Title: Holography at finite radius

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Abstract: <p>I will discuss certain irrelevant operator deformations of holographic conformal field theories that define a one parameter family of quantum field theories which are thought to be dual to quantum gravity in finite regions.
Some examples include the "$T\bar{T}$ deformation of two dimensional holographic CFTs, its generalisations and higher dimensional cousins.
I will emphasise the key role played by emergent radial diffeomorphism invariance in this scenario. In particular, I will present an additional criterion for a theory, in the appropriate regime, to possess a dual description in terms of general relativity coupled to matter in one higher dimension. I will also discuss entanglement properties of such theories, and in a particular example, I will show how the holographic entanglement entropy conjectures still hold in the finite radius setting. This is based on the following work: arXiv:1707.08118 [hep-th], arXiv:1712.07955 [hep-th], arXiv:1806.07444 [hep-th] and arXiv:1808.07760 [hep-th]."</p>

Holography at finite radius

Vasudev Shyam

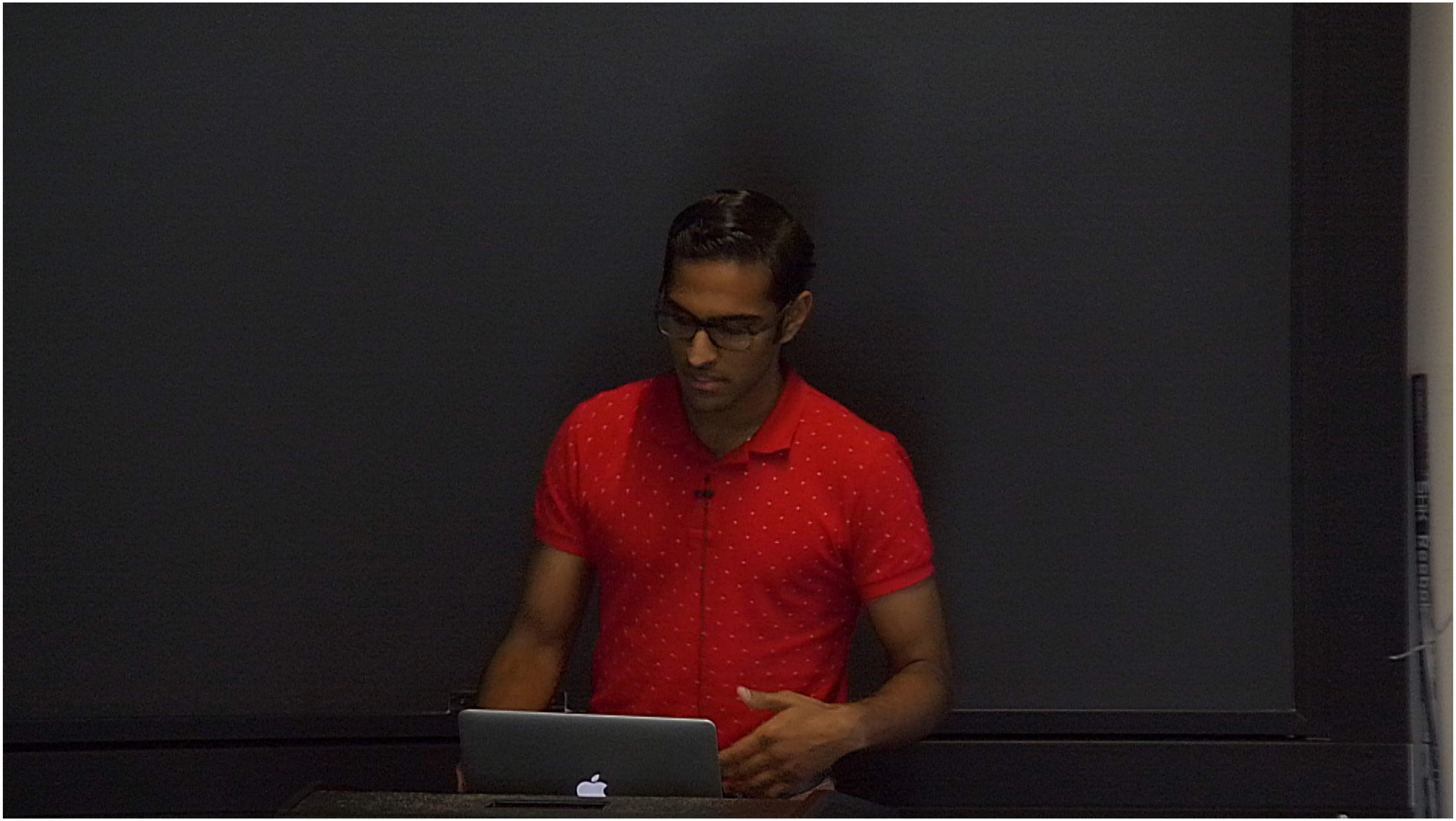
Perimeter Institute

September 20, 2018

PERIMETER PI INSTITUTE FOR THEORETICAL PHYSICS

This is based on the following work:

- Background independent holographic dual to $T\bar{T}$ deformed CFT with large central charge in 2 dimensions, arXiv:1707.08118 [hep-th]- VS.
- Connecting Holographic Wess–Zumino Consistency Conditions to the Holographic Anomaly, arXiv:1712.07955 [hep-th]- VS.
- Entanglement entropy and $T\bar{T}$ deformation, arXiv:1806.07444 [hep-th]-William Donnelly, VS.
- Finite Cutoff AdS_5 Holography and the Generalized Gradient Flow, arXiv:1808.07760 [hep-th]- VS.



Holography and asymptotia

- Holographic duality: Gravity in AdS_{D+1} " = " CFT$_D$ living on the asymptotic boundary of this space.
- The asymptotic boundary and its symmetries take the form:

$$\partial\left(AdS_{D+1}\right) \cong \mathbb{R}^{1,D-1}, \ \frac{Diff(AdS_{D+1})}{Diff^{o}(AdS_{D+1})} = SO(2, D-2)$$

- At finite radius: dual theory must couple to an arbitrary background geometry with metric given by the one induced on that boundary: $g_{\mu\nu}(x, r)$.

