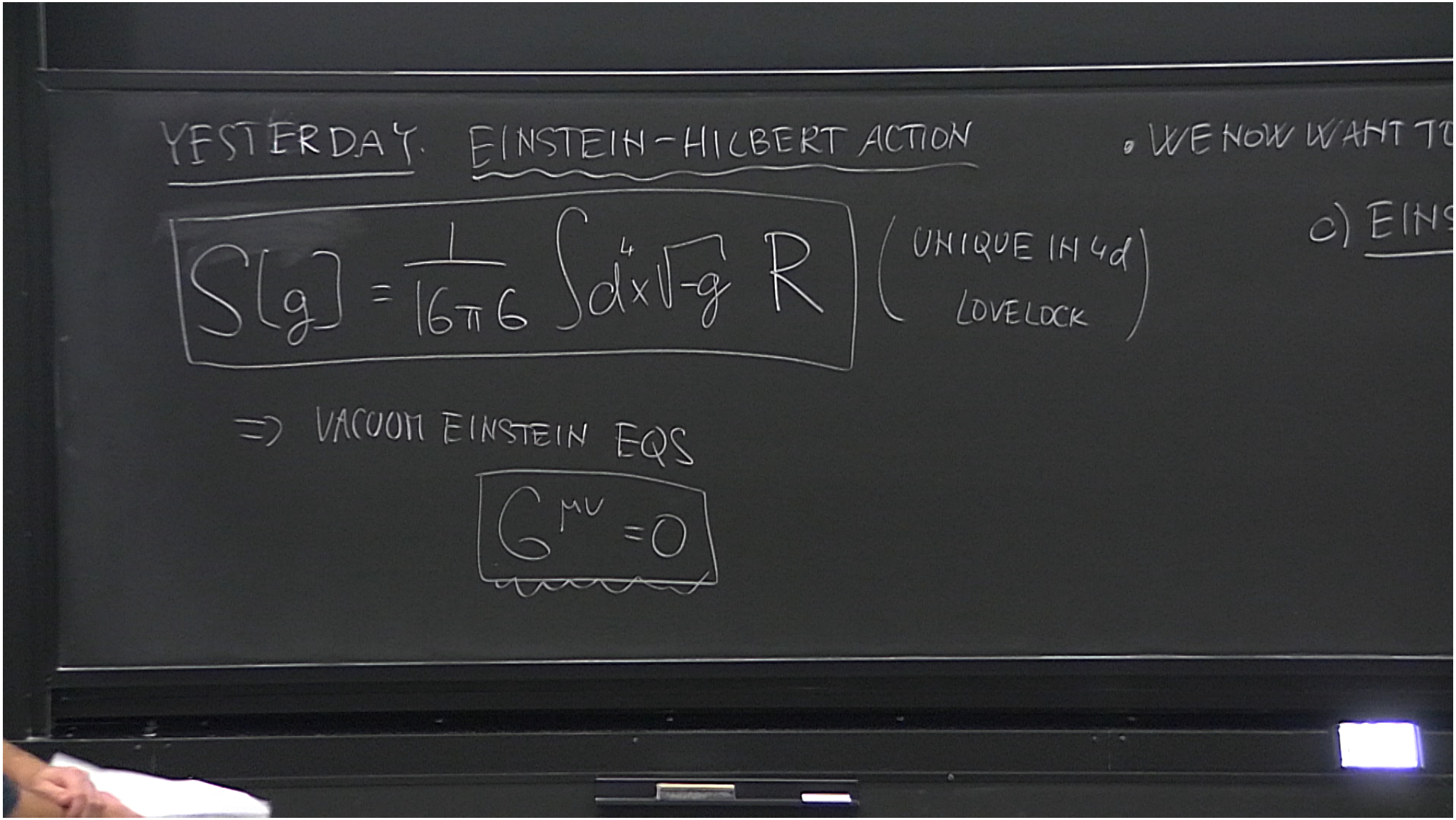
Title: PSI 2018/2019 - Relativity - Lecture 10

Date: Sep 14, 2018 09:00 AM

URL: http://pirsa.org/18090009

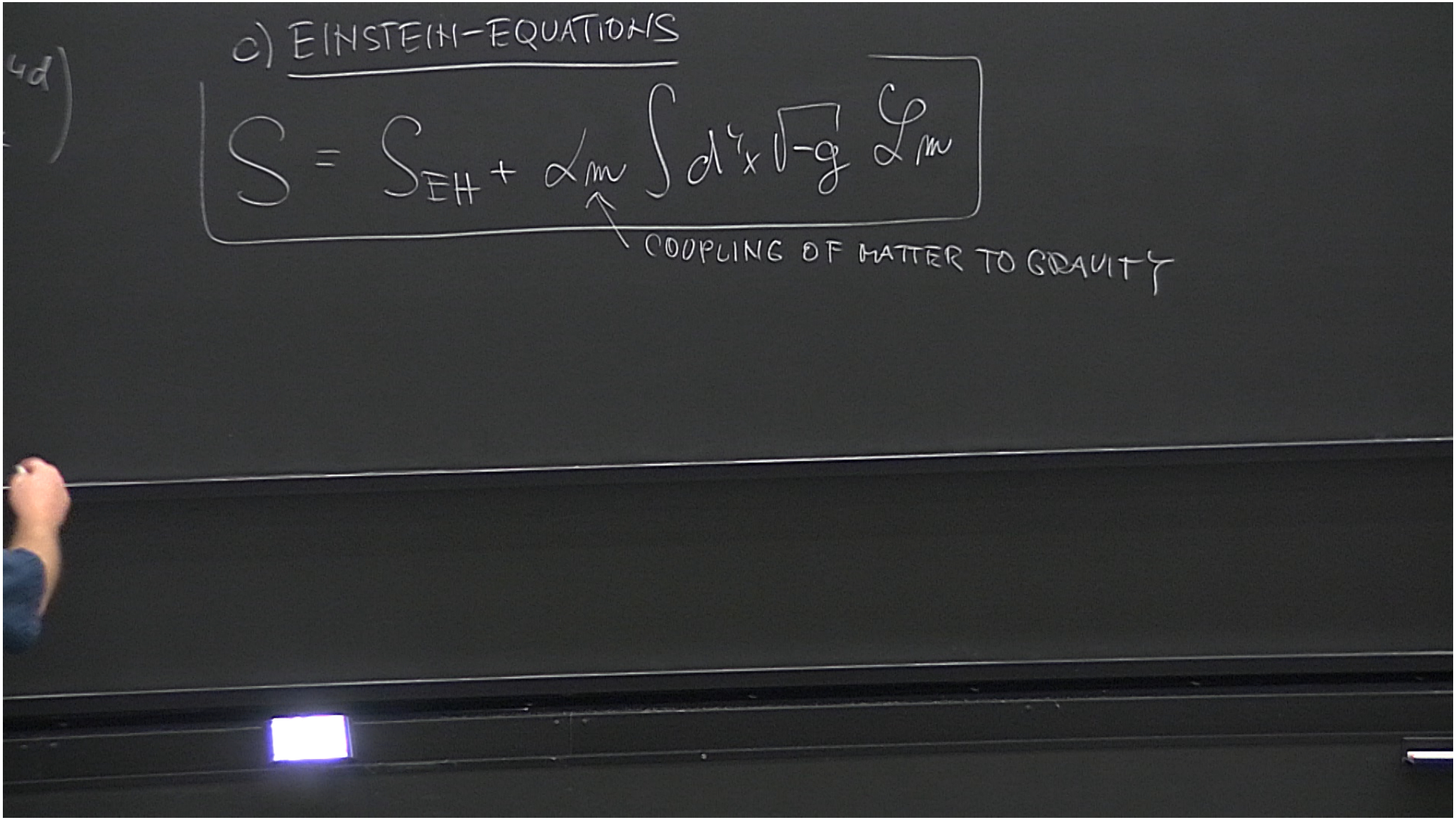
Abstract:

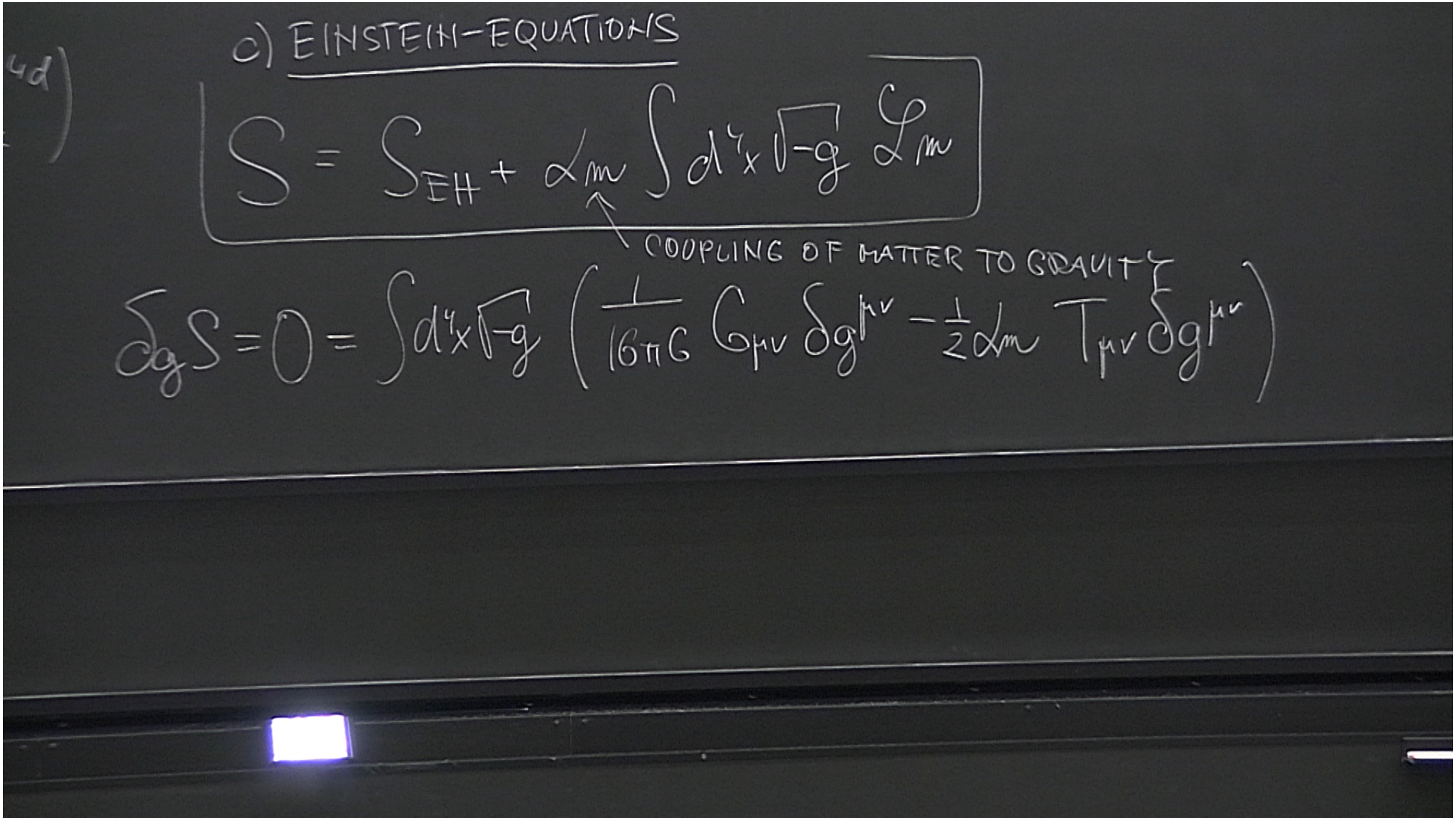

YESTERDAY. EINSTEIN-HILBERT ACTION
$S[g] = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,R$
(UNIQUE IN 4d LOVELOCK)
⇒ VACUUM EINSTEIN EQS
$G^{\mu\nu} = 0$

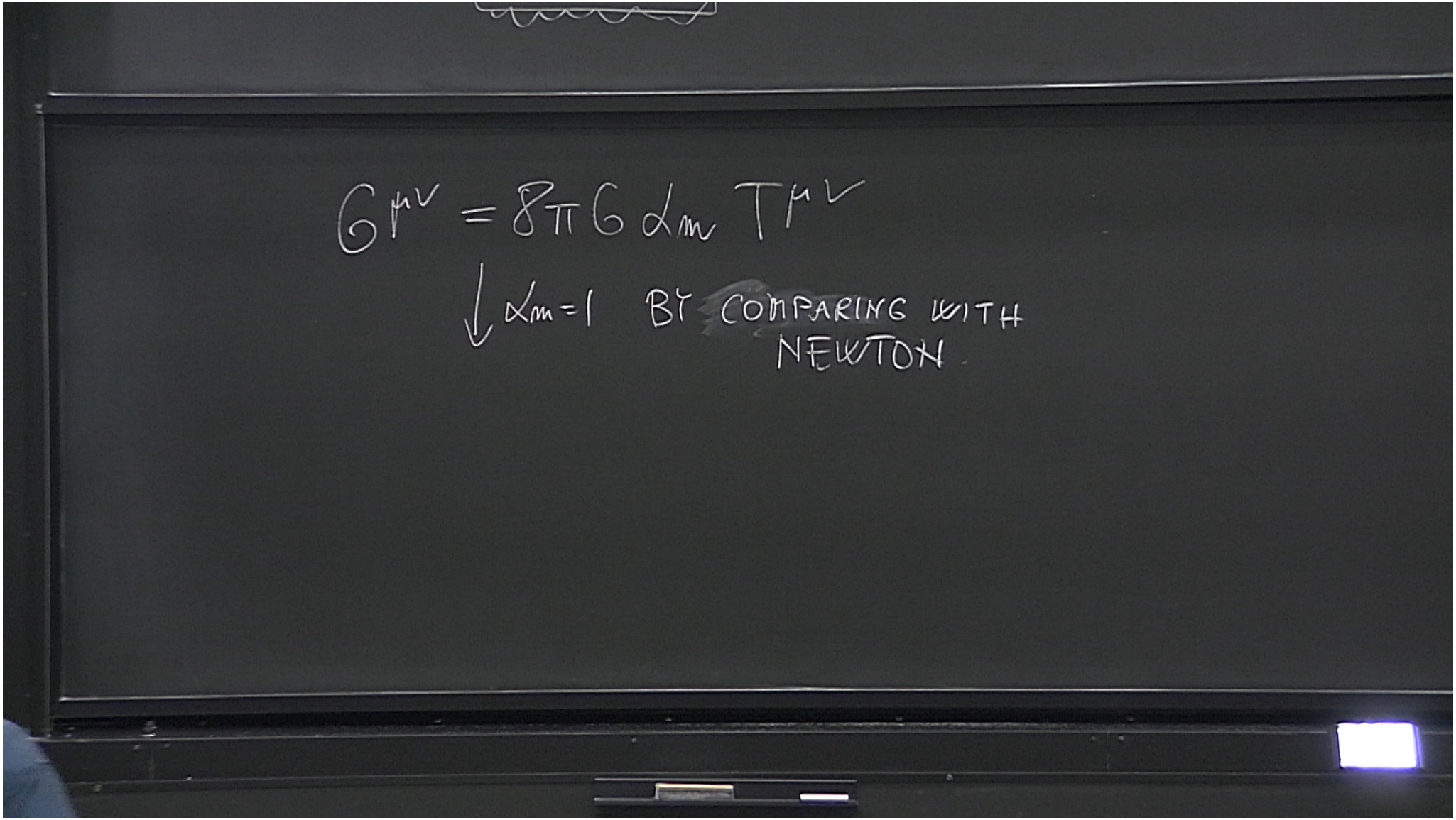
- WE NOW WANT TO COUPLE TO MATTER

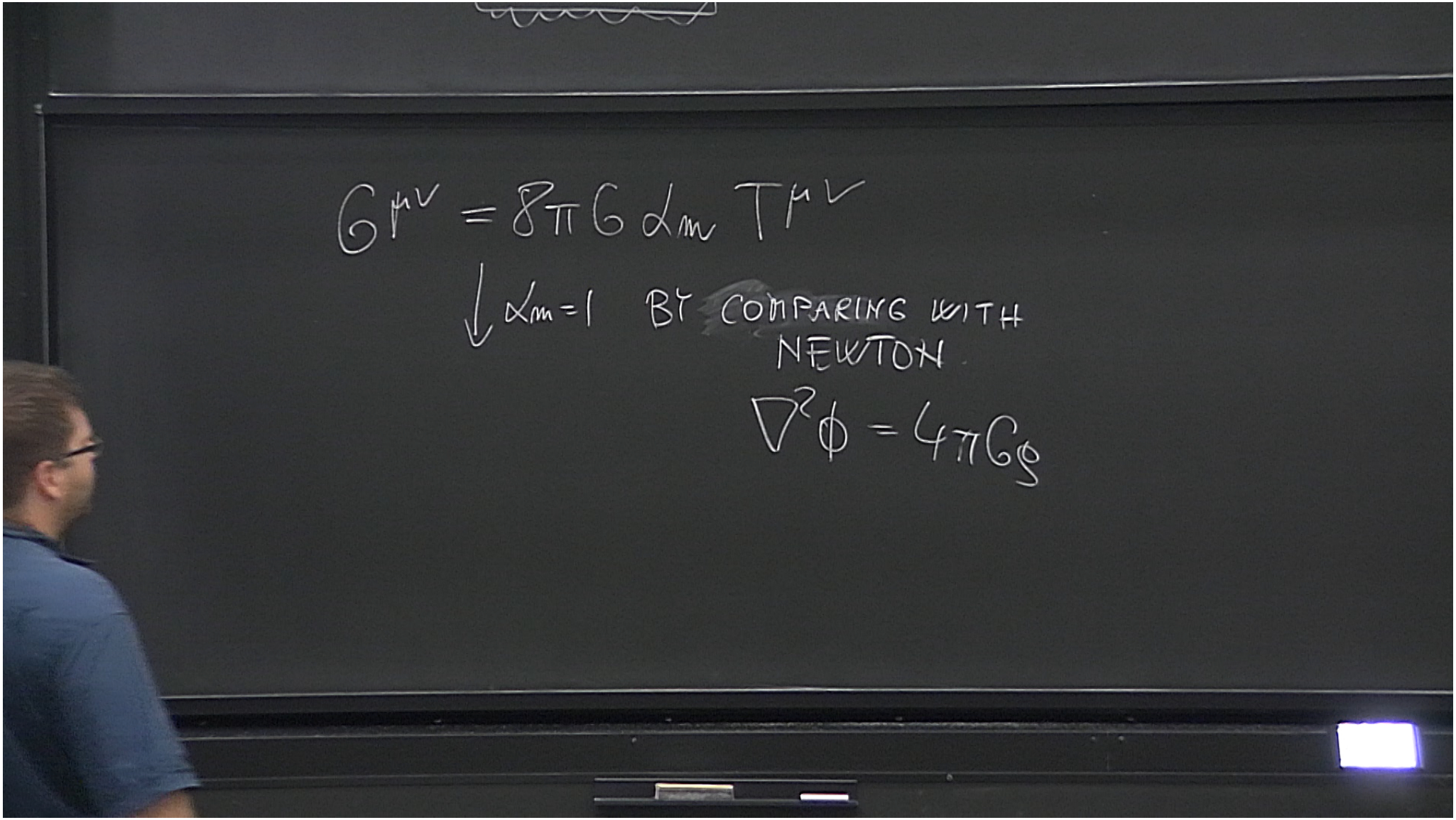
c) EINSTEIN-EQUATIONS

$$S = S_{EH} + \alpha_m \int d^4x \sqrt{-g}\, \mathcal{L}_m$$


c) EINSTEIN-EQUATIONS
$S = S_{EH} + \alpha_m \int d^4x \sqrt{-g}\, \mathcal{L}_m$
COUPLING OF MATTER TO GRAVITY


c) EINSTEIN-EQUATIONS
$S = S_{EH} + \alpha_m \int d^4x \sqrt{-g}\, \mathcal{L}_m$
COUPLING OF MATTER TO GRAVITY
$\delta_g S = 0 = \int d^4x \sqrt{-g} \left(\frac{1}{16\pi G} G_{\mu\nu} \delta g^{\mu\nu} - \frac{1}{2} \alpha_m T_{\mu\nu} \delta g^{\mu\nu} \right)$


