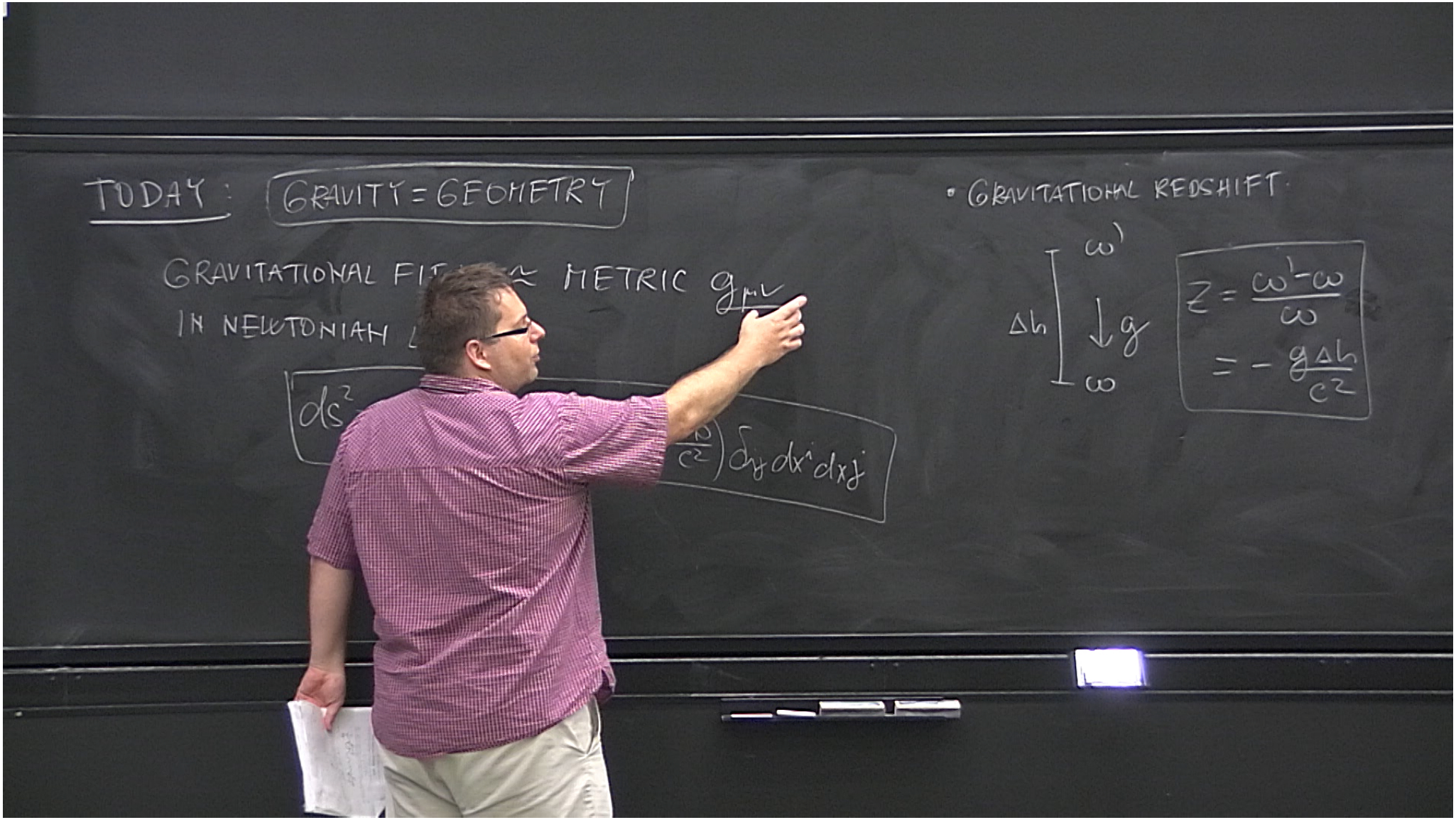
Title: PSI 2018/2019 - Relativity - Lecture 4

Date: Sep 06, 2018 03:45 PM

URL: http://pirsa.org/18090040

Abstract:


TODAY: GRAVITY = GEOMETRY
GRAVITATIONAL FI ≃ METRIC $g_{\mu\nu}$
IN NEWTONIAN
ds^2
$\delta_{ij} dx^i dx^j$
• GRAVITATIONAL REDSHIFT
ω'
Δh
g
ω
$z = \frac{\omega' - \omega}{\omega}$
$= -\frac{g \Delta h}{c^2}$

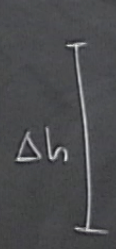
TODAY: GRAVITY = GEOMETRY

GRAVITATIONAL FIELD $\simeq$ METRIC $g_{\mu\nu}$

IN NEWTONIAN LIMIT

$$ds^2 = -\left(1 - \frac{2\Phi}{c^2}\right)dt^2 + \left(1 + \frac{2\Phi}{c^2}\right)\delta_{ij}dx^i dx^j$$

- GRAVITATIONAL REDSHIFT



$$z = \frac{\omega' - \omega}{\omega} = -\frac{g\Delta h}{c^2}$$

L REDSHIFT

$$z = \frac{\omega' - \omega}{\omega} = -\frac{g\Delta h}{c^2}$$

AGAIN

IN STRONG GRAVITATIONAL FIELD

TIME TICKS MORE SLOWLY

$$d\tau = \sqrt{-g_{\mu\nu}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}}\,dt$$

L REDSHIFT

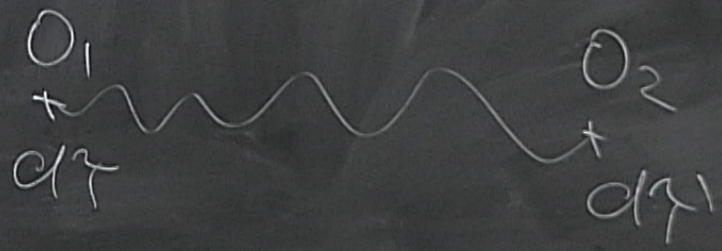
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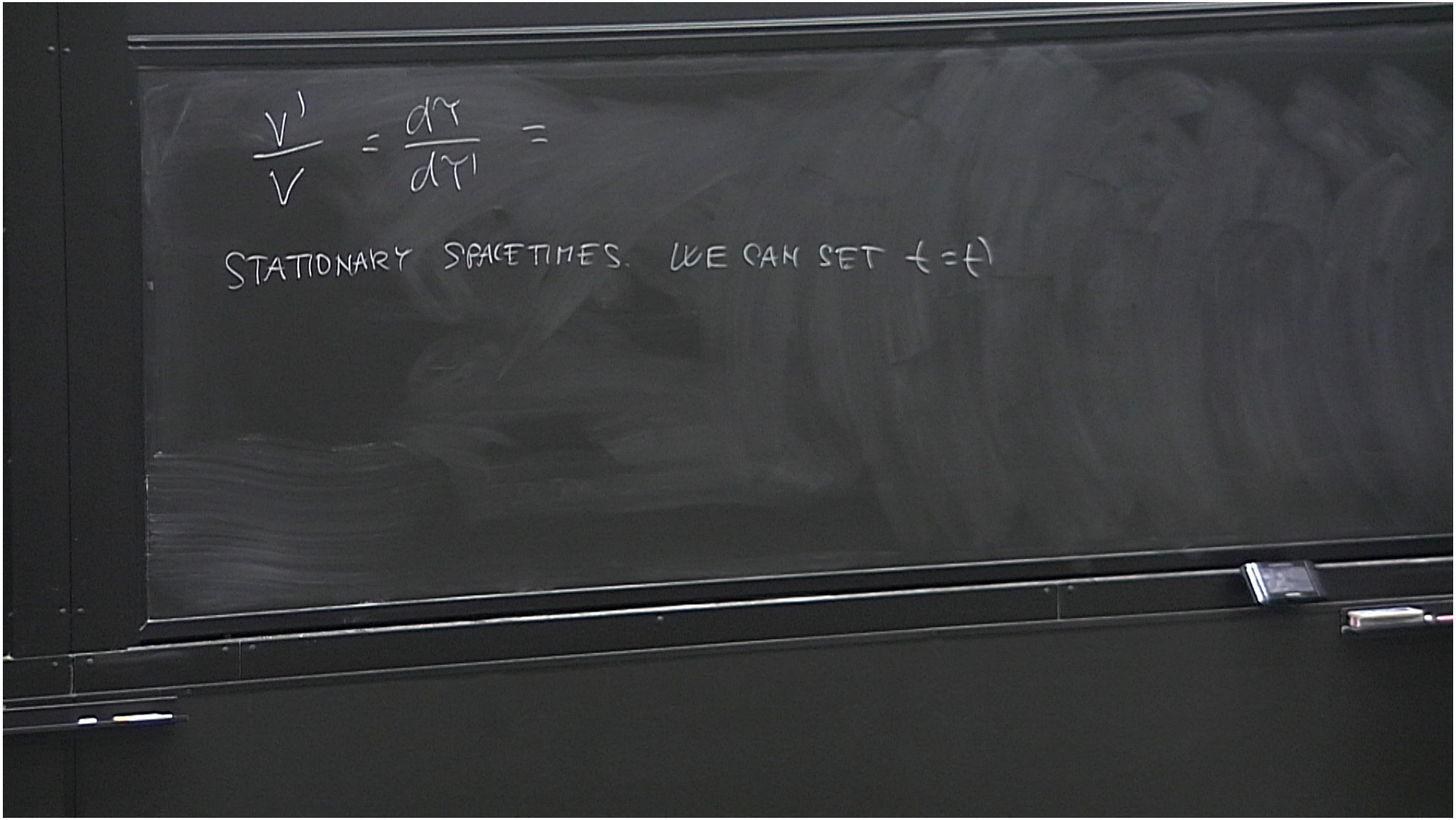
AGAIN

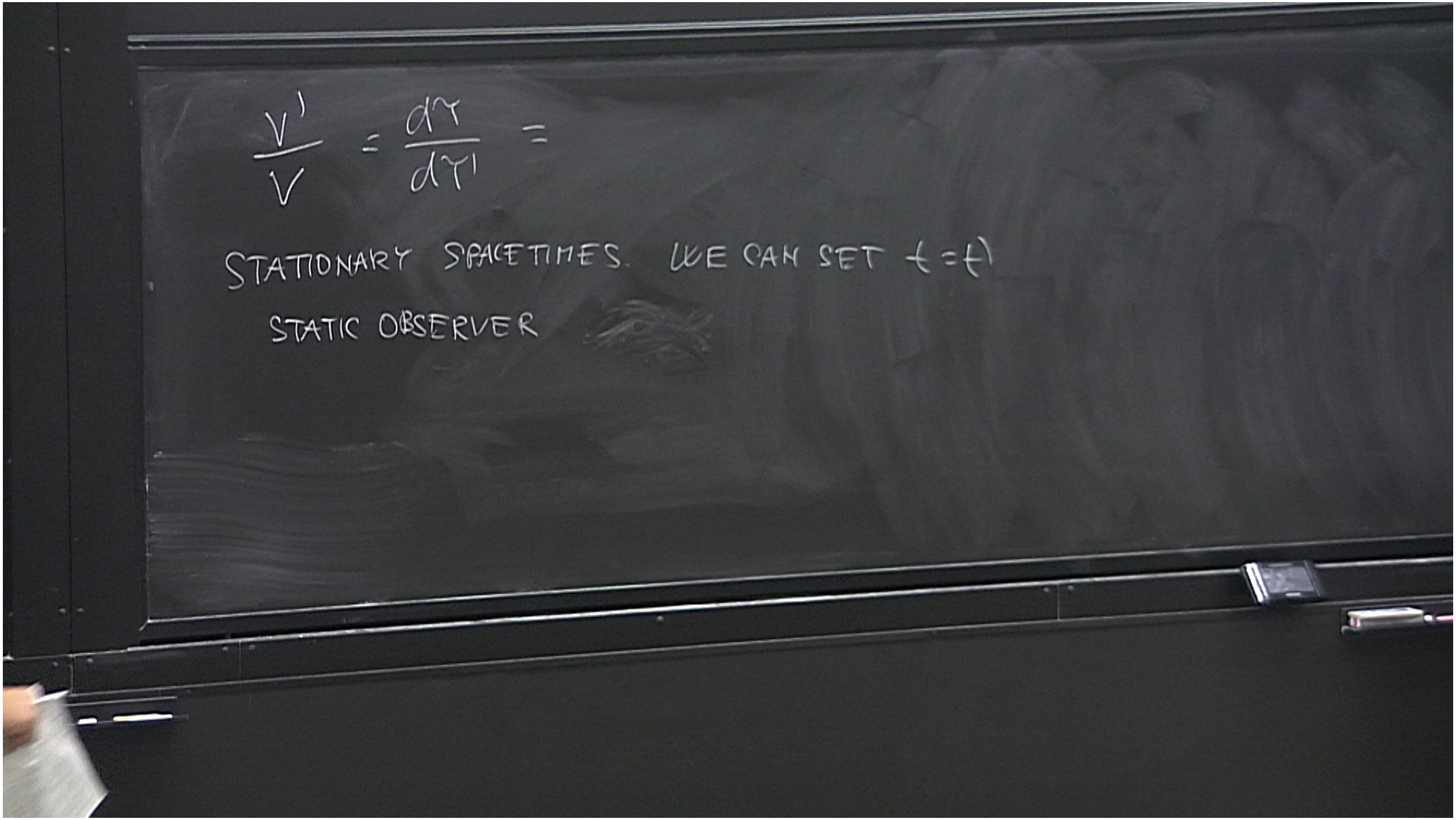
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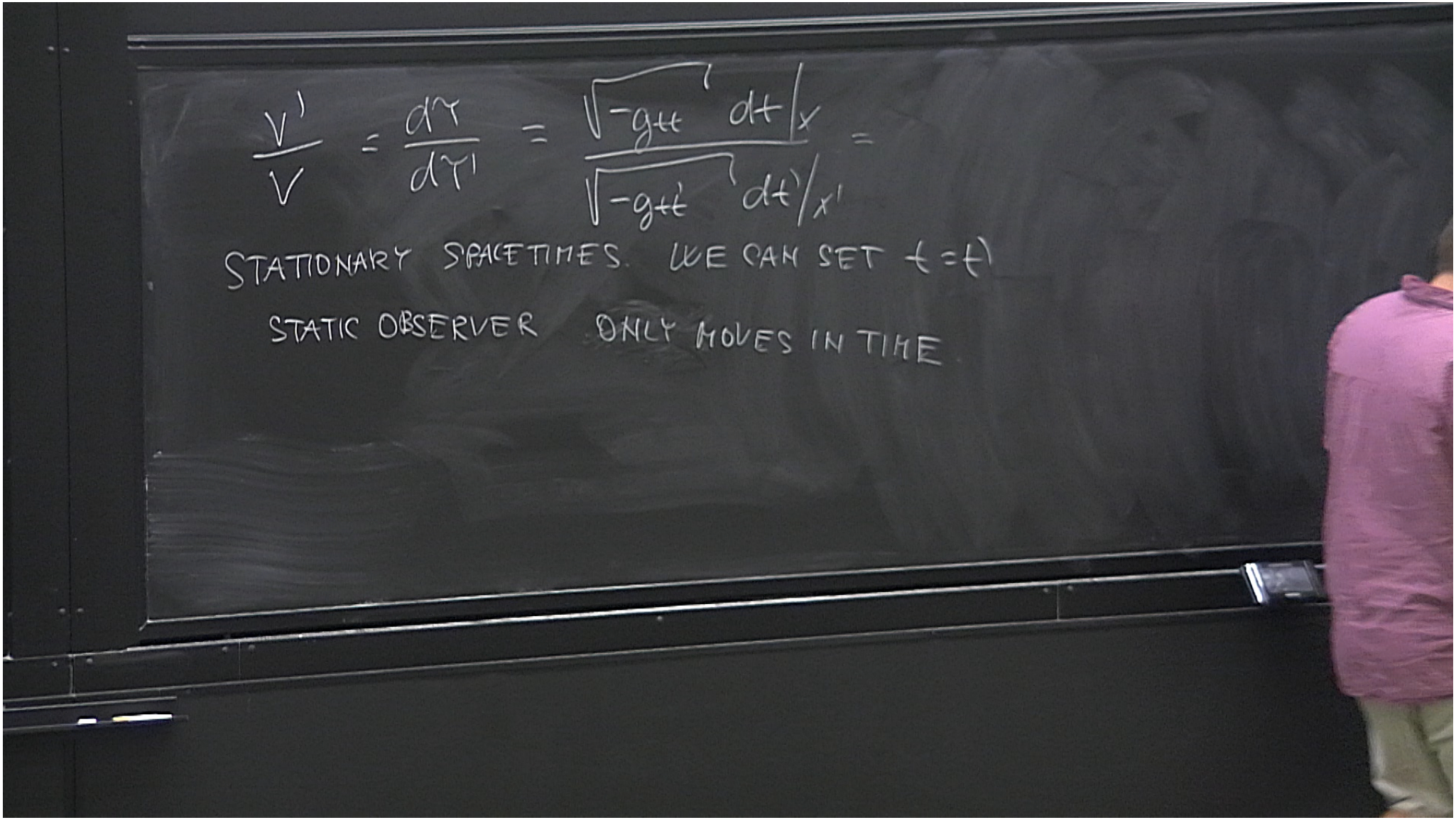
TIME TICKS MORE SLOWLY

