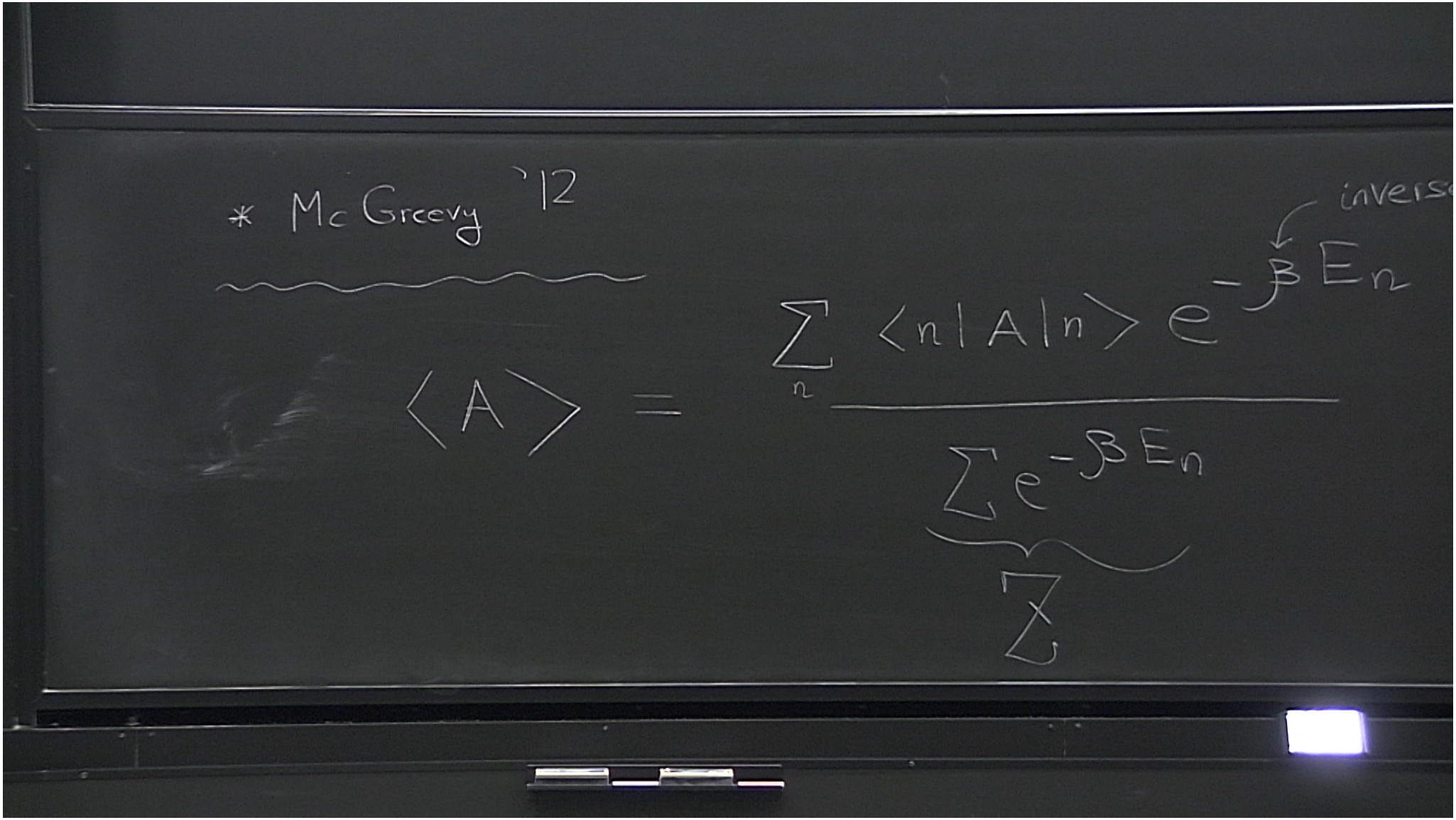
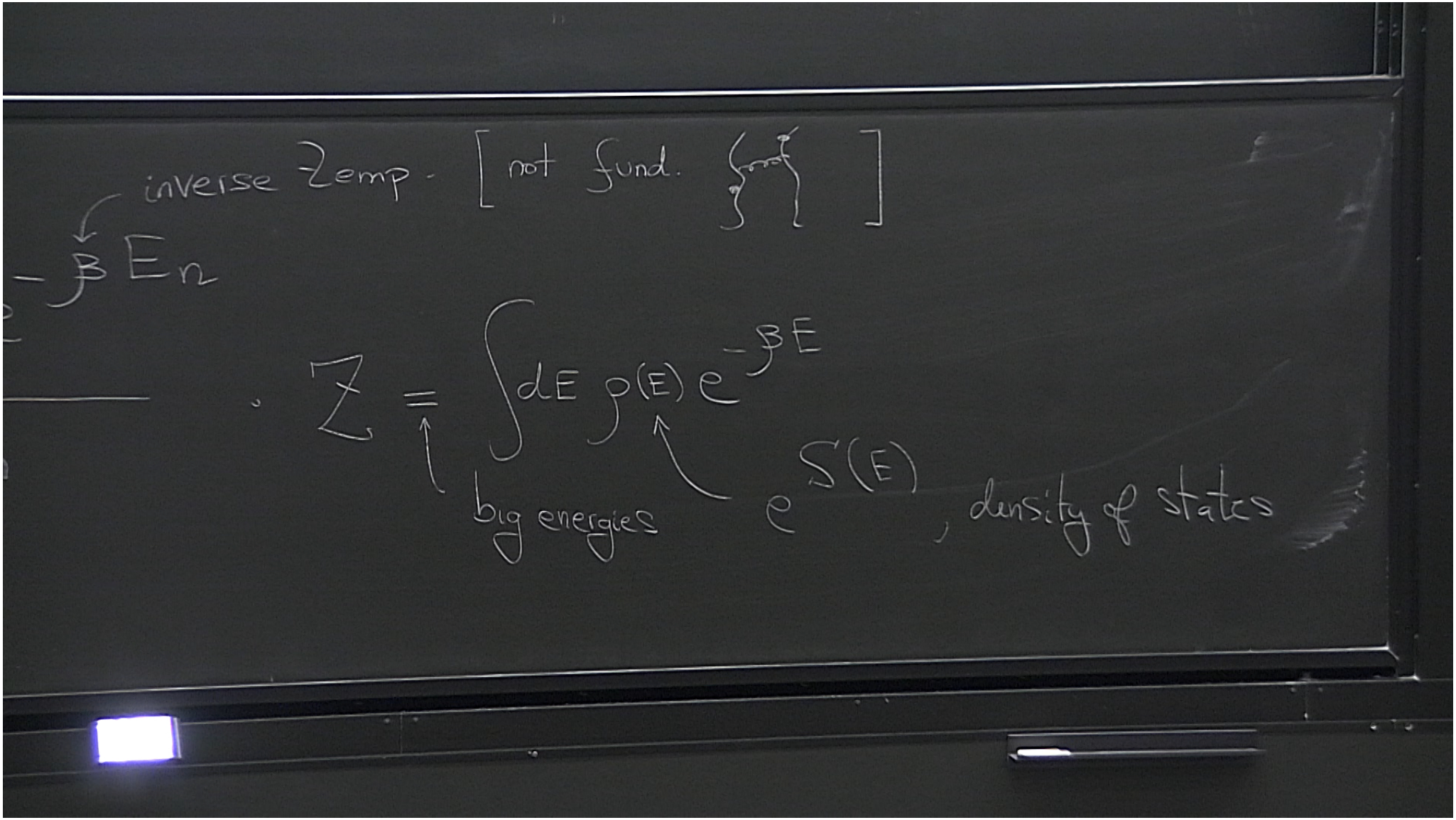
Title: PSI 2018/2019 - Statistical Mechanics - Lecture 1

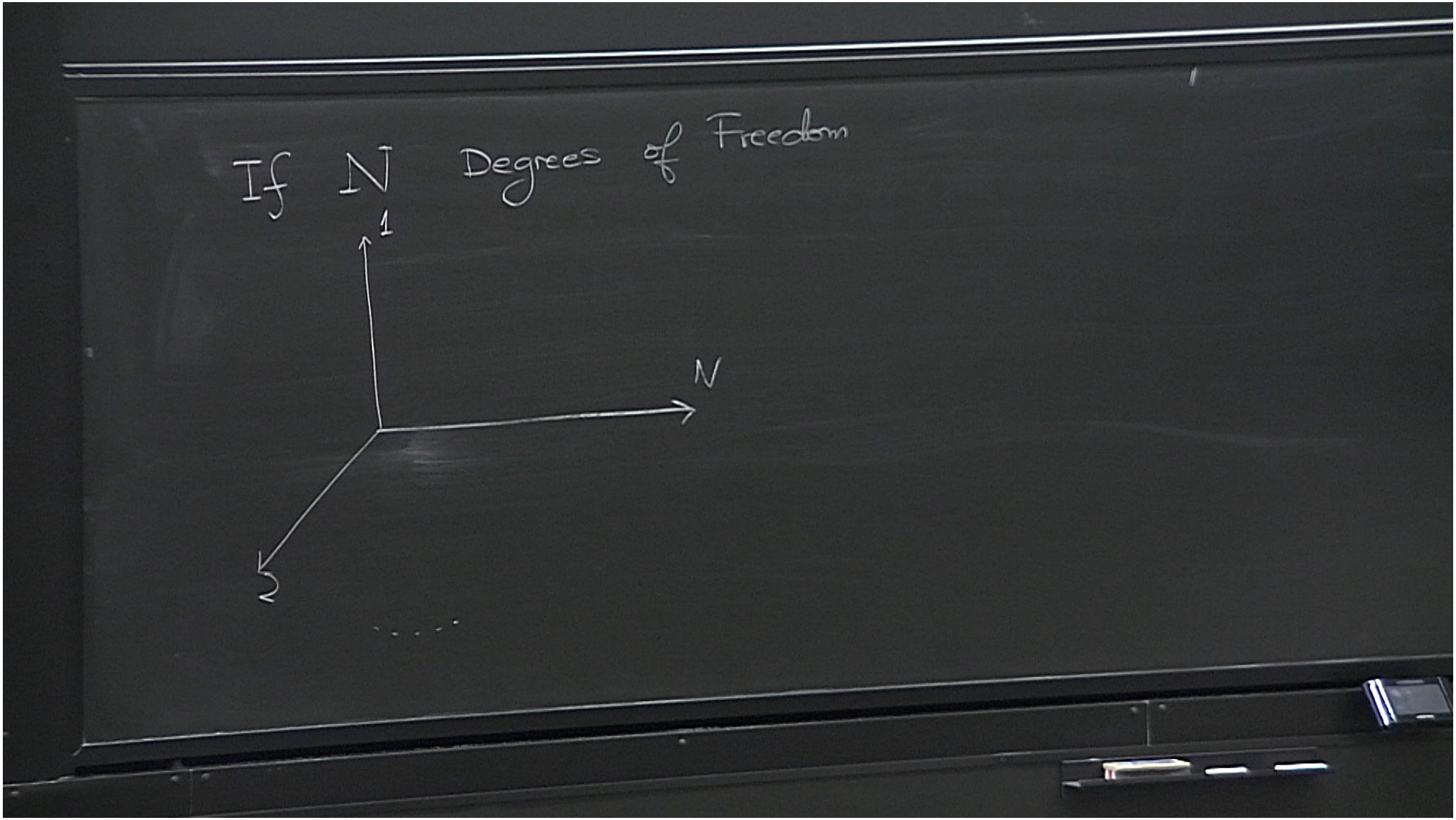
Date: Oct 09, 2018 10:45 AM

URL: http://pirsa.org/18100021

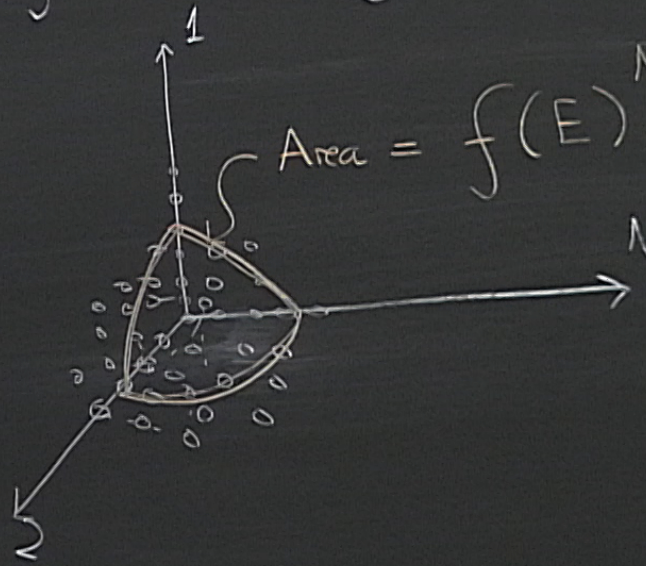
Abstract:


* McGreevy '12
$\langle A \rangle = \frac{\sum_n \langle n|A|n\rangle e^{-\beta E_n}}{\sum e^{-\beta E_n}}$
$\sum e^{-\beta E_n}$
Z


inverse Zemp. [not fund.
$-\beta E_n$
$Z = \int dE\, \rho(E)\, e^{-\beta E}$
big energies
$e^{S(E)}$, density of states


If N Degrees of Freedom
1
N
2
. . . .

