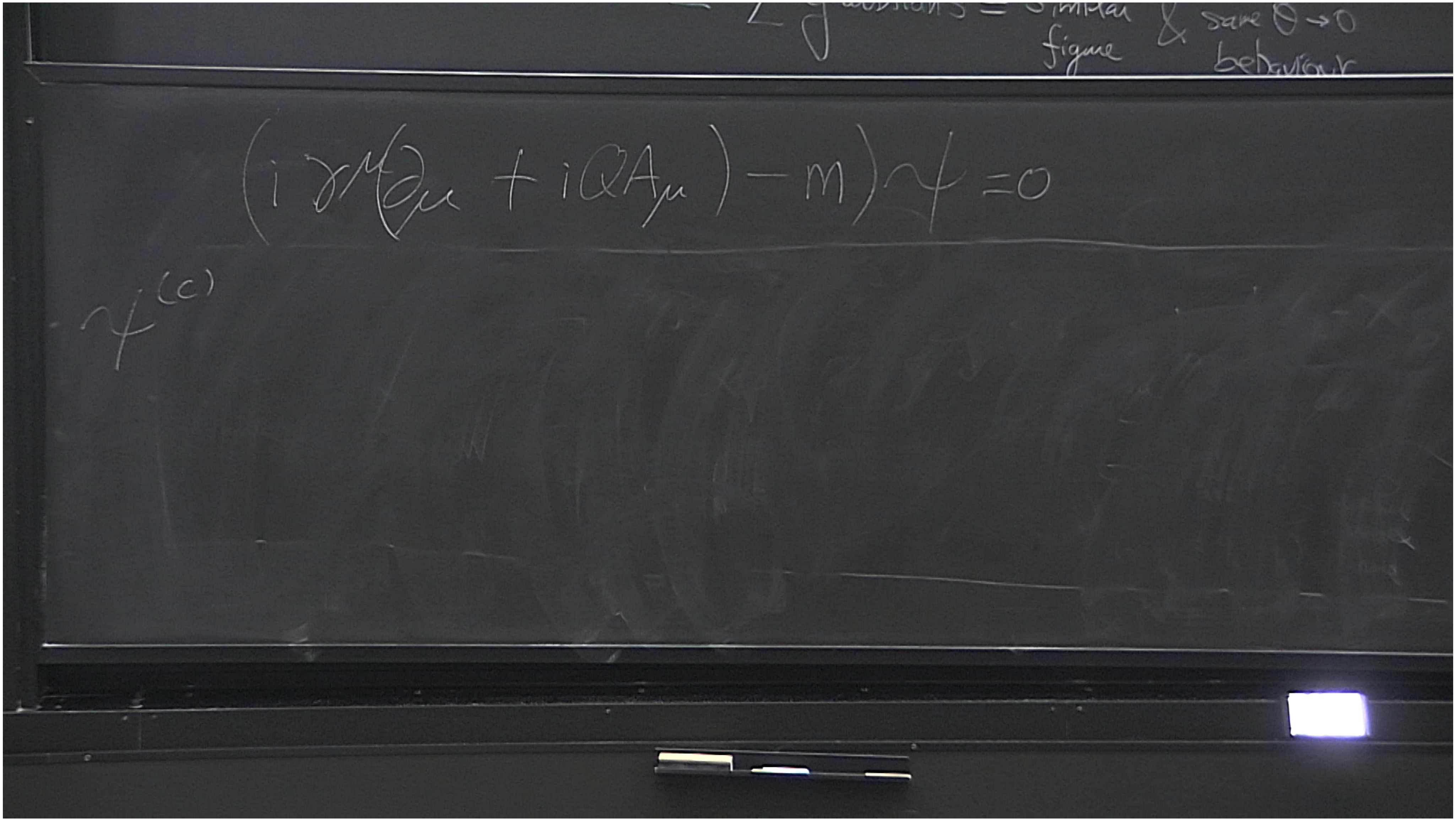
Title: PSI 2018/2019 - Quantum Field Theory I - Lecture 11

Date: Oct 25, 2018 02:00 PM

URL: http://pirsa.org/18100018

Abstract:


similar figure & same θ→0 behaviour
$(i\gamma^\mu(\partial_\mu + iQA_\mu) - m)\psi = 0$
$\psi^{(c)}$

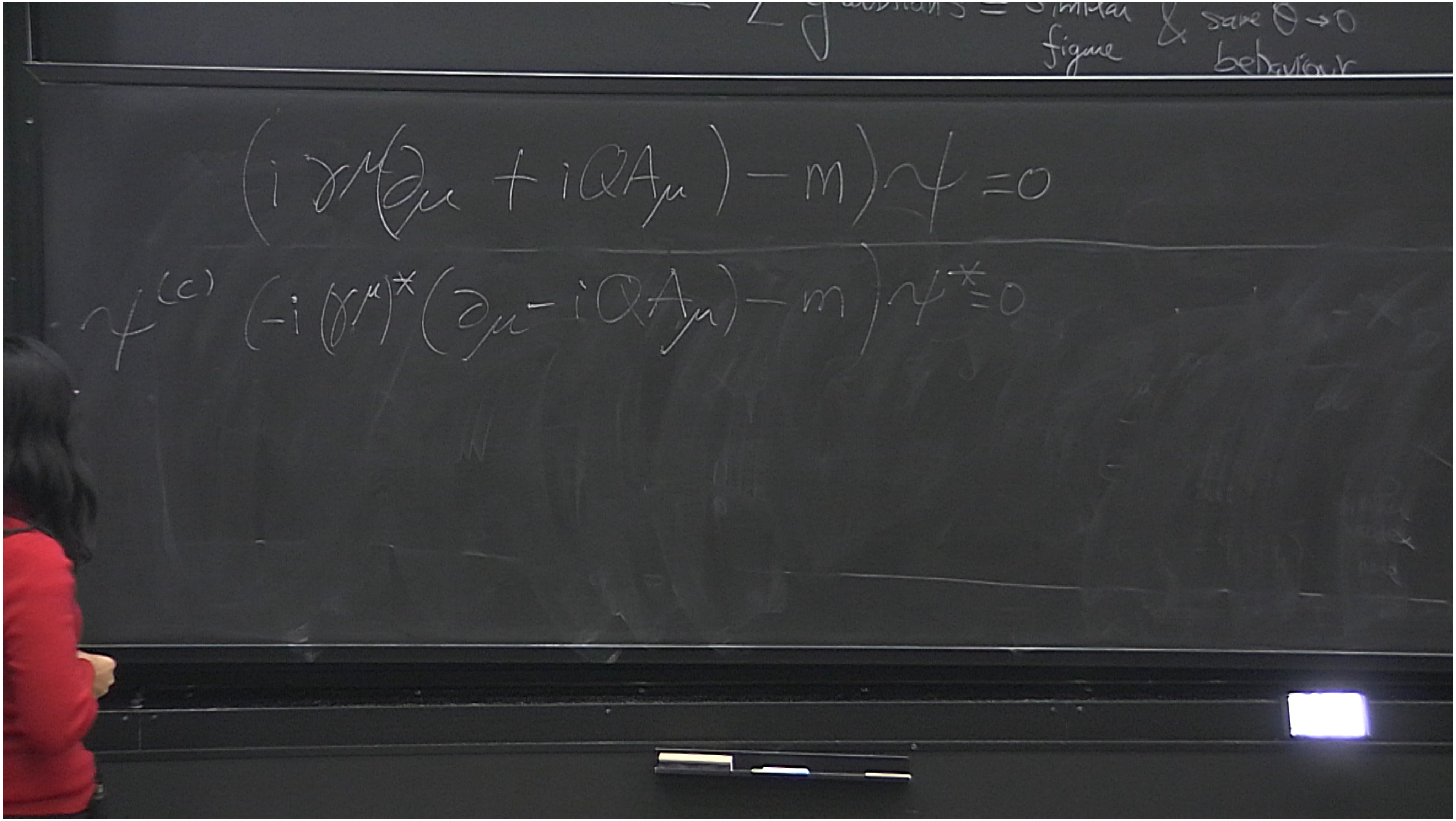


figure
& same θ → 0
behaviour
$(i\gamma^\mu(\partial_\mu + iQA_\mu) - m)\psi = 0$
$\psi^{(c)}$
$(-i(\gamma^\mu)^*(\partial_\mu - iQA_\mu) - m)\psi^* = 0$

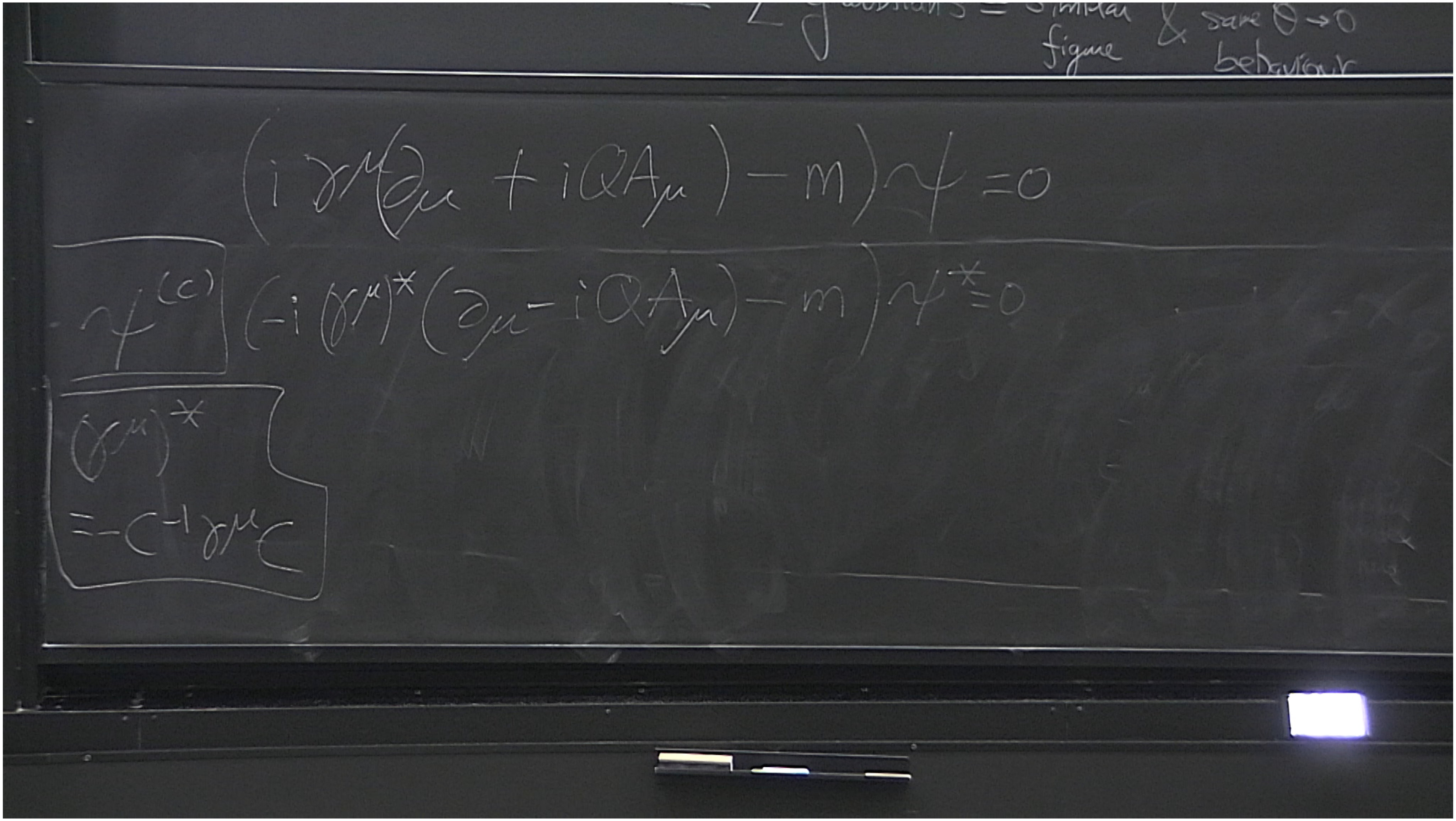
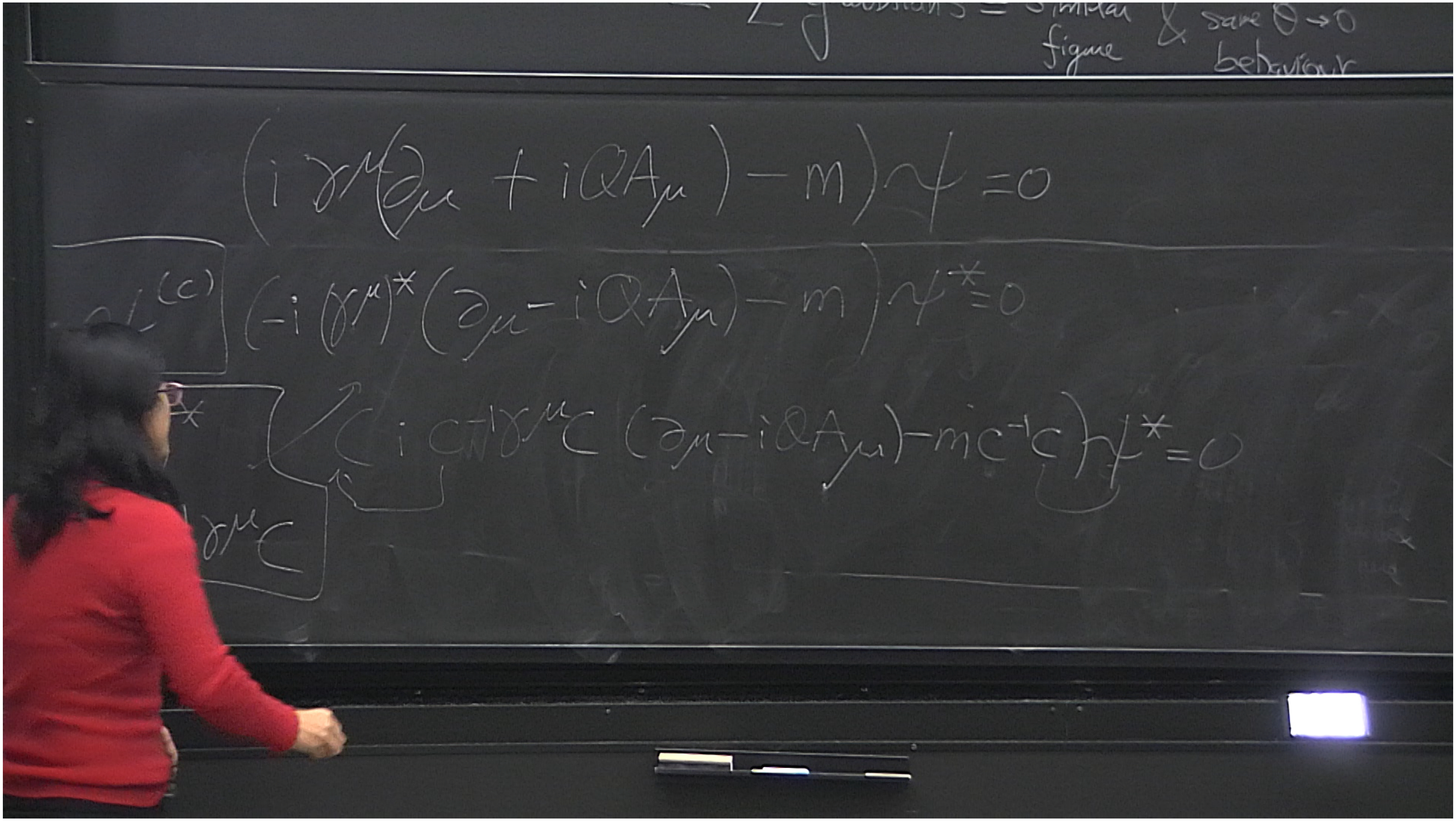


figure & same θ → 0 behaviour
(iγ^μ(∂_μ + iQA_μ) − m)ψ = 0
ψ^(c)
(−i(γ^μ)*(∂_μ − iQA_μ) − m)ψ* = 0
(γ^μ)* = −C^{-1}γ^μ C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γ^μ C

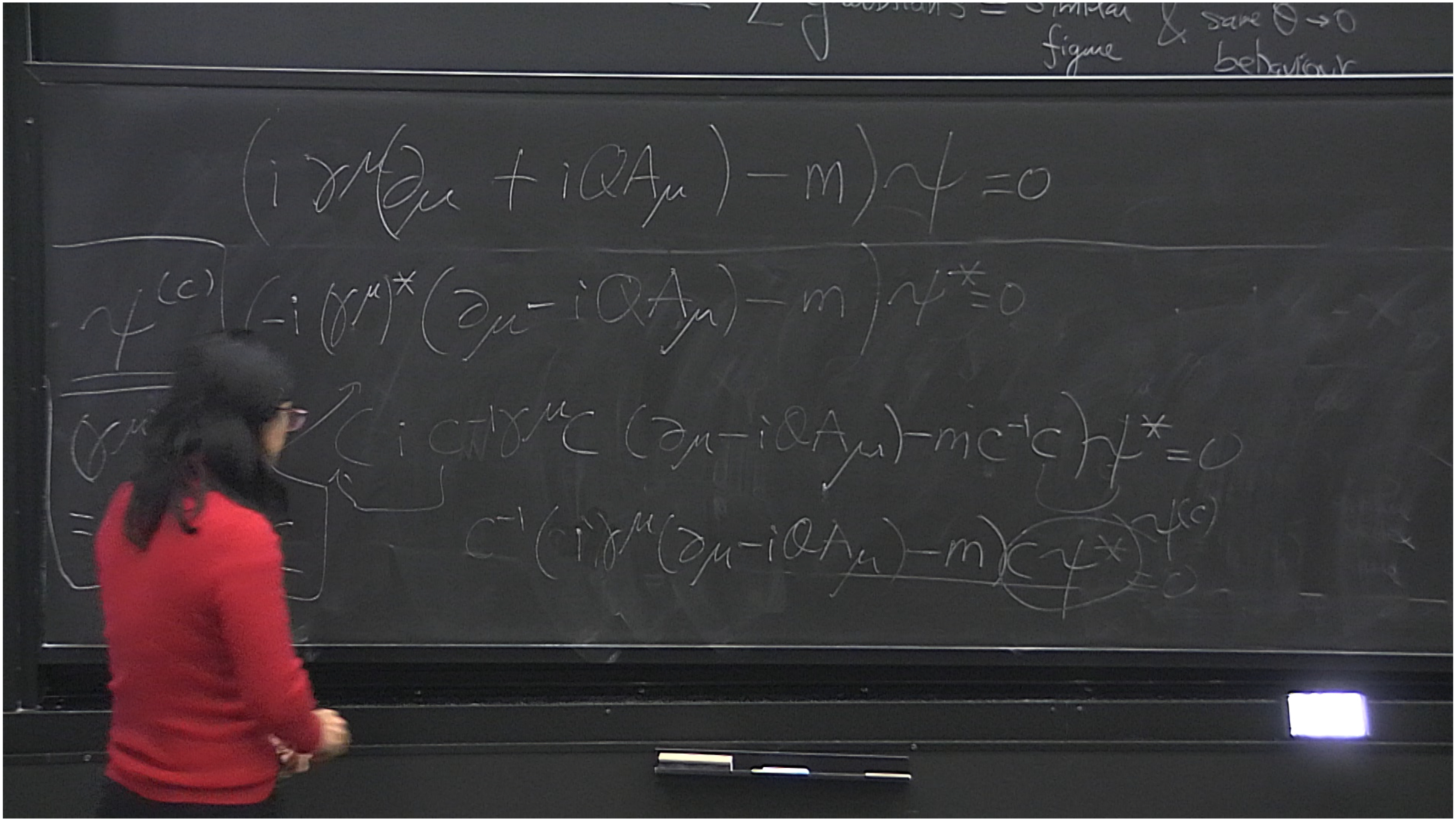
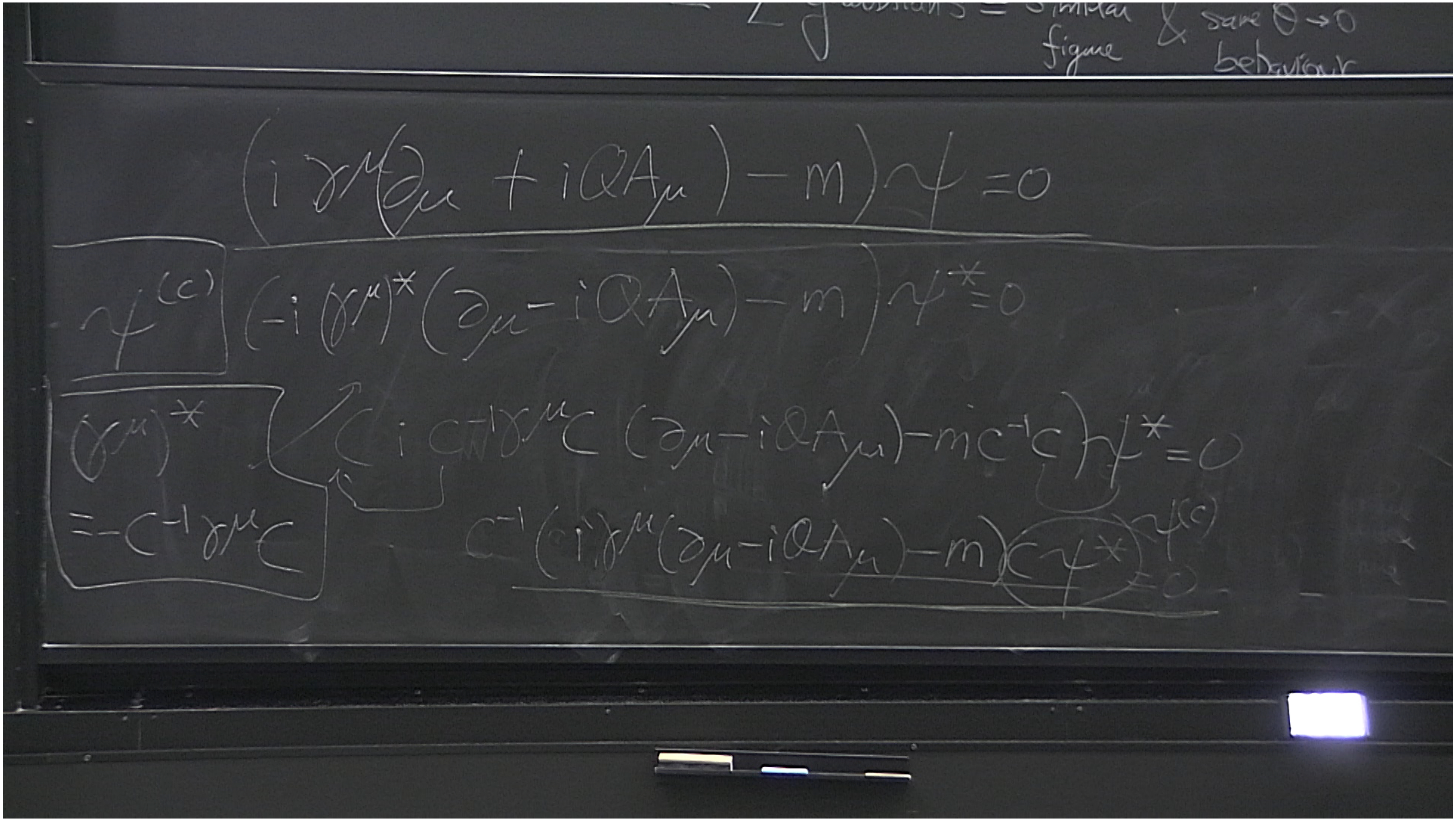
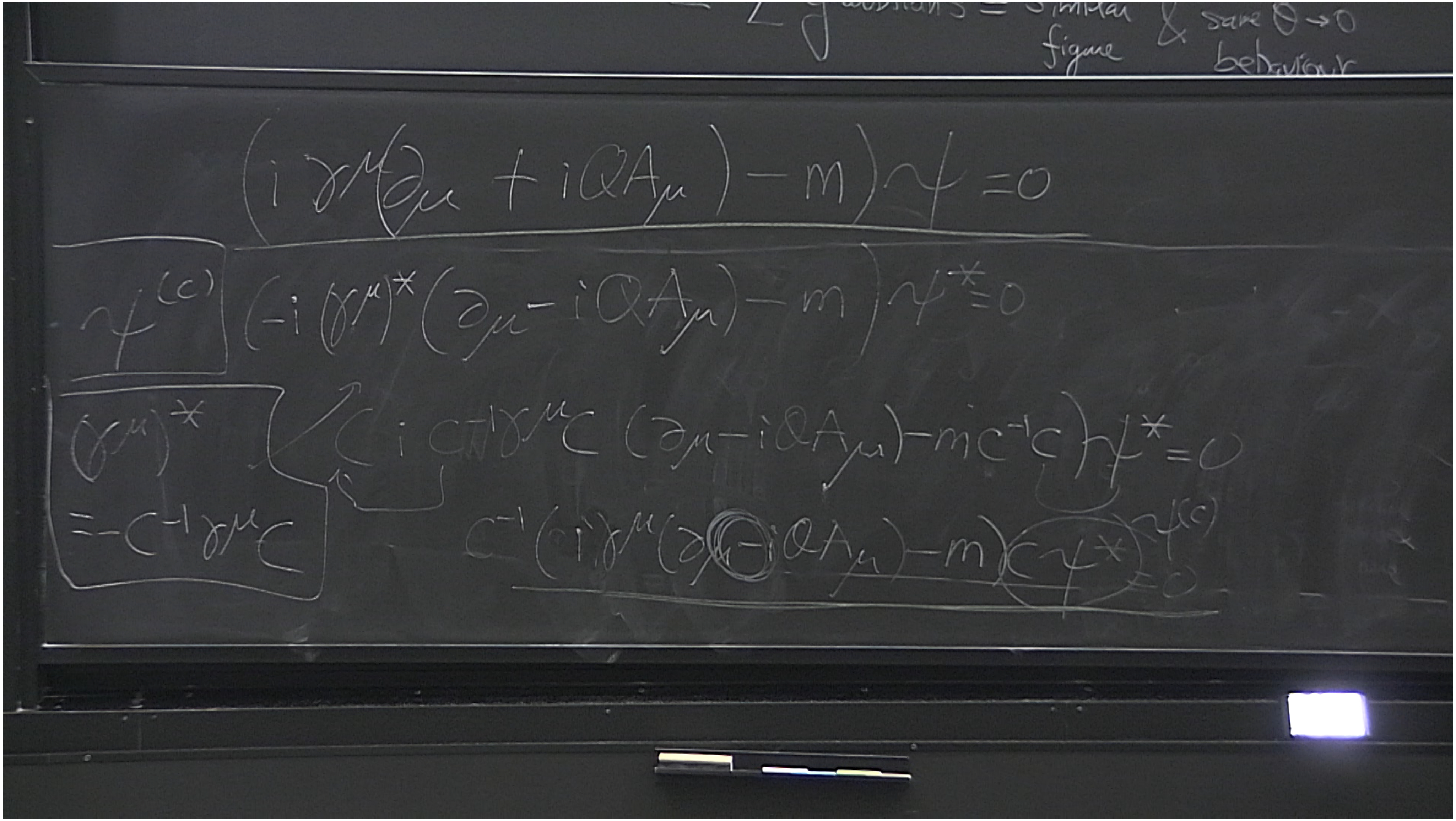


figure
behaviour
(iγ^μ(∂_μ + iQA_μ) − m)ψ = 0
ψ^(c)
(−i(γ^μ)*(∂_μ − iQA_μ) − m)ψ* = 0
C^{-1}(i)γ^μ(∂_μ − iQA_μ) − m)Cψ* = 0


similar figure & same $\theta \to 0$ behaviour
$(i\gamma^\mu(\partial_\mu + iQA_\mu) - m)\psi = 0$
$\psi^{(c)}$
$(-i(\gamma^\mu)^*(\partial_\mu - iQA_\mu) - m)\psi^* = 0$
$(\gamma^\mu)^* = -C^{-1}\gamma^\mu C$
$(i C^{-1}\gamma^\mu C(\partial_\mu - iQA_\mu) - mC^{-1}C)\psi^* = 0$
$C^{-1}(i\gamma^\mu(\partial_\mu - iQA_\mu) - m)C\psi^* = 0$
$\psi^{(c)}$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$\psi^{(c)}$

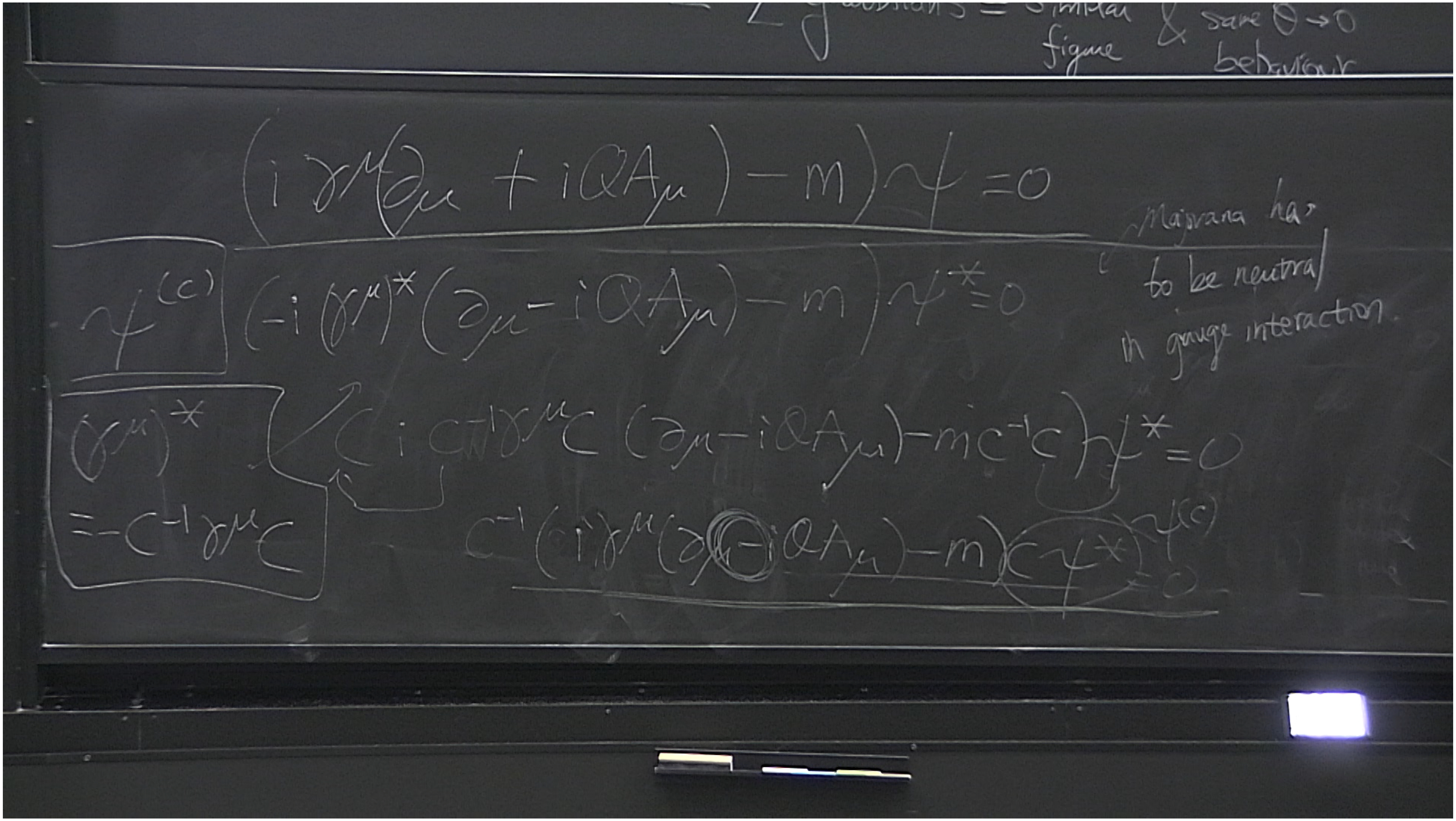
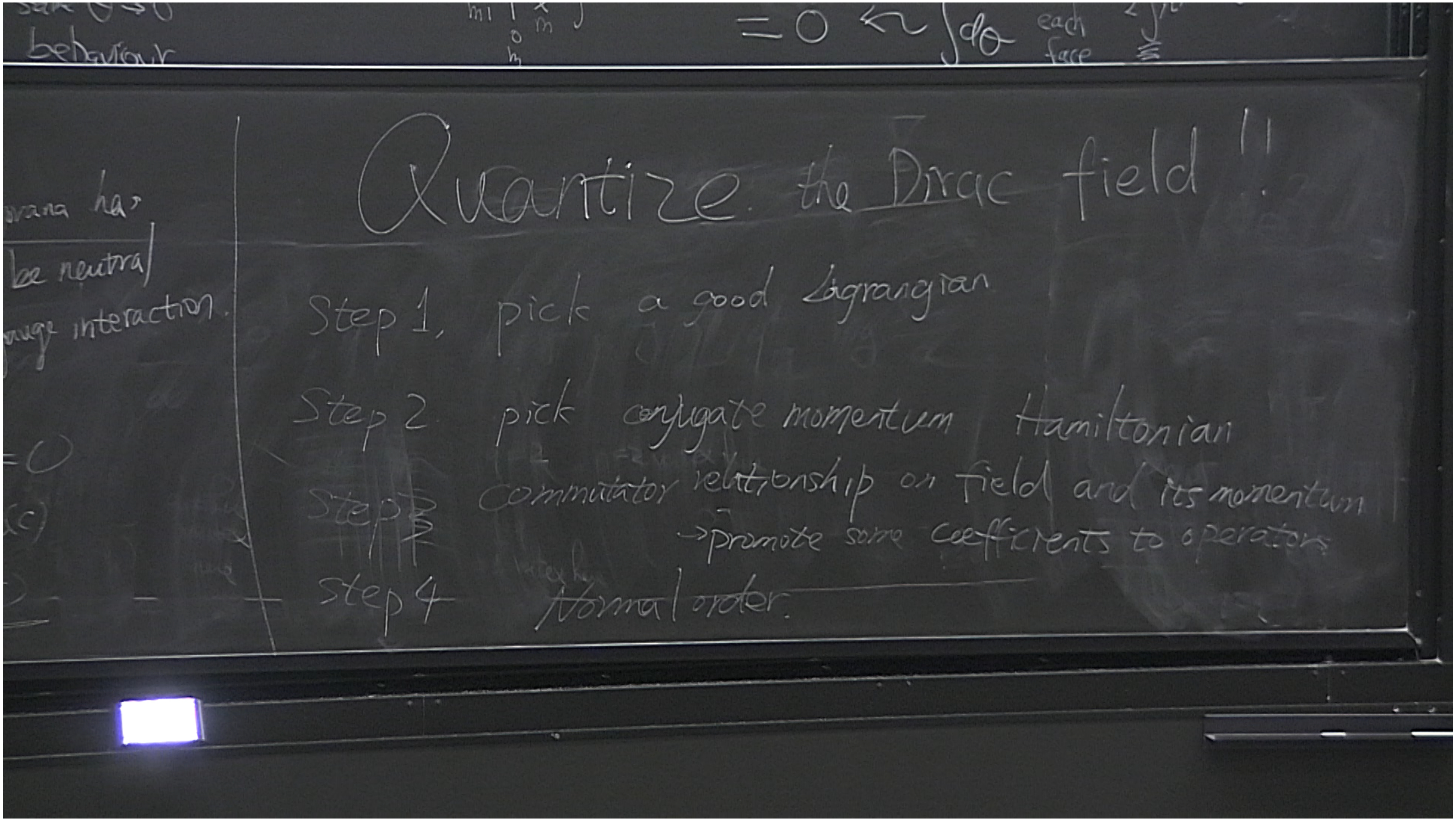


figure & same θ → 0
behaviour
$(i\gamma^\mu(\partial_\mu + iQA_\mu) - m)\psi = 0$
$\psi^{(c)}$
$(-i(\gamma^\mu)^*(\partial_\mu - iQA_\mu) - m)\psi^* = 0$
Majorana has to be neutral in gauge interaction.
$(\gamma^\mu)^* = -C^{-1}\gamma^\mu C$
$(i C^{-1}\gamma^\mu C(\partial_\mu - iQA_\mu) - mC^{-1}C)\psi^* = 0$
$C^{-1}(i)\gamma^\mu(\partial_\mu - iQA_\mu) - m)C\psi^* = 0$
$\psi^{(c)}$


Quantize the Dirac field !!
Step 1, pick a good Lagrangian
Step 2 pick conjugate momentum Hamiltonian
Step 3 commutator relationship on field and its momentum
→promote some coefficients to operators
Step 4 Normal order.

