Title: Holographic entanglement entropy in AdS(4)/BCFT(3) and the Willmore functional

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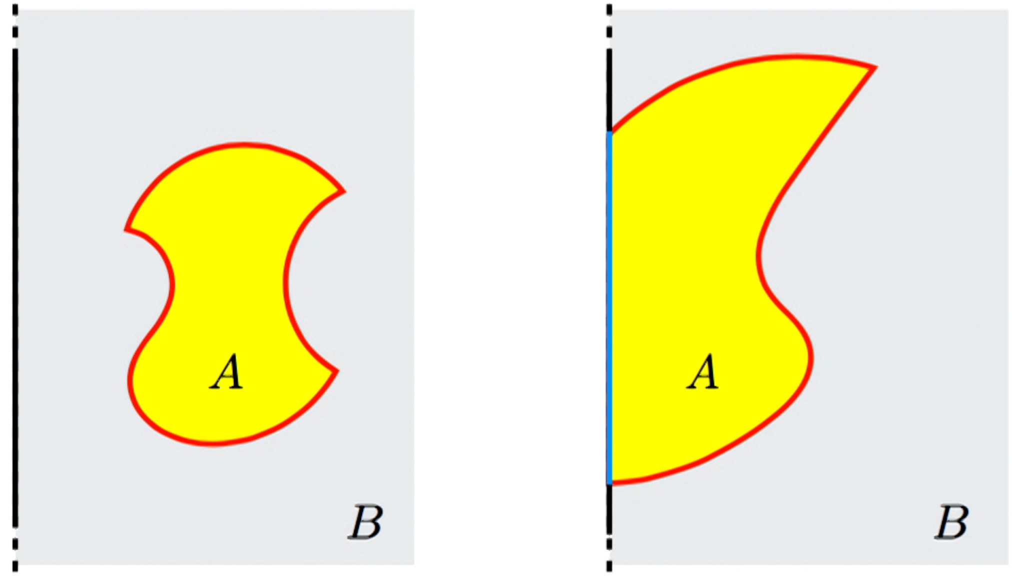
Abstract: <p>In the context of the AdS(4)/BCFT(3) correspondence, we study the holographic entanglement entropy for spatial regions having arbitrary shape. An analytic expression for the subleading term with respect to the area law is discussed. When the bulk spacetime is a part of AdS(4),
this formula becomes the Willmore functional with a proper boundary term evaluated on the minimal surface viewed as a submanifold of the three dimensional flat Euclidean space with a boundary.
Numerical checks of this formula are performed through a code which allows to construct minimal area surfaces anchored to generic curves. For some simple regions like infinite strips and disks, analytic results are obtained and they confirm the general expression for the subleading term. In particular, when the spatial region contains corners adjacent to the boundary, a logarithmic divergence occurs whose coefficient is determined by a so-called corner function which depends on the boundary conditions. An analytic expression for the holographic corner function and its checks are also discussed.</p>

Entanglement entropy in 2+1 CFT's with boundary



- Entanglement entropy: expansion as the UV cutoff $\varepsilon \to 0$

$$S_A = -\operatorname{Tr}_A\left(\rho_A \log \rho_A\right) = \gamma \frac{P_{A,B}}{\varepsilon} - F_A + o(1)$$

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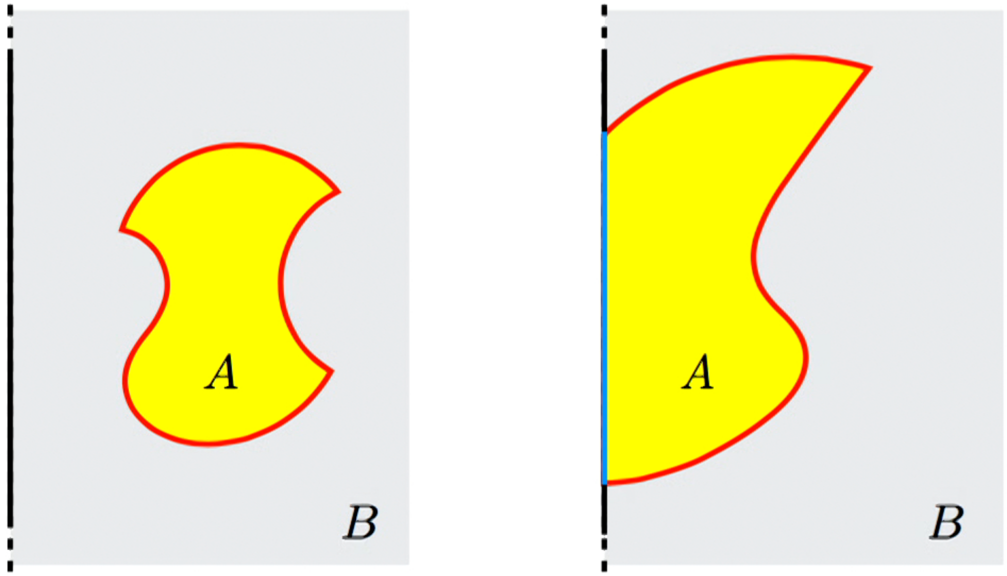
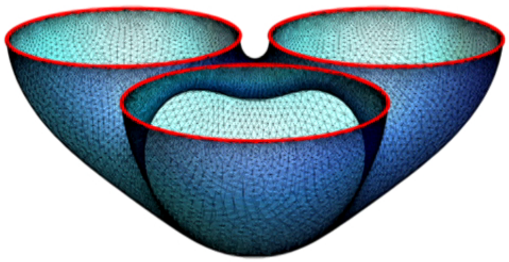


- Entanglement entropy: expansion as the UV cutoff $\varepsilon \to 0$

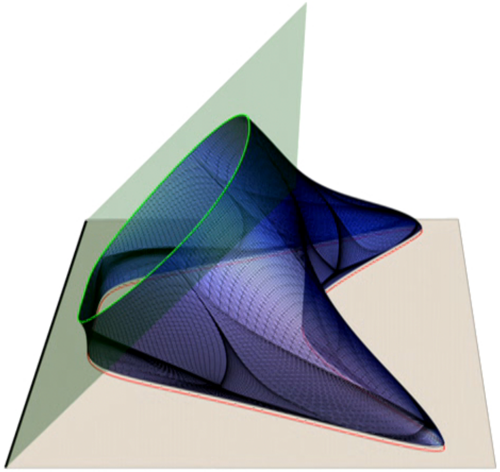
$$S_A = -\,\mathrm{Tr}_A\big(\rho_A \log \rho_A\big) = \gamma\,\frac{P_{A,B}}{\varepsilon} - F_A + o(1)$$

- F_A depends on the b.c.'s of the BCFT_3 and on the shape of A

Outline



- Holographic entanglement entropy in AdS_4/CFT_3 for generic shape regions
  [Fonda, Seminara, E.T., (2015)]
  - Willmore functional
  - Corner function



- Holographic entanglement entropy for generic regions in $AdS_4/BCFT_3$
  [Seminara, Sisti, E.T., (2018)] [Seminara, Sisti, E.T., (2017)]
  - Willmore functional & boundary term
  - Analytic results for strips and disks
  - Corner functions

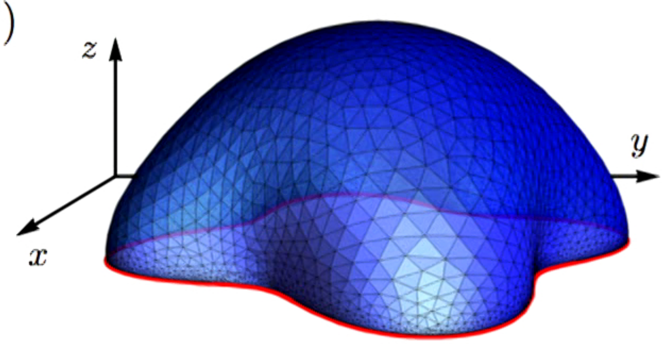
Holographic Entanglement Entropy in AdS(4)/CFT(3)

- Constant time slice in AdS_{d+2}
  Hypersurfaces γ_A s.t. $\partial\gamma_A = \partial A$
  Find the *minimal area* surface $\hat{\gamma}_A$
  [Ryu, Takayanagi, (2006)]

$$S_A = \frac{\mathrm{Area}(\hat{\gamma}_\varepsilon)}{4G_N^{(d+2)}}$$

- Holographic dual of Wilson loops [Maldacena, (1998)]
- E.g.: AdS_4 $\quad ds^2 = \frac{1}{z^2}\left(-dt^2 + dz^2 + d\boldsymbol{x}^2\right)$
- Asymptotically AdS_4 geometries

$$\mathcal{A}[\hat{\gamma}_\varepsilon] = \frac{P_A}{\varepsilon} - F_A + o(1)$$



- Various non trivial checks. E.g. strong subadditivity [Headrick, Takayanagi, (2007)]
- Simply connected domains analytically solved: spheres and infinite strips
- Domains A obtained as small perturbations of the sphere
  [Hubeny, (2012)] [Klebanov, Nishioka, Pufu, Safdi, (2012)] [Allais, Mezei, (2014)]

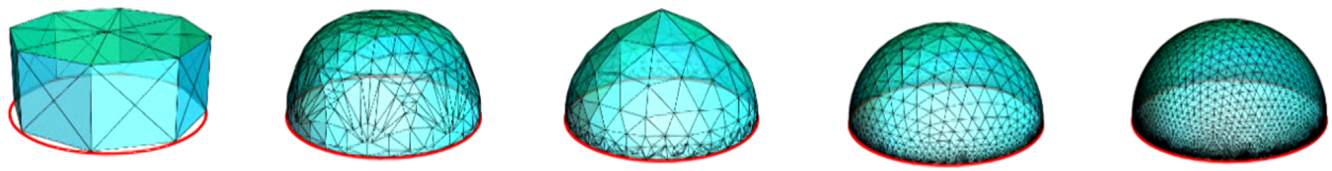
HEE in AdS(4) with Surface Evolver

[Fonda, Giomi, Salvio, E.T., (2014)]
[Fonda, Seminara, E.T., (2015)]

