Title: PSI 2018/2019 - Quantum Field Theory I - Lecture 8

Date: Oct 22, 2018 09:00 AM

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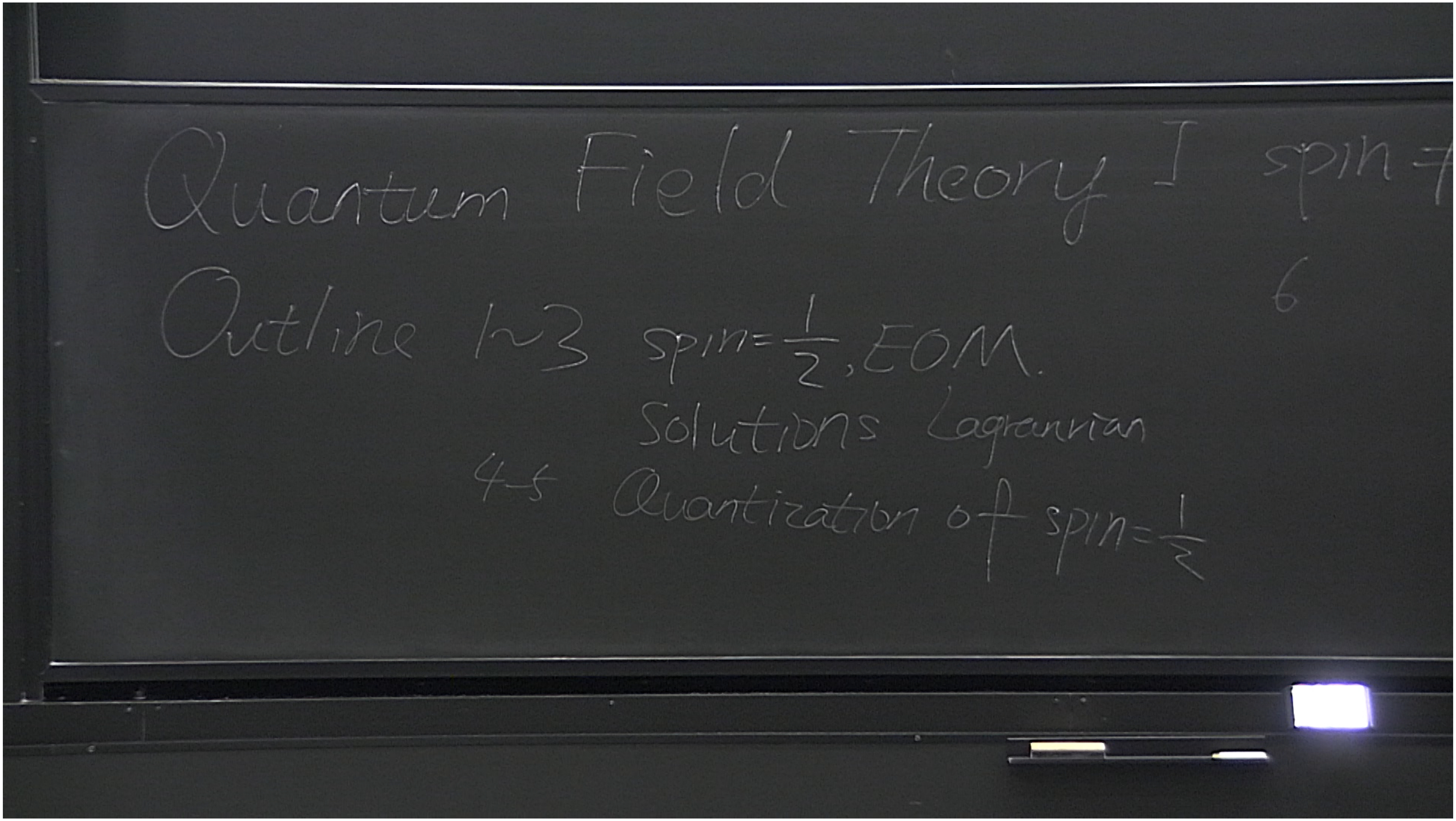
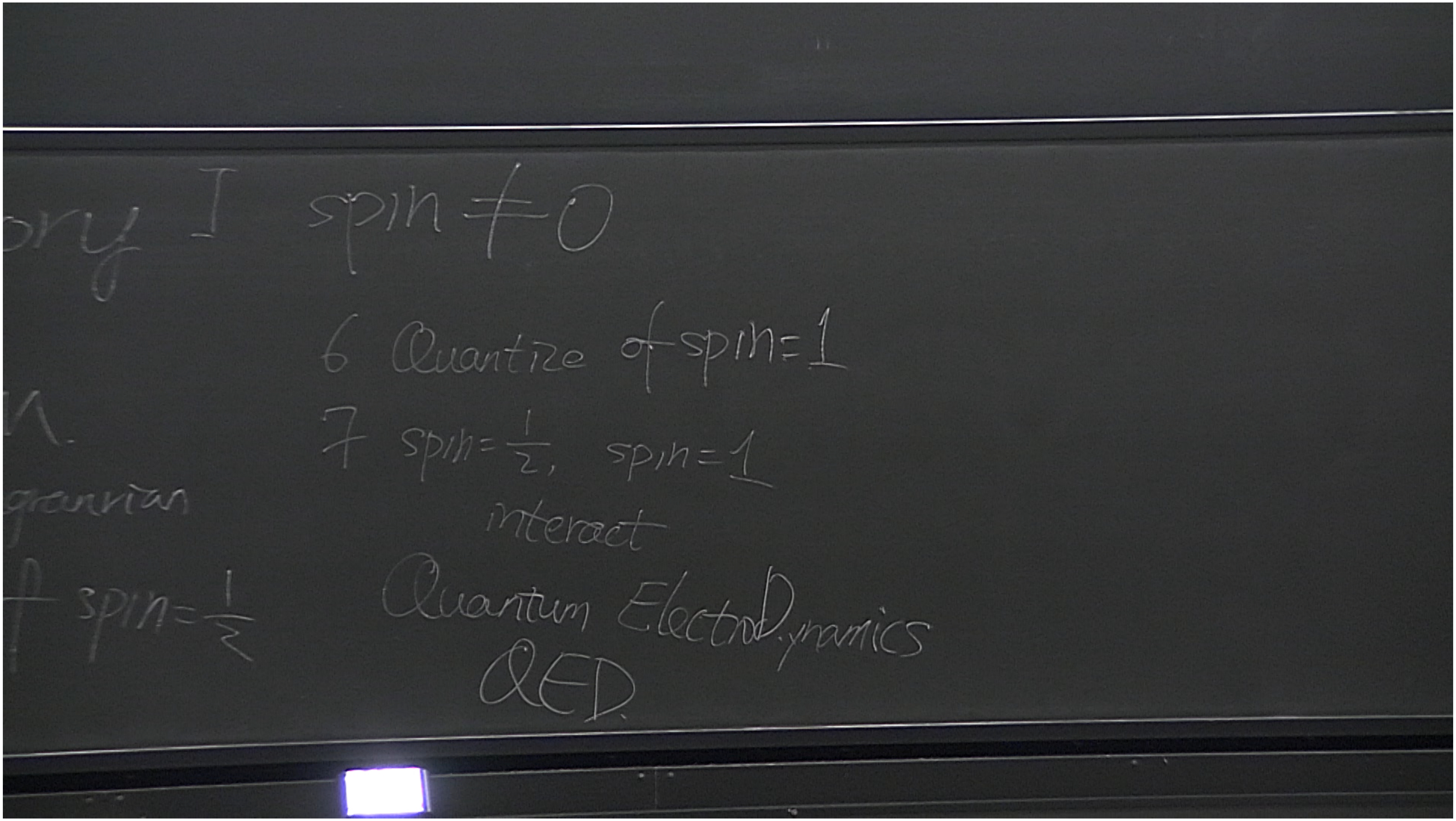
Abstract:


