mu)^2$
$\sum_K^3 \left[\frac{1}{L^2}\right]^2$
which vertex K connects
of the remaining 3 legs who connects to next vertex
who connects to -K of the remaining 3
redifining m^2_{eff}
mass renormalization
expanding exp
$\omega^2 \to \omega^2_{phys} = \omega^2 + 12\frac{\mu}{m}$
k_1 k_2 k_3 K
$\frac{1}{L^2}\sum_q \frac{1}{\cdots q}$

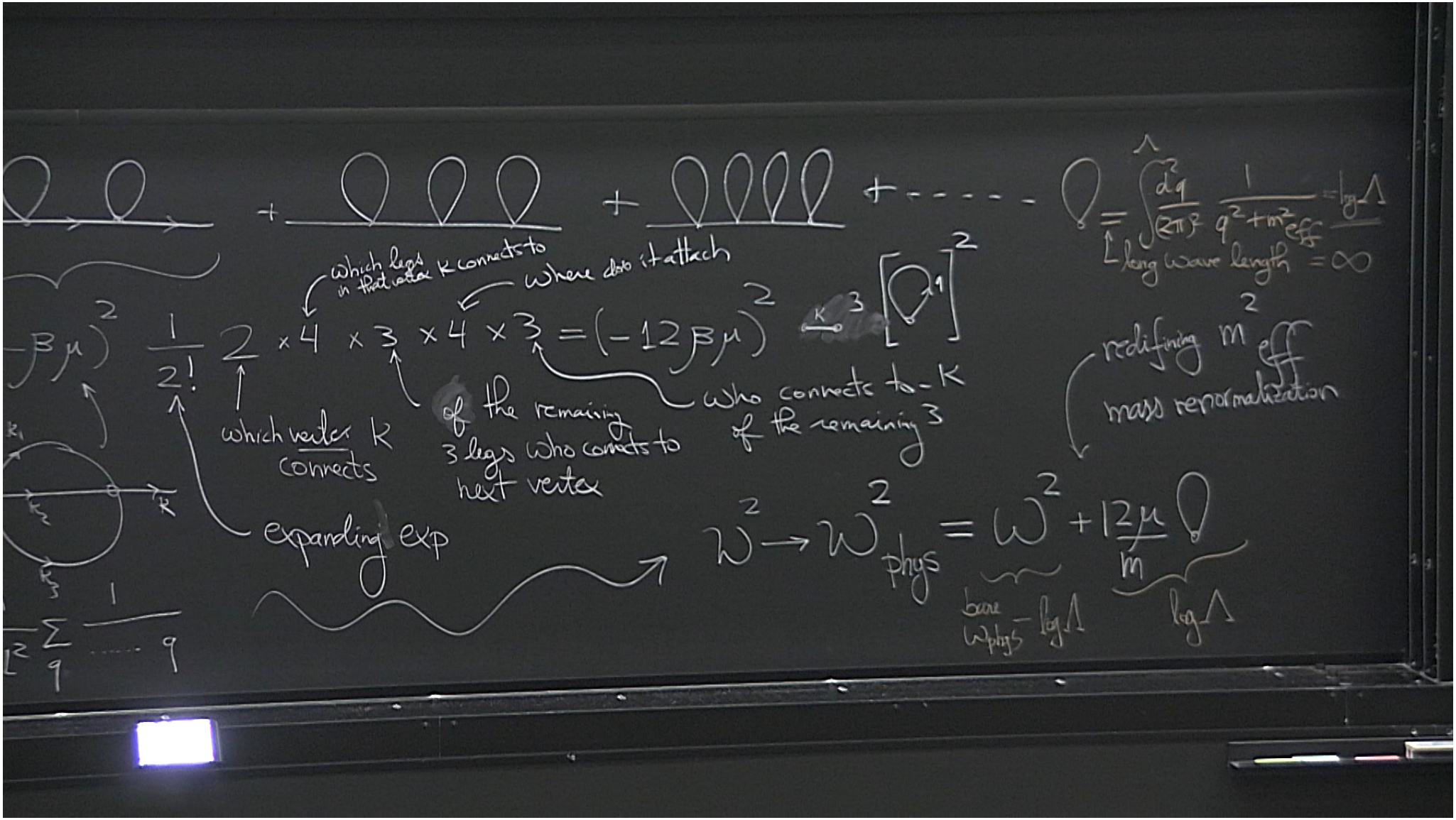

Large Talk,
Large separations
|m| >> 1
details washed out
effective mass or
correlation length
Small mom
Spacial derivative
discrete
large wave lengths
our action


which legs in that vertex K connects to
where does it attach
which vertex K connects
of the remaining 3 legs who connects to next vertex
who connects to -K of the remaining 3
expanding exp
redifining m²eff
mass renormalization
long wave length