If N Degrees of Freedom

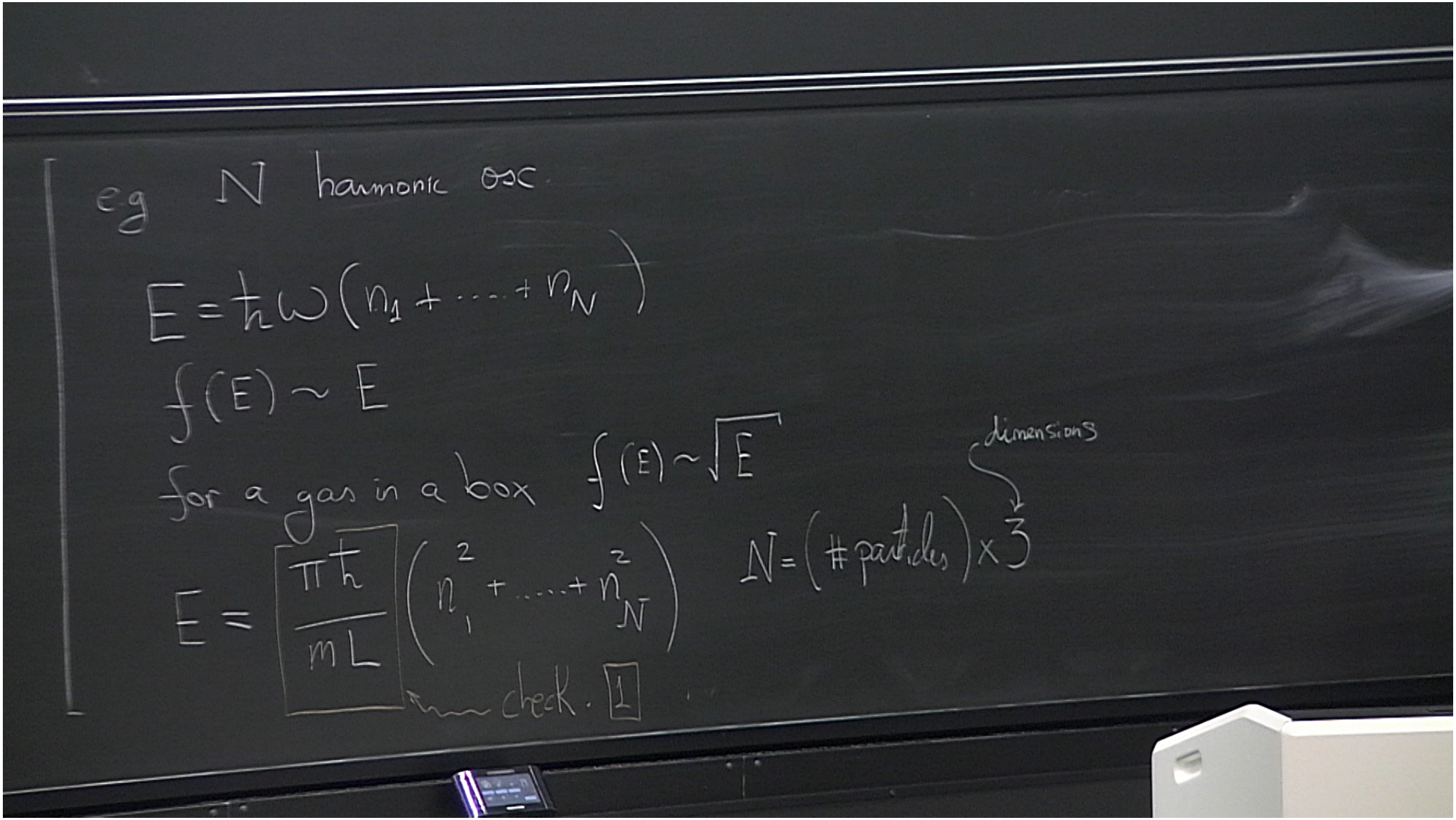


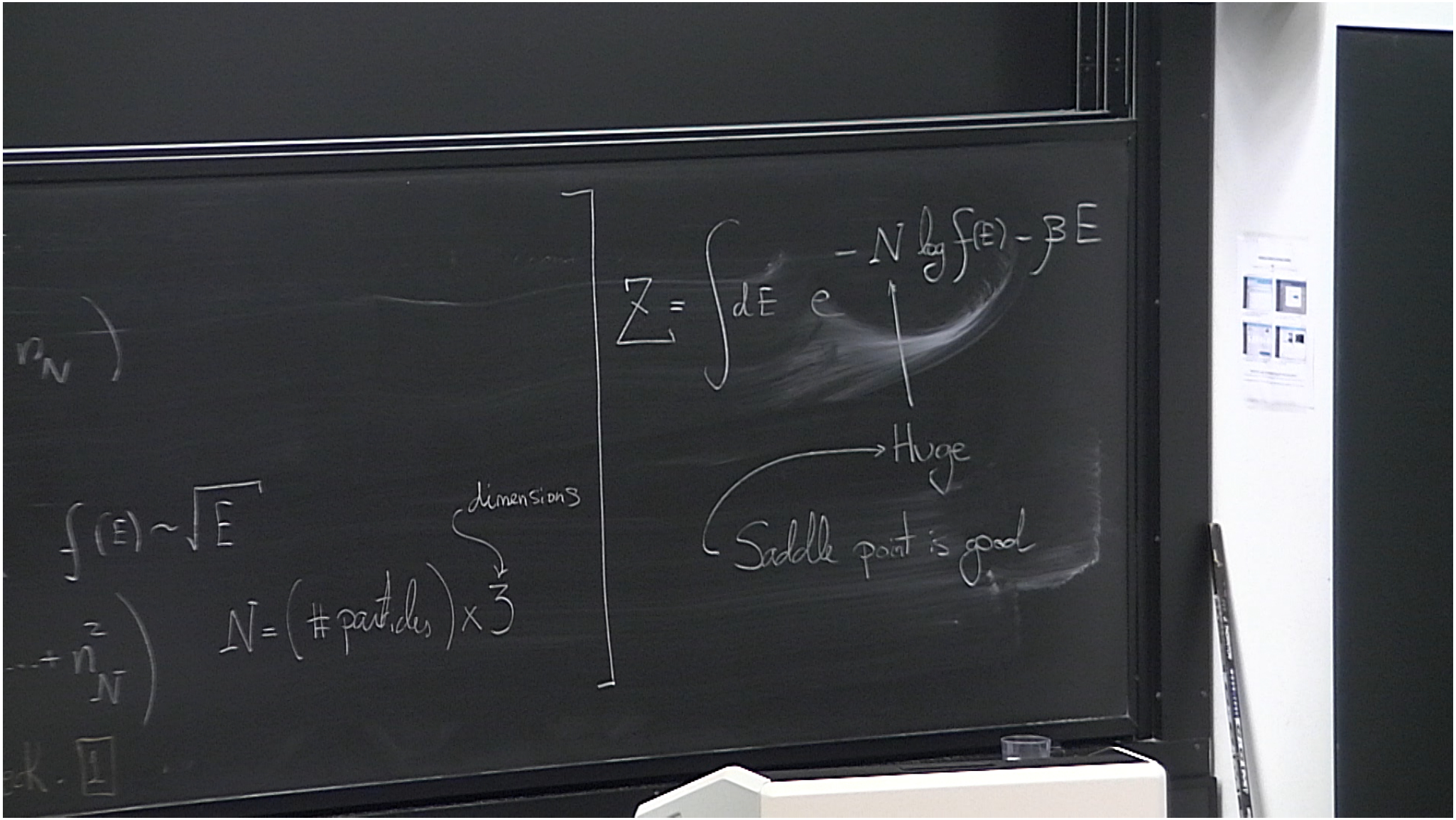
e.g

eg N harmonic osc

$$E = \hbar\omega(n_1 + \cdots + n_N)$$

$$f(E) \sim E$$

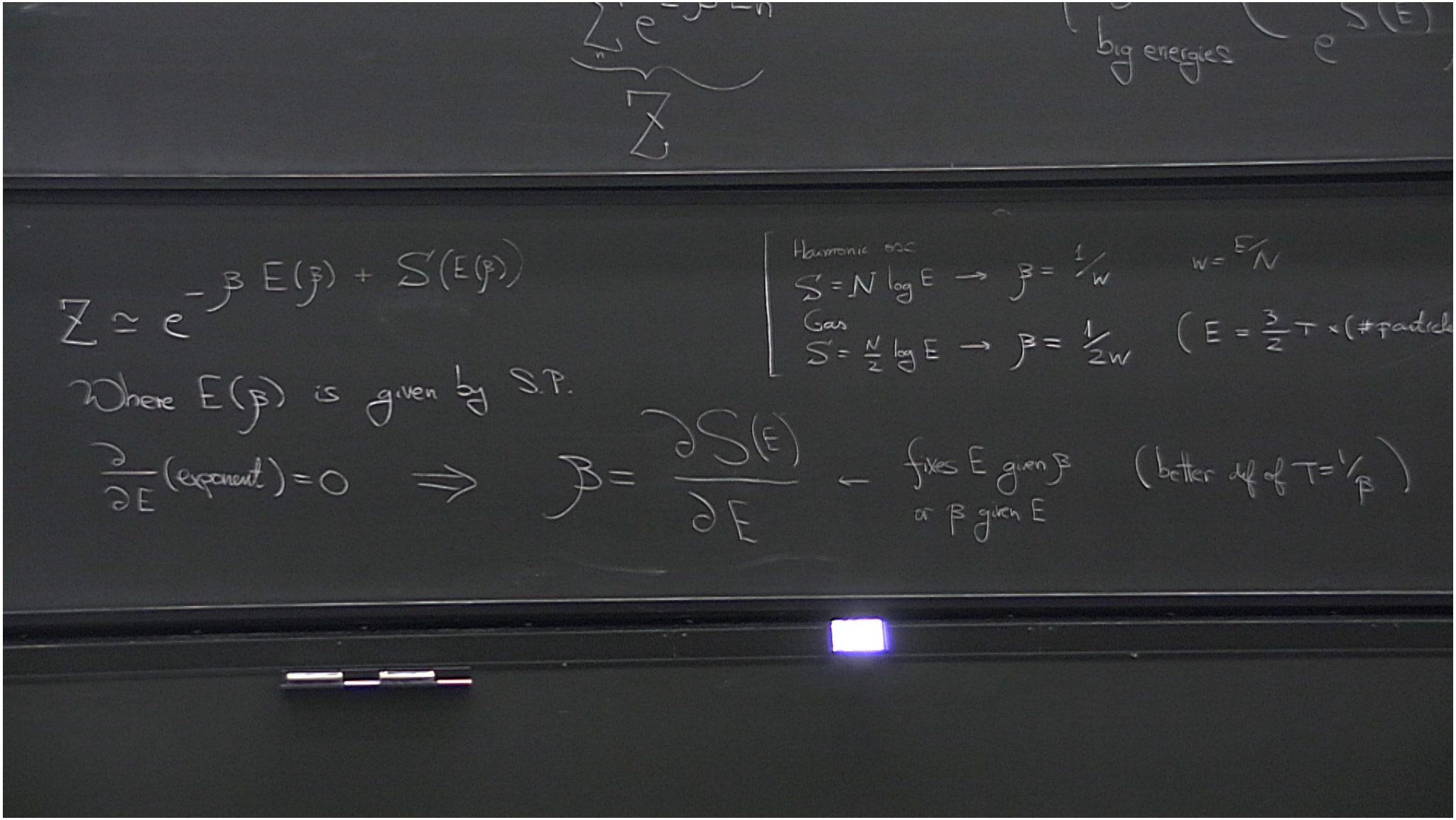

eg N harmonic osc
$E = \hbar\omega(n_1 + \dots + n_N)$
$f(E) \sim E$
for a gas in a box $f(E) \sim \sqrt{E}$
dimensions
$N = (\#\text{particles}) \times 3$
$E = \frac{\pi\hbar}{mL}\left(n_1^2 + \dots + n_N^2\right)$
check. 1


$Z = \int dE\, e^{-N \log f(E) - \beta E}$
Huge
Saddle point is good
$f(E) \sim \sqrt{E}$
$N = (\#\text{particles}) \times 3$
dimensions

$$Z \simeq e^{-\beta E(\beta) + S(E(\beta))}$$

Where $E(\beta)$ is given by S.P.

