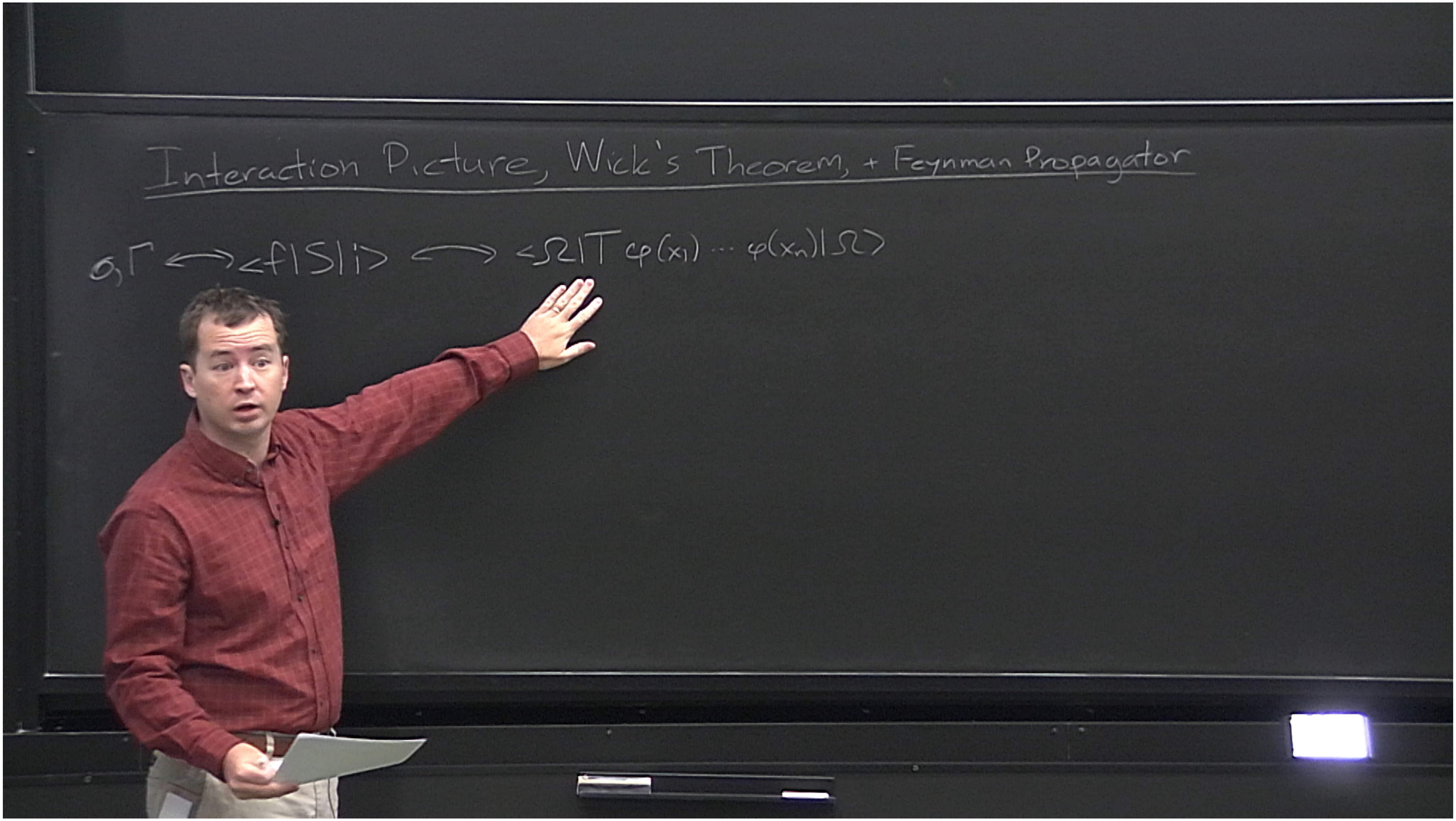
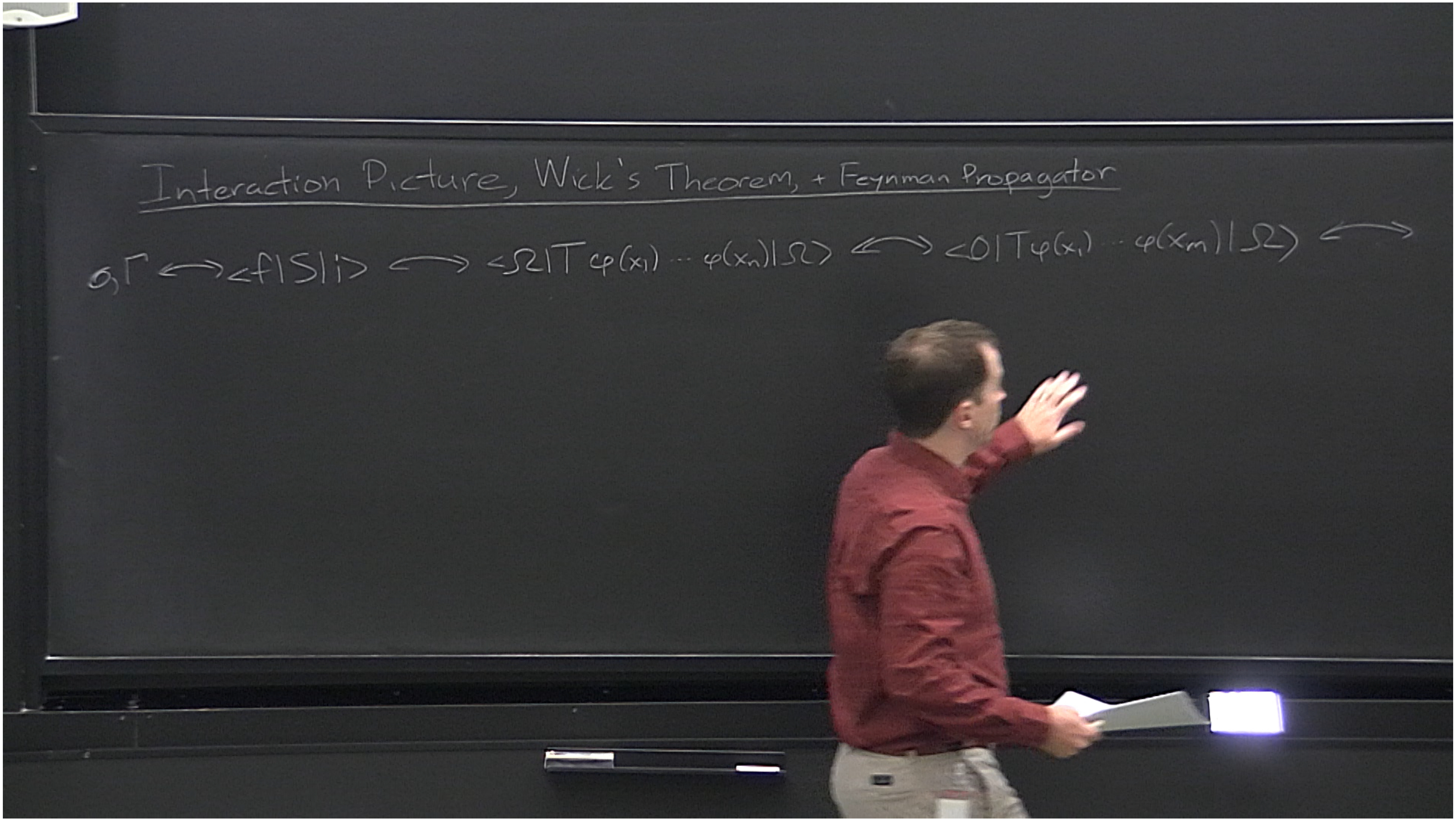
Title: PSI 2018/2019 - Quantum Field Theory I - Lecture 5

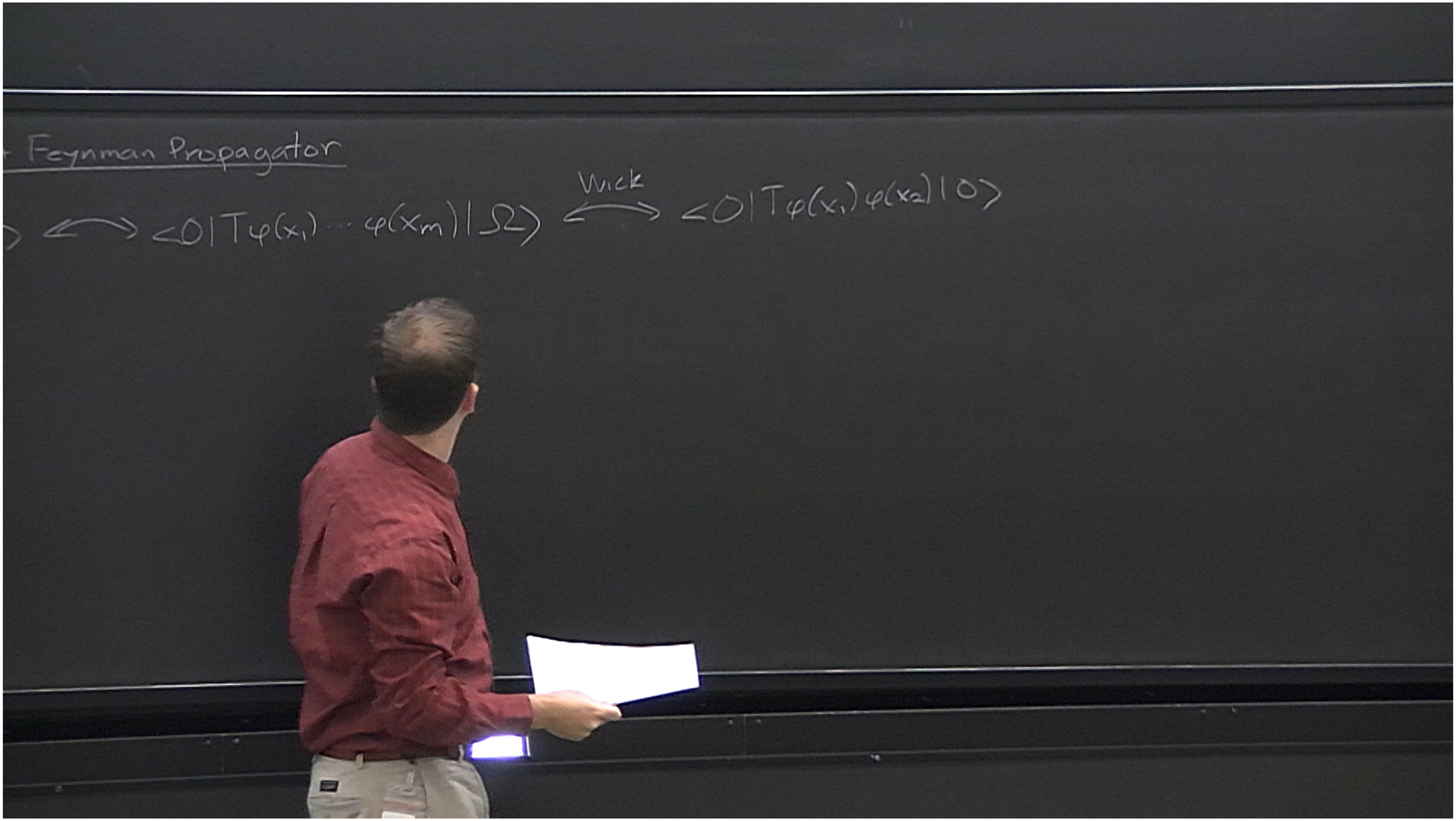
Date: Oct 16, 2018 09:00 AM

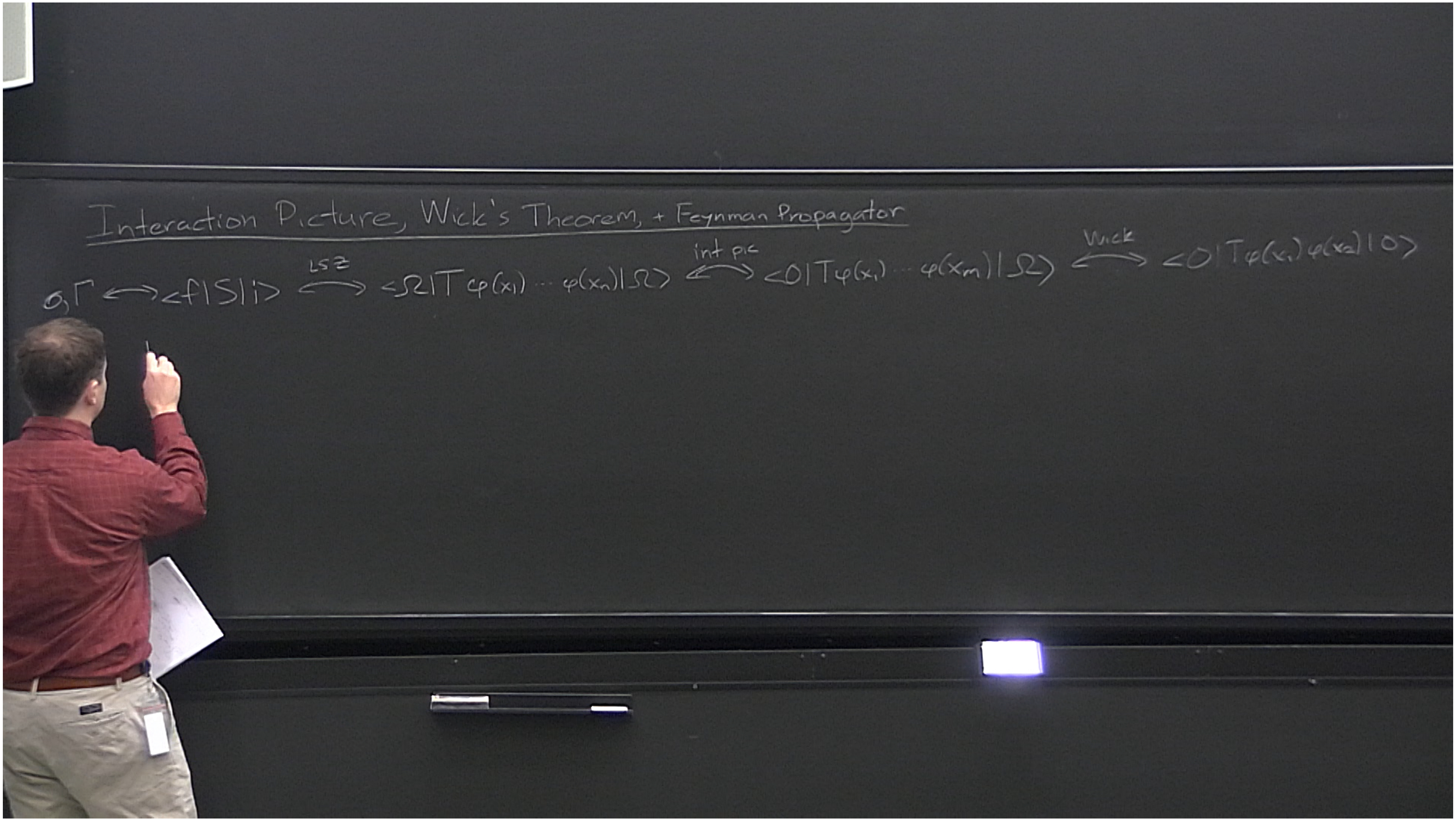
URL: http://pirsa.org/18100010

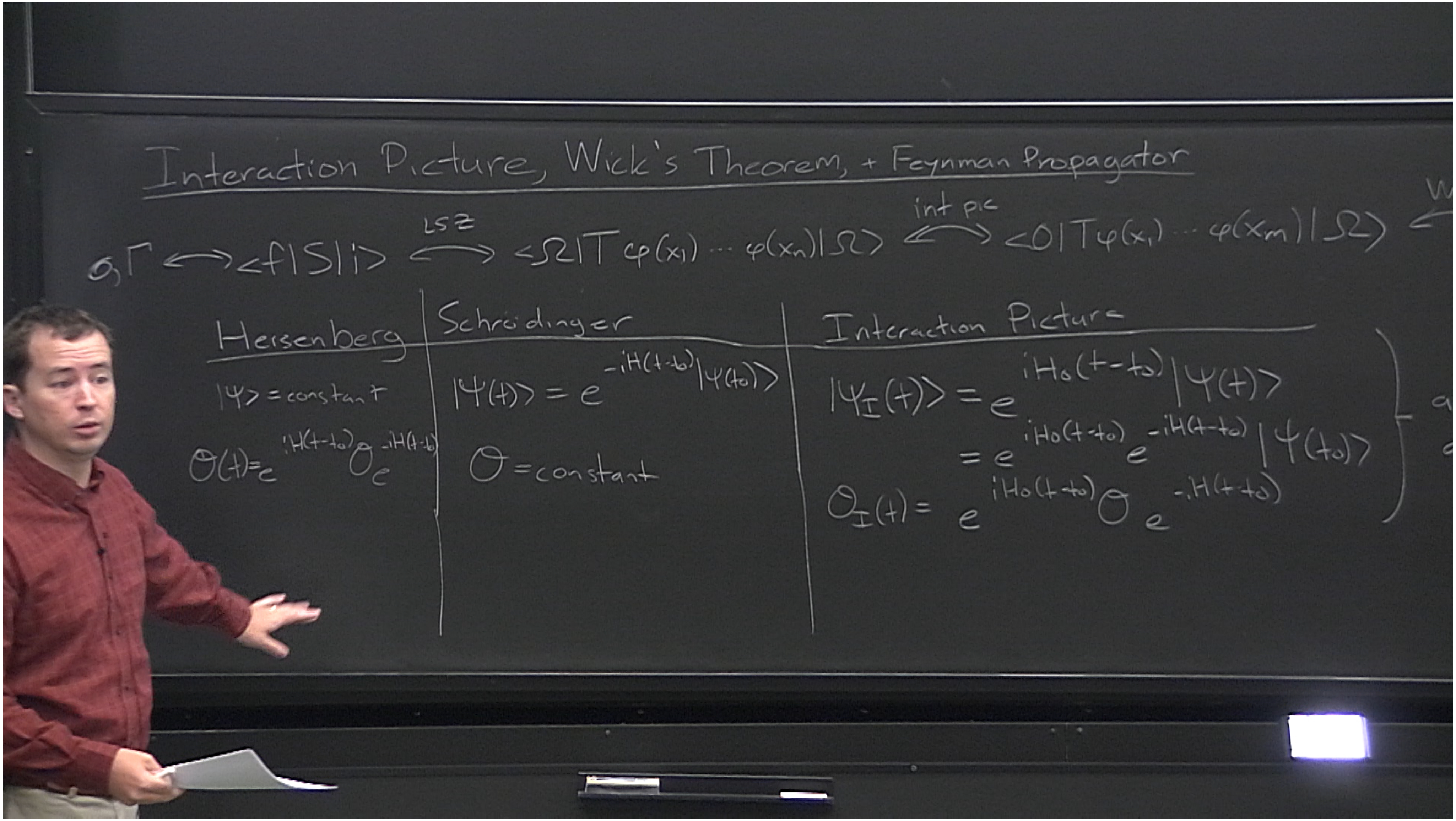
Abstract:

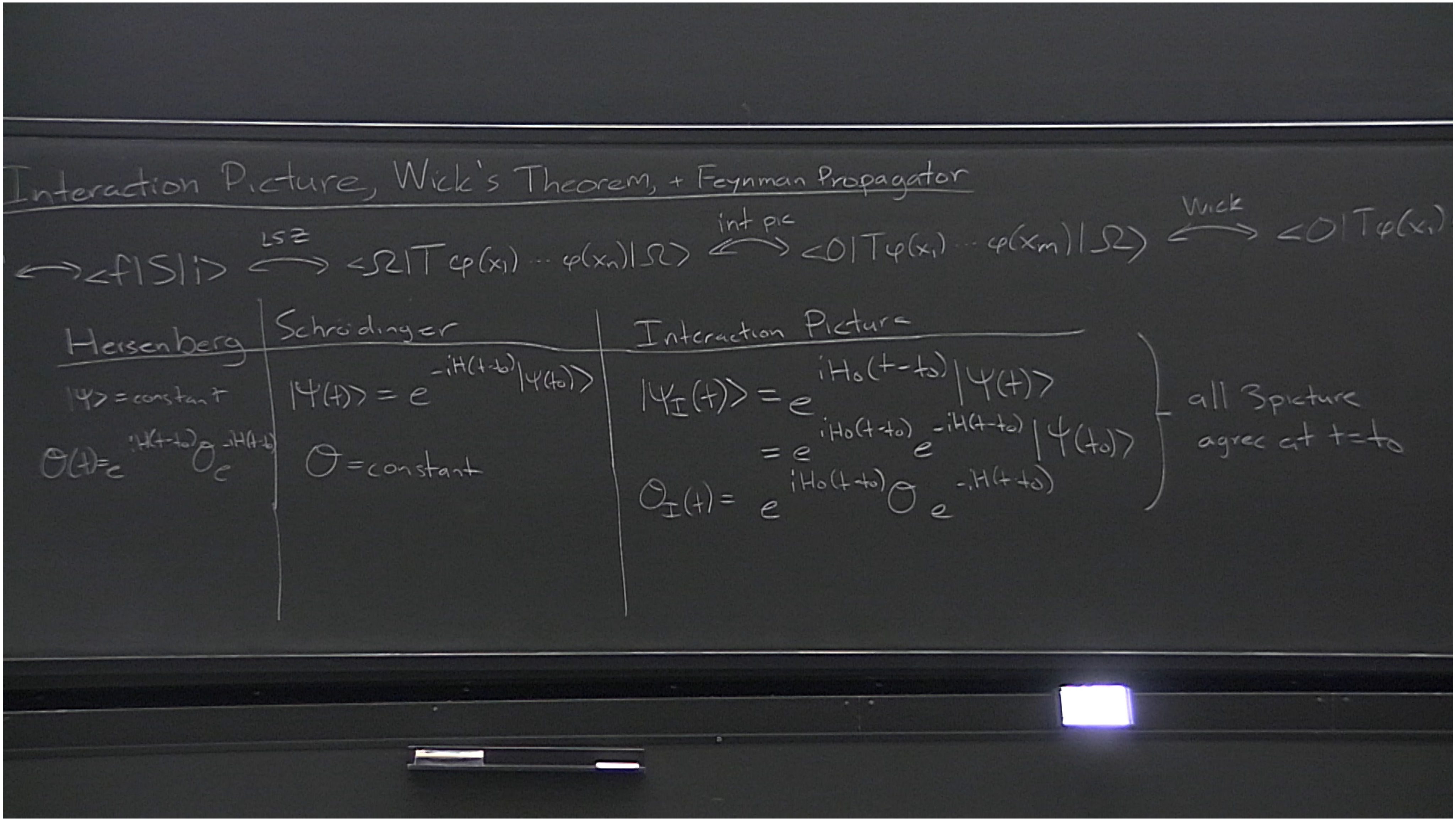

Interaction Picture, Wick's Theorem, + Feynman Propagator
$\sigma, \Gamma \longleftrightarrow \langle f|S|i\rangle \longleftrightarrow \langle\Omega|T\,\varphi(x_1)\cdots\varphi(x_n)|\Omega\rangle$


Interaction Picture, Wick's Theorem, + Feynman Propagator
σ, Γ ⟷ <f|S|i> ⟷ <Ω|T φ(x1) ··· φ(xn)|Ω> ⟷ <0|T φ(x1) ··· φ(xm)|Ω> ⟷


Feynman Propagator
Wick
<0|Tφ(x1)···φ(xm)|Ω>
<0|Tφ(x1)φ(x2)|0>


Interaction Picture, Wick's Theorem, + Feynman Propagator
LSZ
int pic
Wick
$\sigma, \Gamma \longleftrightarrow \langle f|S|i\rangle \longleftrightarrow \langle\Omega|T\,\varphi(x_1)\cdots\varphi(x_n)|\Omega\rangle \longleftrightarrow \langle 0|T\varphi(x_1)\cdots\varphi(x_m)|\Omega\rangle \longleftrightarrow \langle 0|T\varphi(x_1)\varphi(x_2)|0\rangle$


Interaction Picture, Wick's Theorem, + Feynman Propagator
$\sigma, \Gamma \longleftrightarrow \langle f|S|i\rangle \overset{LSZ}{\longleftrightarrow} \langle\Omega|T\varphi(x_1)\cdots\varphi(x_n)|\Omega\rangle \overset{\text{int pic}}{\longleftrightarrow} \langle 0|T\varphi(x_1)\cdots\varphi(x_m)|\Omega\rangle$
Heisenberg
$|\psi\rangle = \text{constant}$
$\mathcal{O}(t) = e^{iH(t-t_0)}\mathcal{O}e^{-iH(t-t_0)}$
Schrödinger
$|\psi(t)\rangle = e^{-iH(t-t_0)}|\psi(t_0)\rangle$
$\mathcal{O} = \text{constant}$
Interaction Picture
$|\psi_I(t)\rangle = e^{iH_0(t-t_0)}|\psi(t)\rangle$
$= e^{iH_0(t-t_0)}e^{-iH(t-t_0)}|\psi(t_0)\rangle$
$\mathcal{O}_I(t) = e^{iH_0(t-t_0)}\mathcal{O}e^{-iH(t-t_0)}$


Interaction Picture, Wick's Theorem, + Feynman Propagator
⟷ <f|S|i> ⟷(LSZ) <Ω|T φ(x₁) ··· φ(xₙ)|Ω> ⟷(int pic) <0|T φ(x₁) ··· φ(xₘ)|Ω> ⟷(Wick) <0|T φ(x₁)
Heisenberg
|ψ> = constant
O(t) = e^{iH(t-t₀)} O e^{-iH(t-t₀)}
Schrödinger
|ψ(t)> = e^{-iH(t-t₀)} |ψ(t₀)>
O = constant
Interaction Picture
|ψ_I(t)> = e^{iH₀(t-t₀)} |ψ(t)>
= e^{iH₀(t-t₀)} e^{-iH(t-t₀)} |ψ(t₀)>
O_I(t) = e^{iH₀(t-t₀)} O e^{-iH(t-t₀)}
all 3 picture agree at t=t₀

Wick's Theorem, + Feynman Propagator

$\langle\Omega| T\varphi(x_1)\cdots\varphi(x_n)|\Omega\rangle \overset{\text{int pic}}{\longleftrightarrow} \langle 0| T\varphi(x_1)\cdots\varphi(x_m)|\Omega\rangle \overset{\text{Wick}}{\longleftrightarrow} \langle 0| T\varphi(x_1)\varphi(x_2)|0\rangle$