$G^{\mu\nu} = 8\pi G \alpha_m T^{\mu\nu}$
$\alpha_m = 1$ BY COMPARING WITH NEWTON


$G^{\mu\nu} = 8\pi G \alpha_m T^{\mu\nu}$
$\alpha_m = 1$ BY COMPARING WITH NEWTON
$\nabla^2\phi = 4\pi G\rho$

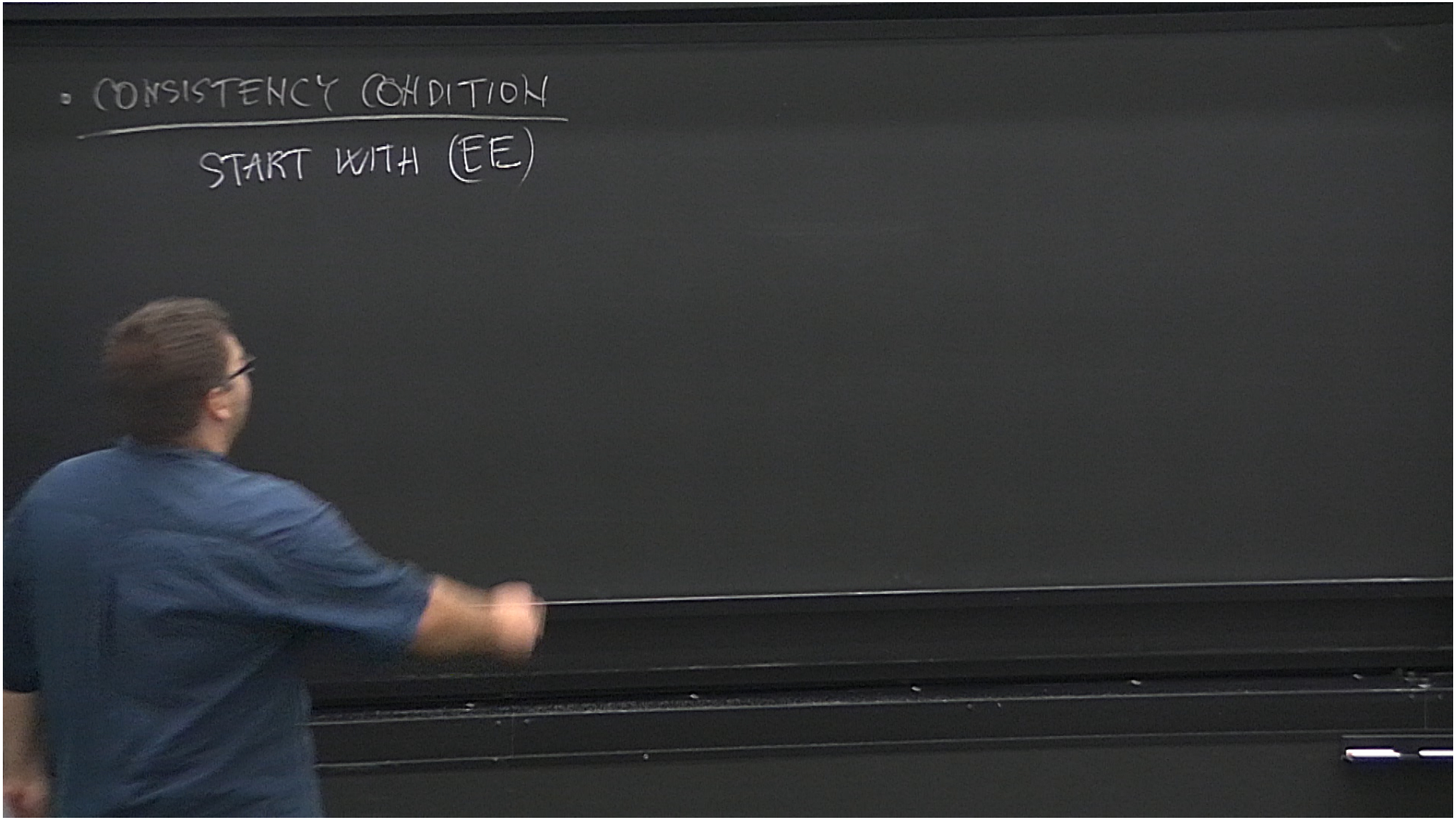
$$G^{\mu\nu} = 8\pi G \alpha_m T^{\mu\nu}$$

$\downarrow \alpha_m = 1$ BY COMPARING WITH NEWTON

$$\nabla^2 \phi = 4\pi G \rho$$

$$\boxed{G^{\mu\nu} = 8\pi G T^{\mu\nu}}$$

EINSTEIN
HILBERT (1915)


CONSISTENCY CONDITION
START WITH (EE)

- CONSISTENCY CONDITION

START WITH (EE) & APPLY D_μ (COVARIANT DIVERGENCE)

L.H.S: $D_\mu G^{\mu\nu} = 0$ IDENTICALLY

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L.H.S: $D_\mu G^{\mu\nu} = 0$ IDENTICALLY

$\Rightarrow$ R.H.S BETTER ALSO BE ZERO

$$\boxed{D_\mu T^{\mu\nu} = 0}$$ CONSISTENCY CONDITION

- CONSISTENCY CONDITION

START WITH (EE) & APPLY D_μ (COVARIANT DIVERGENCE)

L.H.S: $D_\mu G^{\mu\nu} = 0$ IDENTICALLY

$\Rightarrow$ R.H.S BETTER ALSO BE ZERO

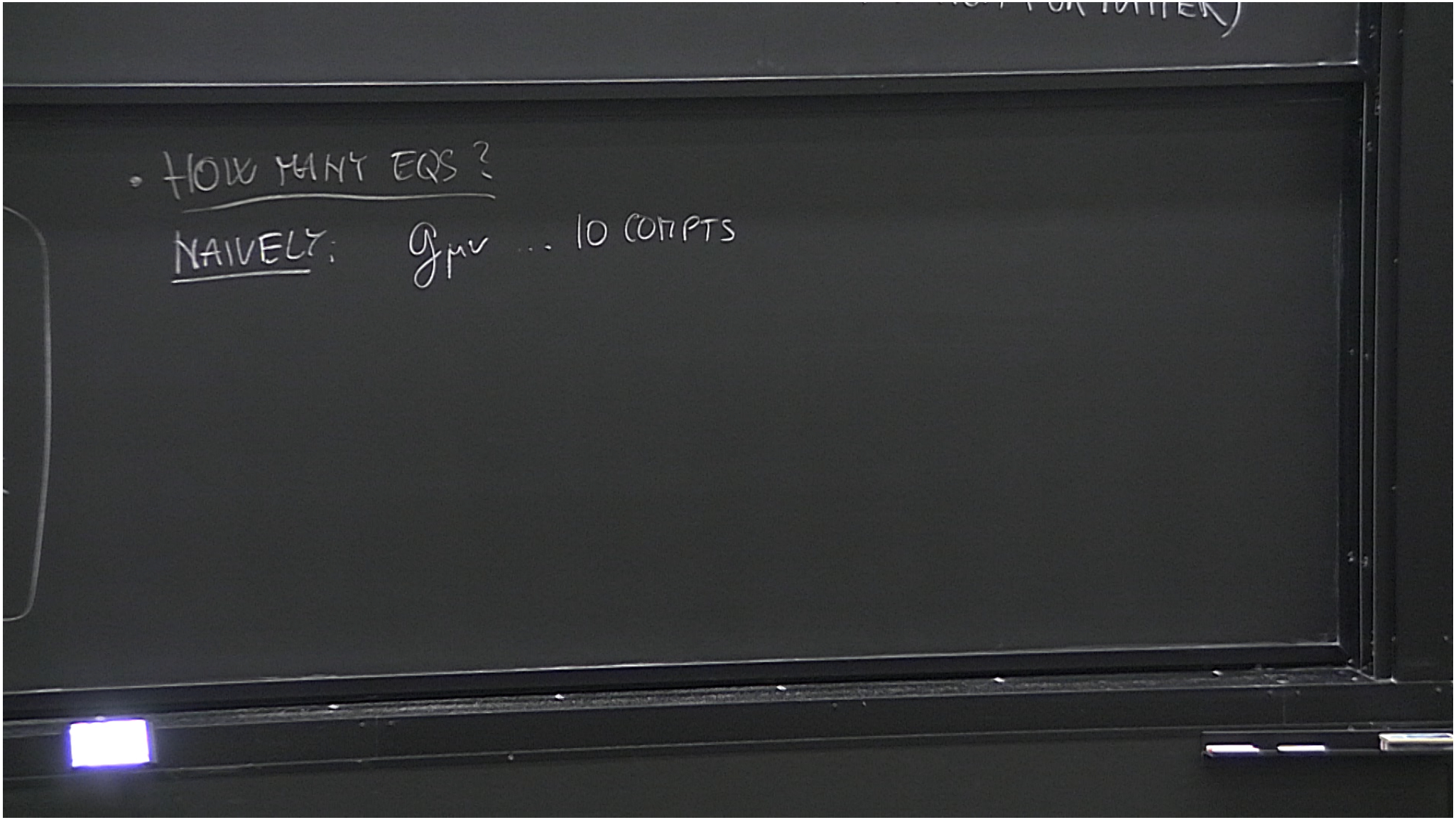
$$\boxed{D_\mu T^{\mu\nu} = 0}$$

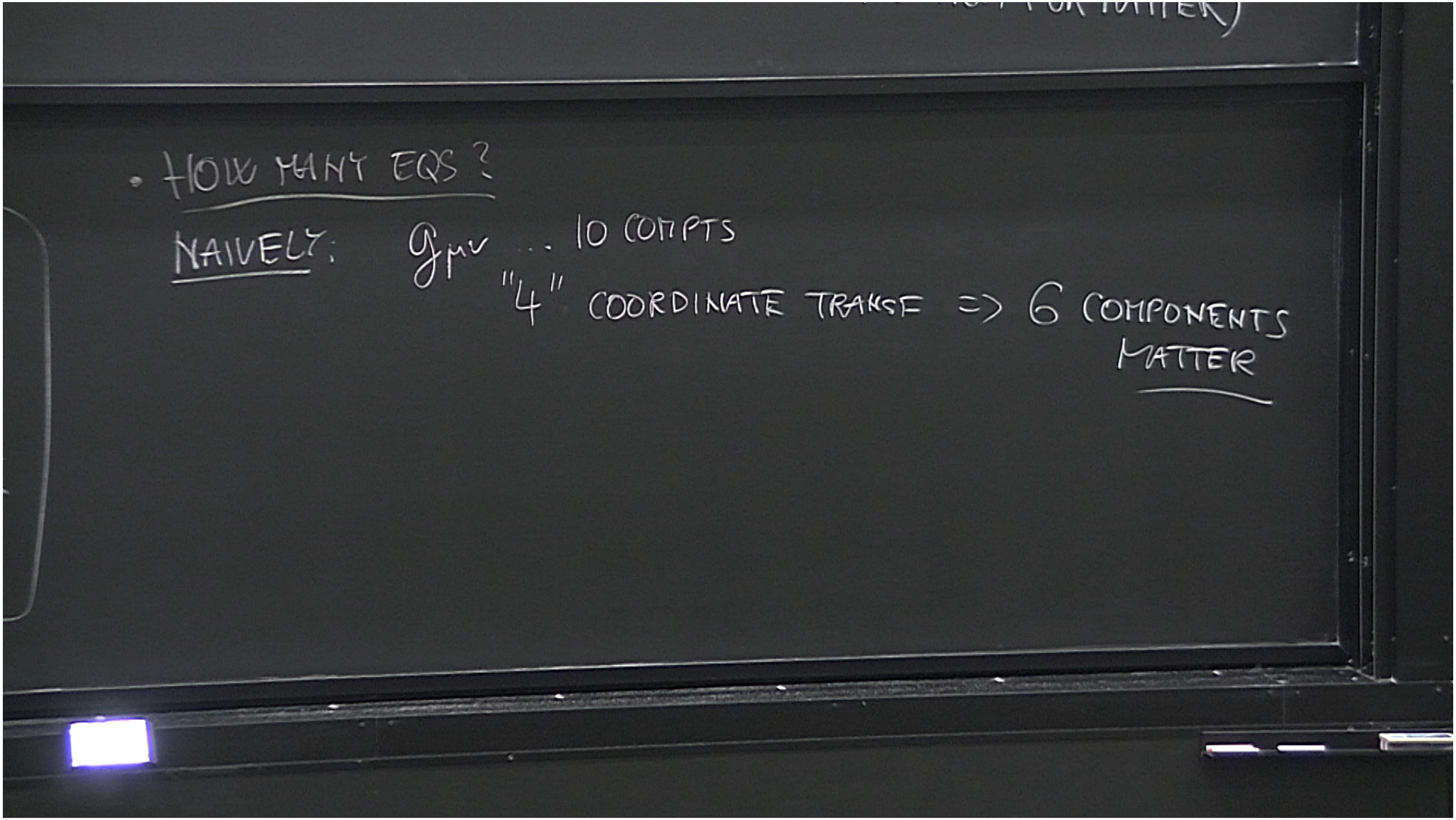
CONSISTENCY CONDITION

(THIS IS EQ. OF MOTION FOR MATTER)

(EE) PLAY A DOUBLE AGENT ROLE:

THEY TELL SPACETIME HOW TO CURVE IN THE PRESENCE OF MATTER (FIELD EQ ROLE)
& TELL MATTER HOW TO MOVE IN CURVED SPACE (EQ OF MOTION ROLE)