$$\frac{\partial}{\partial E}(\text{exponent}) = 0 \Rightarrow \beta = \frac{\partial S(E)}{\partial E}$$


big energies
$Z \simeq e^{-\beta E(\beta) + S(E(\beta))}$
Where $E(\beta)$ is given by S.P.
$\frac{\partial}{\partial E}(\text{exponent}) = 0 \Rightarrow \beta = \frac{\partial S(E)}{\partial E}$
fixes E given β
or β given E
(better off of $T = 1/\beta$)
Harmonic osc
$S = N \log E \rightarrow \beta = 1/w$
$w = E/N$
Gas
$S = \frac{N}{2} \log E \rightarrow \beta = \frac{1}{2w}$
$E = \frac{3}{2} T \times (\#$ partic

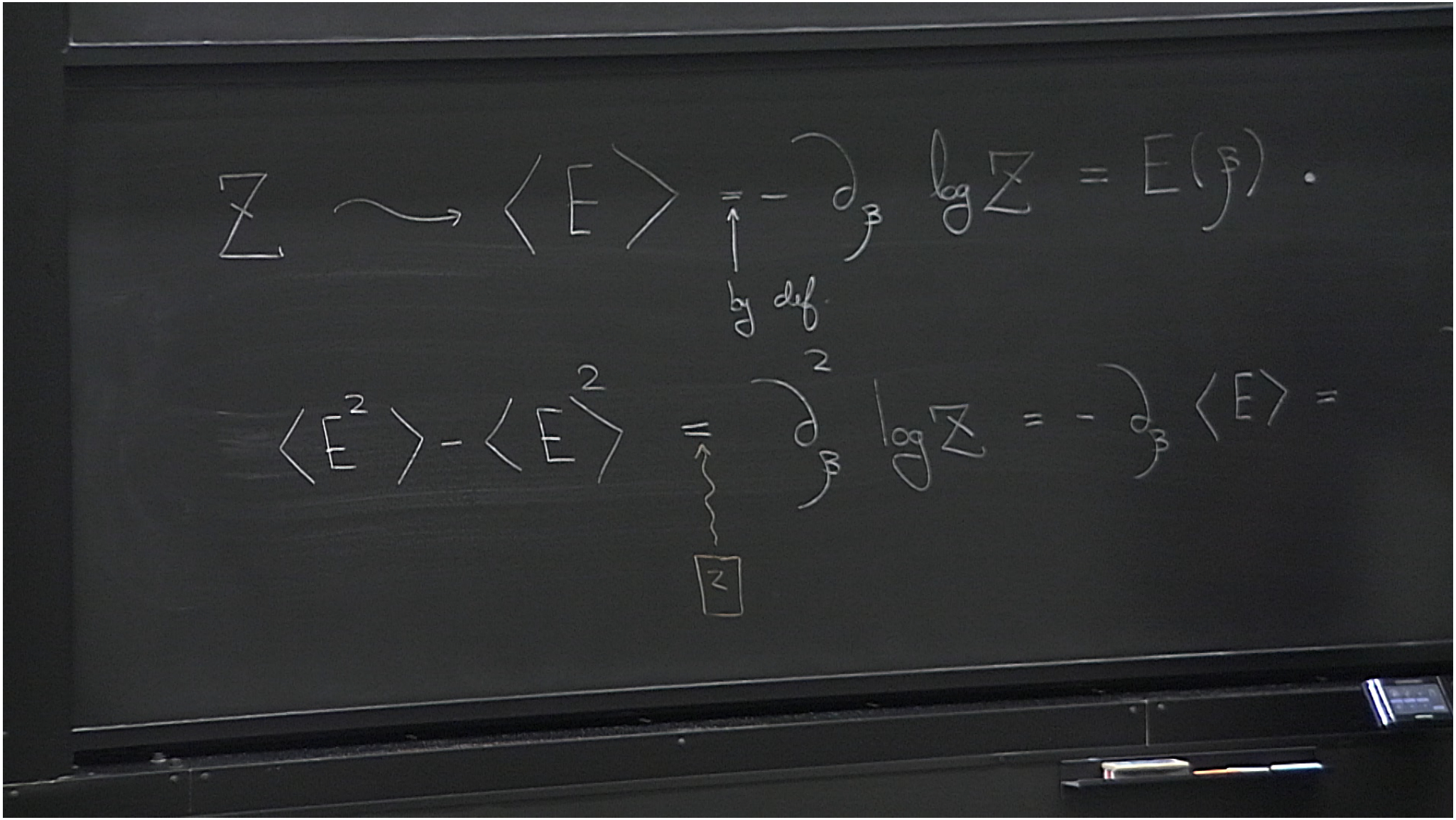
Harmonic osc.

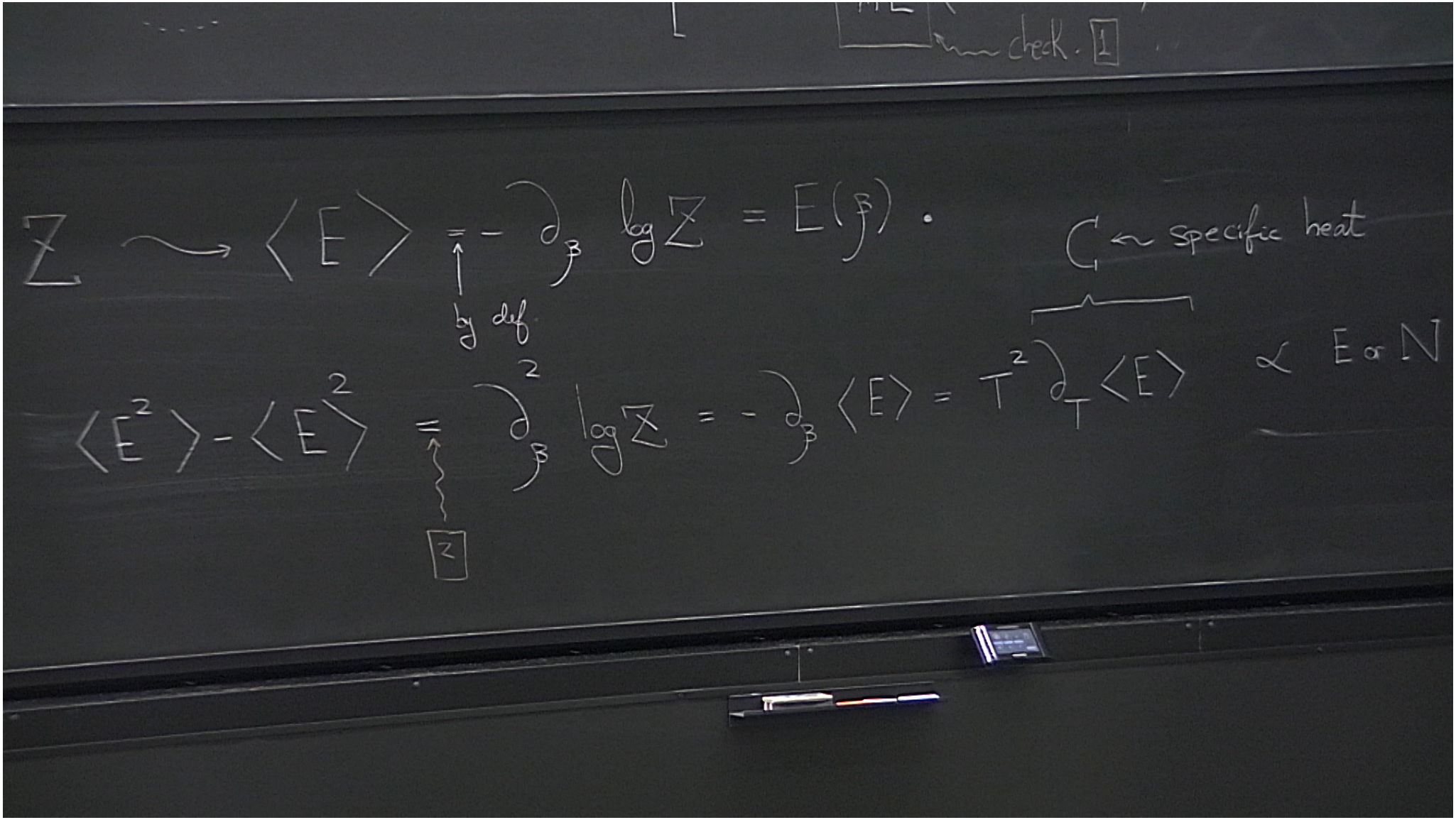
$S = N \log E \rightarrow \beta = 1/w$ $\quad w = E/N$

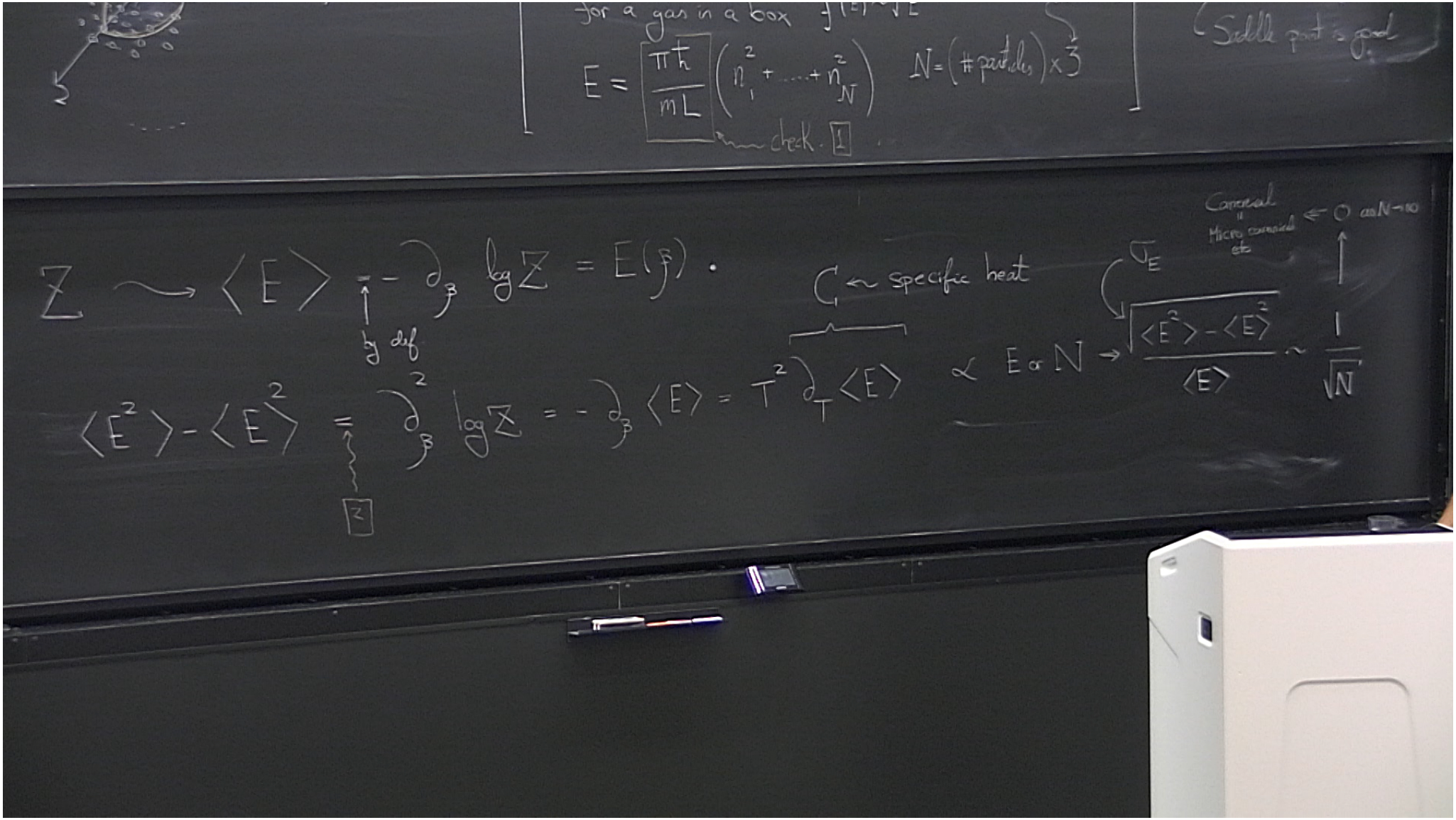
Gas

$S = \frac{N}{2} \log E \rightarrow \beta = \frac{1}{2w}$ $\quad \left(E = \frac{3}{2} T \times (\#\text{particles}) \right)$

$\leftarrow$ fixes E given β or β given E (better def of $T = 1/\beta$)