The Holographic Dictionary
[Gubser et. al: arXiv:hep-th/9802109; Witten: arXiv:hep-th/9802150]

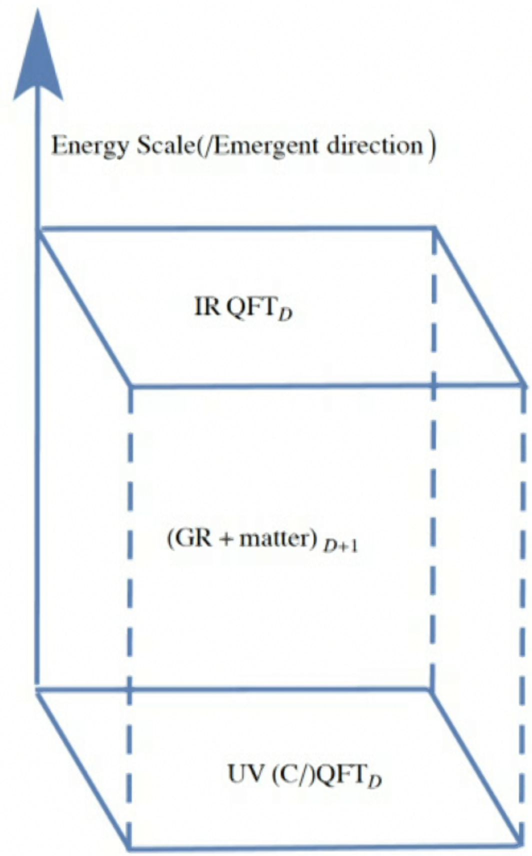
- 'Radial' phase space variables in the bulk identified with pairs of sources and VEVs of single trace operators:

$$(\langle O_n(x)\rangle, j^n(x)) \leftrightarrow (p_n(x,r), \phi^n(x,r))$$

- Bulk dynamics " $=$ " Renormalization group flow of sources for a certain subset (single trace) operators on the boundary (GR $\approx$ RG)
- Scaling dimensions on the boundary are related to masses of bulk fields. As an example, for scalar fields:

$$m^2 \propto \Delta_O(\Delta_O - D)$$

Holographic RG: the picture



Leveraging EFT

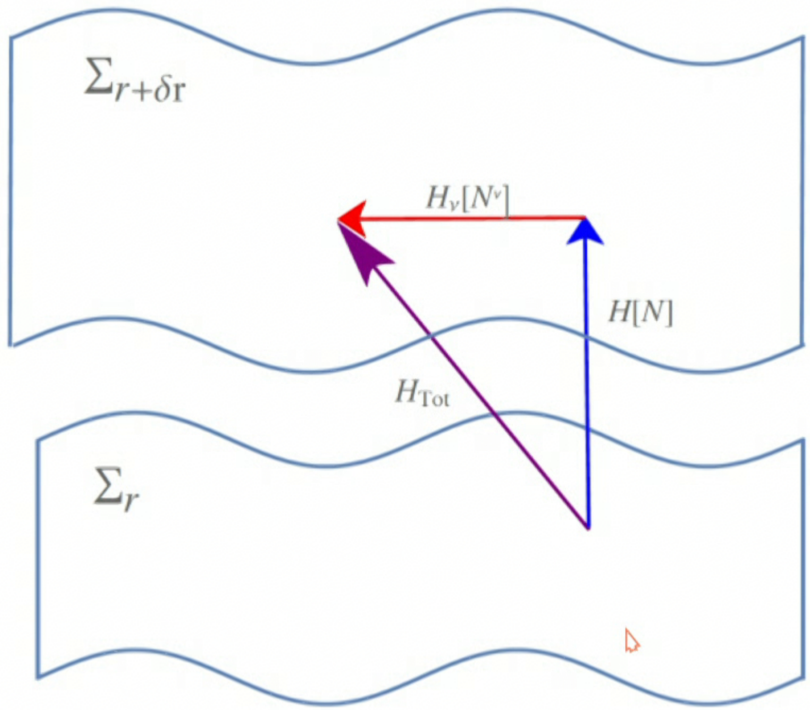
- Einstein (1916): GR in 4D is the unique two derivative diffeomorphism invariant theory of the dynamical metric.
- Lovelock: In higher dimensions, there are more general 'quasi topological' theories that are allowed but on phase space, the dynamics of these theories are given by multi valued Hamiltonian flows.
- Two derivative assumption + single valuedness of Hamiltonian flows $\leftrightarrow$ phase space associated to a codimension 1 hypersurface spanned only by $(g_{\mu\nu}, \pi^{\mu\nu})$.
- Phase space spanned by $(g_{\mu\nu}, \pi^{\mu\nu})$ + manifestation of diffeomorphism invariance $=>$ Einstein's equations.
- Idea: leverage this as a criterion for the emergence of non linear Einstein's equations in the bulk.

Wilsonian holographic RG for the theory on the boundary

[I. Heemskerk, J. Polchinski:arXiv:1010.1264 [hep-th]; T. Faulkner, H. Liu, M. Rangamani: arXiv:1010.4036 [hep-th]]

- Wilson's framework of RG, which is most widely applicable, organises effective field theories by ordering them from high to low energies.
- The scale is typically identified with the cutoff of any interpolating theory along the flow
- The role of the cutoff is played by the radial coordinate in the bulk.
- Interpolating theories along an RG flow live on families of successive constant radius hypersurfaces.
- Wilsonian Holographic RG would realise the duality between an EFT on the boundary theory and a bulk gravitational theory cutoff at a constant radius hypersurface.

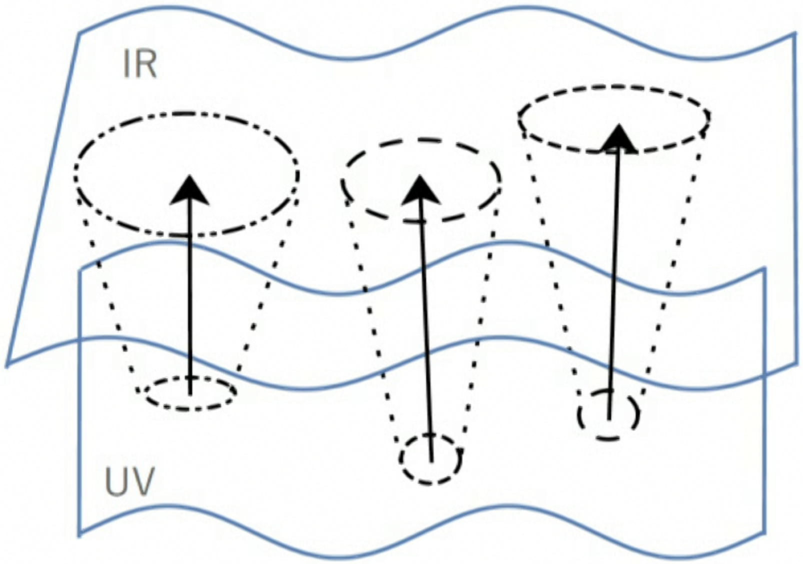
General relativity and embedded cutoff surfaces



