Title: PSI 2018/2019 - Quantum Field Theory I - Lecture 7

Date: Oct 19, 2018 09:00 AM

URL: http://pirsa.org/18100013

Abstract:

Beyond the Trees

tree   loop

$$\mathcal{L}_{\text{naive}} = \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m^2\varphi + \frac{1}{3!}g\varphi^3$$

LSZ requires $\langle\Omega|\varphi(x)|\Omega\rangle = 0$

$$\langle k|\varphi(x)|\Omega\rangle = e^{ik\cdot x}$$

shift, rescale + rename

$$\mathcal{L} = \mathcal{L}_{free} + \mathcal{L}_{int}$$

$$\mathcal{L}_{free} = \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2$$

$$\mathcal{L}_{int} = \frac{1}{3!}Z_g g \varphi^3 + \mathcal{L}_{ct}$$

counterterms

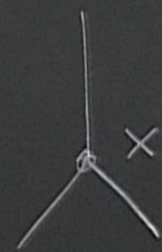
$$\mathcal{L}_{ct} = \frac{1}{2}(Z_\varphi - 1)(\partial\varphi)^2 - \frac{1}{2}m^2(Z_m - 1)\varphi^2 + Y\varphi$$

Determine Z_i, Y perturbatively in g

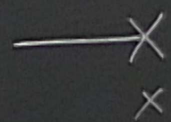
$$Z_i = 1 + \mathcal{O}(g^2)$$

$$Y = 0 + \mathcal{O}(g)$$

$\varphi^2 + Y\varphi$



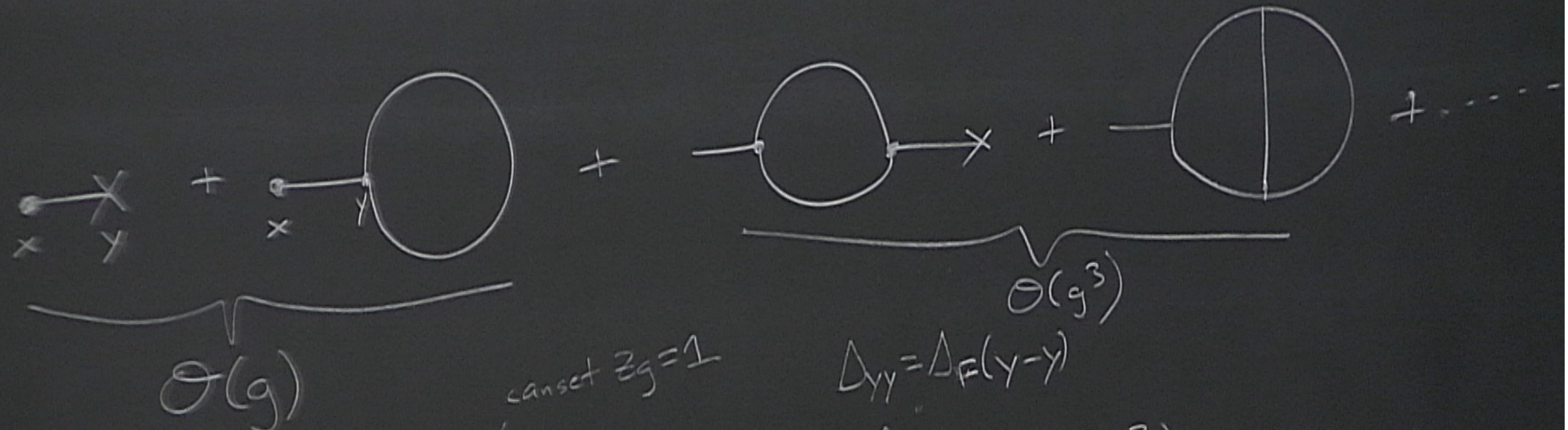
$$= iZgg\int d^4x$$



$$= -iY\int d^4x$$

= easier in momentum space

$=$



can set $Z_g = 1$

$\Delta_{yy} = \Delta_F(y-y)$

$$= iY\int d^4y\,\Delta_{xy} + \frac{1}{2} i Z_g g \int d^4y\,\Delta_{xy}\Delta_{yy} + \mathcal{O}(g^3)$$

$$= \left(iY + \frac{1}{2} i g \Delta_F(0)\right)\int d^4y\,\Delta_{xy} + \mathcal{O}(g^3)$$

