Title: PSI 2018/2019 - Relativity - Lecture 15

Date: Sep 21, 2018 09:00 AM

URL: http://pirsa.org/18090016

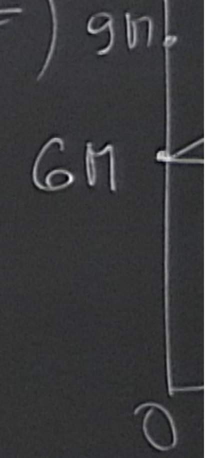
Abstract:

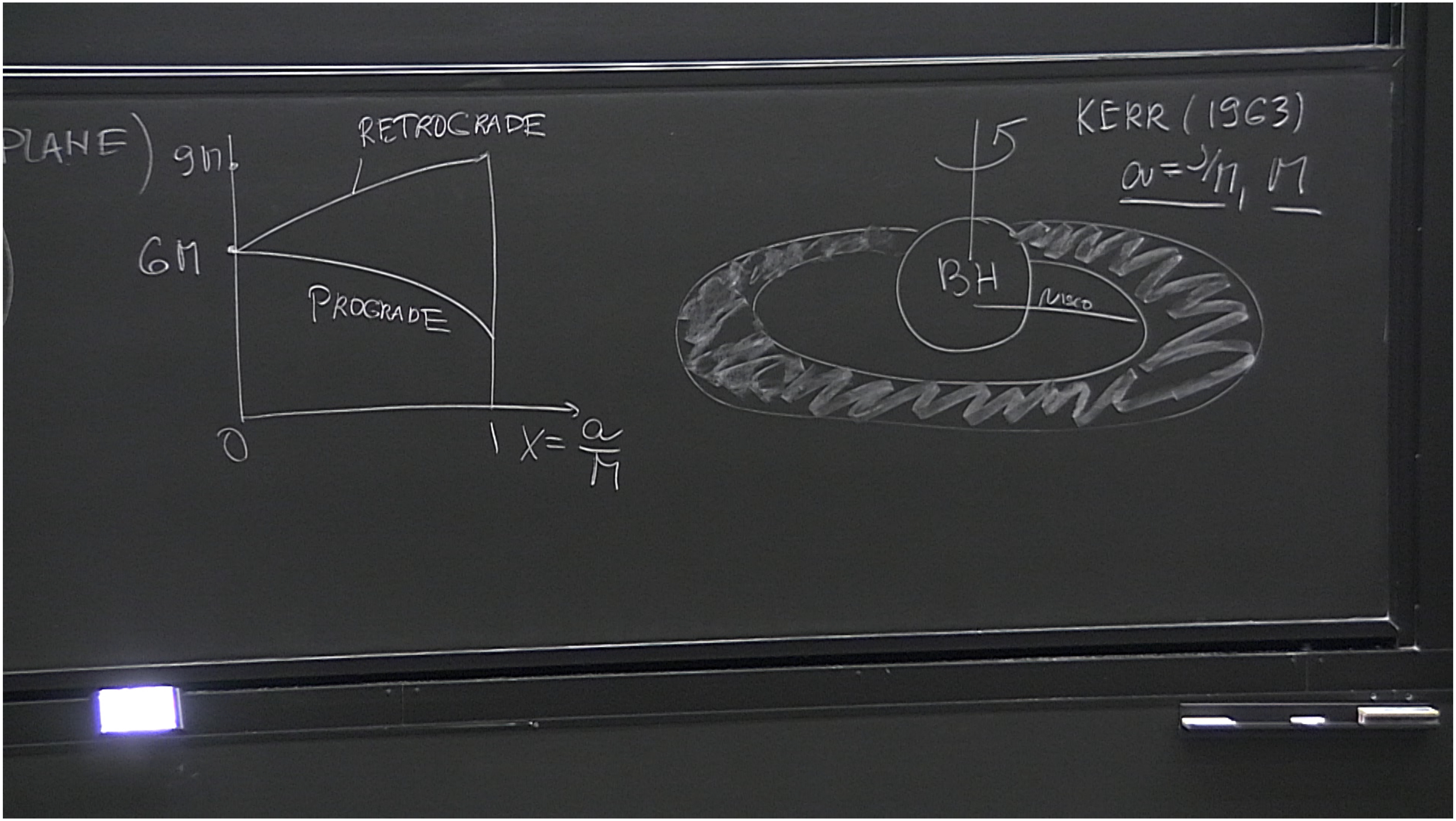
YESTERDAY: ISCO FOR KERR (EQUATORIAL PLANE)