• HOW MANY EQS ?
NAIVELY: $g_{\mu\nu}$ .. 10 COMPTS

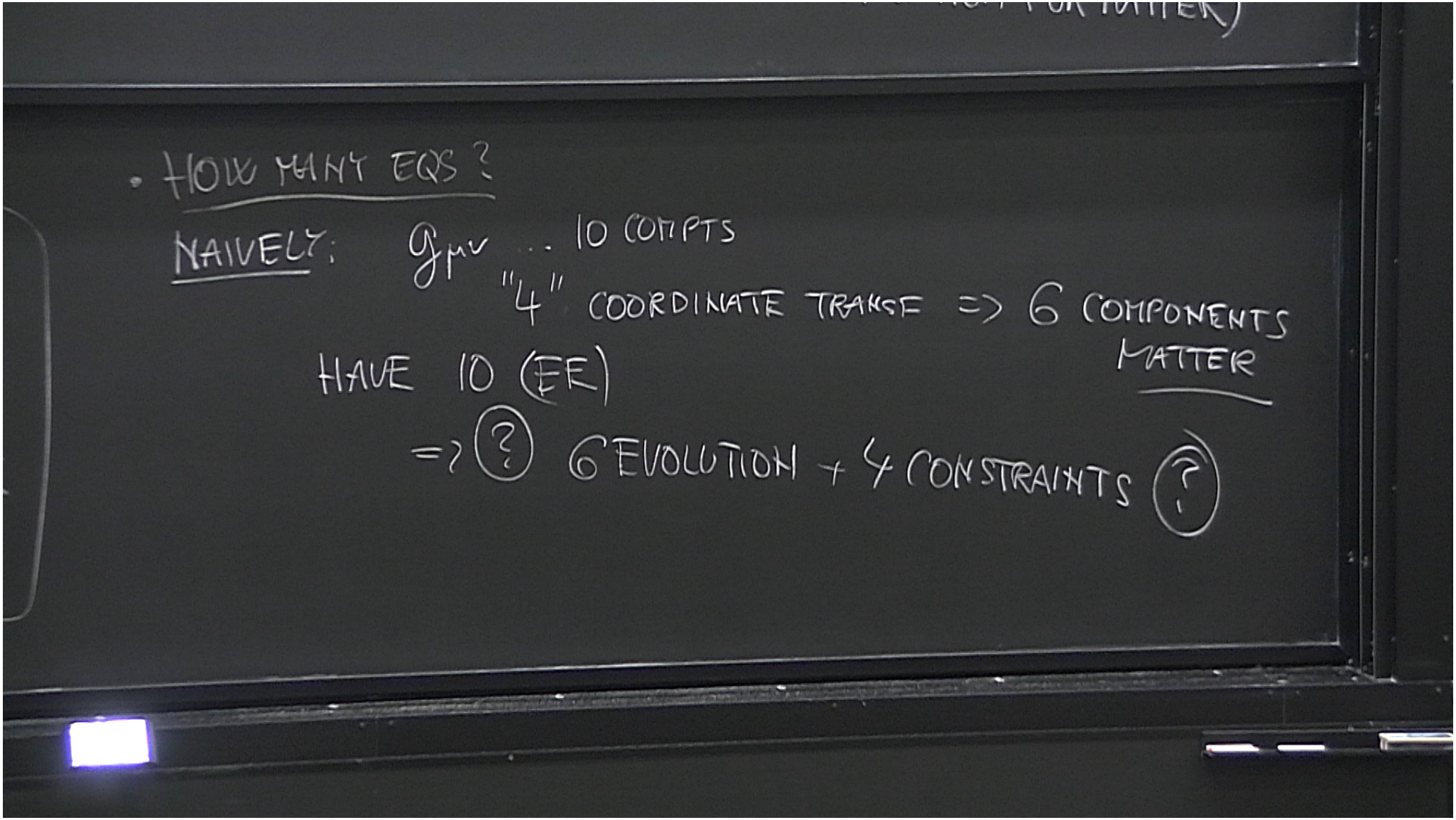

• HOW MANY EQS ?
NAIVELY: $g_{\mu\nu}$ .. 10 COMPTS
"4" COORDINATE TRANSF => 6 COMPONENTS
MATTER

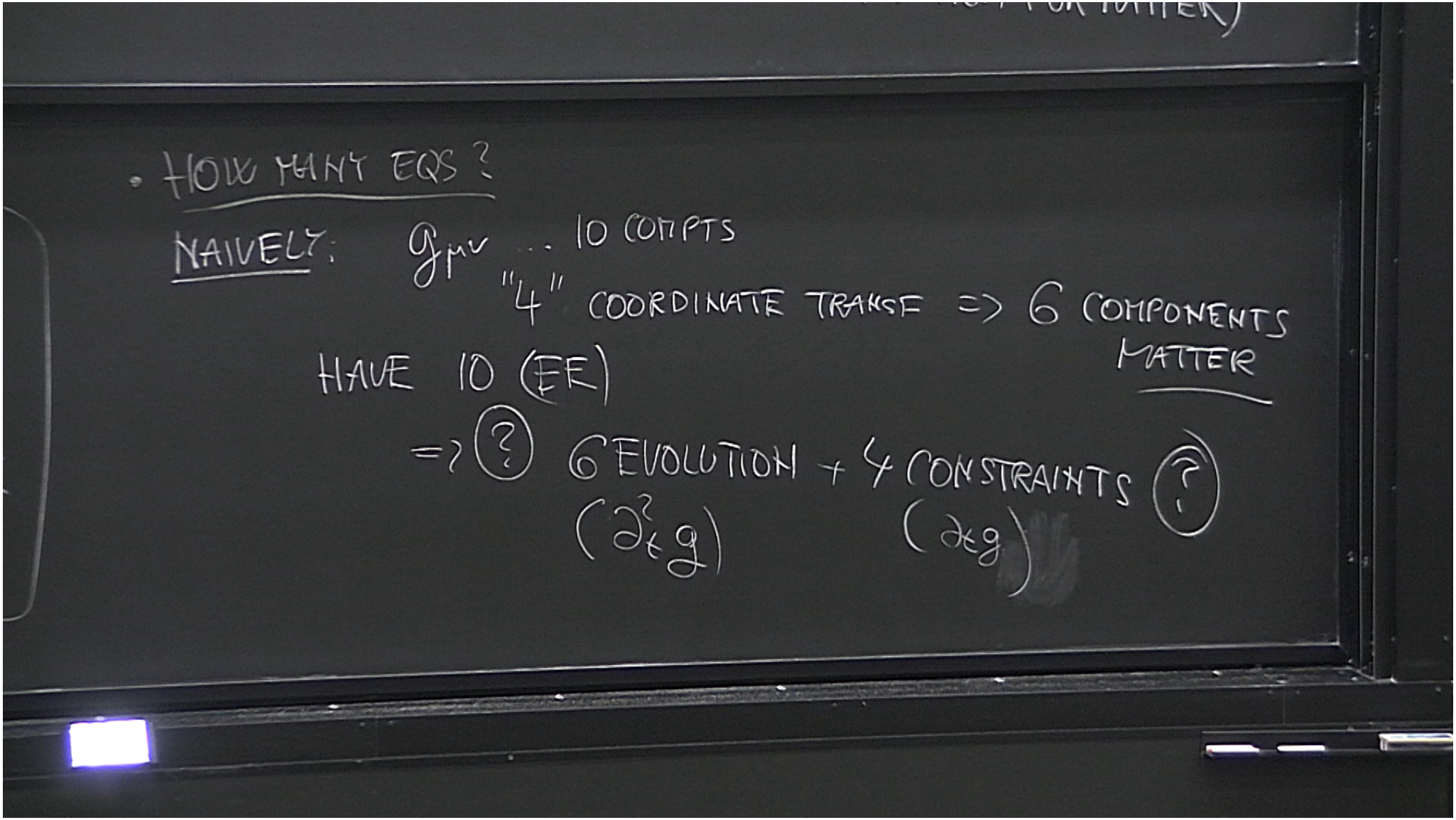
- HOW MANY EQS?

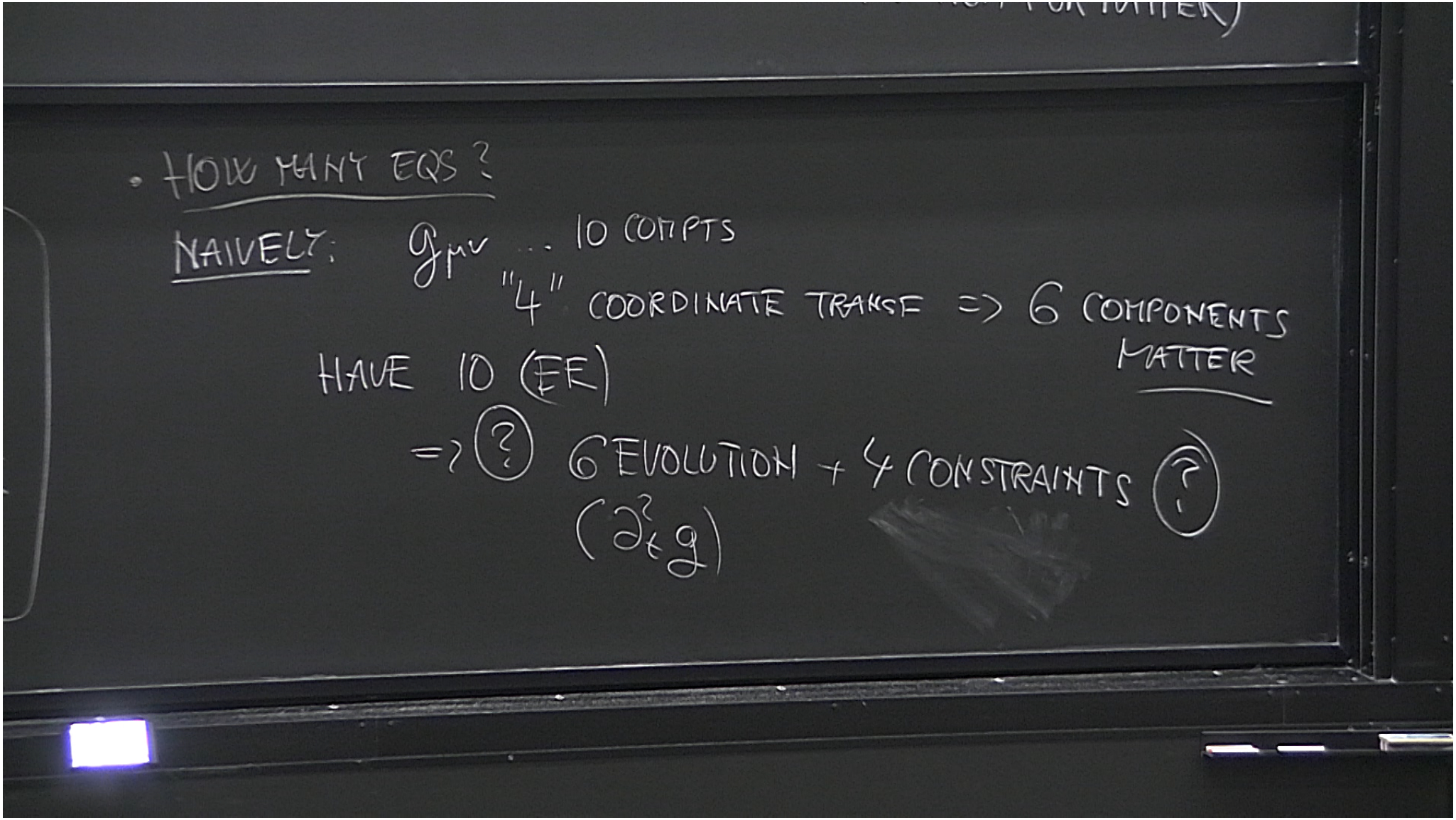
NAIVELY: $g_{\mu\nu}$ .. 10 COMPTS

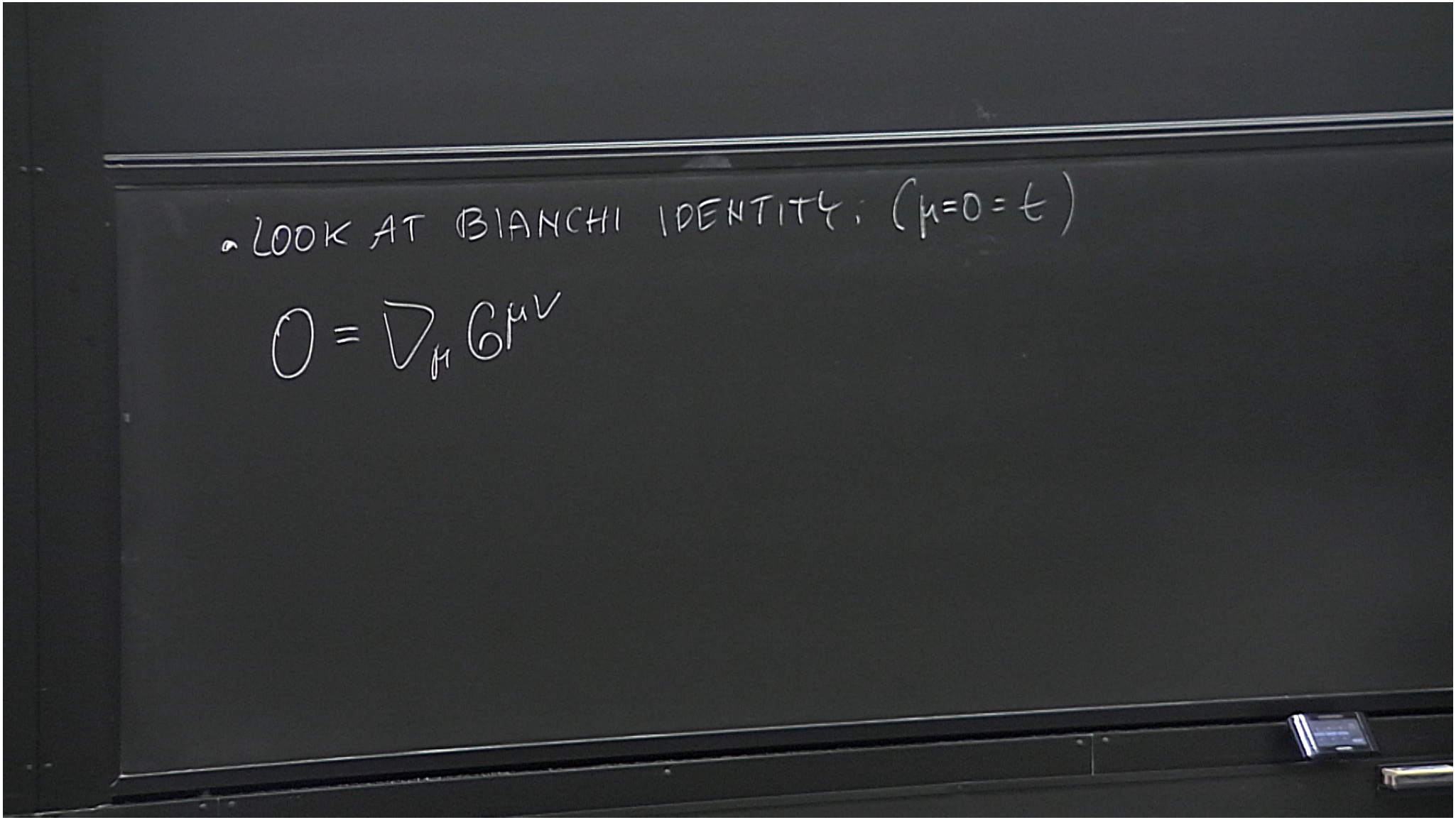
"4" COORDINATE TRANSF => 6 COMPONENTS MATTER

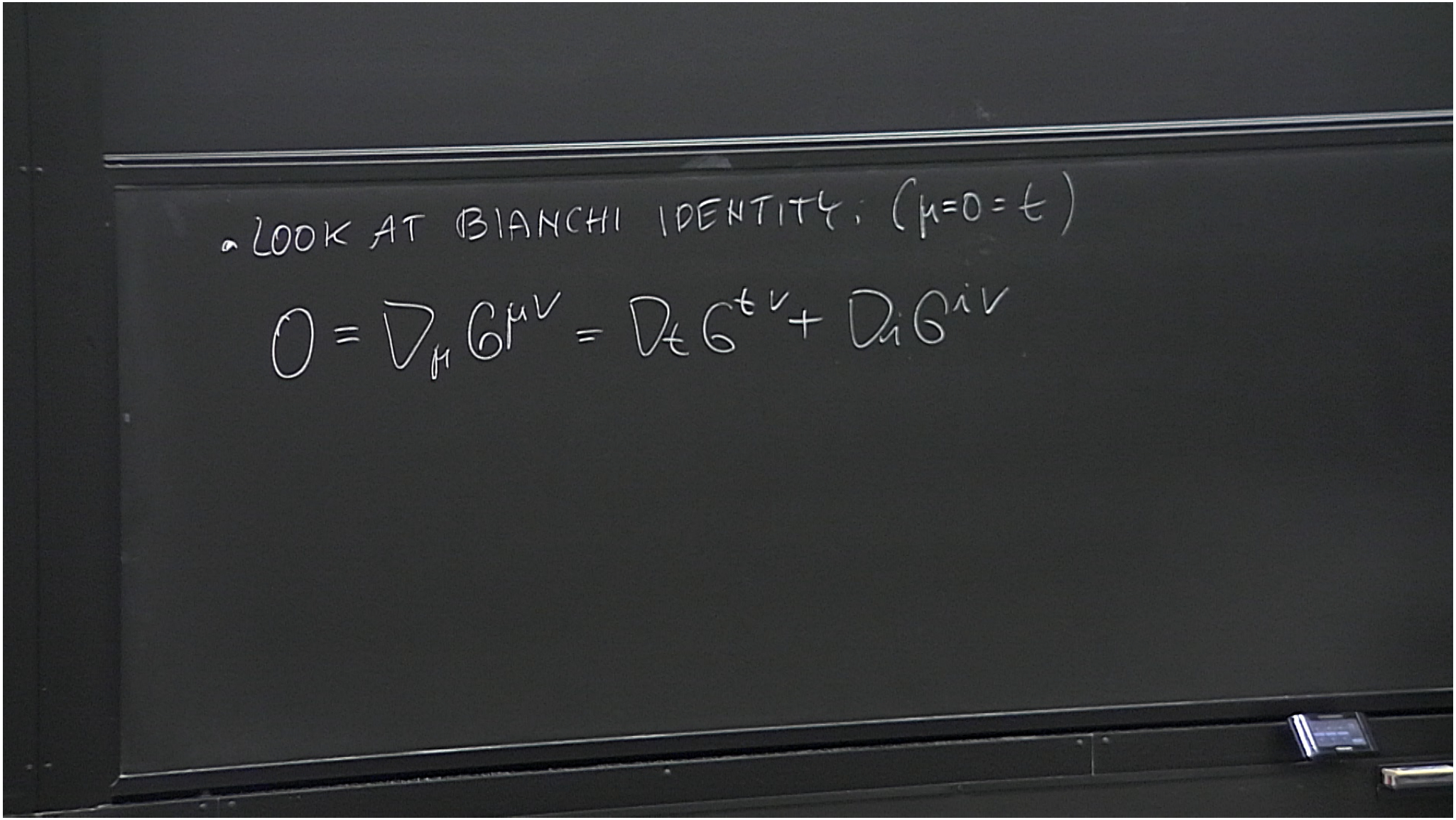
HAVE 10 (FE)