Z ⇝ ⟨E⟩ = − ∂_β log Z = E(β).
by def.
⟨E²⟩ − ⟨E⟩² = ∂²_β log Z = − ∂_β ⟨E⟩ =
Z


check. I
$Z \rightsquigarrow \langle E \rangle = -\partial_\beta \log Z = E(\beta)$.
by def
$C \sim$ specific heat
$\langle E^2 \rangle - \langle E \rangle^2 = \partial_\beta^2 \log Z = -\partial_\beta \langle E \rangle = T^2 \partial_T \langle E \rangle \propto E$ or N
Z


for a gas in a box
$E = \frac{\pi^2\hbar^2}{mL}\left(n_1^2 + \dots + n_N^2\right)$
$N = (\#\text{particles}) \times 3$
Saddle point is good
check.
$Z \rightsquigarrow \langle E \rangle = -\partial_\beta \log Z = E(\beta)$.
by def
$C \sim$ specific heat
$\langle E^2 \rangle - \langle E \rangle^2 = \partial_\beta^2 \log Z = -\partial_\beta \langle E \rangle = T^2 \partial_T \langle E \rangle \propto E \propto N$
σ_E
$\frac{\sqrt{\langle E^2 \rangle - \langle E \rangle^2}}{\langle E \rangle} \sim \frac{1}{\sqrt{N}}$
Canonical ≈ Micro canonical etc.
$\to 0$ as $N \to \infty$

$$Z \simeq e^{-\beta E(\beta) + S(E(\beta))}$$

Where $E(\beta)$ is given by S.P.

$$\frac{\partial}{\partial E}(\text{exponent}) = 0 \Rightarrow \beta = \frac{\partial S(E)}{\partial E}$$

#

fixe

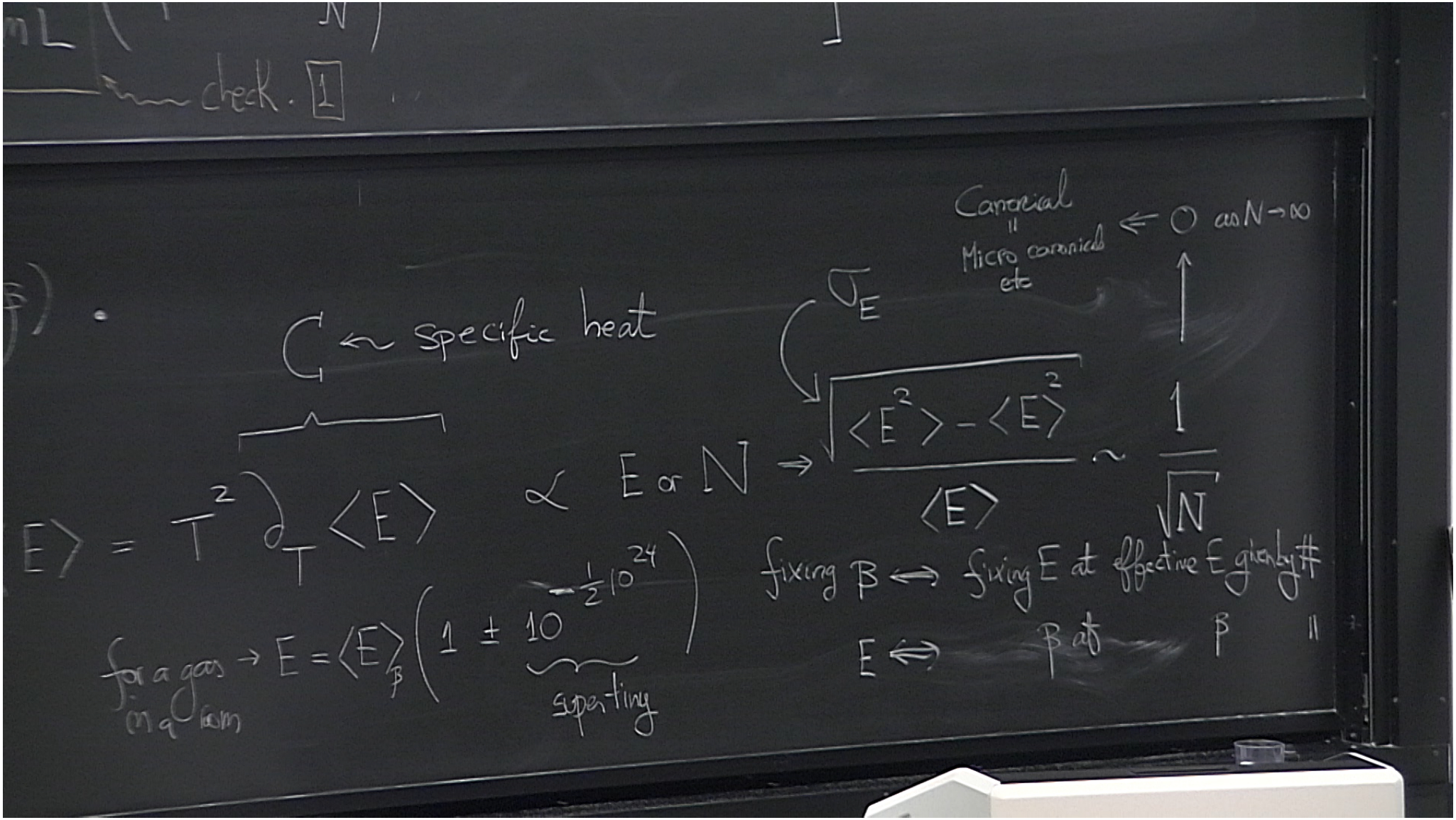
or

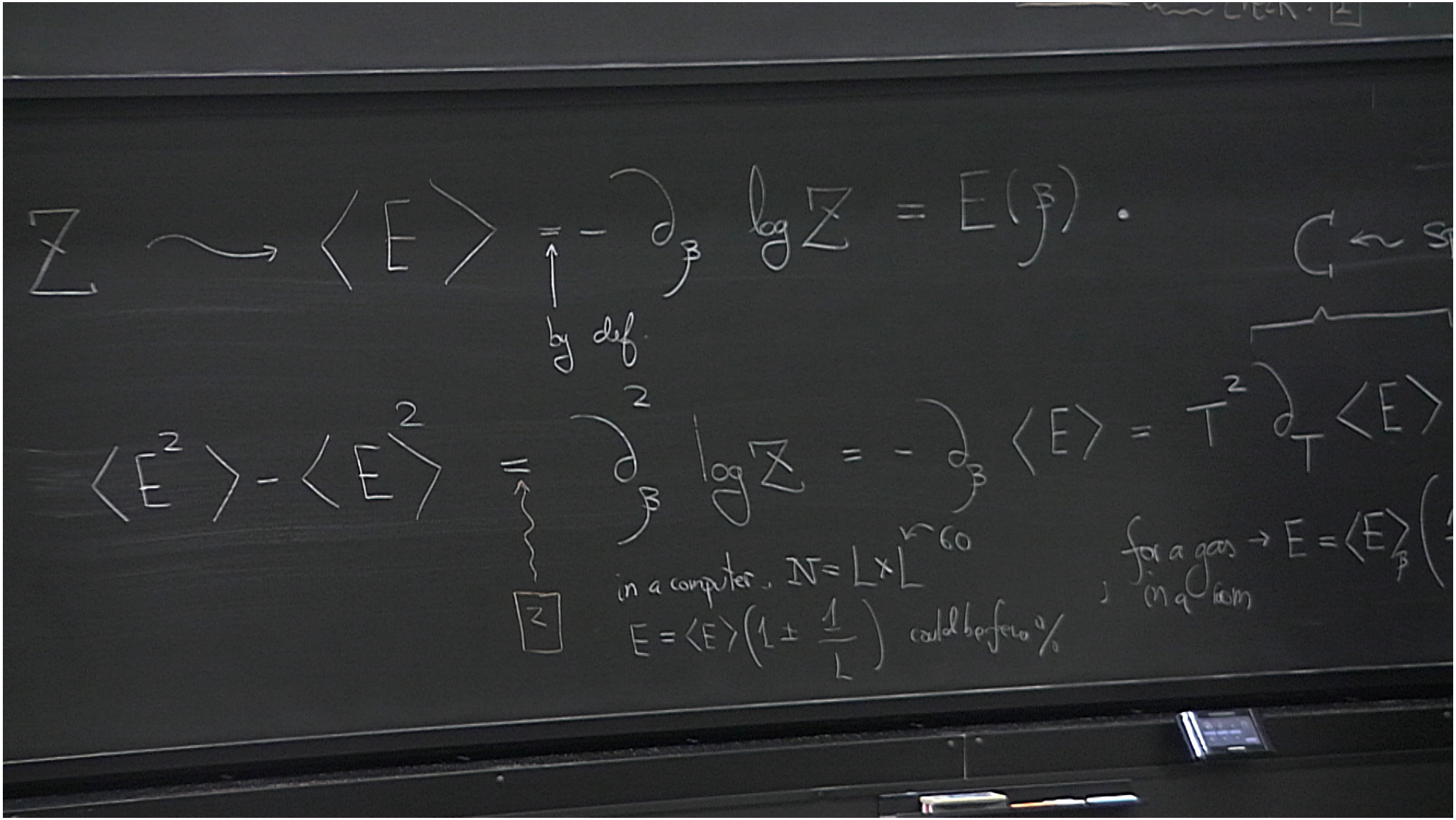
Harmonic osc

$S = N \log E$

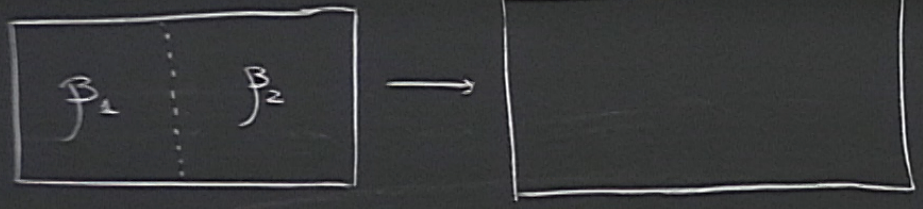
Gas

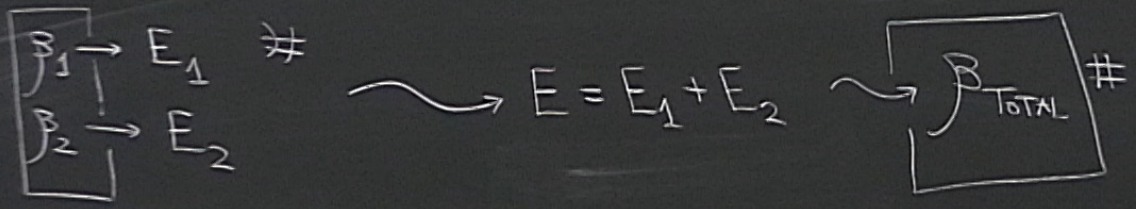
$S = \frac{N}{2} \log E$


check. 1
Canonical
= Micro canonical
etc
$\to 0$ as $N \to \infty$
$C \sim$ specific heat
σ_E
$C = T^2 \partial_T \langle E \rangle \propto E$ or $N \Rightarrow \frac{\sqrt{\langle E^2 \rangle - \langle E \rangle^2}}{\langle E \rangle} \sim \frac{1}{\sqrt{N}}$
fixing $\beta \Leftrightarrow$ fixing E at effective E given by #
$E \Leftrightarrow \beta$ at β =
for a gas $\to E = \langle E \rangle_\beta \left(1 \pm 10^{-\frac{1}{2}10^{24}}\right)$
many dof
super tiny


$Z \rightsquigarrow \langle E \rangle = -\partial_\beta \log Z = E(\beta)$.
by def.
$\langle E^2 \rangle - \langle E \rangle^2 = \partial_\beta^2 \log Z = -\partial_\beta \langle E \rangle = T^2 \partial_T \langle E \rangle$
Z
in a computer, $N = L \times L$ ~60
$E = \langle E \rangle \left(1 \pm \frac{1}{L}\right)$ could be perfect %
for a gas → $E = \langle E \rangle_\beta$ (
many

