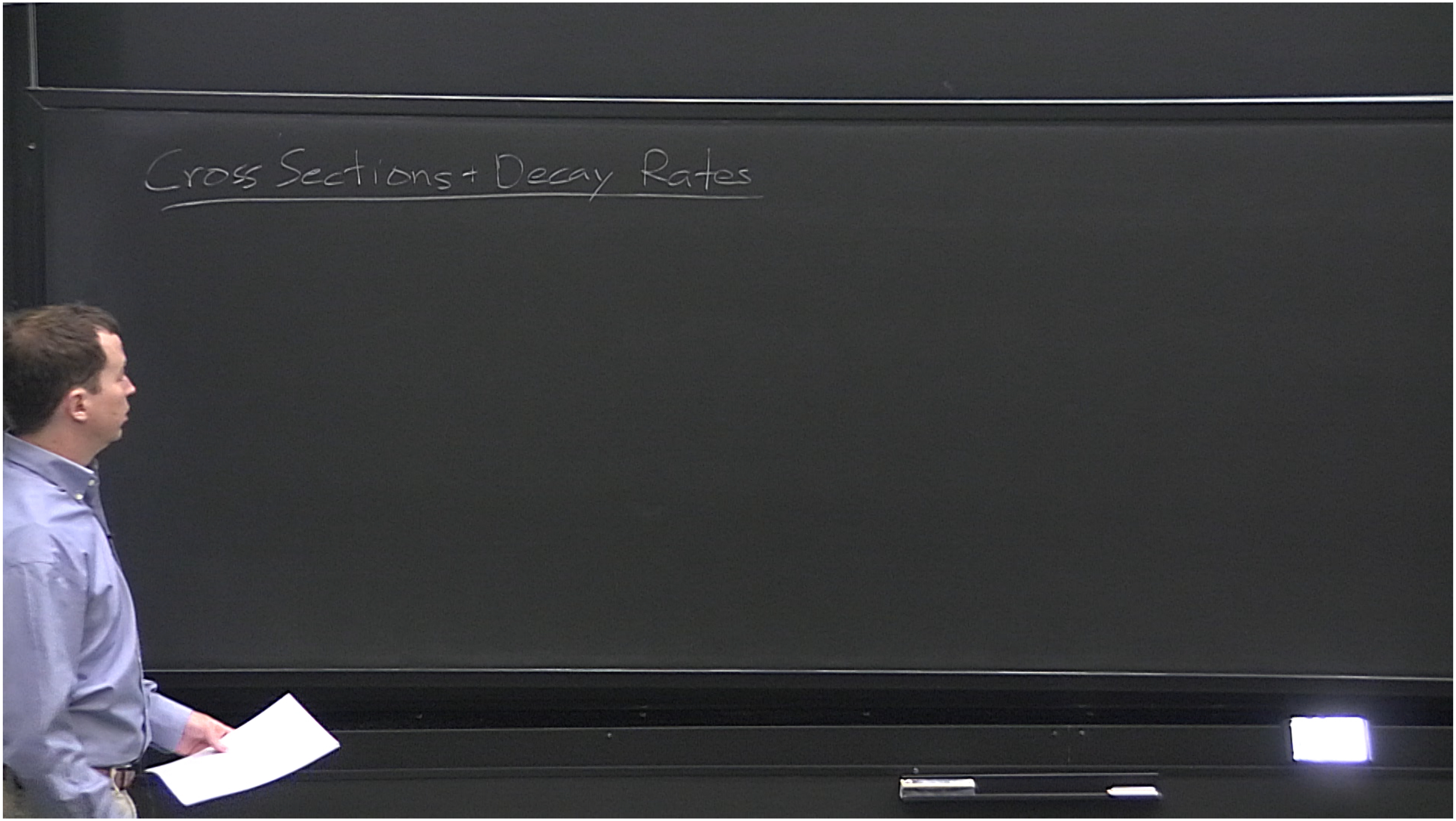
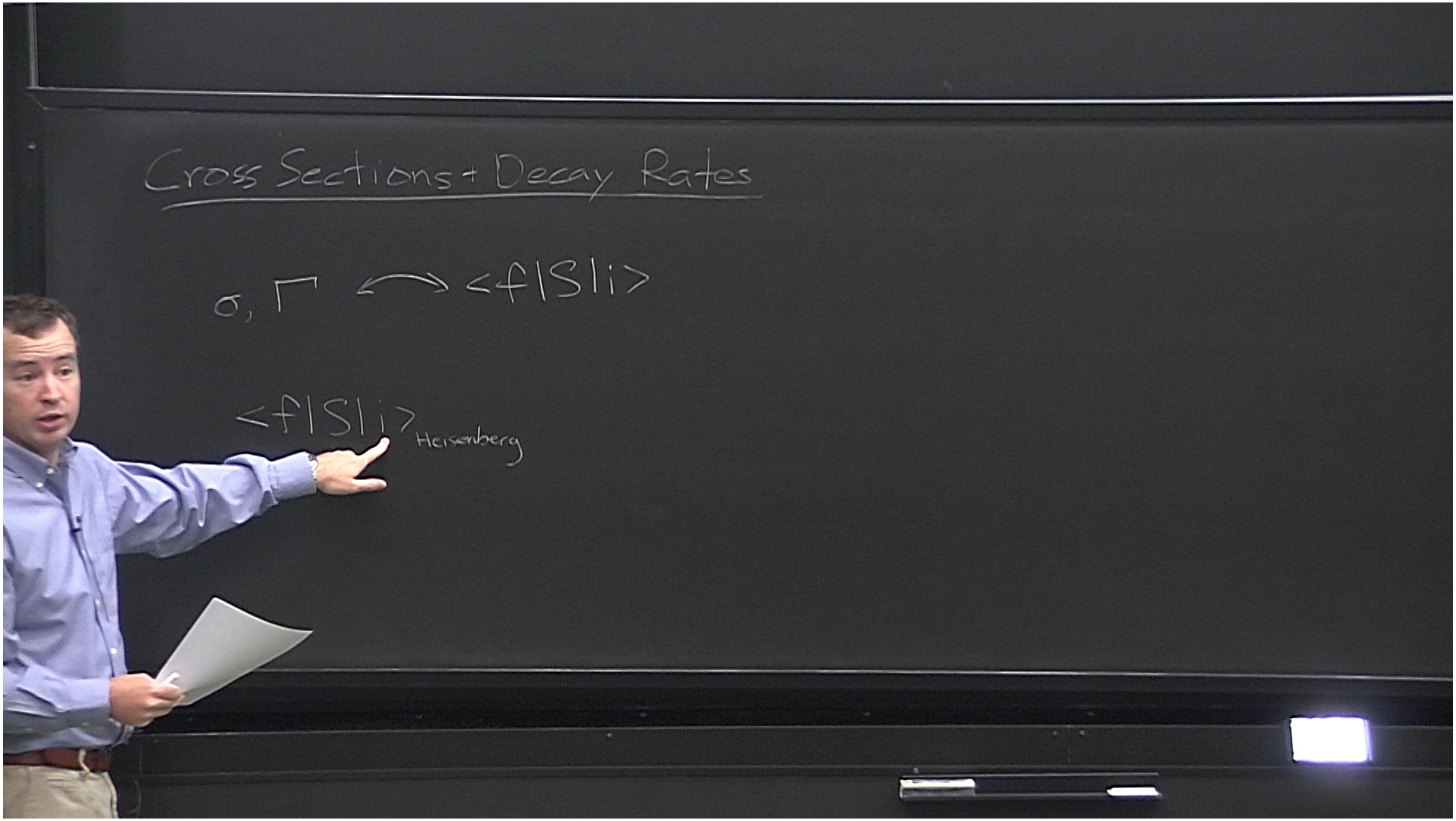
Title: PSI 2018/2019 - Quantum Field Theory I - Lecture 3

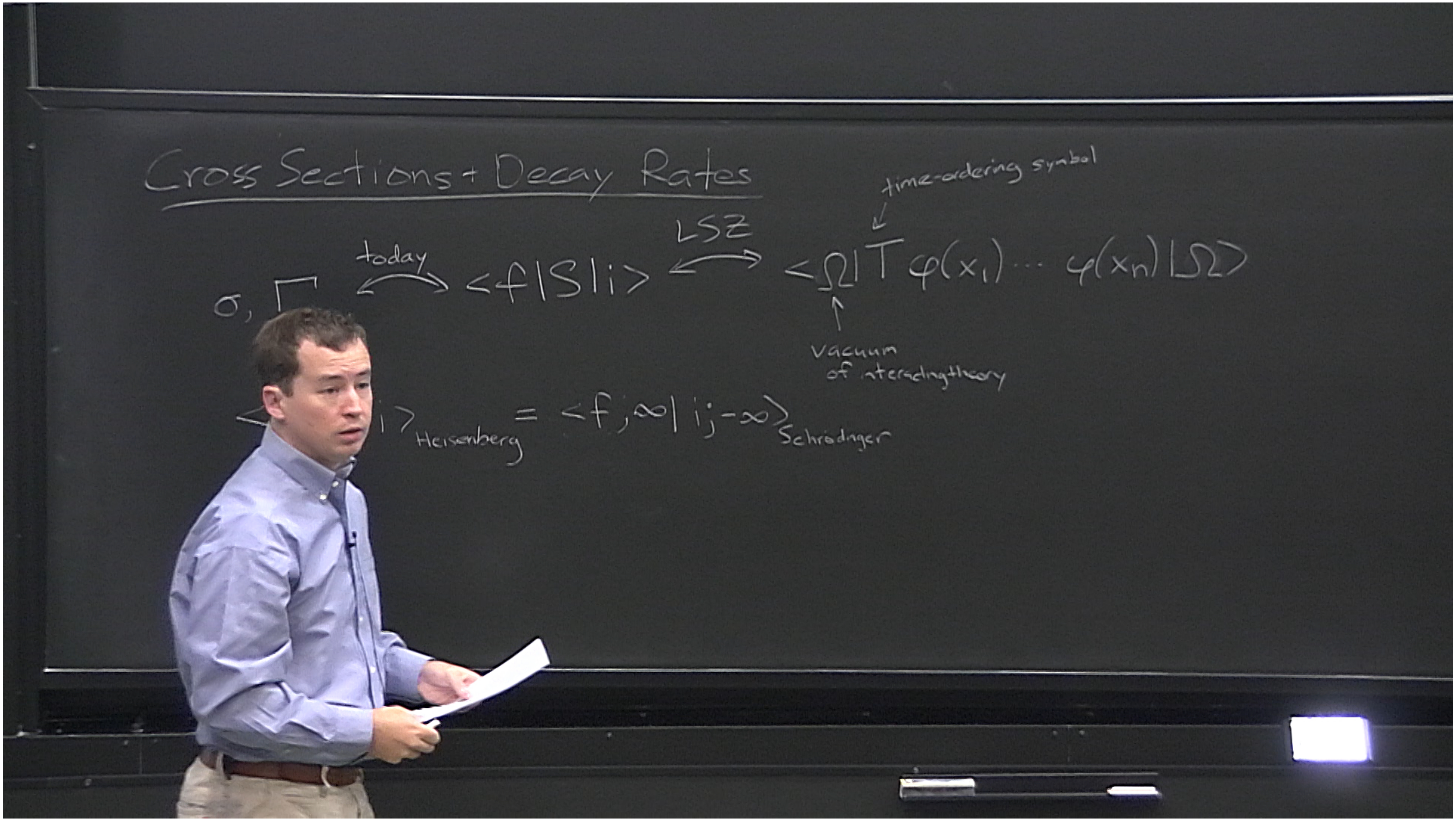
Date: Oct 11, 2018 09:00 AM

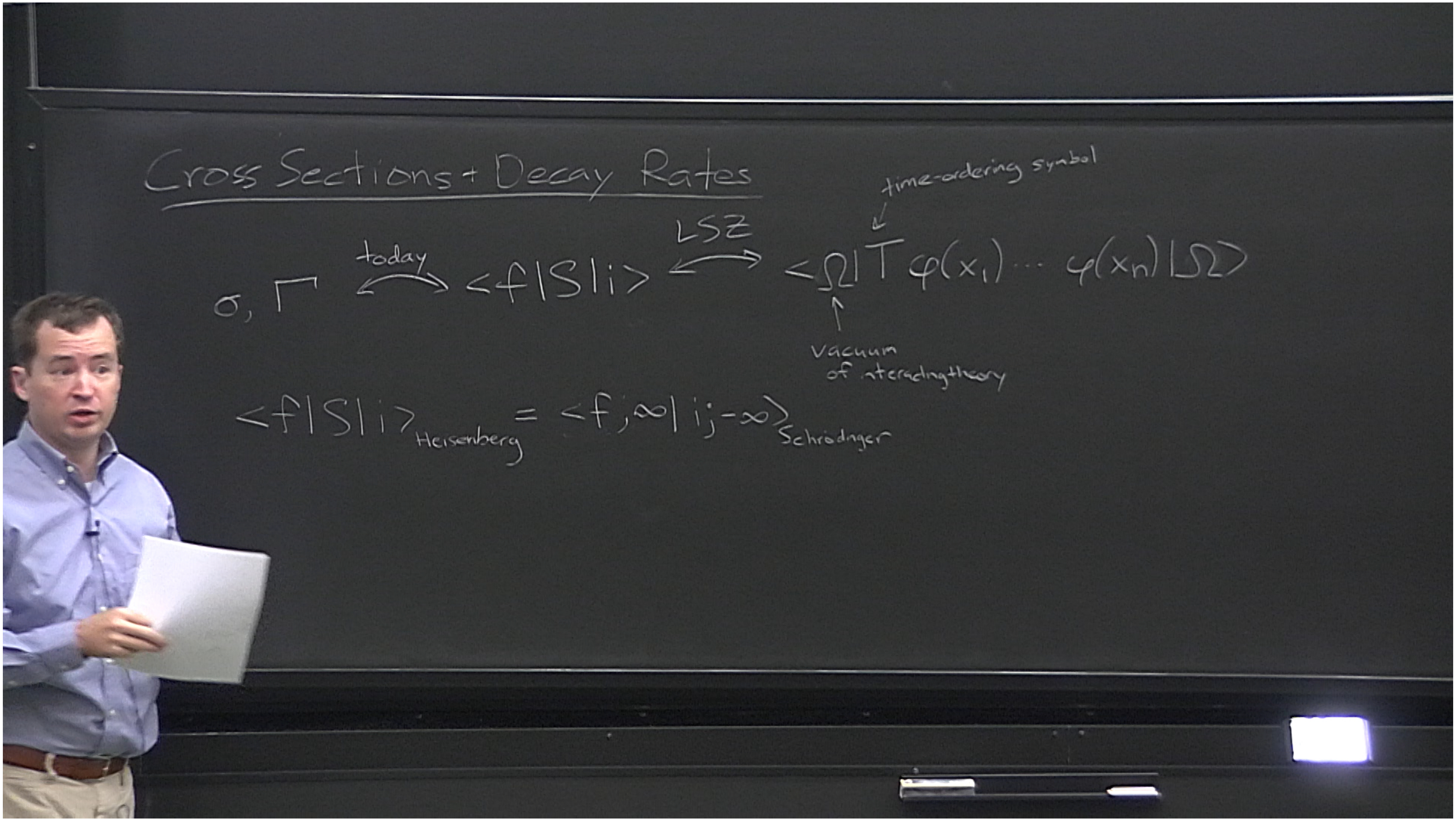
URL: http://pirsa.org/18100007

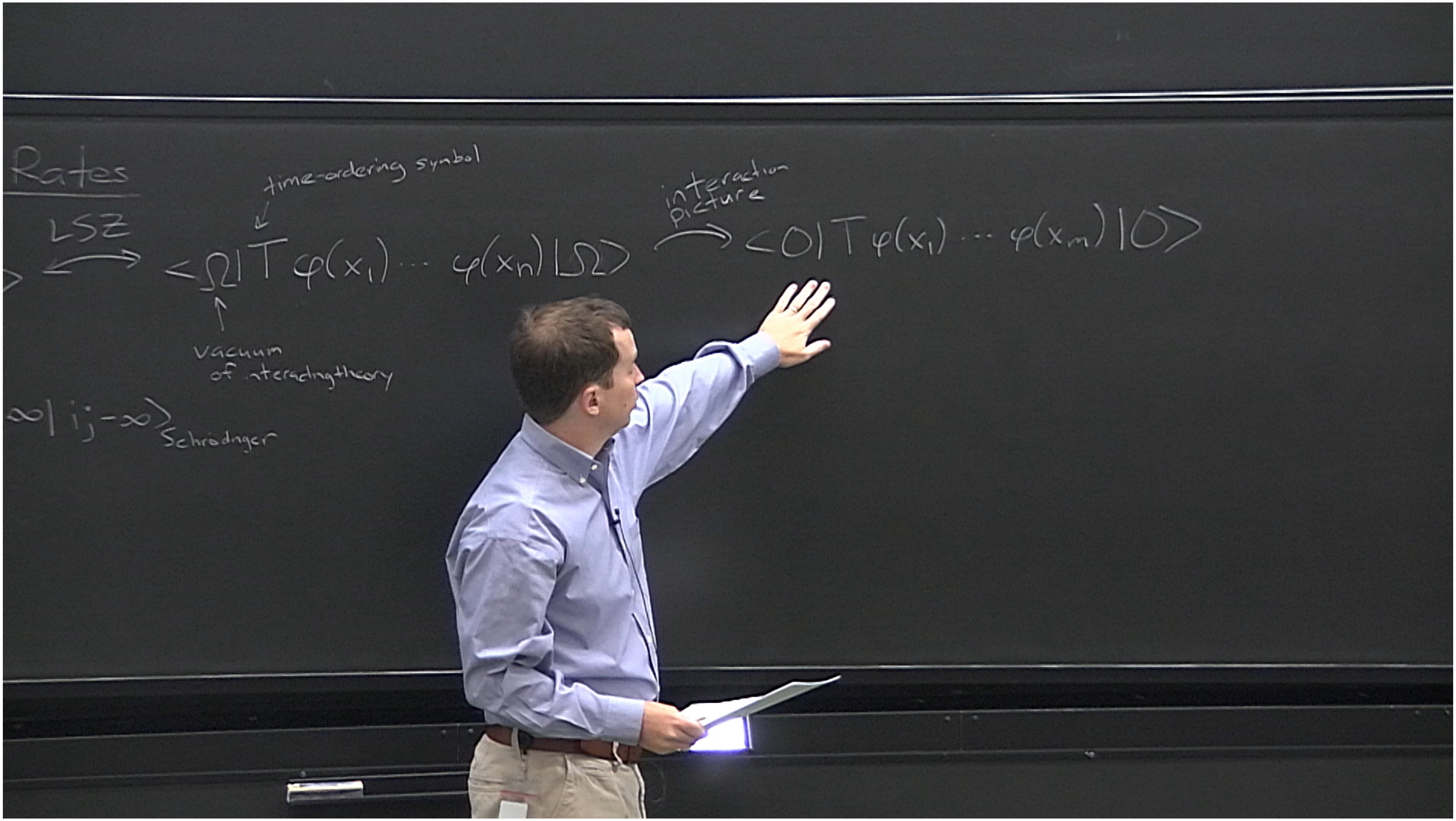
Abstract:

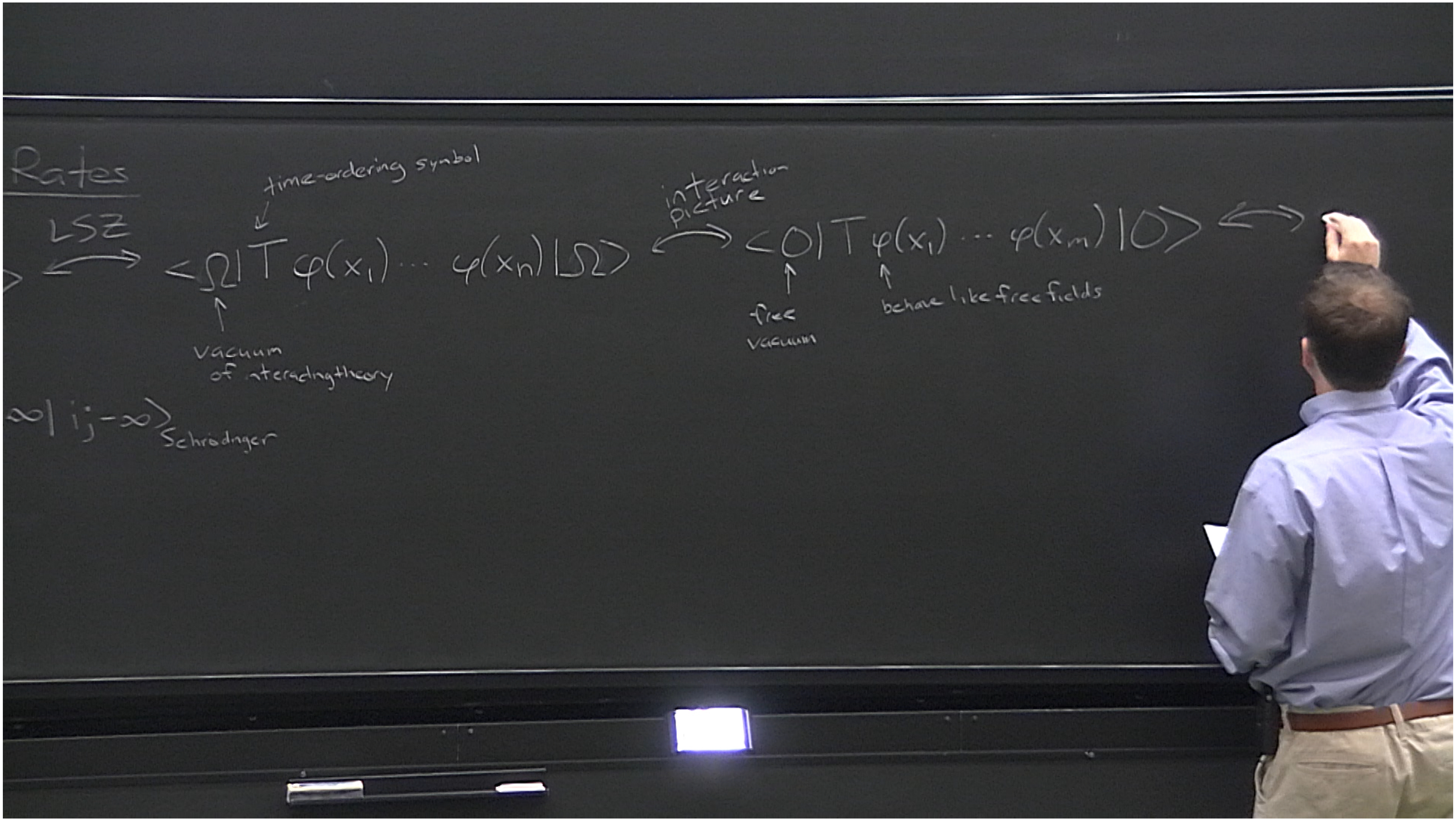

Cross Sections + Decay Rates

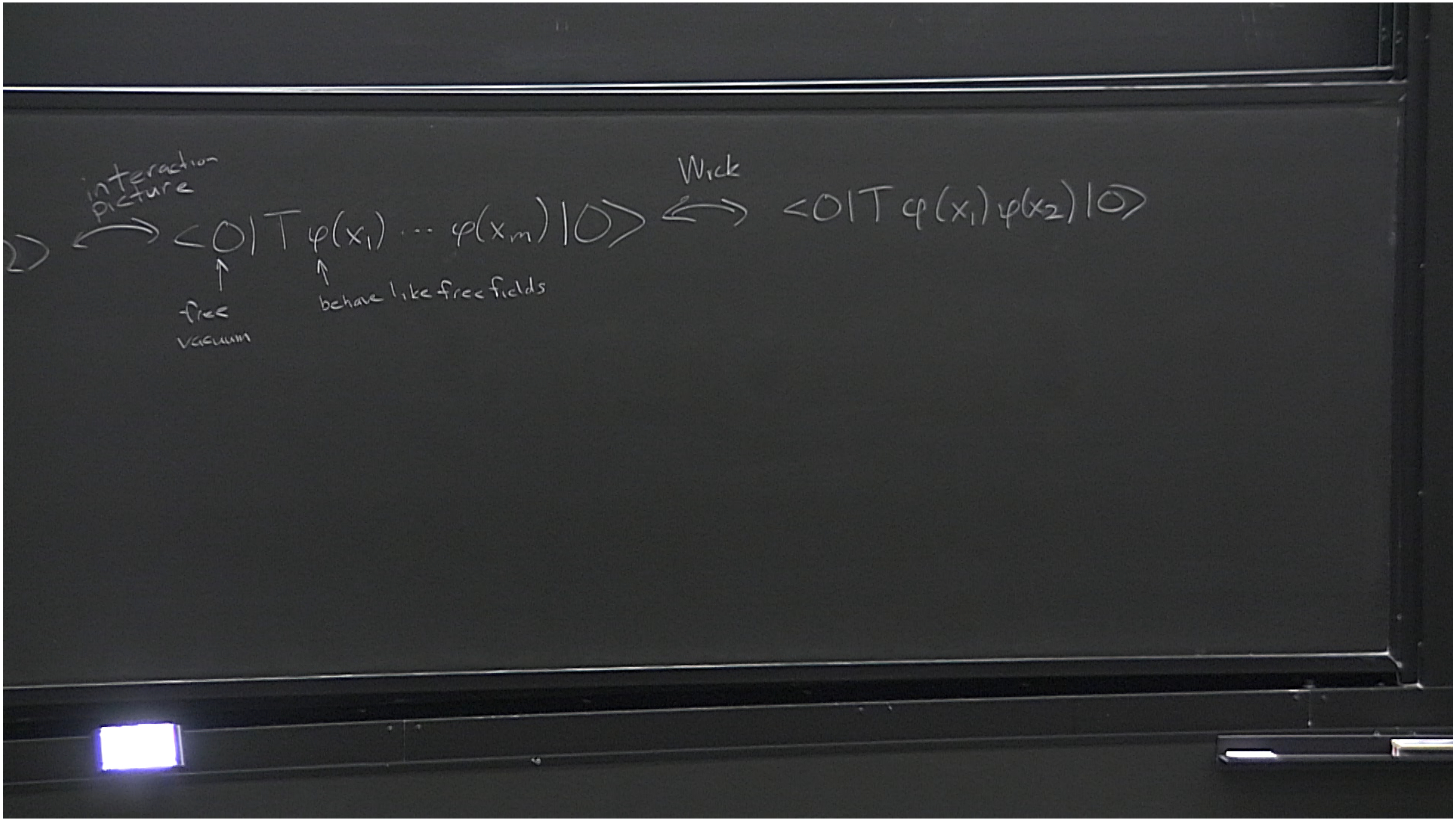

Cross Sections + Decay Rates
σ, Γ ⟷ <f|S|i>
<f|S|i> Heisenberg

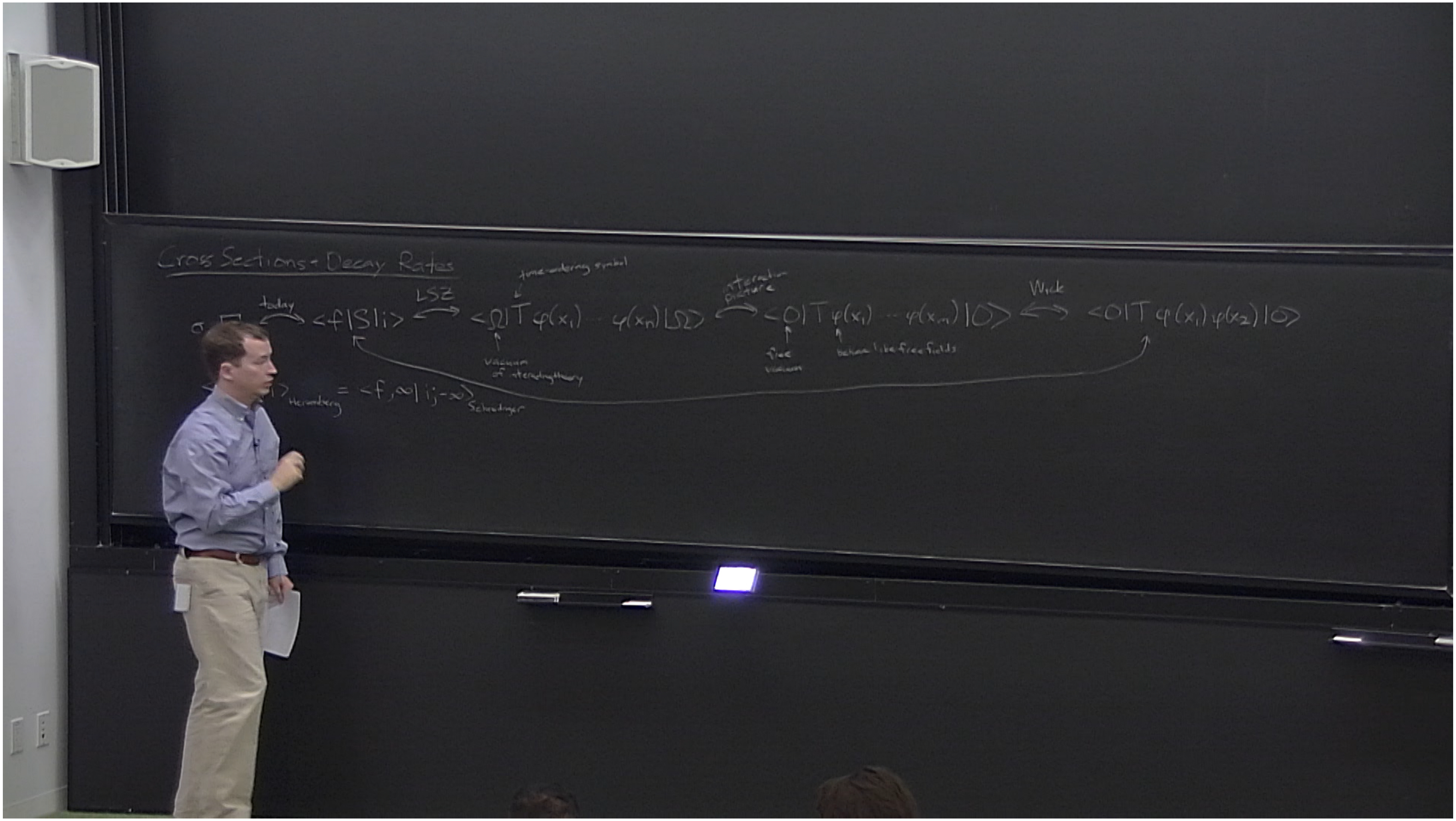

Cross Sections + Decay Rates
time-ordering symbol
LSZ
today
σ, Γ
<f|S|i>
<Ω|T φ(x₁) ··· φ(xn)|Ω>
vacuum
of interacting theory
Heisenberg
= <f;∞|i;-∞>
Schrödinger

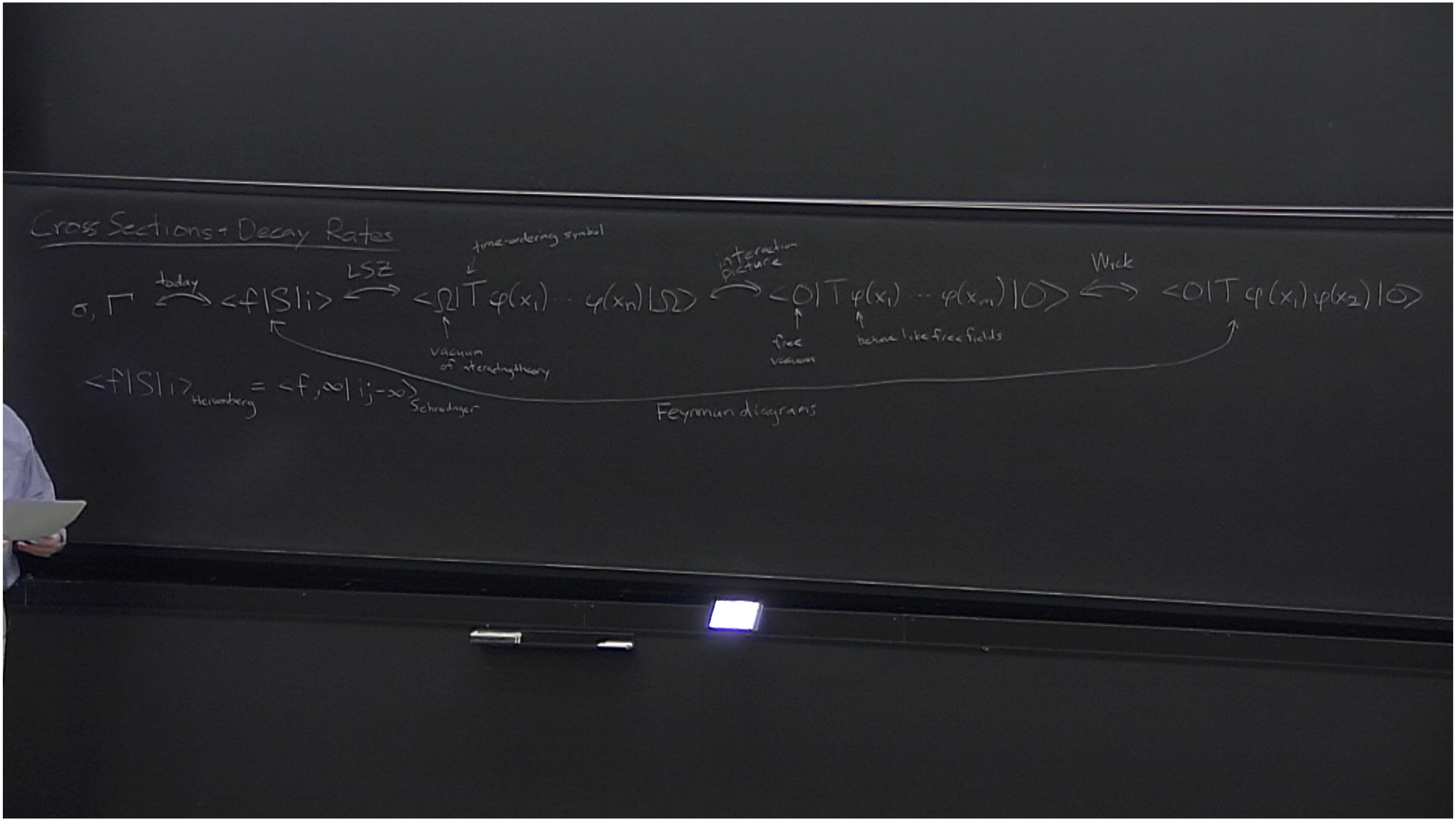

Cross Sections + Decay Rates
time-ordering symbol
LSZ
today
σ, Γ ⟷ <f|S|i> ⟷ <Ω|T φ(x₁) ··· φ(xₙ)|Ω>
vacuum
of interacting theory
<f|S|i>_Heisenberg = <f;∞|i;-∞>_Schrödinger

