

Title: PSI 2018/2019 - Quantum Field Theory I - Lecture 11

Date: Oct 25, 2018 02:00 PM

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Abstract:

γ constant = similar figure & same $\theta \rightarrow 0$ behaviour

$$(i\gamma\mu\partial_\mu + iQA_\mu - m)\psi = 0$$

$\psi^{(c)}$

\int conditions = similar & same $\theta \rightarrow 0$
figure & behaviour

$$(i \gamma^\mu (\partial_\mu + iQA_\mu) - m)\psi = 0$$

$$\psi^{(c)} (-i(\gamma^\mu)^* (\partial_\mu - iQA_\mu) - m)\psi^* = 0$$

\int Lagrangian = similar & same $\theta \rightarrow 0$ behaviour

$$(i \gamma^\mu (\partial_\mu + iQA_\mu) - m) \psi = 0$$

$$\psi^{(c)} \quad (-i (\gamma^\mu)^* (\partial_\mu - iQA_\mu) - m) \psi^* = 0$$

$$(\gamma^\mu)^* = -C^{-1} \gamma^\mu C$$

\int addition = similar & same $\theta \rightarrow 0$ figure behaviour

$$(i \gamma^\mu (\partial_\mu + iQ A_\mu) - m) \psi = 0$$

$\psi^{\dagger(c)}$

$$(-i (\gamma^\mu)^* (\partial_\mu - iQ A_\mu) - m) \psi^* = 0$$

ψ^*

γ^μ

$$(i c^{-1} \gamma^\mu c (\partial_\mu - iQ A_\mu) - m c^{-1} c) \psi^* = 0$$

\int constants = similar & same $\theta \rightarrow 0$ figure behaviour

$$(i \gamma^\mu \partial_\mu + i Q A_\mu - m) \psi = 0$$

$$\psi^{(c)} \quad (-i (\gamma^\mu)^* (\partial_\mu - i Q A_\mu) - m) \psi^* = 0$$

$$C^{-1} (i \gamma^\mu C (\partial_\mu - i Q A_\mu) - m C^{-1} C) \psi^* = 0$$

$$C^{-1} (i \gamma^\mu (\partial_\mu - i Q A_\mu) - m) C \psi^* = 0$$

$\int \dots = 0$ similar & same $\theta \rightarrow 0$
 figure & behaviour

$$(i \gamma^\mu \partial_\mu + i Q A_\mu - m) \psi = 0$$

$$\psi^{(c)} \quad (-i (\gamma^\mu)^* (\partial_\mu - i Q A_\mu) - m) \psi^* = 0$$

$$\begin{aligned}
 & (\gamma^\mu)^* \left[i c^{-1} \gamma^\mu c (\partial_\mu - i Q A_\mu) - m c^{-1} c \right] \psi^* = 0 \\
 & = -c^{-1} \gamma^\mu c \quad c^{-1} (i \gamma^\mu (\partial_\mu - i Q A_\mu) - m) c \psi^{(c)} = 0
 \end{aligned}$$

of conditions = similar & same $\theta \rightarrow 0$
 figure & behaviour

$$(i \gamma^\mu \partial_\mu + i Q A_\mu - m) \psi = 0$$

$\psi^{(c)}$

$$(-i (\gamma^\mu)^* (\partial_\mu - i Q A_\mu) - m) \psi^* = 0$$

$(\gamma^\mu)^* = -C^{-1} \gamma^\mu C$

$$C^{-1} (i \gamma^\mu \partial_\mu + i Q A_\mu - m) C \psi^* = 0$$

$$C^{-1} (i \gamma^\mu (\partial_\mu - i Q A_\mu) - m) C \psi^* = 0$$

$\int \psi \psi = 0$ similar figure & same $\theta \rightarrow 0$ behaviour

$$(i \gamma^\mu \partial_\mu + i Q A_\mu - m) \psi = 0$$

$\psi^{(c)}$

$$(-i (\gamma^\mu)^* (\partial_\mu - i Q A_\mu) - m) \psi^* = 0$$

Majorana has to be neutral in gauge interaction.

$(\gamma^\mu)^* = -C^{-1} \gamma^\mu C$

$$C^{-1} (i \gamma^\mu \partial_\mu + i Q A_\mu - m) C \psi^* = 0$$

$$C^{-1} (i \gamma^\mu (\partial_\mu - i Q A_\mu) - m) C \psi^{(c)} = 0$$

Quantize the Dirac field !!

Step 1, pick a good Lagrangian

Step 2 pick conjugate momentum Hamiltonian

Step 3 commutator relationship on field and its momentum
→ promote some coefficients to operators

Step 4 Normal order.

Step 1 $\mathcal{L} = \bar{\psi} (i \gamma^\mu \partial_\mu - m) \psi$

Step 2 $\Pi = i \psi^\dagger \quad \mathcal{H} = \psi^\dagger (-i \gamma^0 \gamma^i \partial_i + \gamma^0 m) \psi$

Step 1 $\mathcal{L} = \bar{\psi} (i \gamma^\mu \partial_\mu - m) \psi$

Step 2 $\pi = i \psi^\dagger \quad \mathcal{H} = \psi^\dagger (-i \gamma^0 \gamma^i \partial_i + \gamma^0 m) \psi$

Step 3 $H = \int d^3x \mathcal{H} = \int dV_p E_{\vec{p}} (b_{(\vec{p})}^* b_{(\vec{p})} - c_{(\vec{p})} c_{(\vec{p})}^*)$

Step 1 $\mathcal{L} = \bar{\psi} (i \gamma^\mu \partial_\mu - m) \psi$

Step 2 $\Pi = i \psi^\dagger \quad \mathcal{H} = \psi^\dagger (-i \gamma^0 \gamma^i \partial_i + \gamma^0 m) \psi$

Step 3 $H = \int d^3x \mathcal{H} = \int dV_p E_{\vec{p}} \left(b_{\vec{p}}^\dagger \quad b_{\vec{p}} \quad -c_{\vec{p}} \quad c_{\vec{p}}^\dagger \right)$

try to impose $[c_{\vec{p}}, c_{\vec{q}}^\dagger] = (2\pi)^3 2E_{\vec{p}} \delta(\vec{p}-\vec{q})$

$$\begin{aligned} C_{\bar{p}} G_{\bar{p}}^{\dagger} &= C_{\bar{p}} G_{\bar{p}}^{\dagger} - G_{\bar{p}}^{\dagger} C_{\bar{p}} + G_{\bar{p}}^{\dagger} C_{\bar{p}} \\ &= \underbrace{[C_{\bar{p}}, G_{\bar{p}}^{\dagger}] + G_{\bar{p}}^{\dagger} C_{\bar{p}}}_{\text{}} \end{aligned}$$

$$|\vec{x}_i - \vec{x}_j| > a$$

$$-m)c$$

$$= \psi^\dagger (-i\gamma^0 \gamma^i \partial_i + \gamma^0 m) \psi$$

$$\int dV_p E_{\vec{p}} \left(b_{\vec{p}}^\dagger \quad b_{\vec{p}} \right) - C_{\vec{p}} \quad C_{\vec{p}}^\dagger$$

$$[C_{\vec{p}}, C_{\vec{q}}^\dagger] = (2\pi)^3 2E_{\vec{p}} \delta(\vec{p} - \vec{q})$$

$$C_{\vec{p}} C_{\vec{p}}^\dagger = C_{\vec{p}} C_{\vec{p}}^\dagger - C_{\vec{p}}^\dagger C_{\vec{p}} + C_{\vec{p}}^\dagger C_{\vec{p}}$$

