

Title: PSI 2018/2019 - Relativity - Lecture 14

Date: Sep 20, 2018 09:00 AM

URL: <http://pirsa.org/18090015>

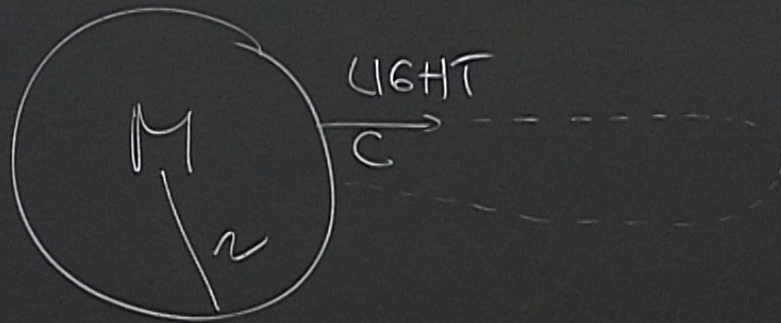
Abstract:

c) BLACK HOLES



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DARK STARS (MICHELL 1783)



$$\frac{1}{2}mc^2 - \frac{GMm}{r} = 0$$

$$r = r_+ = \frac{2GM}{c^2}$$

COINCIDES WITH RADIOS OF BH.

- INTERPRETATION IS COMPLETELY DIFFERENT!
 E.G. INFORMATION CAN 'LEAK OUT'
 ALL THE WAY TO INFINITY.

IN FULL GR. SCHWARZSCHILD SOLUTION

$$ds^2 = -f dt^2 + \frac{dr^2}{f} + r^2 d\Omega^2$$

$$r = 0$$

$$\frac{2GM}{c^2}$$

HORIZON OF BH.

COINCIDES WITH RADIOS OF BH.

• $\Lambda = \Lambda_+ = 2M$... PROBLEM

WHAT IS HAPPENING AT
 $\Lambda = \Lambda_+$?

TO GET SOME INSIGHT LET'S
STUDY RINDLER SPACE

COINCIDES WITH RADII OF BH.

1... PROBLEM

IS HAPPENING AT
 $r = r_+$?

SOME INSIGHT LET'S
RINDLER SPACE

BH DEMYSTIFIED !

$$ds^2 = -(1+ax)^2 dt^2 + dx^2$$

BH DEMYSTIFIED ¹⁰

$$ds^2 = -(1+ax)^2 dT^2 + dx^2$$

• RINDLER HORIZON:

$$X = X_+ = -\frac{1}{a}$$

$$T \in (-\infty, \infty), \quad X \in (X_+, \infty)$$

- CONSIDER TWO STATIC OBSERVERS IN RINDLER SPACE

$$\frac{V_A}{V_B} = \frac{d\tau_B}{d\tau_A} = \frac{\sqrt{-g_{\tau\tau}|_B}}{\sqrt{-g_{\tau\tau}|_A}} = \frac{1+aX_B}{1+aX_A}$$

$$\text{AS } X_B \rightarrow X_+$$

- CONSIDER TWO STATIC OBSERVERS IN RINDLER SPACE

$$\frac{V_A}{V_B} = \frac{d\tau_B}{d\tau_A} = \frac{\sqrt{-g_{\tau\tau}|_B}}{\sqrt{-g_{\tau\tau}|_A}} = \frac{1+aX_B}{1+aX_A}$$