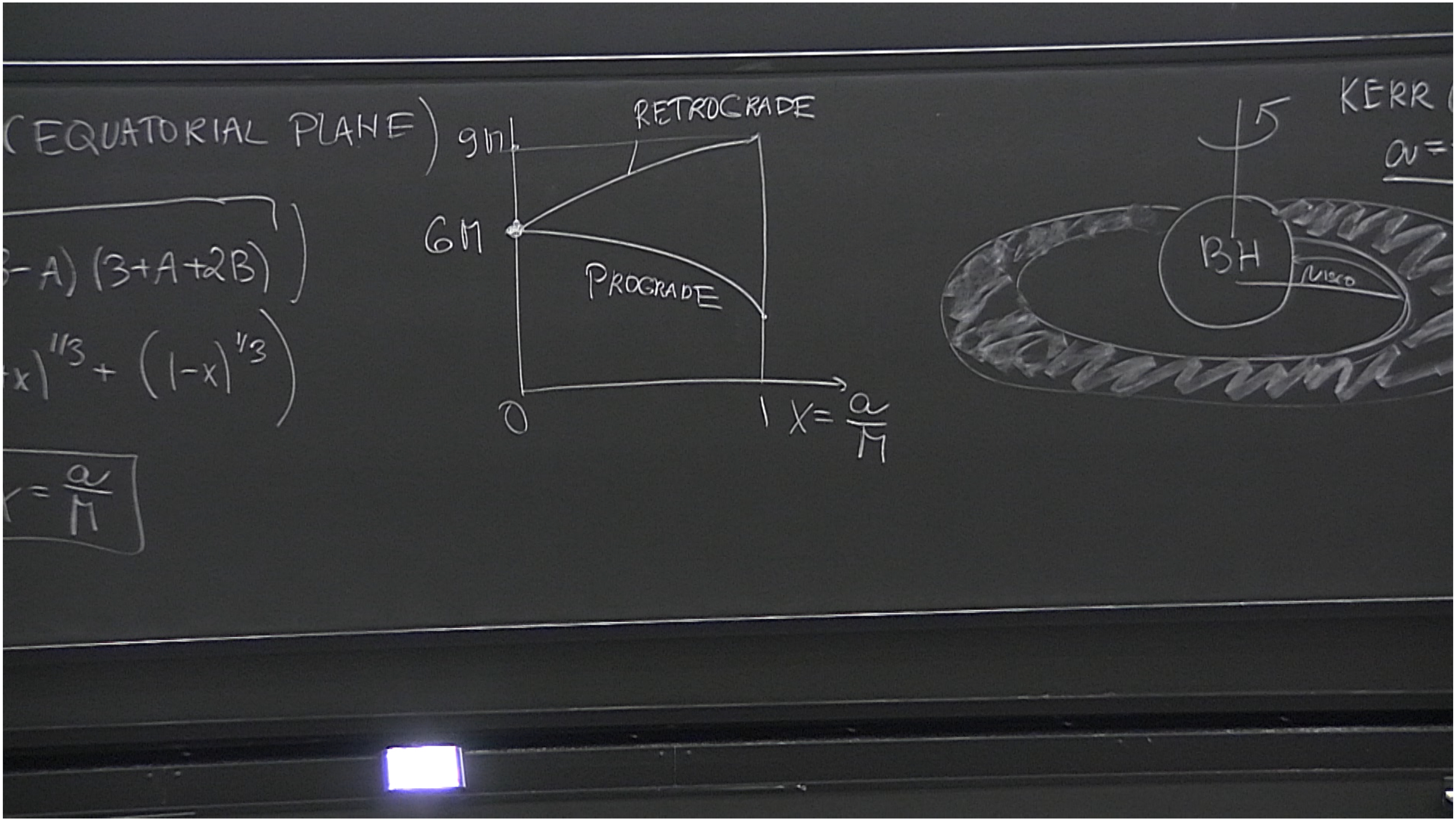
$$r_{ISCO} = M\left(3 + b \pm \sqrt{(3-A)(3+A+2B)}\right)$$