two boxes which we join



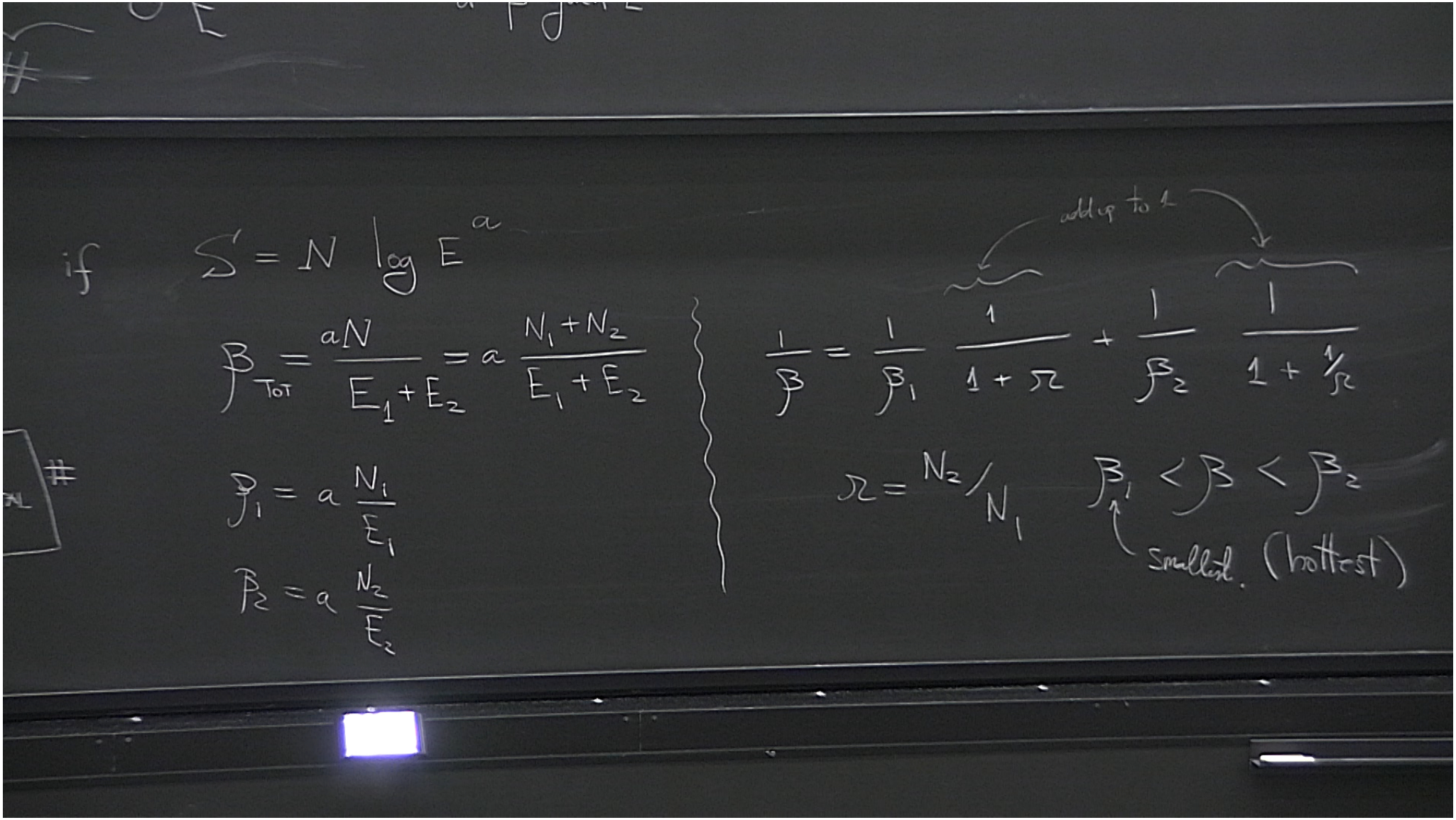


if $S = N \log E^a$

$$\beta_{TOT} = \frac{aN}{E_1 + E_2} = a \frac{N_1 + N_2}{E_1 + E_2}$$

$$\beta_1 = a \frac{N_1}{E_1}$$

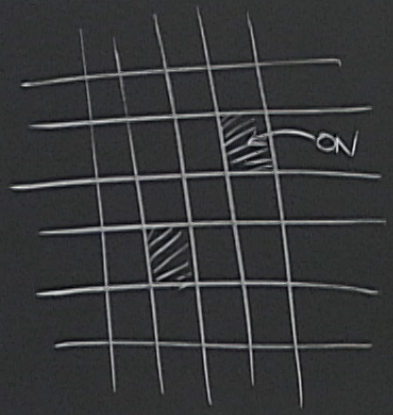
$$\beta_2 = a \frac{N_2}{E_2}$$


if $S = N \log E^a$
$\beta_{Tot} = \frac{aN}{E_1 + E_2} = a \frac{N_1 + N_2}{E_1 + E_2}$
$\beta_1 = a \frac{N_1}{E_1}$
$\beta_2 = a \frac{N_2}{E_2}$
adds to 1
$\frac{1}{\beta} = \frac{1}{\beta_1} \frac{1}{1+\Omega} + \frac{1}{\beta_2} \frac{1}{1+1/\Omega}$
$\Omega = N_2/N_1$
$\beta_1 < \beta < \beta_2$
smallest (hottest)

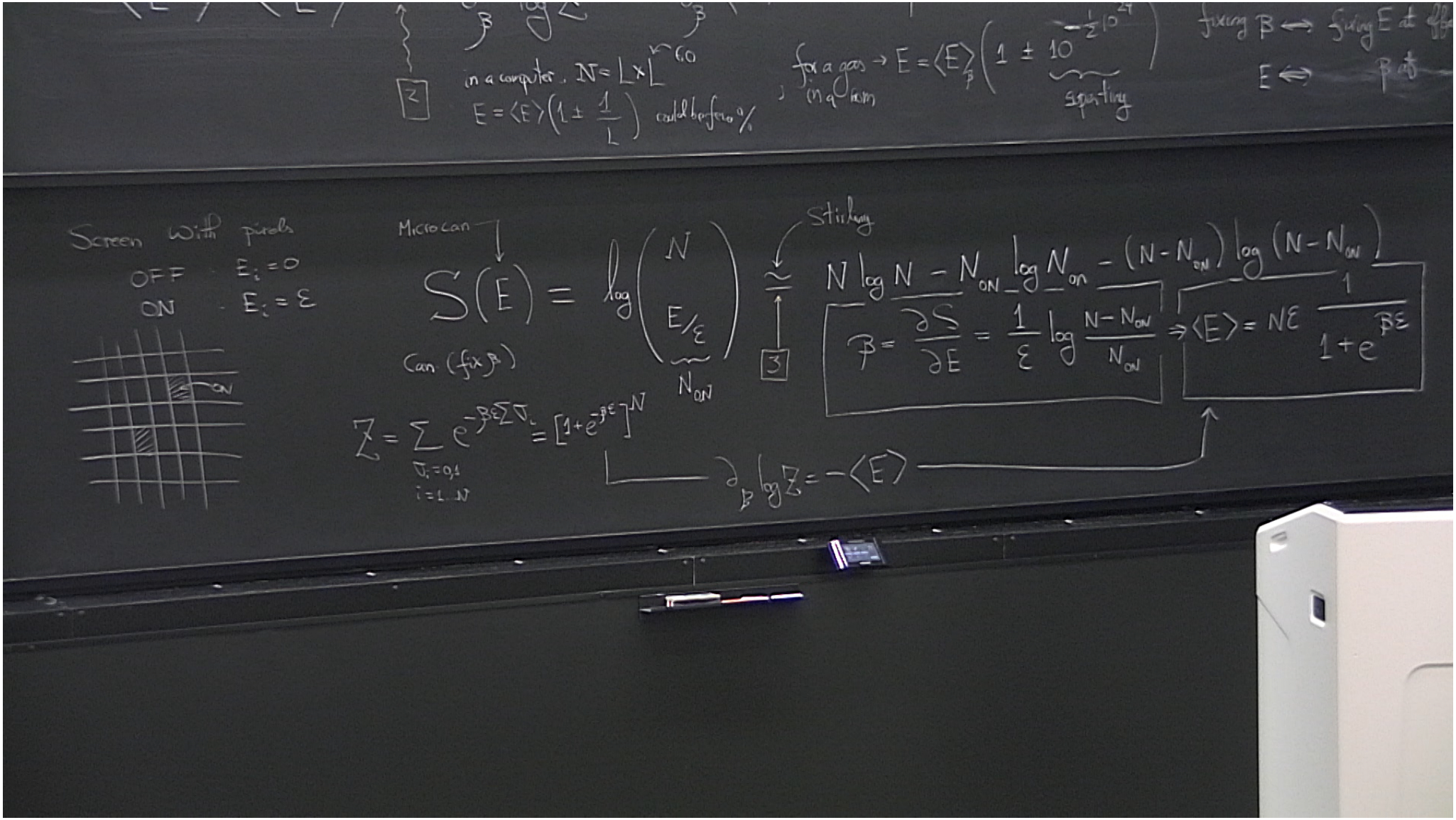
Screen with pixels

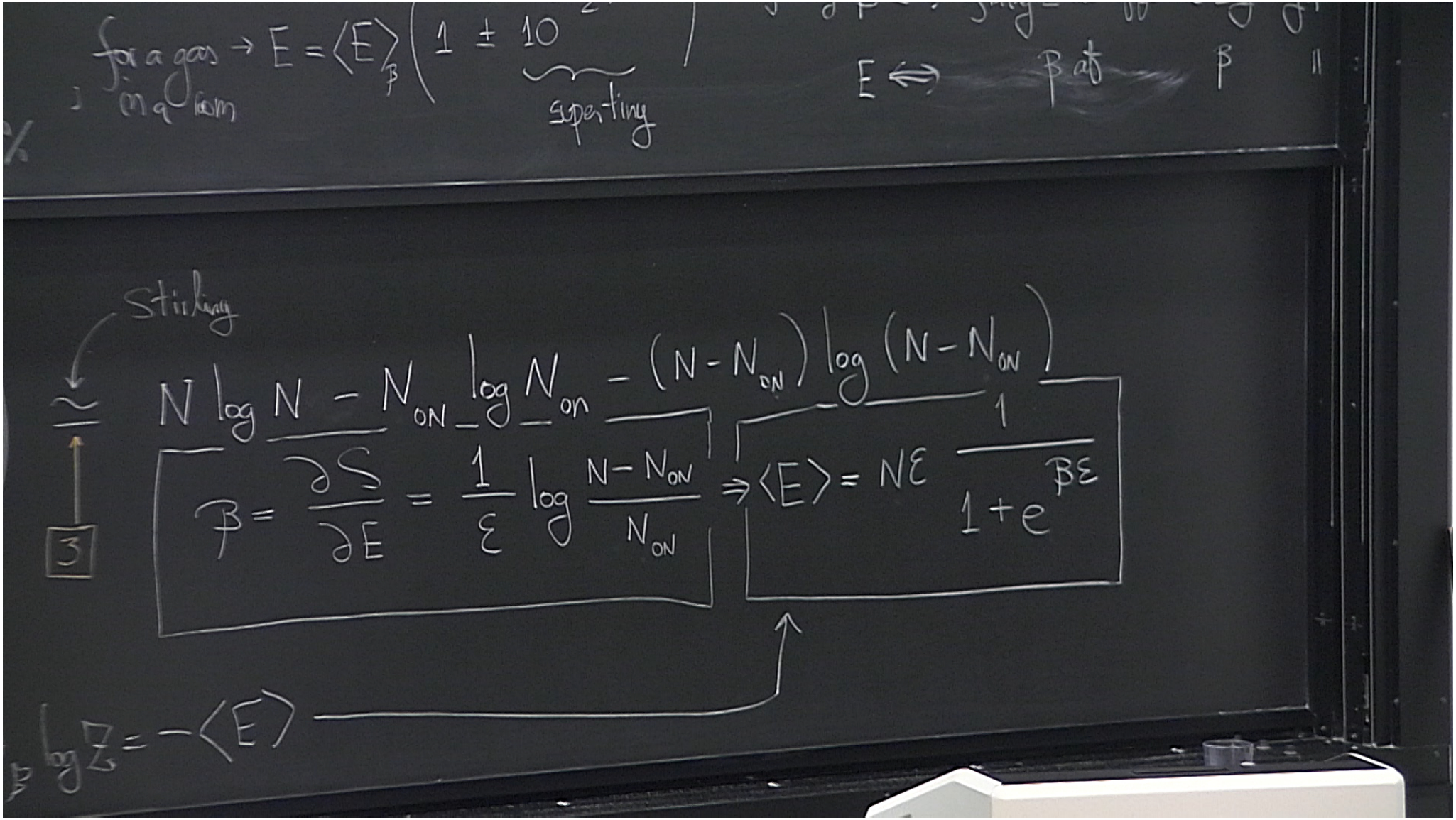
OFF : $E_i = 0$

ON : $E_i = \varepsilon$

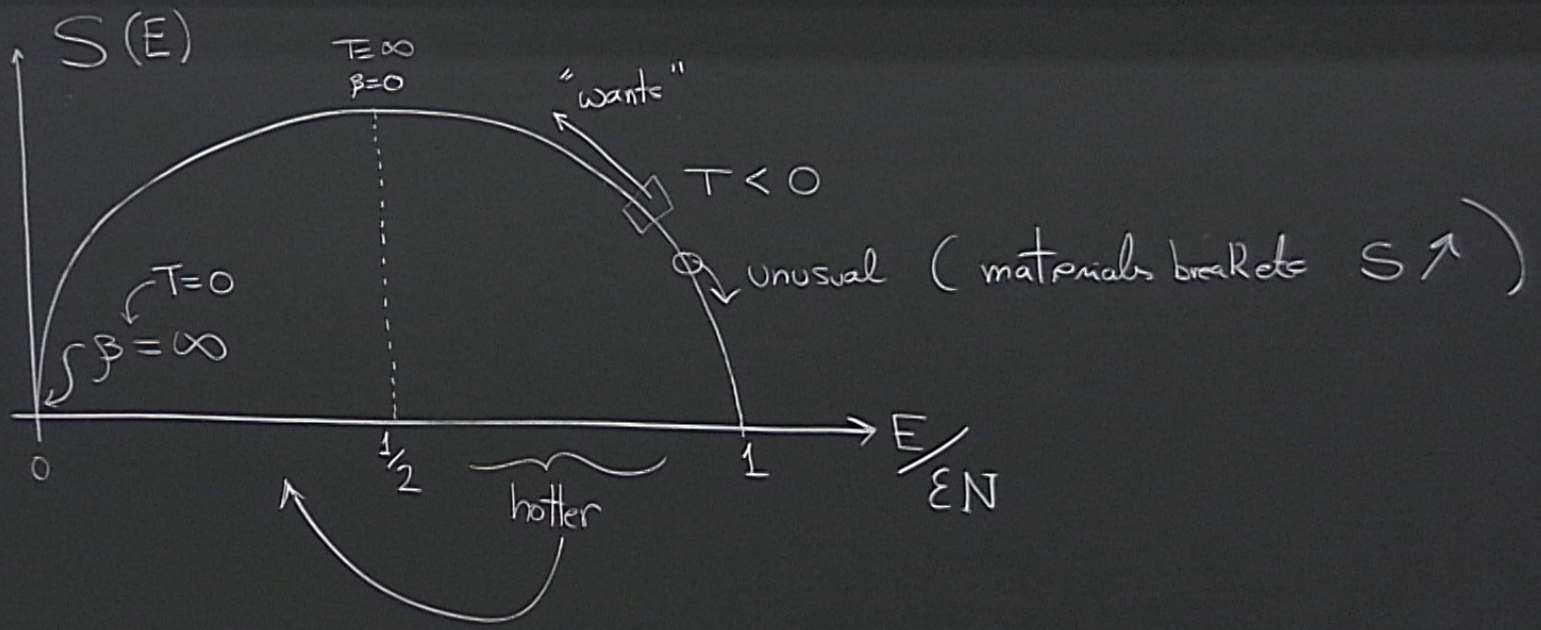


$$S(E) = \log\left(\begin{array}{c} N \\ E/\varepsilon \\ \sim \\ N_{ON} \end{array}\right)$$