$$H[N] = \int \mathrm{d}^D x N(x) G_{nn} = 0$$

$$H_\alpha[N^\alpha] = \int \mathrm{d}^D x N^\nu(x) G_{n\nu} = 0$$

Mimic with local Wilsonian RG



$$\mathcal{R}_\sigma \ln Z[J] = \int_x \left(\sigma(x)\beta^i(J)\frac{\delta \ln Z[J]}{\delta J^i(x)} \right) - \mathcal{A}_\sigma[J] = 0,$$

$$\sigma(x)\beta^i(J) = \frac{\partial J^i(x,r)}{\partial r}$$

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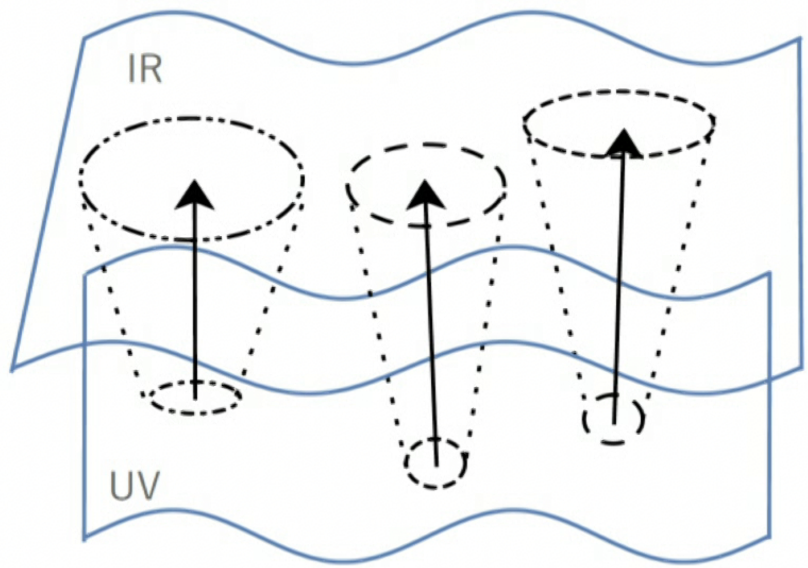

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The $T\bar{T}$ deformation

[A.Zamolodchikov, hep-th/0401146; F. Smirnov, A. Zamolodchikov, arXiv:1608.05499 [hep-th] ; A. Cavaglià et. al: arXiv:1608.05534 [hep-th]]

- The $T\bar{T}$ operator is defined as $\left[\frac{1}{8} : (T_{\mu\nu}T^{\mu\nu} - (T^{\rho}_{\rho})^2)(x) :\right]$
- On a flat background, to leading order in gradients, its expectation value in any translation invariant state factorizes:

$$\langle T\bar{T}\rangle = \frac{1}{8}\left(\langle T^{\mu\nu}\rangle\langle T_{\mu\nu}\rangle - \langle T^{\kappa}_{\kappa}\rangle^2\right).$$

- When a CFT is deformed by it, one finds a one parameter family of quantum field theories:

$$\frac{\partial \ln Z}{\partial r} = -\int_x \mu\langle T\bar{T}\rangle,$$

$$\ln Z(\mu) \sim \ln Z_{CFT} + \mu \int (\langle T^{\mu\nu}\rangle\langle T_{\mu\nu}\rangle - \langle T^{\kappa}_{\kappa}\rangle^2) + O(\mu^2)$$

- This deformation doesn't affect energy momentum conservation: $\partial_\mu\langle T^{\mu\nu}\rangle = 0$.

$T\bar{T}$ deformation as a regulator
[A. Cavaglià et. al.]

- Consider the deformation of a CFT on a cylinder with L being the circumference of the spatial circle,
- The energy levels of the theory are shifted due to the presence of the deformation as follows:

$$E_n(L) \rightarrow E_n(\mu, L) =$$

$$= \frac{2L}{\mu}\left(1 - \sqrt{1 - \frac{2\pi\mu}{L^2}\left(\Delta_n + \bar{\Delta}_n - \frac{c}{12}\right) + \frac{\pi^2\mu^2}{L^4}(\Delta_n - \bar{\Delta}_n)^2}\right)$$

- At some Δ_n this functions hits a square root singularity.
- Discard imaginary eigenvalues and treat E_n at that value of n as a cutoff on the spectrum.

From: L. McGough et. al: arXiv:1611.03470 [hep-th]

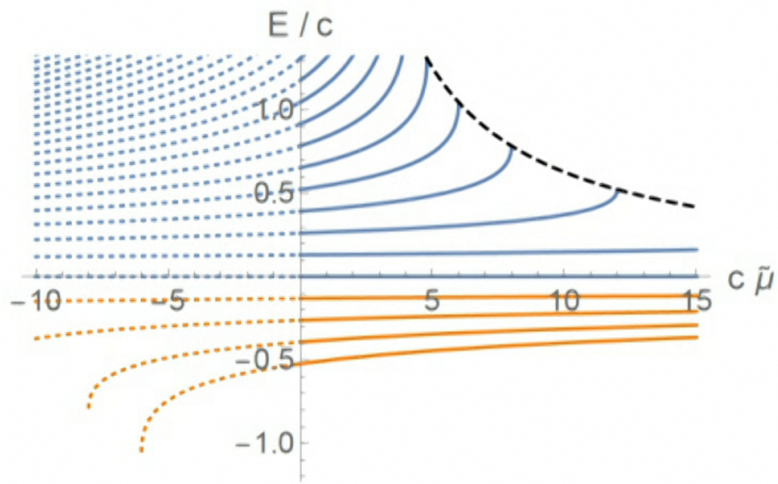

Figure 1: The energy levels E_n at $L = 2\pi$ and $J = 0$ as a function of μ for different values of $E(0) = \Delta_n + \bar{\Delta}_n - \frac{c}{12}$. States with $E(0) > 0$ that correspond to black holes in holographic CFTs are plotted in blue, while low-lying states are plotted in orange. For $\mu > 0$ that is the relevant regime in our study we used solid lines, while for $\mu < 0$ the spectrum is plotted with dotted lines. The levels exhibit a square root singularity at the critical value $\mu E(0) = 2\pi$. This indicates that, for given μ, the energy spectrum of the deformed CFT is bounded by $E < \frac{8}{\mu}$, indicated on the plot by a dashed black line.