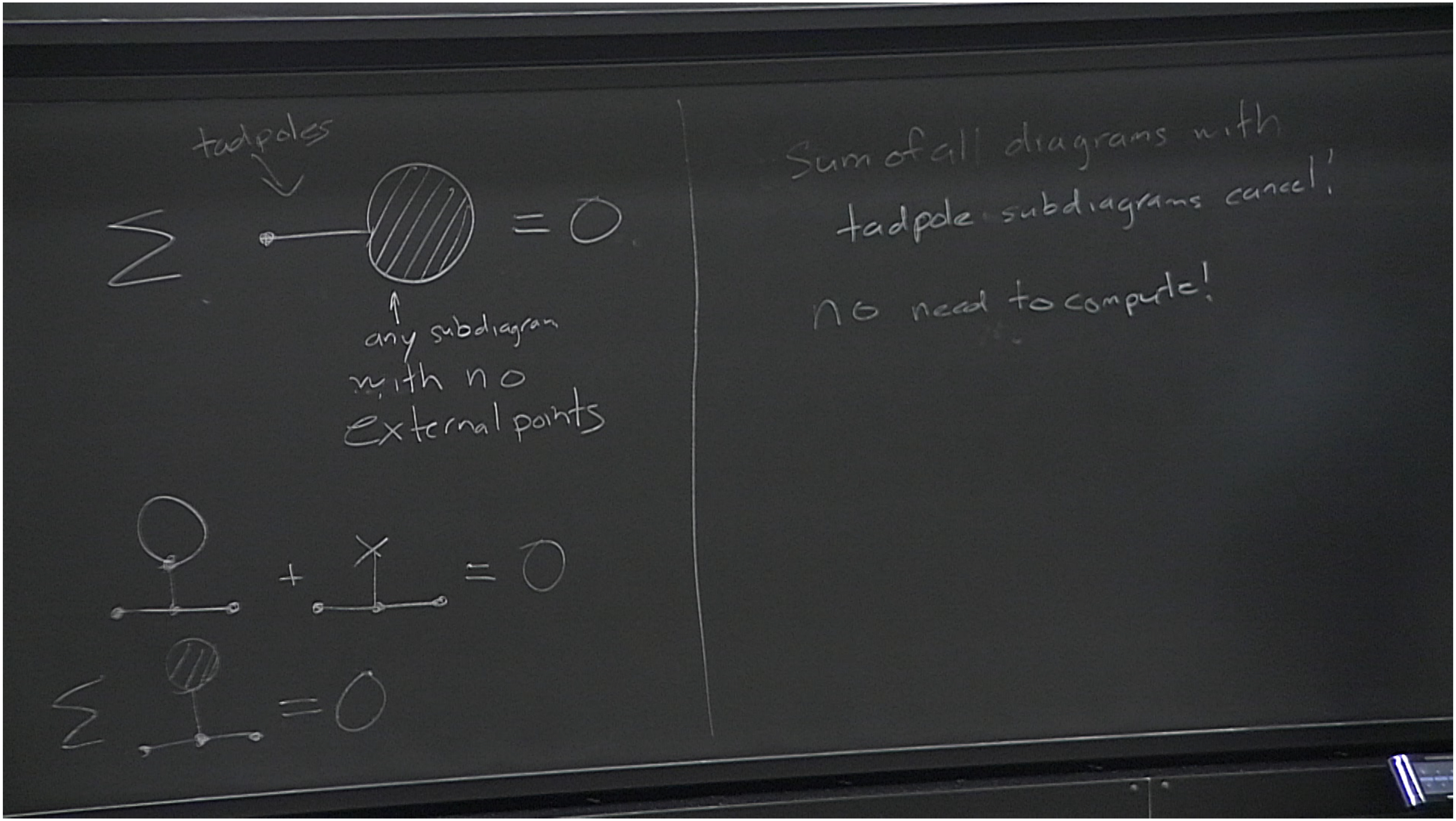
$$\Upsilon = -\frac{1}{2} g \Delta_F(0) = -\frac{1}{2} g \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2-m^2} \cancelto{1}{e^{-ip\cdot 0}} = \infty$$