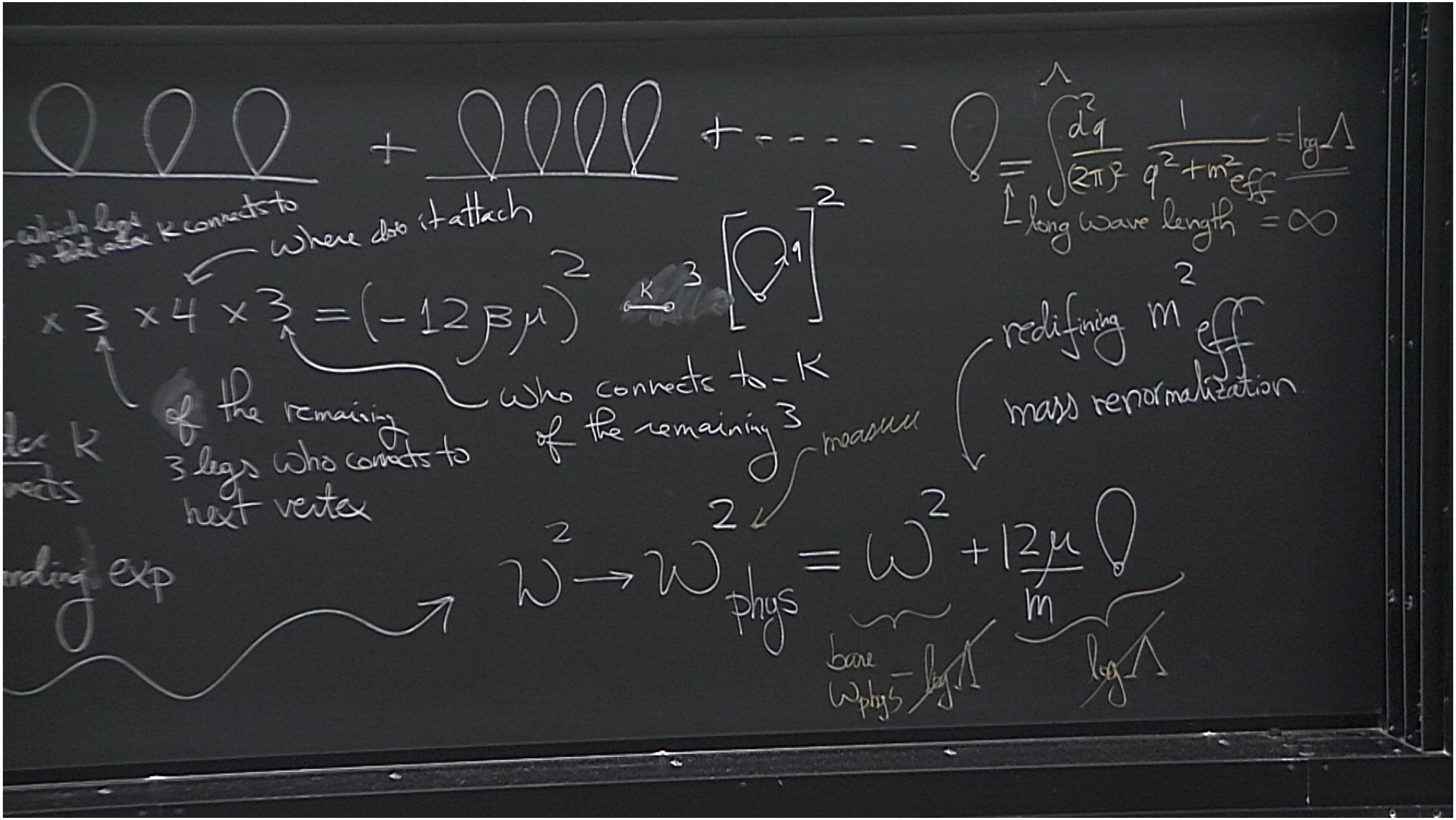

+ - - - - - -
$= \int_{(2\pi)^3}^{\Lambda} \frac{d^3q}{q^2+m^2_{eff}} \frac{1}{} = \log \Lambda$
long wave length
redifining m^2_{eff}
mass renormalization
who connects to -K
of the remaining 3
K
3
q
2


+ - - - - -
$= \int^{\Lambda} \frac{d^2q}{(2\pi)^2} \frac{1}{q^2+m^2_{eff}} = \log \Lambda$
long wave length $= \infty$
K
3
q
2
who connects to $-K$
of the remaining 3
redifining m^2_{eff}
mass renormalization
$+ 12\mu$