- Generic shape for ∂A

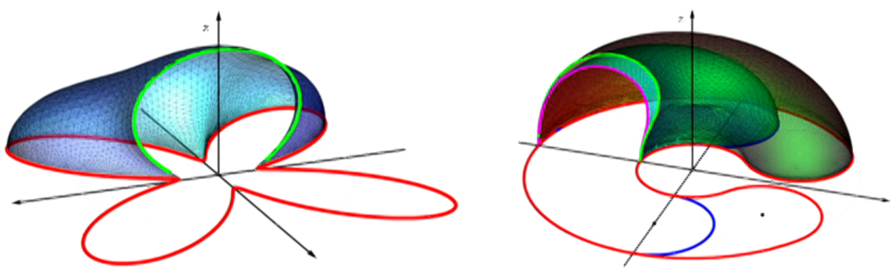
  Numerical analysis based on *Surface Evolver* (developed by Ken Brakke)

  A *Surface Evolver* evolution in AdS_4/CFT_3



  E.g.: When A is a disk, the minimal area surface $\hat{\gamma}_A$ is a hemisphere

- Regions A with more complicated boundary ∂A can be studied



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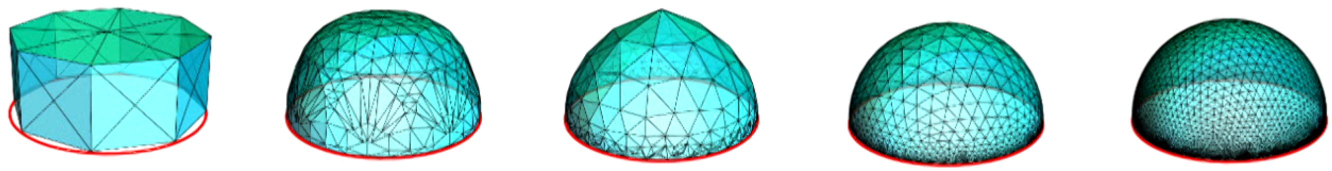
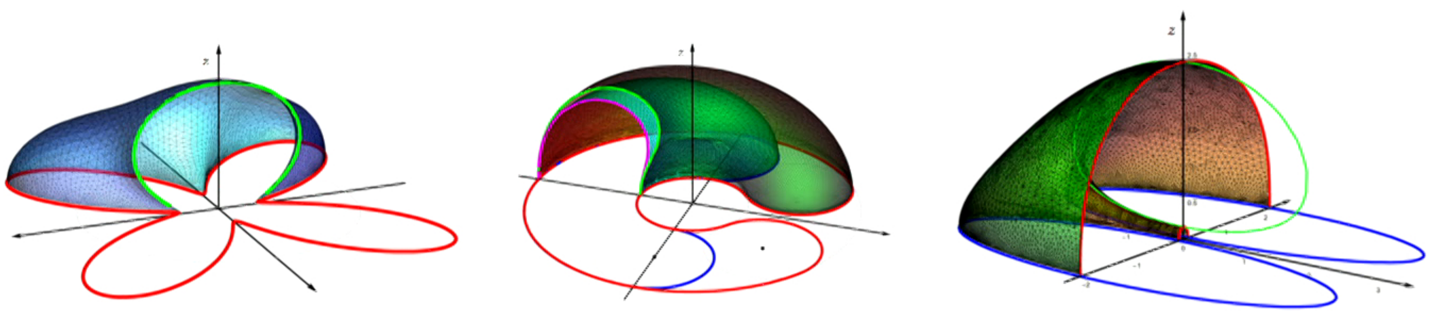


E.g.: When A is a disk, the minimal area surface $\hat{\gamma}_A$ is a hemisphere

- Regions A with more complicated boundary ∂A can be studied
  (e.g. A can be made by disjoint components)



HEE in AdS(4) & Willmore energy

- Willmore energy of a closed smooth surface $\Sigma_g \subset \mathbb{R}^3$

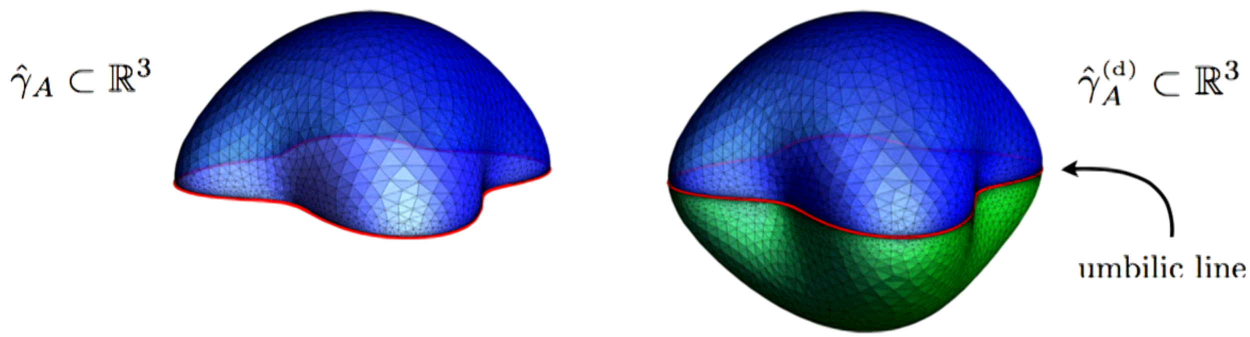
$$\mathcal{W}[\Sigma_g] \equiv \frac{1}{4} \int_{\Sigma_g} \big(\mathrm{Tr}\widetilde{K}\big)^2 \, d\tilde{\mathcal{A}}$$

[Willmore, (1965)]

- Minimal area surface $\hat{\gamma}_A \subset \mathbb{H}^3$ has $\mathrm{Tr}K = 0$
  Consider $\hat{\gamma}_A \subset \mathbb{R}^3$

[Babich, Bobenko, (1993)]
[Alexakis, Mazzeo, (2010)]

$$F_A = \mathcal{W}[\hat{\gamma}_A] = \int_{\hat{\gamma}_A} \frac{(\tilde{n}^z)^2}{z^2} \, d\tilde{\mathcal{A}} = \frac{1}{2} \mathcal{W}[\hat{\gamma}_A^{(\mathrm{d})}]$$

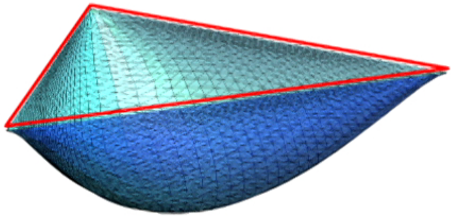


- Since $\mathcal{W}[\Sigma_g] \geqslant 4\pi$ (saturated only by round spheres) [Willmore, (1965)]
  HEE is maximised by the disk for a given perimeter P_A, i.e. $F_A \geqslant 2\pi$

[Fonda, Seminara, E.T., (2015)]

A corner function in AdS(4)/CFT(3)

- When A has corner, a logarithmic divergence occurs in $\mathcal{A}[\hat{\gamma}_\varepsilon]$
  [Drukker, Gross, Ooguri, (1998)] [Hirata, Takayanagi, (2006)]



$$\mathcal{A}[\hat{\gamma}_\varepsilon] = \frac{P_A}{\varepsilon} - \sum_i \widetilde{F}(\theta_i)\, \log(P_A/\varepsilon) + O(1)$$

- The corner function $\widetilde{F}(\theta)$ is known [Drukker, Gross, Ooguri, (1998)]

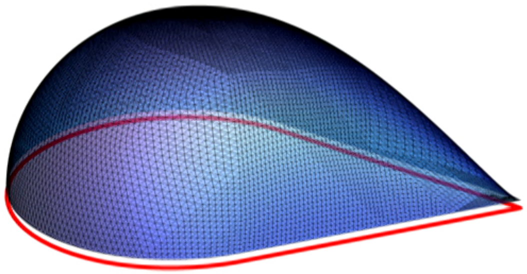
$$\widetilde{F}(\theta) \equiv 2\, F(q_0) \qquad \theta \equiv 2\, P_0(q_0) \qquad\qquad q_0 > 0$$

$$F(q_0) \equiv \frac{\mathbb{E}(\tilde{q}_0^2) - (1-\tilde{q}_0^2)\mathbb{K}(\tilde{q}_0^2)}{\sqrt{1-2\tilde{q}_0^2}} \qquad P_0(q_0) \equiv \tilde{q}_0 \sqrt{\frac{1-2\tilde{q}_0^2}{1-\tilde{q}_0^2}}\, \Big[\Pi(1-\tilde{q}_0^2, \tilde{q}_0^2) - \mathbb{K}(\tilde{q}_0^2)\Big] \qquad \tilde{q}_0^2 \equiv \frac{q_0^2}{1+2q_0^2} \in (0, 1/2)$$

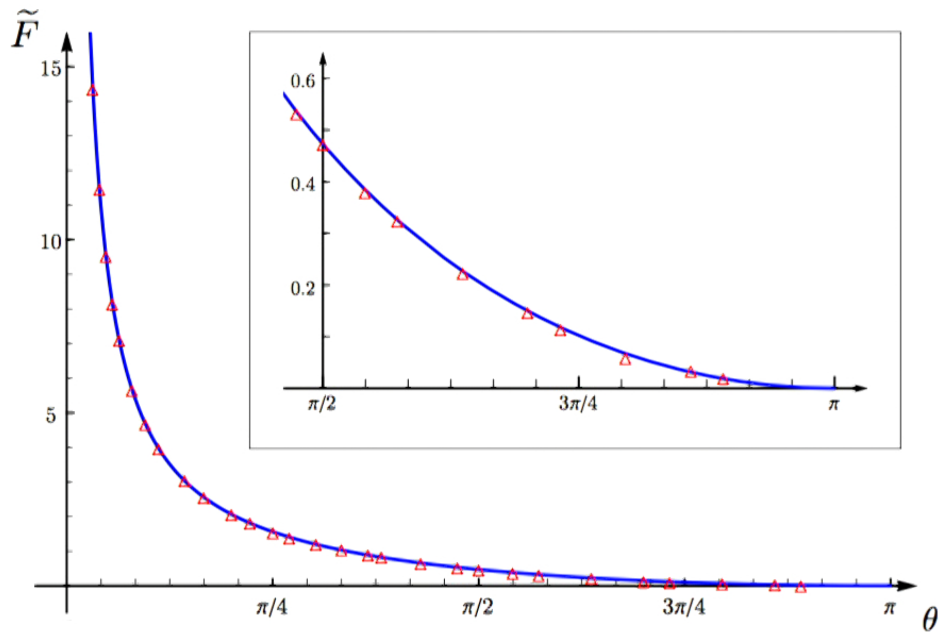
- $\widetilde{F}(\theta)$ has been recovered also through the Willmore functional
  [Fonda, Seminara, E.T., (2015)]