Defining the deformation on a curved background

- On a general background, $\ln Z \to \ln Z[g]$ and the energy momentum tensor is defined as $\langle T^{\mu\nu}\rangle = \frac{\delta \ln Z[g]}{\delta g_{\mu\nu}}$.
- The defining equation for the deformed theory at large c becomes:

$$\frac{\partial \ln Z[g]}{\partial r} = \int_x \sigma(x) g_{\mu\nu} \frac{\delta \ln Z[g]}{\delta g_{\mu\nu}} = \int_x \sigma(x) \langle T^{\mu}_{\mu}\rangle =$$

$$= \int_x \sigma(x) \left(-\frac{\mu}{2} \left(\langle T_{\mu\nu}\rangle \langle T^{\mu\nu}\rangle - \langle T^{\kappa}_{\kappa}\rangle^2 \right) - \frac{c}{24\pi} R \right)$$

- The covariant conservation equation reads

$$\nabla_\mu \langle T^{\mu\nu}\rangle = 0.$$

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Vasudev Shyam | Holography at finite radius

Cutoff AdS_3/CFT_2

[L. McGough et. al: arXiv:1611.03470 [hep-th]]

- The CFT on the boundary in AdS_3/CFT_2 is pushed into the bulk via the $T\bar{T}$ deformation.
- The similarity between the covariant conservation law $\nabla_\mu \langle T^\mu_\nu \rangle = 0$ and the $G_{n\nu}$ constraint in the bulk: $\nabla_\mu \pi^\mu_\nu = 0$ leads us to identify

$$\sqrt{g}\left(\langle T^{\mu\nu}\rangle - \frac{2}{\mu} g^{\mu\nu}\right) = \pi^{\mu\nu}.$$

- Match parameters $c = \frac{3\ell}{2G}$, $\mu = 16\pi G\ell$, and then the flow equation becomes the radial ADM Hamiltonian constraint:

$$H(\sigma) = \int_x \sigma(x)\left(-\frac{8\pi G}{\sqrt{g}} G_{\mu\nu\rho\sigma}\pi^{\mu\nu}\pi^{\rho\sigma} - \frac{\sqrt{g}}{8\pi G}\left(R - \frac{2}{\ell^2}\right)\right) = 0.$$

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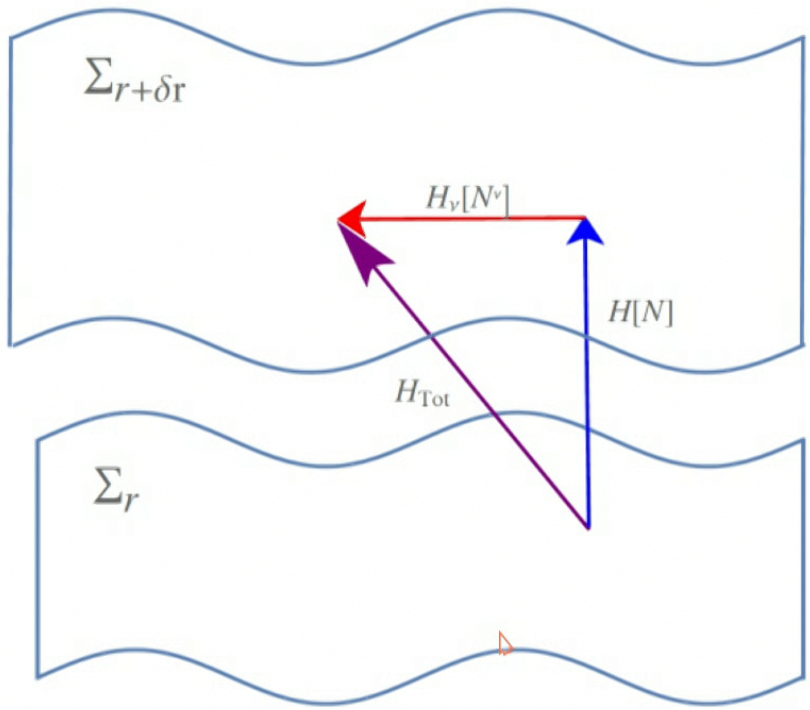

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Diffeomorphism invariance and $T\bar{T}$ on curved space
[VS :arXiv:1707.08118 [hep-th]]

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- The equation needed for the Holographic application must hold on arbitrary curved backgrounds, and requires some further feature of holographic theories to make sense.
- This is a particular feature of the generator of local RG transformations $\mathcal{R}_\sigma$:

$$[\mathcal{R}_\sigma, \mathcal{R}_{\sigma'}]\ln Z[g] =$$

$$= \int \mathrm{d}^2x \sqrt{g} g^{\mu\nu}(\sigma\partial_\nu\sigma' - \sigma'\partial_\nu\sigma)\langle\nabla_\alpha T^\alpha_\mu\rangle = 0$$

- This is the dual statement of the 2+1 D diffeomorphism invariance of the emergent bulk theory.

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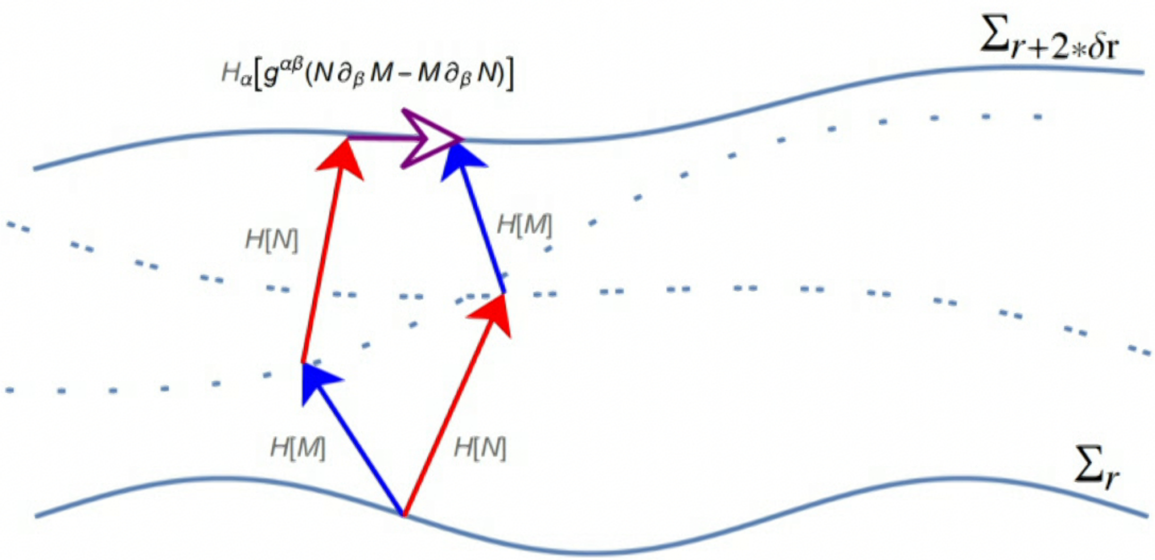
Emergent Bulk Covariance



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- The necessary identifications are: $\sqrt{g}\left(\langle T^{\mu\nu}\rangle - \frac{2}{\mu}g^{\mu\nu}\right) = \pi^{\mu\nu}$, $c = \frac{3\ell}{2G}$, $\mu = \frac{16\pi G\ell}{r_c}$.