$$= [C_{\vec{p}}, C_{\vec{p}}^\dagger] + C_{\vec{p}}^\dagger C_{\vec{p}}$$

$$H = \int dV_p E_{\vec{p}} \left(b_{\vec{p}}^\dagger b_{\vec{p}} \right)$$

$$\begin{aligned}
 G_{\vec{p}} G_{\vec{p}}^{\dagger} &= G_{\vec{p}} G_{\vec{p}}^{\dagger} - G_{\vec{p}}^{\dagger} G_{\vec{p}} + G_{\vec{p}}^{\dagger} G_{\vec{p}} \\
 &= \underbrace{[G_{\vec{p}}, G_{\vec{p}}^{\dagger}]}_{\text{Commutator}} + G_{\vec{p}}^{\dagger} G_{\vec{p}}
 \end{aligned}$$

$$H = \int dV_p E_{\vec{p}} \left(\frac{1}{V_p} b_{\vec{p}}^{\dagger} b_{\vec{p}} - \infty - G_{\vec{p}}^{\dagger} G_{\vec{p}} \right)$$

$$\begin{aligned}
 C_{\vec{p}} G_{\vec{p}}^{\dagger} &= C_{\vec{p}} G_{\vec{p}}^{\dagger} - G_{\vec{p}}^{\dagger} C_{\vec{p}} + G_{\vec{p}}^{\dagger} C_{\vec{p}} \\
 &= \underbrace{[C_{\vec{p}}, G_{\vec{p}}^{\dagger}] + G_{\vec{p}}^{\dagger} C_{\vec{p}}}_{\text{an}}
 \end{aligned}$$

$$H = \int dV_{\vec{p}} E_{\vec{p}} \left(\frac{1}{V_{\vec{p}}} b_{\vec{p}}^{\dagger} - \infty - \underbrace{G_{\vec{p}}^{\dagger} C_{\vec{p}}} \right)$$

$$\begin{aligned}
 \vec{C}_p \vec{C}_p^\dagger &= \vec{C}_p \vec{C}_p^\dagger - \vec{C}_p^\dagger \vec{C}_p + \vec{C}_p^\dagger \vec{C}_p \\
 &= \underbrace{[\vec{C}_p, \vec{C}_p^\dagger]} + \vec{C}_p^\dagger \vec{C}_p
 \end{aligned}$$

$$H = \int dV_p \vec{E}_p \left(\frac{1}{V_p} \vec{b}_p^\dagger \vec{b}_p - \infty \underbrace{\vec{C}_p^\dagger \vec{C}_p} \right)$$

$$= \int dV_p \left(\vec{a}_p^\dagger \vec{a}_p - \vec{b}_p^\dagger \vec{b}_p \right)$$

$$C_{\vec{p}} C_{\vec{p}}^{\dagger} = C_{\vec{p}} C_{\vec{p}}^{\dagger} - C_{\vec{p}}^{\dagger} C_{\vec{p}} + C_{\vec{p}} C_{\vec{p}}$$

$$= [C_{\vec{p}}, C_{\vec{p}}^{\dagger}] + C_{\vec{p}} C_{\vec{p}}$$

$$H = \int dV_p \cdot E_{\vec{p}} \left(b_{\vec{p}}^{\dagger} b_{\vec{p}} - \text{[diagram]} C_{\vec{p}}^{\dagger} C_{\vec{p}} \right)$$

$$= \int_{SW} \sum_{\vec{q}_a} \exp\left(2\pi i \vec{q}_a \cdot \vec{r}_a\right)$$

$$\begin{aligned}
 C_{\vec{p}}^{\dagger} C_{\vec{p}}^{\dagger} &= C_{\vec{p}} C_{\vec{p}}^{\dagger} - \underbrace{C_{\vec{p}}^{\dagger} C_{\vec{p}}^{\dagger} + C_{\vec{p}}^{\dagger} C_{\vec{p}}}_{\text{needs a minus sign}} \\
 &= [C_{\vec{p}}, C_{\vec{p}}^{\dagger}] + C_{\vec{p}}^{\dagger} C_{\vec{p}} \quad \text{had better be plus}
 \end{aligned}$$

$$H = \int dV_p \cdot E_{\vec{p}} \left(\frac{1}{V_p} b_{\vec{p}}^{\dagger} b_{\vec{p}} - \underbrace{C_{\vec{p}}^{\dagger} C_{\vec{p}}}_{\text{wavy line}} \right)$$

\vec{q}

$$C_{\vec{p}} C_{\vec{p}}^{\dagger} = C_{\vec{p}} C_{\vec{p}}^{\dagger} - \underbrace{C_{\vec{p}}^{\dagger} C_{\vec{p}}}_{\text{needs a minus sign}} + C_{\vec{p}}^{\dagger} C_{\vec{p}} \rightarrow \text{had better be plus}$$

$$= [C_{\vec{p}}, C_{\vec{p}}^{\dagger}] + C_{\vec{p}}^{\dagger} C_{\vec{p}}$$

We want anti-commutator

$$\{C_{\vec{p}}, C_{\vec{q}}^{\dagger}\} = 2\pi^3 \epsilon_{\vec{p}} \delta(\vec{p}, \vec{q})$$

$$H = \int dV_p \cdot E_{\vec{p}} \left(\frac{1}{V_p} b_{\vec{p}}^{\dagger} b_{\vec{p}} - \underbrace{C_{\vec{p}}^{\dagger} C_{\vec{p}}}_{\text{wavy line}} \right)$$

\vec{q}

QM problem

$$\{a, a^\dagger\} = 1 \quad \{a, a\} = 0$$

$$\{a^\dagger, a^\dagger\} = 0$$

$a^\dagger a \leftarrow$ what are the eigen values?

$$[a^\dagger a, a]$$

$$[a^\dagger a, a^\dagger]$$

$$(a^\dagger a)^2$$

QM problem

$$\{a, a^\dagger\} = 1 \quad \{a, a\} = 0$$

$$\{a^\dagger, a^\dagger\} = 0$$

$a^\dagger a \leftarrow$ what are the eigen values?

$$[a^\dagger a, a]$$

$$[a^\dagger a, a^\dagger]$$

$$(a^\dagger a)^2$$

0 and 1

QM problem

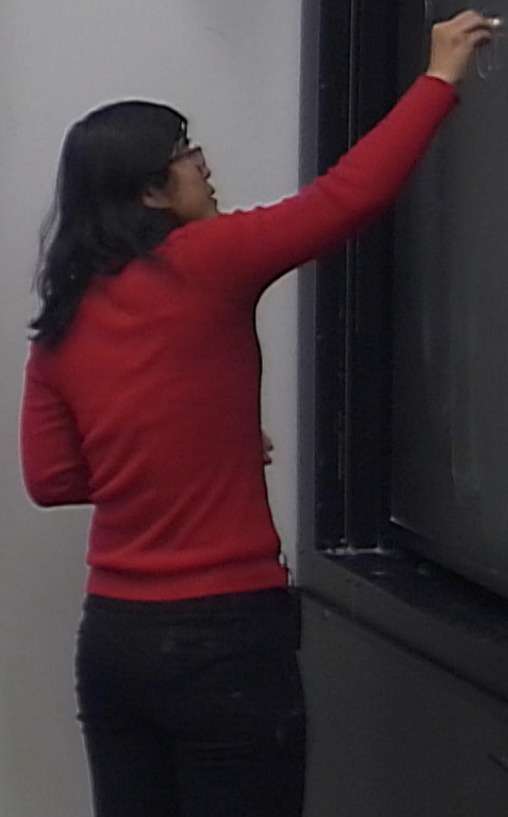
$$\{a, a^\dagger\} = 1 \quad \{a, a\} = 0$$

$$\{a^\dagger, a^\dagger\} = 0$$

$a^\dagger a \leftarrow$ what are the eigen values?