Corner function in AdS(4)/CFT(3) with Surface Evolver

[Seminara, Sisti, E.T., (2017)]



- We tested the numerical approach based on *Surface Evolver* on the corner function $\widetilde{F}(\theta)$
  - A drop-like region has been used



HEE in asymptotically AdS(4) static spacetimes

[Fonda, Seminara, E.T., (2015)]

- Take $ds^2|_{t=\text{const}} = g_{\mu\nu}\, dx^\mu dx^\nu$ with $g_{\mu\nu} = e^{2\varphi}\,\tilde{g}_{\mu\nu}$ and $\varphi = -\log(z) + \dots$

  The metric $\tilde{g}_{\mu\nu}$ is asymptotically flat

  - $\hat{\gamma}_A$ extremal area surface

$$\text{Tr}K = 0 \qquad \Longleftrightarrow \qquad \left(\text{Tr}\widetilde{K}\right)^2 = 4(\tilde{n}^\lambda \partial_\lambda \varphi)^2$$

  The unit vector $\tilde{n}^\mu$ is normal to $\hat{\gamma}_A \subset \tilde{\mathcal{M}}_3$ (defined by $\tilde{g}_{\mu\nu}$)

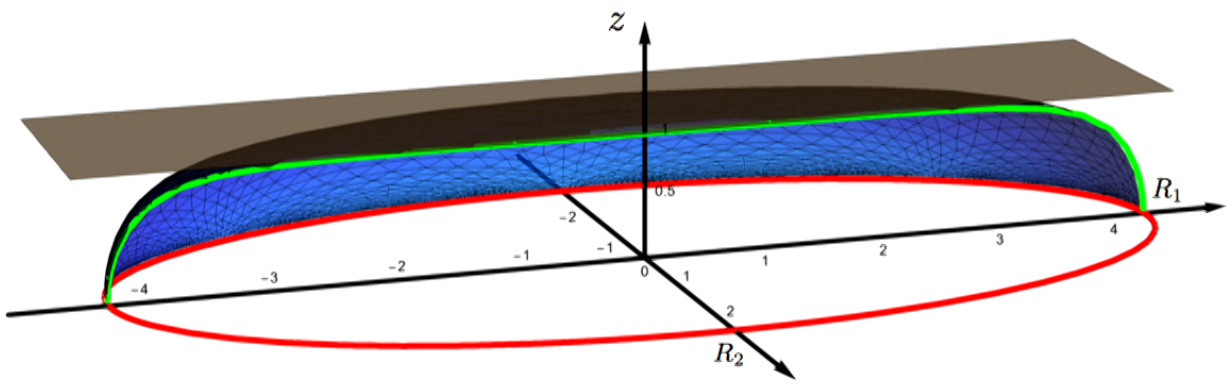
- Generalising the result for AdS_4 to AAdS_4 spacetimes, we find

$$F_A = \int_{\hat{\gamma}_A} \left[\frac{1}{2}\left(\text{Tr}\widetilde{K}\right)^2 + \widetilde{\nabla}^2 \varphi - e^{2\varphi} - \tilde{n}^\mu \tilde{n}^\nu\, \widetilde{\nabla}_\mu \widetilde{\nabla}_\nu \varphi \right] d\tilde{A}$$

- AdS_4: the formula involving the Willmore energy is recovered

HEE in asymptotically AdS(4) black holes

- $ds^2 = \frac{1}{z^2}\left(-f(z)\,dt^2 + \frac{dz^2}{f(z)} + d\boldsymbol{x}^2\right)$ $\qquad f(z) = 1 - Mz^3 + Q^2 z^4$



$$F_A = \int_{\hat{\gamma}_A} \frac{1}{z^2}\left[\left(1 + \frac{z f'(z)}{2f(z)}\right)(\tilde{n}^z)^2 + f(z) - \frac{z f'(z)}{2} - 1\right] d\tilde{\mathcal{A}}$$

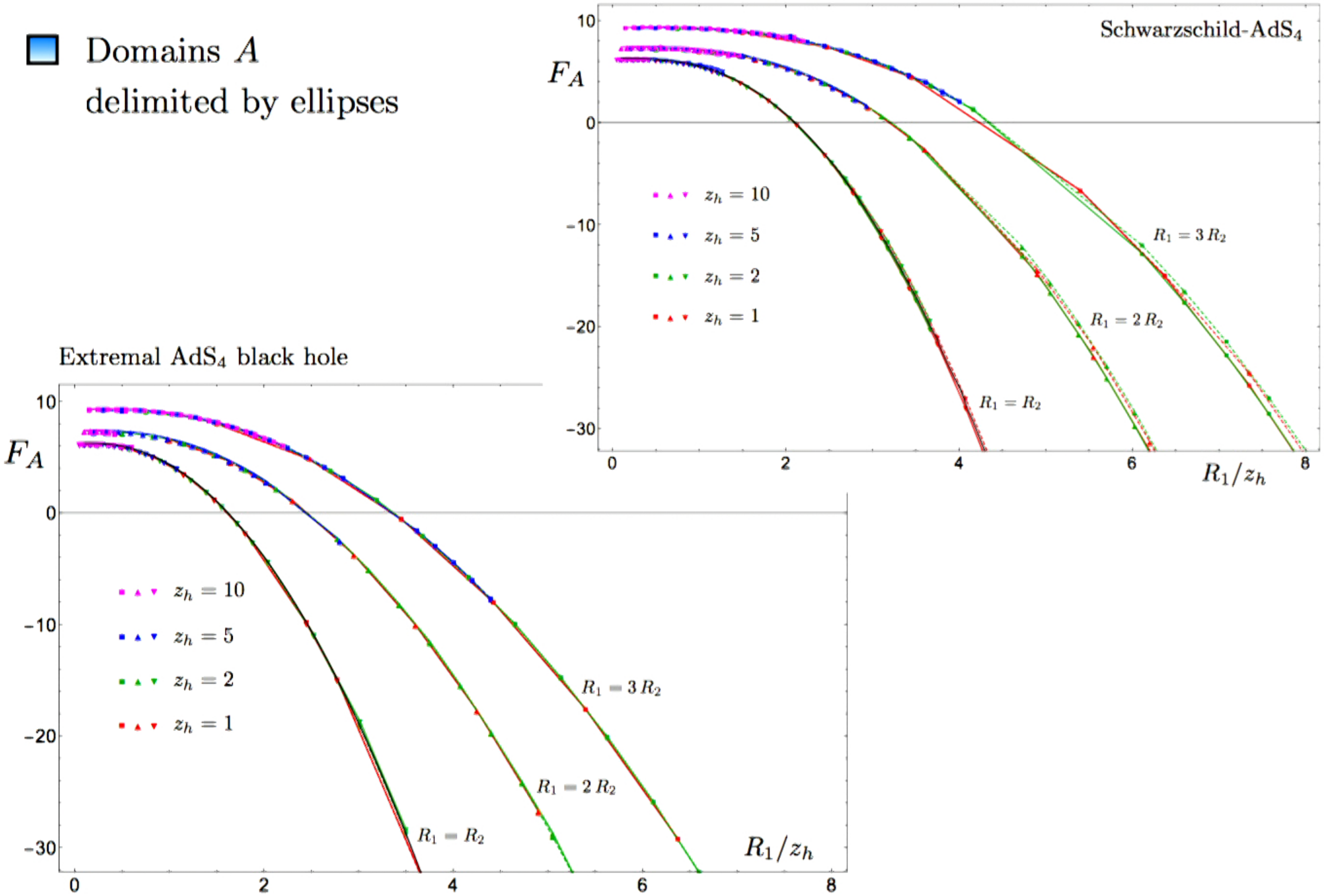
- Large domains A: the highest value of z on $\hat{\gamma}_A$ is $z_* \lesssim z_h$

  $F_A \simeq F_A^{\text{cyl}}$, i.e. F_A evaluated on the cylinder with $0 \leqslant z \leqslant z_*$ built on ∂A

  $\Longrightarrow \quad F_A^{\text{cyl}} = -\,\text{Area}(A)/z_h^2 + \dots$

HEE in asymptotically AdS(4) black holes. Ellipses

- Domains A delimited by ellipses



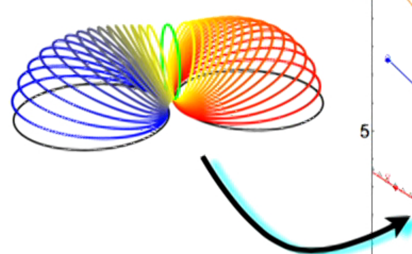
Holographic mutual information in AdS(4)

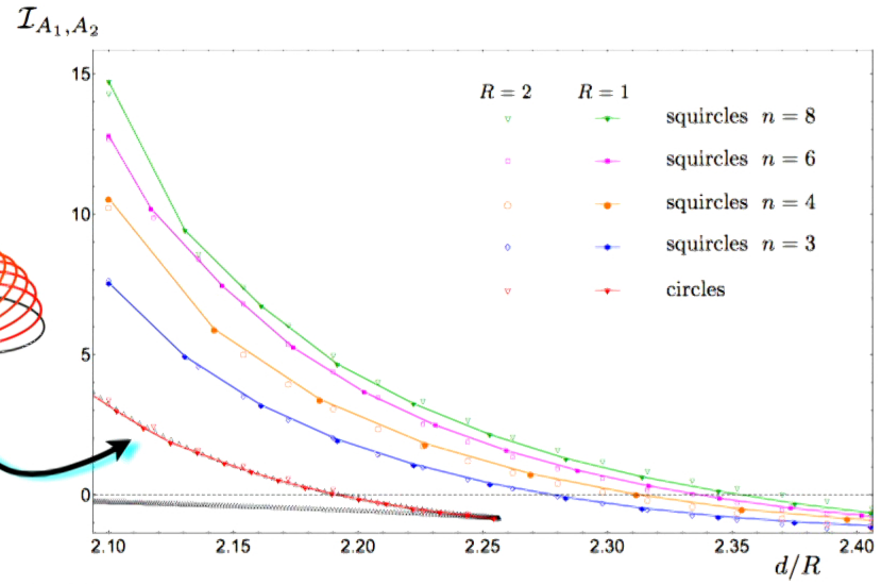
$$I_{A_1,A_2} \equiv S_{A_1} + S_{A_2} - S_{A_1 \cup A_2} \equiv \frac{\mathcal{I}_{A_1,A_2}}{4G_N}$$

$$\mathcal{I}_{A_1,A_2} = F_{A_1 \cup A_2} - F_{A_1} - F_{A_2} + o(1)$$

- Beyond a critical distance $\mathcal{I}_{A_1,A_2} = 0$ and the disconnected configuration is the minimal one

