Title: Aristotelian Supersymmetry

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Abstract: <p>Abstract TBA</p>

Aristotelian Supersymmetry

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T. Griffin, K.T. Grosvenor, P.H. and Z. Yan,
Multicritical Symmetry Breaking and Naturalness of Slow Nambu-Goldstone Bosons,
arXiv:1308.5967, Phys. Rev. **D88** (2013) 101701;

T. Griffin, K.T. Grosvenor, P.H. and Z. Yan,
Scalar Field Theories with Polynomial Shift Symmetries,
arXiv:1412.1046, Commun. Math. Phys. **340** (2015) 1720;

T. Griffin, K.T. Grosvenor, P.H. and Z. Yan,
Cascading Multicriticality in Nonrelativistic Spontaneous Symmetry Breaking,
arXiv:1507.06992, Phys. Rev. Lett. **115** (2015) 241601;

K.T. Grosvenor, P.H., C. Mogni and Z. Yan,
Nonrelativistic Short-Distance Completions of a Naturally Light Higgs,
arXiv:1608.06937;

A brief semi-recent review: P.H., *Surprises with Nonrelativistic Naturalness*, arXiv:1608.06287, Int. J. Mod. Phys. **D25** (2016)1645007;

& in progress w/ Charles Melby-Thompson, Stephen Randall, Alex Frenkel

The Aristotelian Spacetime

By the Aristotelian spacetime, we will mean $\mathbf{R}^{D+1}$ with the Cartesian coordinates (t, x^i), $i = 1, \ldots D$, and with the flat metric $g_{ij} = \delta_{ij}$, $N = 1$, $N_i = 0$, and with the preferred foliation by the flat spatial slices of constant t.

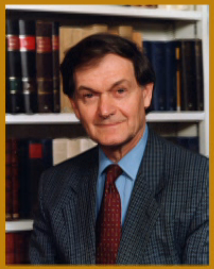
By the Aristotelian symmetry, we will mean the isometries of the Aristotelian spacetime:

$$x^i \to \Lambda^i_j x^j + b^i, \qquad t \to t + b.$$

- These are *derived*, as all foliation-preserving diffeomorphisms that preserve the metric.
- The isometries respect an emergent rest frame.
- Such spacetimes are solutions of HL gravities with zero Λ.
- Often called the "Lifshitz spacetime" in modern literature ...

Lifshitz or Aristotelian Spacetime?

History: in the mid-1960's, Andrzej Trautman, Roger Penrose



talked about the "Aristotelian spacetime": in Penrose's 1968 Structure of Space-Time (*Battelle Rencontres*), he begins with

- Before Einstein (curved relative space-time) and Minkowski (rigid relative space-time), there was
- Galilean spacetime (relative space and absolute time), and before that,
- Aristotelian spacetime (absolute space, absolute time)!

QFT on Aristotelian Spacetimes: Why?

Motivation:

- inherited from nonrelativistic quantum gravity,
- new short-distance completions of relativistic QFTs,
- curiosity about new tools for technical naturalness in Standard Model & beyond, in cosmology,
- spin-off applications to condensed matter,
- interesting from math-ph perspective,
- curiosity about how far can string & M theory extend ...

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Gravity without Relativity

(a.k.a. gravity with anisotropic scaling, or Hořava-Lifshitz gravity)

Gravity on spacetimes with a preferred time foliation (cf. FRW!)

Opens up the possibility of new RG fixed points, with improved UV behavior due to anisotropic scaling.

Field theories with anisotropic scaling:

$$x^i \to \lambda x^i, \quad t \to \lambda^z t.$$

z: dynamical critical exponent – characteristic of RG fixed point. Many interesting examples in condensed matter, dynamical critical phenomena, quantum critical systems, ..., with $z = 1,\ 2,\ \dots,\ n,\ \dots$, or fractions ($z = 3/2$ for KPZ surface growth in $D = 1$), ..., continous families ...

... and now gravity as well, with propagating gravitons, formulated as a quantum field theory of the metric.

Aristotelian QFT

QFT on the Aristotelian spacetime: $\mathbf{R}^{D+1}$ with the Cartesian coordinates (t, x^i), $i = 1, \ldots D$, and with the flat metric $g_{ij} = \delta_{ij}$, $N = 1$, $N_i = 0$, and with the preferred foliation by the flat spatial slices of constant t.

The Aristotelian symmetry is realized on such QFTs at all scales,

$$x^i \to \Lambda^i_j x^j + b^i, \qquad t \to t + b.$$

The isometries respect an emergent rest frame.