$$d\tau = \sqrt{-g_{\mu\nu}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}}\,dt$$




$\frac{V'}{V} = \frac{d\tau}{dT} =$
STATIONARY SPACETIMES. WE CAN SET $t = t$


$\frac{V'}{V} = \frac{d\tau}{dT'} =$
STATIONARY SPACETIMES. WE CAN SET $t = t'$
STATIC OBSERVER


$\frac{V'}{V} = \frac{d\tau}{d\tau'} = \frac{\sqrt{-g_{tt}}\,dt|_x}{\sqrt{-g_{t't'}}\,dt'|_{x'}} =$
STATIONARY SPACETIMES. WE CAN SET $t = t'$
STATIC OBSERVER ONLY MOVES IN TIME

$$\frac{v'}{v} = \frac{d\tau}{d\tau'} = \frac{\sqrt{-g_{tt}}\, dt/x}{\sqrt{-g_{tt}}\, dt'/x'} = \frac{\sqrt{-g_{tt}(x)}}{\sqrt{-g_{tt}(x')}} = \frac{v'}{v}$$

STATIONARY SPACETIMES. WE CAN SET $t = t'$

STATIC OBSERVER ONLY MOVES IN TIME

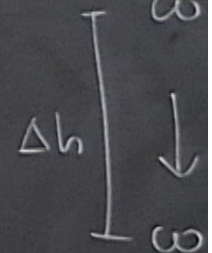
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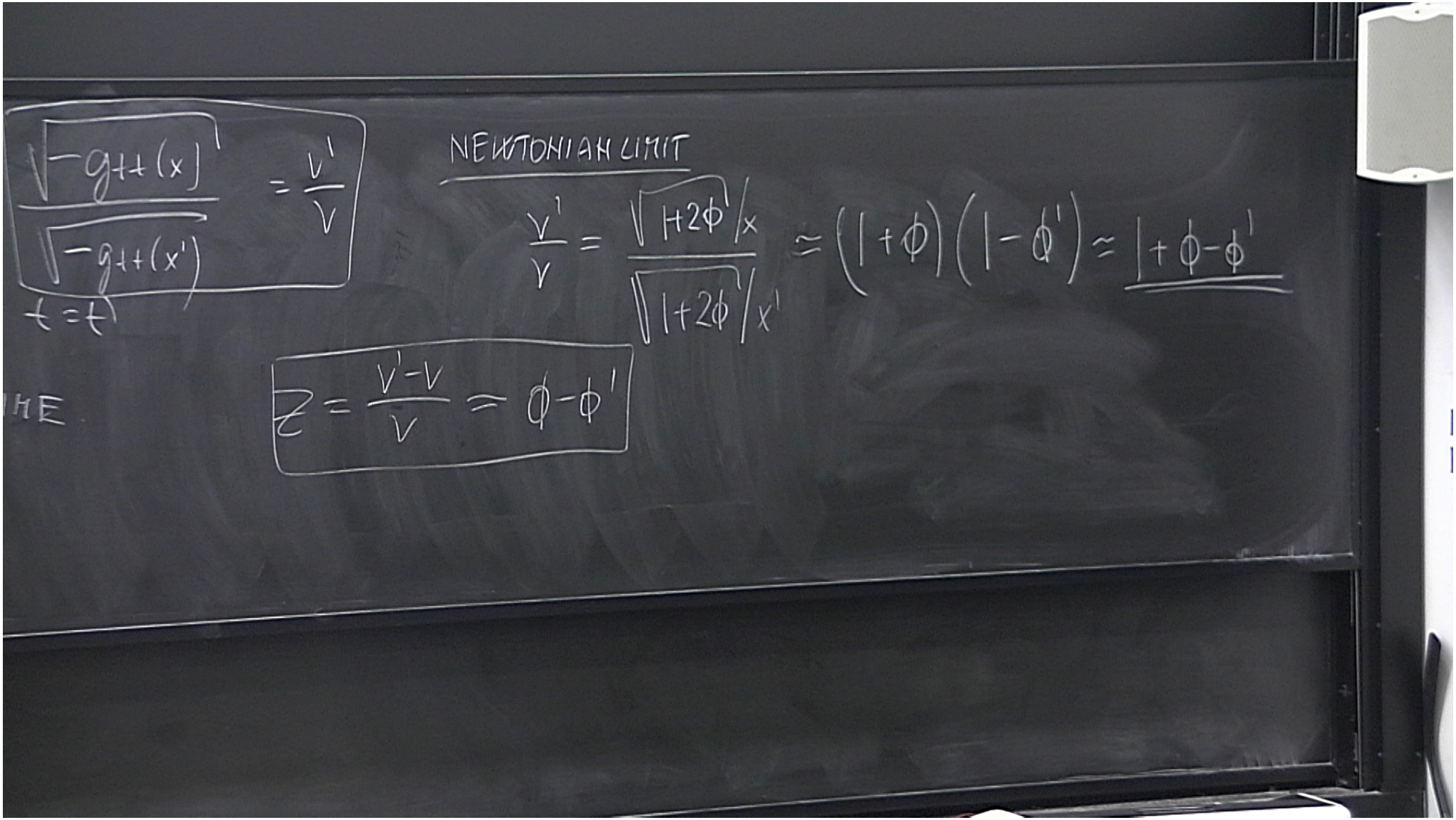
GRAVITATIONAL FIELD ≃ METRIC $g_{\mu\nu}$

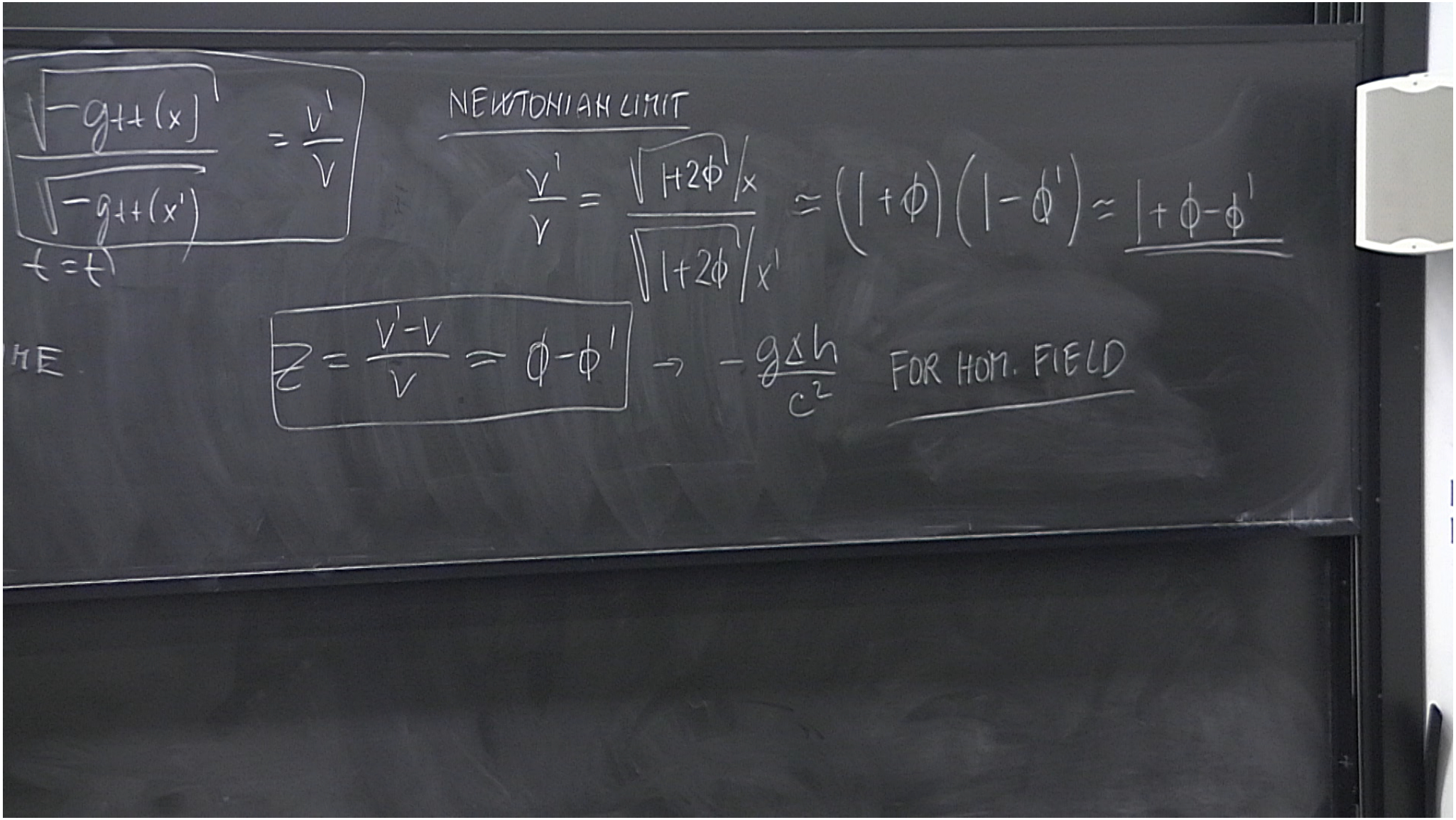
IN NEWTONIAN LIMIT

$$ds^2 = -\left(1+\frac{2\Phi}{c^2}\right)dt^2 + \left(1-\frac{2\Phi}{c^2}\right)\delta_{ij}\,dx^i dx^j$$

• GRAVITATION




$\frac{\sqrt{-g_{tt}(x)}}{\sqrt{-g_{tt}(x')}} = \frac{\nu'}{\nu}$
$t = t'$
NEWTONIAN LIMIT
$\frac{\nu'}{\nu} = \frac{\sqrt{1+2\phi'}/x}{\sqrt{1+2\phi}/x'} \simeq (1+\phi)(1-\phi') \simeq 1+\phi-\phi'$
$z = \frac{\nu'-\nu}{\nu} \simeq \phi-\phi'$


$\frac{\sqrt{-g_{tt}(x)}}{\sqrt{-g_{tt}(x')}} = \frac{\nu'}{\nu}$
$t = t'$
NEWTONIAN LIMIT
$\frac{\nu'}{\nu} = \frac{\sqrt{1+2\phi}/x}{\sqrt{1+2\phi}/x'} \simeq (1+\phi)(1-\phi') \simeq 1+\phi-\phi'$
$z = \frac{\nu'-\nu}{\nu} = \phi-\phi' \rightarrow -\frac{g\Delta h}{c^2}$ FOR HOM. FIELD

FIELD THEORY FOR GRAVITY

	MAXWELL	NEWTON	EINSTEIN
FIELD	A^μ	ϕ	$g_{\mu\nu}$
FIELD EQS	$\Box A^\mu = -4\pi J^\mu$ $\partial_\mu A^\mu = 0$	$\nabla^2\phi = 4\pi G\rho$	(?)
EQS OF MOTION	$\frac{dp^\mu}{d\tau} = e F^\mu{}_\nu$	$\frac{d\vec{p}}{dt} = -m\nabla\phi$	$\frac{dp^\mu}{d\tau} + \Gamma^\mu_{\alpha\beta} p^\alpha p^\beta = 0$

