Title: PSI 2018/2019 - Quantum Field Theory I - Lecture 1

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Abstract:

Quantum Field Theory I

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Goal Unify Special Relativity + Quantum Mechanics

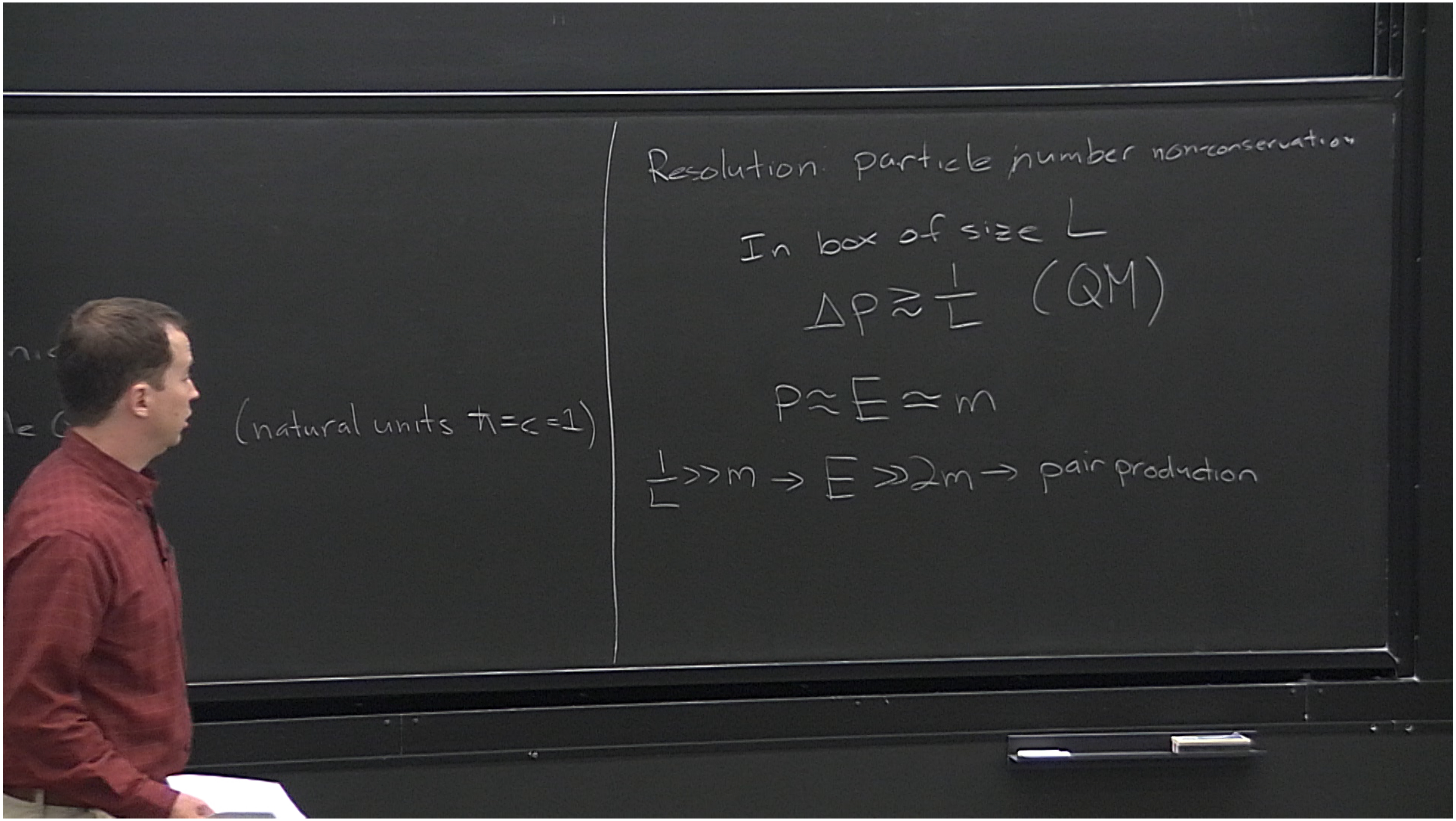
If we assume $H|\vec{p}\rangle = \sqrt{\vec{p}^2 + m^2}\,|\vec{p}\rangle$ in single-particle QM (natural units $\hbar = c = 1$)

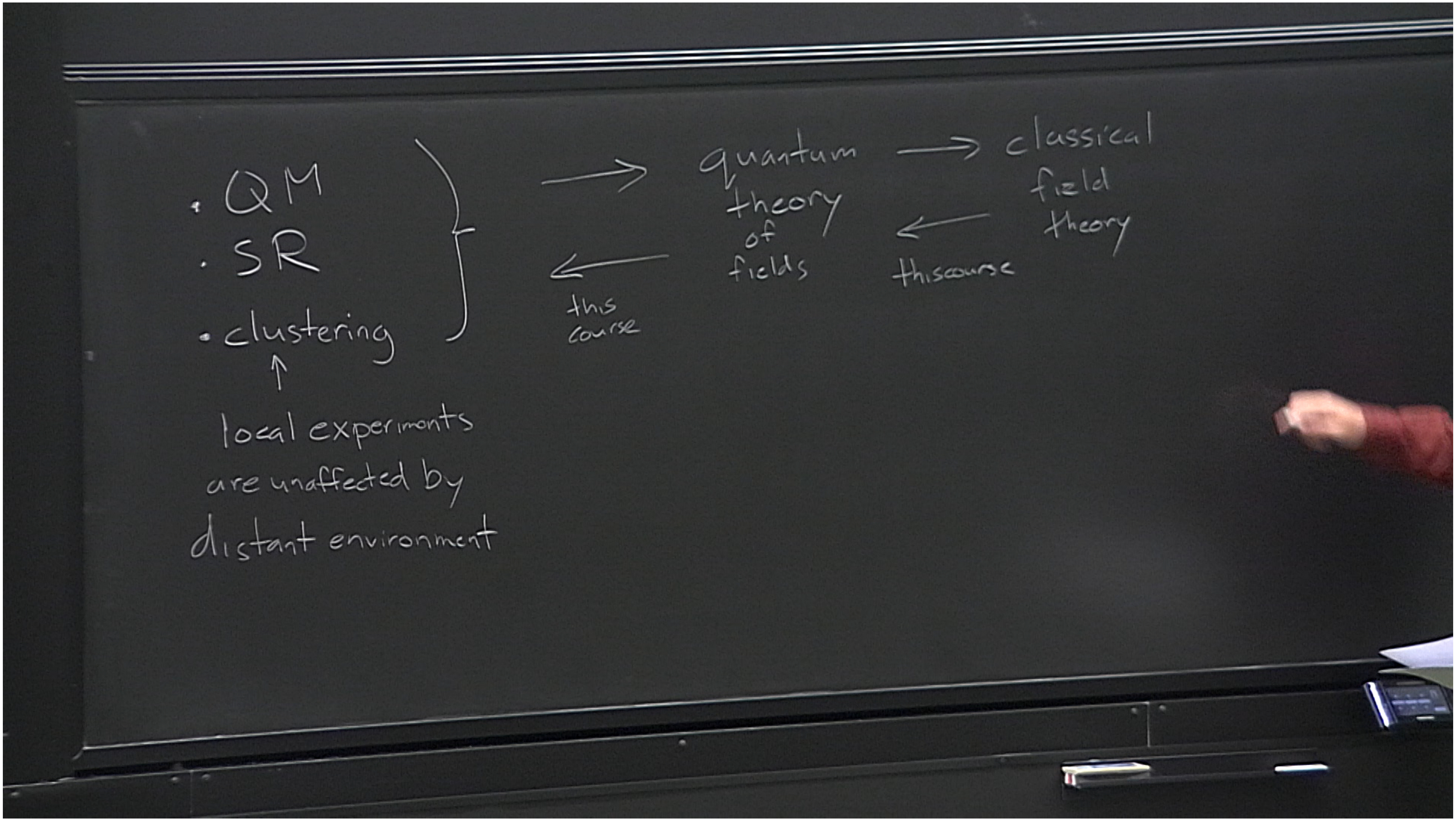
$$\langle \vec{x}|e^{-iHt}|\vec{x}=0\rangle \neq 0 \quad \text{for } |\vec{x}| > t$$