[Gross, Ooguri, (1998)] [Zarembo, (1999)]
[Drukker, Fiol, (2005)]
[Fonda, Giomi, Salvio, E.T., (2014)]





- The Clifford torus minimises the Willmore energy among the genus one surfaces: $\mathcal{W}[\Sigma_1] \geqslant 2\pi^2$ [Willmore, (1965)] [Marques, Neves, (2012)]

→ It cannot be found in this holographic context [Fonda, Seminara, E.T., (2015)]

AdS(4)/BCFT(3) setup

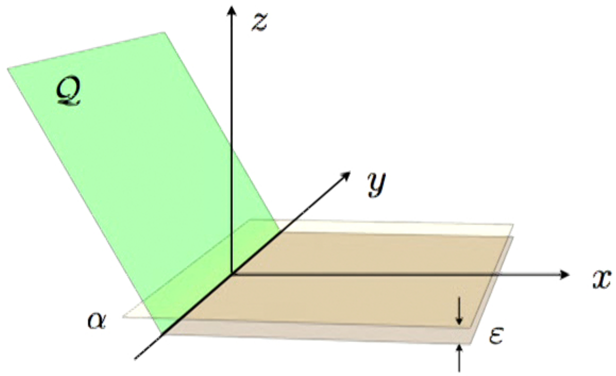
- $\text{AdS}_{d+2}/\text{BCFT}_{d+1}$: Gravitational spacetime bounded by a hypersurface $\mathcal{Q}$ such that $\partial\mathcal{Q}$ is the boundary of the BCFT [Takayanagi, (2011)] [Fujita, Takayanagi, E.T., (2011)] [Nozaki, Takayanagi, Ugajin, (2012)]

$$\mathcal{I} = \frac{1}{16\pi G_{\text{N}}} \int_{\mathcal{M}} \sqrt{-G}\,(R - 2\Lambda) + \frac{1}{8\pi G_{\text{N}}} \int_{\mathcal{Q}} \sqrt{-H}\,(K - \mathcal{L}_{\mathcal{Q}})$$

- More recent discussions in [Astaneh, Berthiere, Fursaev, Solodukhin, (2017)] [Chu, Miao, Guo, (2017)]
- Some releveant papers: [Solodukhin, (2015)] [Fursaev, (2015)] [Herzog, Huang, Jensen, (2015)] [Jensen, O'Bannon, (2013), (2015)] [Erdmenger, Flory, Hoyos, Newrzella, Wu, (2015)] [Bachas, (2002)] [DeWolfe, Freedman, Ooguri, (2001)] [Karch, Randall, (2000)]

- Simplest setup: $\mathcal{L}_{\mathcal{Q}} = T$ is constant and the BCFT_3 is in the ground state.

When the boundary of the BCFT_3 is flat, at $t = \text{const}$ one finds AdS_4 with



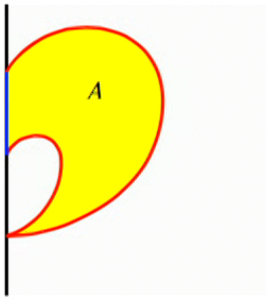
$$\mathcal{Q} \;:\qquad z = -(\tan\alpha)\, x$$

HEE in AdS(4)/BCFT(3): the subleading term

- HEE from the minimal area surface $\hat{\gamma}_A$ anchored to $\partial A \cap \partial B$
  If $\hat{\gamma}_A \cap \mathcal{Q} \neq \emptyset$, then $\hat{\gamma}_A \perp \mathcal{Q}$ along their intersection $\partial\hat{\gamma}_{\mathcal{Q}}$

$$S_A = \frac{L^2_{\mathrm{AdS}}}{4G_{\mathrm{N}}}\,\mathcal{A}[\hat{\gamma}_\varepsilon]$$

$$\mathcal{A}[\hat{\gamma}_\varepsilon] = \frac{P_{A,B}}{\varepsilon} - F_A + o(1)$$



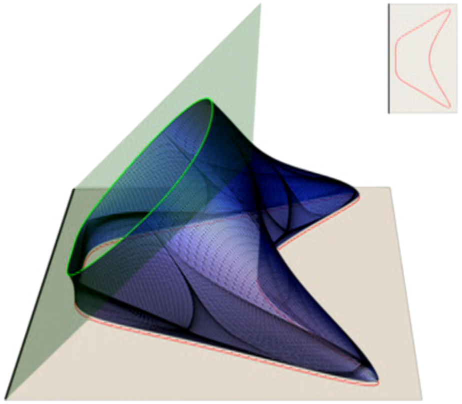
- Static gravitational backgrounds. Subleading term for any kind of region A

$$F_A = \int_{\hat{\gamma}_\varepsilon}\left(\frac{1}{2}(\mathrm{Tr}\widetilde{K})^2 + \widetilde{\nabla}^2\varphi - e^{2\varphi} - \tilde{n}^\mu\tilde{n}^\nu\,\widetilde{\nabla}_\mu\widetilde{\nabla}_\nu\varphi\right)d\tilde{\mathcal{A}} - \int_{\partial\hat{\gamma}_{\mathcal{Q}}}\tilde{b}^\mu\partial_\mu\varphi\,d\tilde{s}$$

This holds for a generic boundary of the BCFT_3.
[Seminara, Sisti, E.T., (2018)]

- Simplest setup with flat boundary
  ($\tilde{g}_{\mu\nu} = \delta_{\mu\nu}$ and $\mathcal{Q}$ is a half-plane)

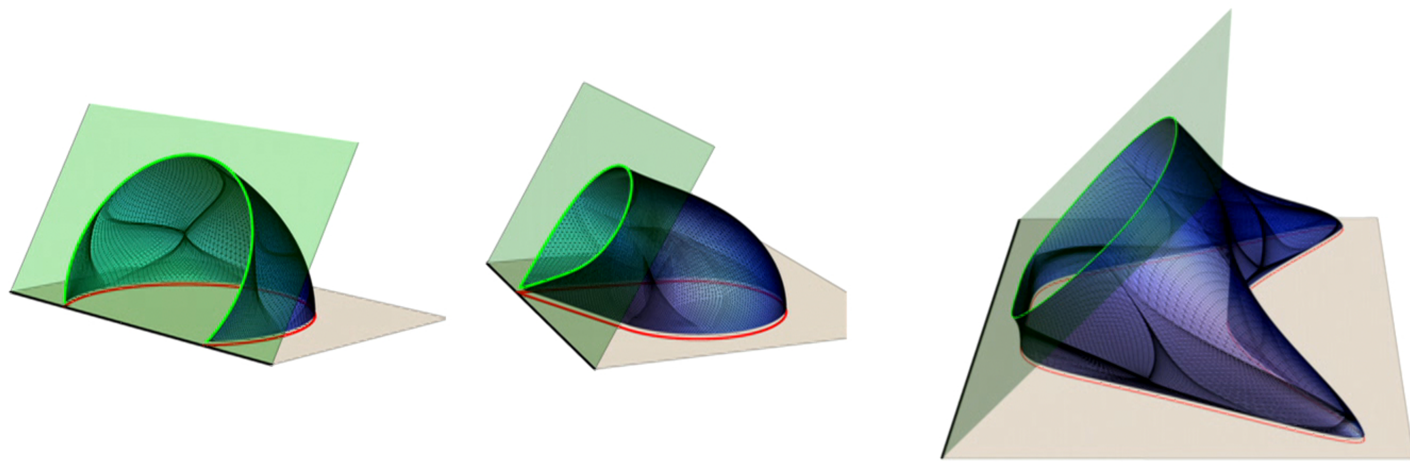
$$F_A = \int_{\hat{\gamma}_\varepsilon}\frac{(\tilde{n}^z)^2}{z^2}\,d\tilde{\mathcal{A}} - (\cos\alpha)\int_{\partial\hat{\gamma}_{\mathcal{Q}}}\frac{1}{z}\,d\tilde{s}$$



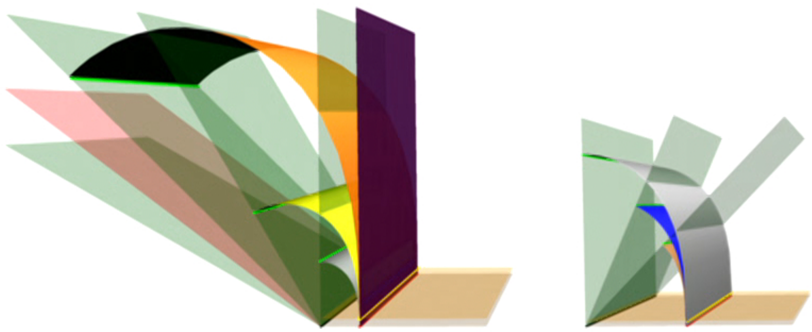
HEE in AdS(4)/BCFT(3) with Surface Evolver

- A *Surface Evolver* evolution in $\mathrm{AdS}_4/\mathrm{BCFT}_3$

- *Surface Evolver* has been employed to study:
  - Corner functions [Seminara, Sisti, E.T., (2017)]
  - Smooth regions disjoint from the boundary [Seminara, Sisti, E.T., (2018)]