Quantum Field Theory I
spin =
Outline 1~3 spin=$\frac{1}{2}$, EOM.
Solutions Lagrannian
4-5 Quantization of spin=$\frac{1}{2}$
6


ory I spin ≠ 0
6 Quantize of spin = 1
7 spin = 1/2, spin = 1
interact
Quantum ElectroDynamics
QED

$= 0$

...ze of spin $= 1$

..., spin $= 1$

...teract

...tum ElectroDynamics

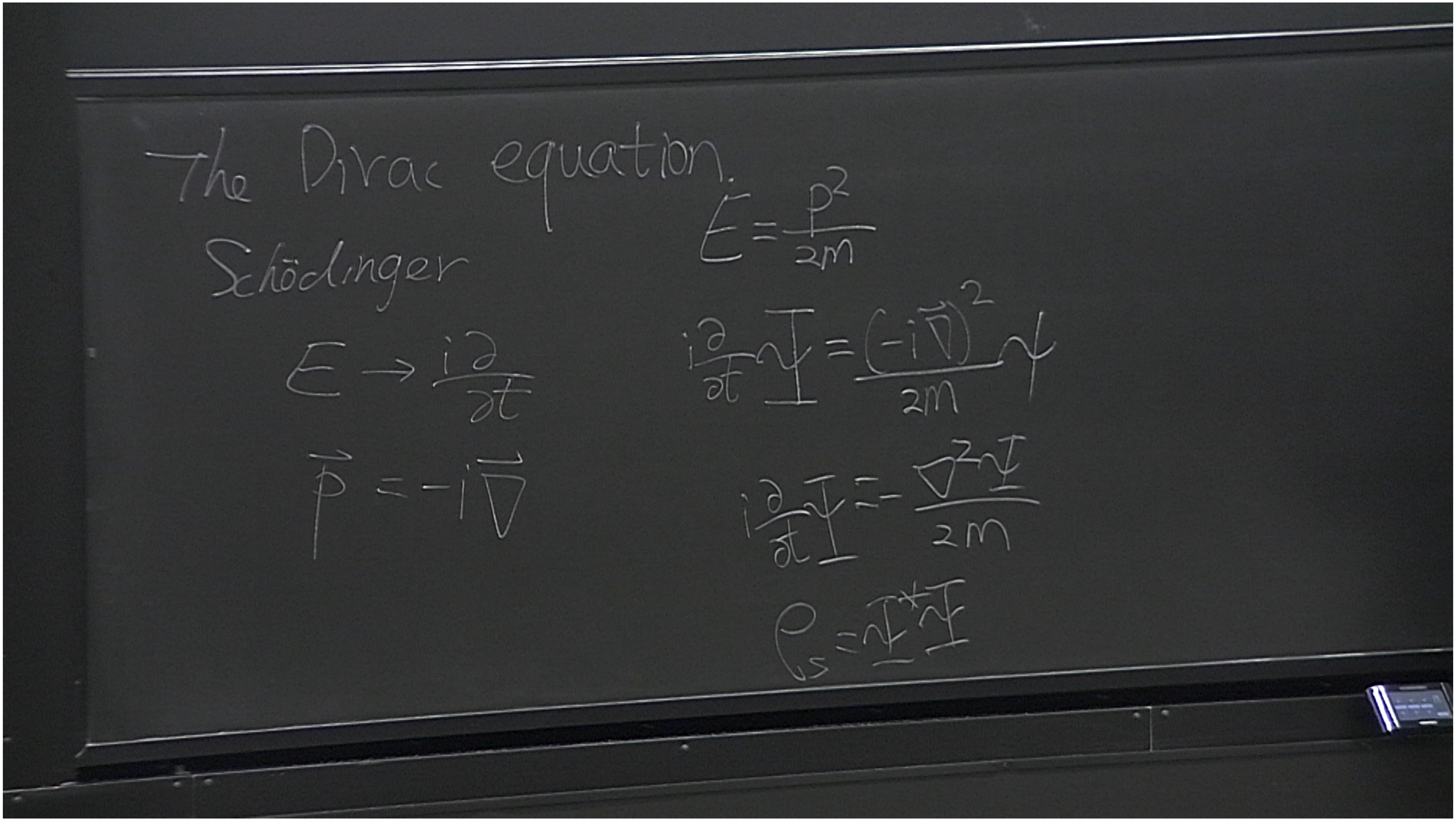
QED.

convention.

