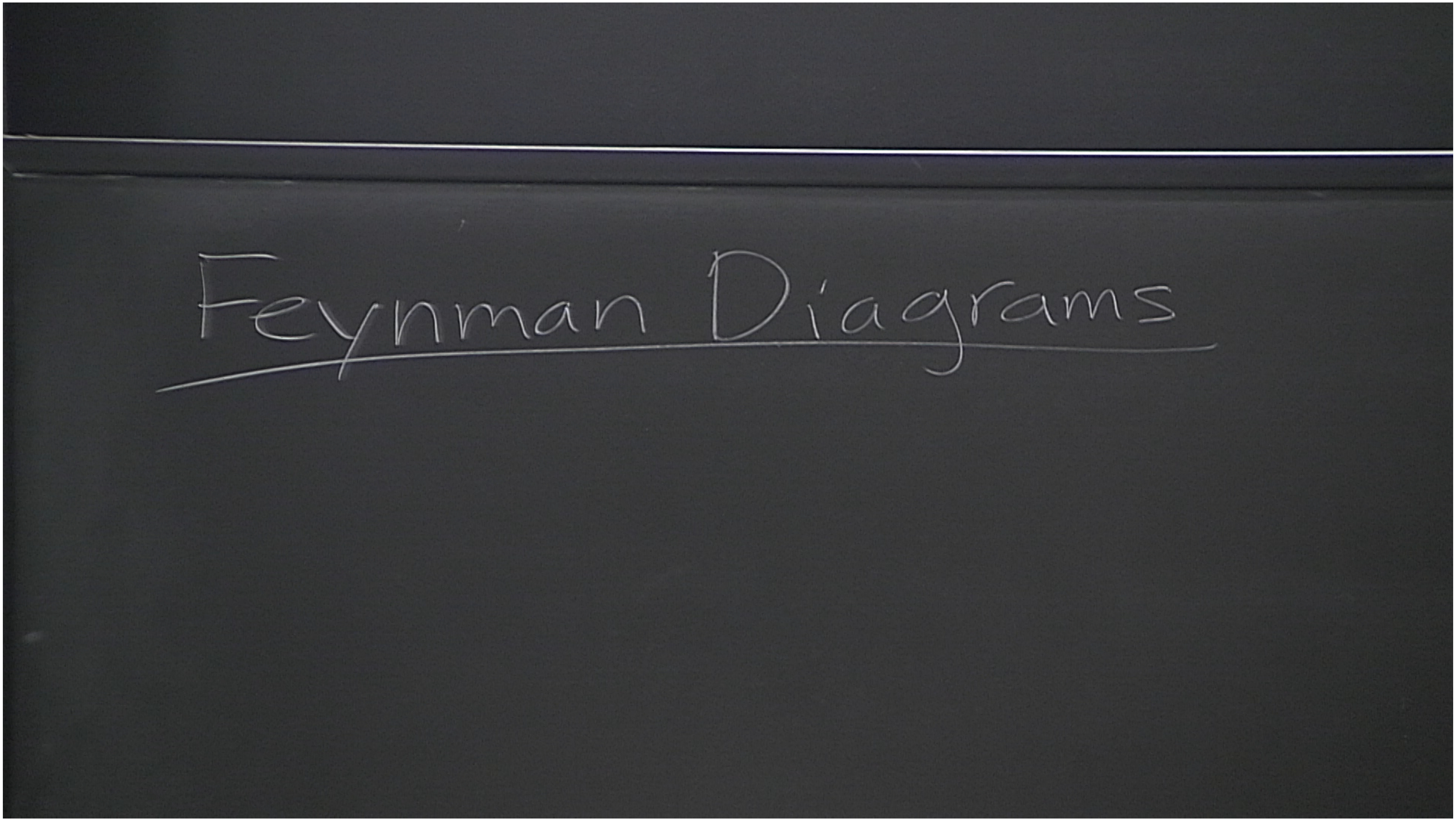
Title: PSI 2018/2019 - Quantum Field Theory I - Lecture 6

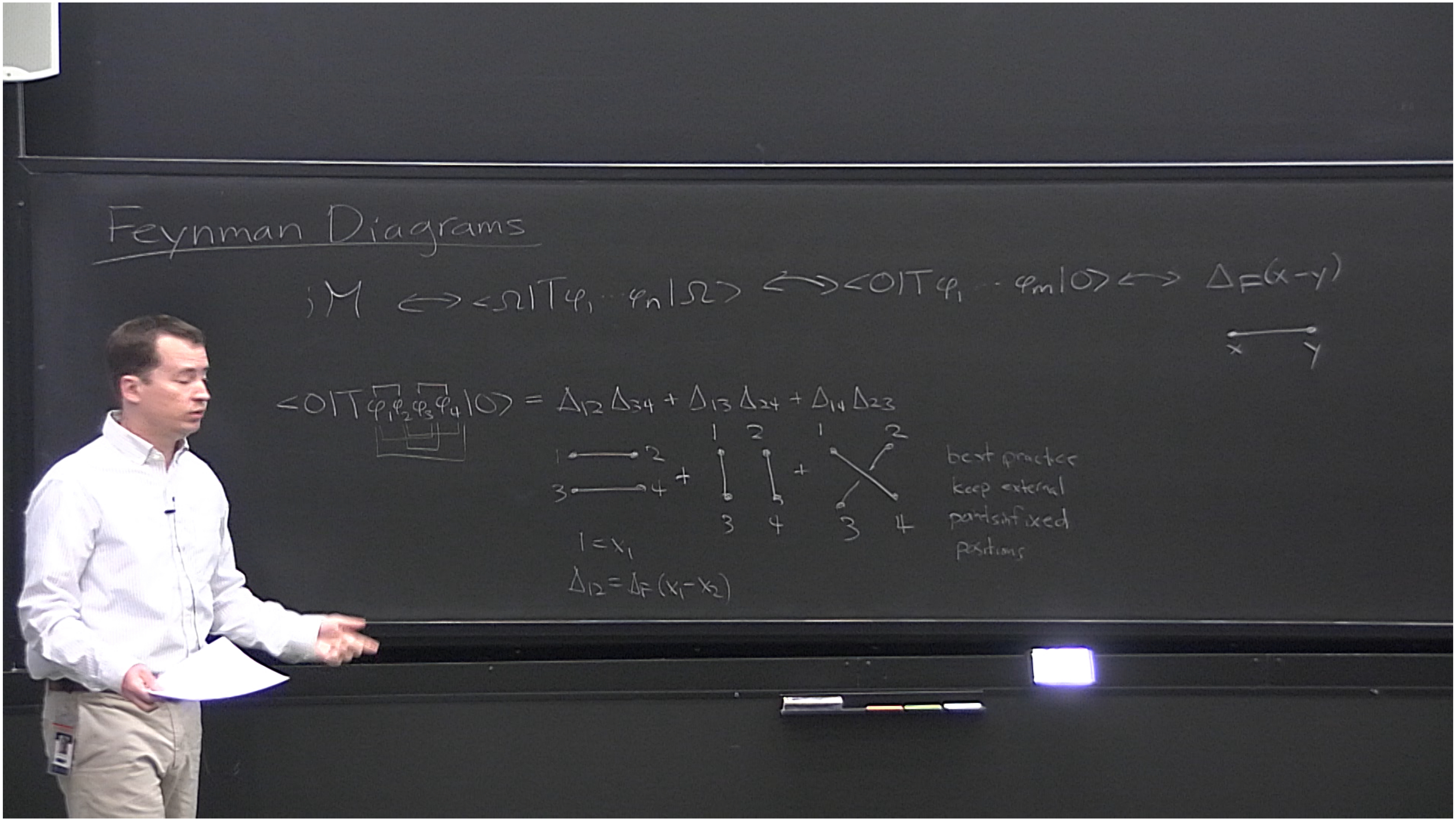
Date: Oct 18, 2018 09:00 AM

URL: http://pirsa.org/18100012

Abstract:



Feynman Diagrams



Feynman Diagrams

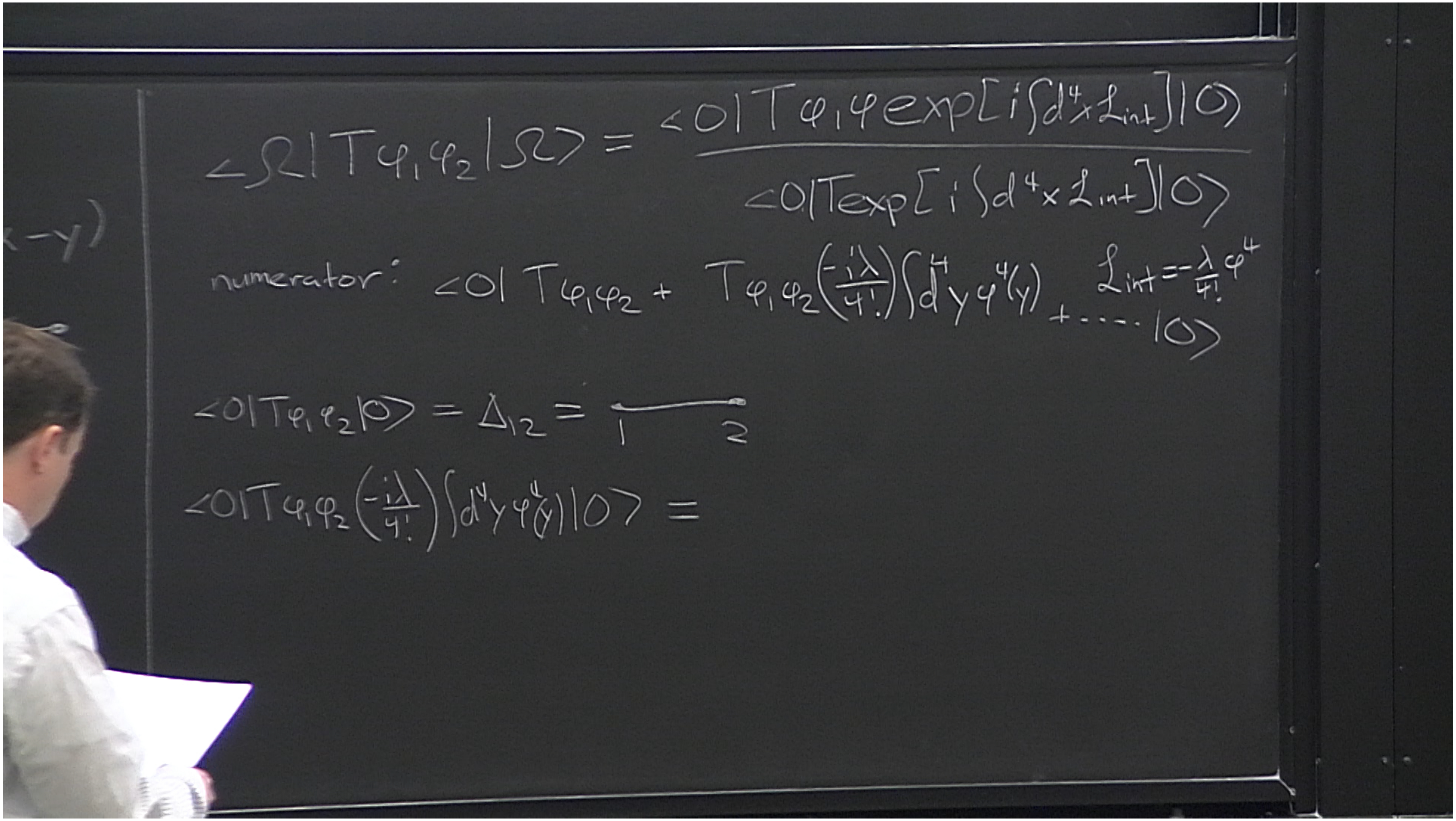
$$iM \longleftrightarrow \langle \Omega | T \varphi_1 \cdots \varphi_n | \Omega \rangle \longleftrightarrow \langle 0 | T \varphi_1 \cdots \varphi_m | 0 \rangle \longleftrightarrow \Delta_F(x-y)$$

$$\langle 0 | T \varphi_1 \varphi_2 \varphi_3 \varphi_4 | 0 \rangle = \Delta_{12}\Delta_{34} + \Delta_{13}\Delta_{24} + \Delta_{14}\Delta_{23}$$

best practice
keep external
parts in fixed
positions

$1 = x_1$

$\Delta_{12} = \Delta_F(x_1 - x_2)$



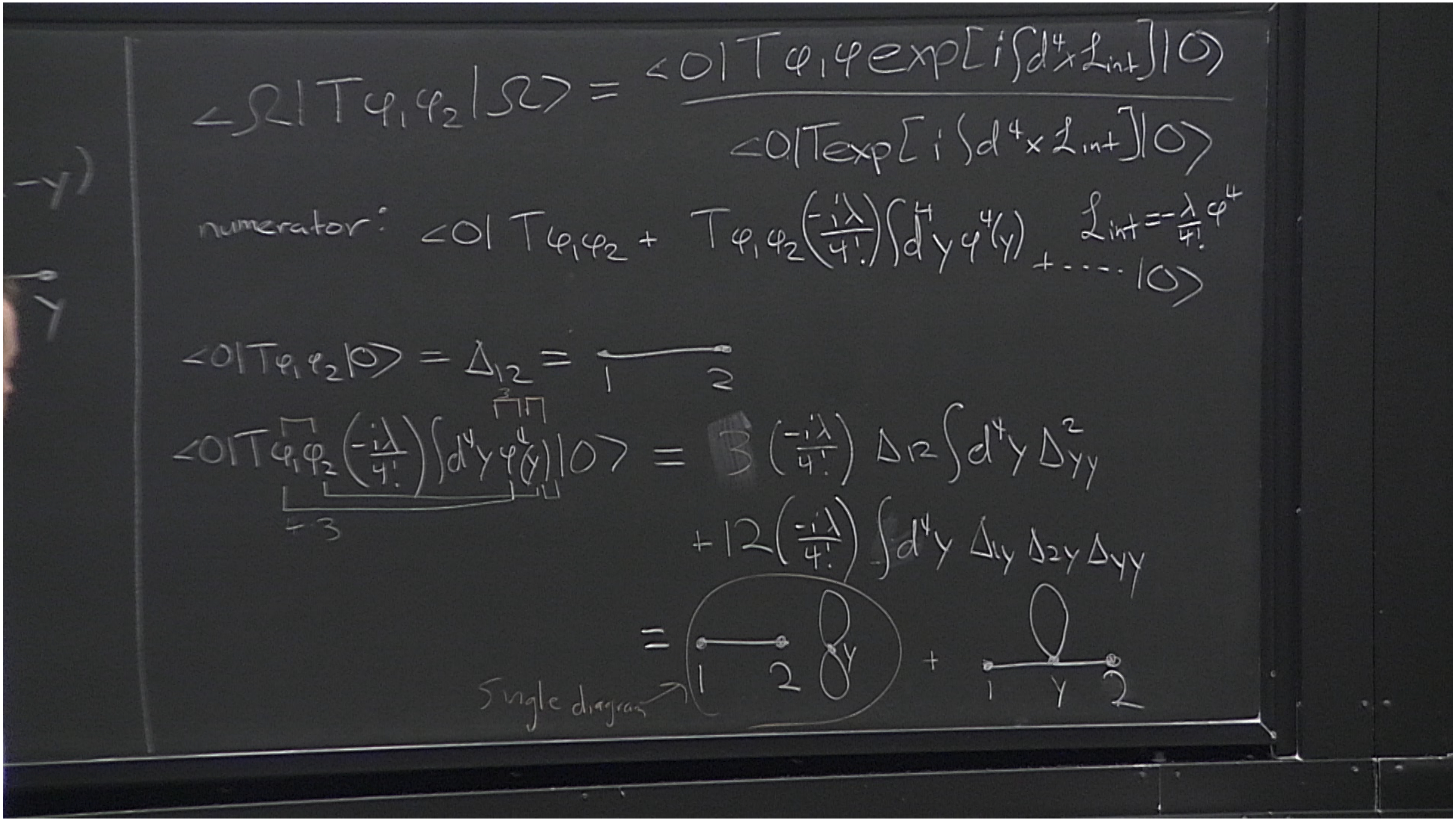
$x-y)$

$$\langle\Omega|T\varphi_1\varphi_2|\Omega\rangle = \frac{\langle 0|T\varphi_1\varphi\exp[i\int d^4x\,\mathcal{L}_{int}]|0\rangle}{\langle 0|T\exp[i\int d^4x\,\mathcal{L}_{int}]|0\rangle}$$

numerator: $\langle 0|\,T\varphi_1\varphi_2 + T\varphi_1\varphi_2\left(\frac{-i\lambda}{4!}\right)\int d^4y\,\varphi^4(y) + \cdots|0\rangle$ $\quad\mathcal{L}_{int} = -\frac{\lambda}{4!}\varphi^4$

$\langle 0|T\varphi_1\varphi_2|0\rangle = \Delta_{12} =$

$\langle 0|T\varphi_1\varphi_2\left(\frac{-i\lambda}{4!}\right)\int d^4y\,\varphi^4(y)|0\rangle =$



$-y)$

y

$$\langle\Omega|T\varphi_1\varphi_2|\Omega\rangle = \frac{\langle 0|T\varphi_1\varphi\exp[i\int d^4x\,\mathcal{L}_{int}]|0\rangle}{\langle 0|T\exp[i\int d^4x\,\mathcal{L}_{int}]|0\rangle}$$

numerator: $\langle 0|\,T\varphi_1\varphi_2 + T\varphi_1\varphi_2\left(\frac{-i\lambda}{4!}\right)\int d^4y\,\varphi^4(y) + \cdots|0\rangle$ $\quad \mathcal{L}_{int} = -\frac{\lambda}{4!}\varphi^4$

$$\langle 0|T\varphi_1\varphi_2|0\rangle = \Delta_{12} =$$

1 2

$$\langle 0|T\varphi_1\varphi_2\left(\frac{-i\lambda}{4!}\right)\int d^4y\,\varphi^4(y)|0\rangle = 3\left(\frac{-i\lambda}{4!}\right)\Delta_{12}\int d^4y\,\Delta_{yy}^2$$