Step 1 $\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi$

Step 2 $\Pi = i\psi^\dagger$ $\quad \mathcal{H} = \psi^\dagger(-i\gamma^0\gamma^i\partial_i + \gamma^0 m)\psi$

Step 1 $\mathcal{L} = \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - m)\psi$

Step 2 $\pi = i\psi^{\dagger}$ $\quad \mathcal{H} = \psi^{\dagger}(-i\gamma^{0}\gamma^{i}\partial_{i} + \gamma^{0}m)\psi$

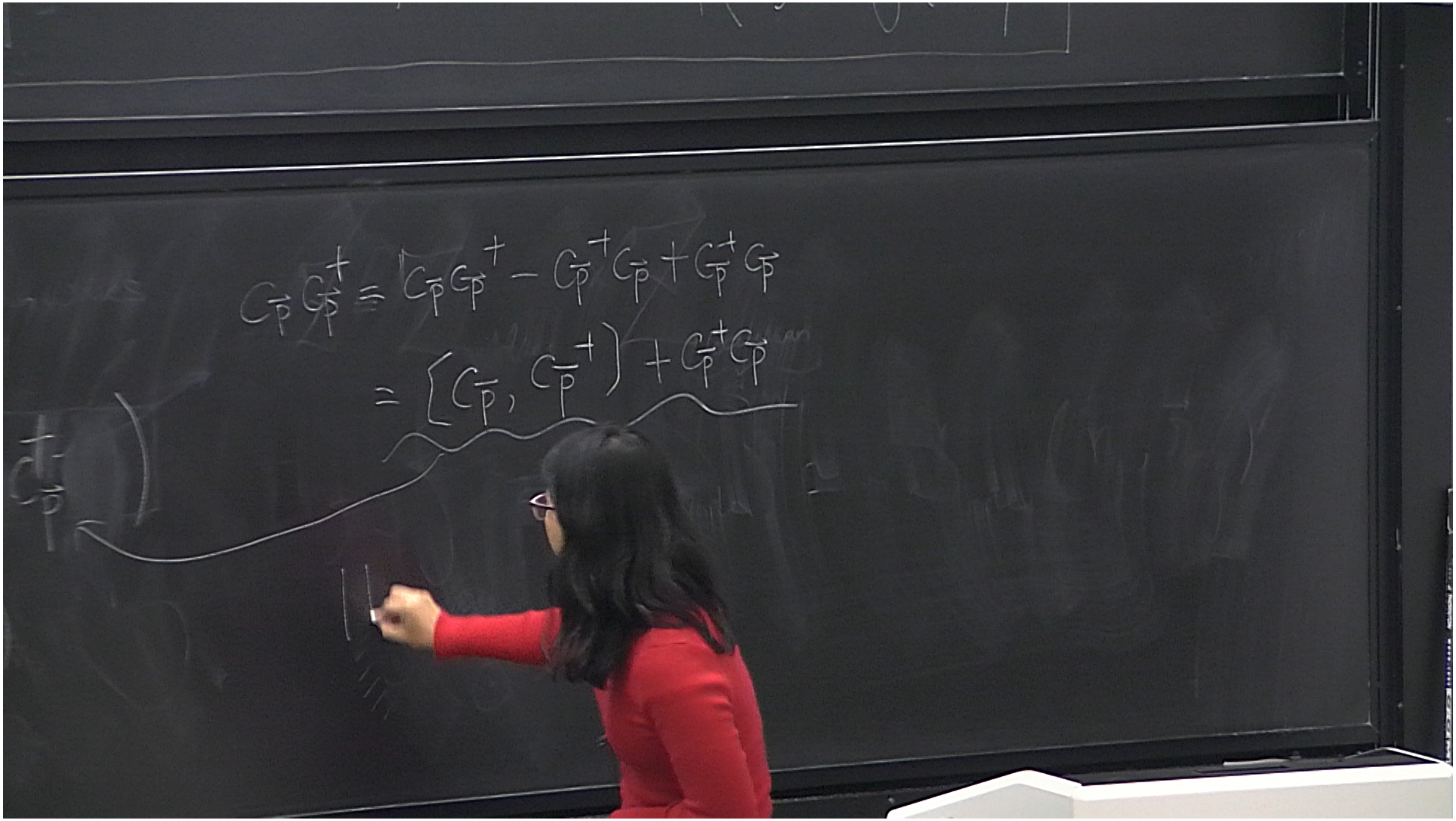
Step 3 $H = \int d^3x\, \mathcal{H} = \int dV_p\, E_{\bar{p}}\left(b^{*}(\bar{p})\, b(\bar{p}) - c(\bar{p})\, c^{*}(\bar{p})\right)$

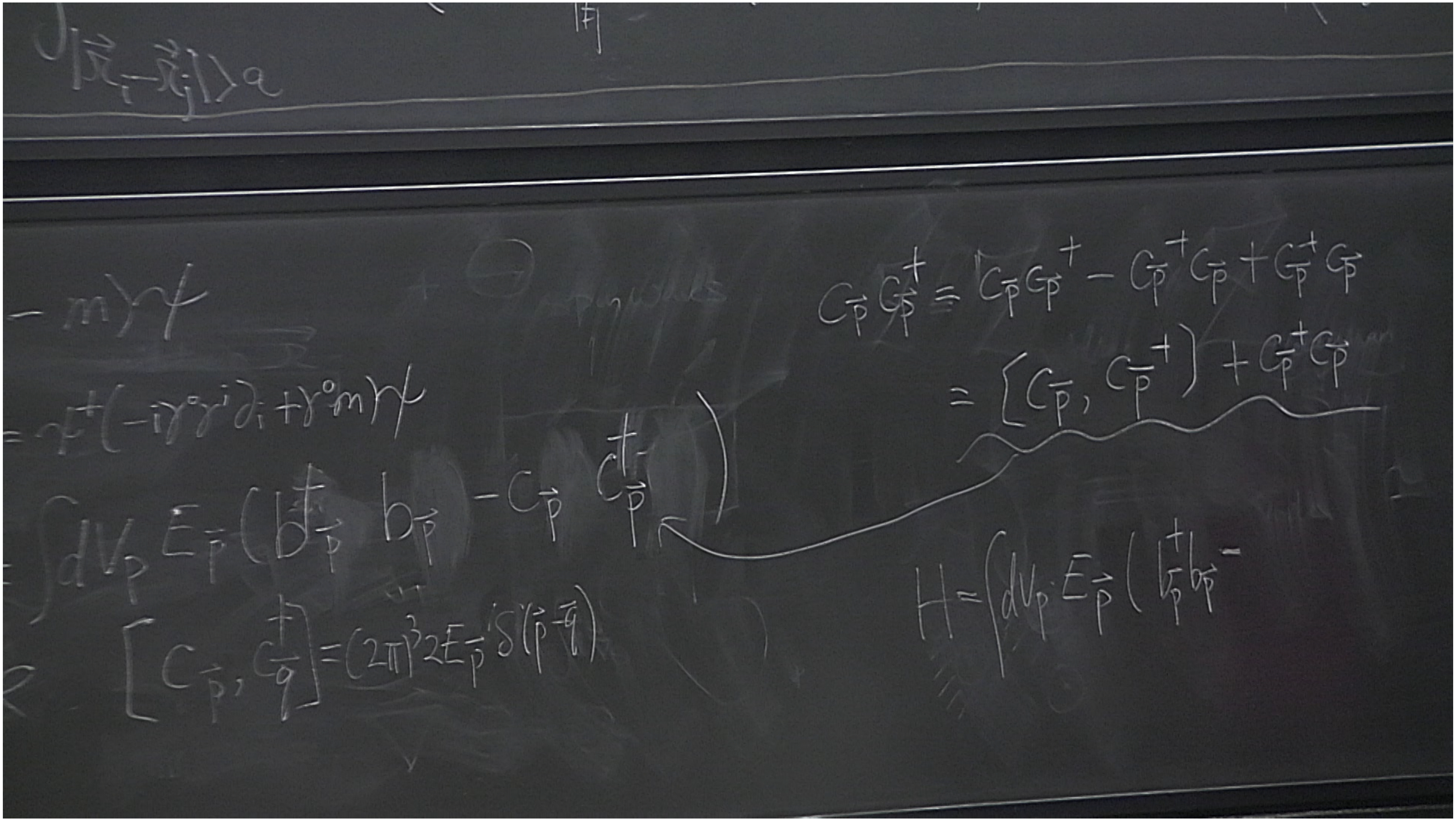
Step 1 $\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi$

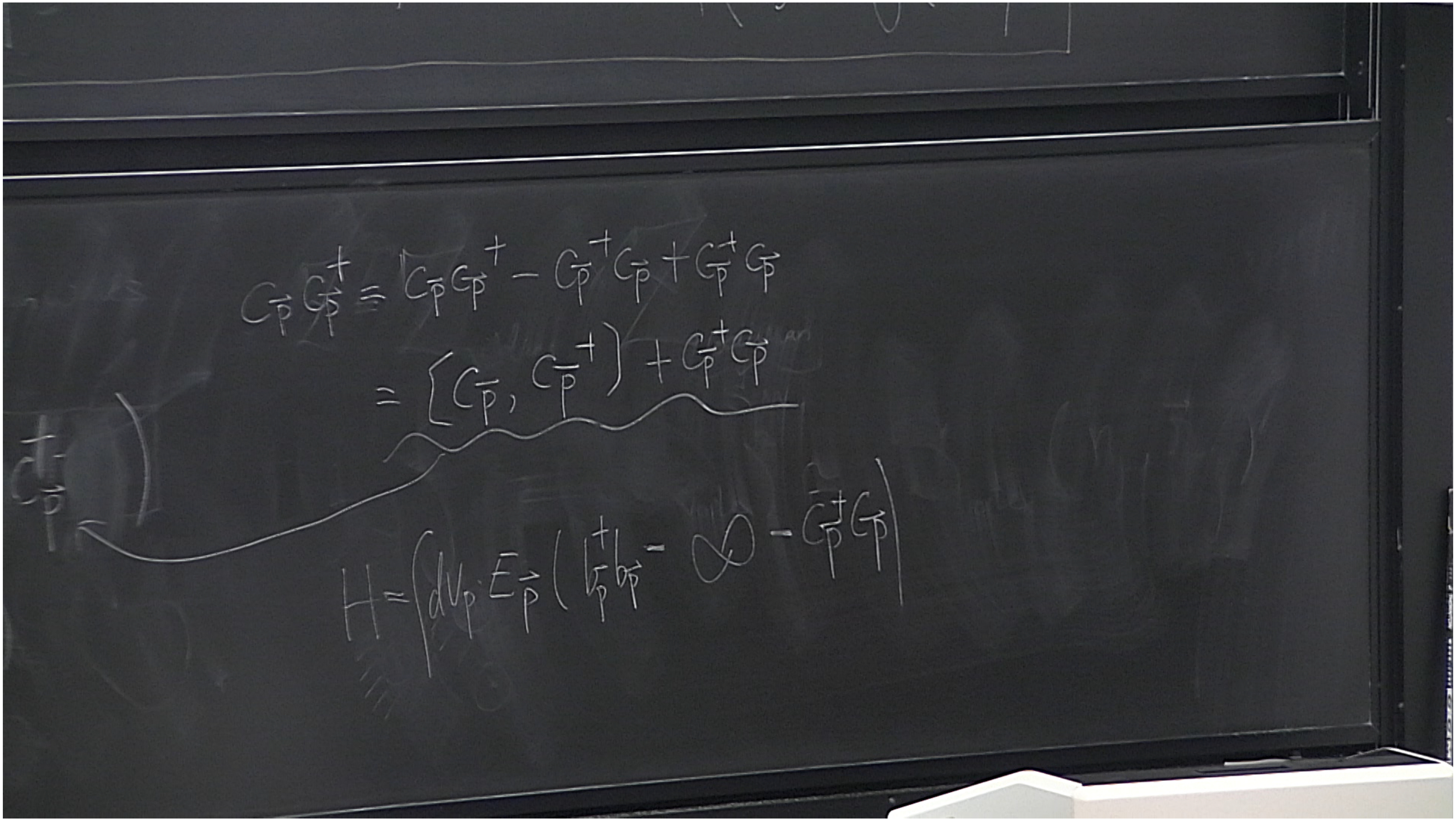
Step 2 $\Pi = i\psi^\dagger$ $\mathcal{H} = \psi^\dagger(-i\gamma^0\gamma^i\partial_i + \gamma^0 m)\psi$

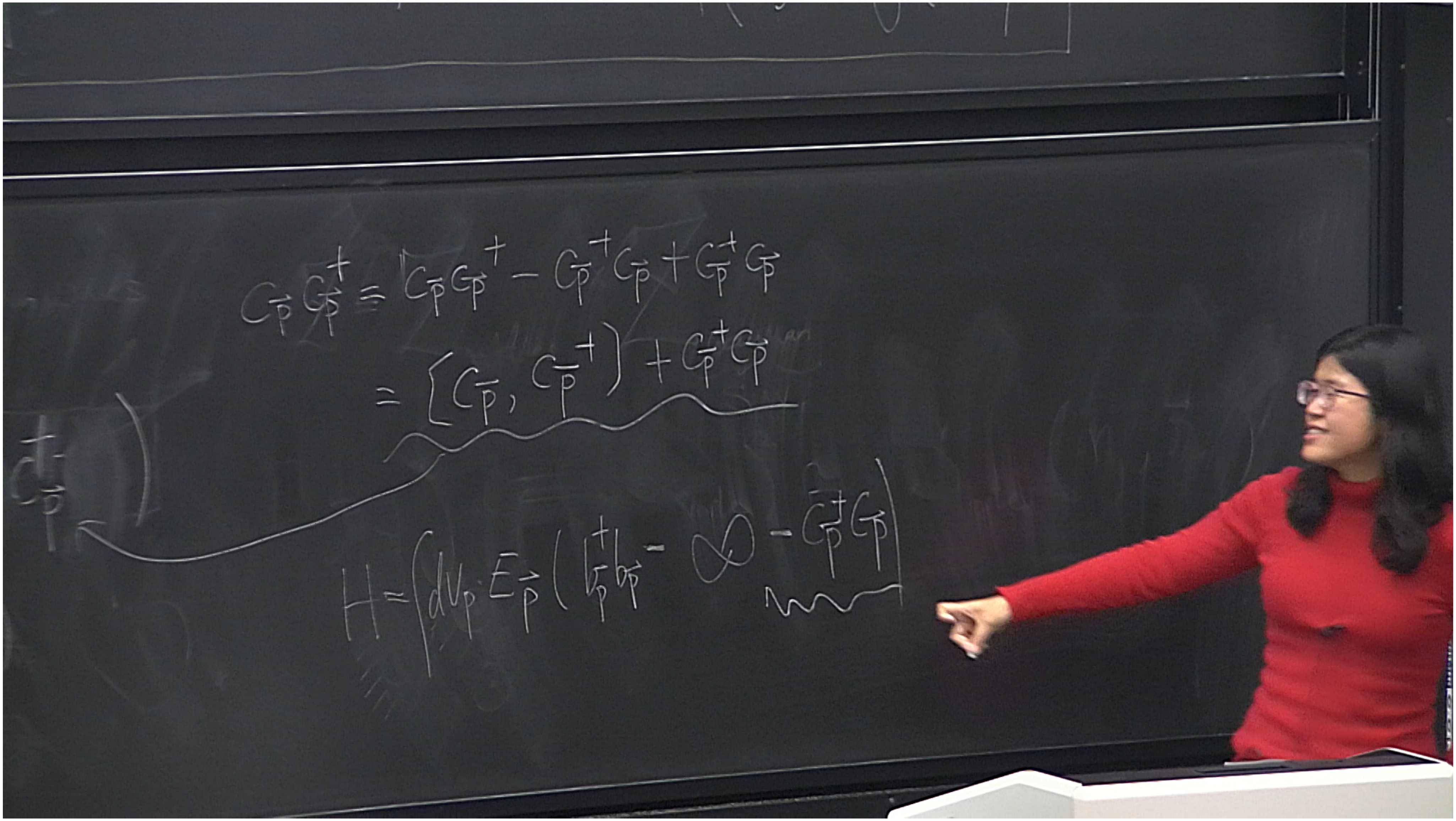
Step 3 $H = \int d^3x\, \mathcal{H} = \int dV_p\, E_{\vec{p}}\left(b^\dagger_{\vec{p}}\, b_{\vec{p}} - c_{\vec{p}}\, c^\dagger_{\vec{p}}\right)$

try to impose $\left[c_{\vec{p}}, c^\dagger_{\vec{q}}\right] = (2\pi)^3 2E_{\vec{p}}\,\delta(\vec{p}-\vec{q})$

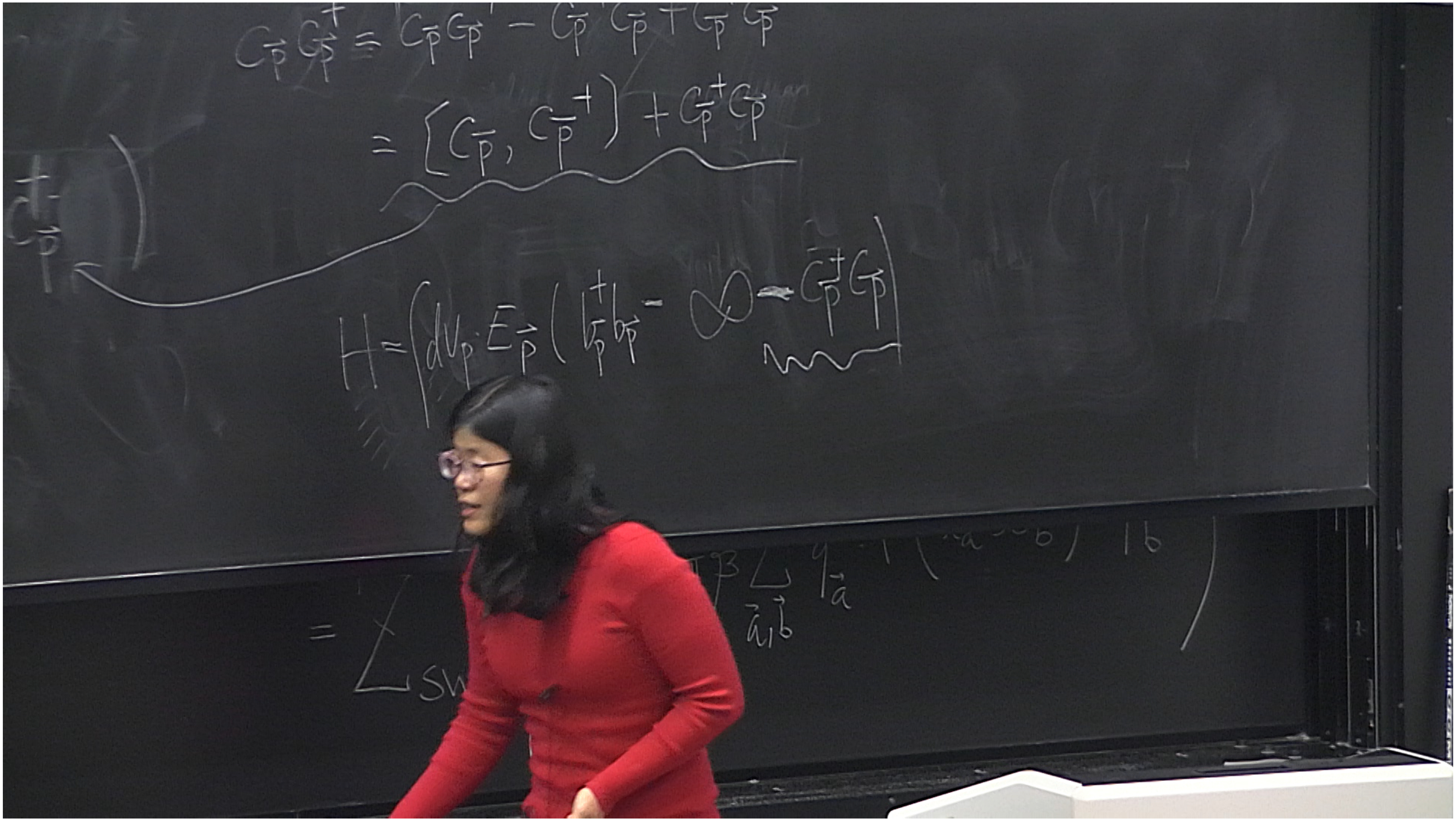

$c_{\vec p} c_{\vec p}^\dagger = c_{\vec p} c_{\vec p}^\dagger - c_{\vec p}^\dagger c_{\vec p} + c_{\vec p}^\dagger c_{\vec p}$
$= \{c_{\vec p}, c_{\vec p}^\dagger\} + c_{\vec p}^\dagger c_{\vec p}$

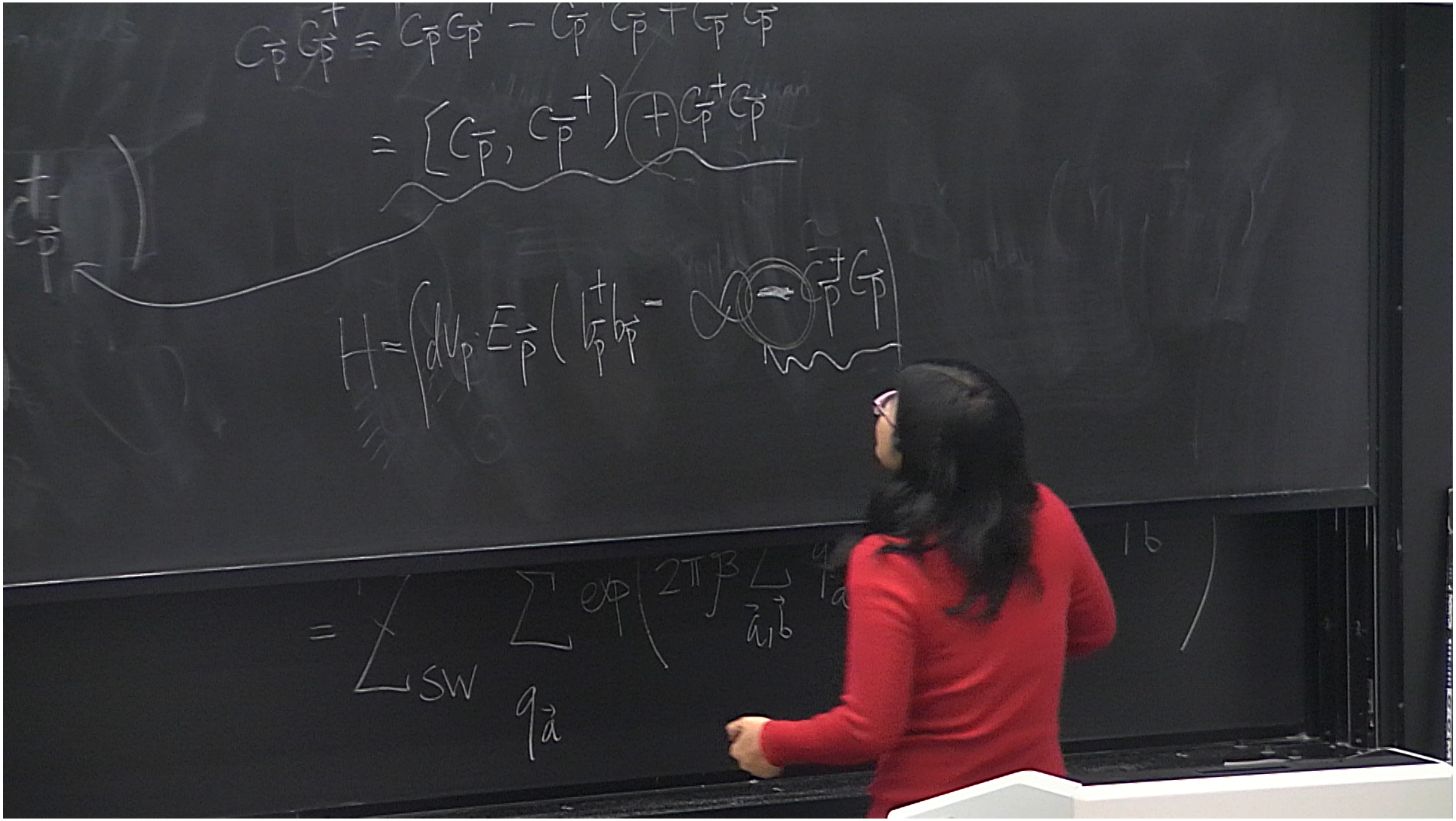

$|\vec{x}_i - \vec{x}_j| > a$
$-m)\psi$
$= \psi^\dagger(-i\gamma^0\gamma^i\partial_i + \gamma^0 m)\psi$
$= \int dV_p\, E_{\vec{p}} (b^\dagger_{\vec{p}}\, b_{\vec{p}} - c_{\vec{p}}\, c^\dagger_{\vec{p}})$
$[c_{\vec{p}}, c^\dagger_{\vec{q}}] = (2\pi)^3 2E_{\vec{p}}\, \delta(\vec{p}-\vec{q})$
$c_{\vec{p}} c^\dagger_{\vec{p}} = c_{\vec{p}} c^\dagger_{\vec{p}} - c^\dagger_{\vec{p}} c_{\vec{p}} + c^\dagger_{\vec{p}} c_{\vec{p}}$
$= [c_{\vec{p}}, c^\dagger_{\vec{p}}] + c^\dagger_{\vec{p}} c_{\vec{p}}$
$H = \int dV_p\, E_{\vec{p}} (b^\dagger_{\vec{p}} b_{\vec{p}} -$

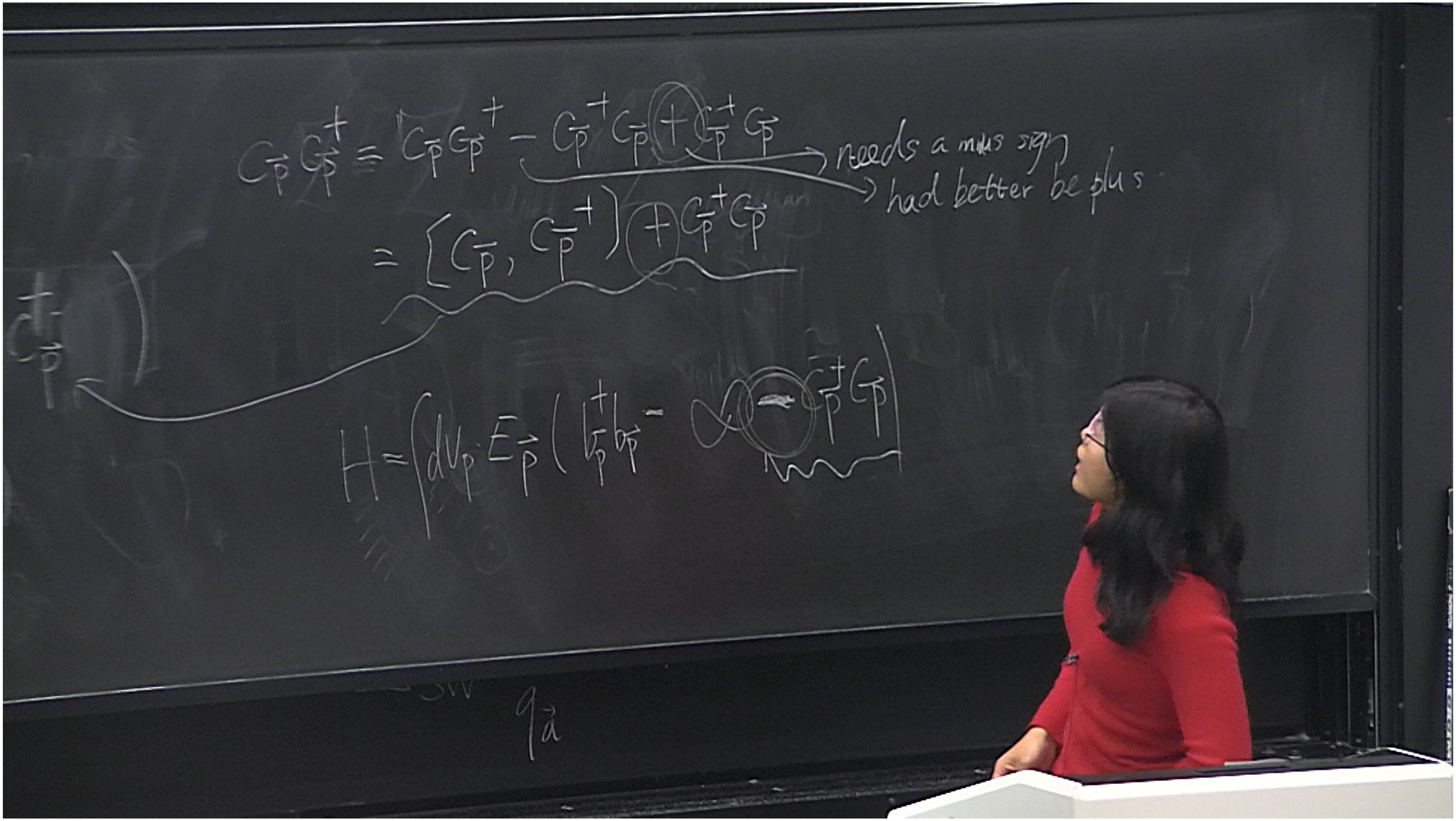

$c_{\vec p} c_{\vec p}^{\dagger} = c_{\vec p} c_{\vec p}^{\dagger} - c_{\vec p}^{\dagger} c_{\vec p} + c_{\vec p}^{\dagger} c_{\vec p}$
$= [c_{\vec p}, c_{\vec p}^{\dagger}] + c_{\vec p}^{\dagger} c_{\vec p}$
$H = \int dV_p\, E_{\vec p} \left(b_{\vec p}^{\dagger} b_{\vec p} - \infty - c_{\vec p}^{\dagger} c_{\vec p} \right)$

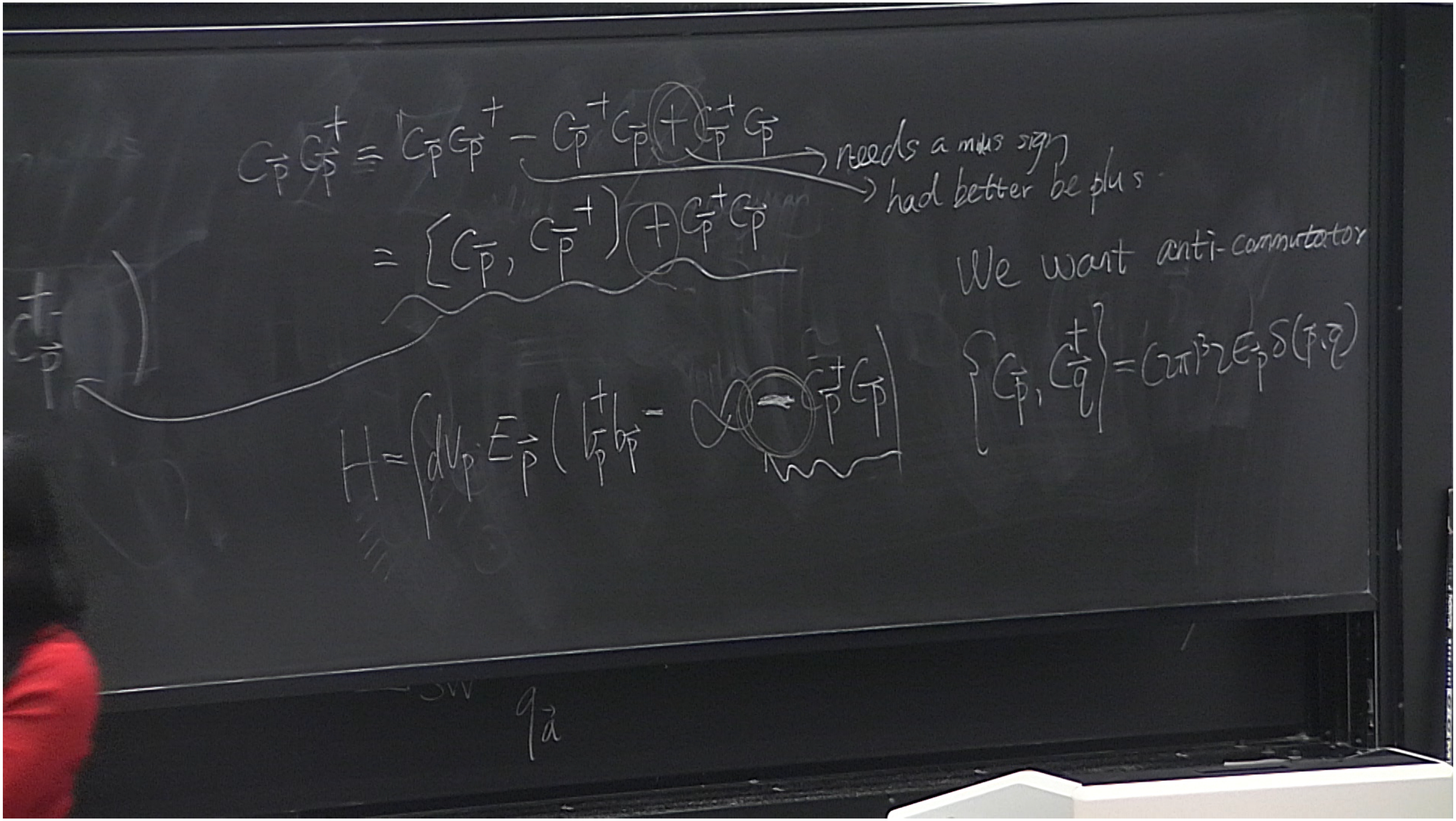


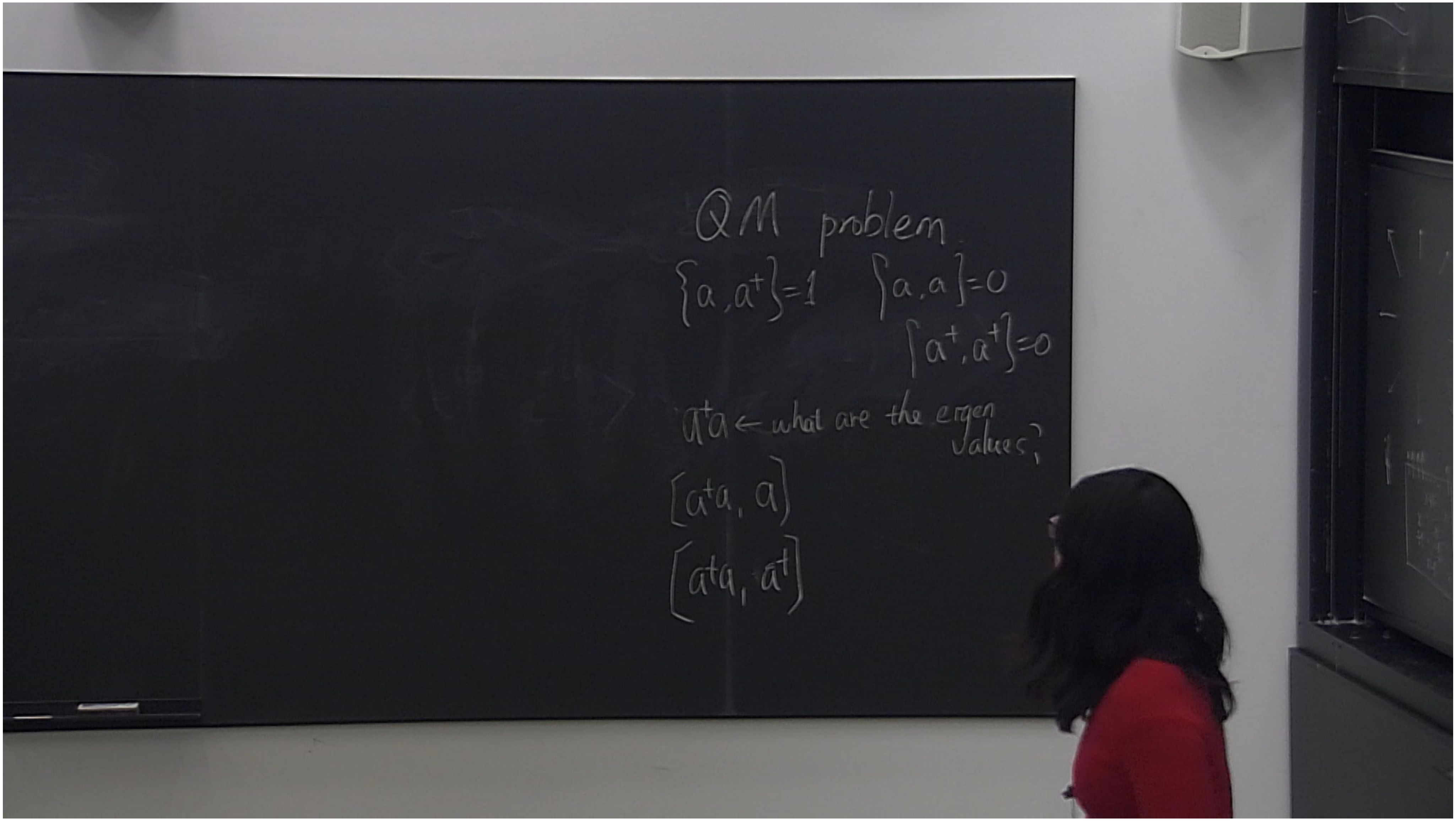
Cp Cp† = Cp Cp† − Cp† Cp + Cp† Cp
= {Cp, Cp†} + Cp† Cp
H = ∫dVp Ep (bp† bp − ∞ − Cp† Cp)