$\varphi(\vec{x},$

...nger

$= e^{-iH(t-t_0)}|\psi(t_0)\rangle$

...onstant

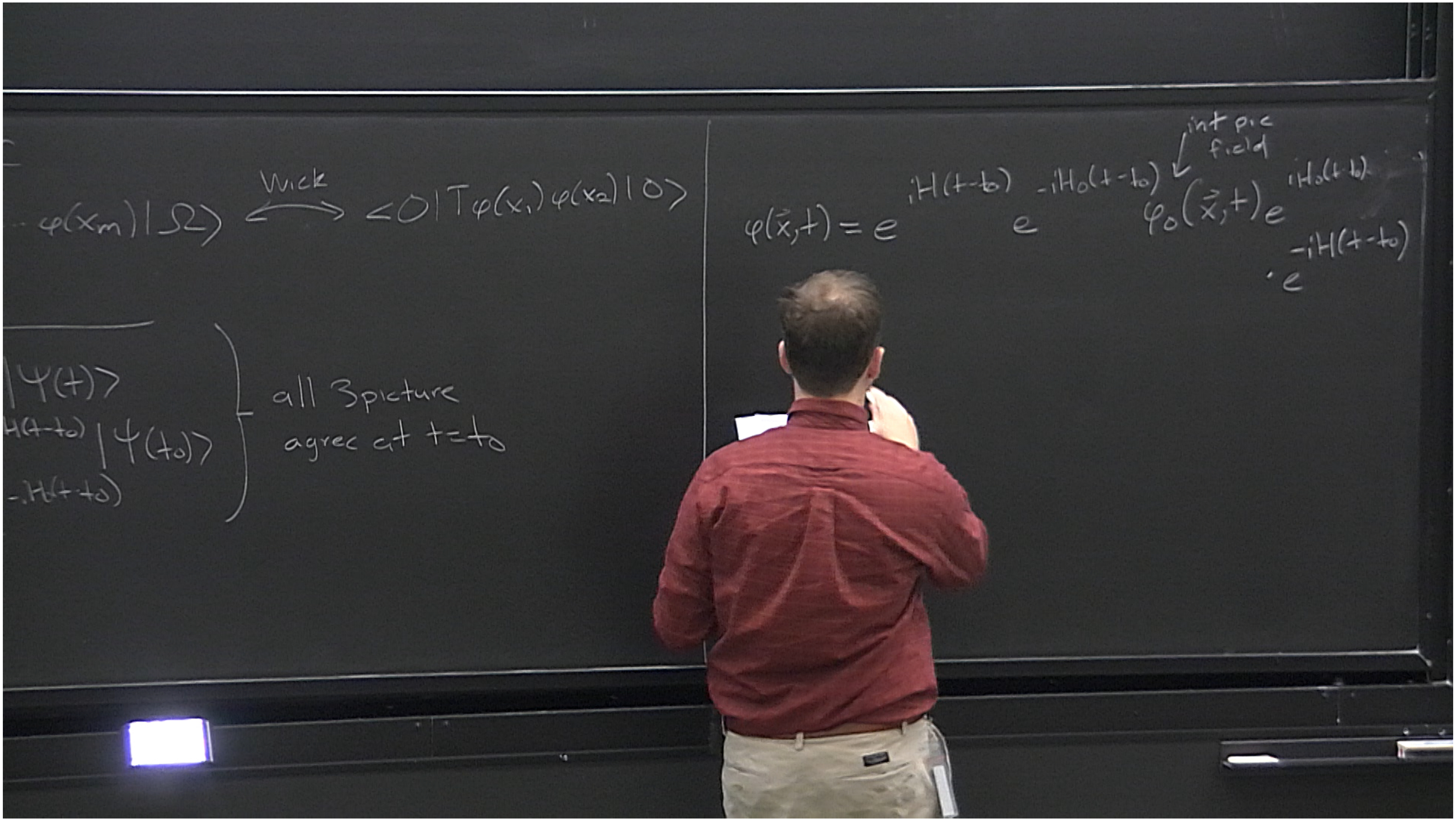
Interaction Picture

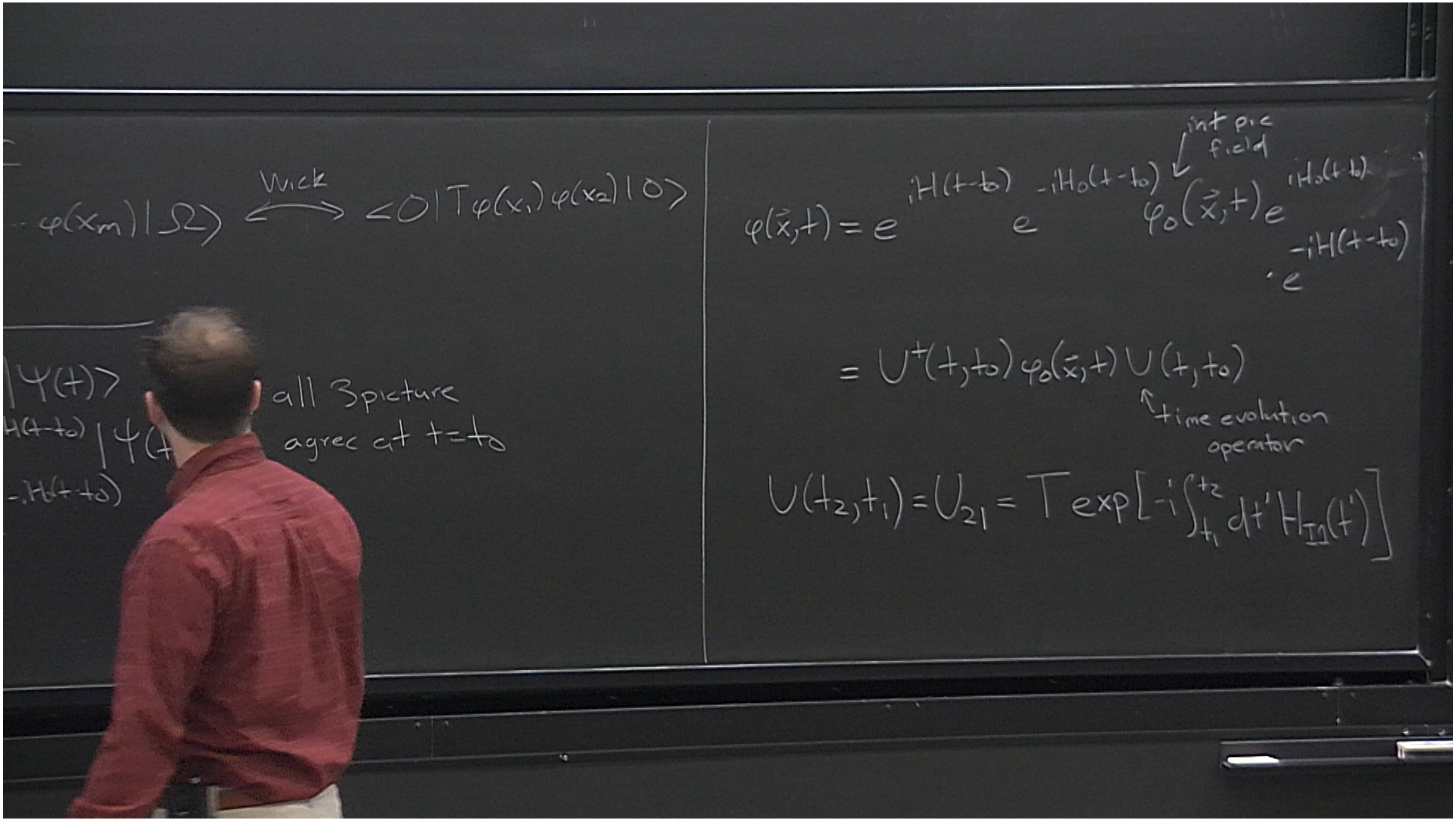
$$|\psi_I(t)\rangle = e^{iH_0(t-t_0)}|\psi(t)\rangle$$

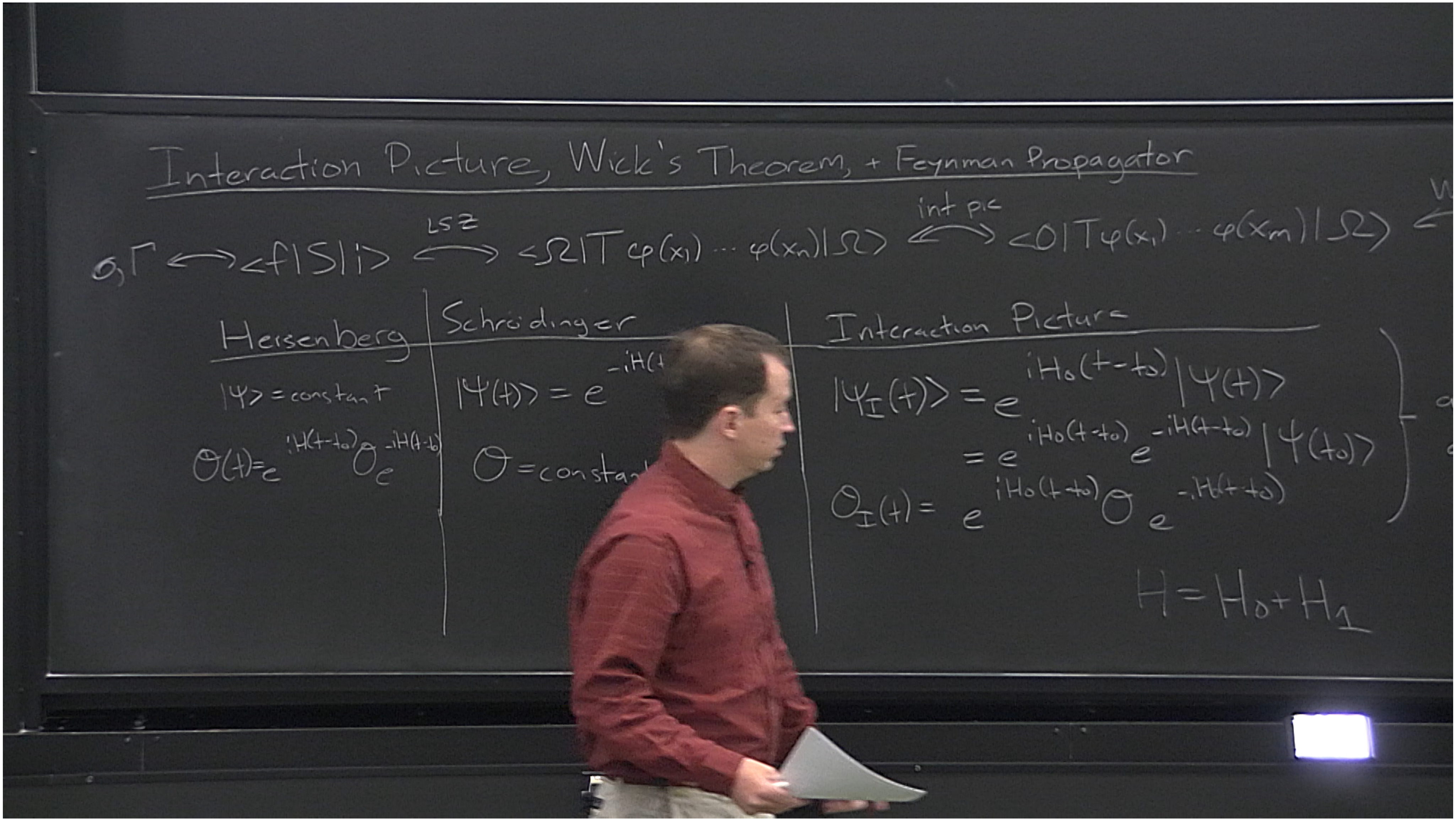
$$= e^{iH_0(t-t_0)} e^{-iH(t-t_0)}|\psi(t_0)\rangle$$

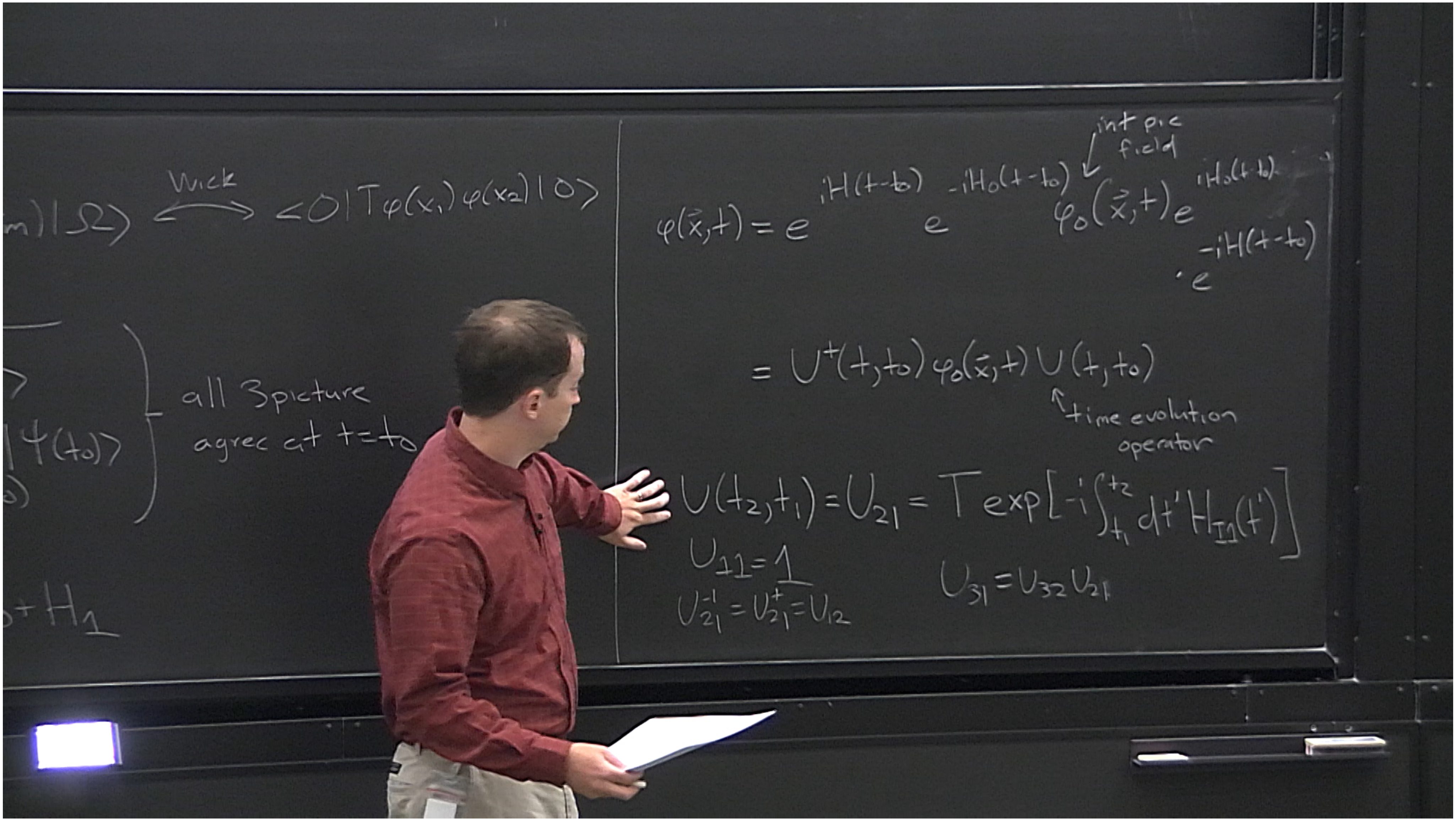
$$O_I(t) = e^{iH_0(t-t_0)}\, O\, e^{-iH_0(t-t_0)}$$

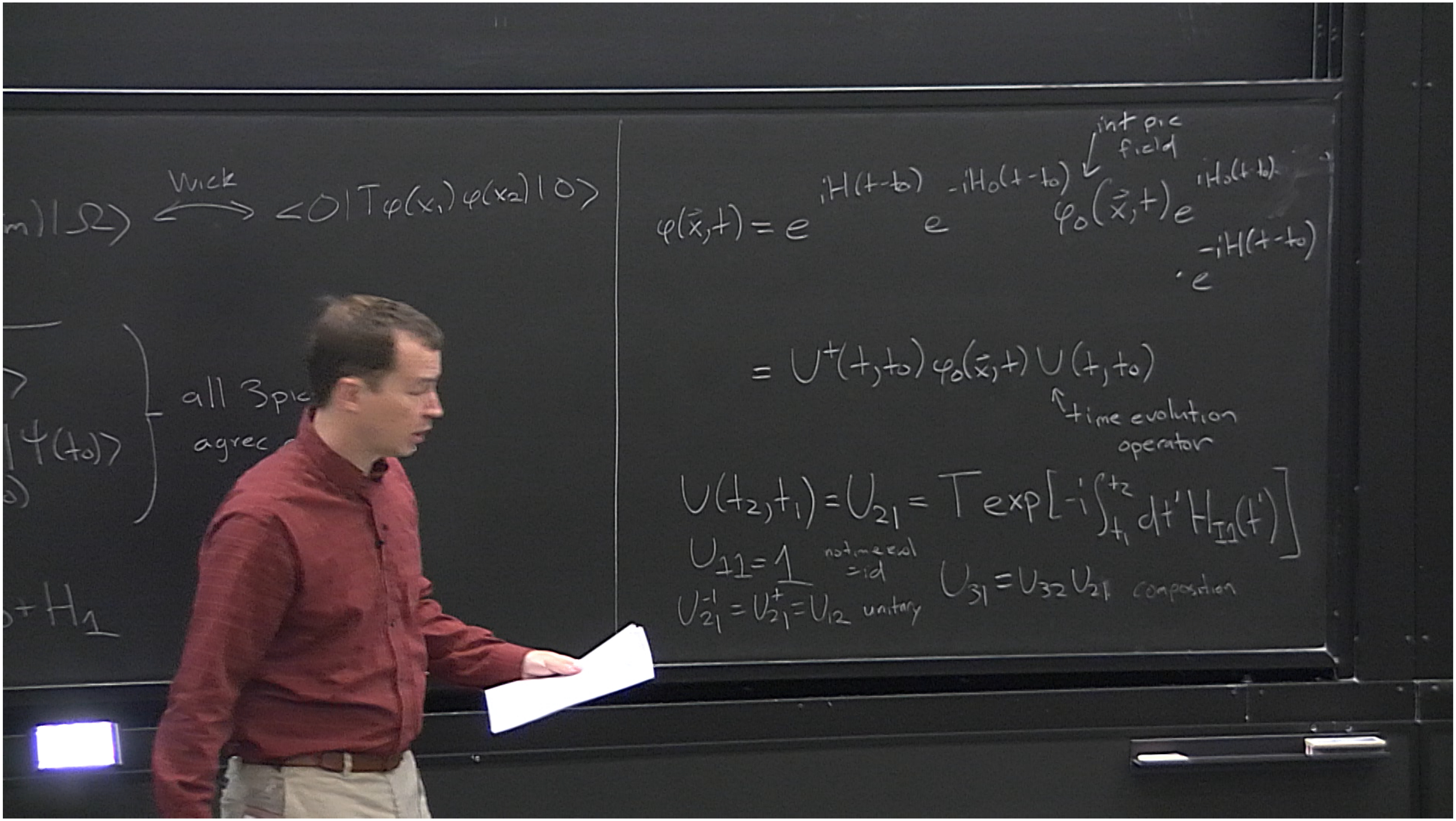
all 3 picture agree at $t=t_0$

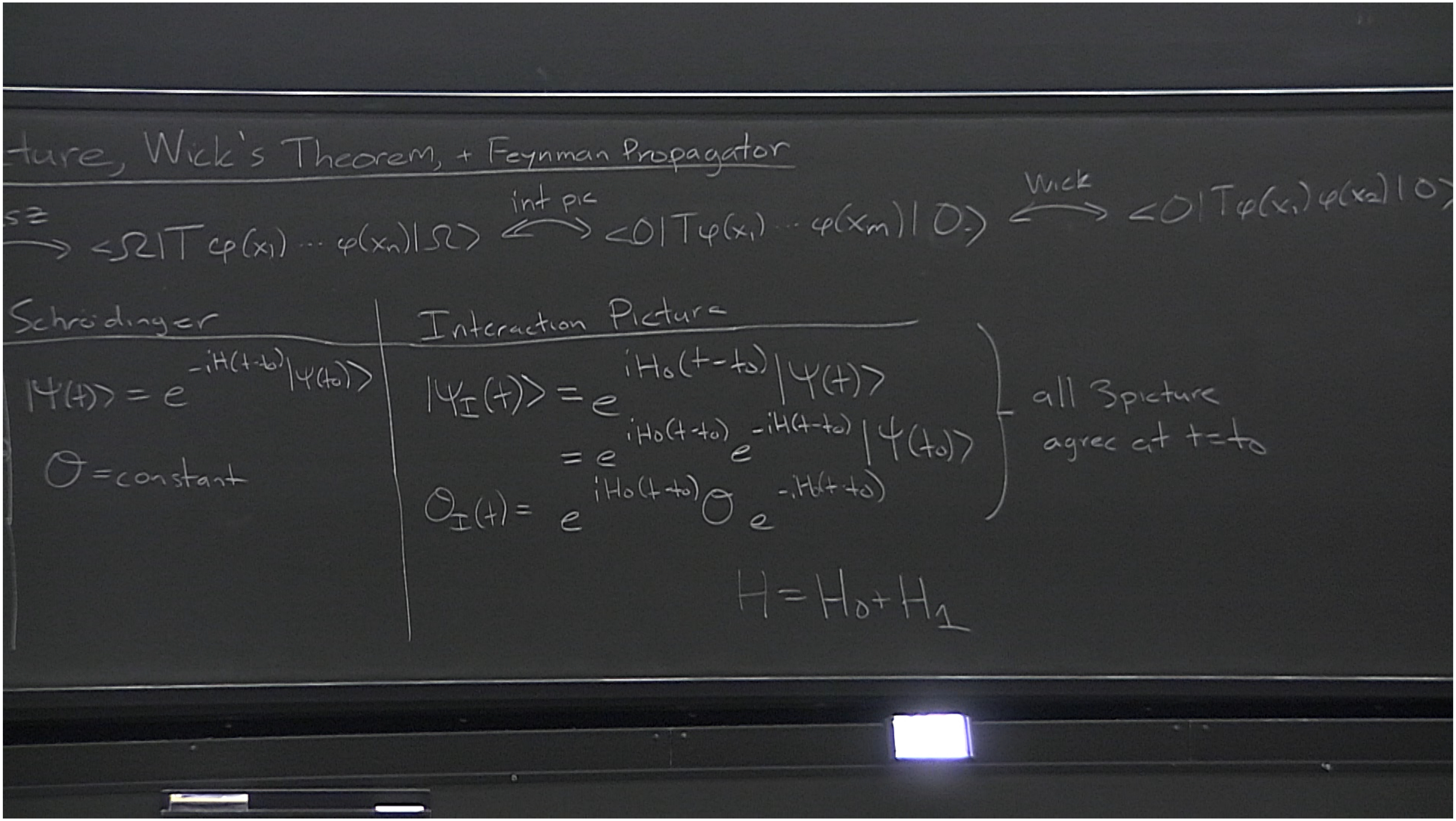

Wick
φ(x_m)|Ω> ⟷ <0|Tφ(x₁)φ(x₂)|0>
|Ψ(t)>
|Ψ(t₀)>
all 3picture
agree at t=t₀
int pic field
φ(x⃗,t) = e^{iH(t-t₀)} e^{-iH₀(t-t₀)} φ₀(x⃗,t) e^{iH₀(t-t₀)} · e^{-iH(t-t₀)}


Wick
φ(x_m)|Ω⟩ ⟷ ⟨0|Tφ(x₁)φ(x₂)|0⟩
|Ψ(t)⟩
all 3 picture
agree at t=t₀
int pic field
φ(x⃗,t) = e^{iH(t−t₀)} e^{−iH₀(t−t₀)} φ₀(x⃗,t) e^{iH₀(t−t₀)} · e^{−iH(t−t₀)}
= U†(t,t₀) φ₀(x⃗,t) U(t,t₀)
time evolution operator
U(t₂,t₁) = U₂₁ = T exp[−i ∫_{t₁}^{t₂} dt′ H_{II}(t′)]


Interaction Picture, Wick's Theorem, + Feynman Propagator
LSZ
int pic
Heisenberg
Schrödinger
Interaction Picture
|ψ> = constant
O = constant
H = H0 + H1


Wick
<0|Tφ(x1)φ(x2)|0>
all 3 picture agree at t=t0
int pic field
φ(x,t) = e^{iH(t-t0)} e^{-iH0(t-t0)} φ0(x,t) e^{iH0(t-t0)} · e^{-iH(t-t0)}
= U†(t,t0) φ0(x,t) U(t,t0)
time evolution operator
U(t2,t1) = U21 = T exp[-i ∫_{t1}^{t2} dt' H_I1(t')]
U11 = 1
U21^{-1} = U21† = U12
U31 = U32 U21


Wick
<0| T φ(x1) φ(x2) |0>
all 3pic
agree
int pic field
φ(x,t) = e^{iH(t-t0)} e^{-iH0(t-t0)} φ0(x,t) e^{iH0(t-t0)} e^{-iH(t-t0)}
= U†(t,t0) φ0(x,t) U(t,t0)
time evolution operator
U(t2,t1) = U21 = T exp[-i ∫_{t1}^{t2} dt' H_I1(t')]
U11 = 1
no time evol = id
U31 = U32 U21 composition
U21^{-1} = U21† = U12 unitary


…ture, Wick's Theorem, + Feynman Propagator
…s ≡ ⟶ $\langle\Omega| T\varphi(x_1)\cdots\varphi(x_n)|\Omega\rangle \overset{\text{int pic}}{\longleftrightarrow} \langle 0| T\varphi(x_1)\cdots\varphi(x_m)|0\rangle \overset{\text{Wick}}{\longleftrightarrow} \langle 0|T\varphi(x_1)\varphi(x_2)|0\rangle$
Schrödinger
$|\psi(t)\rangle = e^{-iH(t-t_0)}|\psi(t_0)\rangle$
$\mathcal{O}$ = constant
Interaction Picture
$|\psi_I(t)\rangle = e^{iH_0(t-t_0)}|\psi(t)\rangle$
$= e^{iH_0(t-t_0)}e^{-iH(t-t_0)}|\psi(t_0)\rangle$
$\mathcal{O}_I(t) = e^{iH_0(t-t_0)}\mathcal{O}\,e^{-iH_0(t-t_0)}$
all 3 picture agree at $t=t_0$
$H = H_0 + H_1$

$|\Omega\rangle$ vs $|0\rangle$

$a_{\vec{p}}(t_0)|\Omega(t)\rangle = 0$ at least for $t = \pm\infty$

$a_{\vec{p}}(t_0)e^{-iH(t-t_0)}|\Omega\rangle = 0$

$a_{\vec{p}}(t_0)|0(t)\rangle = 0$

$a_{\vec{p}}(t_0)e^{-iH_0(t-t_0)}|0\rangle = 0$

$|\Omega\rangle$ vs $|0\rangle$

$a_{\vec{p}}(t_0)|\Omega(t)\rangle = 0$ at least for $t = \pm\infty$

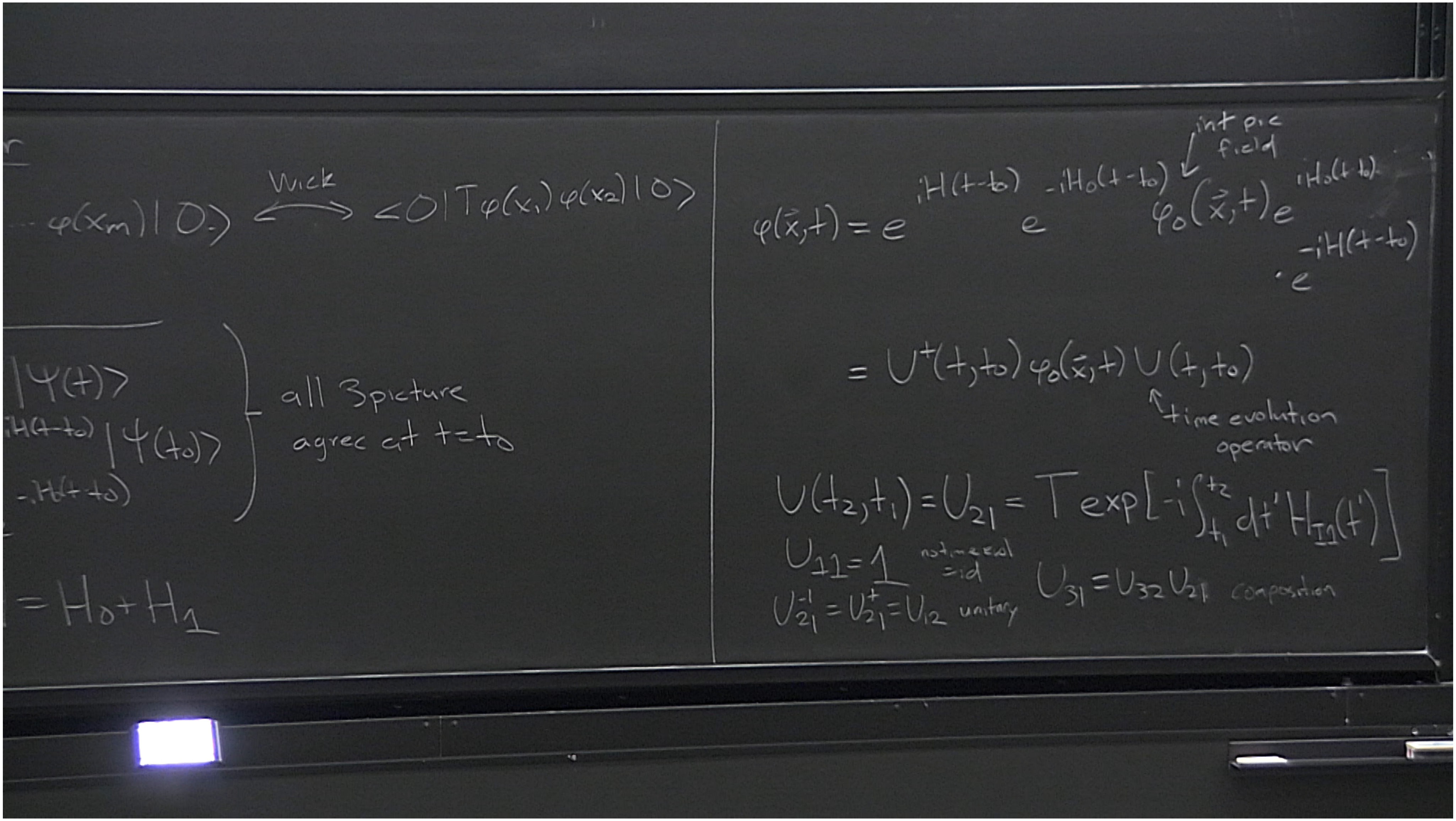
$$a_{\vec{p}}(t_0)\underbrace{e^{-iH(t-t_0)}|\Omega\rangle}=0$$

$$a_{\vec{p}}(t_0)|0(t)\rangle = 0$$

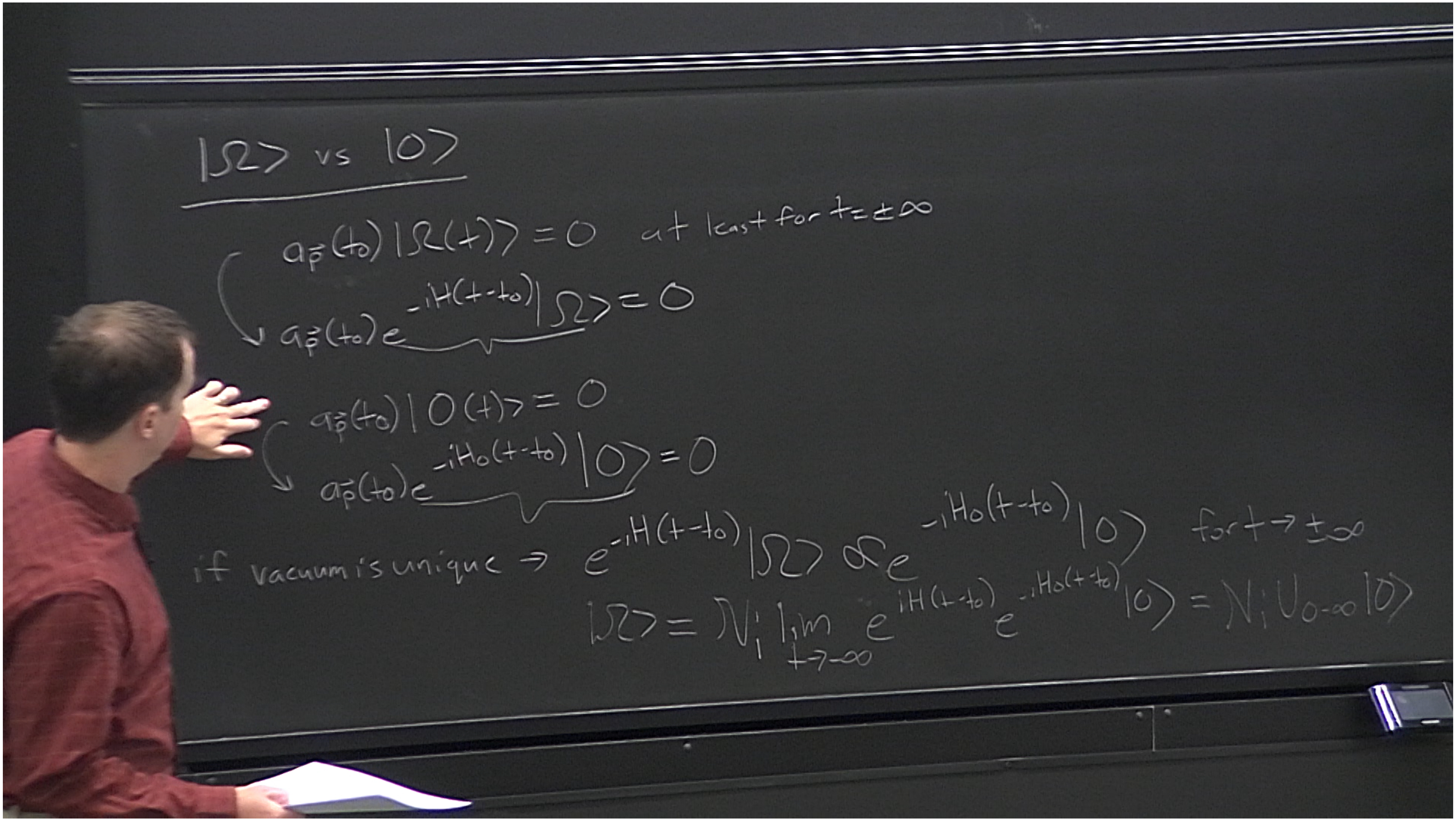
$$a_{\vec{p}}(t_0)\underbrace{e^{-iH_0(t-t_0)}|0\rangle}=0$$