$$A = 1 + (1-\chi^2)^{1/3}\left((1+\chi)^{1/3} + (1-\chi)^{1/3}\right)$$

$$B = \sqrt{3\chi^2 + A^2} \quad , \quad \boxed{\chi = \frac{a}{M}}$$




PLANE)
RETROGRADE
6M
PROGRADE
0
1
x = a/M
KERR (1963)
a = J/M, M
BH
ISCO


(EQUATORIAL PLANE)
9M
6M
RETROGRADE
PROGRADE
0
1
X = a/M
KERR
BH
r_ISCO
x = a/M

d) BLACK HOLE THERMODYNAMICS

MOTIVATION:

$$\boxed{\begin{gathered} ds^2 = -f\,dt^2 + \frac{dr^2}{f} + r^2 d\Omega^2 \\ f = 1 - \frac{2M}{r} \end{gathered}}$$

- METRIC IS STATIC ... $\partial_t = k$ KV

KOMAR MASS

$$M = -\frac{1}{8\pi}\int_{S^2_\infty} \star dk = M \checkmark$$

TOTAL MASS (ENERGY) OF SPACETIME

- SURFACE GRAVITY (ANALOGUE OF GRAV. ACCELARITION ON HORIZON)

BLACK HOLE HORIZON ($r = r_+ = 2M$) IS KILLING HORIZON

(GENERATED BY KV, WHICH IS NULL)

$k^2|_{r=2M} = 0$, k^μ... NORMALL TO $r = 2M$.

$$k^\mu \nabla_\mu k^\nu = \underline{\varkappa} \, k^\nu$$

NULL GEODESIC

MEASURES HOW MUCH THE PARAMETER IS NOT AN AFFINE PARAM.

$\varkappa$ ... SURFACE GRAVITY

$$\kappa = \frac{f'(r_+)}{2} = \frac{1}{4M} = \frac{1}{2r_+}$$

$$\left(g_{\text{NEWTON}} = \frac{M}{r_+^2} = \frac{M}{(2M)^2} = \underline{\frac{1}{4M}} \quad \text{COINCIDENCE} \right)$$

• HORIZON AREA ($t = \text{CONST}$, $r = r_+$) ... INDUCED METRIC

$$d\gamma^2 = r_+^2 d\Omega^2 \qquad \sqrt{\gamma} =$$

$$\text{AREA} = \int \sqrt{\gamma}\, d\theta\, d\varphi = r_+^2 \int \sin\theta\, d\theta\, d\varphi = \underline{4\pi r_+^2 = A}$$

- **OBSERVATION:**

$$dM = \frac{dr_+}{2}, \quad dA = 8\pi r_+ dr_+$$

$$\frac{\varkappa}{2\pi}\frac{dA}{4} = \frac{1}{2r_+}\frac{1}{2\pi}\frac{8\pi r_+ dr_+}{4} = \frac{dr_+}{2} = dM$$

1ST LAW OF BH MECHANICS

$$\boxed{dM = \frac{\varkappa}{2\pi}\frac{dA}{4}}$$

- LAWS OF BLACK HOLE MECHANICS (1973)

GENERAL BH. M, J, Q

0th: SURFACE GRAVITY $\kappa = \mathrm{CONST}$

1ST LAW:

$$dM = \frac{\kappa}{2\pi}\frac{dA}{4} + \underbrace{\underset{\text{ANGULAR VEL.}}{\Omega}\, dJ + \underset{\text{ELSTATIC POTENTIAL}}{\Phi}\, dQ}_{\text{WORK TERMS}}$$

2ND LAW: CLASSICALLY THE AREA OF HORIZON NEVER DECREASES

$$\boxed{dA \geq 0}$$

3RD LAW IT IS IMPOSSIBLE TO REDUCE $\partial\varphi$ TO ZERO IN FINITE # OF STEPS

$\mathcal{H} \sim$ TEMP ?