Generalization to four dimensions

[Bonelli et. al: arXiv:1804.10967 [hep-th] , Hartman et. al. :arXiv:1807.11401 [hep-th], M. Taylor: arXiv:1805.10287 [hep-th]]

- Deforming operators: $T^{\mu\nu}T_{\mu\nu} - \frac{1}{3}(T^{\mu}_{\mu})^2$, $\mathcal{O}^2$ whose one point functions factorize at large N.
- They deform the 4D, large N holographic CFT in the following way:

$$\frac{\partial \ln Z[g,\phi]}{\partial r} = \int_x \left(\beta_{\mu\nu}(g,\phi)\frac{\delta}{\delta g_{\mu\nu}} - \beta_\phi(g,\phi)\frac{\delta}{\delta\phi}\right)\ln Z[g,\phi] =$$

$$\int_x \left(-\mu\left(\langle T^{\mu\nu}\rangle\langle T_{\mu\nu}\rangle - \frac{1}{3}\langle T^{\mu}_{\mu}\rangle^2\right) - \kappa\langle\mathcal{O}\rangle^2\right) + \mathcal{A}^{(a=c)}[g,\phi]$$

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From the deformation equation to the radial scalar constraint
[VS: arXiv:1808.07760 [hep-th]]

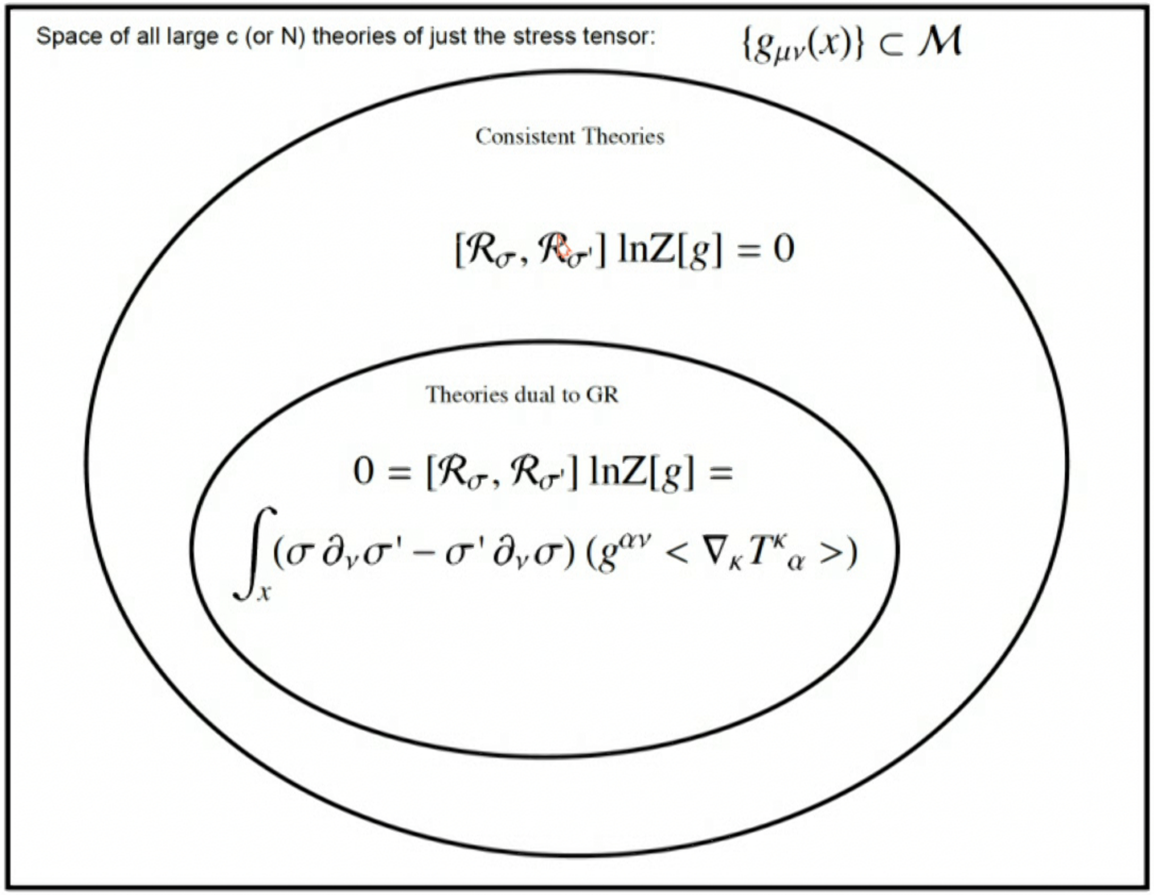
- Obtaining G_{nn} from flow equation => requirement: generalized gradient flows

$$\beta_{\mu\nu}(g,\phi) = \left(g_{\mu(\rho}g_{\sigma)\nu} - \frac{1}{3}g_{\mu\nu}g_{\rho\sigma} \right) \frac{\delta S[g,\phi]}{\delta g_{\mu\nu}}, \;\; \beta_\phi(g,\phi) = \frac{\delta S[g,\phi]}{\delta \phi}$$

- Identify: $\langle T^{\mu\nu} \rangle \sqrt{g} = \pi^{\mu\nu} - \frac{\delta S[g,\phi]}{\delta g_{\mu\nu}}$, $\langle \mathcal{O} \rangle = p_\phi - \frac{\delta S}{\delta \phi}$, $\kappa = \mu$.
- The flow equation becomes

$$\frac{1}{\sqrt{g}} \left(\pi^{\mu\nu}\pi_{\mu\nu} - \frac{(\pi^\mu_\mu)^2}{3} + \frac{p_\phi^2}{2} \right) - \sqrt{g} \left(\frac{(\partial\phi)^2}{2} + R + U(\phi) \right) = 0.$$

Carving out the space of theories dual to GR
[VS: arXiv:1712.07955 [hep-th]]



Holographic Entanglement Entropy

[S. Ryu, T. Takayanagi, arXiv:hep-th/0603001; V.Hubeny, M. Rangamani, T Takayanagi, arXiv:0705.0016 [hep-th]]

- Ryu and Takayanagi proposed a dual quantity to the entanglement entropy of the boundary CFT, which is the area of a minimal surface anchored to the entagling surface of the region of interest.
- This was generalized to the covariant setting by Hubeny, Rangamani and Takayanagi.
- The UV divergence of the CFT's Entanglement entropy is matched by the IR divergence of the area of a minimal surface anchored to the asymptotic boundary.
- Thus EE is sensitive to the UV properties of the boundary QFT and the dual is sensitive to bulk geometry at large distances.