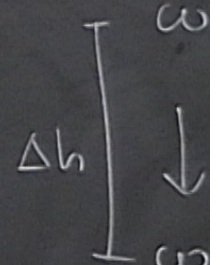
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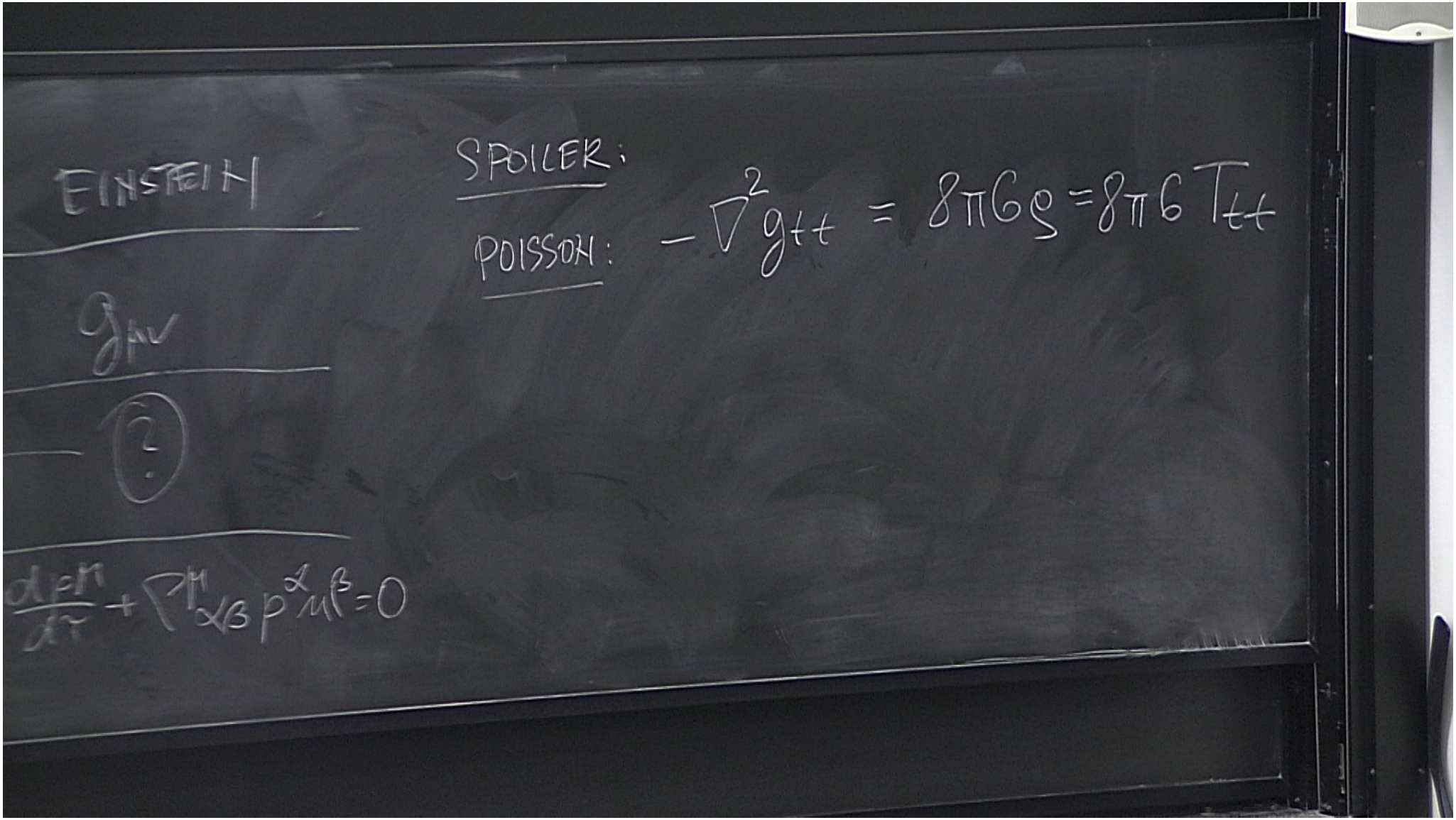
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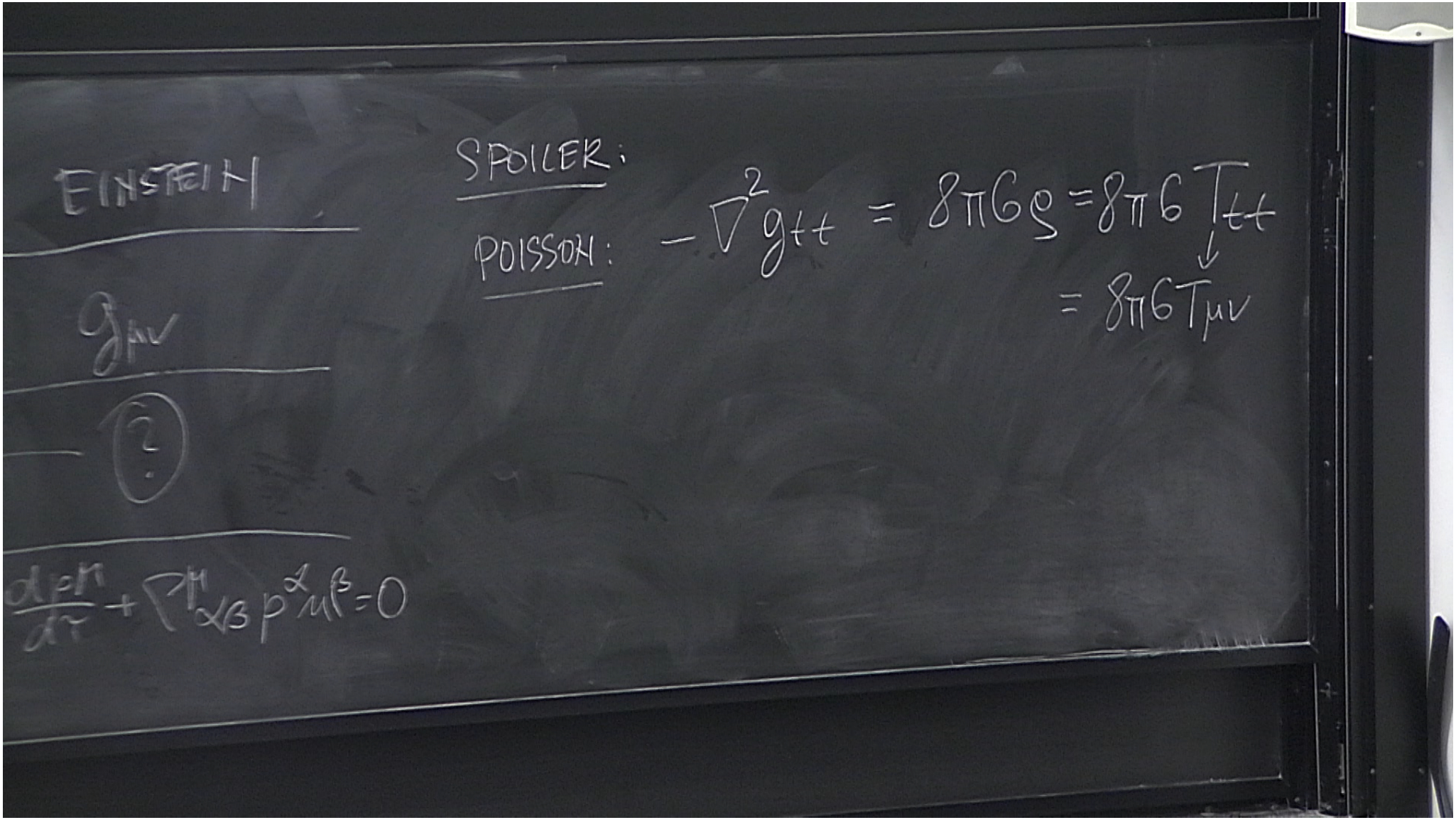
IN NEWTONIAN LIMIT

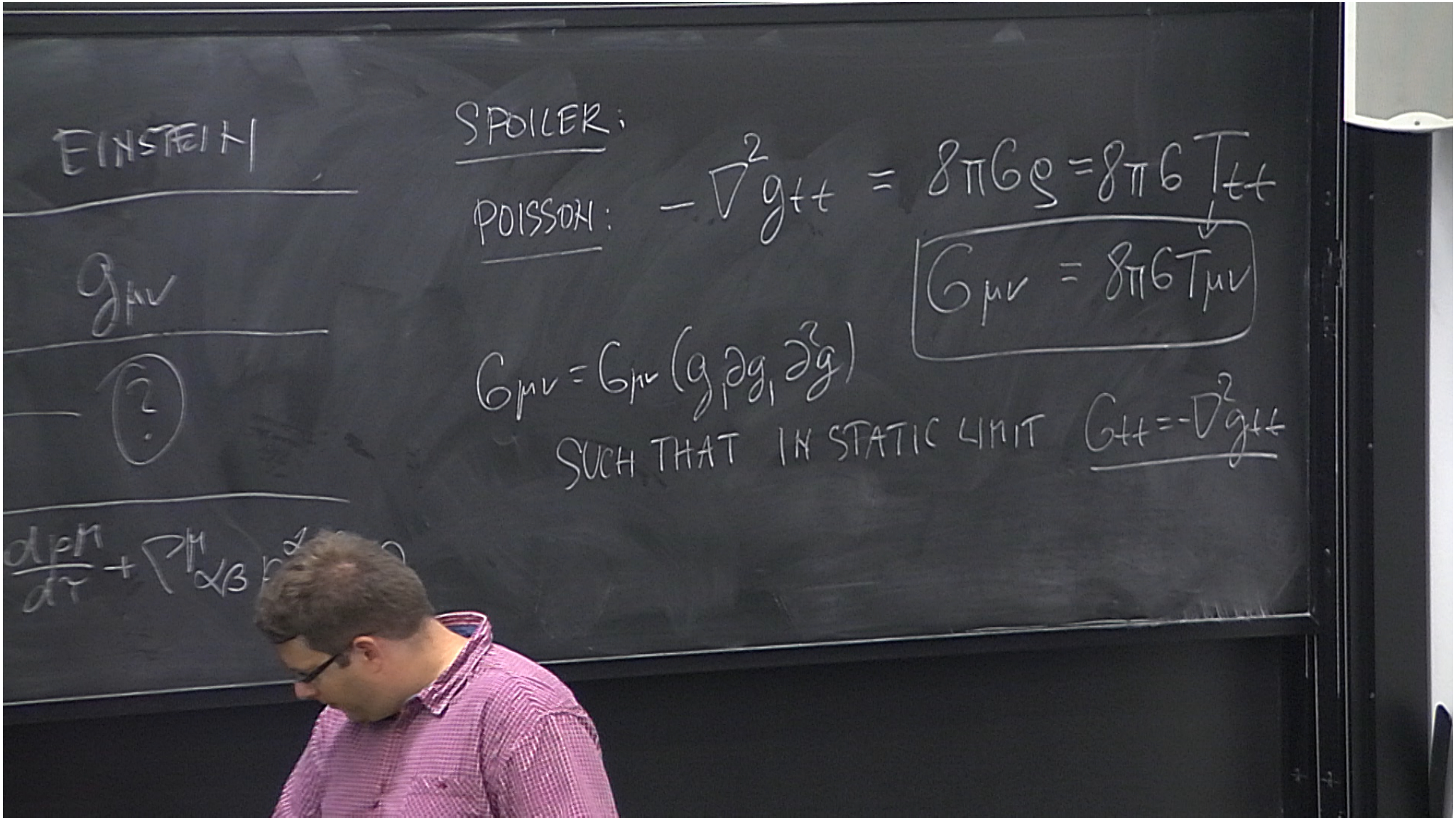
$$ds^2 = -\underbrace{\left(1+\frac{2\Phi}{c^2}\right)}_{g_{tt}} dt^2 + \left(1-\frac{2\Phi}{c^2}\right)\delta_{ij}\,dx^i dx^j$$