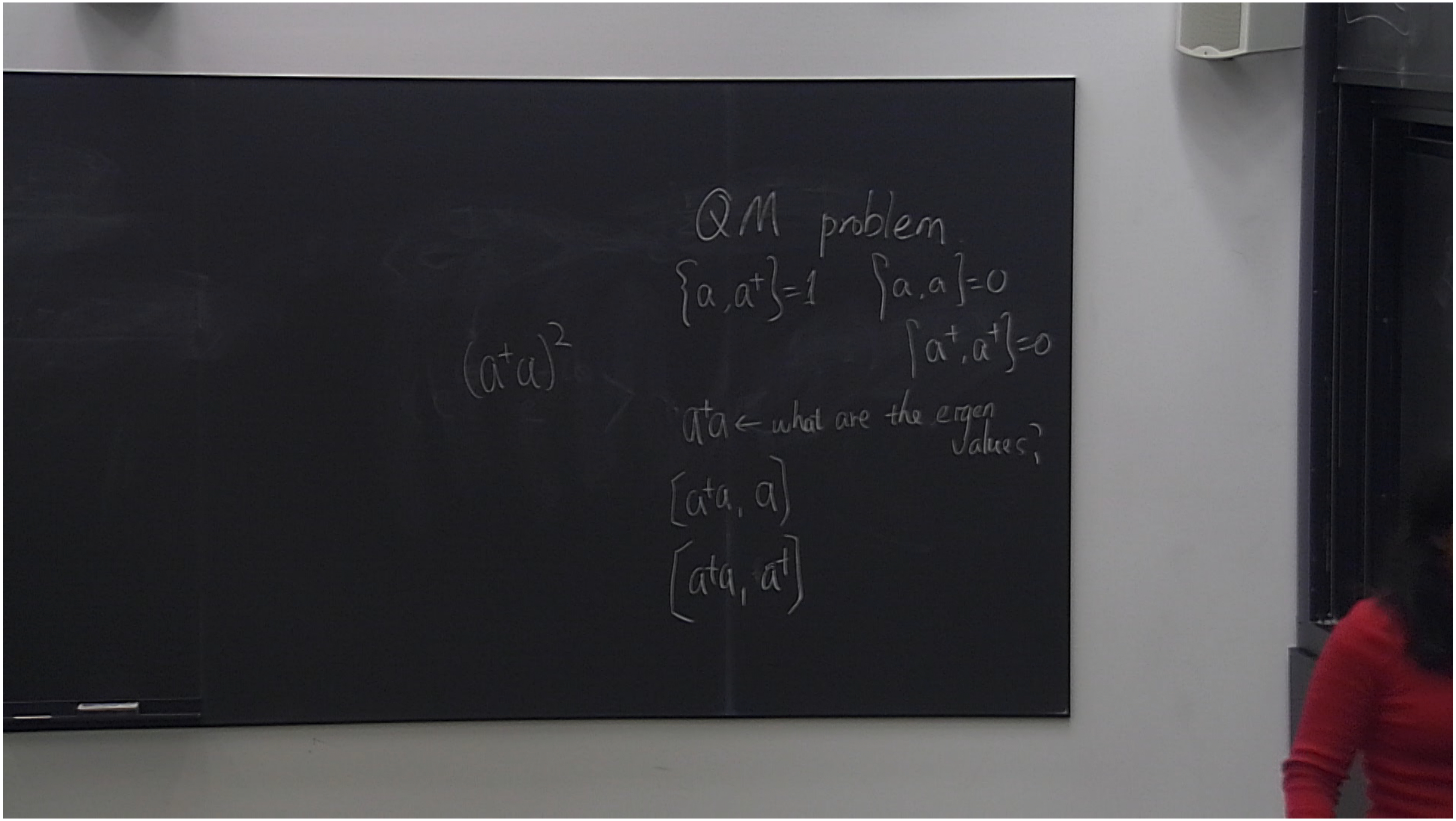


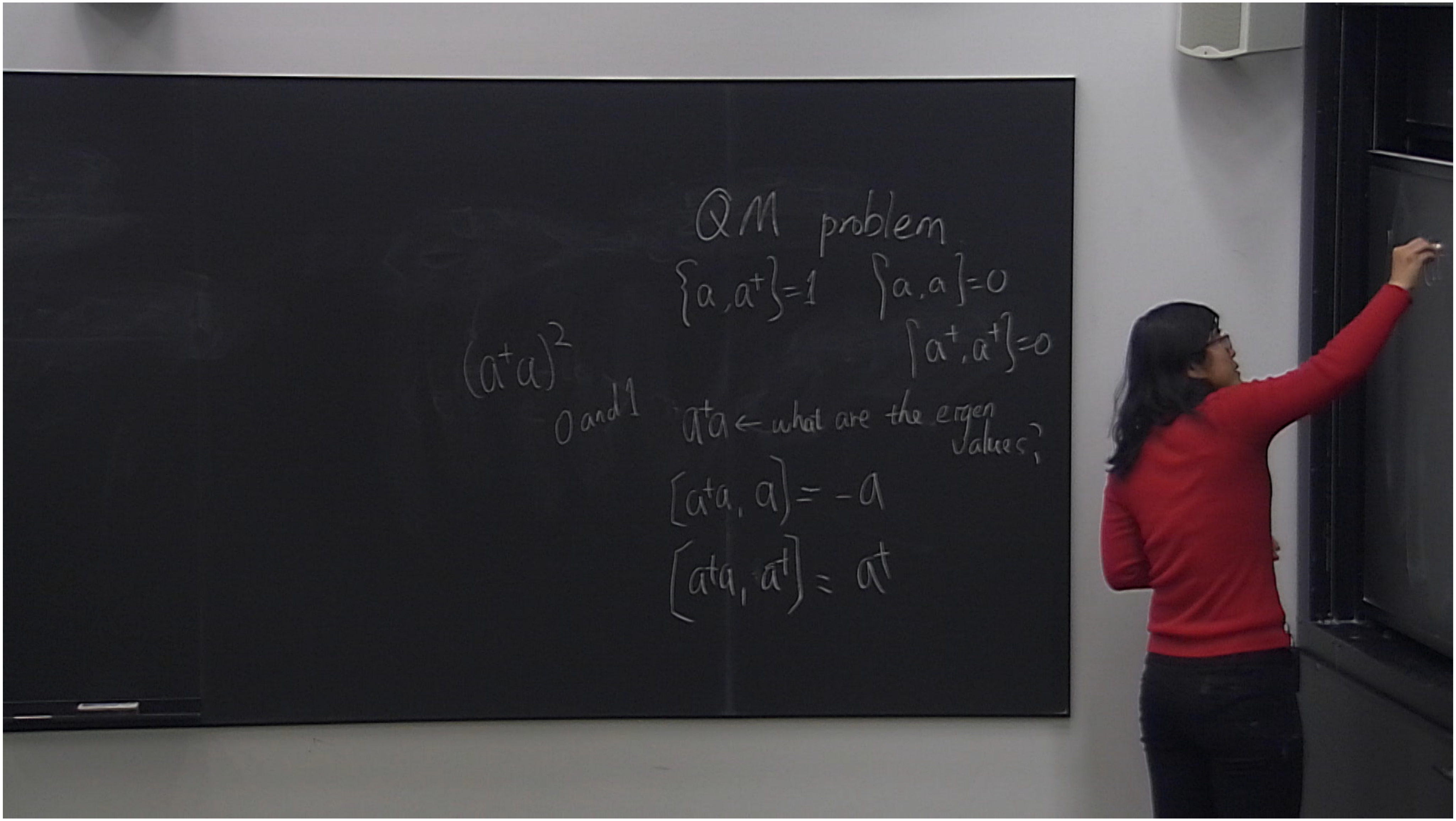


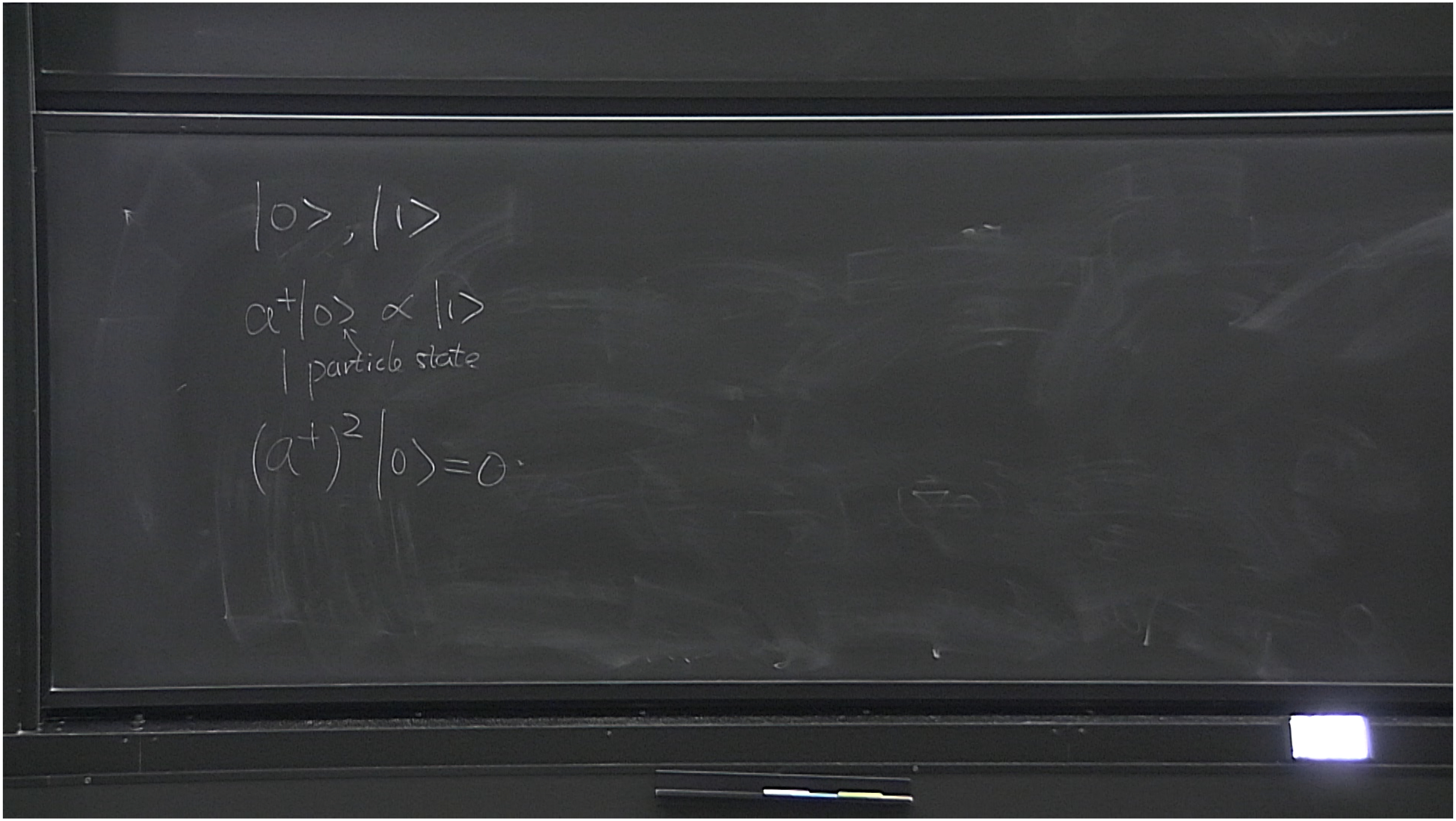

needs a minus sign
had better be plus

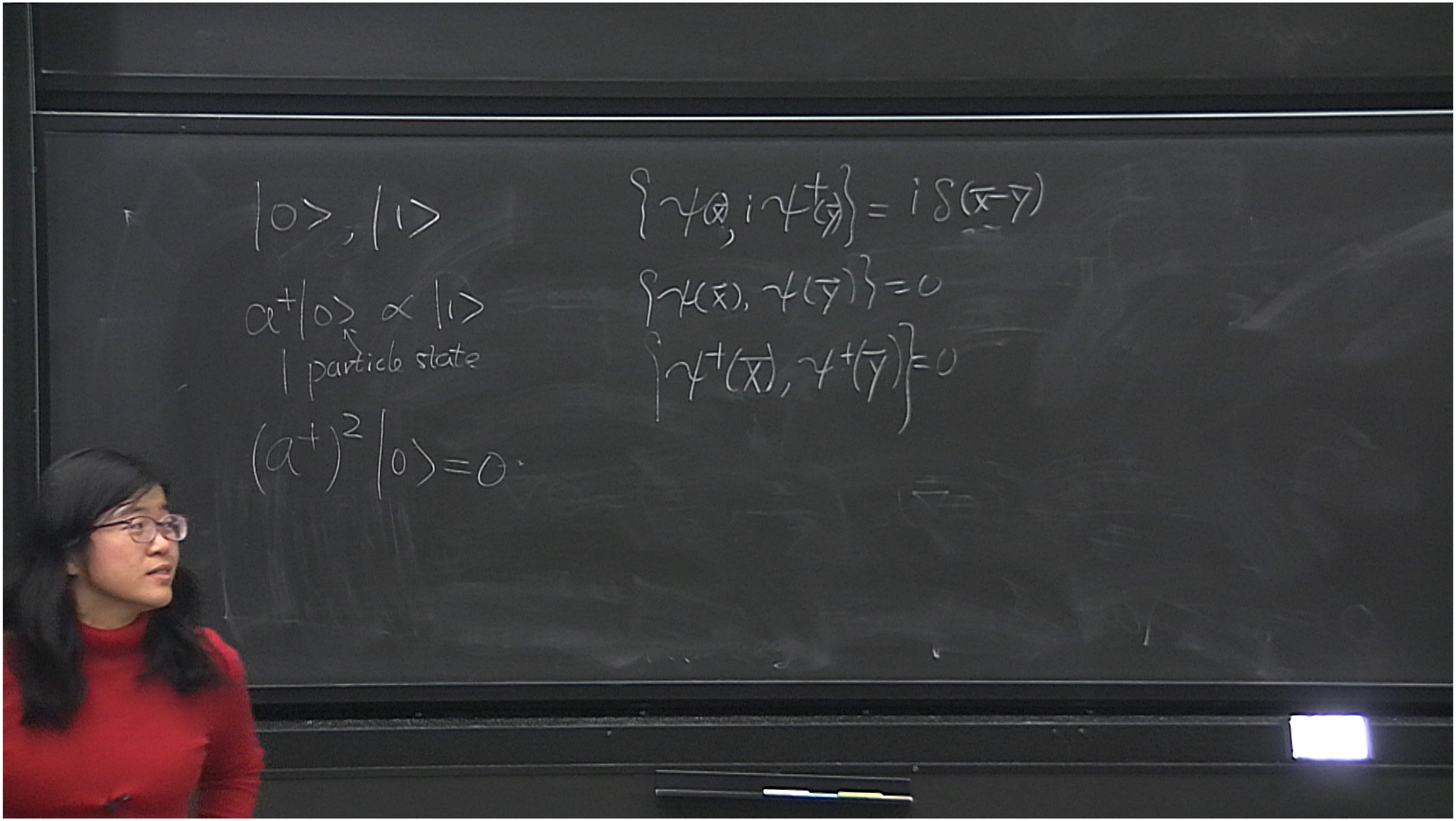

needs a minus sign
had better be plus
We want anti-commutator

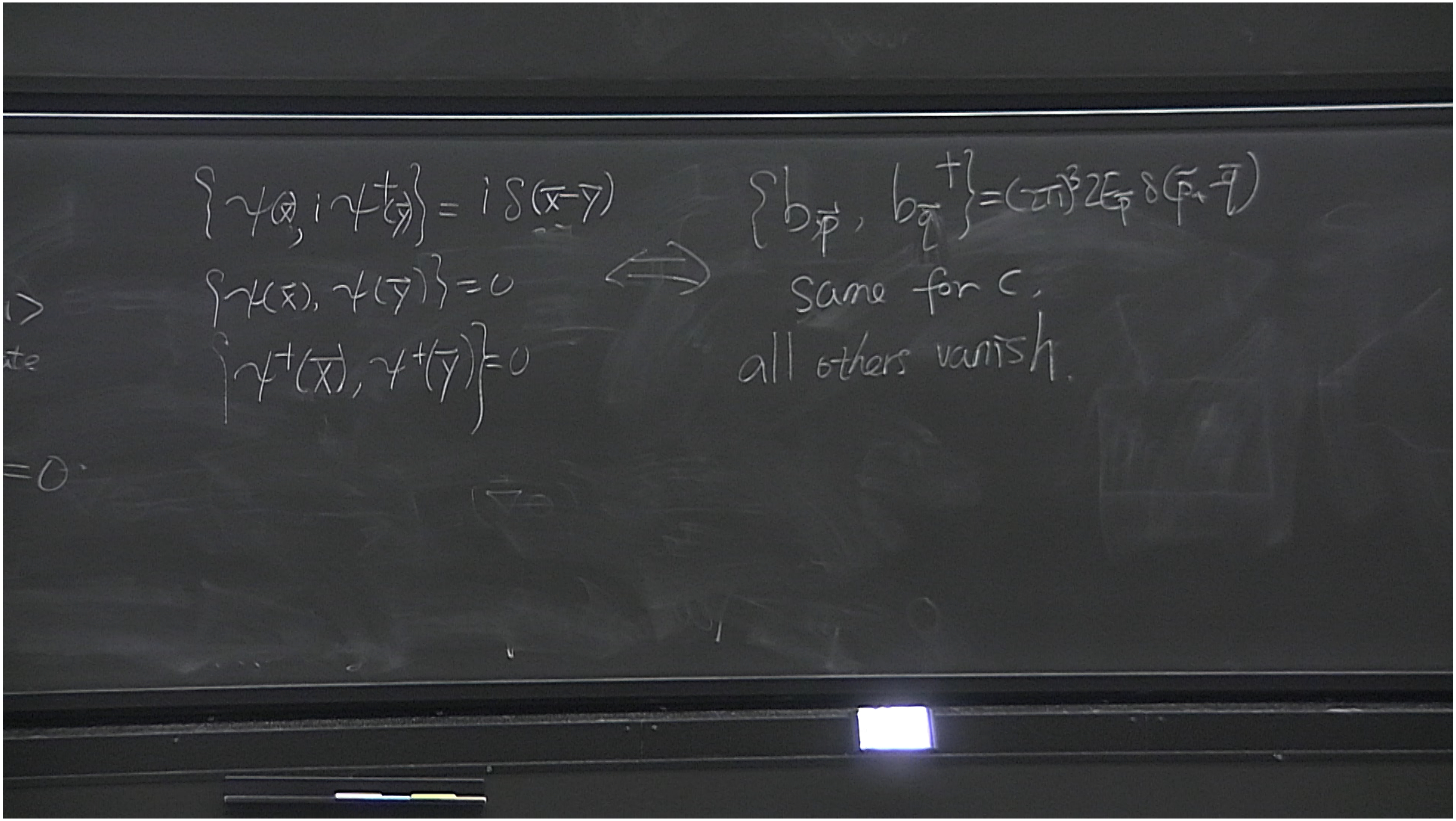

QM problem
{a, a+} = 1
{a, a} = 0
{a+, a+} = 0
a+a ← what are the eigen values?
[a+a, a]
[a+a, a+]


QM problem
{a, a+}=1
{a, a}=0
{a+, a+}=0
(a+a)2
a+a ← what are the eigen values?
[a+a, a]
[a+a, a+]


QM problem
{a, a†} = 1
{a, a} = 0
{a†, a†} = 0
(a†a)²
0 and 1
a†a ← what are the eigen values?
[a†a, a] = −a
[a†a, a†] = a†


|0>, |1>
a+|0> ∝ |1>
1 particle state
(a+)2 |0> = 0


|0>, |1>
a+|0> ∝ |1>
1 particle state
(a+)2|0> = 0
{ψ(x), iψ+(y)} = iδ(x−y)
{ψ(x), ψ(y)} = 0
{ψ+(x), ψ+(y)} = 0


$\{\psi(\vec{x}), i\psi^\dagger(\vec{y})\} = i\delta(\vec{x}-\vec{y})$
$\{\psi(\vec{x}), \psi(\vec{y})\} = 0$
$\{\psi^\dagger(\vec{x}), \psi^\dagger(\vec{y})\} = 0$
$\Longleftrightarrow$
$\{b_{\vec{p}}, b_{\vec{q}}^\dagger\} = (2\pi)^3 2E_{\vec{p}}\, \delta(\vec{p}-\vec{q})$
same for c.
all others vanish.

$$\{b_{\vec p}, b_{\vec q}^{\dagger}\} = (2\pi)^3 2E_{\vec p}\,\delta(\vec p - \vec q)$$

same for c.

all others vanish.

Step 4: $\int dV_{\vec p}\, E_{\vec p} (b^{\dagger}_{\vec p} b_{\vec p} - c_{\vec p} c^{\dagger}_{\vec p})$

Normal ordering

$$:H: \int dV_{\vec p}\, E_{\vec p} (b^{\dagger}_{\vec p} b_{\vec p} + c^{\dagger}_{\vec p} c_{\vec p})$$

$$\{b_{\vec{p}}, b_{\vec{q}}^{\dagger}\} = (2\pi)^3 2E_{\vec{p}}\,\delta(\vec{p}-\vec{q})$$

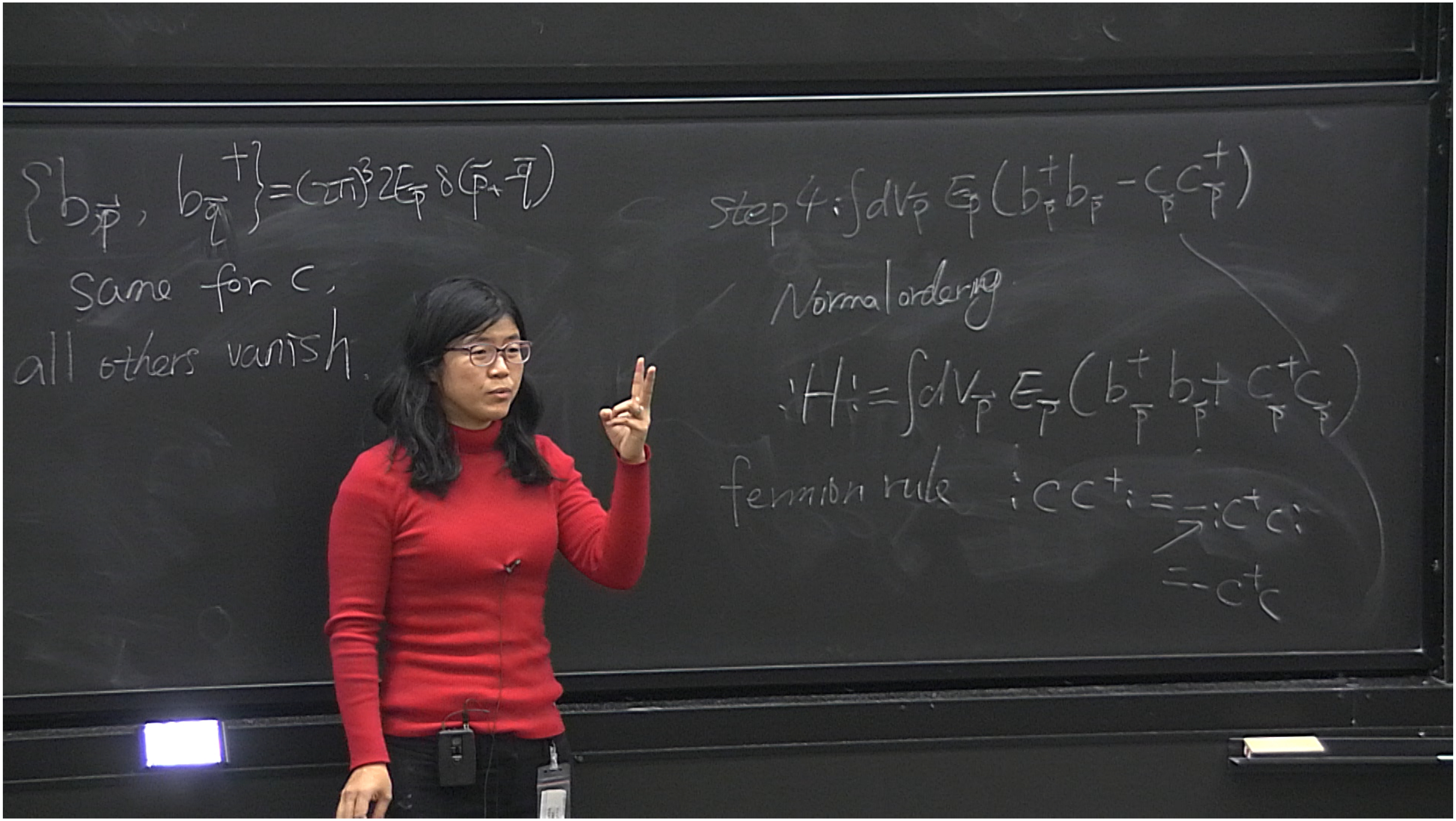
same for c.

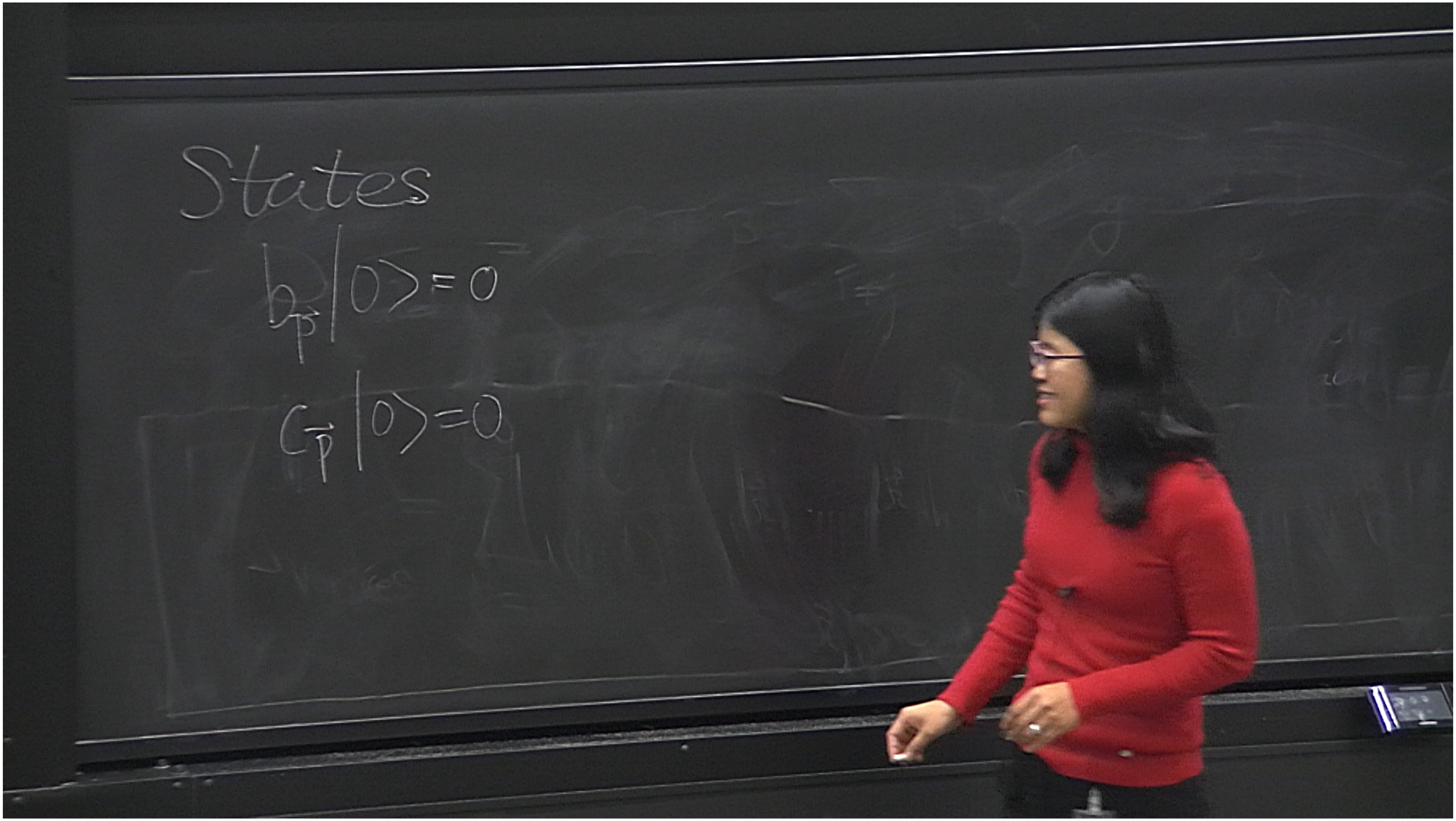
all others vanish.