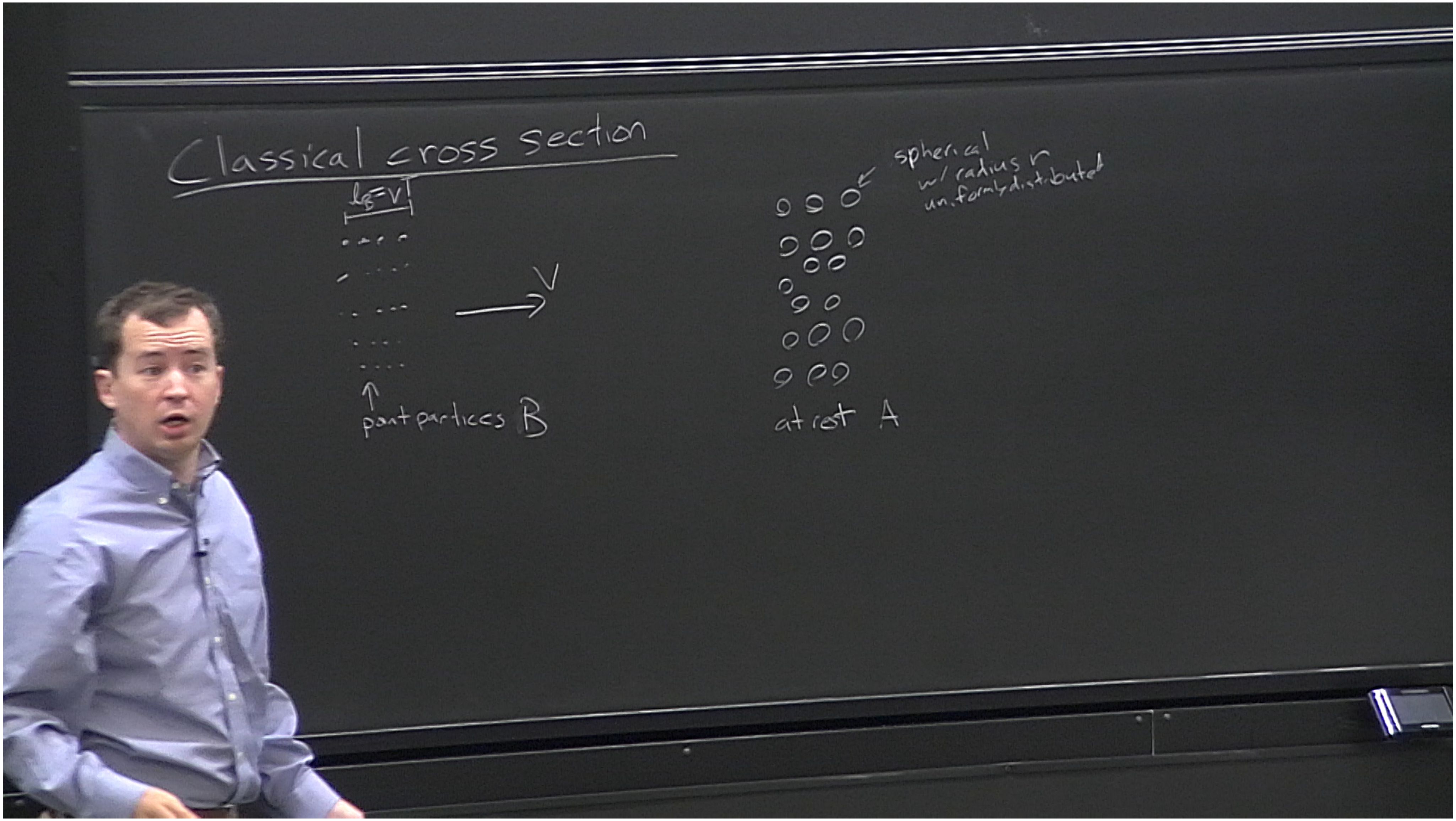

Rates
time-ordering symbol
LSZ
interaction picture
$\langle \Omega | T \varphi(x_1) \cdots \varphi(x_n) | \Omega \rangle$
$\langle 0 | T \varphi(x_1) \cdots \varphi(x_m) | 0 \rangle$
vacuum of interacting theory
Schrödinger

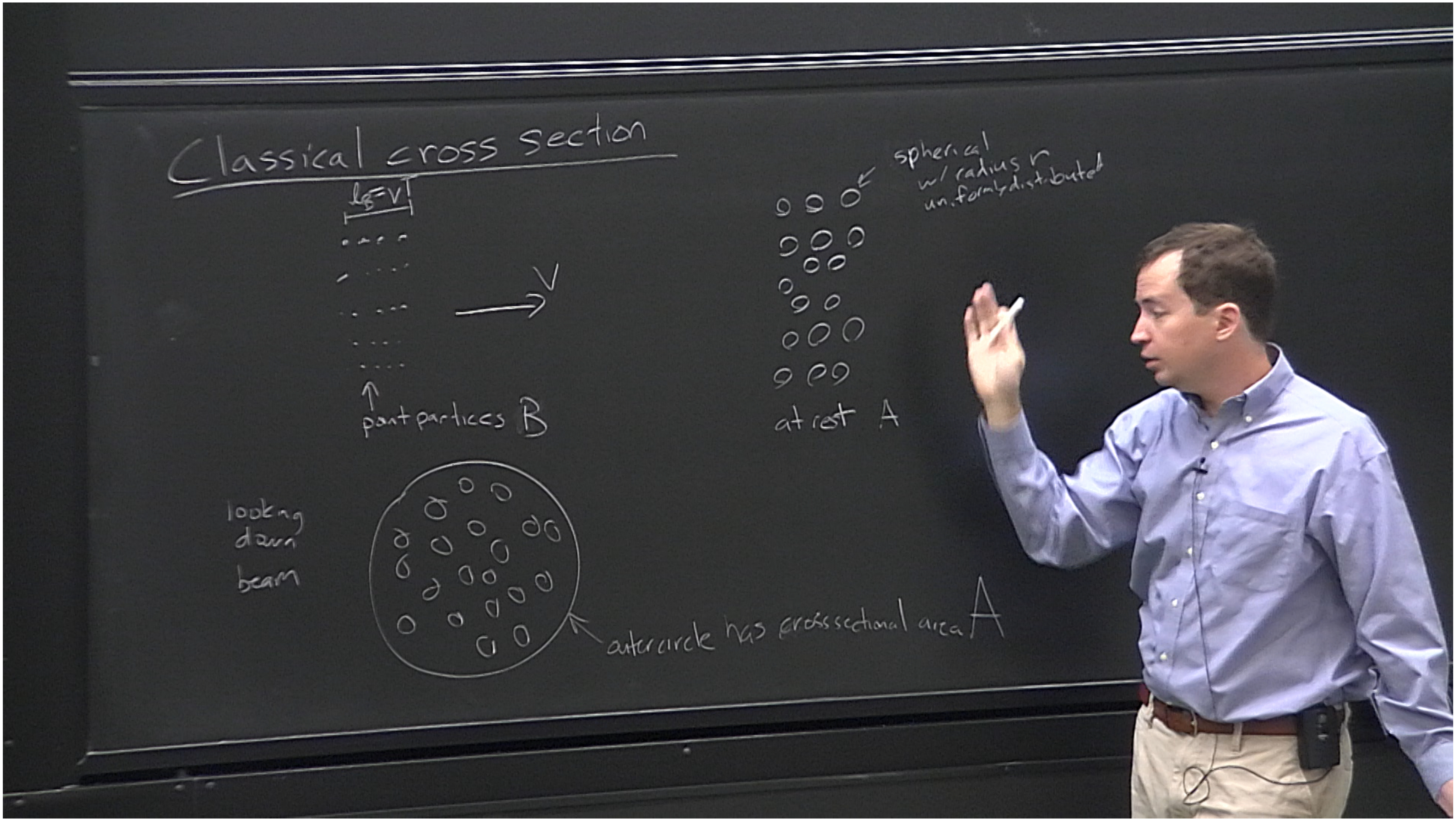

Rates
LSZ
time-ordering symbol
interaction picture
<Ω|T φ(x1) ... φ(xn)|Ω>
<0|T φ(x1) ... φ(xm)|0>
vacuum
of nteractingtheory
free
vacuum
behave like free fields
Schrödinger


interaction picture
<0|T φ(x1) ··· φ(xm)|0>
free vacuum
behave like free fields
Wick
<0|T φ(x1) φ(x2)|0>

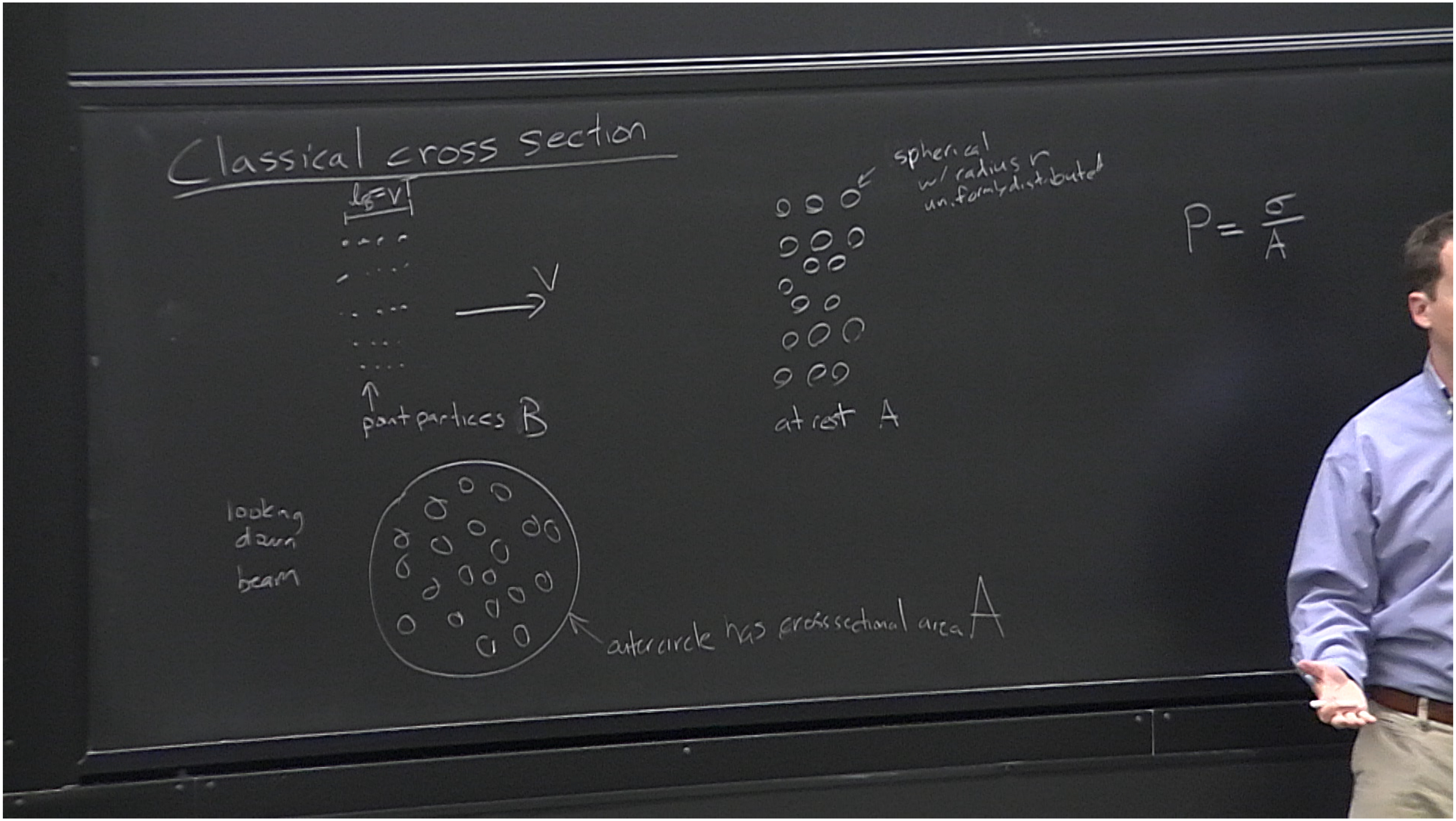

Cross Sections + Decay Rates
today
LSZ
time-ordering symbol
interaction picture
Wick
<f|S|i>
<Ω|T φ(x1) ... φ(xn)|Ω>
<0|T φ(x1) ... φ(xn)|0>
<0|T φ(x1) φ(x2)|0>
vacuum of interacting theory
free vacuum
behave like free fields
= <f,∞|i,-∞>
Heisenberg
Schrodinger

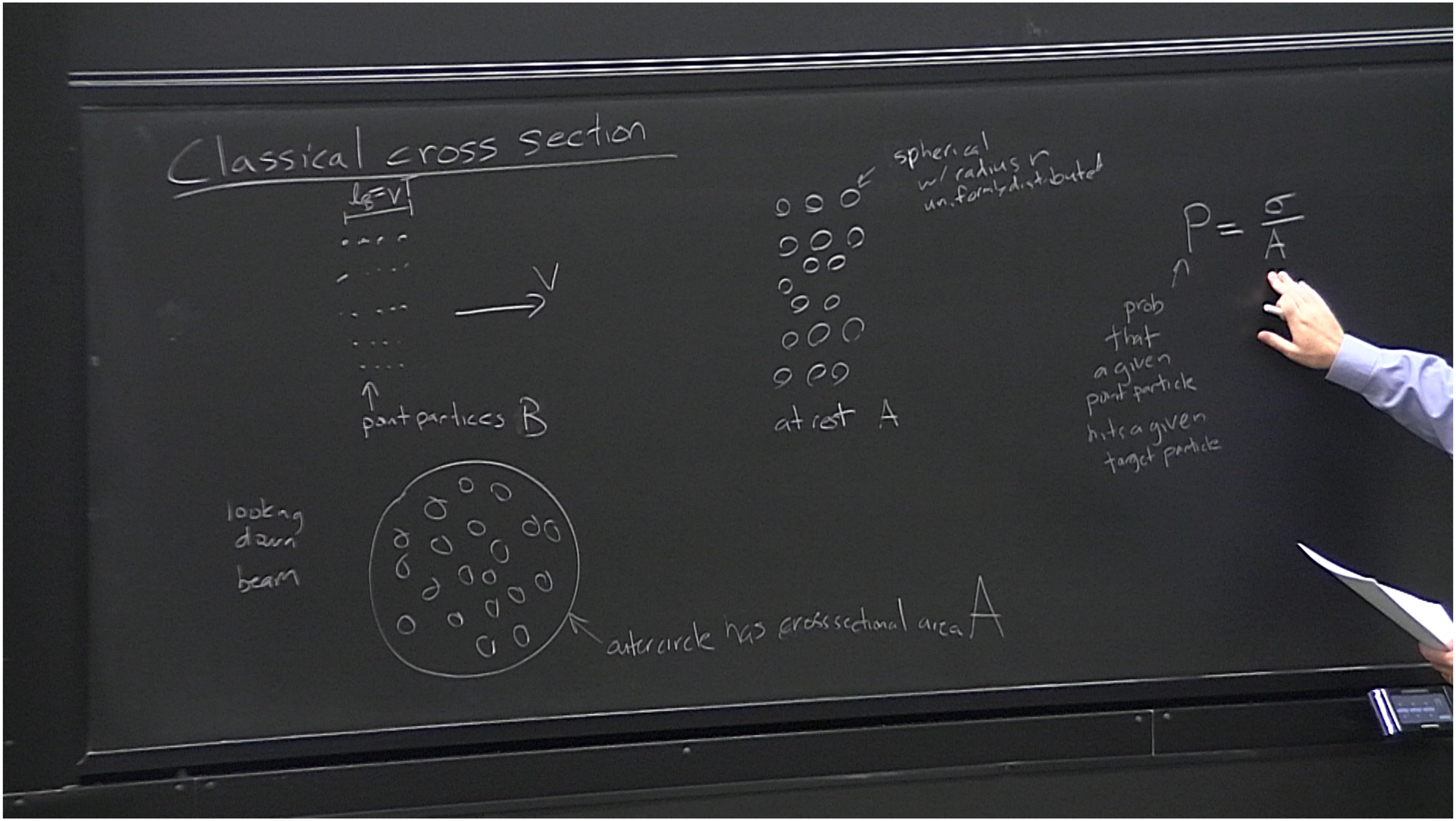

Cross Sections + Decay Rates
time-ordering symbol
today
LSZ
interaction picture
Wick
σ, Γ ⟷ <f|S|i> ⟷ <Ω|T φ(x1) ··· φ(xn)|Ω> ⟷ <0|T φ(x1) ··· φ(xn)|0> ⟷ <0|T φ(x1) φ(x2)|0>
vacuum of interacting theory
free vacuum
behave like free fields
<f|S|i>Heisenberg = <f,∞|i,-∞>Schrodinger
Feynman diagrams

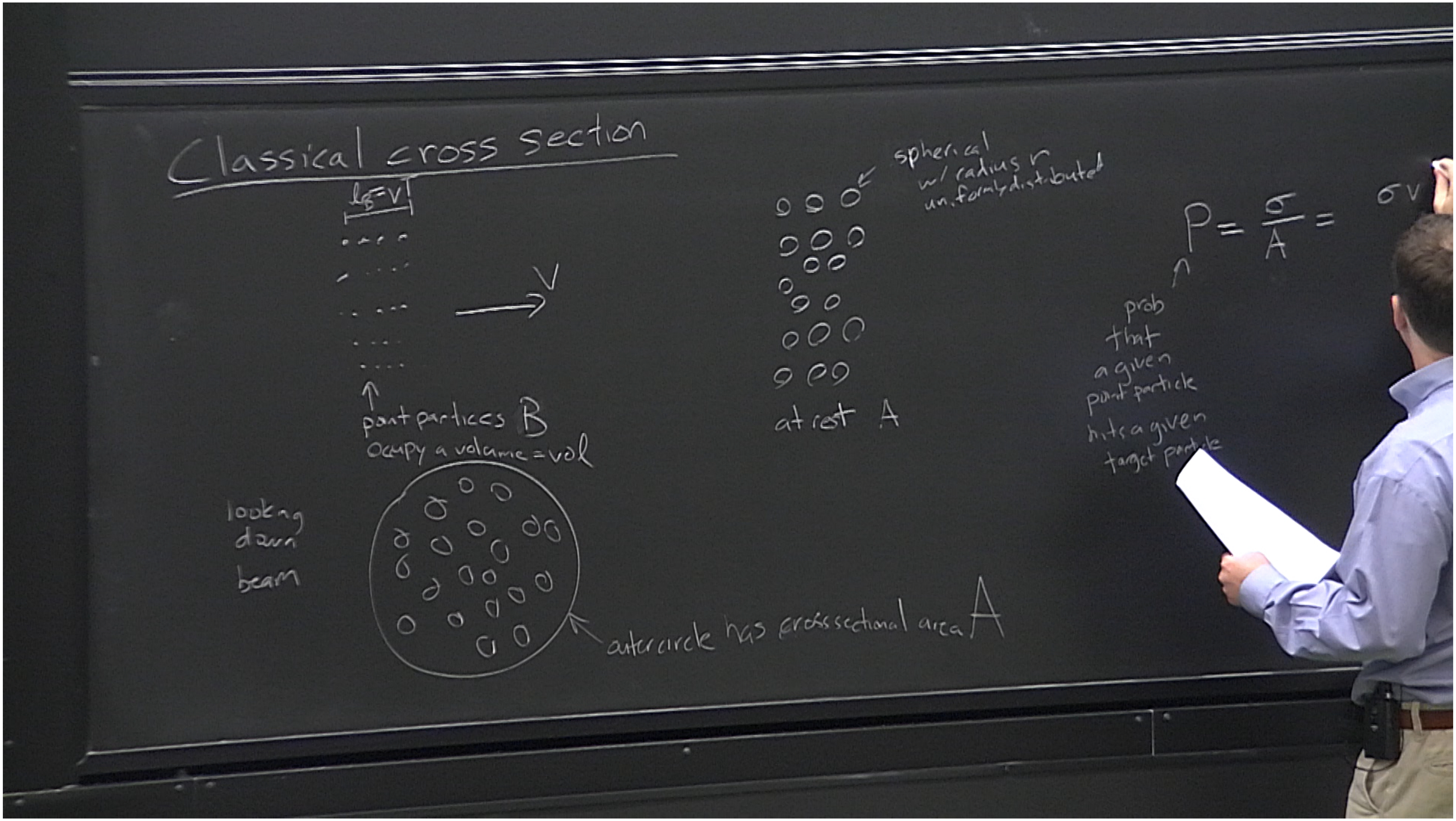

Classical cross section
spherical
w/ radius r
uniformly distributed
V
point particles B
at rest A



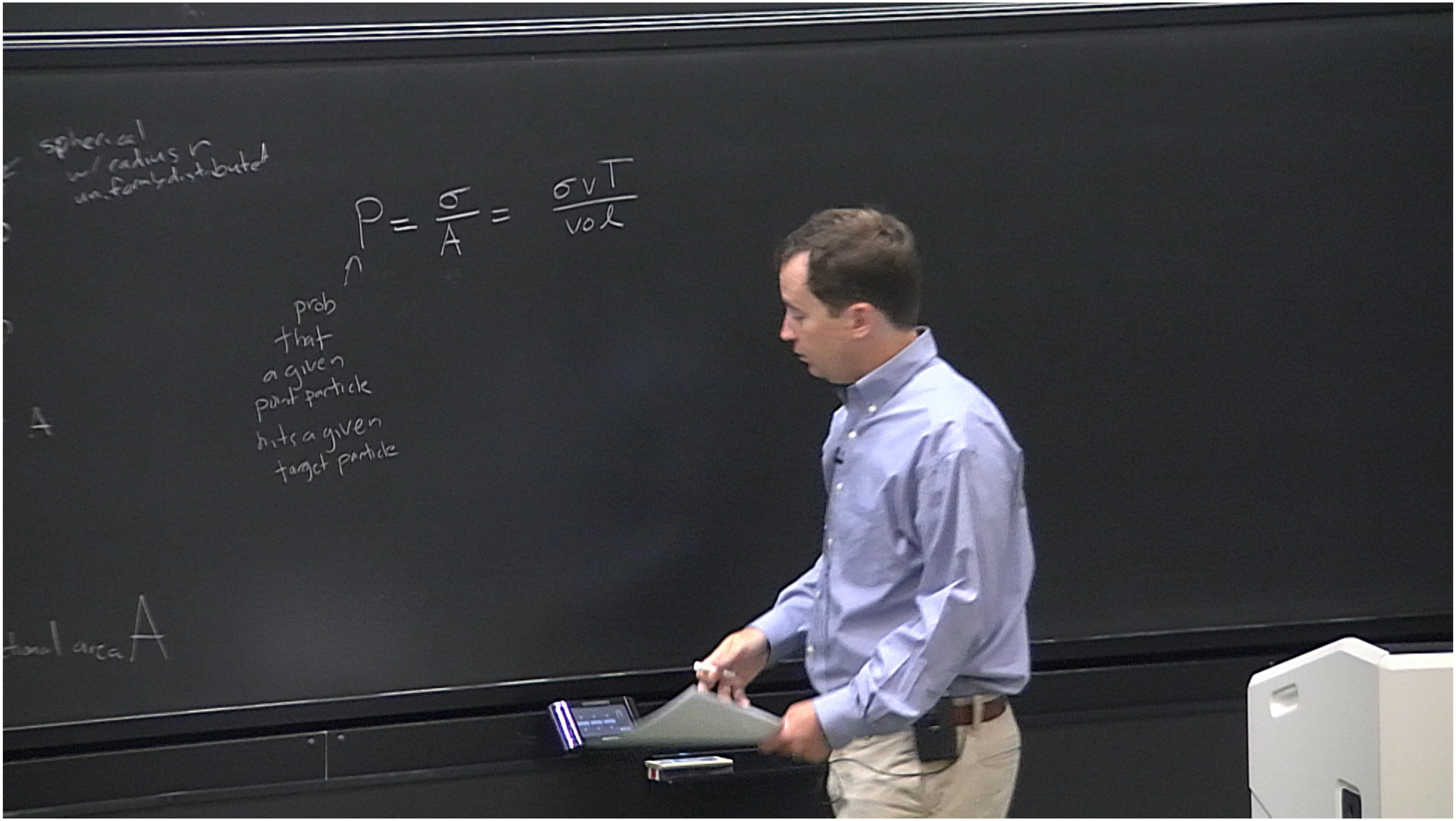
Classical cross section
$l_B = v$
v
point particles B
spherical
w/ radius r
uniformly distributed
at rest A
looking
down
beam
outer circle has cross sectional area A

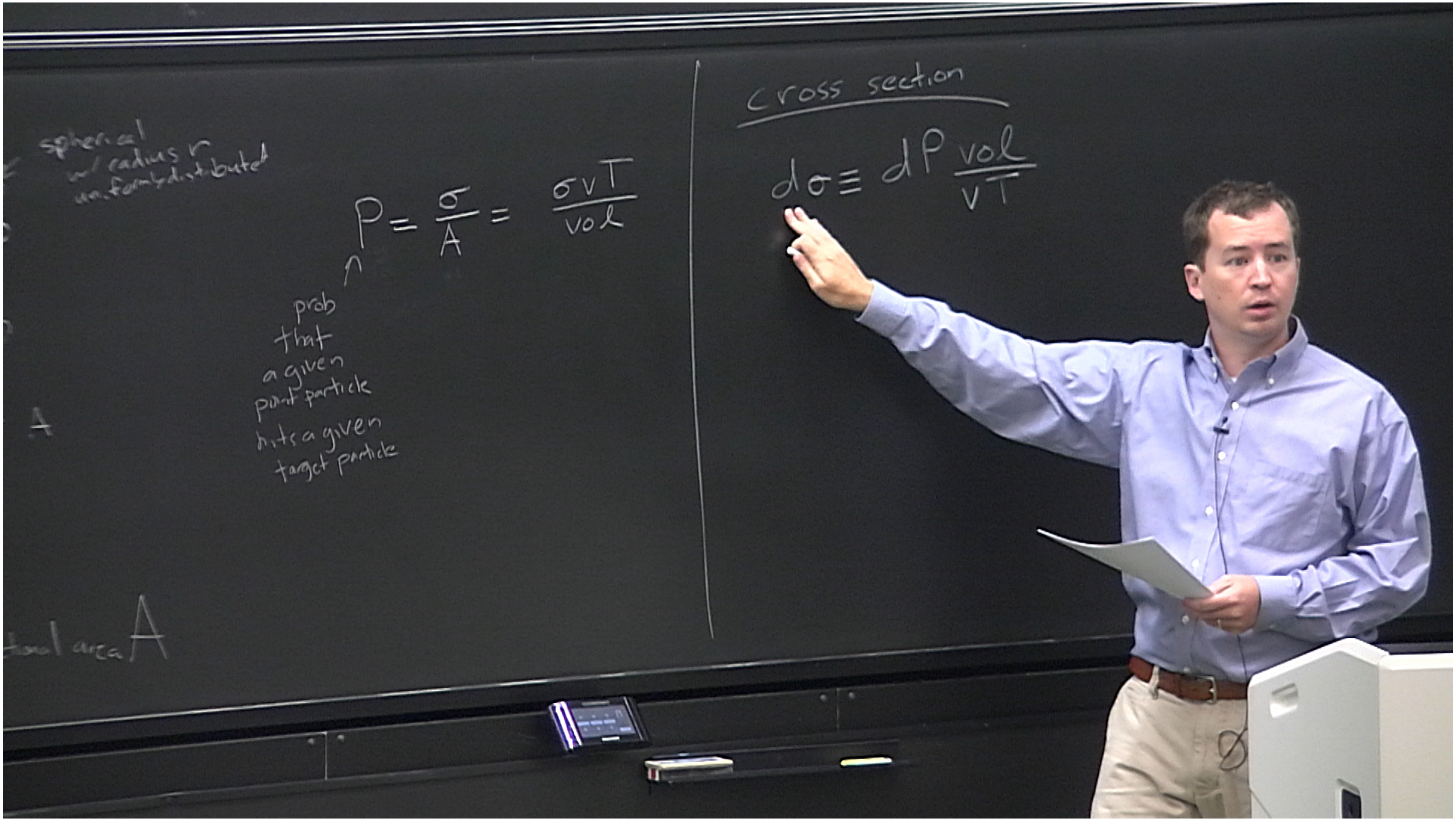

Classical cross section
V
point particles B
spherical
w/ radius r
uniformly distributed
at rest A
P = σ/A
looking down beam
outer circle has cross sectional area A


Classical cross section
point particles B
at rest A
spherical w/ radius r uniformly distributed
P = σ / A
prob that a given point particle hits a given target particle
looking down beam
outer circle has cross sectional area A

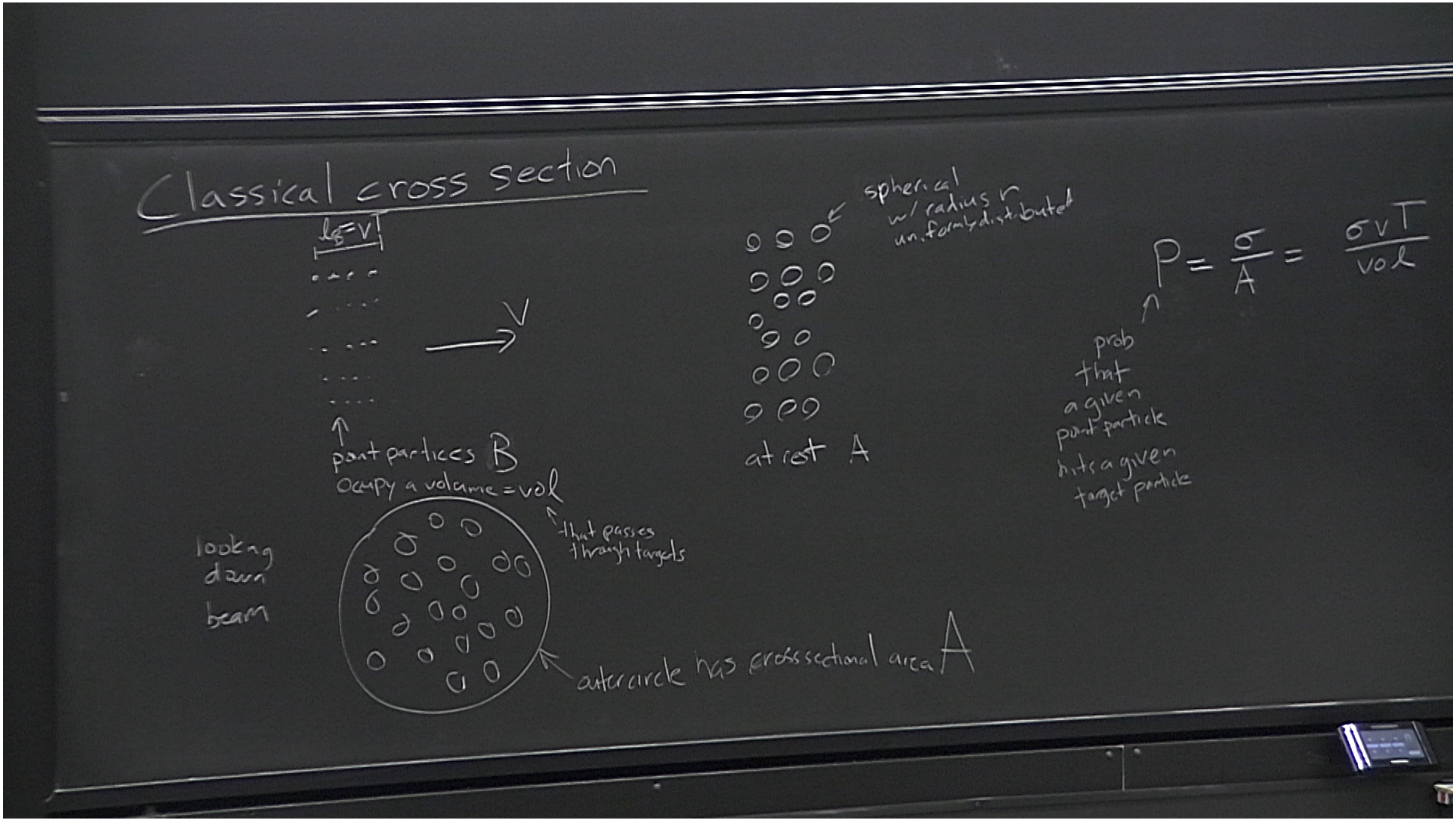


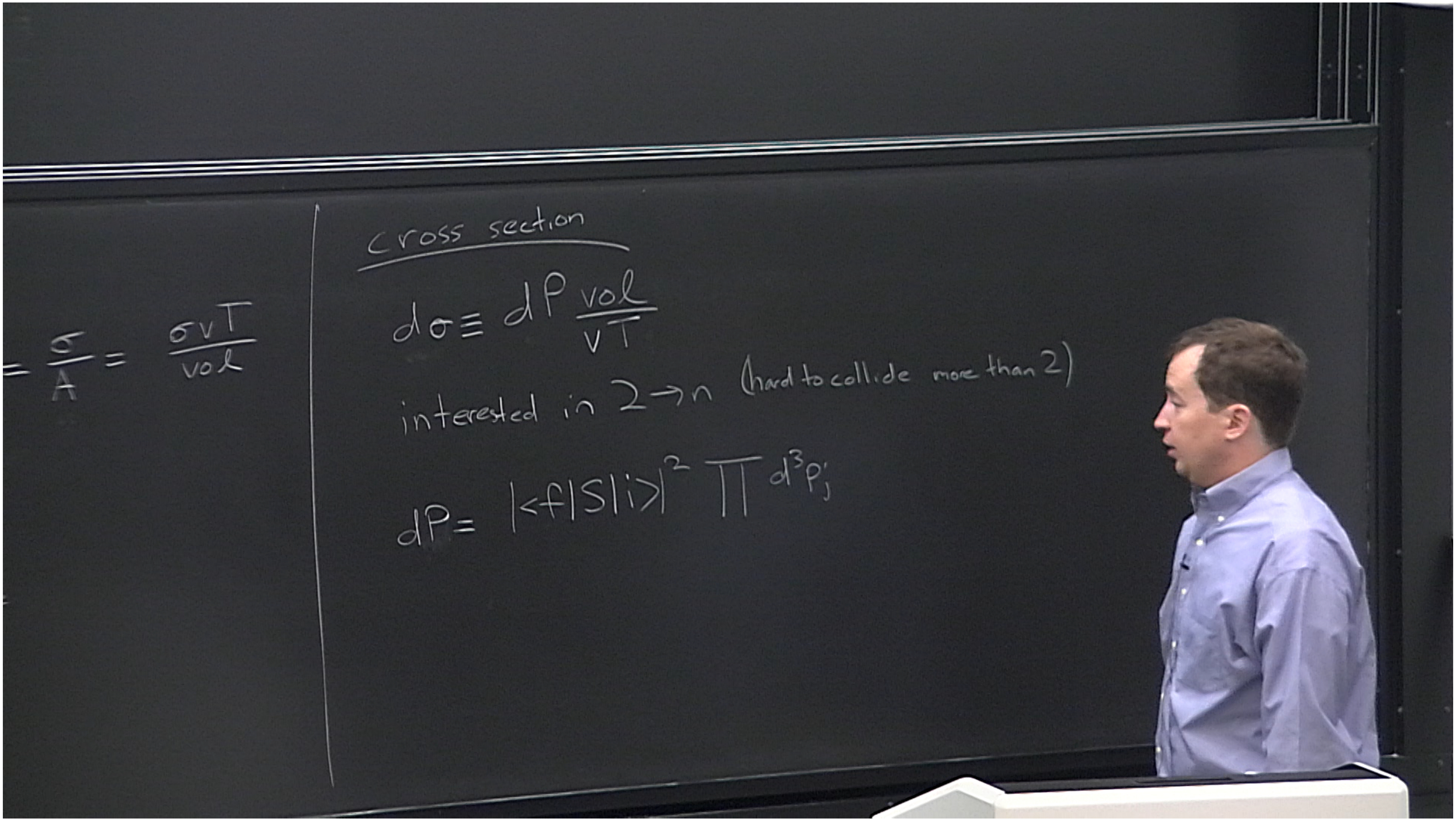
Classical cross section
$l_B = v$
v
point particles B
occupy a volume = vol
looking down beam
outer circle has cross sectional area A
spherical
w/ radius r
uniformly distributed
at rest A
$P = \frac{\sigma}{A} = \sigma v$
prob that a given point particle hits a given target particle


spherical
w/ radius r
uniformly distributed
$P = \frac{\sigma}{A} = \frac{\sigma v T}{vol}$
prob
that
a given
point particle
hits a given
target particle
area A