tutorial

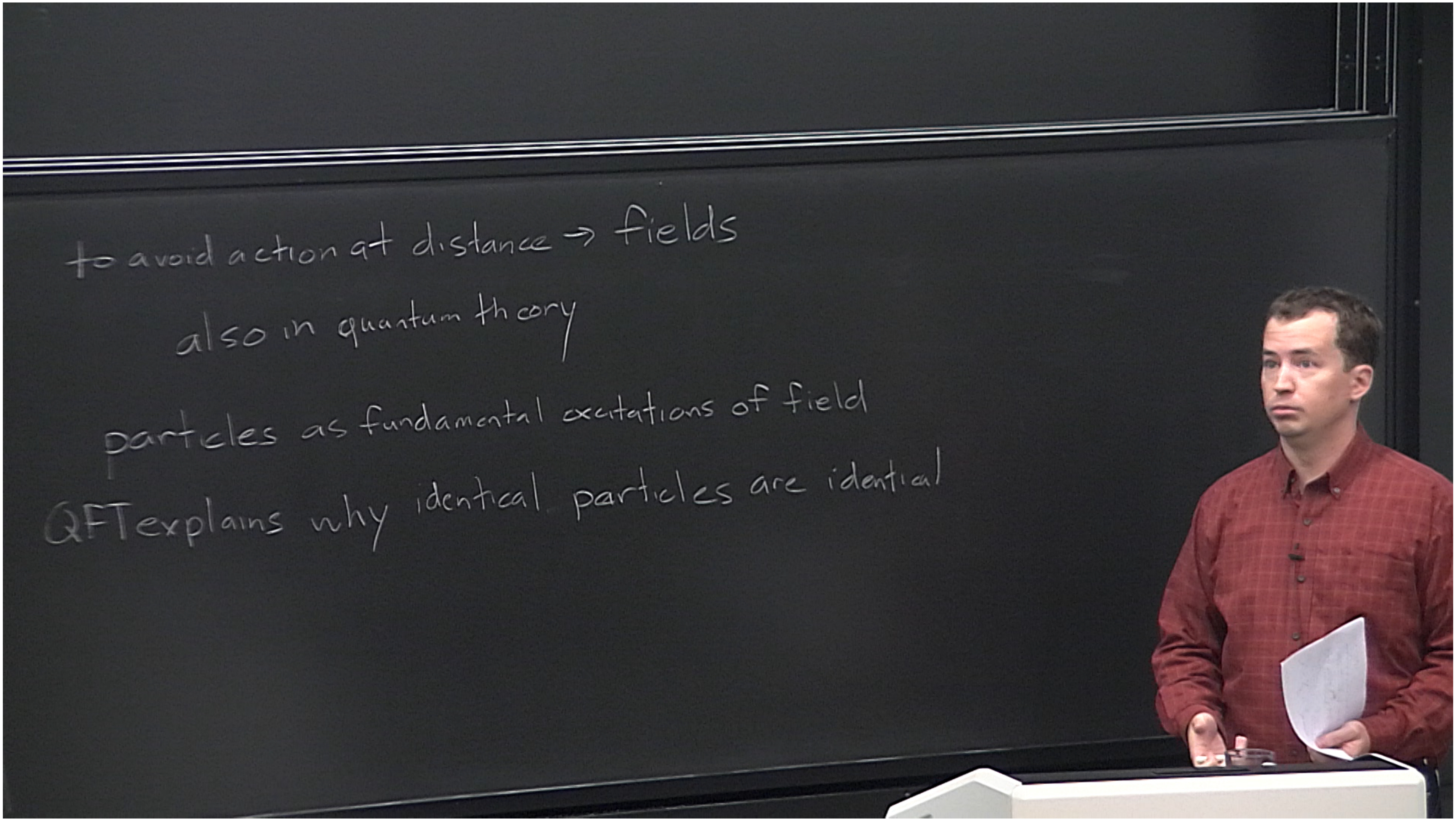
$\vec{x} \to$ 3-vector - bold

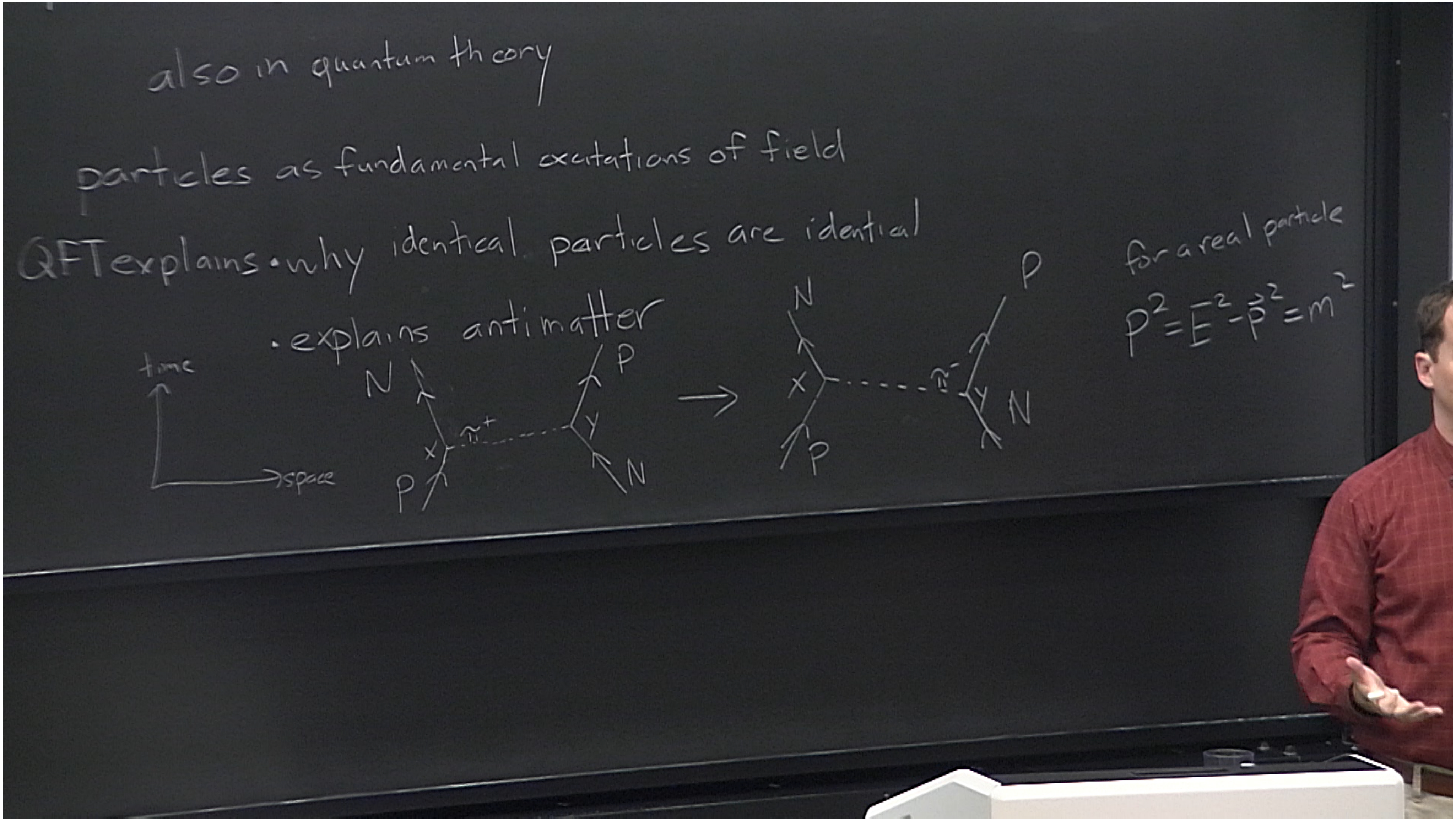
particles can travel faster than light! $x \to$ 4-vector - italic