AS $X_B \rightarrow X_+$

$$\frac{V_A}{V_B} \rightarrow 0$$

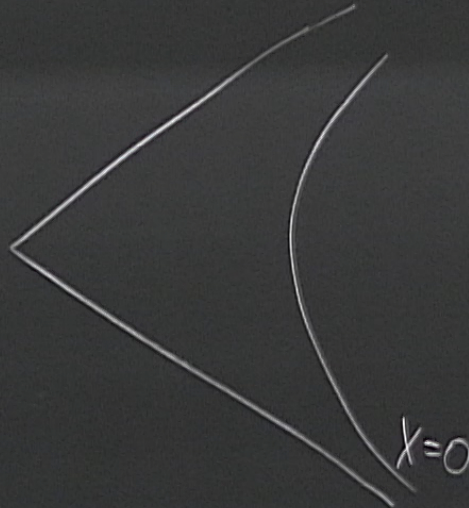
SURFACE OF
INFINITE
REDSHIFT

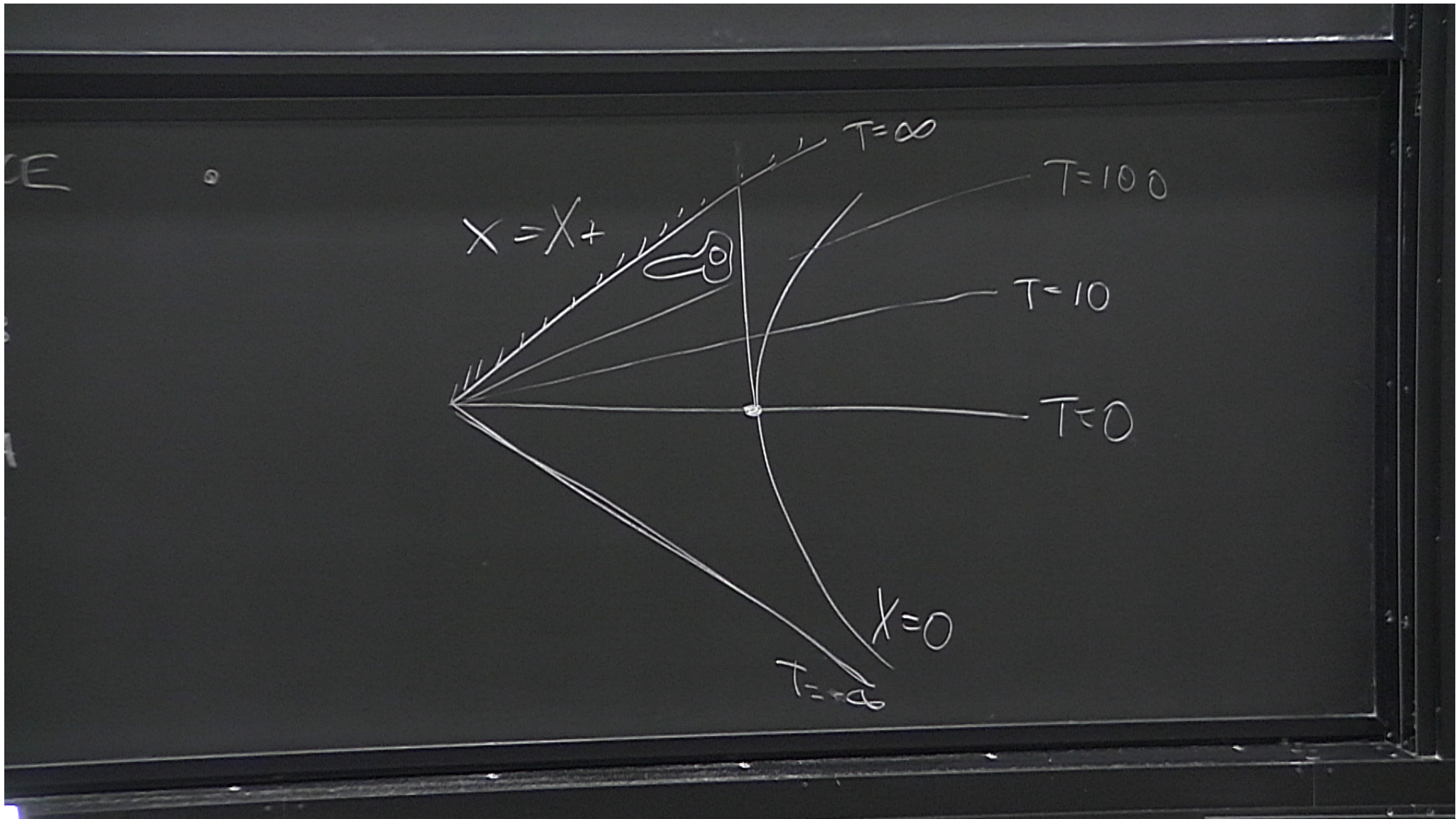
$$x \in (0, \infty), \quad x \in (x_+, \infty)$$