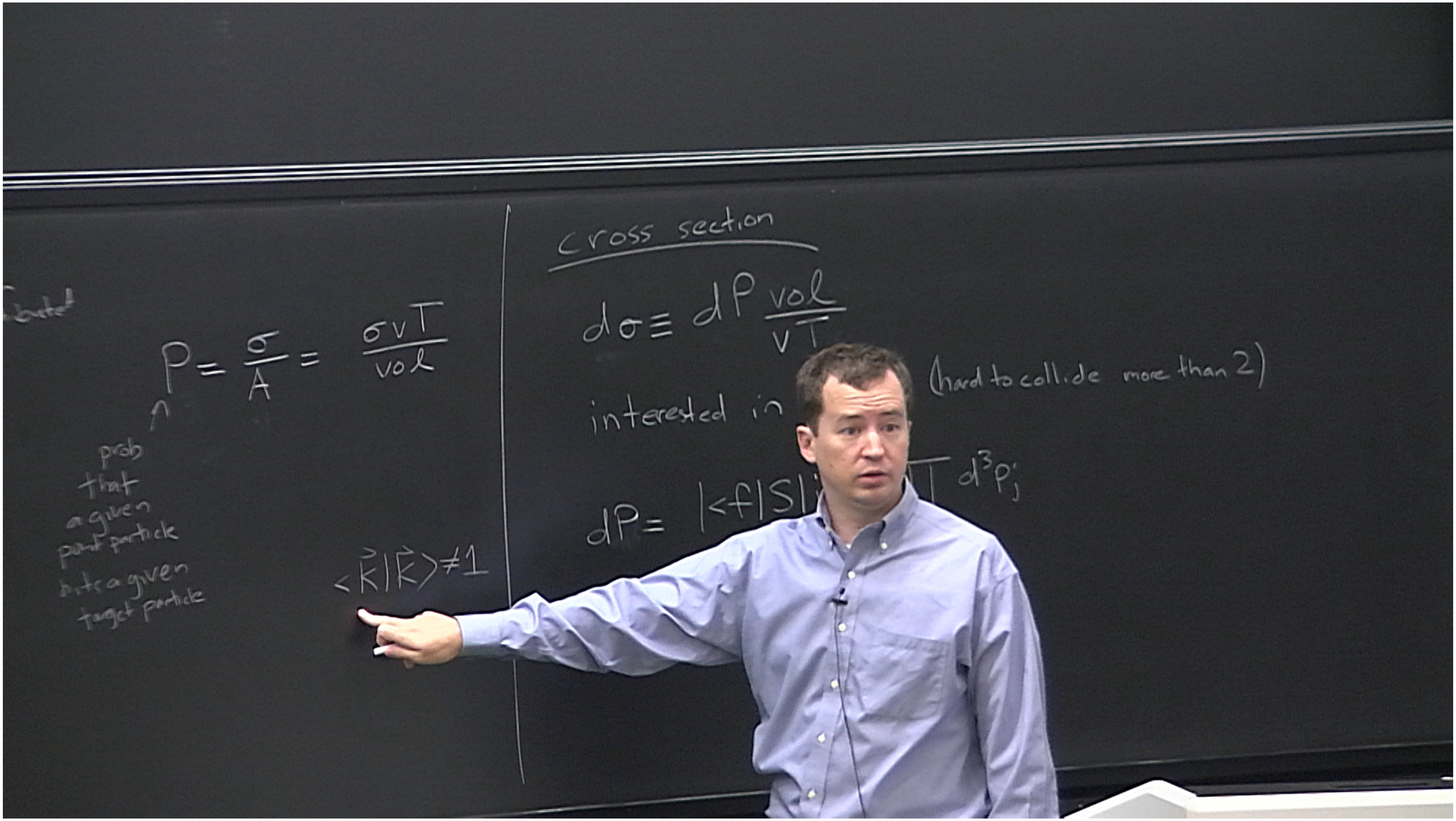


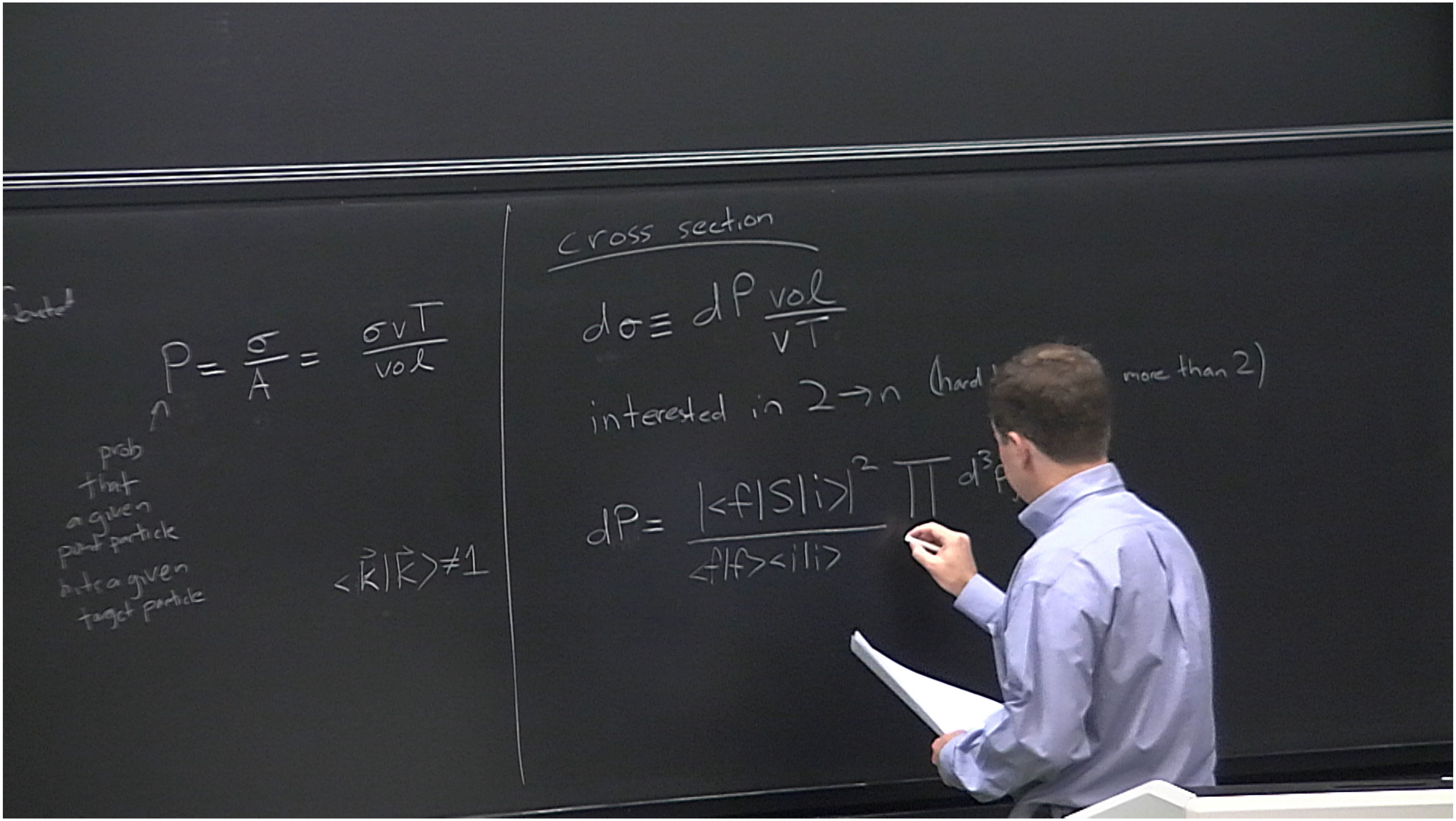
cross section
$d\sigma \equiv dP \frac{vol}{vT}$
$P = \frac{\sigma}{A} = \frac{\sigma v T}{vol}$
prob that a given point particle hits a given target particle
spherical w/ radius r uniformly distributed

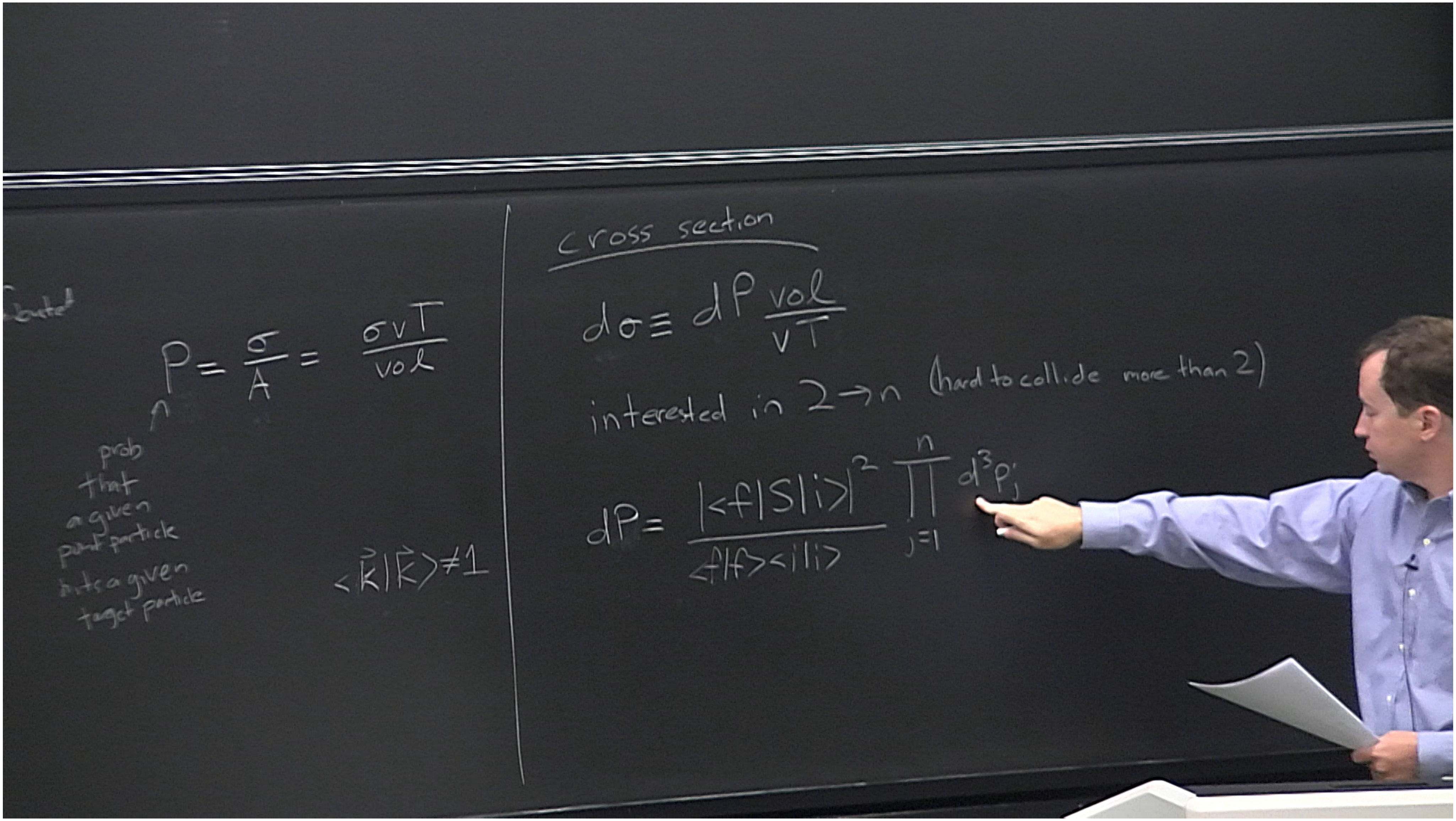

Classical cross section
$l_B = vT$
V
point particles B
occupy a volume = vol
that passes through targets
looking down beam
outer circle has cross sectional area A
spherical w/ radius r uniformly distributed
at rest A
$P = \frac{\sigma}{A} = \frac{\sigma v T}{vol}$
prob that a given point particle hits a given target particle

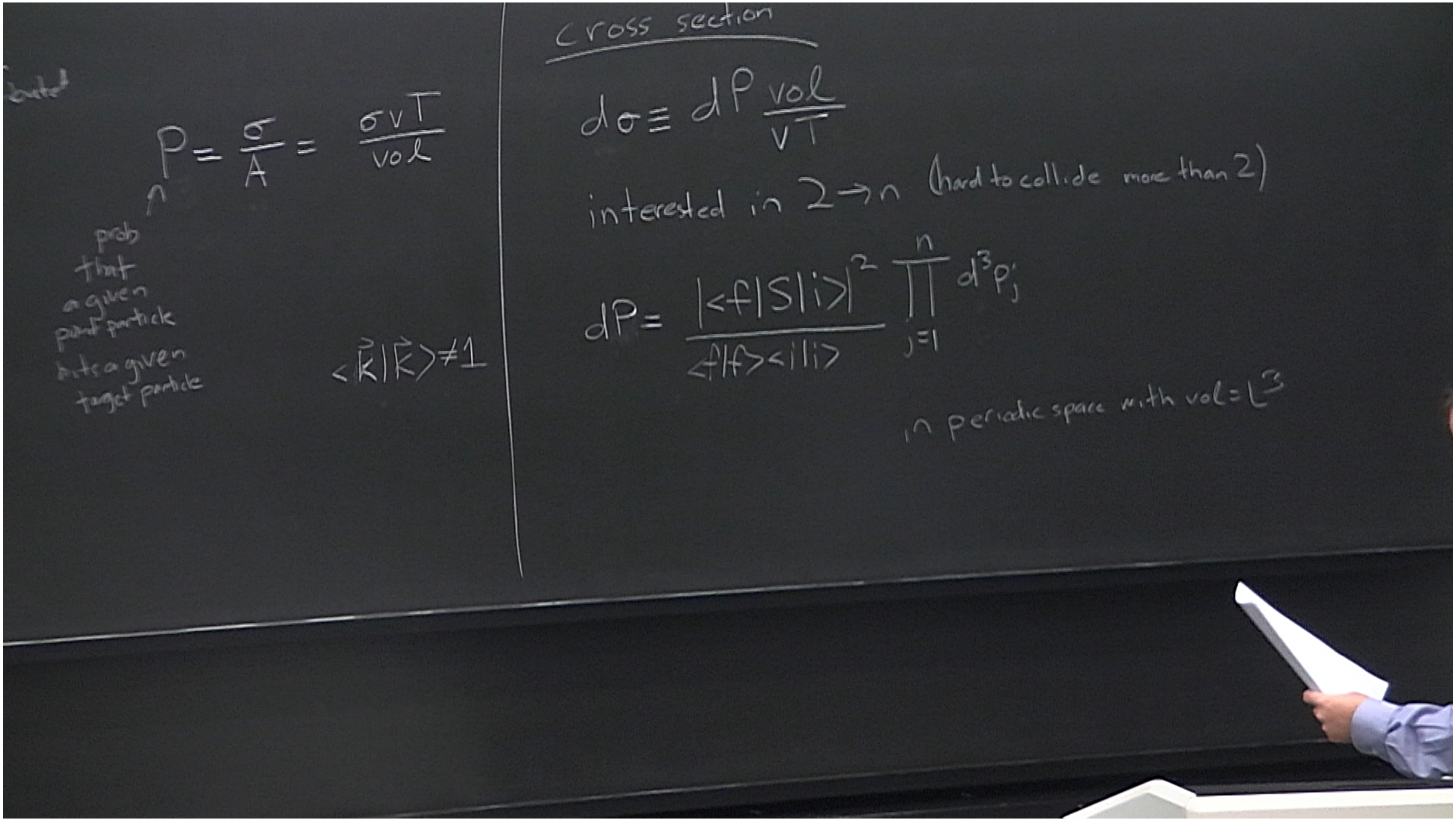

= σ/A = σvT/vol
cross section
dσ ≡ dP vol/vT
interested in 2→n (hard to collide more than 2)
dP = |<f|S|i>|² ∏ d³p_j

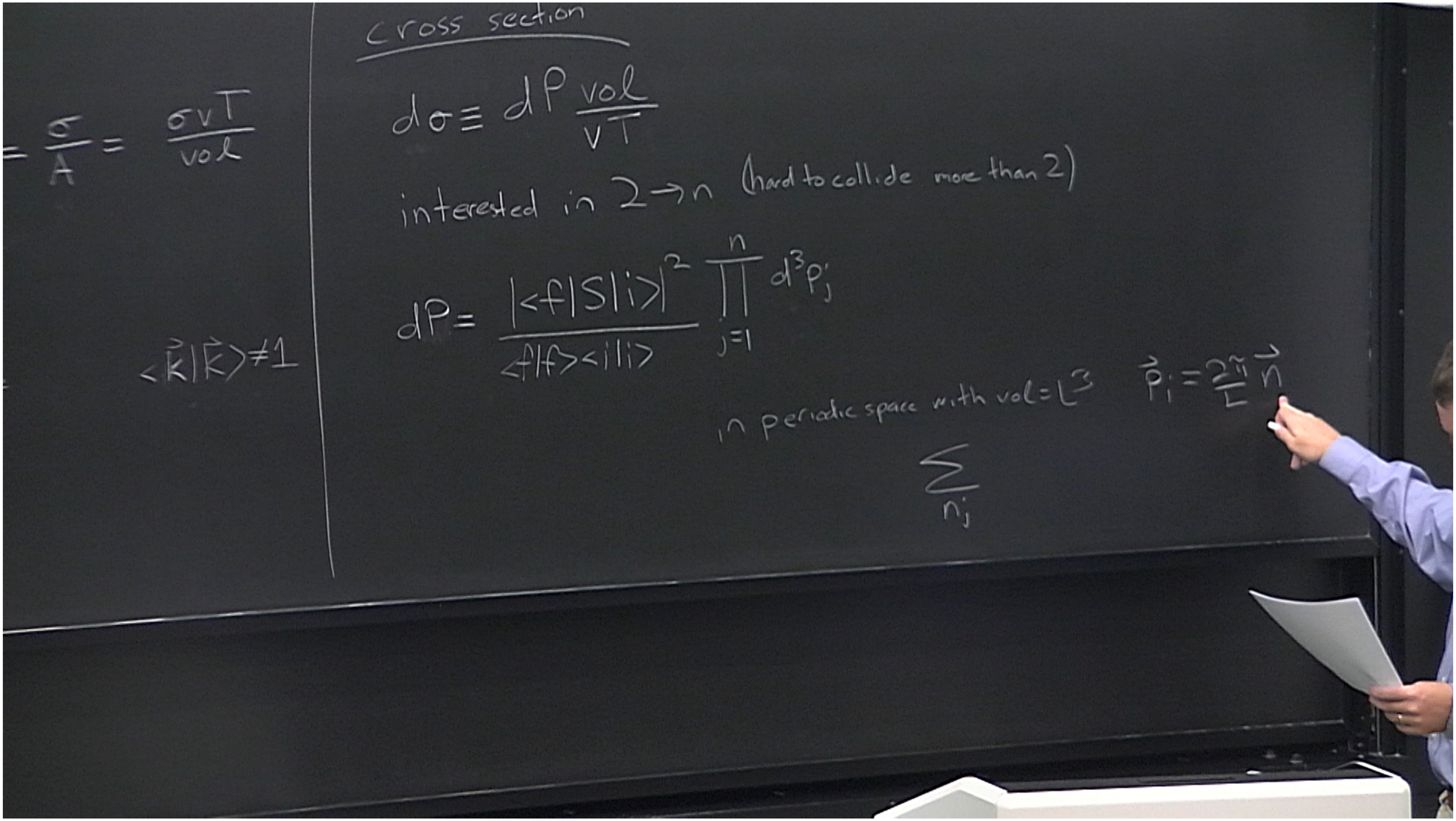
cross section
$d\sigma \equiv dP \frac{vol}{vT}$
interested in 2
hard to collide more than 2)
$dP = |\langle f|S|i\rangle|$
d^3p_j
$P = \frac{\sigma}{A} = \frac{\sigma v T}{vol}$
prob that a given point particle hits a given target particle
$\langle \vec{k}|\vec{k}\rangle$

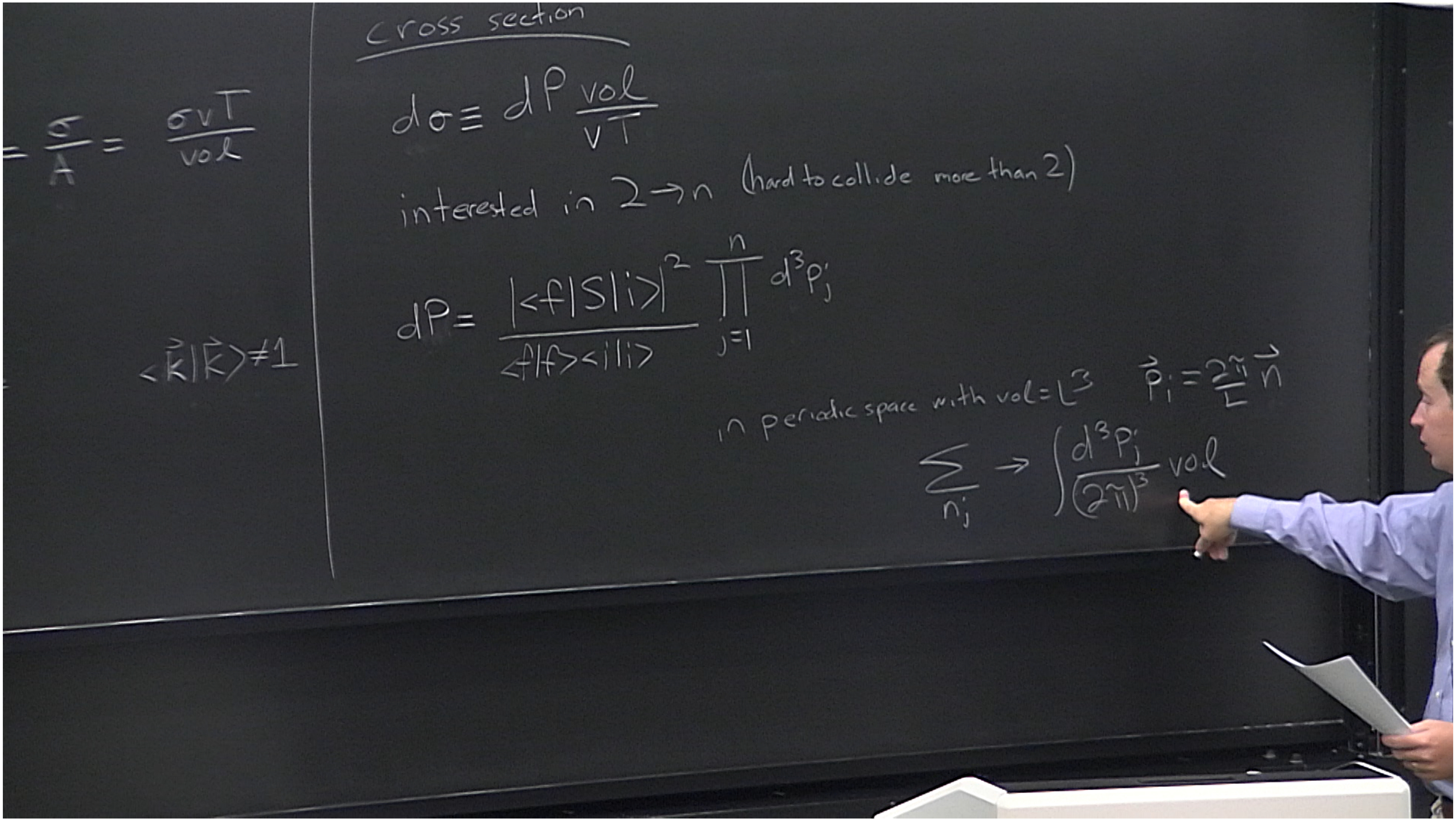

cross section
$d\sigma \equiv dP \frac{vol}{vT}$
(hard to collide more than 2)
interested in
d^3p_j
dP= |<f|S|
$P = \frac{\sigma}{A} = \frac{\sigma v T}{vol}$
prob that a given point particle hits a given target particle
$\langle \vec{k} | \vec{k} \rangle \neq 1$


cross section
dσ ≡ dP vol/vT
interested in 2→n
more than 2)
dP = |<f|S|i>|² ∏ d³p
<f|f><i|i>
P = σ/A = σvT/vol
prob
that
a given
particle
hits a given
target particle
<k|k>≠1

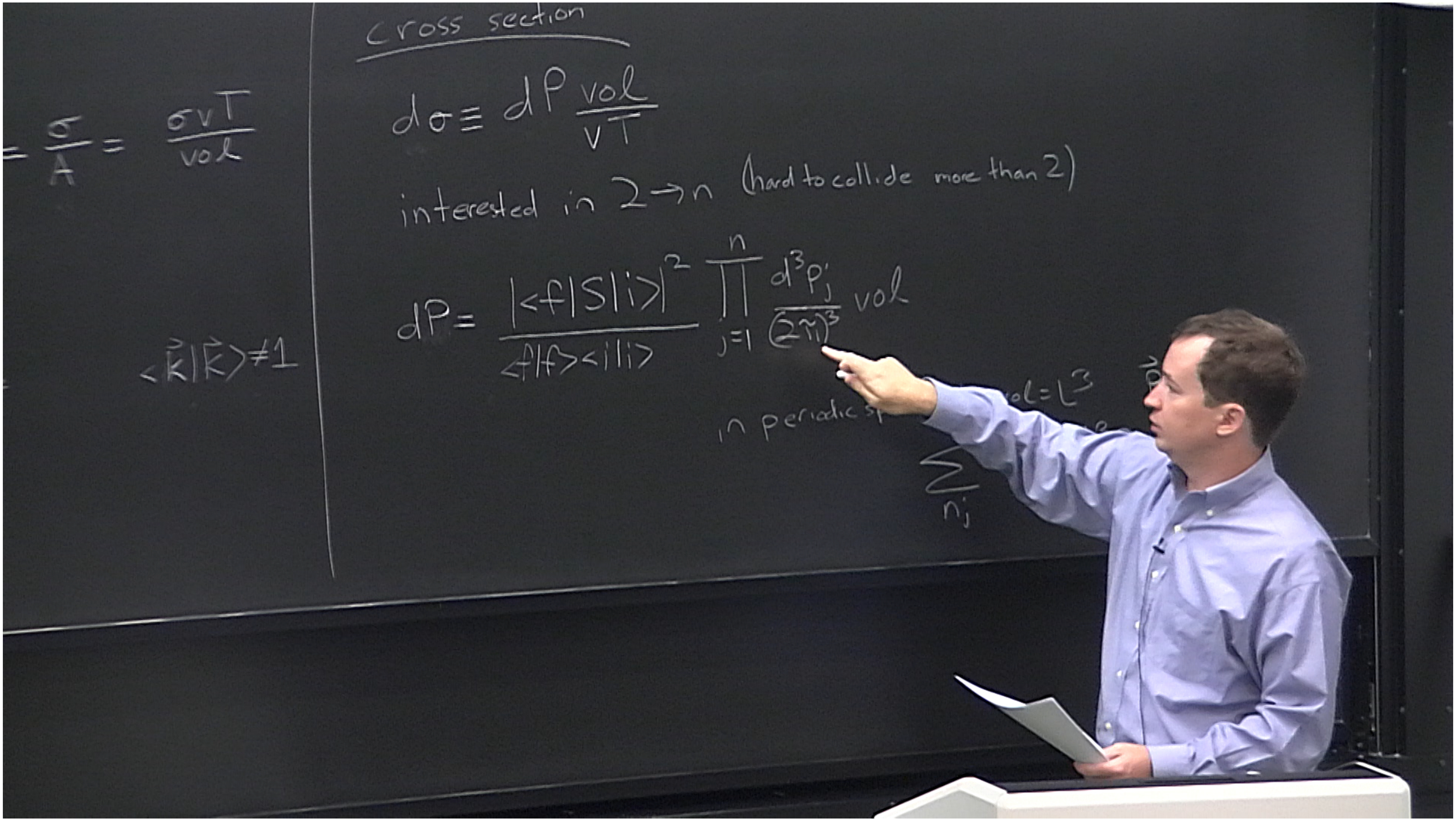

cross section
dσ ≡ dP vol/vT
interested in 2→n (hard to collide more than 2)
dP = |<f|S|i>|² / <f|f><i|i> ∏ d³p_j (j=1 to n)
P = σ/A = σvT/vol
prob that a given point particle hits a given target particle
<k|k> ≠ 1

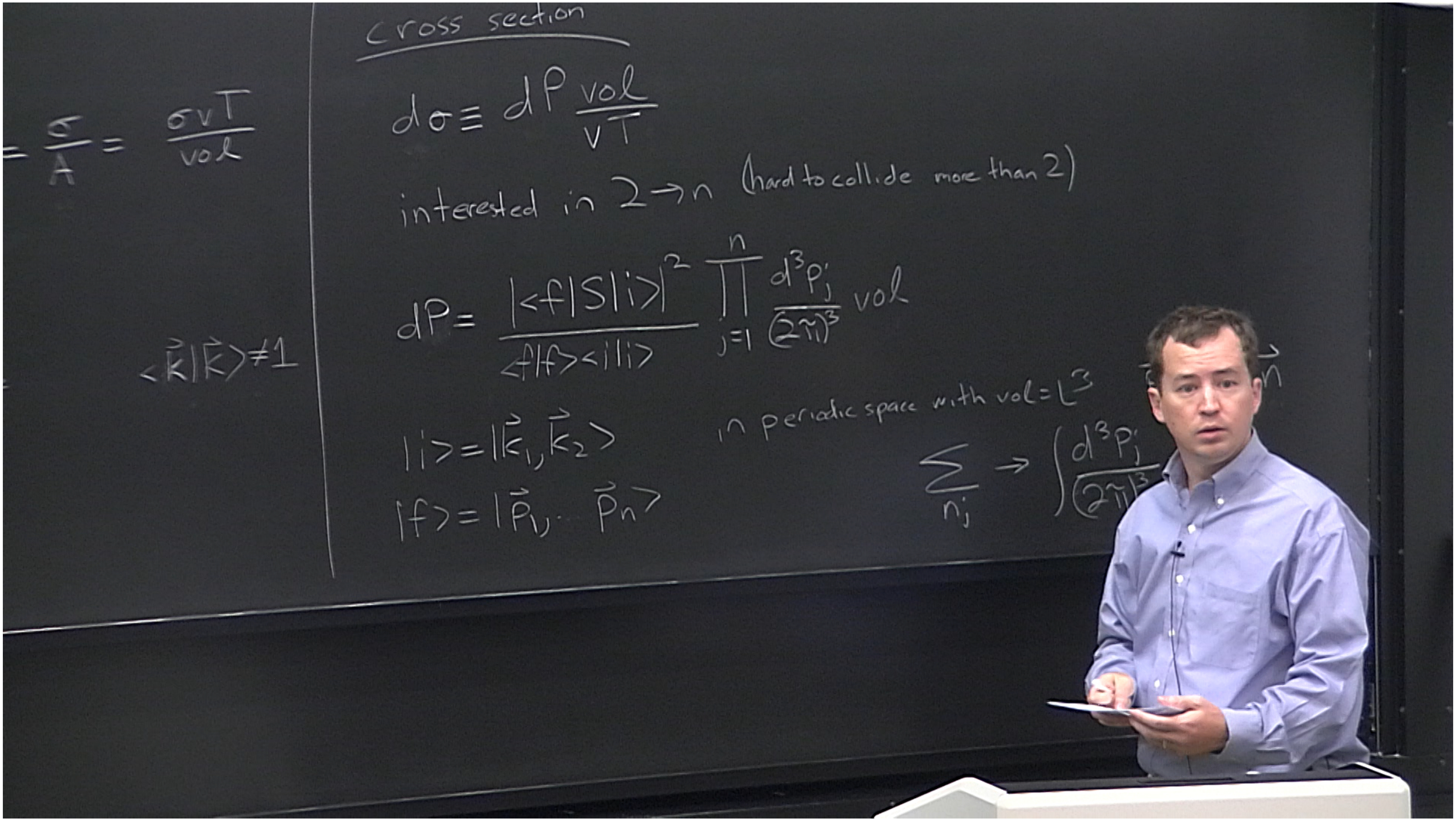

Cross section
$d\sigma \equiv dP \frac{vol}{vT}$
interested in $2 \to n$ (hard to collide more than 2)
$dP = \frac{|\langle f|S|i\rangle|^2}{\langle f|f\rangle\langle i|i\rangle} \prod_{j=1}^{n} d^3p_j$
in periodic space with $vol = L^3$
$P = \frac{\sigma}{A} = \frac{\sigma v T}{vol}$
prob that a given pnt particle hits a given target particle
$\langle \vec{k}|\vec{k}\rangle \neq 1$


cross section
$d\sigma \equiv dP \frac{vol}{vT}$
interested in $2 \to n$ (hard to collide more than 2)
$dP = \frac{|\langle f|S|i\rangle|^2}{\langle f|f\rangle\langle i|i\rangle} \prod_{j=1}^{n} d^3p_j$
in periodic space with $vol = L^3$
$\vec{p}_i = \frac{2\pi}{L}\vec{n}$
$\sum_{n_j}$
$= \frac{\sigma}{A} = \frac{\sigma v T}{vol}$
$\langle \vec{k}|\vec{k}\rangle \neq 1$

