Title: Renormalization and Effective Field Theory - Lecture 2

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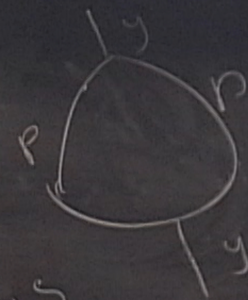
Abstract:

Last time

$$\int e^{\frac{1}{2\hbar}x_i A^{ij} x_j}\, e^{\frac{1}{\hbar}F(x_i+y)}\, dx = e^{\frac{\hbar}{2}\partial_p}\, e^{\frac{1}{\hbar}F(y)}$$

$$= \sum_{\gamma \text{ w. external legs}}$$



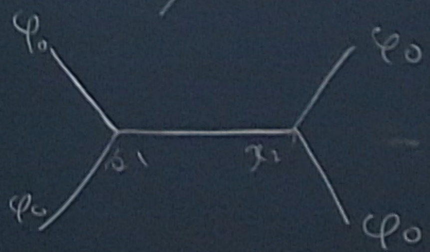
Consider the path integral

$$\hbar \log \int_{\varphi \in C^\infty(\mathbb{R}^n)} e^{-\frac{1}{2\hbar}\int \varphi \Delta \varphi + \frac{1}{6\hbar}(\varphi + \varphi_0)^3}$$

$$= \sum_{\substack{\text{connected} \\ \text{diagrams} \\ \text{w. external lines}}} \hbar^{\#\text{loops}} \frac{W_\gamma(\varphi_0)}{|\mathrm{Aut}(\gamma)|}$$

$\hbar^0$ terms

$$= \frac{1}{3!}\int_x \varphi_0(x)^3$$



$$= \frac{1}{8}\int_{x_1,x_2\in\mathbb{R}^n} \varphi_0(x_1)^2\,\varphi_0(x_2)^2\; P(x_1,x_2)\; dVol$$

$$= \frac{1}{8}\int_{x_1,x_2} \varphi_0(x_1)^2\,\varphi_0(x_2)^2\;\frac{1}{\|x_1-x_2\|^{(n-2)}}$$

$\varphi_0(x_1)^2 \varphi_0(x_2)^2 \quad P(x_1, x_2)$

$\in \mathbb{R}^n$

$\varphi_0(x_2)^2 \dfrac{1}{\|x_1 - x_2\|^{(n-2)}} \, dVol$

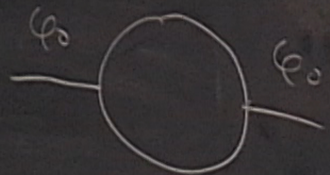
r^{n-1}

$\int_r \dfrac{1}{r^{n-2}} \, Vol(S^{n-1})$

1 Loop:

Order $\hbar$

$$= \int_x \varphi_0(x)\, P(x-x) \quad \leftarrow \text{makes}$$



$$= \int_{x_1, x_2} \varphi_0(x_1)\, \varphi_0(x_2)\, \frac{1}{\|x_1 - x_2\|^{2n-4}}$$

Dim. Analysis

$y = x_1 - x_2$

$$\int \frac{1}{y^{2n-4}}\, d^n y \sim \text{diverges}$$

$$\int_x \varphi_0(x)\, P(x-x) \longleftarrow \text{makes no sense}$$

$$\int_{x_1, x_2} \varphi_0(x_1)\, \varphi_0(x_2)\, \frac{1}{\|x_1 - x_2\|^{2n-4}}\, dVol$$

$_1 - x_2$

$$\int \frac{1}{y^{2n-4}}\, d^n y \quad \text{— diverges,} \quad \int r^{3-n}\, dr$$

Regularization Schemes

1) "Dimensional Regularization"

doesn't make sense on general manifolds

2) Point - splitting

$$\int_{\|x_1-x_2\|\geq\varepsilon} \varphi_0(x_1)\,\varphi_0(x_2)\,\frac{1}{\|x_1-x_2\|^{2n-4}}$$