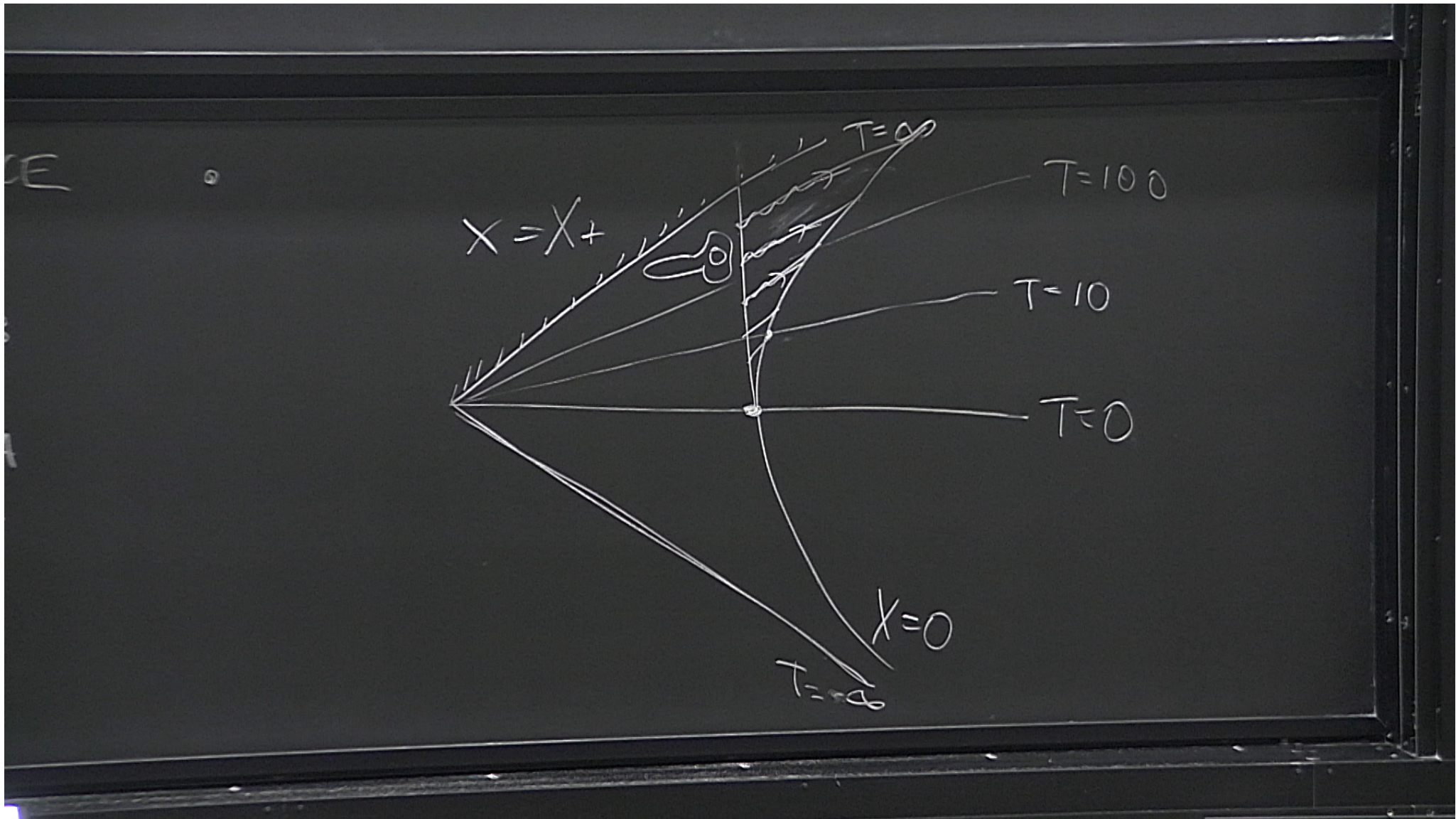
IN RINDLER SPACE

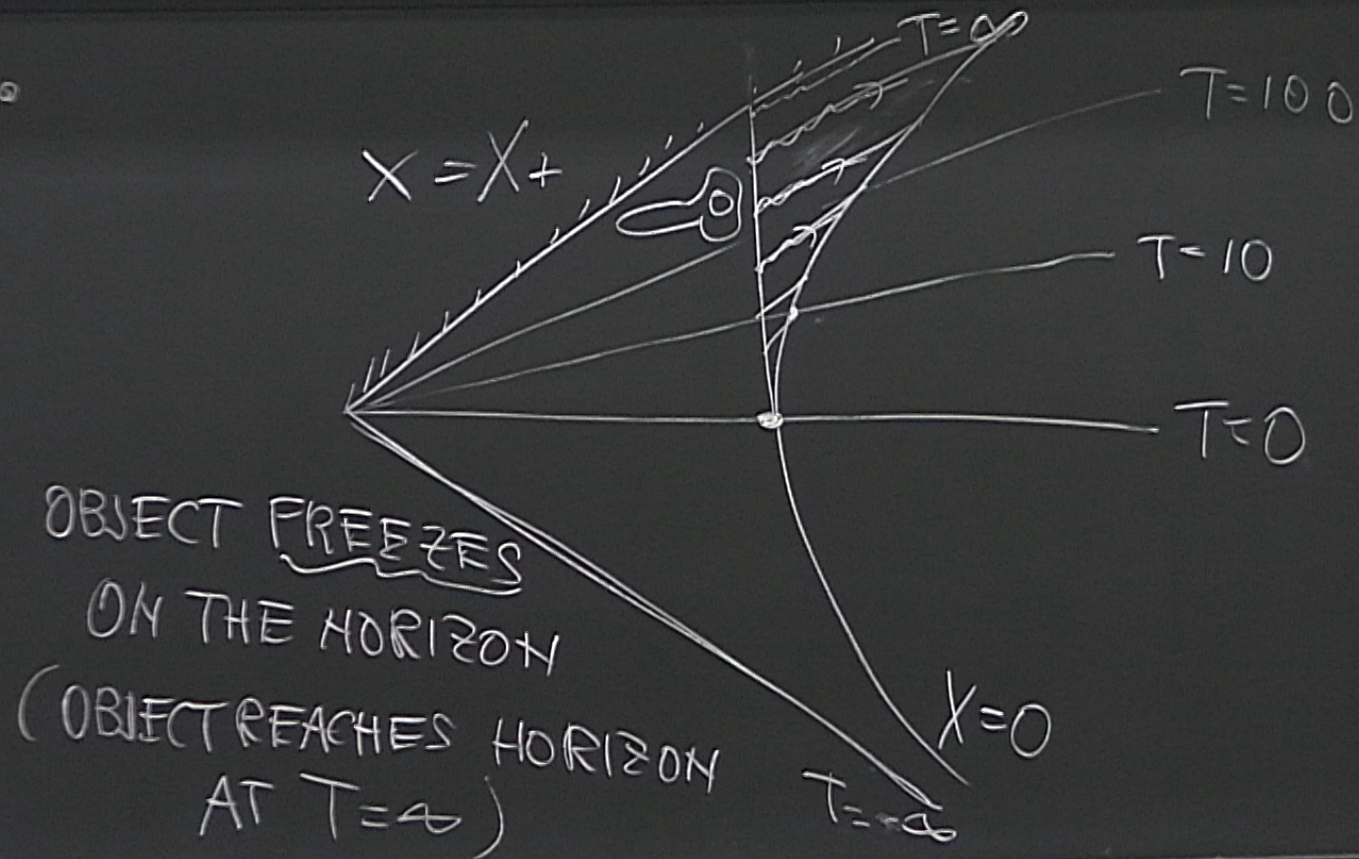
$$\frac{\tau_B}{\tau_A} = \frac{1 + a x_B}{1 + a x_A}$$

SURFACE OF
INFINITE
REDSHIFT



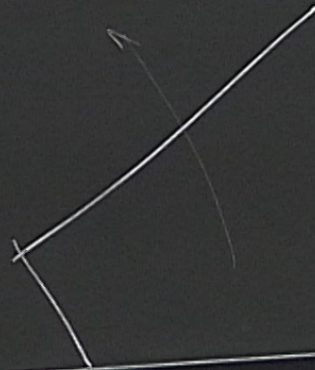






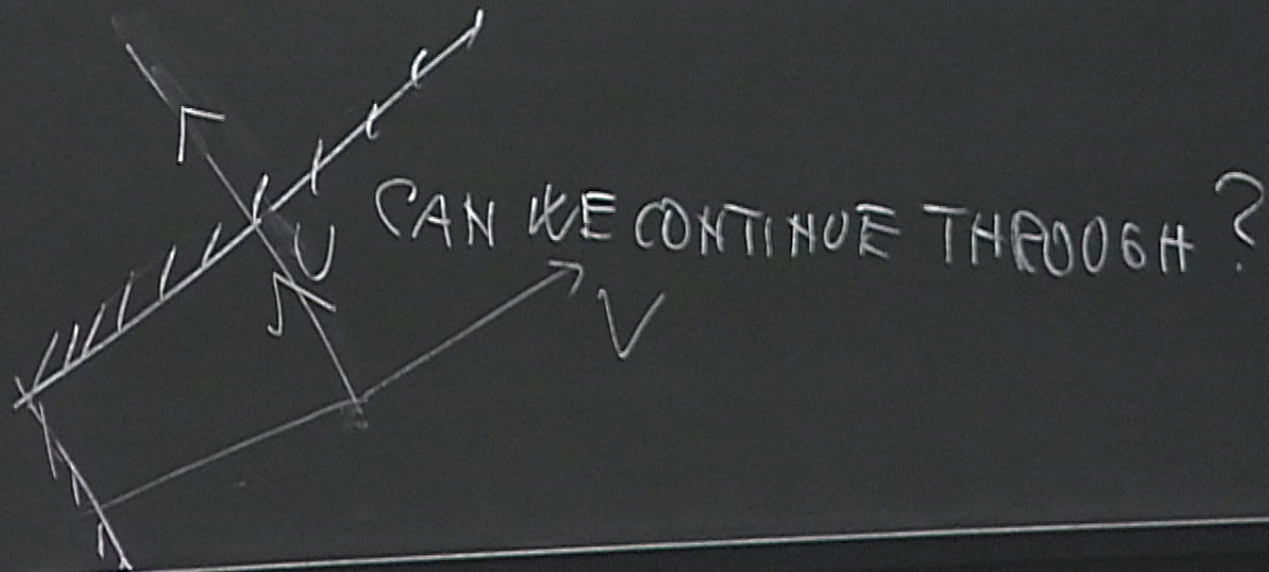
- WHAT IS HAPPENING ON THE HORIZON?
MAXIMAL EXTENSION OF RINDLER