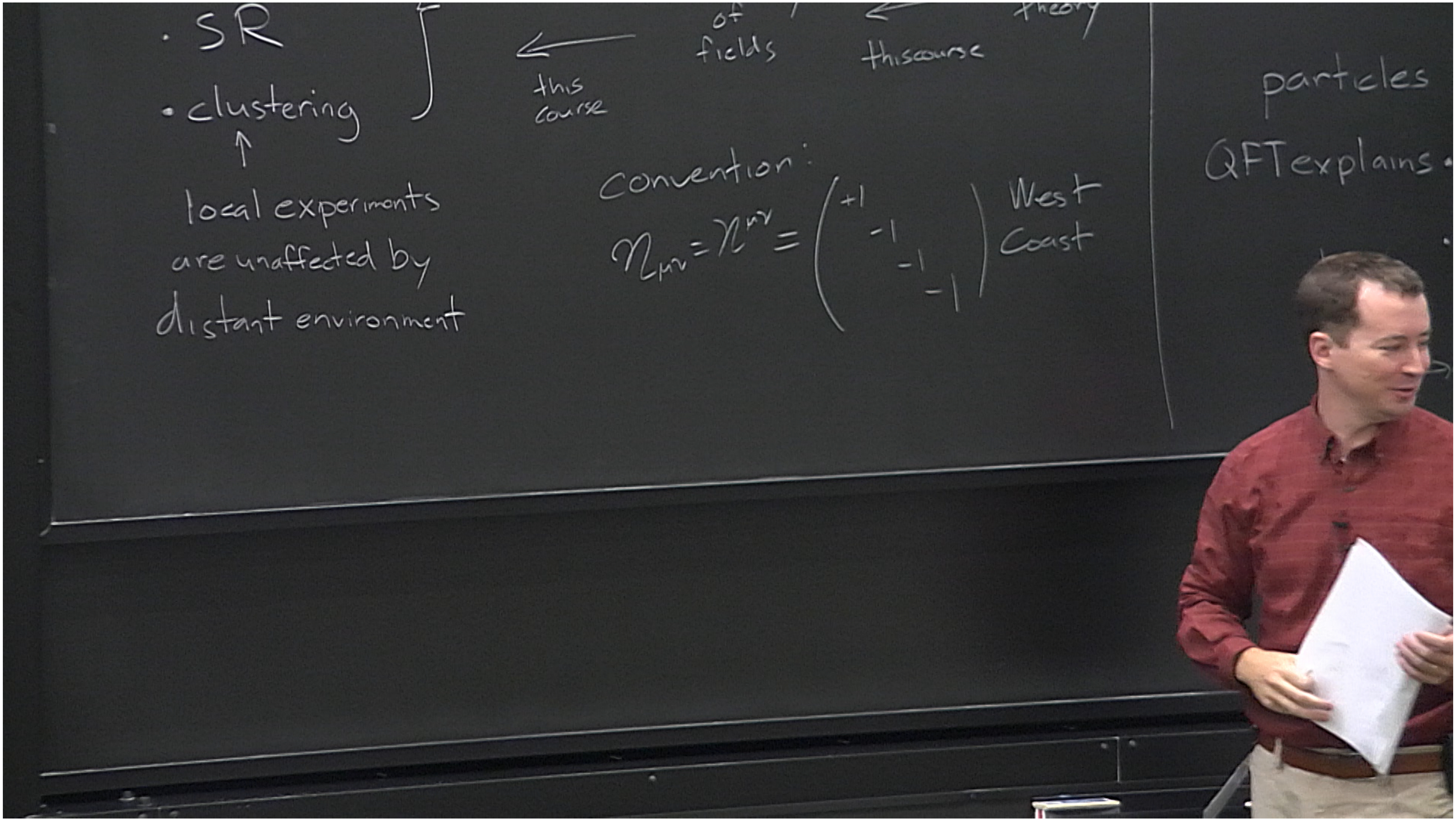

(natural units $\hbar = c = 1$)
Resolution: particle number non-conservation
In box of size L
$\Delta p \gtrsim \frac{1}{L}$ (QM)
$p \approx E \simeq m$
$\frac{1}{L} >> m \rightarrow E \gg 2m \rightarrow$ pair production



· QM
· SR
· clustering
local experiments are unaffected by distant environment
quantum theory of fields
this course
classical field theory
this course


to avoid action at distance → fields
also in quantum theory
particles as fundamental excitations of field
QFT explains why identical particles are identical


also in quantum theory
particles as fundamental excitations of field
QFT explains • why identical particles are identical
• explains antimatter
time
space
N
P
x
y
π+
→
π−
for a real particle
$p^2 = E^2 - \vec{p}^2 = m^2$


· SR
· clustering
local experiments
are unaffected by
distant environment
this course
of
fields
this course
particles
QFT explains
convention:
$\eta_{\mu\nu} = \eta^{\mu\nu} = \begin{pmatrix} +1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$
West
Coast

QFT explains: Spin-statistics Theorem
integer spin $\Leftrightarrow$ Bose-Einstein
half-integer spin $\Leftrightarrow$ Fermi-Dirac