Heat Kernel Method

Write $P = \int_{t=0}^{\infty} K_t(x,x')$

$$P_\varepsilon^L(x,x') = \int_{t=\varepsilon}^{L} K_t(x,x')$$

Feynman diagrams have no singularities using $P_\varepsilon^L(x,x')$

If M is a manifold

and Δ is Laplacian

A heat kernel $K_t(x,x')$ is a smooth funct

satisfies

$$\partial_t K_t(x,x') + \Delta_x K_t(x,x') = 0 \;; \quad \lim_{t \to}$$

On flat space, $\mathbb{R}^n$, $\exists$ unique K_t

$$K_t(x,x') = t^{-n/2} e^{-\|x-x'\|^2/t}$$

manifold

is Laplacian

rnel $K_t(x,x')$ is a smooth function on $M \times M$

$x') + \Delta_x K_t(x,x') = 0$; $\lim_{t \to 0} K_t(x,x') = \delta_{x=x'}$

), $\exists$ unique K_t

$e^{-\|x-x'\|^2/t}$

M is compact manifold,

$\exists!$ heat kernel

M is not compact, $\exists$ a heat kernel, not unique

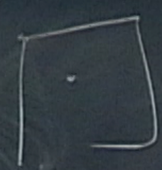
$M = S^1$, θ, $\theta + 2\pi = \theta$

$$K_t(\theta, \theta') = t^{-1/2} \sum_{n \in \mathbb{Z}} e^{-(\theta - \theta' + 2\pi n)^2/t}$$

Heat Kernel Method

Physical Interpretation

1) $K_t(x, x') =$ prob. distribution for a Brownian particle which at $t=0$ is at x