IDEA: SHOOT GEODESICS AT THE HORIZON
& SEE WHAT HAPPENS



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MAXIMAL EXTENSION OF RINDLER

IDEA: SHOOT GEODESICS AT THE HORIZON
 ∞ SEE WHAT HAPPENS



$$ds^2=0 = -(1+ax)^2 \dot{T}^2 + \dot{X}^2 \Rightarrow \underbrace{T = \pm \frac{1}{a} \log(1+ax) + \text{CONST}}_{\text{ }}$$

$$U = T - \frac{1}{a} \log(1+ax), \quad V = T + \frac{1}{a} \log(1+ax)$$

$\lambda \neq 0$
AFFINE PARAMETER.

$$\lambda_{OUT} = e^V, \quad \lambda_{IN} = -e^{-U}$$

$\downarrow$
 $\lambda_{IN}(X_+) = 0 \dots$ CAN CONTINUE
THROUGH THIS

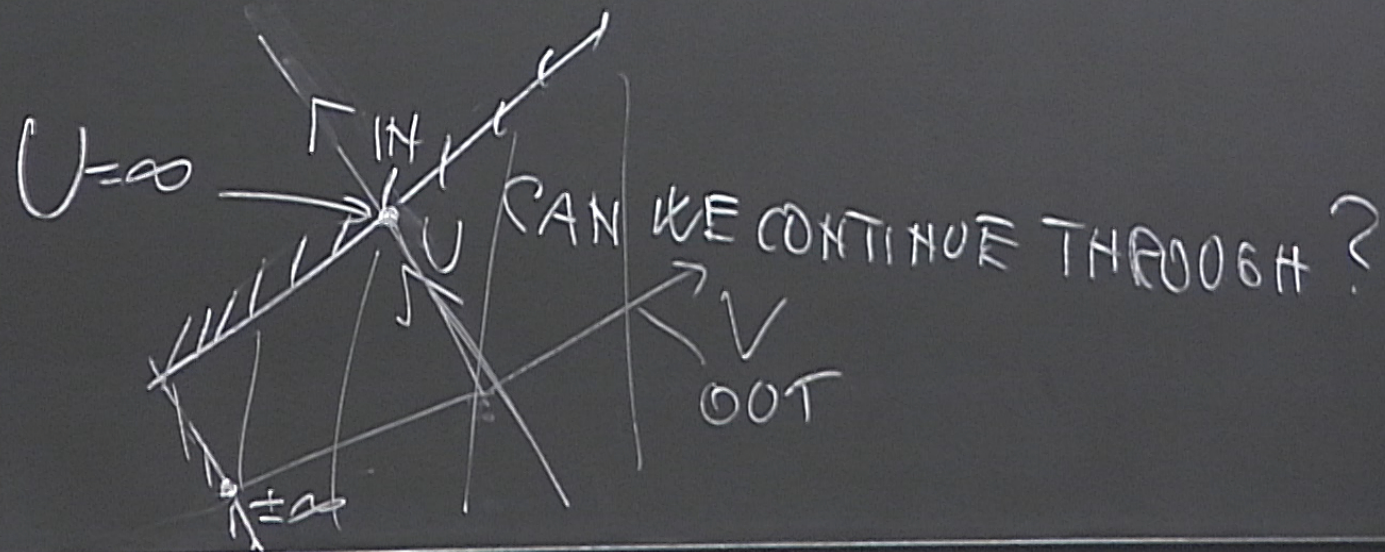
$$\Gamma = \lambda_{OUT} = e^V$$

$$\mathcal{W} = \lambda_{IN} = -e^{-U}$$

• WHAT IS HAPPENING ON THE HORIZON?