$$[a^\dagger a, a] = -a$$

$$[a^\dagger a, a^\dagger] = a^\dagger$$



$$|0\rangle, |1\rangle$$

$$a^+|0\rangle \propto |1\rangle$$

↑
1 particle state

$$(a^+)^2|0\rangle = 0.$$

$$|0\rangle, |1\rangle$$

$$a^+|0\rangle \propto |1\rangle$$

↑
1 particle state

$$(a^+)^2|0\rangle = 0$$

$$\{\psi(x), \psi^\dagger(y)\} = i\delta(x-y)$$

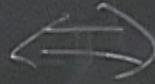
$$\{\psi(x), \psi(y)\} = 0$$

$$\{\psi^\dagger(x), \psi^\dagger(y)\} = 0$$

$$\{\psi(\vec{x}), \psi^\dagger(\vec{y})\} = i\delta(\vec{x}-\vec{y})$$

$$\{\psi(\vec{x}), \psi(\vec{y})\} = 0$$

$$\{\psi^\dagger(\vec{x}), \psi^\dagger(\vec{y})\} = 0$$



$$\{b_{\vec{p}}, b_{\vec{q}}^\dagger\} = (2\pi)^3 2E_{\vec{p}} \delta(\vec{p}-\vec{q})$$

same for c ,

all others vanish.

$$\{b_{\vec{p}}, b_{\vec{q}}^{\dagger}\} = (2\pi)^3 2E_{\vec{p}} \delta(\vec{p} - \vec{q})$$

same for c ,
all others vanish.

$$\text{step 4: } \int dV_{\vec{p}} E_{\vec{p}} (b_{\vec{p}}^{\dagger} b_{\vec{p}} - c_{\vec{p}} c_{\vec{p}}^{\dagger})$$

Normal ordering

$$:H: \int dV_{\vec{p}} E_{\vec{p}} (b_{\vec{p}}^{\dagger} b_{\vec{p}} + c_{\vec{p}}^{\dagger} c_{\vec{p}})$$

$$\{b_{\vec{p}}, b_{\vec{q}}^{\dagger}\} = (2\pi)^3 2E_{\vec{p}} \delta(\vec{p} - \vec{q})$$

same for c ,
all others vanish.

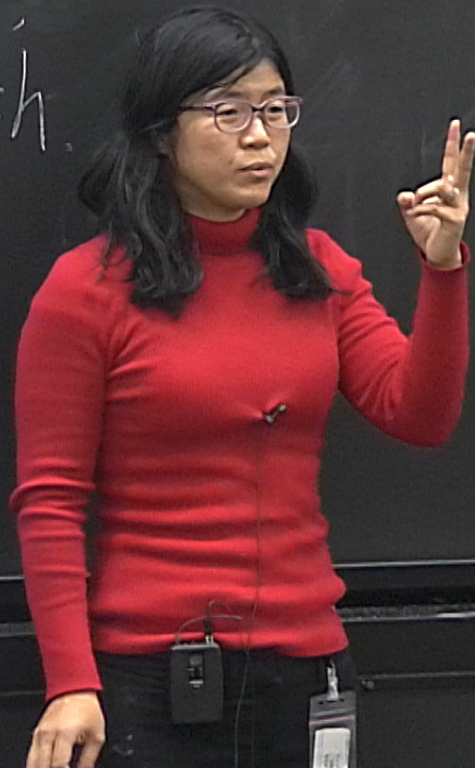
$$\text{step 4: } \int dV_{\vec{p}} E_{\vec{p}} (b_{\vec{p}}^{\dagger} b_{\vec{p}} - c_{\vec{p}} c_{\vec{p}}^{\dagger})$$

Normal ordering

$$\therefore H_{\vec{p}} = \int dV_{\vec{p}} E_{\vec{p}} (b_{\vec{p}}^{\dagger} b_{\vec{p}} + c_{\vec{p}}^{\dagger} c_{\vec{p}})$$

$$\{b_{\vec{p}}, b_{\vec{q}}^{\dagger}\} = (2\pi)^3 2E_p \delta(\vec{p} - \vec{q})$$

same for c ,
all others vanish.



$$\text{step 4: } \int dV_p E_p (b_p^{\dagger} b_p - c_p c_p^{\dagger})$$

Normal ordering

$$:H: = \int dV_p E_p (b_p^{\dagger} b_p + c_p^{\dagger} c_p)$$

fermion rule $:c c^{\dagger}: = - :c^{\dagger} c:$
 $= -c c$

States

$$b_{\vec{p}} |0\rangle = 0$$

$$c_{\vec{p}} |0\rangle = 0$$

States

$$:H: |0\rangle = 0$$

$$b_{\vec{p}} |0\rangle = 0$$

$$c_{\vec{p}} |0\rangle = 0$$

$$\langle 0|0\rangle = 1$$

States

$$:H: |0\rangle = 0$$

1-particle state

$$b_{\vec{p}} |0\rangle = 0$$

$$b_{\vec{p}}^{+s} |0\rangle = |\vec{p}, s\rangle$$

$$c_{\vec{p}} |0\rangle = 0$$

$$\langle 0|0\rangle = 1$$

States

$$:H: |0\rangle = 0$$

$$\langle p, s | q, r \rangle = \delta_{p, q} \delta_{s, r}$$

$$b_{\vec{p}} |0\rangle = 0$$

1-particle state

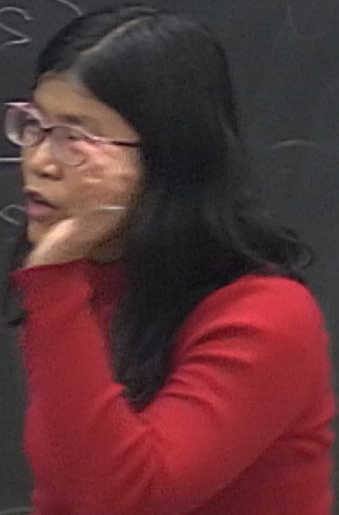
$$b_{\vec{p}}^{+s} |0\rangle = |\vec{p}, s\rangle$$

$$c_{\vec{p}} |0\rangle = 0$$

$$c_{\vec{p}}^{+s} |0\rangle = |\vec{p}, s\rangle$$

$$\langle 0 | 0 \rangle = 1$$

$$\langle \vec{p}, s | \vec{q}, r \rangle = \frac{(2\pi)^3 2E_{\vec{p}}}{2\pi} \delta(\vec{p}-\vec{q}) \delta^{rs}$$



$$\langle \vec{p}, s | \vec{q}, r \rangle = (2\pi)^3 2E_{\vec{p}} \delta(\vec{p}-\vec{q}) \delta^{rs}$$

$$:H: |\vec{p}, s\rangle = :H: b_{\vec{p}}^{\dagger s} |0\rangle$$

$$[:H:, b_{\vec{p}}^{\dagger s}] =$$

$$\langle \vec{p}, s | \vec{q}, r \rangle = (2\pi)^3 2E_{\vec{p}} \delta(\vec{p}-\vec{q}) \delta^{rs}$$