Strategy for computing the entropy in the QFT

- Obtain $\langle T_{\mu\nu}\rangle$ by solving the following equations on a replicated manifold:

$$\int_x \langle T^\mu_\mu\rangle = \int_x \left(-\frac{\mu}{2}\left(\langle T^{\mu\nu}\rangle\langle T_{\mu\nu}\rangle - \langle T^\kappa_\kappa\rangle^2\right) - \frac{c}{24\pi}R(g)\right),$$

$$\nabla_\mu\langle T^{\mu\nu}\rangle = 0.$$

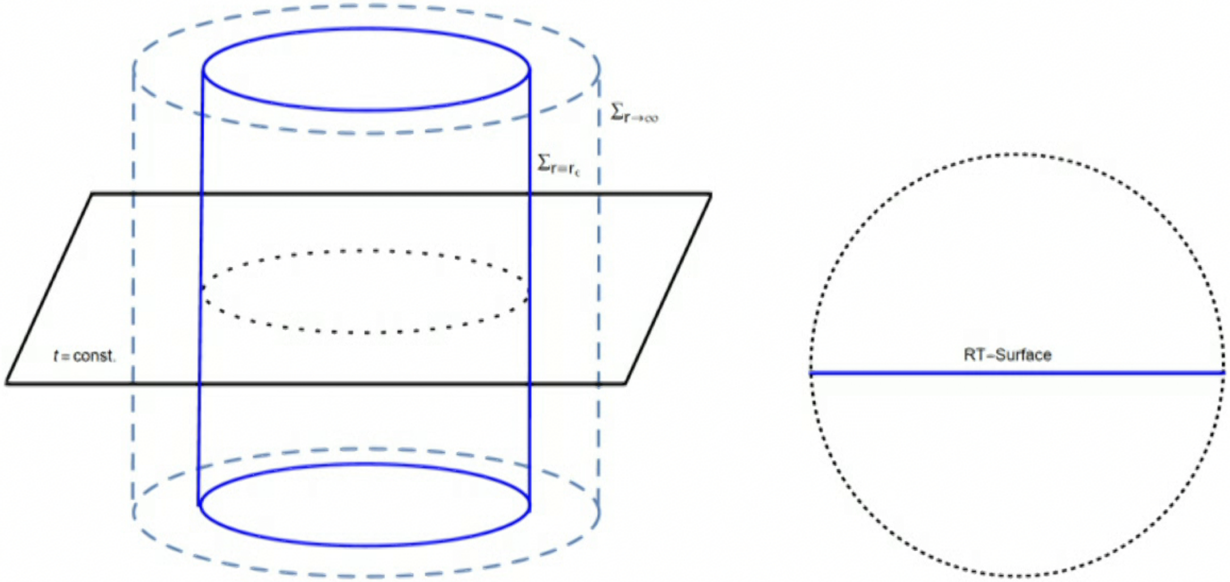
- Obtain generating function in terms of r and n by integrating $\langle T_{\mu\nu}\rangle$:

$$r\frac{d\ln Z_n(r)}{dr} = -\int_x \langle T^\mu_\mu\rangle$$

- von Neumann entropy obtained in as follows:

$$S_{EE} = \lim_{n\to 1}\left[\left(1 - n\frac{\partial}{\partial n}\right)\ln Z_n\right].$$

Finite radius RT prescription
[W. Donnelly, VS]



$$S_{EE} = \frac{c}{3}\sinh^{-1}\left(\frac{\sqrt{24\pi}r}{\sqrt{\mu c}}\right) = \frac{L}{4G} = Ar(\text{RT-Surface})$$

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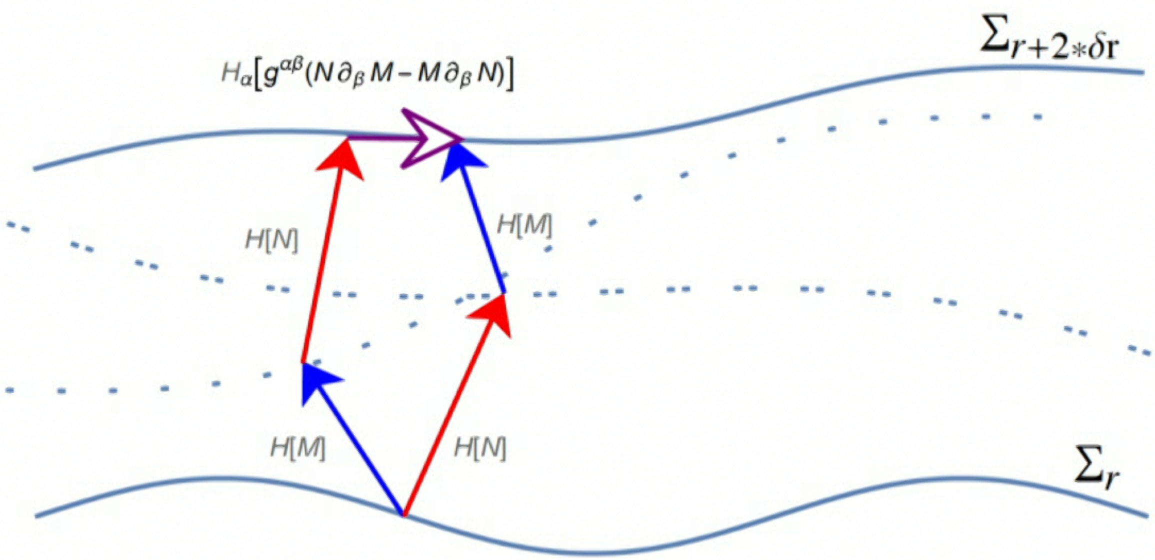
Emergent Bulk Covariance



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Finite radius RT prescription
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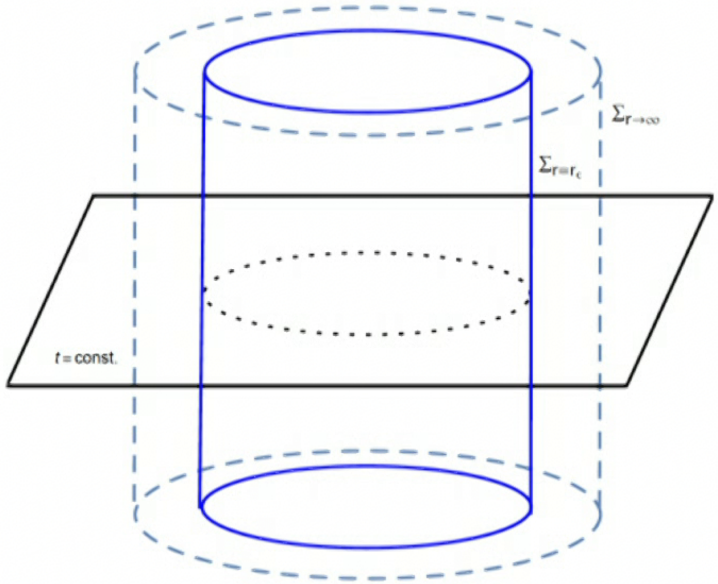
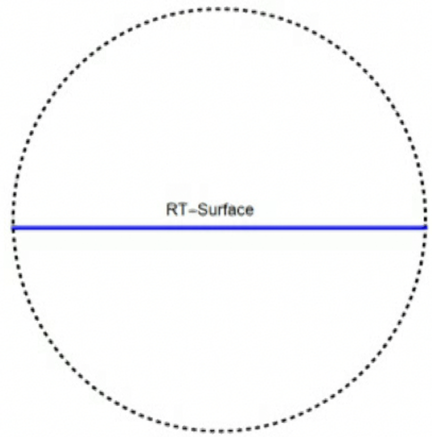