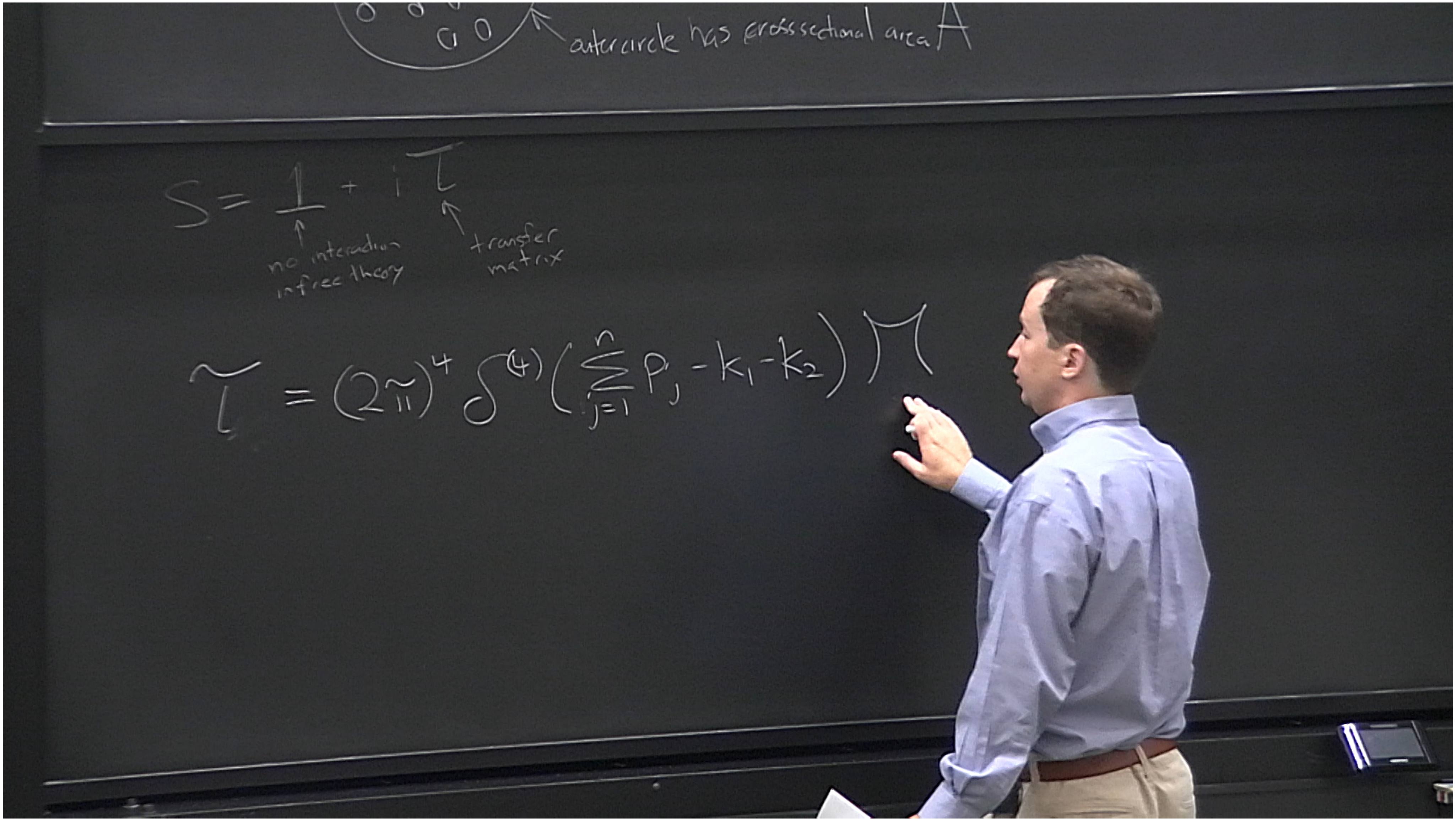


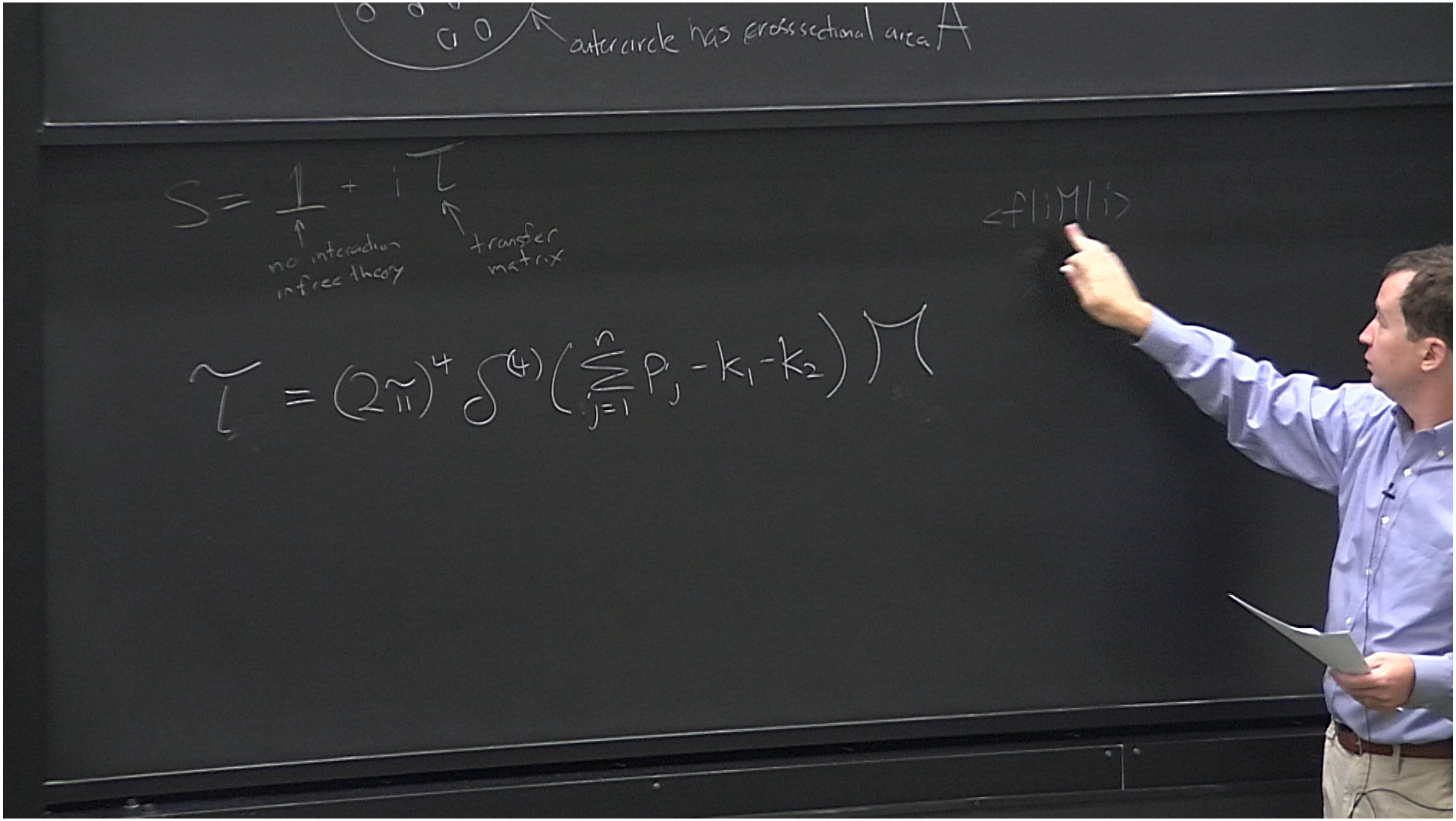
= σ/A = σvT/vol
⟨k|k⟩ ≠ 1
Cross section
dσ ≡ dP vol/(vT)
interested in 2 → n (hard to collide more than 2)
dP = |⟨f|S|i⟩|² / (⟨f|f⟩⟨i|i⟩) ∏_{j=1}^{n} d³p'_j
in periodic space with vol = L³   p_i = (2π/L) n
Σ_{n'_j} → ∫ d³p'_j/(2π)³ vol

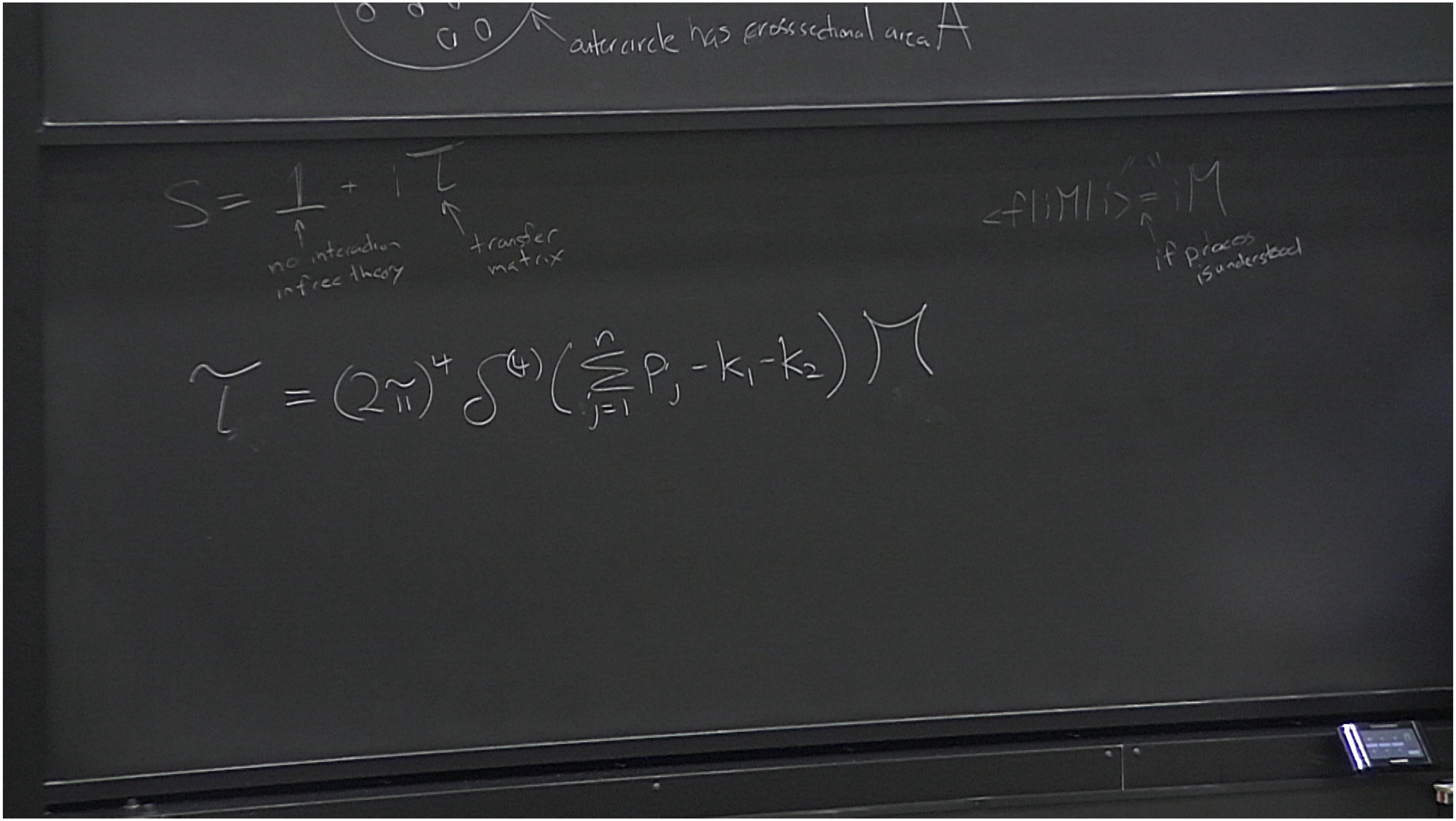

cross section
dσ ≡ dP vol / vT
= σ/A = σvT / vol
interested in 2 → n (hard to collide more than 2)
dP = |<f|S|i>|² / <f|f><i|i> ∏ d³p'_j / (2π)³ vol
<k|k> ≠ 1
in periodic sp

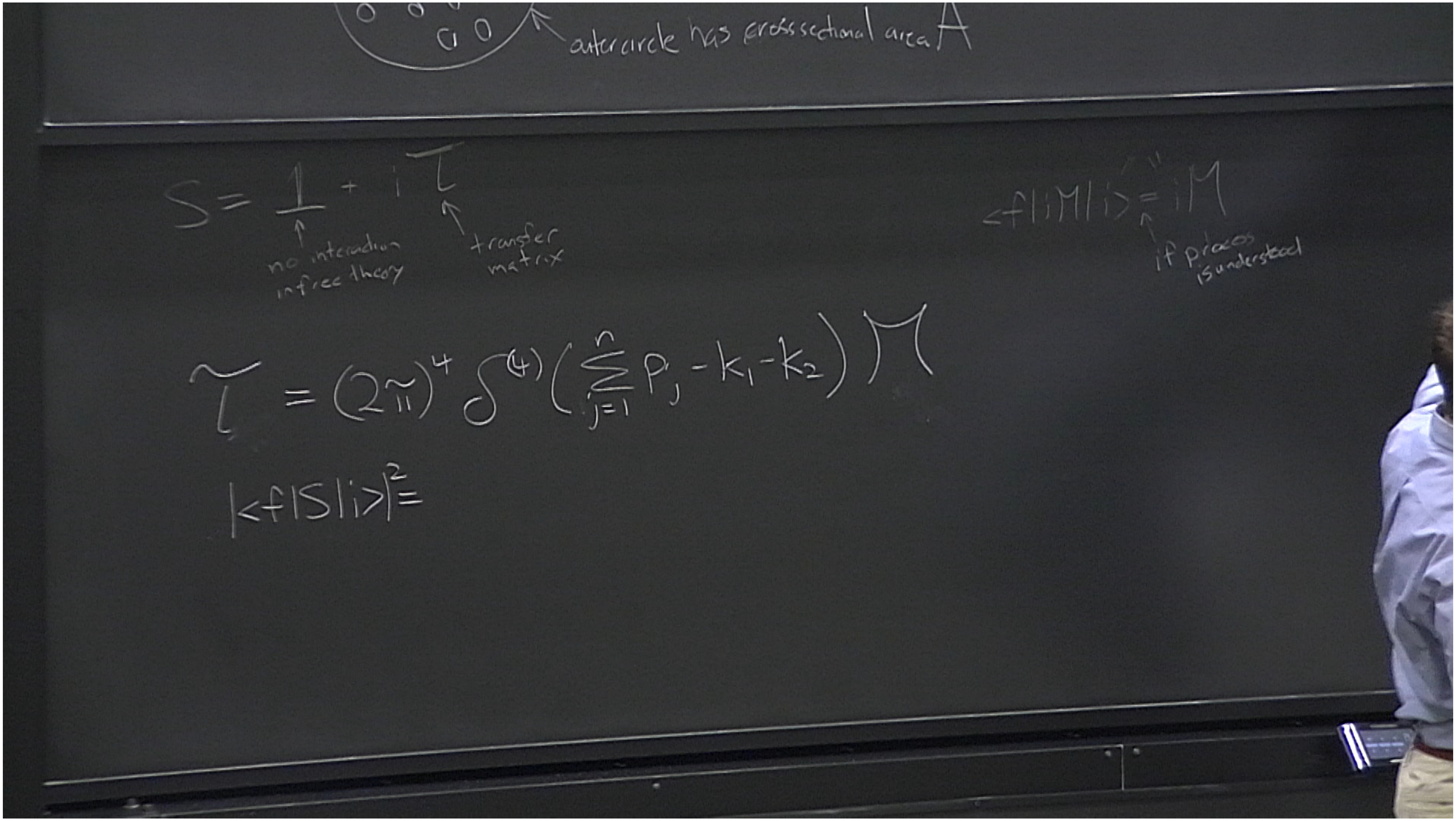

cross section
dσ ≡ dP vol / vT
interested in 2→n (hard to collide more than 2)
dP = |<f|S|i>|2 / <f|f><i|i> ∏ j=1 n d3pj / (2π)3 vol
in periodic space with vol = L3
|i> = |k1, k2>
|f> = |p1, ... pn>
∑ nj → ∫ d3pj / (2π)3
<k|k> ≠ 1
σ / A = σvT / vol

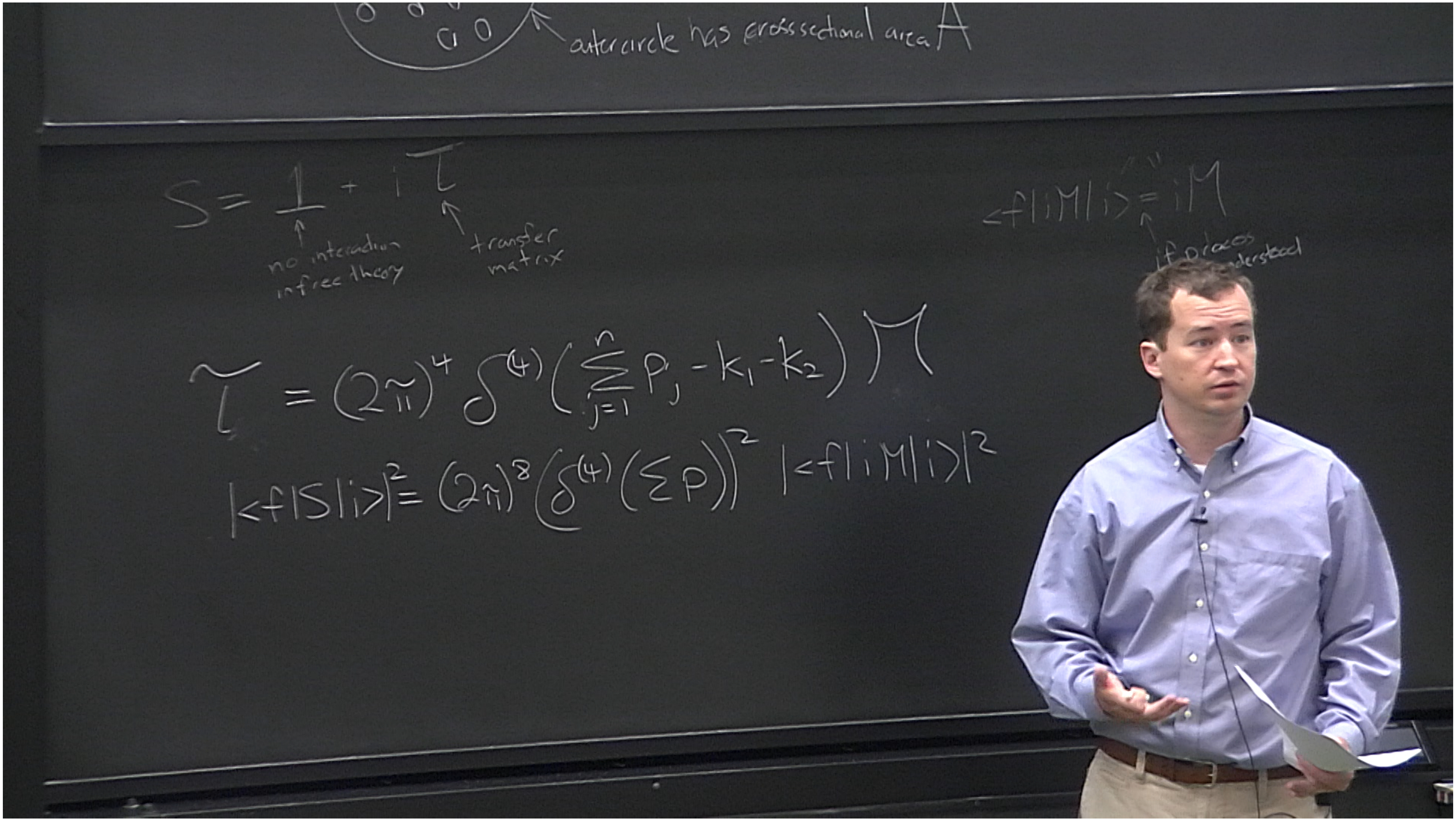

outer circle has cross sectional area A
S = 1 + i T
no interaction in free theory
transfer matrix


outer circle has cross sectional area A
S = 1 + i T
no interaction in free theory
transfer matrix
T = (2π)^4 δ^(4)(Σ_{j=1}^n p_j - k_1 - k_2) M

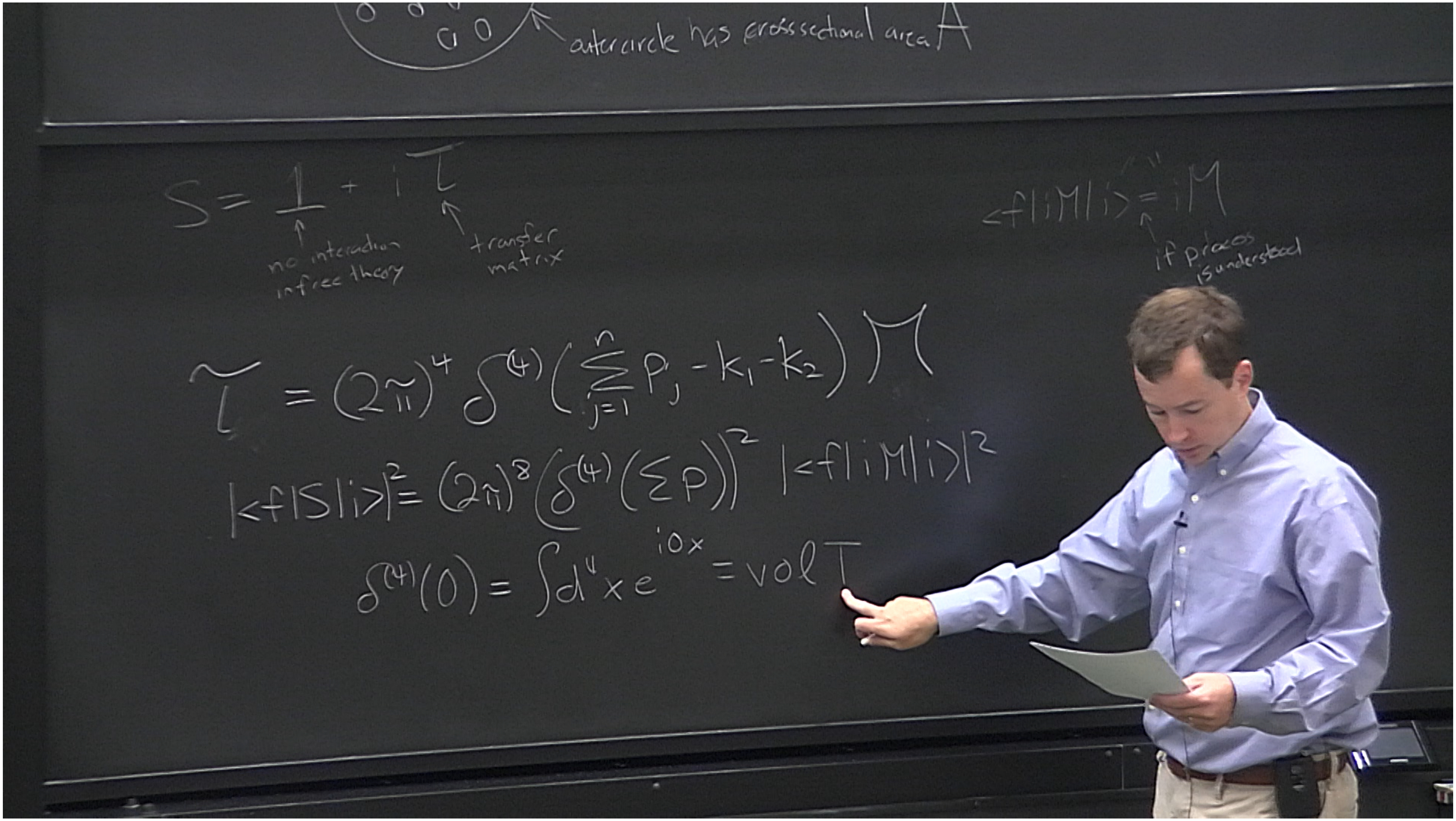

outer circle has cross sectional area A
S = 1 + i T
no interaction in free theory
transfer matrix
<f|iM|i>
T = (2π)^4 δ^(4)(Σ_{j=1}^n p_j - k_1 - k_2) M

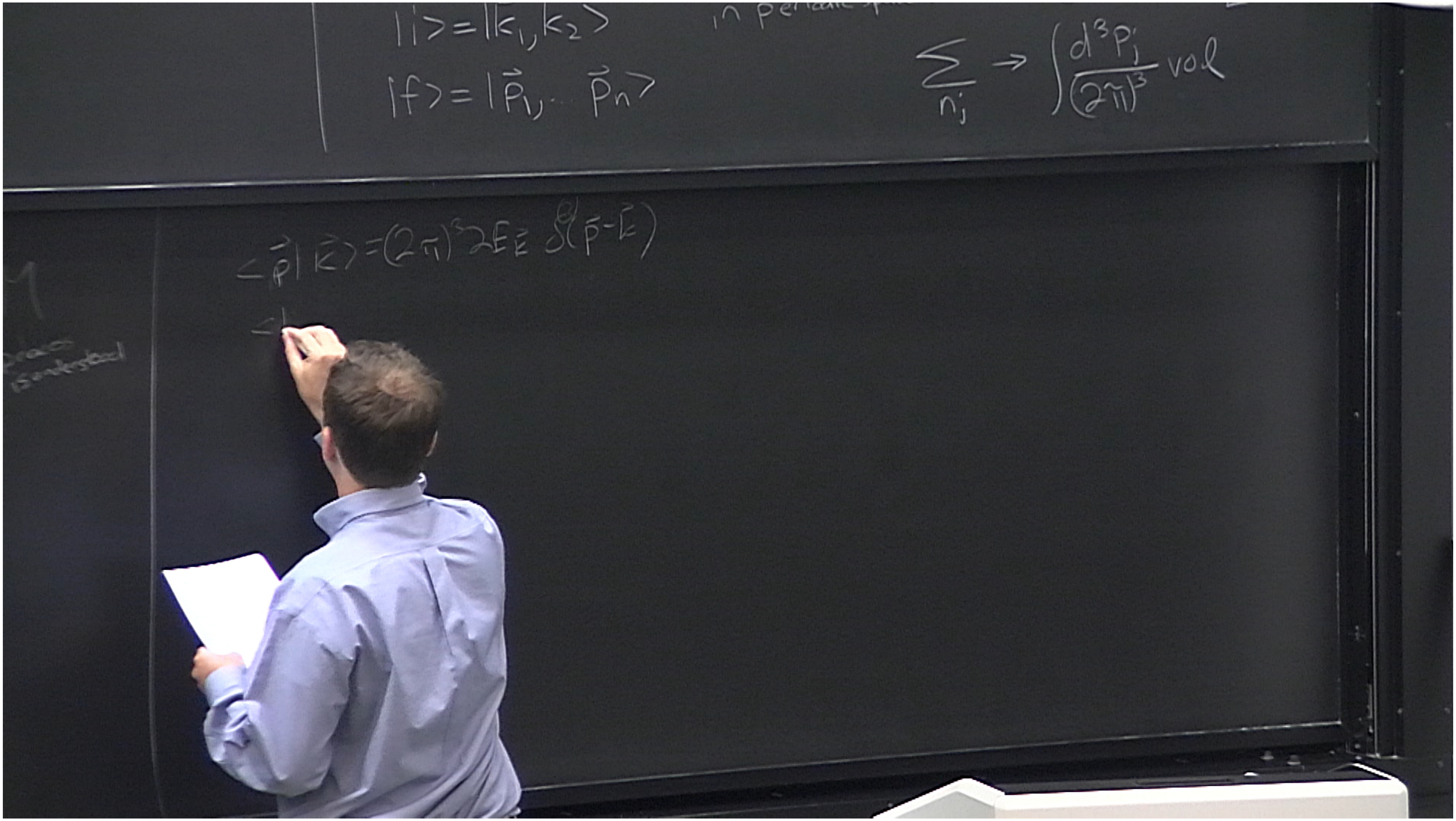

outer circle has cross sectional area A
S = 1 + iT
no interaction in free theory
transfer matrix
<f|iM|i> = iM
if process is understood
T = (2π)^4 δ^(4)(Σ_{j=1}^{n} p_j - k_1 - k_2) M

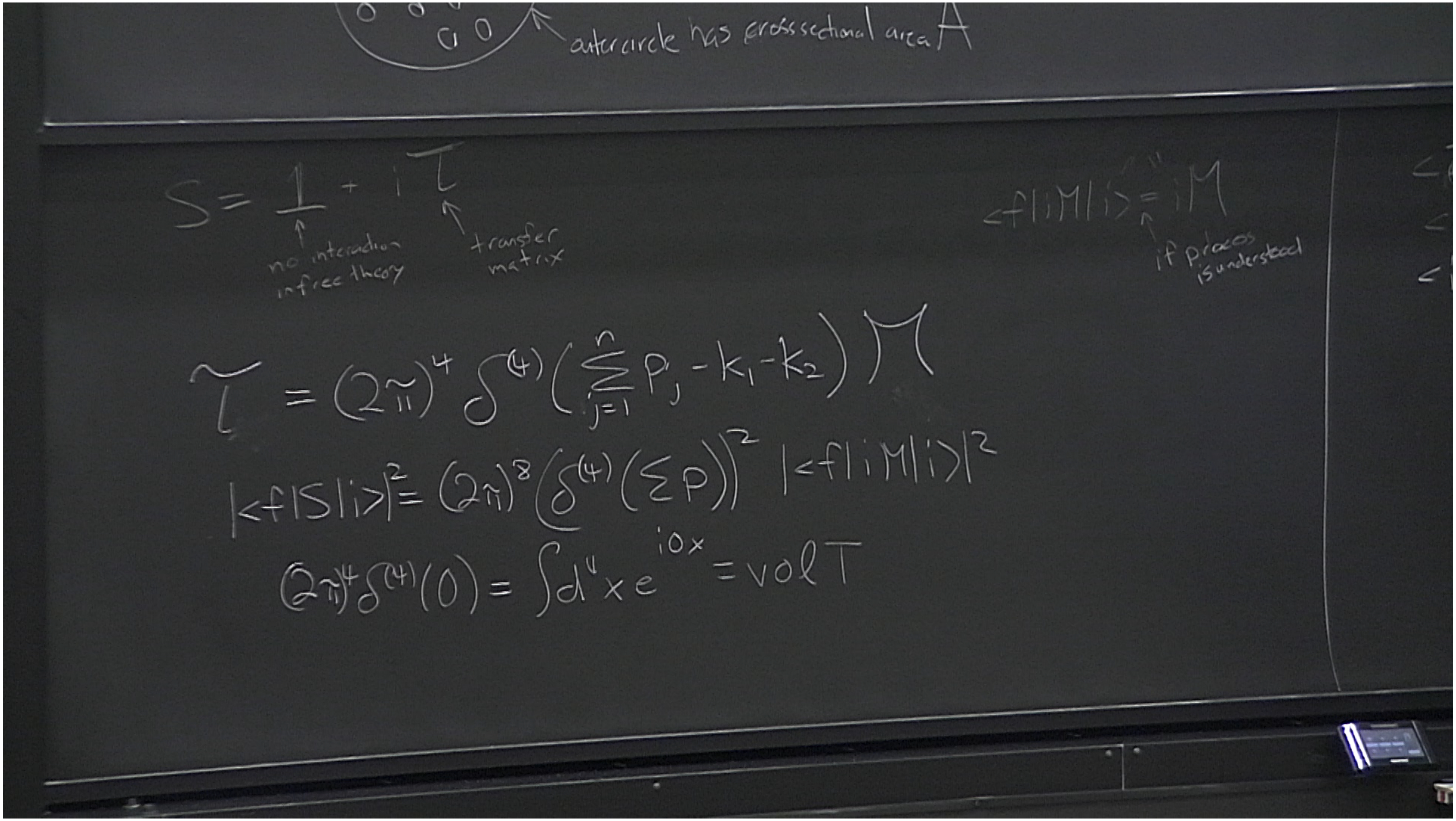

outer circle has cross sectional area A
S = 1 + i T
no interaction in free theory
transfer matrix
<f|iM|i> = iM
if process is understood
T = (2π)^4 δ^(4)(Σ_{j=1}^n p_j - k_1 - k_2) M
|<f|S|i>|^2 =

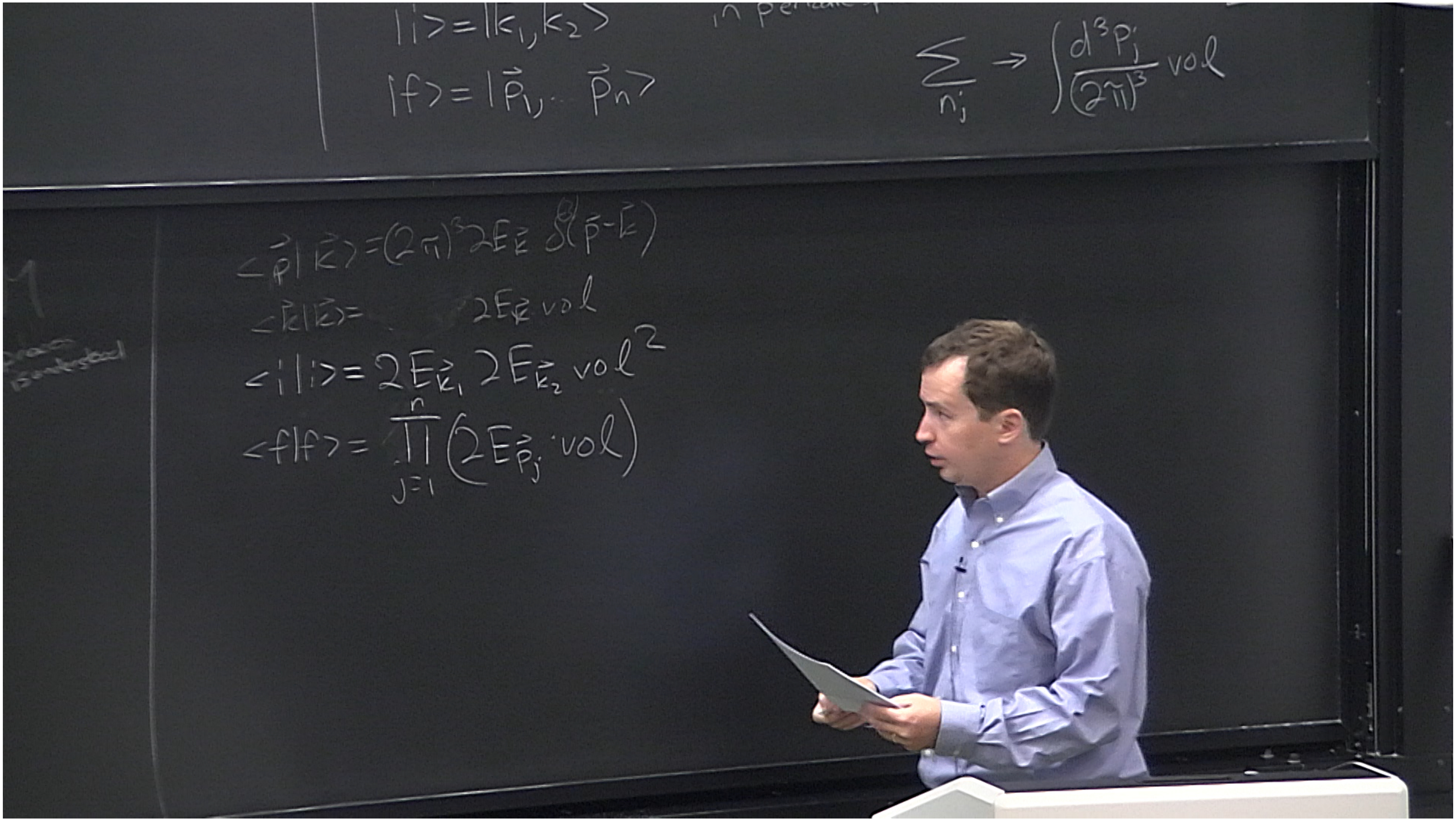


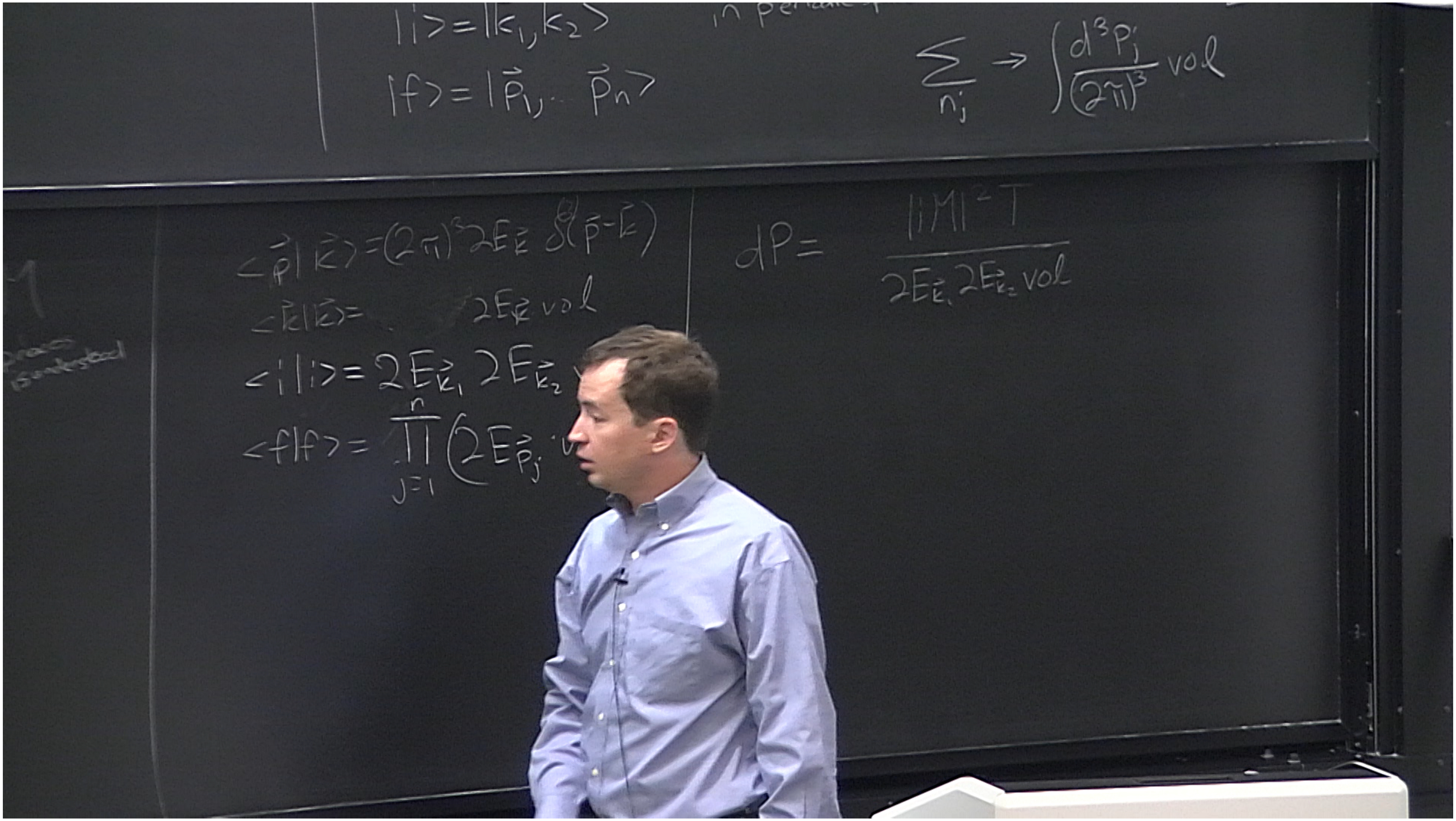
outercircle has crosssectional area A
$S = 1 + iT$
no interaction in free theory
transfer matrix
$\langle f | iM | i \rangle = iM$
$T = (2\pi)^4 \delta^{(4)}\left(\sum_{j=1}^{n} p_j - k_1 - k_2\right) M$
$|\langle f | S | i \rangle|^2 = (2\pi)^8 \left(\delta^{(4)}(\sum p)\right)^2 |\langle f | iM | i \rangle|^2$