$$:H: |\vec{p}, s\rangle = :H: b_{\vec{p}}^{+s} |0\rangle = [:H:, b_{\vec{p}}^{+s}] |0\rangle$$

$$[:H:, b_{\vec{p}}^{+s}] = E_{\vec{p}} b_{\vec{p}}^{+s} = E_{\vec{p}} | \vec{p}, s \rangle$$

$$|0\rangle, |1\rangle$$

$$a^+|0\rangle \propto |1\rangle$$

↑
1 particle state

$$(a^+)^2|0\rangle = 0$$

$$\{\psi_a(x), \psi_b^+(\bar{y})\} = i\delta(\underline{x}-\bar{y})\delta_{ab}$$

$$\{\psi_a(x), \psi_b(\bar{y})\} = 0$$

$$\{\psi_a^+(x), \psi_b^+(\bar{y})\} = 0$$

$$\{b_{\vec{p}}, b_{\vec{q}}^+\} = 0$$

same for c

all others vanish

$$\mathcal{L}_{\text{QED}} = \bar{\psi} (i \gamma^\mu D_\mu - m) \psi$$

$$\psi \rightarrow e^{-iQ\alpha(x)} \psi$$

global subgroup $\psi \rightarrow e^{-iQ\alpha} \psi$ ↓ const again

$$i \delta \psi = -iQ\alpha \psi$$

$$j^\mu = \bar{\psi} \gamma^\mu \psi$$

$$(\not{D}_\mu - m)\psi$$

$$\alpha(x)\psi$$

$$e^{-iQ\alpha} \begin{matrix} \downarrow \text{const} \\ \downarrow \text{again} \end{matrix} \psi$$

$$\psi = -iQ\alpha\psi$$

$$j^\mu = \bar{\psi}\gamma^\mu\psi$$

new current

$$j^\mu = Q\bar{\psi}\gamma^\mu\psi$$

$$\int d^3x j^0 = Q \int d^3x \bar{\psi}\gamma^0\psi$$

$$(\not{D}_\mu - m)\psi$$

$$\alpha(x)\psi$$

$$\psi \rightarrow e^{-iQ\alpha} \psi$$

const again

$$\not{\partial}\psi = -iQ\alpha\psi$$

$$j^\mu = \bar{\psi}\gamma^\mu\psi$$

new current

$$j^\mu = Q\bar{\psi}\gamma^\mu\psi$$

$$\int d^3x j^0 = Q \int d^3x \psi^\dagger\psi$$

$T_{\mu\nu}$

current

$\partial_\mu T^{\mu\nu}$

$\partial \int d^3x \psi^\dagger \psi$

$$\psi(x) = \sum_s \int dV_p b_p^s u_a^s(p) e^{-ip \cdot x} + \sum_s \int dV_p c_p^{ts} v_a^s(p) e^{+ip \cdot x}$$

$\overline{\psi} \gamma^\mu \psi$

current

$\overline{\psi} \gamma^\mu \psi$

$\int d^3x \overline{\psi} \gamma^\mu \psi$

$$\psi_a(\vec{x}) = \int dV_p \left(b_p^s u_a^s(\vec{p}) e^{-i\vec{p}\cdot\vec{x}} + c_p^{ts} v_a^s(\vec{p}) e^{+i\vec{p}\cdot\vec{x}} \right)$$

$$\psi_a^\dagger(\vec{x}) = \int dV_q \left(b_q^\dagger u_a^\dagger(\vec{q}) e^{+i\vec{q}\cdot\vec{x}} + c_q^\dagger v_a^\dagger(\vec{q}) e^{-i\vec{q}\cdot\vec{x}} \right)$$

$$j^\mu = \bar{\psi} \gamma^\mu \psi$$

new current

$$j^\mu = Q \bar{\psi} \gamma^\mu \psi$$

$$\int d^3x j^0 = Q \int d^3x \psi^\dagger \psi = Q \int d^3x$$

$$\psi(\vec{x}) = \int \frac{d^3p}{(2\pi)^3} b_p^s u^s(\vec{p}) e^{-ip \cdot x} + \int \frac{d^3p}{(2\pi)^3} c_p^{\dagger s} v^s(\vec{p}) e^{+ip \cdot x}$$

$$\psi^\dagger(\vec{x}) = \int \frac{d^3q}{(2\pi)^3} b_q^{\dagger s} u^{\dagger s}(\vec{q}) e^{+iq \cdot x} + \int \frac{d^3q}{(2\pi)^3} c_q^s v^s(\vec{q}) e^{-iq \cdot x}$$

$$j^\mu = \bar{\psi} \gamma^\mu \psi$$

new current

$$j^\mu = Q \bar{\psi} \gamma^\mu \psi$$

$$\int d^3x j^0 = Q \int d^3x \psi^\dagger \psi = Q \int d^3x \int dV_p \int dV_q \left(b_q^\dagger b_p u(q) u^\dagger(p) e^{-ix(p-q)} + c_q c_p^\dagger v(q) v^\dagger(p) e^{ix(p-q)} \right) = Q \int dV_p \left(b_p^\dagger b_p + c_q c_q^\dagger \right)$$

$$\psi(x) = \int dV_p \left(b_p^s u(p) e^{-ip \cdot x} + c_p^{\dagger s} v(p) e^{ip \cdot x} \right)$$

$$\psi^\dagger(x) = \int dV_q \left(b_q^\dagger u^\dagger(q) e^{iq \cdot x} + c_q v^\dagger(q) e^{-iq \cdot x} \right)$$

$$: \int d^3x j^0 : = Q \int dV_{\vec{p}} \left(b_{\vec{p}}^\dagger b_{\vec{p}} - c_{\vec{p}}^\dagger c_{\vec{p}} \right)$$

normal ordering



Causality.

$$\psi(\vec{x}, t) = \int dV_p \left[b_{\vec{p}} u(\vec{p}) e^{-ip \cdot x} + c_{\vec{p}}^{\dagger} u(\vec{p}) e^{ip \cdot x} \right]$$

Causality.

$$\psi(\vec{x}, t) = \int dV_p b_{\vec{p}} u(\vec{p}) e^{-ip \cdot x} + C_{\vec{p}}^{\dagger} u(\vec{p}) e^{ip \cdot x}$$

Scalar $[\phi(x), \phi(y)] = 0$ spacelike.

Causality.

$$\phi(\vec{x}, t) = \int dV_p b_{\vec{p}} u(\vec{p}) e^{-ip \cdot x} + C_{\vec{p}}^{\dagger} u(\vec{p}) e^{ip \cdot x}$$

Scalar $[\phi(x), \phi(y)] = 0$ spacelike

$$[\psi(x), \bar{\psi}(y)] \neq 0$$

Causality.