$$S_{EE} = \frac{c}{3}\sinh^{-1}\left(\frac{\sqrt{24\pi}r}{\sqrt{\mu c}}\right) = \frac{L}{4G} = Ar(\text{RT-Surface})$$

Holographic Conical Entropies-Dong's conjecture
[X. Dong- arXiv:1601.06788[hep-th]]

- Dong proposed a dual quantitiy to what is called the Conical entropy, $\tilde{S}_n$ (which is closely related to the Renyi entropy S_n) which is the area of a cosmic brane in the bulk $AdS_{d+1}/\mathbb{Z}_n$ anchored to the entangling surface.
- This quantity is defined as

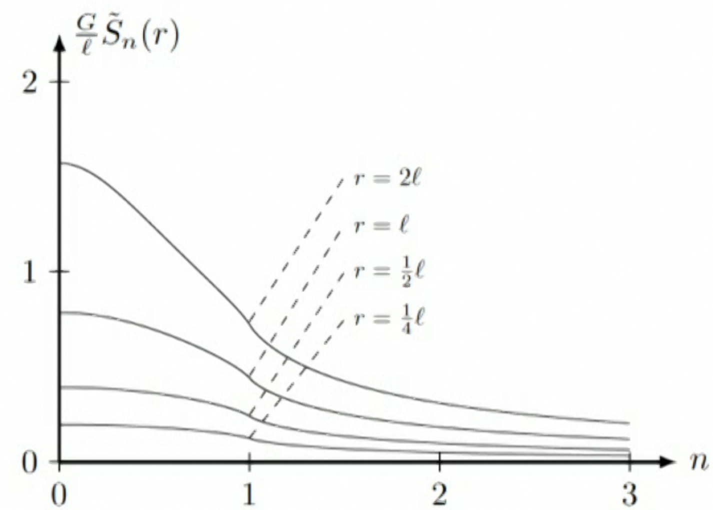
$$\tilde{S}_n = -n^2 \frac{\mathrm{d}}{\mathrm{d}n}\left(\frac{\ln Z_n}{n}\right) = n^2 \partial_n \left(\frac{n-1}{n} S_n\right)$$

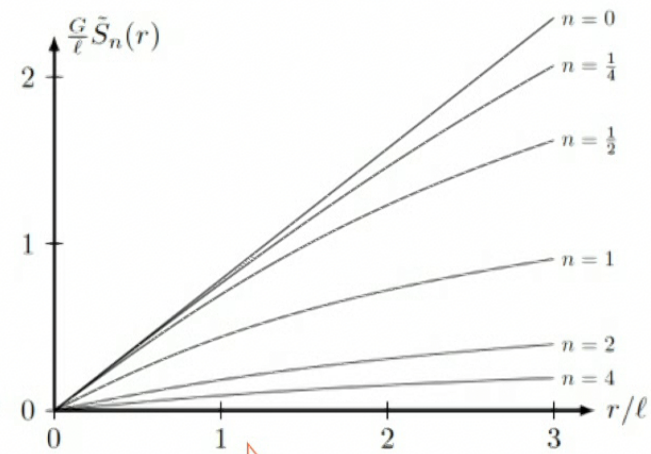
- The tension of the dual brane is $T_n = \frac{n-1}{4\pi G n}$, and it backreacts to create a conical deficit with opening angle $\Delta\phi = 2\pi \frac{n-1}{n}$.
- It limits to the entanglement entropy as $n \to 1$:

$$\lim_{n\to 1} \tilde{S}_n = S_{EE}$$

The Conical entropy in the $T\bar{T}$ deformed theory

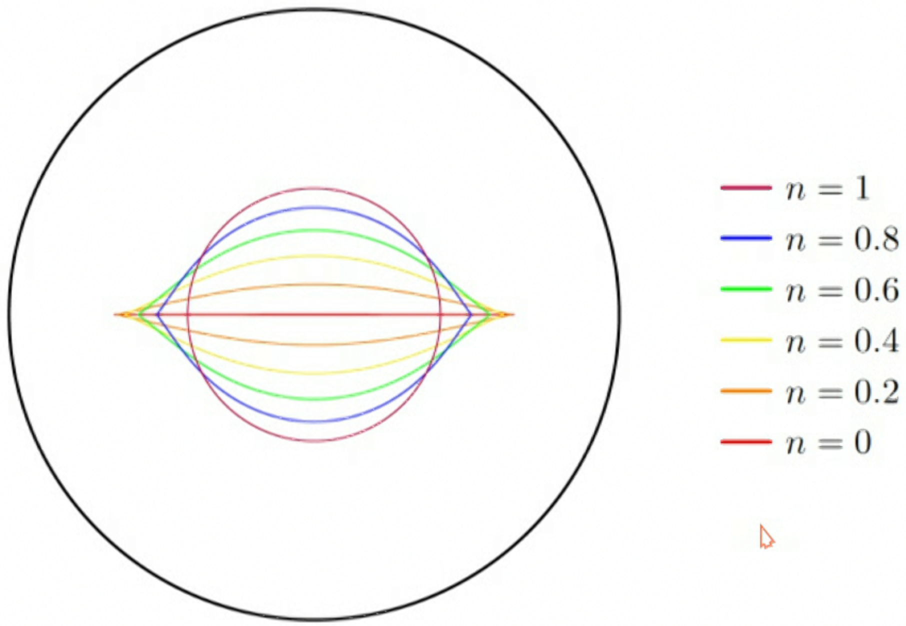
$$\tilde{S}_n = \frac{c}{3}\frac{(1-n^2)}{\sqrt{\frac{c\mu}{24\pi r^2}+n^2}}\Pi\left(n^2\middle|\frac{r^2+\frac{c\mu}{24\pi}}{r^2+\frac{c\mu}{24\pi n^2}}\right).$$