2) $K_t(x, x') =$ heat at x' if at $t=0$ point source of heat at x

3) K_{it}

prob. distribution
for a Brownian particle
which at $t=0$ is at x

heat at x' if at $t=0$
point source of heat at x

ve fn for a quantum particle which at $t=0$ is at x

Propagator from the heat kernel

Kinetic term is $\varphi(\Delta + m^2)\varphi$

then, P satisfies

$$(\Delta_x + m^2) P(x,x') = \delta_{x=x'}$$

Then

$$P(x,x') = \int_{t=0}^{\infty} e^{-m^2 t} K_t(x,x')$$

$$(\Delta_x + m^2)\int_{t=0}^{\infty} e^{-m^2 t} K_t(x,x')$$

$$= \int m^2 e^{-m^2 t} K_t(x,x')$$

$$- \int e^{-m^2 t} \partial_t K_t(x,x')$$

IBP

$$\int_{t=0}^{\infty} \left((m^2 + \partial_t) e^{-m^2 t} \right) K_t(x,x')$$

$$+ \; K_0(x,x')$$

$$= \delta_{x,x'}$$

$K_t(x,x')$

Massless case

$$\Delta_x \int K_t(x,x')$$

$$= \delta_{x=x'} + \text{harmonic representative of } \delta\text{-fn}$$

$K_t(x, x')$

Massless case

$$\Delta_x \int K_t(x, x')$$

$$= \delta_{x=x'} + \text{harmonic representative of } \delta\text{-fn}$$

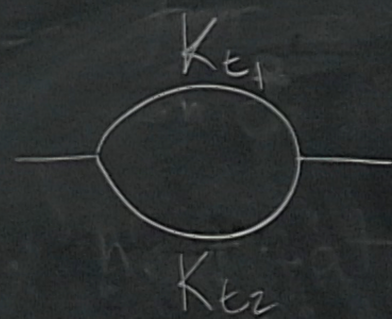
On $\mathbb{R}^n$

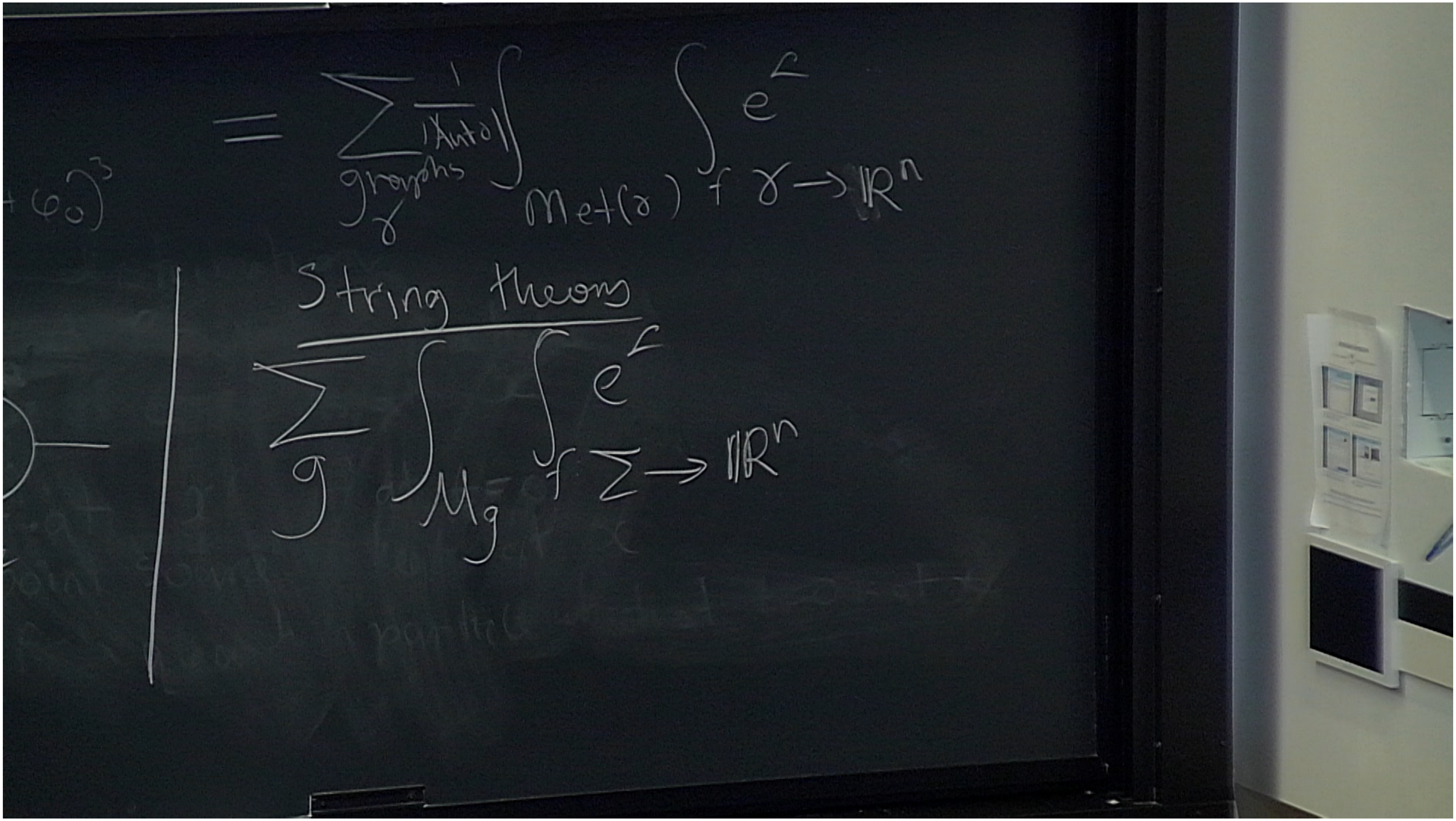
$$\int t^{-n/2} e^{-\|x-x'\|^2/t} \, dt = \frac{1}{\|x-x'\|^{n-2}}$$

In φ^3 theory on $\mathbb{R}^n$

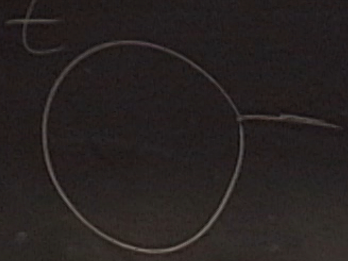
$$\hbar \log \int_{\varphi \in C^\infty(\mathbb{R}^n)} e^{-\int \varphi \Delta \varphi / 2\hbar} \, e^{\frac{1}{6\hbar}\int (\varphi + \varphi_0)^3}$$

$$= \sum_{\substack{\text{connected} \\ \text{graphs}, \gamma}} \int_{\substack{t_e, \\ e \text{ edge} \\ \text{of } \gamma}}$$




$+\varphi_0)^3$
$= \sum_{\text{graphs } \gamma} \frac{1}{|\mathrm{Aut}\,\gamma|} \int_{\mathrm{Met}(\gamma)} \int_{f\,\gamma \to \mathbb{R}^n} e^{\mathcal{L}}$
String theory
$\sum_{g} \int_{M_g} \int_{f\,\Sigma \to \mathbb{R}^n} e^{\mathcal{L}}$

As before



$$\longrightarrow \int_{x\in\mathbb{R}^n} \varphi_0(x)\, K_t(x,x)$$

$$= \int_{x\in\mathbb{R}^n} \varphi_0(x)\, t^{-n/2}$$

$\varphi_0(x)\ K_t(x,x_0)$

$\mathbb{R}^n$

$t^{-n/2}$

If we integrate from

$t = \varepsilon \longrightarrow \infty$

we get

$$\frac{1}{1-n/2}\,\varepsilon^{1-n/2}\int \varphi_0(x)$$

We'll proceed by introducing a counter term

$n=2,$

$$\int_\varepsilon^{L} t^{-1}\int_x \varphi_0(x) = (\log L - \log \varepsilon)\int_x \varphi_0(x)$$

Counter term $\log \varepsilon \int_x \varphi_0(x)$