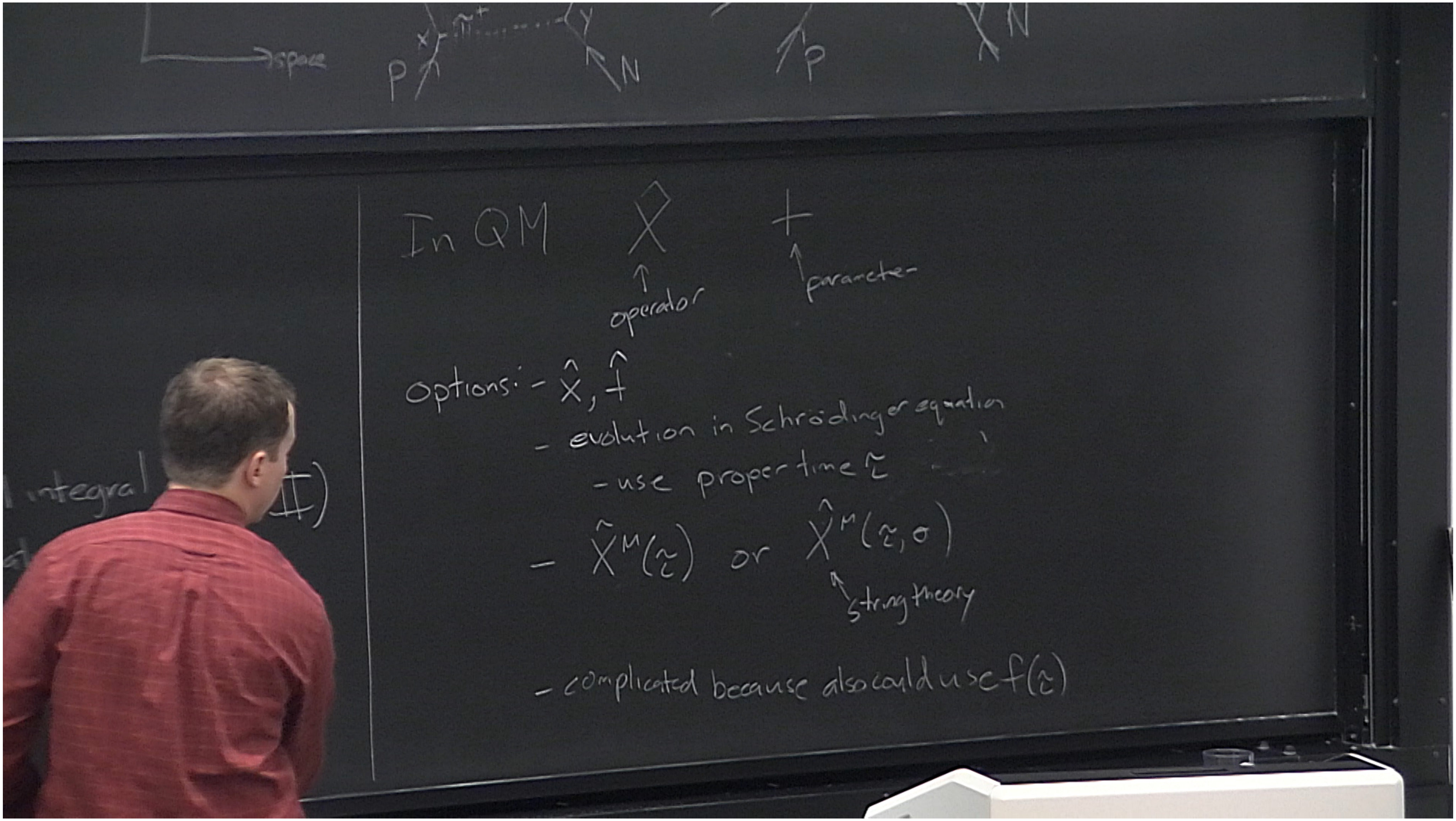
Possible approaches:

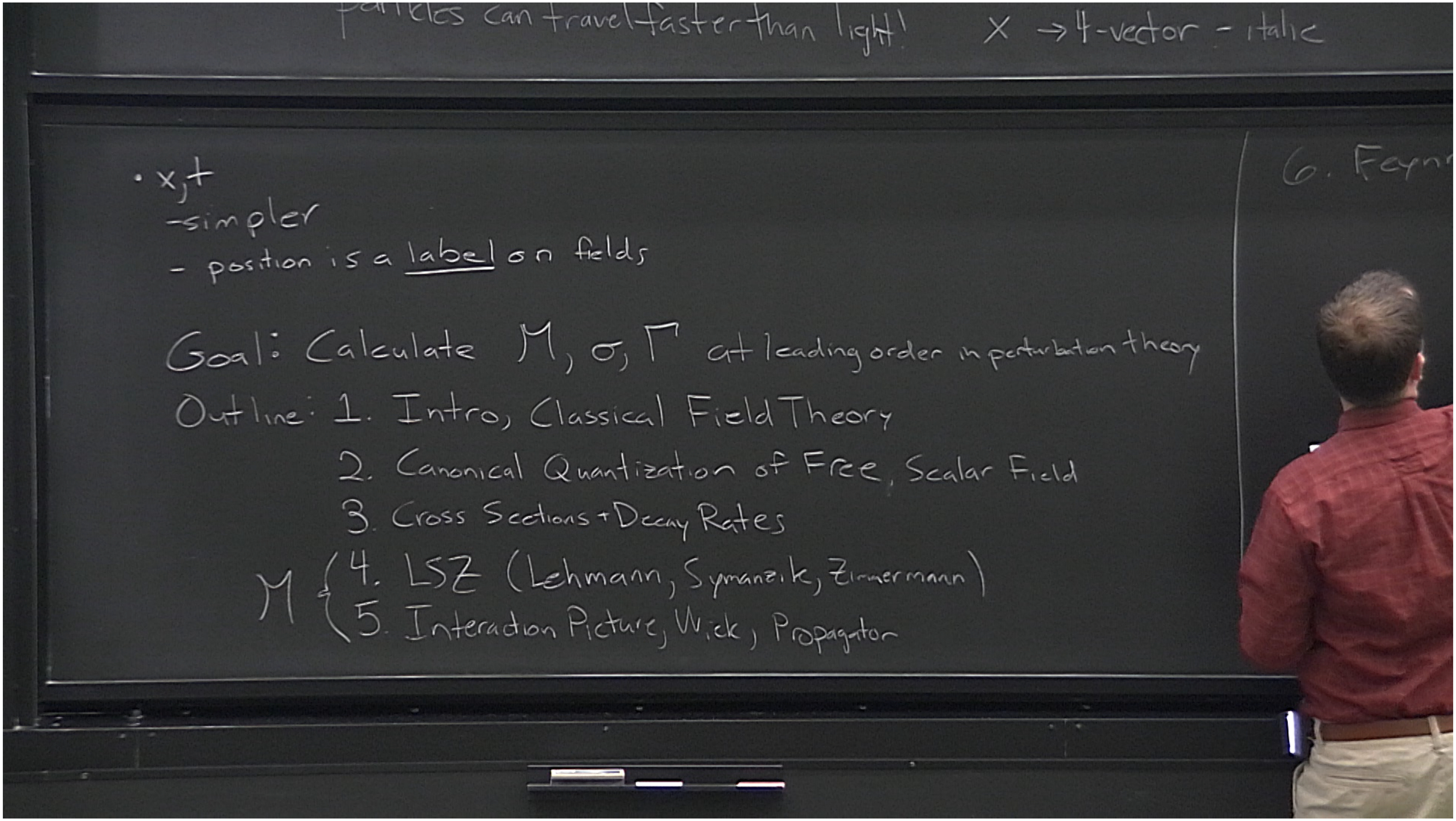
classical field theory

(this course) canonical quantization

functional integral quantization (QFT II)

