

Title: PSI 2018/2019 - Relativity - Lecture 11

Date: Sep 17, 2018 09:00 AM

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Abstract:

LAST TIME LINEARIZED GRAVITY

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + O(h^2)$$

PLUG TO MACHINERY...

$$R_{\mu\nu} = \frac{1}{2} (\partial_\sigma \partial_\nu h^\sigma_\mu + \partial_\sigma \partial_\mu h^\sigma_\nu - \partial_\sigma \partial_\nu h^\sigma_\sigma - \partial_\sigma \partial_\mu h^\sigma_\sigma)$$

$$G_{\mu\nu} = \frac{1}{2} (\partial_\sigma \partial_\nu h^\sigma_\mu + \partial_\sigma \partial_\mu h^\sigma_\nu - \partial_\sigma \partial_\nu h^\sigma_\sigma - \partial_\sigma \partial_\mu h^\sigma_\sigma - \eta_{\mu\nu} \partial_\sigma \partial_\lambda h^{\sigma\lambda} + \eta_{\mu\nu} \square h)$$

NEWTONIAN LIMIT

• NEXT-ORDER

SIMPLIFIES IF $\boxed{\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} h \eta_{\mu\nu}} \Leftrightarrow \boxed{h_{\mu\nu} = \bar{h}_{\mu\nu} - \frac{1}{2} \bar{h} \eta_{\mu\nu}}$

• GAUGE FREEDOM $x^\mu \rightarrow x^\mu - \xi^\mu$

$$h_{\mu\nu} \rightarrow h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

ALLOWS TO SET

$$\boxed{\partial_\mu \bar{h}^{\mu\nu} = 0}$$

LORENZ, HARMONIC

DE DONDER GAUGE

$\Rightarrow$ EE

$$\boxed{\square \bar{h}^{\mu\nu} = -16\pi T^{\mu\nu}}$$

NEWTONIAN LIMIT

$$T_{\mu\nu} = \begin{pmatrix} \rho & 0 \\ 0 & 0 \end{pmatrix}$$

$$\Rightarrow \begin{cases} \nabla^2 \phi = 4\pi G \rho \\ a = -\nabla \phi \end{cases}$$

"GRAVITOSTATICS"

NEXT-ORDER

$$T_{\mu\nu} = \begin{pmatrix} \rho & \vec{j} \\ \vec{j} & 0 \end{pmatrix} \quad \text{SLOWLY MOVING SOURCES}$$

$$\Rightarrow \begin{cases} \square A^\mu = -4\pi j^\mu \\ \partial_\mu A^\mu = 0 \end{cases}$$

STATIC FIELD

$$a = E - 4\pi \times B$$

$$A^\mu = -\frac{1}{4} \nabla^{\mu} \phi$$

GRAVITOMAGNETISM

L WAVES

EE, WITHOUT SOURCES

$$= 0 \Rightarrow \boxed{\square h^{\mu\nu} = 0} \text{ (EE)}$$

$\mu, \nu = 0, 1, 2, 3$

SEEK $\boxed{h_{\mu\nu} = \epsilon_{\mu\nu} e^{i k_\alpha x^\alpha}}$

$\uparrow$ POLARIZATION TENSOR

PLUG TO (EE)

$$\boxed{k^2 = 0}$$

"GRAVITY WAVES PROPAGATE AT
THE SPEED OF LIGHT"

5) APPLICATIONS

a) GRAVITATIONAL WAVES

LINEARIZED EE, WITHOUT SOURCES

$$\square \overline{h}^{\mu\nu} = 0 \Rightarrow \boxed{\square h^{\mu\nu} = 0} \text{ (EE)}$$

$\mu, \nu \neq 0$
 $\partial_\alpha \partial_\beta$

SEEK $\boxed{h\mu}$

PLUG TO

"GRAV

ALLOWS TO SET

$$\partial_\mu \bar{h}^{\mu\nu} = 0$$

LORENZ, HARTONIC

DE DONDER GAUGE

$\Rightarrow$ EE

$$\square \bar{h}^{\mu\nu} = -16\pi T^{\mu\nu}$$

$$\nabla^\mu \alpha \beta g^{\alpha\beta} = 0$$

$$\nabla^2 \chi^\mu = 0$$

$$h_{\mu\nu} = \epsilon_{\mu\nu} e^{i k_\alpha x^\alpha}$$

$\uparrow$ POLARIZATION TENSOR

TO (EE)

$$k^2 = 0$$

GRAVITY WAVES PROPAGATE AT

SEEK $\boxed{h_{\mu\nu} = \epsilon_{\mu\nu} e^{i k_\alpha x^\alpha}}$

$\uparrow$ POLARIZATION TENSOR (10 COMPONENTS)