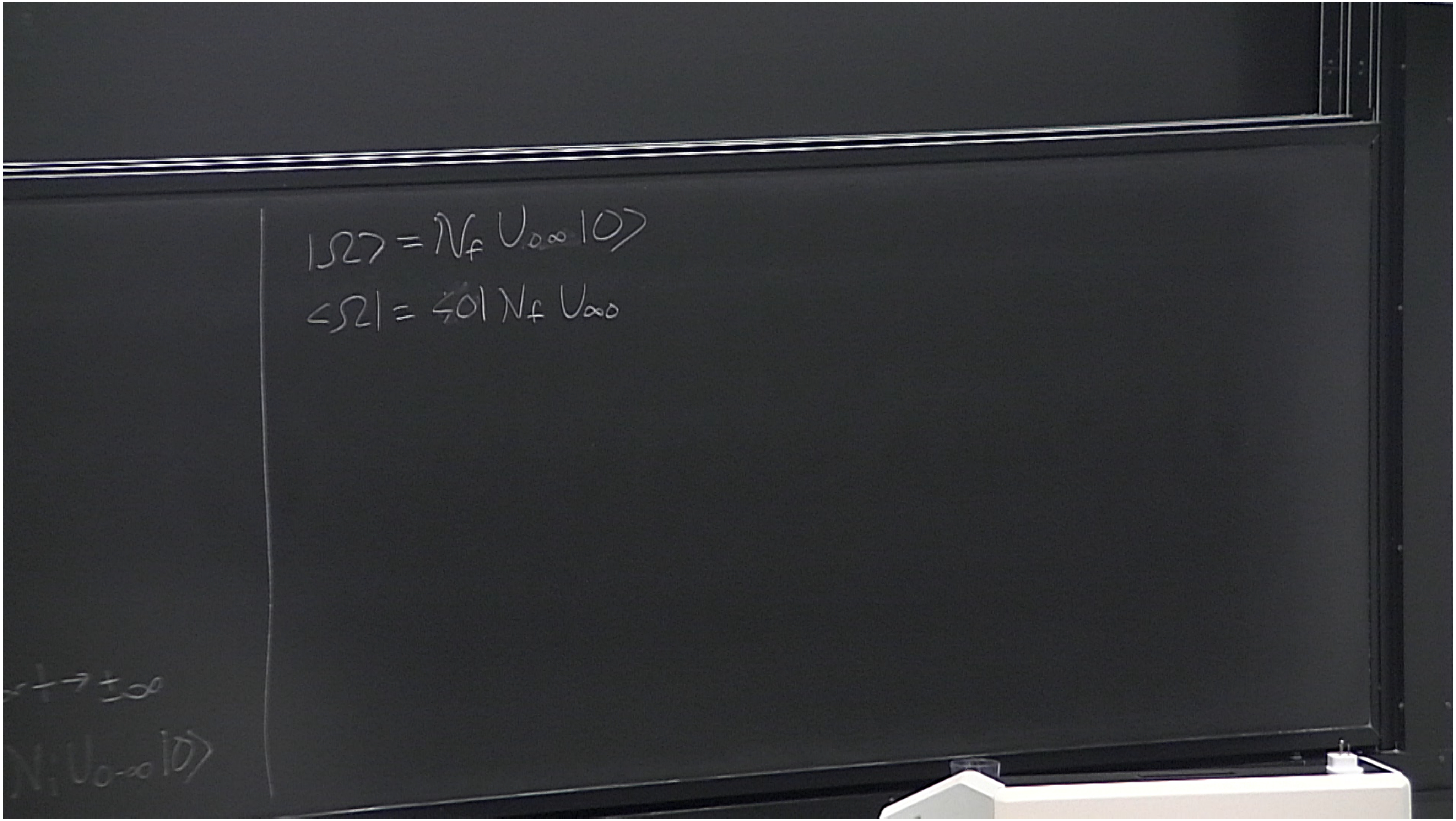
if vacuum is unique $\Rightarrow$ $e^{-iH(t-t_0)}|\Omega\rangle \propto e^{-iH_0(t-t_0)}|0\rangle$ for $t\to\pm\infty$

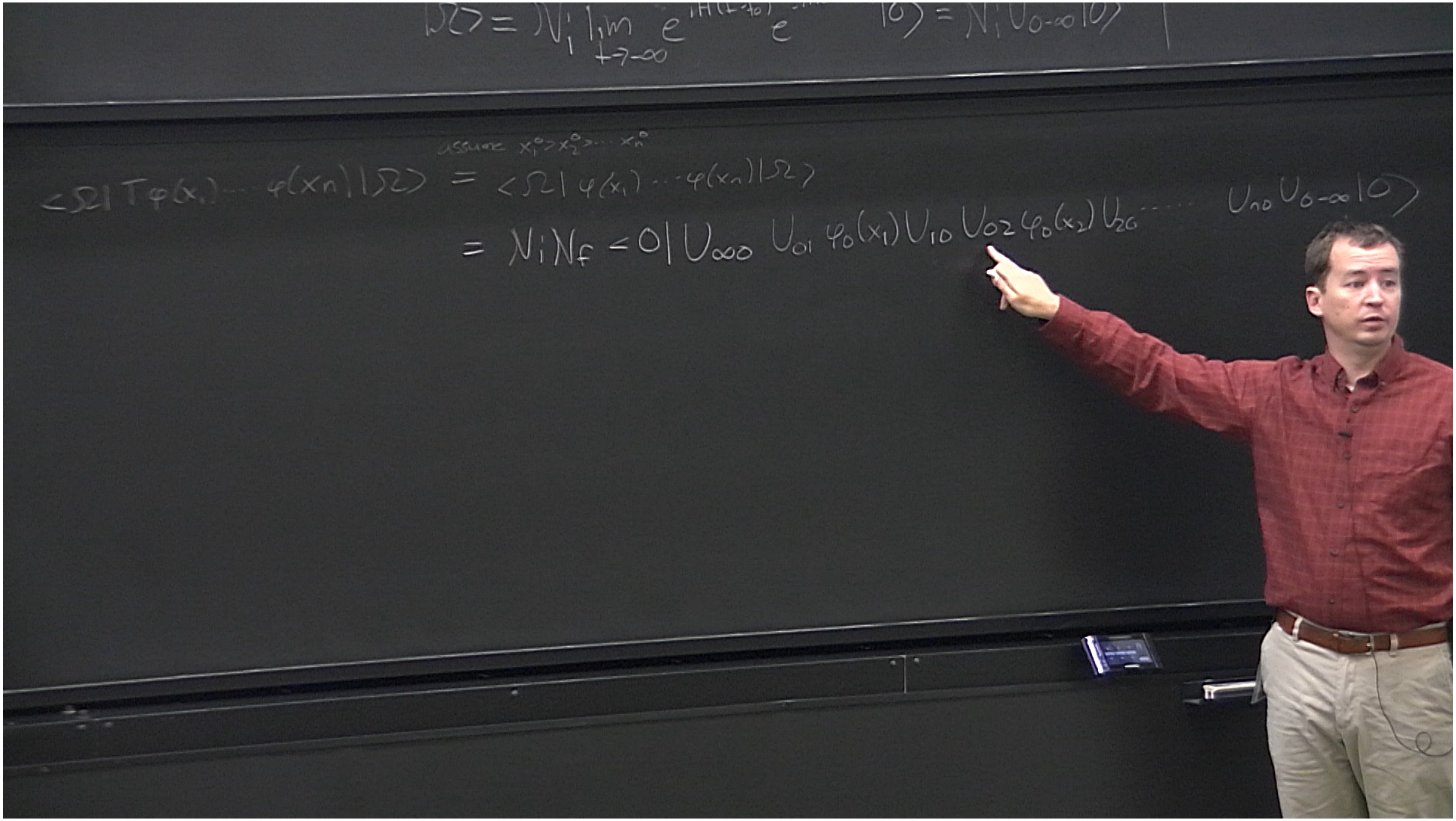
$$|\Omega\rangle = N_i \lim_{t\to-\infty} e^{iH(t-t_0)} e^{-iH_0(t-t_0)}|0\rangle$$

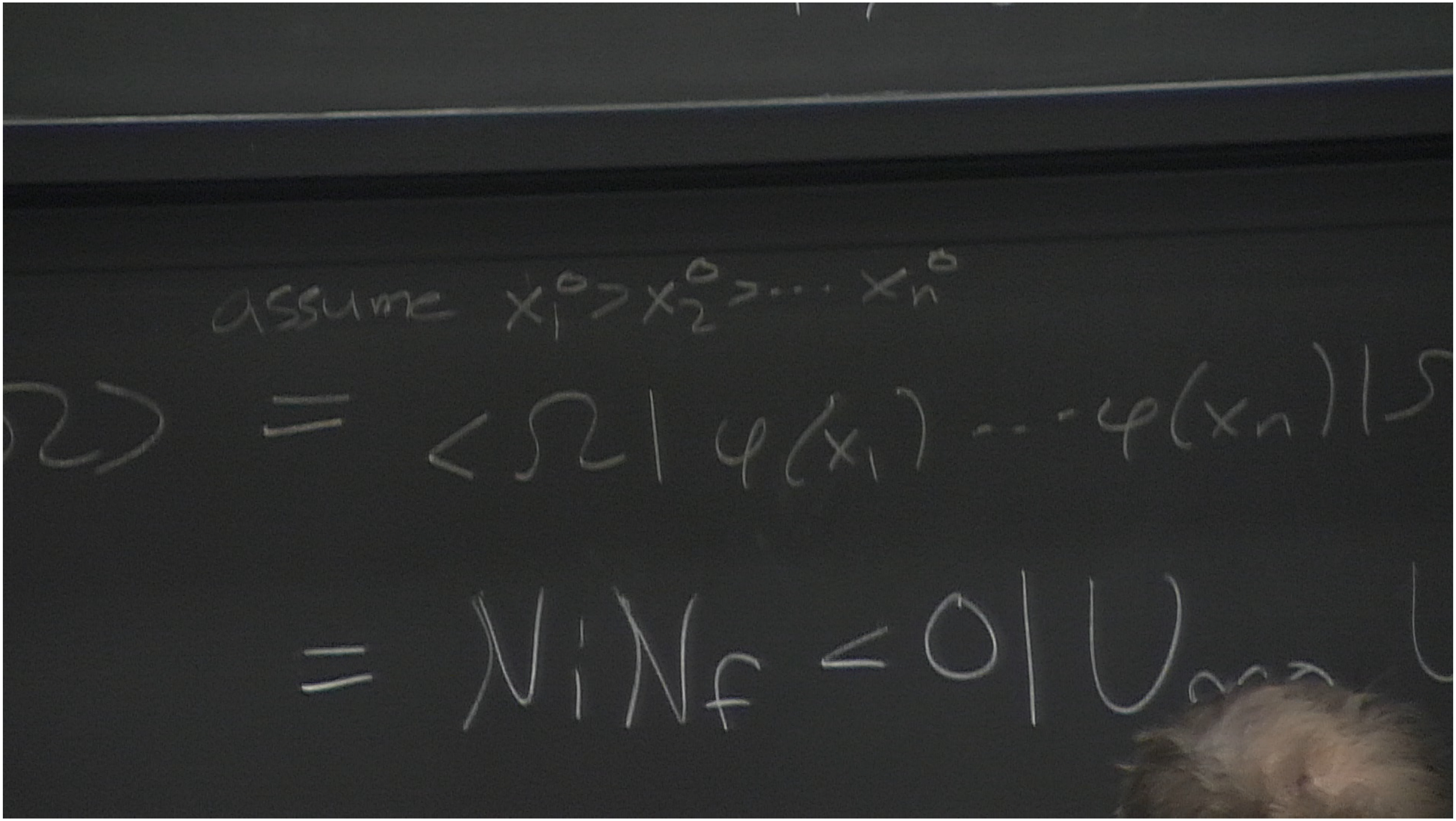


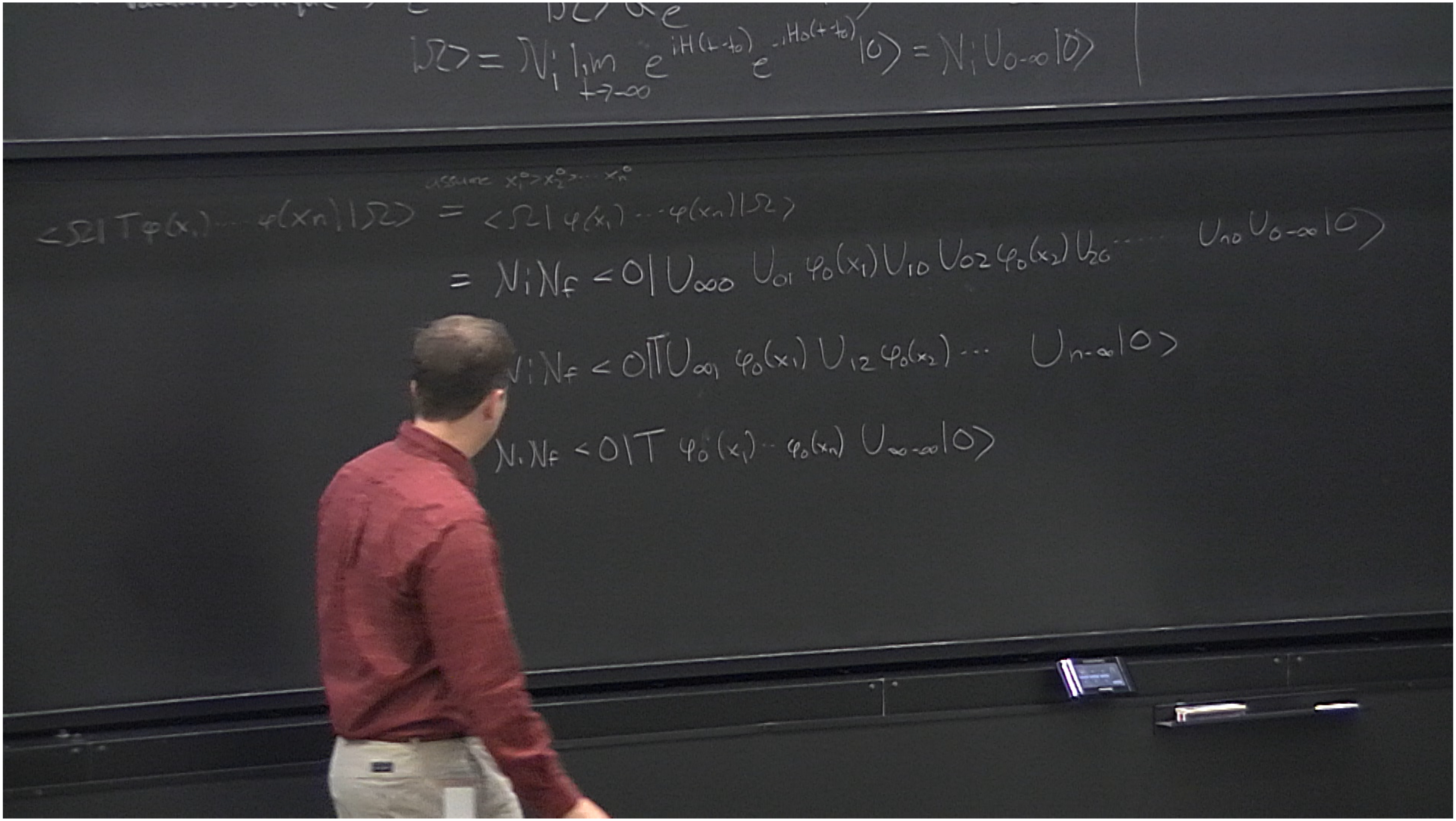
Wick
... φ(x_m)|0> ⟷ <0|Tφ(x_1)φ(x_2)|0>
|Ψ(t)>
iH(t-t_0) |Ψ(t_0)>
-iH_0(t-t_0)
all 3 picture agree at t=t_0
= H_0 + H_1
int pic field
φ(x⃗,t) = e^{iH(t-t_0)} e^{-iH_0(t-t_0)} φ_0(x⃗,t) e^{iH_0(t-t_0)} · e^{-iH(t-t_0)}
= U^†(t,t_0) φ_0(x⃗,t) U(t,t_0)
time evolution operator
U(t_2,t_1) = U_21 = T exp[-i ∫_{t_1}^{t_2} dt' H_I1(t')]
U_11 = 1
not merely = id
U_31 = U_32 U_21 composition
U_21^{-1} = U_21^† = U_12 unitary

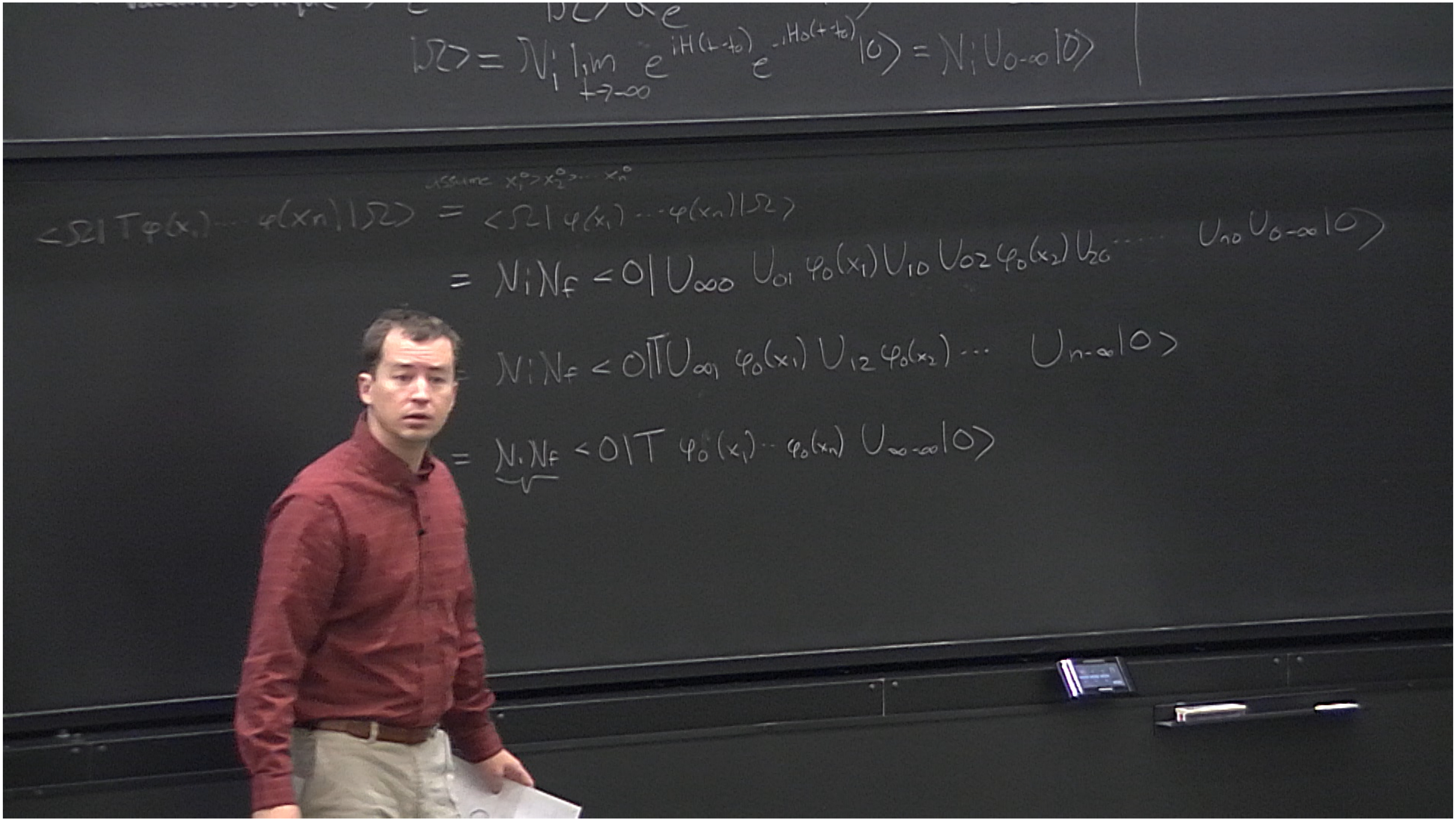

|Ω⟩ vs |0⟩
a_p(t₀)|Ω(t)⟩ = 0 at least for t = ±∞
a_p(t₀) e^{-iH(t-t₀)}|Ω⟩ = 0
a_p(t₀)|0(t)⟩ = 0
a_p(t₀) e^{-iH₀(t-t₀)}|0⟩ = 0
if vacuum is unique → e^{-iH(t-t₀)}|Ω⟩ ∝ e^{-iH₀(t-t₀)}|0⟩ for t → ±∞
|Ω⟩ = N_i lim_{t→-∞} e^{iH(t-t₀)} e^{-iH₀(t-t₀)}|0⟩ = N_i U_{0-∞}|0⟩

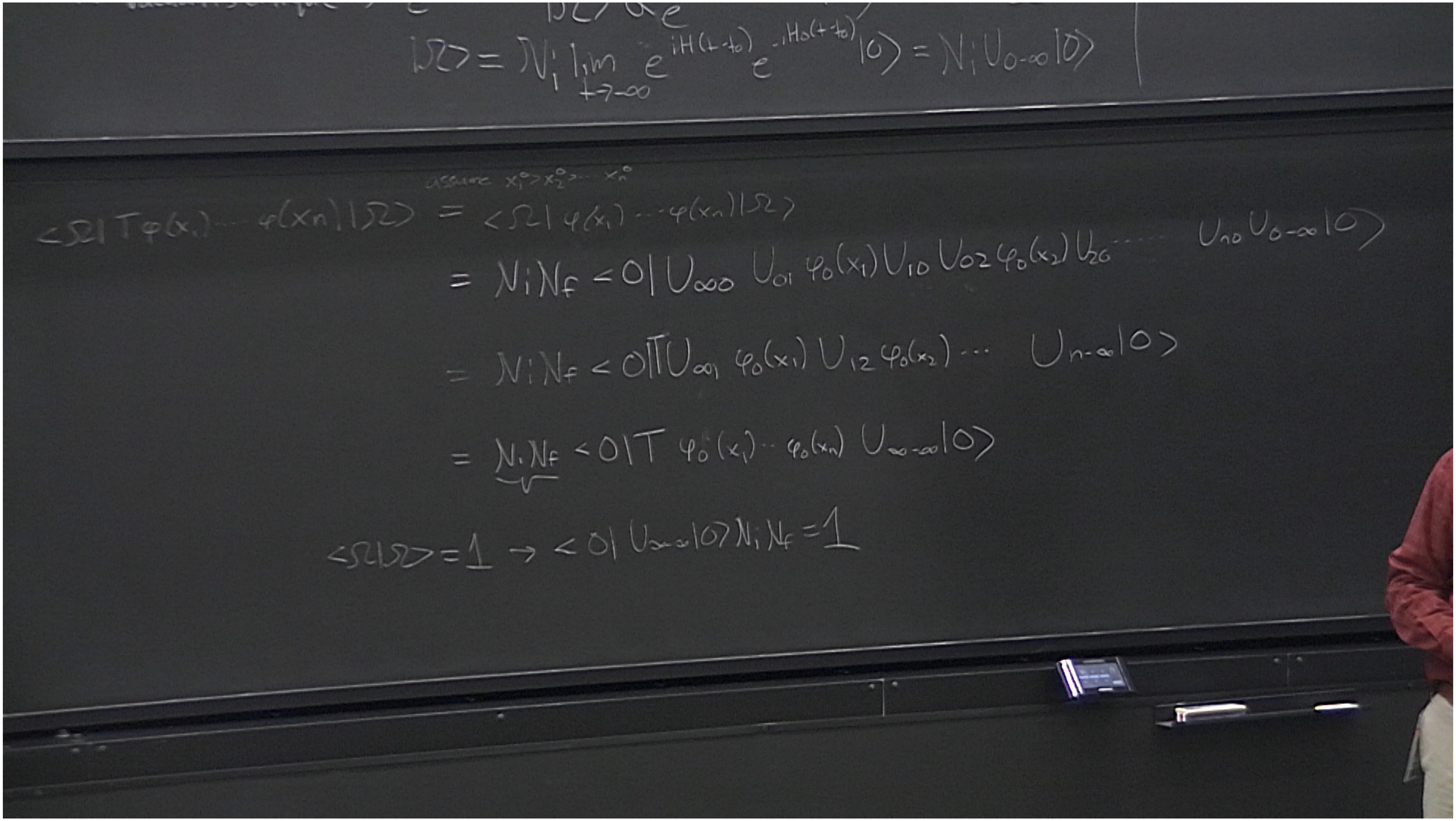

$|\Omega\rangle = N_f U_{0\infty} |0\rangle$
$\langle\Omega| = \langle 0| N_f U_{\infty 0}$
$t \to \pm\infty$
$N_i U_{0-\infty} |0\rangle$