$A \sim S$?

PROOF: ALL GEOMETRY

HOW CAN THIS BE ?

CLASSICAL TEMP OF BH

IS ABSOLUTE ZERO (SPONGE)

- WHEELER'S CUP OF TEA — BEKENSTEIN

$$\boxed{S \sim A}$$

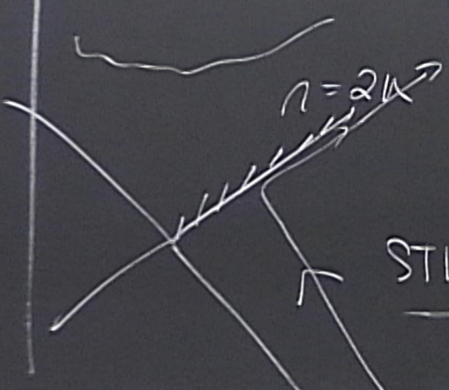
BH CARRIES ENTROPY.

WORK TERMS

HAWKING 1974 : IF QUANTUM EFFECTS TAKEN INTO ACCOUNT

BH RADIATES. AS BLACK BODY WITH

$$\boxed{T = \frac{\kappa}{2\pi}\frac{\hbar}{k_B}} \Rightarrow \boxed{S = \frac{A}{4}\frac{1}{\hbar G}}$$



STIMULATED EMISSION (QFT IN CURVED)

BLACK HOLE THERMODYNAMICS

• $k^\mu \nabla_\mu k^\nu = \underline{\kappa\, k^\nu}$ $\qquad \kappa = \frac{\pm \ldots}{2}$

DERIVE THIS USING EUCLIDEAN TRICK

• THERMAL GREEN FUNCTION HAS PERIODICITY IN IMAGINARY TIME $\tau = it$

$$\boxed{G(\tau) = G(\tau + \beta), \quad \beta = \frac{1}{T}}$$

- PARTITION FUNCTION

$$Z = \int Dg\, e^{-S_E[g]} \overset{WKB}{\simeq} e^{-S_E[g_c]}$$

FREE ENERGY $\underline{F} = -\frac{1}{\beta} \log Z$

$$S = -\frac{\partial F}{\partial T}$$

LET'S DO IT FOR RINDLER

$$ds^2 = -(1+ax)^2 dT^2 + dx^2 + dy^2 + dz^2$$

- WICK ROTATE $\tau = iT$

$$ds_E^2 = (1+ax)^2 d\tau^2 + dx^2 + \ldots$$

$$\rho = \frac{1+aX}{a}, \quad d\rho = dX$$

$$d^2 S_E = \rho^2 \underbrace{a^2 d\tau^2}_{d\varphi^2} + d\rho^2 + \cdots = d\rho^2 + \rho^2 d\varphi^2 + \cdots$$

$$\boxed{\varphi = a\tau}$$

HORIZON $\propto$ $\rho = 0$

POLAR COORDINATES

WE DEMAND THERE IS NO SING.

TO AVOID CONICAL SINGULARITY

$$\varphi \sim \varphi + 2\pi \Rightarrow \tau \sim \tau + \frac{2\pi}{a} \quad \beta$$

$$\boxed{T = \frac{a}{2\pi}}$$

$$\varphi \sim \varphi + 2\pi \Rightarrow \tau \sim \tau + \left(\frac{2\pi}{a}\right)\beta \qquad \boxed{T = \frac{a}{2\pi}}$$

UNRUH TEMPERATURE

$$= \frac{1}{4M} = \frac{1}{2r_+}$$

$$= \frac{M}{r_+^2} = \frac{M}{(2M)^2} = \frac{1}{4M}$$ (COINCIDENCE)