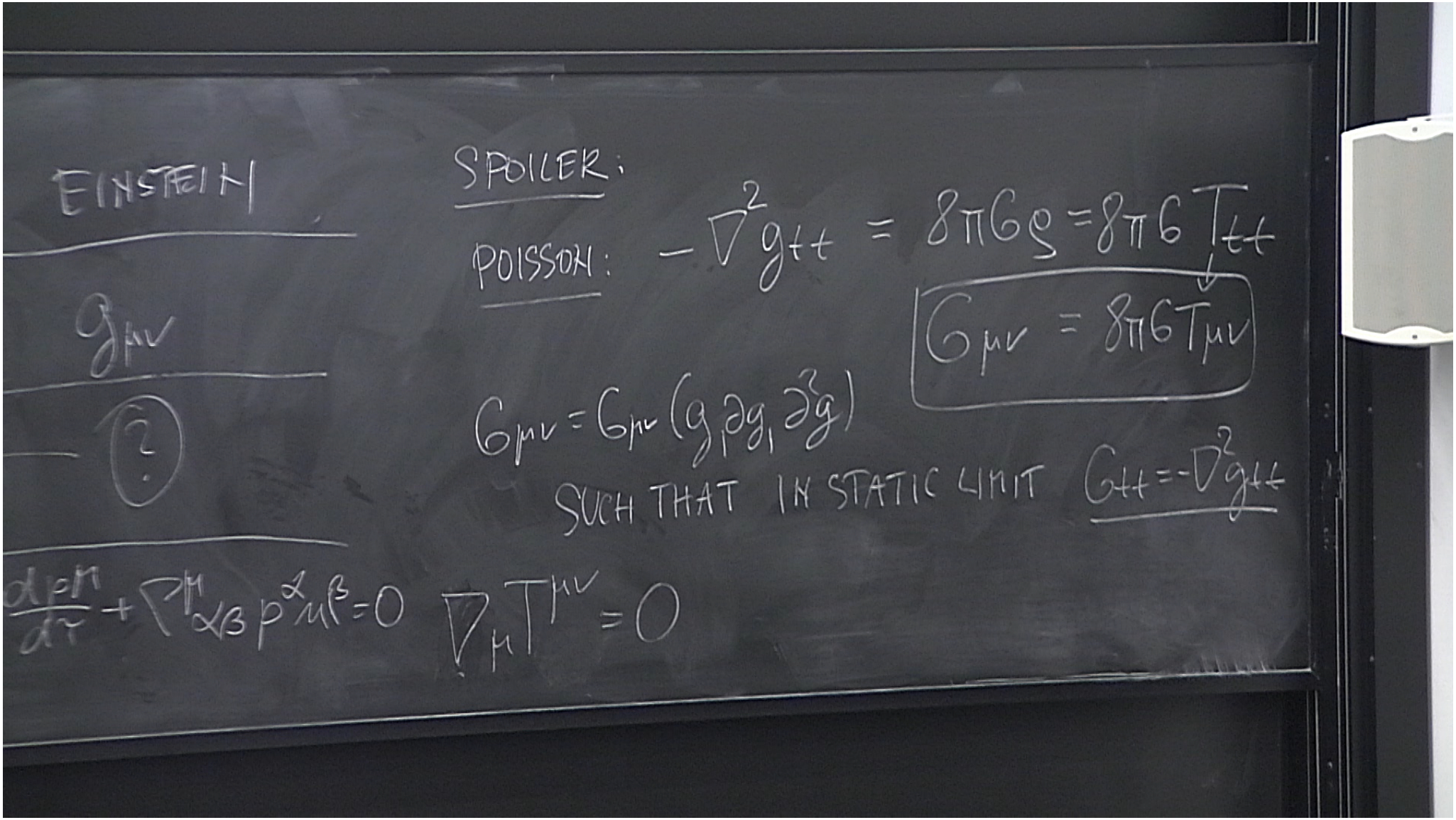
• GRAVITATIO

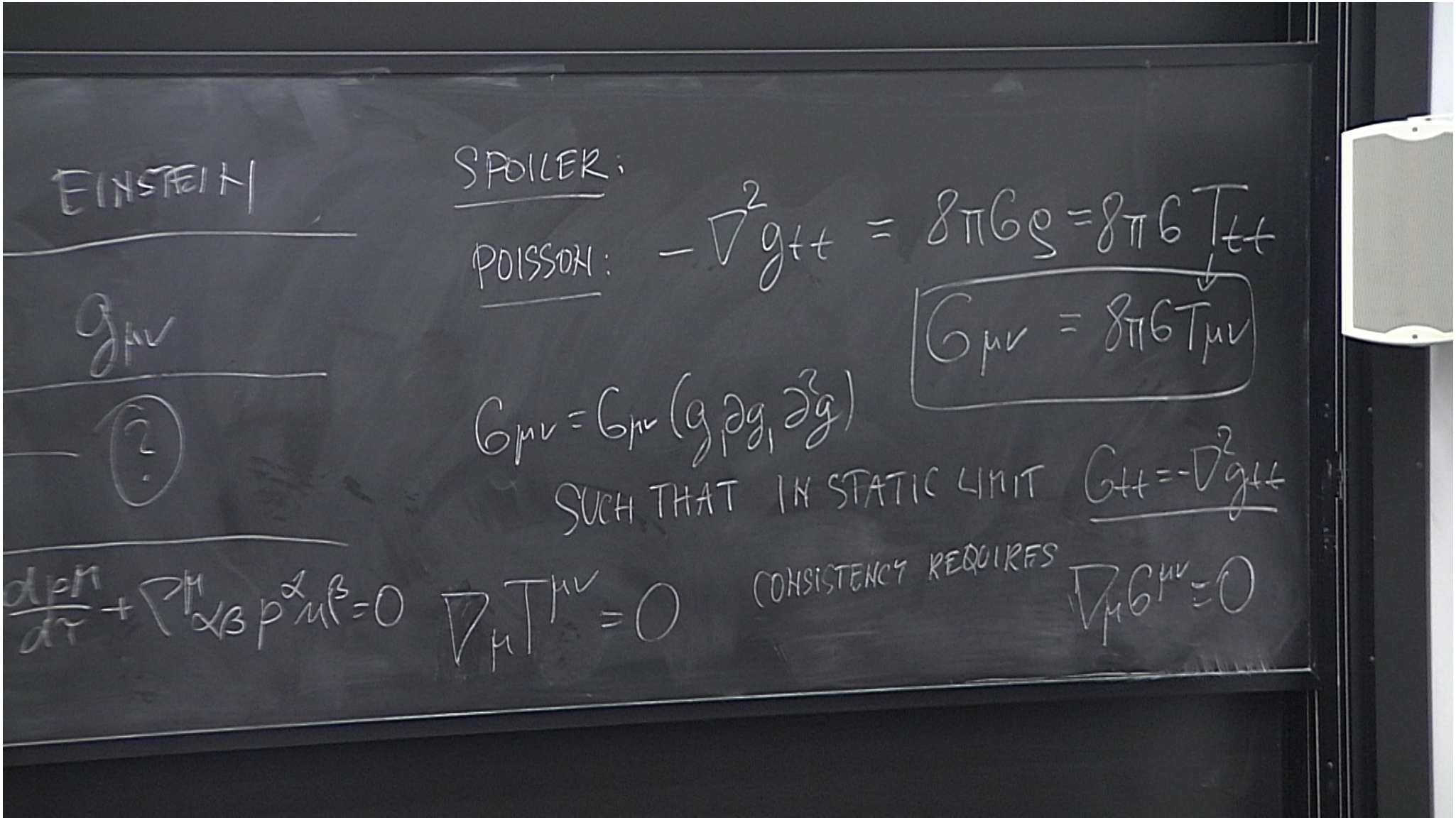



EINSTEIN
SPOILER:
POISSON: $-\nabla^2 g_{tt} = 8\pi G \rho = 8\pi G T_{tt}$
$g_{\mu\nu}$
?
$\frac{dp^\mu}{d\tau} + \Gamma^\mu_{\alpha\beta} p^\alpha p^\beta = 0$


EINSTEIN
$g_{\mu\nu}$
?
$\frac{d p^\mu}{d\tau} + \Gamma^\mu_{\alpha\beta} p^\alpha p^\beta = 0$
SPOILER:
POISSON: $-\nabla^2 g_{tt} = 8\pi G \rho = 8\pi G T_{tt}$
$= 8\pi G T_{\mu\nu}$


EINSTEIN
SPOILER:
POISSON: $-\nabla^2 g_{tt} = 8\pi G\rho = 8\pi G T_{tt}$
$G_{\mu\nu} = 8\pi G T_{\mu\nu}$
$G_{\mu\nu} = G_{\mu\nu}(g, \partial g, \partial^2 g)$
SUCH THAT IN STATIC LIMIT $G_{tt} = -\nabla^2 g_{tt}$
$g_{\mu\nu}$
?

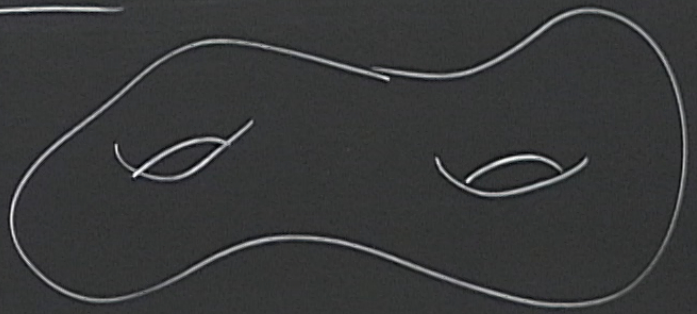

EINSTEIN
$g_{\mu\nu}$
?
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POISSON: $-\nabla^2 g_{tt} = 8\pi G\rho = 8\pi G T_{tt}$
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SUCH THAT IN STATIC LIMIT $G_{tt} = -\nabla^2 g_{tt}$
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$\nabla_\mu T^{\mu\nu} = 0$


EINSTEIN
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SUCH THAT IN STATIC LIMIT $G_{tt} = -\nabla^2 g_{tt}$
$\nabla_\mu T^{\mu\nu} = 0$
CONSISTENCY REQUIRES
$\nabla_\mu G^{\mu\nu} = 0$

GROSSMANN → $G^{\mu\nu}$.. EINSTEIN TENSOR

3) GEOMETRIC TOOLS

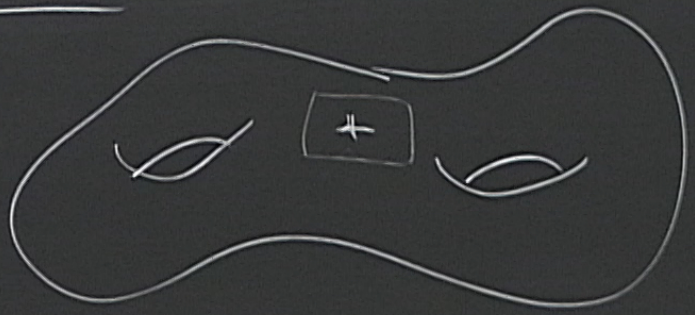
a) MANIFOLDS



GROSSMANN → $G^{\mu\nu}$.. EINSTEIN TENSOR

3) GEOMETRIC TOOLS

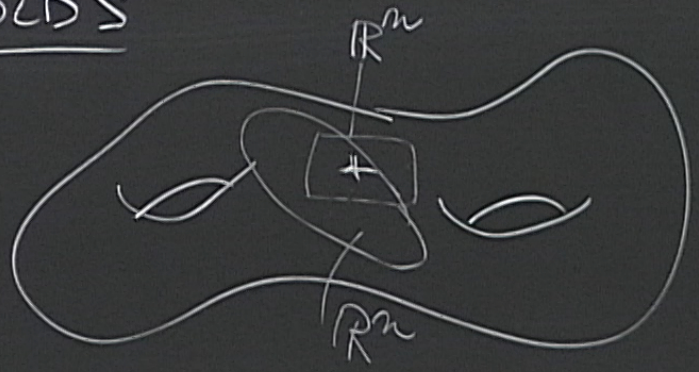
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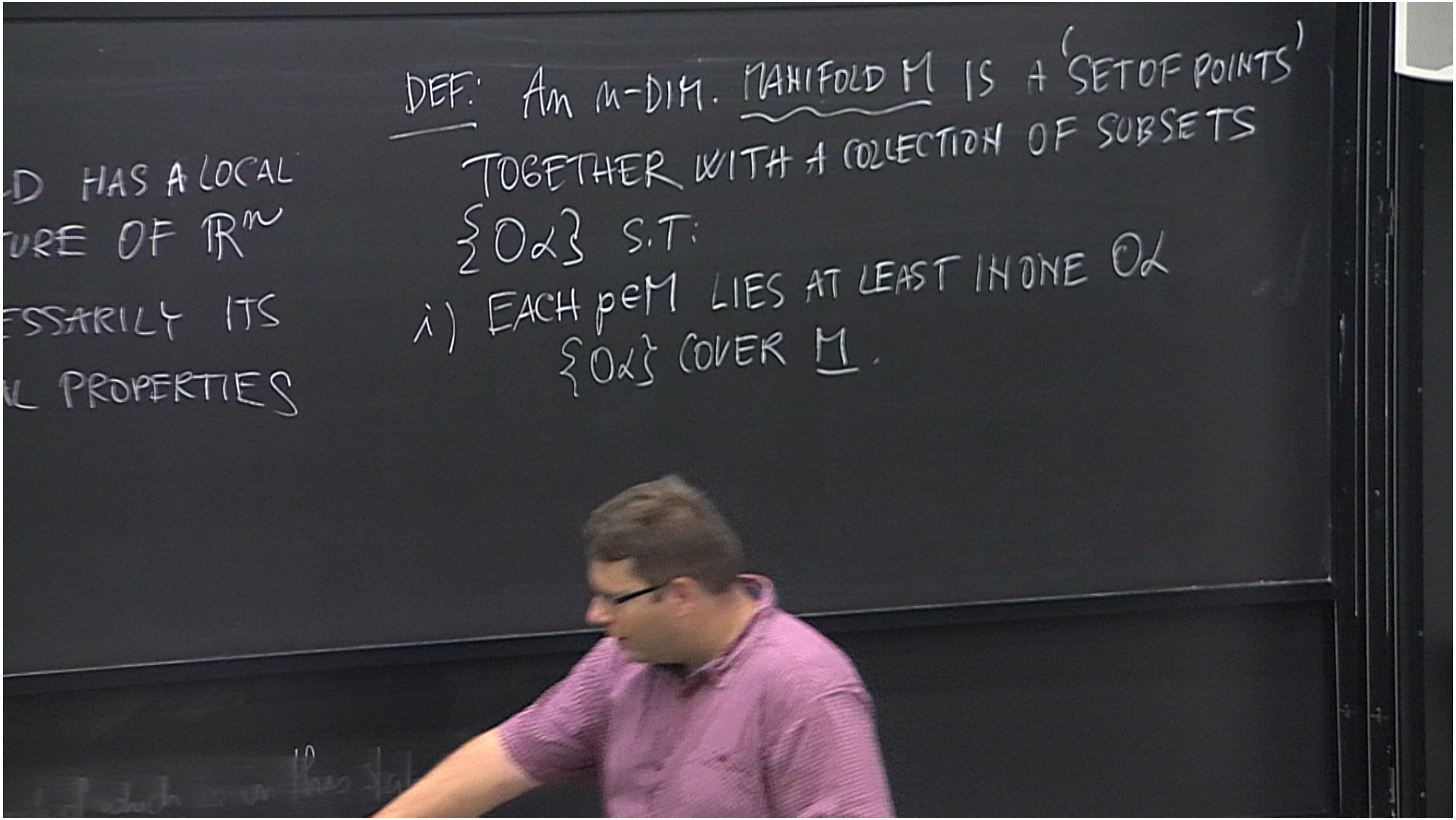
GROSSMANN → $G^{\mu\nu}$ .. EINSTEIN TENSOR

3) GEOMETRIC TOOLS

a) MANIFOLDS



n-DIM MANIFOLD HAS A LOCAL DIFF STRUCTURE OF $\mathbb{R}^n$ BUT NOT NECESSARILY ITS GLOBAL PROPERTIES

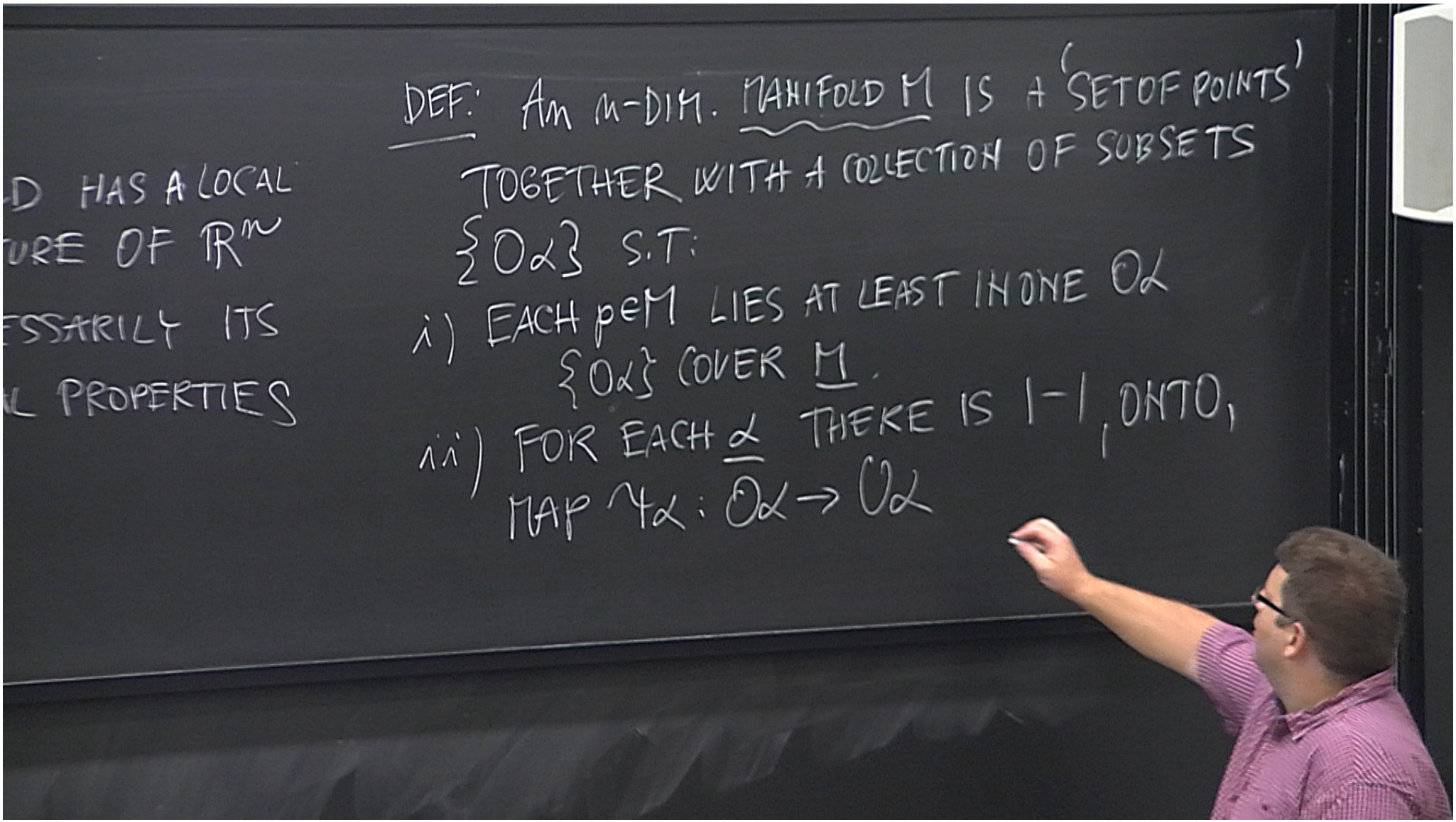

D HAS A LOCAL
URE OF $\mathbb{R}^n$
ESSARILY ITS
L PROPERTIES
DEF: An n-DIM. MANIFOLD M IS A (SET OF POINTS)
TOGETHER WITH A COLLECTION OF SUBSETS
$\{O_\alpha\}$ S.T:
i) EACH $p \in M$ LIES AT LEAST IN ONE O_α
$\{O_\alpha\}$ COVER M.