MAXIMAL EXTENSION OF RINDLER

IDEA: SHOOT GEODESICS AT THE HORIZON
& SEE WHAT HAPPENS

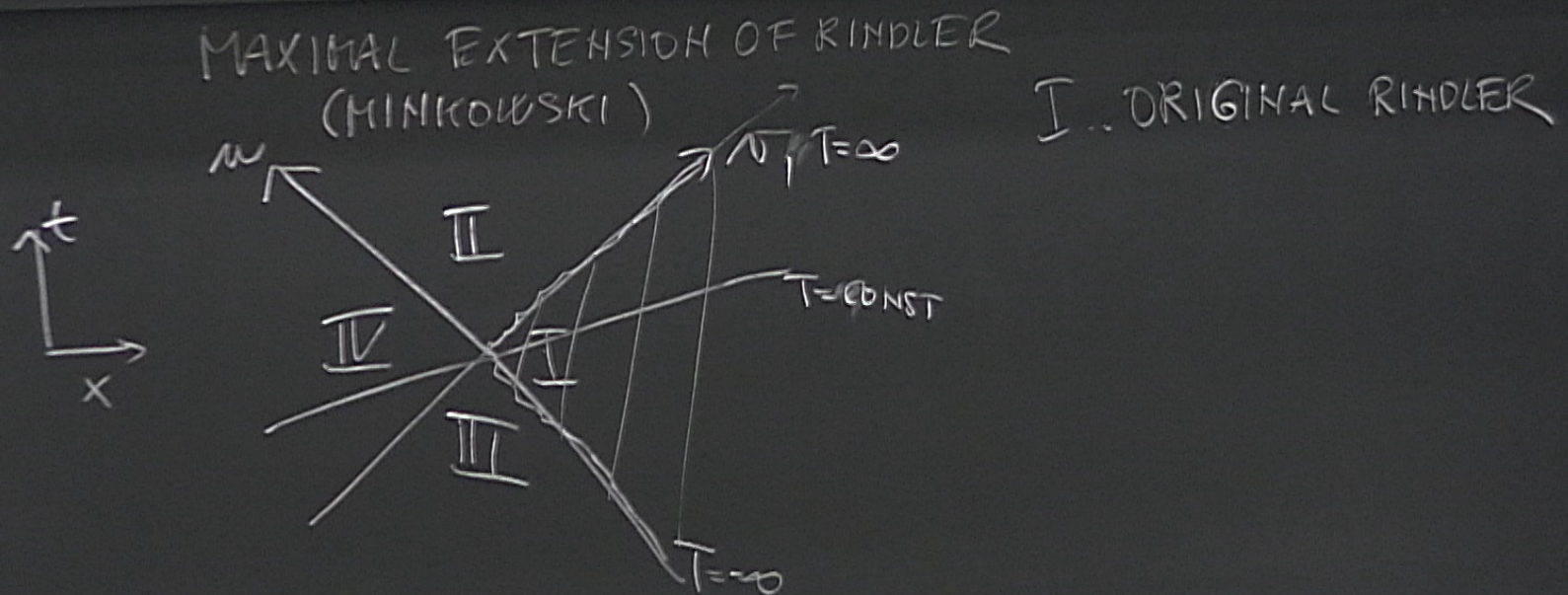


$$\underline{ds^2 = -du dv} \quad u, v \in (-\infty, \infty)$$

$$t = \frac{u+v}{2}, \quad x = \frac{v-u}{2}$$

$$\underline{ds^2 = -dt^2 + dx^2}$$

$$u = \lambda_{IH} = -e^{-U}$$



BH CASE IS EXACTLY THE SAME
BUT NOW WE HAVE SINGULARITY.
IN II & III.

BACK TO SCHWARZSCHILD

HORIZON

$$r = r_+ = 2M$$

— SURFACE OF INFINITE REDSHIFT.

— "FROZEN OBJECTS"

→ FROZEN STAR → BLACK HOLE.

IN RINDLER SPACE

~~$T=0$~~

- $r=r_+$ BOUNDARY OF BLACK HOLE
(CANNOT SEE ANYTHING BELOW)
- MAXIMAL EXTENSION (KRUSKAL 1960)

$$u = -e^{\frac{r^* - t}{4M}}, \quad v = e^{\frac{r^* + t}{4M}}, \quad r^* = \int \frac{dr}{f} = r + 2M \lg \left| \frac{r}{2M} - 1 \right|$$