in a computer, $N = L \times L$
$E = \langle E \rangle \left(1 \pm \frac{1}{L}\right)$
for a gas → $E = \langle E \rangle_\beta \left(1 \pm 10^{-\frac{1}{2}10^{24}}\right)$
fixing β ⟷ fixing E at ff
$E \Leftrightarrow \beta$
Screen with pixels
OFF $E_i = 0$
ON $E_i = \varepsilon$
Micro can
$S(E) = \log \binom{N}{E/\varepsilon}$
N_{ON}
Can. (fix β)
Stirling
$N \log N - N_{ON} \log N_{ON} - (N - N_{ON}) \log (N - N_{ON})$
$\beta = \frac{\partial S}{\partial E} = \frac{1}{\varepsilon} \log \frac{N - N_{ON}}{N_{ON}}$
$\Rightarrow \langle E \rangle = N\varepsilon \frac{1}{1 + e^{\beta\varepsilon}}$
$Z = \sum e^{-\beta\varepsilon \sum \sigma_i} = [1 + e^{-\beta\varepsilon}]^N$
$\sigma_i = 0, 1$
$i = 1 \dots N$
$\partial_\beta \log Z = -\langle E \rangle$

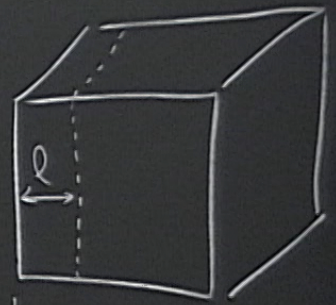

for a gas → E = ⟨E⟩_β (1 ± 10)
splitting
Stirling
≃ N log N − N_ON log N_on − (N − N_ON) log (N − N_ON)
β = ∂S/∂E = 1/ε log (N − N_ON)/N_ON
⇒ ⟨E⟩ = Nε 1/(1 + e^{βε})
3
log Z = −⟨E⟩

$P_2 = a \frac{N_2}{E_2}$



Gas in a box @ fixed E

- $\text{Prob}(x_i \in]-L/2, -L/2 + \ell[)$ — open interval
- $\text{Prob}(\vec{v}_i)$



$$= \frac{\int_{-L/2}^{-L/2+\ell} dq_1 \int_{-L/2}^{L/2} dq_{3N} \int_{-\infty}^{+\infty} dp_1 \cdots dp_{3N} \times \delta_\Delta\left(E = \frac{p_1^2}{2m} + \cdots + \frac{p_{3N}^2}{2m}\right)}{\Omega = \int dq_1 \cdots dq_{3N}\, dp_1 \cdots dp_{3N}\, \delta_\Delta(\cdots)}$$

any bracket works

$\delta_\Delta \to 1$ if $\in]E-\Delta, E+\Delta[$

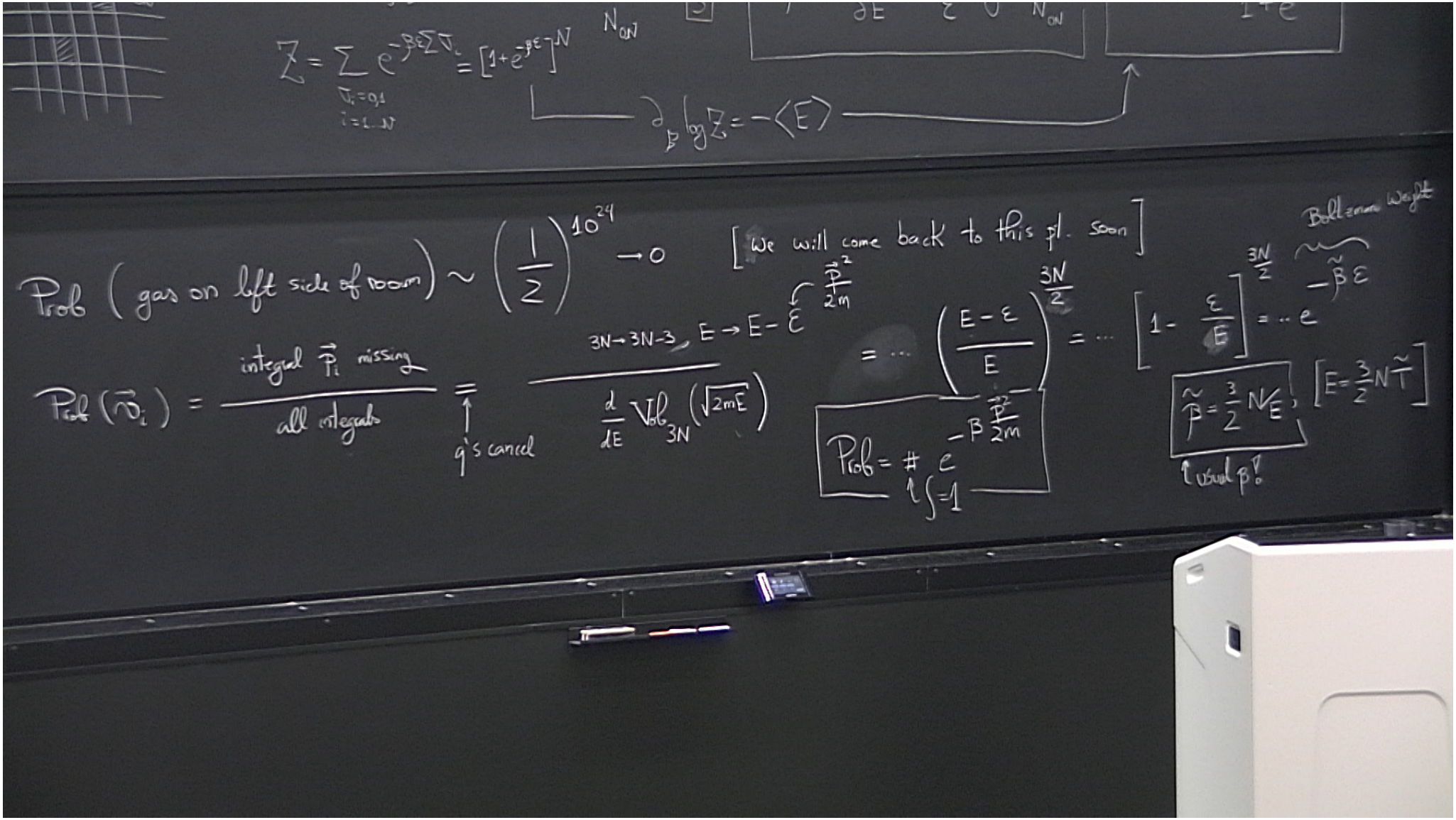
$$= \frac{\ell}{L}$$

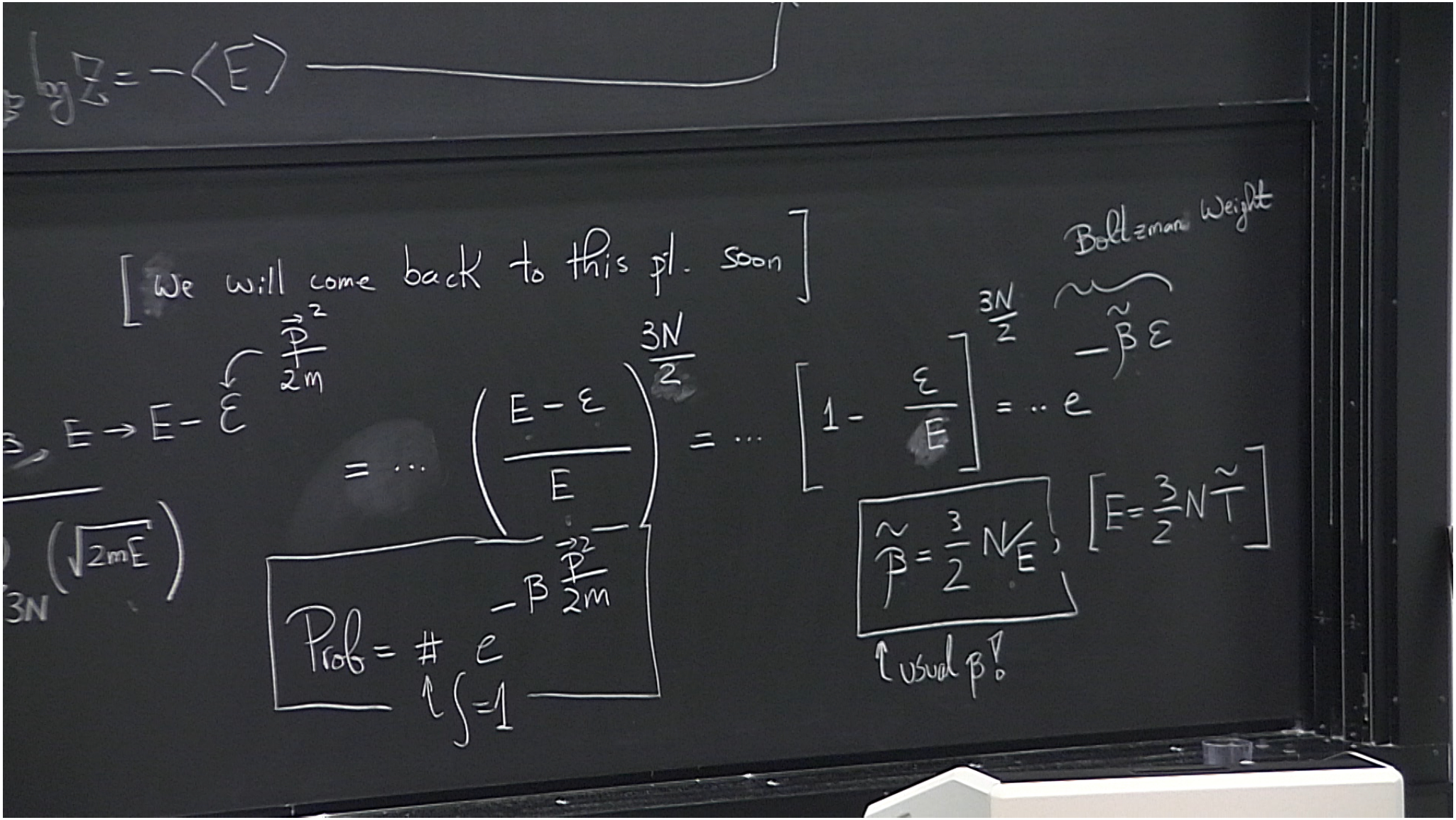
$$\text{Prob}\left(\text{gas on left side of room}\right) \sim \left(\frac{1}{2}\right)^{10^{24}} \to 0$$

[We will come ba

$$\text{Prob}(\vec{v}_i) = \frac{\text{integral } \vec{p}_i \text{ missing}}{\text{all integrals}} = \frac{3N \to 3N-3,\ E \to E - \varepsilon \left(\frac{\vec{p}^2}{2m}\right)}{\frac{d}{dE}\,\text{Vol}_{3N}\left(\sqrt{2mE}\right)}$$