$\eta_{\mu\nu} = (+,-,-,-)$ $\quad p^2 = m^2$

$\hbar = c = 1$.

pairs of indices $\Rightarrow$ sum.


The Dirac equation
Schödinger
$E \to i\frac{\partial}{\partial t}$
$\vec{p} = -i\vec{\nabla}$
$E = \frac{p^2}{2m}$
$i\frac{\partial}{\partial t}\Psi = \frac{(-i\vec{\nabla})^2}{2m}\Psi$
$i\frac{\partial}{\partial t}\Psi = -\frac{\nabla^2\Psi}{2m}$
$\rho_s = \Psi^*\Psi$

$$\longrightarrow \quad p^\mu p_\mu = p^2 = m^2 .$$

$$p_\mu = i\frac{\partial}{\partial x^\mu} = i\partial_\mu$$

$$(i\partial^\mu)(i\partial_\mu)\Phi = m^2\Phi$$

$$(\partial^2 + m^2)\Phi = 0$$

$$\rho_{KG} = i\left((\partial_0\Phi^*)\Phi - \Phi^*(\partial_0\Phi)\right)$$

$$= i\frac{\partial}{\partial x^\mu} = i\partial_\mu$$

$$(i\partial^\mu)(i\partial_\mu)\Phi = m^2\Phi$$

$$(\partial^2 + m^2)\Phi = 0$$

$$\Phi \propto e^{-ik\cdot x}$$

$$\Phi^* \propto e^{ik\cdot x}$$

$$\rho_{KG} = i\Big(\underbrace{(\partial_0\Phi^*)}_{ik^0}\Phi - \Phi^*(\partial_0\Phi)\Big)$$

$$= -2k_0$$

What Dirac wants

$$\sqrt{\partial^2 + m^2}$$

① one time derivative.

② one space derivative.

③ $p^2 - m^2 = 0 \quad (\partial^2 + m^2)\psi = 0$

$$\sqrt{\partial^2 + m^2}$$

$$(a+ib)(a-ib) = a^2 + b^2$$

$$\gamma^\nu \gamma^\mu \partial_\nu$$

$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$

$$(-i\gamma^\nu \partial_\nu - m)(i\gamma^\mu \partial_\mu - m)\psi = 0$$

$$(\gamma^\nu \gamma^\mu \partial_\nu \partial_\mu + m^2)\psi = 0$$

$$+ib)(a-ib)=a^2+b^2$$

$$-m)\psi=0$$

$$\gamma^\nu\gamma^\mu\partial_\nu\partial_\mu=\partial_\nu\partial_\mu\eta^{\nu\mu}$$