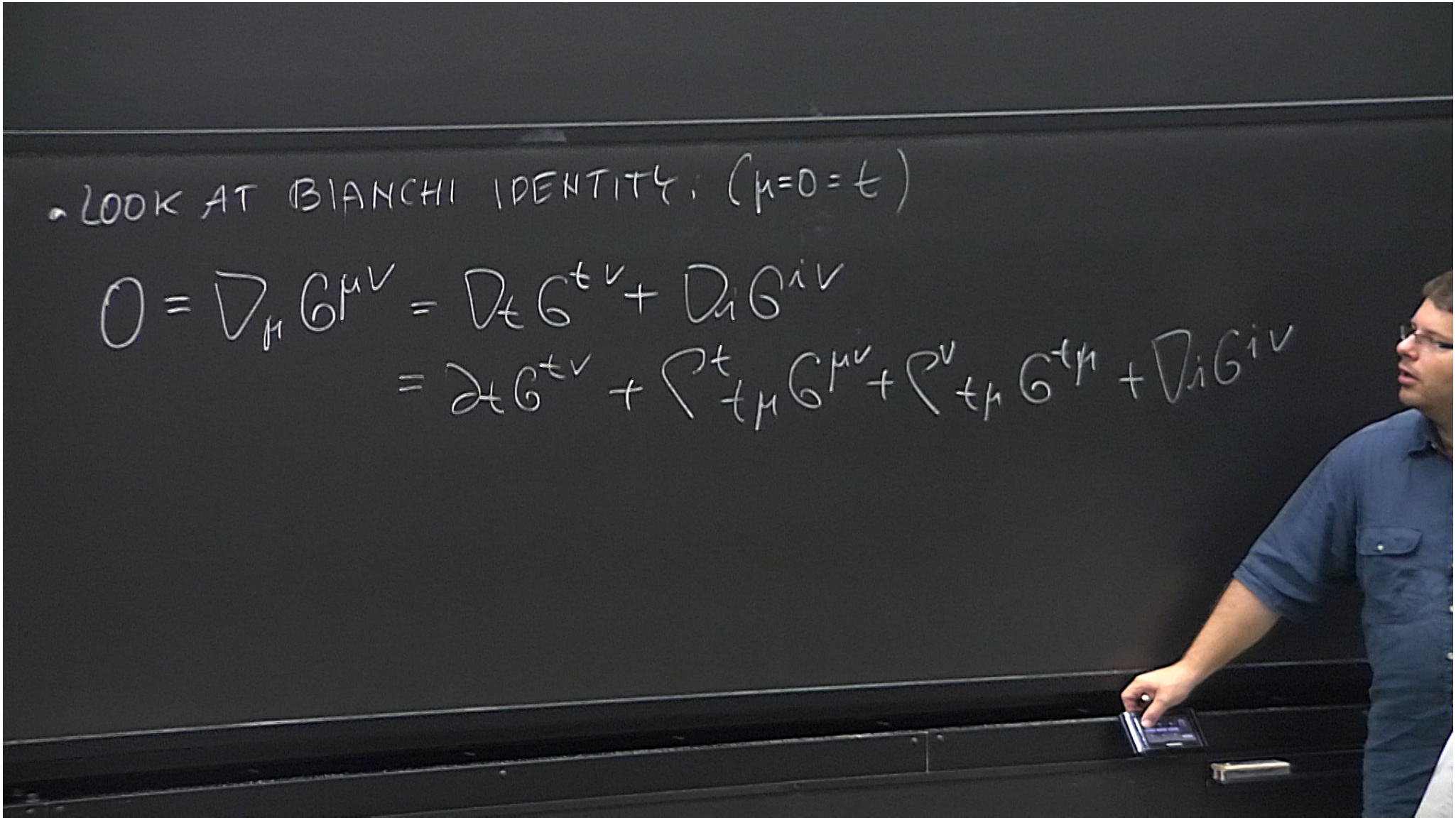

• HOW MANY EQS ?
NAIVELY: $g_{\mu\nu}$ .. 10 COMPTS
"4" COORDINATE TRANSF => 6 COMPONENTS MATTER
HAVE 10 (EE)
=> ? 6 EVOLUTION + 4 CONSTRAINTS ?

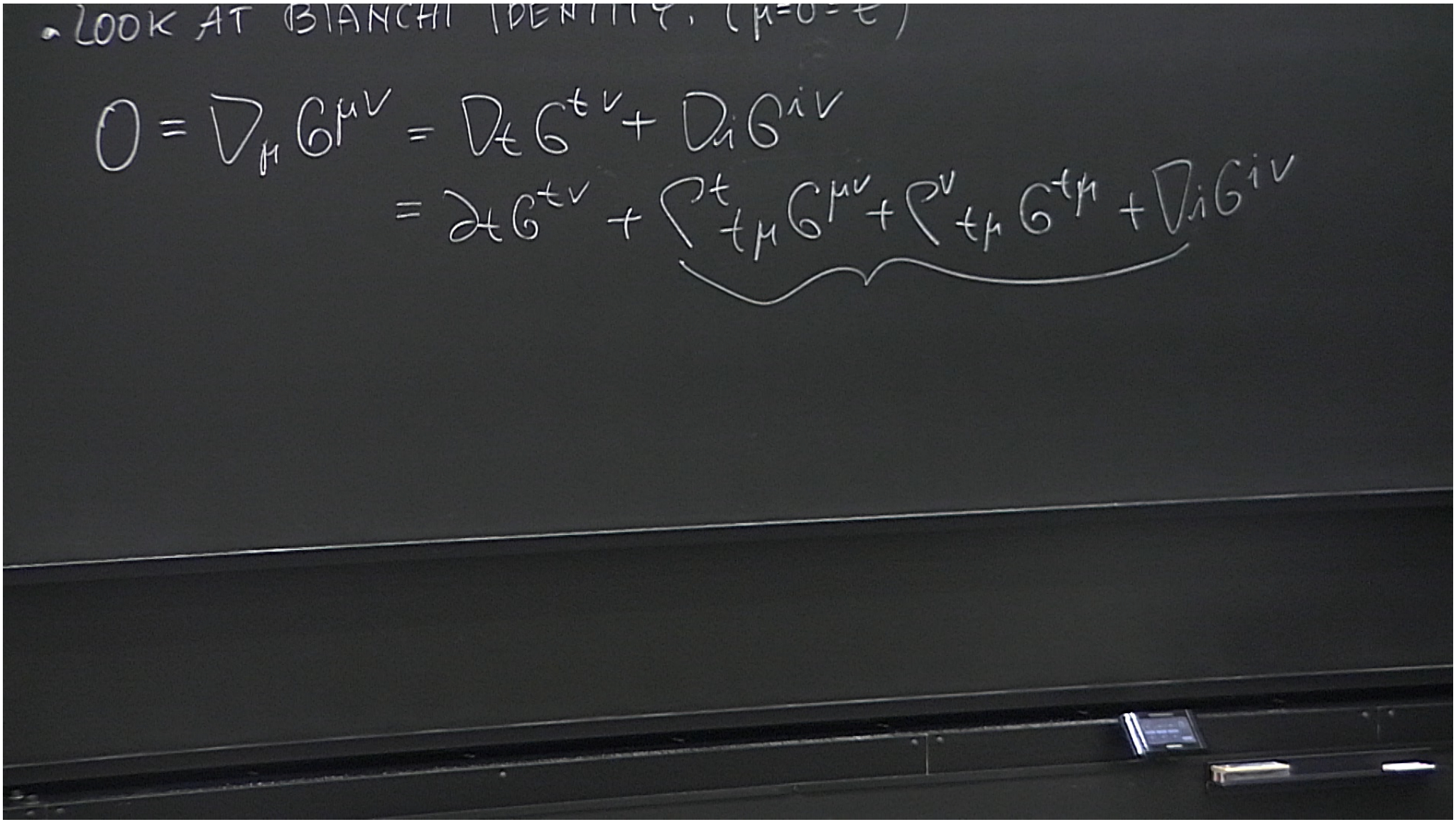

• HOW MANY EQS?
NAIVELY: $g_{\mu\nu}$ .. 10 COMPTS
"4" COORDINATE TRANSF => 6 COMPONENTS MATTER
HAVE 10 (EE)
=> ? 6 EVOLUTION + 4 CONSTRAINTS ?
$(\partial_t^2 g)$ $(\partial_t g)$

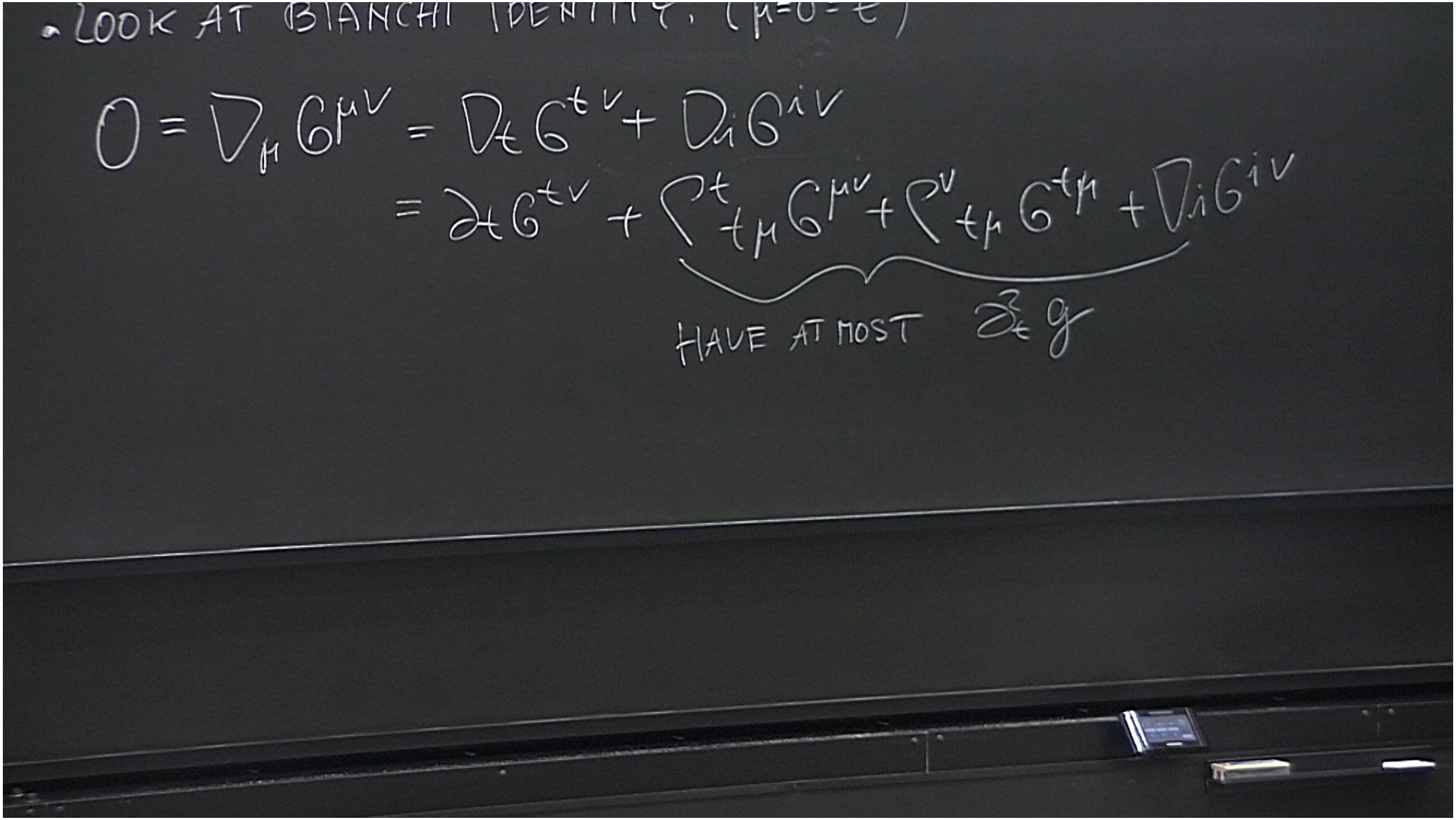

• HOW MANY EQS?
NAIVELY: $g_{\mu\nu}$ .. 10 COMPTS
"4" COORDINATE TRANSF => 6 COMPONENTS
MATTER
HAVE 10 (EE)
=> (?) 6 EVOLUTION + 4 CONSTRAINTS (?)
$(\partial_t^2 g)$

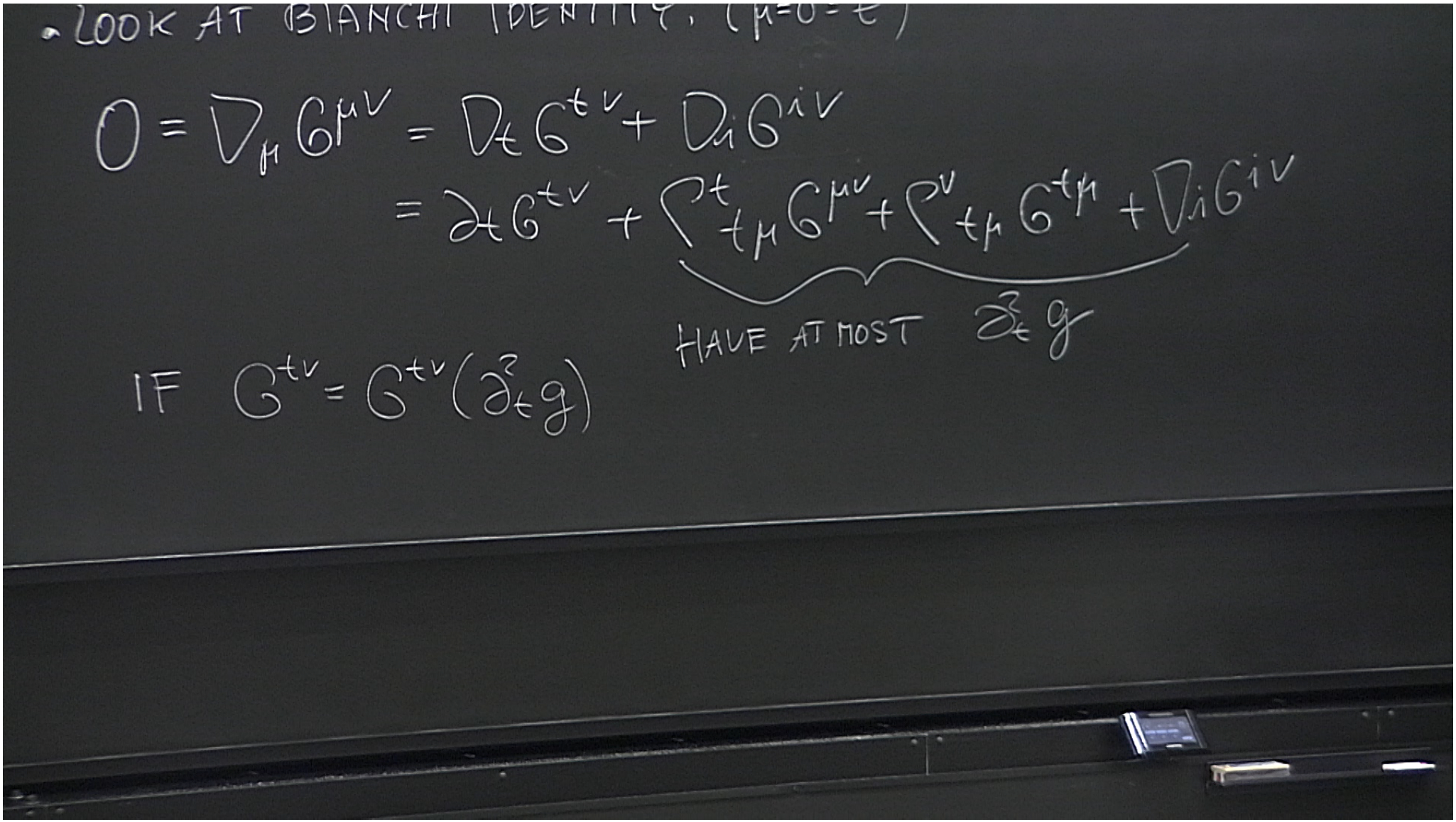

• LOOK AT BIANCHI IDENTITY: (μ=0=t)
$0 = \nabla_\mu G^{\mu\nu}$


• LOOK AT BIANCHI IDENTITY: ($\mu=0=t$)
$0 = D_\mu G^{\mu\nu} = D_t G^{t\nu} + D_i G^{i\nu}$


• LOOK AT BIANCHI IDENTITY: ($\mu=0=t$)
$0 = \nabla_\mu G^{\mu\nu} = \nabla_t G^{t\nu} + \nabla_i G^{i\nu}$
$= \partial_t G^{t\nu} + \Gamma^t_{t\mu} G^{\mu\nu} + \Gamma^\nu_{t\mu} G^{t\mu} + \nabla_i G^{i\nu}$


• LOOK AT BIANCHI IDENTITY: ($\mu=0=t$)
$0 = \nabla_\mu G^{\mu\nu} = \nabla_t G^{t\nu} + \nabla_i G^{i\nu}$
$= \partial_t G^{t\nu} + \Gamma^t_{t\mu} G^{\mu\nu} + \Gamma^\nu_{t\mu} G^{t\mu} + \nabla_i G^{i\nu}$


• LOOK AT BIANCHI IDENTITY: ($\mu=0-3$)
$0 = \nabla_\mu G^{\mu\nu} = \nabla_t G^{t\nu} + \nabla_i G^{i\nu}$
$= \partial_t G^{t\nu} + \Gamma^t_{t\mu} G^{\mu\nu} + \Gamma^\nu_{t\mu} G^{t\mu} + \nabla_i G^{i\nu}$
HAVE AT MOST $\partial_t^3 g$