Infinite strip adjacent to the boundary



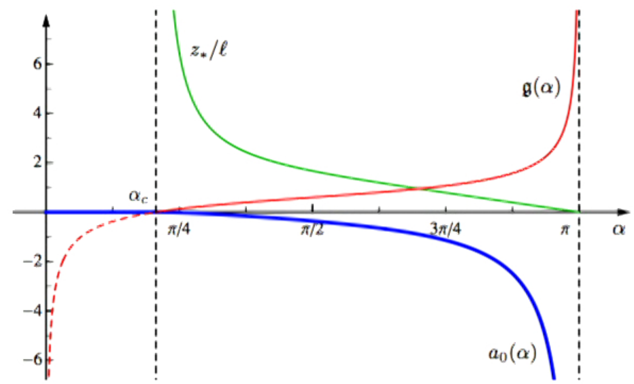
- Standard approach

[Nagasaki, Tanida, Yamaguchi, (2011)]
[Chu, Miao, Guo, (2017)]
[Seminara, Sisti, E.T., (2017)]

$$z_* = \frac{\sqrt{\sin\alpha}}{\mathfrak{g}(\alpha)}\,\ell$$

$$\mathfrak{g}(\alpha) \equiv \mathbb{E}(\pi/4 - \alpha/2\,|\,2) - \frac{\cos\alpha}{\sqrt{\sin\alpha}} + \frac{\Gamma\left(\frac{3}{4}\right)^2}{\sqrt{2\pi}}$$

$\mathfrak{g}(\alpha_c) = 0$ defines a critical slope



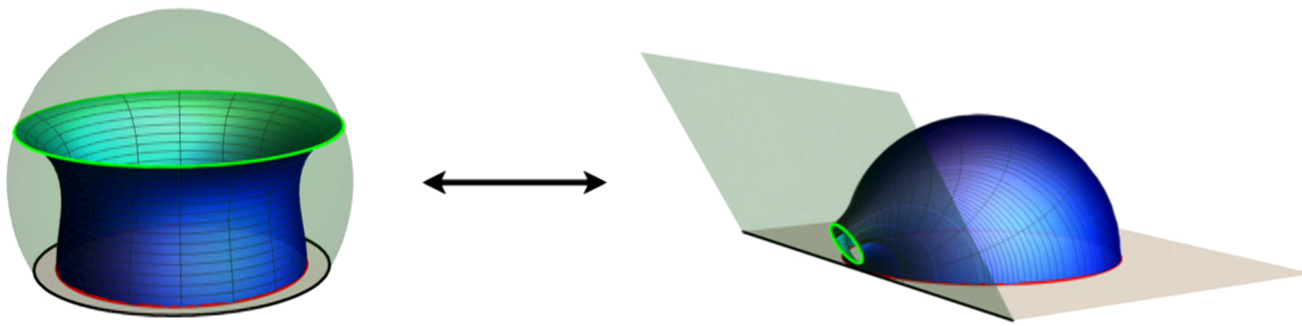
- $F_A = L_{\parallel}\,\dfrac{a_0(\alpha)}{\ell}$ $\qquad a_0(\alpha) = \begin{cases} -\,\mathfrak{g}(\alpha)^2 & \alpha \geqslant \alpha_c \\ 0 & \alpha \leqslant \alpha_c \end{cases}$

- The general formula for F_A specified to this case reproduces this result. The integral over the line $\partial\hat{\gamma}_Q$ gives a non vanishing contribution.

[Seminara, Sisti, E.T., (2018)]

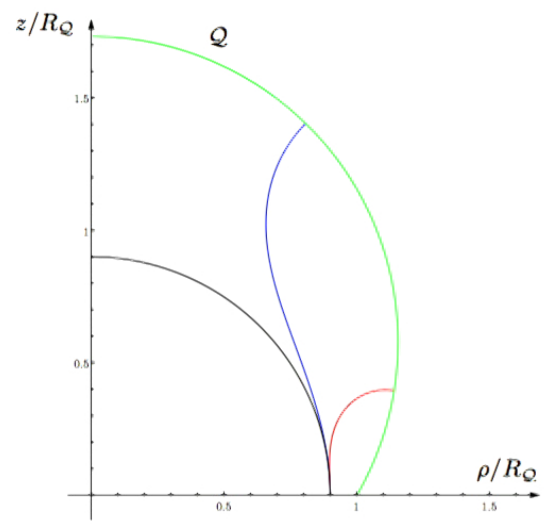
Disk disjoint from a (either flat or circular) boundary

- This configuration can be treated analytically. [Seminara, Sisti, E.T., (2018)]

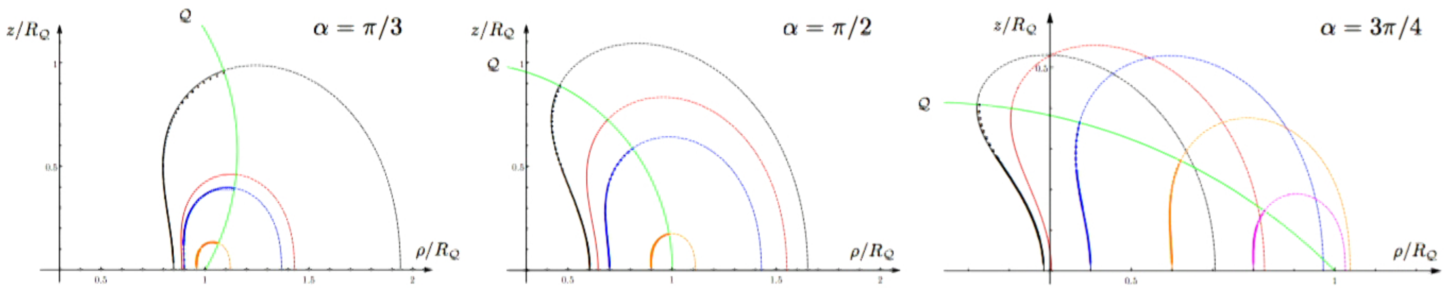


Conformal invariance $\Longrightarrow$ consider the disk concentric to a circular boundary

- We find at most three extremal surfaces:
  - the hemisphere
  - at most two extremal surfaces that intersect $\mathcal{Q}$ orthogonally

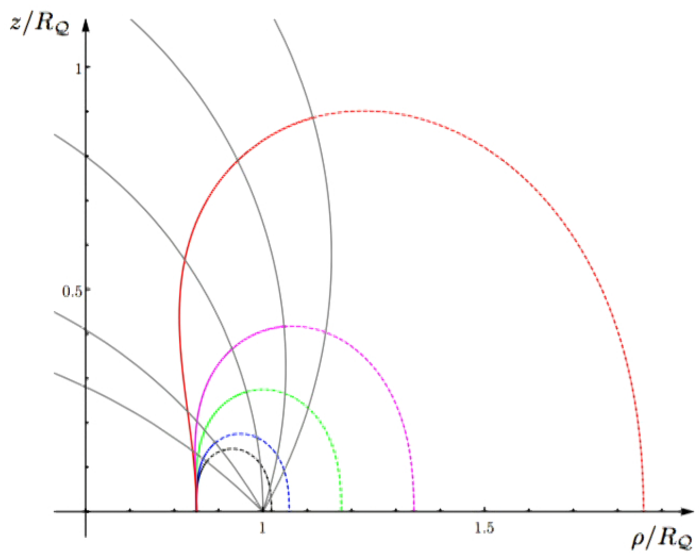


Disk disjoint from a circular boundary: extremal surfaces



- $\hat{\gamma}_A^{\text{con}}$ can be viewed as part of an extremal surface in $\mathbb{H}_3$ anchored to a proper annulus

- Analytic expression for the profiles checked against the numerical data obtained with *Surface Evolver*



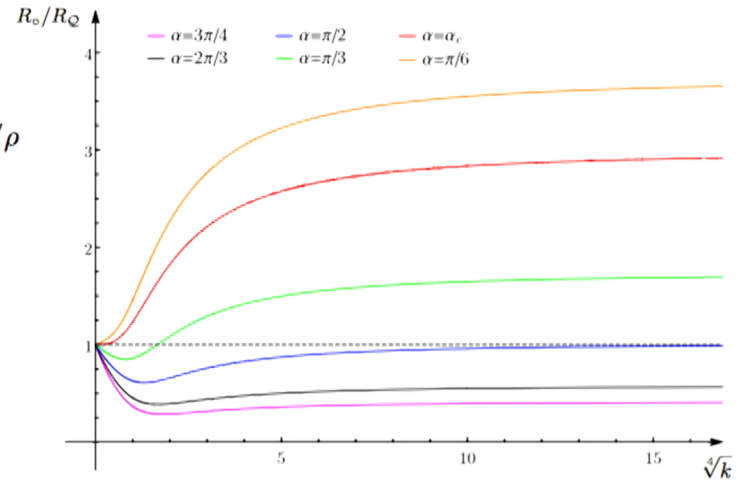
Disk disjoint from a circular boundary: extremal surfaces

- Radial profiles

$$\rho_\gamma(\theta) = \begin{cases} R_\circ \, e^{-q_{-,k}(\zeta)} \\ R_{\text{aux}} \, e^{-q_{+,k}(\zeta)} \end{cases} \qquad \zeta \equiv \tan\theta = z/\rho$$

$$\zeta_m^2 = \frac{k + \sqrt{k(k+4)}}{2}$$

$$\Omega(\zeta) \equiv \arcsin\left(\frac{\zeta/\zeta_m}{\sqrt{1+\kappa^2(\zeta^2/\zeta_m^2 - 1)}} \right)$$