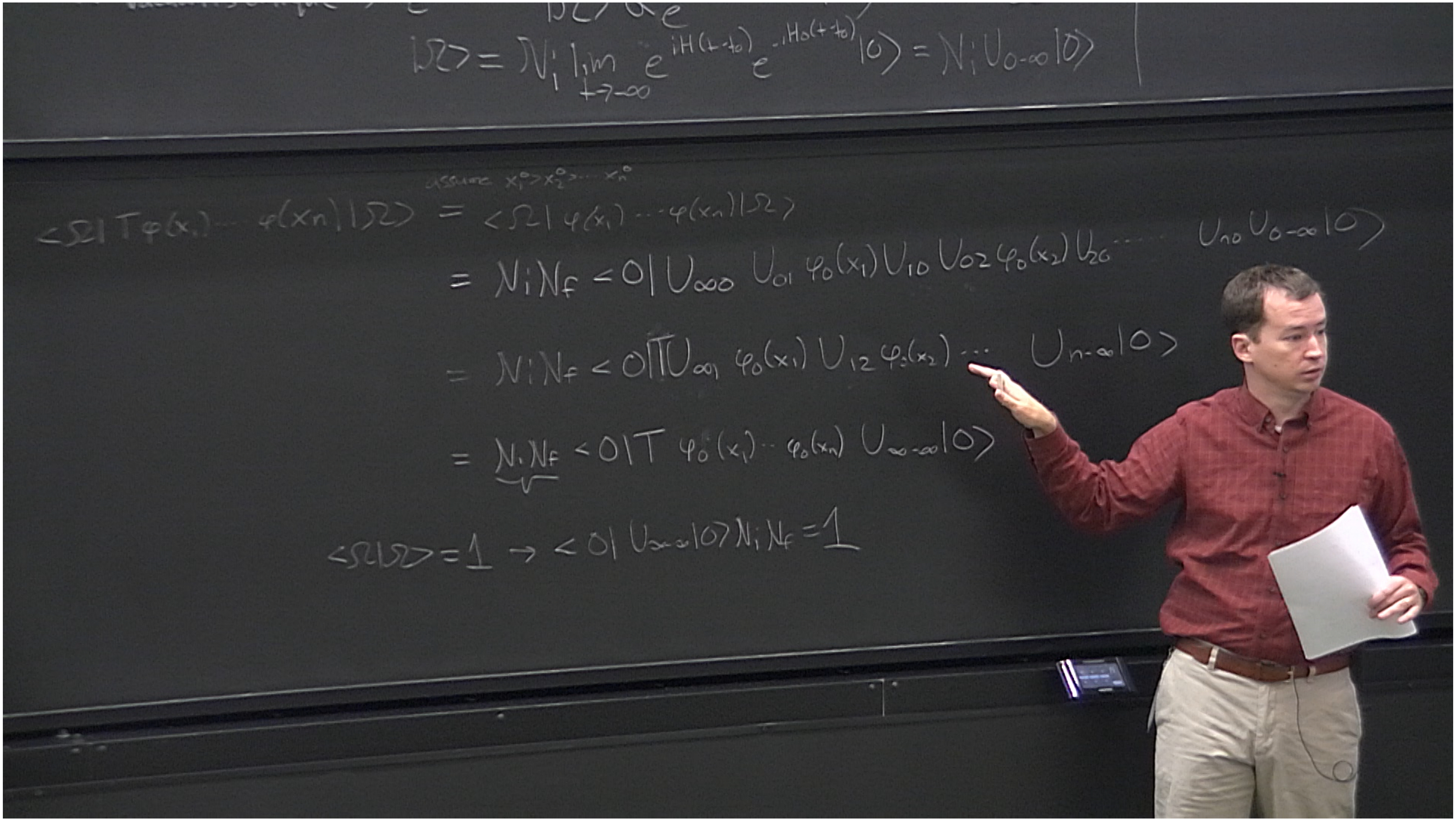

assume x1° > x2° > ... xn°
<Ω| T φ(x1) ... φ(xn) |Ω> = <Ω| φ(x1) ... φ(xn) |Ω>
= Ni Nf <0| U∞0 U01 φ0(x1) U10 U02 φ0(x2) U20 ...... Un0 U0-∞ |0>

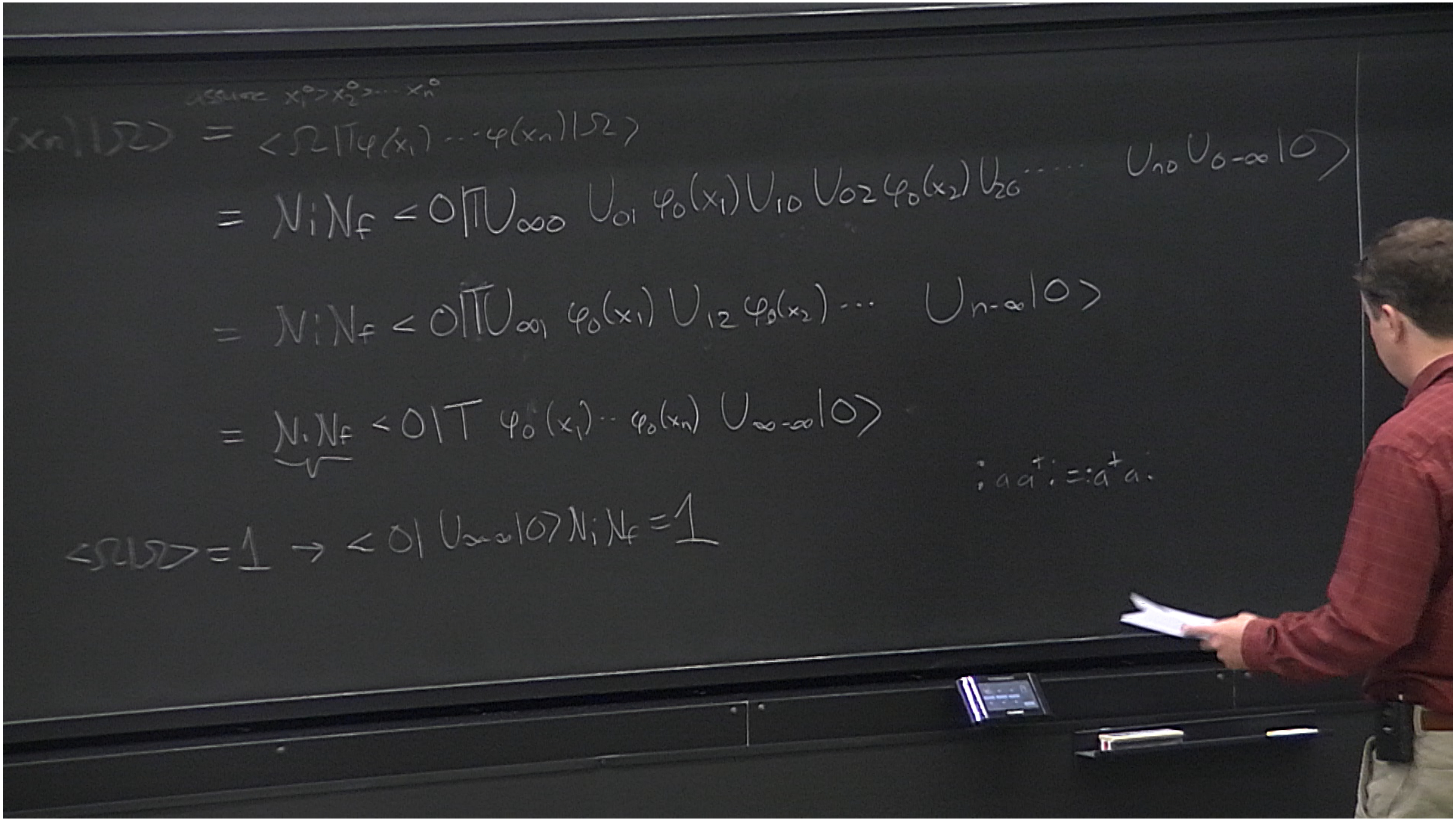

assume $x_1^0 > x_2^0 > \cdots x_n^0$
$= \langle \Omega | \varphi(x_1) \cdots \varphi(x_n) | \Omega$
$= N_i N_f \langle 0 | U$

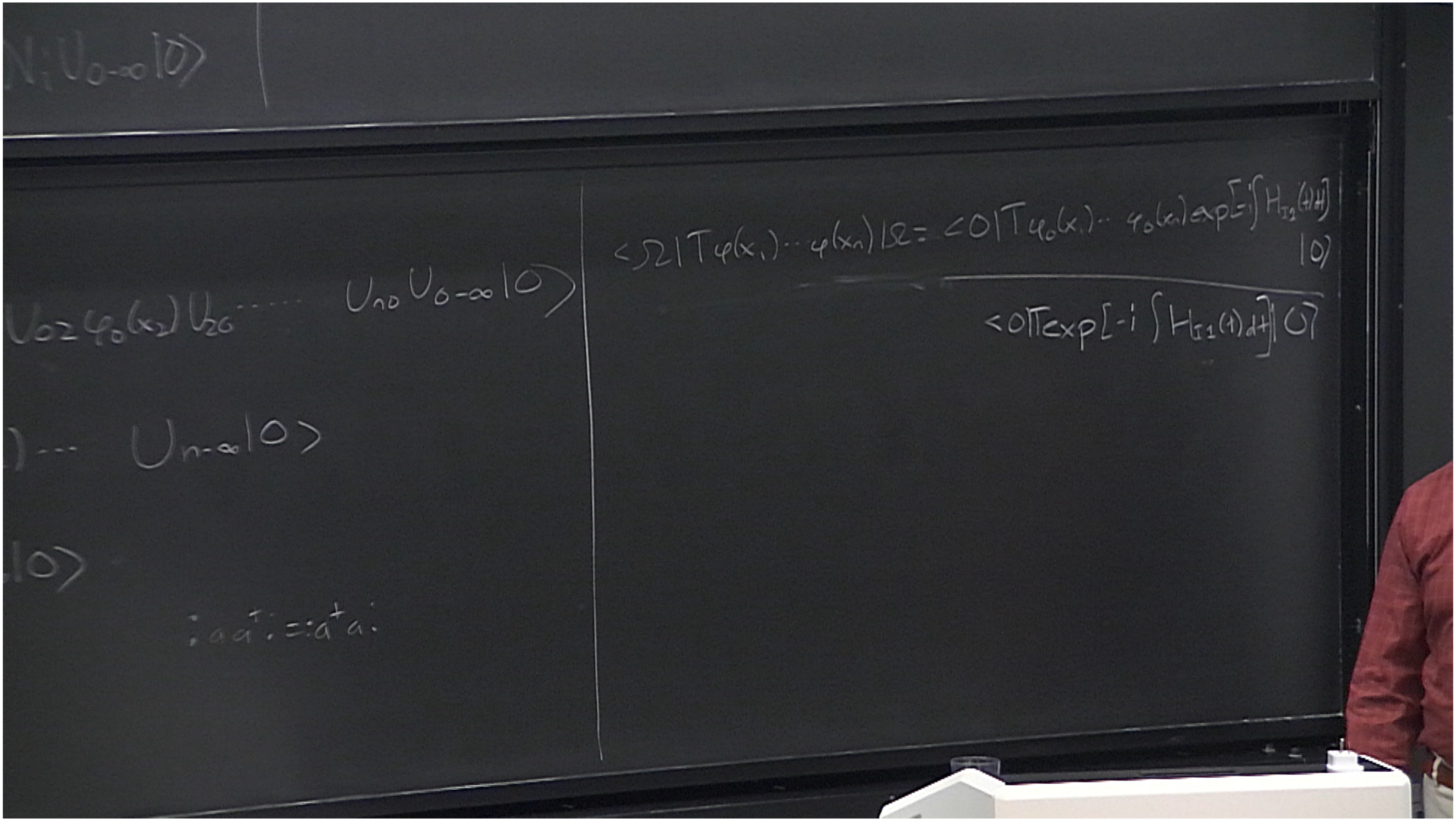

|Ω> = N_i lim_{t→-∞} e^{iH(t-t_0)} e^{-iH_0(t-t_0)} |0> = N_i U_{0-∞} |0>
assume x_1^0 > x_2^0 > ... x_n^0
<Ω| T φ(x_1) ... φ(x_n) |Ω> = <Ω| φ(x_1) ... φ(x_n) |Ω>
= N_i N_f <0| U_{∞0} U_{01} φ_0(x_1) U_{10} U_{02} φ_0(x_2) U_{20} ... U_{n0} U_{0-∞} |0>
N_i N_f <0| T U_{∞1} φ_0(x_1) U_{12} φ_0(x_2) ... U_{n-∞} |0>
N_i N_f <0| T φ_0(x_1) ... φ_0(x_n) U_{∞-∞} |0>

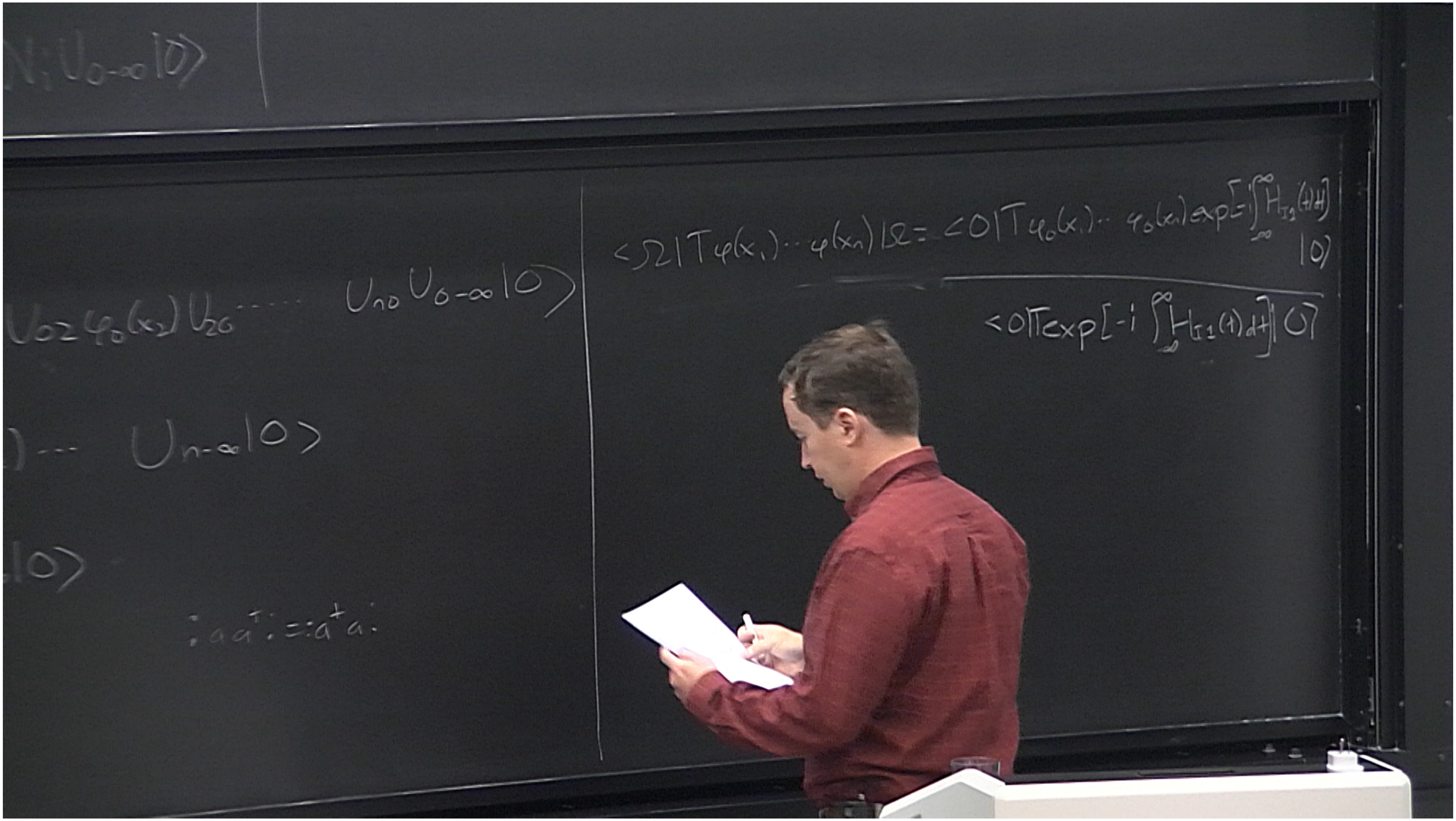

$|\Omega\rangle = N_i \lim_{t\to-\infty} e^{iH(t-t_0)} e^{-iH_0(t-t_0)} |0\rangle = N_i U_{0-\infty}|0\rangle$
assume $x_1^0 > x_2^0 > \cdots x_n^0$
$\langle\Omega| T\varphi(x_1)\cdots\varphi(x_n)|\Omega\rangle = \langle\Omega|\varphi(x_1)\cdots\varphi(x_n)|\Omega\rangle$
$= N_i N_f \langle 0| U_{\infty 0} U_{01}\varphi_0(x_1) U_{10} U_{02}\varphi_0(x_2) U_{20} \cdots U_{n0} U_{0-\infty}|0\rangle$
$= N_i N_f \langle 0| T U_{\infty 1}\varphi_0(x_1) U_{12}\varphi_0(x_2)\cdots U_{n-\infty}|0\rangle$
$= N_i N_f \langle 0| T \varphi_0(x_1)\cdots\varphi_0(x_n) U_{\infty-\infty}|0\rangle$

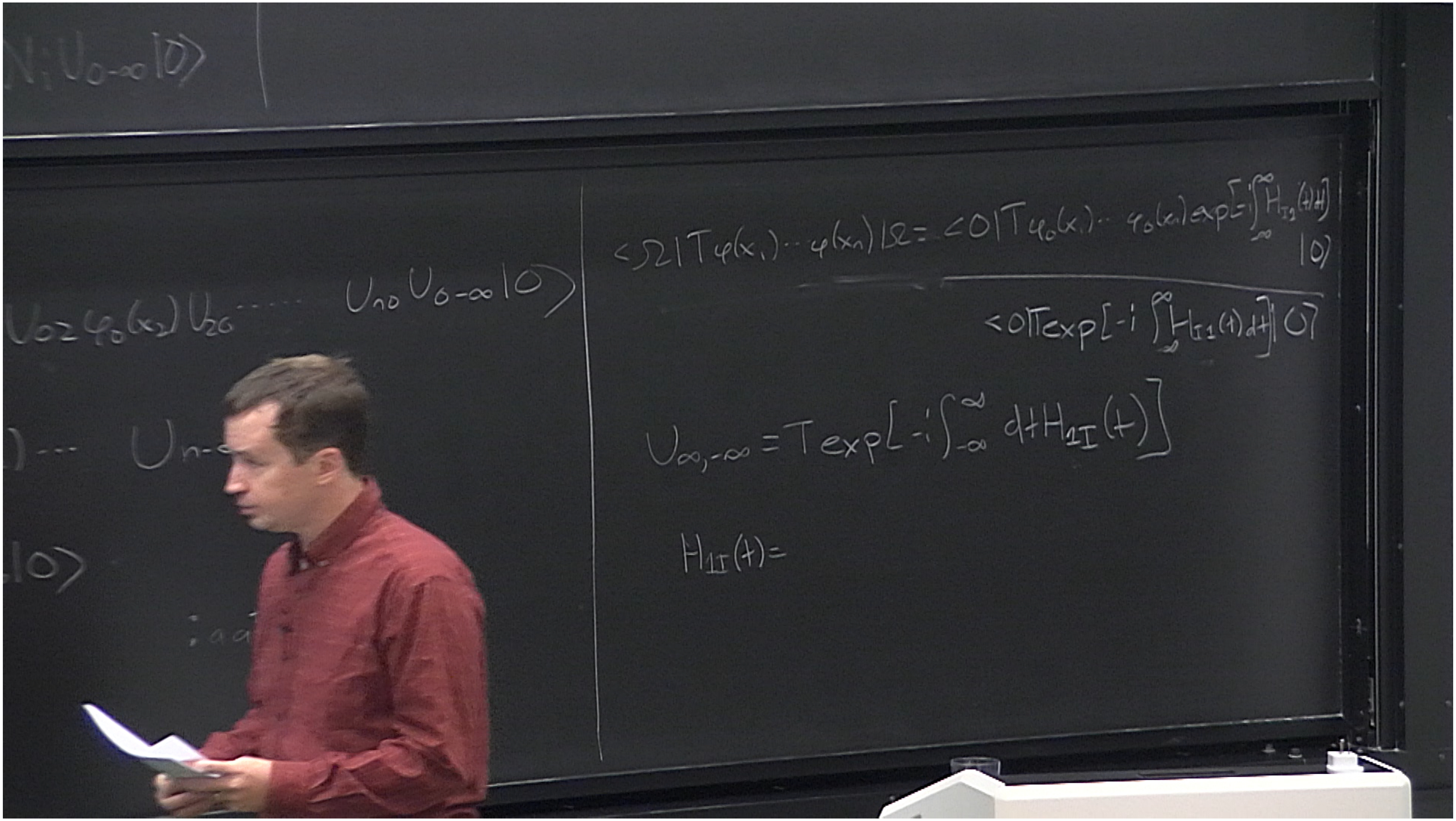

$|\Omega\rangle = N_i \lim_{t\to-\infty} e^{iH(t-t_0)} e^{-iH_0(t-t_0)}|0\rangle = N_i U_{0-\infty}|0\rangle$
assume $x_1^0 > x_2^0 > \cdots x_n^0$
$\langle\Omega|T\varphi(x_1)\cdots\varphi(x_n)|\Omega\rangle = \langle\Omega|\varphi(x_1)\cdots\varphi(x_n)|\Omega\rangle$
$= N_i N_f \langle 0|U_{\infty 0} U_{01}\varphi_0(x_1)U_{10}U_{02}\varphi_0(x_2)U_{20}\cdots U_{n0}U_{0-\infty}|0\rangle$
$= N_i N_f \langle 0|T U_{\infty 1}\varphi_0(x_1)U_{12}\varphi_0(x_2)\cdots U_{n-\infty}|0\rangle$
$= N_i N_f \langle 0|T\varphi_0(x_1)\cdots\varphi_0(x_n)U_{\infty-\infty}|0\rangle$
$\langle\Omega|\Omega\rangle = 1 \to \langle 0|U_{\infty-\infty}|0\rangle N_i N_f = 1$

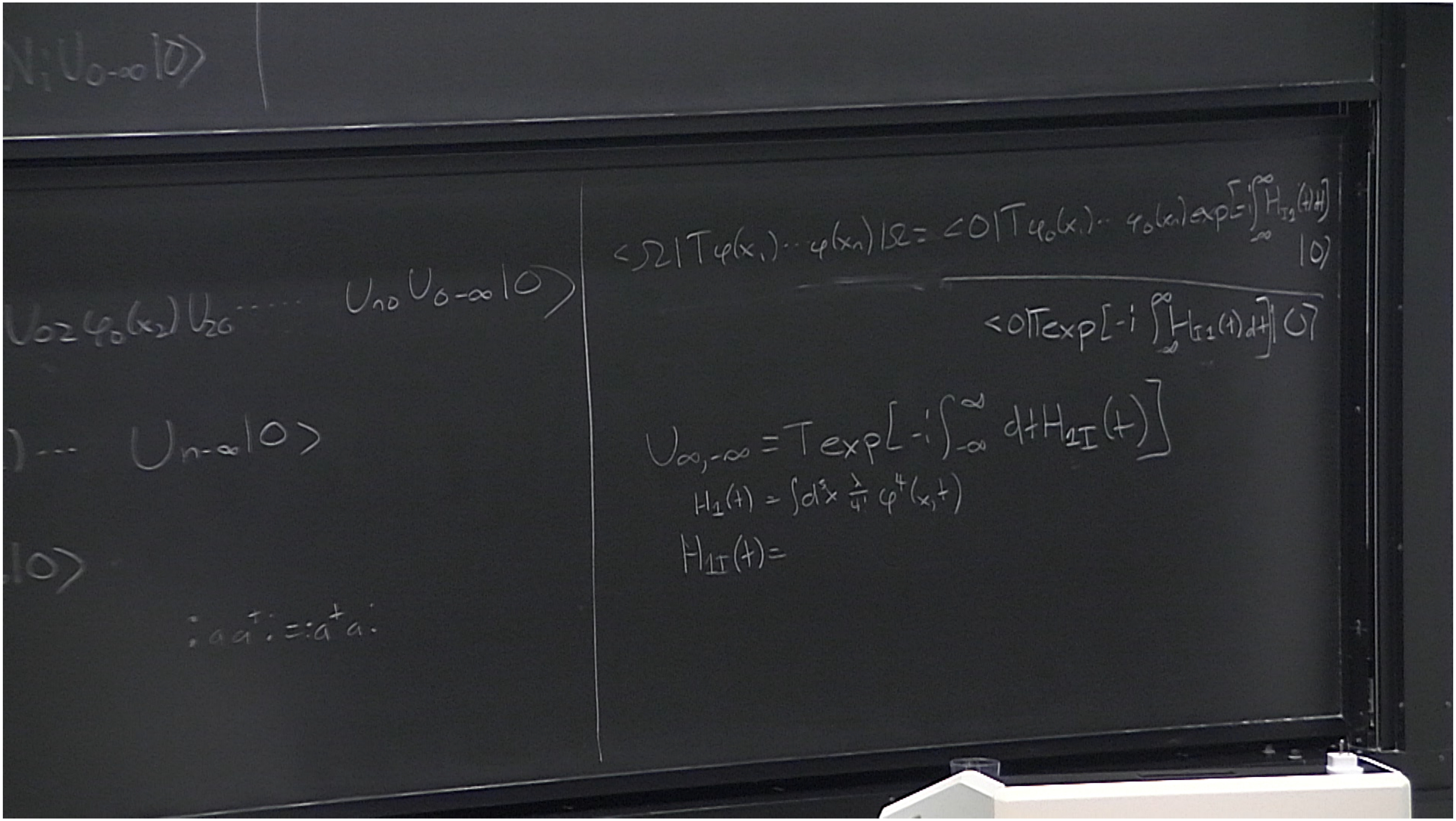

$|\Omega\rangle = N_i \lim_{t\to-\infty} e^{iH(t-t_0)} e^{-iH_0(t-t_0)}|0\rangle = N_i U_{0,-\infty}|0\rangle$
assume $x_1^0 > x_2^0 > \cdots x_n^0$
$\langle\Omega|T\varphi(x_1)\cdots\varphi(x_n)|\Omega\rangle = \langle\Omega|\varphi(x_1)\cdots\varphi(x_n)|\Omega\rangle$
$= N_i N_f \langle 0|U_{\infty 0}\, U_{01}\, \varphi_0(x_1) U_{10}\, U_{02}\, \varphi_0(x_2) U_{20} \cdots U_{n0}\, U_{0,-\infty}|0\rangle$
$= N_i N_f \langle 0|U_{\infty 1}\, \varphi_0(x_1)\, U_{12}\, \varphi_0(x_2) \cdots U_{n,-\infty}|0\rangle$
$= N_i N_f \langle 0|T\, \varphi_0(x_1)\cdots\varphi_0(x_n)\, U_{\infty,-\infty}|0\rangle$
$\langle\Omega|\Omega\rangle = 1 \rightarrow \langle 0|U_{\infty,-\infty}|0\rangle N_i N_f = 1$