• LOOK AT BIANCHI IDENTITY: ($\mu=0=t$)
$0 = \nabla_\mu G^{\mu\nu} = \nabla_t G^{t\nu} + \nabla_i G^{i\nu}$
$= \partial_t G^{t\nu} + \Gamma^t_{t\mu} G^{\mu\nu} + \Gamma^\nu_{t\mu} G^{t\mu} + \nabla_i G^{i\nu}$
HAVE AT MOST $\partial_t^3 g$
IF $G^{t\nu} = G^{t\nu}(\partial_t^2 g)$

- LOOK AT BIANCHI IDENTITY: ($\mu = 0 - 3$)

$$0 = \nabla_\mu G^{\mu\nu} = \nabla_t G^{t\nu} + \nabla_i G^{i\nu}$$

$$= \partial_t G^{t\nu} + \underbrace{\Gamma^t_{t\mu} G^{\mu\nu} + \Gamma^\nu_{t\mu} G^{t\mu} + \nabla_i G^{i\nu}}_{\text{HAVE AT MOST } \partial_t^3 g}$$

IF $G^{t\nu} = G^{t\nu}(\partial_t^2 g)$

$\Rightarrow \partial_t G^{t\nu}$ HAS $\partial_t^3 g$

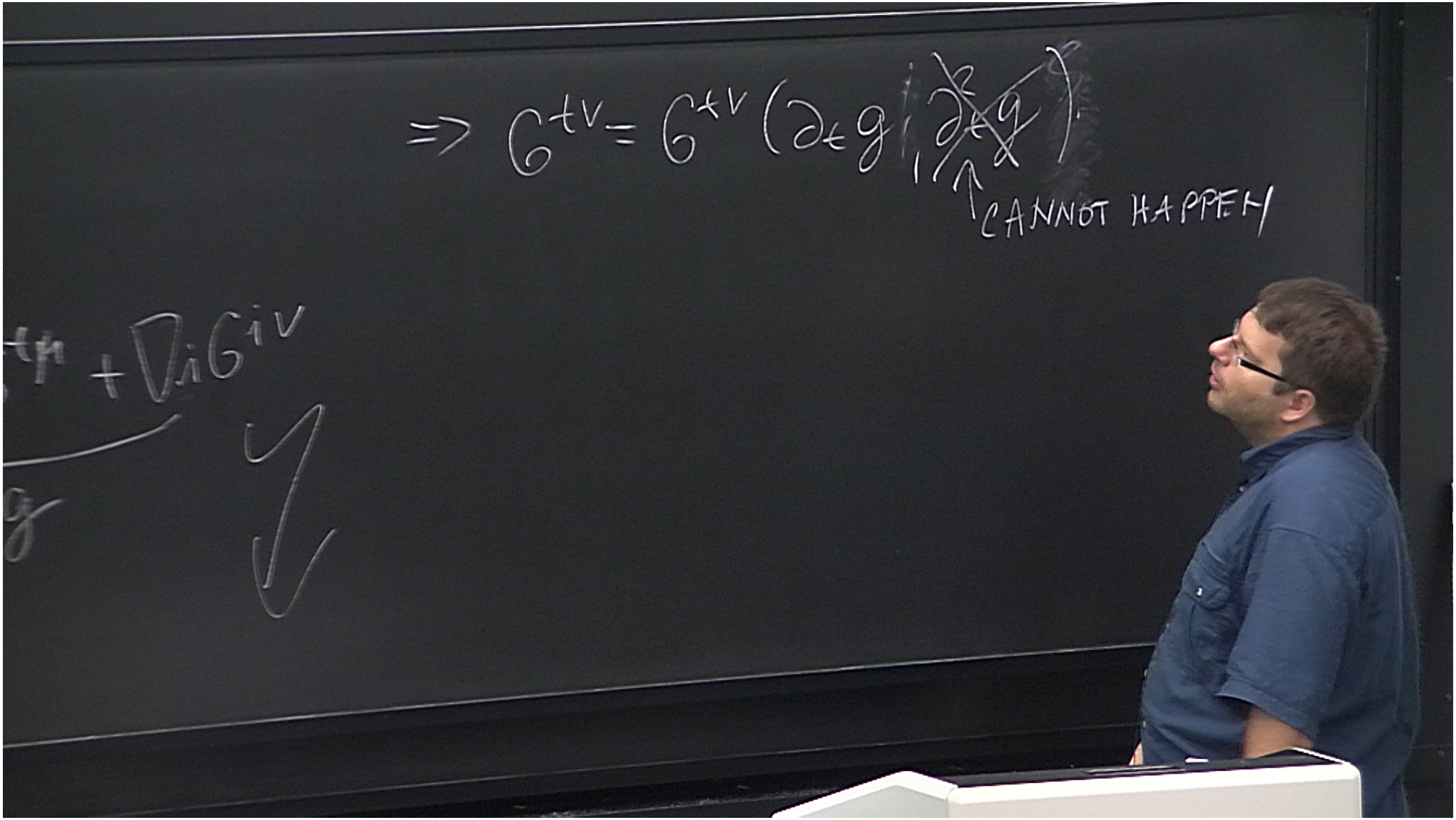
- LOOK AT BIANCHI IDENTITY: ($\mu = 0 - t$)

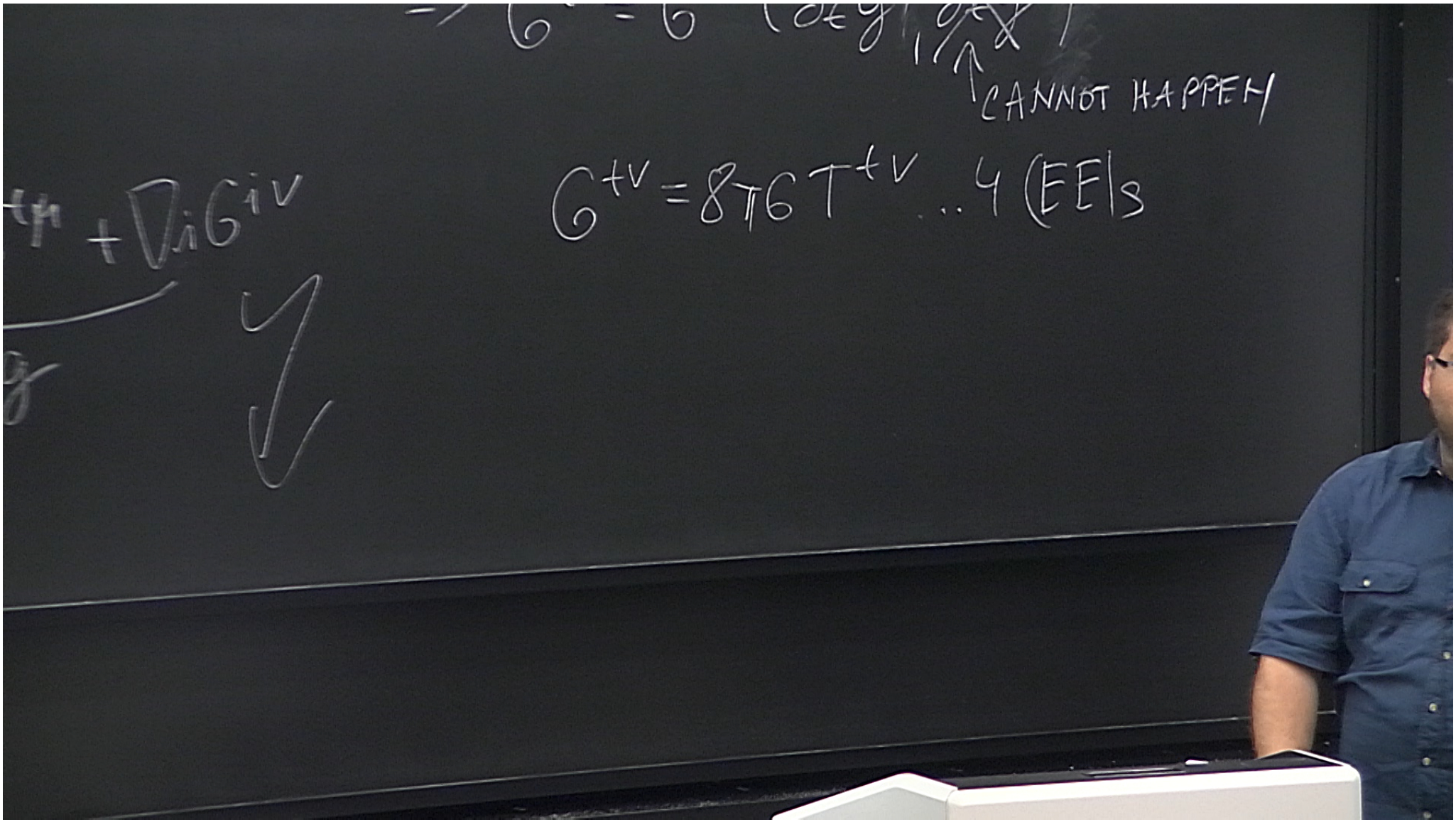
$$0 = \nabla_\mu G^{\mu\nu} = \nabla_t G^{t\nu} + \nabla_i G^{i\nu}$$

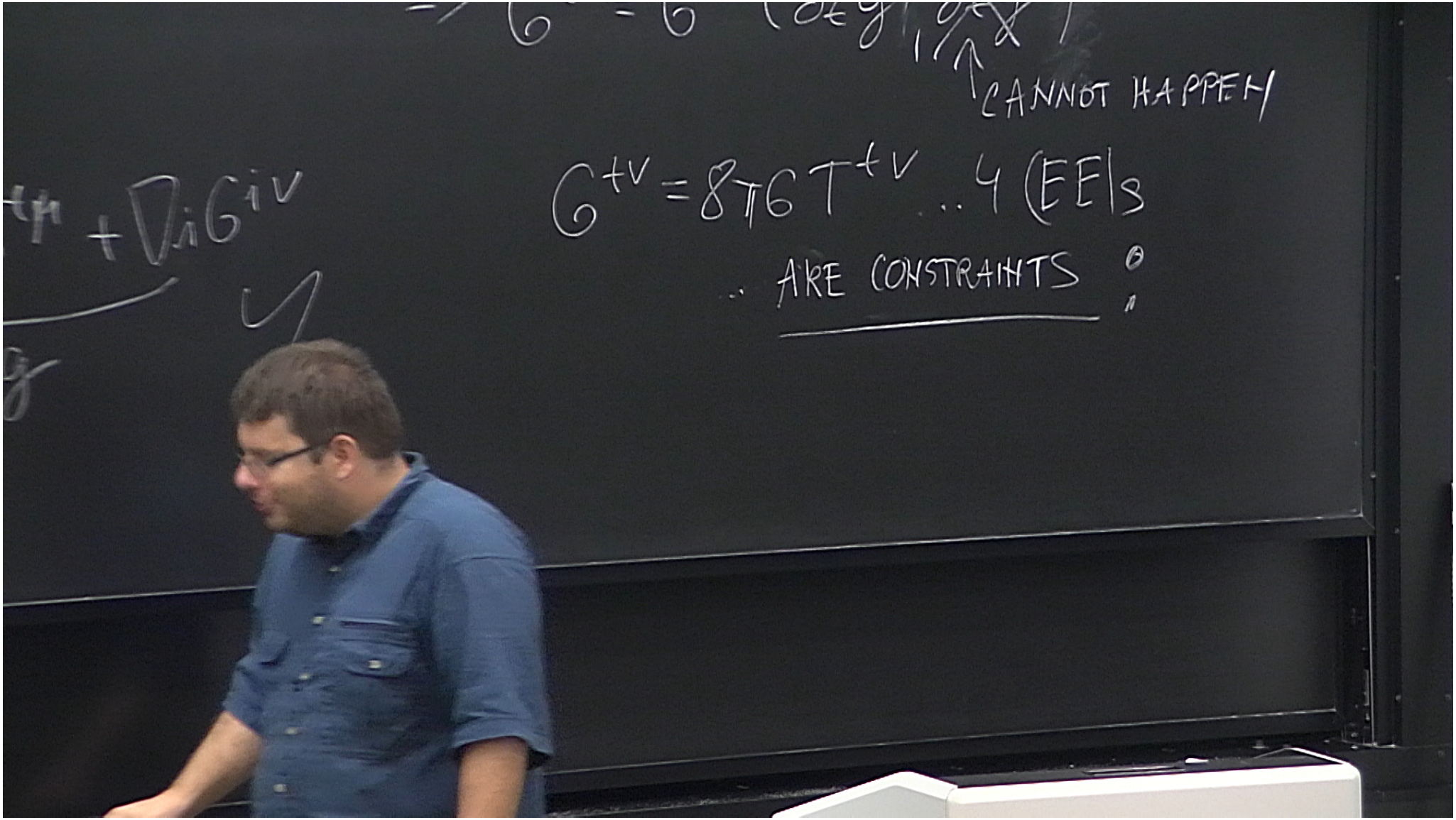
$$= \partial_t G^{t\nu} + \underbrace{\Gamma^t_{t\mu} G^{\mu\nu} + \Gamma^\nu_{t\mu} G^{t\mu} + \nabla_i G^{i\nu}}_{\text{HAVE AT MOST } \partial_t^3 g}$$

IF $G^{t\nu} = G^{t\nu}(\partial_t^2 g)$

$\Rightarrow \partial_t G^{t\nu}$ HAS $\partial_t^3 g$


$\Rightarrow G^{\mu\nu} = G^{\mu\nu}(\partial_\mu g, \partial^2_{\mu\nu} g)$
CANNOT HAPPEN
$+D_i G^{iv}$


CANNOT HAPPEN
$G^{\mu\nu} = 8\pi G T^{\mu\nu}$ ..4 (EE)s
$+ D_i G^{iv}$


CANNOT HAPPEN
$G^{\mu\nu} = 8\pi G T^{\mu\nu}$ .. 4 (EE)s
.. ARE CONSTRAINTS !

$\Rightarrow \partial + G^{tv}$ HAS $\partial^3 + g$

- CONSERVATION LAWS

$$D_\mu T^{\mu\nu} = 0$$

$\Rightarrow \partial_t \theta^{tv}$ HAS $\partial_t^3 g$

- CONSERVATION LAWS

$$\boxed{\nabla_\mu T^{\mu\nu} = 0} \overset{?}{\Leftrightarrow} \partial_\mu T^{\mu\nu} = 0$$

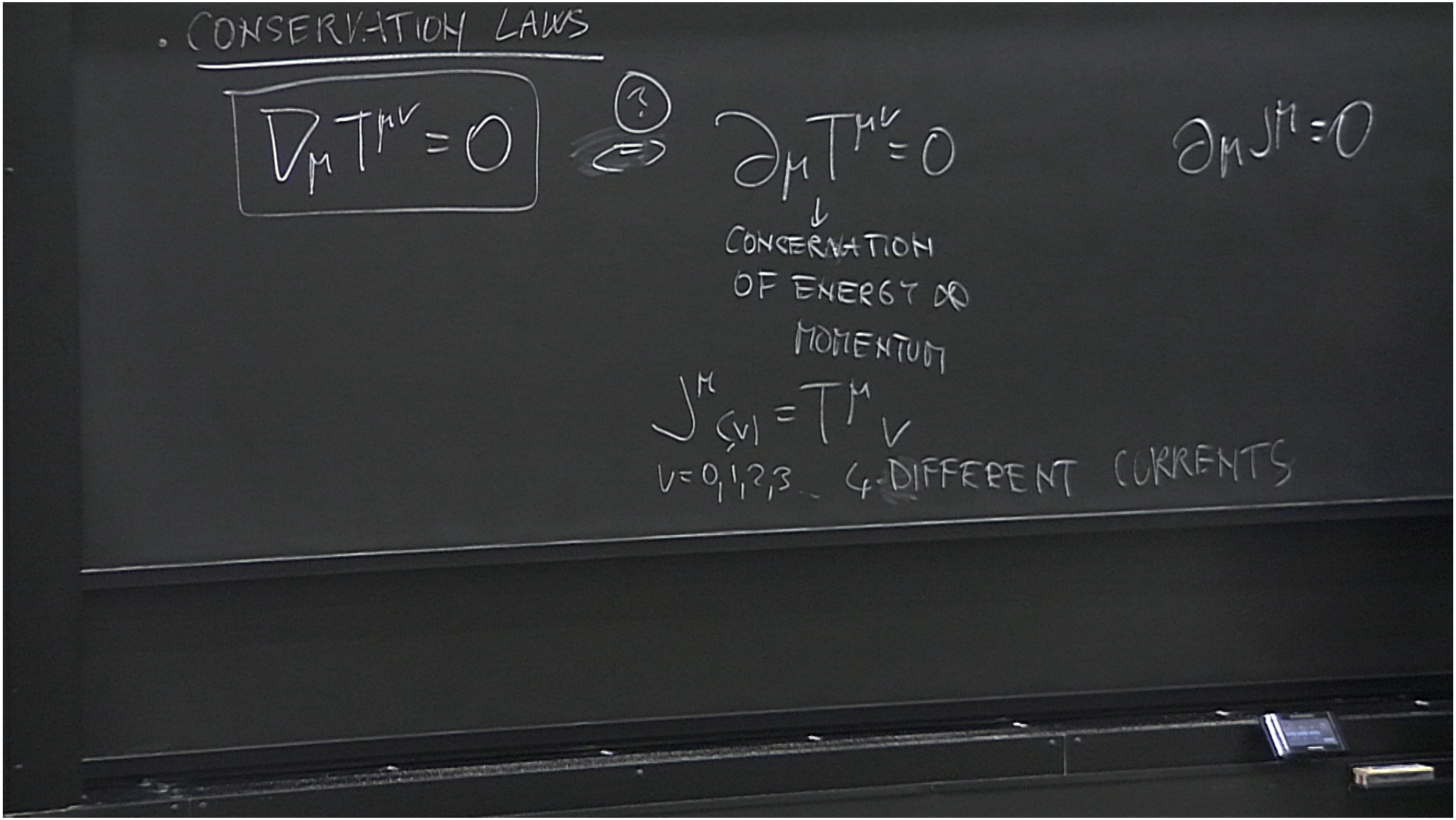
$\Rightarrow \partial + G^{tv}$ HAS $\partial^3 + g$