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ii) FOR EACH α THERE IS 1-1, ONTO,
MAP $\varphi_\alpha : O_\alpha \to U_\alpha$

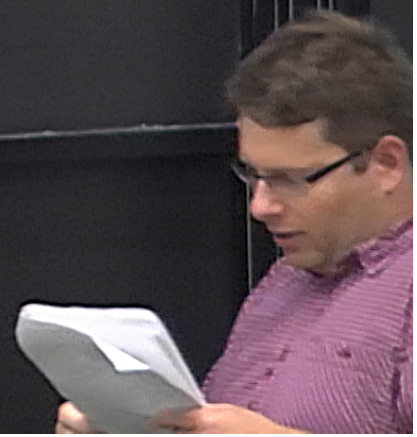


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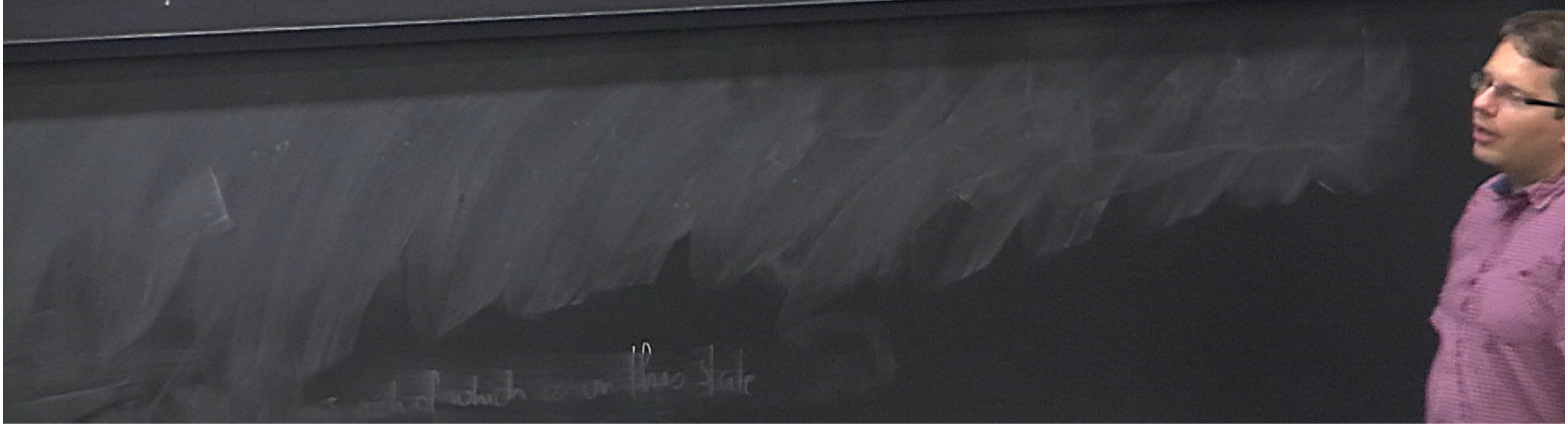
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TOGETHER WITH A
$\{O_\alpha\}$ S.T:

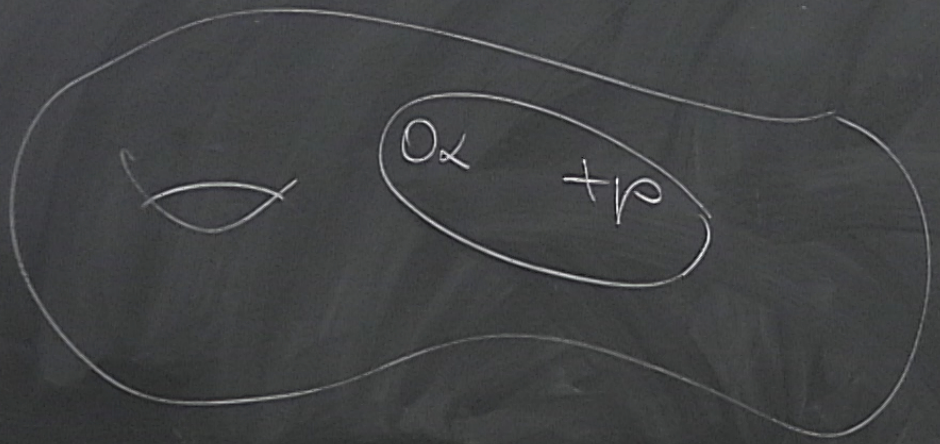
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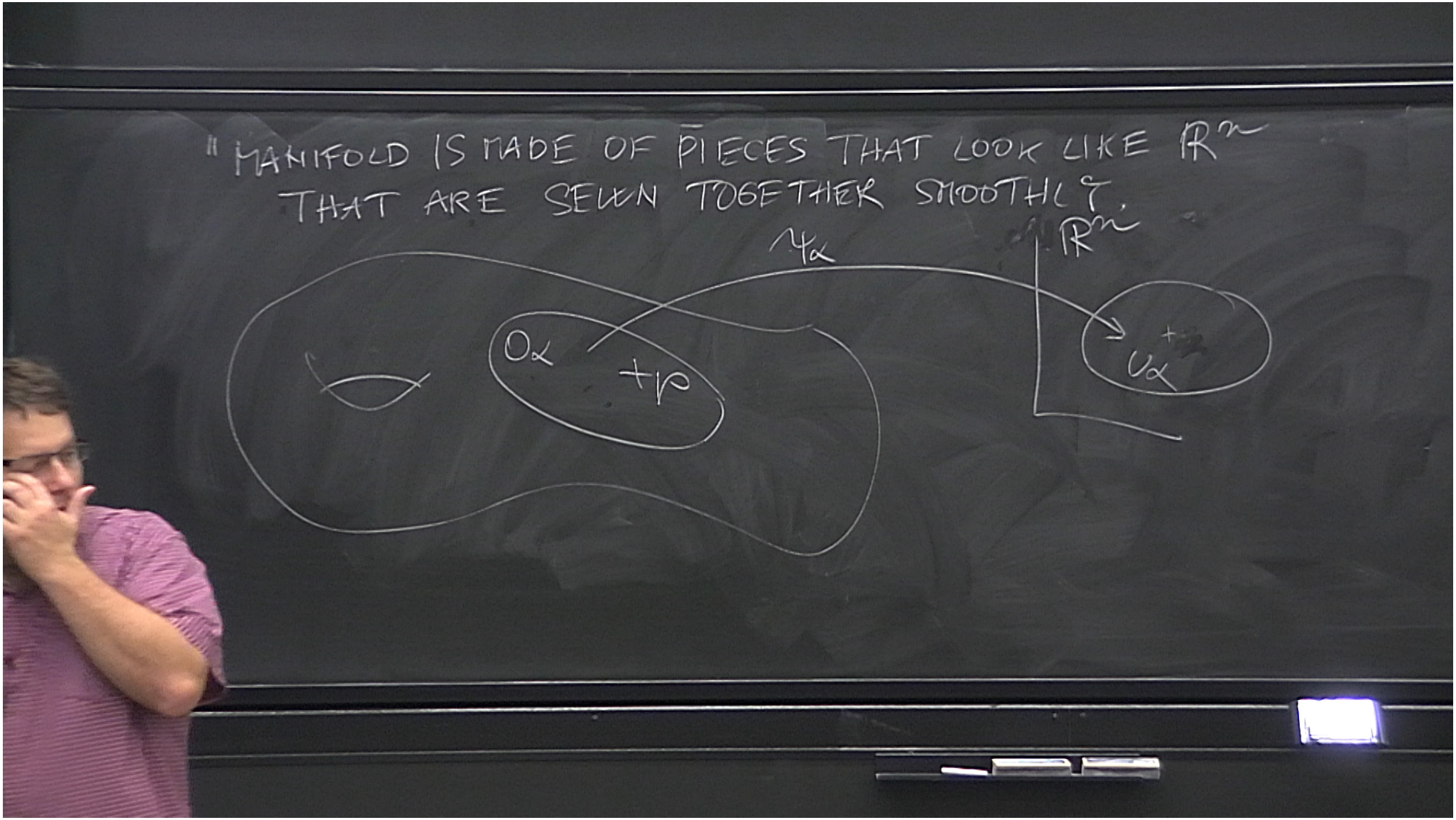
ii) FOR EACH α THERE IS 1-1, ONTO,
MAP $\psi_\alpha : \bar{O}_\alpha \to O_\alpha$ .. OPEN SUBSETS OF

iii) IF ANY 2 SETS $O_\alpha \& O_\beta$ OVERLAP, $O_\alpha \cap O_\beta \neq 0$, $\psi_\beta \circ \psi_\alpha^{-1}$ IS C^∞.

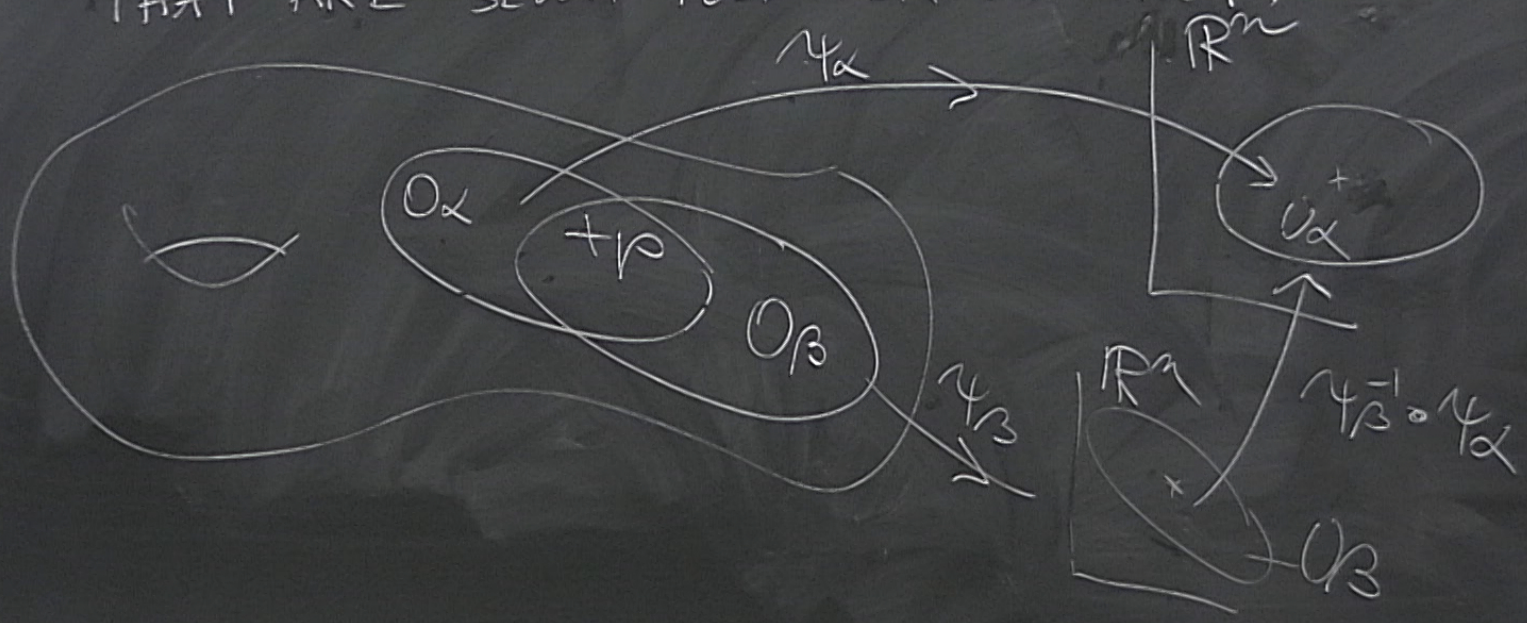


"MANIFOLD IS MADE OF PIECES THAT LOOK LIKE $\mathbb{R}^n$ THAT ARE SEWN TOGETHER SMOOTHLY.