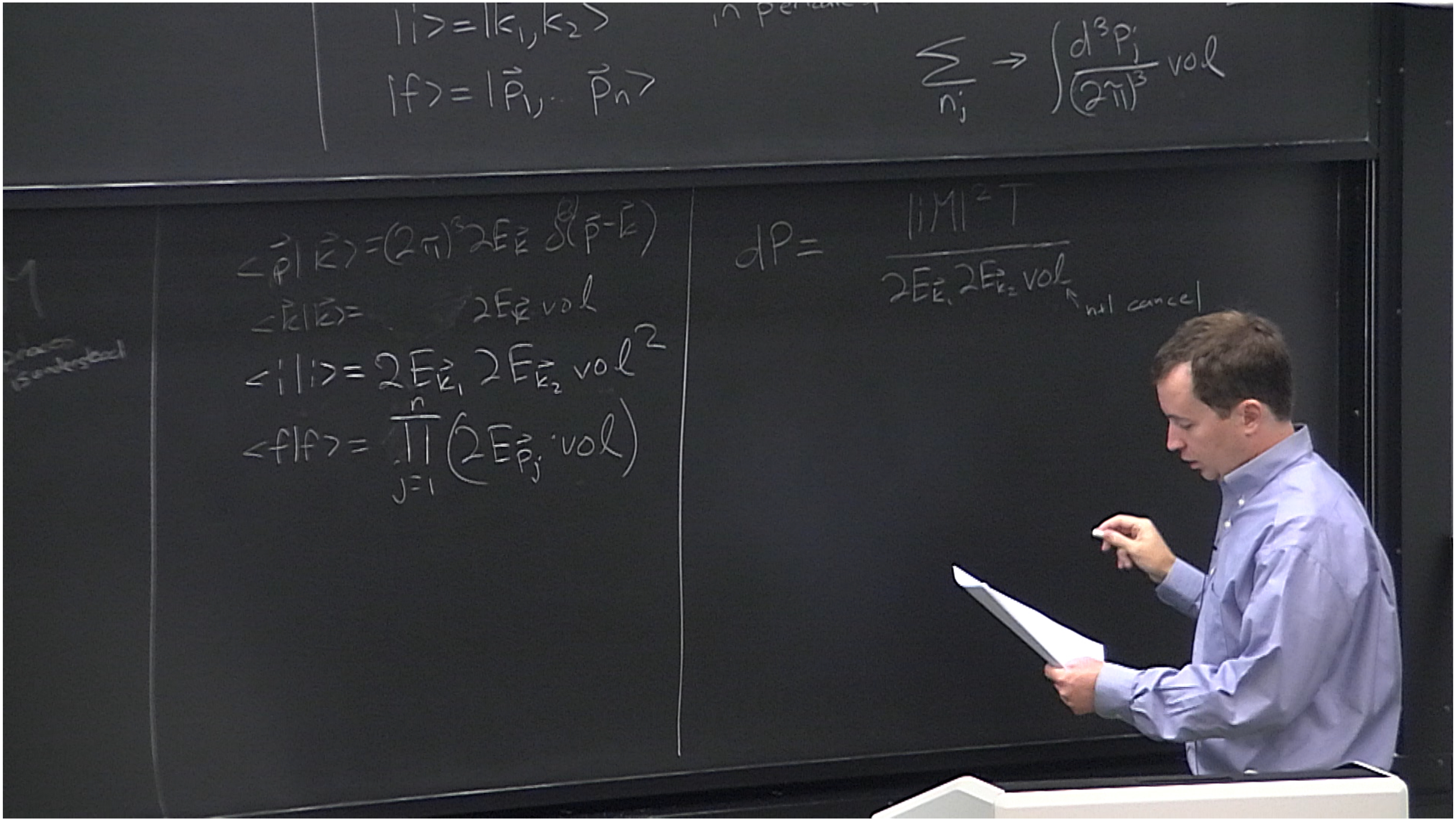

outer circle has cross sectional area A
S = 1 + iT
no interaction in free theory
transfer matrix
<f|iM|i> = iM
if process is understood
T = (2π)^4 δ^(4)(Σ_{j=1}^n p_j - k_1 - k_2) M
|<f|S|i>|^2 = (2π)^8 (δ^(4)(Σp))^2 |<f|iM|i>|^2
δ^(4)(0) = ∫d^4x e^{i0x} = vol T

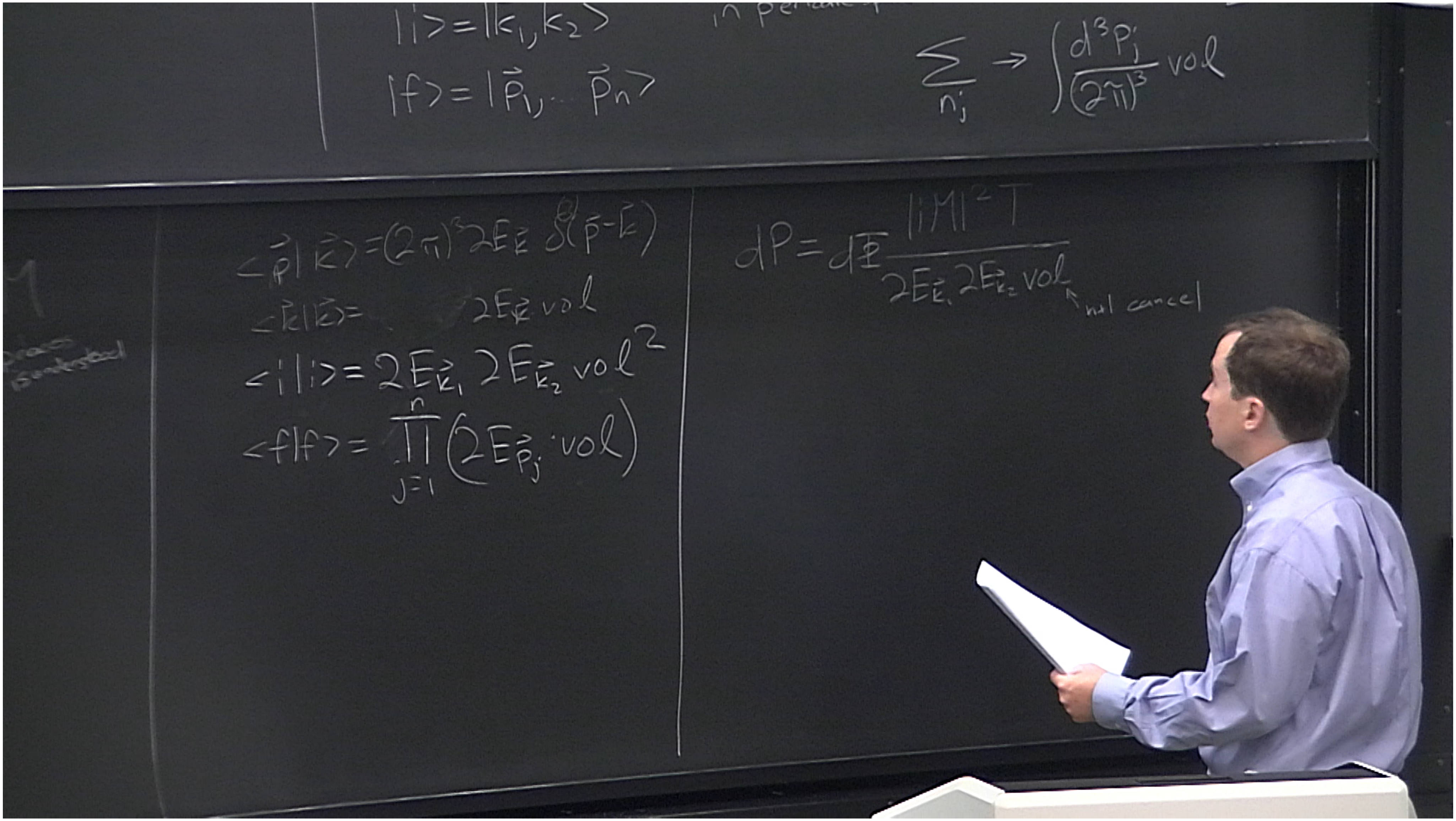

|i> = |k1, k2>
|f> = |p1, … pn>
Σ_nj → ∫ d^3 pj / (2π)^3 vol
<p|k> = (2π)^3 2E_k δ(p - k)

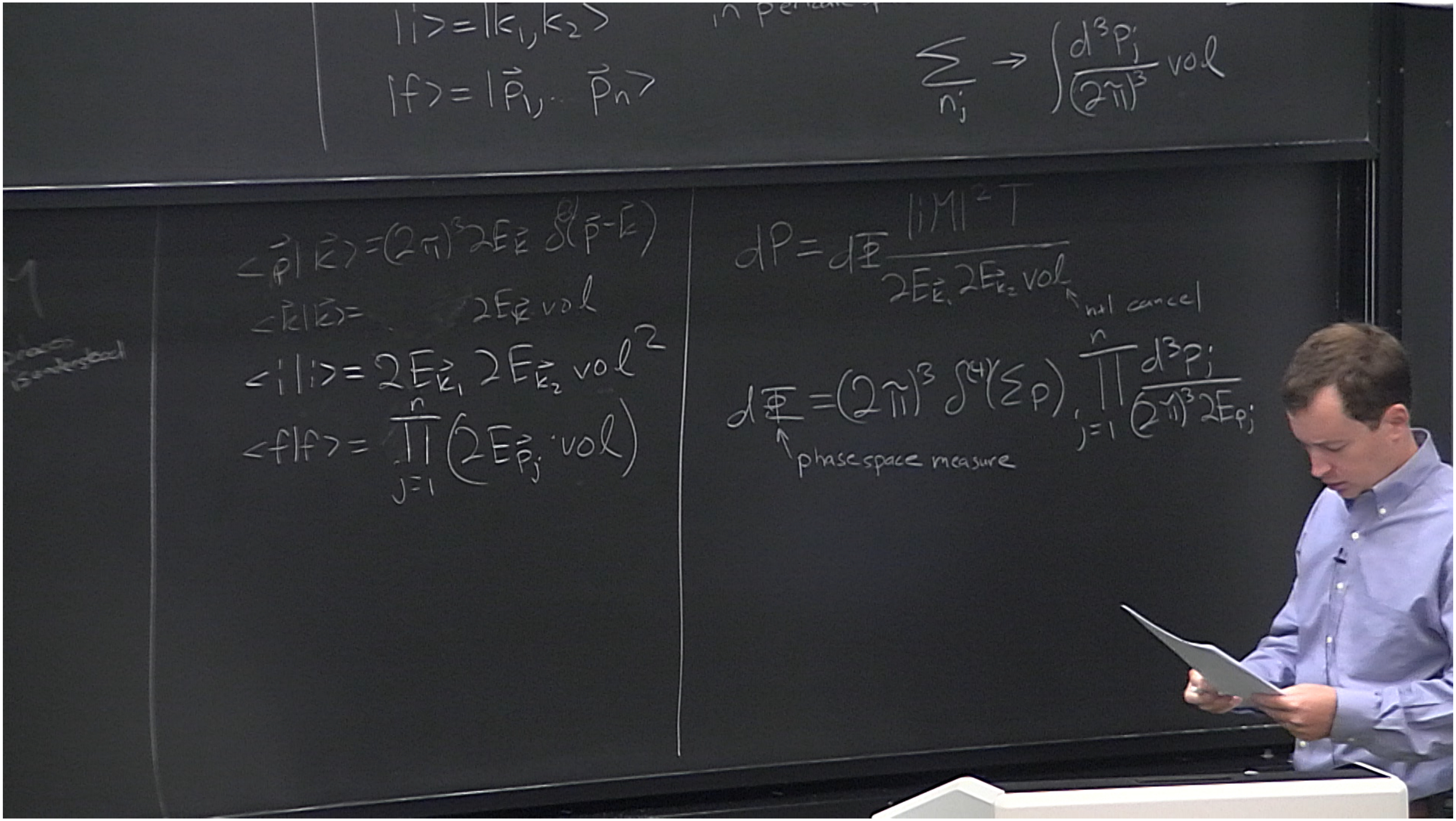

outer circle has cross sectional area A
$S = 1 + iT$
no interaction in free theory
transfer matrix
$\langle f | iM | i \rangle = iM$
if process is understood
$T = (2\pi)^4 \delta^{(4)}\left(\sum_{j=1}^{n} p_j - k_1 - k_2\right) M$
$|\langle f | S | i \rangle|^2 = (2\pi)^8 \left(\delta^{(4)}(\Sigma p)\right)^2 |\langle f | iM | i \rangle|^2$
$(2\pi)^4 \delta^{(4)}(0) = \int d^4x \, e^{i0x} = \mathrm{vol}\, T$

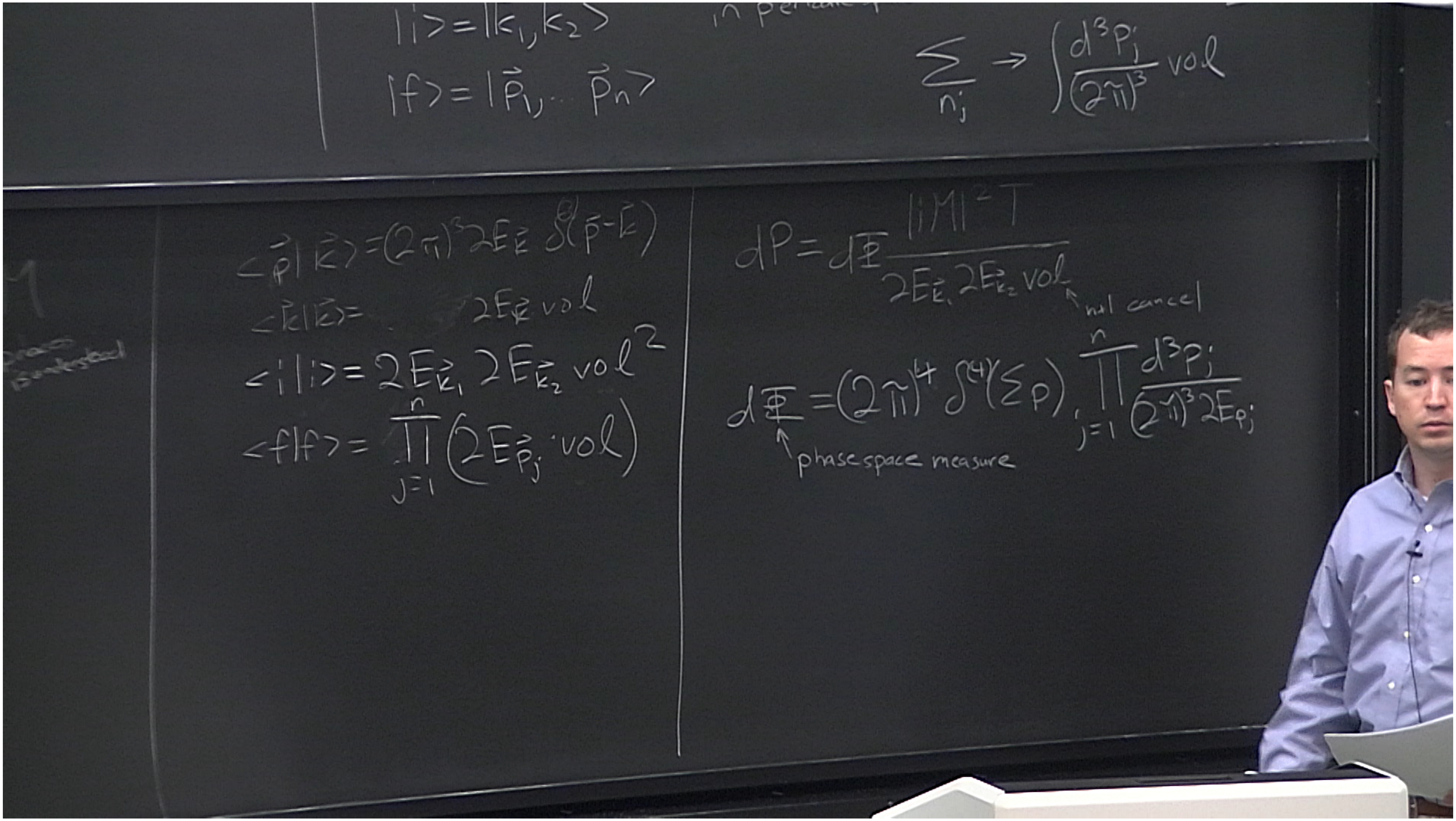

$|i\rangle = |k_1, k_2\rangle$
$|f\rangle = |\vec{p}_1, \ldots \vec{p}_n\rangle$
$\sum_{n_j} \rightarrow \int \frac{d^3 p_j}{(2\pi)^3} \, vol$
$\langle \vec{p} | \vec{k} \rangle = (2\pi)^3 2E_{\vec{k}} \, \delta^3(\vec{p} - \vec{k})$
$\langle \vec{k} | \vec{k} \rangle = 2E_{\vec{k}} \, vol$
$\langle i | i \rangle = 2E_{\vec{k}_1} 2E_{\vec{k}_2} \, vol^2$
$\langle f | f \rangle = \prod_{j=1}^{n} (2E_{\vec{p}_j} \cdot vol)$


|i> = |k1, k2>
|f> = |p1, ... pn>
dP = |iM|^2 T / 2E_k1 2E_k2 vol
<i|i> = 2E_k1 2E_k2

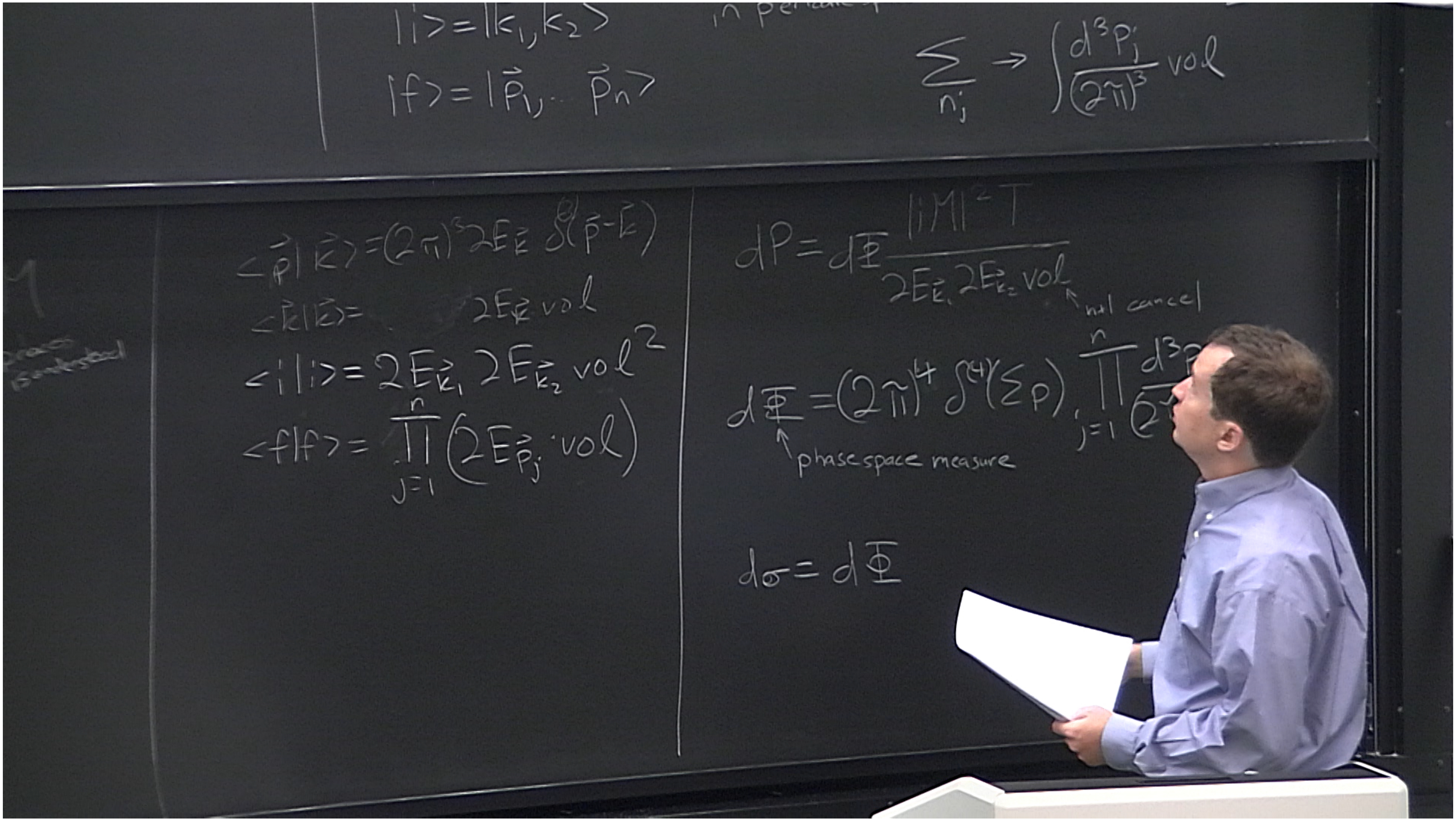

|i> = |k1, k2>
|f> = |p1, ... pn>
dP =
not cancel

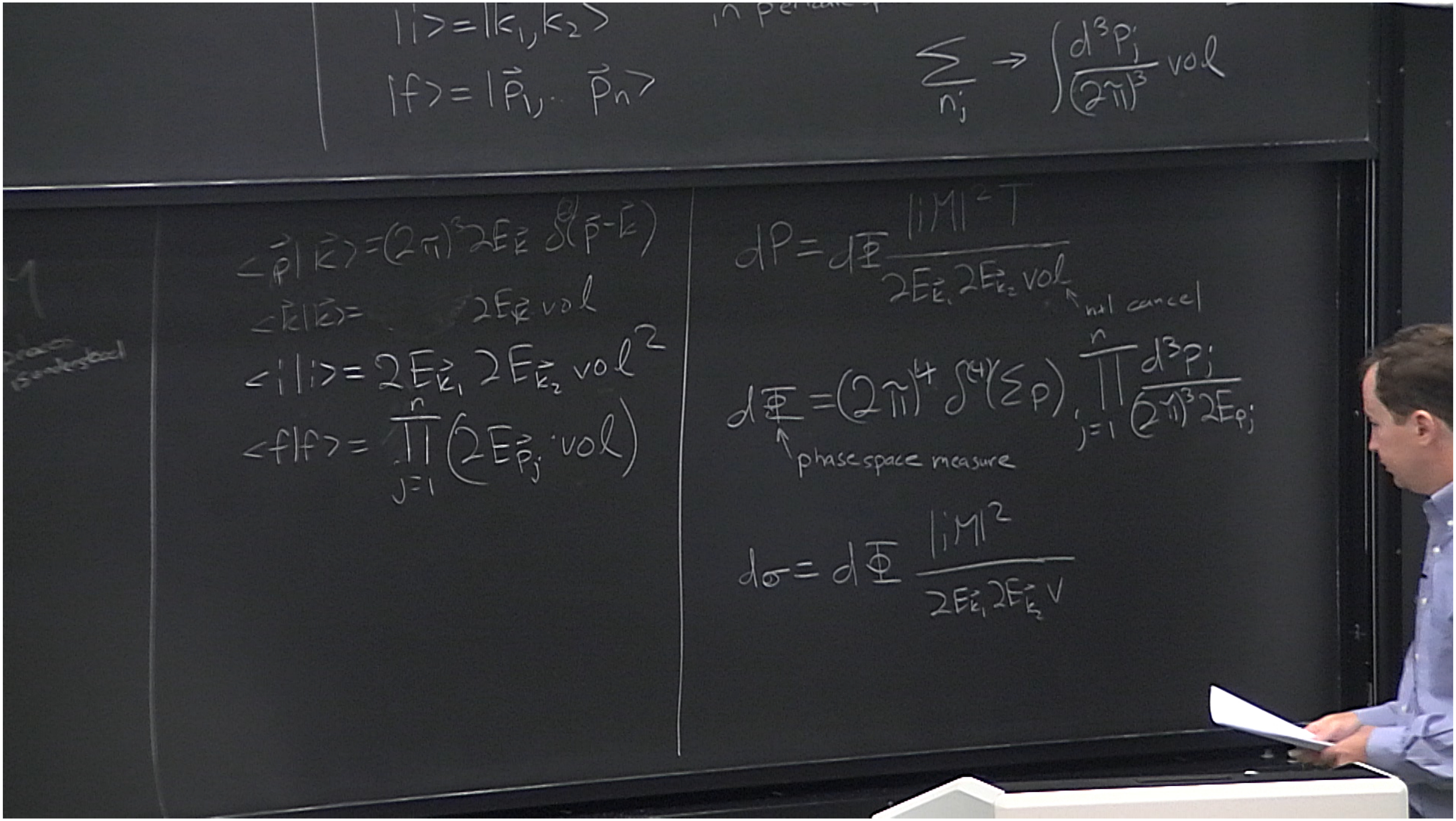

$|i\rangle = |k_1, k_2\rangle$
$|f\rangle = |\vec{p}_1, \ldots \vec{p}_n\rangle$
$\sum_{n_j} \rightarrow \int \frac{d^3 p_j}{(2\pi)^3} \, vol$
$\langle \vec{p} | \vec{k} \rangle = (2\pi)^3 2E_{\vec{k}} \, \delta^{(3)}(\vec{p} - \vec{k})$
$\langle \vec{k} | \vec{k} \rangle = 2E_{\vec{k}} \, vol$
$\langle i | i \rangle = 2E_{\vec{k}_1} 2E_{\vec{k}_2} \, vol^2$
$\langle f | f \rangle = \prod_{j=1}^{n} (2E_{\vec{p}_j} \cdot vol)$
$dP = d\Phi \frac{|iM|^2 T}{2E_{\vec{k}_1} 2E_{\vec{k}_2} \, vol}$
n+1 cancel


|i> = |k1, k2>
|f> = |p1, ... pn>
dP = dΦ |iM|² T / 2E 2E vol
not cancel
phase space measure

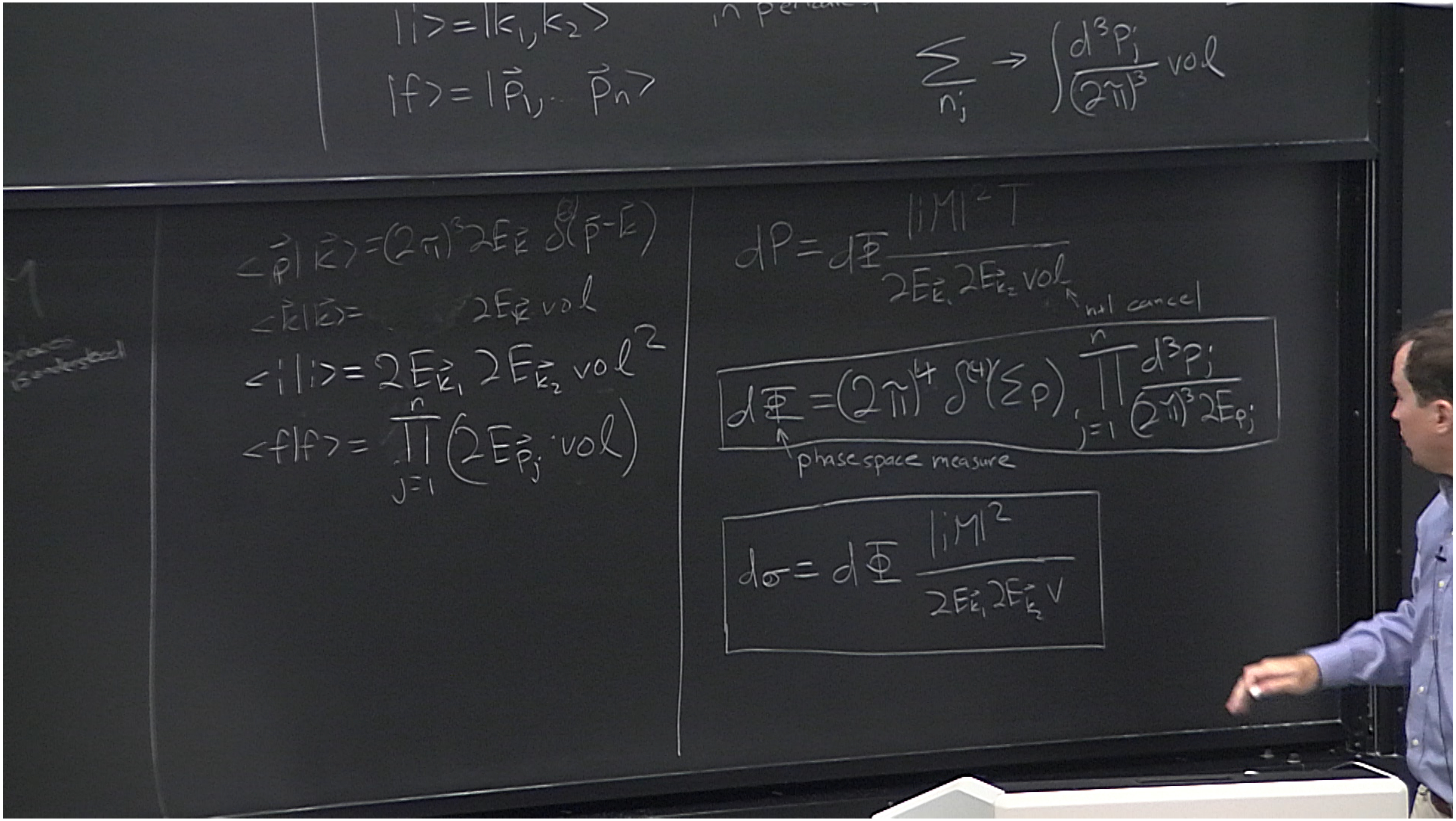


|i> = |k1, k2>
|f> = |p1, ... pn>
Σ_{n_j} → ∫ d³p_j/(2π)³ vol
<p|k> = (2π)³ 2E_k δ³(p − k)
<k|k> = 2E_k vol
<i|i> = 2E_k1 2E_k2 vol²
<f|f> = ∏_{j=1}^{n} (2E_pj · vol)
dP = dΦ |iM|² T / (2E_k1 2E_k2 vol)
not cancel
dΦ = (2π)⁴ δ⁽⁴⁾(Σp) ∏_{j=1}^{n} d³p_j / ((2π)³ 2E_pj)
phase space measure

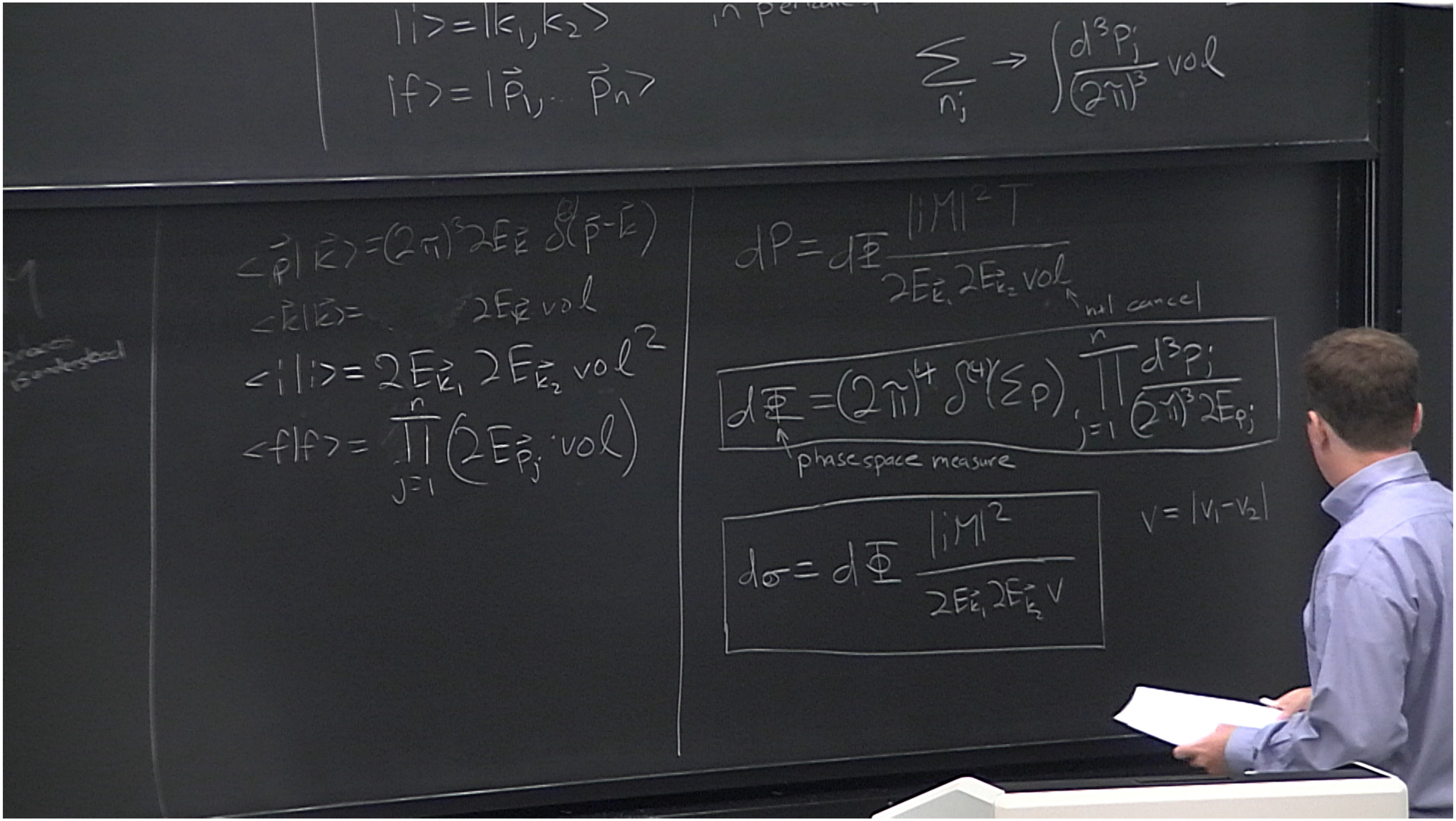

|i> = |k1, k2>
|f> = |p1, ... pn>
vol
<p|k> = (2π)^3 2E_k δ(p − k)
<k|k> = 2E_k vol
<i|i> = 2E_k1 2E_k2 vol^2
<f|f> = ∏_{j=1}^{n} (2E_pj · vol)
dP = dΦ |iM|^2 T / (2E_k1 2E_k2 vol)
not cancel
dΦ = (2π)^4 δ^(4)(Σp) ∏_{j=1}^{n}
phase space measure
dσ = dΦ