$\uparrow$ q's cancel


$Z = \sum e^{-\beta\varepsilon\sum n_i} = [1+e^{-\beta\varepsilon}]^N$
$n_i = 0,1$
$i = 1 \dots N$
N_{QM}
$\partial_\beta \log Z = -\langle E \rangle$
Prob (gas on left side of room) $\sim \left(\frac{1}{2}\right)^{10^{24}} \to 0$
[We will come back to this pt. soon]
Boltzmann Weight
$\text{Prob}(\vec{v}_i) = \frac{\text{integral } \vec{p}_i \text{ missing}}{\text{all integrals}} = \frac{3N \to 3N-3,\ E \to E-\varepsilon}{\frac{d}{dE} Vol_{3N}(\sqrt{2mE})}$
$= \dots \left(\frac{E-\varepsilon}{E}\right)^{\frac{3N}{2}} = \dots \left[1-\frac{\varepsilon}{E}\right]^{\frac{3N}{2}} \dots e^{-\tilde{\beta}\varepsilon}$
$\tilde{\beta} = \frac{3}{2}\frac{N}{E}$, $\left[E = \frac{3}{2}N\tilde{T}\right]$
↑ usual β!
q's cancel
$\text{Prob} = \#\, e^{-\beta \frac{\vec{p}^2}{2m}}$
↑ $\int = 1$

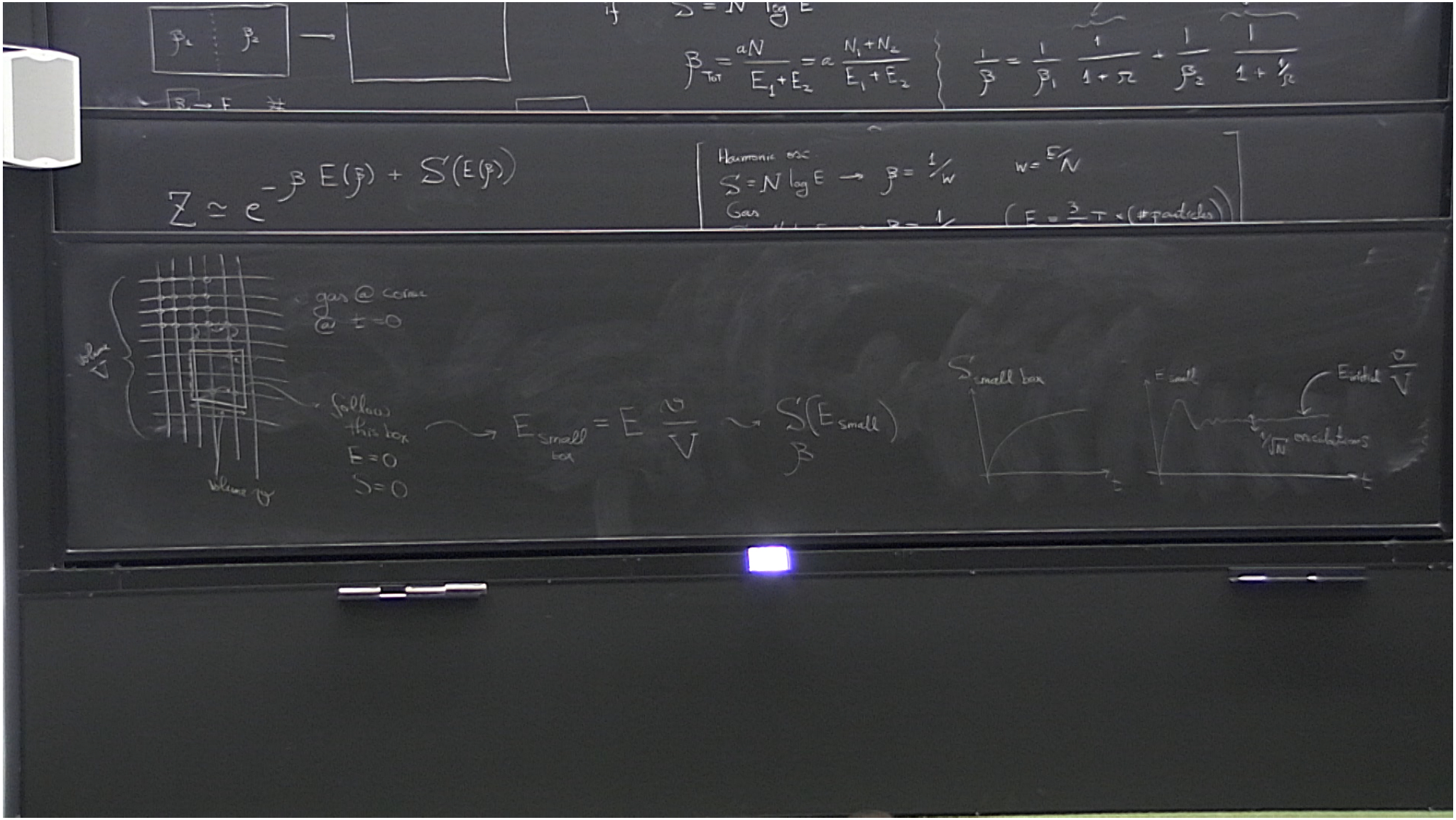

$\ln Z = -\langle E \rangle$
[We will come back to this pt. soon]
$E \to E - \vec{\varepsilon}$
$\frac{\vec{p}^2}{2m}$
$\left(\sqrt{2mE}\right)$
$3N$
$= \cdots \left(\frac{E-\varepsilon}{E}\right)^{\frac{3N}{2}} = \cdots \left[1 - \frac{\varepsilon}{E}\right]^{\frac{3N}{2}} = \cdots e^{-\tilde{\beta}\varepsilon}$
Boltzman Weight
$\tilde{\beta} = \frac{3}{2}\frac{N}{E}$, $\left[E = \frac{3}{2}N\tilde{T}\right]$
↑ usual β!
Prob = # $e^{-\beta \frac{\vec{p}^2}{2m}}$
↑ $\int = 1$

$$\text{Prob}\left(\text{gas on left side of room}\right) \sim \left(\frac{1}{2}\right)^{10^{24}} \to 0$$

[We wil

$$\text{Prob}\left(\vec{v}_i\right) = \frac{\text{integral } \vec{p}_i \text{ missing}}{\text{all integrals}} = \frac{3N \to 3N-3,\ E \to E-\varepsilon}{\frac{d}{dE}\,\text{Vol}_{3N}\left(\sqrt{2mE}\right)}$$

q's cancel


$\beta_{Tot} = \frac{aN}{E_1+E_2} = a\frac{N_1+N_2}{E_1+E_2}$
$\frac{1}{\beta} = \frac{1}{\beta_1}\frac{1}{1+\pi} + \frac{1}{\beta_2}\frac{1}{1+1/\pi}$
$Z \simeq e^{-\beta E(\beta) + S(E(\beta))}$
Harmonic osc
$S = N \log E \rightarrow \beta = 1/w$
$w = E/N$
Gas
$E = \frac{3}{2} T \times (\#\text{particles})$
gas @ const @ $t=0$
follows this box
$E = 0$
$S = 0$
$E_{small\ box} = E\frac{v}{V} \rightsquigarrow S(E_{small})$
$S_{small\ box}$
E_{small}
$E_{orbital}$
$1/\sqrt{N}$ oscillations

Gas

$E = \frac{3}{2} T \times (\#\text{particles})$

$\rightsquigarrow S(E_{small})$

β

$S_{small\ box}$

t

E_{small}

$E_{initial} \frac{\sigma}{V}$

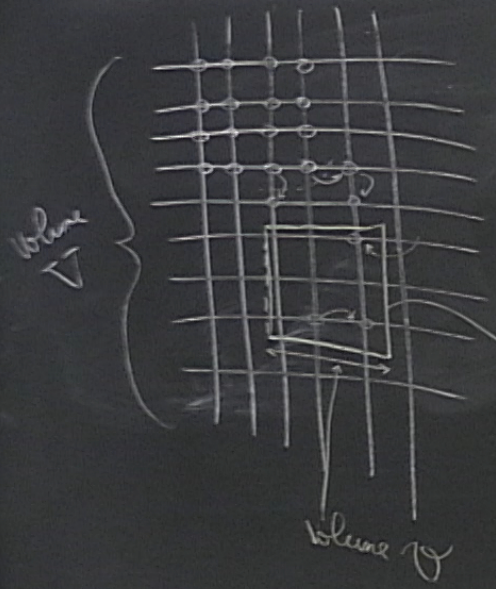
$\frac{1}{\sqrt{N}}$ oscillations

t

$$Z \simeq e^{-\beta E(\beta) + S(E(\beta))}$$

$S = N \log E \rightarrow \beta =$

Gas



gas @ corner @ $t = 0$

follow this box

$E = 0$

$S = 0$

$$\left[\text{QM ly} \quad \Psi_{\text{Box}}(t=0) = \sum^{\dim \mathcal{H}} C_n \Psi_n \right.$$

$$E_{\substack{\text{small} \\ \text{box}}} = E \frac{v}{V} \rightsquigarrow S(E_{\text{small}})_{\beta}$$

(β))

Harmonic osc.

$S = N \log E \rightarrow \beta = 1/w \qquad w = E/N$

Gas

$(E = \frac{3}{2} T \times (\#\text{particles}))$

[QM ly

$\Psi_{Box}(t \neq 0) = \sum_{\dim \mathcal{H}} C_n \Psi_n e^{-i E_n t/\hbar}$

$[\sum |C_n|^2 E_n = E_{initial}]$

$t \cdot (\text{number}) \leftarrow$ huge rational number

$1 = e^{t=0} = e$

$(t \rightarrow t + \underbrace{\text{Conv. den}}_{T_{big}})$

$5753 \cdots\cdots\cdots 0$

$1 \cdots\cdots\cdots\cdots 0$

$\rightarrow E_{small\ box} = E \frac{v}{V} \rightsquigarrow S(E_{small})$

$S_{small\ box}$ vs t

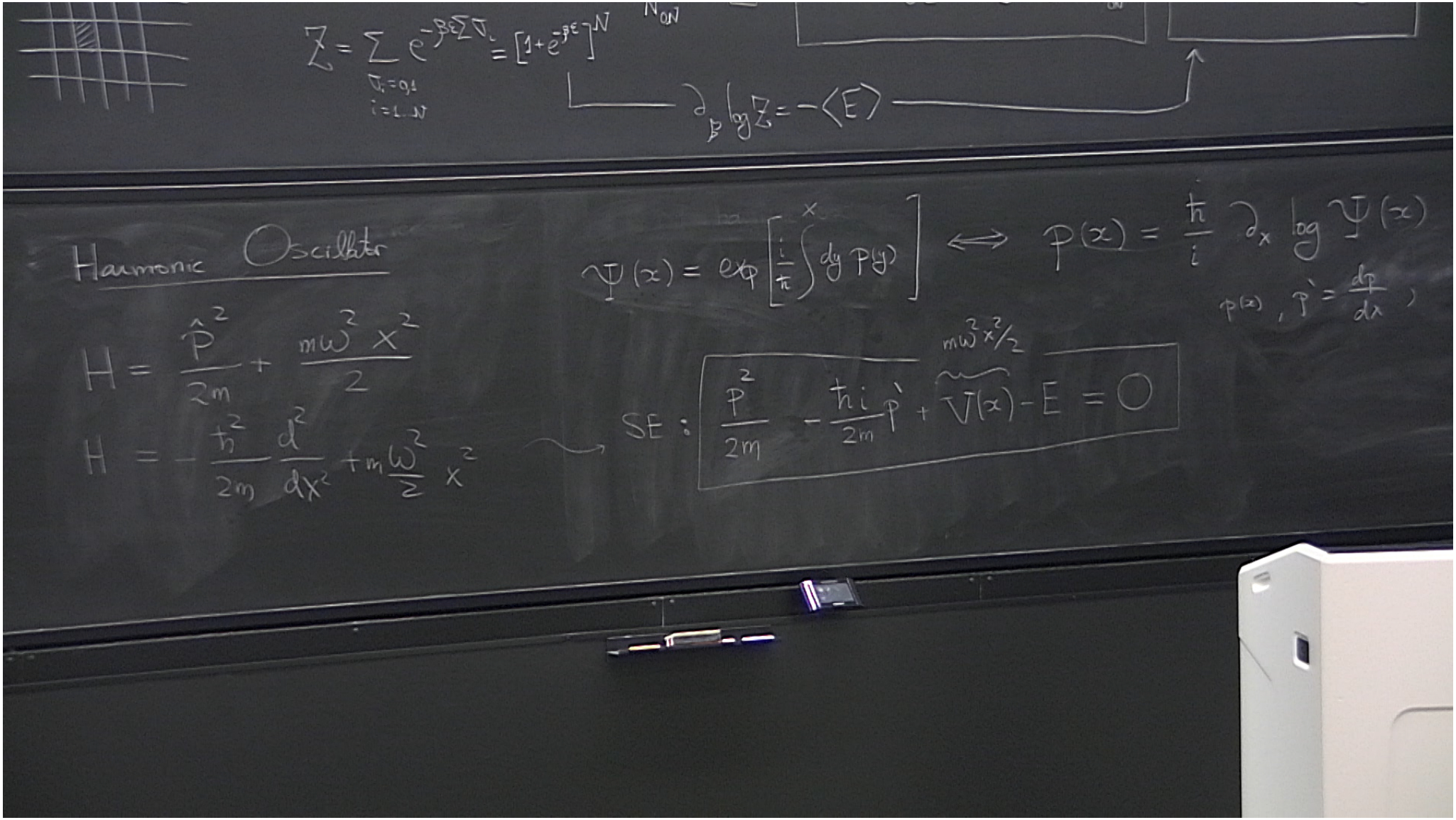
E_{small} vs t, $E_{initial}$, $\frac{v}{V}$, $1/\sqrt{N}$ oscillations