$$\psi(\vec{x}, t) = \int dV_p b_{\vec{p}} u(\vec{p}) e^{-ip \cdot x} + C_{\vec{p}}^{\dagger} u(\vec{p}) e^{ip \cdot x}$$

$$[\mathcal{O}_1, \mathcal{O}_2] = 0$$

Scalar $[\phi(x), \phi(y)] = 0$ spacelike

$$[\psi(x), \bar{\psi}(y)] \neq 0$$

$$[\mathcal{O}_1, \mathcal{O}_2] = 0 \quad \text{spacelike separation}$$

$$[O_1, O_2] = 0 \quad \text{spacelike separation}$$

$$[\psi_a \Gamma_{ab} \psi_b, \psi_c \Gamma_{cd} \psi_d] = 0$$

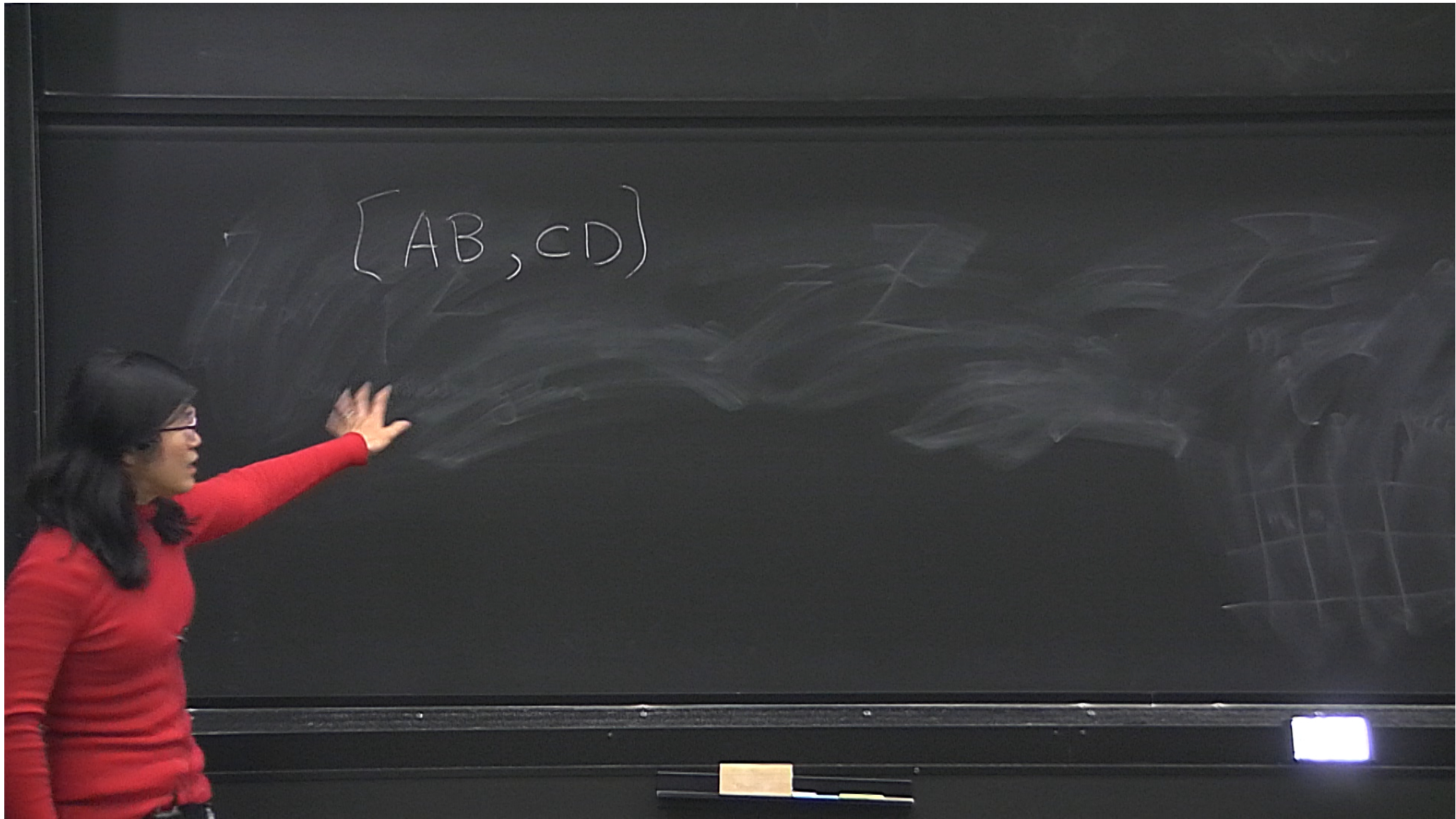
$$[\psi_a \psi_b, \psi_c \psi_d] \Gamma_{ab} \Gamma_{cd} = 0$$

$$[O_1, O_2] = 0 \quad \text{spacelike separation}$$

$$[\bar{\psi}_a \Gamma_{ab} \psi_b, \bar{\psi}_c \Gamma_{cd} \psi_d] = 0$$

$$[\bar{\psi}_a \psi_b, \bar{\psi}_c \psi_d] \Gamma_{ab} \Gamma_{cd} = 0$$

$$[\bar{\psi}_a \psi_b, \bar{\psi}_c \psi_d] = 0 \quad \underline{\text{spacelike}}$$



$$[AB, CD]$$

$$\begin{aligned} [AB, \Delta] &= A[B, \Delta] \\ &= AB\Delta - \Delta AB + [A, \Delta]B \\ &= AB\Delta - A\Delta B + A\Delta B \end{aligned}$$

$$[AB, CD] = A[B, CD] + [A, CD]B$$

$$\begin{aligned} [AB, \Delta] &= A[B, \Delta] \\ &= AB\Delta - \Delta AB + [A, \Delta]B \\ &= AB\Delta - A\Delta B + A\Delta B \end{aligned}$$

$$+ C \bar{q} \bar{p}^T v(\bar{q}) v(\bar{p}) e$$

B

$$[A, CD] = [A, C]D - [A, D]C$$

$$+C \bar{q} \bar{p}^T v(\bar{q}) v(\bar{p}) e$$

B

$$[A, CD] = \{A, C\}D - \{A, D\}C$$

$$= ACD - CDA$$

$$= ACD + CAD - CAD - CDA$$

$$= A[B, CD] + [A, CD]B$$

$$= A[B, C] + [A, C]B$$

$$= A(\{B, C\}D - C\{B, D\}) + (\{A, C\}D - C\{A, D\})B$$

$$[A, CD] = \{A, C\}D - C\{A, D\}$$

$$= ACD - CDA$$

$$= ACD + CAD - CAD - CDA$$

$$A[B, CD] + [A, CD]B$$

$$= \overline{A}(\{B, \overline{C}\}D - \overline{C}\{B, D\}) + (\overline{A}, \overline{C}\}D - \overline{C}\{A, D\})B$$

$$[A, CD] = \{A, C\}D - C\{A, D\}$$

$$= ACD - CDA$$

$$= ACD + CAD - CAD - CDA$$

$$A[B, CD] + [A, CD]B$$

$$= A(\overline{B, C}D - C\overline{B, D}) + (\overline{A, C}D - C\overline{A, D})B$$

$$[A, CD] = \{A, C\}D - C\{A, D\}$$

$$= ACD - CDA$$

$$= ACD + CAD - CAD - CDA$$

$$+ C \bar{p} C^T v(\bar{p}) v(\bar{p}) e$$

B
 $\bar{C} \{B, D\}$
 $\bar{A}, D \} B$

$$[A, CD] = \{A, C\}D - \{A, D\}C$$

$$= ACD - CDA$$

$$= ACD + CAD - CAD - CDA$$

$\{\psi, \bar{\psi}\}$



$$\{\gamma(x), \Psi(y)\}$$

m

$$S = \int (a(x)^2 - 2im)^2 dx$$

lets to be
 \approx integer

Substitution

$$E = \int \dots dx$$

$= -c^{\dagger}c$

$\psi^{\dagger}\psi$
current
 $\psi^{\dagger}\psi$

$$\psi_a(x) = \int dV_p \left[b_p^s u_a^s(p) e^{-ip \cdot x} + c_p^{\dagger s} v_a^s(p) e^{+ip \cdot x} \right]$$