Step 4: $\int dV_{\vec{p}}\, E_{\vec{p}} (b_{\vec{p}}^{\dagger} b_{\vec{p}} - c_{\vec{p}} c_{\vec{p}}^{\dagger})$

Normal ordering

$$:H: = \int dV_{\vec{p}}\, E_{\vec{p}} (b_{\vec{p}}^{\dagger} b_{\vec{p}} + c_{\vec{p}}^{\dagger} c_{\vec{p}})$$


{b_p, b_q^+} = (2π)^3 2E_p δ(p − q)
same for c,
all others vanish.
Step 4: ∫dV_p E_p (b_p^+ b_p − c_p c_p^+)
Normal ordering
:H: = ∫dV_p E_p (b_p^+ b_p + c_p^+ c_p)
fermion rule :cc^+: = − :c^+c:
= −c^+c


States
$b_{\vec{p}}|0\rangle = 0$
$c_{\vec{p}}|0\rangle = 0$

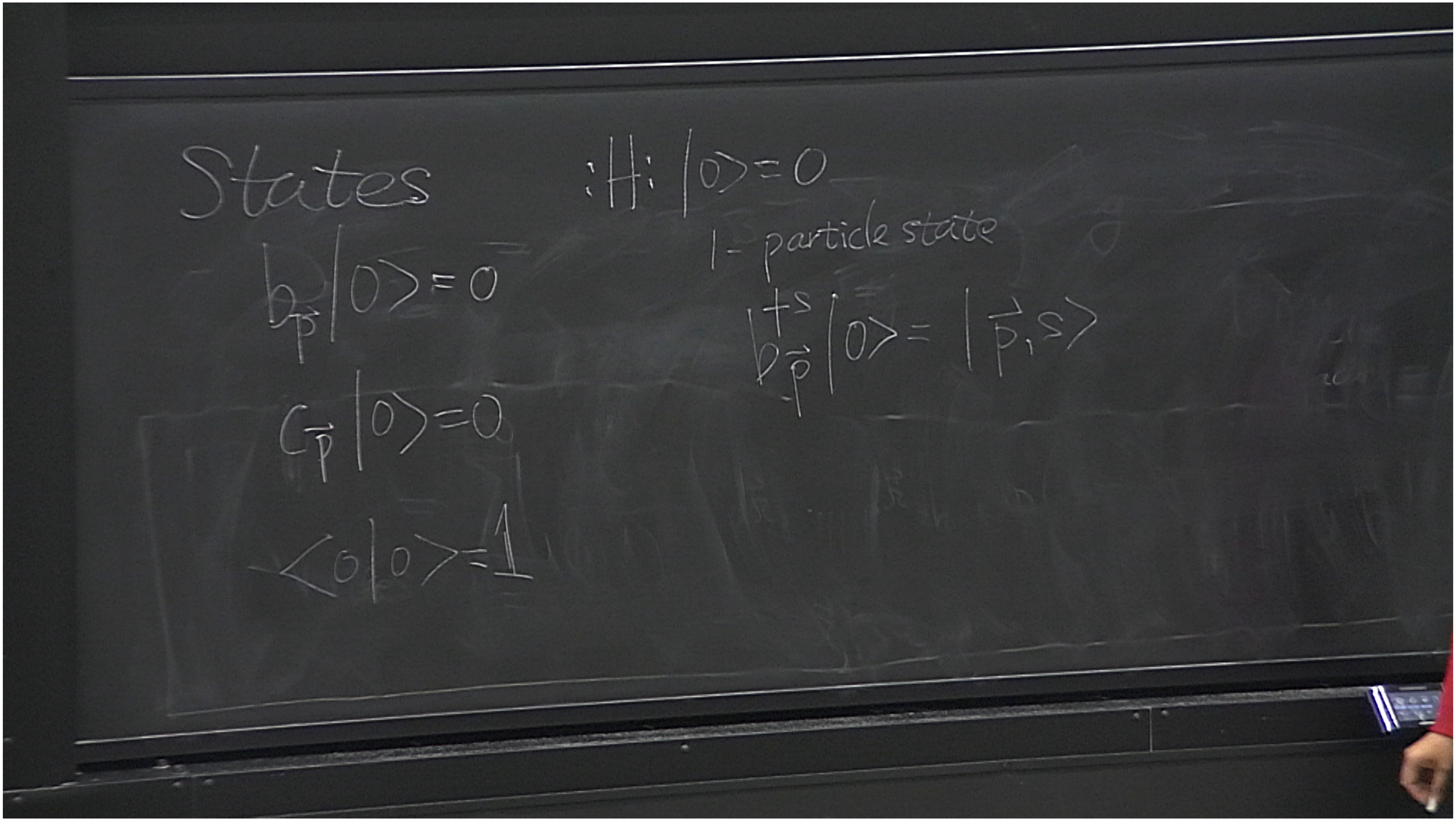
States

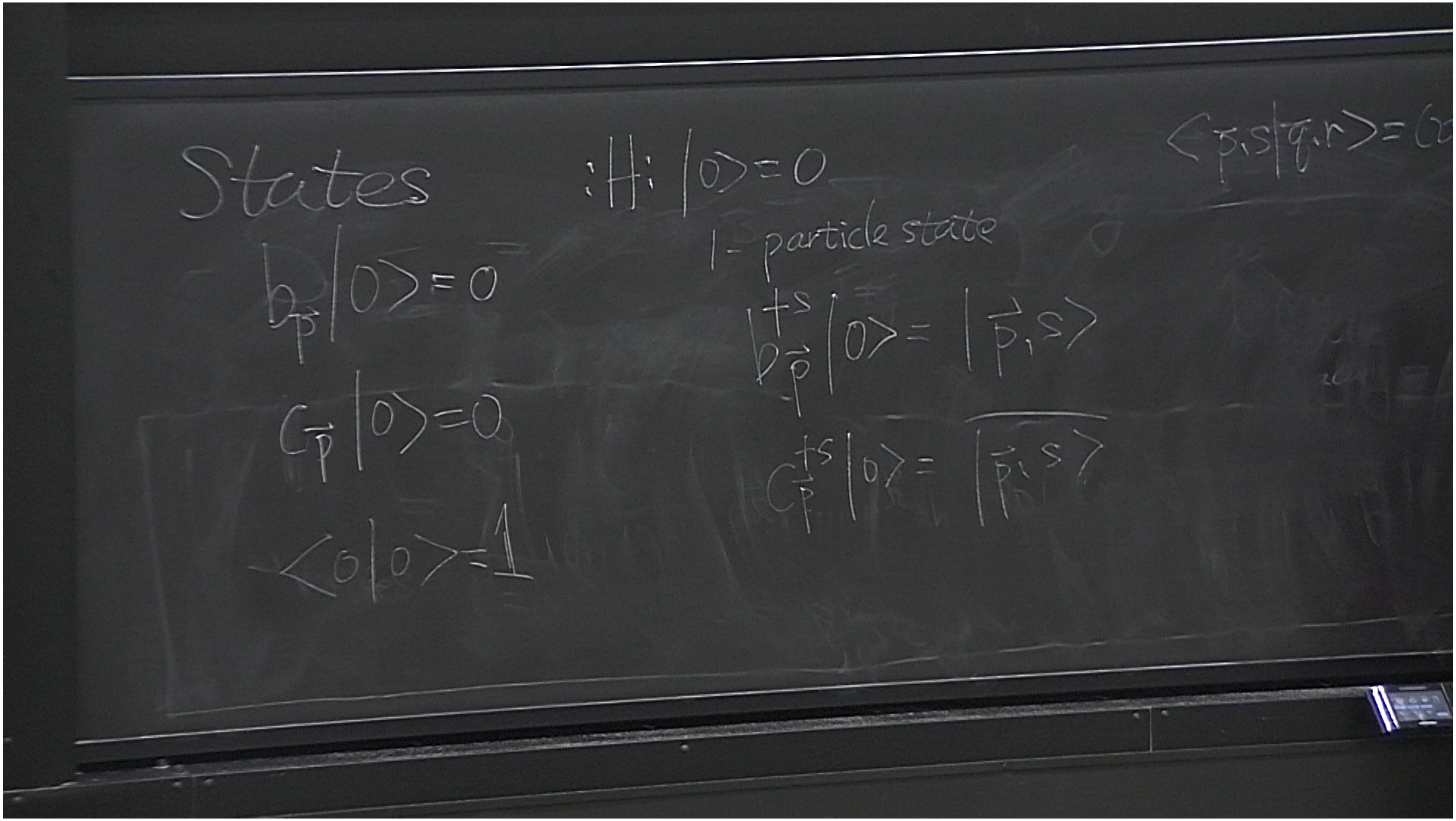
$:H:|0\rangle = 0$

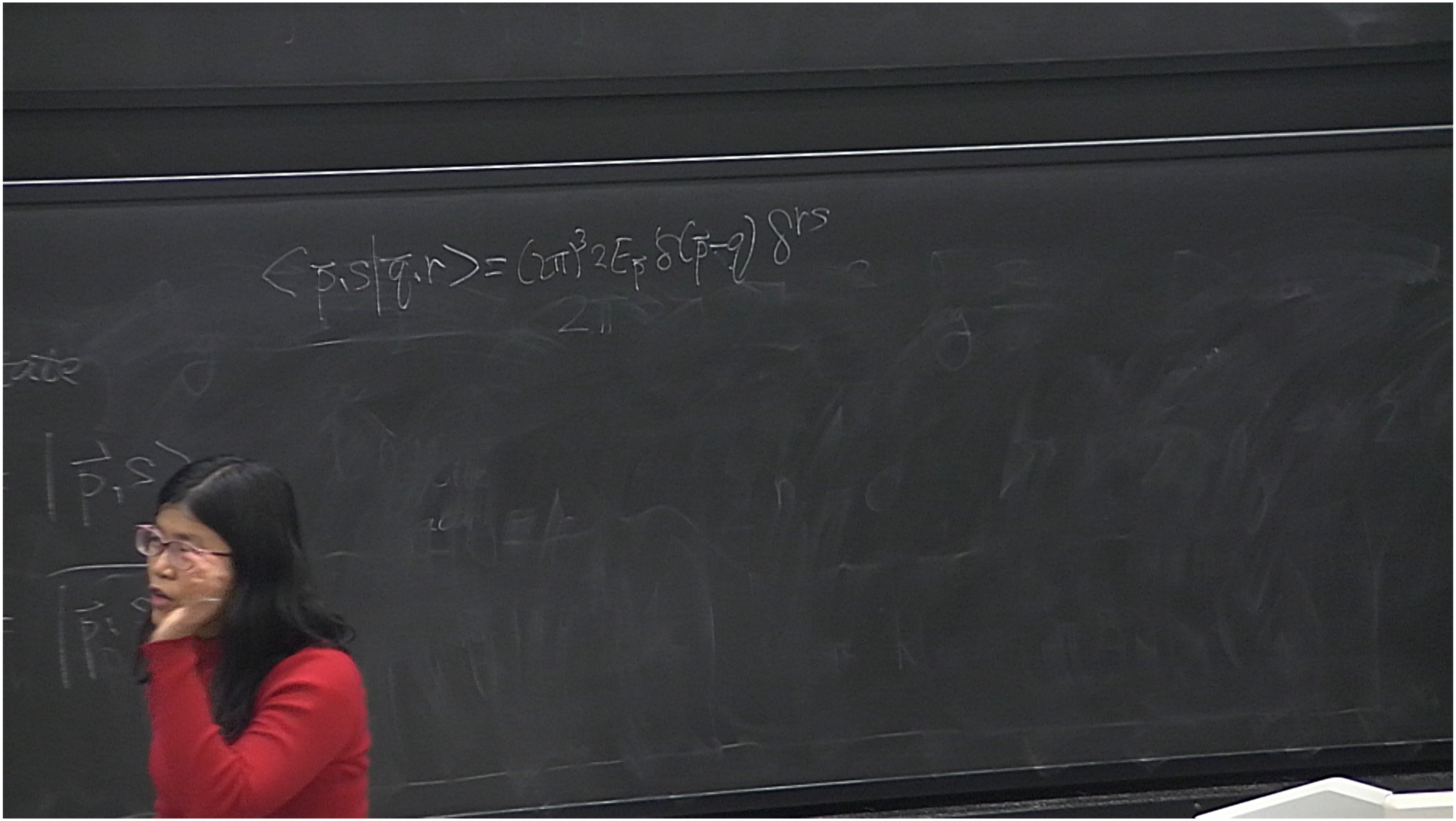
$b_{\vec{p}}|0\rangle = 0$

$c_{\vec{p}}|0\rangle = 0$

$\langle 0|0\rangle = 1$


States
:H: |0> = 0
b_p |0> = 0
1- particle state
b†s_p |0> = |p, s>
c_p |0> = 0
<0|0> = 1


States
$:H: |0\rangle = 0$
$\langle \vec{p}, s | \vec{q}, r \rangle =$
1- particle state
$b_{\vec{p}} |0\rangle = 0$
$b^{\dagger s}_{\vec{p}} |0\rangle = |\vec{p}, s\rangle$
$c_{\vec{p}} |0\rangle = 0$
$c^{\dagger s}_{\vec{p}} |0\rangle = \overline{|\vec{p}, s\rangle}$
$\langle 0|0\rangle = 1$


$\langle \vec{p}, s | \vec{q}, r \rangle = (2\pi)^3 2E_{\vec{p}} \delta(\vec{p}-\vec{q}) \delta^{rs}$

ate

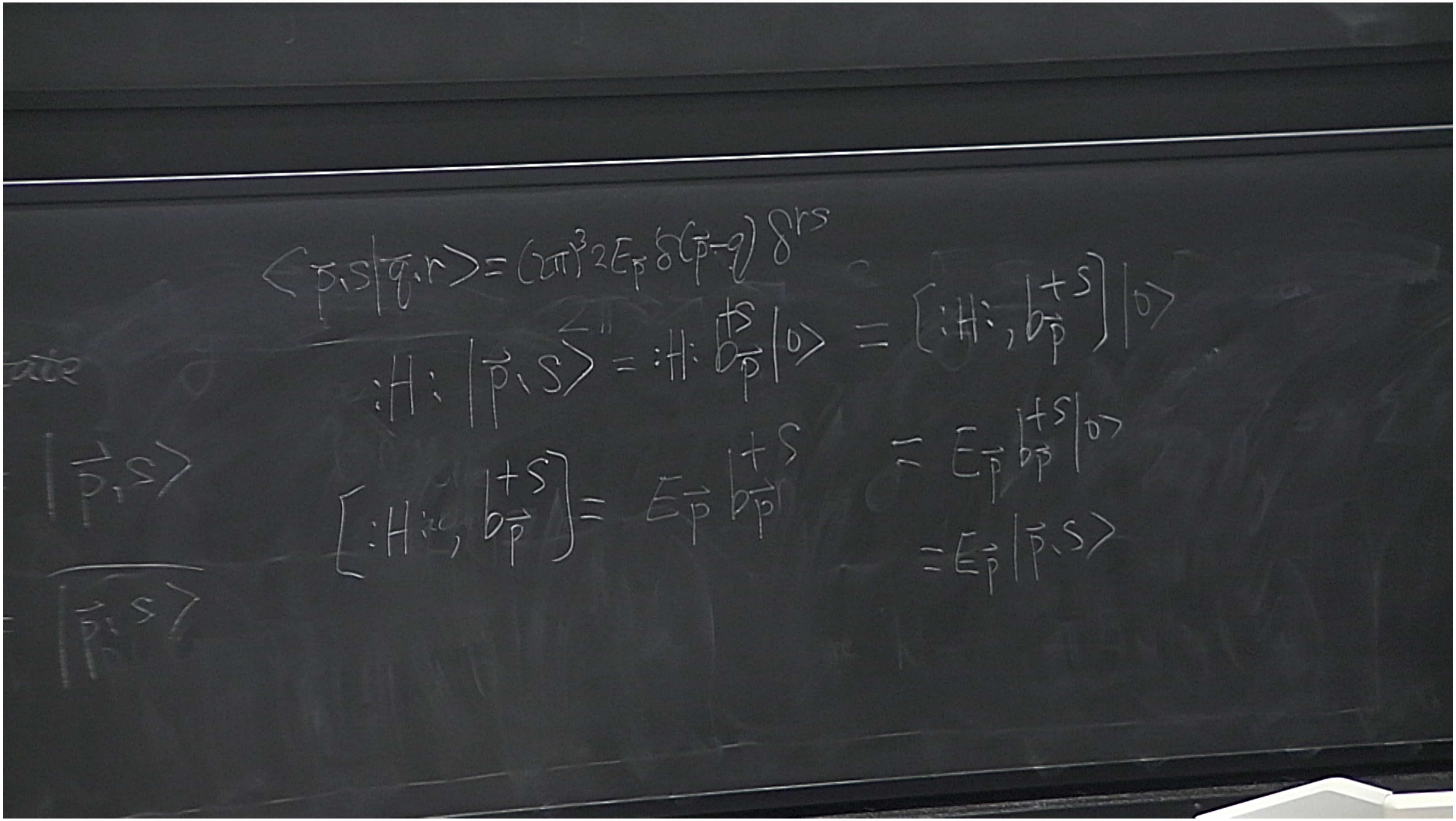
$= |\vec{p}, s\rangle$

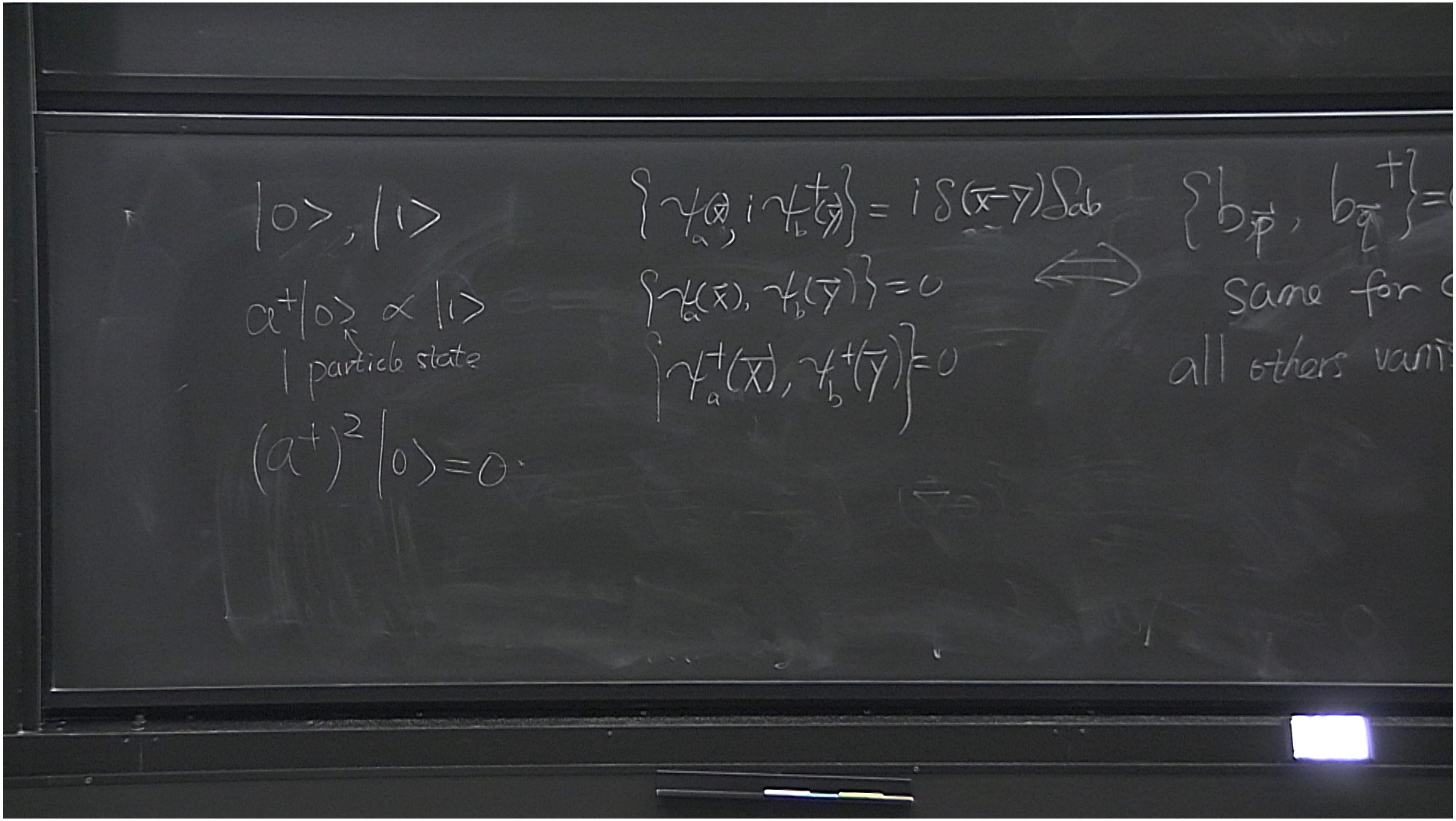
$= |\vec{p}, s\rangle$

$$\langle \vec{p}, s | \vec{q}, r \rangle = (2\pi)^3 \, 2E_{\vec{p}} \, \delta(\vec{p}-\vec{q}) \, \delta^{rs}$$

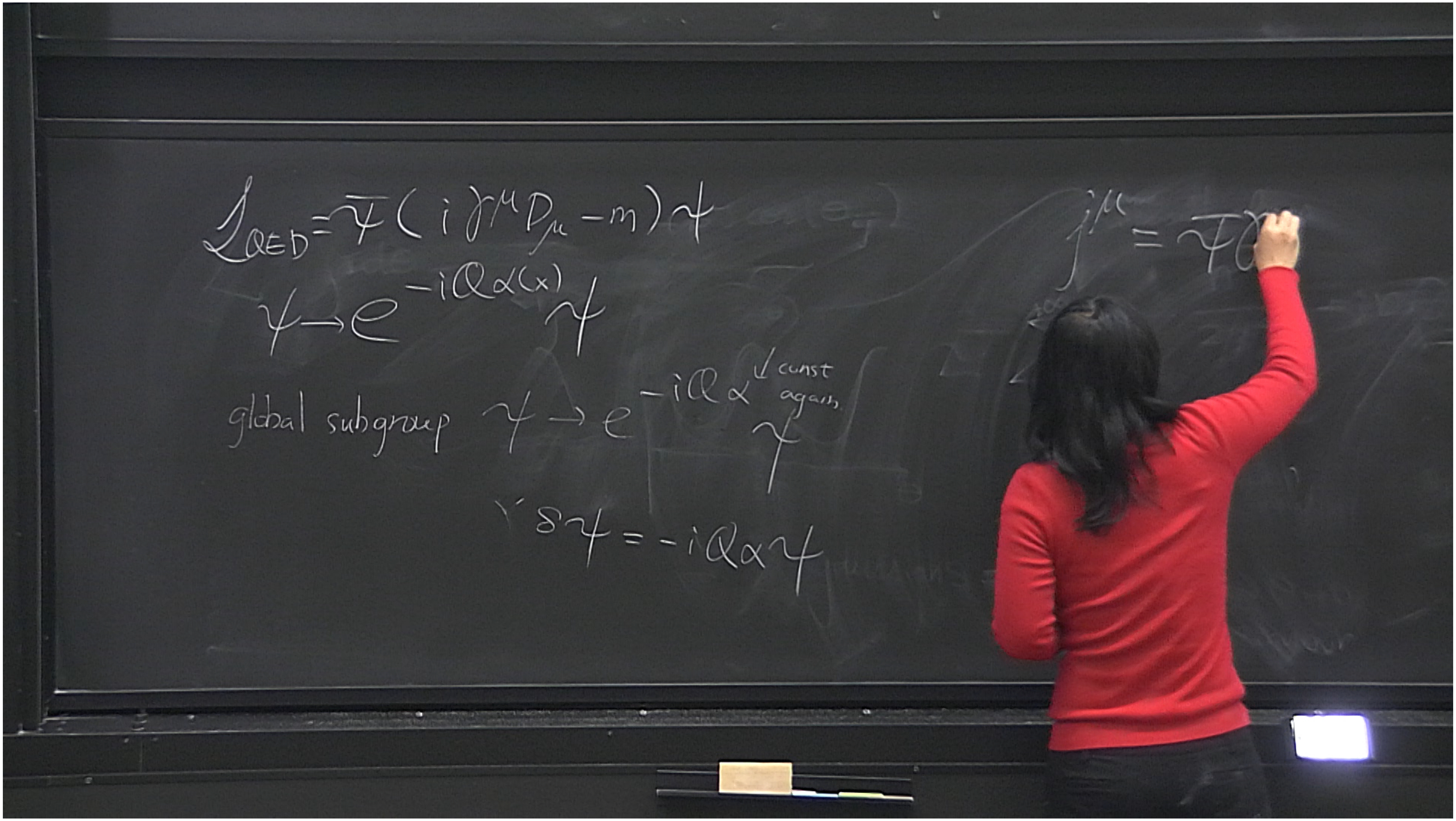
$$:H: |\vec{p}, s\rangle = :H: b_{\vec{p}}^{\dagger s} |0\rangle$$

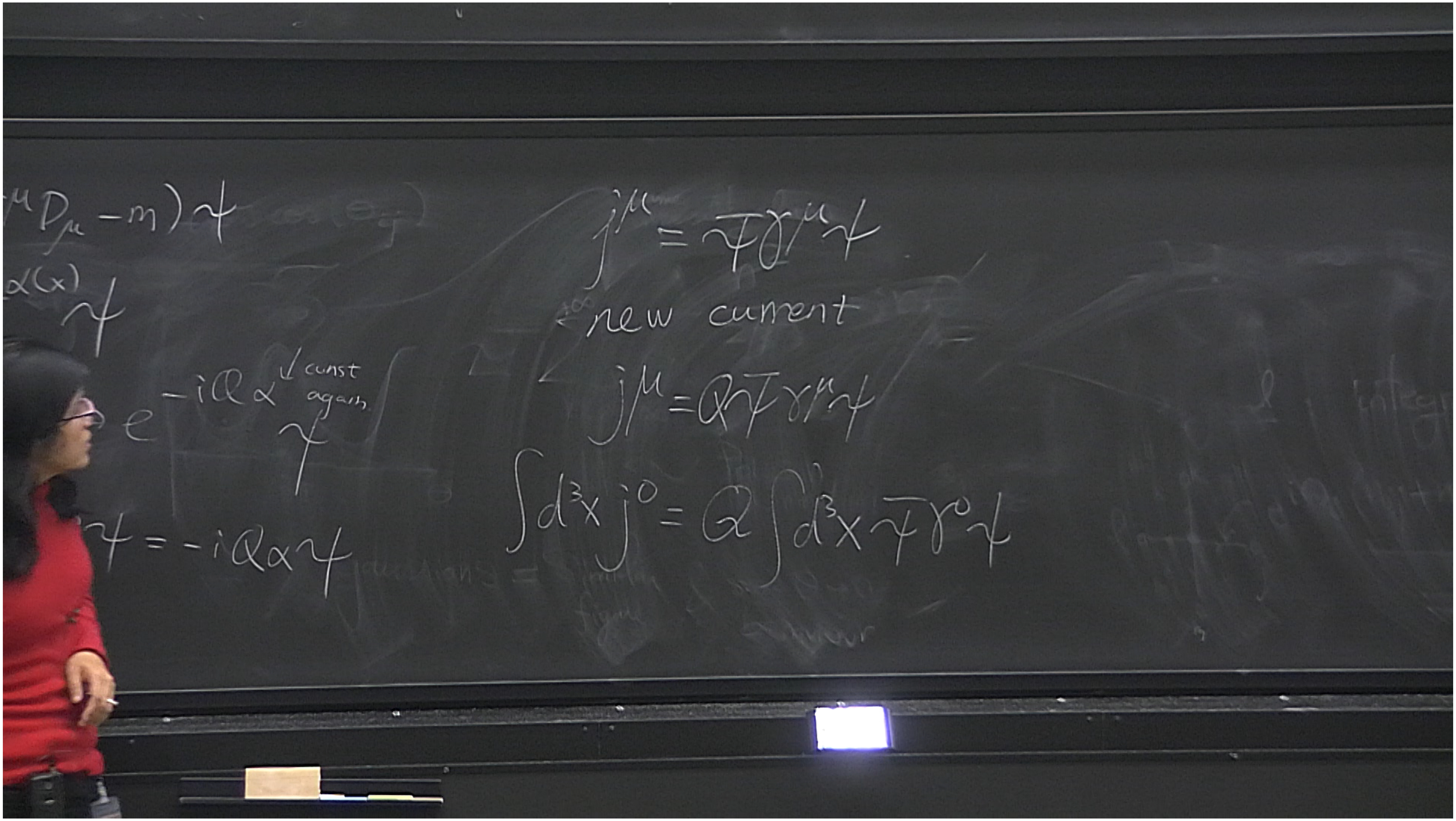
$$[:H:, b_{\vec{p}}^{\dagger s}] =$$

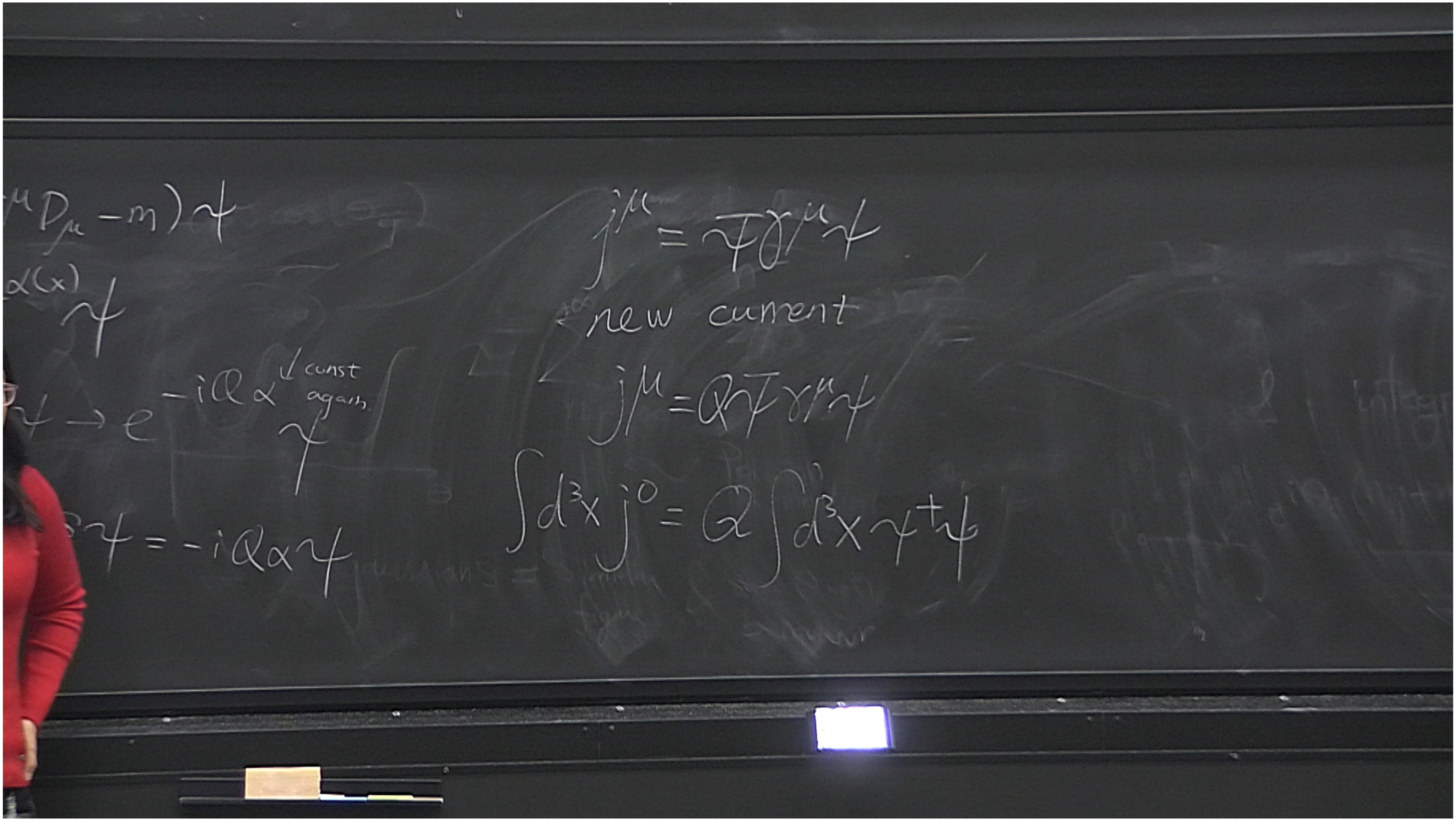

⟨p⃗, s | q⃗, r⟩ = (2π)³ 2E_p δ(p⃗ − q⃗) δ^rs
:H: |p⃗, s⟩ = :H: b†s_p⃗ |0⟩ = [:H:, b†s_p⃗] |0⟩
[:H:, b†s_p⃗] = E_p⃗ b†s_p⃗
= E_p⃗ b†s_p⃗ |0⟩
= E_p⃗ |p⃗, s⟩
|p⃗, s⟩
|p⃗, s⟩

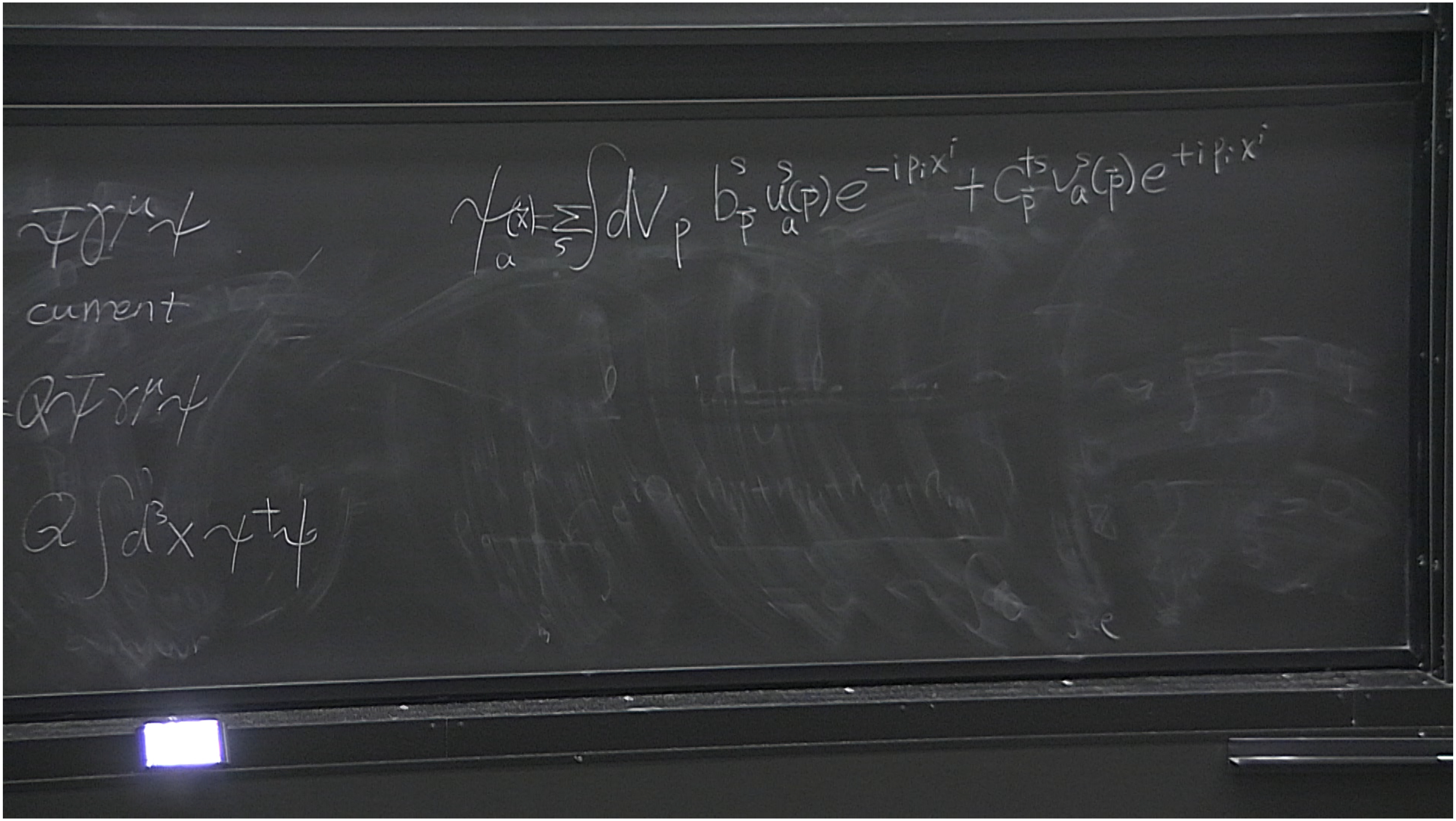


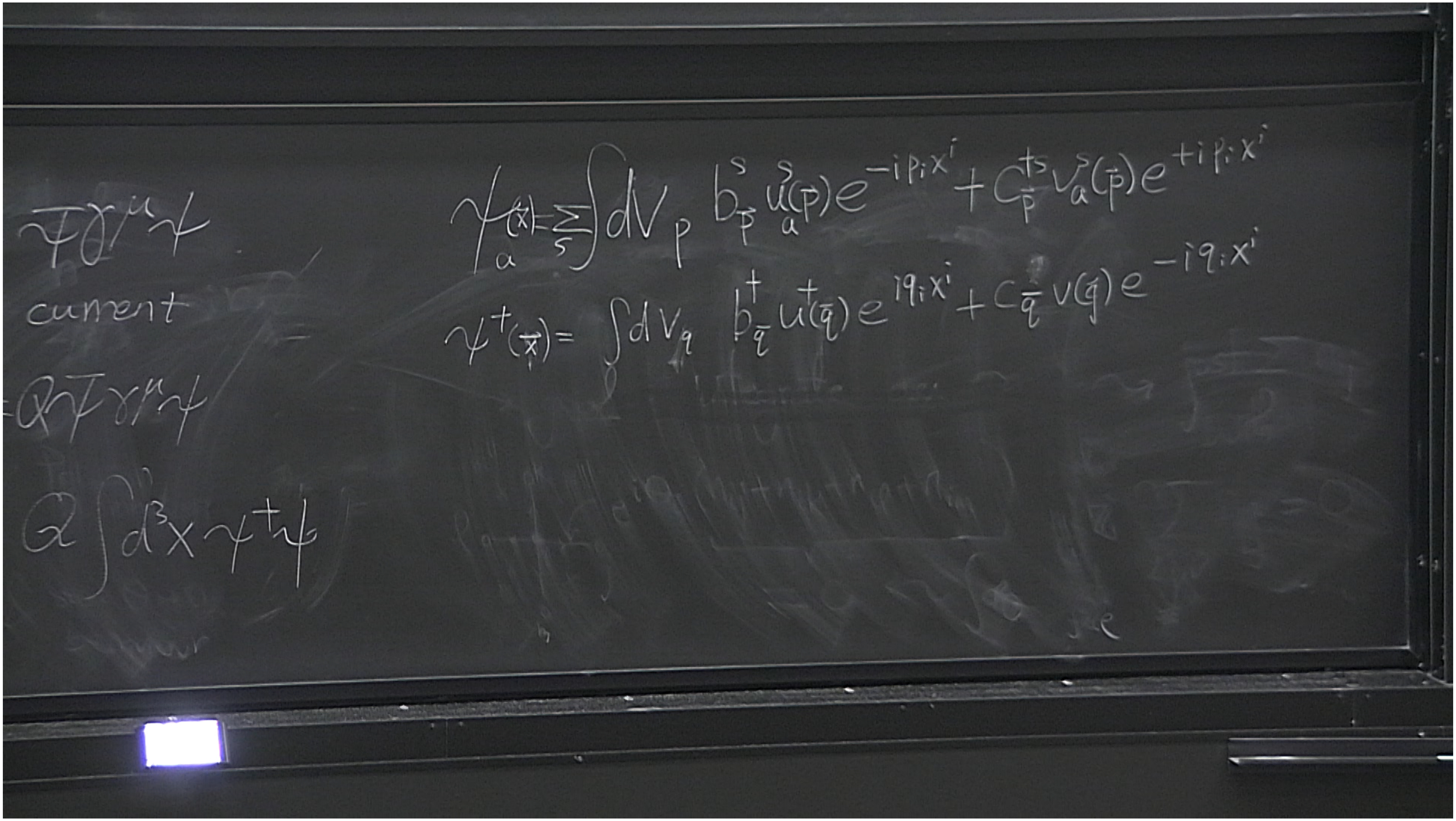
|0>, |1>
a+|0> ∝ |1>
1 particle state
(a+)^2 |0> = 0
{ψa(x), i ψb+(y)} = i δ(x−y) δab
{ψa(x), ψb(y)} = 0
{ψa+(x), ψb+(y)} = 0
{bp, bq+} =
same for
all others vani