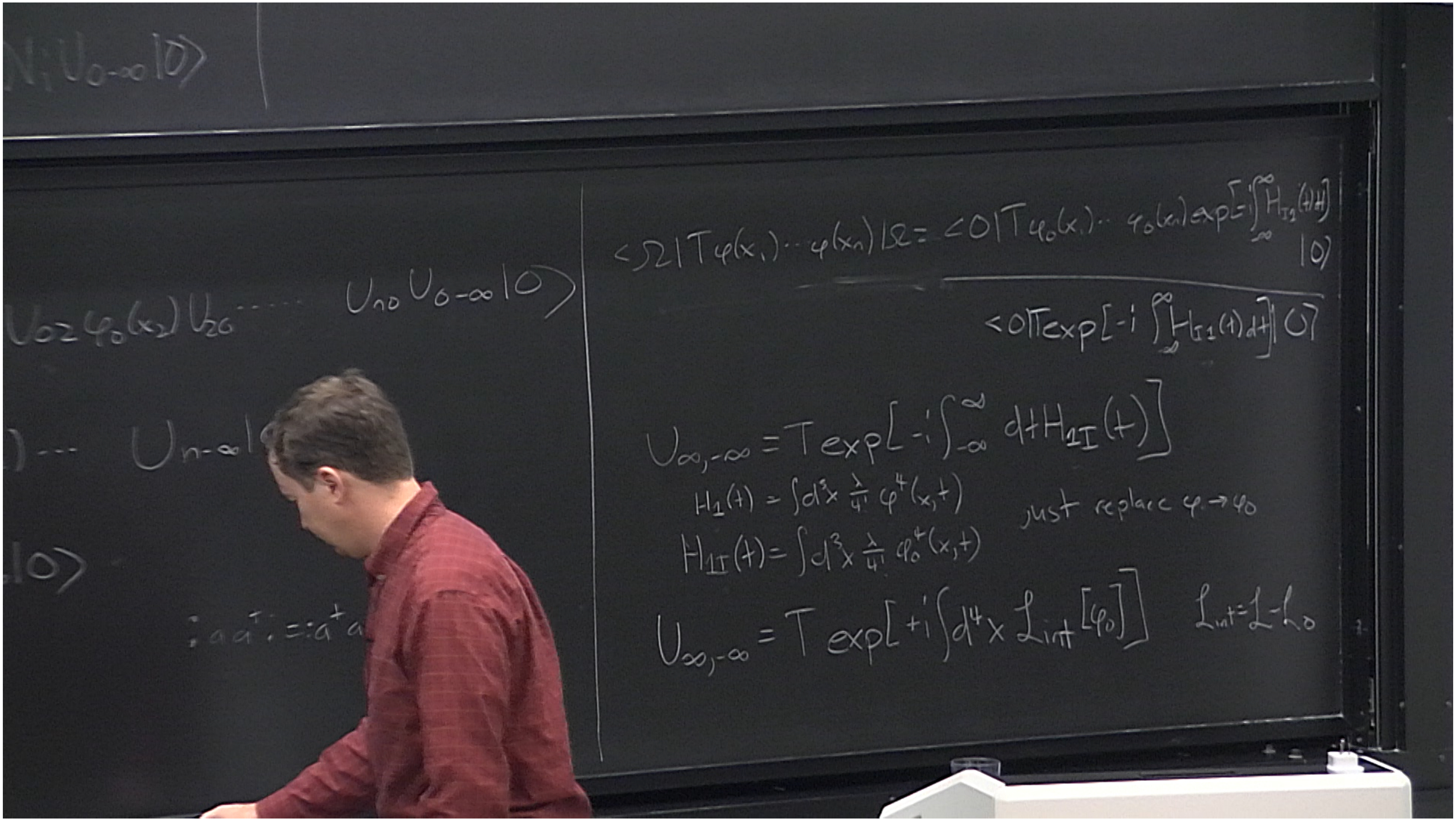

assume $x_1^0 > x_2^0 > \cdots x_n^0$
$(x_n)|\Omega\rangle = \langle\Omega|T\varphi(x_1)\cdots\varphi(x_n)|\Omega\rangle$
$= N_i N_f \langle 0|T U_{\infty 0} U_{01} \varphi_0(x_1) U_{10} U_{02} \varphi_0(x_2) U_{20} \cdots U_{n0} U_{0-\infty}|0\rangle$
$= N_i N_f \langle 0|T U_{\infty 1} \varphi_0(x_1) U_{12} \varphi_0(x_2) \cdots U_{n-\infty}|0\rangle$
$= N_i N_f \langle 0|T \varphi_0(x_1)\cdots\varphi_0(x_n) U_{\infty-\infty}|0\rangle$
$:aa^\dagger: = :a^\dagger a:$
$\langle\Omega|\Omega\rangle = 1 \rightarrow \langle 0|U_{\infty-\infty}|0\rangle N_i N_f = 1$

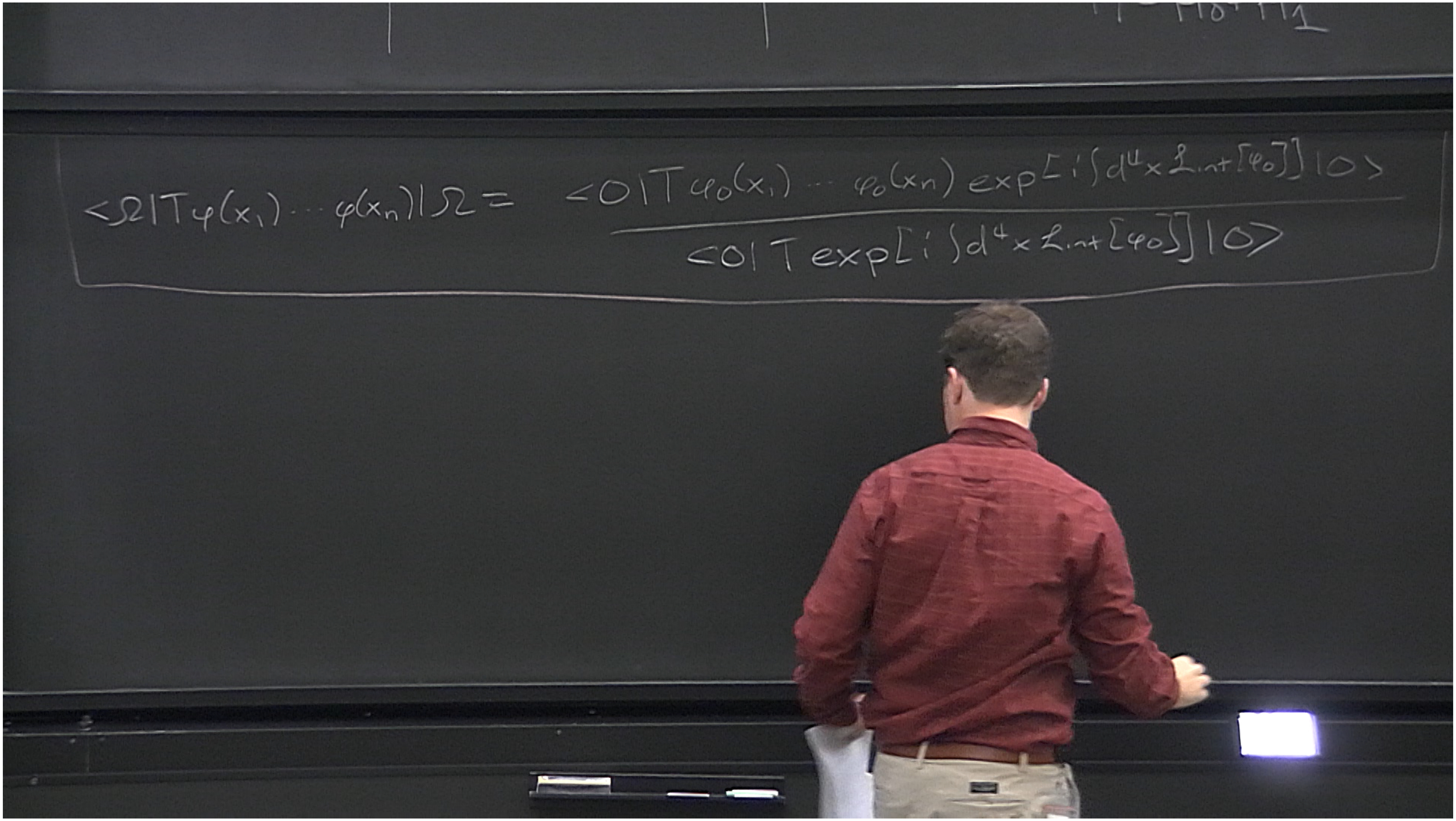

N_i U_{0-∞}|0⟩
U_{02} φ_0(x_2) U_{20} ······ U_{n0} U_{0-∞}|0⟩
) ··· U_{n-∞}|0⟩
|0⟩
:aa†: = :a†a:
⟨Ω|Tφ(x_1)··φ(x_n)|Ω⟩ = ⟨0|Tφ_0(x_1)·· φ_0(x_n) exp[-i∫H_{I1}(t)dt]|0⟩
⟨0|T exp[-i∫H_{I1}(t)dt]|0⟩

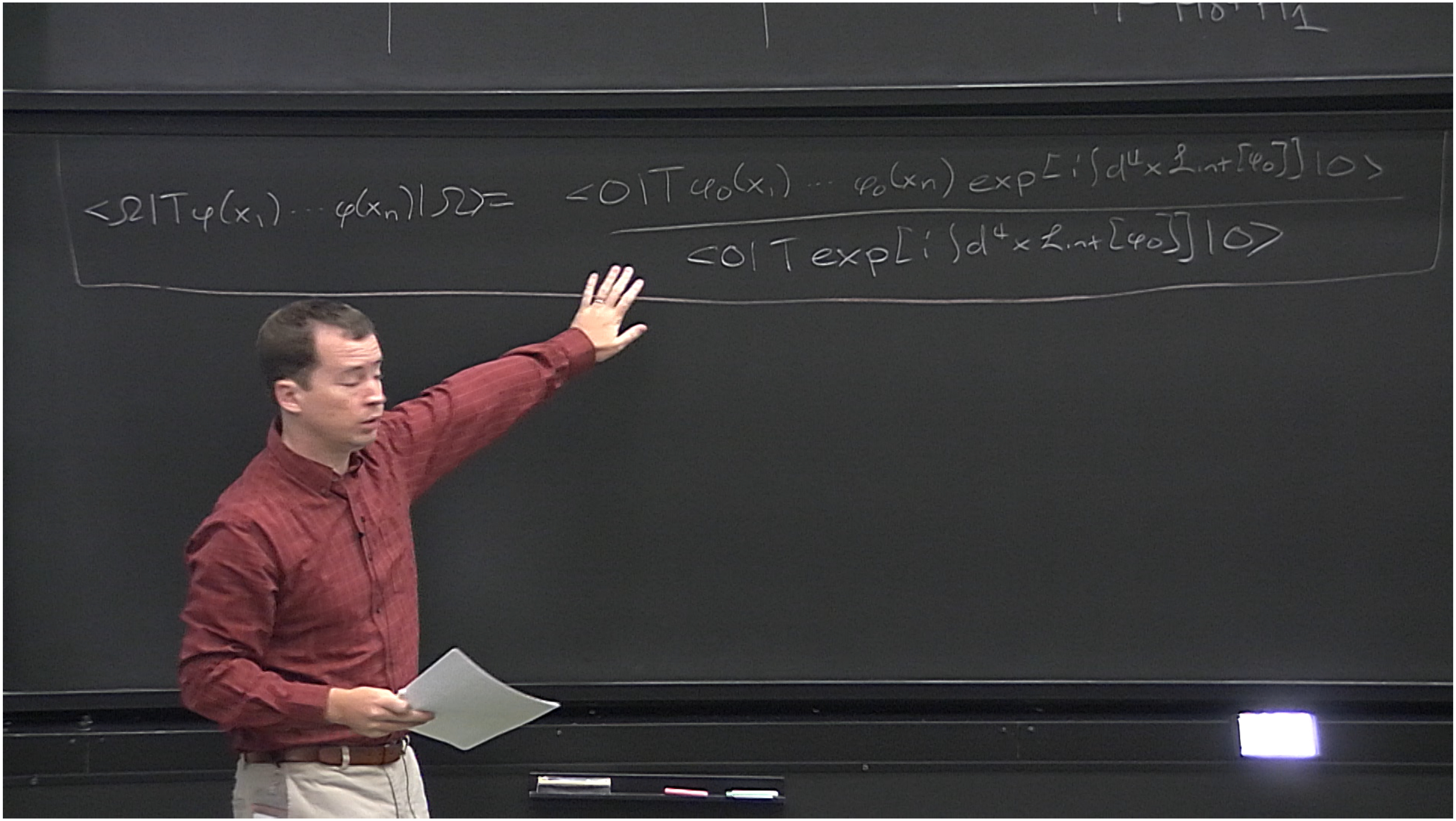

⟨Ω|T φ(x₁)···φ(xₙ)|Ω⟩ = ⟨0|T φ₀(x₁)···φ₀(xₙ) exp[−i ∫ H_I(t)dt]|0⟩ / ⟨0|T exp[−i ∫ H_I(t)dt]|0⟩
:aa†: = :a†a:

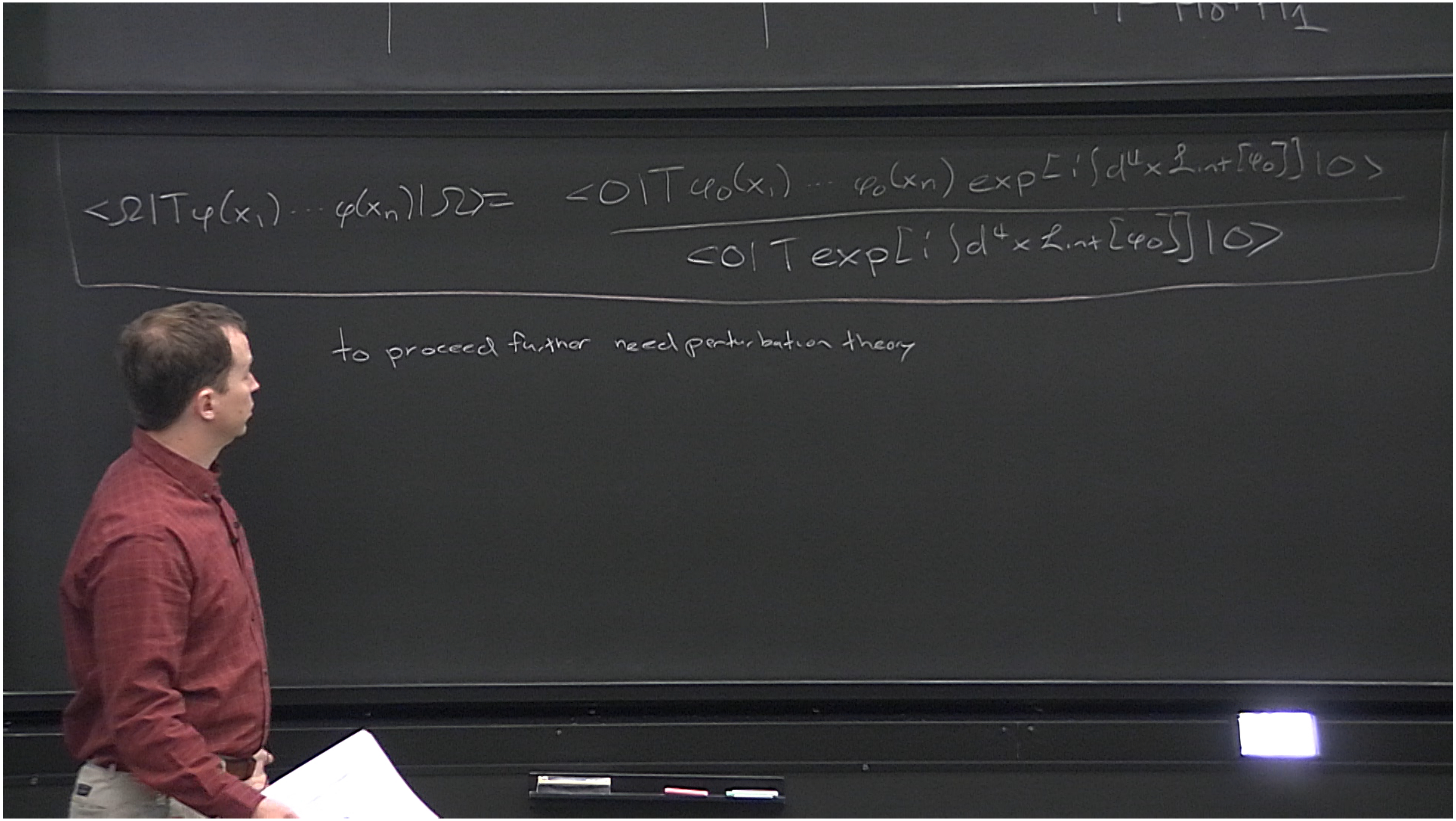

⟨Ω|T φ(x₁)⋯φ(xₙ)|Ω⟩ = ⟨0|T φ₀(x₁)⋯φ₀(xₙ) exp[−i∫ H_{1I}(t) dt]|0⟩ / ⟨0|T exp[−i∫ H_{I1}(t) dt]|0⟩
U_{∞,−∞} = T exp[−i∫_{−∞}^{∞} dt H_{1I}(t)]
H_{1I}(t) =
U_{n0} U_{0−∞}|0⟩

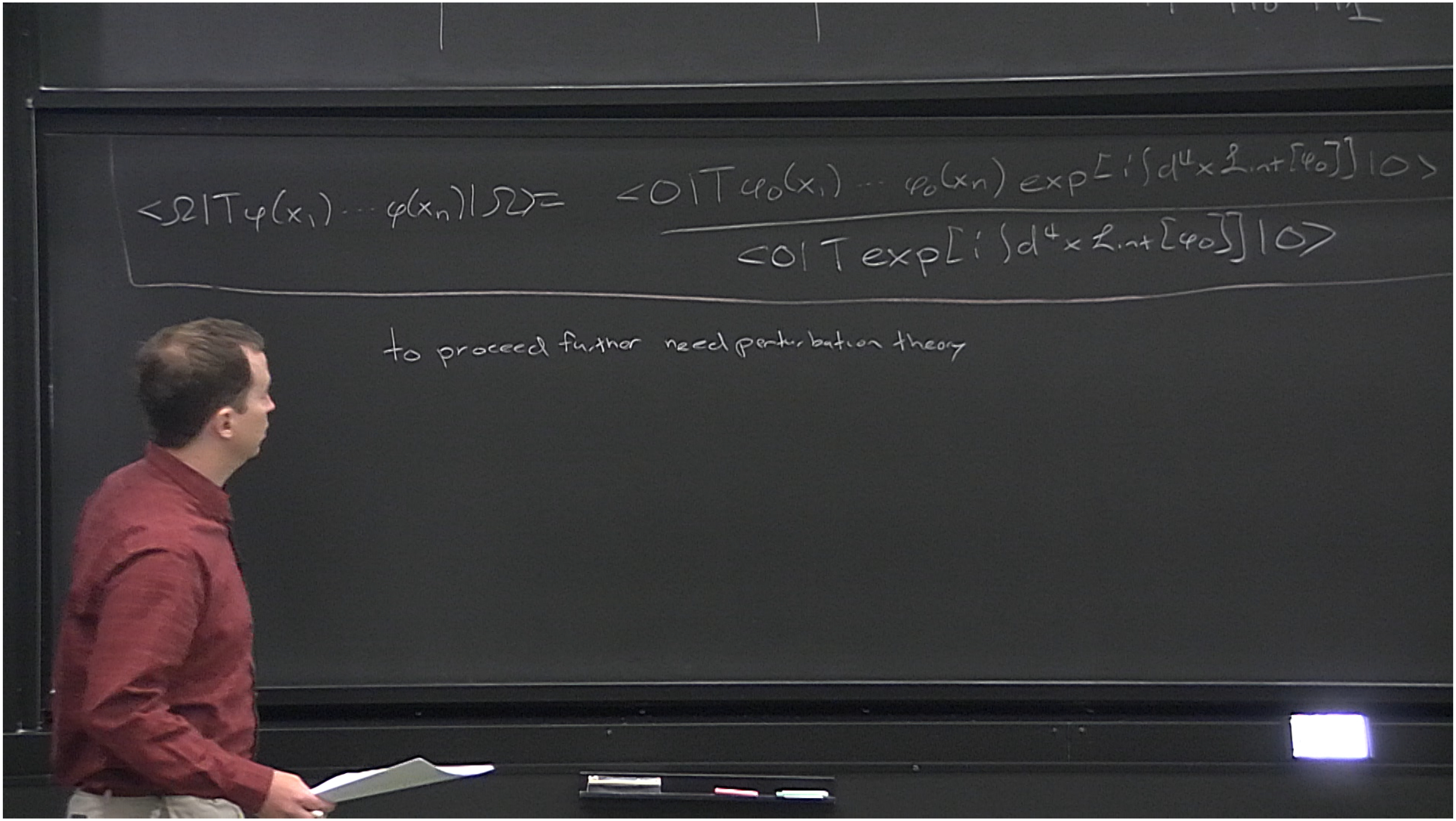

$N_i U_{0-\infty}|0\rangle$
$U_{02}\varphi_0(x_2)U_{20} \cdots\cdots U_{n0}U_{0-\infty}|0\rangle$
$) \cdots U_{n-\infty}|0\rangle$
$|0\rangle$
$:aa^\dagger: = :a^\dagger a:$
$\langle\Omega|T\varphi(x_1)\cdots\varphi(x_n)|\Omega\rangle = \dfrac{\langle 0|T\varphi_0(x_1)\cdots\varphi_0(x_n)\exp[-i\int_{-\infty}^{\infty}H_{I1}(t)dt]|0\rangle}{\langle 0|T\exp[-i\int_{-\infty}^{\infty}H_{I1}(t)dt]|0\rangle}$
$U_{\infty,-\infty} = T\exp[-i\int_{-\infty}^{\infty}dt\,H_{1I}(t)]$
$H_1(t) = \int d^3x\,\frac{\lambda}{4!}\varphi^4(x,t)$
$H_{1I}(t) =$

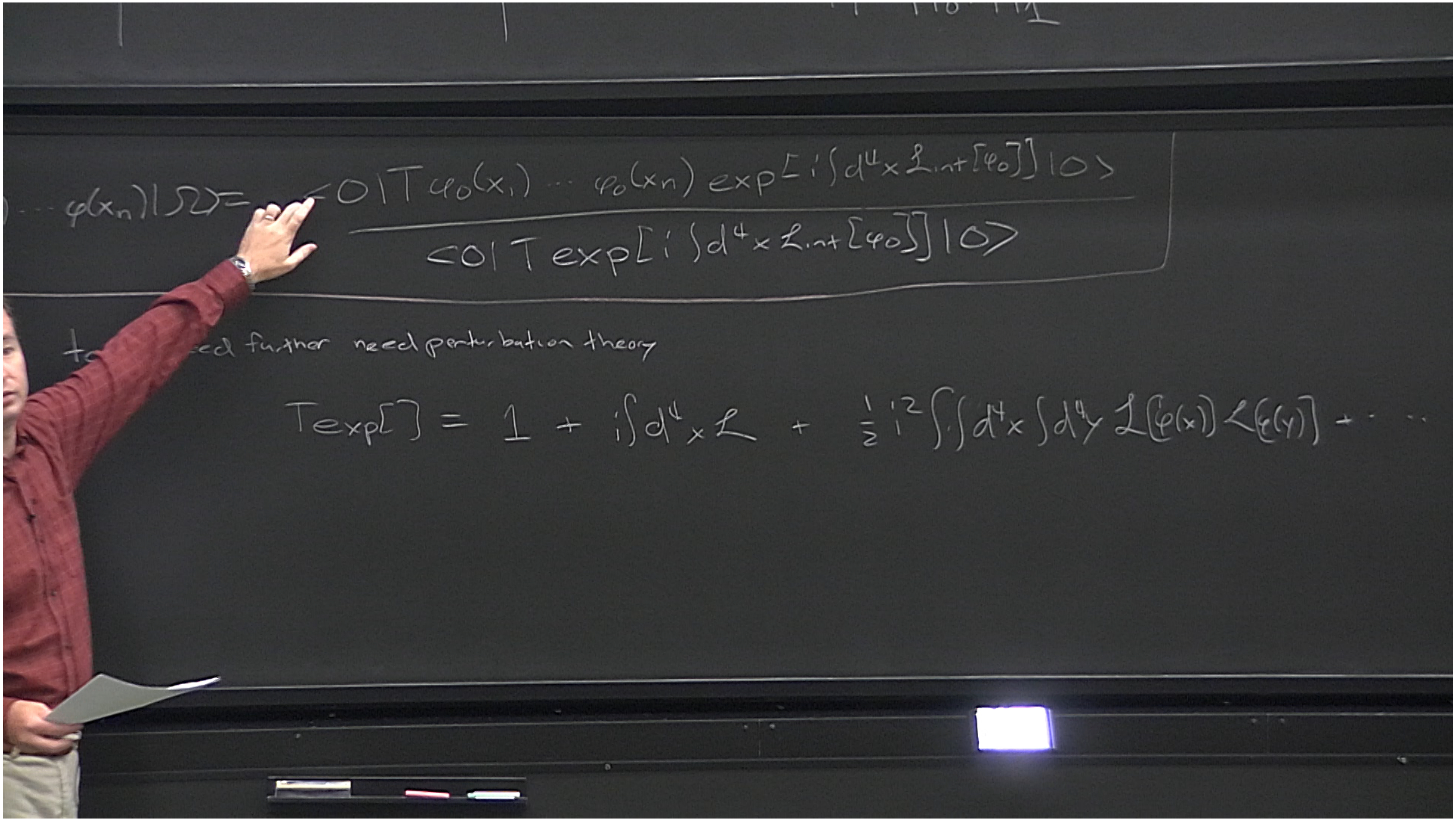

$\langle\Omega|T\varphi(x_1)\cdots\varphi(x_n)|\Omega\rangle = \dfrac{\langle 0|T\varphi_0(x_1)\cdots\varphi_0(x_n)\exp[-i\int_{-\infty}^{\infty}H_{I1}(t)dt]|0\rangle}{\langle 0|T\exp[-i\int_{-\infty}^{\infty}H_{I1}(t)dt]|0\rangle}$
$U_{\infty,-\infty} = T\exp[-i\int_{-\infty}^{\infty} dt\, H_{1I}(t)]$
$H_1(t) = \int d^3x \frac{\lambda}{4!}\varphi^4(x,t)$
just replace $\varphi \to \varphi_0$
$H_{1I}(t) = \int d^3x \frac{\lambda}{4!}\varphi_0^4(x,t)$
$U_{\infty,-\infty} = T\exp[+i\int d^4x\, \mathcal{L}_{int}[\varphi_0]]$
$\mathcal{L}_{int} = \mathcal{L} - \mathcal{L}_0$


$\langle\Omega|T\varphi(x_1)\cdots\varphi(x_n)|\Omega\rangle = \dfrac{\langle 0|T\varphi_0(x_1)\cdots\varphi_0(x_n)\exp[i\int d^4x\,\mathcal{L}_{int}[\varphi_0]]|0\rangle}{\langle 0|T\exp[i\int d^4x\,\mathcal{L}_{int}[\varphi_0]]|0\rangle}$


$\langle\Omega|T\varphi(x_1)\cdots\varphi(x_n)|\Omega\rangle = \frac{\langle 0|T\varphi_0(x_1)\cdots\varphi_0(x_n)\exp[i\int d^4x\,\mathcal{L}_{int}[\varphi_0]]|0\rangle}{\langle 0|T\exp[i\int d^4x\,\mathcal{L}_{int}[\varphi_0]]|0\rangle}$


$\langle\Omega|T\varphi(x_1)\cdots\varphi(x_n)|\Omega\rangle = \dfrac{\langle 0|T\varphi_0(x_1)\cdots\varphi_0(x_n)\exp[i\int d^4x\,\mathcal{L}_{int}[\varphi_0]]|0\rangle}{\langle 0|T\exp[i\int d^4x\,\mathcal{L}_{int}[\varphi_0]]|0\rangle}$
to proceed further need perturbation theory


<Ω|T φ(x_1) ··· φ(x_n)|Ω> = <0|T φ_0(x_1) ··· φ_0(x_n) exp[i ∫d^4x L_int[φ_0]]|0> / <0|T exp[i ∫d^4x L_int[φ_0]]|0>
to proceed further need perturbation theory



$\cdots\varphi(x_n)|\Omega\rangle = \dfrac{\langle 0|T\varphi_0(x_1)\cdots\varphi_0(x_n)\exp[i\int d^4x\,\mathcal{L}_{int}[\varphi_0]]|0\rangle}{\langle 0|T\exp[i\int d^4x\,\mathcal{L}_{int}[\varphi_0]]|0\rangle}$
to ...ed further need perturbation theory
$T\exp[\,] = 1 + i\int d^4x\,\mathcal{L} + \frac{1}{2}i^2\int\int d^4x\int d^4y\,\mathcal{L}[\varphi(x)]\mathcal{L}[\varphi(y)] + \cdots$