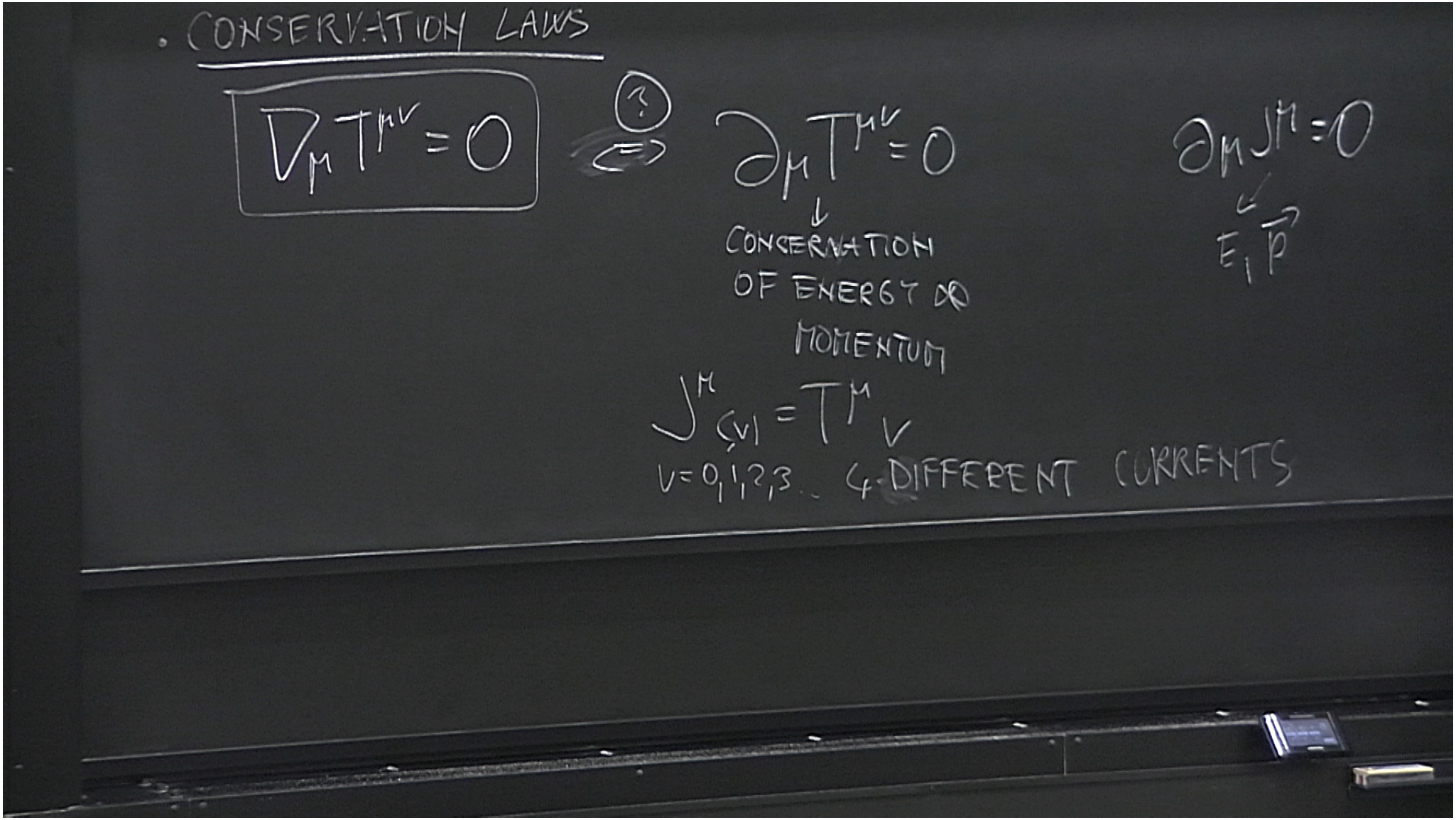
- CONSERVATION LAWS

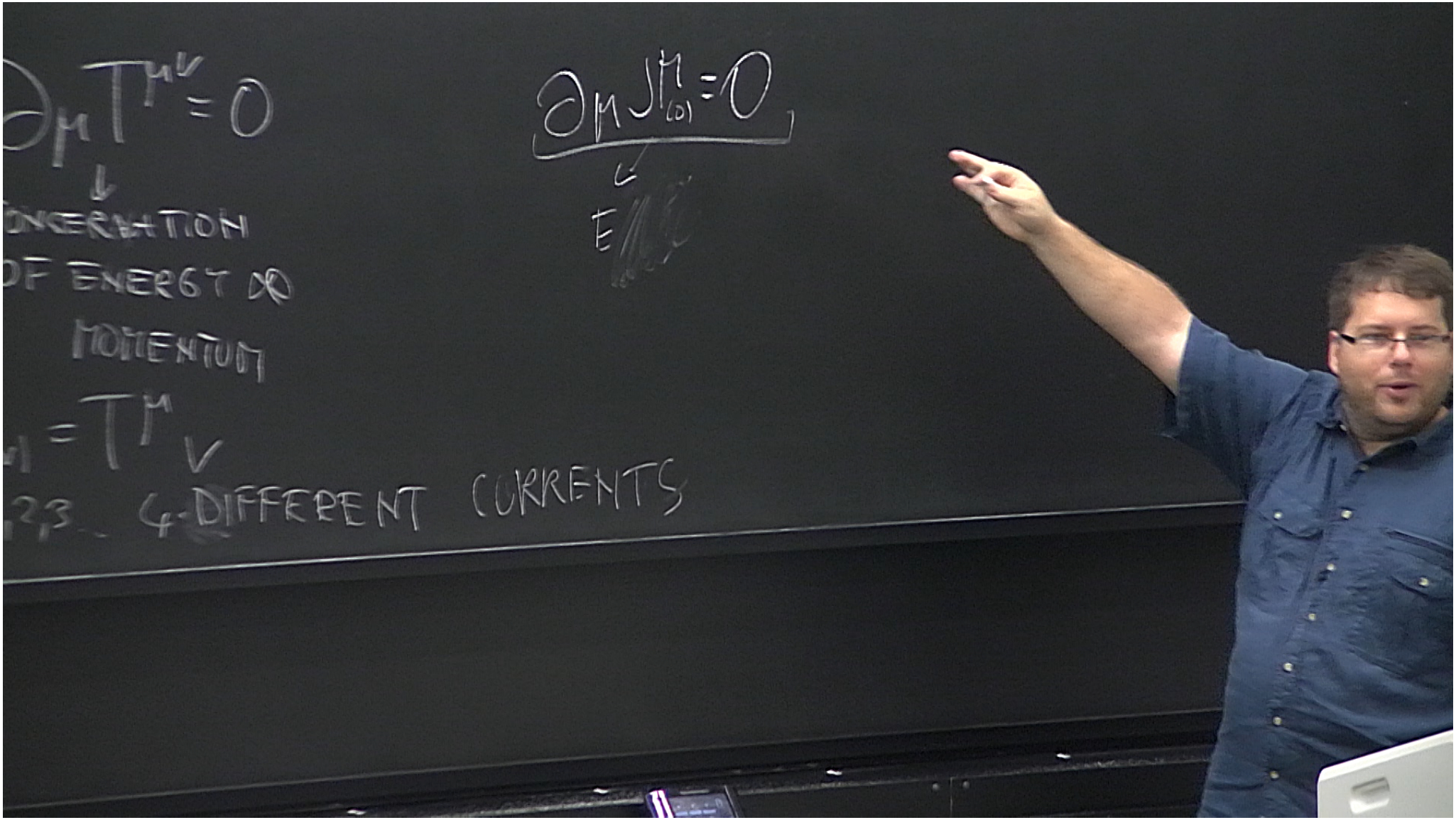
$$\boxed{\nabla_\mu T^{\mu\nu} = 0} \overset{?}{\Leftrightarrow} \partial_\mu T^{\mu\nu} = 0$$

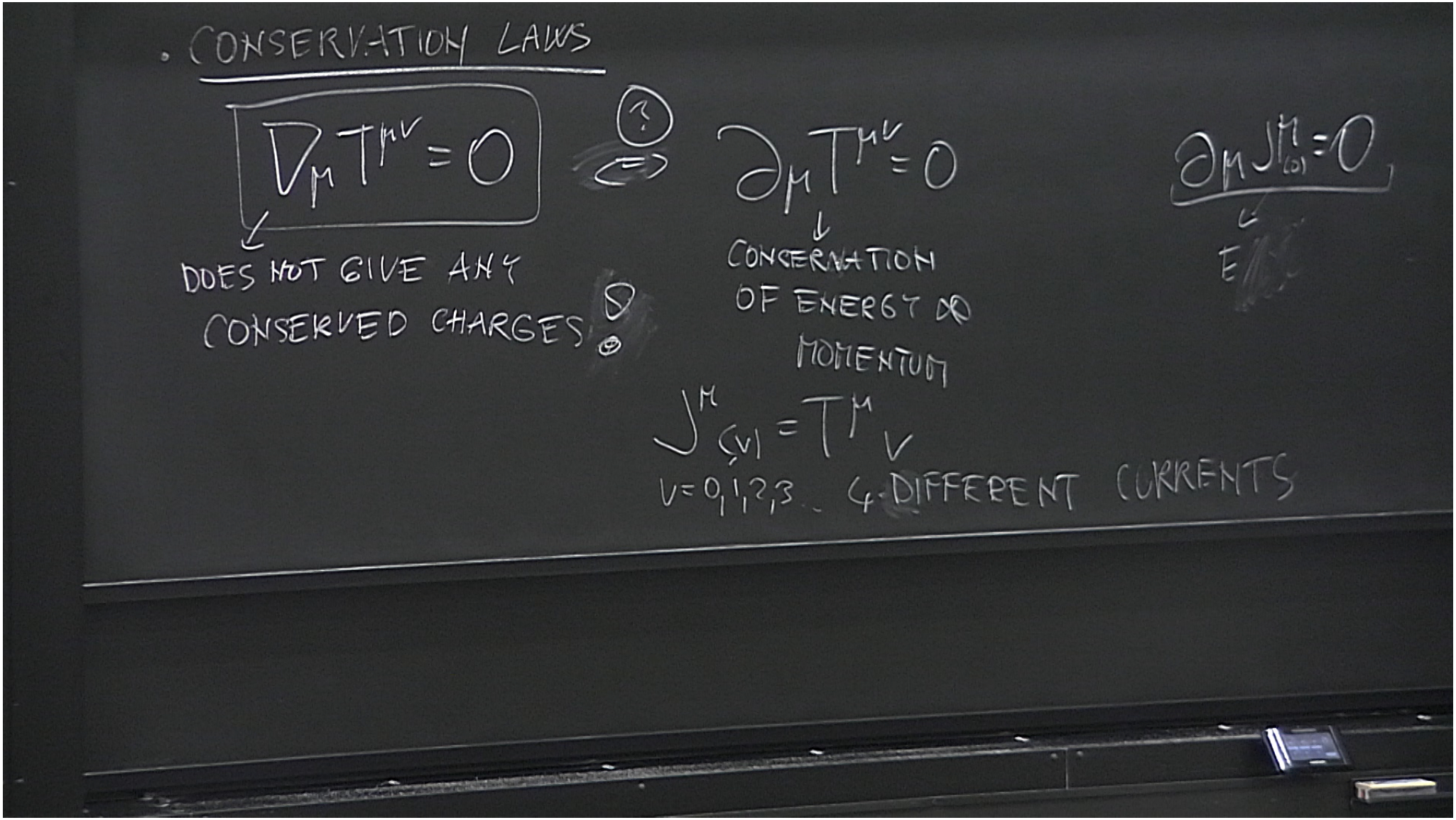
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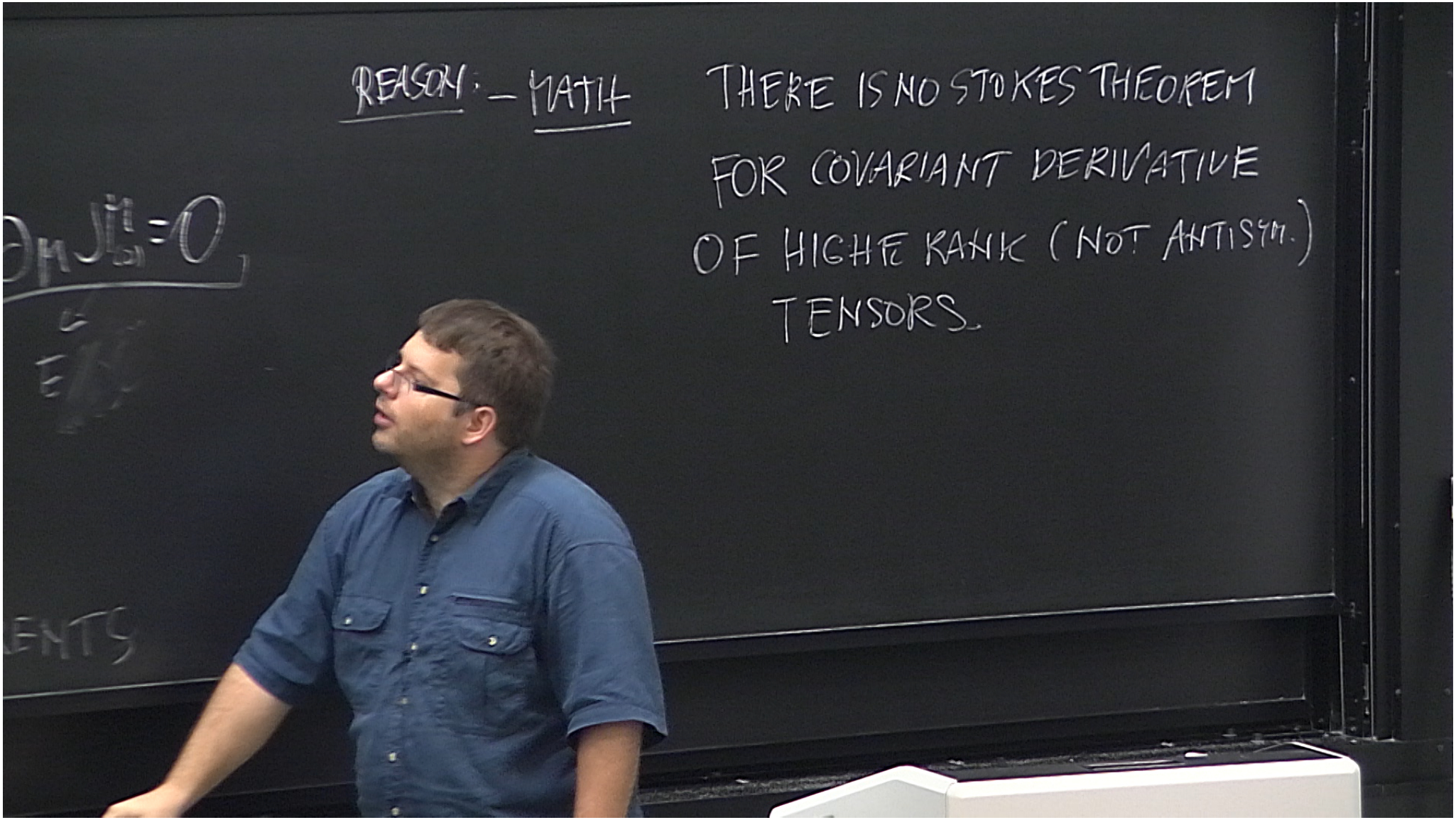
CONSERVATION OF ENERGY & MOMENTUM