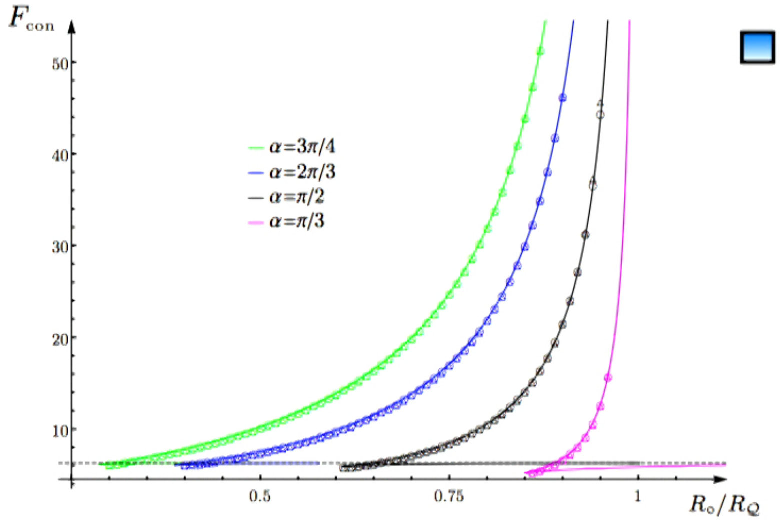
$$q_{\pm,k}(\zeta) = \frac{1}{2}\log(1+\zeta^2) \pm \kappa \sqrt{\frac{1-2\kappa^2}{\kappa^2-1}} \left[\Pi\big(1-\kappa^2, \Omega(\zeta)|\kappa^2\big) - \mathbb{F}\big(\Omega(\zeta)|\kappa^2\big) \right] \qquad \kappa \equiv \sqrt{\frac{1+\zeta_m^2}{2+\zeta_m^2}}$$

- Imposing $\hat{\gamma}_A \perp \mathcal{Q}$ leads to $\qquad \zeta_*^2 = \dfrac{k + \sqrt{k(k+4(\sin\alpha)^2)}}{2} \qquad \eta_\alpha \equiv -\,\text{sign}(\cot\alpha)$

$$\frac{R_\circ}{R_Q} = \frac{\sqrt{\zeta_*^2 + (\sin\alpha)^2} + \zeta_* \cos\alpha}{(\zeta_*^2+1)\sin\alpha} \left(\frac{1+\eta_\alpha}{2}\, e^{q_{-,k}(\zeta_*)} + \frac{1-\eta_\alpha}{2}\, e^{q_{-,k}(\zeta_m) - q_{+,k}(\zeta_m)}\, e^{q_{+,k}(\zeta_*)} \right)$$

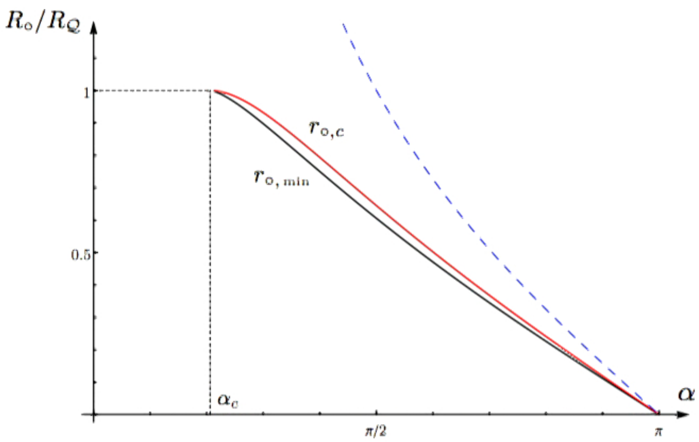
- As $R_\circ/R_Q \to 1$ we find $\qquad \dfrac{R_\circ}{R_Q} = 1 - \mathfrak{g}(\alpha)\, \sqrt[4]{k} + \mathcal{O}(\sqrt{k}\,)$

Disk disjoint from a boundary: subleading term in HEE



- Subleading term F_A for the extremal surfaces intersecting $\mathcal{Q}$.

  Two checks with *Surface Evolver*:
  - $\hat{\mathcal{A}}_\varepsilon^{\rm SE} - 2\pi R_\circ/\varepsilon$
  - Willmore-like formula for F_A



- Critical values of $R_\circ/R_Q$:
  - at $r_{\circ,c}$ the transition between $\hat{\gamma}_A^{\rm con}$ and $\hat{\gamma}_A^{\rm dis}$ occurs
  - below $r_{\circ,\rm min}$ the solutions $\hat{\gamma}_A^{\rm con} \perp \mathcal{Q}$ do not exist

Disk disjoint from a boundary: subleading term in HEE

- $$\mathcal{A}[\hat{\gamma}_\varepsilon] = \frac{2\pi R_\circ}{\varepsilon} - \max\big(2\pi, \widehat{F}_{\text{con}}\big) + \mathcal{O}(\varepsilon)$$

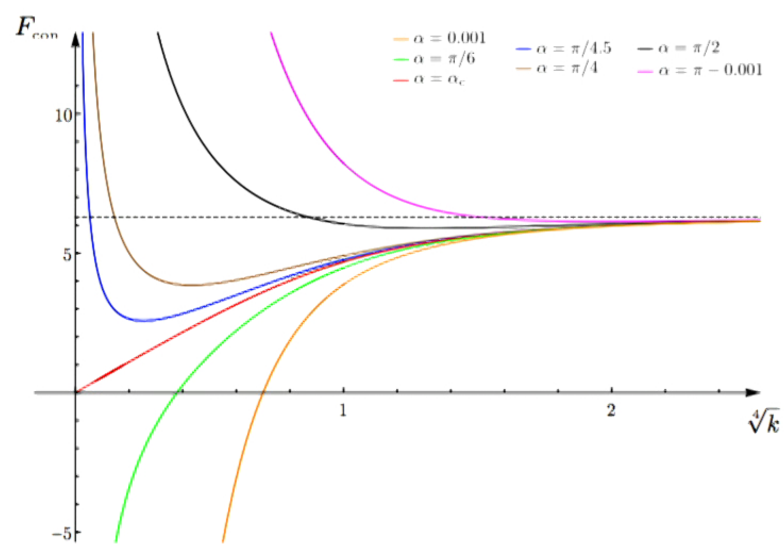
$$F_{\text{con}} = 2\pi \left[\frac{1+\eta_\alpha}{2}\, \mathcal{F}_k(\zeta_*) + \frac{1-\eta_\alpha}{2} \Big(2\,\mathcal{F}_k(\zeta_m) - \mathcal{F}_k(\zeta_*) \Big) \right]$$

$$\mathcal{F}_k(\zeta) \equiv \frac{\sqrt{k(1+\zeta^2)-\zeta^4}}{\sqrt{k}\,\zeta} - \frac{\mathbb{F}(\arcsin(\zeta/\zeta_m)\,|\,-\zeta_m^2-1) - \mathbb{E}(\arcsin(\zeta/\zeta_m)\,|\,-\zeta_m^2-1)}{\zeta_m}$$

- As $R_\circ/R_Q \to 1$ we find

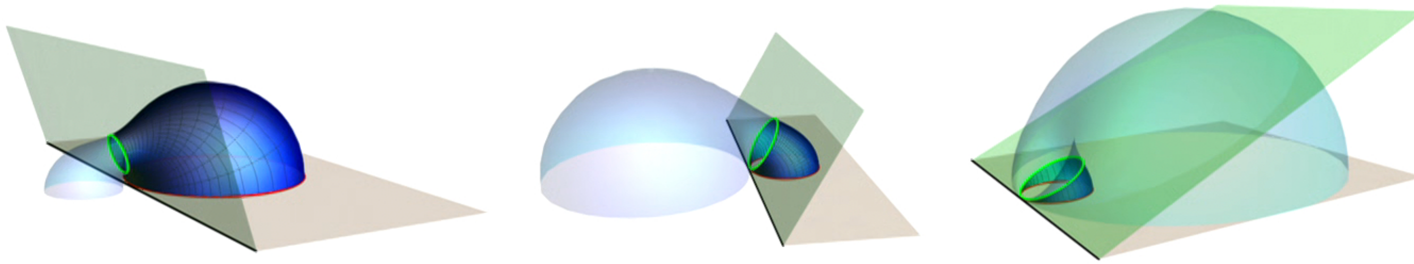
$$F_{\text{con}} = \frac{2\pi\,\mathfrak{g}(\alpha)}{\sqrt[4]{k}} + O(\sqrt[4]{k})$$

- The analytic result for F_A has been obtained also through the analytic formula holding for every region A.



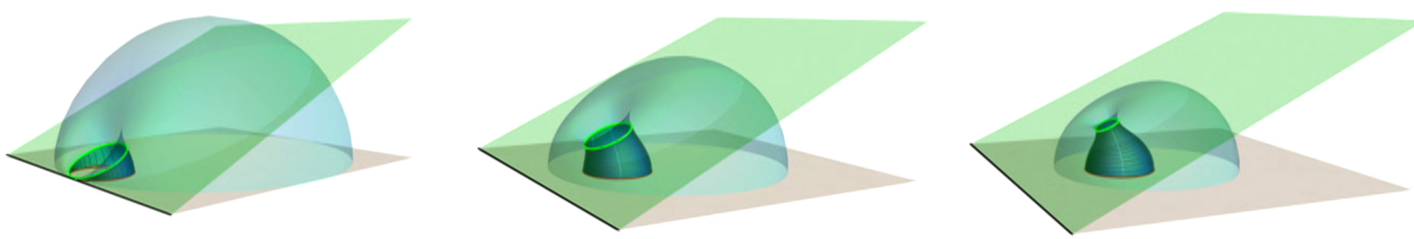
Disk disjoint from a flat boundary

[Seminara, Sisti, E.T., (2018)]



- This configuration can be studied analytically by composing the expressions for the disk concentric to a circular boundary with a well know map
  [Berenstein, Corrado, Fischler, Maldacena, (1998)]