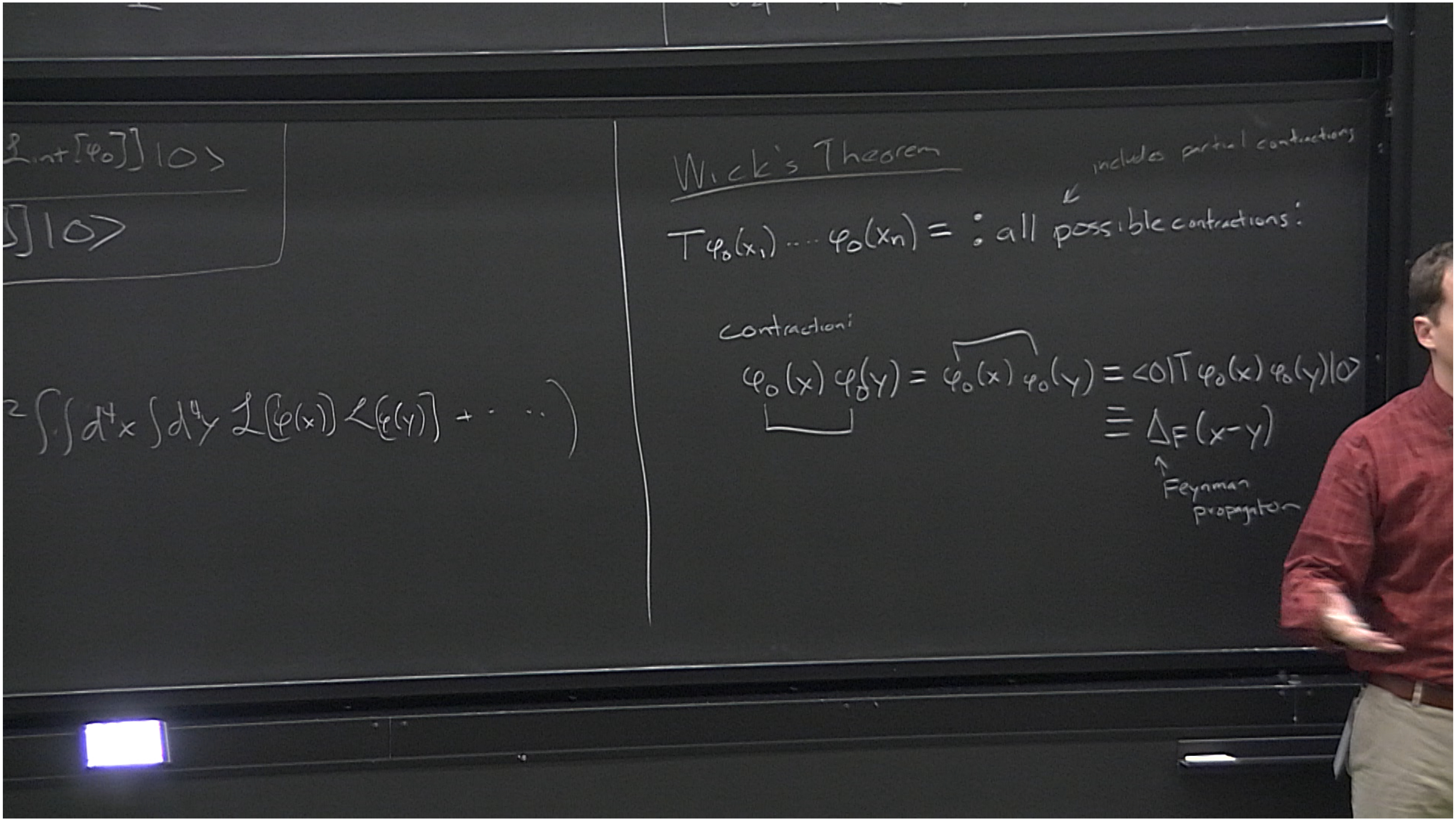
$H = H_0 + H_1$

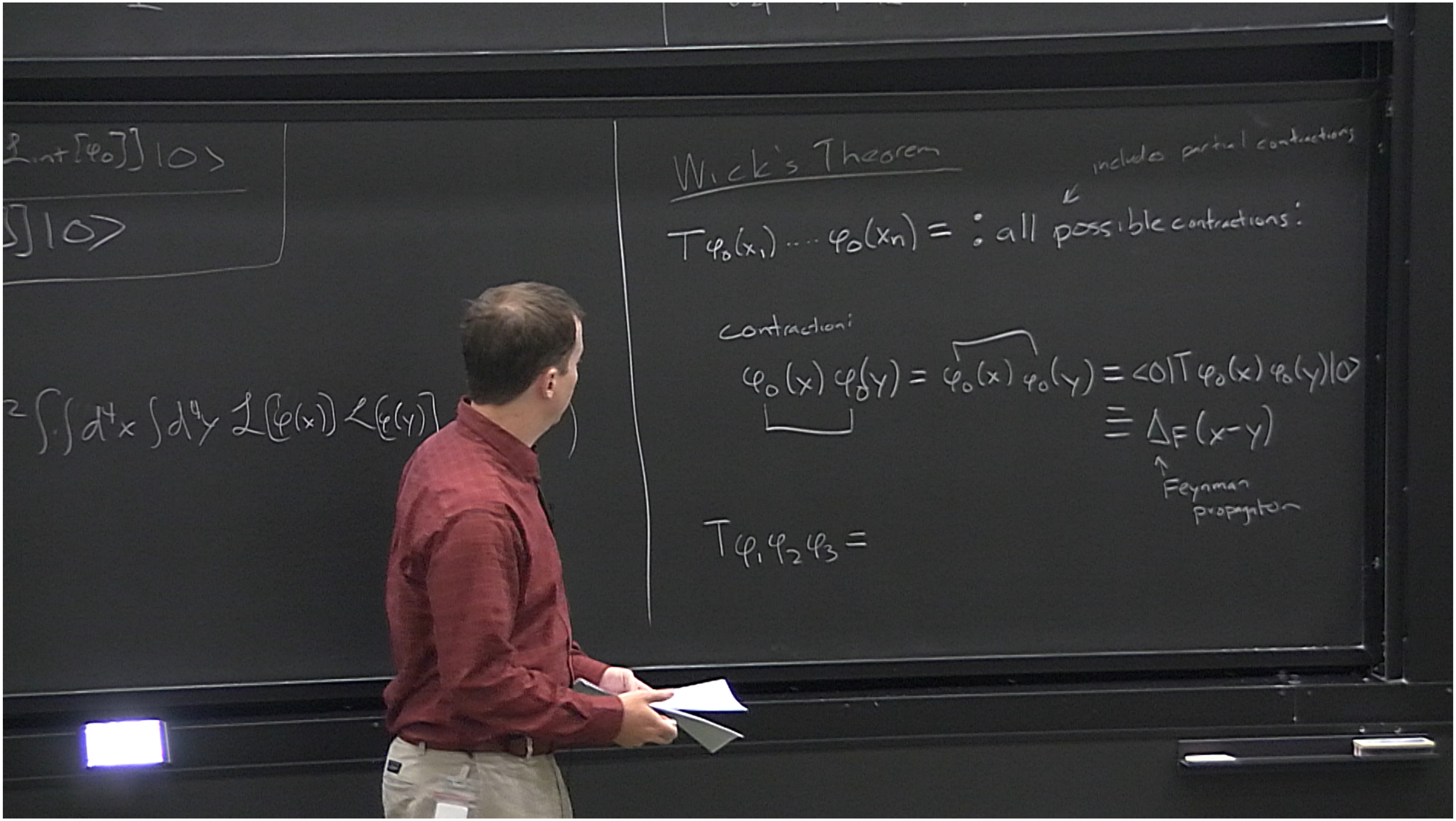
$$\cdots \varphi(x_n)|\Omega\rangle = \frac{\langle 0| T\, \varphi_0(x_1) \cdots \varphi_0(x_n) \exp[i \int d^4x\, \mathcal{L}_{int}[\varphi_0]] |0\rangle}{\langle 0| T \exp[i \int d^4x\, \mathcal{L}_{int}[\varphi_0]] |0\rangle}$$

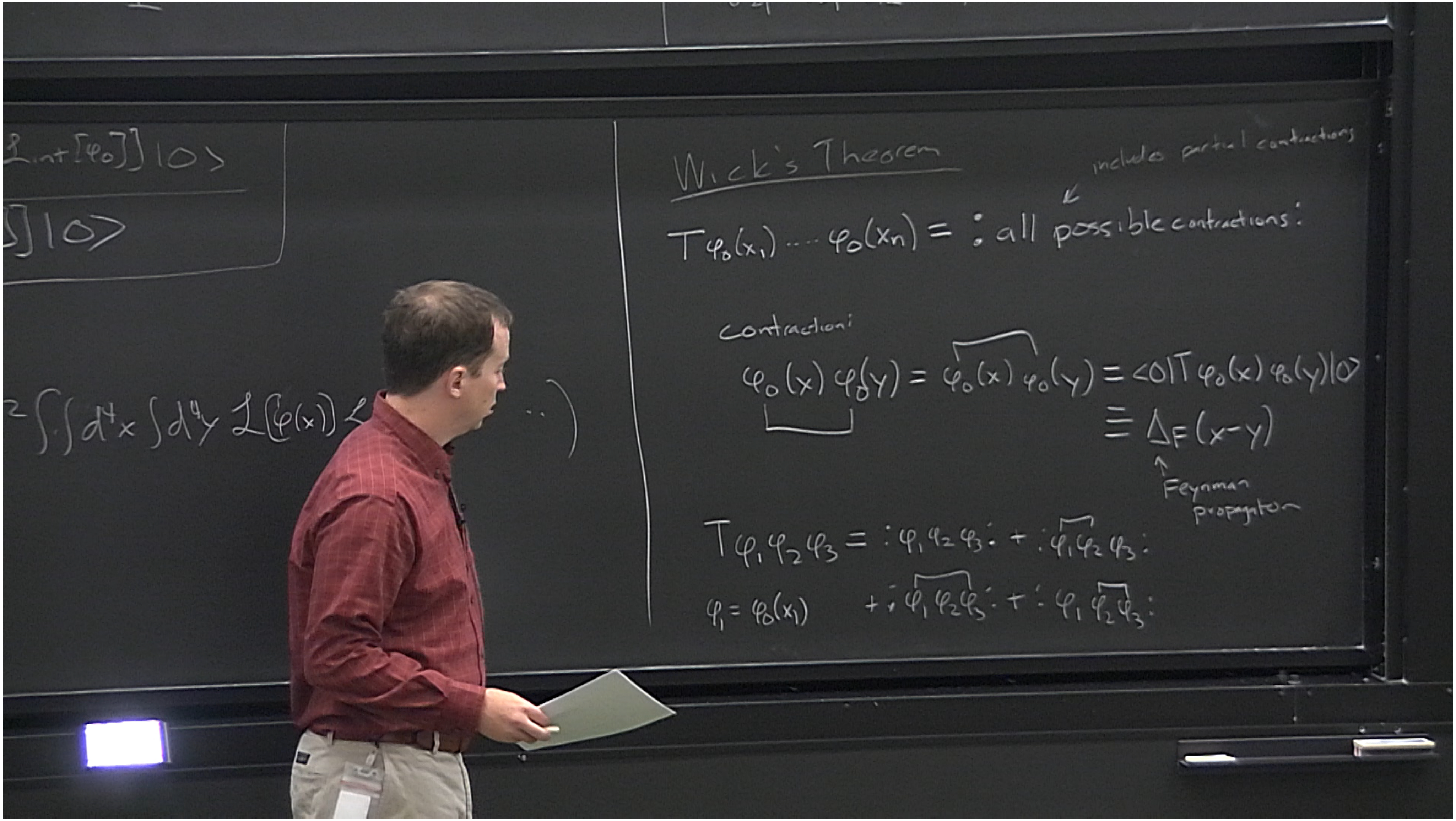
to proceed further need perturbation theory

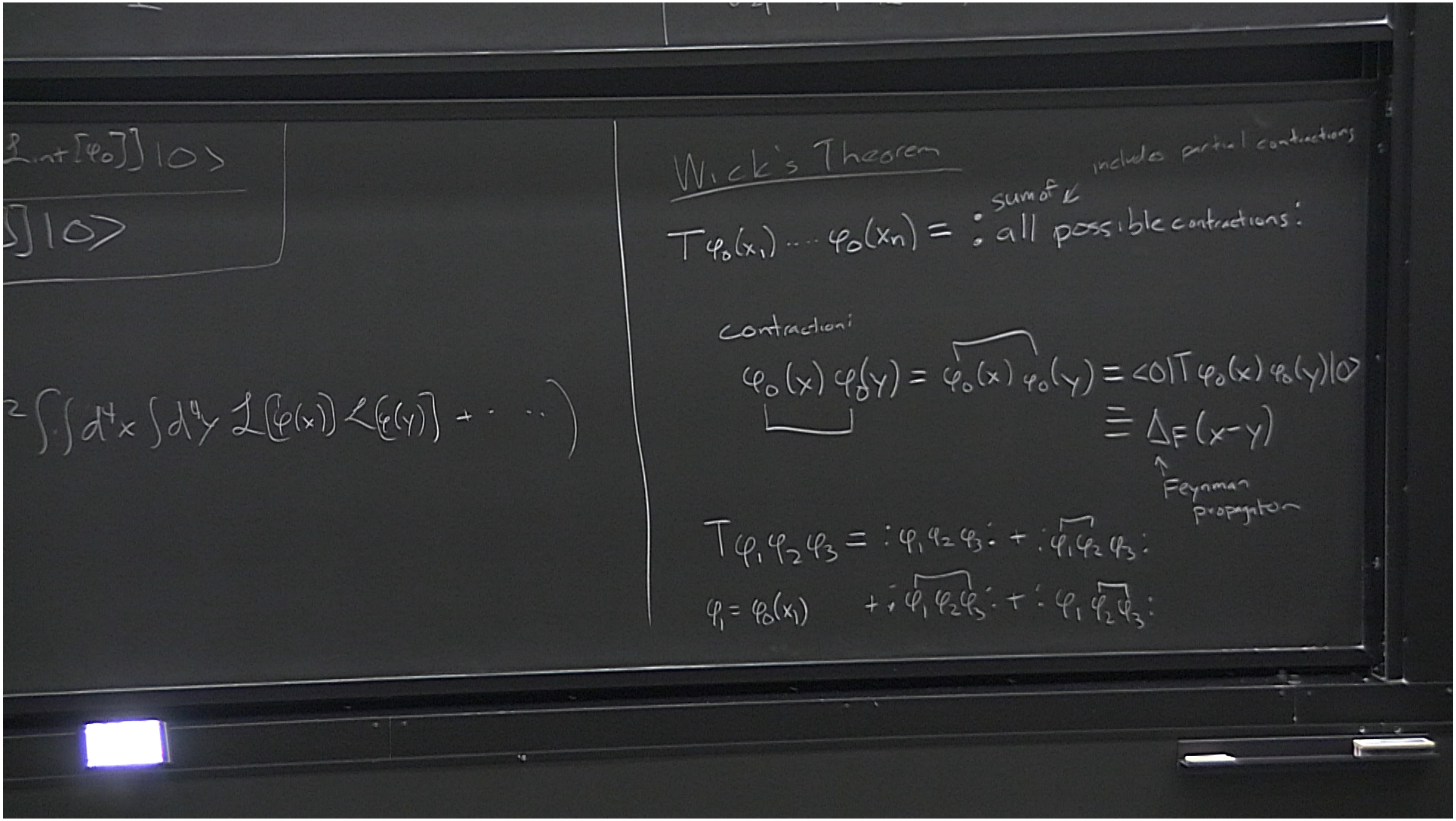
$$T\exp[\,] = T\left(1 + i\int d^4x\, \mathcal{L} + \frac{1}{2} i^2 \int\!\!\int d^4x \int d^4y\, \mathcal{L}[\varphi(x)]\, \mathcal{L}[\varphi(y)] + \cdots\right)$$

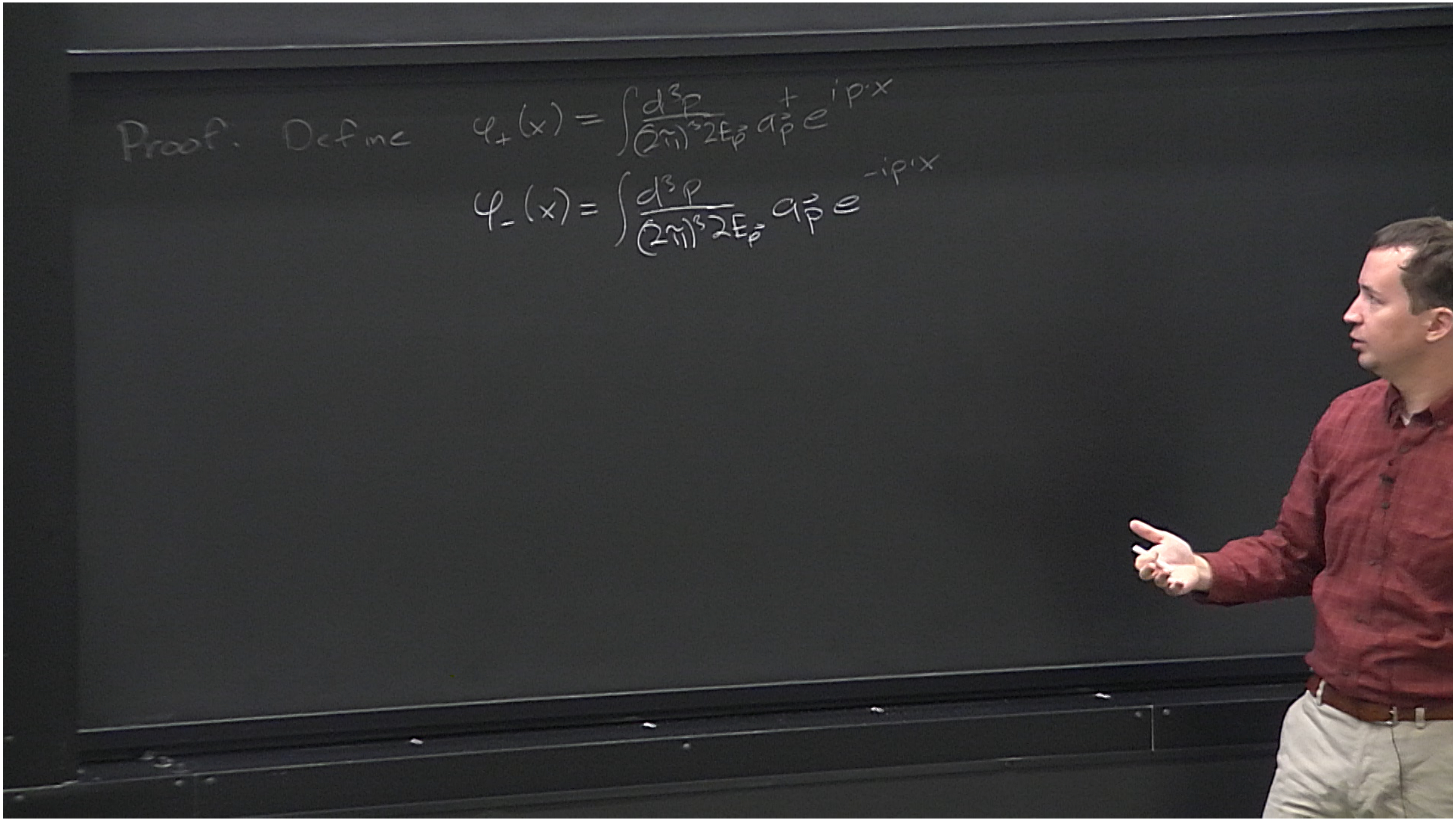
each term $\langle 0| T\, \varphi_0(x_1) \cdots \varphi_0(x_m) |0\rangle$


Wick's Theorem
includes partial contractions
T φ0(x1) ... φ0(xn) = : all possible contractions :
contraction:
φ0(x) φ0(y) = φ0(x) φ0(y) = <0|T φ0(x) φ0(y)|0>
≡ ΔF (x−y)
Feynman
propagator


Wick's Theorem
includes partial contractions
T φ0(x1) .... φ0(xn) = : all possible contractions :
contraction:
φ0(x) φ0(y) = φ0(x) φ0(y) = <0|T φ0(x) φ0(y)|0>
≡ ΔF (x−y)
Feynman
propagator
T φ1 φ2 φ3 =


Wick's Theorem
include partial contractions
T φ0(x1) ... φ0(xn) = : all possible contractions :
contraction:
φ0(x) φ0(y) = φ0(x) φ0(y) = <0|T φ0(x) φ0(y)|0>
≡ ΔF (x−y)
Feynman propagator
T φ1 φ2 φ3 = : φ1 φ2 φ3 : + : φ1 φ2 φ3 :
φ1 = φ0(x1)
+ : φ1 φ2 φ3 : + : φ1 φ2 φ3 :


Wick's Theorem
includes partial contractions
sum of
T φ₀(x₁) ... φ₀(xₙ) = : all possible contractions :
contraction:
φ₀(x) φ₀(y) = φ₀(x) φ₀(y) = <0| T φ₀(x) φ₀(y) |0>
≡ Δ_F (x−y)
Feynman propagator
T φ₁ φ₂ φ₃ = : φ₁ φ₂ φ₃ : + : φ₁ φ₂ φ₃ :
φ₁ = φ₀(x₁)
+ : φ₁ φ₂ φ₃ : + : φ₁ φ₂ φ₃ :
∫d⁴x ∫d⁴y ℒ[φ(x)] ℒ[φ(y)] + ...


Proof: Define
$\varphi_+(x) = \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} a^\dagger_{\vec{p}} e^{ip\cdot x}$
$\varphi_-(x) = \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} a_{\vec{p}} e^{-ip\cdot x}$

Proof: Define $\varphi_+(x) = \int \frac{d^3p}{(2\pi)^3 2E_{\vec p}} a^\dagger_{\vec p}\, e^{ip\cdot x}$

$$\varphi_-(x) = \int \frac{d^3p}{(2\pi)^3 2E_{\vec p}} a_{\vec p}\, e^{-ip\cdot x}$$

If $x_1^0 > x_2^0$

$$T\varphi_1\varphi_2 = \varphi_+(x_1)\varphi_+(x_2) + \varphi_+(x_1)\varphi_-(x_2) + \varphi_-(x_1)\varphi_+(x_2) + \varphi_-(x_1)\varphi_-(x_2)$$

Proof: Define $\varphi_+(x) = \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} a^\dagger_{\vec{p}} e^{ip\cdot x}$

$$\varphi_-(x) = \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} a_{\vec{p}} e^{-ip\cdot x}$$

If $x_1^0 > x_2^0$

(not normal ordered → $\varphi_-(x_1)\varphi_+(x_2)$)

$$T\varphi_1\varphi_2 = \varphi_+(x_1)\varphi_+(x_2) + \varphi_+(x_1)\varphi_-(x_2) + \varphi_-(x_1)\varphi_+(x_2) + \varphi_-(x_1)\varphi_-(x_2)$$

$$= :\varphi_1\varphi_2: + [\varphi_-(x_1), \varphi_+(x_2)]$$

$$T\varphi_1\varphi_2 = :\varphi_1\varphi_2: + \underbrace{[\varphi_-(x_1), \varphi_+(x_2)]\,\Theta(x_1^0 - x_2^0) + [\varphi_-(x_2), \varphi_+(x_1)]\,\Theta(x_2^0 - x_1^0)}_{= \Delta_F(x_1 - x_2)}$$

Proof: Define $\varphi_+(x) = \int \frac{d^3p}{(2\pi)^3 2E_{\vec p}} a^\dagger_{\vec p} e^{ip\cdot x}$

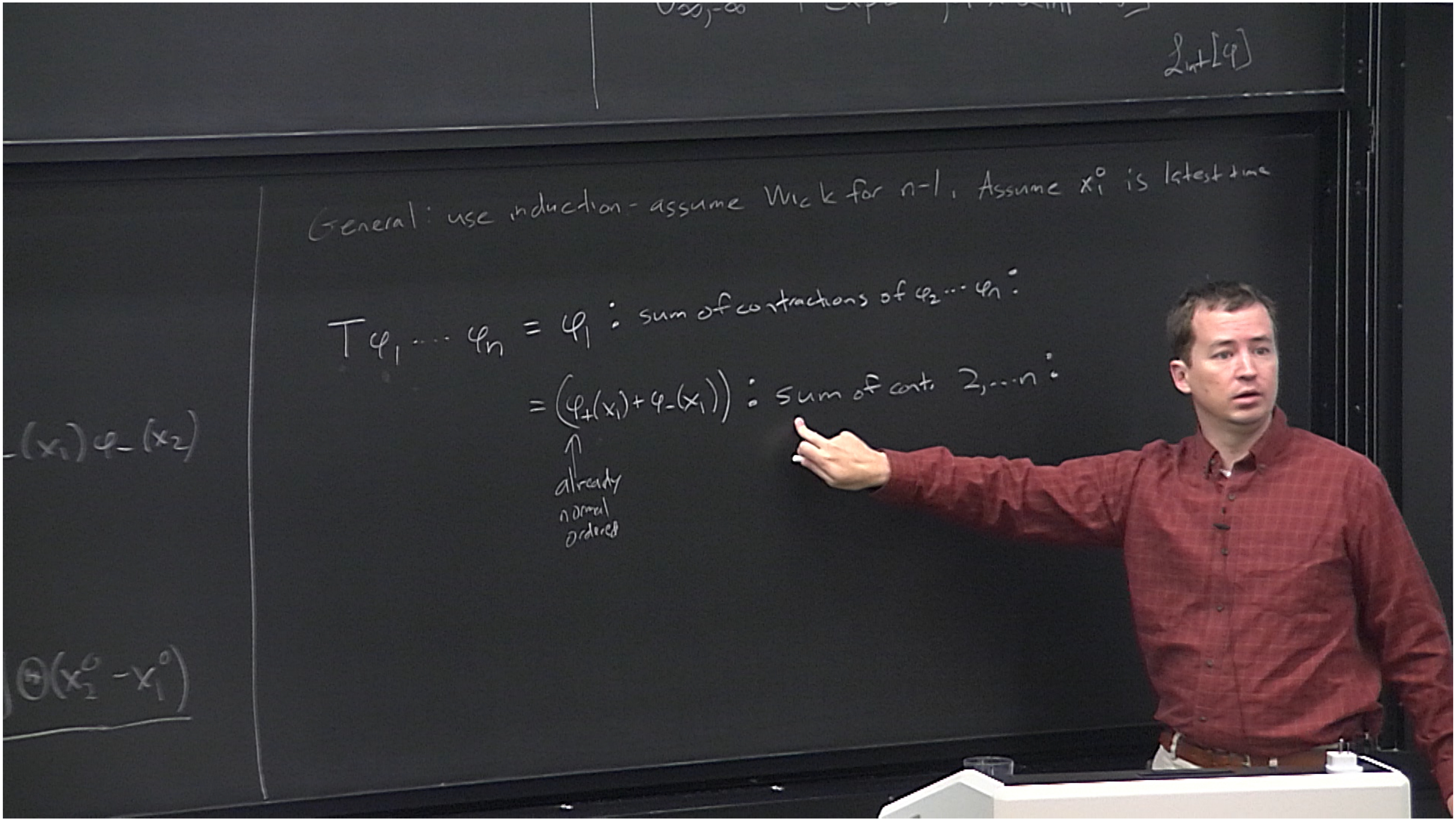
$$\varphi_-(x) = \int \frac{d^3p}{(2\pi)^3 2E_{\vec p}} a_{\vec p} e^{-ip\cdot x}$$