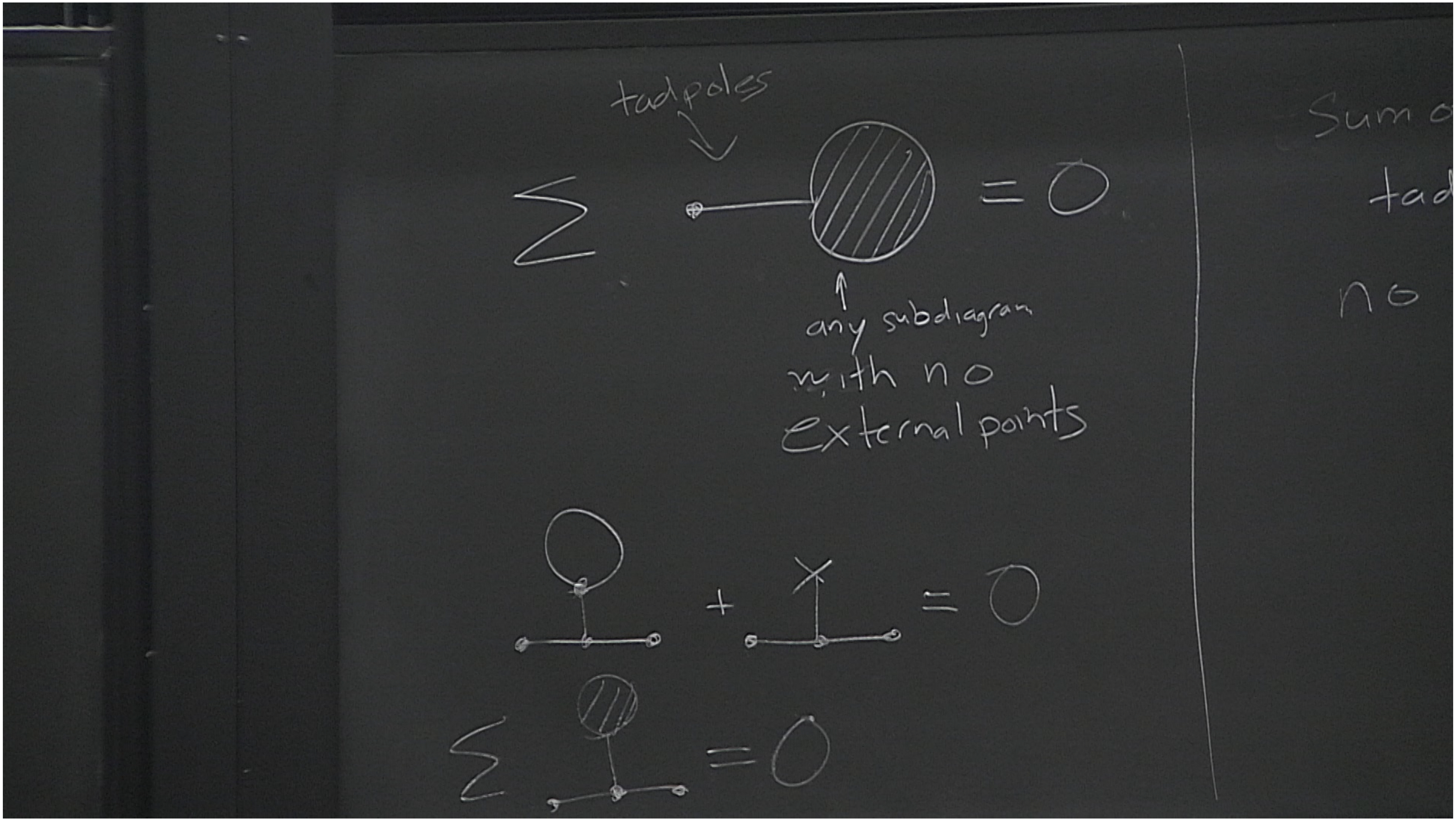
Introduce a regulator: UV cutoff Λ

dim reg $d = 4 - \varepsilon$

Pauli-Villars $\left(\frac{\Lambda^2}{p^2-\Lambda^2+i\varepsilon}\right)^2$ into integrand

introduce regulator $\rightarrow$ compute observable $\rightarrow$ remove regulator


tadpoles
$\sum$ = 0
any subdiagram
with no
external points
+ = 0
$\sum$ = 0
Sum of all diagrams with
tadpole subdiagrams cancel!
no need to compute!


tadpoles
= 0
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external points
+
= 0
= 0
Sum o
tad
no

Sum of all diagrams with tadpole subdiagrams cancel!

no need to compute!