PLUG TO (EE) $\boxed{k^2 = 0}$

"GRAVITY WAVES PROPAGATE AT
THE SPEED OF LIGHT"

- FIXING POLARIZATION TENSOR

(WE WANT TO SHOW THAT ONLY 2 TRUE DOF)

- HARMONIC GAUGE:

$$k^\mu \epsilon_{\mu\nu} - \frac{1}{2} \epsilon k_\nu = 0$$

RESTRICTS 4 COMPTS OF $\epsilon_{\mu\nu}$.

- RESIDUAL FREEDOM

$$X^\mu \rightarrow X^\mu - \xi^\mu \quad \text{AS LONG AS } \nabla^2 X^\mu = 0$$

$$\Rightarrow \nabla^2 \xi^\mu = 0$$

$$\cdot \xi^\mu = \tilde{\xi}^\mu \quad \text{with } i x^\alpha \partial_\alpha$$

$\nabla^2 \xi^\mu = 0$ IS AUTOMATICALLY SATISFIED FOR ANY $\tilde{\xi}^\mu$
 $\Rightarrow$ STILL HAVE 4 FUNCTIONS AT OUR DISPOSE

$$\epsilon_{\mu\nu} \rightarrow \epsilon_{\mu\nu} + i \partial_\mu \tilde{\xi}_\nu + i \partial_\nu \tilde{\xi}_\mu$$

CAN ELIMINATE 4 MORE $\epsilon_{\mu\nu}$ 'S

$$\cdot \xi^\mu = \tilde{\xi}^\mu \quad e^{i x^\alpha \partial_\alpha}$$

$\nabla^2 \xi^\mu = 0$ IS AUTOMATICALLY SATISFIED FOR ANY $\tilde{\xi}^\mu$
 $\Rightarrow$ STILL HAVE 4 FUNCTIONS AT OUR DISPOSE

$$\epsilon_{\mu\nu} \rightarrow \epsilon_{\mu\nu} + i \partial_\mu \tilde{\xi}_\nu + i \partial_\nu \tilde{\xi}_\mu$$

CAN ELIMINATE 4 MORE $\epsilon_{\mu\nu}$ 'S

$\Rightarrow$ 2 DOF

BETTER WAY

LIGHT CONE GAUGE
 SVT DECOMP.

SIMPLIFIES IF $\boxed{\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} h \gamma_{\mu\nu}} \Leftrightarrow \boxed{h_{\mu\nu} = \bar{h}_{\mu\nu} - \frac{1}{2} \bar{h} \gamma_{\mu\nu}}$

$\boxed{h = -\bar{h}}$

• GAUGE FREEDOM $X^\mu \rightarrow X^\mu - \xi^\mu$

$h_{\mu\nu} \rightarrow h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$

ALLOWS TO SET

$\boxed{\partial_\mu \bar{h}^{\mu\nu} = 0}$

LORENZ, HARMONIC

DE DONDER GAUGE

$\Rightarrow$ EE

$\boxed{\square \bar{h}^{\mu\nu} = -16\pi T^{\mu\nu}}$

$\boxed{\int d^3x g^{\alpha\beta} = 0}$

$\boxed{\nabla^2 X^\mu = 0}$

SPEC: CONSIDER A WAVE PROPAGATING IN X-DIRECTION

$$k^\mu = (k, k, 0, 0)$$

$$\epsilon_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \epsilon_+ & \epsilon_- \\ 0 & 0 & \epsilon_- & -\epsilon_+ \end{pmatrix}$$

TRANSVERSE WAVE

TRACELESS

TRANSVERSE-TRACELESS
GAUGE

(ONLY IN VACUUM)