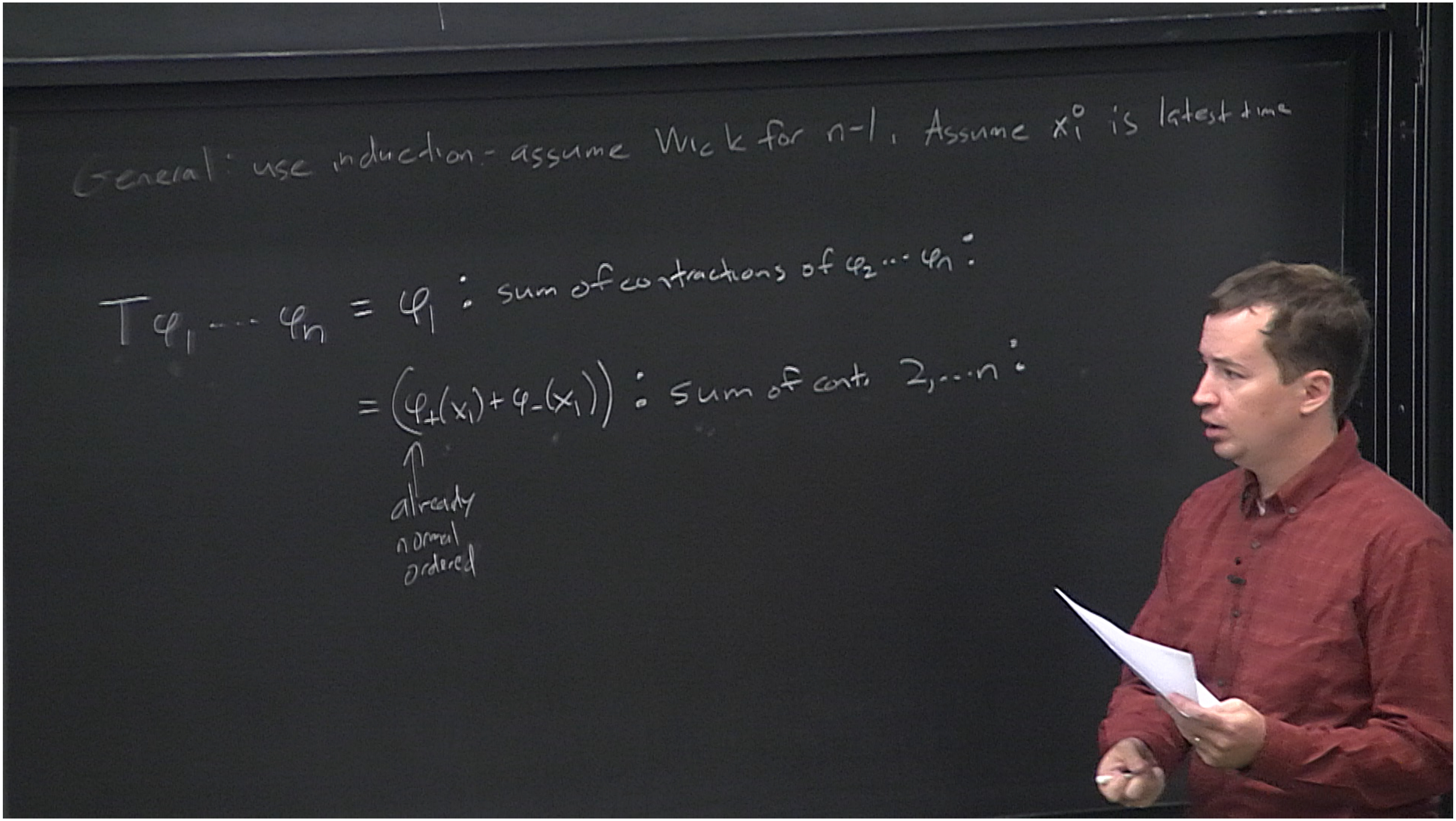
If $x_1^0 > x_2^0$

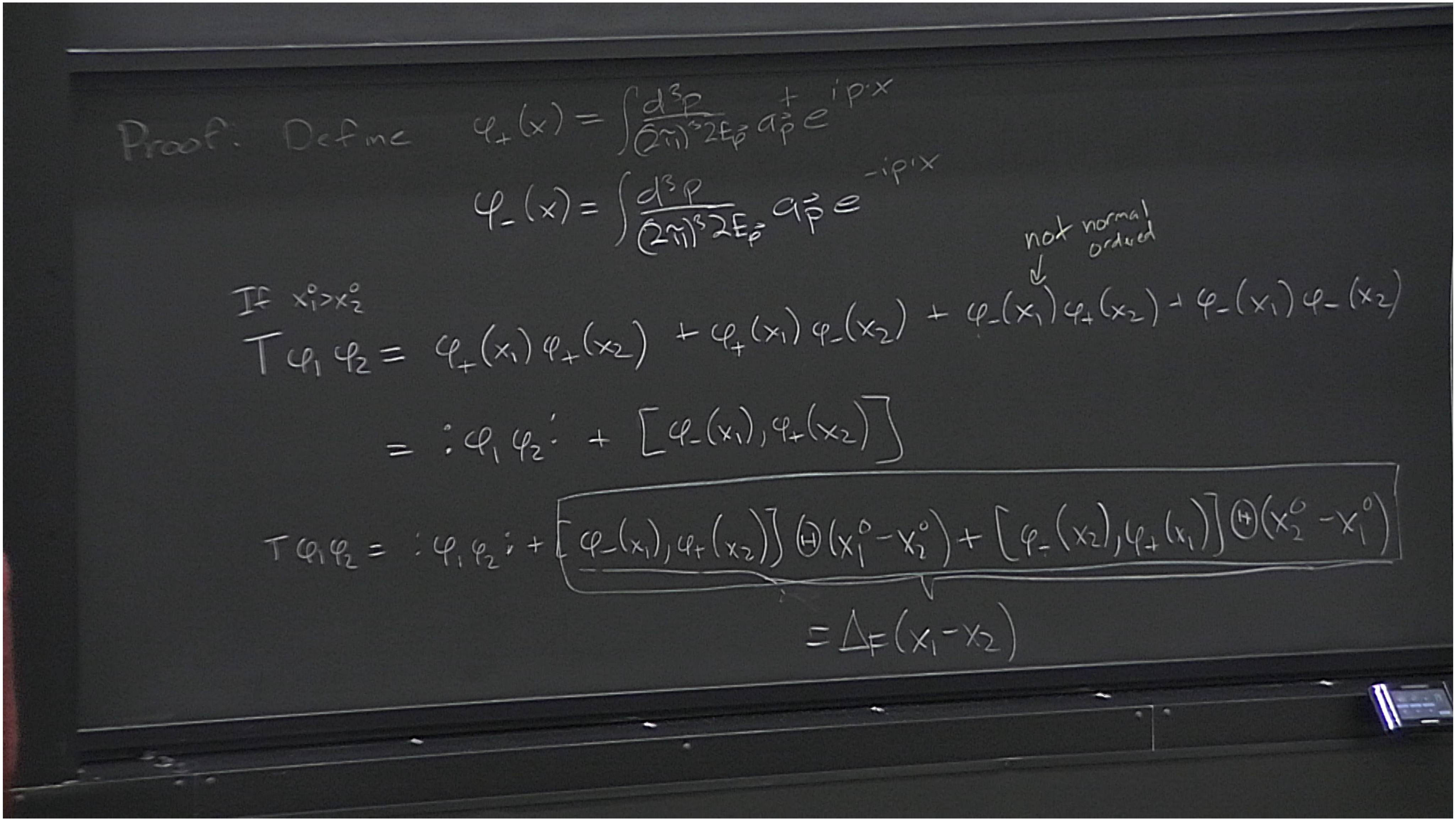
$$T\varphi_1\varphi_2 = \varphi_+(x_1)\varphi_+(x_2) + \varphi_+(x_1)\varphi_-(x_2) + \underbrace{\varphi_-(x_1)\varphi_+(x_2)}_{\text{not normal ordered}} + \varphi_-(x_1)\varphi_-(x_2)$$

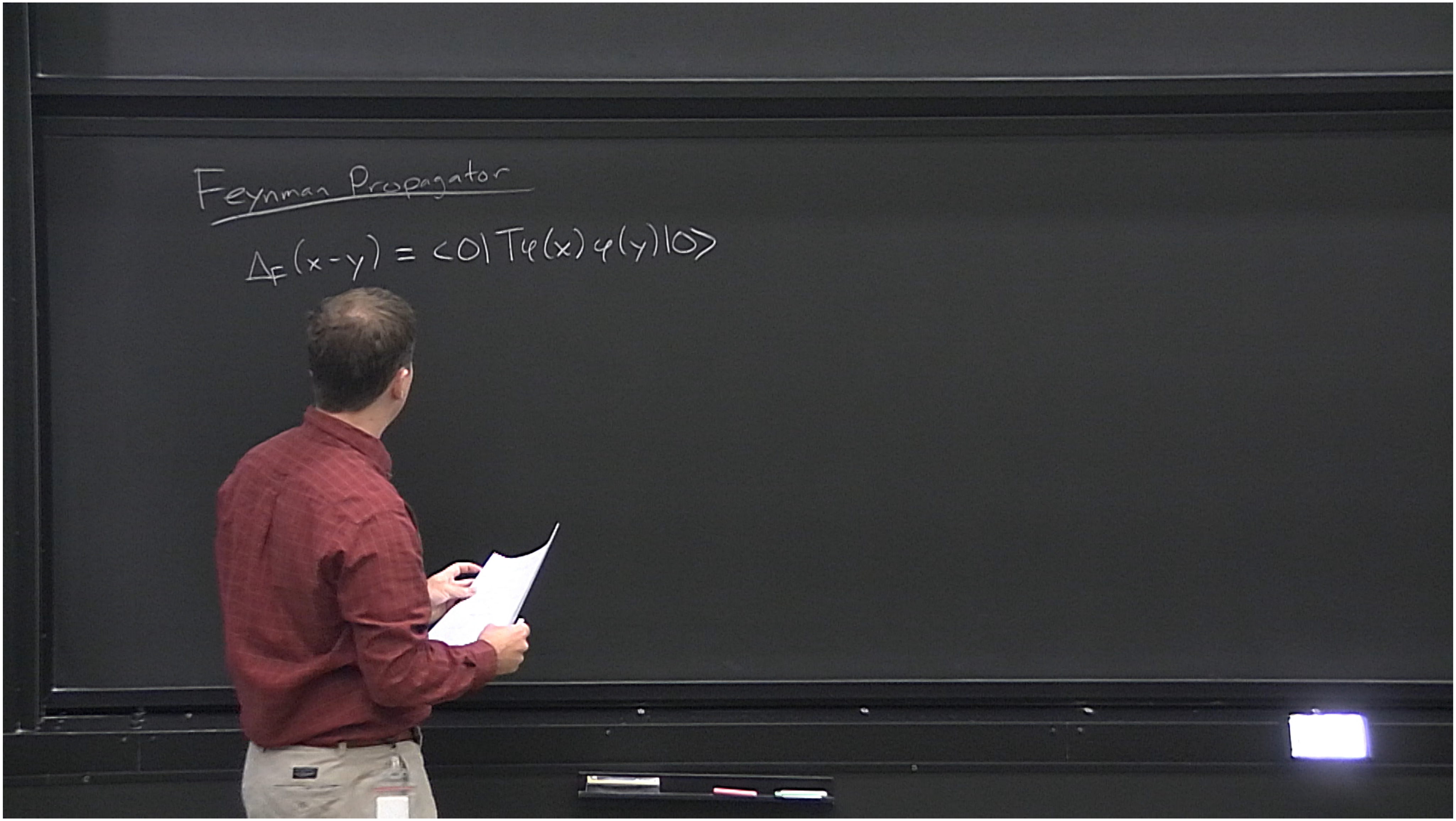
$$= :\varphi_1\varphi_2: + [\varphi_-(x_1), \varphi_+(x_2)]$$

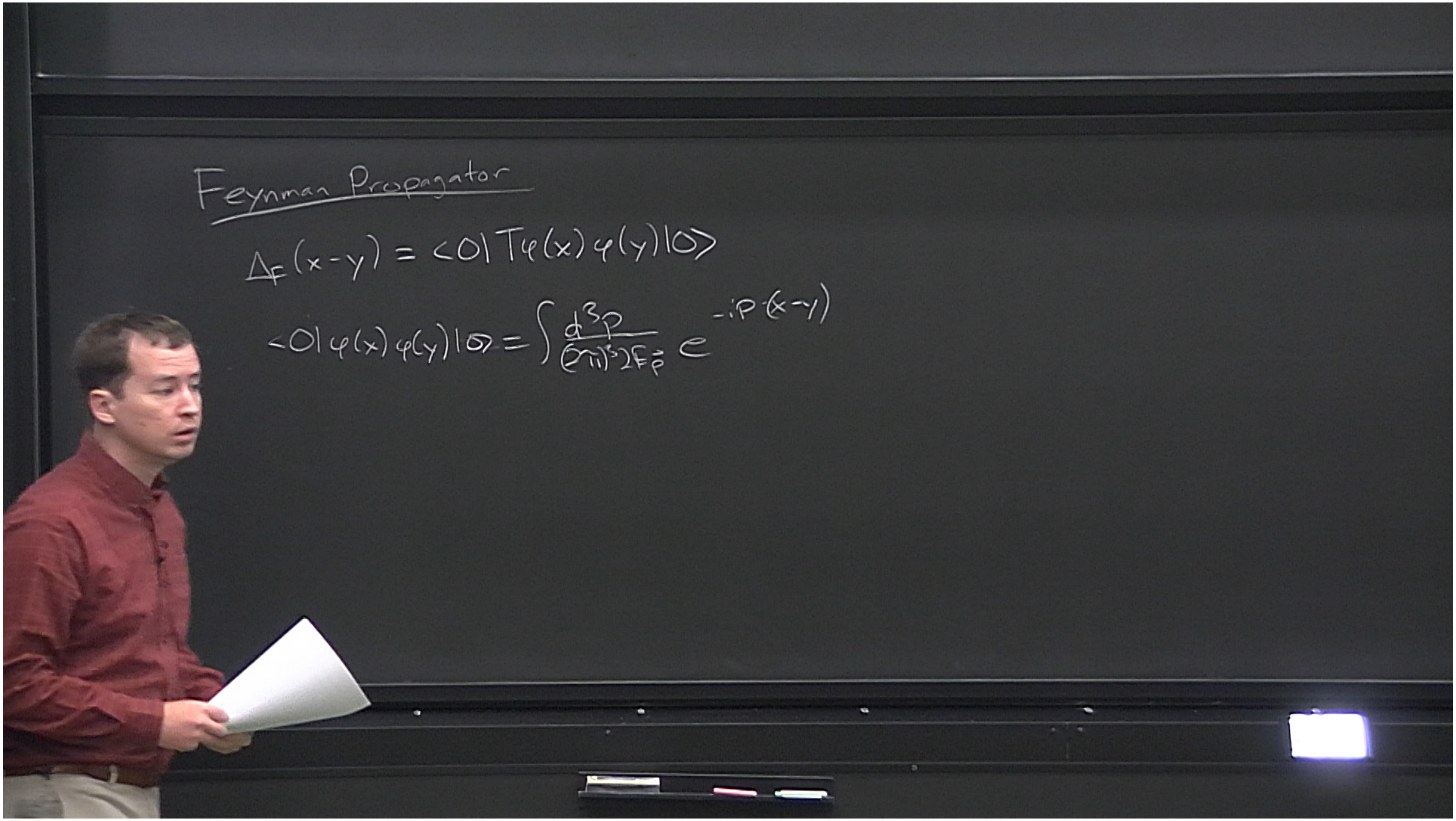
$$T\varphi_1\varphi_2 = :\varphi_1\varphi_2: + \underbrace{[\varphi_-(x_1), \varphi_+(x_2)]\,\Theta(x_1^0 - x_2^0) + [\varphi_-(x_2), \varphi_+(x_1)]\,\Theta(x_2^0 - x_1^0)}_{= \Delta_F(x_1 - x_2)}$$


L_int[φ]
General: use induction - assume Wick for n-1. Assume x_1^0 is latest time
T φ_1 ⋯ φ_n = φ_1 : sum of contractions of φ_2 ⋯ φ_n :
= (φ_+(x_1) + φ_-(x_1)) : sum of cont. 2,…n :
already normal ordered
Θ(x_2^0 - x_1^0)


General: use induction - assume Wick for n-1. Assume x_1^0 is latest time
$T\varphi_1 \cdots \varphi_n = \varphi_1 :$ sum of contractions of $\varphi_2 \cdots \varphi_n :$
$= (\varphi_+(x_1) + \varphi_-(x_1)) :$ sum of cont. $2, \ldots n :$
already normal ordered


Proof: Define $\varphi_+(x) = \int \frac{d^3p}{(2\pi)^3 2E_{\vec p}} a^\dagger_{\vec p} e^{ip\cdot x}$
$\varphi_-(x) = \int \frac{d^3p}{(2\pi)^3 2E_{\vec p}} a_{\vec p} e^{-ip\cdot x}$
not normal ordered
If $x_1^0 > x_2^0$
$T\varphi_1\varphi_2 = \varphi_+(x_1)\varphi_+(x_2) + \varphi_+(x_1)\varphi_-(x_2) + \varphi_-(x_1)\varphi_+(x_2) + \varphi_-(x_1)\varphi_-(x_2)$
$= :\varphi_1\varphi_2: + [\varphi_-(x_1), \varphi_+(x_2)]$
$T\varphi_1\varphi_2 = :\varphi_1\varphi_2: + [\varphi_-(x_1), \varphi_+(x_2)]\Theta(x_1^0 - x_2^0) + [\varphi_-(x_2), \varphi_+(x_1)]\Theta(x_2^0 - x_1^0)$
$= \Delta_F(x_1 - x_2)$


Feynman Propagator
$\Delta_F(x-y) = \langle 0|T\varphi(x)\varphi(y)|0\rangle$


Feynman Propagator
$\Delta_F(x-y) = \langle 0| T\varphi(x)\varphi(y)|0\rangle$
$\langle 0|\varphi(x)\varphi(y)|0\rangle = \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} e^{-ip\cdot(x-y)}$

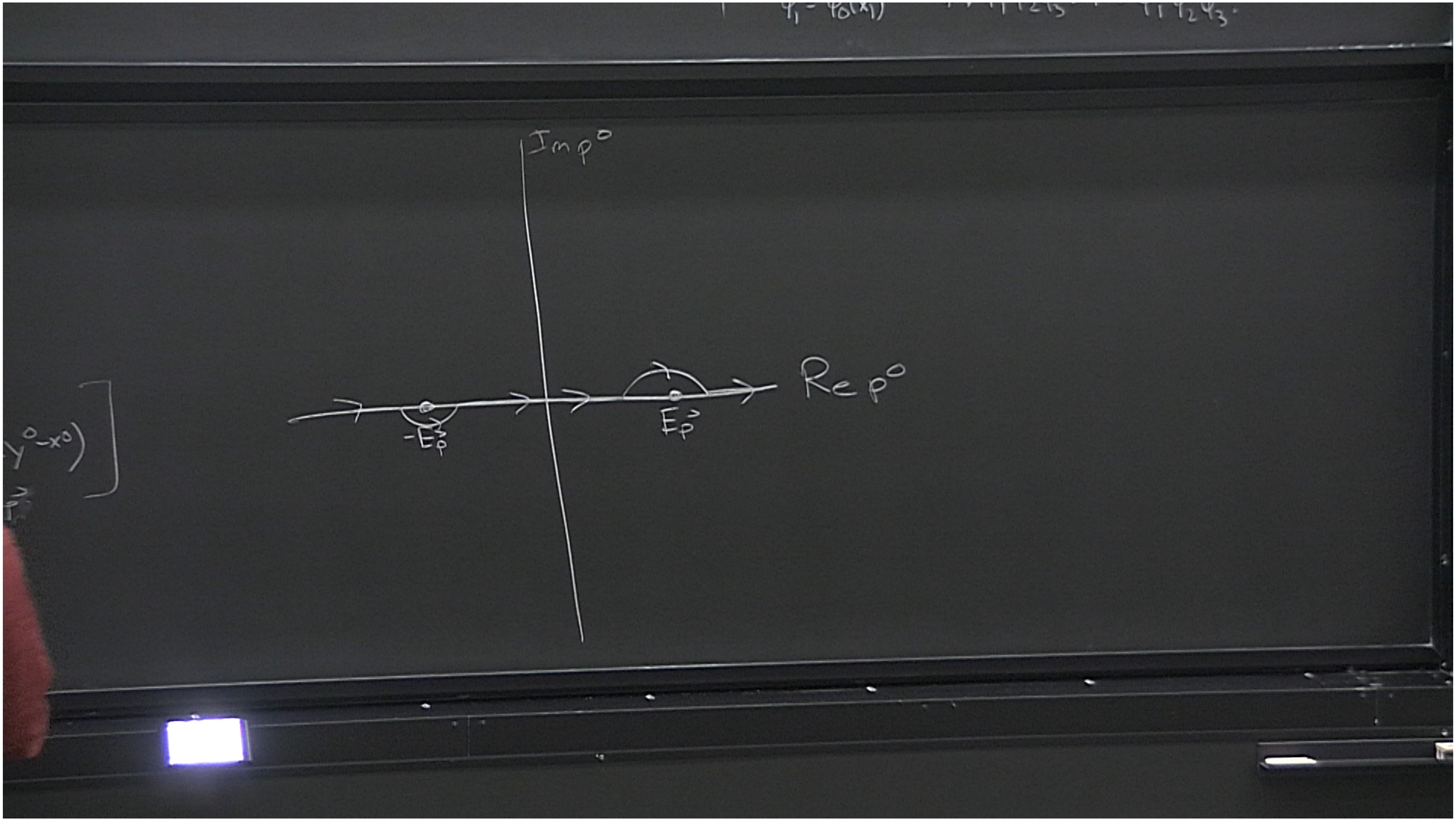
Feynman Propagator

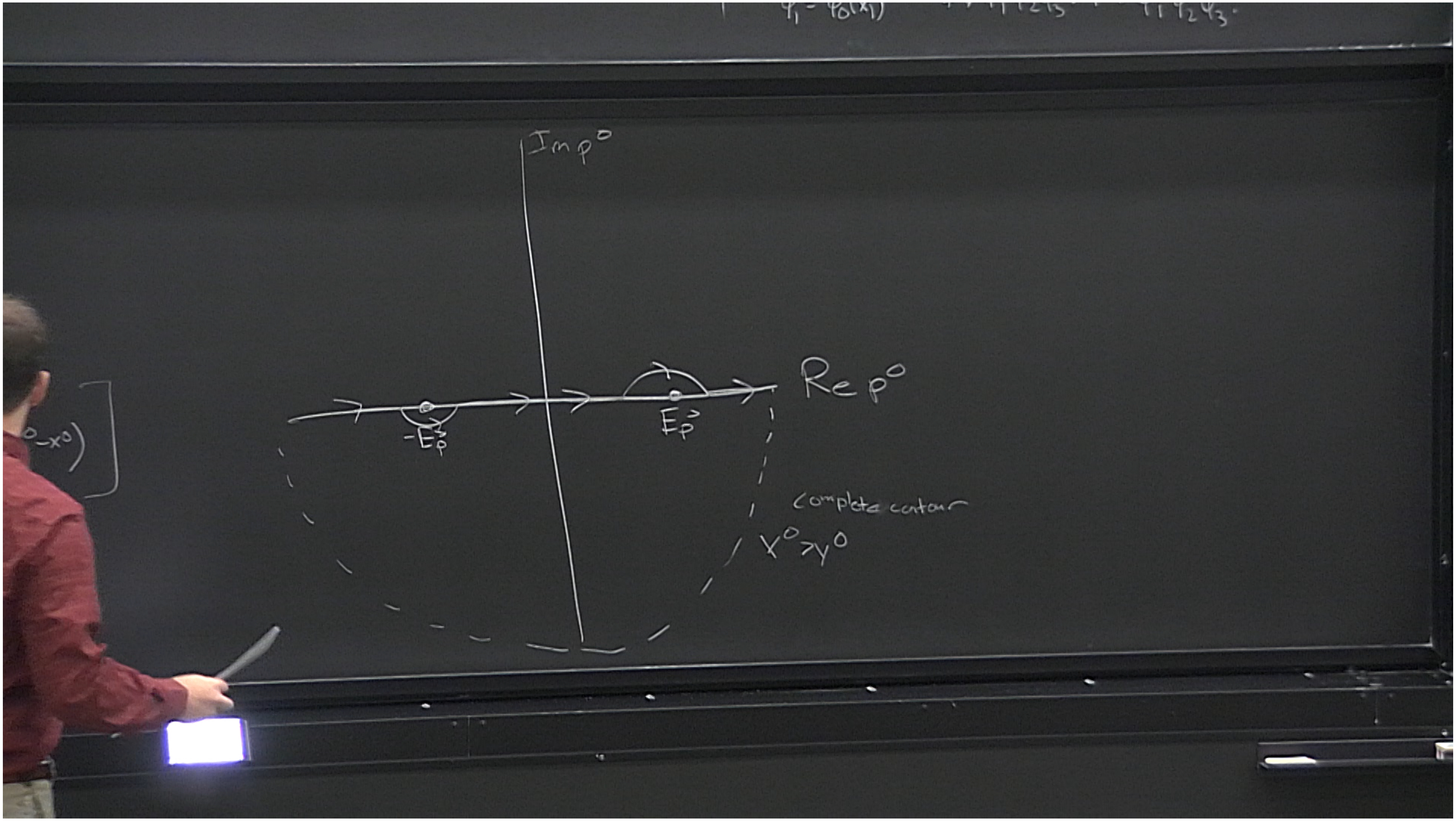
$$\Delta_F(x-y) = \langle 0 | T\varphi(x)\varphi(y) | 0 \rangle$$

$$\langle 0 | \varphi(x)\varphi(y) | 0 \rangle = \int \frac{d^3p}{(2\pi)^3 2E_{\vec{p}}} e^{-ip\cdot(x-y)}$$

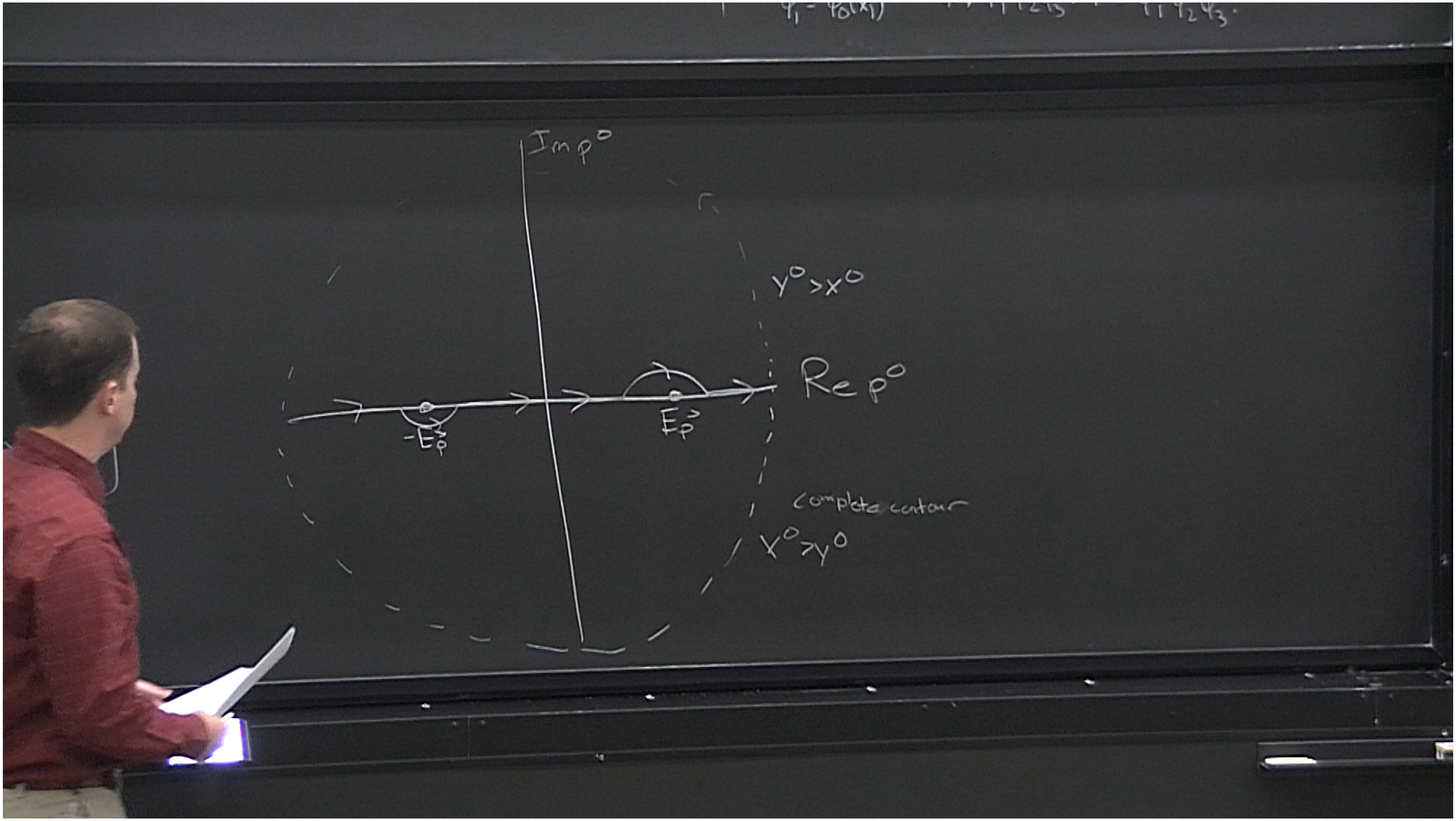
$$\Delta_F(x-y) = \int \frac{d^3p}{(2\pi)^3} \left[\frac{1}{2E_{\vec{p}}} e^{-ip\cdot(x-y)}\Big|_{p^0=E_{\vec{p}}} \Theta(x^0-y^0) + \frac{1}{2E_{\vec{p}}} e^{-ip\cdot(x-y)}\Big|_{p^0=-E_{\vec{p}}} \Theta(y^0-x^0) \right]$$