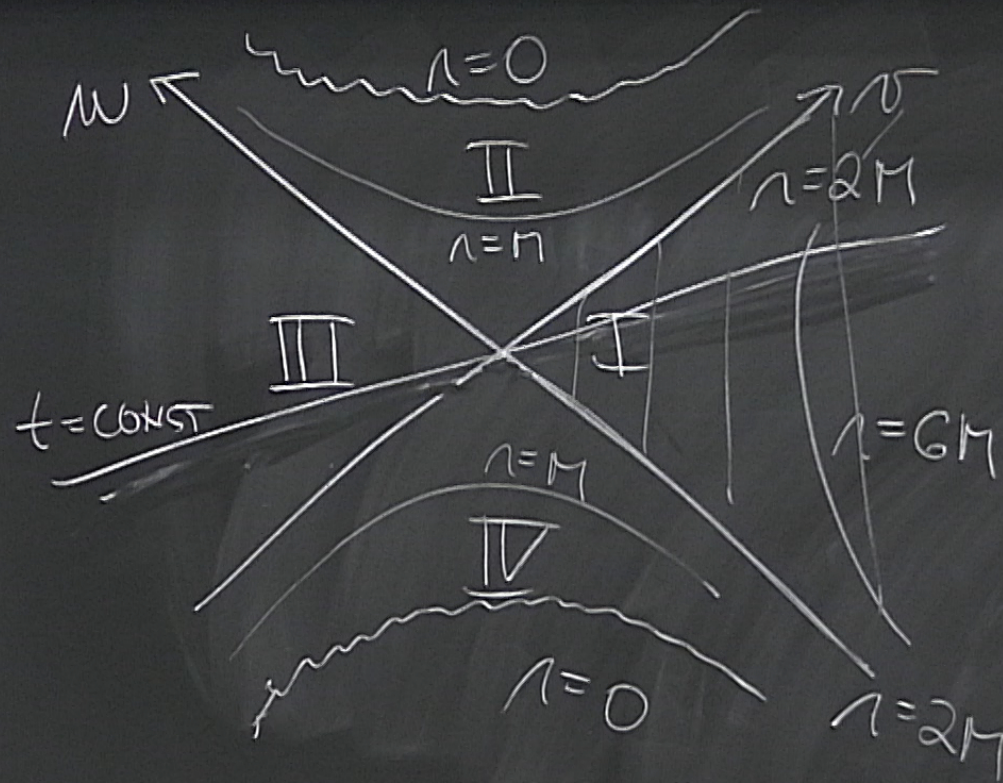
BH DEMYSTIFIED ¹⁰

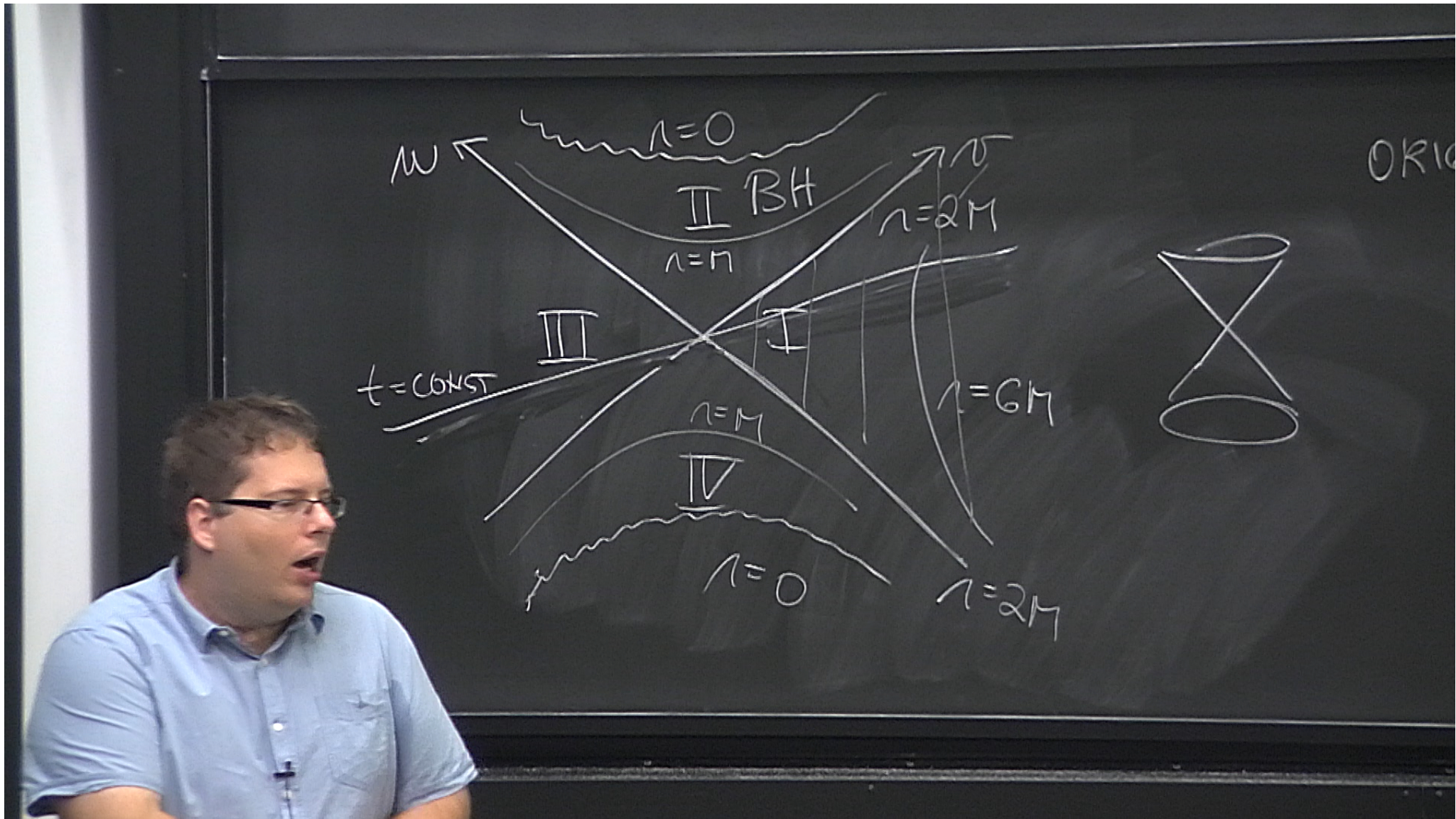
$$ds^2 = -(1+ax)^2 dt^2 + dx^2$$

• RINDLER HORIZON

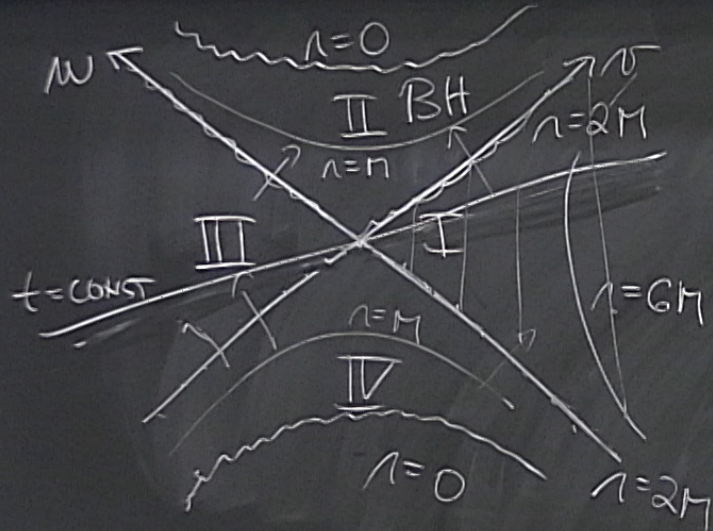
$$X = X_+ = -\frac{1}{a}$$

$$T \in (-\infty, \infty), \quad X \in (X_+, \infty)$$





→ FROZEN STAR → BLACK HOLE.



ORIGINAL SCHW. I

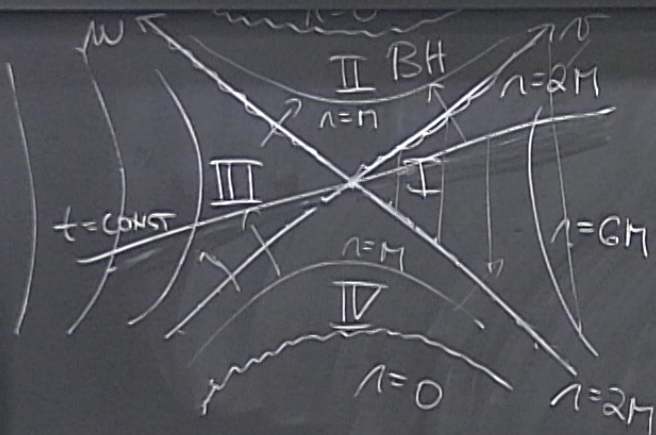
REGION II BH REGION

PERHAPS UNEXPECTED

REGION IV ... WHITE HOLE.