QFT


space
integral (II)
In QM X t
operator
parameter
Options: - $\hat{x}$, $\hat{t}$
- evolution in Schrödinger equation
- use proper time τ
- $\hat{X}^{\mu}(\tau)$ or $\hat{X}^{\mu}(\tau,\sigma)$
string theory
- complicated because also could use $f(\tau)$


particles can travel faster than light!
X → 4-vector - italic
• x, t
-simpler
- position is a label on fields
Goal: Calculate M, σ, Γ at leading order in perturbation theory
Outline: 1. Intro, Classical Field Theory
2. Canonical Quantization of Free, Scalar Field
3. Cross Sections + Decay Rates
M { 4. LSZ (Lehmann, Symanzik, Zimmermann)
5. Interaction Picture, Wick, Propagator
6. Feyn

particles can travel faster than light! $x \to$ 4-vector - italic

- x, t
  - simpler
  - position is a <u>label</u> on fields

Goal: Calculate $\mathcal{M}$, σ, Γ at leading order in perturbation theory

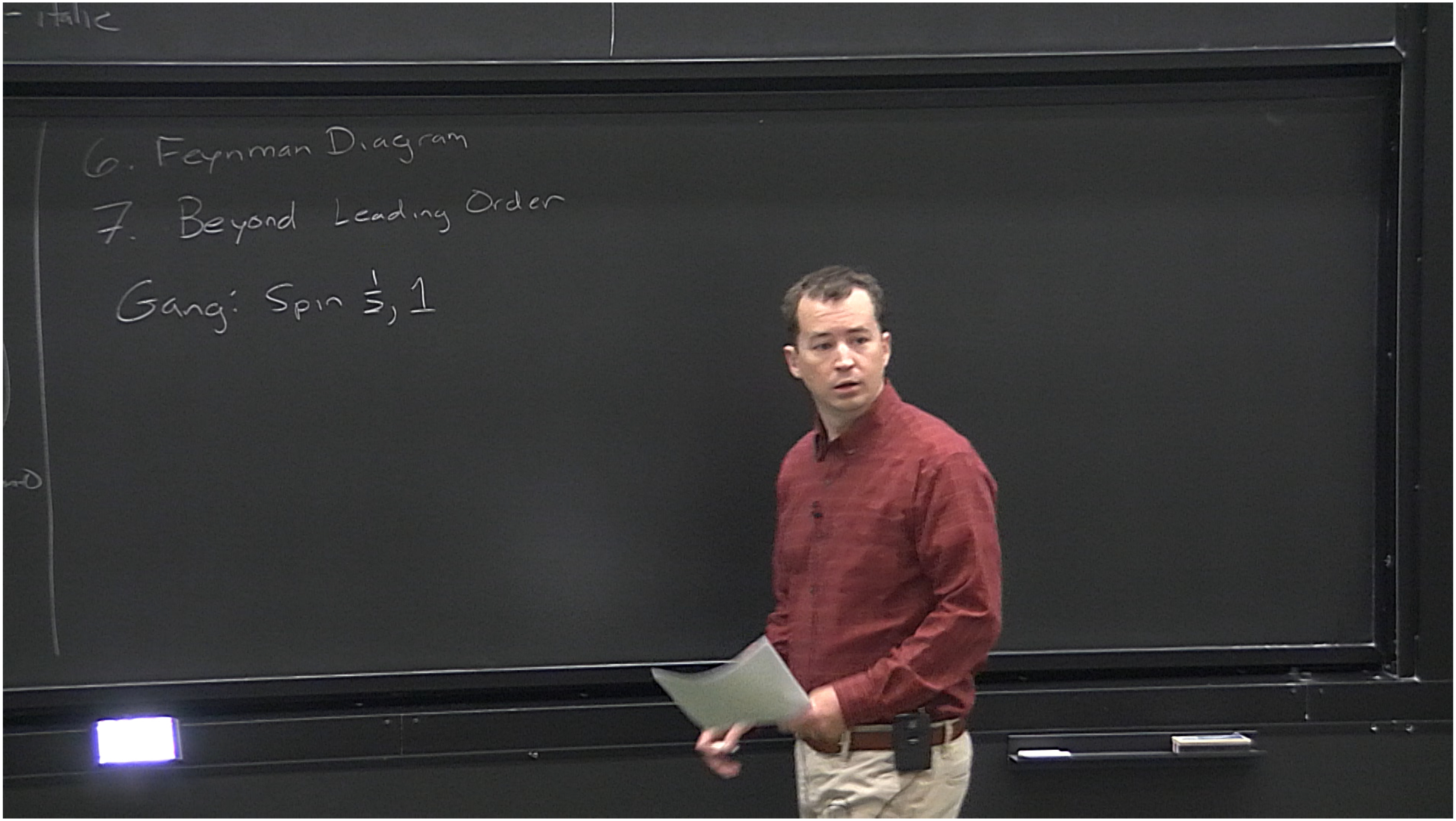
Outline: 1. Intro, Classical Field Theory

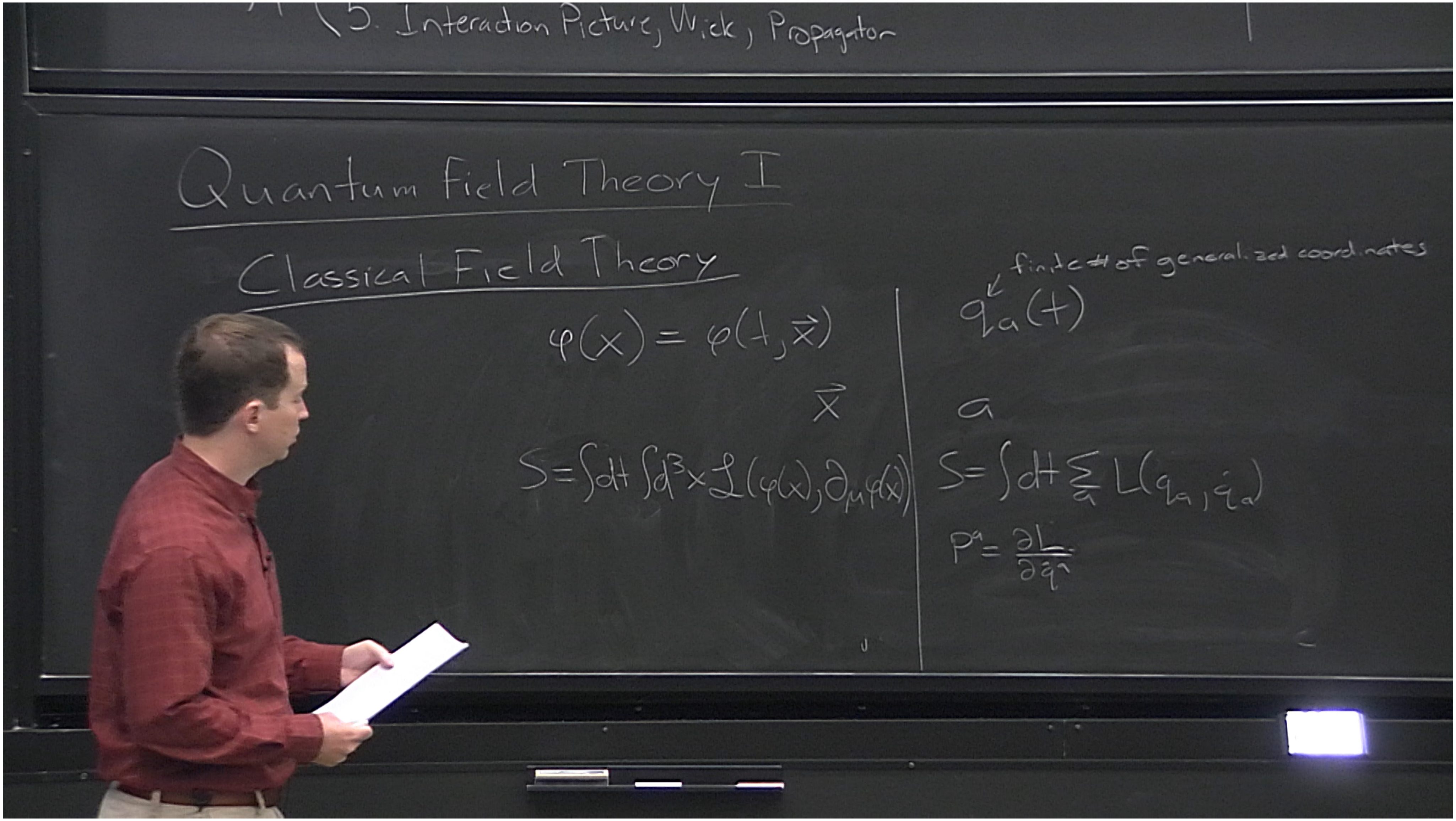
2. Canonical Quantization of Free, Scalar Field   Spin-0
3. Cross Sections + Decay Rates