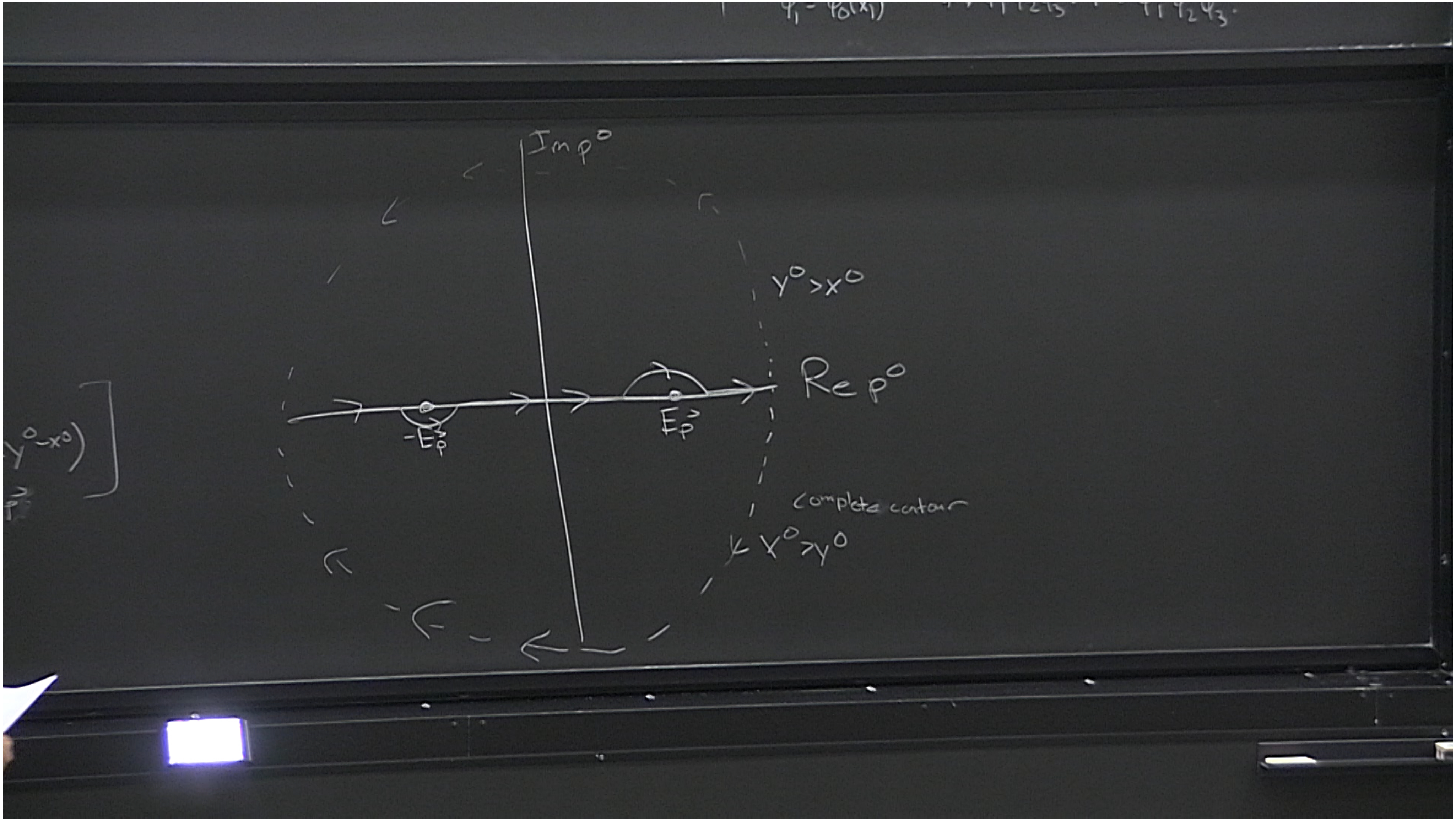
$$= \int \frac{d^3p}{(2\pi)^3} \int_{C_F} \frac{dp^0}{2\pi i} \frac{-1}{p^2-m^2} e^{-ip\cdot(x-y)}$$

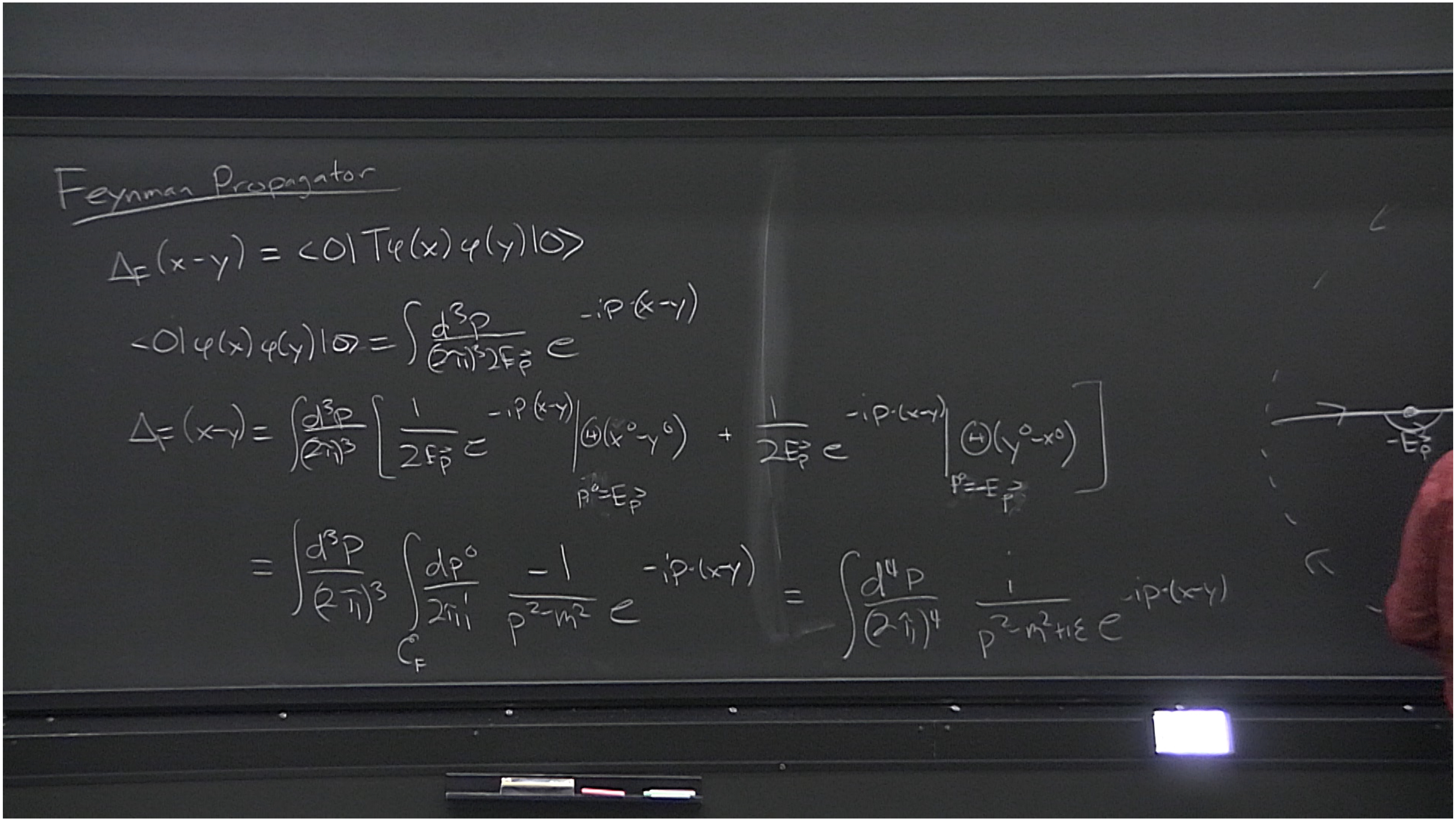

Im p⁰
Re p⁰
-E_p
E_p

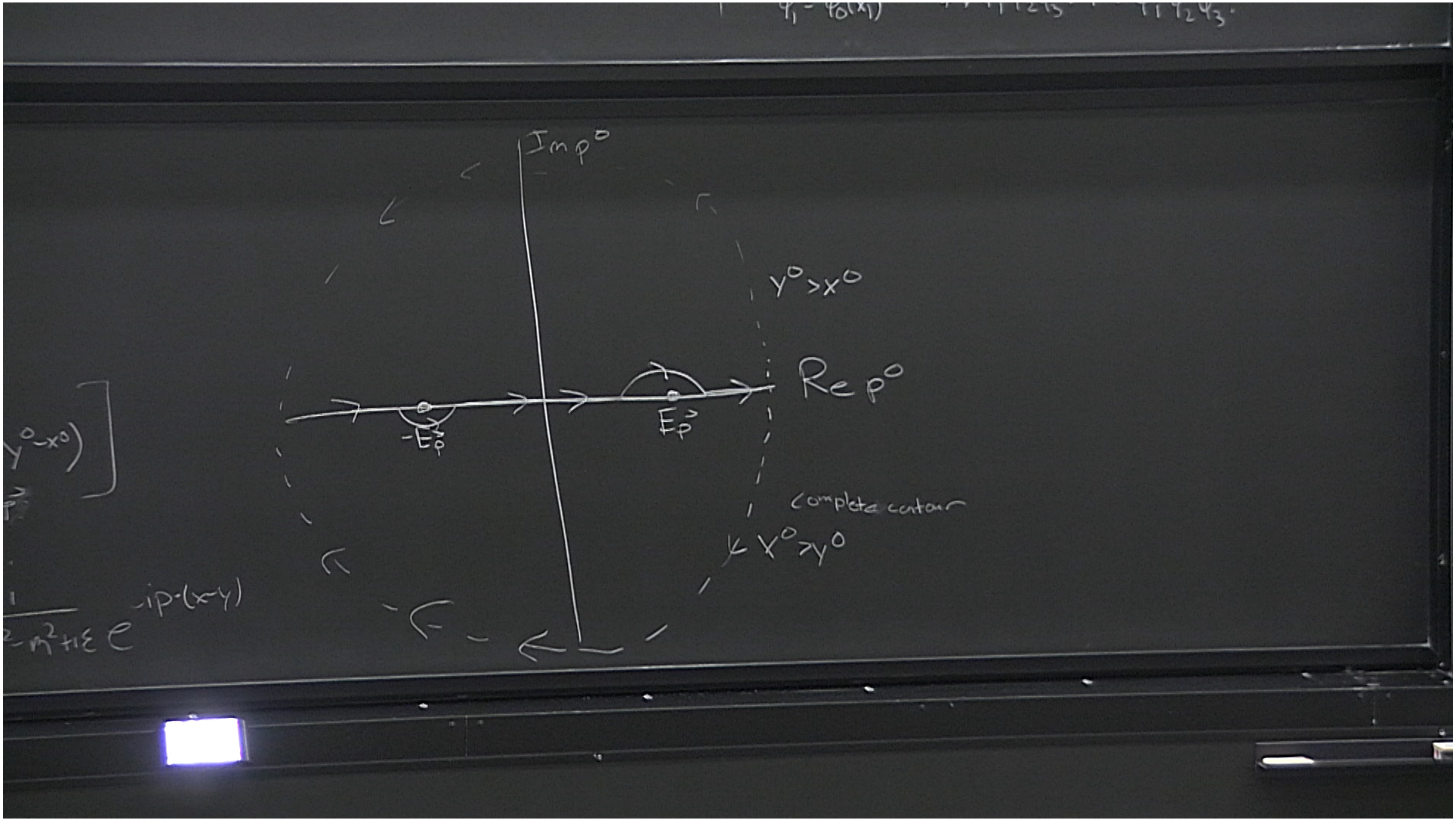


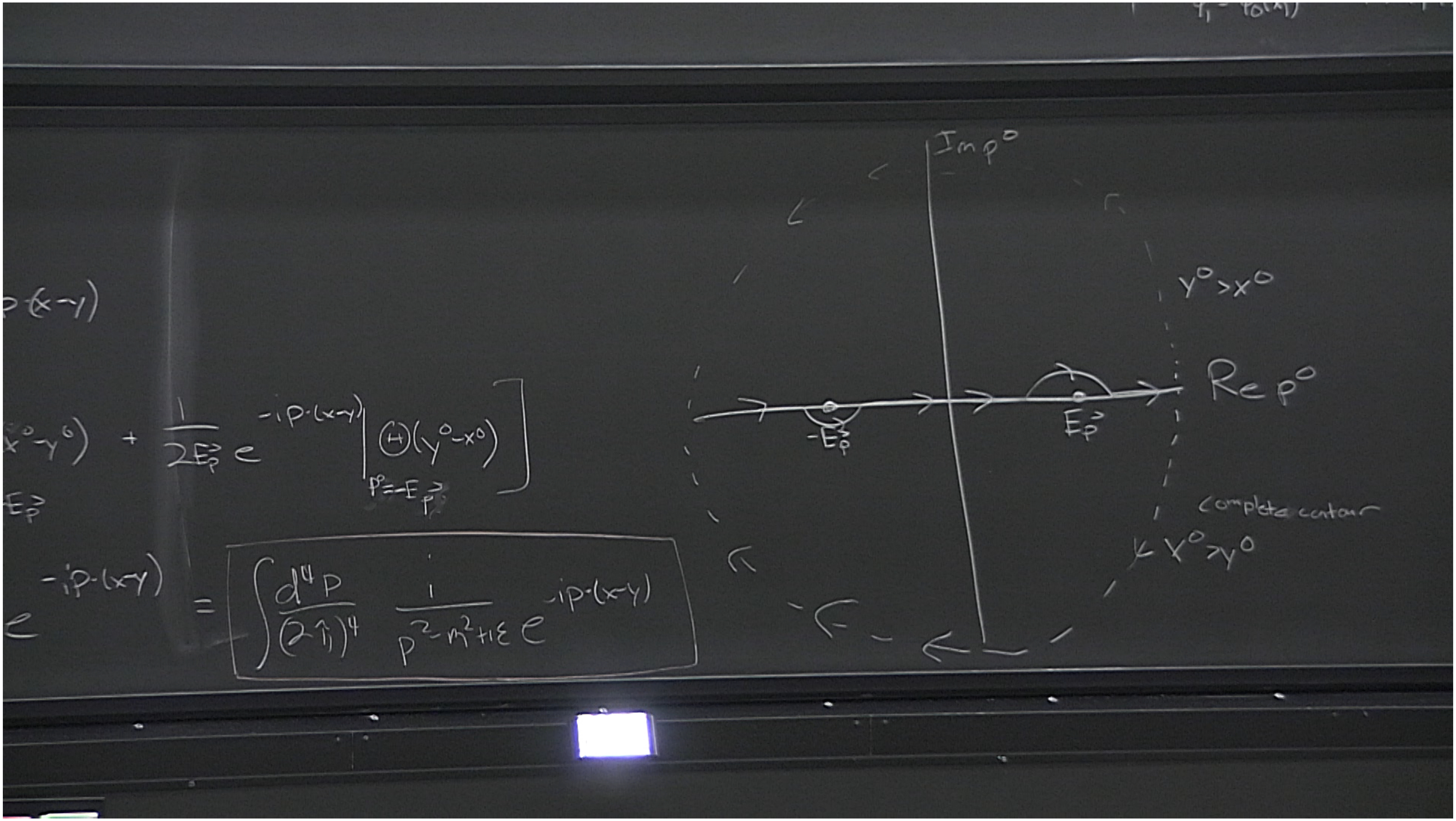
Im p⁰
Re p⁰
-Ep
Ep
complete contour
x⁰ > y⁰

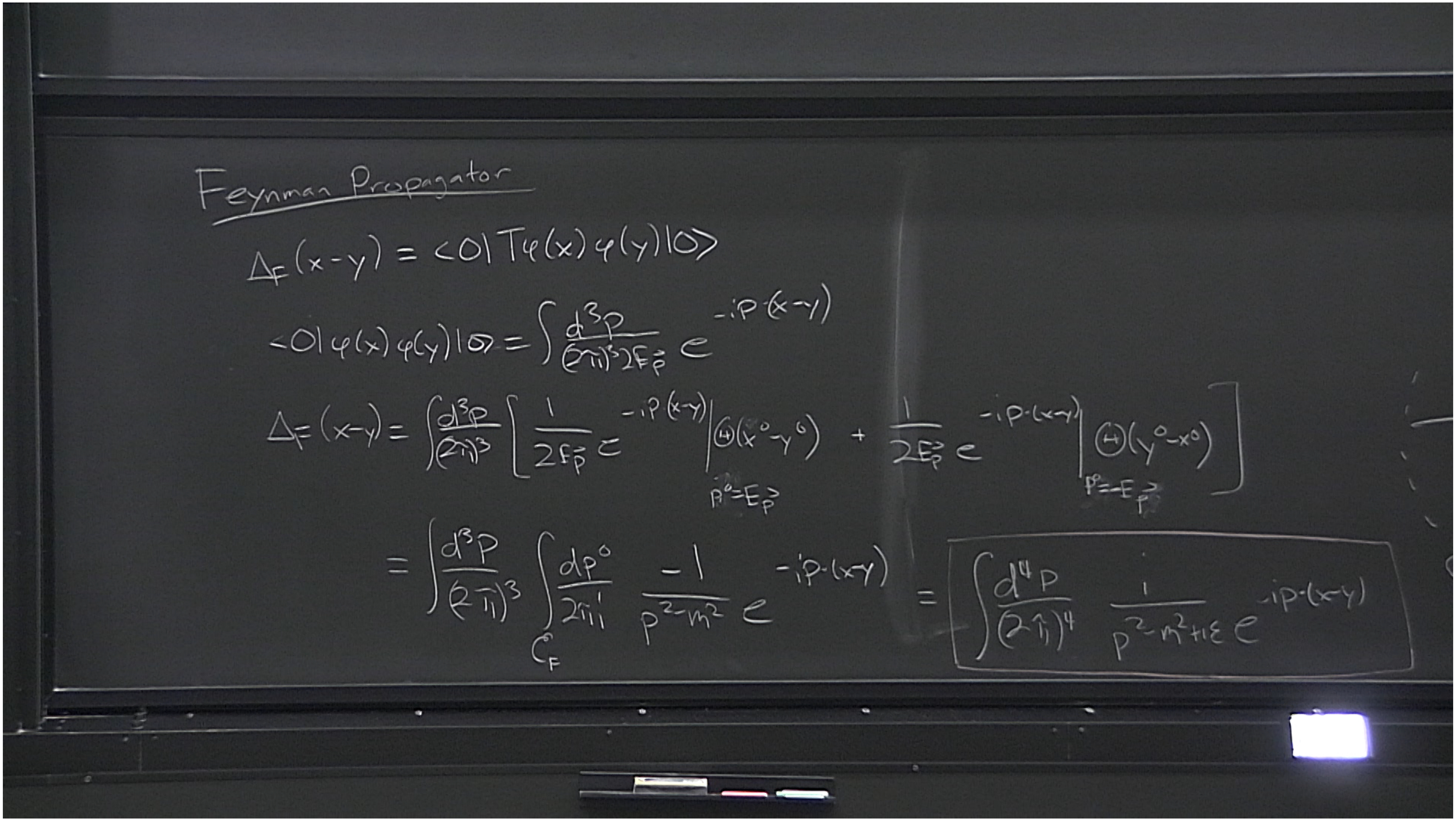

Im p⁰
y⁰ > x⁰
Re p⁰
$-E_p$
E_p
complete contour
x⁰ > y⁰

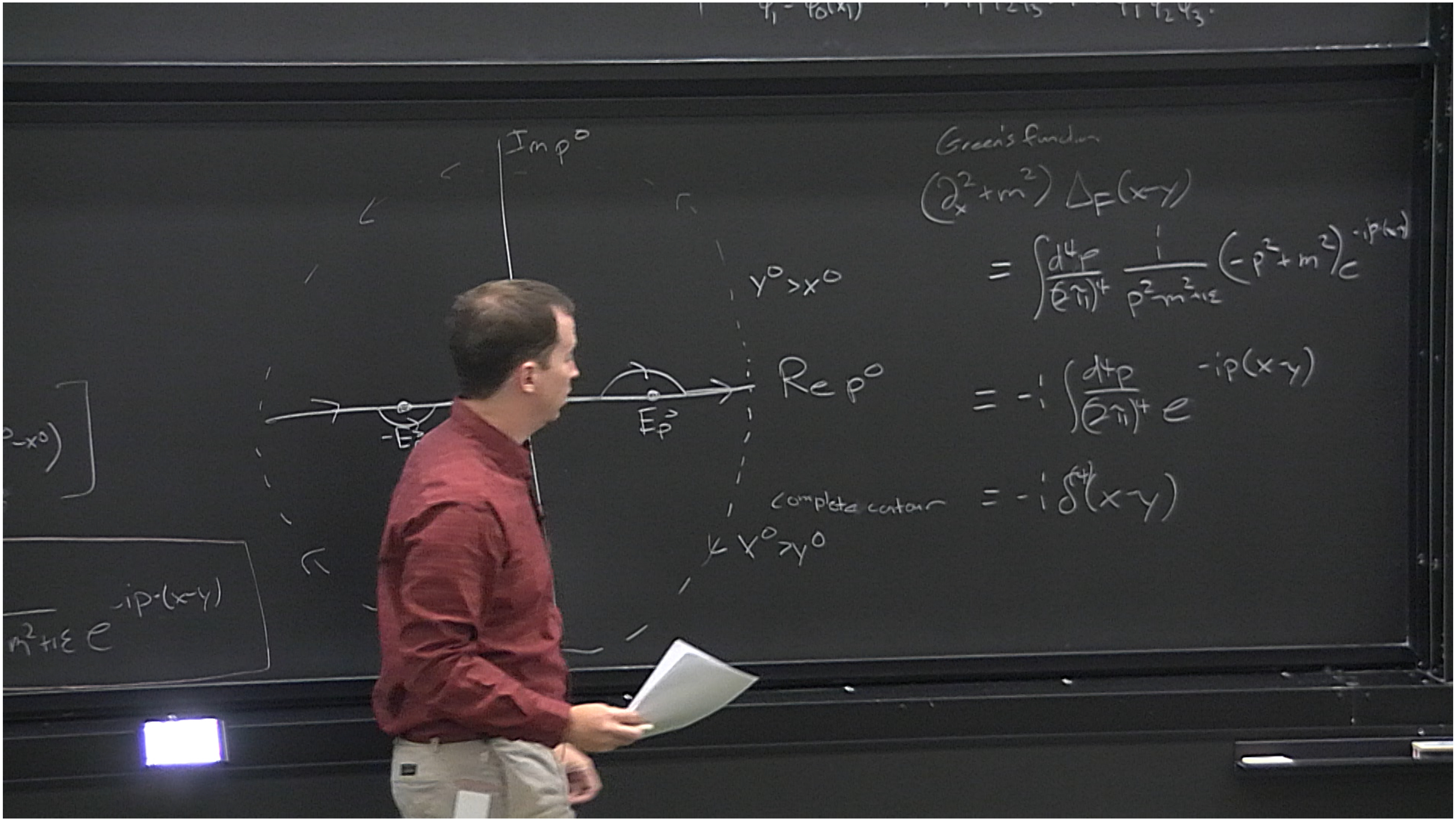

Im p⁰
Re p⁰
y⁰ > x⁰
x⁰ > y⁰
complete contour
-Ep
Ep

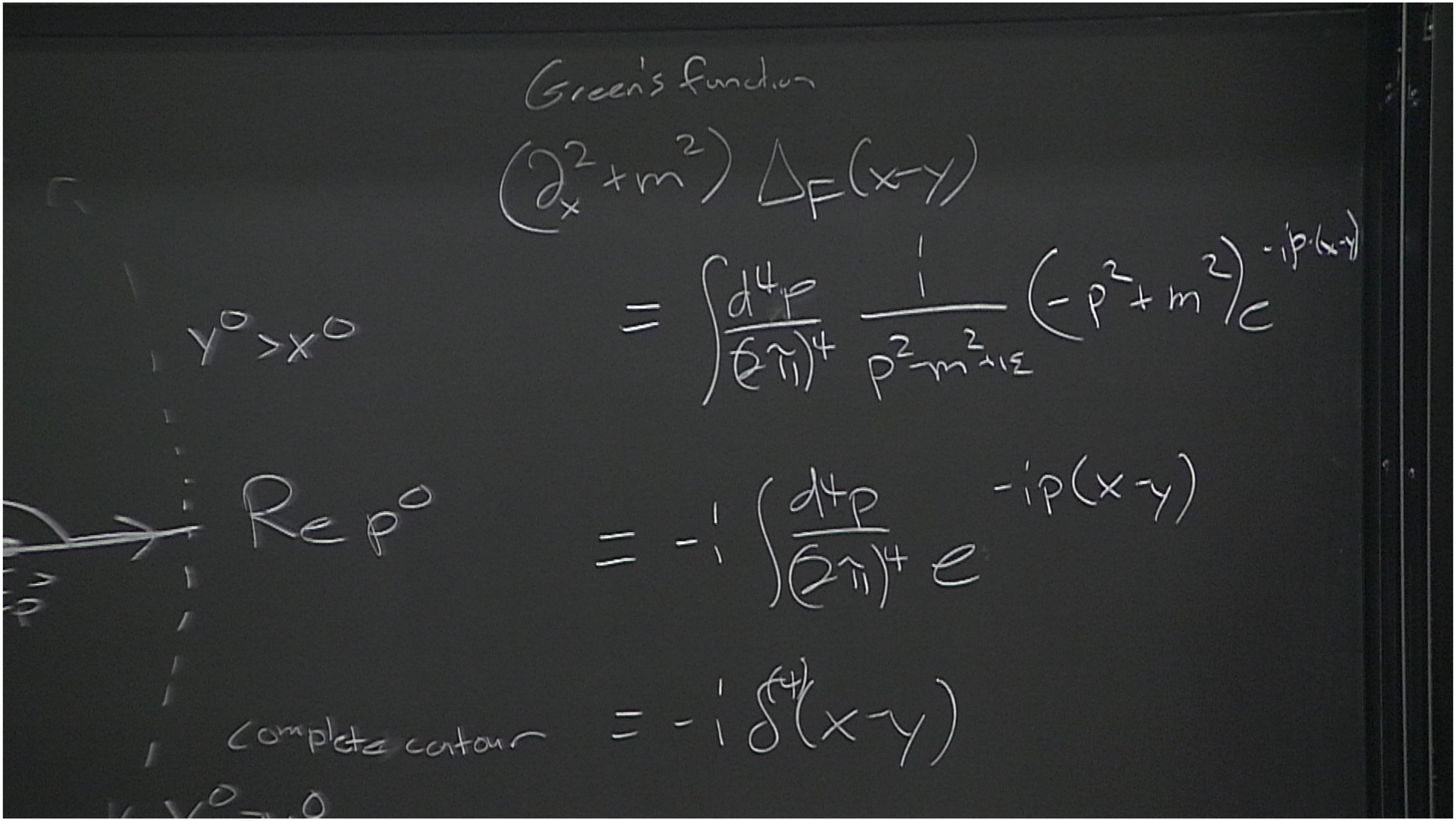

Feynman Propagator
$\Delta_F(x-y) = \langle 0| T\varphi(x)\varphi(y)|0\rangle$
$\langle 0|\varphi(x)\varphi(y)|0\rangle = \int \frac{d^3p}{(2\pi)^3 2E_{\vec p}} e^{-ip\cdot(x-y)}$
$\Delta_F(x-y) = \int \frac{d^3p}{(2\pi)^3}\left[\frac{1}{2E_{\vec p}} e^{-ip\cdot(x-y)}\Big|_{p^0=E_{\vec p}} \Theta(x^0-y^0) + \frac{1}{2E_{\vec p}} e^{-ip\cdot(x-y)}\Big|_{p^0=-E_{\vec p}} \Theta(y^0-x^0)\right]$
$= \int \frac{d^3p}{(2\pi)^3} \int_{C_F} \frac{dp^0}{2\pi i} \frac{-1}{p^2-m^2} e^{-ip\cdot(x-y)} = \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2-m^2+i\varepsilon} e^{-ip\cdot(x-y)}$
$-E_{\vec p}$


Im p⁰
y⁰>x⁰
Re p⁰
-E_p
E_p
complete contour
x⁰>y⁰


Im p°
Re p°
y° > x°
x° > y°
complete contour
-Ep
Ep
p° = -Ep


Feynman Propagator
$\Delta_F(x-y) = \langle 0| T\varphi(x)\varphi(y)|0\rangle$
$\langle 0|\varphi(x)\varphi(y)|0\rangle = \int \frac{d^3p}{(2\pi)^3 2E_{\vec p}} e^{-ip\cdot(x-y)}$
$\Delta_F(x-y) = \int \frac{d^3p}{(2\pi)^3}\left[\frac{1}{2E_{\vec p}} e^{-ip\cdot(x-y)}\Big|_{p^0=E_{\vec p}} \Theta(x^0-y^0) + \frac{1}{2E_{\vec p}} e^{-ip\cdot(x-y)}\Big|_{p^0=-E_{\vec p}} \Theta(y^0-x^0)\right]$
$= \int \frac{d^3p}{(2\pi)^3} \int_{C_F} \frac{dp^0}{2\pi i} \frac{-1}{p^2-m^2} e^{-ip\cdot(x-y)} = \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2-m^2+i\varepsilon} e^{-ip\cdot(x-y)}$


Im p^0
Re p^0
$y^0 > x^0$
$x^0 > y^0$
$-E_p$
E_p
Green's function
$(\partial_x^2 + m^2)\Delta_F(x-y)$
$= \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2-m^2+i\epsilon}(-p^2+m^2)e^{-ip\cdot(x-y)}$
$= -i\int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)}$
complete contour
$= -i\delta^{(4)}(x-y)$
$m^2+i\epsilon\ e^{-ip\cdot(x-y)}$


Green's function
$(\partial_x^2 + m^2)\Delta_F(x-y)$
$= \int \frac{d^4p}{(2\pi)^4} \frac{i}{p^2-m^2+i\varepsilon}(-p^2+m^2)e^{-ip\cdot(x-y)}$
$y^0 > x^0$
Re p^0
$= -i\int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)}$
complete contour
$= -i\delta^{(4)}(x-y)$