Harmonic Oscillator

$$H = \frac{\hat{p}^2}{2m} + \frac{m\omega^2 x^2}{2}$$

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + m\frac{\omega^2}{2}x^2$$

$$\psi(x) = \exp\left[\frac{i}{\hbar}\int^{x} dy\, p\right]$$


$Z = \sum e^{-\beta\varepsilon\sum n_i} = [1+e^{-\beta\varepsilon}]^N$
$n_i = 0,1$
$i = 1 \dots N$
N_{0N}
$\partial_\beta \log Z = -\langle E \rangle$
Harmonic Oscillator
$\Psi(x) = \exp\left[\frac{i}{\hbar}\int^x dy\, p(y)\right] \iff p(x) = \frac{\hbar}{i}\partial_x \log \Psi(x)$
$p(x)$, $p' = \frac{dp}{dx}$
$H = \frac{\hat{p}^2}{2m} + \frac{m\omega^2 x^2}{2}$
$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + m\frac{\omega^2}{2}x^2$
$m\omega^2 x^2/2$
SE: $\frac{p^2}{2m} - \frac{\hbar i}{2m}p' + V(x) - E = 0$

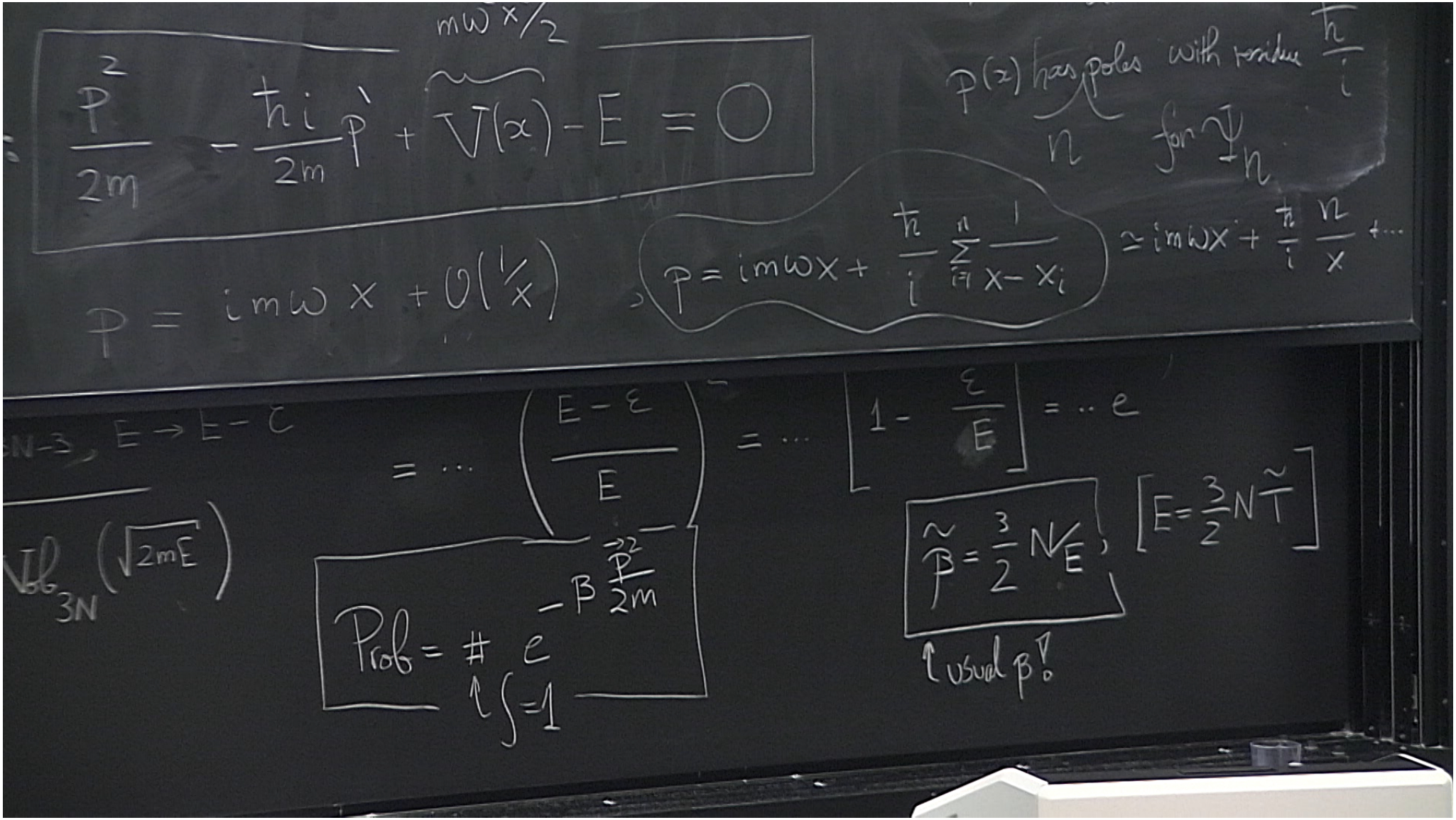
$\log Z = -\langle E \rangle$

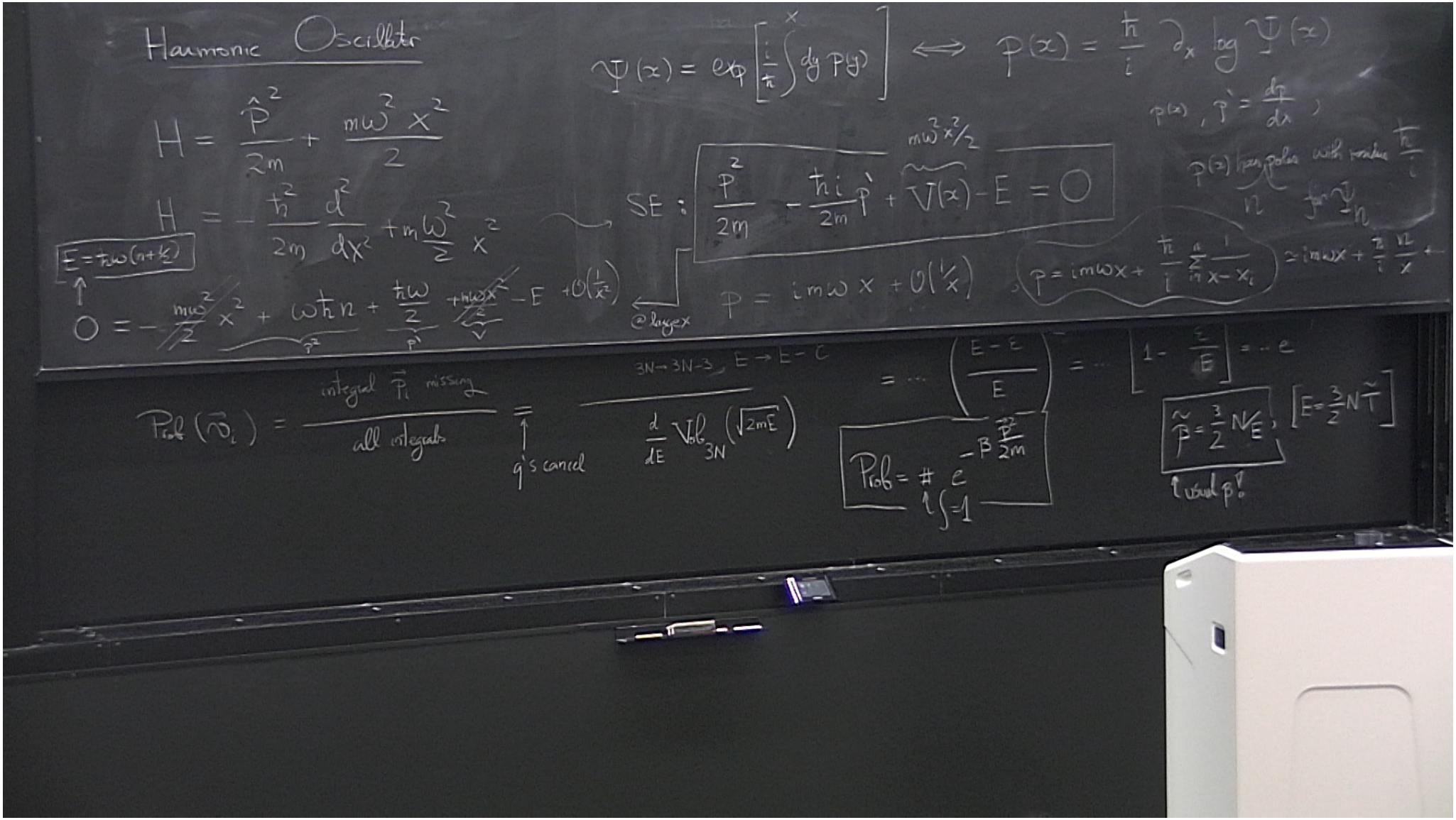
$$\exp\left[\frac{i}{\hbar}\int^{x} dy\, p(y)\right] \iff p(x) = \frac{\hbar}{i}\partial_x \log \Psi(x)$$

$p(x)$, $p' = \frac{dp}{dx}$,

$p(x)$ has n poles with residue $\frac{\hbar}{i}$ for Ψ_n

$$\frac{p^2}{2m} - \frac{\hbar i}{2m}p' + \underbrace{V(x)}_{m\omega^2 x^2/2} - E = 0$$


$\frac{p^2}{2m} - \frac{\hbar i}{2m} p' + \widetilde{V(x)} - E = 0$
$m\omega x/2$
$p(z)$ has poles with residue $\frac{\hbar}{i}$
n for Ψ_n
$p = im\omega x + O(1/x)$
$p = im\omega x + \frac{\hbar}{i} \sum_{i=1}^{n} \frac{1}{x - x_i} \simeq im\omega x + \frac{\hbar}{i} \frac{n}{x} + \cdots$
$N-3$, $E \to E - \varepsilon$
$= \cdots \left(\frac{E-\varepsilon}{E} \right)^{-} = \cdots \left[1 - \frac{\varepsilon}{E} \right] = \cdots e$
$\text{Vol}_{3N}\left(\sqrt{2mE}\right)$
$\text{Prob} = \# e^{-\beta \frac{\vec{p}^2}{2m}}$
$\uparrow \int = 1$
$\tilde{\beta} = \frac{3}{2} N/E$, $\left[E = \frac{3}{2} N \tilde{T} \right]$
$\uparrow$ usual β!


Harmonic Oscillator
$\Psi(x) = \exp\left[\frac{i}{\hbar}\int^x dy\, p(y)\right] \Leftrightarrow p(x) = \frac{\hbar}{i}\partial_x \log \Psi(x)$
$H = \frac{\hat{p}^2}{2m} + \frac{m\omega^2 x^2}{2}$
$p(x)$, $p' = \frac{dp}{dx}$
$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + m\frac{\omega^2}{2}x^2$
$p(x)$ has poles with residue $\frac{\hbar}{i}$ in Ψ_n for Ψ_n
$E = \hbar\omega(n+\frac{1}{2})$
SE: $\frac{p^2}{2m} - \frac{\hbar i}{2m}p' + V(x) - E = 0$
$m\omega^2 x^2/2$
$0 = -\frac{m\omega^2}{2}x^2 + \omega\hbar n + \frac{\hbar\omega}{2} + \frac{m\omega^2 x^2}{2} - E + O(\frac{1}{x^2})$
@large x
$p = im\omega x + O(\frac{1}{x})$
$p = im\omega x + \frac{\hbar}{i}\sum_{i=1}^{n}\frac{1}{x - x_i} = im\omega x + \frac{n}{i}\frac{\hbar}{x} + \ldots$
$\text{Prob}(\vec{v}_i) = \frac{\text{integral } \vec{p}_i \text{ missing}}{\text{all integrals}} = \frac{3N \to 3N-3,\ E \to E - \epsilon}{\frac{d}{dE}\text{Vol}_{3N}(\sqrt{2mE})}$
$= \ldots \left(\frac{E-\epsilon}{E}\right)^{\cdots} = \ldots \left[1 - \frac{\epsilon}{E}\right]^{\cdots} = \ldots e$
$\vec{p}$'s cancel
$\text{Prob} = \#\, e^{-\beta\frac{\vec{p}^2}{2m}}$
$\int = 1$
$\tilde{p} = \frac{3}{2}N\tilde{E}$, $[E = \frac{3}{2}N\tilde{T}]$
$\tilde{p}$ usual p!