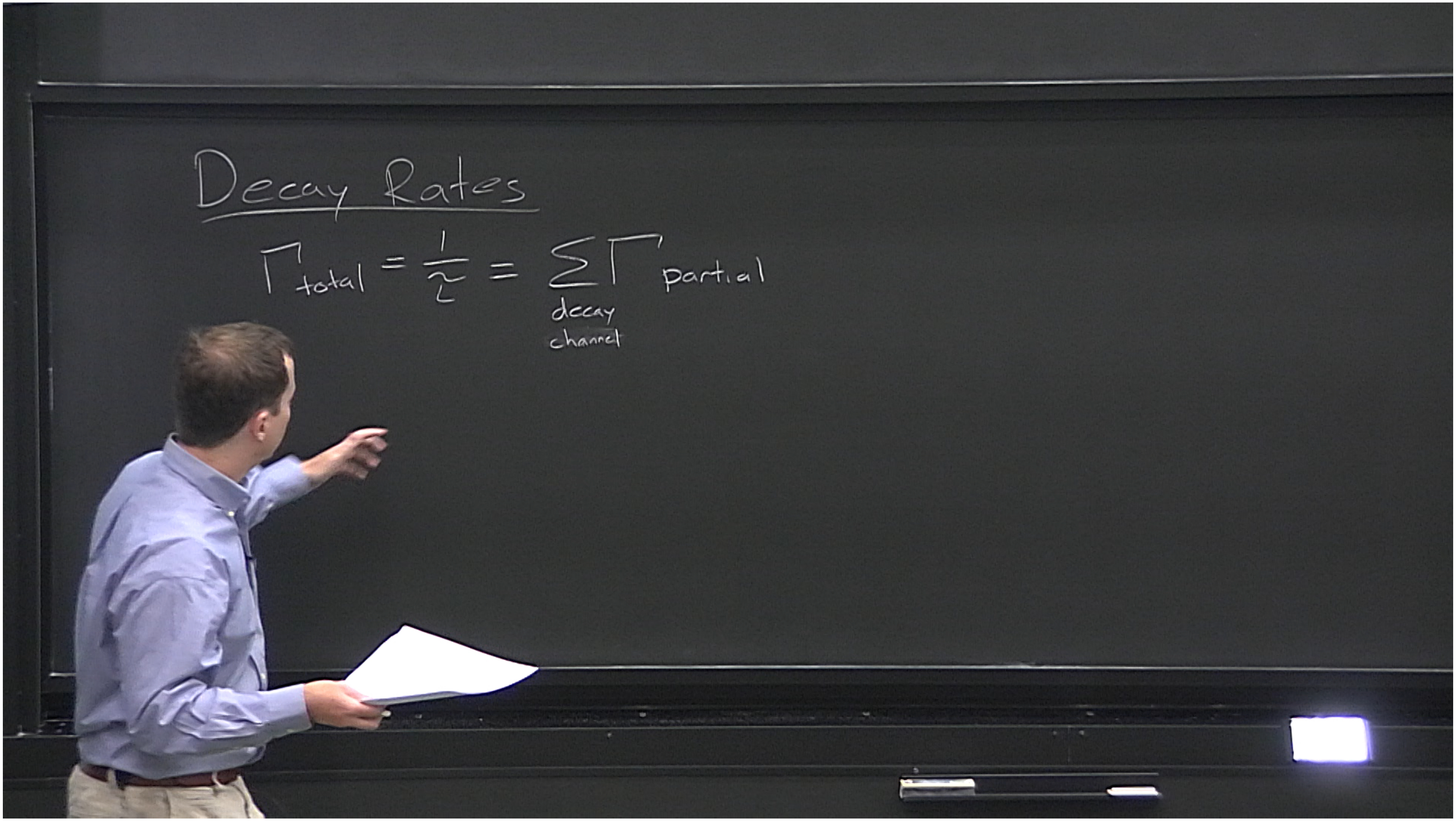
$|i\rangle = |k_1, k_2\rangle$
$|f\rangle = |\vec{p}_1, \ldots \vec{p}_n\rangle$
$\sum_{n_j} \rightarrow \int \frac{d^3p_j}{(2\pi)^3} \mathrm{vol}$
$\langle \vec{p} | \vec{k} \rangle = (2\pi)^3 2E_{\vec{k}} \delta^{(3)}(\vec{p}-\vec{k})$
$\langle \vec{k} | \vec{k} \rangle = \quad 2E_{\vec{k}} \mathrm{vol}$
$\langle i | i \rangle = 2E_{\vec{k}_1} 2E_{\vec{k}_2} \mathrm{vol}^2$
$\langle f | f \rangle = \prod_{j=1}^{n} (2E_{\vec{p}_j} \cdot \mathrm{vol})$
$dP = d\Phi \frac{|iM|^2 T}{2E_{\vec{k}_1} 2E_{\vec{k}_2} \mathrm{vol}}$
not cancel
$d\Phi = (2\pi)^4 \delta^{(4)}(\Sigma p) \prod_{j=1}^{n} \frac{d^3p_j}{(2\pi)^3 2E_{p_j}}$
phase space measure
$d\sigma = d\Phi \frac{|iM|^2}{2E_{\vec{k}_1} 2E_{\vec{k}_2} V}$

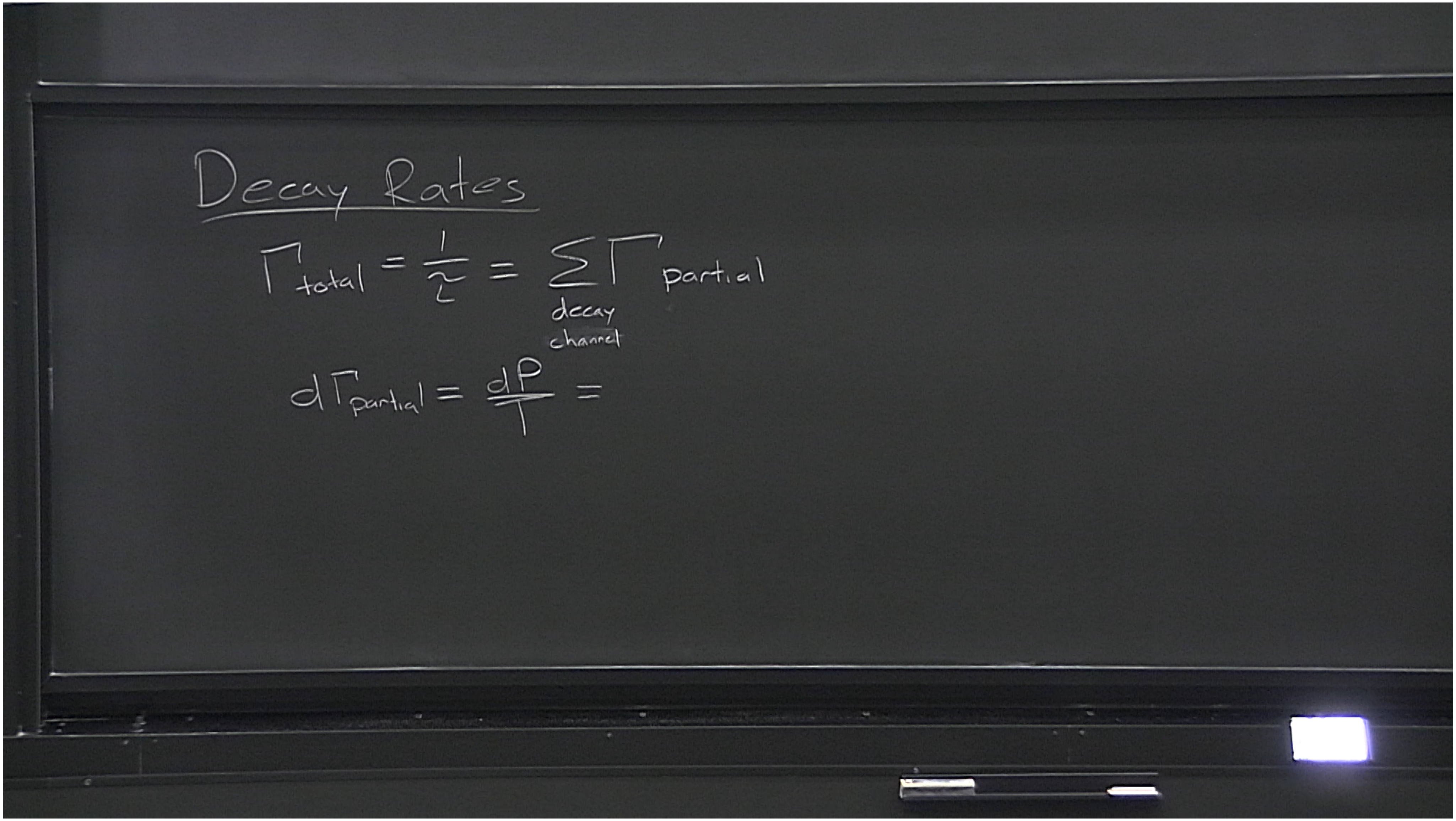


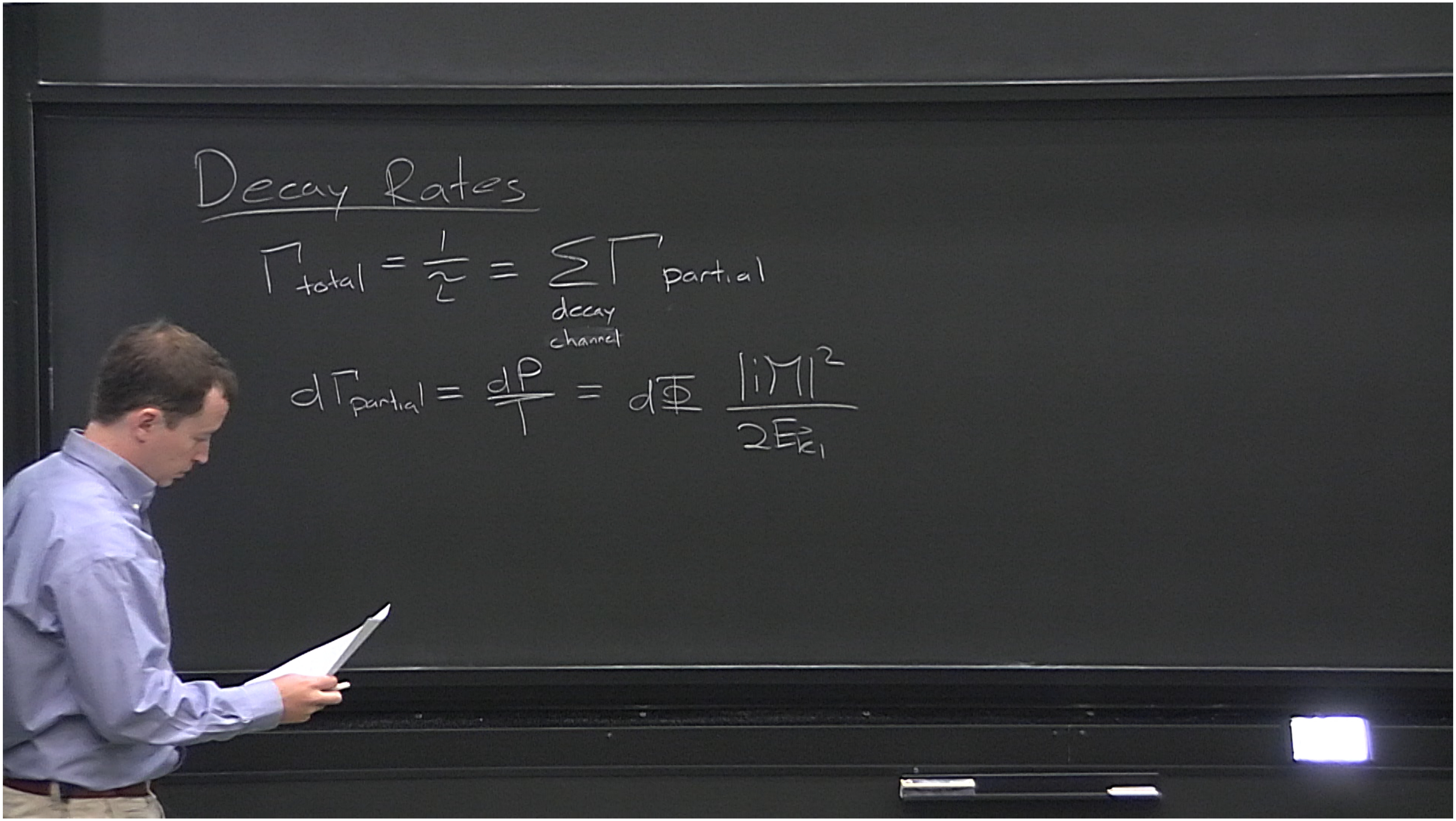
|i> = |k1, k2>
|f> = |p1, ... pn>
Σ_{n_j} → ∫ d³p_j / (2π)³ vol
<p|k> = (2π)³ 2E_k δ⁽³⁾(p − k)
<k|k> = 2E_k vol
<i|i> = 2E_k1 2E_k2 vol²
<f|f> = ∏_{j=1}^{n} (2E_pj · vol)
dP = dΦ |iM|² T / (2E_k1 2E_k2 vol)
not cancel
dΦ = (2π)⁴ δ⁽⁴⁾(Σp) ∏_{j=1}^{n} d³p_j / ((2π)³ 2E_pj)
phase space measure
dσ = dΦ |iM|² / (2E_k1 2E_k2 V)

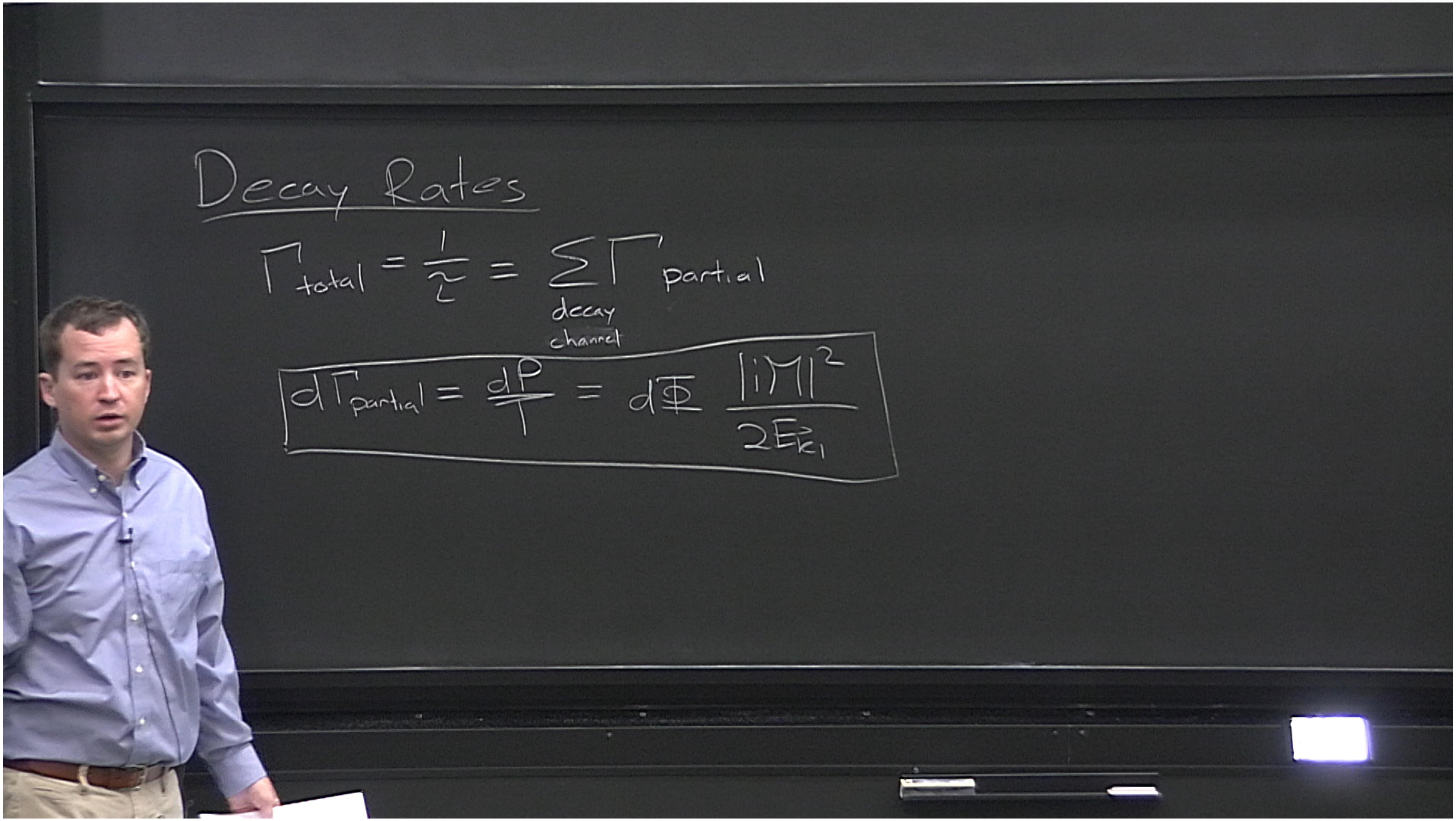


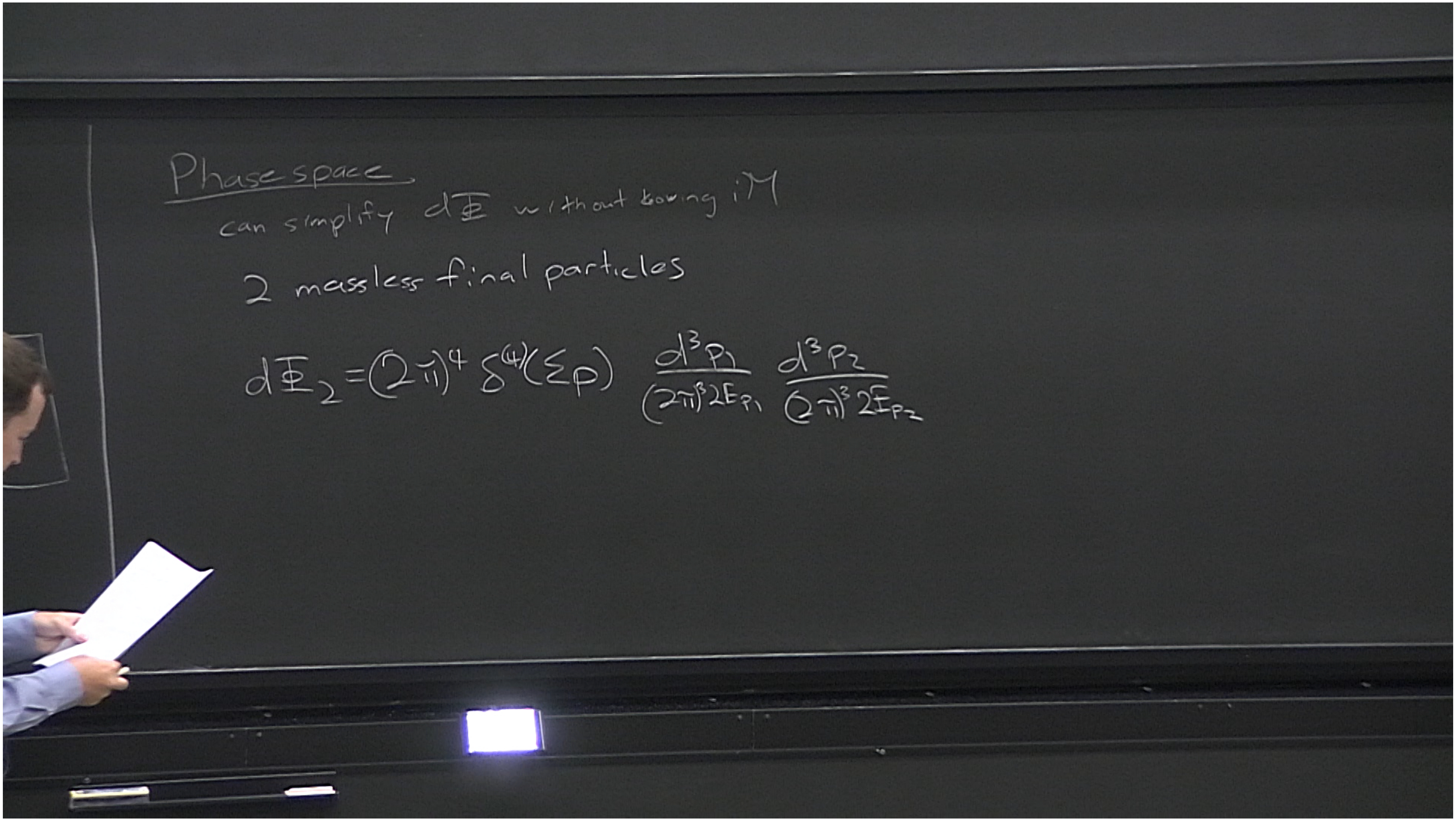
|i> = |k1, k2>
|f> = |p1, ... pn>
Σ over n_j → ∫ d³p_j / (2π)³ vol
<p|k> = (2π)³ 2E_k δ³(p − k)
<k|k> = 2E_k vol
<i|i> = 2E_k1 2E_k2 vol²
<f|f> = ∏_{j=1}^{n} (2E_pj · vol)
dP = dΦ |iM|² T / (2E_k1 2E_k2 vol)
n+1 cancel
dΦ = (2π)⁴ δ⁽⁴⁾(Σp) ∏_{j=1}^{n} d³p_j / ((2π)³ 2E_pj)
phase space measure
dσ = dΦ |iM|² / (2E_k1 2E_k2 V)
V = |v1 − v2|

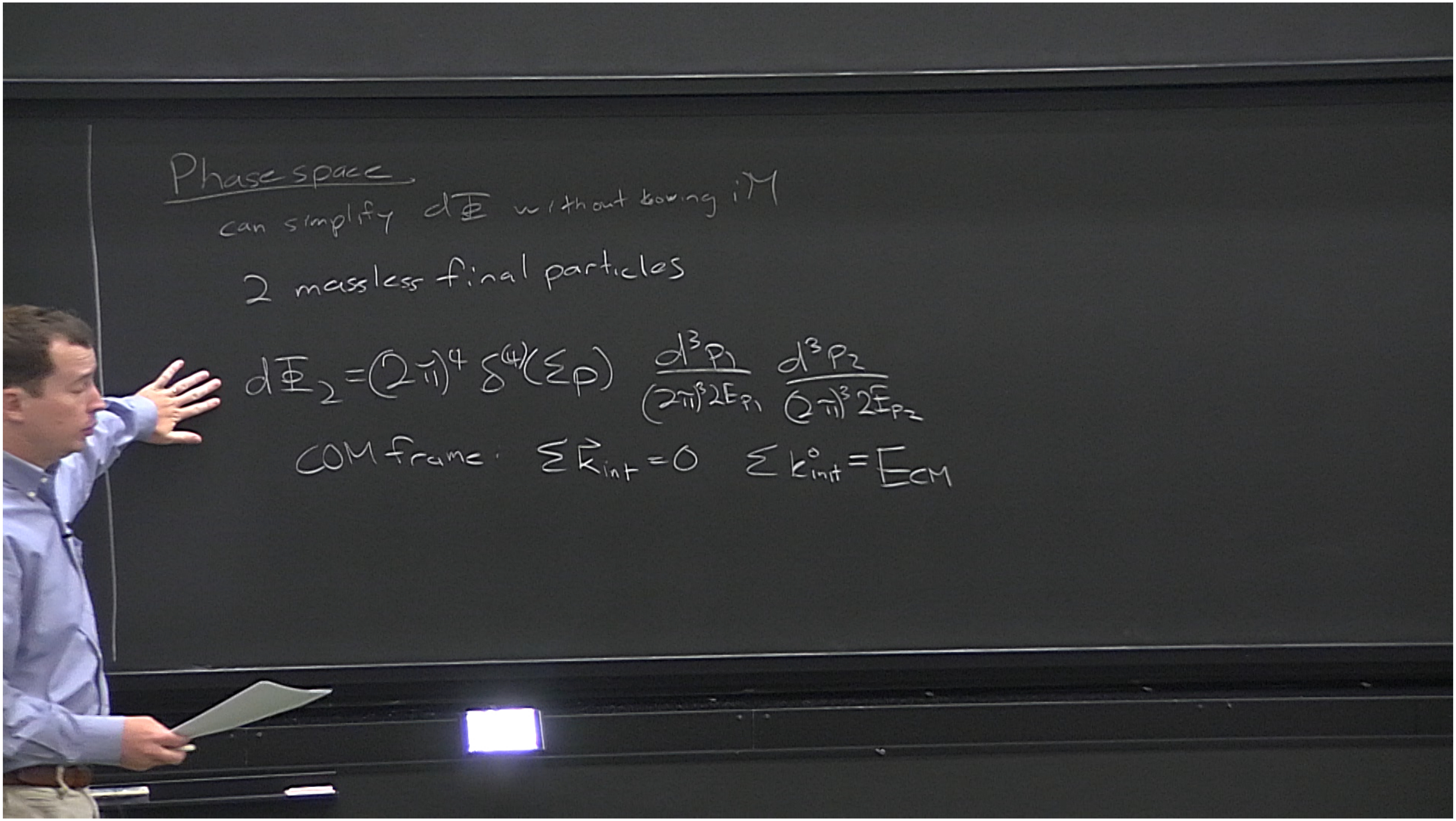

Decay Rates
$\Gamma_{total} = \frac{1}{\tau} = \sum_{\text{decay channel}} \Gamma_{partial}$


Decay Rates
$\Gamma_{total} = \frac{1}{\tau} = \sum_{\text{decay channel}} \Gamma_{partial}$
$d\Gamma_{partial} = \frac{dP}{T} =$


Decay Rates
$\Gamma_{total} = \frac{1}{\tau} = \sum_{\text{decay channel}} \Gamma_{partial}$
$d\Gamma_{partial} = \frac{dP}{T} = d\Phi \frac{|iM|^2}{2E_{\vec{k}_1}}$


Decay Rates
$\Gamma_{total} = \frac{1}{\tau} = \sum_{\text{decay channel}} \Gamma_{partial}$
$d\Gamma_{partial} = \frac{dP}{T} = d\Phi \, \frac{|iM|^2}{2E_{\vec{k}_1}}$


Phase space
can simplify $d\Phi$ without knowing $i\mathcal{M}$
2 massless final particles
$d\Phi_2 = (2\pi)^4 \delta^{(4)}(\Sigma p) \frac{d^3p_1}{(2\pi)^3 2E_{p_1}} \frac{d^3p_2}{(2\pi)^3 2E_{p_2}}$


Phase space
can simplify $d\Phi$ without knowing $i\mathcal{M}$
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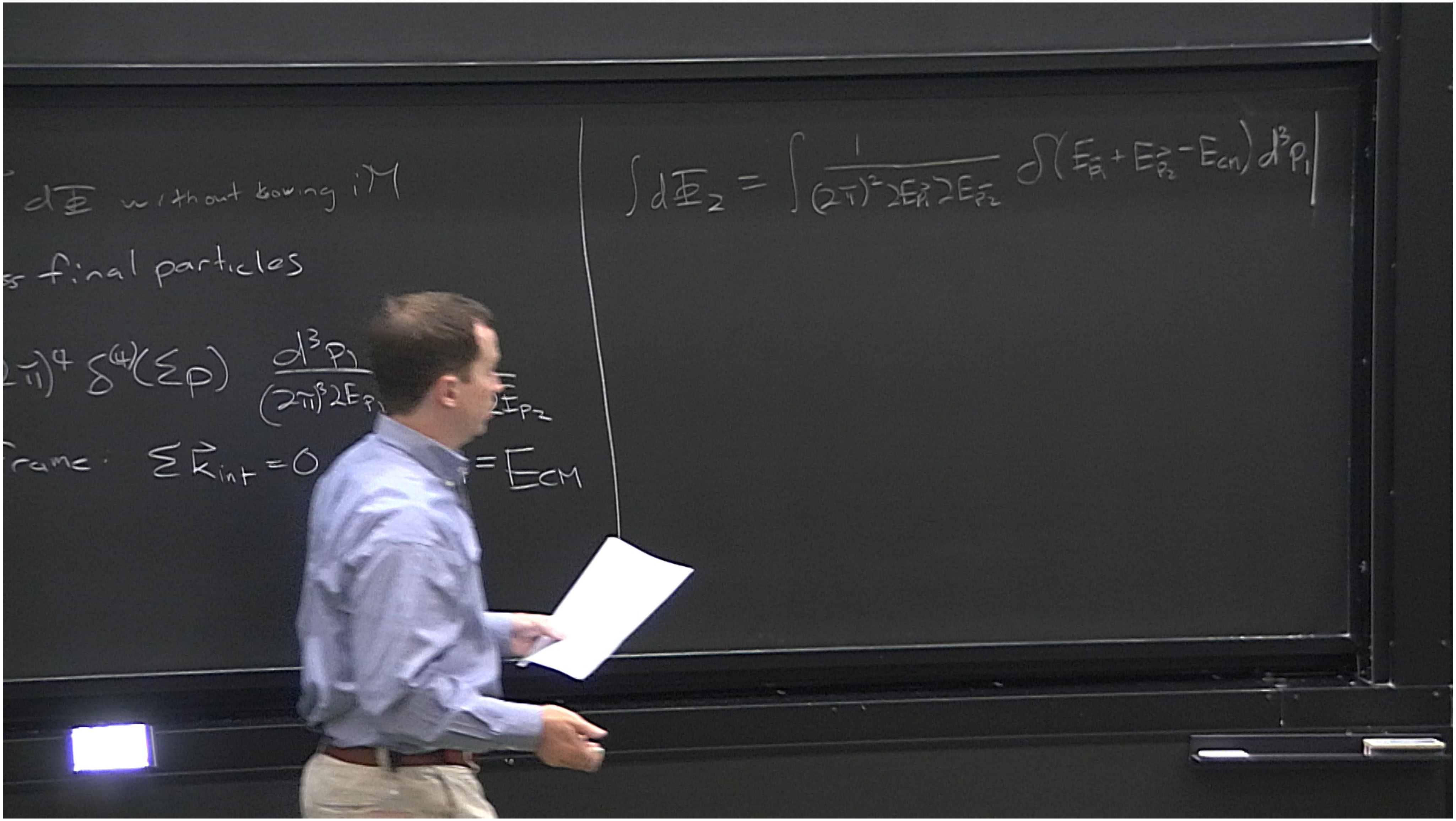
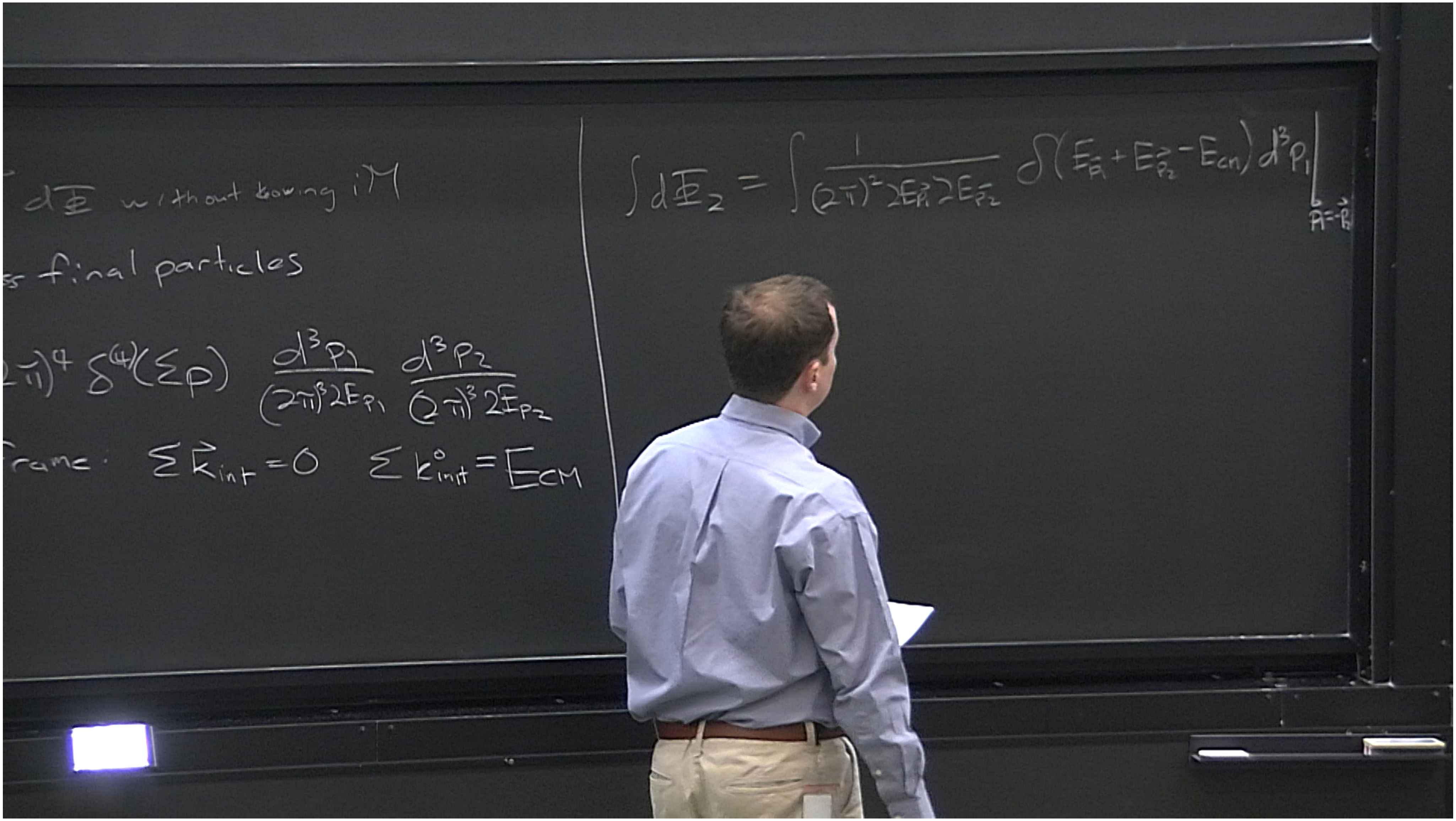
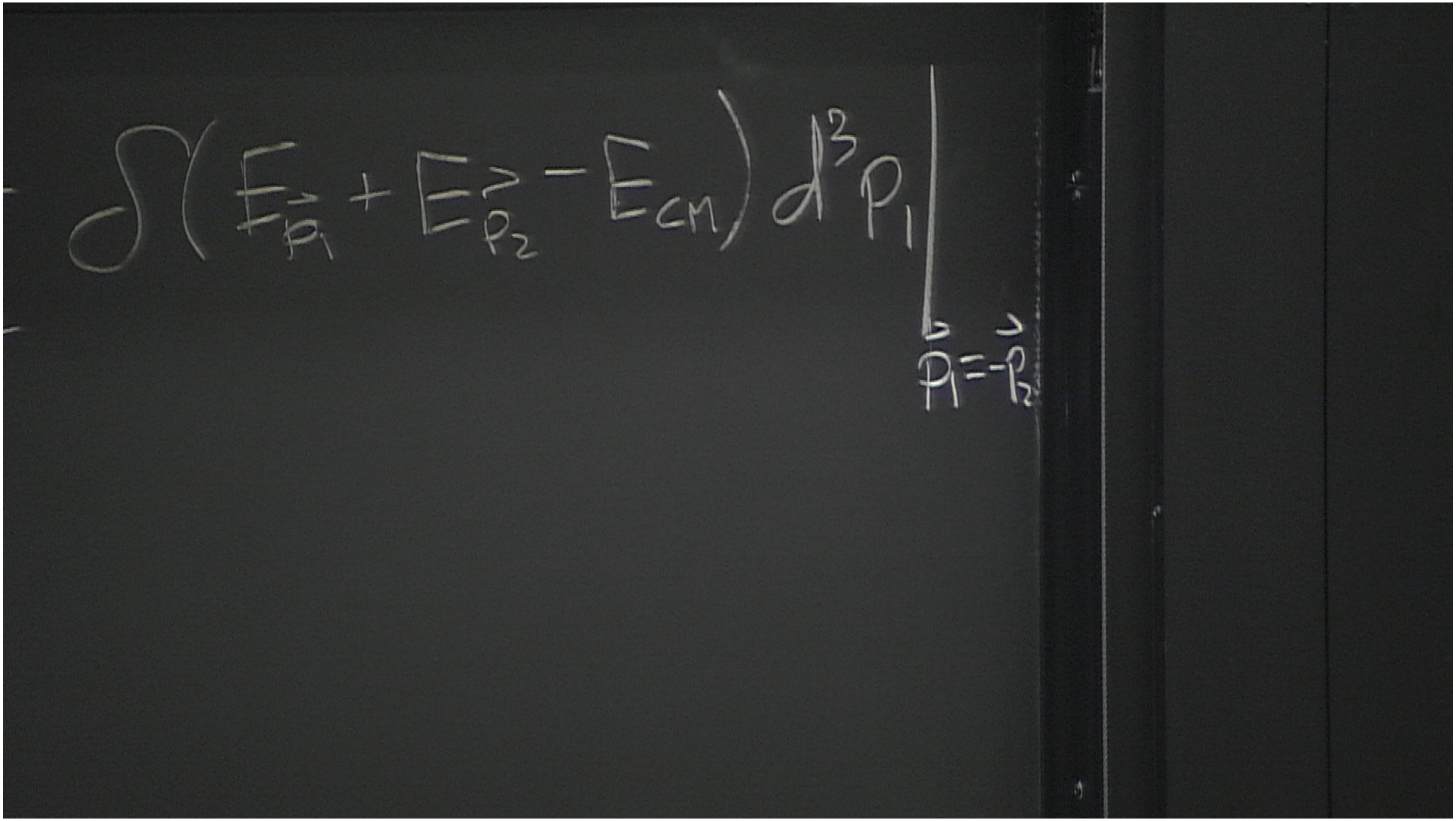
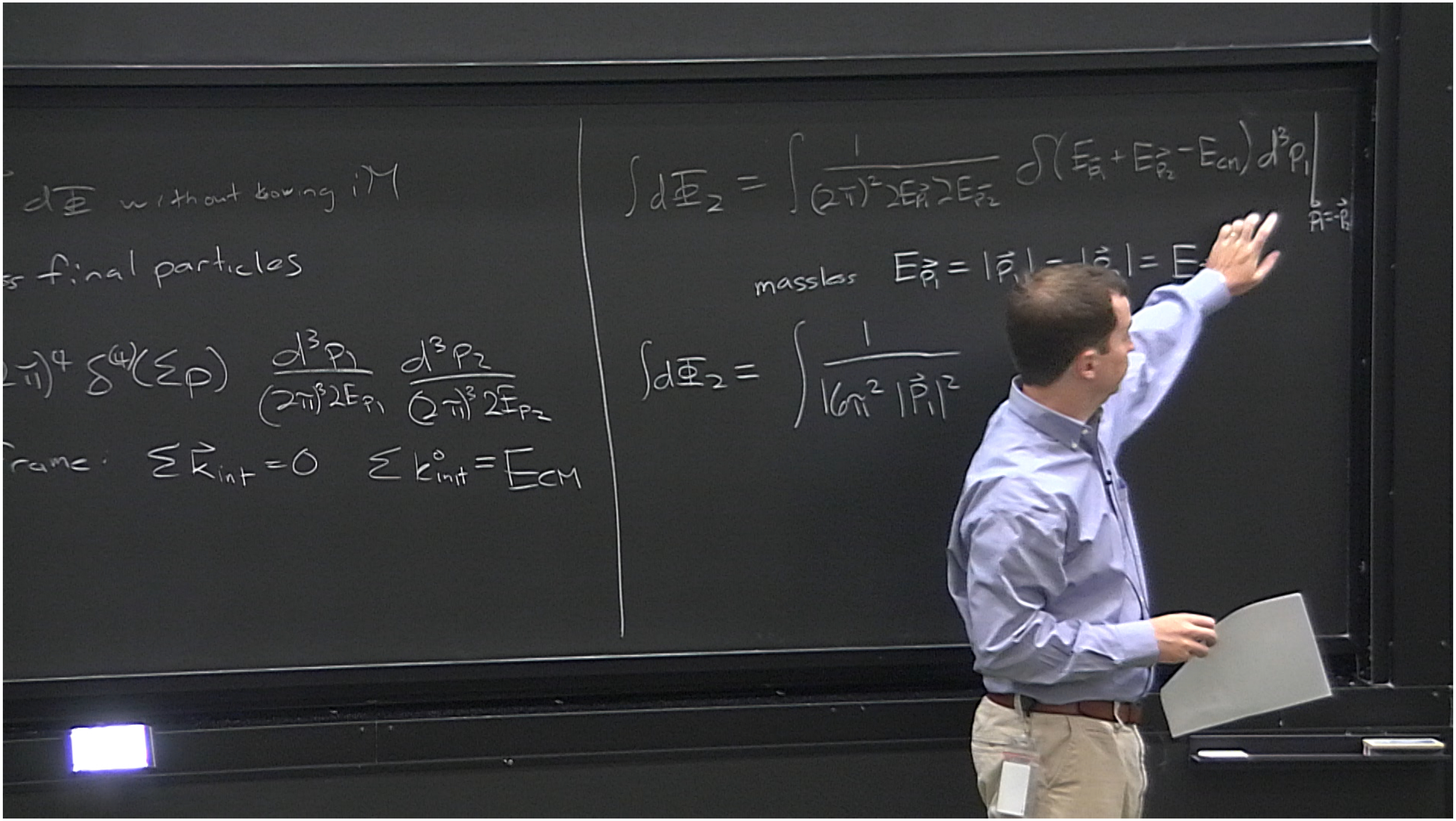
$d\Phi_2 = (2\pi)^4 \delta^{(4)}(\Sigma p) \frac{d^3p_1}{(2\pi)^3 2E_{p_1}} \frac{d^3p_2}{(2\pi)^3 2E_{p_2}}$
COM frame: $\Sigma \vec{k}_{int} = 0$ $\Sigma k^0_{int} = E_{CM}$