Propagator

$$\langle \Omega | \varphi(x)\, \varphi(y) | \Omega \rangle$$

Källen-Lehmann spectral representation

zero-particle

1-particle

multi particle

$$\mathbb{1} = |\Omega\rangle\langle\Omega| + \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} |\vec{k}\rangle\langle\vec{k}| + \sum_{\sigma} \int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} |\vec{k},\sigma\rangle\langle\vec{k},\sigma|$$

$$\int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} = \int \frac{d^4k}{(2\pi)^4} 2\pi\, \delta(k^2 - M^2)\, \Theta(k^0)$$

$$E_{\vec{k}}^2 = \vec{k}^2 + M^2$$

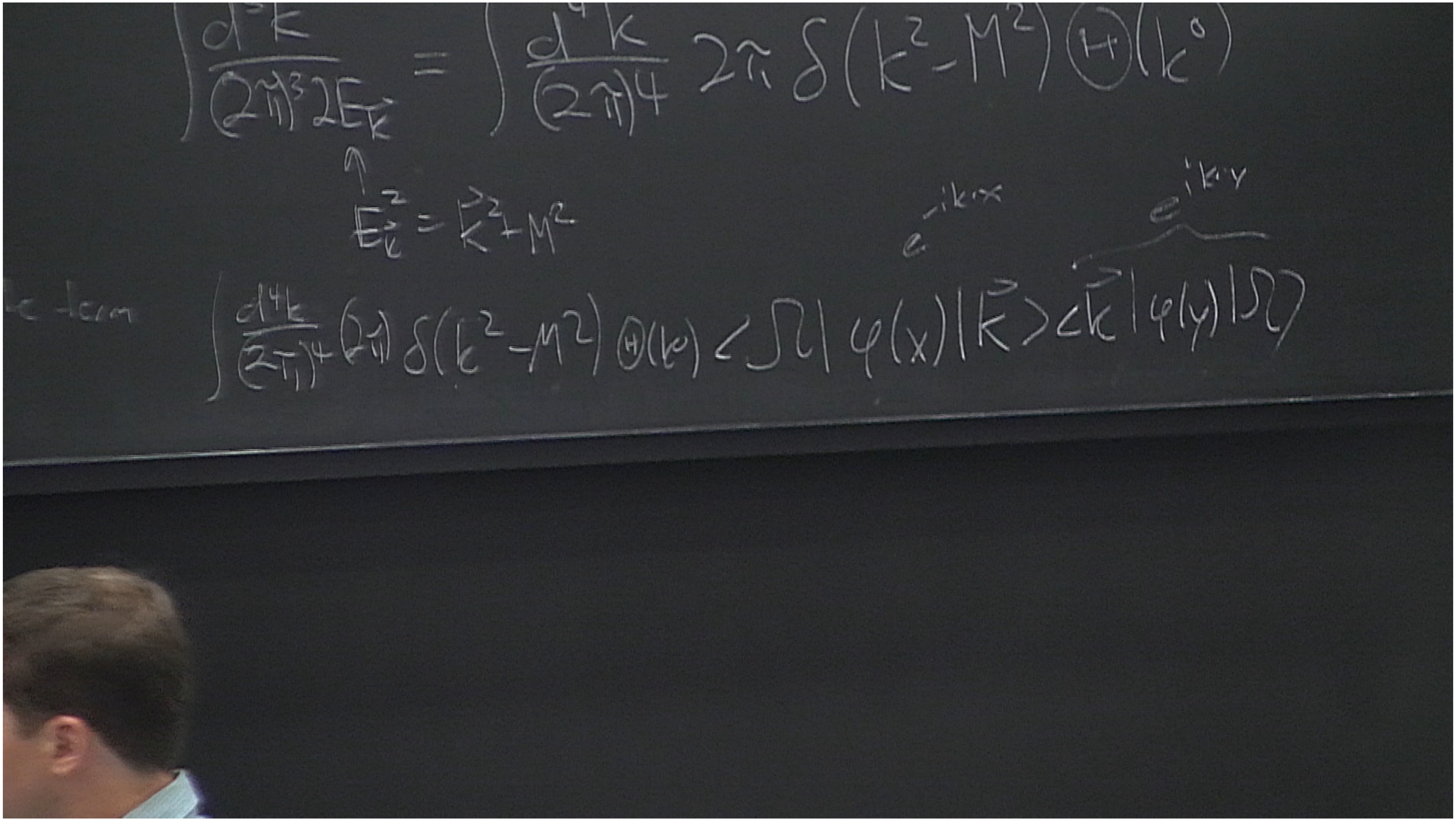
$|\vec{k}$

$$\int \frac{d^3k}{(2\pi)^3 2E_{\vec{k}}} = \int \frac{d^4k}{(2\pi)^4} 2\pi \delta(k^2-M^2)\Theta(k^0)$$

$$E_{\vec{k}}^2 = \vec{k}^2 + M^2$$

1-particle term

$$\int \frac{d^4k}{(2\pi)^4}(2\pi)\delta(k^2-M^2)\Theta(k^0)\langle\Omega|\varphi(x)|\vec{k}\rangle\langle\vec{k}|\varphi(y)|\Omega\rangle$$