If a QFT with Aristotelian symmetries is at an RG fixed point, it develops an extra symmetry, anisotropic conformal symmetry:

$$x^i \to \lambda x^i, \qquad t \to \lambda^z t.$$

Warmup: Spontaneous Symmetry Breaking

Global internal symmetry breaking leads to Nambu-Goldstone modes. Phenomenon is remarkably universal, across many fields dealing with many-body systems.

But how many NG modes, and what is their low-energy dispersion relation?

- Relativistic case: All questions answered by Goldstone's theorem: One NG per broken generator, gapless=massless, $z = 1$ dispersion $\omega = k$. Constant shift symmetry.

- Nonrelativistic case: Classify by classifying their low-energy effective QFTs [Murayama&Watanabe, '12,'13].
  This appears to lead to Type A (linear dispersion), Type B (quadratic dispersion).

Symmetries of Nambu-Goldstone Modes

Example 1: Massive $\lambda\phi^4$ in $3+1$ dimensions.

$$S = \frac{1}{2}\int d^4x \left(\partial_\mu\phi\partial^\mu\phi - m^2\phi^2 - \frac{1}{12}\lambda\phi^4\right)$$

Recall technical naturalness:

$$\lambda \sim \varepsilon, \quad m^2 \sim \mu^2\varepsilon, \quad \mu \sim m/\sqrt{\lambda}.$$

Symmetry: The constant shift $\phi \to \phi + a$.

Aristotelian case:

$$S = \frac{1}{2}\int dt\, d^D x \left[(\dot{\phi})^2 - (\partial_i \ldots \partial_k\phi)(\partial_i \ldots \partial_k\phi) + \ldots\right]$$

Extended symmetries?

Polynomial Shift Symmetry

In Aristotelian QFT: An infinite hierarchy of new symmetries, which generalize the constant shift symmetry.

"Polynomial shift symmetry,"

$$\pi^I(t, x^i) \to \pi^I(t, x^i) + a^I_{j_1 j_2 \cdots j_{2z-2}} x^{j_1} x^{j_2} \ldots x^{j_{2z-2}} + \ldots$$

protects the $\omega \sim k^z$ low-energy dispersion for Type A modes (and the $\omega \sim k^{2z}$ low-energy dispersion for Type B modes).

Note: It depends only on spatial coordinates, not on time. compare the Galileon cosmology: linear spacetime shifts)

Refining the Classification of Nonrelativistic NG Modes

Refined classification of technically natural NG modes with Aristotelian spacetime symmetries:

- Type A tower of multicritical NG modes with $z = 1, 2, \ldots$, until one hits against the multicritical analog of the Coleman Hohenberg-Mermin-Wagner theorem at $z = D$;

- Type B tower of multicritical NG modes with $z = 2, 4, \ldots$ (and no analog of the MCW theorem).

These IR fixed points describe the free limit of multicritical NG modes, and imply low-energy theorems for scattering etc.

Generic interactions break the polynomial shift symmetry to the constant shift. But: Corrections are controllably small, if couplings are small.

Field Theories with Polynomial Shift Symmetries

So, we found examples where a new symmetry

$$\phi(t, x^i) \to \phi(t, x^i) + a_{j_1 j_2 \dots j_P} x^{j_1} x^{j_2} \dots x^{j_P} + \dots$$

protects the smallness of leading terms in the dispersion relation, and protects hierarchies.

In the examples shown, the symmetry is broken by interactions.

Now we can turn this around, and ask for the classification of scalar theories in which the polynomial shift symmetry is exact.

This is a very cute mathematical problem!

The simplest case of linear shift is related to the Galileon.

Polynomial Shift Invariants

It is natural to organize the invariants by their dimension at the free RG fixed point.

Task: Classify all terms in the Lagrangian containing n fields and $\Delta \equiv 2m$ derivatives, invariant under the degree-P shift symmetry up to a total derivative:

$$\delta_P L = \partial_i L_i.$$

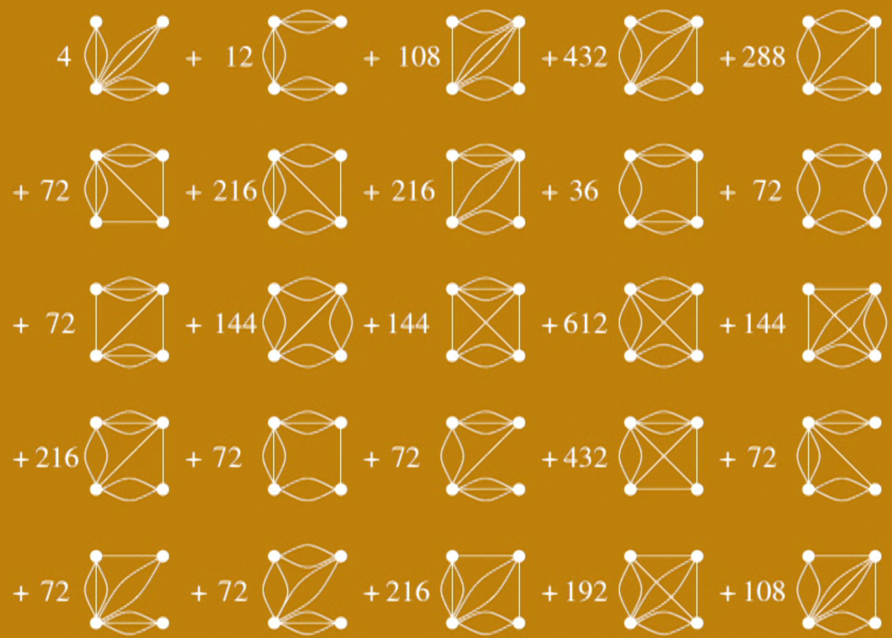
This is essentially a cohomological problem.
It defines a new graph-theory cohomology:
Vector spaces $H_{P,n,\Delta,D}$ of invariants, labeled by P, n and Δ.

How to solve it?

use Graph Theory!

$P > 0$ Invariants: Superposition of Trees

Example: The most relevant quintic-shift 4-pt invariant is



Summary of Applications to Graph Cohomology

Polynomial shift symmetries of Aristotelian QFT of gapless scalars defines naturally an infinite sequence of novel graph-theory cohomology groups, $H_{P,n,\Delta}$.

For a range of low values of P, n, Δ, these groups have been computed explicitly in:

T. Griffin, K.T. Grosvenor, P.H. and Z. Yan, *Scalar Field Theories with Polynomial Shift Symmetries*, arXiv:1412.1046, Commun. Math. Phys. **340** (2015) 1720.

Cascading Multicriticality: Examples

$2+1$ dimensions:

$$S = \frac{1}{2}\int dt\, d^2\mathbf{x} \left\{ \dot{\phi}^2 - (\partial_i\partial_j\phi)^2 - c^2(\partial_i\phi)^2 - m^2\phi^2 - g(\partial_i\phi\partial_i\phi)^2 \right\}$$

$$g \sim \varepsilon_1, \quad c^2 \sim \varepsilon_1\mu^2, \quad m^2 \sim \varepsilon_0\mu^4, \quad \varepsilon_0 \ll \varepsilon_1 \ll 1.$$

$3+1$ dimensions:

$$S = \frac{1}{2}\int dt\, d^3\mathbf{x} \quad \left\{ \dot{\phi}^2 - (\partial_i\partial_j\partial_k\phi)^2 - \zeta_2^2(\partial_i\partial_j\phi)^2 - c^2(\partial_i\phi)^2 - m^2\phi^2 \right.$$
$$\left. - \lambda\epsilon_{ijk}\epsilon_{\ell mp}\partial_i\phi\partial_j\partial_\ell\phi\partial_k\partial_m\phi\partial_p\phi \right\}$$

$$\zeta_2^2 \sim \varepsilon_2\mu^2, \quad \lambda \sim \varepsilon_1, \quad c^2 \sim \varepsilon_1\mu^4, \quad m^2 \sim \varepsilon_0\mu^6,$$
$$\varepsilon_0 \ll \varepsilon_1 \ll \varepsilon_2 \ll 1.$$

Now, Higgs Naturalness

Let's start with a simple scalar field theory first.

Recall one of our earlier examples of a cascading hierarchy:

$$S = \frac{1}{2}\int dt\, d^3\mathbf{x}\left\{\dot{\phi}^2 - (\partial_i\partial_j\partial_k\phi)^2 - \zeta_2^2(\partial_i\partial_j\phi)^2 - c^2(\partial_i\phi)^2 - m^2\phi^2 - \lambda\epsilon_{ijk}\epsilon_{\ell mp}\partial_i\phi\partial_j\partial_\ell\phi\partial_k\partial_m\phi\partial_p\phi\right\}$$

$$\zeta_2^2 \sim \varepsilon_2\mu^2, \quad \lambda \sim \varepsilon_1, \quad c^2 \sim \varepsilon_1\mu^4, \quad m^2 \sim \varepsilon_0\mu^6,$$
$$\varepsilon_0 \ll \varepsilon_1 \ll \varepsilon_2 \ll 1.$$

Aristotelian, Wilsonian and Lorentzian Observers

Different observers: Different ways how to relate space to time.

Examples:

- Aristotelian observers. Choose t and y^i once and for all, regardless of the dynamics of fields in spacetime. Renormalization generates $\zeta_3^2(\mu)$, but $\lambda(\mu) = \lambda_{\rm bare}$. Running dispersion relation.