EA $(t = \text{CONST}, r = r_+)$ ... INDUCED METRIC

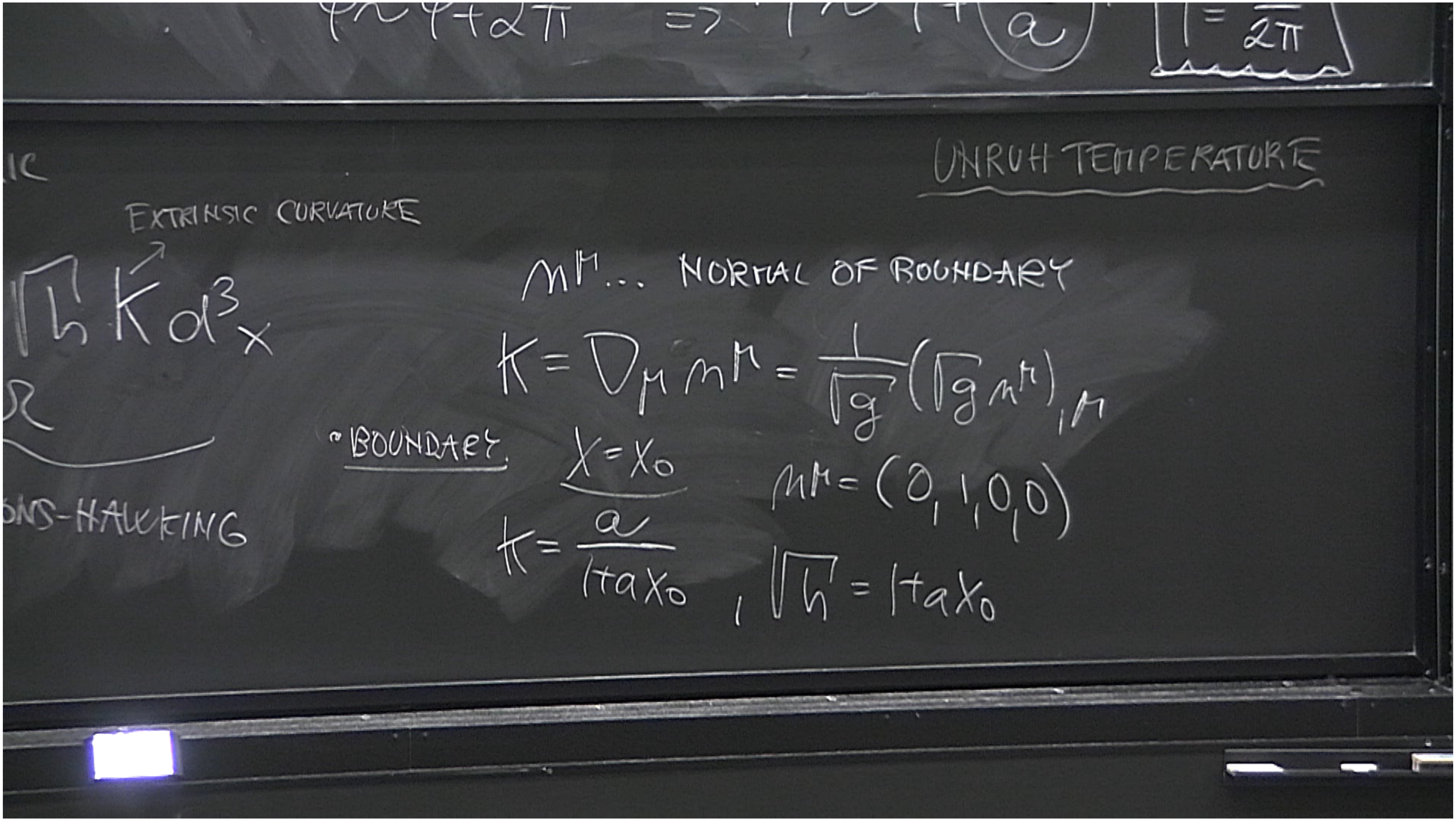
Ω^2 $\qquad \sqrt{\gamma} = r_+^2 \sin\theta$