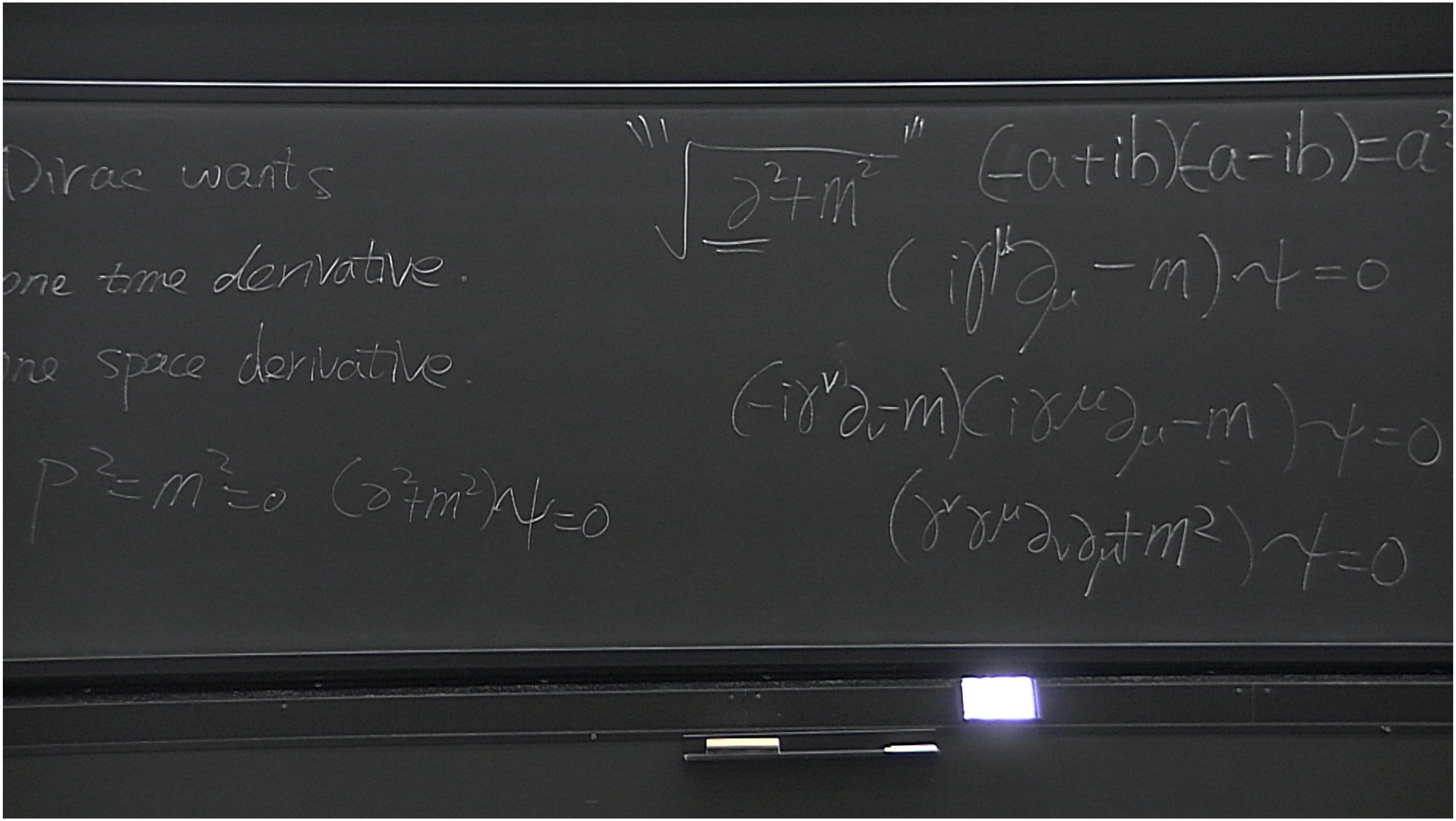
$$\frac{\gamma^\nu\gamma^\mu+\gamma^\mu\gamma^\nu}{2}=\eta^{\nu\mu}$$

$$^\mu)\partial_\mu-m)\psi=0$$

$$\{a,b\}=ab+ba$$

$$\partial_\mu+m^2)\psi=0$$

$$\{\gamma^\nu,\gamma^\mu\}=2\eta^{\nu\mu}$$


Dirac wants
one time derivative.
one space derivative.
$p^2 = m^2 \Rightarrow (\partial^2 + m^2)\psi = 0$
"$\sqrt{\partial^2 + m^2}$"
$(a+ib)(a-ib) = a^2$
$(i\gamma^\mu \partial_\mu - m)\psi = 0$
$(-i\gamma^\nu \partial_\nu - m)(i\gamma^\mu \partial_\mu - m)\psi = 0$
$(\gamma^\nu \gamma^\mu \partial_\nu \partial_\mu + m^2)\psi = 0$