- Wilsonian observers. Anticipating $z \approx 3$ in UV, redefine $\tilde{t} = t$, $\tilde{y}^i$ by setting $\zeta_3^2 = 1$; equivalent to μ-dependent rescaling of space: Running spacetime. Now ϕ develops an anomalous dimension, $\lambda(\mu)$ depends on RG scale.

Cascading Multicriticality: Examples

$2+1$ dimensions:

$$S=\frac{1}{2}\int dt\, d^2\mathbf{x}\left\{\dot{\phi}^2-(\partial_i\partial_j\phi)^2-c^2(\partial_i\phi)^2-m^2\phi^2-g(\partial_i\phi\partial_i\phi)^2\right\}$$

$$g\sim\varepsilon_1,\quad c^2\sim\varepsilon_1\mu^2,\quad m^2\sim\varepsilon_0\mu^4,\quad \varepsilon_0\ll\varepsilon_1\ll 1.$$

$3+1$ dimensions:

$$S=\frac{1}{2}\int dt\, d^3\mathbf{x}\quad\left\{\dot{\phi}^2-(\partial_i\partial_j\partial_k\phi)^2-\zeta_2^2(\partial_i\partial_j\phi)^2-c^2(\partial_i\phi)^2-m^2\phi^2\right.$$
$$\left.-\lambda\epsilon_{ijk}\epsilon_{\ell mp}\partial_i\phi\partial_j\partial_\ell\phi\partial_k\partial_m\phi\partial_p\phi\right\}$$

$$\zeta_2^2\sim\varepsilon_2\mu^2,\quad \lambda\sim\varepsilon_1,\quad c^2\sim\varepsilon_1\mu^4,\quad m^2\sim\varepsilon_0\mu^6,$$
$$\varepsilon_0\ll\varepsilon_1\ll\varepsilon_2\ll 1.$$

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Aristotelian, Wilsonian and Lorentzian Observers

... and finally,

- Lorentzian observers. The low-energy observer anticipates Lorentz invariance, and sets $c = 1$.

This is equivalent to redefining the coordinates to $x^\mu =\equiv (x^0, x^i)$, with $x^0 = t$ and $x^i = y^i/c$.

Dimensions: Measuring in the units of energy,

$$[t] = -1, \quad [y^i] = -1/3, \quad [c] = 2/3, \quad [x^i] = -1.$$

Technical Naturalness: Imposed in the UV, "microscopic" theory. Cascading hierarchy of scales is possible, protected by the pattern of partial breakings of polynomial shift symmetries.

The Low-Energy Lorentzian Perspective

Compare the perspective of the UV Aristotelian observer:

$$S_{\rm UV} = \frac{1}{2}\int dt\, d^3\mathbf{y} \left\{ \dot{\phi}^2 - (\partial_i\partial_j\partial_k\phi)^2 - \zeta_2^2(\partial_i\partial_j\phi)^2 - c^2(\partial_i\phi)^2 + m^2\phi^2 \right.$$
$$\left. - g\phi^4 - \lambda\epsilon_{ijk}\epsilon_{\ell mp}\partial_i\phi\partial_j\partial_\ell\phi\partial_k\partial_m\phi\partial_p\phi \right\}$$

$$\zeta_2^2 \sim \varepsilon_2\mu^2, \quad \lambda \sim \varepsilon_1, \quad c^2 \sim \varepsilon_1\mu^4, \quad m^2 \sim \varepsilon_0\mu^6, \quad g \sim \varepsilon_0\mu^6,$$
$$\varepsilon_0 \ll \varepsilon_1 \ll \varepsilon_2 \ll 1.$$

and the IR Lorentzian observer:

$$S_{\rm IR} = \frac{1}{2}\int d^4x \left\{ \nabla_\mu\Phi\nabla^\mu\Phi + m^2\Phi^2 - \lambda_h\Phi^4 \right.$$
$$\left. - \tilde{\zeta}_3^2(\nabla_i\nabla_j\nabla_k\Phi)^2 - \tilde{\zeta}_2^2(\nabla_i\nabla_j\Phi)^2 - \tilde{\lambda}\nabla_i\Phi\ldots\Phi \right\}$$

where $\Phi = c^{3/2}\phi$ and $\nabla_\mu = \partial/\partial x^\mu$.