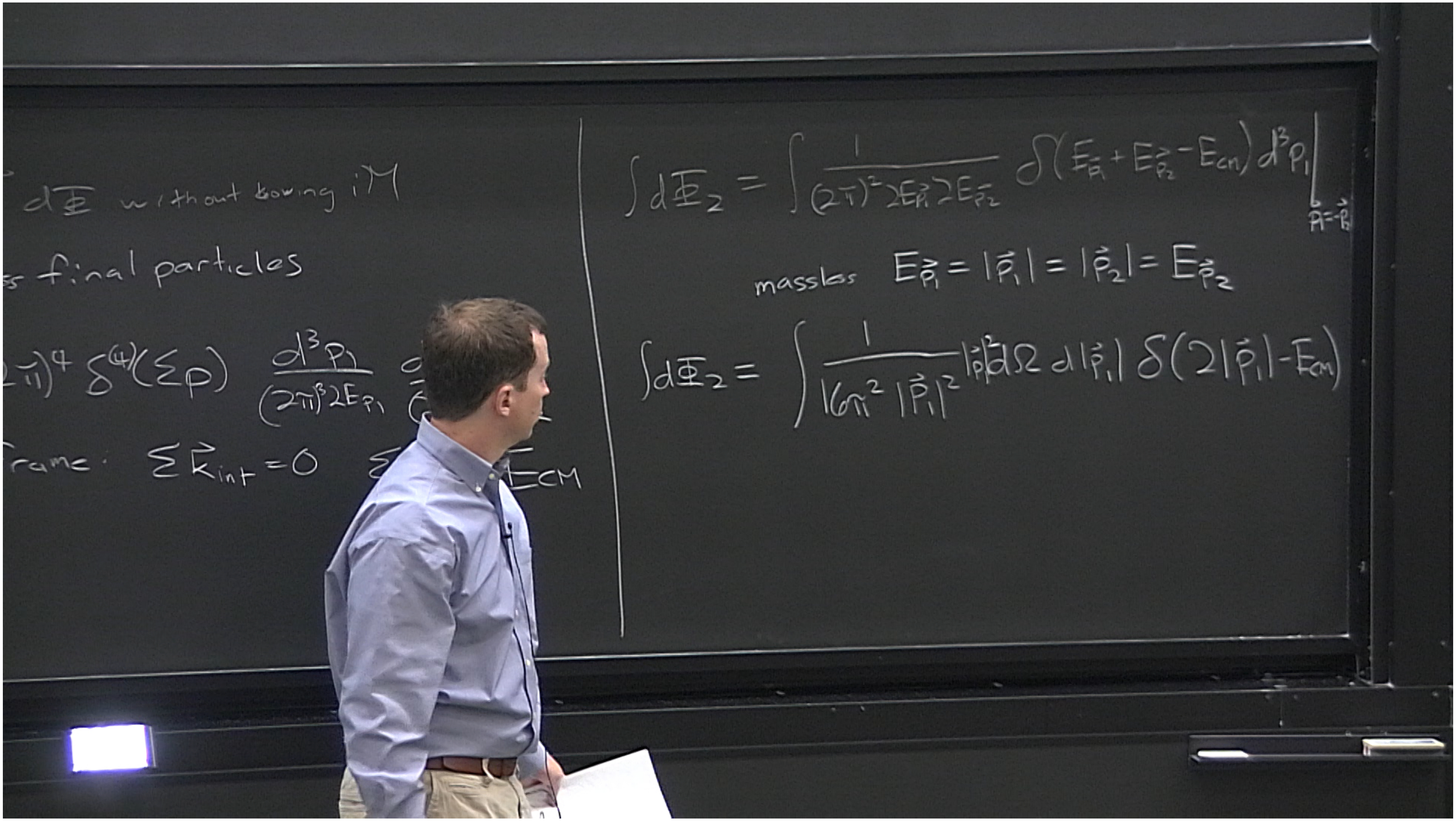

dΦ without knowing iM
final particles
(2π)^4 δ^(4)(Σp) d^3p_1 / (2π)^3 2E_p1 ... E_p2
frame: Σ k_int = 0 ... = E_CM
∫dΦ_2 = ∫ 1/((2π)^2 2E_p1 2E_p2) δ(E_p1 + E_p2 − E_cm) d^3p_1

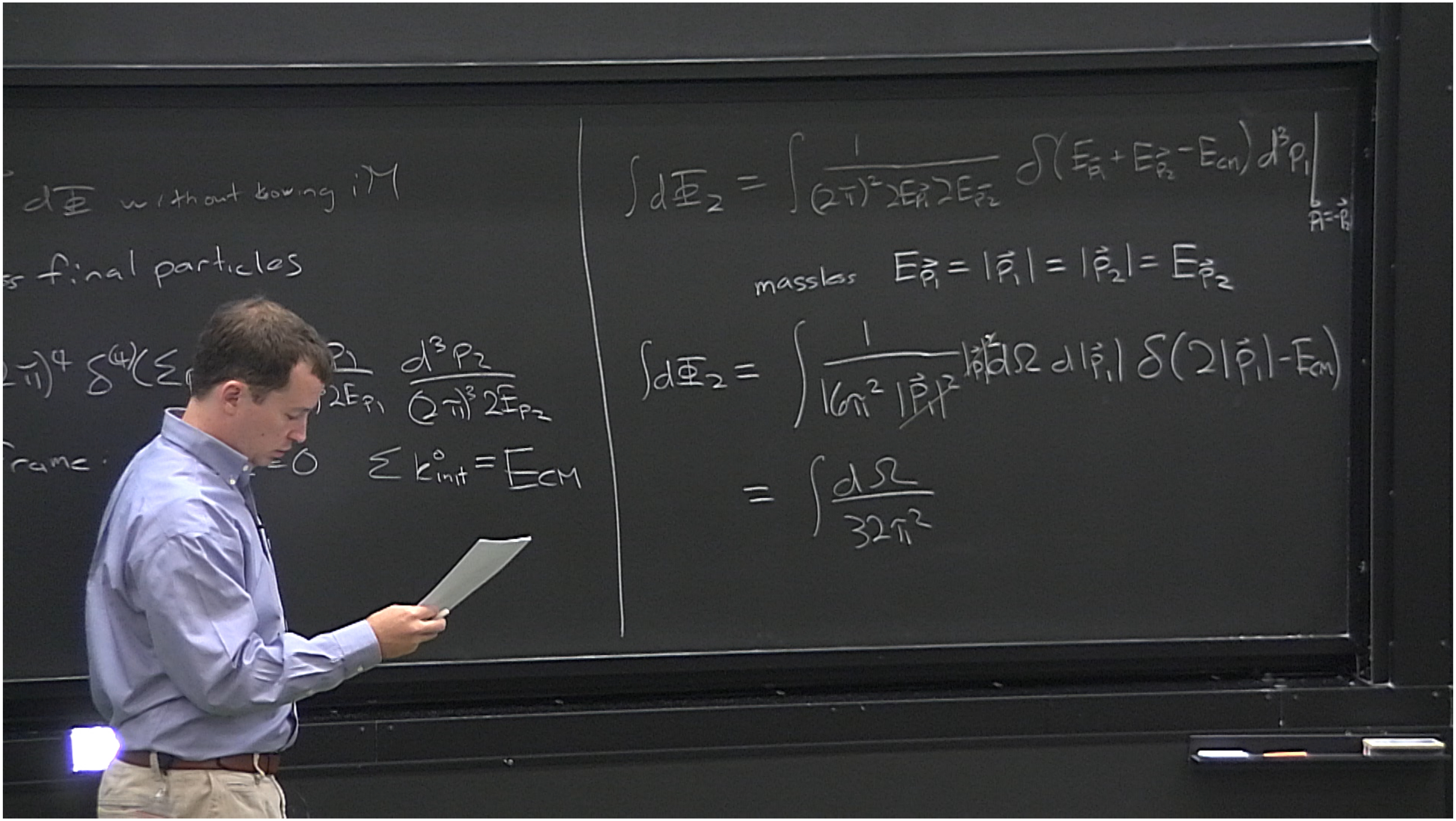


$d\Phi$ without knowing iM
final particles
$(2\pi)^4 \delta^{(4)}(\Sigma p) \frac{d^3p_1}{(2\pi)^3 2E_{p_1}} \frac{d^3p_2}{(2\pi)^3 2E_{p_2}}$
rame: $\Sigma \vec{k}_{init} = 0$ $\Sigma k^0_{init} = E_{CM}$
$\int d\Phi_2 = \int \frac{1}{(2\pi)^2 2E_{\vec{p}_1} 2E_{\vec{p}_2}} \delta(E_{\vec{p}_1} + E_{\vec{p}_2} - E_{cm}) d^3p_1 \Big|_{\vec{p}_1 = -\vec{p}_2}$


$\delta(E_{\vec{p}_1} + E_{\vec{p}_2} - E_{cm})\, d^3p_1 \Big|_{\vec{p}_1 = -\vec{p}_2}$

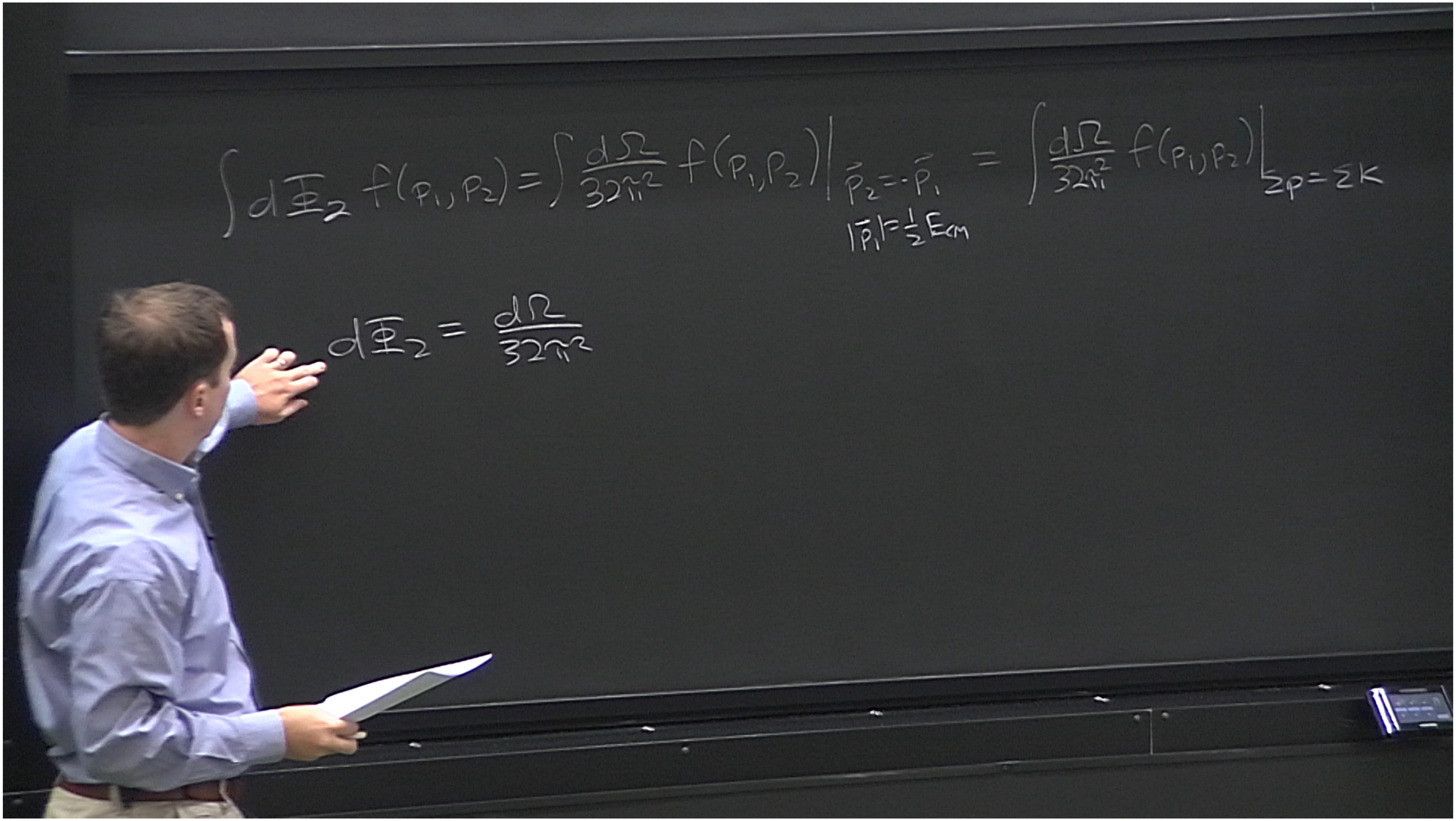

without knowing iM
final particles
massless

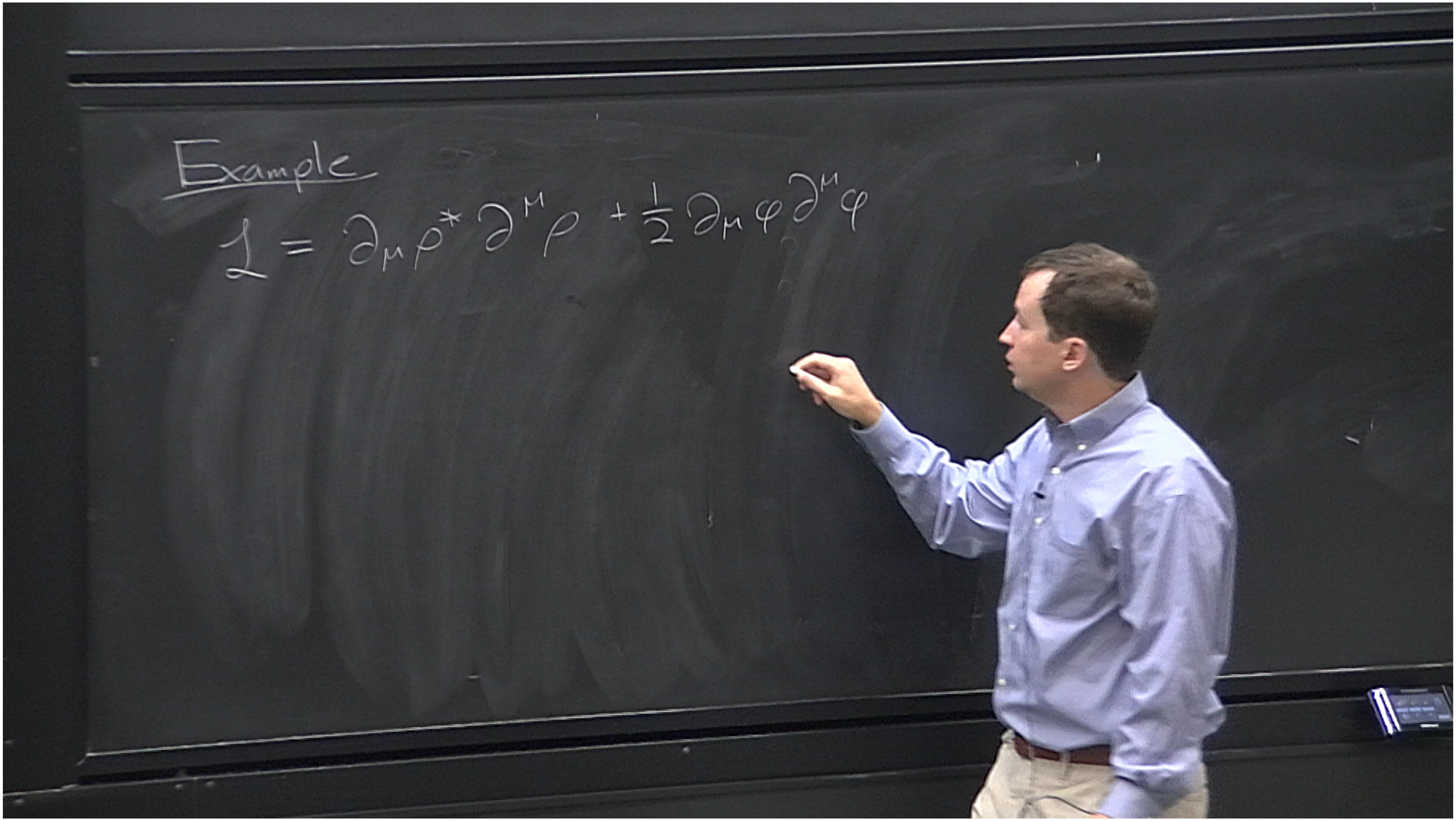

dΦ without knowing iM
final particles
frame: Σ k_int = 0
∫dΦ₂ = ∫ 1/((2π)² 2E_p1 2E_p2) δ(E_p1 + E_p2 − E_cm) d³p₁ |p₁=−p₂
massless E_p1 = |p₁| = |p₂| = E_p2
∫dΦ₂ = ∫ 1/(16π² |p₁|²) |p₁|² dΩ d|p₁| δ(2|p₁| − E_cm)

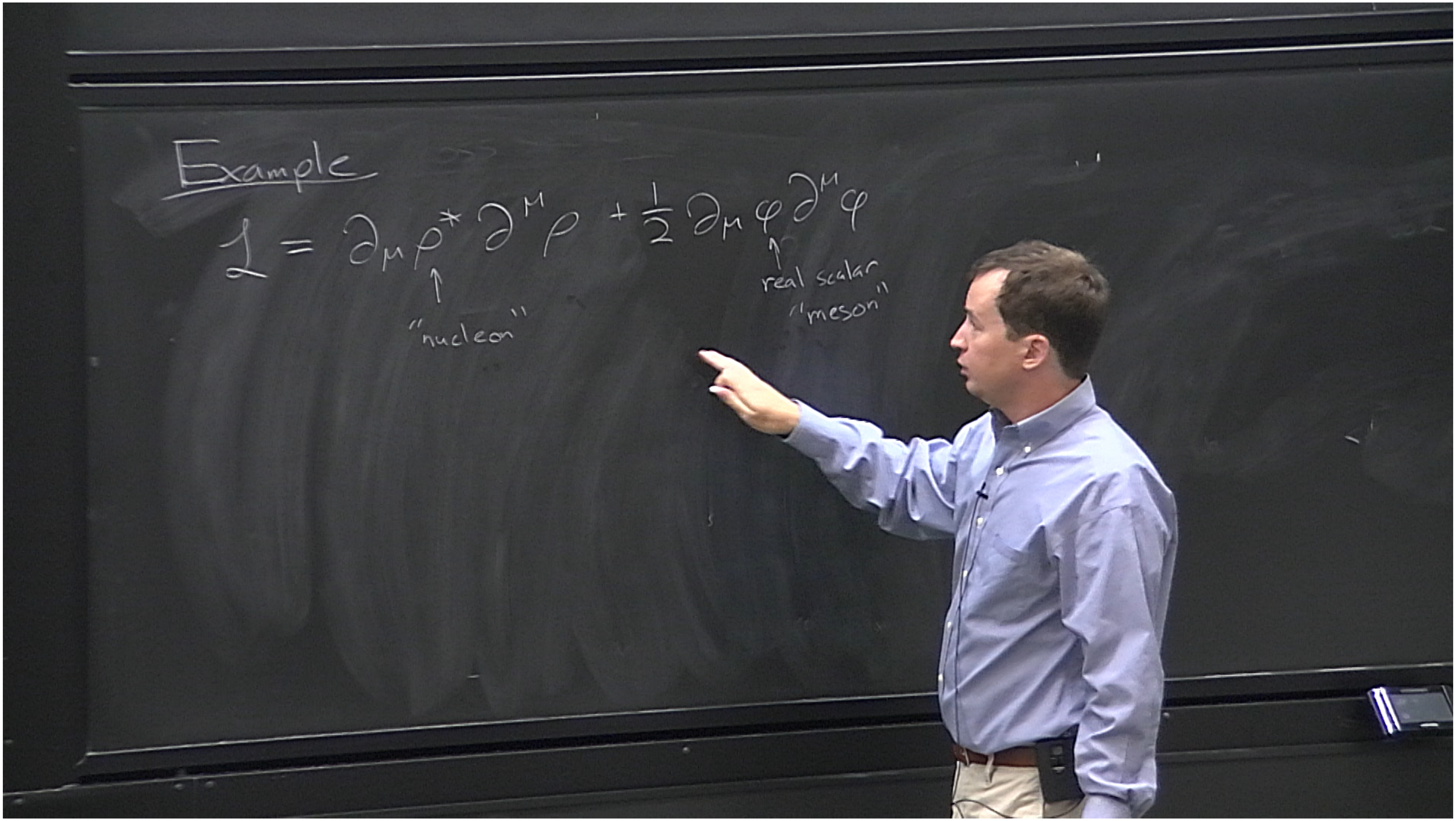


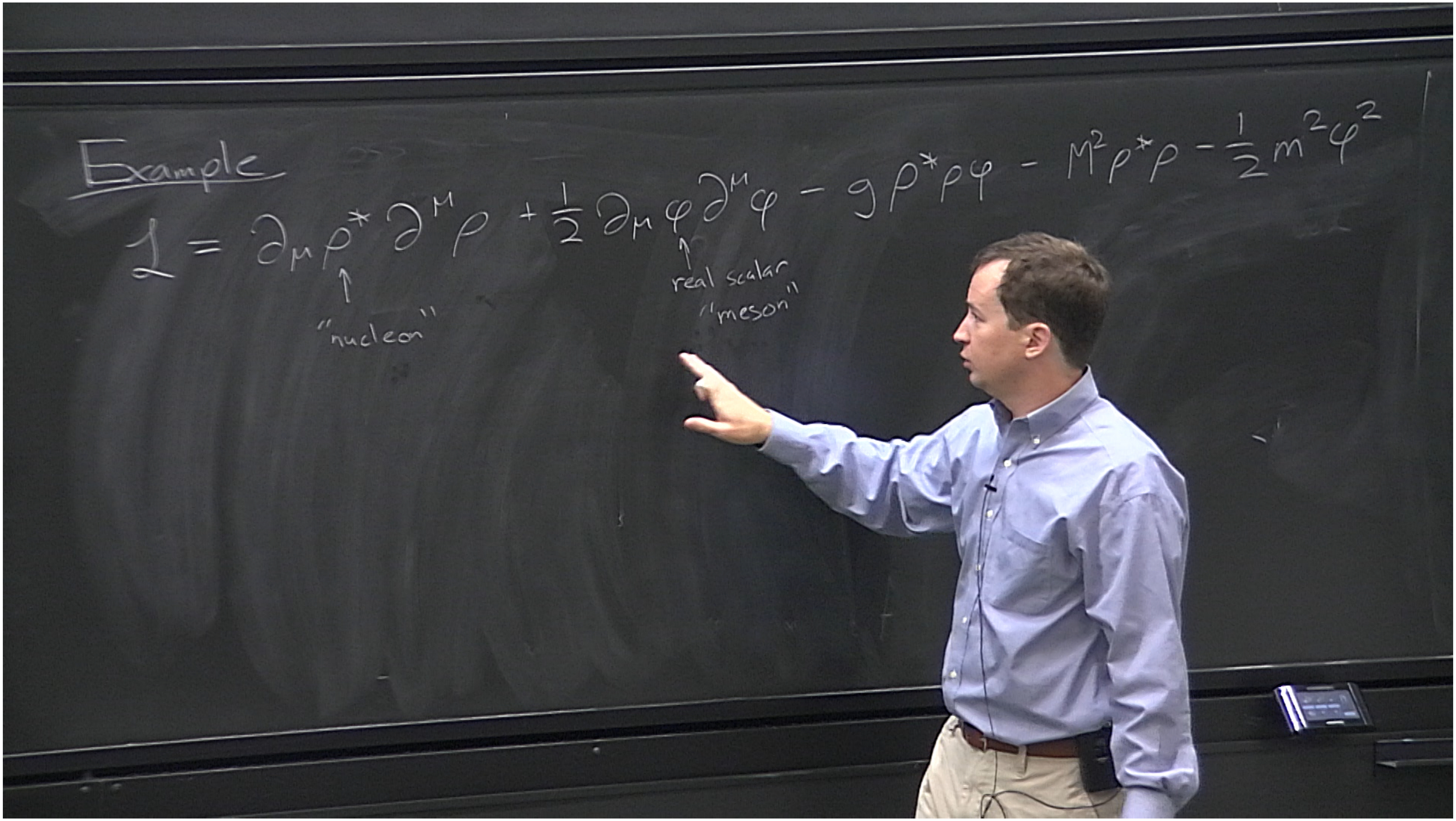
dΦ without knowing iM
final particles
frame
$\int d\Phi_2 = \int \frac{1}{(2\pi)^2 2E_{\vec{p}_1} 2E_{\vec{p}_2}} \delta(E_{\vec{p}_1} + E_{\vec{p}_2} - E_{cm}) d^3p_1 \Big|_{\vec{p}_1 = -\vec{p}_2}$
massless $E_{\vec{p}_1} = |\vec{p}_1| = |\vec{p}_2| = E_{\vec{p}_2}$
$\int d\Phi_2 = \int \frac{1}{16\pi^2 |\vec{p}_1|^2} |\vec{p}_1|^2 d\Omega \, d|\vec{p}_1| \, \delta(2|\vec{p}_1| - E_{cm})$
$= \int \frac{d\Omega}{32\pi^2}$

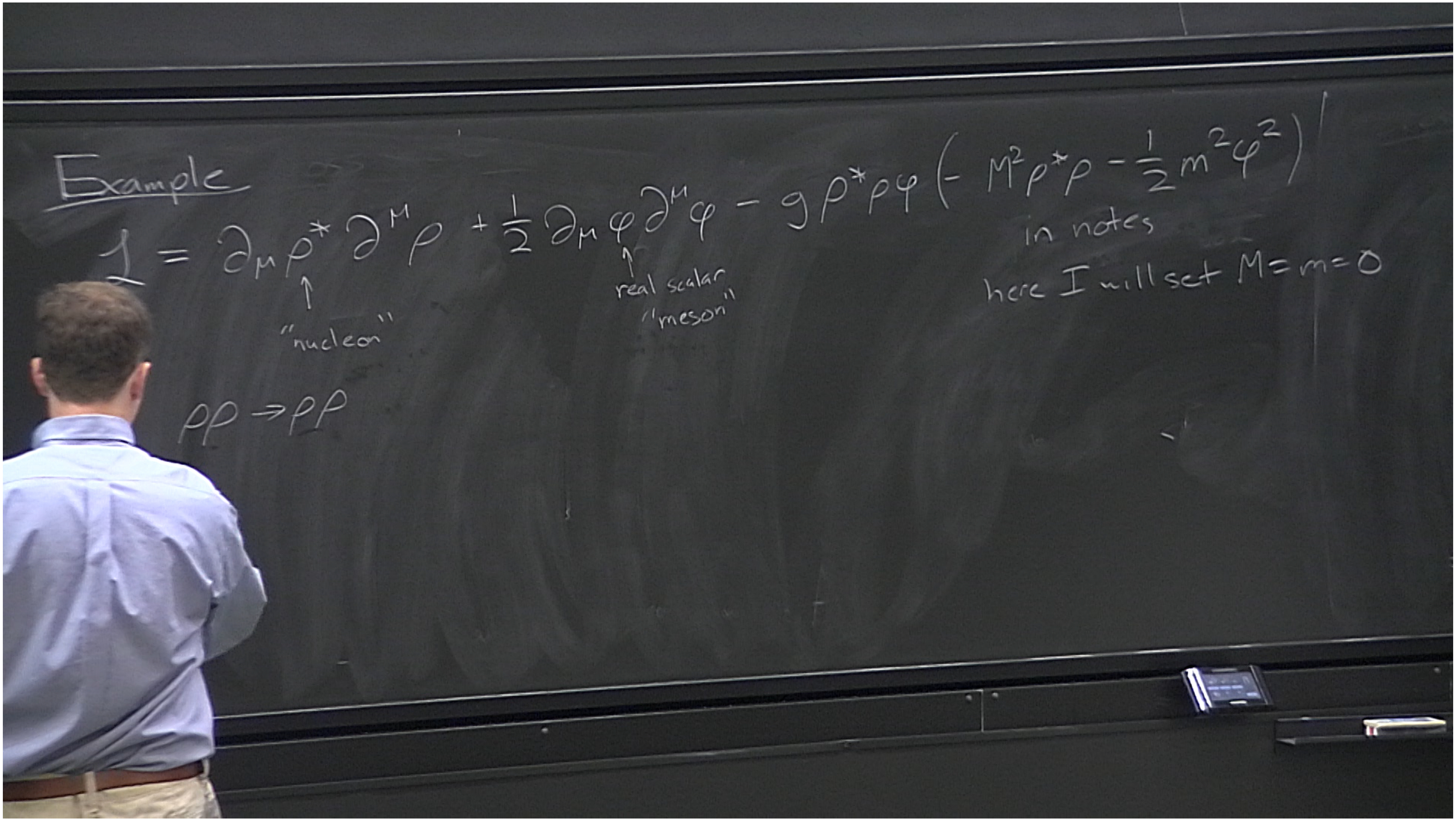
$$\int d\Phi_2\, f(p_1, p_2) = \int \frac{d\Omega}{32\pi^2} f(p_1, p_2)\Big|_{\substack{\vec{p}_2 = -\vec{p}_1 \\ |\vec{p}_1| = \frac{1}{2} E_{cm}}} = \int \frac{d\Omega}{32\pi^2} f(p_1, p_2)\Big|_{\Sigma p = \Sigma K}$$