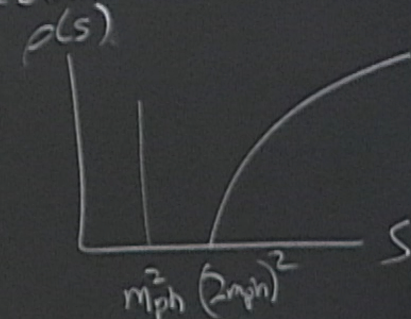


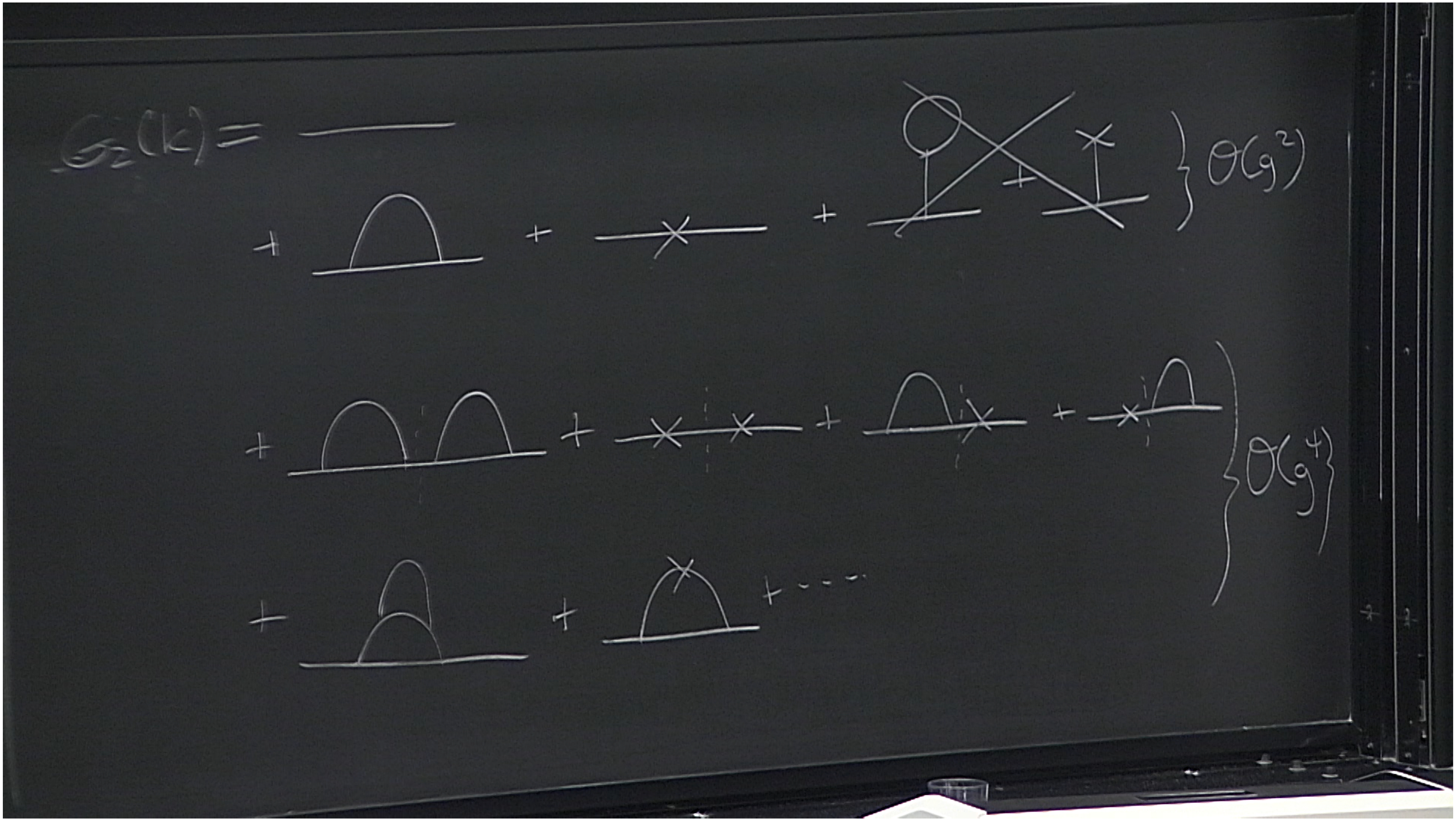
$\int \frac{d^3k}{(2\pi)^3 2E_{\vec k}} = \int \frac{d^4k}{(2\pi)^4} 2\pi \delta(k^2-M^2)\Theta(k^0)$
$E_{\vec k}^2 = \vec k^2 + M^2$
$e^{-ik\cdot x}$
$e^{ik\cdot y}$
le term
$\int \frac{d^4k}{(2\pi)^4}(2\pi)\delta(k^2-M^2)\Theta(k^0)\langle\Omega|\varphi(x)|\vec k\rangle\langle\vec k|\varphi(y)|\Omega\rangle$

$$\boxed{G_2(k) = \frac{i}{k^2 - m_{ph}^2 + i\varepsilon} + \int_{4m_{ph}^2}^{\infty} \frac{ds}{2\pi}\, \rho(s)\, \frac{i}{k^2 - s + i\varepsilon}}$$ exact