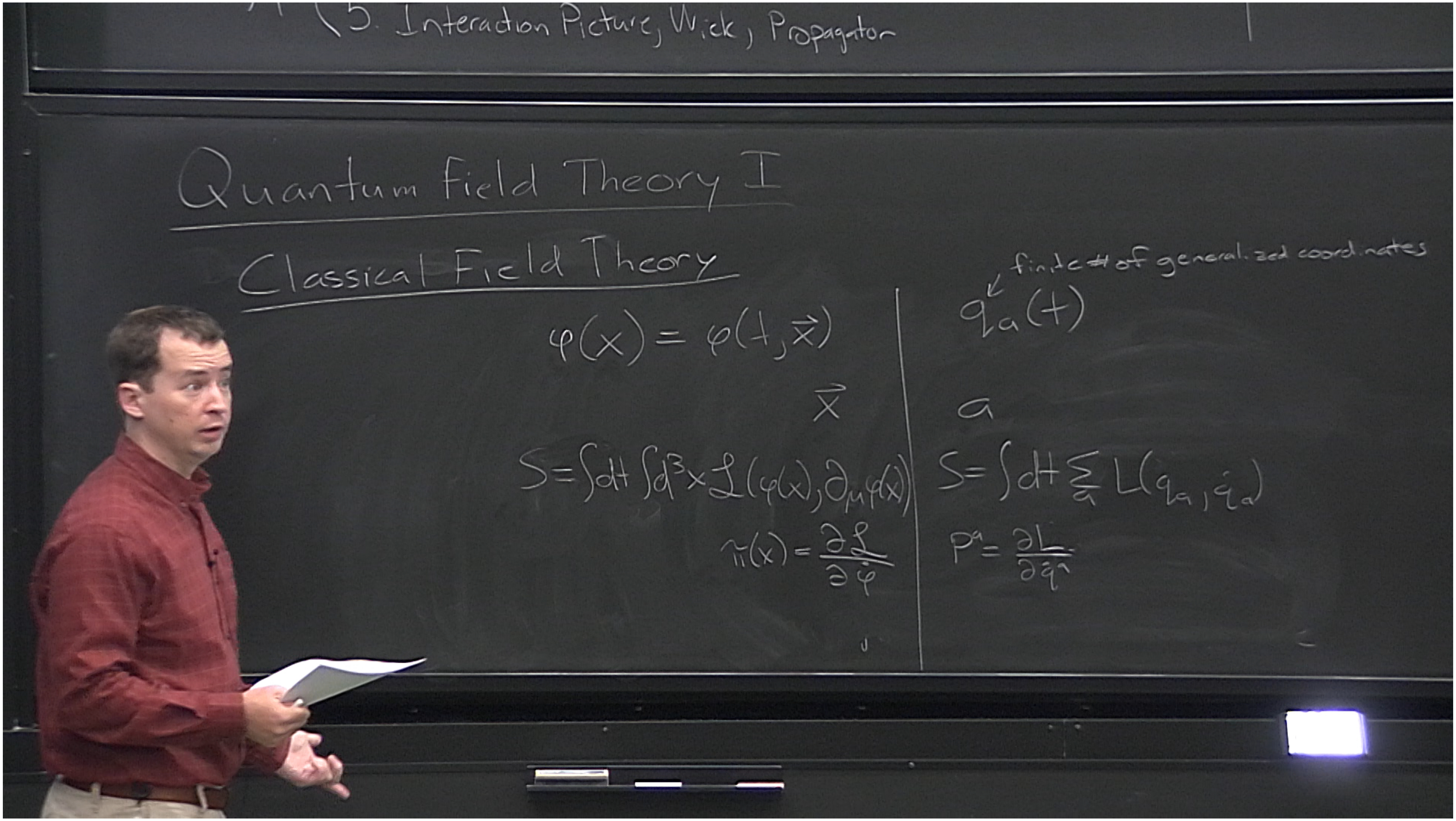
$\mathcal{M}$
4. LSZ (Lehmann, Symanzik, Zimmermann)
5. Interaction Picture, Wick, Propagator

6. Feyn
7. Beyo

Gang:


6. Feynman Diagram
7. Beyond Leading Order
Gang: Spin 1/2, 1


5. Interaction Picture, Wick, Propagator
Quantum Field Theory I
Classical Field Theory
$\varphi(x) = \varphi(t, \vec{x})$
$\vec{x}$
$S = \int dt \int d^3x \, \mathcal{L}(\varphi(x), \partial_\mu \varphi(x))$
finite # of generalized coordinates
$q_a(t)$
a
$S = \int dt \sum_a L(q_a, \dot{q}_a)$
$p^a = \frac{\partial L}{\partial \dot{q}^a}$


5. Interaction Picture, Wick, Propagator
Quantum Field Theory I
Classical Field Theory
φ(x) = φ(t, x⃗)
x⃗
S = ∫dt ∫d³x ℒ(φ(x), ∂_μ φ(x))
π(x) = ∂ℒ/∂φ̇
finite # of generalized coordinates
q_a(t)
a
S = ∫dt Σ_a L(q_a, q̇_a)
p^a = ∂L/∂q̇^a

eralized coordinates

$,\dot{q}_a)$

Klein-Gordon Theory

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi - \frac{1}{2}m^2\varphi^2$$

$$= \frac{1}{2}\dot{\varphi}^2 - \frac{1}{2}(\vec{\nabla}\varphi)^2 - \frac{1}{2}m^2\varphi^2$$