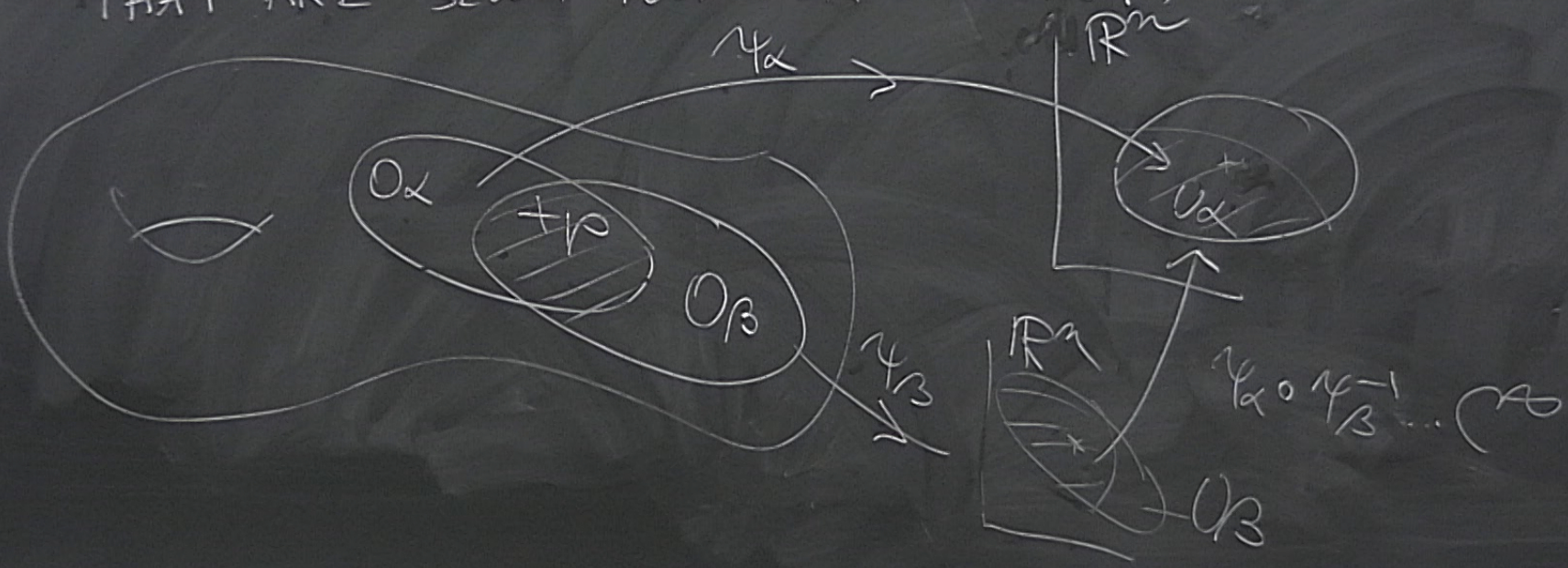


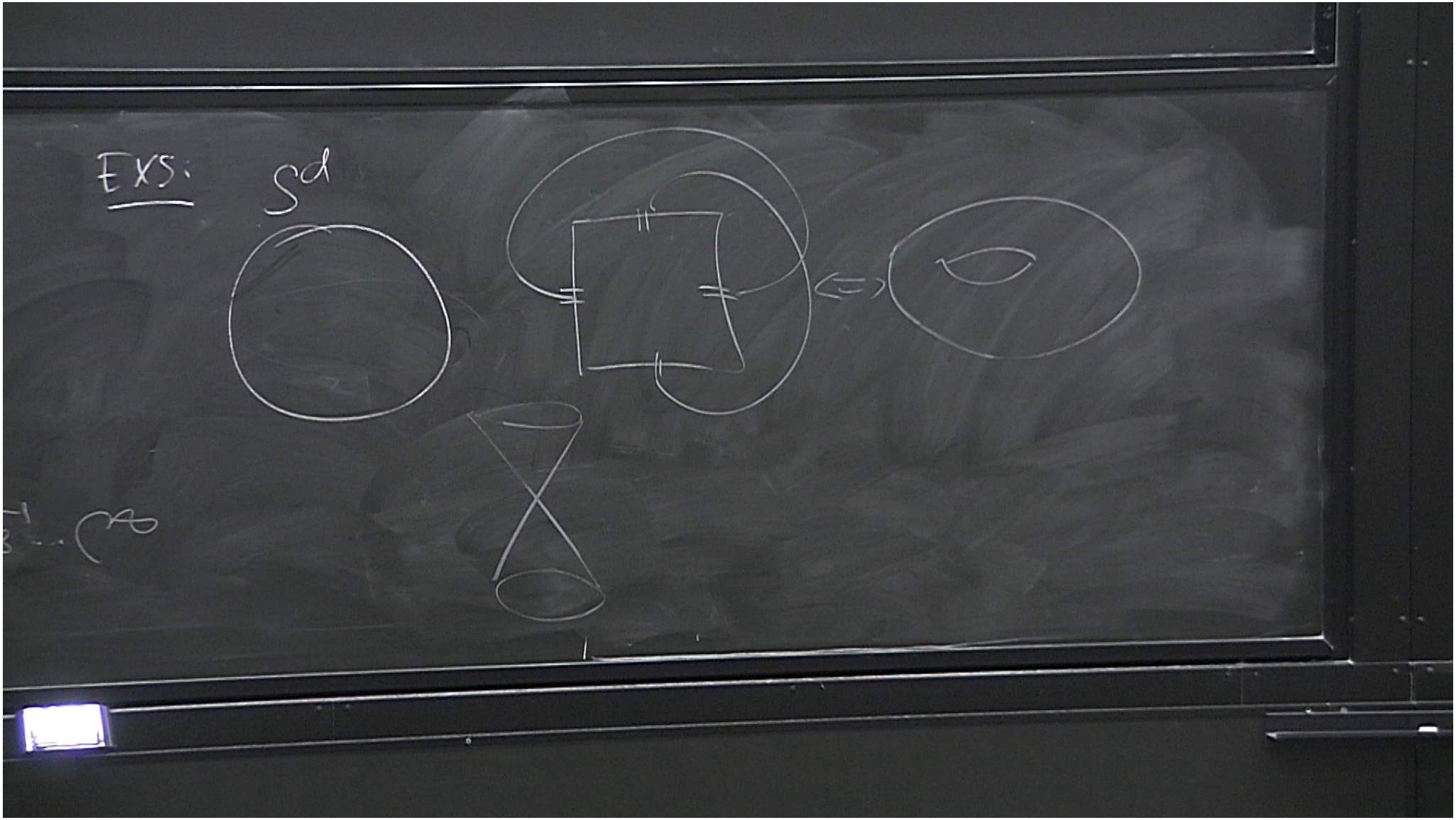

"MANIFOLD IS MADE OF PIECES THAT LOOK LIKE $\mathbb{R}^n$
THAT ARE SEWN TOGETHER SMOOTHLY.
φ_α
$\mathbb{R}^n$
O_α
p
U_α

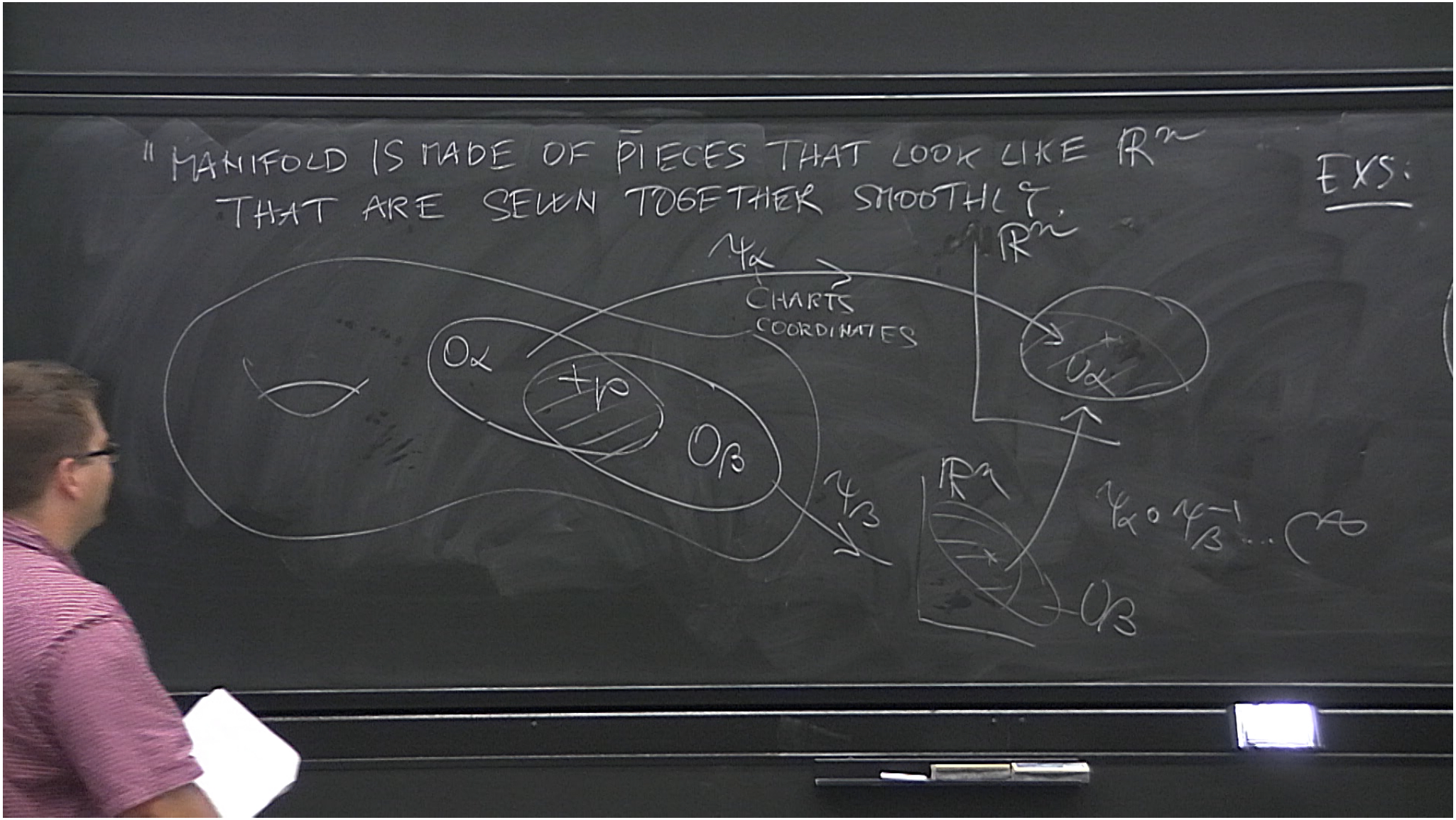
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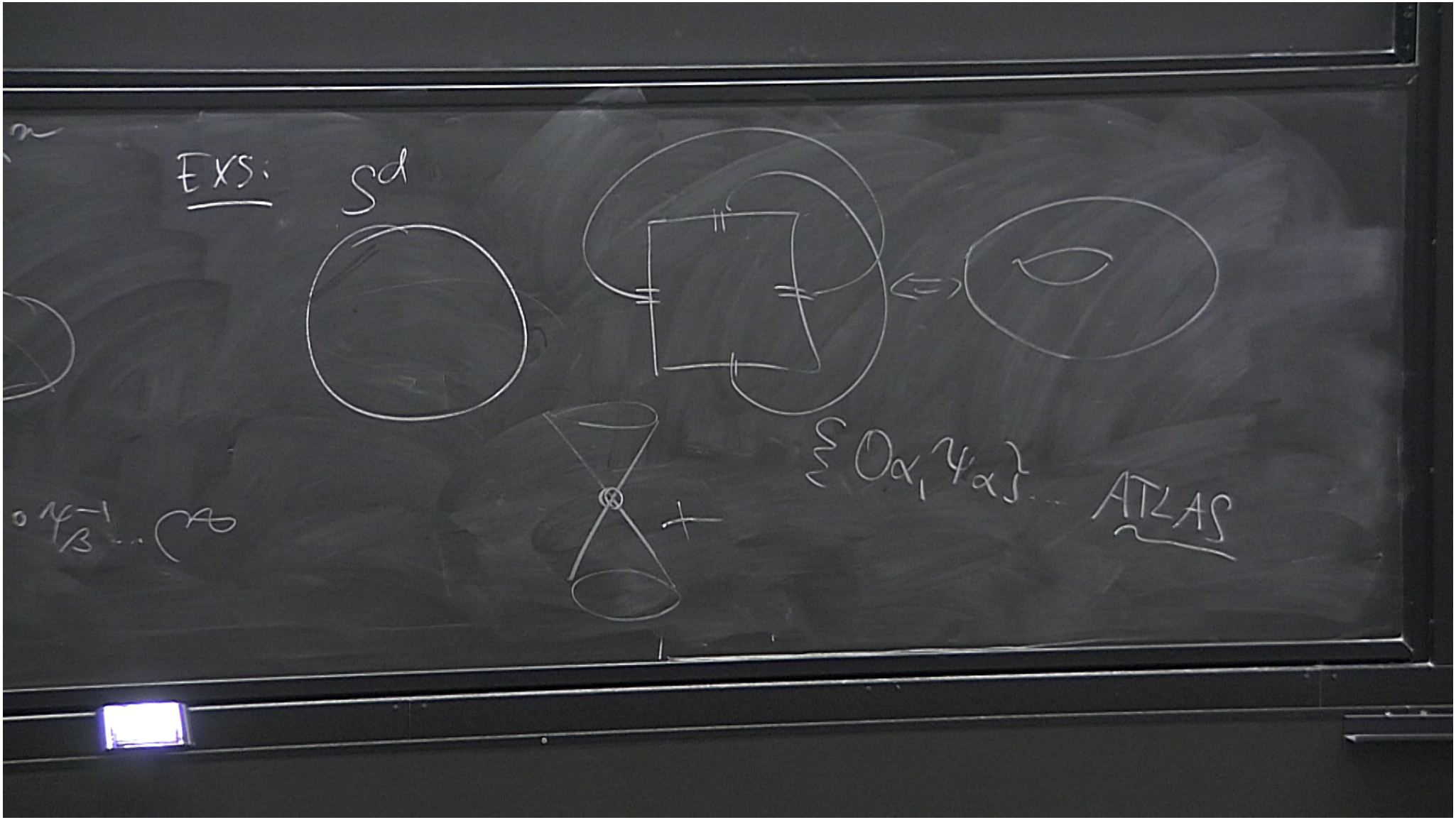


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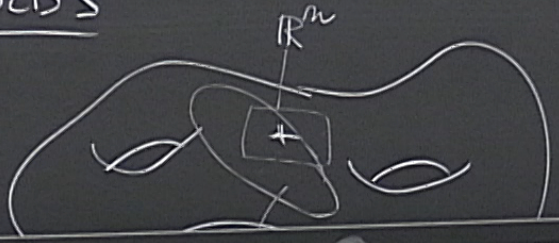



EXS:
S^d


"MANIFOLD IS MADE OF PIECES THAT LOOK LIKE $\mathbb{R}^n$ THAT ARE SEWN TOGETHER SMOOTHLY."
EXS:
$\mathbb{R}^n$
ψ_α
CHARTS
COORDINATES
U_α
p
U_β
ψ_β
$\mathbb{R}^n$
U_α
$\psi_\alpha \circ \psi_\beta^{-1} \dots C^\infty$
U_β