$$\bar{\psi}(\bar{x}) = \int dV_q \left[b_q^{\dagger} \bar{u}(\bar{q}) e^{iq \cdot \bar{x}} + c_q v(\bar{q}) e^{-iq \cdot \bar{x}} \right]$$

$$\alpha \int d^3x \psi^{\dagger} \psi = \alpha \int d^3x dV_p dV_q \left[b_q^{\dagger} b_p u^{\dagger}(\bar{q}) u^s(p) e^{-ix^i(p-q)_i} + c_q c_p^{\dagger} v(\bar{q}) v^s(p) e^{ix^i(p-q)_i} \right] = \alpha \int dV_p \left[b_p^{\dagger} b_p + c_q c_q^{\dagger} \right]$$

$$\{\Psi(x), \bar{\Psi}(y)\} \\ = \int dV_p dV_q \{b, b^\dagger\} u(p)\bar{u}(q) e^{-ipx + iqy}$$

$$S = \int d^4x \bar{\psi} (\not{\partial} - 2im) \psi$$

$= -c^{\dagger}c$

$\psi^{\dagger} \psi$

current

$\psi^{\dagger} \psi$

$$\psi(x) = \sum_s \int dV_p b_{\vec{p}}^s u^s(p) e^{-ip \cdot x} + c_{\vec{p}}^{\dagger s} v^s(p) e^{+ip \cdot x}$$

$$\bar{\psi}(y) = \int dV_q b_{\vec{q}}^{\dagger} \bar{u}(q) e^{iq \cdot y} + c_{\vec{q}}^{\dagger} \bar{v}(q) e^{-iq \cdot y}$$

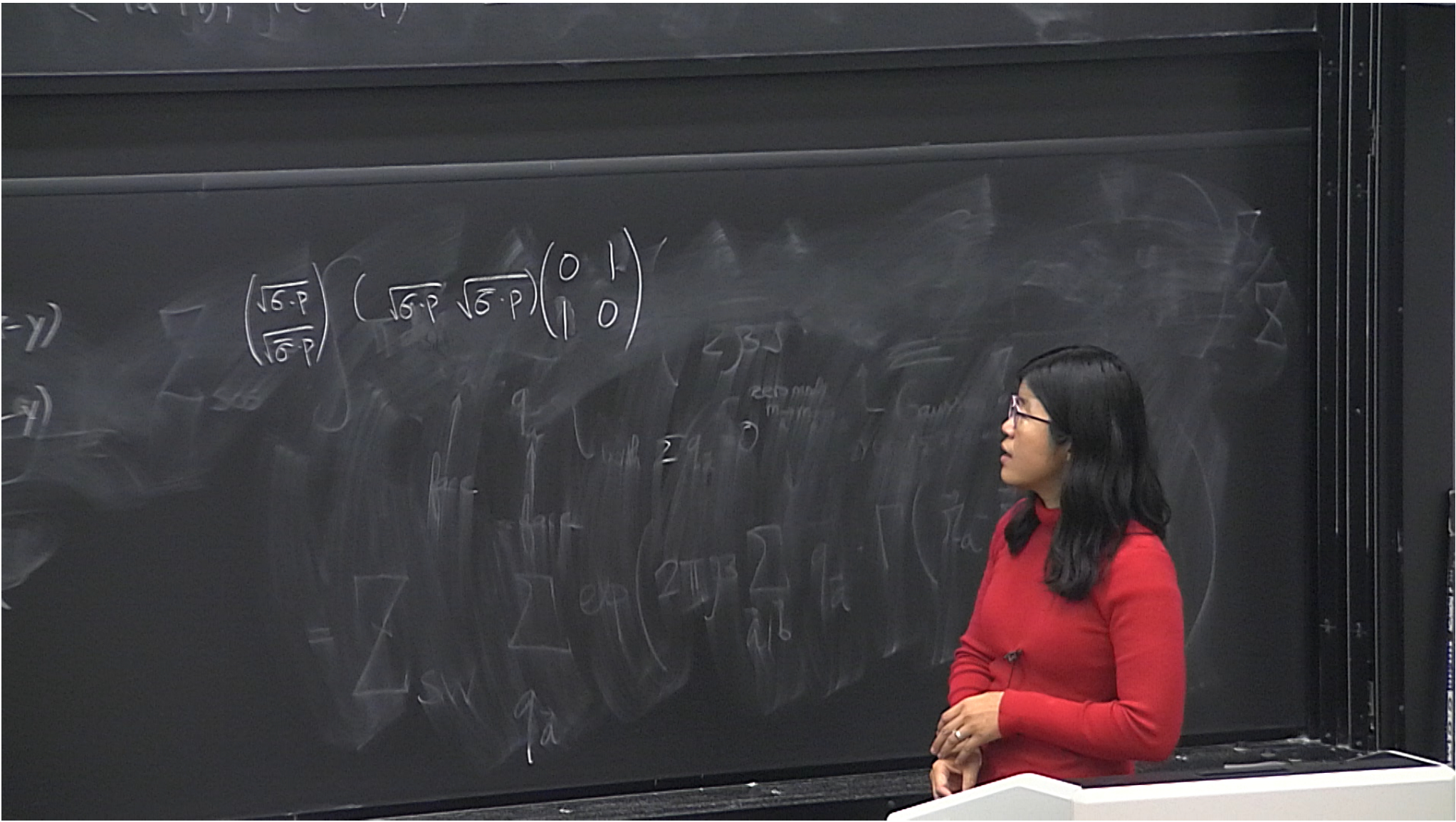
$$\int d^3x \psi^{\dagger} \psi = \int d^3x \int dV_p \int dV_q b_{\vec{q}}^{\dagger} b_{\vec{p}} u^{\dagger}(q) u^s(p) e^{-ix \cdot (p-q)} + c_{\vec{q}} c_{\vec{p}}^{\dagger} v^{\dagger}(q) v(p) e^{ix \cdot (p-q)}$$

$$= \int dV_p \left(b_{\vec{p}}^{\dagger} b_{\vec{p}} + c_{\vec{q}} c_{\vec{q}}^{\dagger} \right)$$

$$\begin{aligned}
 & \{ \psi(x), \bar{\psi}(y) \} \\
 &= \int d^4p d^4q \{ b, b^\dagger \} u(p) \bar{u}(q) e^{-ipx + iq \cdot y} \\
 &+ \int d^4p d^4q \{ c, c^\dagger \} v(p) \bar{v}(q) e^{+ipx - iq \cdot y}
 \end{aligned}$$

$$\begin{aligned}
 & \{ \psi(x), \bar{\psi}(y) \} \\
 &= \int dV_p dV_q \underbrace{\{ b, b^\dagger \}}_{(2\pi)^3 \delta^3(\vec{p}-\vec{q})} u(\vec{p}) \bar{u}(\vec{q}) e^{-ipx + iq \cdot y} = \int dV_p u(\vec{p}) \bar{u}(\vec{p}) e^{-ip(x-y)} \\
 &+ \int dV_p dV_q \underbrace{\{ c, c^\dagger \}}_{(2\pi)^3 \delta^3(\vec{p}-\vec{q})} v(\vec{p}) \bar{v}(\vec{q}) e^{+ipx - iq \cdot y}
 \end{aligned}$$

$$\begin{aligned}
 & \{ \psi(x), \bar{\psi}(y) \} \\
 &= \int dV_p dV_q \left\{ \begin{array}{l} \text{---} \\ \text{---} \end{array} \right\} \begin{array}{l} \xrightarrow{(2\pi)^3 \delta^3(\vec{p}-\vec{q})} \\ \psi(p) \bar{u}(q) e^{-ipx + iq \cdot y} \\ \psi(q) \bar{v}(p) e^{+ipx - iq \cdot y} \end{array} = \int dV_p \begin{array}{l} u(p) \bar{u}(p) e^{-ip(x-y)} \\ v(p) \bar{v}(p) e^{ip(x-y)} \end{array}
 \end{aligned}$$