Technically Natural Mass Hierarchy

$$m^2 \sim \varepsilon_0 M^2, \qquad \lambda_h \sim \frac{\varepsilon_0}{\varepsilon_1^{3/2}},$$

$$\tilde{\zeta}_3^2 \sim \frac{\varepsilon_0^2}{\varepsilon_1^3}\frac{1}{m^4}, \qquad \tilde{\zeta}_2^2 \sim \frac{\varepsilon_0 \varepsilon_2}{\varepsilon_1^2}\frac{1}{m^2}, \qquad \tilde{\lambda} \sim \frac{\varepsilon_0^3 \varepsilon_2}{\varepsilon_1^{9/2}}\frac{1}{m^6}.$$

We want $m = M_{\rm EW} \sim 1{\rm TeV}$, $M = M_P \sim 10^{18}{\rm GeV}$. We also want $\lambda_h \sim 0.1$ or 1. Take the "10-20-30" model:

$$\varepsilon_0 \sim 10^{-30}, \qquad \varepsilon_1 \sim 10^{-20}, \qquad \varepsilon_2 \sim 10^{-10}.$$

The nonrelativistic corrections are small,

$$\tilde{\zeta}_2^2 \sim \frac{1}{m^2}, \quad \tilde{\zeta}_3^2 \sim \frac{1}{m^4}, \quad \tilde{\lambda} \sim 10^{-10}\frac{1}{m^6}.$$

Fermions and Yukawa Couplings

Microscopic theory vs. low-energy relativistic picture:

$$\sum_f Y_f \int d^3\mathbf{y}\, dt\, \phi \Psi_f^\dagger \Psi_f = \sum_f y_f \int d^4x\, \Phi \bar{\psi}_f \psi_f,$$

where $[Y_f] = 1$, $\psi_f = c^{3/2}\Psi_f$ and $y_f = Y_f/c^{3/2}$.

Microscopic naturalness: $Y_f \sim \varepsilon_0 \mu^3$? Actually, there is more wiggling room:

$$\varepsilon_0 \mu^3 \leq Y_f \leq \sqrt{\varepsilon_0}\mu^3.$$

The low-energy observer sees this window of naturalness as

$$\varepsilon_0/\varepsilon_1^{3/4} \leq y_f \leq \varepsilon_0^{1/2}/\varepsilon_1^{3/4}.$$

In the 10-20-30 model, this gives the Yukawa range $10^{-15} \leq y_f \leq 1$, accommodating all the fermions of the SM!

Gauging

The grain of salt?

So far, we found the technically natural light scalar with non-derivative self-coupling, but only in the "gaugeless" limit of the SM.

Gauging, in the microscopic theory:

$\partial_i \to \partial_i + ie\mathcal{A}_i, \quad \partial_t \to \partial_t + ie\mathcal{A}_0;$ go to $\mathcal{A}_0 = 0$ gauge.

Action: $\int d^3\mathbf{y}\, dt\, \dot{\mathcal{A}}_i\dot{\mathcal{A}}_i + \ldots$; implies $[\mathcal{A}_i] = 0$, $[e] = 1/3$.
Low-energy relativistic perspective:

$$A_i = c^{3/2}\mathcal{A}_i, \qquad g_{\rm YM} = e/c^{1/2}.$$

If $e^2 \sim \varepsilon_0\mu^2$, then $g_{\rm YM}^2 \sim \varepsilon_0/\varepsilon_1^{1/2}$ is still way too small.

But: Bottom-up pheno approach followed by Berthier, Grosvenor, Yan; encouraging natural hierarchy by 2 OofM.

Summary on Naturally Light Scalars

- Technical naturalness exhibits surprising features in the nonrelativistic settings of Aristotelian spacetime.
- We presented a new mechanism for a naturally light scalar with non-derivative self-couping:

$$m^2 \sim \varepsilon M^2, \quad \lambda_h \sim \varepsilon/\varepsilon_1^{3/2},$$

in contrast with the relativistic naturalness:

$$m^2 \sim \varepsilon M^2, \qquad \lambda_h \sim \varepsilon.$$

- The crucial new small parameter ε_1 controls the size of the speed of light in the microscopic theory with $z > 1$.
- Higgs phenomenology looks quite promising, a large hierarchy with $m = M_{EW}$ and $M = M_P$ is possible at least in the "gaugeless" limit of SM. All fermion masses also natural! Need to learn more about the gauge sector.

Simplest Aristotelian Supersymmetry

Supercharges can square to the Hamiltonian:

$$\{Q, Q\} = H.$$

- Fermions don't even have to be spacetime spinors;
- Theories easy to construct (e.g., by borrowing the scalar SUSY of stochastic quantization):

$$\int (\dot{\phi})^2 - (\partial_i \partial_j \phi)^2 + \bar{\psi}\dot{\psi} + \bar{\psi}\partial^2\psi + \text{interactions;}$$

But for particle physics, we want something more interesting ...
something that flows to a relativistic SUSY theory in IR!

Let's study an example: $N = 1$ scalar supermultiplet in $2 + 1$ dimensions.