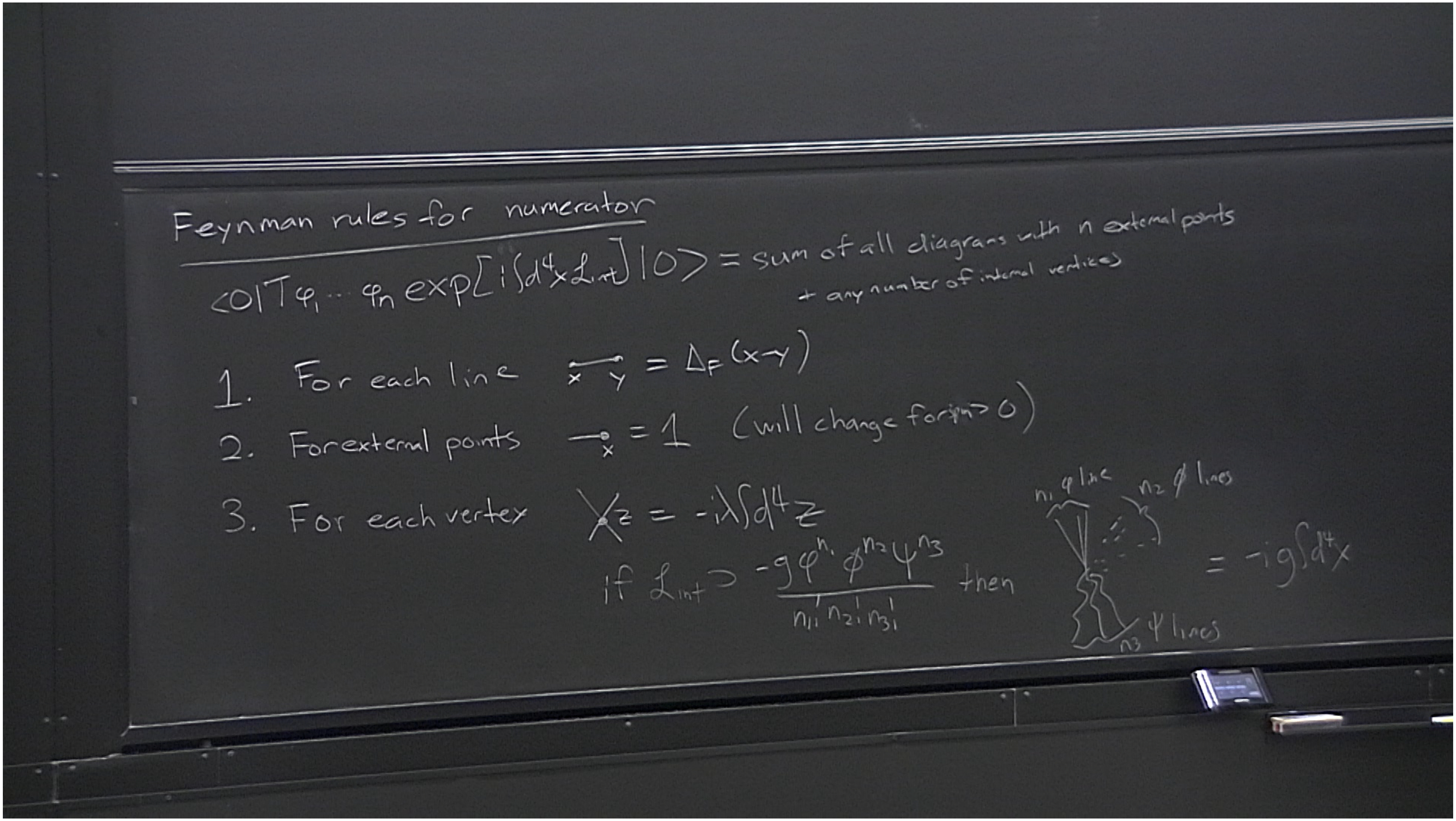
3

+3

$$+12\left(\frac{-i\lambda}{4!}\right)\int d^4y\,\Delta_{1y}\Delta_{2y}\Delta_{yy}$$

$=$ 1 2 y $+$ 1 y 2

Single diagram



Feynman rules for numerator

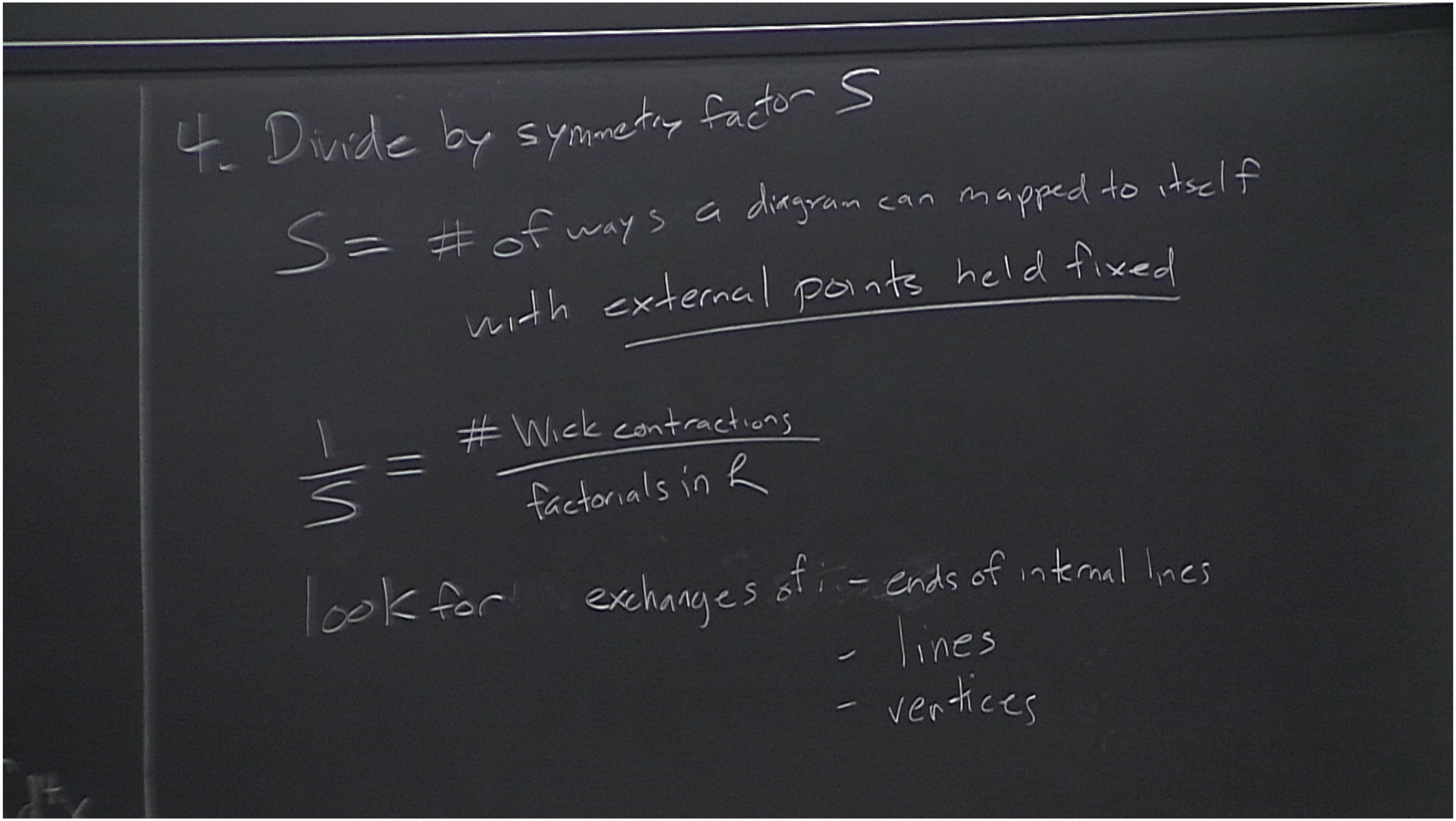
$\langle 0 | T \varphi_1 \cdots \varphi_n \exp[i \int d^4x \mathcal{L}_{int}] | 0 \rangle$ = sum of all diagrams with n external points + any number of internal vertices

1. For each line $\overset{\bullet\!\!-\!\!-\!\!\bullet}{x \quad y} = \Delta_F(x-y)$

2. For external points $-\!\!\!\bullet_{x} = 1$ (will change for spin > 0)

3. For each vertex $\times_z = -i\lambda \int d^4z$

if $\mathcal{L}_{int} \supset -\dfrac{g \varphi^{n_1} \phi^{n_2} \psi^{n_3}}{n_1! \, n_2! \, n_3!}$ then $= -ig \int d^4x$