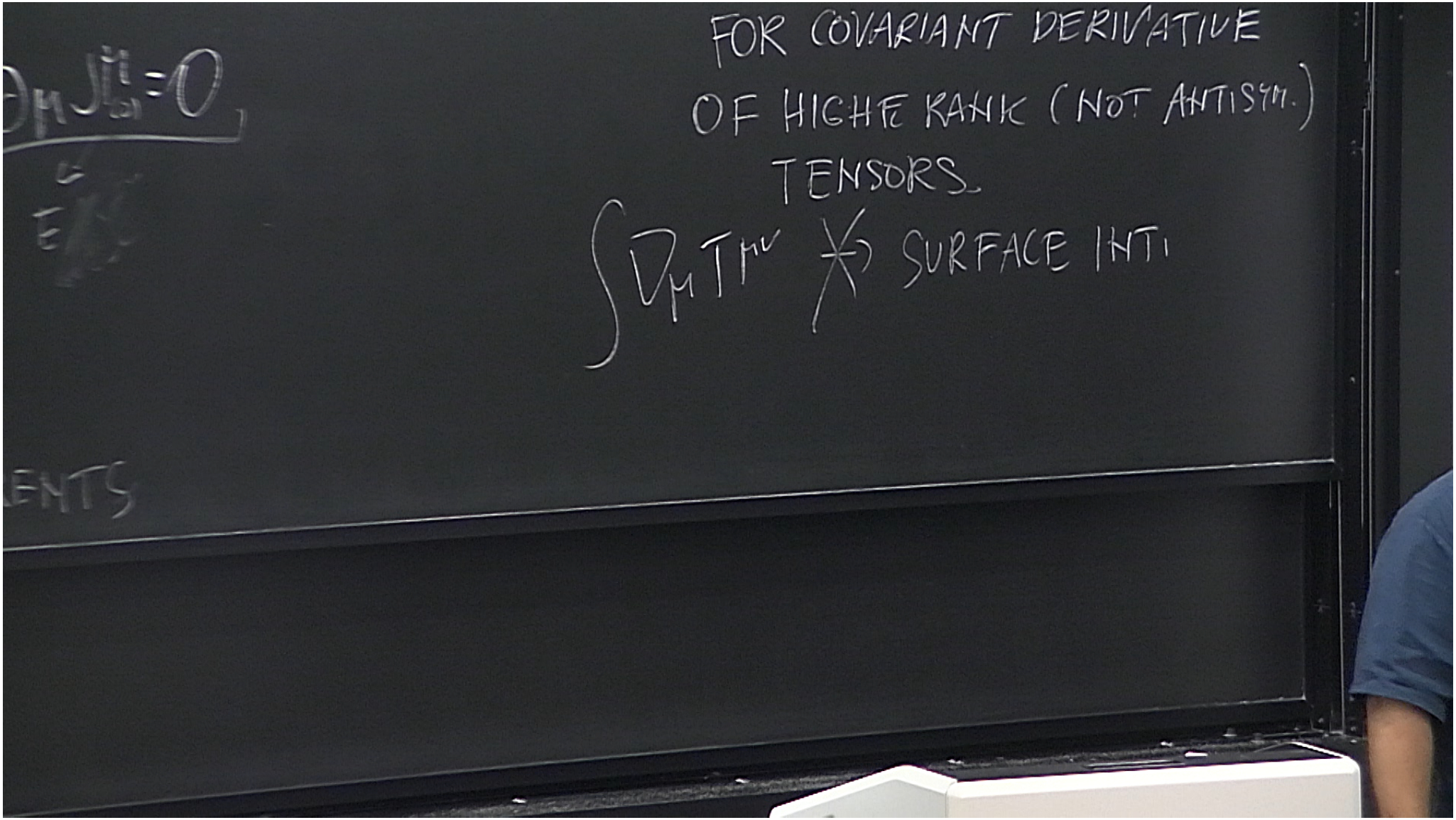

CONSERVATION LAWS
$D_\mu T^{\mu\nu} = 0$
?
$\partial_\mu T^{\mu\nu} = 0$
$\partial_\mu J^\mu = 0$
CONSERVATION OF ENERGY & MOMENTUM
$J^\mu_{(\nu)} = T^\mu{}_\nu$
$\nu = 0,1,2,3$ 4-DIFFERENT CURRENTS

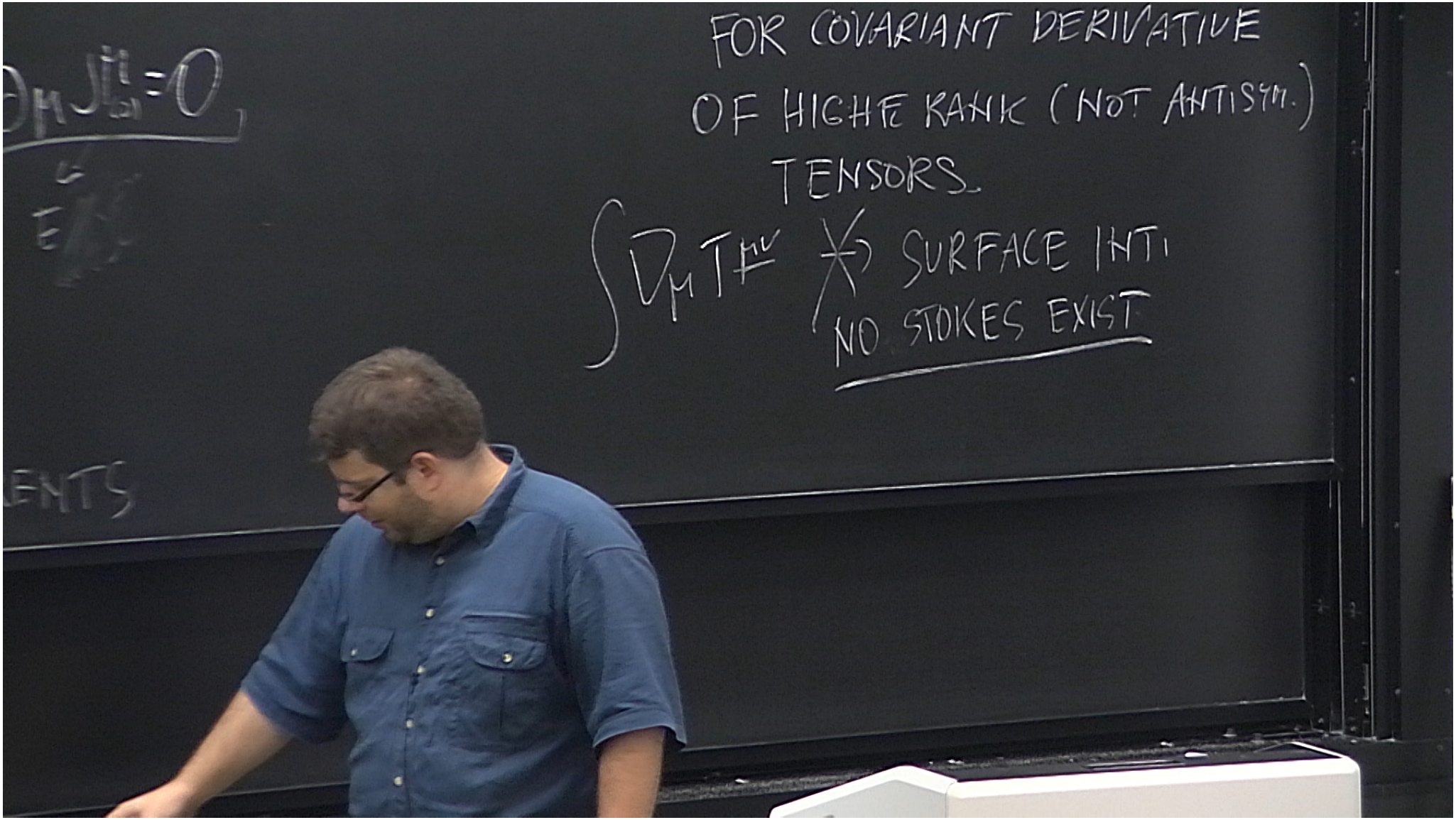

CONSERVATION LAWS
$D_\mu T^{\mu\nu} = 0$
?
$\partial_\mu T^{\mu\nu} = 0$
CONSERVATION OF ENERGY & MOMENTUM
$\partial_\mu J^\mu = 0$
$E, \vec{p}$
$J^\mu_{(\nu)} = T^\mu{}_\nu$
$\nu = 0,1,2,3$ 4 DIFFERENT CURRENTS


$\partial_\mu T^{\mu\nu} = 0$
$\partial_\mu J^\mu_{(a)} = 0$
CONSERVATION
OF ENERGY
MOMENTUM
$= T^\mu{}_\nu$
4-DIFFERENT CURRENTS


CONSERVATION LAWS
$D_\mu T^{\mu\nu} = 0$
DOES NOT GIVE ANY CONSERVED CHARGES!
$\partial_\mu T^{\mu\nu} = 0$
CONSERVATION OF ENERGY & MOMENTUM
$\partial_\mu J^\mu_{(a)} = 0$
E
$J^\mu_{(\nu)} = T^\mu{}_\nu$
$\nu = 0,1,2,3$ 4 DIFFERENT CURRENTS


REASON: – MATH
THERE IS NO STOKES THEOREM
FOR COVARIANT DERIVATIVE
OF HIGHE RANK (NOT ANTISYM.)
TENSORS.


FOR COVARIANT DERIVATIVE
OF HIGHER RANK (NOT ANTISYM.)
TENSORS.
SURFACE INT.

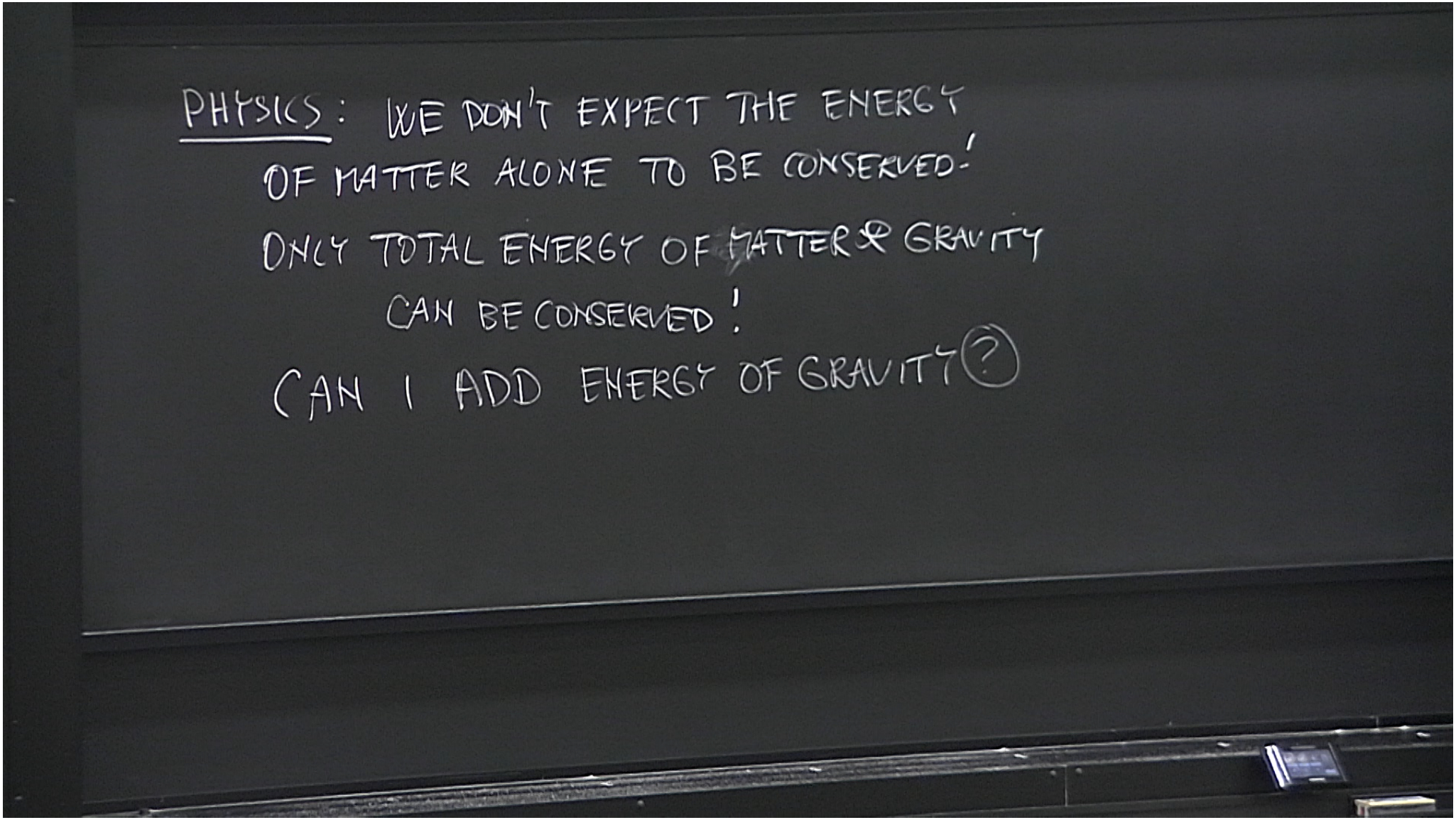

FOR COVARIANT DERIVATIVE
OF HIGHE RANK (NOT ANTISYM.)
TENSORS.
SURFACE INT.
NO STOKES EXIST

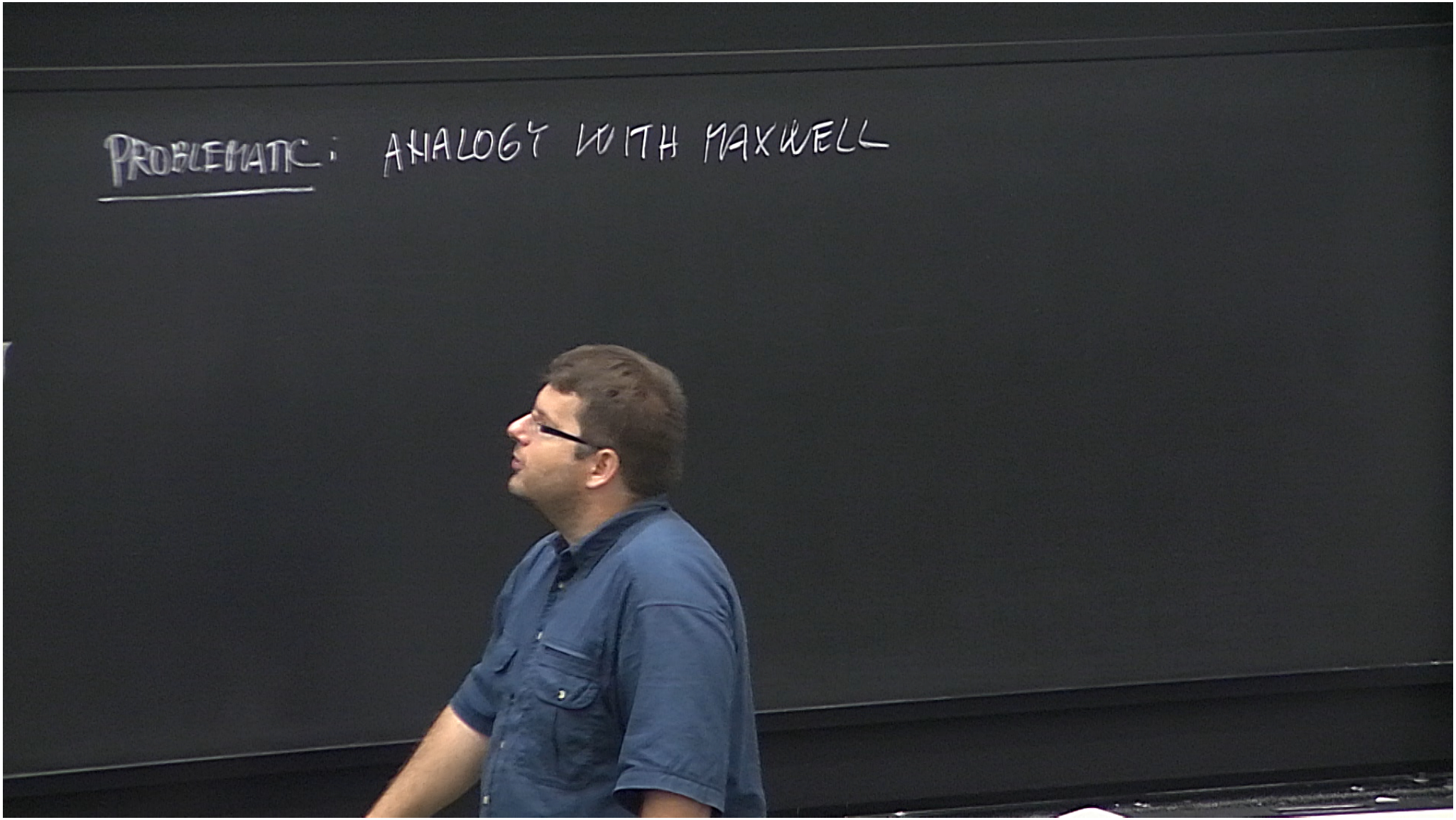
$J^{\mu}_{(\nu)} = T^{\mu}{}_{\nu}$

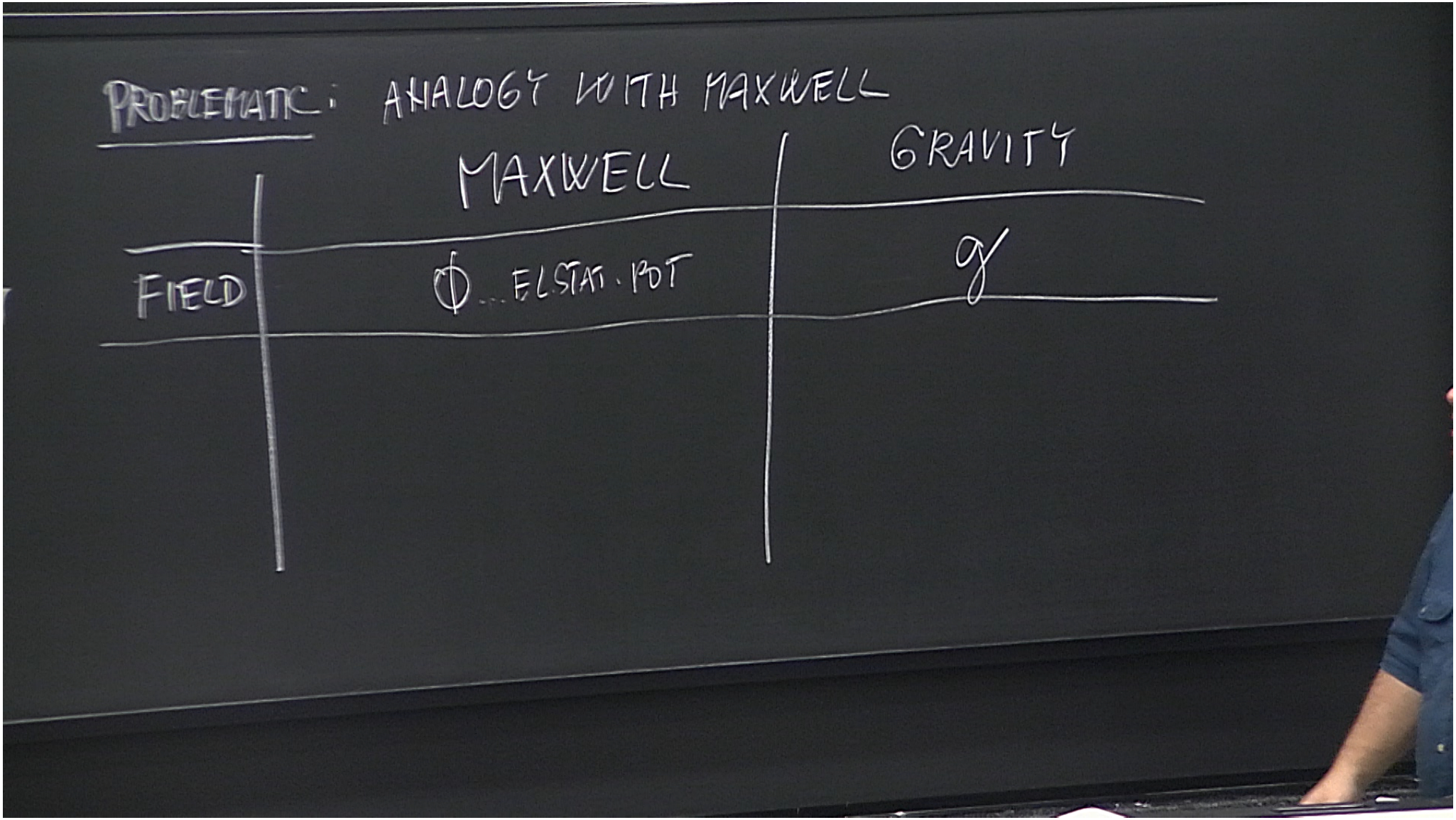
$\nu = 0,1,2,3$ ... 4 DIFFERENT CURRENTS