III

→ FROZEN STAR → BLACK HOLE,



ORIGINAL SCHW. I

REGION II BH REGION

PERHAPS UNEXPECTED

REGION IV ... WHITE HOLE

III ... ASYMPTOTICALLY FLAT REGION
(CANNOT COMMUNICATE WITH I)

• CONSIDER TWO STAT

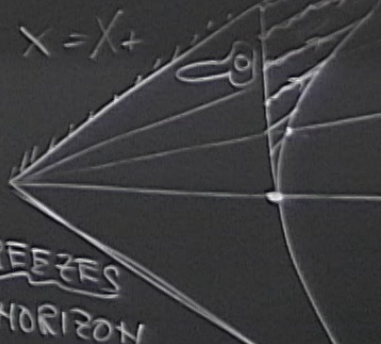
IN RINDLER SPACE

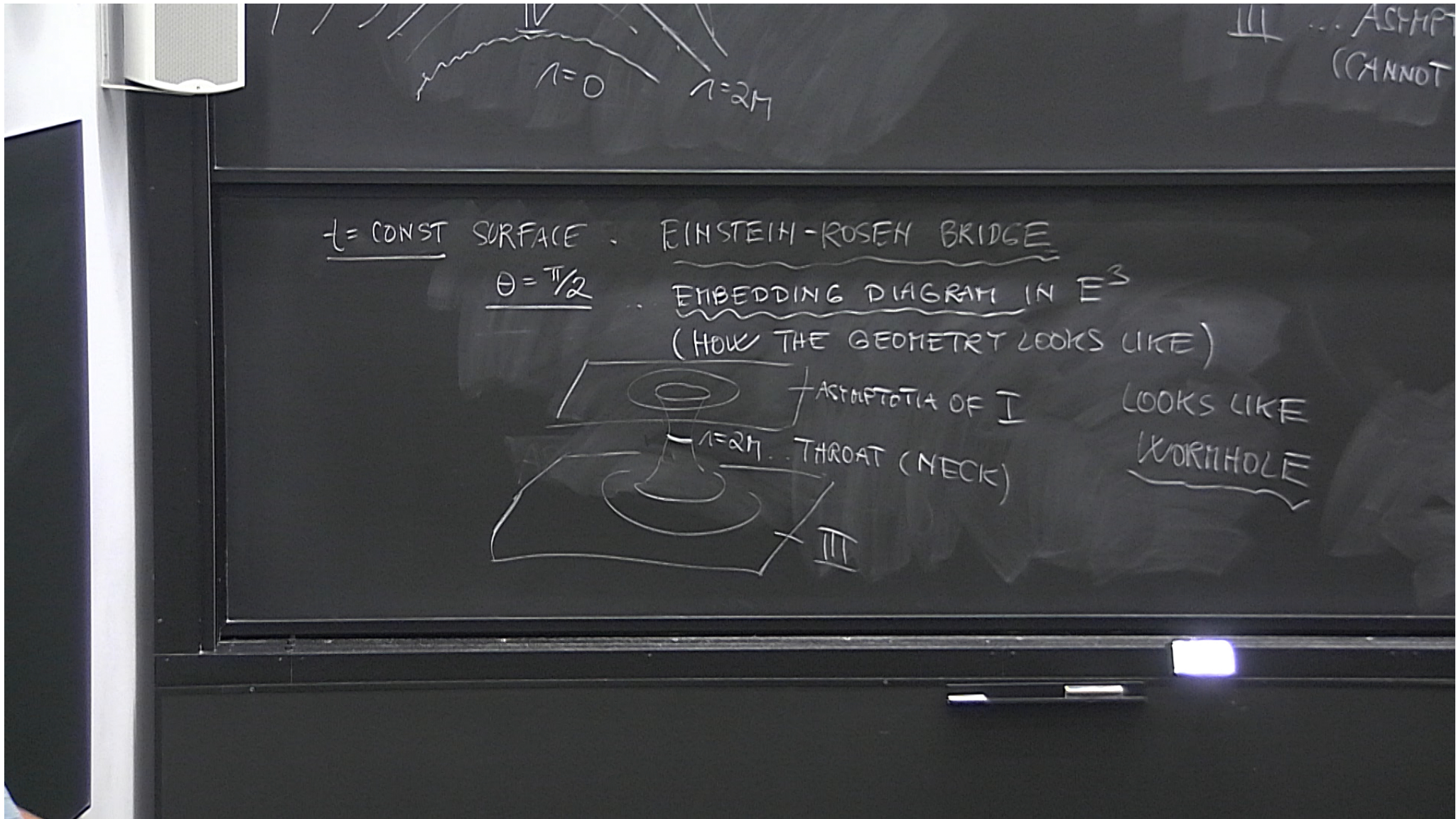
$$\frac{V_A}{V_B} = \frac{d\tau_B}{d\tau_A}$$

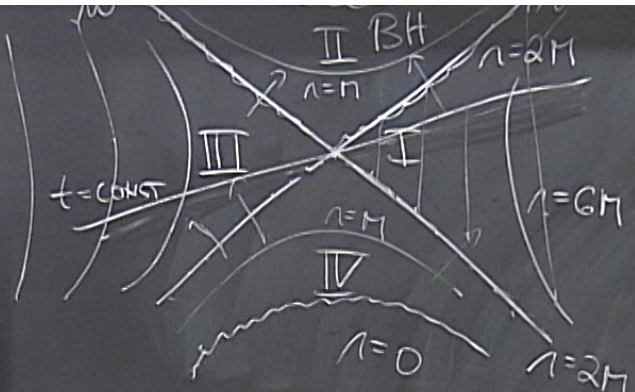
AS $X_B \rightarrow X_+$

$$\frac{1}{\sqrt{1+a^2 X_B^2}} = \frac{1}{\sqrt{1+a^2 X_A^2}}$$

SURFACE OF INFINITE



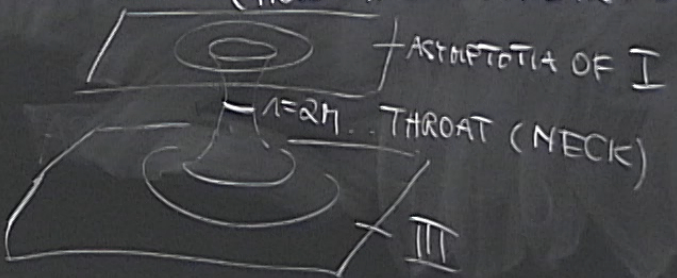




$t = \text{CONST}$
 $r = 0$
 $r = M$
 $r = 2M$
 REGION II BH REGION
 REGION IV ... WHITE HOLE
 REGION III ... ASYMPTOTICALLY FLAT
 (CANNOT COMMUNICATE)