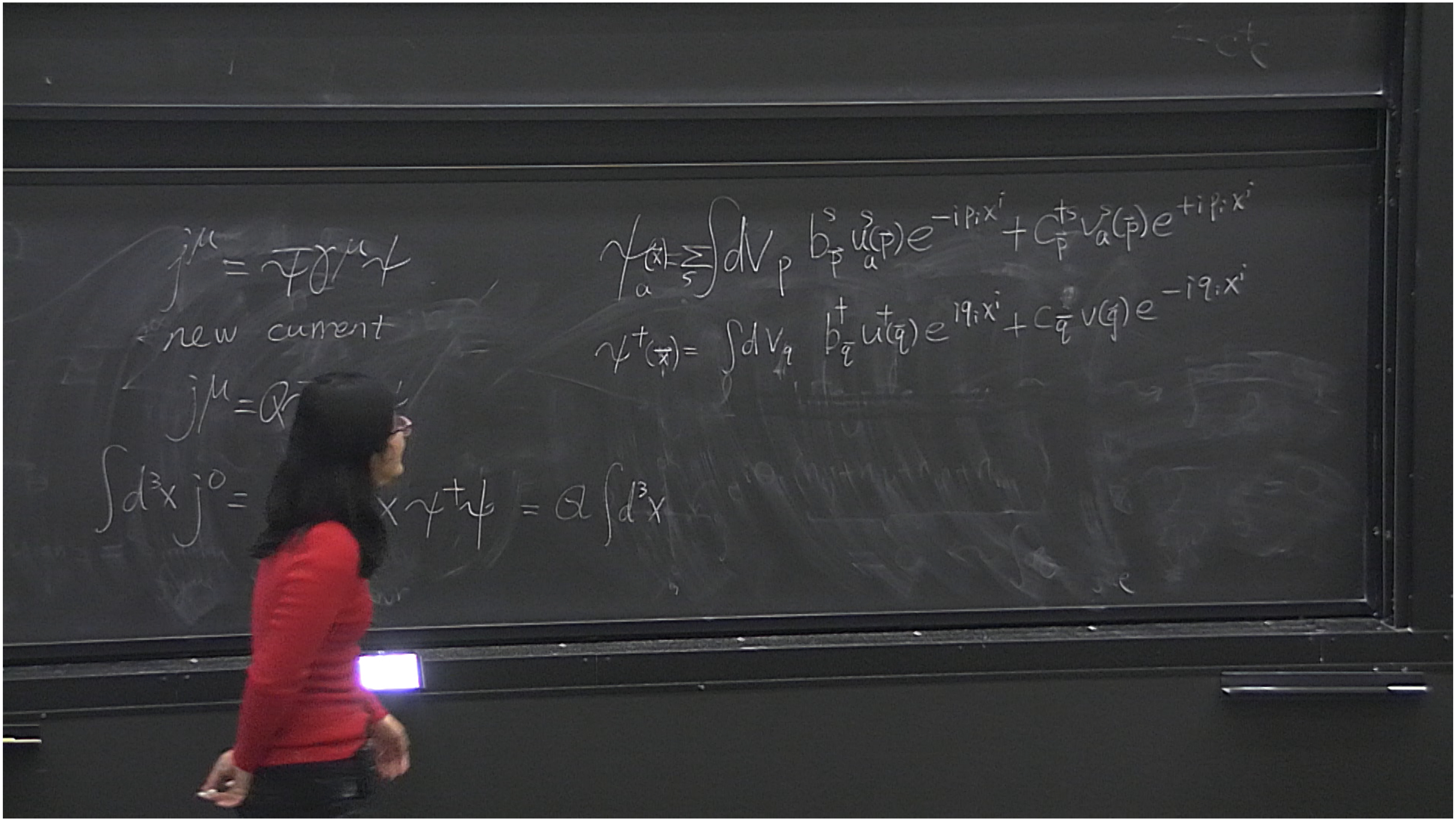

L_QED = ψ̄ (i γ^μ D_μ − m) ψ
ψ → e^{−iQα(x)} ψ
global subgroup ψ → e^{−iQα} ψ
const again
δψ = − i Q α ψ
j^μ = ψ̄ γ


j^μ = ψ̄ γ^μ ψ
new current
j^μ = Q ψ̄ γ^μ ψ
∫d³x j⁰ = Q ∫d³x ψ̄ γ⁰ ψ
const
again


new current
const
again


$\bar{\psi}\gamma^\mu\psi$
current
$=Q\bar{\psi}\gamma^\mu\psi$
$Q\int d^3x\, \psi^\dagger\psi$
$\psi_a(x) = \sum_s \int dV_p\, b^s_{\vec{p}}\, u^s_a(\vec{p})\, e^{-ip_i x^i} + c^{\dagger s}_{\vec{p}}\, v^s_a(\vec{p})\, e^{+ip_i x^i}$


$\bar{\psi}\gamma^\mu\psi$
current
$= Q\bar{\psi}\gamma^\mu\psi$
$Q\int d^3x\, \psi^\dagger\psi$
$\psi_a(\vec{x}) = \sum_s \int dV_p\, b^s_{\vec{p}}\, u^s_a(\vec{p})\, e^{-ip_i x^i} + c^{\dagger s}_{\vec{p}}\, v^s_a(\vec{p})\, e^{+ip_i x^i}$
$\psi^\dagger(\vec{x}) = \int dV_q\, b^\dagger_{\vec{q}}\, u^\dagger(\vec{q})\, e^{iq_i x^i} + c_{\vec{q}}\, v(\vec{q})\, e^{-iq_i x^i}$


jμ = ψ̄ γμ ψ
new current
jμ = Q
∫d³x j⁰ =
x ψ† ψ = Q ∫d³x
ψa(x) = Σs ∫dVp bsp usa(p) e−ipixi + c†sp vsa(p) e+ipixi
ψ†(x) = ∫dVq b†q u†(q) eiqixi + cq v(q) e−iqixi

$$j^\mu = \bar{\psi}\gamma^\mu\psi$$

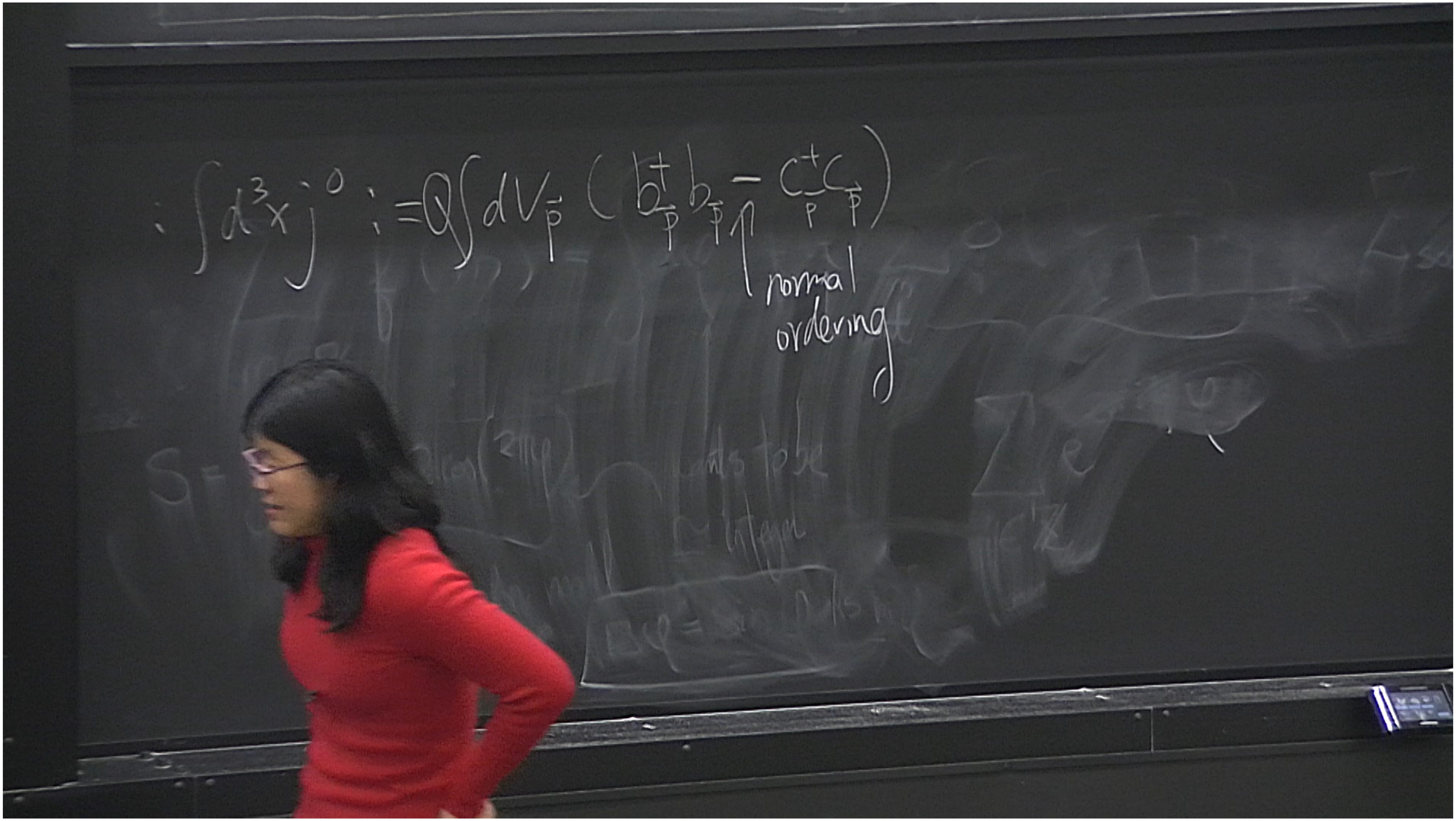
new current

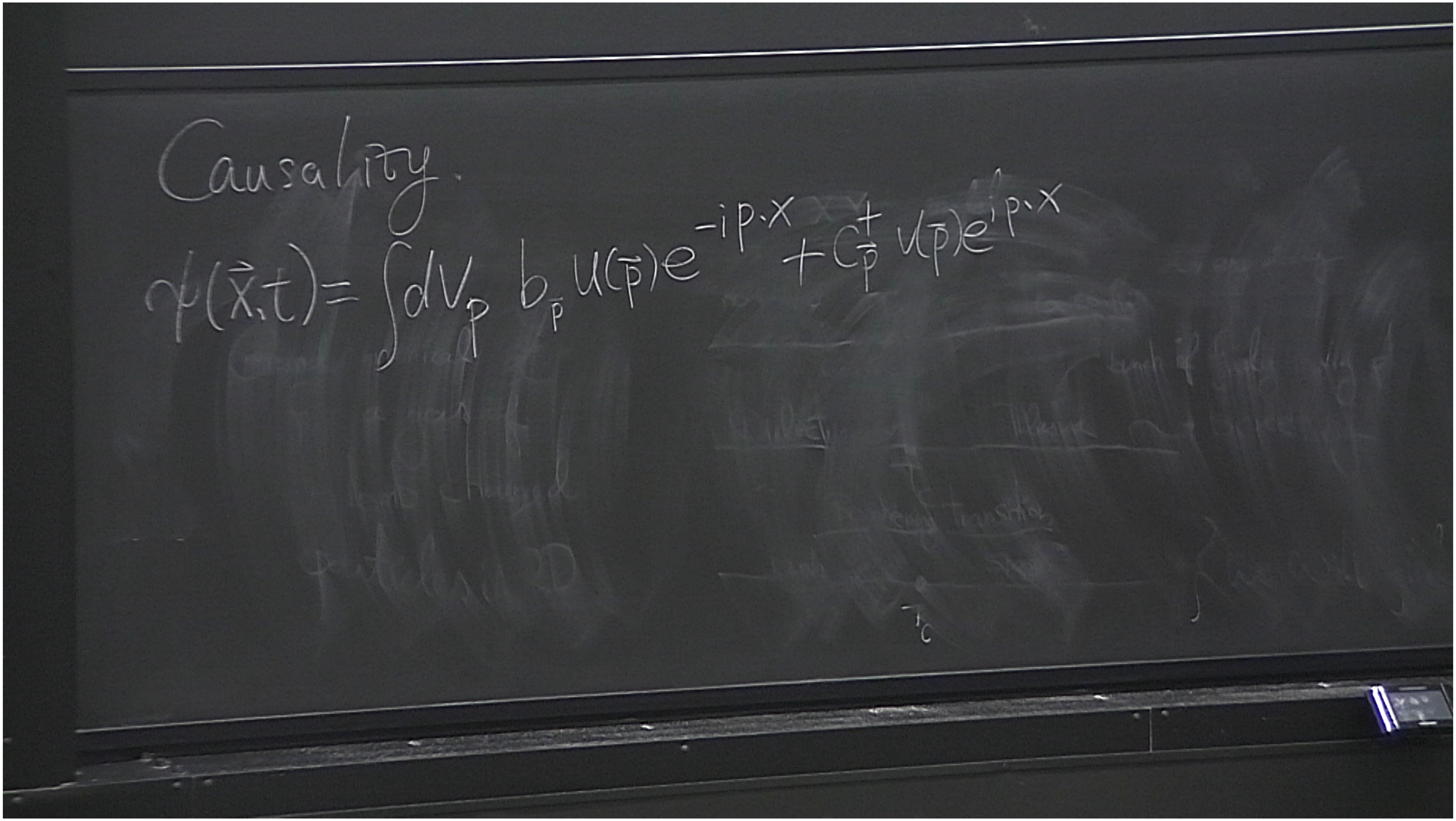
$$j^\mu = Q\bar{\psi}\gamma^\mu\psi$$

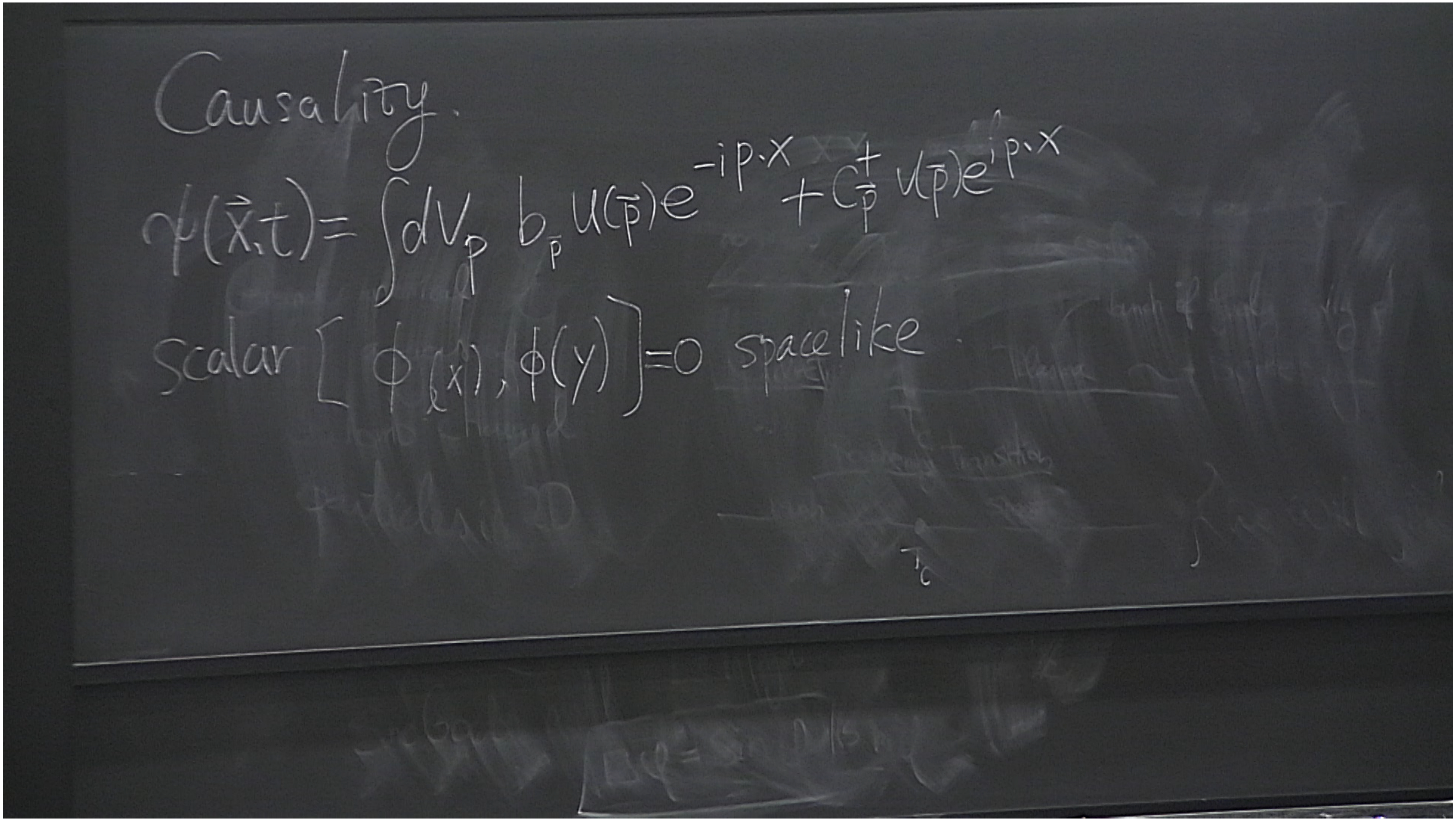
$$\psi_a(\vec{x}) = \sum_s \int dV_p \; b^s_{\vec{p}}\, u^s_a(\vec{p})\, e^{-ip_i x^i} + c^{\dagger s}_{\vec{p}}\, v^s_a(\vec{p})\, e^{+ip_i x^i}$$

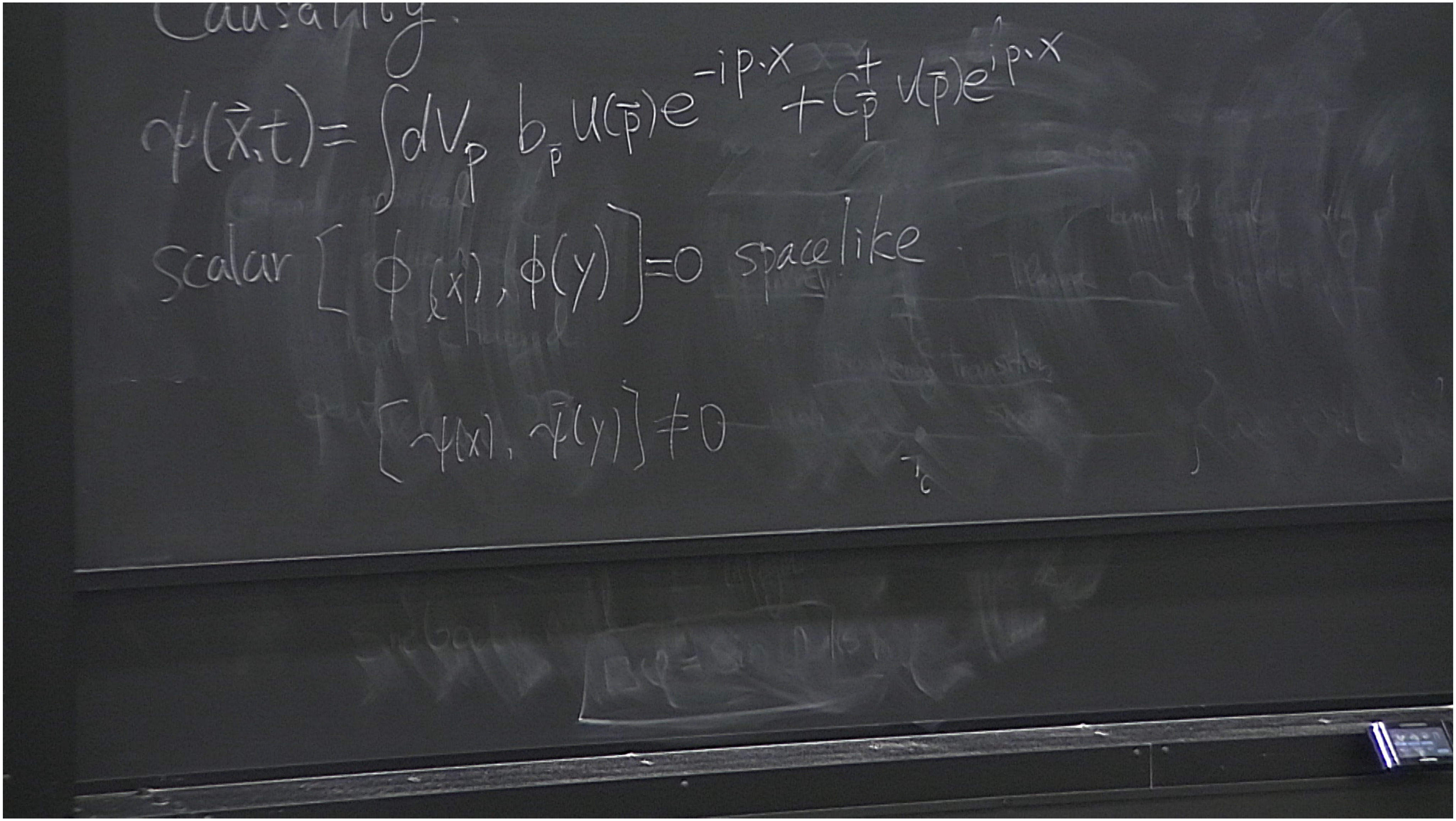
$$\psi^\dagger(\vec{x}) = \int dV_q \; b^\dagger_{\vec{q}}\, u^\dagger(\vec{q})\, e^{iq_i x^i} + c_{\vec{q}}\, v^\dagger(\vec{q})\, e^{-iq_i x^i}$$

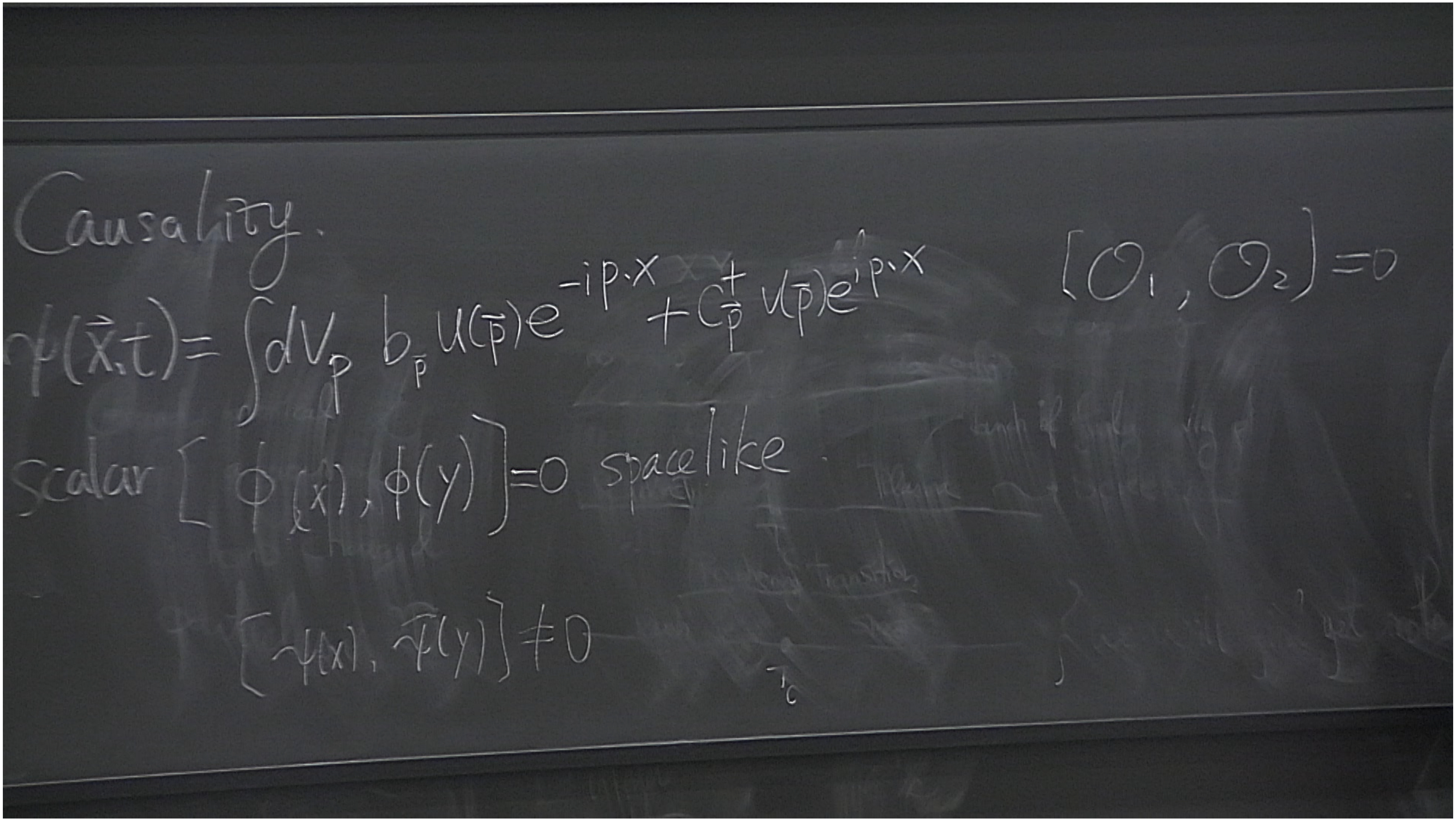
$$\int d^3x\, j^0 = Q\int d^3x\, \psi^\dagger\psi = Q\int d^3x\, dV_p\, dV_q\; b^\dagger_{\vec{q}} b_{\vec{p}}\, u^\dagger(\vec{q})\, u(\vec{p})\, e^{-ix^i(p-q)_i} + c_{\vec{q}} c^\dagger_{\vec{p}}\, v^\dagger(\vec{q})\, v(\vec{p})\, e^{ix^i(p-q)_i} = Q\int dV_{\vec{p}} \left(b^\dagger_{\vec{p}} b_{\vec{p}} + c_{\vec{q}} c^\dagger_{\vec{q}} \right)$$

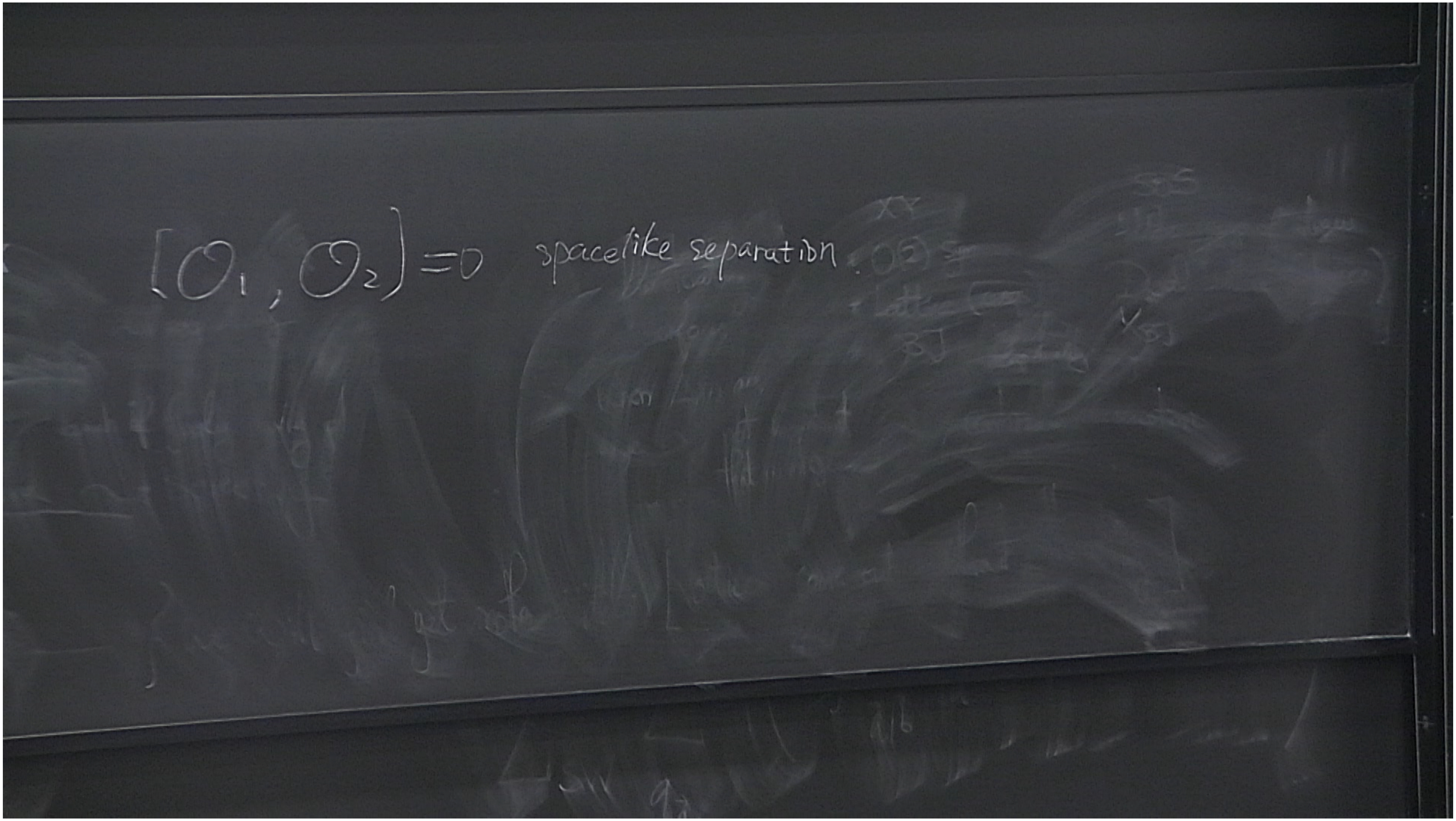

:∫d³x j⁰: = Q∫dV_p (b†_p b_p − c†_p c_p)
normal ordering

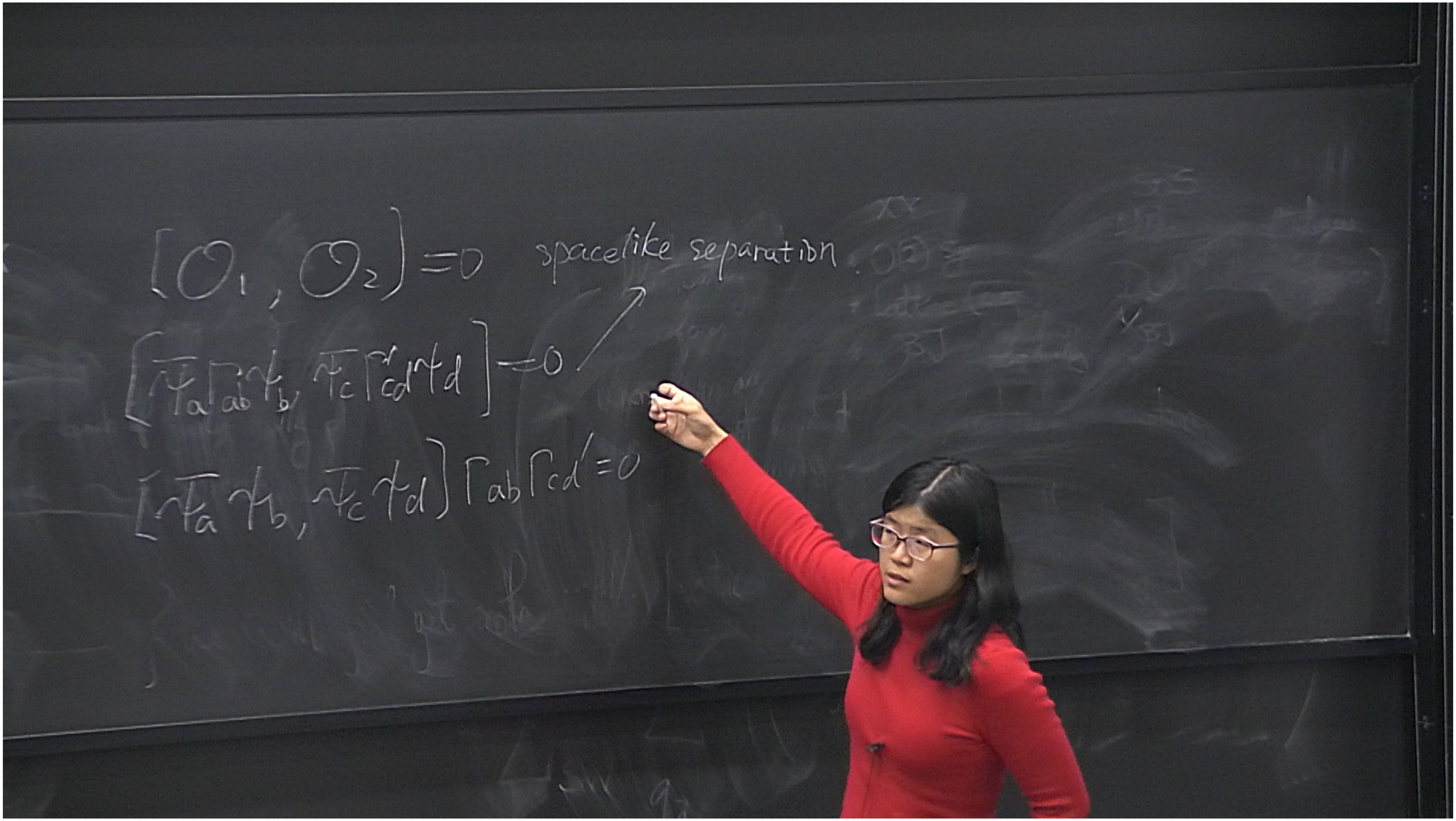

Causality.
$\psi(\vec{x},t) = \int dV_p \, b_{\vec{p}} \, u(\vec{p}) e^{-ip\cdot x} + c^\dagger_{\vec{p}} \, u(\bar{p}) e^{ip\cdot x}$

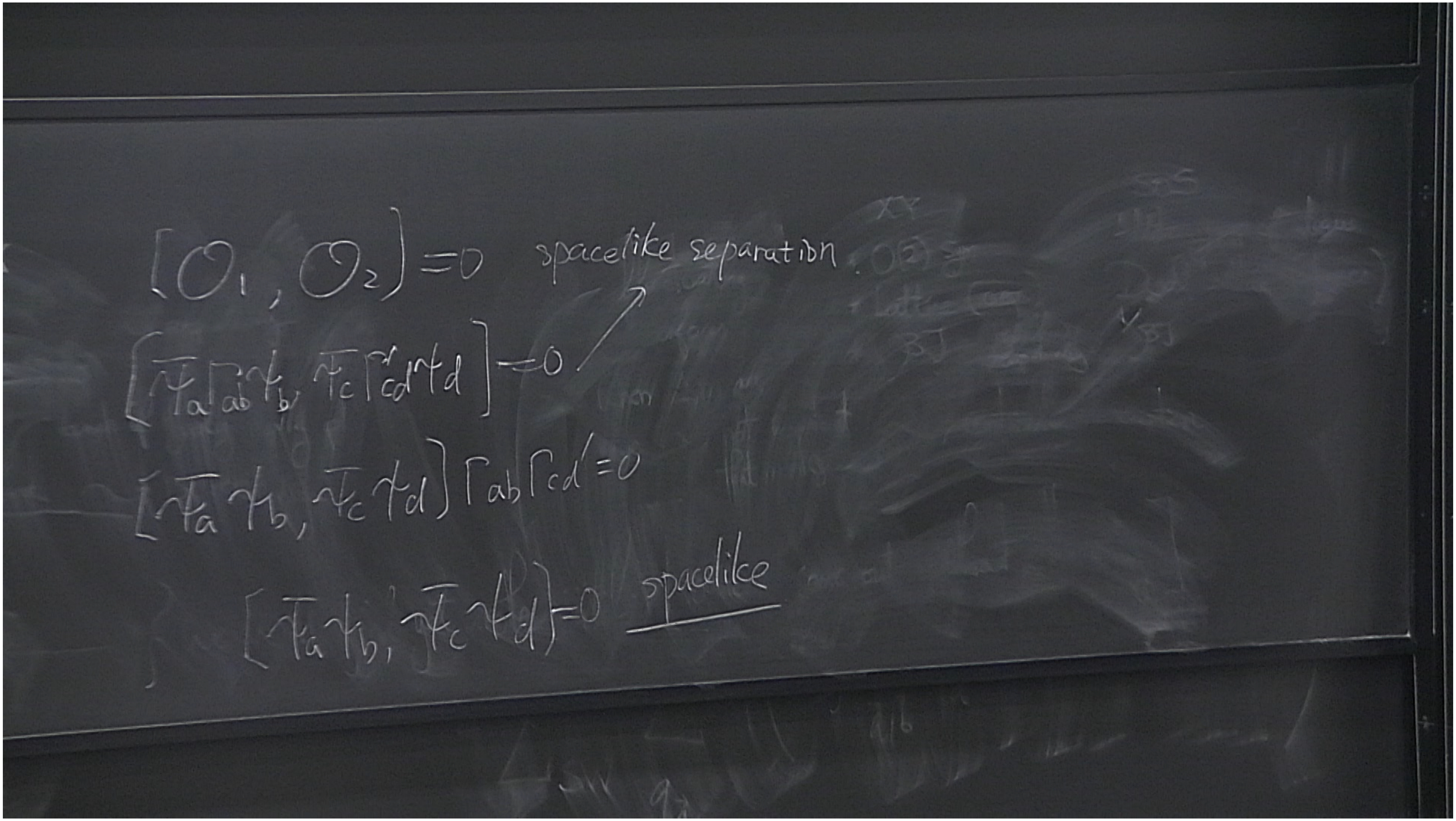

Causality.
$\psi(\vec{x},t) = \int dV_p \, b_{\bar{p}} \, u(\bar{p}) e^{-ip\cdot x} + c^{\dagger}_{\bar{p}} \, u(\bar{p}) e^{ip\cdot x}$
scalar $[\phi_\ell(x), \phi(y)] = 0$ spacelike.