DIRECTION

AVE

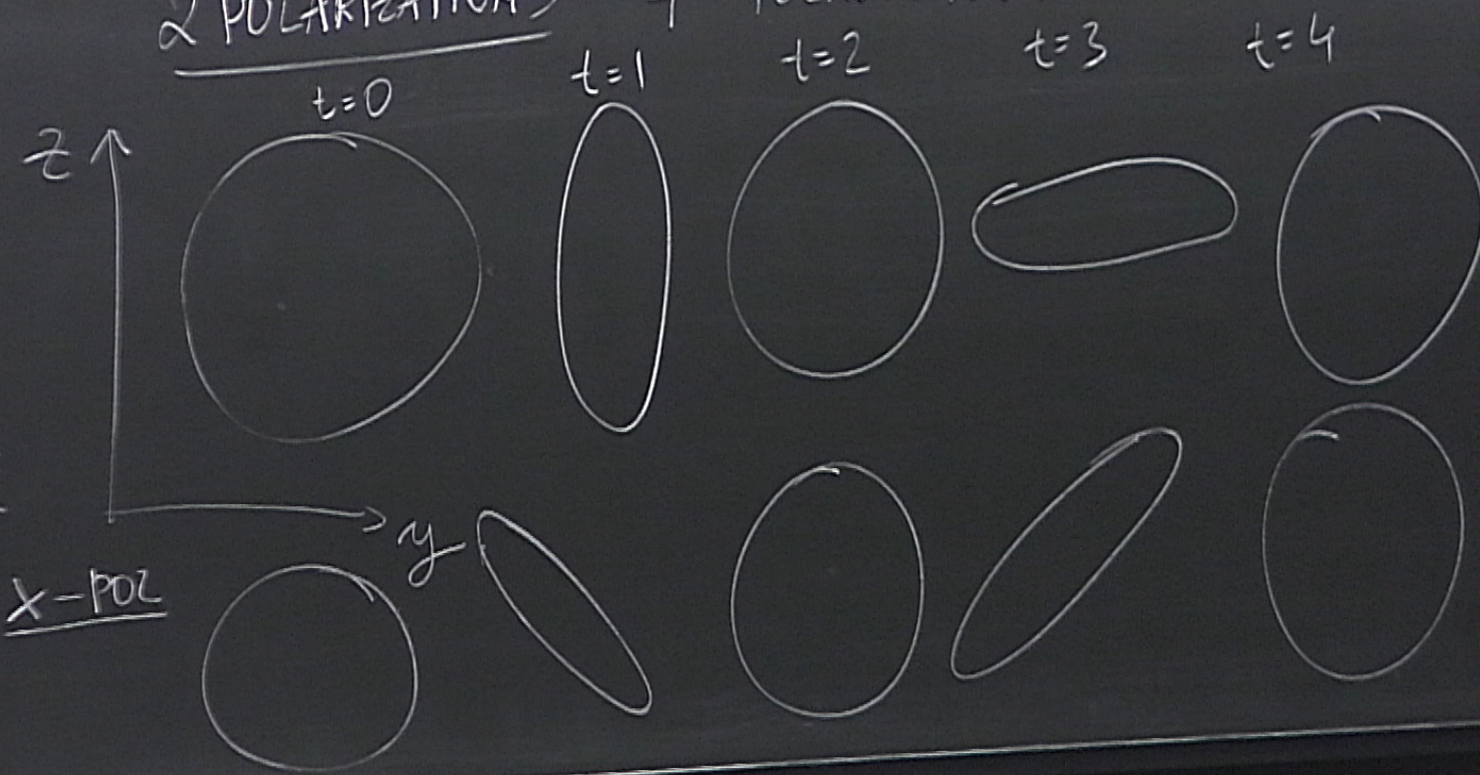
TRACELESS

GAUGE

ON)

2 POLARIZATIONS

+ POLARIZATION



$$h_{\mu\nu} = \epsilon_{\mu\nu} e^{i k_\alpha x^\alpha}$$

↑
POLARIZATION TENSOR (10 COMPONENTS)

$$G_{\mu\nu} = 0$$

↑
MATTER

UG TO (EE)

$$k^2 = 0$$

"GRAVITY WAVES PROPAGATE AT
THE SPEED OF LIGHT"

PROOF:

CENTER "JUST STAYS"

$$\dot{X}^M = (1, 0, 0, 0)$$

PARTICLES MOVE ON GEODESICS

WE OBSERVE GEOD. DEVIATIONS

$$\left[\frac{D^2 \delta x^\mu}{d\tau^2} = R^\mu_{\alpha\beta\gamma} \dot{x}^\alpha \dot{x}^\beta \delta x^\gamma \right]$$

$$\frac{\partial^2 \delta x^\mu}{\partial t^2} = R^\mu_{ttv} \delta x^v$$

$$R_{\mu tt v} = \frac{1}{2} \left(\partial_t^2 h_{\mu v} + \cancel{\partial_\nu \partial_\mu h_{tt}} - \cancel{\partial_\nu \partial_t h_{\mu t}} - \cancel{\partial_\mu \partial_t h_{\nu t}} \right) = \frac{1}{2} \partial_t^2 h_{\mu v}$$