EXS:
S^d
$\{U_\alpha, \varphi_\alpha\}$...
ATLAS

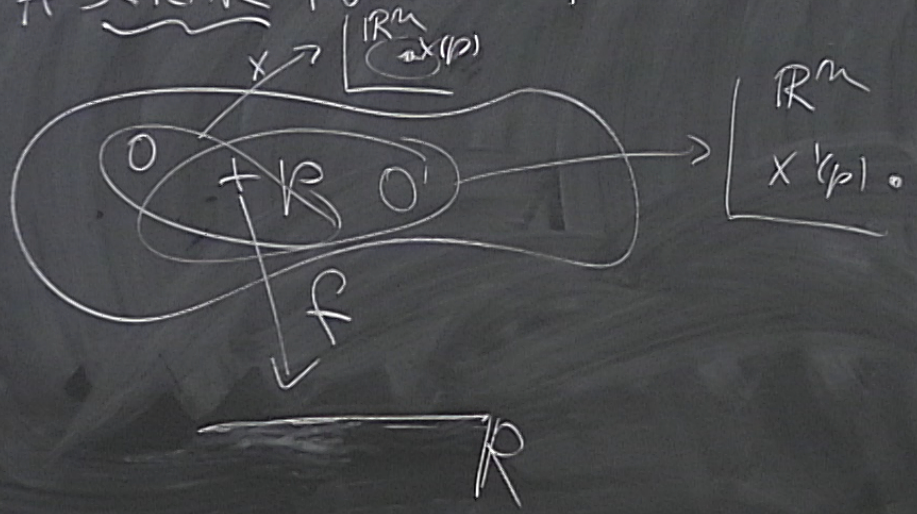
a) MANIFOLDS

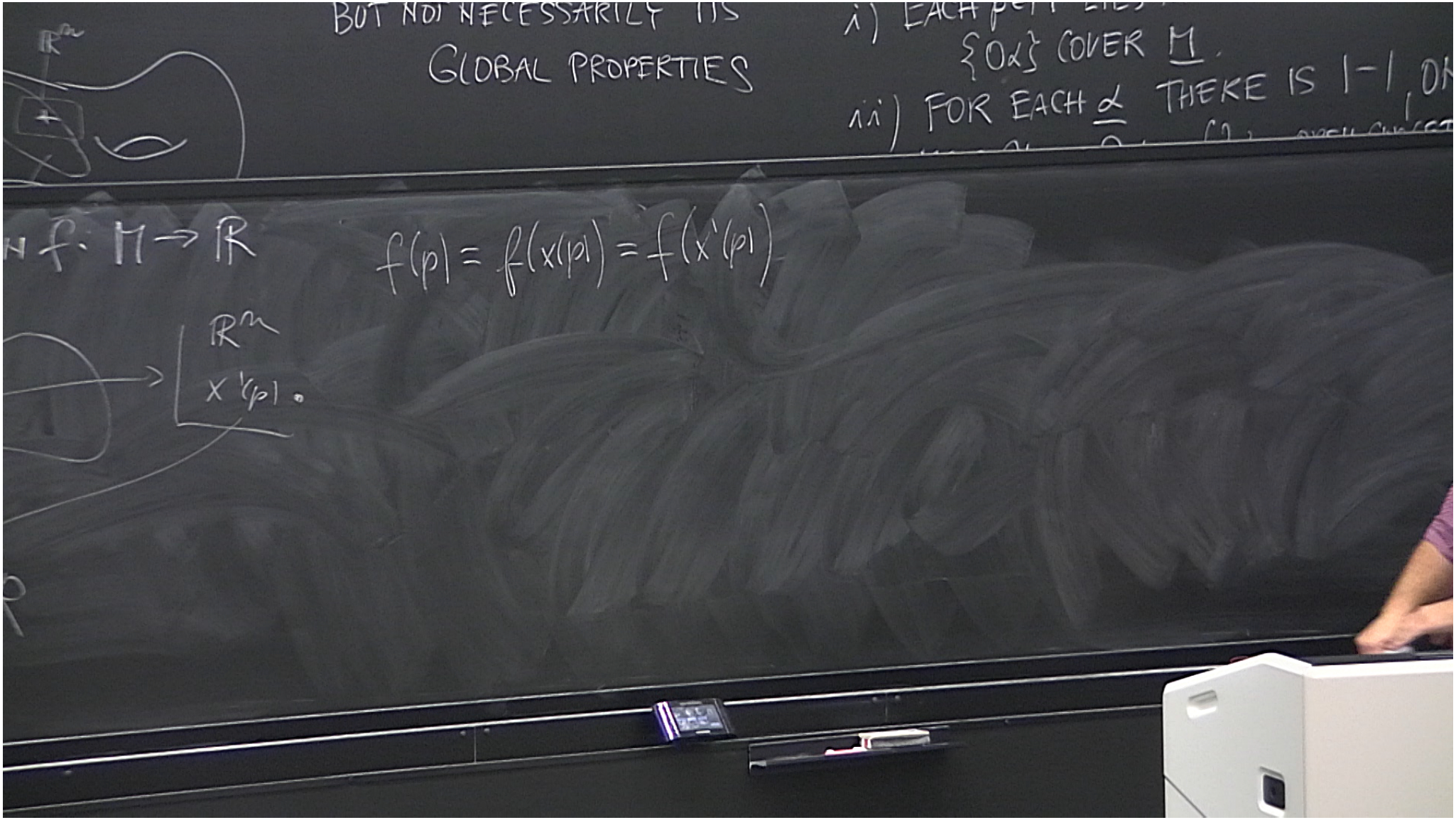


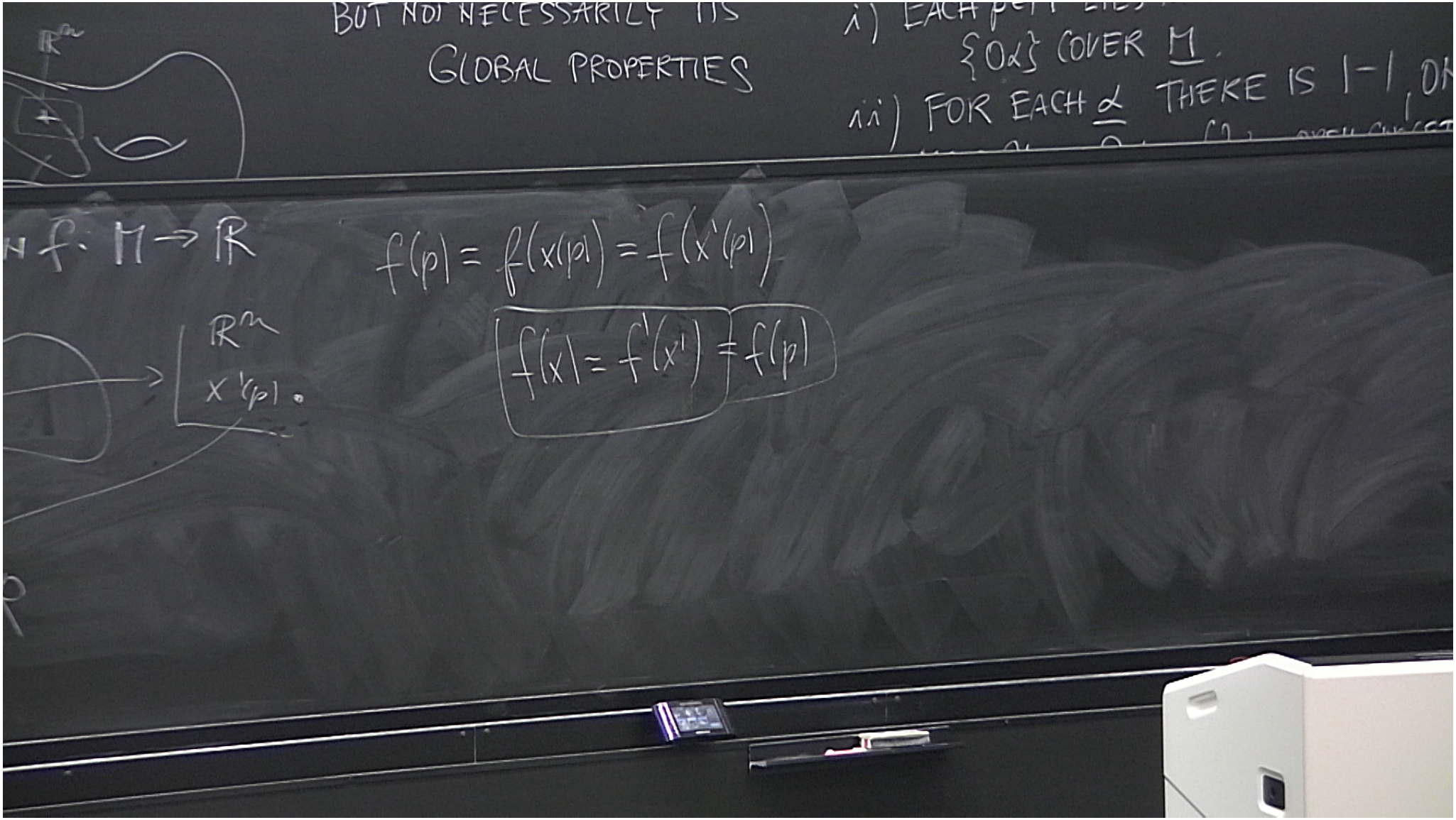
BUT NOT NECESSARILY ITS

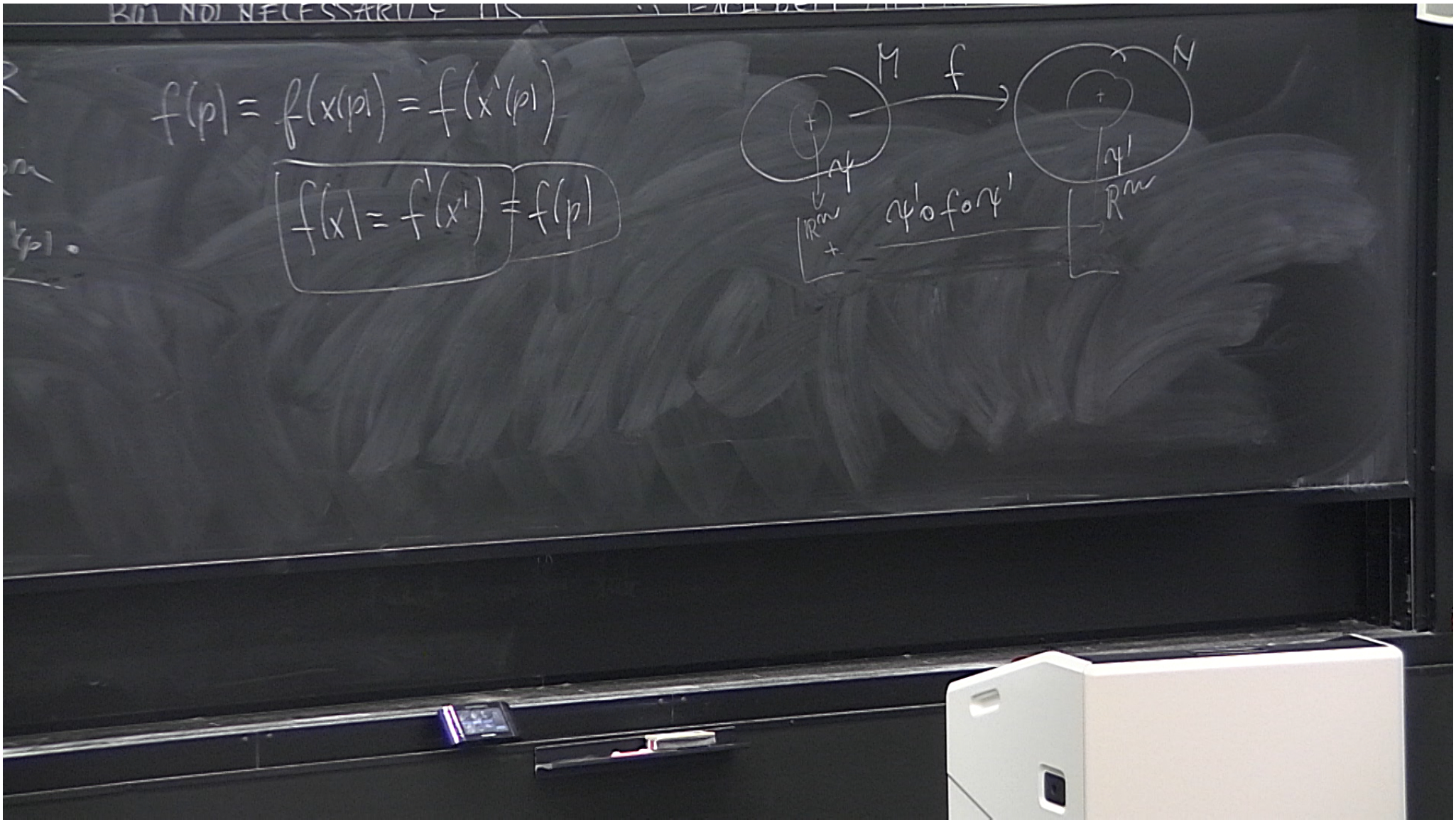
GLOBAL PROPERTIES

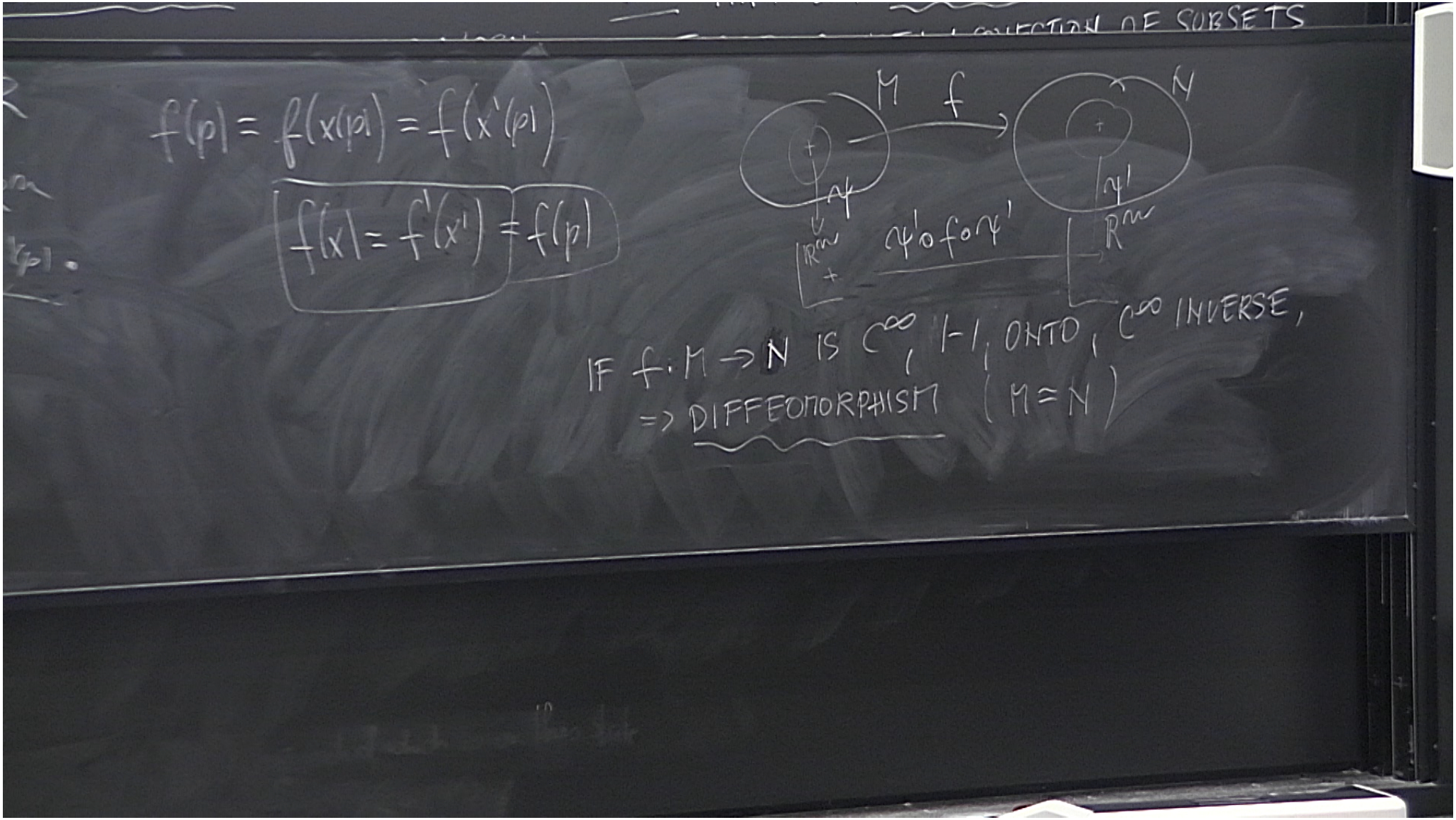
- A SCALAR FUNCTION $f: M \to \mathbb{R}$