The Top-Down Perspective

(the perspective of a high-energy Aristotelian observer)

Supermultiplet: $\phi(t,\mathbf{x})$ real, $\psi_\alpha(t,\mathbf{x})$ Majorana; auxiliary real scalar B. We also define $\bar{\psi}^\alpha = (\psi^\dagger\gamma^0)^\alpha$, with $\gamma^0 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.

$$S_{\rm UV} = \int dt d^2x \left\{ \frac{1}{2}\left(\dot{\phi}^2 + \bar{\psi}\gamma^0\dot{\psi} + B^2\right) - B\partial^2\phi - \partial_i\bar{\psi}\partial_i\psi \right\},$$

is invariant under $N=1$ supersymmetry:

$$\begin{aligned} \delta\phi &= \bar{\varepsilon}\psi, \\ \delta\psi &= -\dot{\phi}\gamma^0\varepsilon + B\varepsilon, \\ \delta B &= \bar{\varepsilon}\gamma^0\dot{\psi} \end{aligned}$$

with the superalgebra

$$\{\bar{Q}, Q\} = (\gamma^0)H.$$

Relevant Deformations

(the perspective of a high-energy Aristotelian observer)

$$S_{\rm UV} = \int dt d^2x \left\{ \frac{1}{2}\left(\dot\phi^2 + \bar\psi\gamma^0\dot\psi + B^2\right) - B\partial^2\phi - \bar\psi\partial^2\psi \right\},$$

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Relevant Deformations

(the perspective of a high-energy Aristotelian observer)

$$S_{\rm UV} = \int dt d^2x \left\{ \frac{1}{2}\left(\dot{\phi}^2 + \bar{\psi}\gamma^0\dot{\psi} + B^2\right) - B\partial^2\phi - \bar{\psi}\partial^2\psi - \frac{c^2}{2}\partial_i\phi\partial_i\phi - \frac{c}{2}\bar{\psi}\gamma^i\partial_i\psi \right\}.$$

$N = 1$ supersymmetry deformed to:

$$\begin{aligned} \delta\phi &= \bar{\varepsilon}\psi, \\ \delta\psi &= -\dot{\phi}\gamma^0\varepsilon + B\varepsilon - c\partial_i\gamma^i\varepsilon, \\ \delta B &= \bar{\varepsilon}\gamma^0\dot{\psi} + c\bar{\varepsilon}\gamma^i\partial_i\phi, \end{aligned}$$

$$\{\bar{Q}, Q\} = (\gamma^0)H + c\gamma^i P_i.$$

Top-down: Emergent relativistic SUSY at low energies!

The Bottom-Up Perspective

(the perspective of a low-energy Poincaré relativistic observer)

In $\mathbf{R}^{D+1}$ spacetime dimensions, consider supercharges Q_α, in your favorite spinor rep of the Lorentz group. Write:

$$\{\bar{Q}^\alpha, Q_\beta\} = (\gamma^\mu)_\beta{}^\alpha P_\mu$$

Is this relativistic or not? Not necessarily!

Depends on how many of the Lorentz generators we add to this supertranslation algebra.

- Adding $J_{\mu\nu}$ gives standard Poincaré supersymmetry;
- Adding just J_{ij} gives Aristotelian supersymmetry.

To see more, it will be convenient to switch to $N = 1$ superspace description.

$N = 1$ Superspace

(the perspective of a low-energy Poincaré relativistic observer)

Superspace coordinates: (t, x^i, θ_α).

Supercharges: $Q = \frac{\partial}{\partial\theta} - \gamma^0\theta\frac{\partial}{\partial t} - c\gamma^i\theta\partial_i$.

Derivatives: $D = \frac{\partial}{\partial\theta} + \gamma^0\theta\frac{\partial}{\partial t} + c\gamma^i\theta\partial_i$.

Real superfield: $\Phi(t, \mathbf{x}, \theta) = \phi(t, \mathbf{x}) + \bar{\theta}\psi(t, \mathbf{x}) + \frac{1}{2}\bar{\theta}\theta\, B(t, \mathbf{x})$.

Action:

$$S = \int dt d^2\mathbf{x} d^2\theta \left\{ \bar{D}^\alpha\Phi\, D_\alpha\Phi - \partial_i\Phi\partial_i\Phi + \text{interactions} \right\}.$$

Bottom-up: Aristotelian UV completions of relativistic SUSY theories!

$N = 1$ SUSY Aristotelian σ-Models

Easily generalized (still in $2+1$ dimensions) to $N = 1$ non-linear σ-models.

Target $\mathcal{M}$ with coordinates Φ^a.

$$S = \int dt d^2\mathbf{x} d^2\theta \left\{ g_{ab}(\Phi) \bar{D}^\alpha \Phi^a \, D_\alpha \Phi^b - h_{ab}(\Phi) \partial_i \Phi^a \partial_i \Phi^b \right\} .$$

The target space $\mathcal{M}$ is a real manifold with *two* metrics: g_{ab} and h_{ab}.