$$\boxed{\frac{\partial^2 \delta x^\mu}{\partial t^2} = \frac{1}{2} \delta x^v \frac{\partial^2}{\partial t^2} h^\mu_v}$$

$$\delta x^{\mu} = (0, 0, \delta y, \delta z)$$

• + POLARIZATION . ($\epsilon_x = 0$)

$$\frac{\partial^2 \delta y}{\partial t^2} = \frac{1}{2} \delta y \frac{\partial^2}{\partial t^2} \left(\epsilon + \ell^{i2x} \right)$$

$$\frac{\partial^2 \delta z}{\partial t^2} = \frac{1}{2} \delta z \frac{\partial^2}{\partial t^2} \left(-\epsilon + \ell^{i2x} \right)$$

$\Rightarrow$ SOLUTION (TO LEADING ORDER)

$$\begin{aligned}\delta y &= \left(1 + \frac{1}{2}\varepsilon + e^{i2x}\right) \delta y(0) \\ \delta z &= \left(1 - \frac{1}{2}\varepsilon + e^{i2x}\right) \delta z(0)\end{aligned}$$

WHY SPIN 2 ? (GRAVITON)

SPIN S IS RELATED TO THE ANGLE θ
UNDER WHICH THE POLARIZATIONS ARE
INVARIANT BY

$$S = \frac{360^\circ}{\theta}$$

ANGLE Θ
ONS ARE

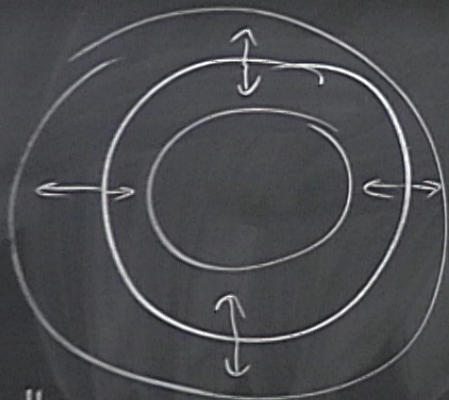
FOR GRAVITY $\Theta = 180^\circ \Rightarrow \underline{S=2}$

FOR EM $\Theta = 360^\circ \Rightarrow \underline{S=1}$

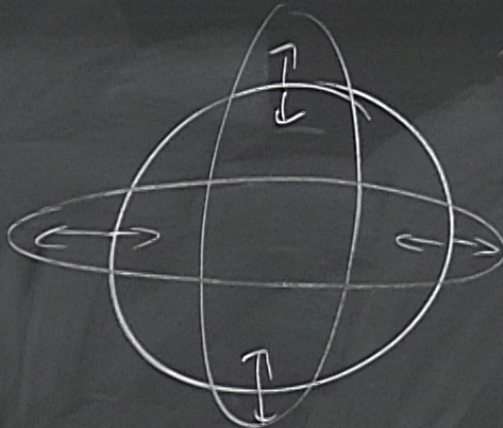
SPINOR $\Theta = 720^\circ \Rightarrow \underline{S=\frac{1}{2}}$

$\frac{0^\circ}{\Theta}$

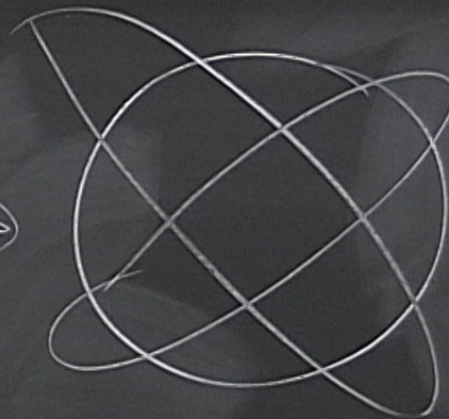
REMARK, VIBRATIONS OF CLOSED STRING



"BREATHING MODE"
($S=0$)



SPIN 2 (+, x)



→ SCALAR (DILATON) + GRAVITON
↓
NOT OBSERVED SO FAR !

$$h_{\mu\nu} = (h, h, 0, 0)$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