$\varphi \sim \varphi$

- LET'S CALCULATE THE ACTION FOR RINDLER METRIC

EXTRINSIC CURVATURE

$$S = \frac{1}{16\pi G}\int_{\Omega}\sqrt{g}\,R\,d^4x + \underbrace{\frac{1}{8\pi G}\int_{\partial\Omega}\sqrt{h}\,K\,d^3x}_{\text{GIBBONS-HAWKING}}$$


UNRUH TEMPERATURE
EXTRINSIC CURVATURE
$\sqrt{h}\,K\,d^3x$
n^μ ... NORMAL OF BOUNDARY
$K = \nabla_\mu n^\mu = \frac{1}{\sqrt{g}}(\sqrt{g}\,n^\mu)_{,\mu}$
BOUNDARY
$X = X_0$
$n^\mu = (0,1,0,0)$
$K = \frac{a}{1+aX_0}$, $\sqrt{h} = 1+aX_0$
ONS-HAWKING

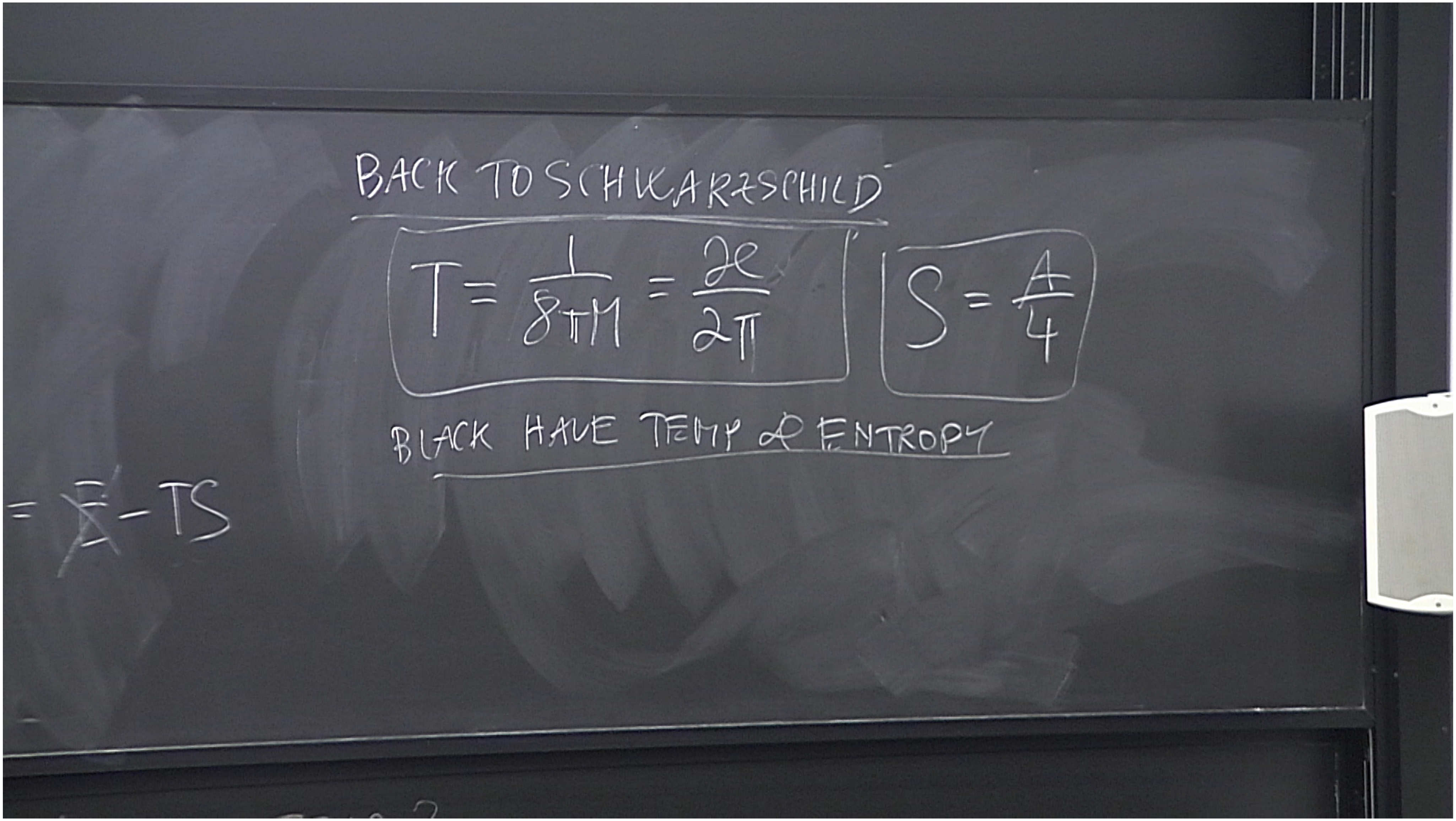
• WHEELER'S CUP OF TEA — BEKENSTEIN | HAWKING 1974 :