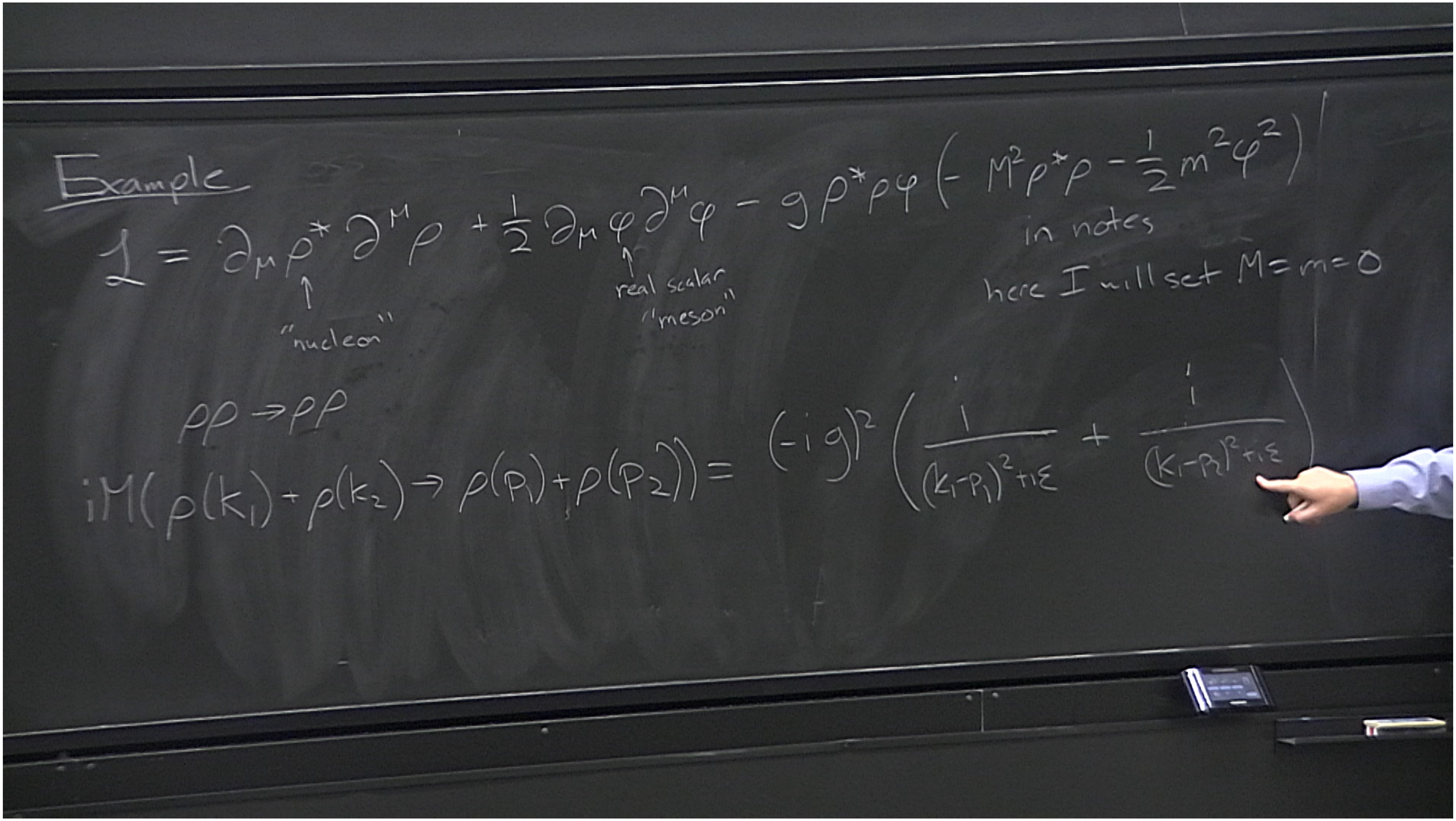

$\int d\Phi_2 \, f(p_1,p_2) = \int \frac{d\Omega}{32\pi^2} f(p_1,p_2)\Big|_{\vec{p}_2=-\vec{p}_1} = \int \frac{d\Omega}{32\pi^2} f(p_1,p_2)\Big|_{\Sigma p=\Sigma K}$
$|\vec{p}_1| = \frac{1}{2}E_{cm}$
$d\Phi_2 = \frac{d\Omega}{32\pi^2}$

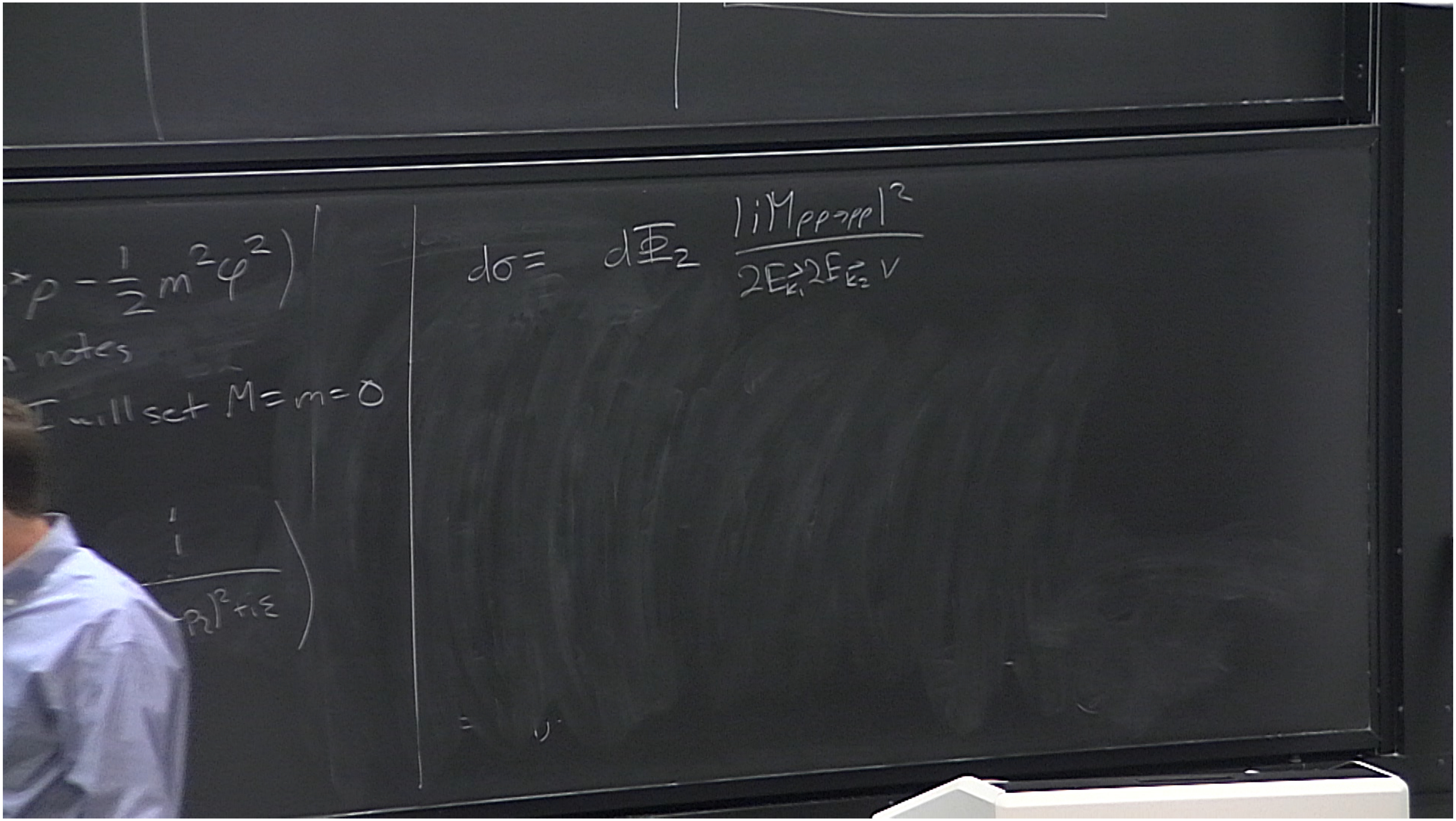

Example
$\mathcal{L} = \partial_\mu \rho^* \partial^\mu \rho + \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi$

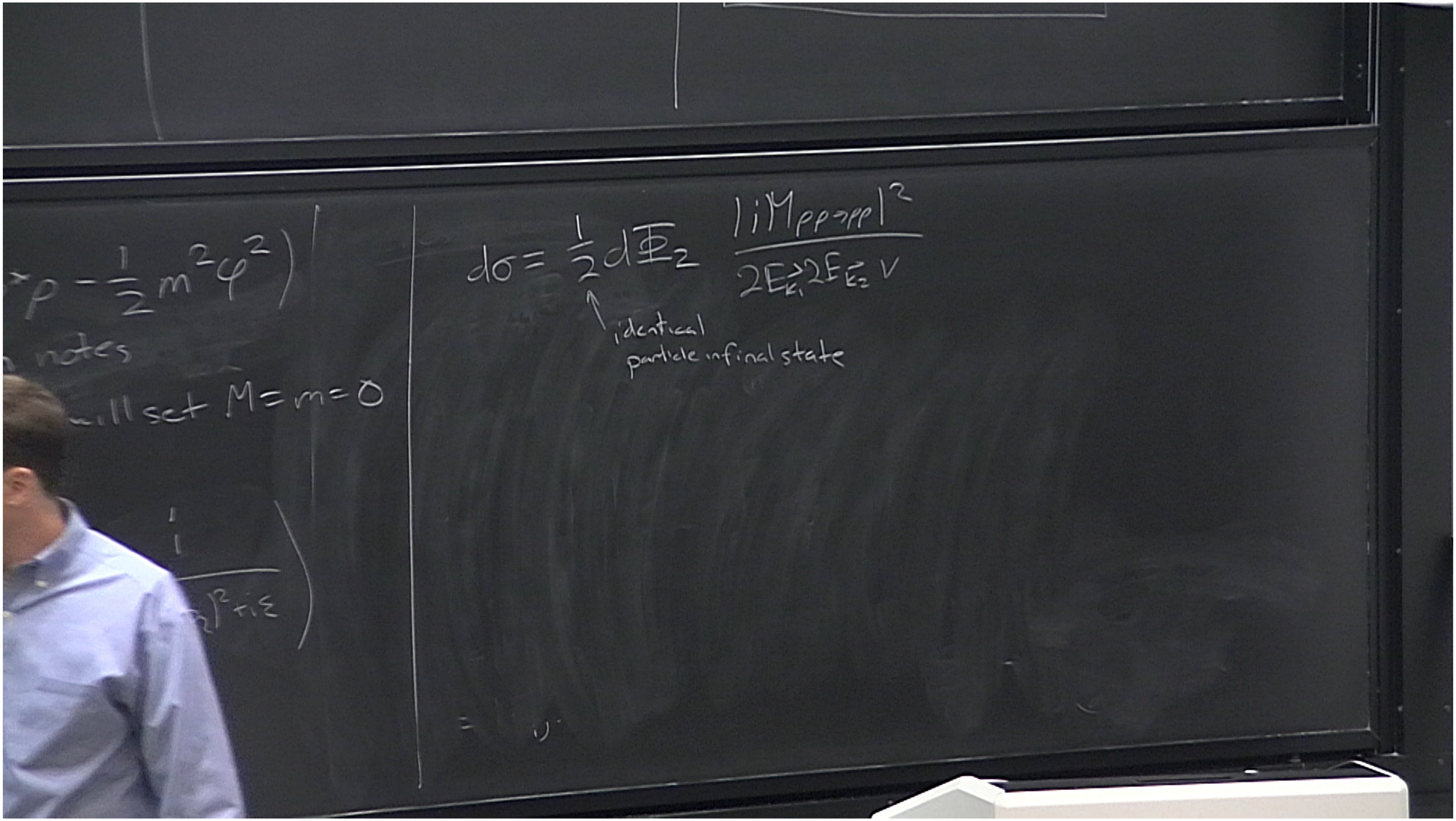

Example
$\mathcal{L} = \partial_\mu \rho^* \partial^\mu \rho + \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi$
"nucleon"
real scalar
"meson"

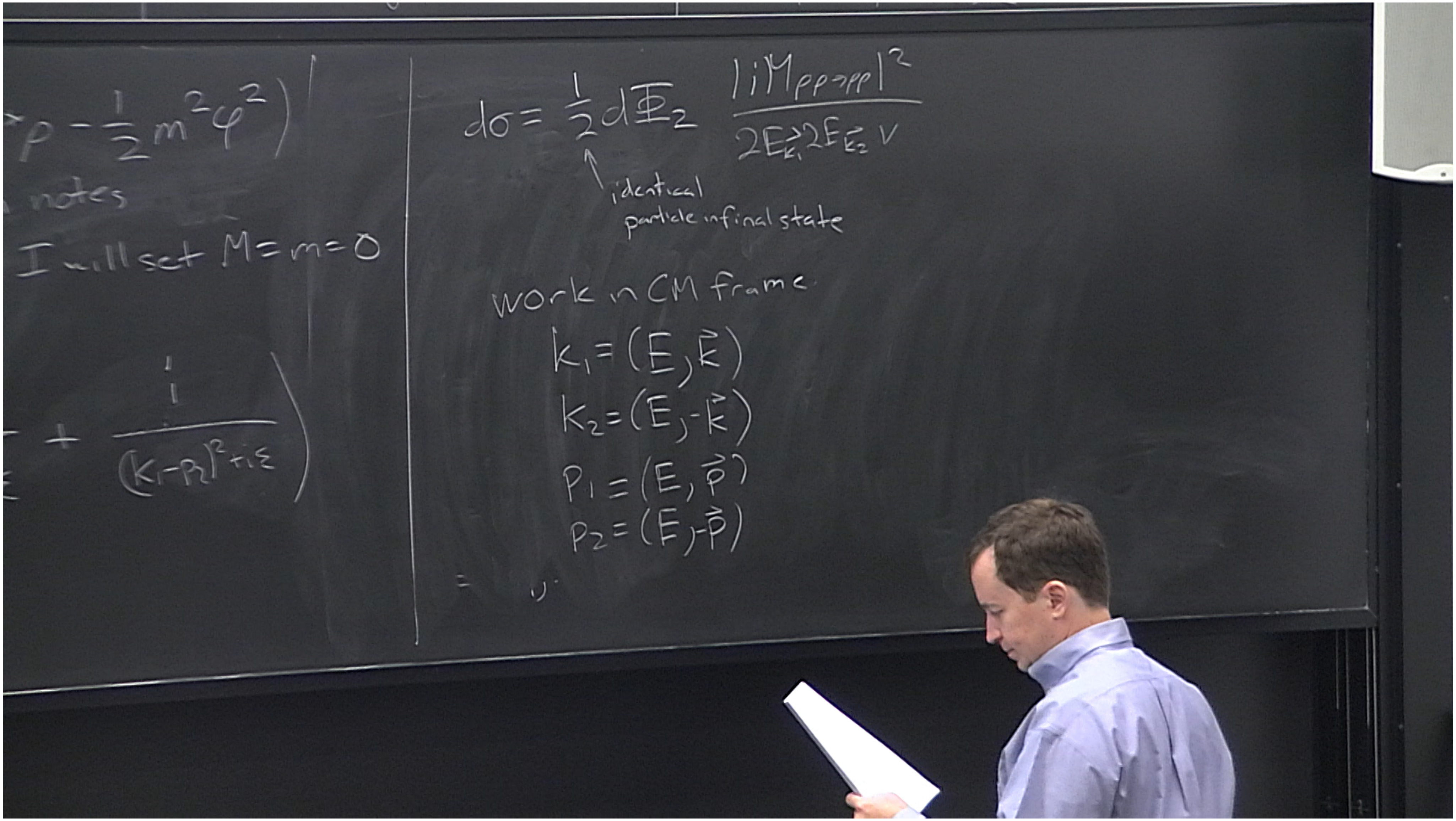

Example
$\mathcal{L} = \partial_\mu \rho^* \partial^\mu \rho + \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - g \rho^* \rho \varphi - M^2 \rho^* \rho - \frac{1}{2} m^2 \varphi^2$
"nucleon"
real scalar
"meson"


Example
$\mathcal{L} = \partial_\mu \rho^* \partial^\mu \rho + \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi - g\rho^*\rho\varphi \left(- M^2\rho^*\rho - \frac{1}{2}m^2\varphi^2\right)$
"nucleon"
real scalar
"meson"
in notes
here I will set M = m = 0
$\rho\rho \to \rho\rho$

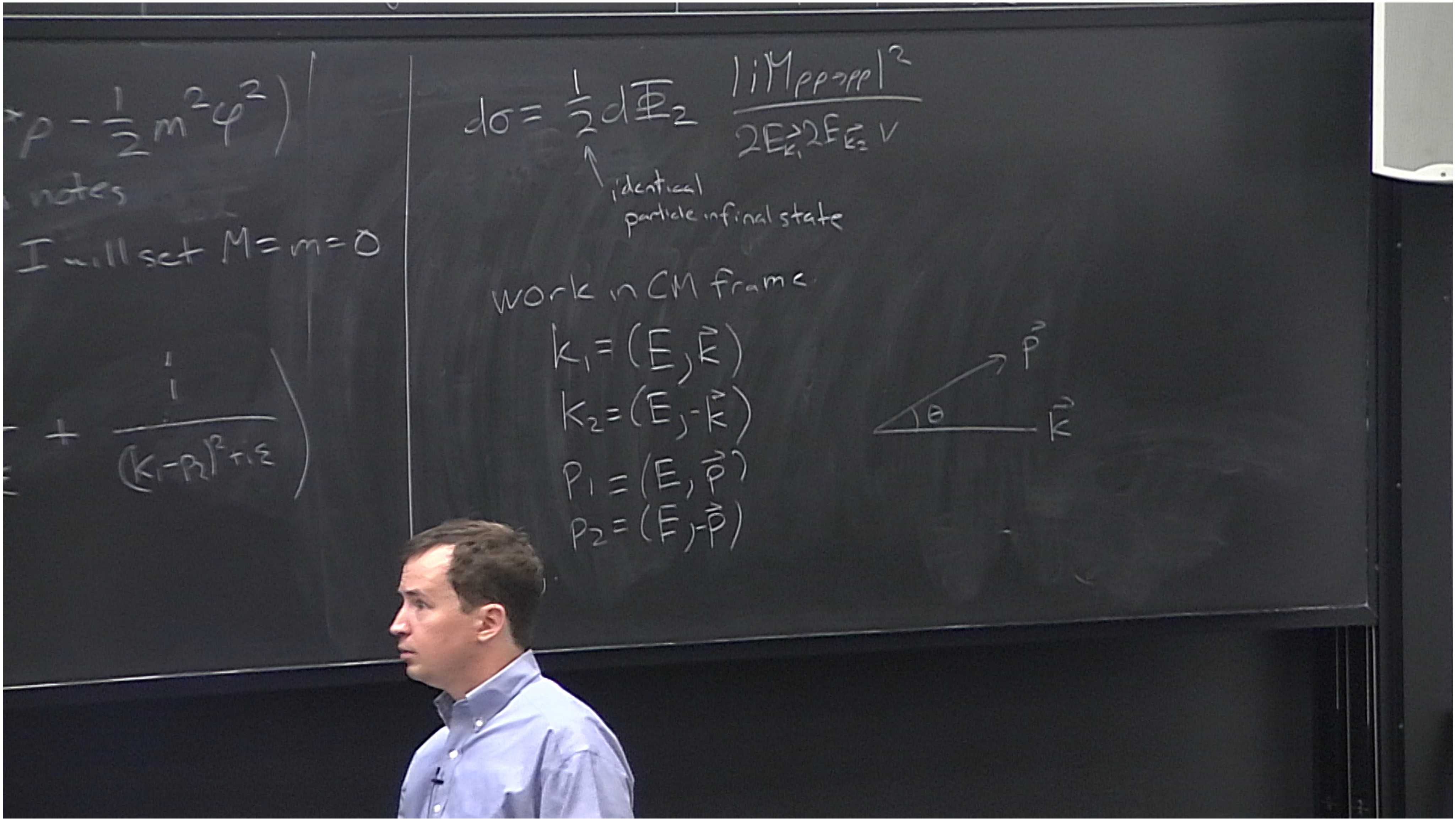

Example
$\mathcal{L} = \partial_\mu \rho^* \partial^\mu \rho + \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi - g\rho^*\rho\varphi \left(- M^2\rho^*\rho - \frac{1}{2}m^2\varphi^2\right)$
"nucleon"
real scalar "meson"
in notes
here I will set M=m=0
$\rho\rho \to \rho\rho$
$iM(\rho(k_1)+\rho(k_2)\to\rho(p_1)+\rho(p_2)) = (-ig)^2\left(\frac{i}{(k_1-p_1)^2+i\epsilon} + \frac{i}{(k_1-p_2)^2+i\epsilon}\right)$


notes
I will set M=m=0
dσ= dΦ2 |iM φφ→φφ|^2 / 2E 2E V

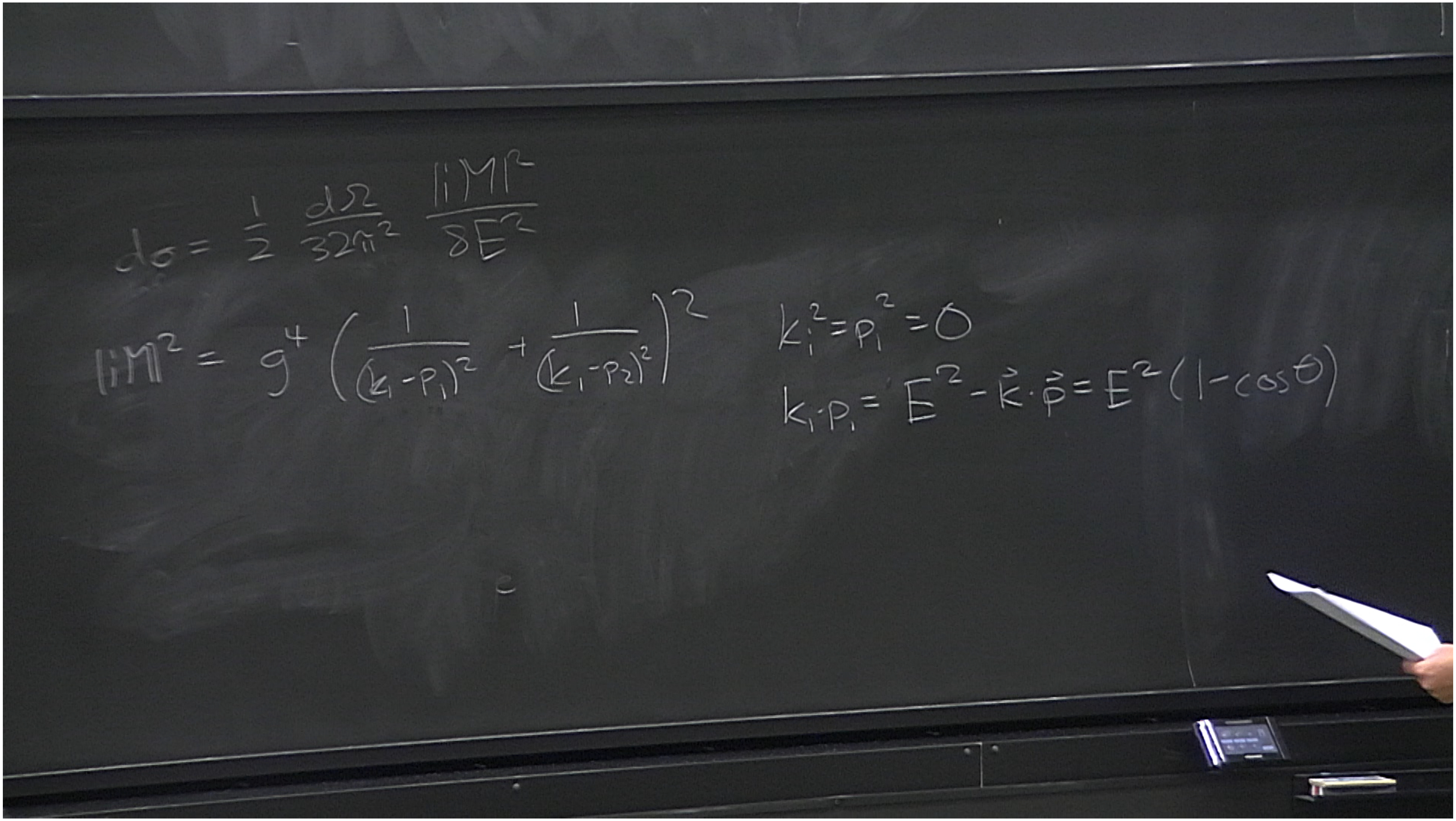

dσ = 1/2 dΦ₂ |iM_pp→pp|² / 2E 2E V
identical
particle in final state
notes
will set M = m = 0

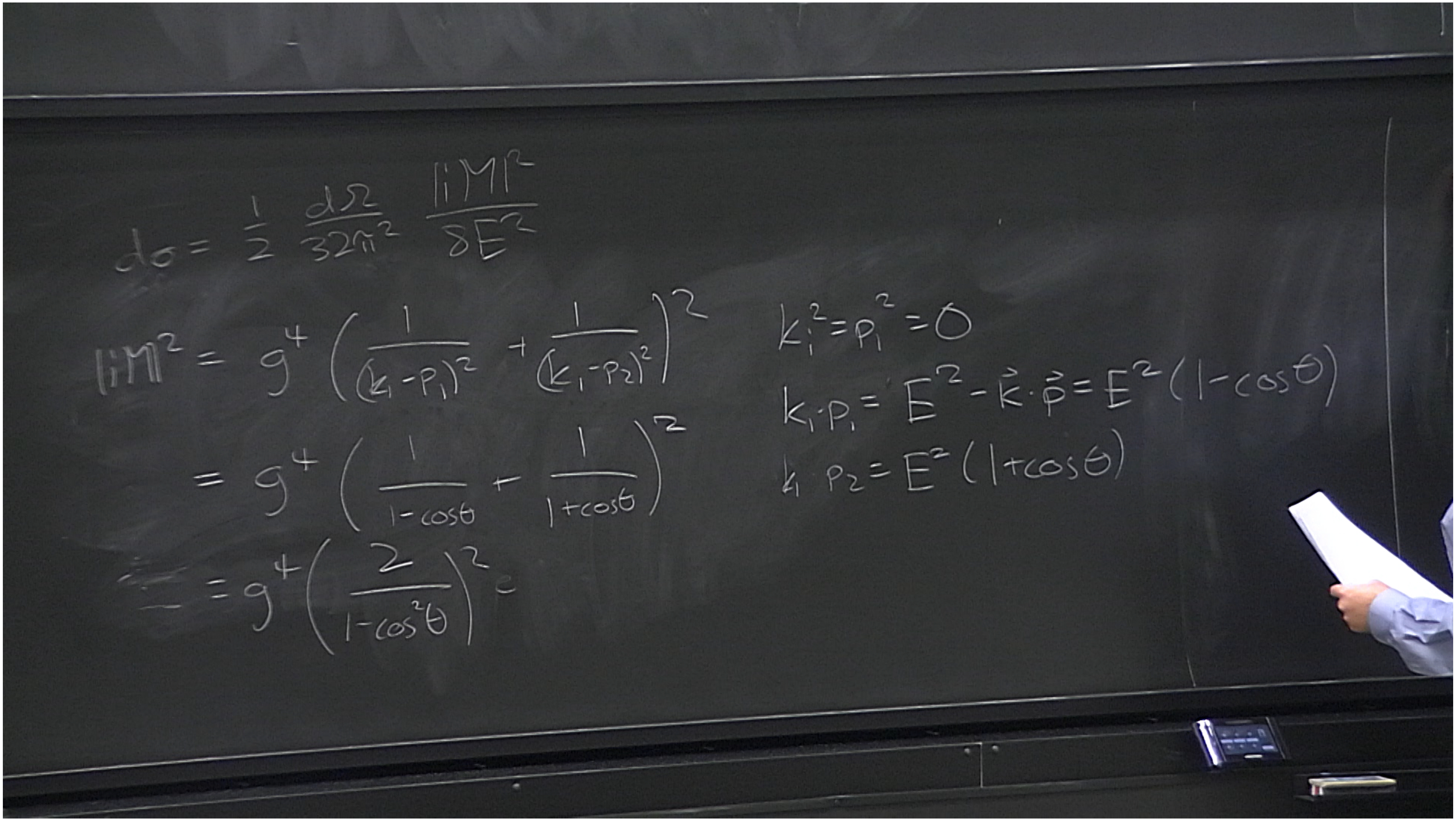


$d\sigma = \frac{1}{2} d\Phi_2 \frac{|iM_{pp\to pp}|^2}{2E_{\vec{k}_1} 2E_{\vec{k}_2} V}$
identical particle in final state
work in CM frame
$k_1 = (E, \vec{k})$
$k_2 = (E, -\vec{k})$
$p_1 = (E, \vec{p})$
$p_2 = (E, -\vec{p})$
$-\frac{1}{2}m^2\varphi^2)$
notes
I will set $M = m = 0$
$+ \frac{i}{(k_1-p_2)^2 + i\varepsilon}$



dσ = 1/2 dΦ_2 |iM_pp→pp|^2 / 2E_k1 2E_k2 V
identical particle in final state
I will set M = m = 0
work in CM frame
k1 = (E, k)
k2 = (E, -k)
p1 = (E, p)
p2 = (E, -p)
θ


$d\sigma = \frac{1}{2} \frac{d\Omega}{32\pi^2} \frac{|iM|^2}{8E^2}$
$|iM|^2 = g^4 \left(\frac{1}{(k_1-p_1)^2} + \frac{1}{(k_1-p_2)^2} \right)^2$
$k_1^2 = p_1^2 = 0$
$k_1 \cdot p_1 = E^2 - \vec{k}\cdot\vec{p} = E^2(1-\cos\theta)$



$d\sigma = \frac{1}{2} \frac{d\Omega}{32\pi^2} \frac{|iM|^2}{8E^2}$
$|iM|^2 = g^4 \left(\frac{1}{(k_1 - p_1)^2} + \frac{1}{(k_1 - p_2)^2} \right)^2$
$= g^4 \left(\frac{1}{1-\cos\theta} + \frac{1}{1+\cos\theta} \right)^2$
$= g^4 \left(\frac{2}{1-\cos^2\theta} \right)^2$
$k_1^2 = p_1^2 = 0$
$k_1 \cdot p_1 = E^2 - \vec{k} \cdot \vec{p} = E^2 (1 - \cos\theta)$
$k_1 \cdot p_2 = E^2 (1 + \cos\theta)$

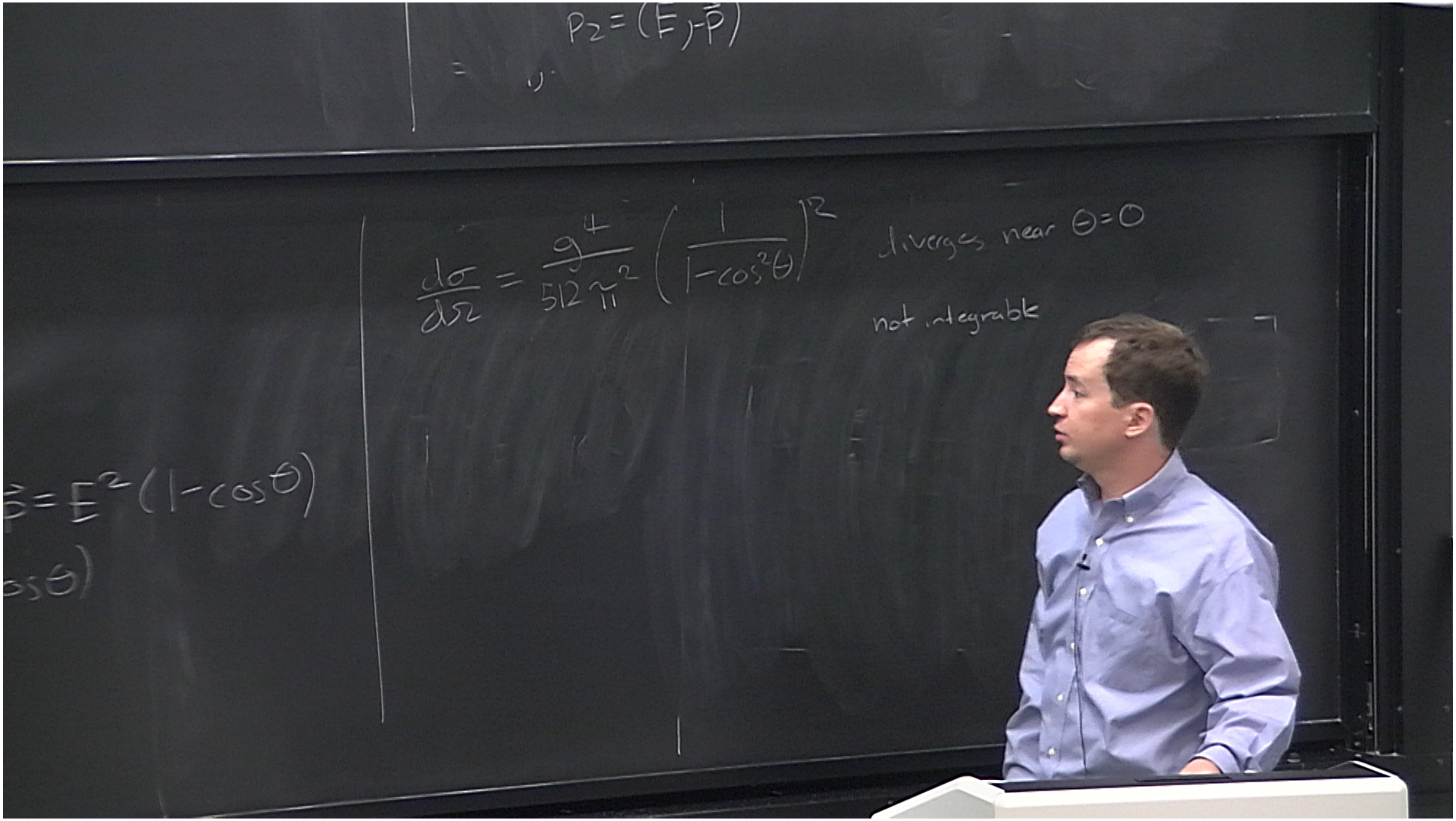
$$d\sigma = \frac{1}{2}\frac{d\Omega}{32\pi^2}\frac{|iM|^2}{8E^2} \qquad |v_1 - v_2| = 2$$

$$|iM|^2 = g^4\left(\frac{1}{(k_1-p_1)^2} + \frac{1}{(k_1-p_2)^2}\right)^2 \qquad k_i^2 = p_i^2 = 0$$

$$k_1 \cdot p_1 = E^2 - \vec{k}\cdot\vec{p} = E^2(1-\cos\theta)$$

$$= \frac{g^4}{4E^4}\left(\frac{1}{1-\cos\theta} + \frac{1}{1+\cos\theta}\right)^2 \qquad k_1\, p_2 = E^2(1+\cos\theta)$$

$$= \frac{g^4}{4E^4}\left(\frac{2}{1-\cos^2\theta}\right)^2$$



$p_2 = (E, -\vec{p})$
$\frac{d\sigma}{d\Omega} = \frac{g^4}{512\pi^2}\left(\frac{1}{1-\cos^2\theta}\right)^2$
diverges near $\theta = 0$
not integrable
$= E^2(1-\cos\theta)$