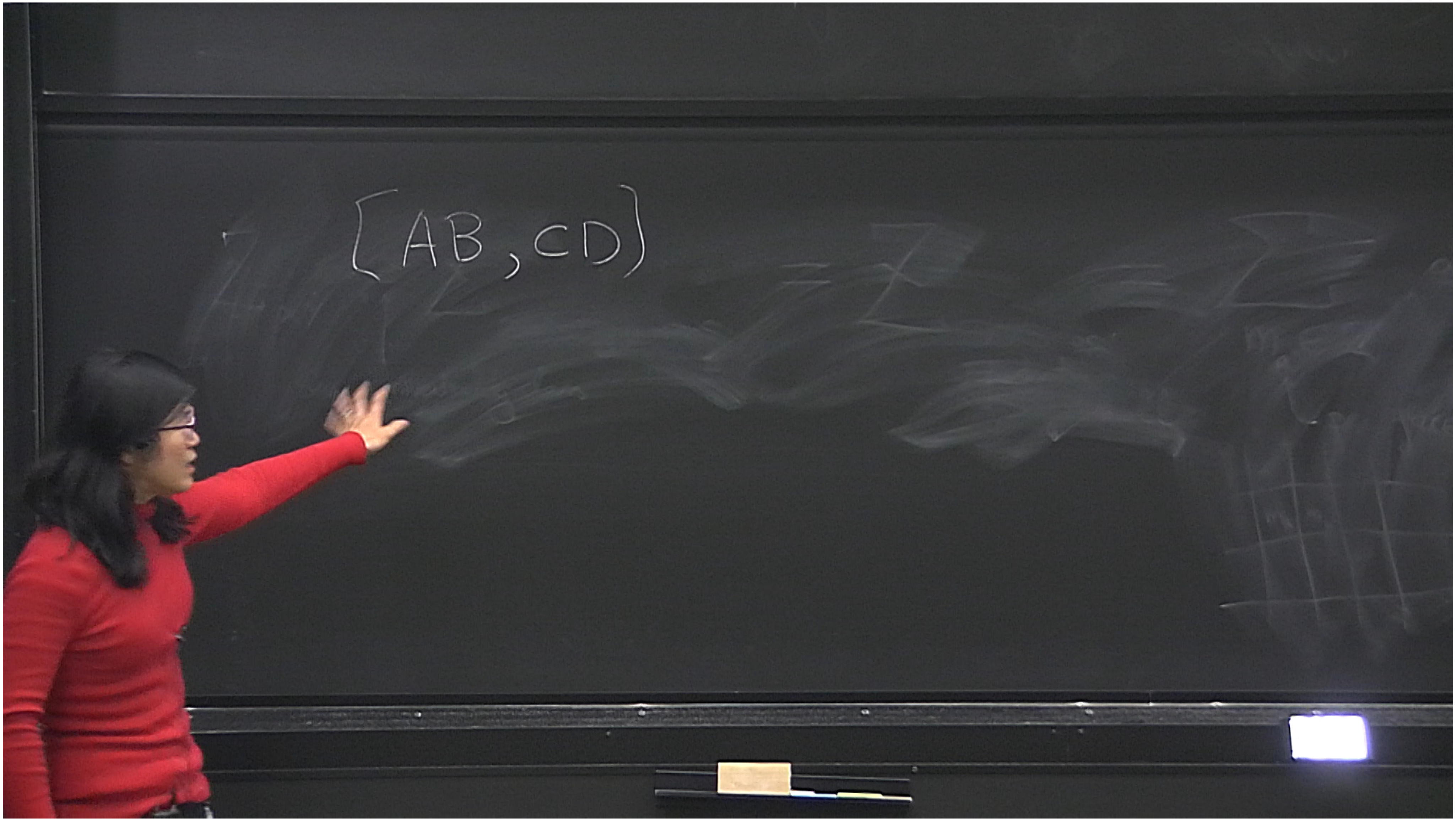

Causality:
$\psi(\vec{x},t) = \int dV_p \, b_{\bar{p}} u(\bar{p}) e^{-ip\cdot x} + c^{\dagger}_{\bar{p}} u(\bar{p}) e^{ip\cdot x}$
scalar $[\phi(x), \phi(y)] = 0$ spacelike
$[\psi(x), \bar{\psi}(y)] \neq 0$

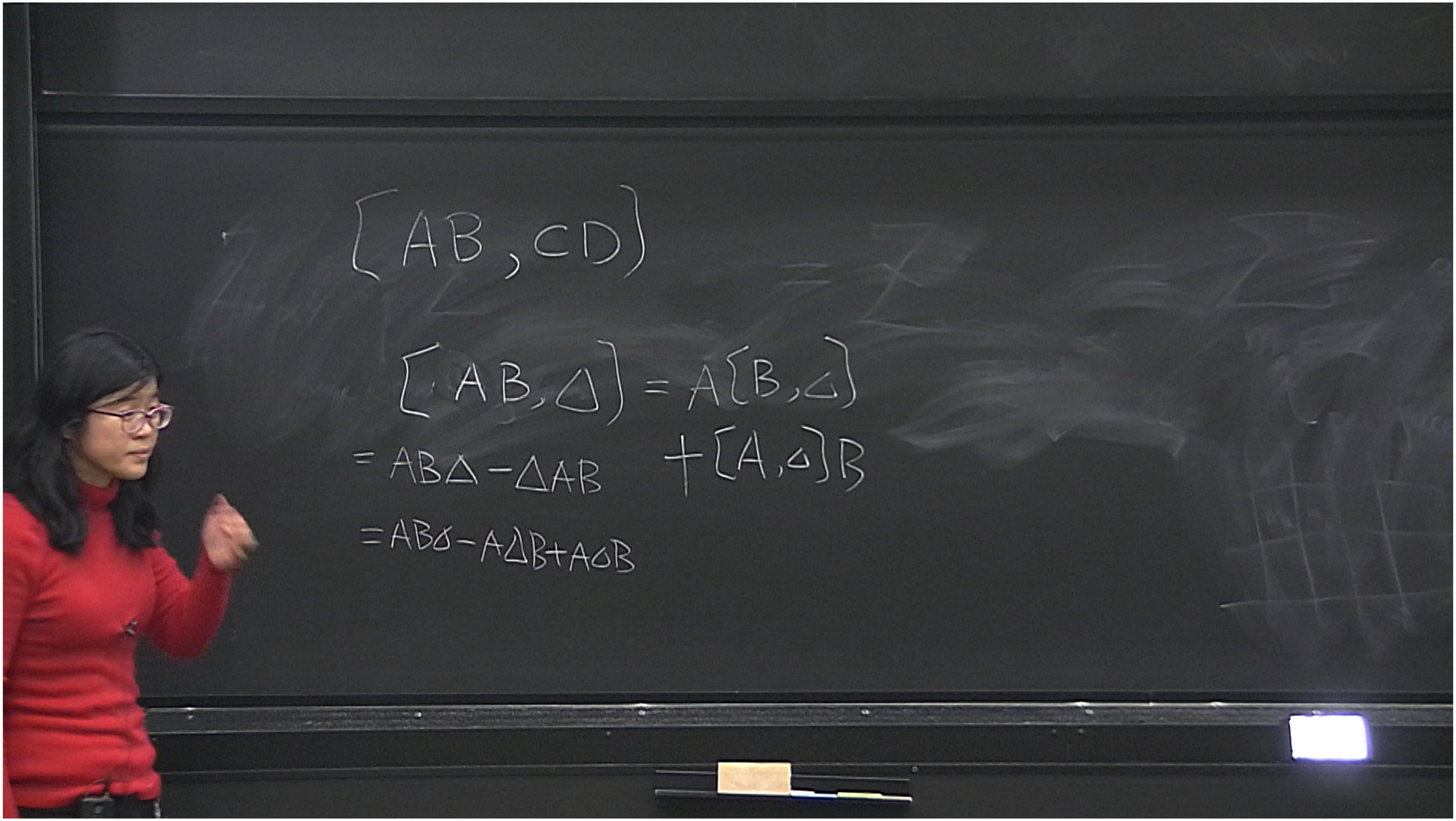

Causality.
ψ(x⃗,t) = ∫dV_p b_p u(p⃗) e^{-ip·x} + c_p† v(p⃗) e^{ip·x}
[O₁, O₂] = 0
scalar [φ(x), φ(y)] = 0 spacelike
[ψ(x), ψ̄(y)] ≠ 0

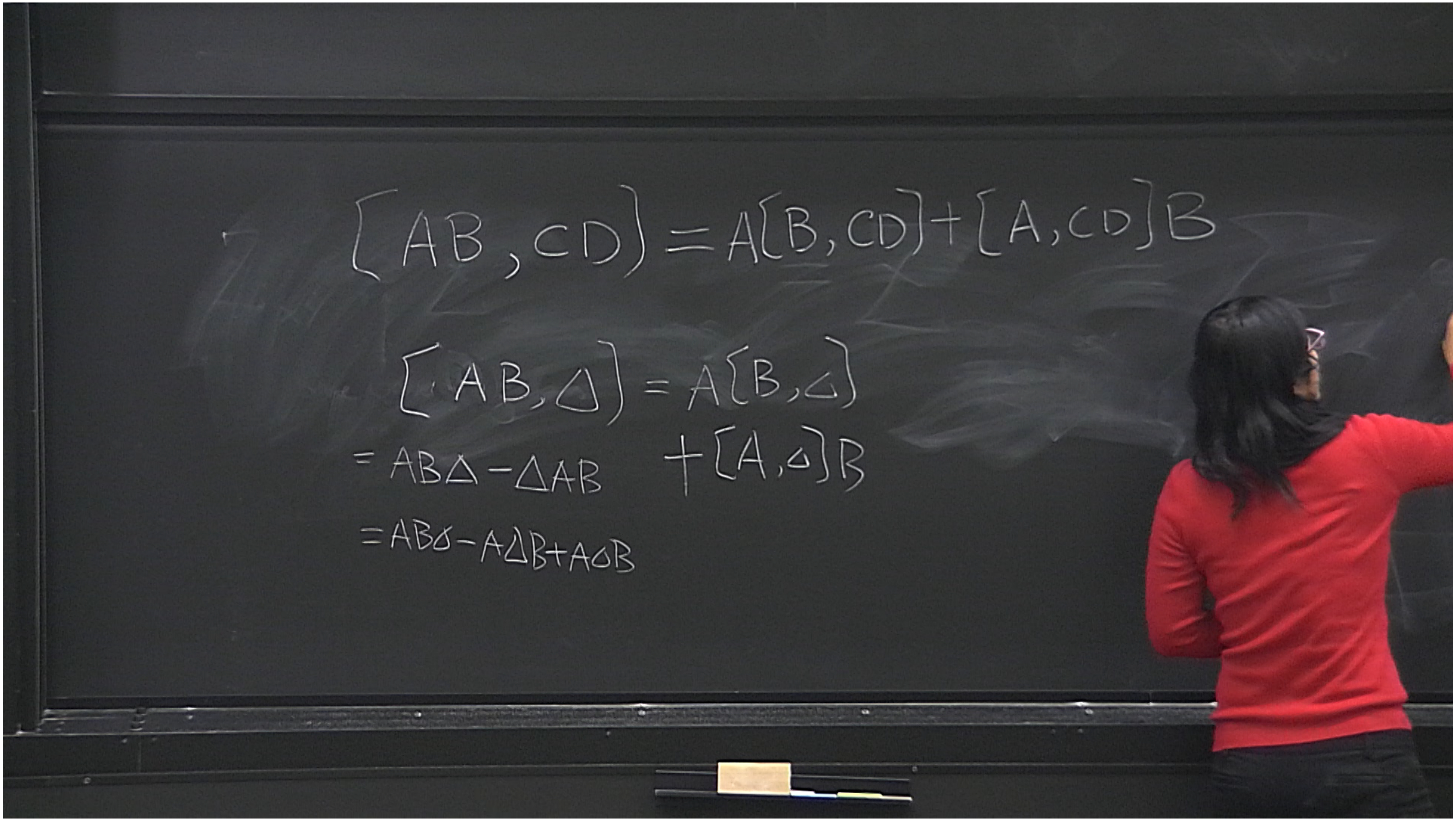

$[\mathcal{O}_1, \mathcal{O}_2) = 0$ spacelike separation

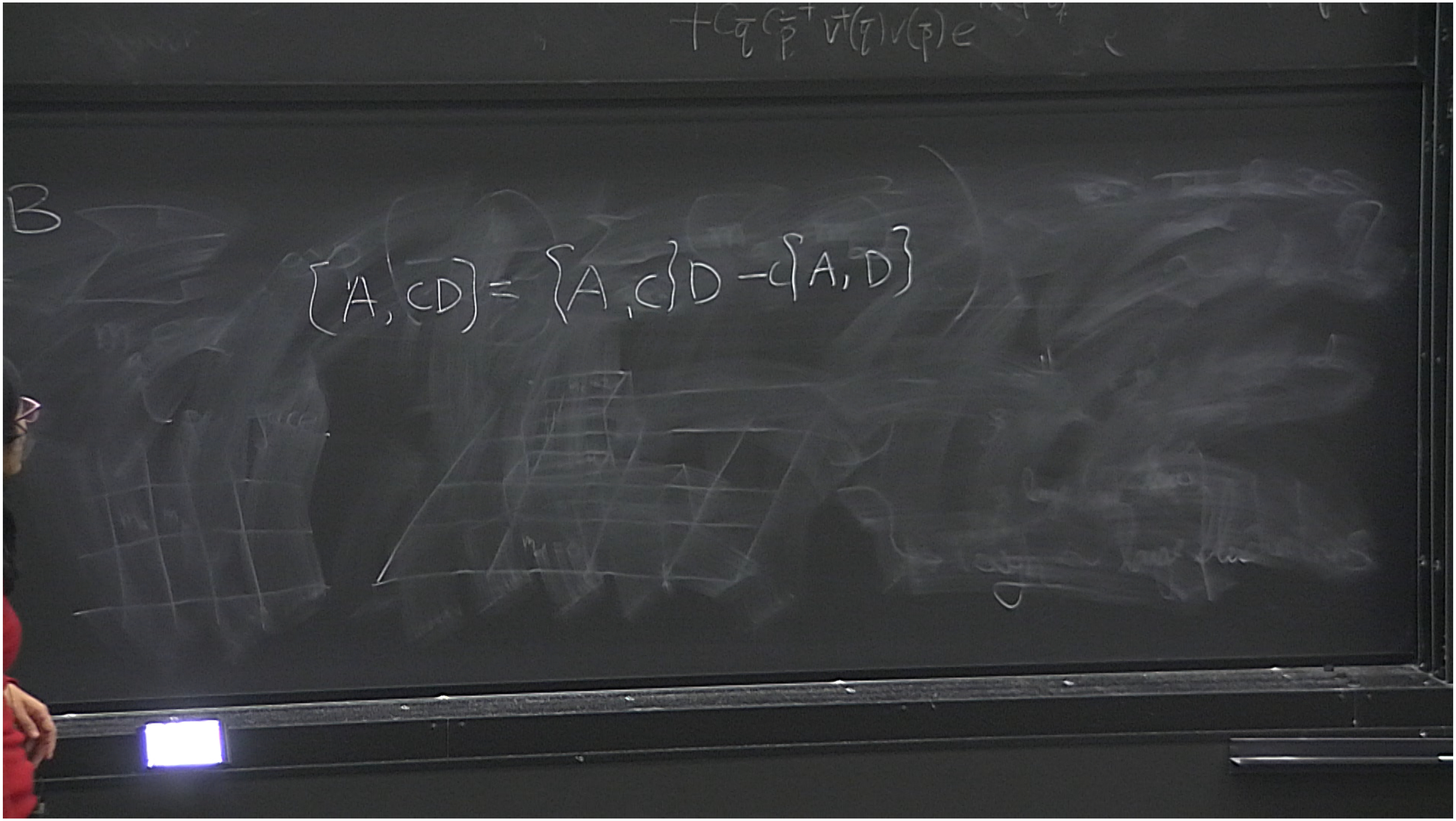

$[\mathcal{O}_1, \mathcal{O}_2] = 0$ spacelike separation
$[\tilde{\psi}_a \Gamma_{ab} \psi_b, \tilde{\psi}_c \Gamma_{cd} \psi_d] = 0$
$[\tilde{\psi}_a \psi_b, \tilde{\psi}_c \psi_d] \Gamma_{ab} \Gamma_{cd} = 0$

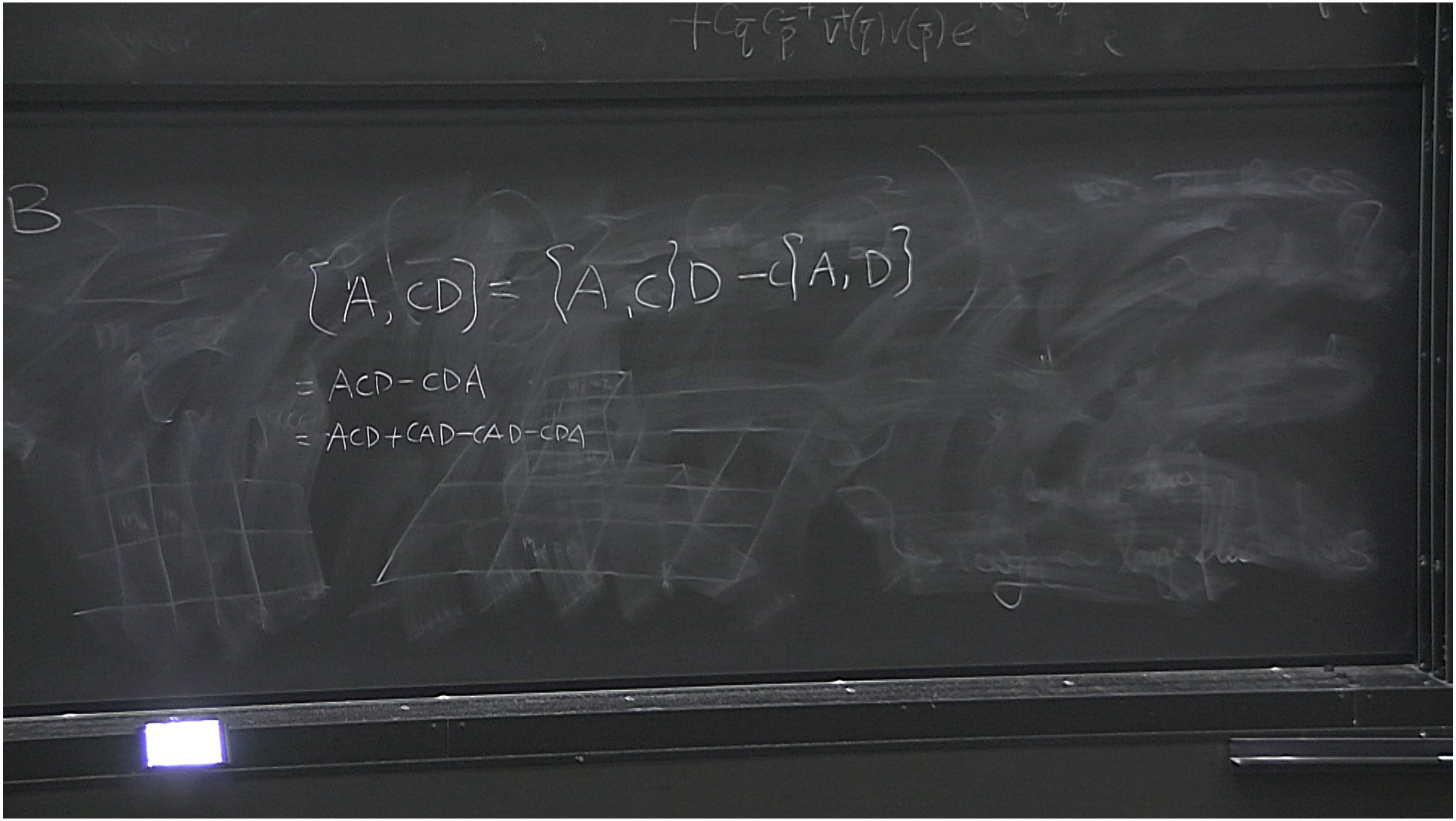

$[\mathcal{O}_1, \mathcal{O}_2) = 0$ spacelike separation
$[\bar{\Psi}_a \Gamma_{ab} \Psi_b, \bar{\Psi}_c \Gamma_{cd} \Psi_d] = 0$
$[\bar{\Psi}_a \Psi_b, \bar{\Psi}_c \Psi_d] \Gamma_{ab} \Gamma_{cd} = 0$
$[\bar{\Psi}_a \Psi_b, \bar{\Psi}_c \Psi_d] = 0$ spacelike

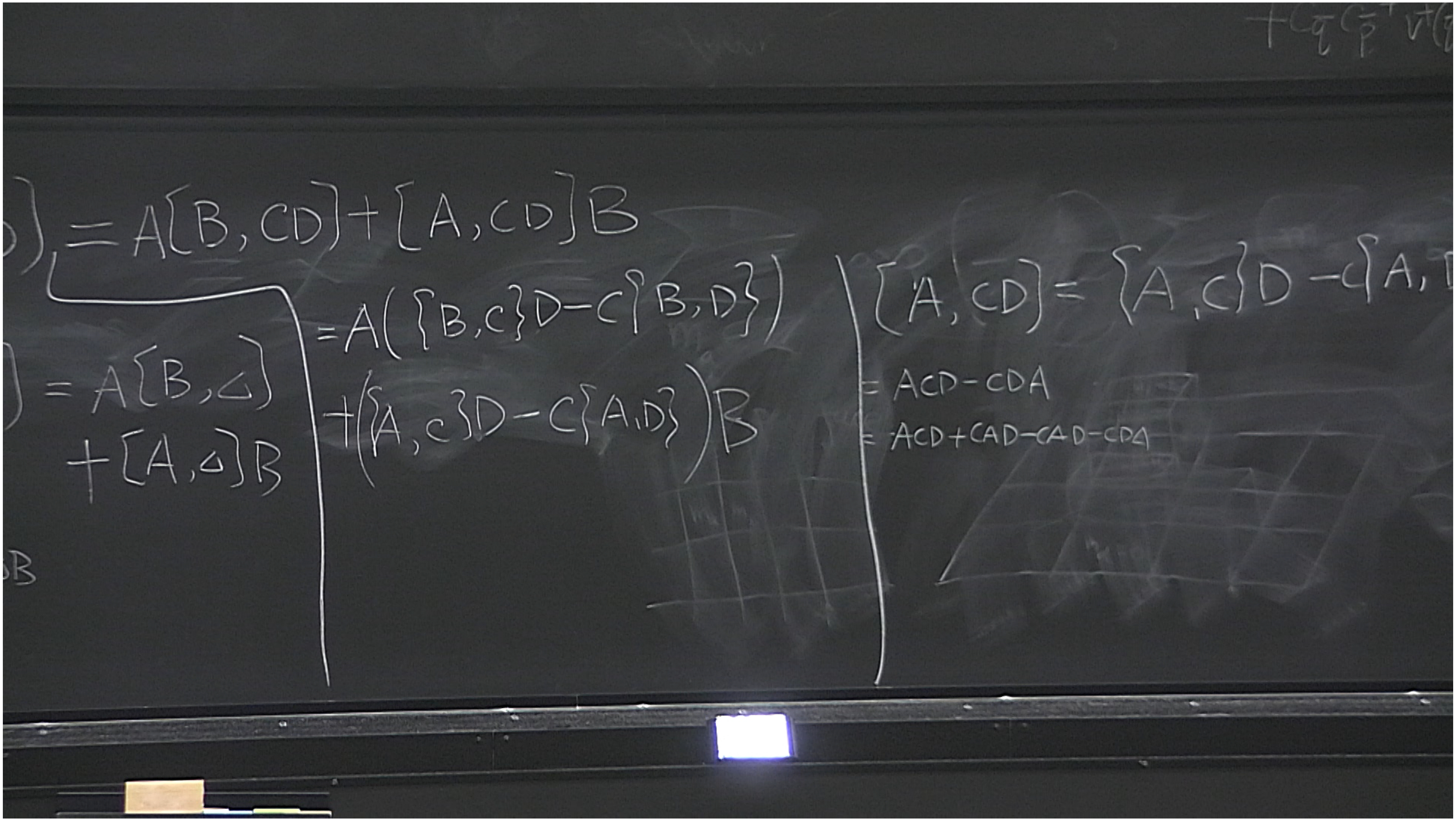

[AB, CD]