Fourier transform of $\langle \Omega | T\varphi(x)\varphi(y) | \Omega \rangle$

spectral function



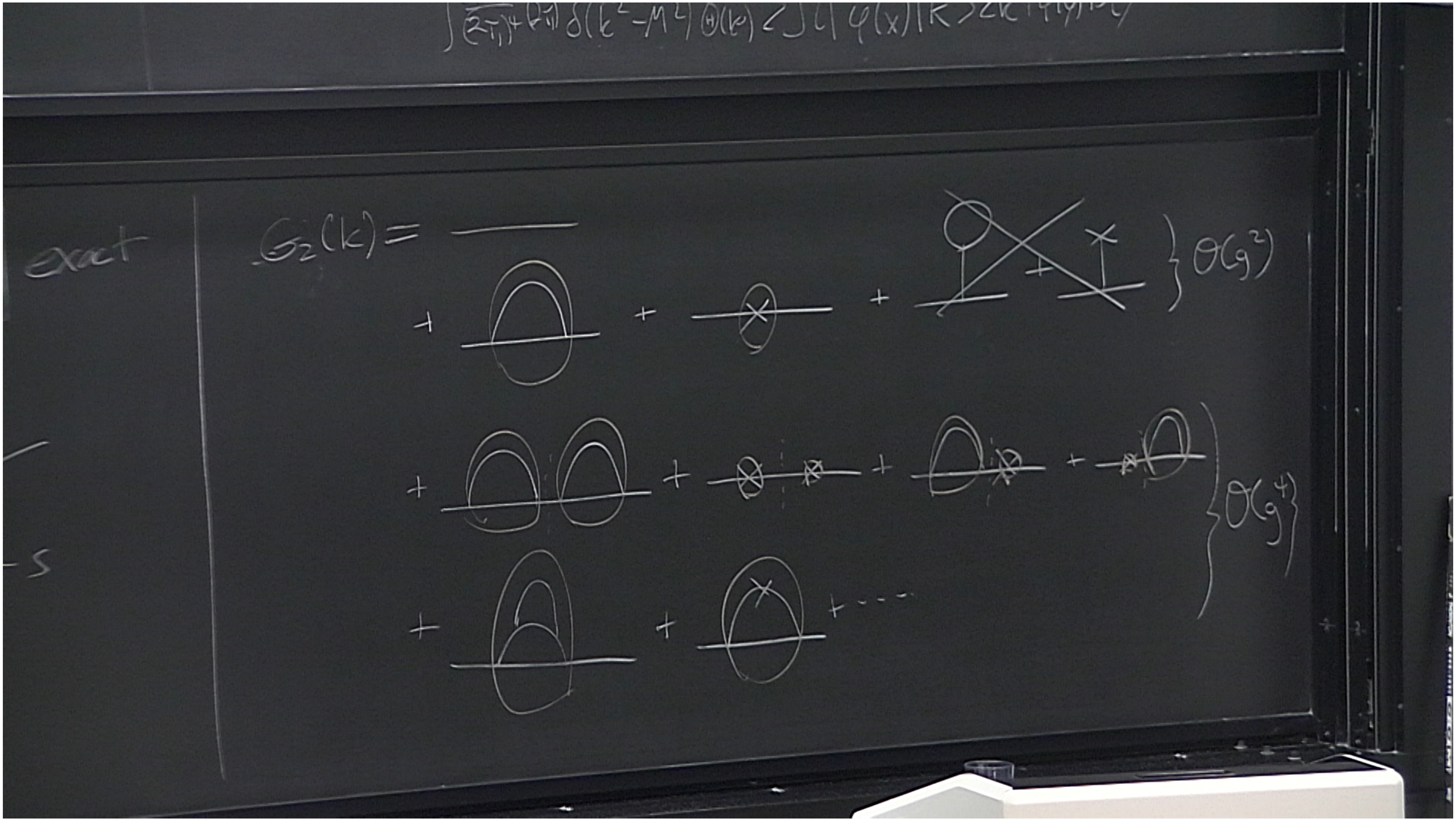

G₂(k) =
O(g²)
O(g⁴)

$Y = 0 + \mathcal{O}(g)$

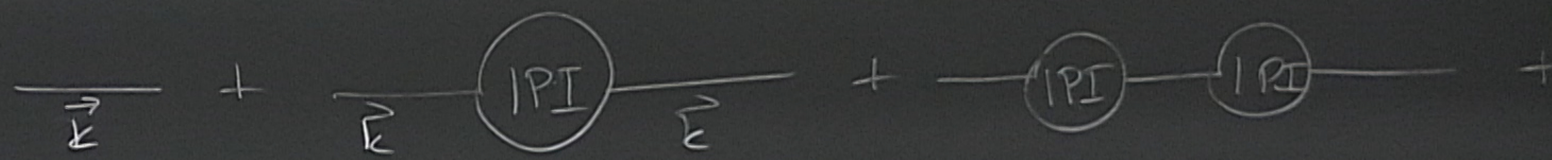
$$G_2(k) = \text{——} + \text{—}\,1PI\,\text{—} + \text{—}\,1PI\,\text{—}\,1PI\,\text{—} + \text{—}\,1PI\,\text{—}\,1PI\,\text{—}\,1PI\,\text{—}$$

1PI = one-particle irreducible

= remains connected if any one line is cut


exact
G2(k) =
O(g2)
O(g4)

$G_2(k) =$



$+ \cdots$

1PI = one-particle irreducible

= remains connected if any one line is cut

$$\sum \text{1PI} = -i\Sigma(k^2)$$

self-energy

$\Upsilon = 0 + \mathcal{O}(g)$

ducible $\quad + \cdots$

ted if

s cut

$$G_2(k) = \frac{i}{k^2 - m^2 + i\varepsilon} + \left(\frac{i}{k^2 - m^2 + i\varepsilon}\right)^2 \Big($$