$$\pi = \frac{\partial\mathcal{L}}{\partial\dot{\varphi}} = \dot{\varphi}$$

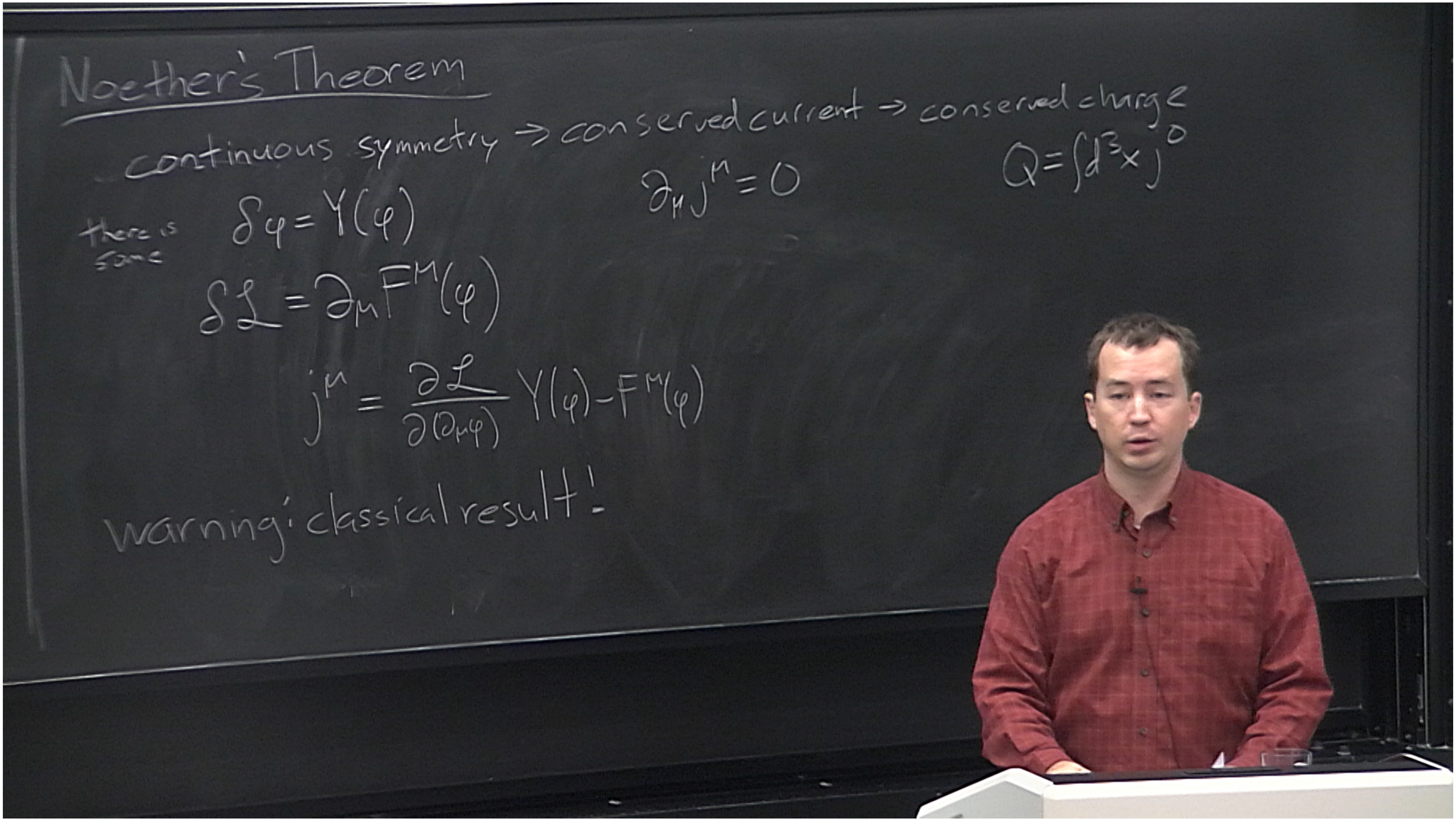
$$\mathcal{H} = \pi\dot{\varphi} - \mathcal{L}$$

$$= \underbrace{\pi^2 - \frac{1}{2}\pi^2}_{\frac{1}{2}\pi^2} + \frac{1}{2}(\nabla\varphi)^2 + \frac{1}{2}m^2\varphi^2$$

$$\frac{\partial \mathcal{L}}{\partial \varphi} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \right) = -m^2 \varphi - \partial_\mu \partial^\mu \varphi = 0$$

$$\boxed{(\partial_\mu \partial^\mu + m^2)\varphi = 0}$$

Noether's


Noether's Theorem
continuous symmetry → conserved current → conserved charge
$\partial_\mu j^\mu = 0$
$Q = \int d^3x \, j^0$
there is some
$\delta\varphi = Y(\varphi)$
$\delta\mathcal{L} = \partial_\mu F^\mu(\varphi)$
$j^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} Y(\varphi) - F^\mu(\varphi)$
warning: classical result!

continuous symmetry → conserved current →

$$\partial_\mu j^\mu = 0 \qquad Q = \int d^3x\, j^0$$

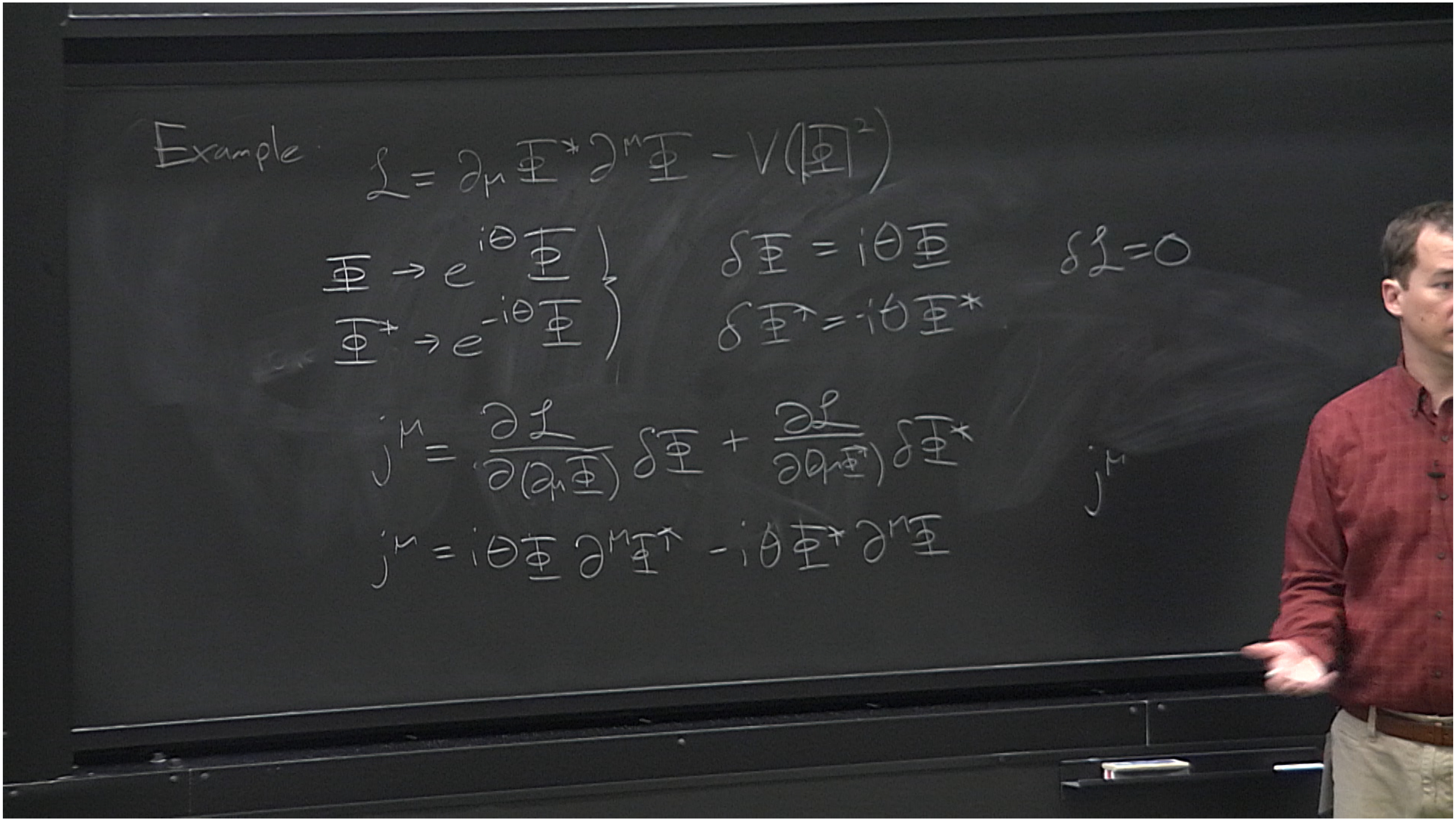
there is some $\delta\varphi = Y(\varphi)$

$$\delta\mathcal{L} = \partial_\mu F^\mu(\varphi)$$

$$j^\mu = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi)} Y(\varphi) - F^\mu(\varphi)$$

quantum effects that spoil classical symmetries
↓

warning: classical result! In quantum theory there can be anomalies!