This theory is a UV completion of the effective relativistic $N = 1$ σ-model in $2+1$ dimensions.

RG flows? A bi-metric theory of gravity!

Extended Aristotelian SUSY?

Try to generalize to $N = 2$ non-linear σ-models.

Recall the relativistic case in $1 + 1$: $N = 1$ extends to $N = 2$ when $\mathcal{M}$ is complex, with a Kähler metric.

Try the same Ansatz in Aristotelian case in $2 + 1$ dimensions: Doesn't work!

The complex structure on $\mathcal{M}$ with a Kähler metric is not sufficient to construct $N = 2$. Naive SUSY transformations don't close to translations:

$$\{\bar{Q}^A, Q^B\} = \begin{pmatrix} \gamma^0 \partial t + c\gamma^i \partial_i & \partial^2 \\ \partial^2 & \gamma^0 \partial t + c\gamma^i \partial_i \end{pmatrix}.$$

only the free theory, $g_{I\bar{J}}$ must be flat.

We can see the way out in the $N = 2$ superspace.

$N = 2$ SUSY Aristotelian σ-Models

The $N = 2$ superspace: $(t, \mathbf{x}, \bar{\theta}, \theta)$.

Target $\mathcal{M}$ with complex coordinates Φ^I, chiral superfields.

$$S = \int dt d^2\mathbf{x} d^2\theta d^2\bar{\theta}\, K(\Phi^I, \bar{\Phi}^J) - \int dt d^2\mathbf{x} d^2\theta\, h_{IJ}(\Phi)\partial_i\Phi^I\partial_i\Phi^J + \text{h.c.}$$

The target space $\mathcal{M}$ is a complex manifold with *two* metrics: a Kähler $g_{I\bar{J}} = \partial_I\bar{\partial}_{\bar{J}}K(\Phi, \bar{\Phi})$ and a holomorphic $h_{IJ}(\Phi)$.

This structure is much more rigid than in $1 + 1$ relativistic $N = 2$.

It allows an uplift to $N = 1$ in $3 + 1$ dimensions, with $z = 2$ (and 4, $\ldots$).

However: Gauging? Extension to SUSY Yang-Mills??

Recall: $S_{\text{SYM}} = \frac{1}{g^2}\int dt\, d^2x\, d^2\theta\, W_\alpha W^\alpha$ ...

Aristotelian Super Yang-Mills: $2+1$

First, try to construct $N=1$ super Yang-Mills with $z=2$ in $2+1$ dimensions.

Strategy: Go back to the basic ingredients of super-geometry. Superconnection on superspace (t, x^i, θ_α):

$$\mathcal{D}_t = \partial_t - ie\Gamma_t, \mathcal{D}_i = \partial_i - ie\Gamma_i,\ \mathcal{D}_\alpha = D_\alpha - ie\Gamma_\alpha.$$

Field strengths:

$$W_{\alpha i} = D_\alpha \Gamma_i \partial_i \Gamma_\alpha + e[\Gamma_\alpha, \Gamma_i],$$

and similarly for $W_{\alpha t}, W_{ij}, W_{it}, W_{\alpha\beta}$.

"Conventional" constraint: $W_{\alpha\beta} = 0$.

Solving the conventional constraint leads to an irreducible real spinor superfield strength W_α.

Aristotelian Super Yang-Mills: $2+1$

Long story short: The relativistic case goes through W_α.
In the Aristotelian case, the independent ingredients are W_α and W_{ij}:

$$W_{ij} = F_{ij} + \bar{\theta}(\gamma_j \partial_i \lambda - \gamma_i \partial_j \lambda) + (\bar{\theta}\theta)(\varepsilon_{ik}\partial_j E_k - \varepsilon_{jk}\partial_i E_k),$$

$$W_{ij}W_{ij} = \ldots + (\bar{\theta}\theta)(F\partial_i E_i + \partial_i\bar{\lambda}\partial_i\lambda - \varepsilon_{ij}\partial_i\bar{\lambda}\partial_j\lambda)$$

This yields a $z=2$ super Yang-Mills action in $2+1$ dimensions, with a somewhat unexpected bosonic part, leading to a modified Gauss' law.

A canonical transformation exists, which takes the bosonic part of the action and Gauss's law to the more conventional $z=2$ form, at the cost of obscuring the simplicity of the supersymmetry transformations.