E.g.: $$\frac{d}{R} = \frac{(R_\circ/R_Q - 1)^2}{2\,R_\circ/R_Q}$$



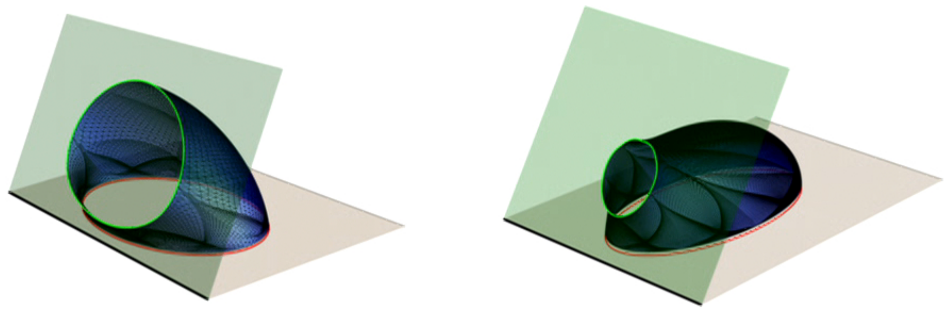
On regions with generic shape

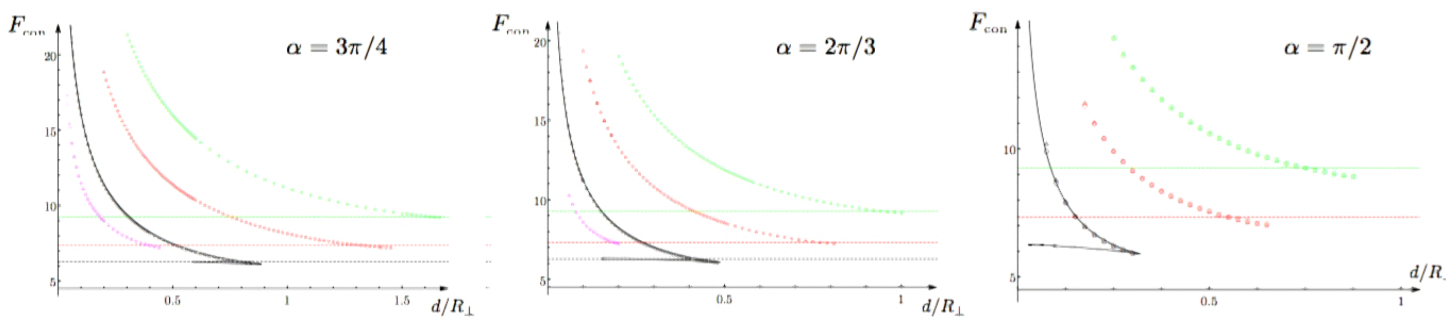
- *Surface Evolver* allows to study numerically any kind of region A.
  F_A can be found in two ways:
  - $\hat{\mathcal{A}}_{\varepsilon}^{\mathrm{SE}} - P_{A,B}/\varepsilon$
  - Willmore-like formula for F_A

- Given the BCFT$_3$ in its ground state and a region A disjoint from the boundary, we find

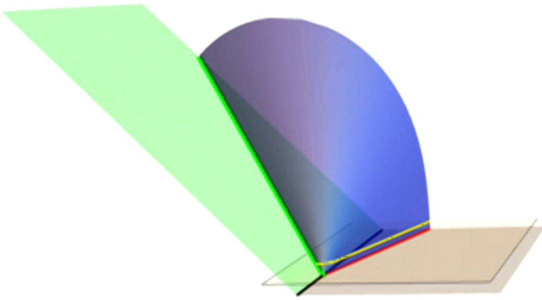
$$F_A \geqslant 2\pi$$

- E.g.: Ellipsis





Regions with corners adjacent to the boundary



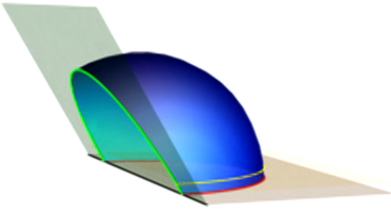
- Infinite wedge adjacent to the boundary

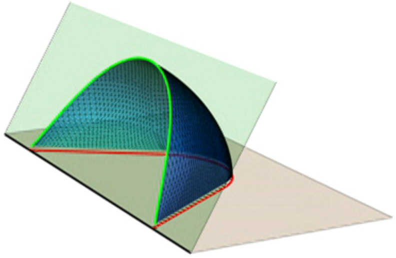
$$\mathcal{A}[\hat{\gamma}_\varepsilon] = \frac{L}{\varepsilon} - F_\alpha(\gamma)\,\log(L/\varepsilon) + O(1)$$

Corner function $F_\alpha(\gamma)$ computed through the standard approach [Seminara, Sisti, E.T., (2017)]

- Special case: half disk adjacent to the flat boundary

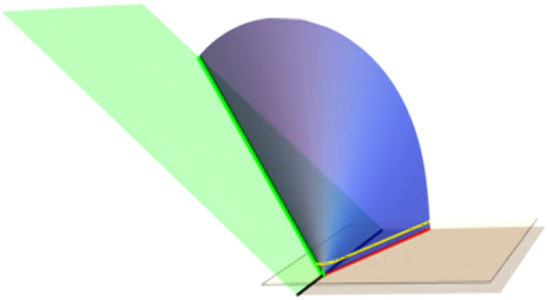
$$\mathcal{A}[\hat{\gamma}_\varepsilon] = \frac{\pi R}{\varepsilon} + 2\cot\alpha\,\log(R/\varepsilon) + O(1)$$





- *Surface Evolver* has been employed to check the analytic expression of the corner function $F_\alpha(\gamma)$

Regions with corners adjacent to the boundary



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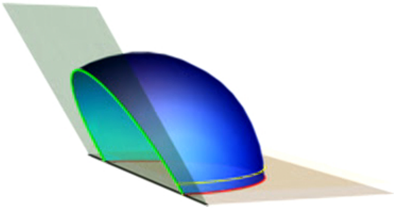
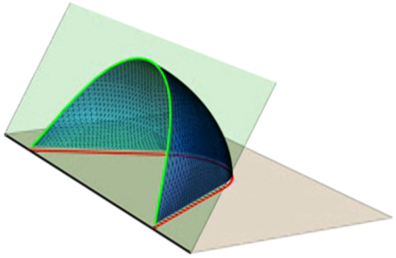




- *Surface Evolver* has been employed to check the analytic expression of the corner function $F_\alpha(\gamma)$

- Specifying the Willmore-like formula for F_A to the infinite wedge, the corner function is recovered. The integral over the line $\partial\hat{\gamma}_Q$ is non vanishing. [Seminara, Sisti, E.T., (2018)]

Infinite wedge adjacent to the boundary: Corner function

[Seminara, Sisti, E.T., (2017)]

- The corner function can be written in a parametric form

$$\begin{cases} F_\alpha = F(q_0) + \eta_\alpha \, \mathcal{G}\big(q_*(\alpha, q_0), q_0\big) \\ \gamma = P_0(q_0) + \eta_\alpha \Big(\arcsin[s_*(\alpha, q_0)] - P\big(q_*(\alpha, q_0), q_0\big)\Big) \end{cases}$$

$$P(q, q_0) \equiv \frac{1}{q_0(1+q_0^2)} \Big\{ (1+2q_0^2)\, \Pi\big(-1/Q_0^2\,,\, \sigma(q,q_0)\,\big|\, -Q_0^2\big) - q_0^2\, \mathbb{F}\big(\sigma(q,q_0)\,\big|\, -Q_0^2\big) \Big\}$$

$$\mathcal{G}(q, q_0) \equiv \sqrt{1+q_0^2}\, \left\{ \mathbb{F}\big(\sigma(q,q_0)\,\big|\, -Q_0^2\big) - \mathbb{E}\big(\sigma(q,q_0)\,\big|\, -Q_0^2\big) + \sqrt{\frac{(q^2+1)(q^2-q_0^2)}{(q_0^2+1)(q^2+q_0^2+1)}} \right\}$$

$$s_*(\alpha, q_0) \equiv$$

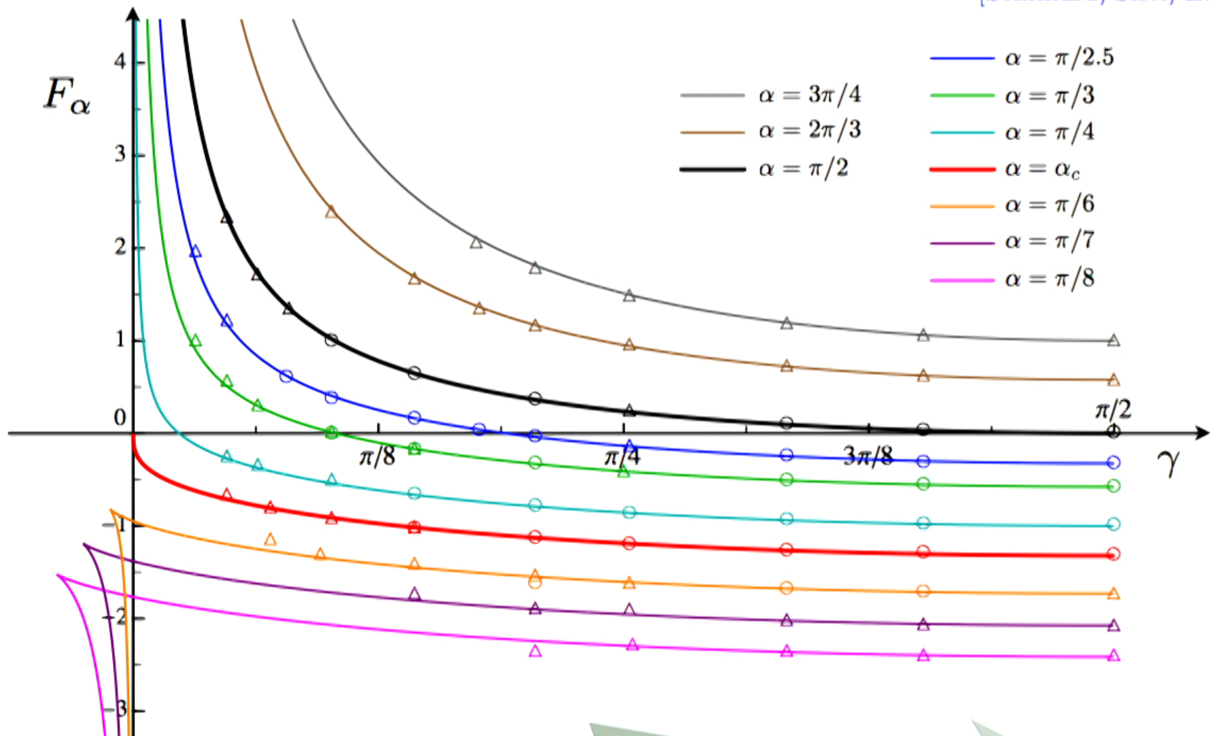
$$\frac{1}{\sqrt{2}} \left\{ \left(1 + \frac{q_0^4+q_0^2}{(\cos\alpha)^2}\right)^{-1} \left[1 - (\cot\alpha)^2 + \sqrt{\Big[1-(\cot\alpha)^2\Big]^2 + 4\left(1+\frac{q_0^4+q_0^2}{(\cos\alpha)^2}\right)(\cot\alpha)^2}\; \right] \right\}^{\frac{1}{2}}$$

$$q_*(\alpha, q_0) = \frac{|\cot\alpha\,|}{s_*(\alpha, q_0)} \qquad \sigma(q, q_0) \equiv \arctan\sqrt{\frac{q^2-q_0^2}{1+2q_0^2}} \qquad Q_0^2 \equiv \frac{q_0^2}{1+q_0^2}$$