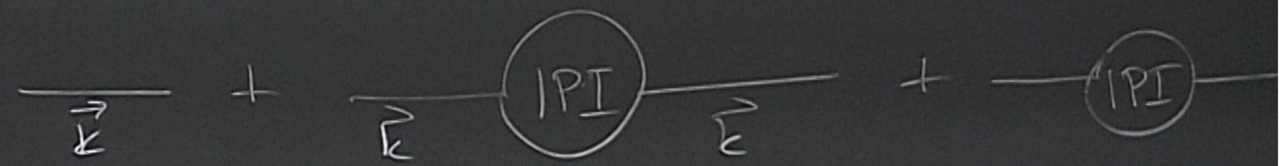
$$G_2(k) = \frac{i}{k^2-m^2+i\varepsilon} + \left(\frac{i}{k^2-m^2+i\varepsilon}\right)^2\left(-i\Sigma(k^2)\right) + \left(\frac{i}{k^2-m^2+i\varepsilon}\right)^3\left(-i\Sigma(k^2)\right)^2 + \cdots$$

$$= \frac{i}{k^2-m^2+i\varepsilon} \sum_{n=0}^{\infty} \left(\frac{\Sigma(k^2)}{k^2-m^2+i\varepsilon}\right)^n$$

$$= \frac{i}{k^2-m^2+i\varepsilon} \frac{1}{1-\frac{\Sigma(k^2)}{k^2-m^2+i\varepsilon}}$$

$$\boxed{G_2(k) = \frac{i}{k^2-m^2-\Sigma(k^2)+i\varepsilon}}$$

all orders in perturbation theory

$G_2(k) =$ 

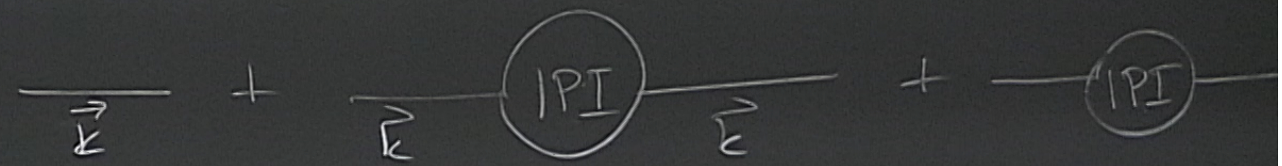
FT of Δ_F

1PI = one-particle irreducible

= remains connected if any one line is cut

$$\Sigma \; (1PI) = -i\Sigma(k^2)$$

self-energy

$G_2(k) =$ 

$\vec{k}$ ↑ FT of Δ_{F_2} involves m_{Lag}

1PI = one-particle irreducible

= remains connected if any one line is cut

$$\sum \text{1PI} = -i\Sigma(k^2)$$

self-energy

If we want $m^2_{Lag} = m^2_{ph}$

$$\boxed{\begin{aligned} \Sigma(m^2) &= 0 \\ \Sigma'(m^2) &= 0 \end{aligned}}$$

location of pole is correct

residue is correct

$' = \frac{d}{dk^2}$

$k^2 - m^2 - c|k|^2 + i\varepsilon$

Vertex

So far 3 conditions | 1 tadpole + 2 propagator don't fix Z_g

need 1 more

Need to define coupling strength

$k^2 - m^2 - \Sigma(k^2) + i\epsilon$

ions | tadpole + 2 propagators don't fix Z_g at leading order

pling strength

tex

so far 3 conditions | 1 tadpole + 2 propagator don't

need 1 more

Need ~~to~~ define coupling strength

impose a condition at some reference momentum → effec adpole + 2 propagator   don't fix Z_g at leading order

th

reference momentum $\rightarrow$ effective coupling strength "runs"

or varies with p or E

Summary

$$\Gamma, \sigma \longrightarrow iM \xrightarrow{LSZ} \langle \Omega | T \varphi_1 \cdots \varphi_n | \Omega \rangle \longrightarrow \langle 0 | T \varphi_1 \cdots \varphi_n | 0 \rangle \rightarrow \Delta_F$$