$$S_E = 0 - \frac{1}{8\pi G}\,|1+ax_0|^{\sqrt{\beta}}\,\frac{a}{1+ax_0}\underbrace{\int d\tau\, dy\, dz}_{\beta \underbrace{\int dy\, dz}_{A}}$$

$$= -\frac{a\beta}{8\pi G} A$$

$$F = -\frac{1}{\beta}\log e^{-S_E} = \frac{S_E}{\beta} = -\overset{T}{\frac{a}{2\pi}}\,\frac{A}{4} = -T\frac{A}{4} = \cancel{E} - TS$$

$$S = -\frac{\partial F}{\partial T} = \frac{A}{4} \ldots$$ BEKENSTEIN


BACK TO SCHWARZSCHILD
$T = \frac{1}{8\pi M} = \frac{\kappa}{2\pi}$
$S = \frac{A}{4}$
BLACK HAVE TEMP & ENTROPY
$= \cancel{E} - TS$

HAWKING-PAGE TRANSITION

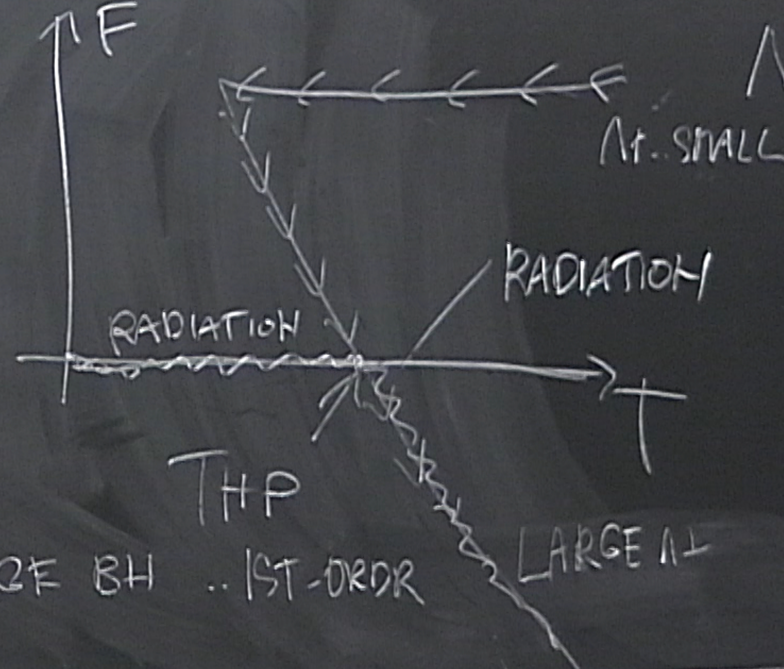
$$F = M - TS$$

... IN EQ. THE STATE CORRESPONDING TO GLOBAL MINIMUM OF FREE ENERGY IS PREFFERED

SCH-AdS $(\Lambda<0)$ $\quad F=F(T,\Lambda)$

. THE STATE
…SPONDING TO GLOBAL
…MUM OF FREE ENERGY
IS PREFFERED.

Λ=FIXED



PHASE TRANSITION RAD→LARGE BH .. 1ST-ORDR