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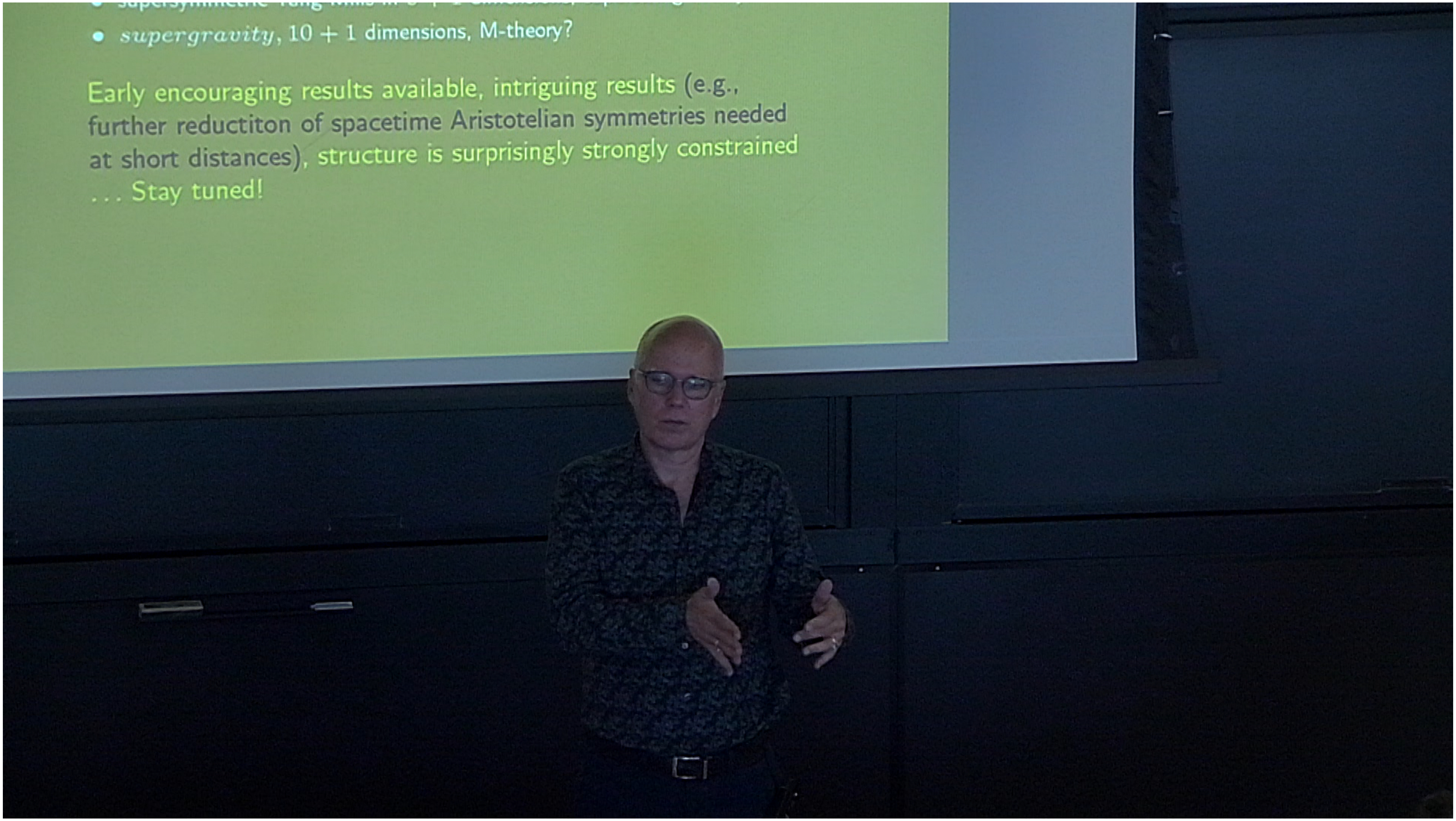
Aristotelianization Program

For various relativistic QFTs with various degrees of SUSY, can we construct Aristotelian UV-improved theories (with $z = 2, 3, \ldots$) which would flow in the long-distance limit automatically to the relativistic theory?

Holy Grails:

- $N = 1$ supersymmetric Yang-Mills in $4 + 1$ dimensions (knot theory)
- $N = (1, 0), (1, 1), (2, 0)$ supersymmetric Yang-Mills in $5 + 1$ dimensions
- supersymmetric Yang-Mills in $9 + 1$ dimensions, superstring theory?
- $supergravity$, $10 + 1$ dimensions, M-theory?

Early encouraging results available, intriguing results (e.g., further reductiton of spacetime Aristotelian symmetries needed at short distances), structure is surprisingly strongly constrained ... Stay tuned!



• supergravity, 10 + 1 dimensions, M-theory?
Early encouraging results available, intriguing results (e.g., further reductiton of spacetime Aristotelian symmetries needed at short distances), structure is surprisingly strongly constrained
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