Example: $\mathcal{L} = \partial_\mu \Phi^* \partial^\mu \Phi - V(|\Phi|^2)$
$\Phi \to e^{i\theta}\Phi$
$\Phi^* \to e^{-i\theta}\Phi$
$\delta\Phi = i\theta\Phi$
$\delta\Phi^* = -i\theta\Phi^*$
$\delta\mathcal{L} = 0$
$j^\mu = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\Phi)}\delta\Phi + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\Phi^*)}\delta\Phi^*$
j^μ
$j^\mu = i\theta\Phi\,\partial^\mu\Phi^* - i\theta\Phi^*\partial^\mu\Phi$

Example

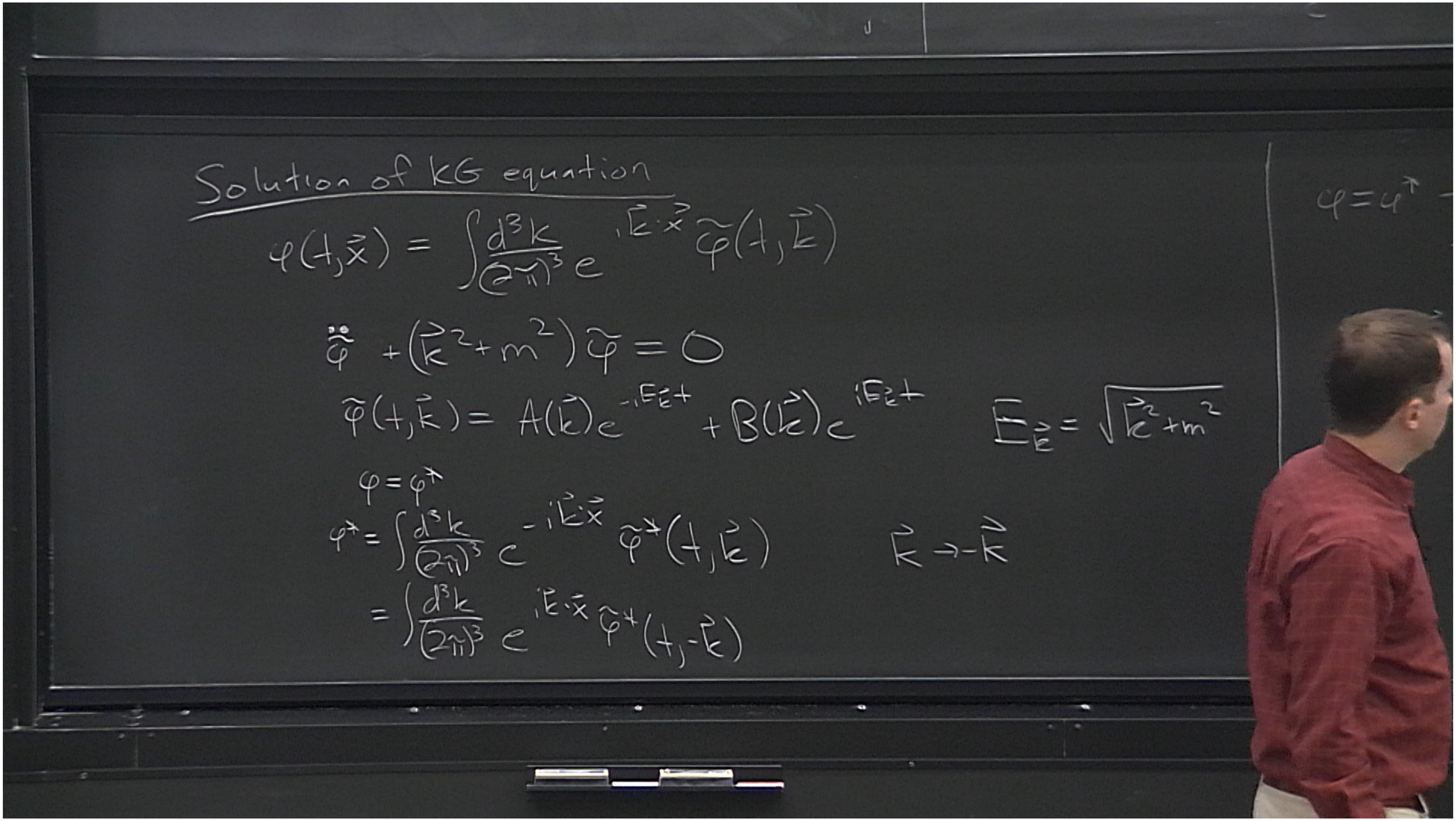
$$\mathcal{L} = \partial_\mu \Phi^* \partial^\mu \Phi - V(|\Phi|^2)$$

$$\left.\begin{aligned} \Phi &\to e^{i\theta}\Phi \\ \Phi^* &\to e^{-i\theta}\Phi \end{aligned}\right\} \qquad \begin{aligned} \delta\Phi &= i\theta\Phi \\ \delta\Phi^* &= -i\theta\Phi^* \end{aligned} \qquad \delta\mathcal{L} = 0$$

$$j^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \Phi)}\delta\Phi + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \Phi^*)}\delta\Phi^*$$

$$j^\mu = i\theta\Phi\,\partial^\mu\Phi^* - i\theta\Phi^*\partial^\mu\Phi$$

$$\boxed{j^\mu = \Phi\,\partial^\mu\Phi^* - \Phi^*\partial^\mu\Phi}$$


Solution of KG equation
$\varphi(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \tilde{\varphi}(t,\vec{k})$
$\ddot{\tilde{\varphi}} + (\vec{k}^2 + m^2)\tilde{\varphi} = 0$
$\tilde{\varphi}(t,\vec{k}) = A(\vec{k}) e^{-iE_{\vec{k}}t} + B(\vec{k}) e^{iE_{\vec{k}}t}$
$E_{\vec{k}} = \sqrt{\vec{k}^2 + m^2}$
$\varphi = \varphi^*$
$\varphi^* = \int \frac{d^3k}{(2\pi)^3} e^{-i\vec{k}\cdot\vec{x}} \tilde{\varphi}^*(t,\vec{k})$
$\vec{k} \to -\vec{k}$
$= \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \tilde{\varphi}^*(t,-\vec{k})$
$\varphi = \varphi^*$

$$\varphi = \varphi^\dagger \to \tilde{\varphi}(t,\vec{k}) = \tilde{\varphi}^*(t,-\vec{k})$$

$$\to B(\vec{k}) = A^*(-\vec{k})$$

$$\varphi(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^3}\left[A(\vec{k})\, \underbrace{e^{-iE_{\vec{k}}t + i\vec{k}\cdot\vec{x}}}_{e^{ik\cdot x}} + \underbrace{A^*(-\vec{k})\, e^{iE_{\vec{k}}t + i\vec{k}\cdot\vec{x}}}_{\vec{k}\to-\vec{k}} \right]$$

$$k = (E_{\vec{k}}, \vec{k})$$

$$\boxed{\varphi(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^3}\left[A(\vec{k})\, e^{-ik\cdot x} + A^*(\vec{k})\, e^{ik\cdot x} \right]}$$