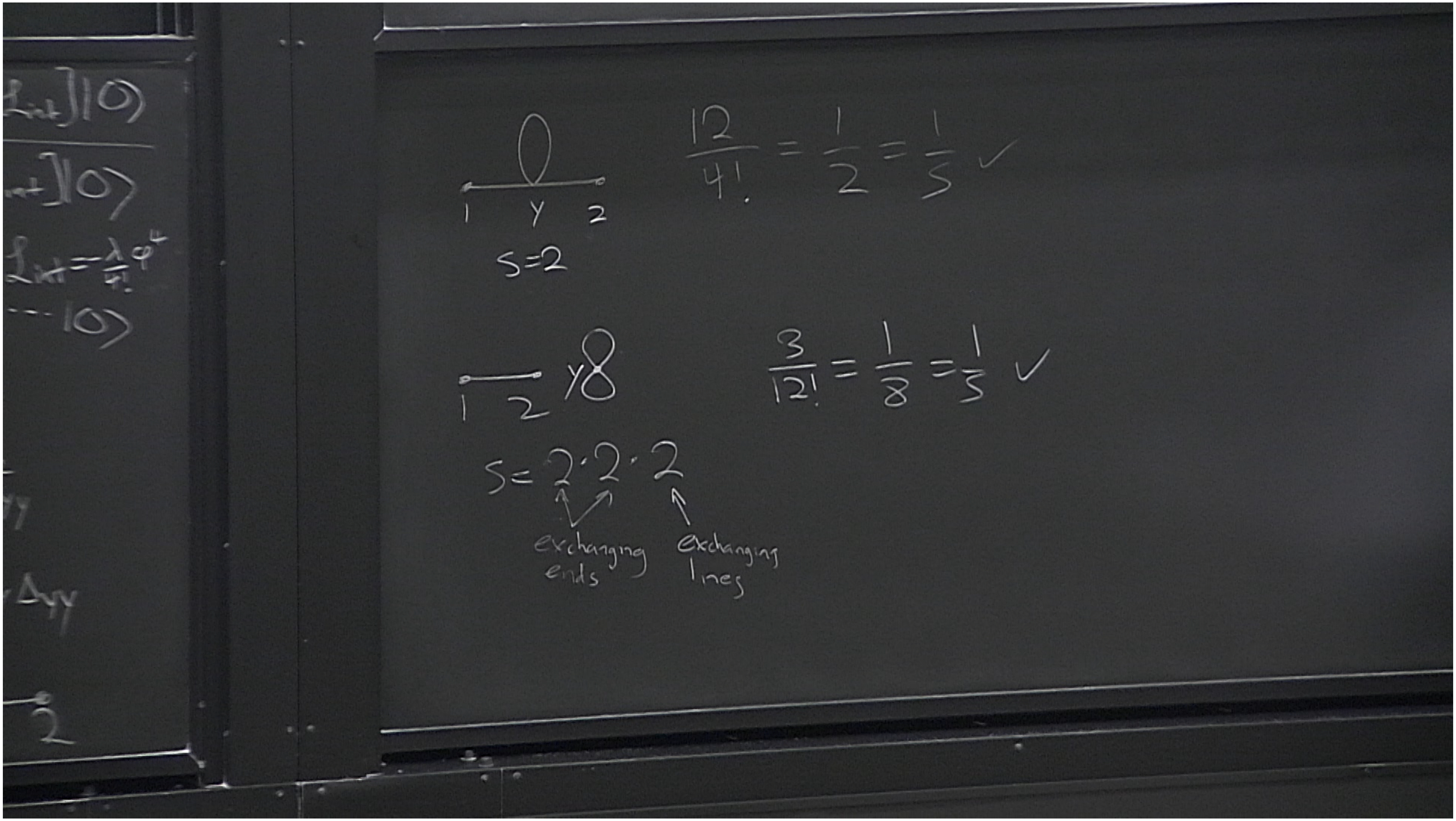
4. Divide by symmetry factor S

$S =$ # of ways a diagram can mapped to itself with external points held fixed

$$\frac{1}{S} = \frac{\#\text{ Wick contractions}}{\text{factorials in } \mathcal{L}}$$

look for exchanges of - ends of internal lines

- lines
- vertices



$\mathcal{L}_{int}]|0\rangle$

$_{int}]|0\rangle$

$\mathcal{L}_{int} = -\frac{\lambda}{4!}\varphi^4$

$\cdots|0\rangle$

yy

Δ_{yy}

2

$S=2$

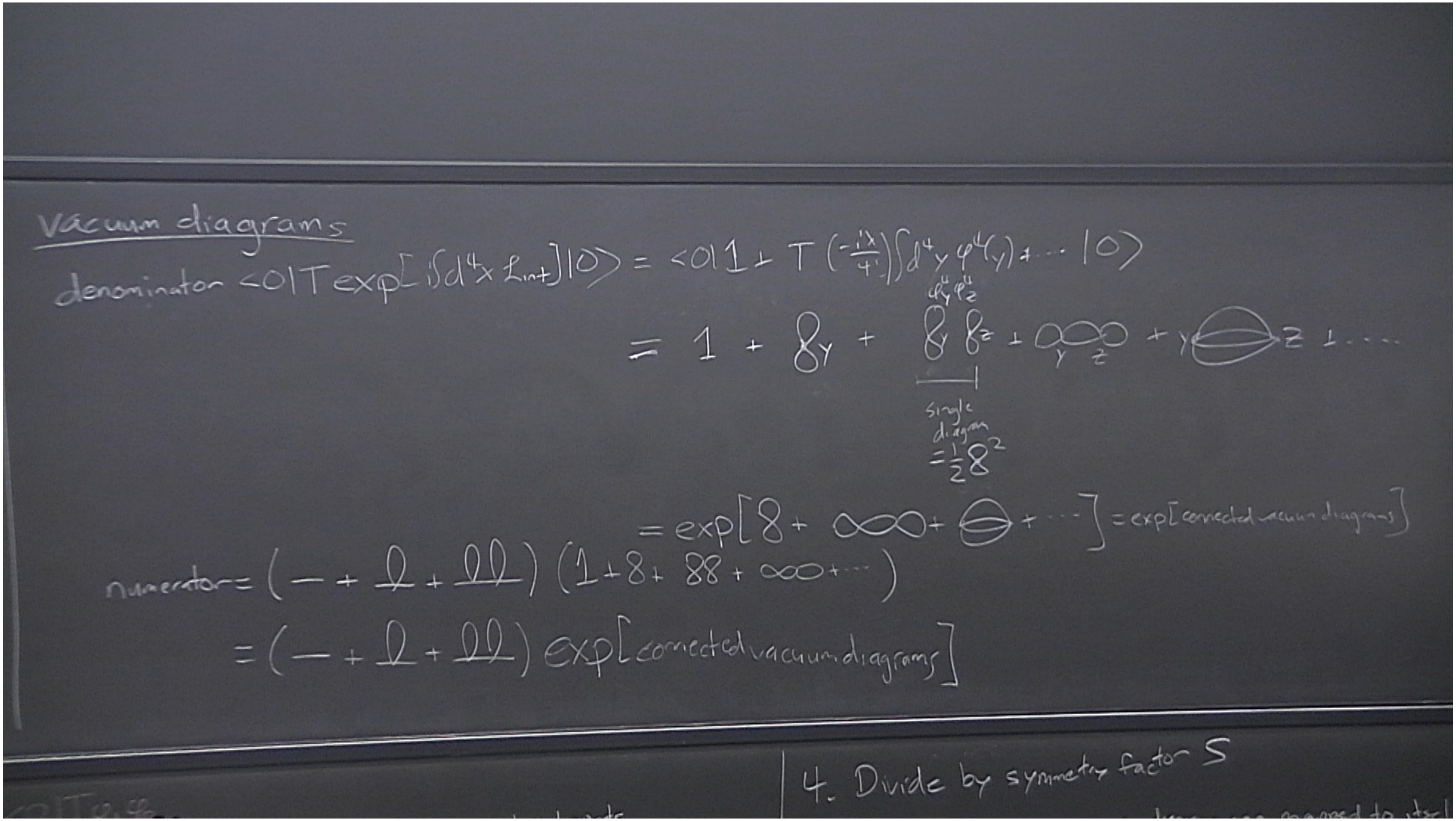
$\frac{12}{4!} = \frac{1}{2} = \frac{1}{S}$ ✓

$\frac{3}{12!} = \frac{1}{8} = \frac{1}{S}$ ✓

$S = 2 \cdot 2 \cdot 2$

exchanging ends

exchanging lines



vacuum diagrams

denominator $\langle 0|T\exp[i\int d^4x\,\mathcal{L}_{int}]|0\rangle = \langle 0|1 + T\left(\frac{-i\lambda}{4!}\right)\int d^4y\,\varphi^4(y) + \cdots|0\rangle$

$\varphi_y^4\varphi_z^4$

$= 1 + 8_y + \underbrace{8_y\,8_z}_{} + \underset{y\quad z}{\infty\!\infty} + y\ominus z + \cdots$

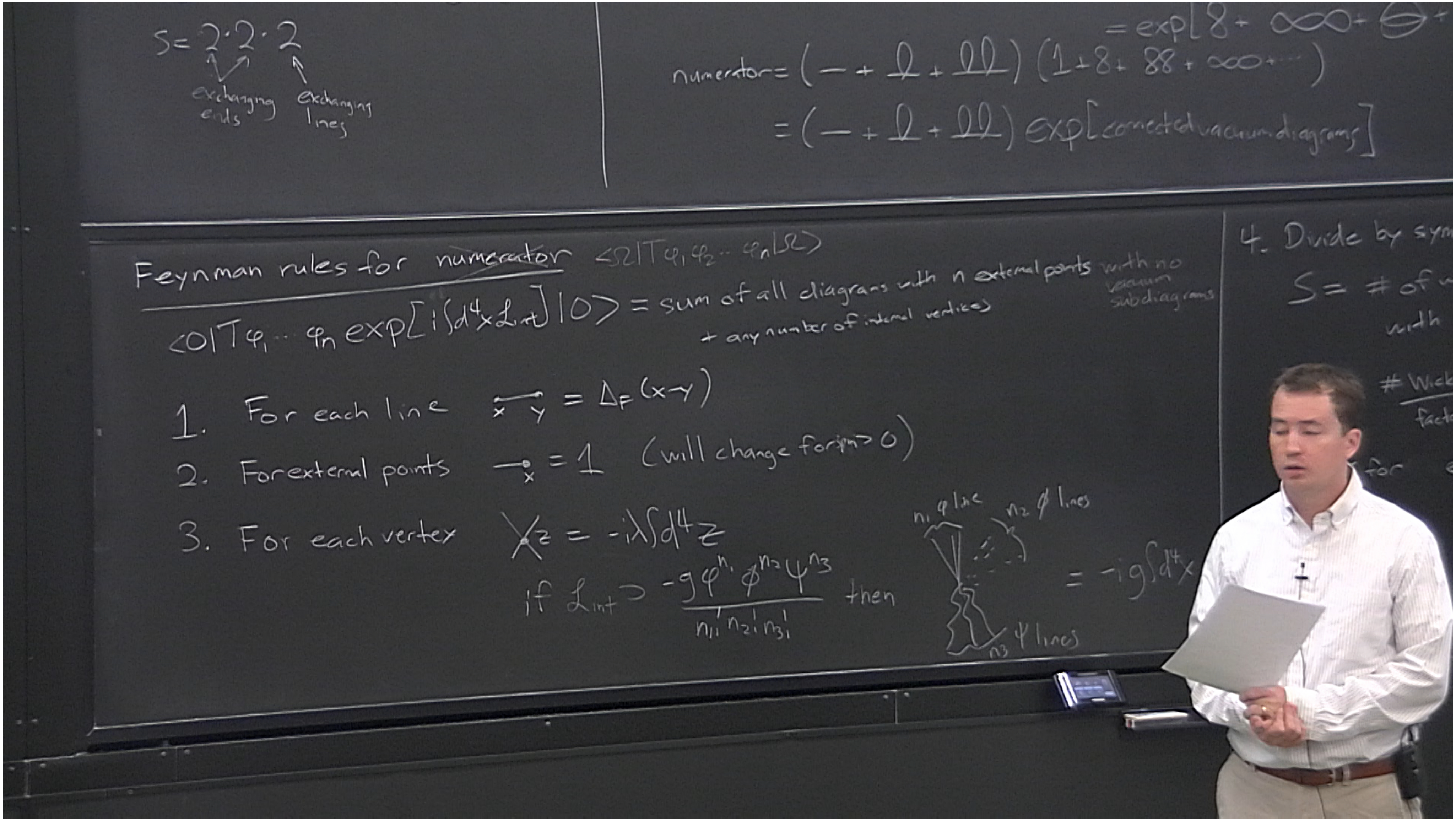
single diagram $= \frac{1}{2}8^2$

$= \exp[8 + \infty\!\infty + \ominus + \cdots] = \exp[\text{connected vacuum diagrams}]$