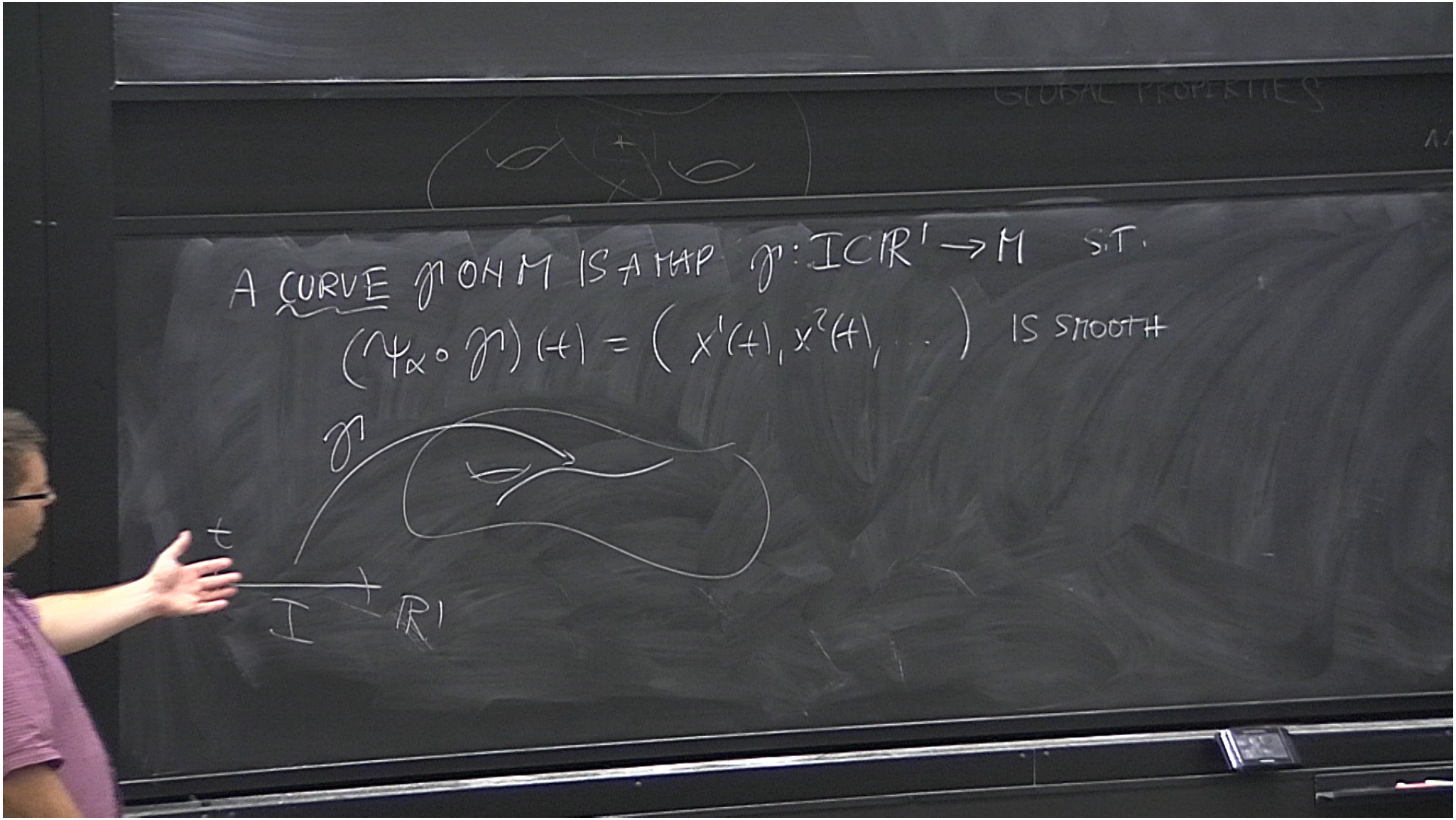



BUT NOT NECESSARILY ITS
GLOBAL PROPERTIES
$\{O_\alpha\}$ COVER M.
FOR EACH α THERE IS 1-1
$f: M \to \mathbb{R}$
$f(p) = f(x(p)) = f(x'(p))$
$\mathbb{R}^m$
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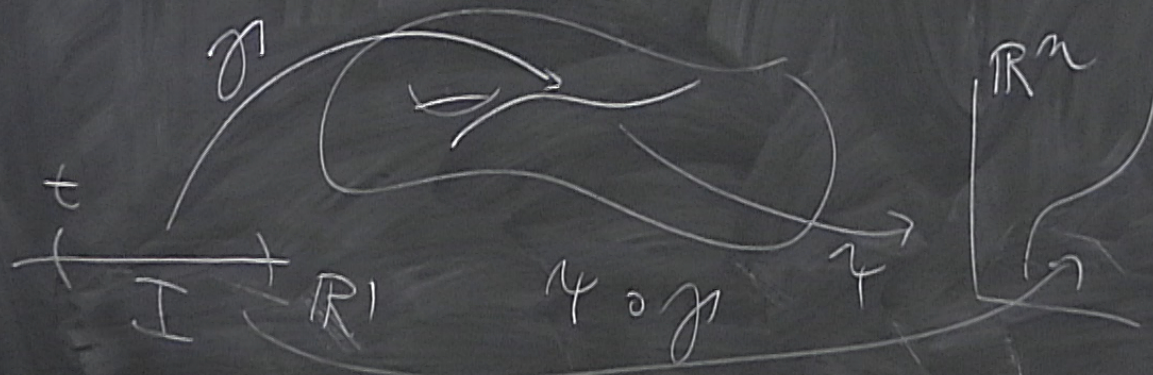

$f(p) = f(x(p)) = f(x'(p))$
$f(x) = f'(x') = f(p)$
M
f
N
φ
φ'
$\mathbb{R}^m$
$\psi \circ f \circ \varphi'$
$\mathbb{R}^m$

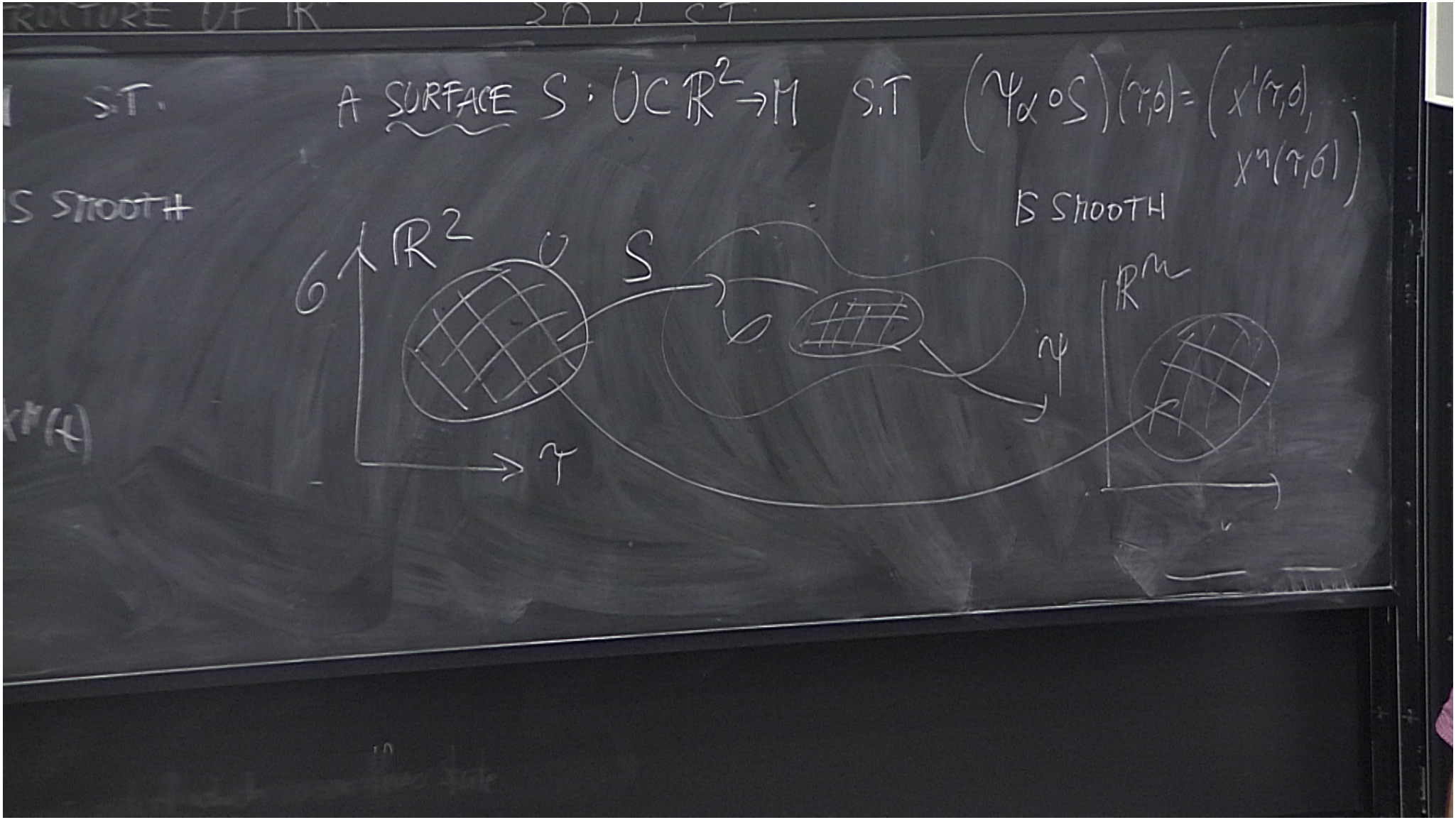

COLLECTION OF SUBSETS
$f(p) = f(x(p)) = f(x'(p))$
$f(x) = f'(x') = f(p)$
M
f
N
φ
φ'
$\mathbb{R}^m$
$\psi \circ f \circ \varphi'$
$\mathbb{R}^m$
IF $f: M \to N$ IS C^∞, 1-1, ONTO, C^∞ INVERSE,
$\Rightarrow$ DIFFEOMORPHISM $(M = N)$

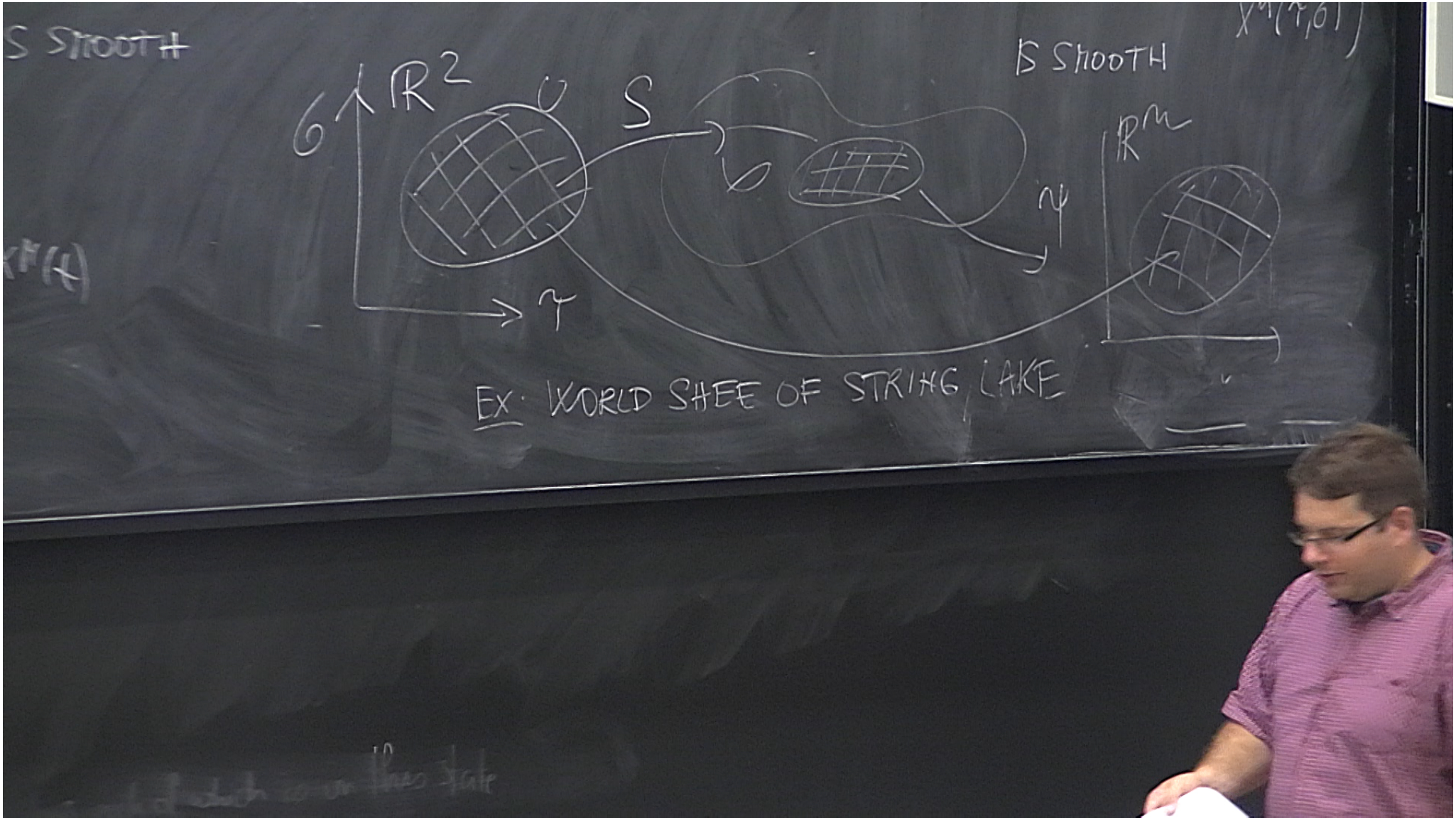

GLOBAL PROPERTIES
A CURVE γ ON M IS A MAP $\gamma: I \subset \mathbb{R}^1 \to M$ S.T.
$(\psi_\alpha \circ \gamma)(t) = (x^1(t), x^2(t), \ldots)$ IS SMOOTH
γ
t
I
$\mathbb{R}^1$

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A SURFACE $S : U \subset \mathbb{R}^2 \to M$ S.T $(\psi_\alpha \circ S)(\tau,\theta) = (x^1(\tau,\theta), \ldots, x^n(\tau,\theta))$
S.T.
S SMOOTH
IS SMOOTH
$\mathbb{R}^2$
U
S
$\mathbb{R}^m$
ψ
τ
θ


S SMOOTH
$\mathbb{R}^2$
S
S SMOOTH
$\mathbb{R}^m$
EX: WORLD SHEE OF STRING LAKE