$t = \text{CONST}$ SURFACE · EINSTEIN-ROSEN BRIDGE
 $\theta = \pi/2$ · EMBEDDING DIAGRAM IN E^3
 (HOW THE GEOMETRY LOOKS LIKE)



ASYMPTOTICALLY FLAT OF I
 $r = 2M$ THROAT (NECK)
 REGION III

LOOKS LIKE
WORMHOLE
 (CANNOT TRAVEL THROUGH)

(CANNOT COMMUNICATE WITH I)

AT PRESENT WORMHOLES LIKE THIS

AdS/CFT COR,

LENGTH OF NECK $\sim$ HOLOGRAPHIC
COMPLEXITY

KE)
LOOKS LIKE
WORMHOLE

NOT TRAVEL THROUGH)

NOTE ON ASTROPHYSICAL BHs.

- FORMED BY GRAVITATIONAL COLLAPSE
(REGIONS III & IV DO NOT EXIST)

- TYPICALLY ROTATE

(a, M) ← MASS
ROTATION PARAM

NO HAIR THEOREM

(UNIQUENESS THEOREM)

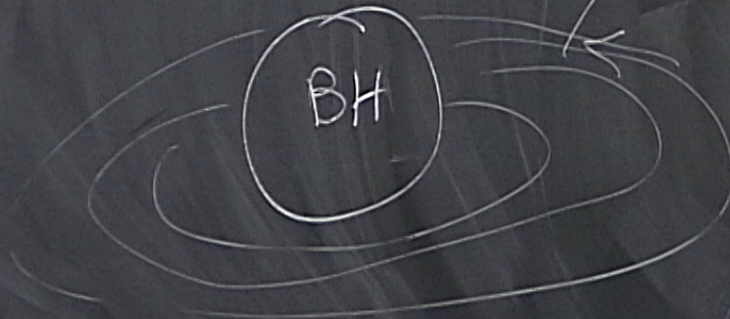
KERR (1963)

CHARGE IS NOT ASTROPHYSICALLY INTERESTING

• NOT ISOLATED ... ACCRETION MATTER DISCS

→ DUE FRICTION

← RADIATES A LOT

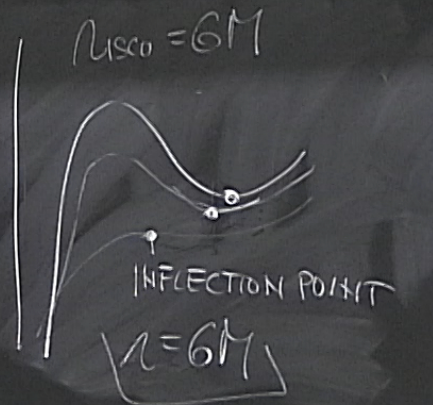


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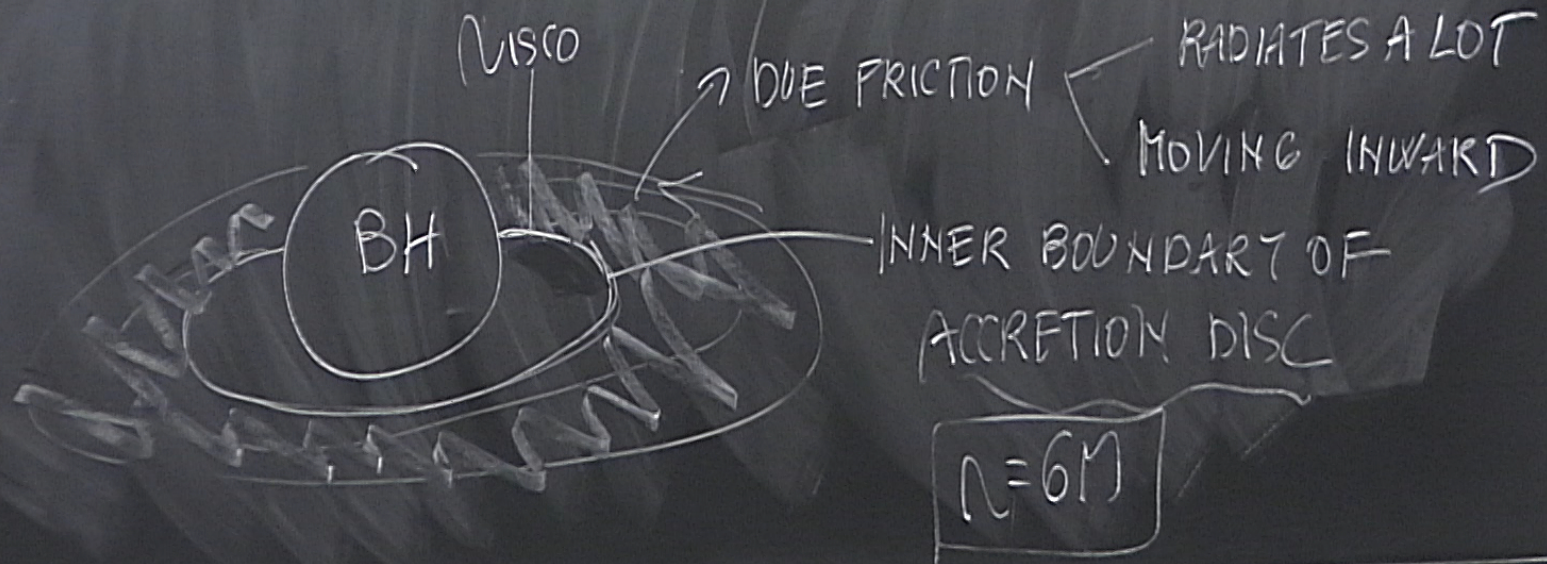
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