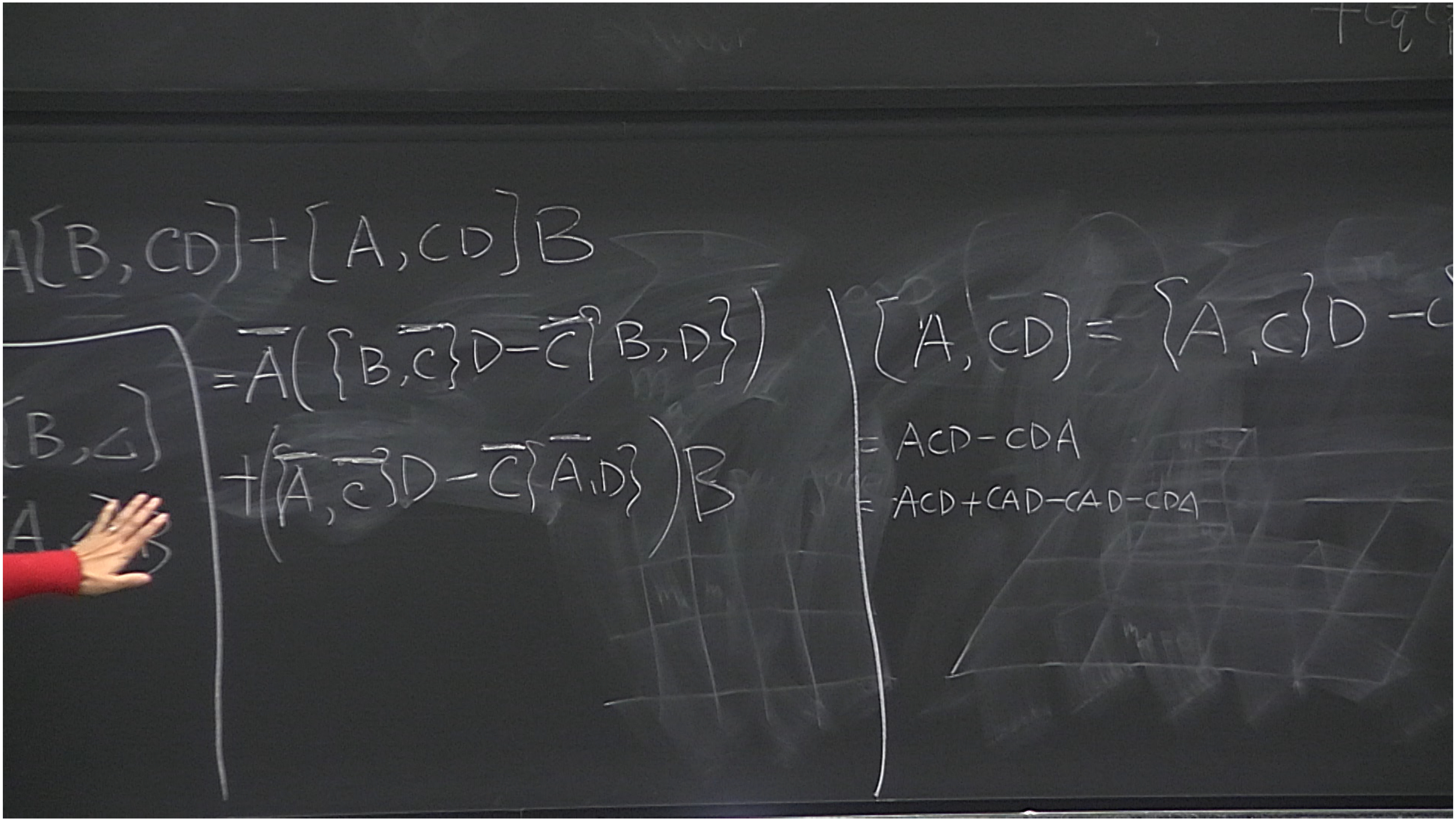

[AB, CD]
[AB, Δ] = A[B, Δ]
= ABΔ − ΔAB + [A, Δ]B
= ABΔ − AΔB + AΔB

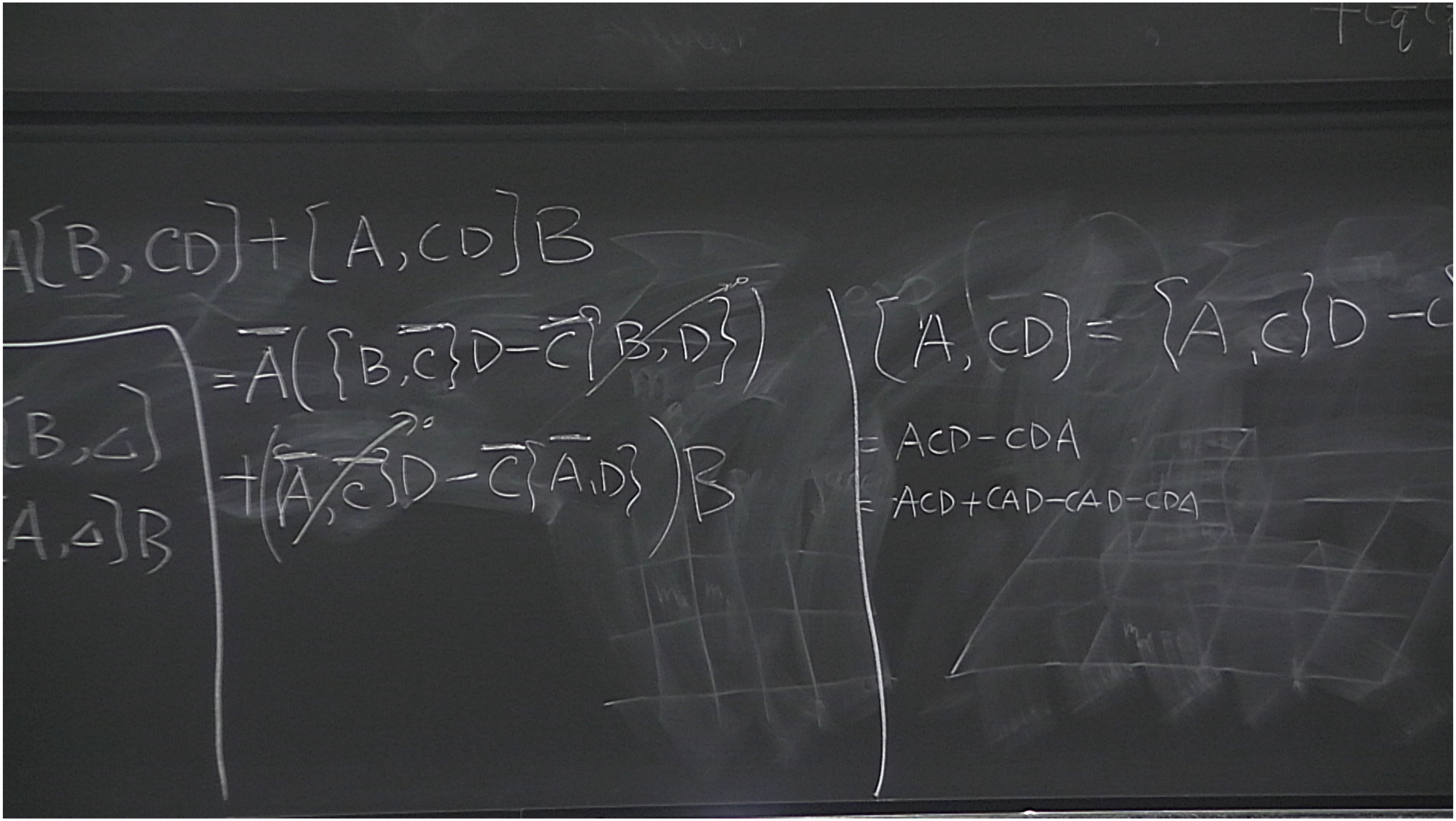

[AB, CD] = A[B, CD] + [A, CD]B
[AB, Δ] = A[B, Δ] + [A, Δ]B
= ABΔ − ΔAB
= ABΔ − AΔB + AΔB

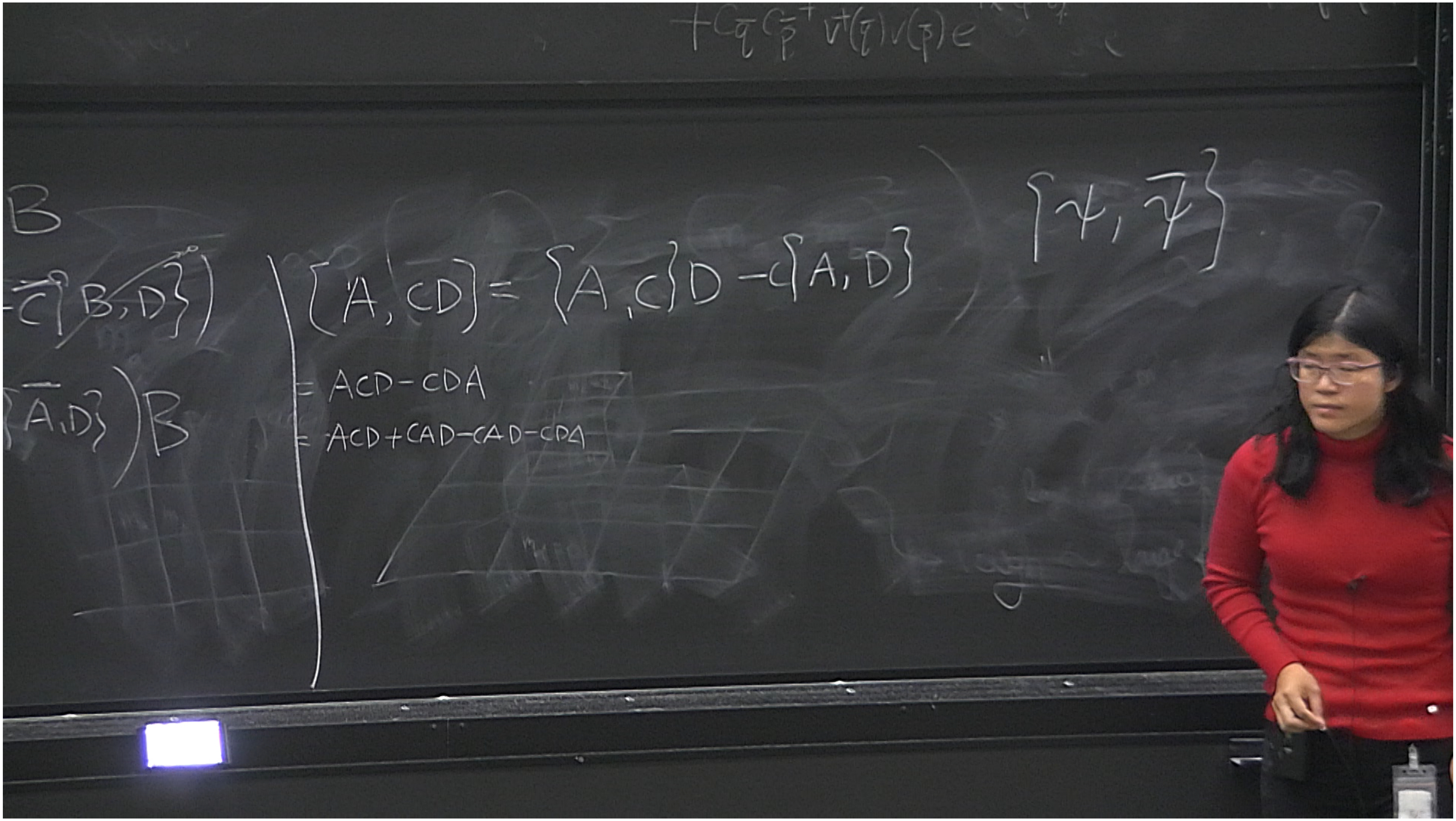

{A, CD} = {A, C}D − C{A, D}

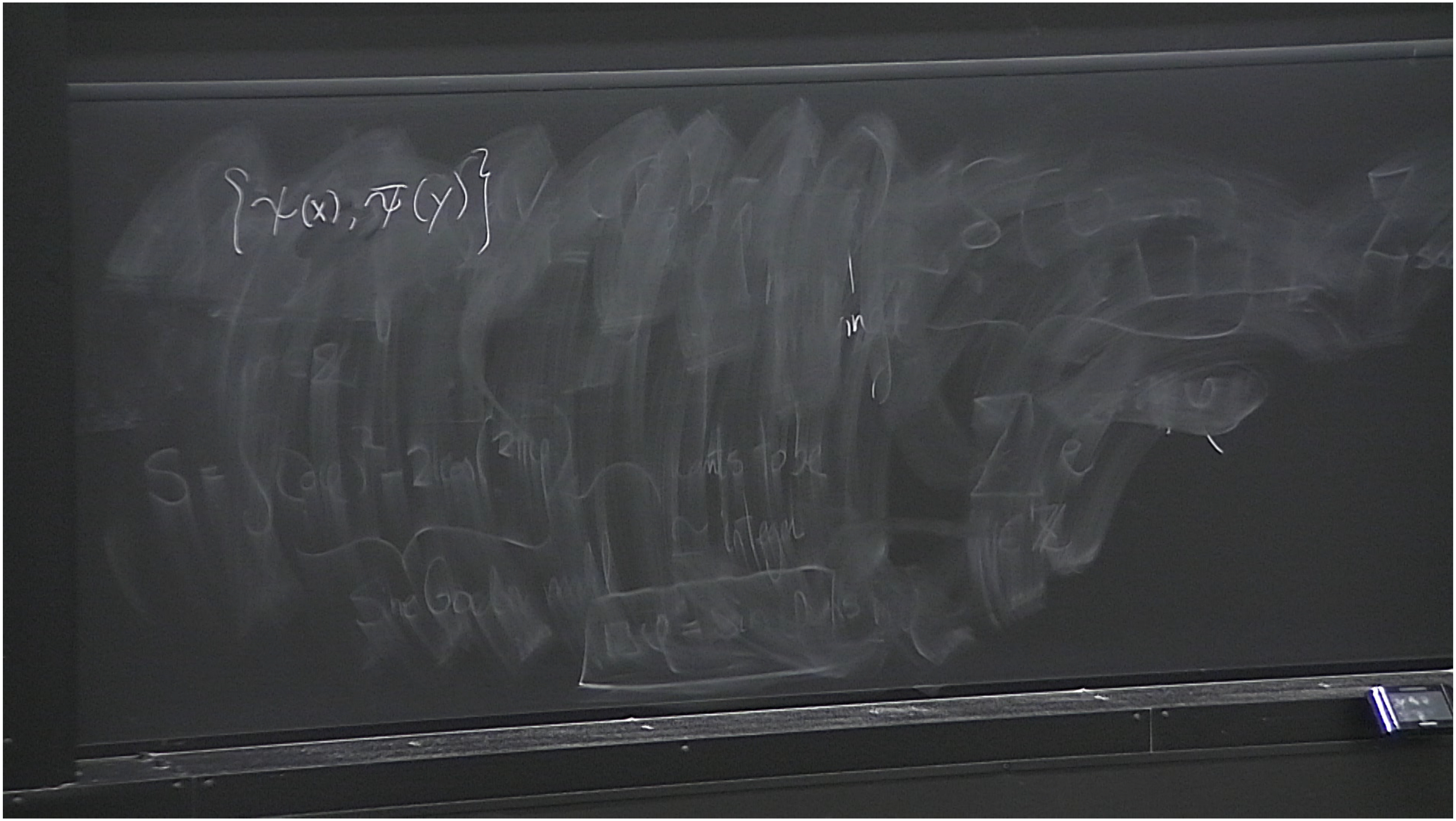

[A, CD] = {A, C}D − C{A, D}
= ACD − CDA
= ACD + CAD − CAD − CDA

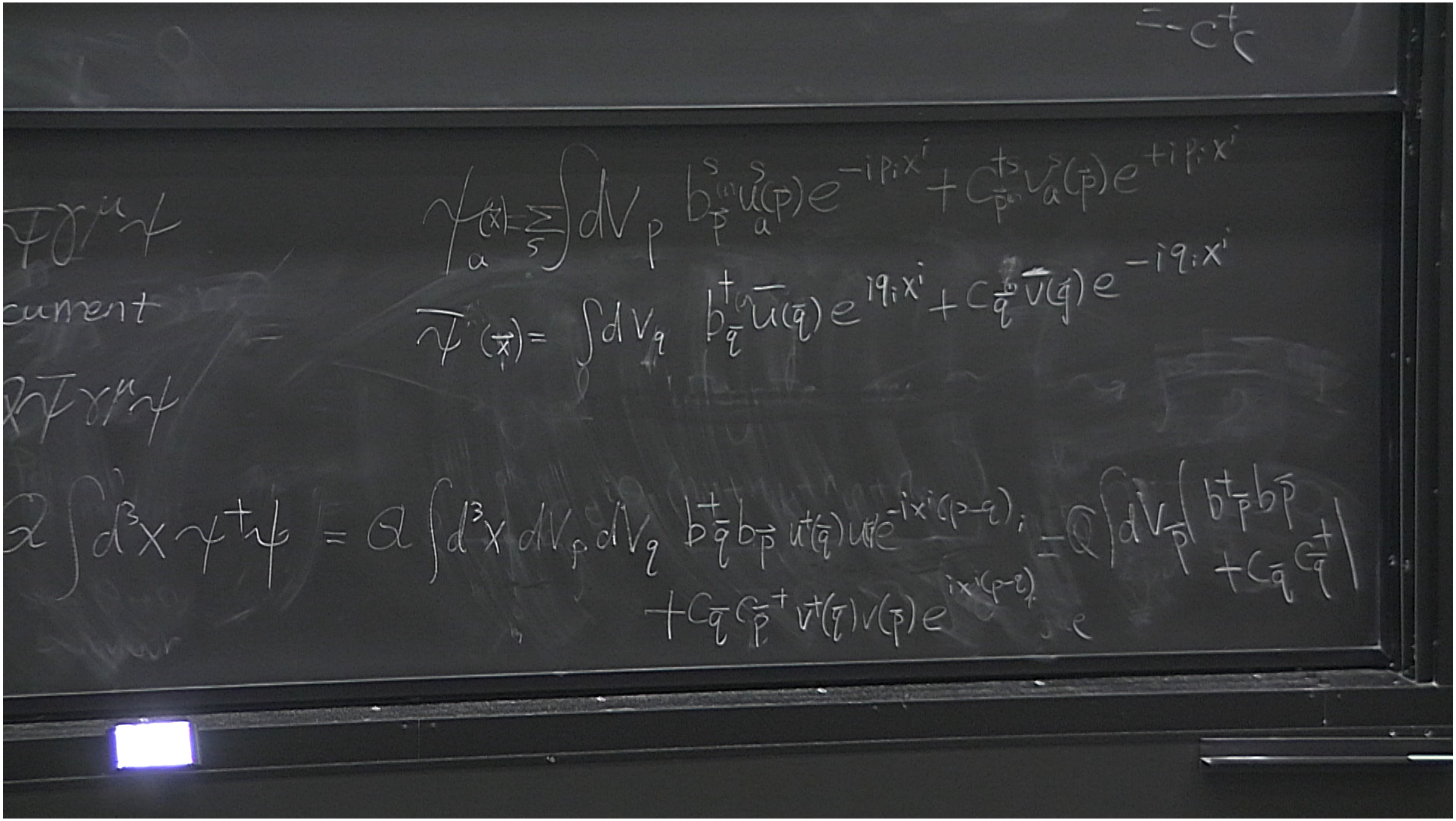

$= A[B, CD] + [A, CD]B$
$= A(\{B,C\}D - C\{B,D\})$
$+ (\{A,C\}D - C\{A,D\})B$
$= A\{B,\Delta\}$
$+ [A,\Delta]B$
$[A, CD] = \{A,C\}D - C\{A,$
$= ACD - CDA$
$= ACD + CAD - CAD - CDA$

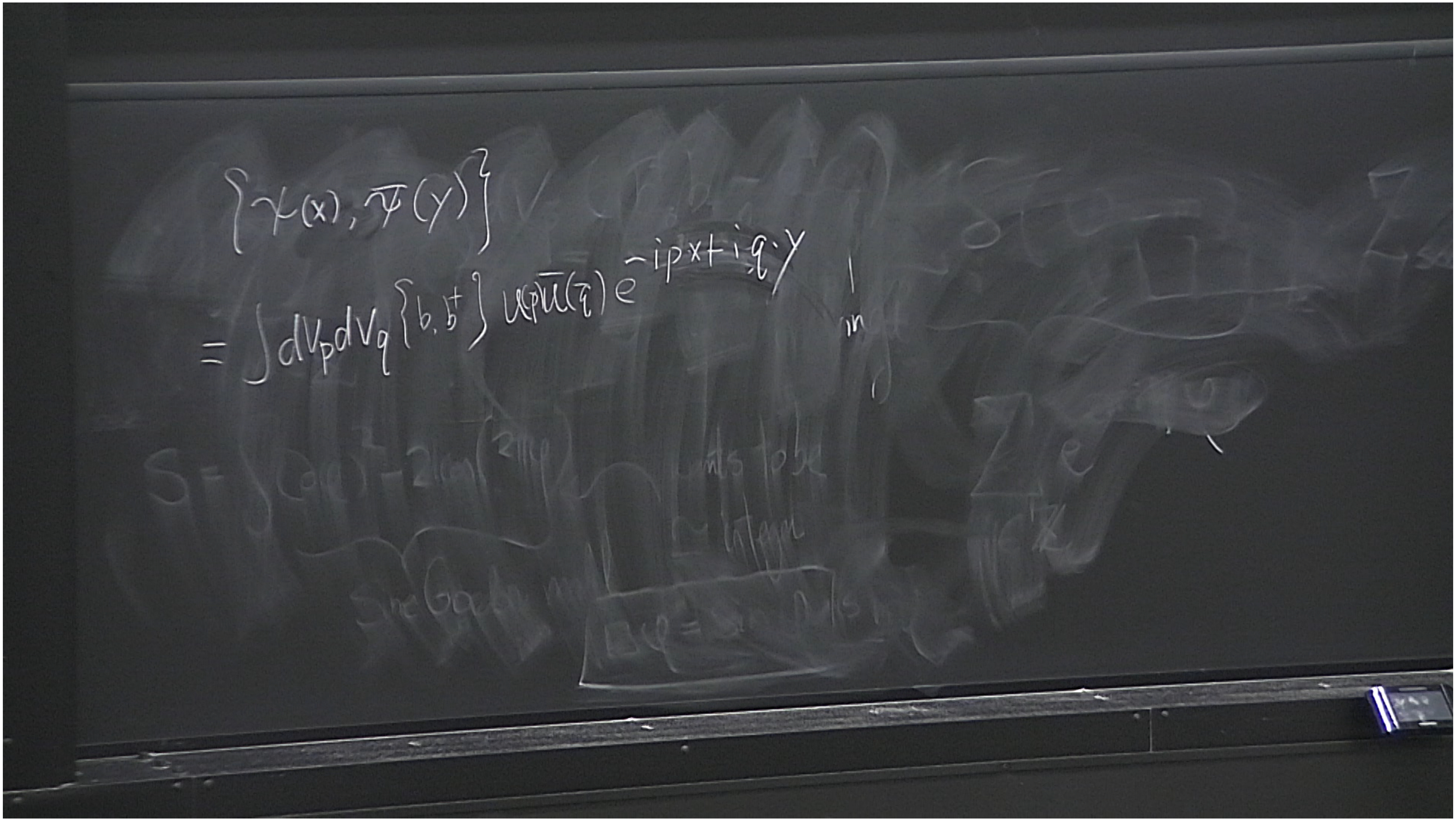

A[B, CD] + [A, CD]B
= Ā({B, C̄}D − C̄{B, D})
+ ({Ā, C̄}D − C̄{Ā, D})B
[B, Δ]
[A, CD] = {A, C}D − C
= ACD − CDA
= ACD + CAD − CAD − CDA

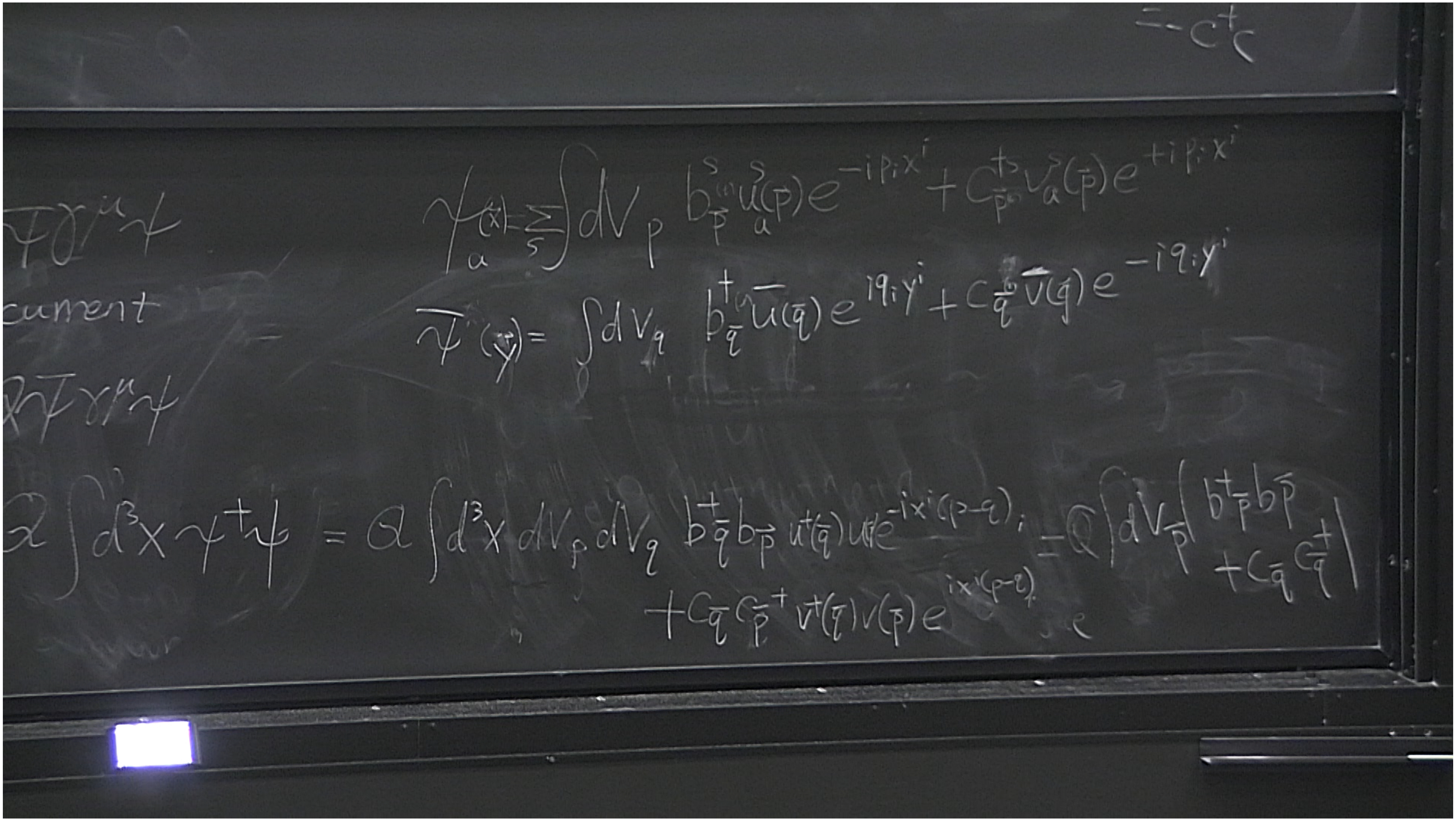

A[B, CD] + [A, CD]B
= Ā({B, C̄}D − C̄{B, D})
+ ({Ā, C̄}D − C̄{Ā, D})B
[B, Δ]
[A, Δ]B
[A, CD] = {A, C}D −
= ACD − CDA
= ACD + CAD − CAD − CDA


{A, CD} = {A, C}D − C{A, D}
= ACD − CDA
= ACD + CAD − CAD − CDA
{ψ, ψ̄}

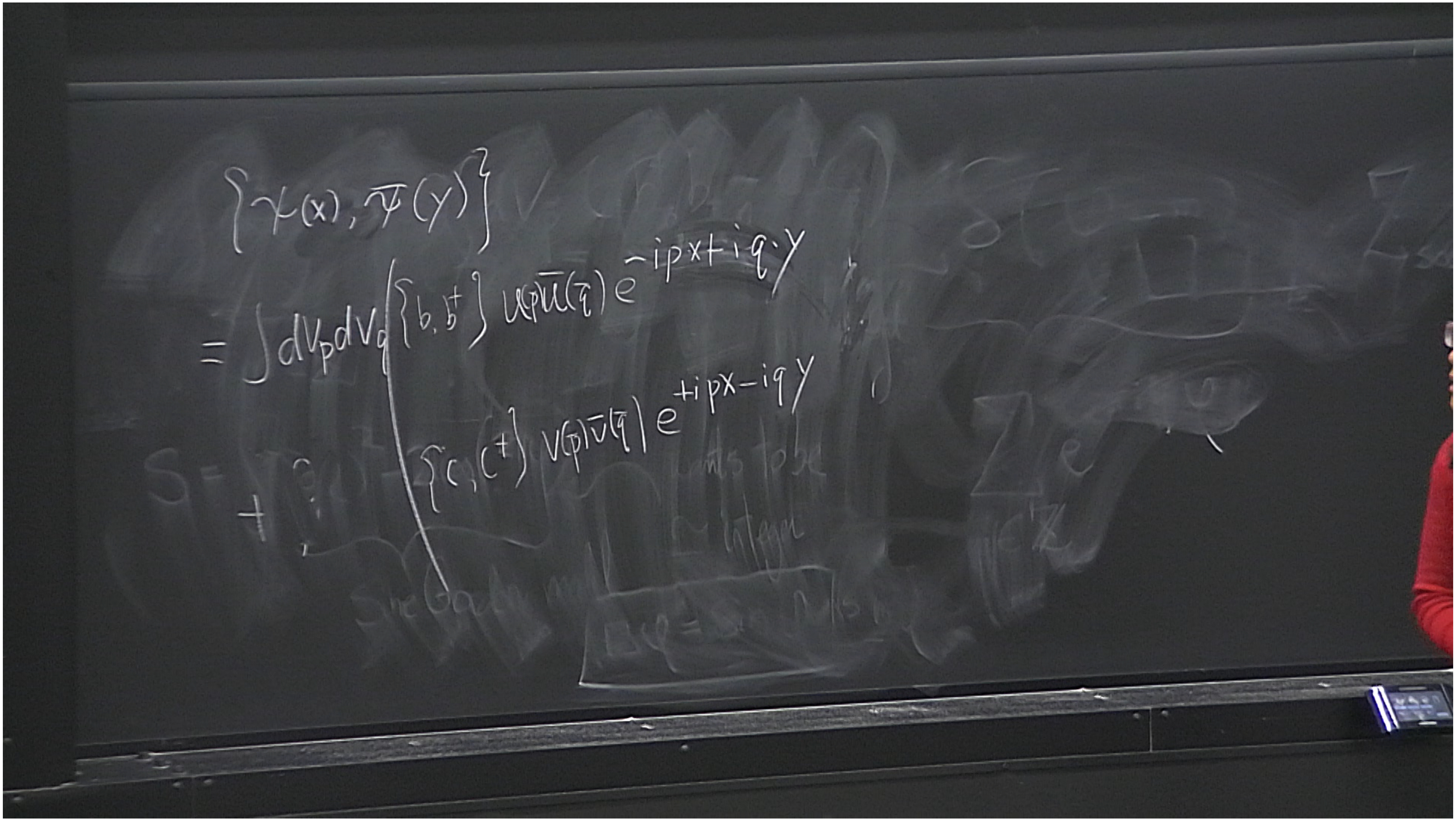

$\{\chi(x), \bar{\Psi}(y)\}$


current

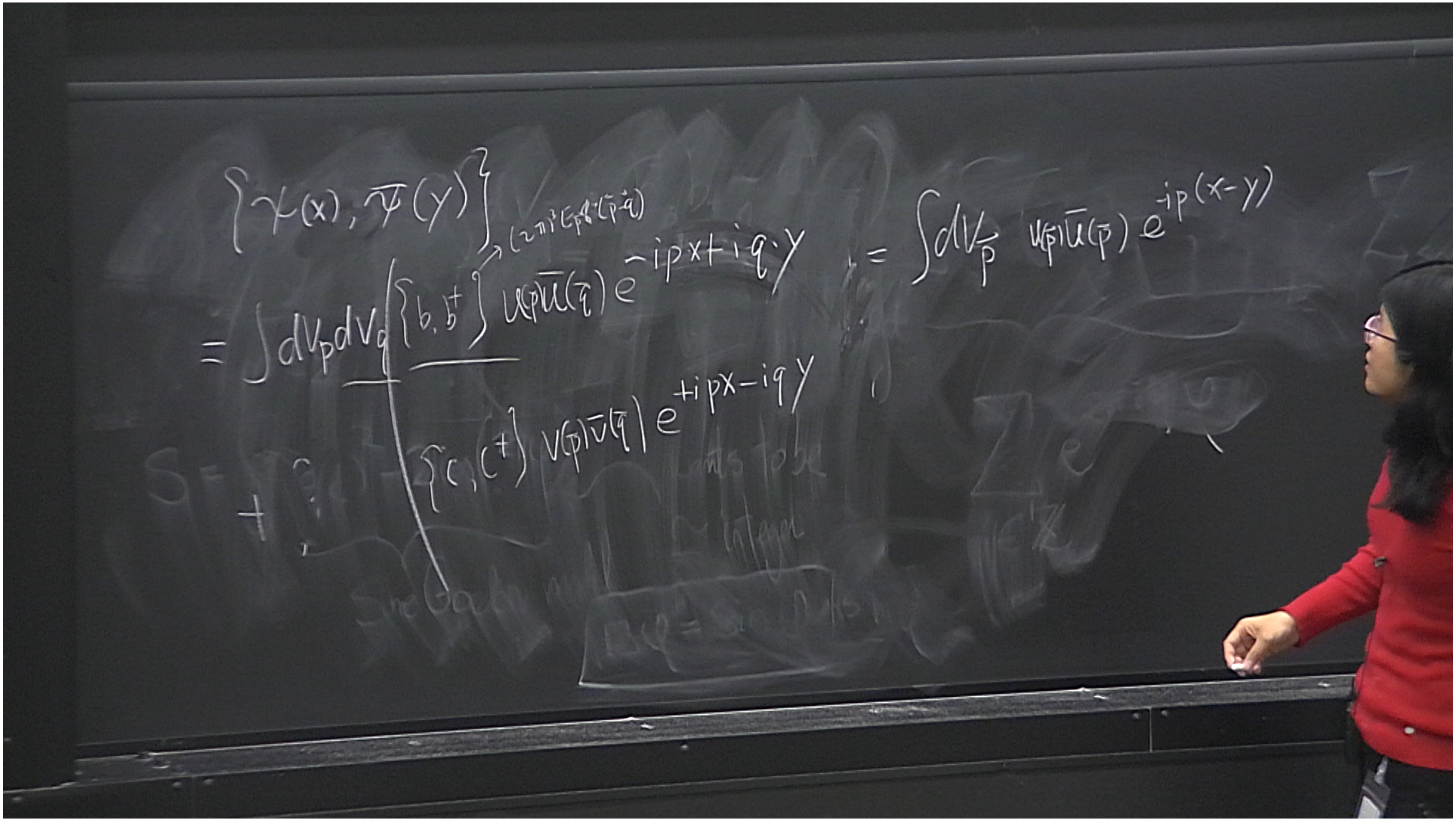

{ψ(x), Ψ̄(y)}
= ∫ dV_p dV_q {b, b†} u(p) ū(q) e^{-ip·x + iq·y}



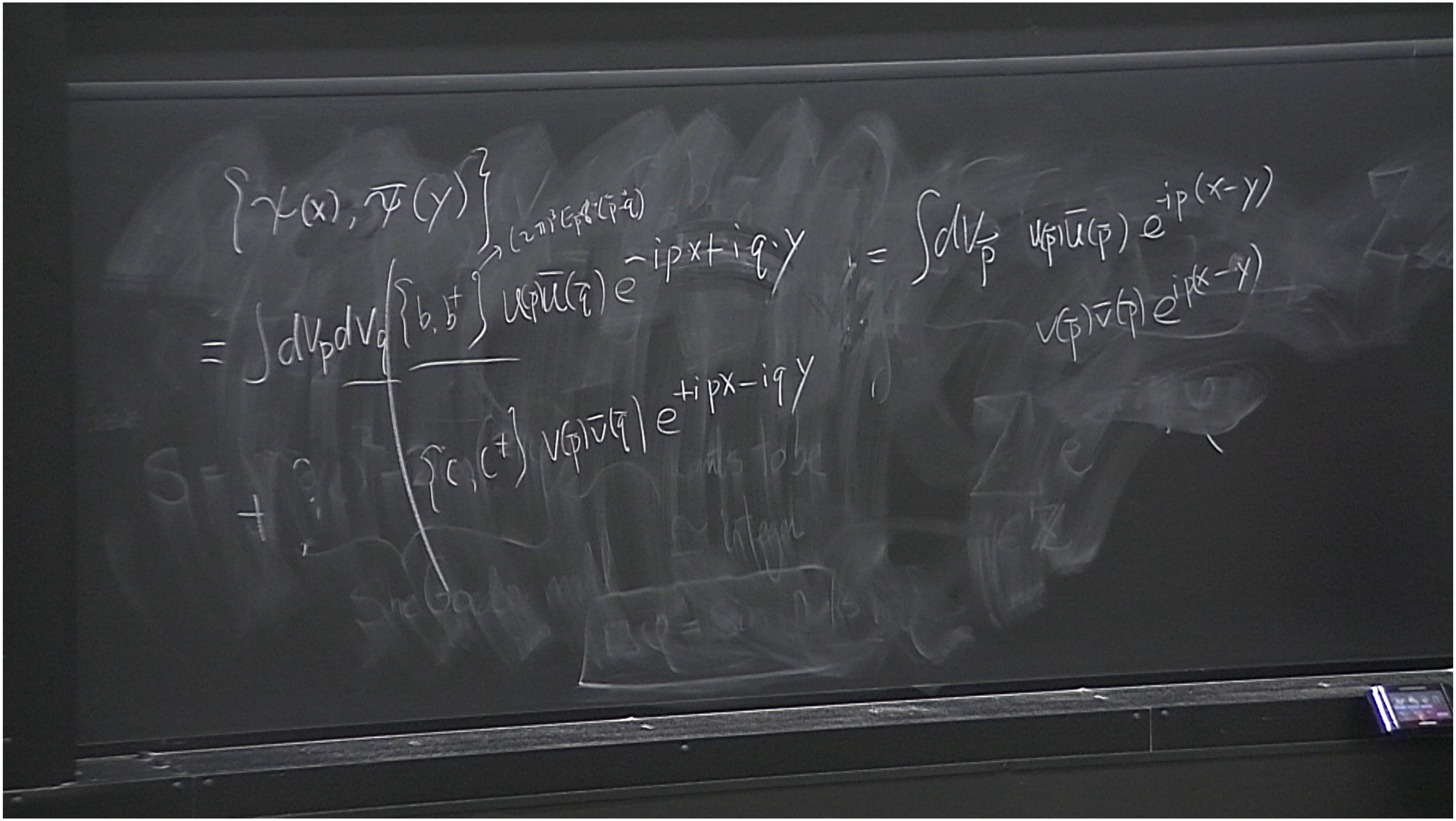
$= -c^\dagger c$
$\bar\psi\gamma^\mu\psi$
current
$Q\bar\psi\gamma^\mu\psi$
$\psi_a(x) = \sum_s \int dV_p\, b^s_{\vec p}\, u^s_a(\vec p)\, e^{-ip_i x^i} + c^{\dagger s}_{\vec p}\, v^s_a(\vec p)\, e^{+ip_i x^i}$
$\bar\psi(y) = \int dV_q\, b^\dagger_{\vec q}\, \bar u(\vec q)\, e^{iq_i y^i} + c_{\vec q}\, \bar v(\vec q)\, e^{-iq_i y^i}$
$Q\int d^3x\, \psi^\dagger\psi = Q\int d^3x\, dV_p\, dV_q\, b^\dagger_{\vec q} b_{\vec p}\, u^\dagger(\vec q) u(\vec p)\, e^{-ix^i(p-q)_i} + c_{\vec q} c^\dagger_{\vec p}\, v^\dagger(\vec q) v(\vec p)\, e^{ix^i(p-q)_i}$
$= Q\int dV_{\vec p}\, \left(b^\dagger_{\vec p} b_{\vec p} + c_{\vec q} c^\dagger_{\vec q} \right)$