numerator $= (— + \ell + \ell\ell)(1 + 8 + 88 + \infty\!\infty + \cdots)$

$= (— + \ell + \ell\ell)\exp[\text{connected vacuum diagrams}]$

4. Divide by symmetry factor S



$S = 2 \cdot 2 \cdot 2$

exchanging ends

exchanging lines

$= \exp[\,8 + \infty\!\!\!\infty + \ominus +$

numerator $= (\,— + \ell + \ell\ell\,)\,(1 + 8 + 88 + \infty\!\!\!\infty + \cdots)$

$= (\,— + \ell + \ell\ell\,)\,\exp[$connected vacuum diagrams$]$

Feynman rules for ~~numerator~~ $\langle\Omega|T\varphi_1\varphi_2\cdots\varphi_n|\Omega\rangle$

$\langle 0|T\varphi_1\cdots\varphi_n \exp[i\int d^4x\,\mathcal{L}_{int}]|0\rangle$ = sum of all diagrams with n external points + any number of internal vertices) with no vacuum subdiagrams

1. For each line $\underset{x\quad y}{\bullet\!\!-\!\!\!-\!\!\bullet} = \Delta_F(x-y)$
2. For external points $\underset{x}{-\!\!\bullet} = 1$ (will change for $\langle\varphi\rangle > 0$)
3. For each vertex $\times_z = -i\lambda\int d^4z$

if $\mathcal{L}_{int} \supset \dfrac{-g\varphi^{n_1}\phi^{n_2}\psi^{n_3}}{n_1!\,n_2!\,n_3!}$ then