$$i\frac{\partial}{\partial t}\psi = H\psi$$

$$(i\gamma^\mu\partial_\mu - m)\psi = 0$$

$$(i\gamma^0\partial_0 + i\gamma^i\partial_i)\psi - m\psi = 0$$

$$i\gamma^0\frac{\partial}{\partial t}\psi = (-i\gamma^i\partial_i + m)\psi$$

$$i\frac{\partial}{\partial t}\psi = (-i\gamma^0\gamma^i\partial_i + m\gamma^0)\psi = H\psi$$

$$(\gamma^0)^\dagger = \gamma^0\gamma^0\gamma^0 \qquad (\gamma^\mu)^\dagger = \gamma^0\gamma^\mu\gamma^0$$

$$\{\gamma^\nu, \gamma^\mu\} = 2\eta^{\nu\mu}$$

$$(\gamma^0\gamma^i)^\dagger = \gamma^0\gamma^i$$

$$= \gamma^{i\dagger}\gamma^0 = \gamma^0\gamma^i$$

$$\psi \propto e^{ikx}$$

$$\gamma^{i\dagger} = \gamma^0\gamma^i\gamma^0$$

$$i\frac{\partial}{\partial t}\psi = (i)(ik_0) = \underline{-k_0}$$

$$= H\psi$$

$$\left(i\gamma^\mu \frac{\partial}{\partial x^\mu} - m\right)\psi(x) = 0$$

$$\left(i\gamma^\nu \frac{\partial}{\partial x'^\nu} - m\right)\psi'(x') = 0$$

$$x'^\mu = \Lambda^\mu{}_\nu x^\nu$$

$$\psi(x) = S^{-1}(\Lambda)\psi'(x')$$

$$\ldots m)\,\psi(x)=0$$

$$\ldots m)\,\psi'(x')=0$$

$$\ldots x^{\nu}$$

$$\ldots) = S^{-1}(\Lambda)\,\psi'(x')$$

$$S(\Lambda)\left(i\gamma^{\mu}\frac{\partial x'^{\nu}}{\partial x^{\mu}}\frac{\partial}{\partial x'^{\nu}}-m\right)S^{-1}(\Lambda)\,\psi'(x')=0$$

$$S(\Lambda)\,\gamma^{\mu}\,\Lambda^{\nu}{}_{\mu}\,S^{-1}(\Lambda)=\gamma^{\nu}$$

$$S(\Lambda)\,\gamma^\mu \Lambda_{\nu\mu}\, S^{-1}(\Lambda) = \gamma_\nu$$

$$\boxed{\frac{4\times 3}{2} = 6}$$

$$\Lambda^\mu{}_\nu = \delta^\mu{}_\nu + \varepsilon^\mu{}_\nu$$

$$\Lambda_{\mu\nu} = \eta_{\mu\nu} + \underline{\varepsilon_{\mu\nu}}$$ ← anti-symmetric

$$x'^\mu = x^\mu + \varepsilon^\mu{}_\rho x^\rho$$

$$x'_\mu = x_\mu + \varepsilon_{\mu\nu} x^\nu$$

$$\underline{\varepsilon_{\mu\nu} x^\nu x^\mu} + \varepsilon_{\mu\rho} x^\rho x^\mu = 0$$

$$S(\Lambda) = e^{\frac{1}{2}\varepsilon_{\mu\nu} S^{\mu\nu}} \sim 1 + \frac{1}{2}\varepsilon_{\mu\nu} S^{\mu\nu}$$

$$S^{\alpha\beta}$$

$$\left(1+\frac{1}{2}\varepsilon_{\alpha\beta} S^{\alpha\beta}\right)\gamma^{\mu}\left(\eta_{\nu\mu} + \varepsilon_{\nu\mu}\right)\left(1-\frac{1}{2}\varepsilon_{\rho\sigma} S^{\rho\sigma}\right) = \gamma_{\nu}$$

$$\frac{1}{2}\varepsilon_{\alpha\beta} S^{\alpha\beta}\gamma_{\nu} + \gamma^{\beta}\varepsilon_{\alpha\beta}\delta^{\alpha}_{\nu} - \frac{1}{2}\gamma_{\nu}\varepsilon_{\alpha\beta} S^{\alpha\beta} = 0$$

$$\varepsilon_{\alpha\beta}\,\frac{\gamma^{\beta}\delta^{\alpha}_{\nu} - \gamma^{\alpha}\delta^{\beta}_{\nu}}{2}$$