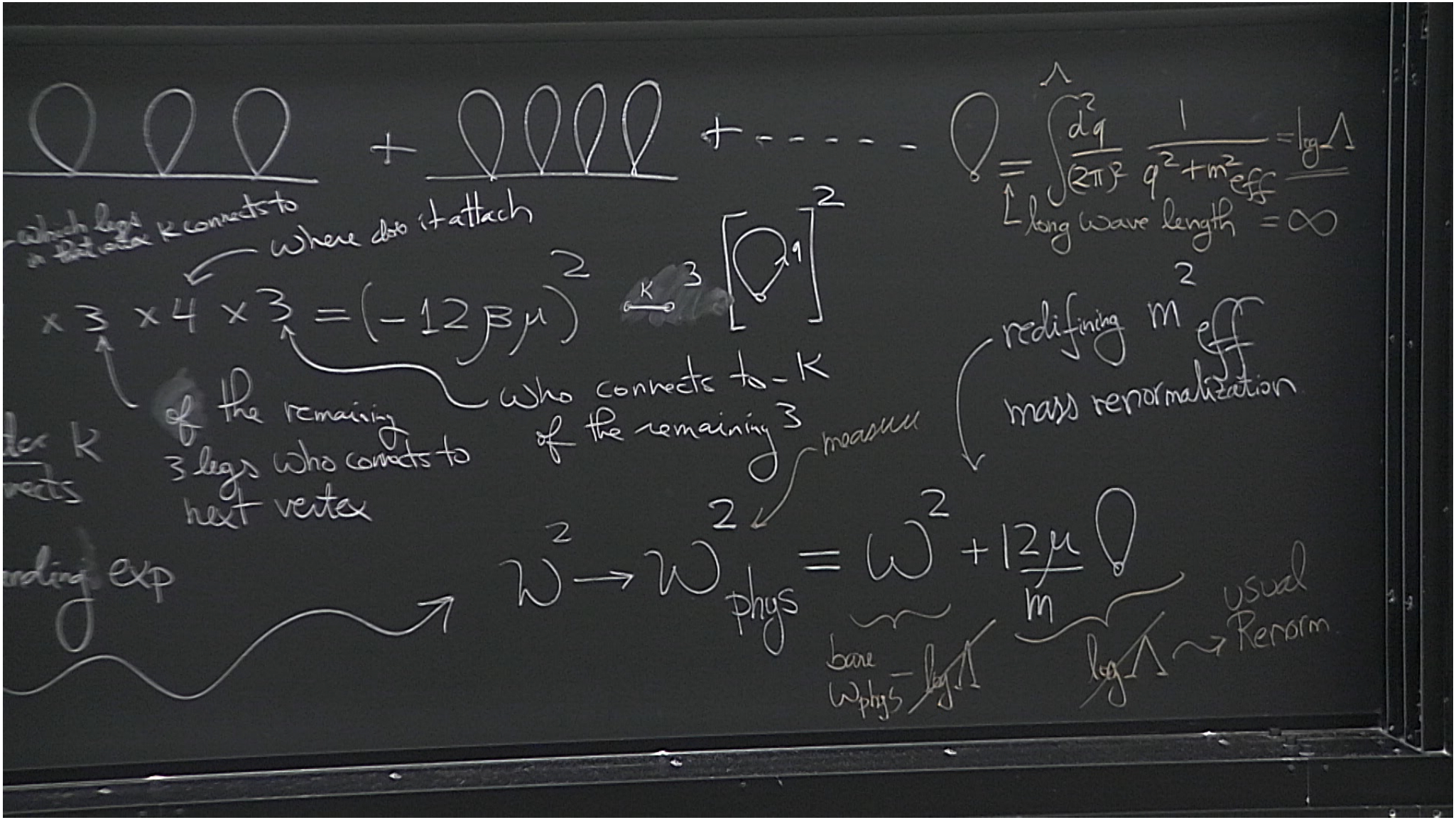

which legs in that vertex K connects to
where does it attach
which vertex K connects
of the remaining 3 legs who connects to next vertex
who connects to -K of the remaining 3
expanding exp
redifining m eff
mass renormalization
long wave length = ∞
log Λ



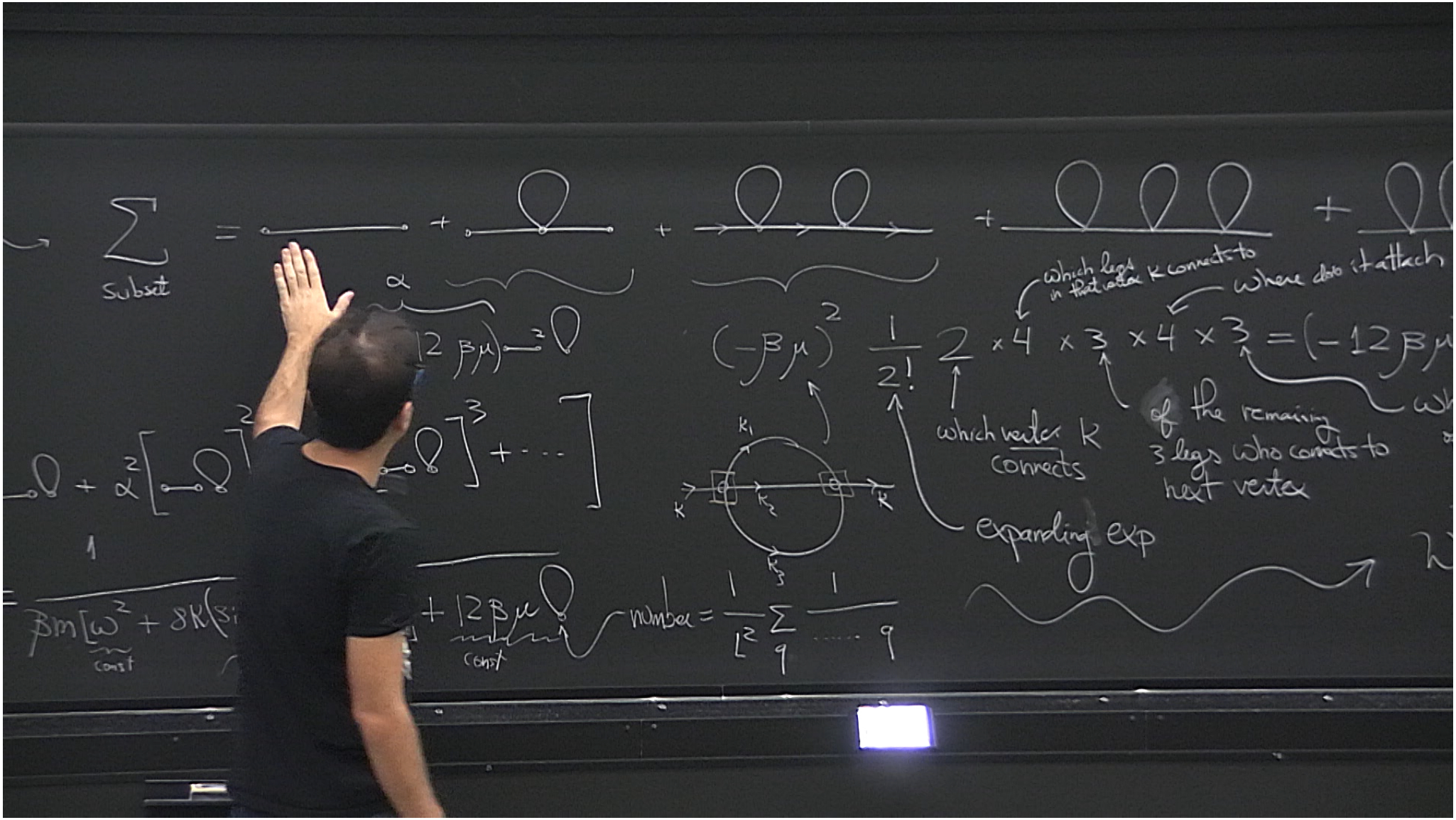
which legs in that vertex K connects to
where does it attach
$\frac{1}{2!}\, 2 \times 4 \times 3 \times 4 \times 3 = (-12\beta\mu)^2$
which vertex K connects
of the remaining 3 legs who connects to next vertex
who connects to -K of the remaining 3
expanding exp
$= \int^{\Lambda} \frac{d^2q}{(2\pi)^2} \frac{1}{q^2+m^2_{eff}} = \log\Lambda$
long wave length $= \infty$
redifining m^2_{eff}
mass renormalization
$W^2 \to W^2_{phys} = W^2 + \frac{12\mu}{m}$
bare W_{phys} $-\log\Lambda$
$\log\Lambda$