n_1 φ line

n_2 ϕ lines

n_3 ψ lines

$= -ig\int d^4x$

4. Divide by sym

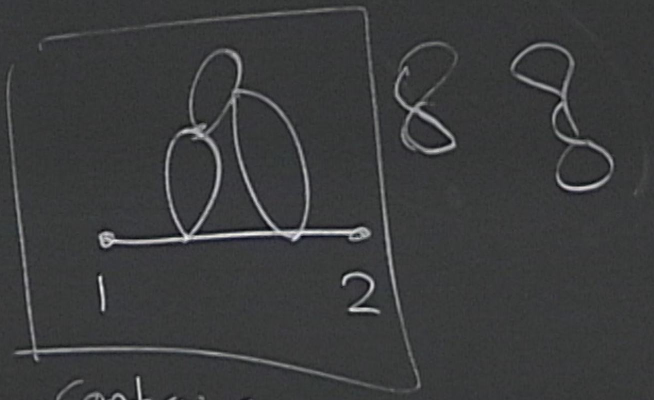
$S = \#$ of w

with

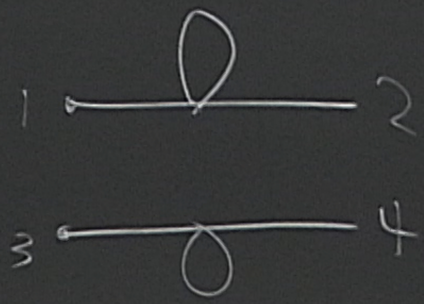
Wick

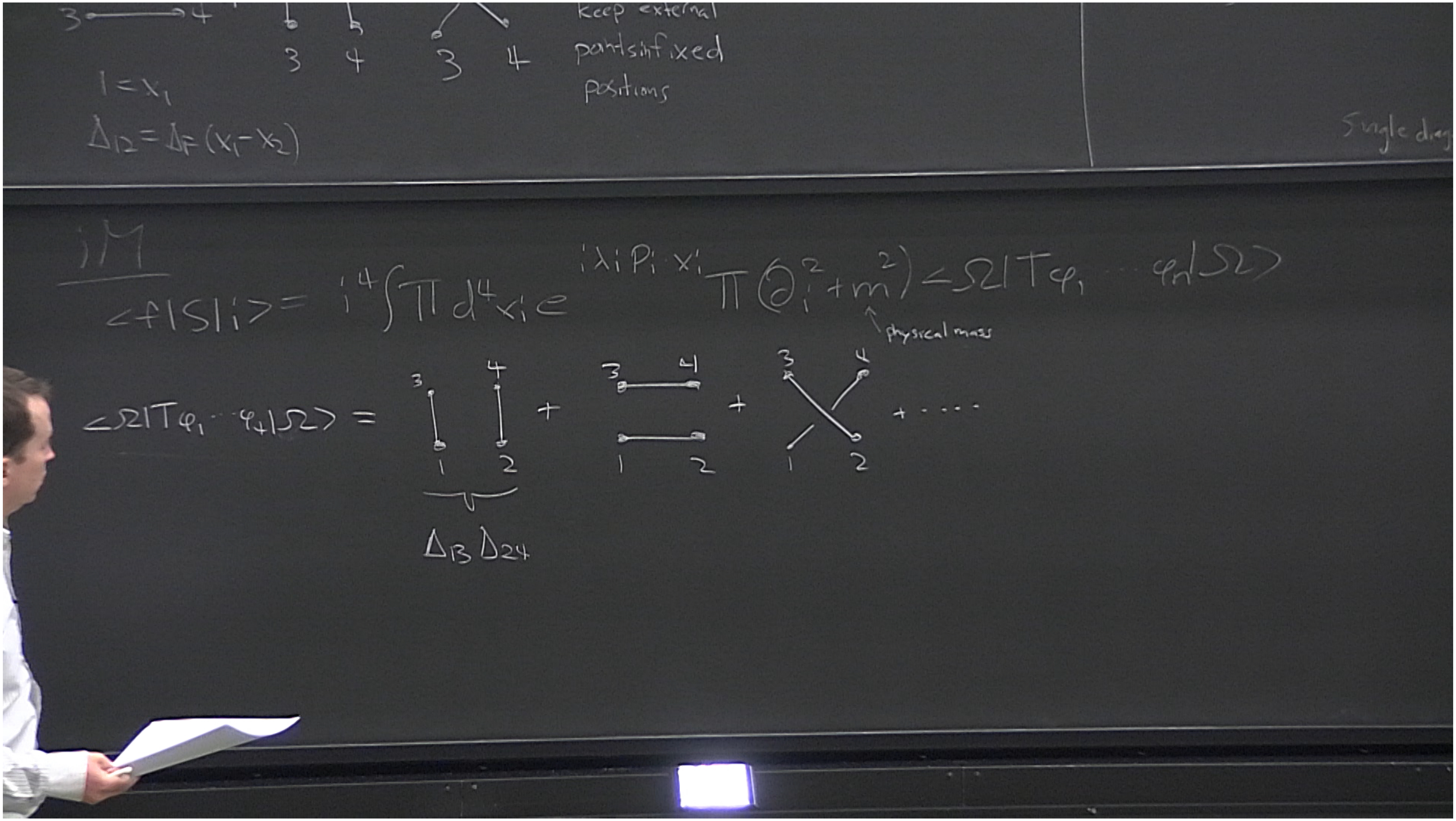
fact

for



Contains no vacuum subdiagram





keep external points in fixed positions

$1 = x_1$

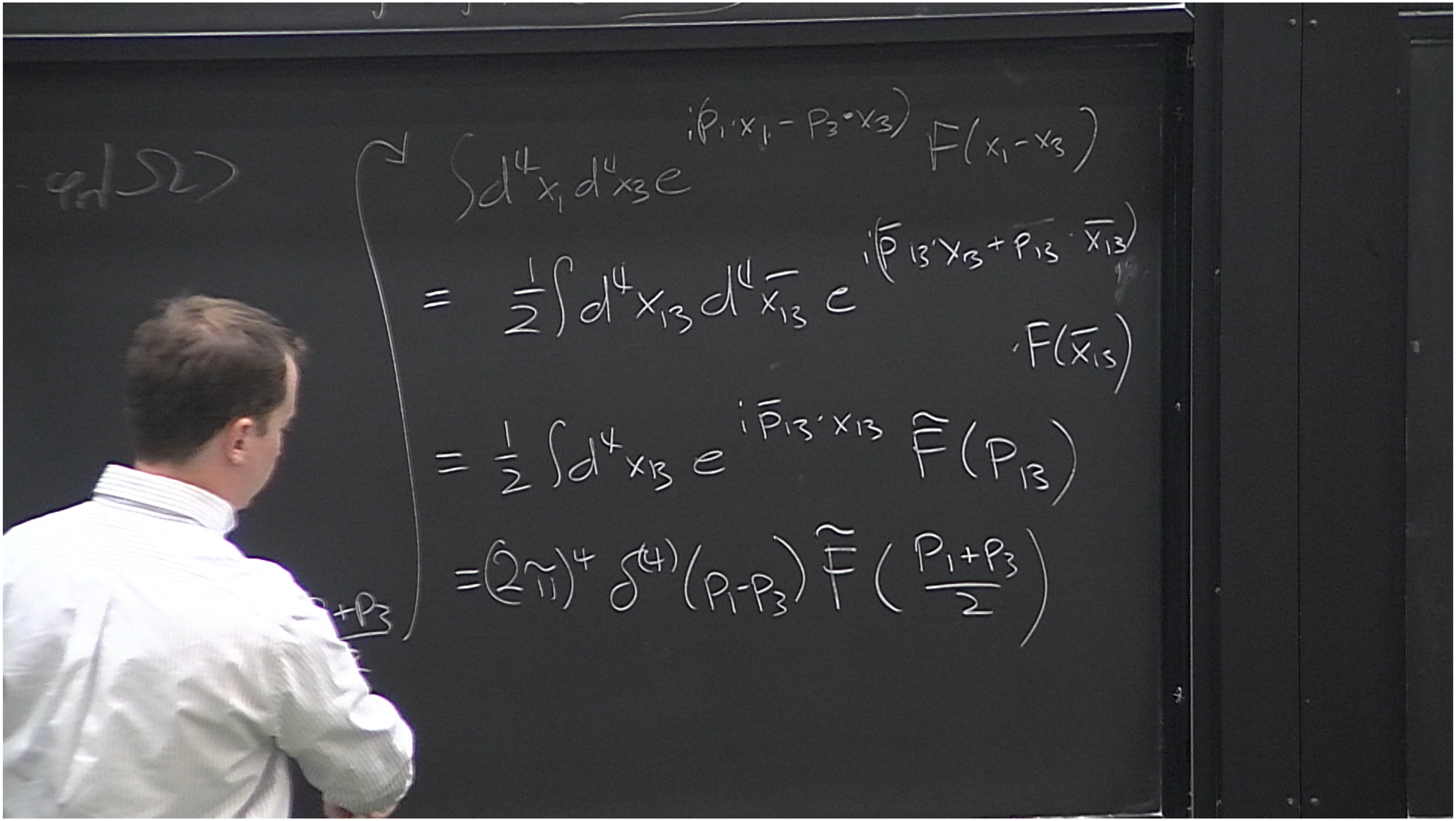
$\Delta_{12} = \Delta_F(x_1 - x_2)$

Single diag

$$\overset{iM}{\langle f|S|i\rangle} = i^4 \int \prod d^4x_i \, e^{i\lambda_i p_i \cdot x_i} \prod (\partial_i^2 + m^2) \langle \Omega | T\varphi_1 \cdots \varphi_n | \Omega \rangle$$

↑ physical mass

$$\langle \Omega | T\varphi_1 \cdots \varphi_4 | \Omega \rangle = \underbrace{\;}_{\Delta_{13}\Delta_{24}} + \; + \; + \cdots$$



$\varphi_{cl}|\Omega\rangle$

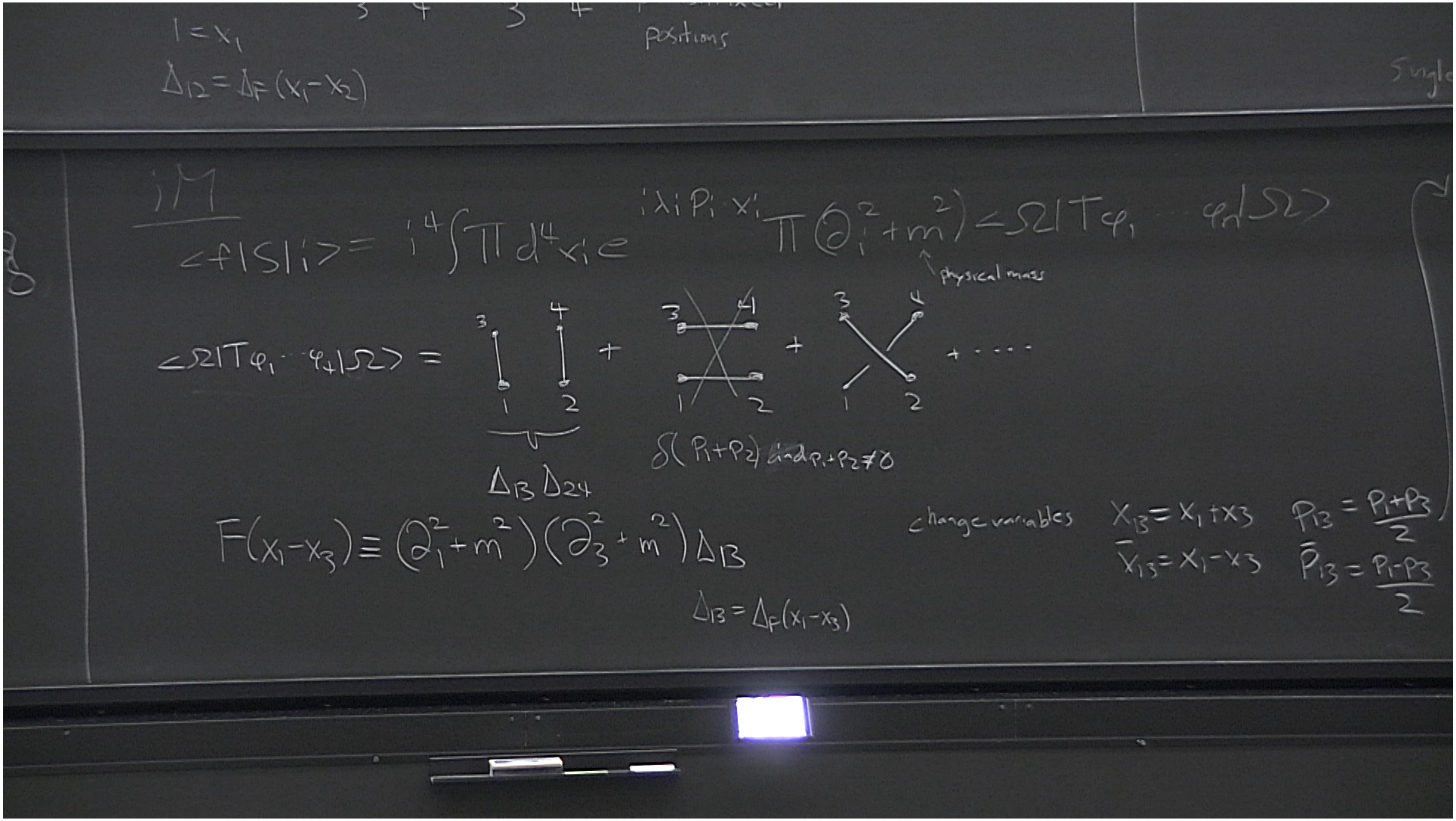
$$\int d^4x_1\, d^4x_3\, e^{i(p_1\cdot x_1 - p_3\cdot x_3)}\, F(x_1-x_3)$$

$$= \frac{1}{2}\int d^4x_{13}\, d^4\bar{x}_{13}\, e^{i(\bar{p}_{13}\cdot x_{13} + p_{13}\cdot \bar{x}_{13})} \cdot F(\bar{x}_{13})$$

$$= \frac{1}{2}\int d^4x_{13}\, e^{i\bar{p}_{13}\cdot x_{13}}\, \tilde{F}(p_{13})$$

$$= (2\pi)^4\, \delta^{(4)}(p_1-p_3)\, \tilde{F}\left(\frac{p_1+p_3}{2}\right)$$

$+p_3$



$1 = x_1$

positions

$\Delta_{12} = \Delta_F(x_1 - x_2)$

$$\frac{iM}{\langle f|S|i\rangle} = i^4 \int \prod d^4x_i \, e^{i\lambda_i p_i \cdot x_i} \prod (\partial_i^2 + m^2) \langle \Omega | T\varphi_1 \cdots \varphi_n | \Omega \rangle$$