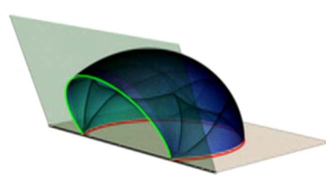
$$\eta_\alpha \equiv -\,\mathrm{sign}(\cot\alpha)$$

Infinite wedge adjacent to the boundary: Corner function

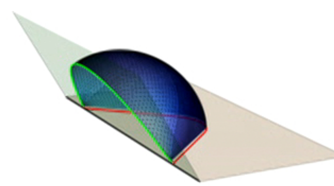
[Seminara, Sisti, E.T., (2017)]



Two kinds of domains employed in the numerical analysis based on *Surface Evolver*



(empty circles)



(empty triangles)

Corner function & the stress tensor one-point function

- Expansions of the corner function $F_\alpha(\gamma)$ as $\gamma \to 0$ and $\gamma \to \pi/2$

$$\begin{cases} F_\alpha(\gamma) = \dfrac{\mathfrak{g}(\alpha)^2}{\gamma} + O(\gamma) & \alpha \in (\alpha_c, \pi) \\ F_\alpha(\gamma) = -\cot\alpha + \dfrac{(\pi/2-\gamma)^2}{2(\pi-\alpha)} + O\big((\pi/2-\gamma))^4\big) \end{cases}$$

- One-point function of the stress tensor in a BCFT_3 near a curved boundary as the proper distance (from the boundary) $X \to 0$

[Deutsch, Candelas, (1979)]

$$\langle T_{ij} \rangle = \frac{A_T}{X^2}\,\kappa_{ij} + \dots$$

traceless part of the extrinsic curvature of the boundary

- Relation between $F_\alpha(\gamma)$ and $\langle T_{ij} \rangle$ in this holographic setup

[Seminara, Sisti, E.T., (2017)]

$$\boxed{\frac{f''_\alpha(\pi/2)}{A_T} = -2\pi} \qquad f_\alpha(\gamma) \equiv \frac{L^2_{\mathrm{AdS}}}{4G_{\mathrm{N}}}\,F_\alpha(\gamma)$$

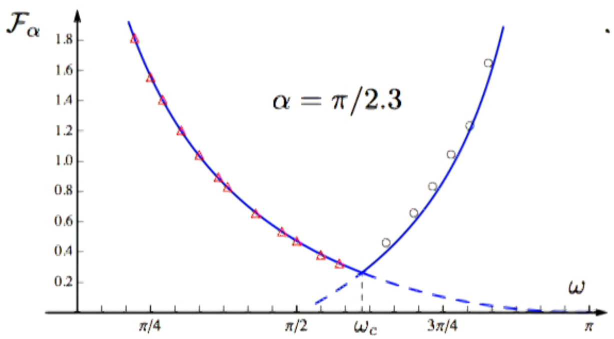
It would be interesting to test this ratio in BCFT_3 models.

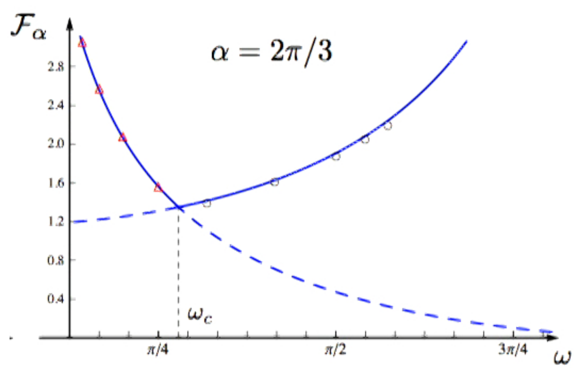
Corners with only the tip on the boundary (I)

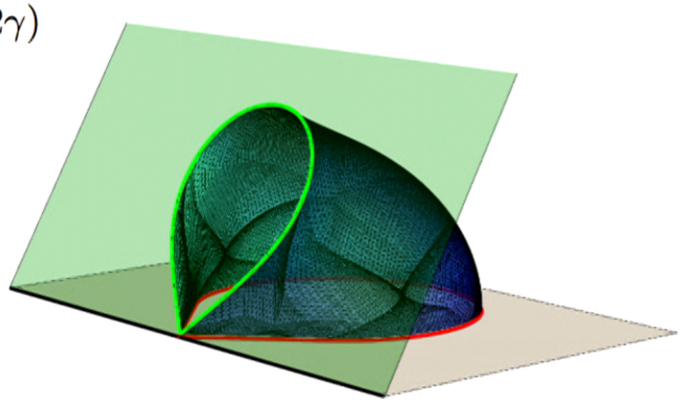
- Symmetric case $(\gamma = \tilde{\gamma} \Rightarrow \omega = \pi - 2\gamma)$

$$\mathcal{F}_\alpha(\omega, \gamma) = \max\left\{ \widetilde{F}(\omega), 2F_\alpha(\gamma) \right\}$$

- Transition at $\omega = \omega_c$



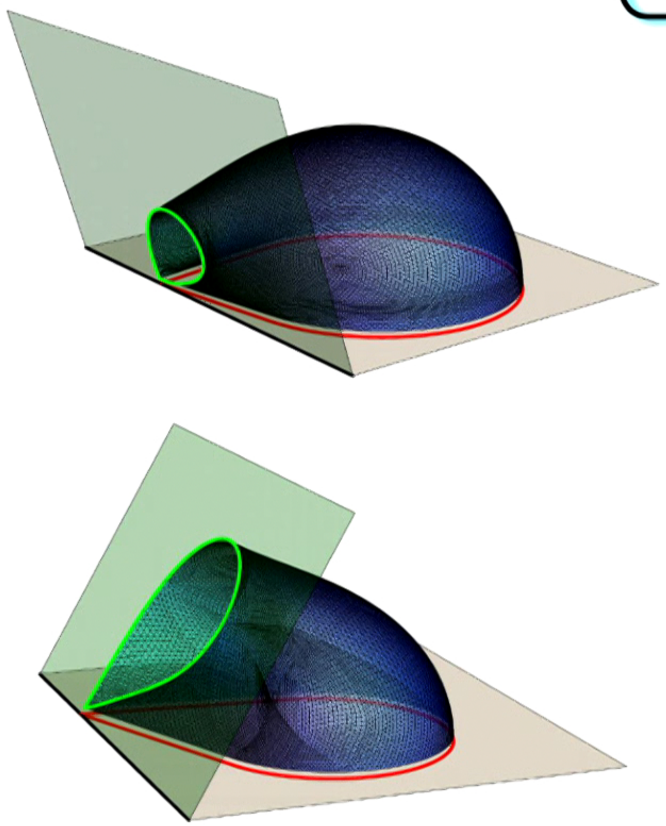


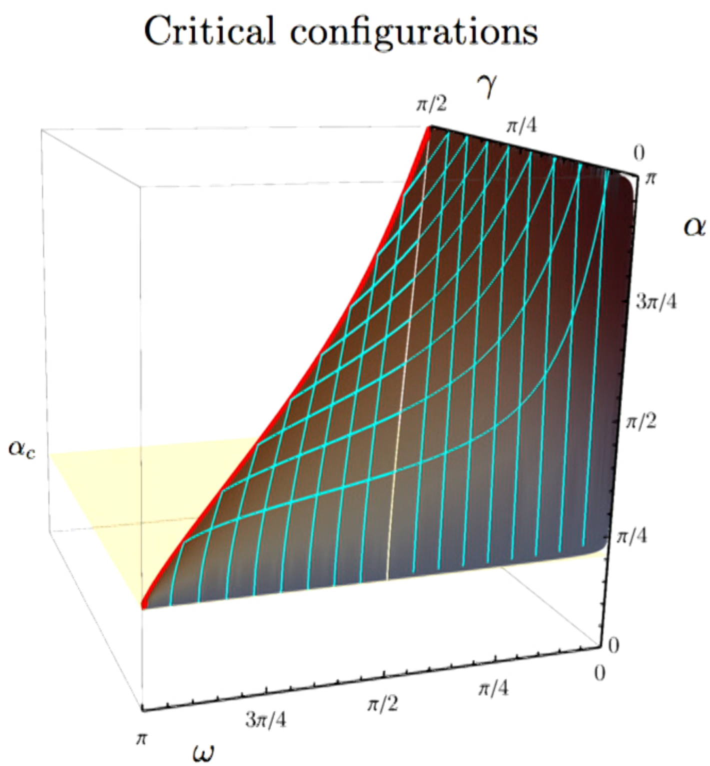


Corners with only the tip on the boundary (II)

- Generic case $(\gamma + \omega + \tilde{\gamma} = \pi)$

$$\mathcal{F}_\alpha(\omega, \gamma) = \max\left\{\widetilde{F}(\omega), F_\alpha(\gamma) + F_\alpha(\tilde{\gamma})\right\}$$





Conclusions & some open issues

- Holographic entanglement entropy in $AdS_4/BCFT_3$:
  - analytic expression for F_A through the Willmore functional
  - analytic results for a disk disjoint from the (flat or circular) boundary
  - analytic expression for the corner function & its properties

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- Some open problems:
  - $BCFT_3$ interpretation of α
  - $BCFT_3$ calculations of entanglement entropy for non trivial regions and how to extract information about the $BCFT_3$ data
  - Time-dependent backgrounds
  - Higher dimensions
  - Extensions to the context of AdS/dCFT
  - Connections with mathematics