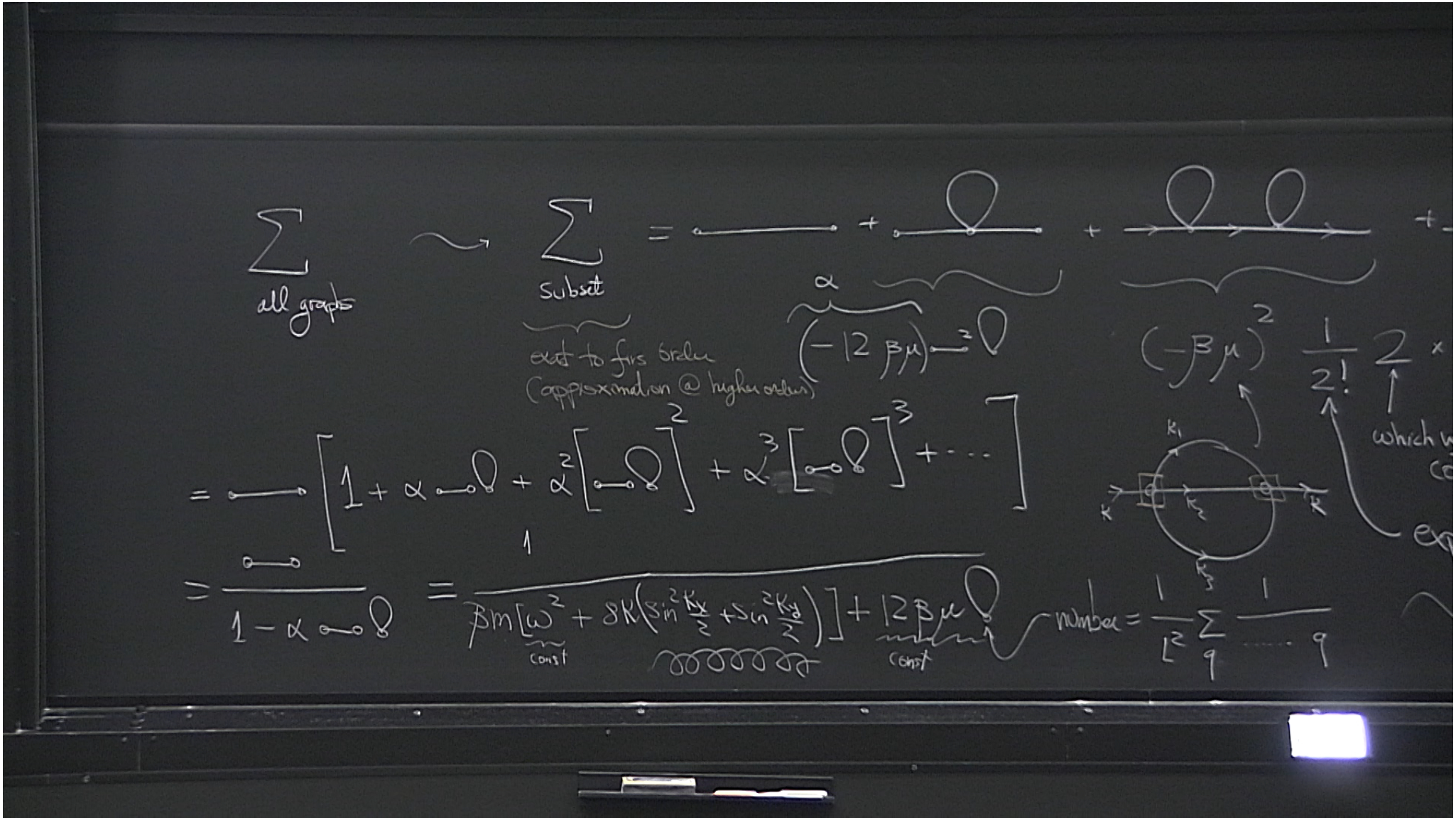


+ + - - - - -
$= \int^{\Lambda} \frac{d^2q}{(2\pi)^2} \frac{1}{q^2+m^2_{eff}} = \log\Lambda$
long wave length $= \infty$
which legs in that vertex K connects to
where does it attach
$\times 3 \times 4 \times 3 = (-12\beta\mu)^2$
K
3
q
2
of the remaining 3 legs who connects to next vertex
who connects to -K of the remaining 3
redifining m^2_{eff}
mass renormalization
measure
$W^2 \to W^2_{phys} = W^2 + 12\frac{\mu}{m}$
bare W_{phys} $-\cancel{\log\Lambda}$
$\cancel{\log\Lambda}$



+ + - - - - -
$= \int^{\Lambda} \frac{d^2q}{(2\pi)^2} \frac{1}{q^2+m^2_{eff}} = \log\Lambda$
long wave length $= \infty$
which legs in that vertex K connects to
where does it attach
$\times 3 \times 4 \times 3 = (-12\beta\mu)^2$
K
3
q
2
of the remaining 3 legs who connects to next vertex
who connects to -K of the remaining 3
redifining m^2_{eff}
mass renormalization
measure
$W^2 \to W^2_{phys} = W^2 + 12\frac{\mu}{m}$
bare W_{phys} $-\cancel{\log\Lambda}$
$\cancel{\log\Lambda}$ ~ usual Renorm


Subset
which leg in that vertex K connects to
where does it attach
which vertex K connects
of the remaining 3 legs who connects to next vertex
expanding exp
number


all graphs
subset
exact to first order
(approximation @ higher orders)
α
(−12 βμ)
(−βμ)²
1/2!
2
which
number = 1/L² Σ q 1/q
const
const