physical mass

$$\langle \Omega | T\varphi_1 \cdots \varphi_4 | \Omega \rangle =$$

$\Delta_{13}\Delta_{24}$

$\delta(p_1+p_2)$ and $p_1+p_2 \neq 0$

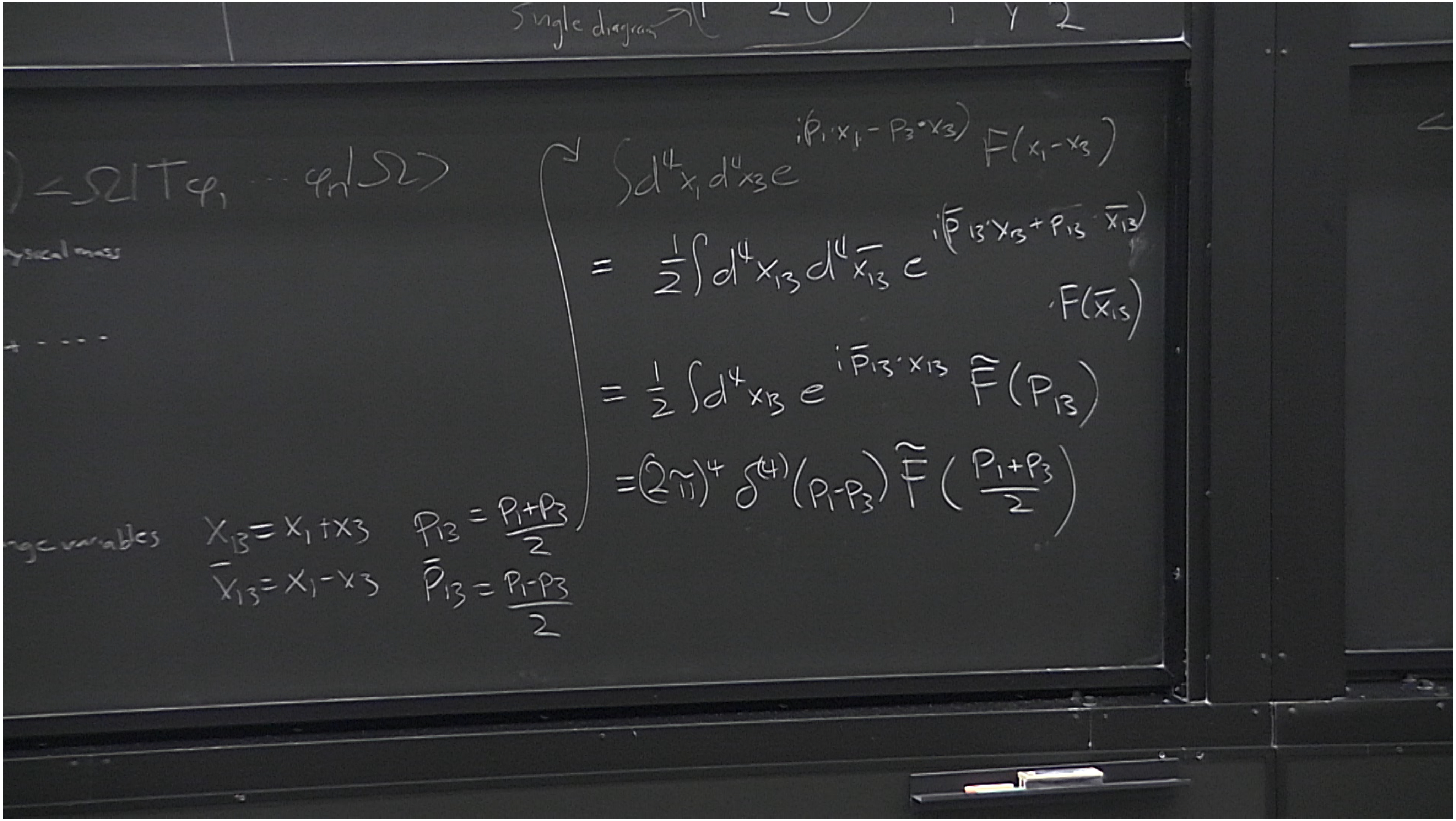
$$F(x_1 - x_3) \equiv (\partial_1^2 + m^2)(\partial_3^2 + m^2)\Delta_{13}$$

$\Delta_{13} = \Delta_F(x_1 - x_3)$

change variables

$X_{13} = x_1 + x_3$ $\quad P_{13} = \frac{p_1 + p_3}{2}$

$\bar{X}_{13} = x_1 - x_3$ $\quad \bar{P}_{13} = \frac{p_1 - p_3}{2}$



Single diagram →

$)\langle\Omega|T\varphi_1\cdots\varphi_n|\Omega\rangle$

ysical mass

$+ - - - -$

ge variables $X_{13}=x_1+x_3 \quad p_{13}=\frac{p_1+p_3}{2}$

$\bar{x}_{13}=x_1-x_3 \quad \bar{p}_{13}=\frac{p_1-p_3}{2}$

$$\int d^4x_1\, d^4x_3\, e^{i(p_1\cdot x_1-p_3\cdot x_3)}F(x_1-x_3)$$

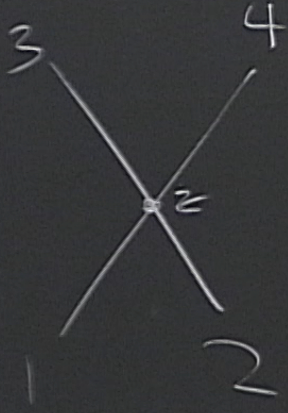
$$=\frac{1}{2}\int d^4x_{13}\, d^4\bar{x}_{13}\, e^{i(\bar{p}_{13}\cdot x_{13}+p_{13}\cdot\bar{x}_{13})}\cdot F(\bar{x}_{13})$$

$$=\frac{1}{2}\int d^4x_{13}\, e^{i\bar{p}_{13}\cdot x_{13}}\tilde{F}(p_{13})$$

$$=(2\pi)^4\delta^{(4)}(p_1-p_3)\tilde{F}\left(\frac{p_1+p_3}{2}\right)$$

$$\langle f|S|i\rangle_{11} \propto \delta^{(4)}(p_1-p_3)\,\delta^{(4)}(p_2-p_4)$$

→ no interference between disconnected + $\underbrace{\text{connected diagrams}}_{\text{only interesting ones}}$



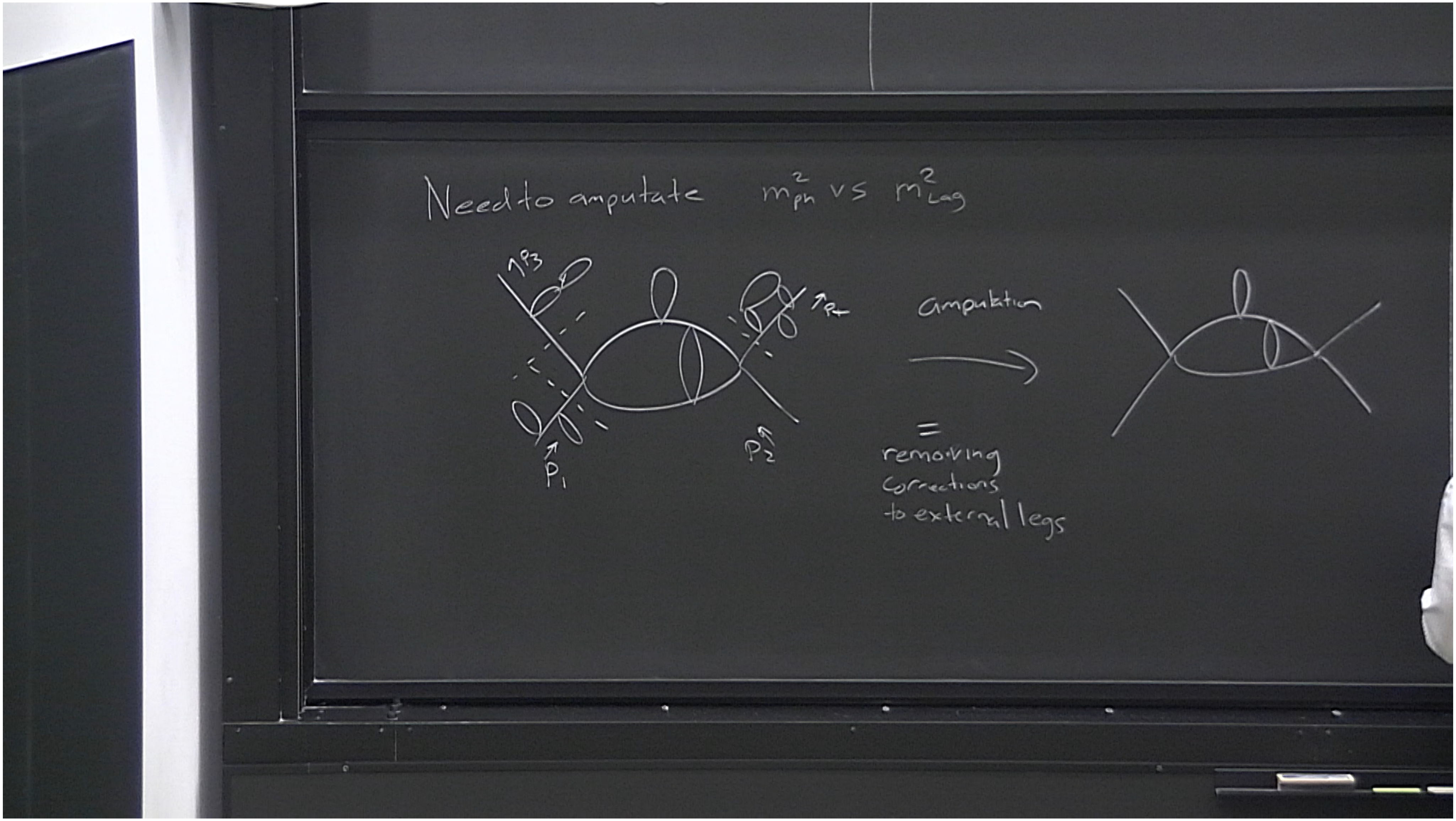
$$= -i\lambda \int d^4y\ \Delta_{1y}\,\Delta_{2y}\,\Delta_{3y}\,\Delta_{4y}$$

$$(\partial_1^2+m^2)\Delta_{1y} = -i\,\delta^{(4)}(x_1-y)$$ use to do x-integrals

$$\langle f | S | i \rangle_{\cancel{x}} = -i\lambda \int d^4y \, e^{i(p_1+p_2-p_3-p_4)\cdot y}$$