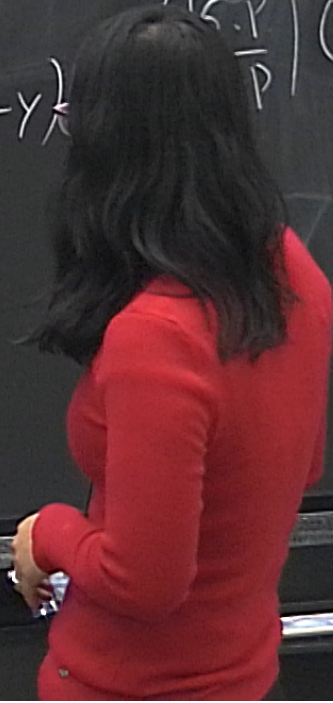
$$\begin{aligned}
 & \begin{pmatrix} \sqrt{5 \cdot p} \\ \sqrt{5 \cdot p} \end{pmatrix} \begin{pmatrix} \sqrt{5 \cdot p} & \sqrt{5 \cdot p} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\
 & = \begin{pmatrix} \sqrt{5 \cdot p} \\ \sqrt{5 \cdot p} \end{pmatrix} \begin{pmatrix} \sqrt{5 \cdot p} & \sqrt{5 \cdot p} \end{pmatrix} = \begin{pmatrix} m & \sigma \cdot p \\ \sigma \cdot p & m \end{pmatrix} = p \gamma^\mu + m
 \end{aligned}$$



$$\begin{aligned}
 &= \int dV_{\vec{p}} \left(\begin{array}{c} u(\vec{p}) \bar{u}(\vec{p}) e^{-ip(x-y)} \\ v(\vec{p}) \bar{v}(\vec{p}) e^{ip(x-y)} \end{array} \right) \\
 &= \int dV_{\vec{p}} \left(\begin{array}{c} p_{\mu} \gamma^{\mu} + m \\ \dots \end{array} \right) e^{-ip(x-y)}
 \end{aligned}$$

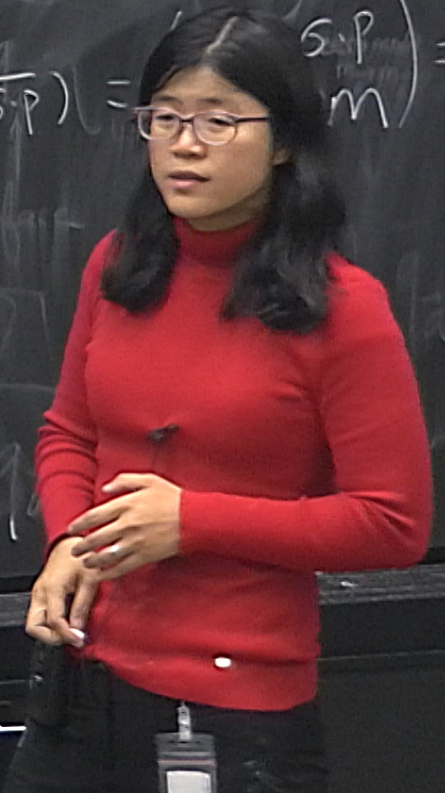
$$\left(\frac{\sqrt{\sigma \cdot p}}{\sqrt{\sigma \cdot p}} \right) \left(\begin{array}{cc} \sqrt{\sigma \cdot p} & \sqrt{\sigma \cdot p} \\ \sqrt{\sigma \cdot p} & \sqrt{\sigma \cdot p} \end{array} \right) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\left(\frac{\sqrt{\sigma \cdot p}}{\sqrt{\sigma \cdot p}} \right) \left(\begin{array}{cc} \sqrt{\sigma \cdot p} & \sqrt{\sigma \cdot p} \\ \sqrt{\sigma \cdot p} & \sqrt{\sigma \cdot p} \end{array} \right) = \begin{pmatrix} m & \sigma \cdot p \\ \sigma \cdot p & m \end{pmatrix} = \not{p} + m$$



$$\begin{aligned}
 &= \int dV_{\vec{p}} \begin{pmatrix} u(\vec{p}) \\ v(\vec{p}) \end{pmatrix} \bar{u}(\vec{p}) e^{-ip(x-y)} \\
 &= \int dV_{\vec{p}} \begin{pmatrix} i\alpha_x \\ p_\mu \gamma^\mu + m \end{pmatrix} e^{-ip(x-y)} \\
 &\quad \begin{pmatrix} -i\alpha_x \\ p_\mu \gamma^\mu - m \end{pmatrix} e^{-ip(y-x)}
 \end{aligned}$$

$$\begin{pmatrix} \sqrt{E \cdot p} \\ \sqrt{E \cdot p} \end{pmatrix} \begin{pmatrix} \sqrt{E \cdot p} & \sqrt{E \cdot p} \\ \sqrt{E \cdot p} & -\sqrt{E \cdot p} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