6

$\ _{\rho} x^{\rho}$

$=0$

$$\ldots_{\mu\nu} S^{\mu\nu} \sim 1 + \frac{1}{2}\varepsilon_{\mu\nu} S^{\mu\nu}$$

$$\ldots^{\mu} (\eta_{\nu\mu} + \underline{\varepsilon_{\nu\mu}})\left(1 - \frac{1}{2}\varepsilon_{\rho\sigma} S^{\rho\sigma}\right) = \gamma_\nu$$

$$+\gamma^\beta \underline{\varepsilon_{\alpha\beta}\delta^\alpha_\nu} - \frac{1}{2}\gamma_\nu \varepsilon_{\alpha\beta} S^{\alpha\beta} = 0$$

$$\varepsilon_{\alpha\beta} \frac{\gamma^\beta \delta^\alpha_\nu - \gamma^\alpha \delta^\beta_\nu}{2}$$

$$S^{\alpha\beta}\gamma_\nu - \gamma_\nu S^{\alpha\beta} = -\gamma^\beta \delta^\alpha_\nu + \gamma^\alpha \delta^\beta_\nu$$

$$[S^{\alpha\beta}, \gamma_\nu] = -\gamma^\beta \delta^\alpha_\nu + \gamma^\alpha \delta^\beta_\nu$$

$$S_{\alpha\beta} = \frac{1}{4}[\gamma_\alpha, \gamma_\beta]$$

xy-plane rotation

$$\begin{pmatrix} v_1' \\ v_2' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

$$\approx \begin{pmatrix} 1 & \theta \\ -\theta & 1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

what is $S(\Lambda)$

$$\Lambda^\mu{}_\nu = e^{\frac{1}{2}\varepsilon^{\alpha\beta}(M_{\alpha\beta})^\mu{}_\nu}$$

generators for vector rep

$$(M_{\alpha\beta})^\mu{}_\nu = \delta_\alpha{}^\mu \eta_{\beta\nu} - \eta_{\alpha\nu}\delta_\beta{}^\mu$$