$$= -i\lambda (2\pi)^4 \delta^{(4)}(p_1+p_2-p_3-p_4)$$

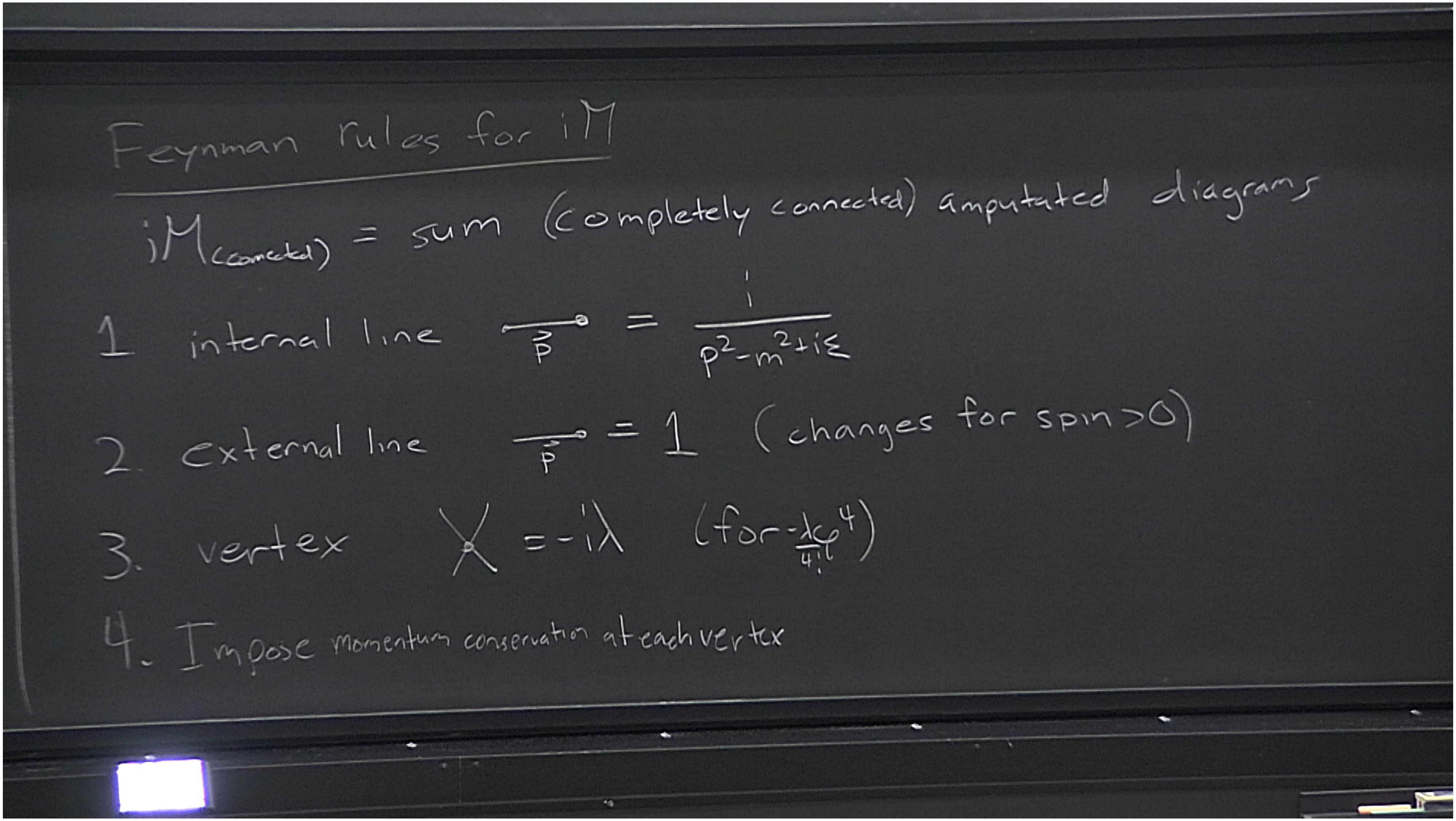
$$i\mathcal{M}_{i\to f} = -i\lambda + \mathcal{O}(\lambda^2)$$



Need to amputate m^2_{ph} vs m^2_{Lag}

amputation

=
removing
corrections
to external legs



Feynman rules for iM

$iM_{(\text{connected})}$ = sum (completely connected) amputated diagrams

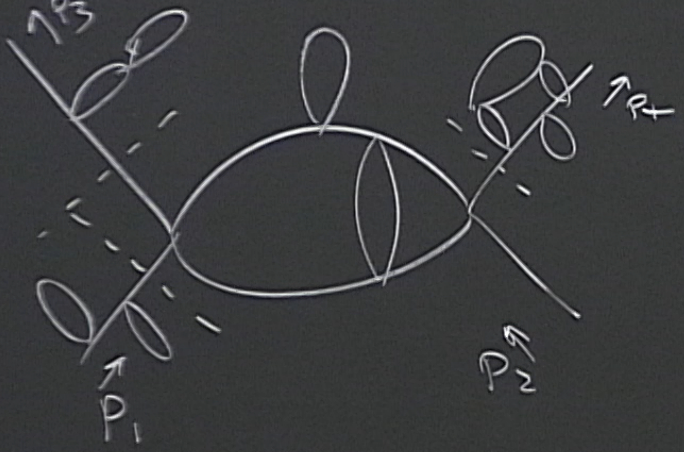
1 internal line $\underset{\vec{p}}{\circ\!\!-\!\!-\!\!-\!\!\circ} = \dfrac{i}{p^2-m^2+i\varepsilon}$

2. external line $\underset{\vec{p}}{\circ\!\!-\!\!-\!\!-\!\!\circ} = 1$ (changes for spin>0)

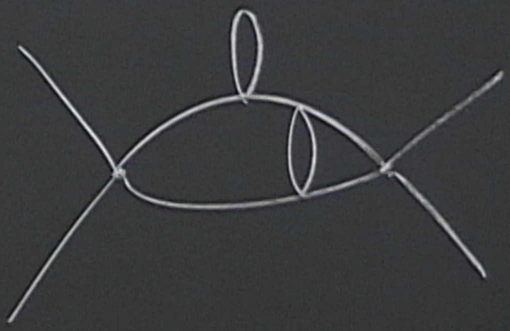
3. vertex $\times = -i\lambda$ (for $-\frac{\lambda}{4!}\varphi^4$)

4. Impose momentum conservation at each vertex

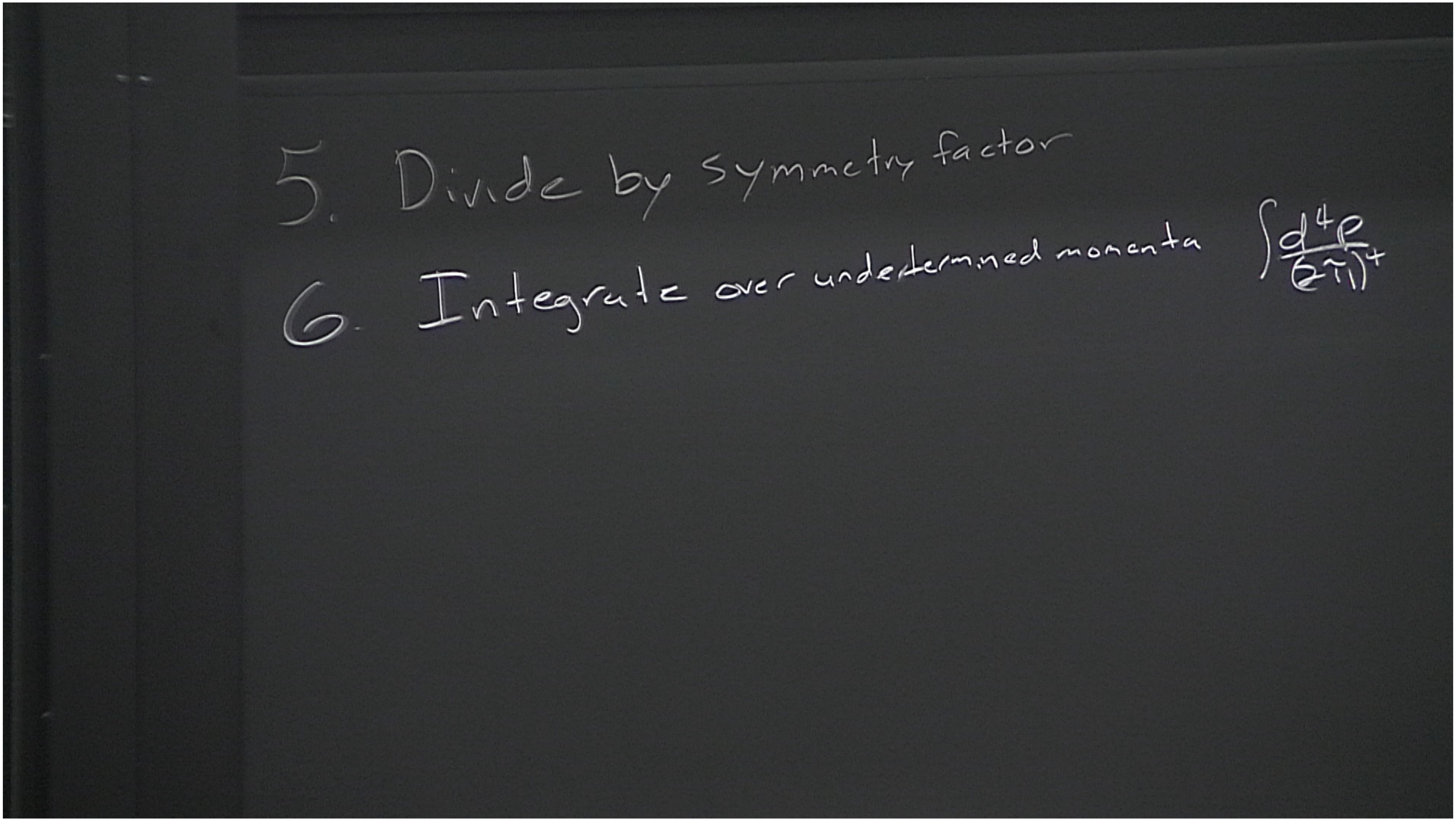
Need to amputate m^2_{ph} vs m^2_{Lag}



$\xrightarrow{\text{amputation}}$



= removing corrections to external legs



5. Divide by symmetry factor

6. Integrate over undetermined momenta $\int \frac{d^4p}{(2\pi)^4}$