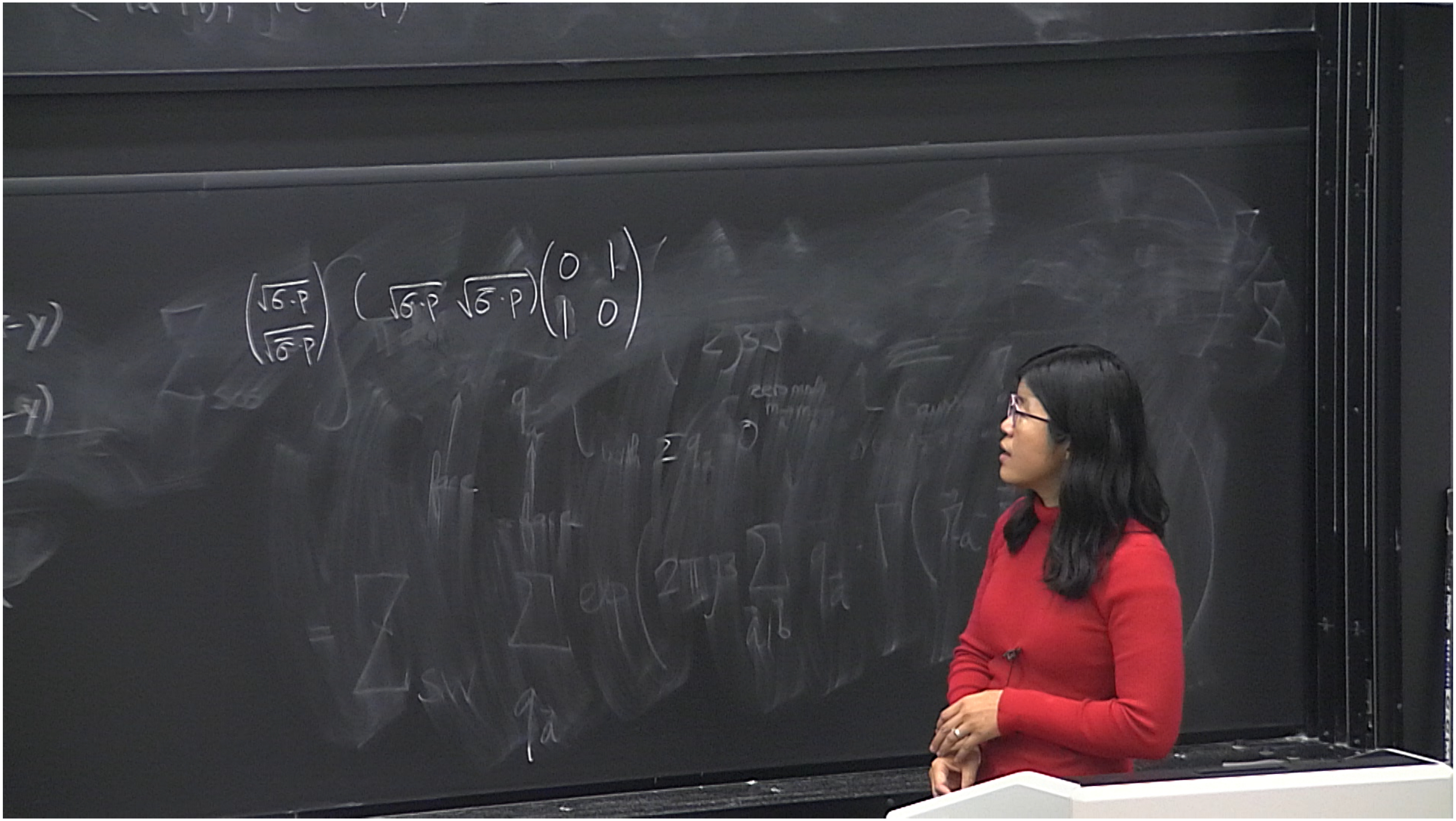


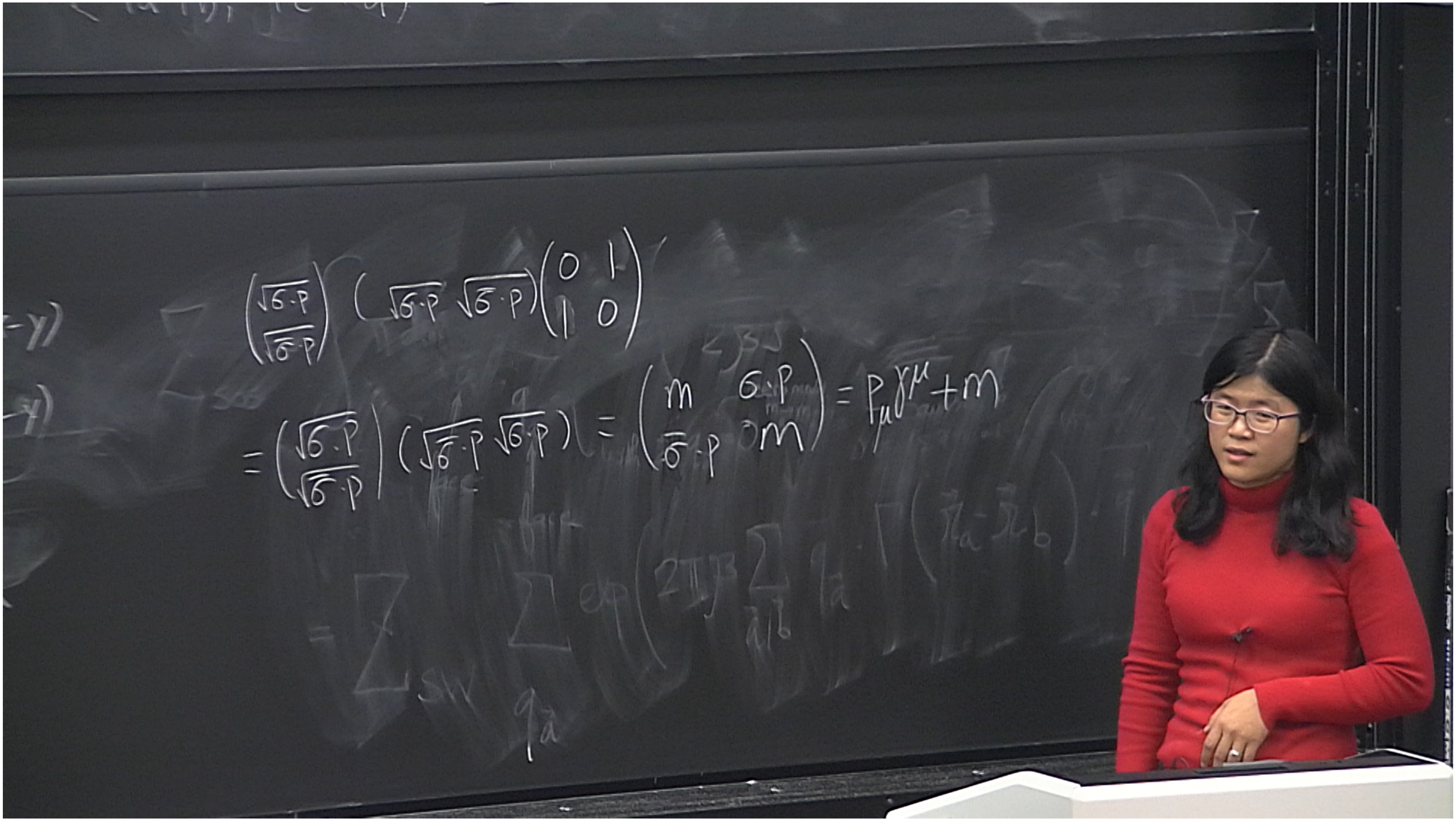
$\{\psi(x), \bar{\psi}(y)\}$
$= \int dV_p dV_q \Big(\{b, b^\dagger\} u(p)\bar{u}(q) e^{-ipx+iq\cdot y}$
$+ \{c, c^\dagger\} v(p)\bar{v}(q) e^{+ipx-iqy}$

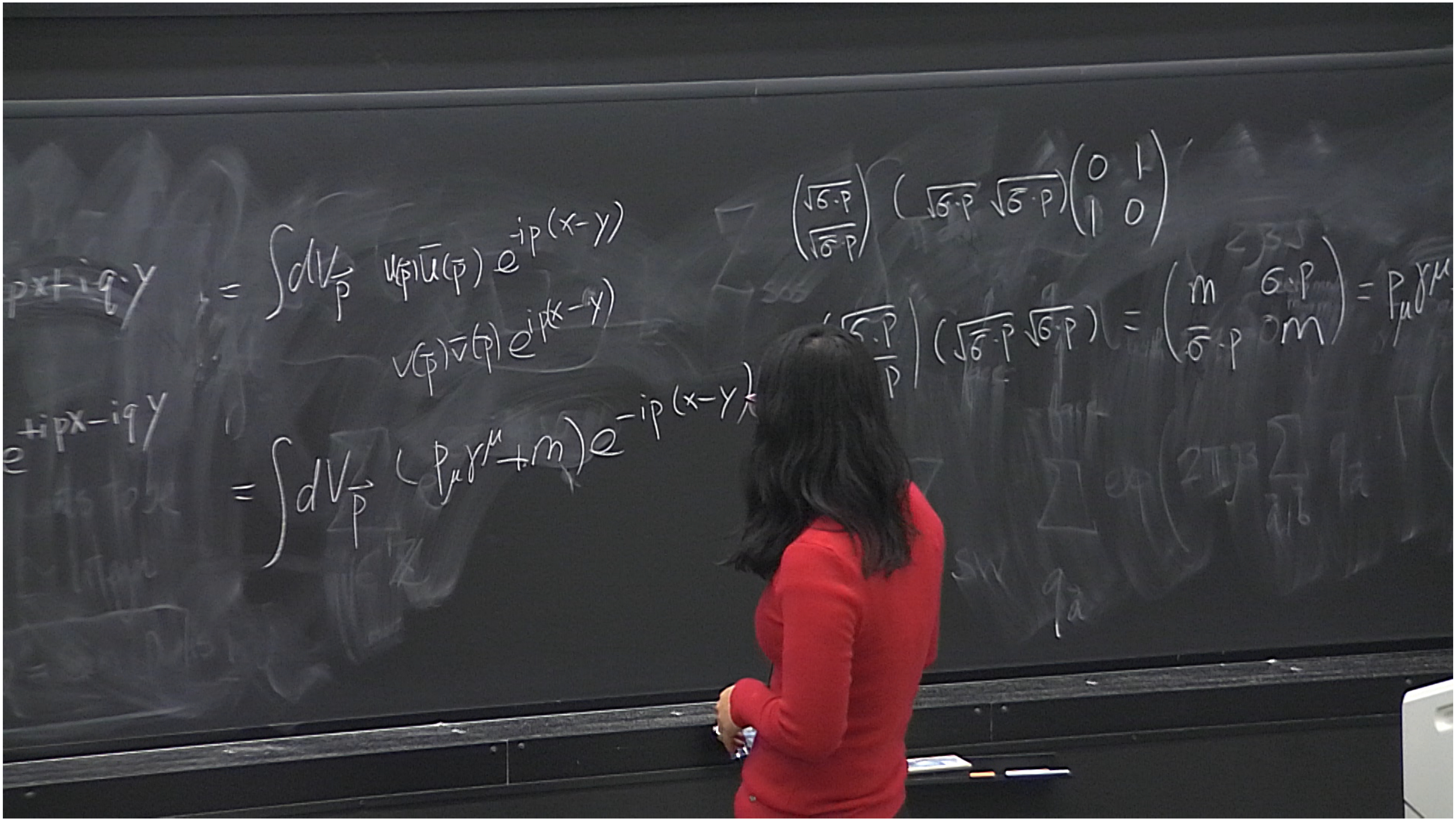


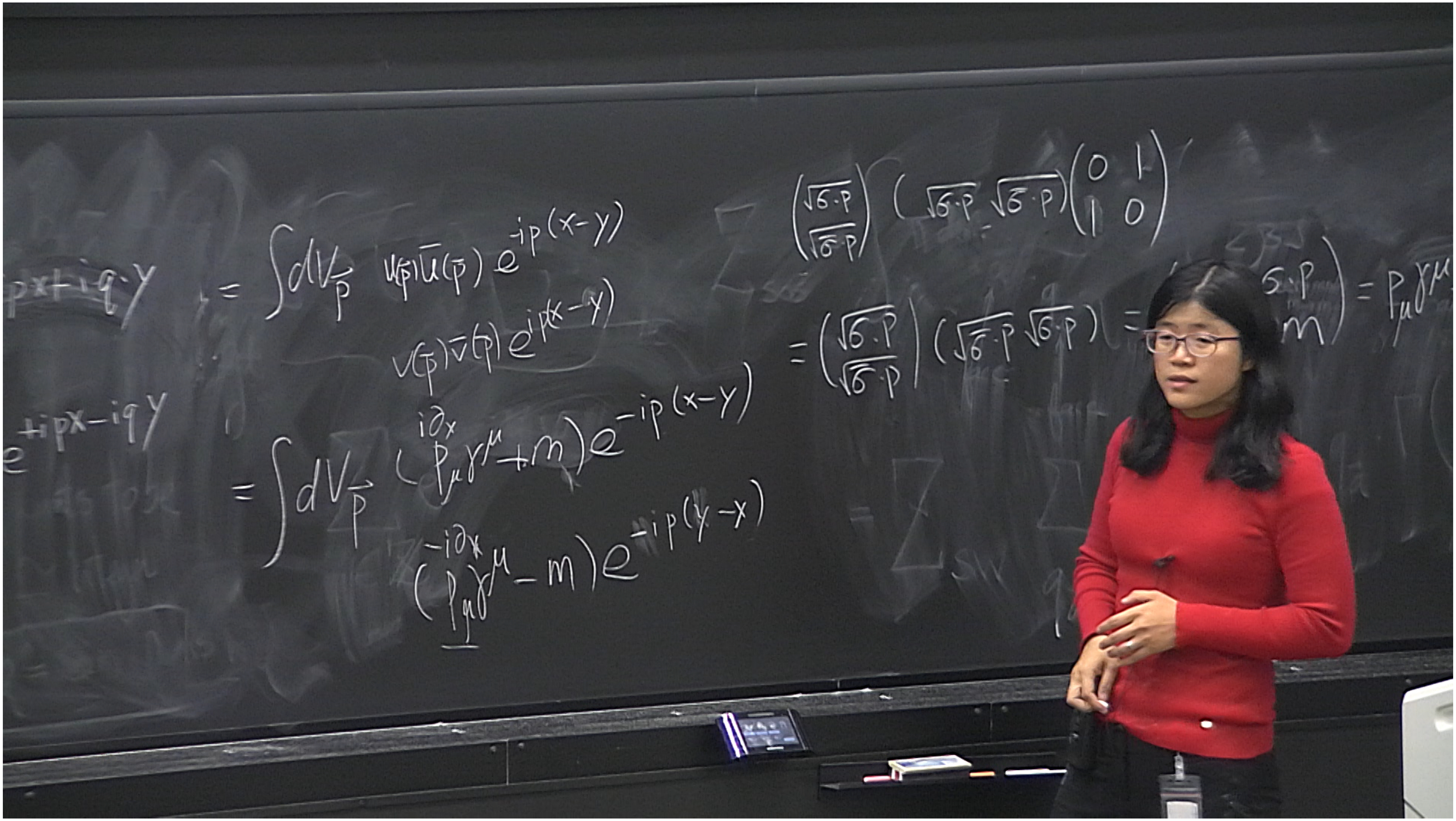
{ψ(x), ψ̄(y)} = ∫dV_p u(p)ū(p) e^{-ip(x-y)}
= ∫dV_p dV_q ({b, b†} u(p)ū(q) e^{-ipx+iq·y}
(2π)^3 2E_p δ^3(p-q)
+ {c, c†} v(p)v̄(q) e^{+ipx-iqy}

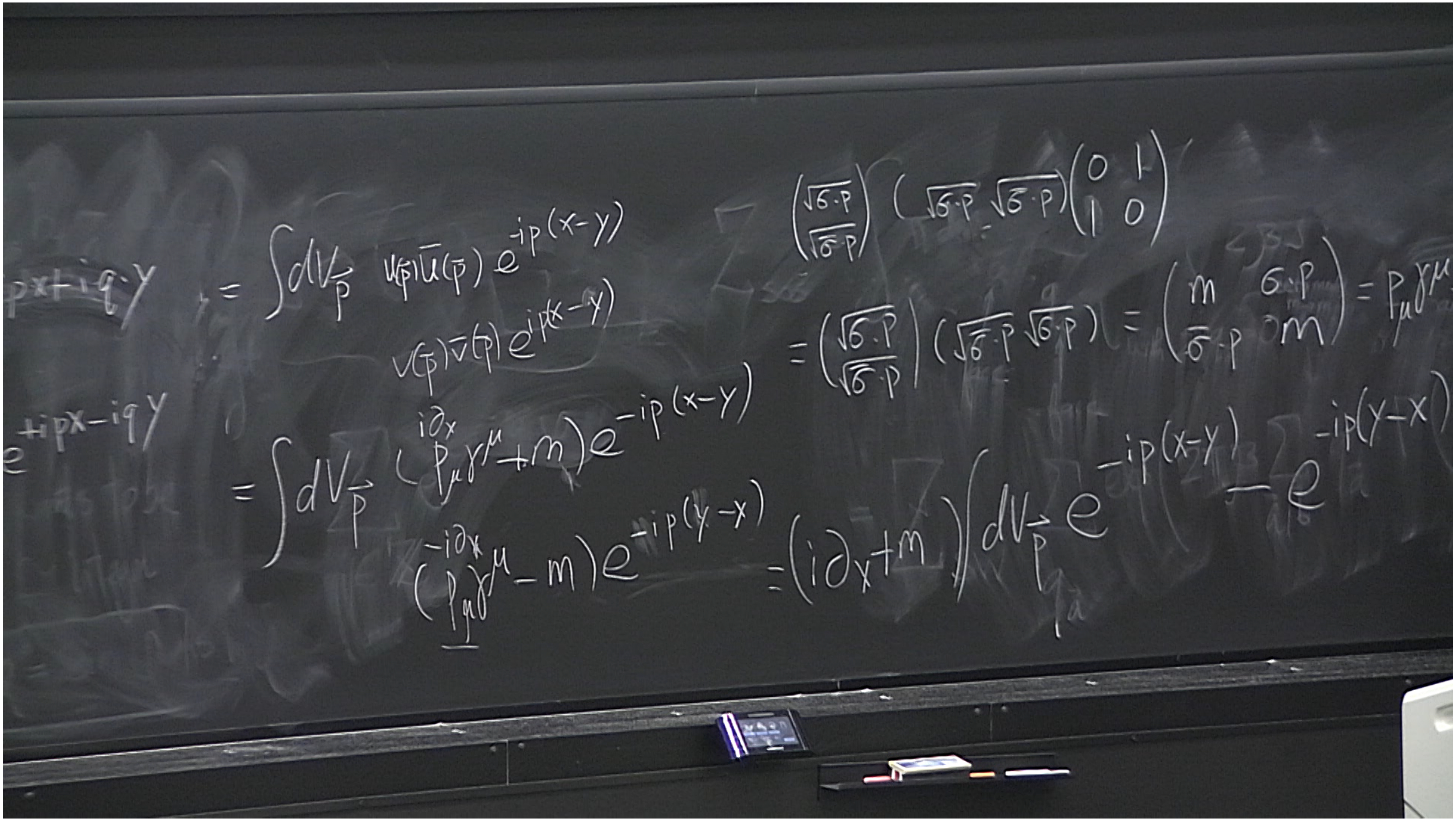

{ψ(x), ψ̄(y)}
(2π)³ 2E_p δ³(p⃗ − q⃗)
= ∫ dV_p dV_q ({b, b†} u(p) ū(q) e^{−ipx + iq·y}
+ {c, c†} v(p) v̄(q) e^{+ipx − iqy})
= ∫ dV_p u(p) ū(p) e^{−ip(x−y)}
v(p) v̄(p) e^{ip(x−y)}

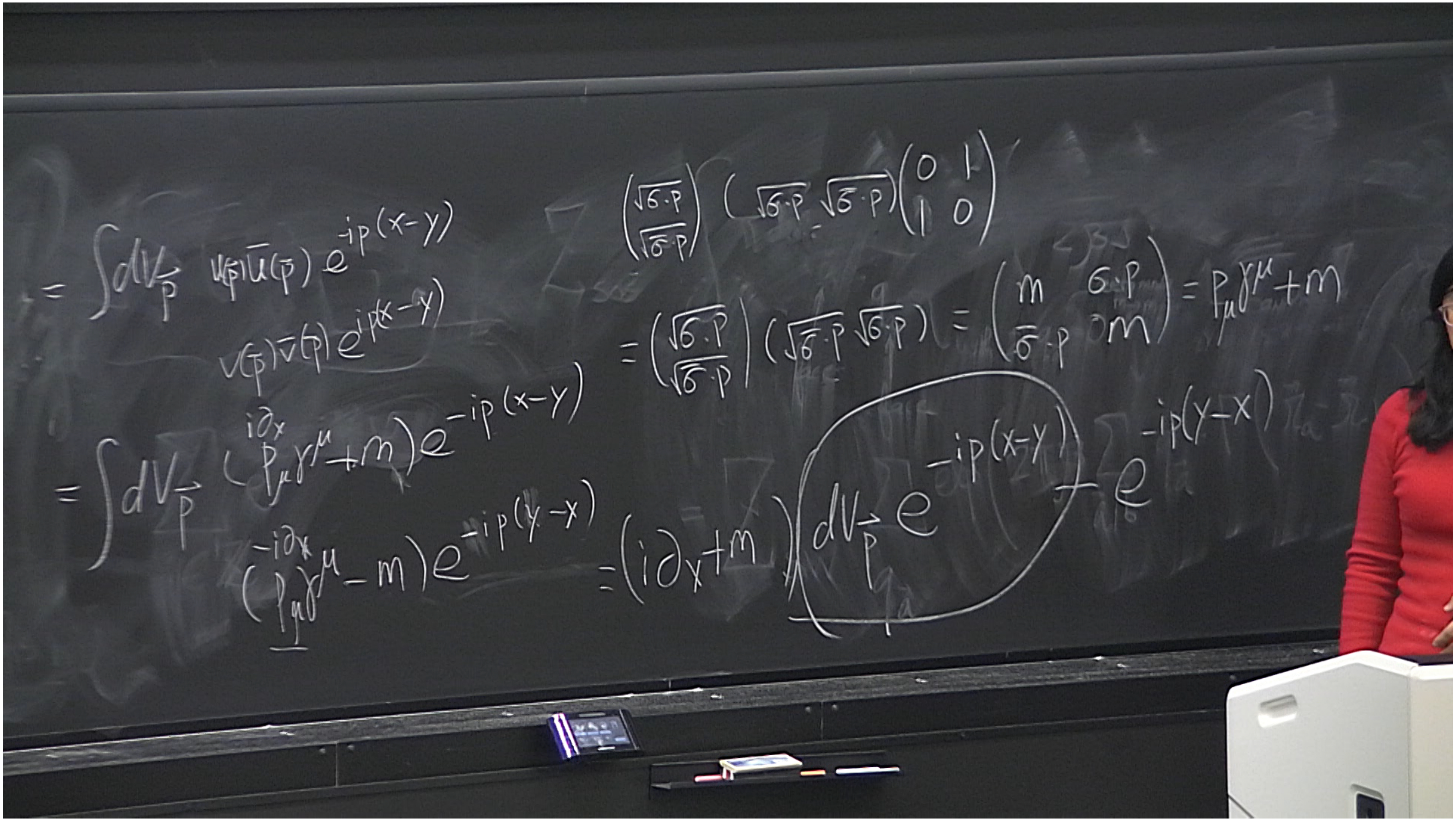

$\begin{pmatrix}\sqrt{\sigma\cdot p}\\ \sqrt{\bar\sigma\cdot p}\end{pmatrix}\left(\sqrt{\sigma\cdot p}\ \sqrt{\bar\sigma\cdot p}\right)\begin{pmatrix}0&1\\1&0\end{pmatrix}$

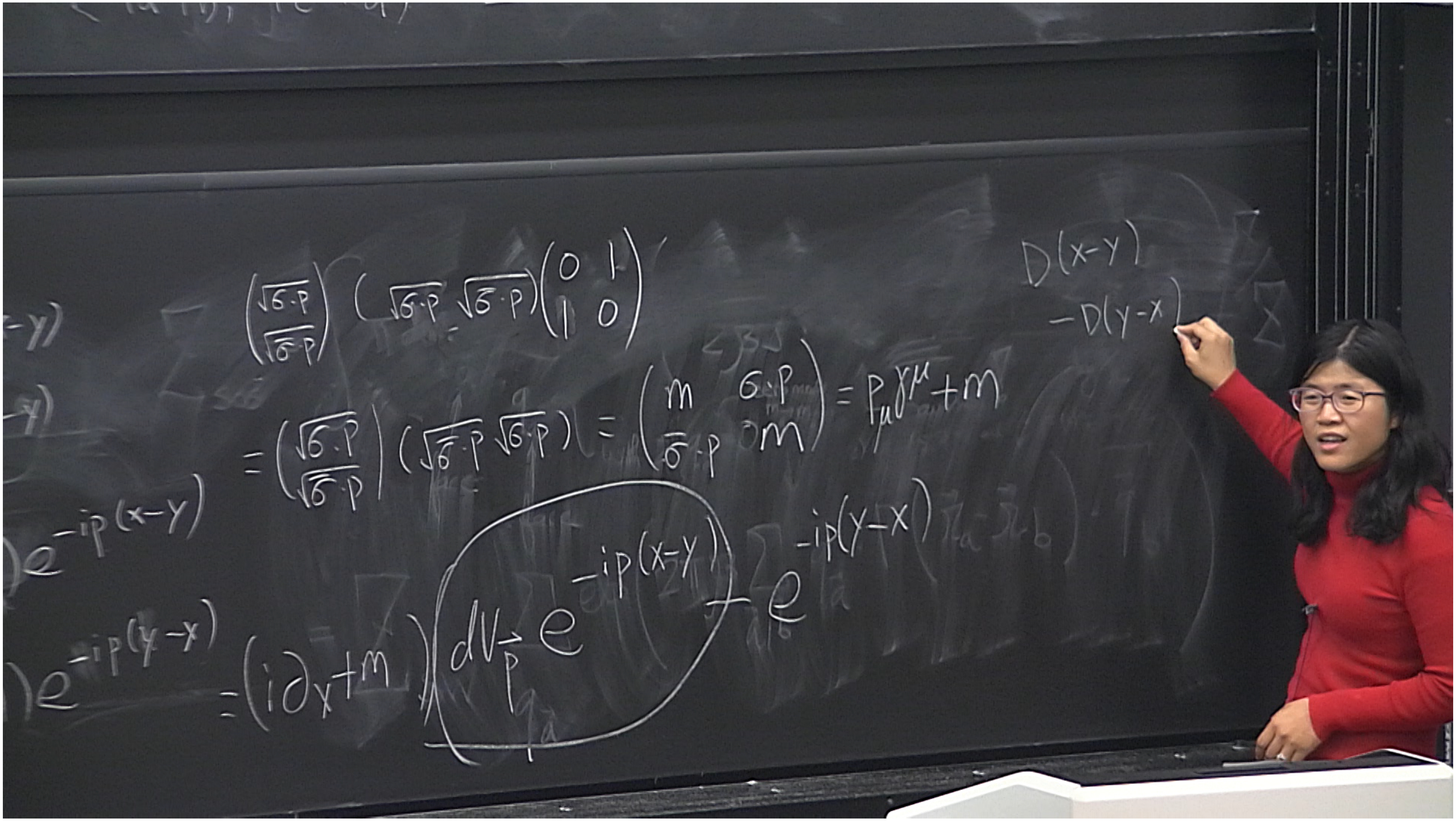

$\begin{pmatrix}\sqrt{\sigma\cdot p}\\ \sqrt{\bar\sigma\cdot p}\end{pmatrix}\left(\sqrt{\sigma\cdot p}\ \sqrt{\bar\sigma\cdot p}\right)\begin{pmatrix}0&1\\1&0\end{pmatrix}$
$=\begin{pmatrix}\sqrt{\sigma\cdot p}\\ \sqrt{\bar\sigma\cdot p}\end{pmatrix}\left(\sqrt{\bar\sigma\cdot p}\ \sqrt{\sigma\cdot p}\right)=\begin{pmatrix}m&\sigma\cdot p\\ \bar\sigma\cdot p&m\end{pmatrix}=p_\mu\gamma^\mu+m$

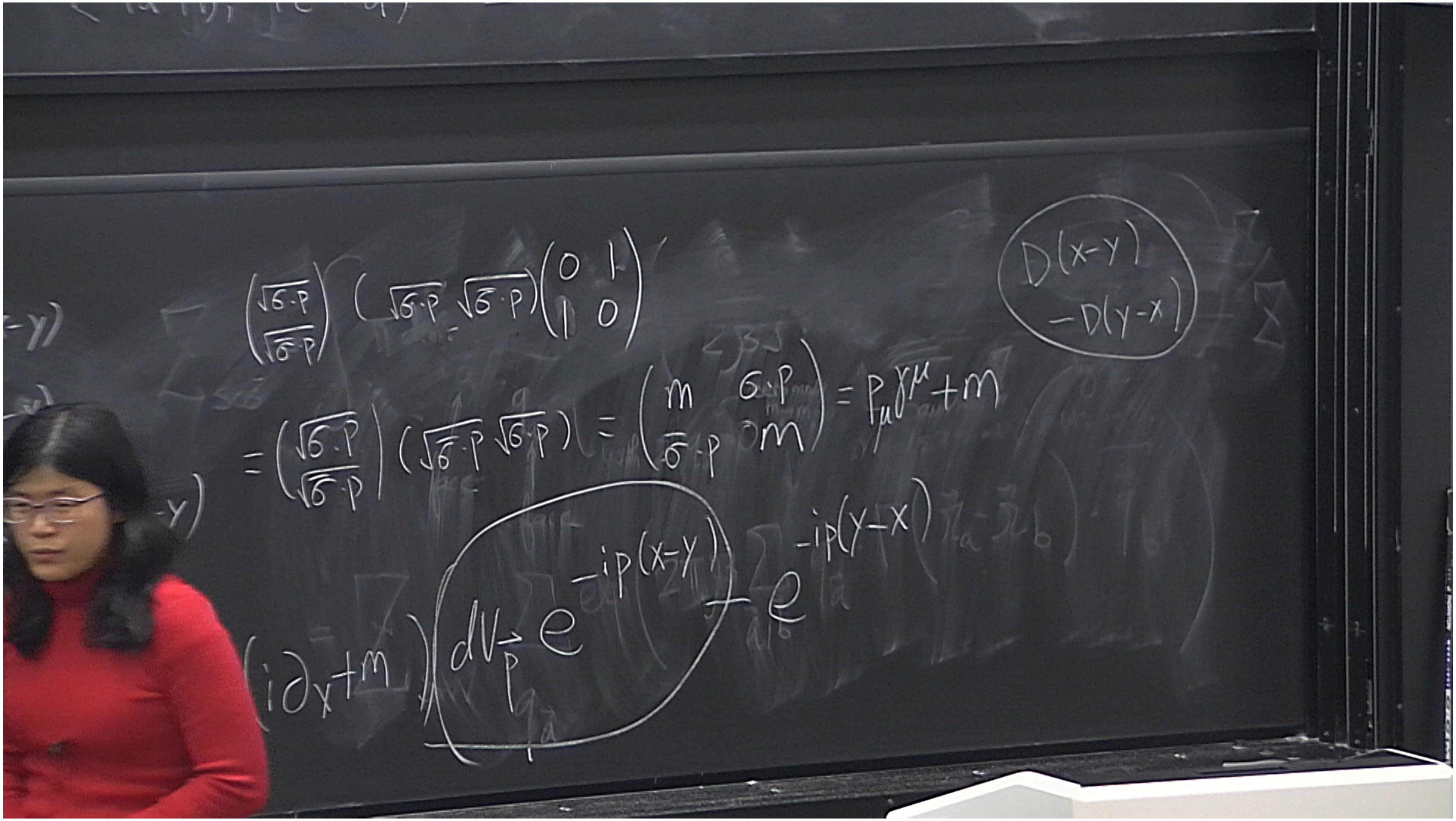


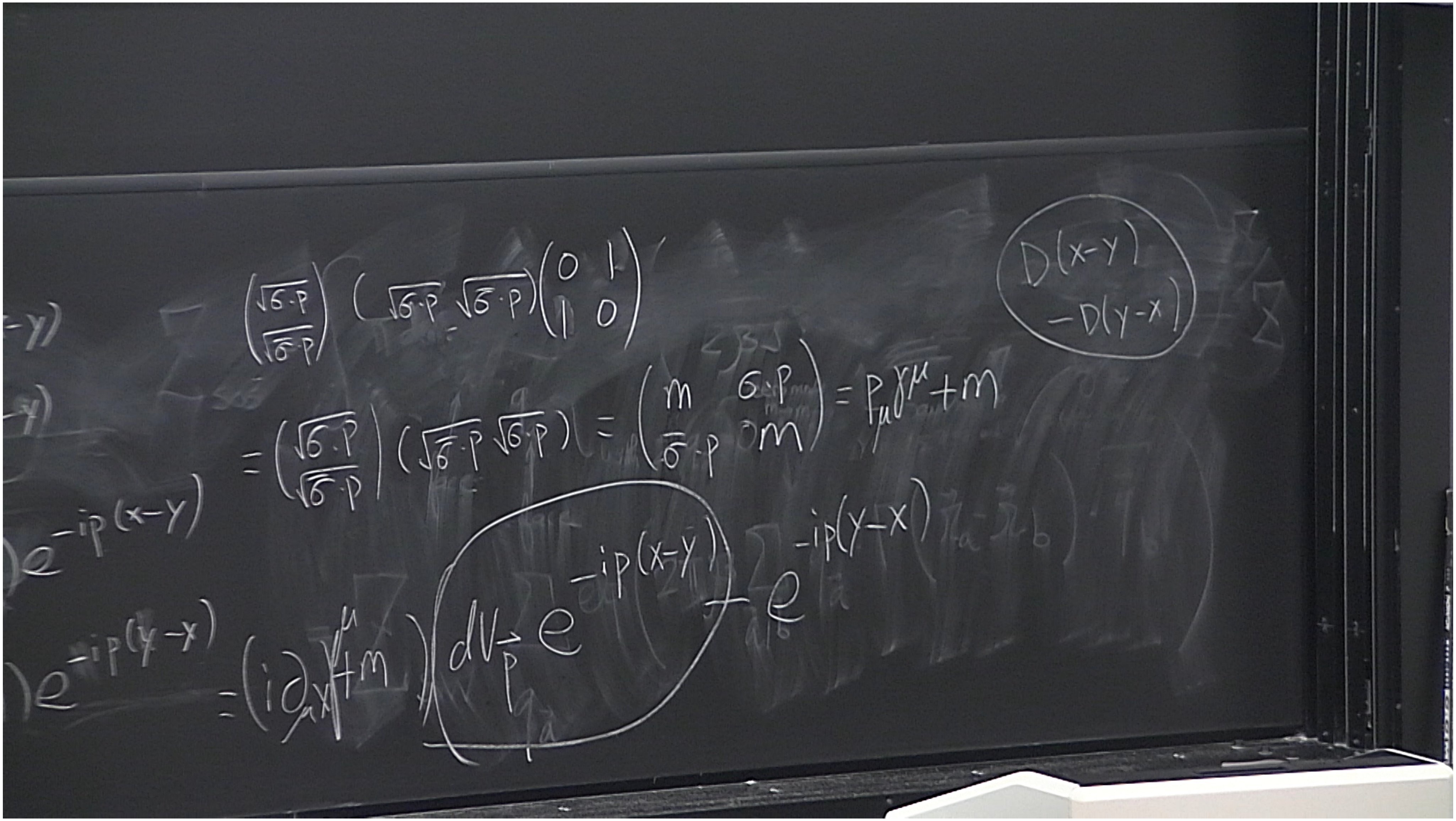



= ∫dV_p u(p) ū(p) e^{-ip(x-y)}
v(p) v̄(p) e^{ip(x-y)}
(√σ·p, √σ̄·p) (√σ·p √σ̄·p) (0 1; 1 0)
= (√σ·p, √σ̄·p) (√σ̄·p √σ·p) = (m σ·p; σ̄·p m) = p_μ γ^μ
= ∫dV_p (p_μ γ^μ + m) e^{-ip(x-y)}
(p_μ γ^μ - m) e^{+ip(y-x)}
= (i∂_x + m) ∫dV_p e^{-ip(x-y)} - e^{-ip(y-x)}


= ∫dV_p u(p)ū(p) e^{-ip(x-y)}
v(p)v̄(p) e^{ip(x-y)}
= ∫dV_p (p_μγ^μ + m) e^{-ip(x-y)}
(p_μγ^μ - m) e^{-ip(y-x)}
= (√σ·p, √σ̄·p)(√σ·p √σ̄·p) = (m σ·p; σ̄·p m) = p_μγ^μ + m
= (i∂_x + m) ∫dV_p e^{-ip(x-y)}


D(x-y)
-D(y-x)


D(x-y)
-D(y-x)
= p_μ γ^μ + m
(i∂_x + m)
e^{-ip(x-y)}
e^{-ip(y-x)}


D(x-y)
-D(y-x)