Finite radius generalization of Dong's conjecture

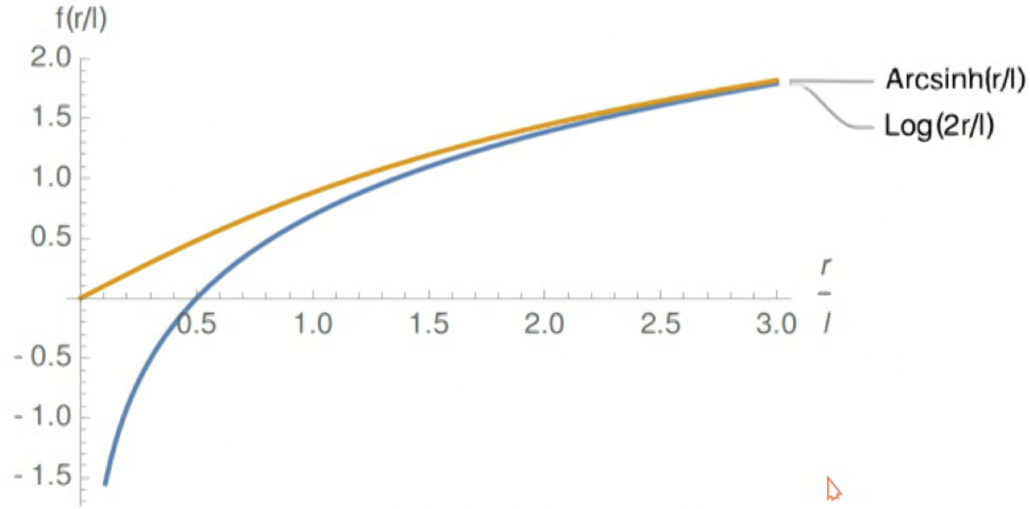
Through a suitable rescaling, the conical deficit can be shifted from the bulk to the boundary and the area of the brane reduced to computing the length of a geodesic.



Similarity with cMERA

[A. Rubio-Franco and G. Vidal: arXiv:1706.02841[quant-ph]]

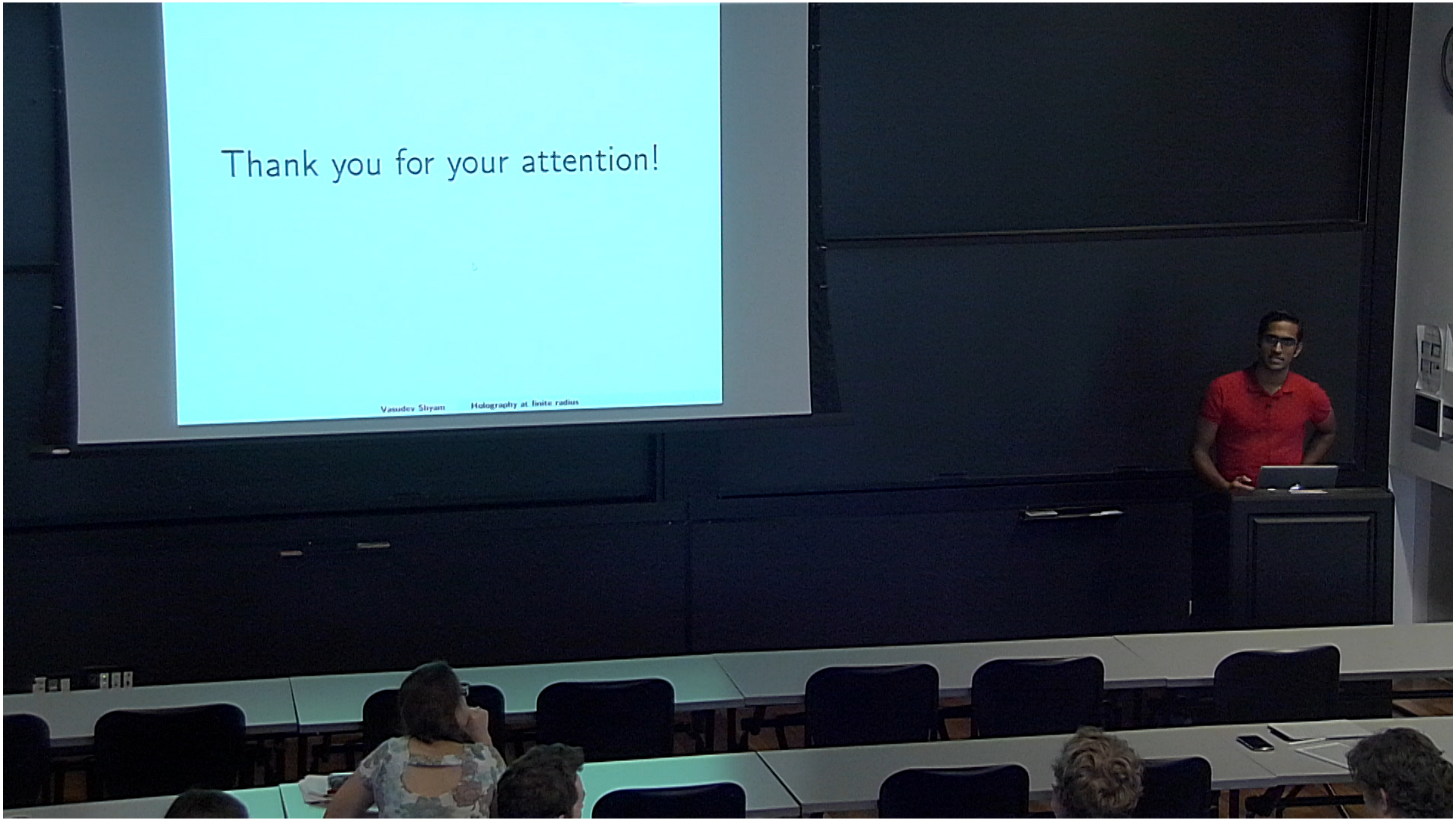
- The entanglement entropy of the deformed theory suggests it triggers a flow from a trivial theory in the UV to a CFT in the IR



- This is analogous to the picture the cMERA suggests.

Conclusions and Outlook

- Operator deformations involving the energy momentum tensor and scalar double trace operators provide the right cutoff prescription to match with a sharp radial cutoff in the bulk.
- The UV properties of entanglement measures such as the von Neumann and conical entropies for the deformed theory reflect the regularization one would anticipate from putting a radial cutoff in the bulk.
- Understanding what happens to these deformed theories away from the large N or c limit is of crucial importance.
- The $T\bar{T}$ deformed theory can be seen as arising from coupling the CFT to a certain topological gravity theory (Jackiw–Teitelboim theory). It would help to understand how this fits into the holographic story.


Thank you for your attention!
Vasudev Shyam
Holography at finite radius