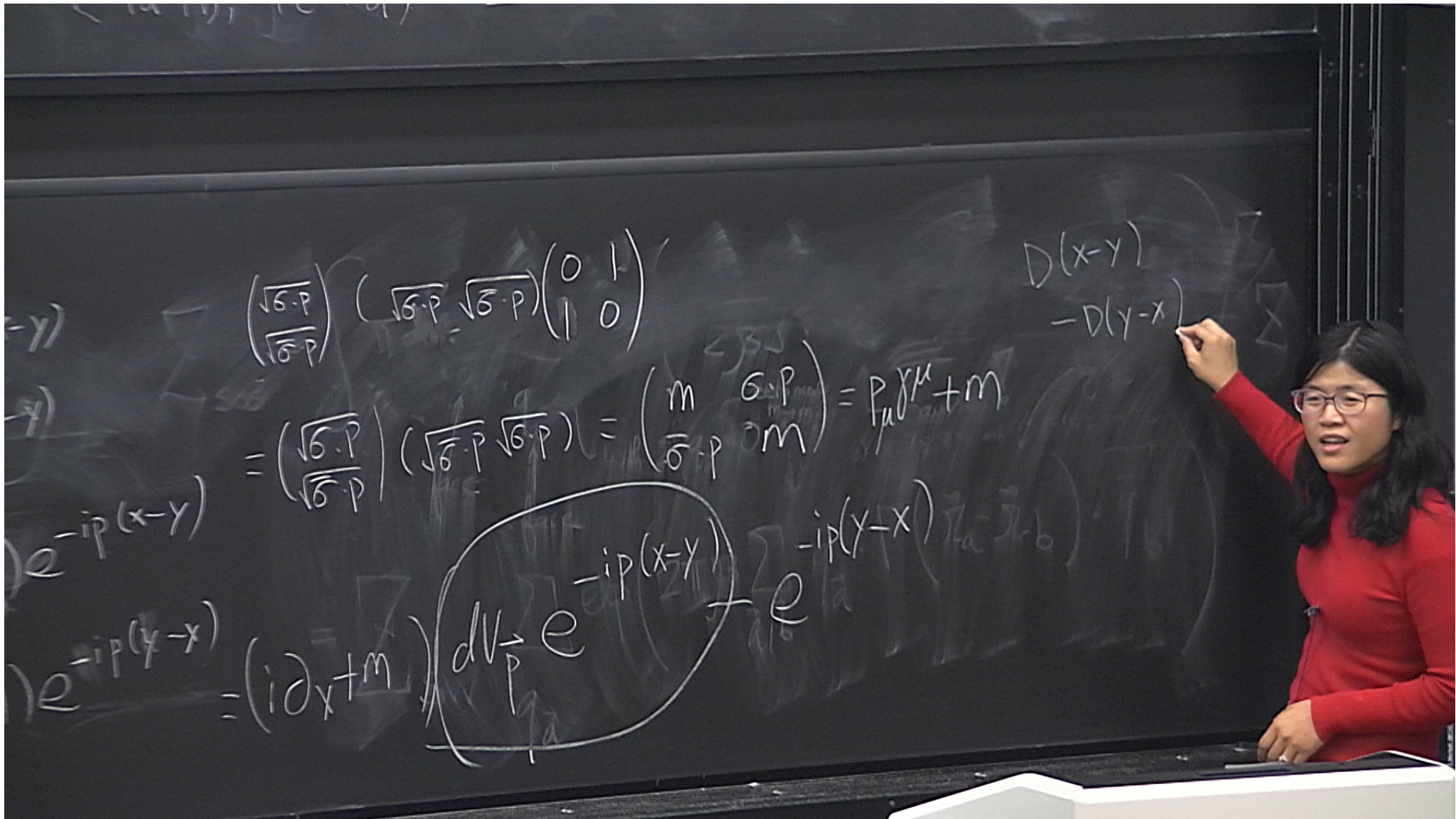


$$\begin{aligned}
 &= \int dV_{\vec{p}} \begin{pmatrix} u(\vec{p}) \\ v(\vec{p}) \end{pmatrix} \begin{pmatrix} \bar{u}(\vec{p}) \\ \bar{v}(\vec{p}) \end{pmatrix} e^{-ip(x-y)} \\
 &= \int dV_{\vec{p}} \begin{pmatrix} i\partial_x \\ p_\mu \gamma^\mu + m \end{pmatrix} e^{-ip(x-y)} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\
 &\quad \begin{pmatrix} -i\partial_x \\ p_\mu \gamma^\mu - m \end{pmatrix} e^{-ip(y-x)} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = (i\partial_x + m) \int dV_{\vec{p}} e^{-ip(x-y)} - e^{-ip(y-x)}
 \end{aligned}$$

$\begin{pmatrix} \sqrt{\sigma \cdot p} \\ \sqrt{\sigma \cdot p} \end{pmatrix} \begin{pmatrix} \sqrt{\sigma \cdot p} & \sqrt{\sigma \cdot p} \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} m & \sigma \cdot p \\ \sigma \cdot p & m \end{pmatrix} = p_\mu \gamma^\mu$

$$\begin{aligned}
 &= \int dV_{\vec{p}} \begin{pmatrix} u(\vec{p}) \\ v(\vec{p}) \end{pmatrix} \bar{u}(\vec{p}) e^{-ip(x-y)} \\
 &= \int dV_{\vec{p}} \begin{pmatrix} i\partial_x \\ \not{p} \end{pmatrix} \begin{pmatrix} \chi^\mu + m \\ \chi^\mu - m \end{pmatrix} e^{-ip(x-y)} \\
 &= (i\partial_x + m) \left(\int dV_{\vec{p}} e^{-ip(x-y)} - e^{-ip(y-x)} \right)
 \end{aligned}$$

$$\begin{pmatrix} \sqrt{\sigma \cdot p} \\ \sqrt{\sigma \cdot p} \end{pmatrix} \begin{pmatrix} \sqrt{\sigma \cdot p} & \sqrt{\sigma \cdot p} \\ 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} m & \sigma \cdot p \\ \sigma \cdot p & m \end{pmatrix} = \not{p} \chi^\mu + m$$



$$\begin{pmatrix} \sqrt{\sigma \cdot p} \\ \sqrt{\sigma \cdot p} \end{pmatrix} \begin{pmatrix} \sqrt{\sigma \cdot p} & \sqrt{\sigma \cdot p} \\ \sqrt{\sigma \cdot p} & \sqrt{\sigma \cdot p} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt{\sigma \cdot p} \\ \sqrt{\sigma \cdot p} \end{pmatrix} \begin{pmatrix} \sqrt{\sigma \cdot p} & \sqrt{\sigma \cdot p} \\ \sqrt{\sigma \cdot p} & \sqrt{\sigma \cdot p} \end{pmatrix} = \begin{pmatrix} m & \sigma \cdot p \\ \sigma \cdot p & m \end{pmatrix} = p_\mu \gamma^\mu + m$$

$$D(x-y) - D(y-x)$$

$$e^{-ip(x-y)} - e^{-ip(y-x)} = (i\partial_x + m) \left(\frac{dV}{p} e^{-ip(x-y)} - e^{-ip(y-x)} \right)$$

$$\begin{pmatrix} \sqrt{\sigma \cdot p} \\ \sqrt{\sigma \cdot p} \end{pmatrix} \begin{pmatrix} \sqrt{\sigma \cdot p} & \sqrt{\sigma \cdot p} \\ \sqrt{\sigma \cdot p} & \sqrt{\sigma \cdot p} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt{\sigma \cdot p} \\ \sqrt{\sigma \cdot p} \end{pmatrix} \begin{pmatrix} \sqrt{\sigma \cdot p} & \sqrt{\sigma \cdot p} \\ \sqrt{\sigma \cdot p} & \sqrt{\sigma \cdot p} \end{pmatrix} = \begin{pmatrix} m & \sigma \cdot p \\ \sigma \cdot p & m \end{pmatrix} = p_{\mu} \gamma^{\mu} + m$$

$$\begin{pmatrix} D(x-y) \\ -D(y-x) \end{pmatrix}$$

$$(i\partial_x + m) \left(dV_{\vec{p}} e^{-ip(x-y)} - e^{-ip(y-x)} \right)$$

$$\begin{pmatrix} \sqrt{\sigma \cdot p} \\ \sqrt{\sigma \cdot p} \end{pmatrix} \begin{pmatrix} \sqrt{\sigma \cdot p} & \sqrt{\sigma \cdot p} \\ \sqrt{\sigma \cdot p} & \sqrt{\sigma \cdot p} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt{\sigma \cdot p} \\ \sqrt{\sigma \cdot p} \end{pmatrix} \begin{pmatrix} \sqrt{\sigma \cdot p} & \sqrt{\sigma \cdot p} \\ \sqrt{\sigma \cdot p} & \sqrt{\sigma \cdot p} \end{pmatrix} = \begin{pmatrix} m & \sigma \cdot p \\ \sigma \cdot p & m \end{pmatrix} = p \gamma^\mu + m$$

$$\begin{pmatrix} D(x-y) \\ -D(y-x) \end{pmatrix}$$

$$e^{-ip(x-y)}$$

$$e^{-ip(y-x)} = \left(i \frac{\gamma^\mu}{\not{x} + m} \right) \left(\frac{dV_{\vec{p}}}{p_0} e^{-ip(x-y)} - e^{-ip(y-x)} \right)$$