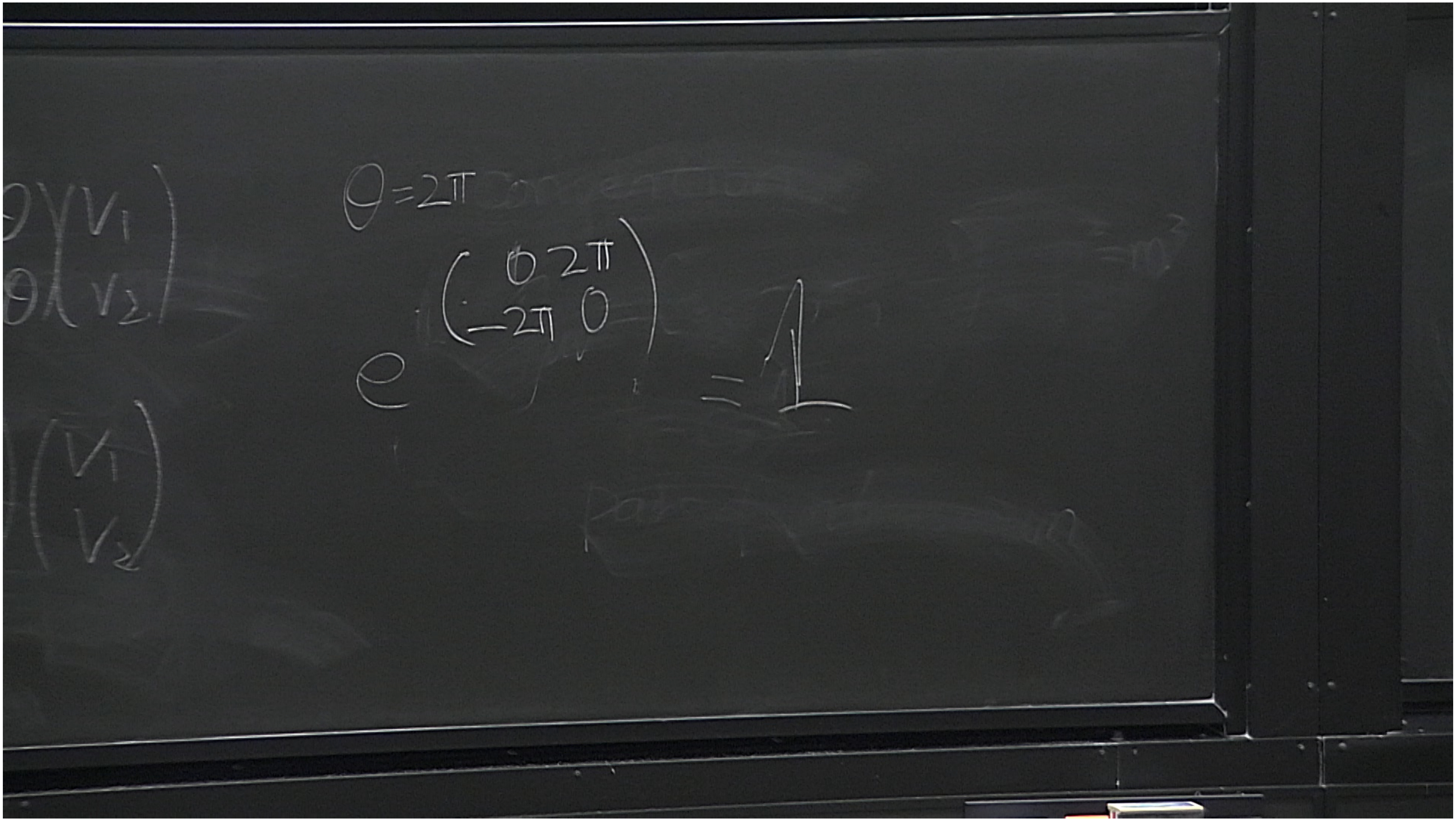
$$M_{12} = \begin{pmatrix} & -1 \\ 1 & \end{pmatrix}$$

$$\varepsilon^{12} = \theta = -\varepsilon^{21}$$

xy-plane rota

$$\begin{pmatrix} V_1' \\ V_2' \end{pmatrix} = \begin{pmatrix} \cos \\ -\sin \end{pmatrix}$$

$$\approx \Big($$


$\theta = 2\pi$
$e^{\begin{pmatrix} 0 & 2\pi \\ -2\pi & 0 \end{pmatrix}} = 1$

Weyl/Chiral base

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \gamma^1 = \begin{pmatrix} & \sigma^1 \\ -\sigma^1 & \end{pmatrix} \quad \gamma^2 = \begin{pmatrix} & \sigma^2 \\ -\sigma^2 & \end{pmatrix} \quad \gamma^3 = \begin{pmatrix} & \sigma^3 \\ -\sigma^3 & \end{pmatrix}$$

$$\varepsilon^{12} = \theta$$

$$S_{12} = \frac{1}{4}[\gamma_1, \gamma_2] = \frac{1}{2}\gamma_1\gamma_2$$

$$= \frac{1}{2}\begin{pmatrix} & \sigma^1 \\ -\sigma^1 & \end{pmatrix}\begin{pmatrix} & \sigma^2 \\ -\sigma^2 & \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} -\sigma^1\sigma^2 & \\ & -\sigma^1\sigma^2 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -i\sigma^3 & \\ & -i\sigma^3 \end{pmatrix}$$