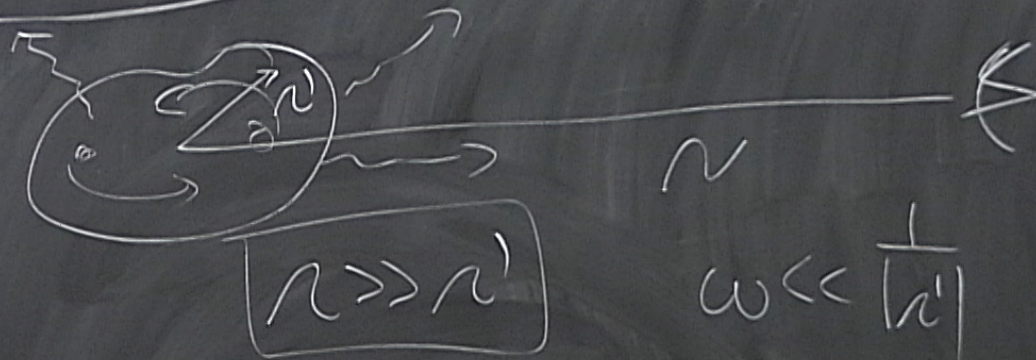
TRANSVERSE WAVE

TRACELESS

$z \uparrow$

$t=0$

GRAVITATIONAL RADIATION FROM AN ISOLATED SYSTEM



$$\omega < \frac{1}{|r'|}$$

SLOW MOTION

ISOLATED SYSTEM

WE HAVE TO SOLVE

$$\square \bar{h}^{\mu\nu} = -16\pi G T^{\mu\nu} \quad \text{SOURCE IMPORTANT!}^0$$

SPECIAL SOLUTION:

$$\bar{h}^{\mu\nu}(t, \vec{r}) = 4G \int d^3r' \frac{T^{\mu\nu}(t_R, \vec{r}')}{|\vec{r} - \vec{r}'|}, \quad t_R = t - |\vec{r} - \vec{r}'|$$



PROOF:

CENTER "JUST STAYS"

$$\frac{D^2 \delta x^\mu}{d\tau^2} = R^\mu{}_{\alpha\beta\gamma} \dot{x}^\alpha \dot{x}^\beta \delta x^\gamma$$

$$\begin{aligned} \tilde{h}_{\mu\nu}(\omega, \vec{n}) &= \frac{1}{\sqrt{2\pi}} \int dt e^{i\omega t} h_{\mu\nu}(t, \vec{n}) = \frac{4G}{\sqrt{2\pi}} \int dt d^3n' e^{i\omega t} \frac{T_{\mu\nu}(t_R, \vec{n}')}{|\vec{n} - \vec{n}'|} \\ &= \frac{4G}{\sqrt{2\pi}} \int dt_R d^3n' e^{i\omega t_R + i\omega |\vec{n} - \vec{n}'|} \frac{T_{\mu\nu}(t_R, \vec{n}')}{|\vec{n} - \vec{n}'|} \\ &= 4G \int d^3n' e^{i\omega |\vec{n} - \vec{n}'|} \frac{\tilde{T}_{\mu\nu}(\omega, \vec{n}')}{|\vec{n} - \vec{n}'|} \approx \frac{4G}{N} e^{i\omega r} \int d^3n' \tilde{T}_{\mu\nu}(\omega, \vec{n}') \end{aligned}$$

NOTE LORENZ GAUGE $\partial_\mu \bar{h}^{\mu\nu} = 0$

$$\boxed{\tilde{h}^{\mu\nu} = \frac{1}{\omega} \partial_i \tilde{h}^{i\mu\nu}}$$

$\mu=j$ $\tilde{h}^{tj} = \frac{1}{\omega} \partial_i \tilde{h}^{ij}$

$$\tilde{h}^{tt} = -\frac{1}{\omega^2} \partial_i \partial_j \tilde{h}^{ij}$$

ONLY $\tilde{T}_{ij}$ ARE INDEPENDENT

$$\begin{aligned}
 \text{KAT} \quad \int d^3x' \tilde{T}^{ij}(\omega, x') &= \int \underbrace{\partial_k' (x'^i \tilde{T}^{kj})}_{\substack{0 \\ \text{(BOUNDARY TERM)}}} - \int x'^i (\partial_k' \tilde{T}^{kj}) \\
 &= i\omega \int x'^i T^{tj} d^3x' \\
 &= -\frac{\omega^2}{2} \underbrace{\int x'^i x'^j T^{tt} d^3x'}_{\substack{\text{QUADRUPOLE MOMENT} \\ I_{ij}}}
 \end{aligned}$$

$$\tilde{h}_{ij}(\omega, \vec{r}) = -2G\omega^2 \frac{e^{i\omega r}}{r} \tilde{I}_{ij}(\omega)$$

$$\boxed{h_{ij}(t, \vec{r}) = \frac{2G}{r} \left. \frac{d^2 I_{ij}}{dt^2} \right|_{t=t_R}}$$

QUADRUPOLE FORMULA