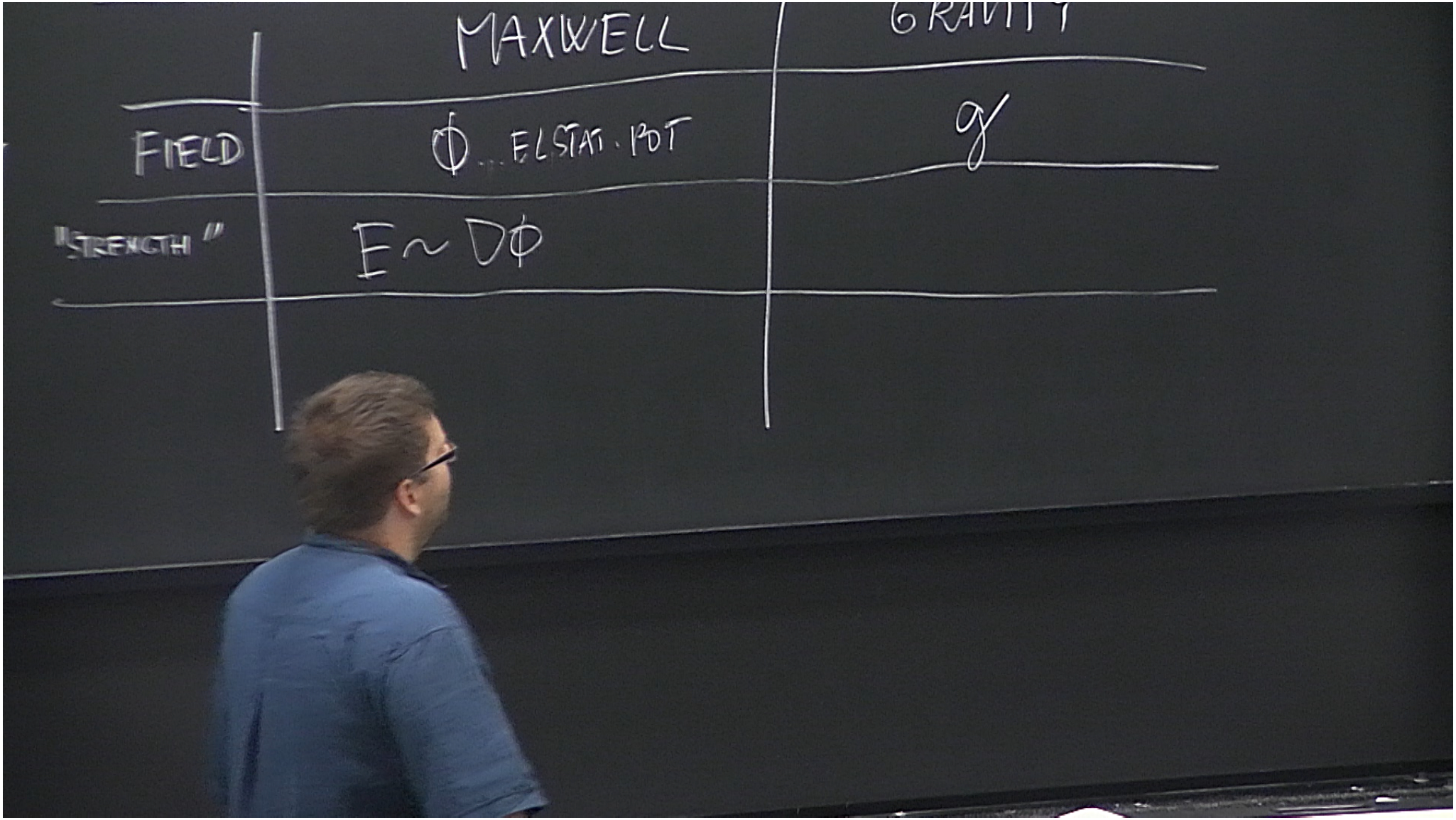
PHYSICS: WE DON'T EXPECT THE ENERGY OF MATTER ALONE TO BE CONSERVED!

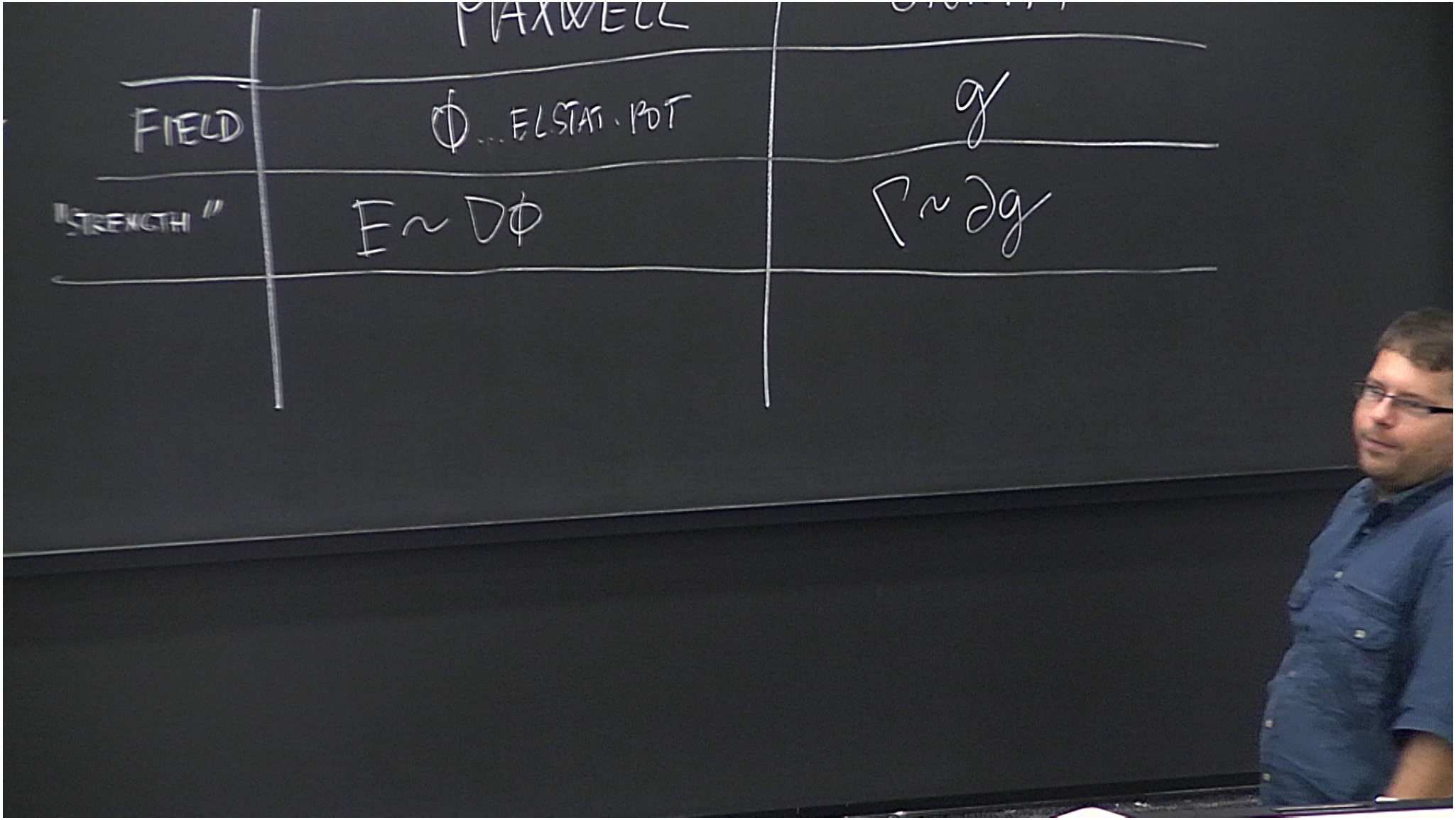
ONLY TOTAL ENERGY OF MATTER & GRAVITY CAN BE CONSERVED!

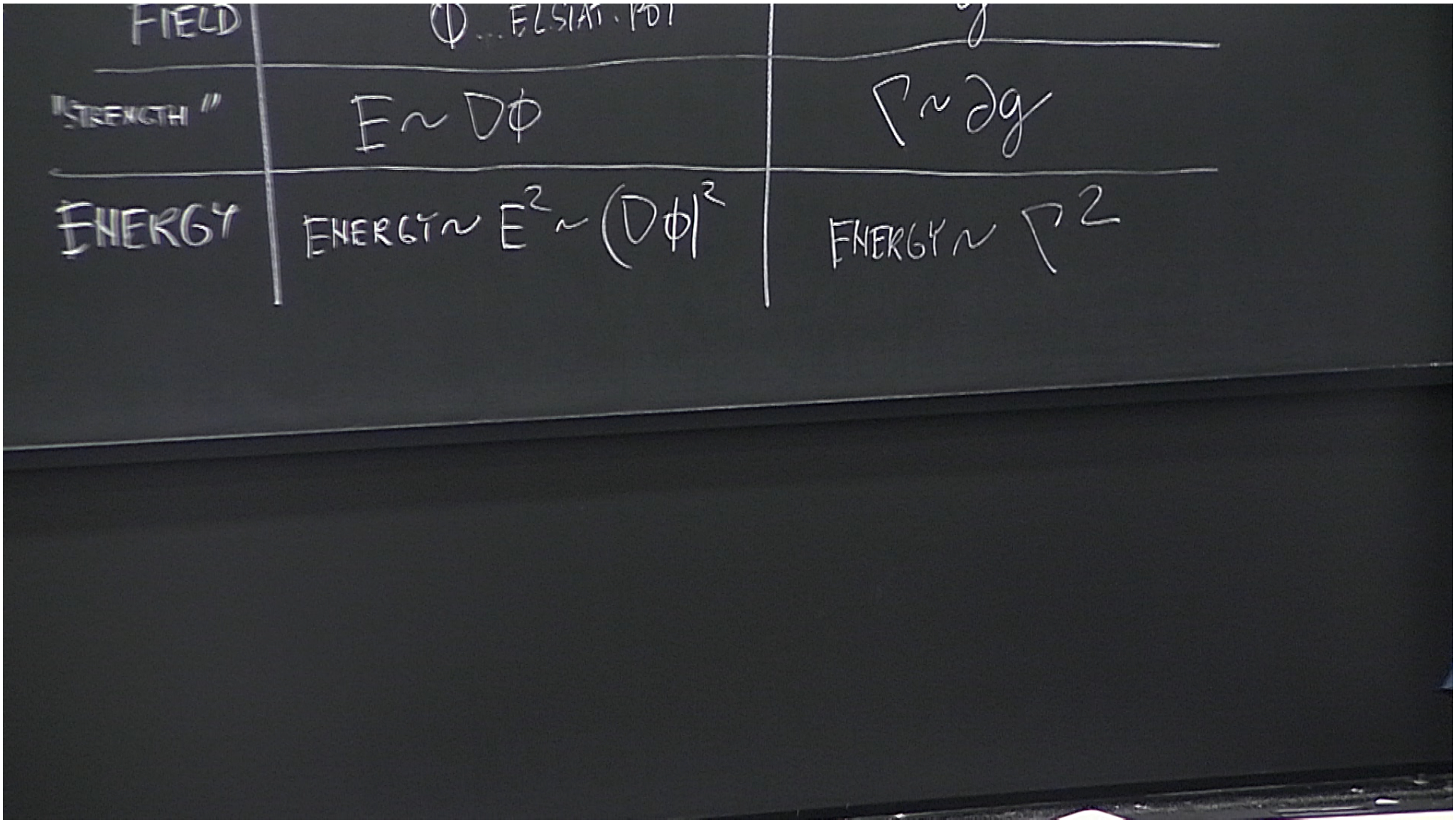

PHYSICS: WE DON'T EXPECT THE ENERGY
OF MATTER ALONE TO BE CONSERVED!
ONLY TOTAL ENERGY OF MATTER & GRAVITY
CAN BE CONSERVED!
CAN I ADD ENERGY OF GRAVITY?

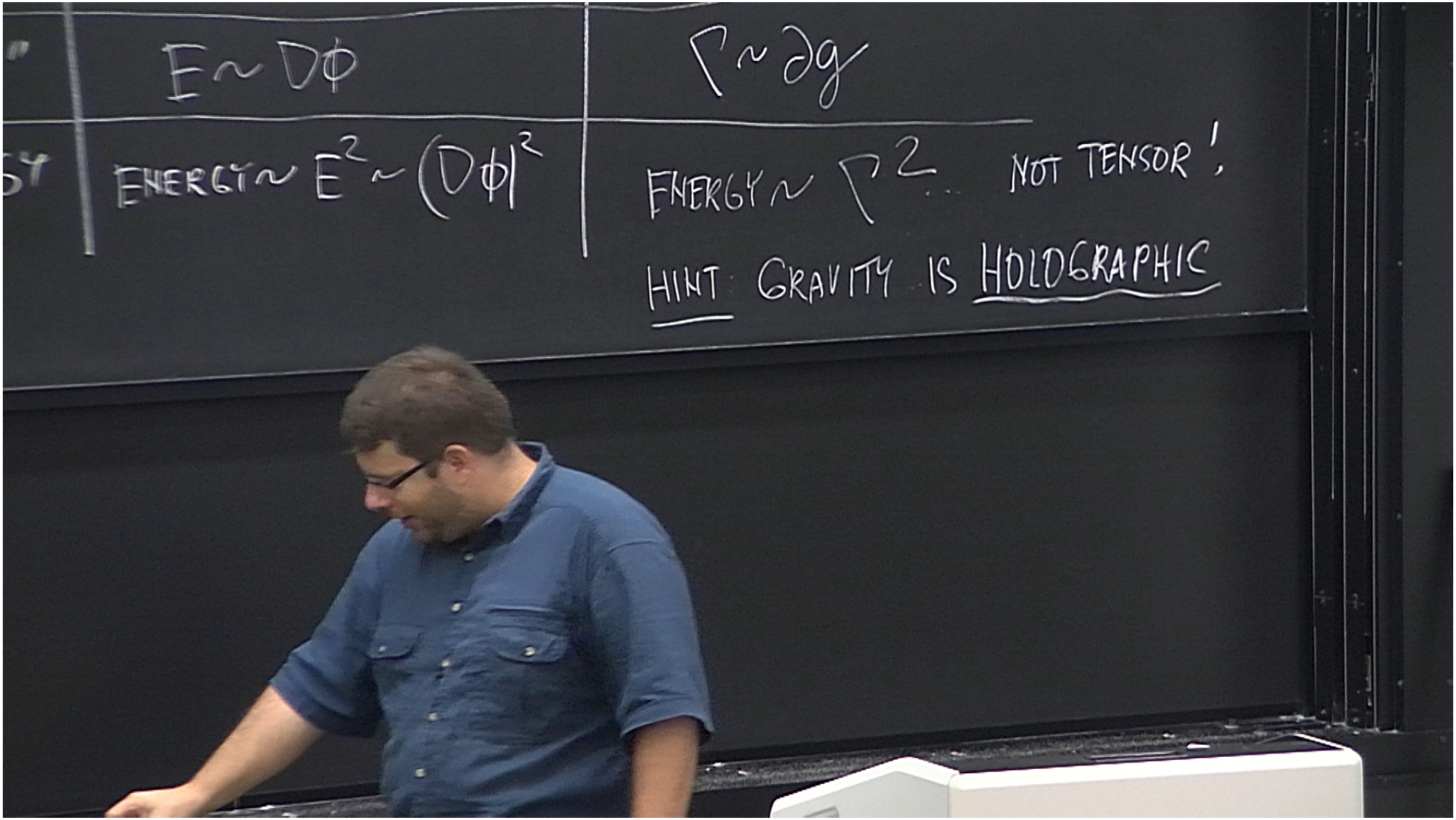

PROBLEMATIC: ANALOGY WITH MAXWELL

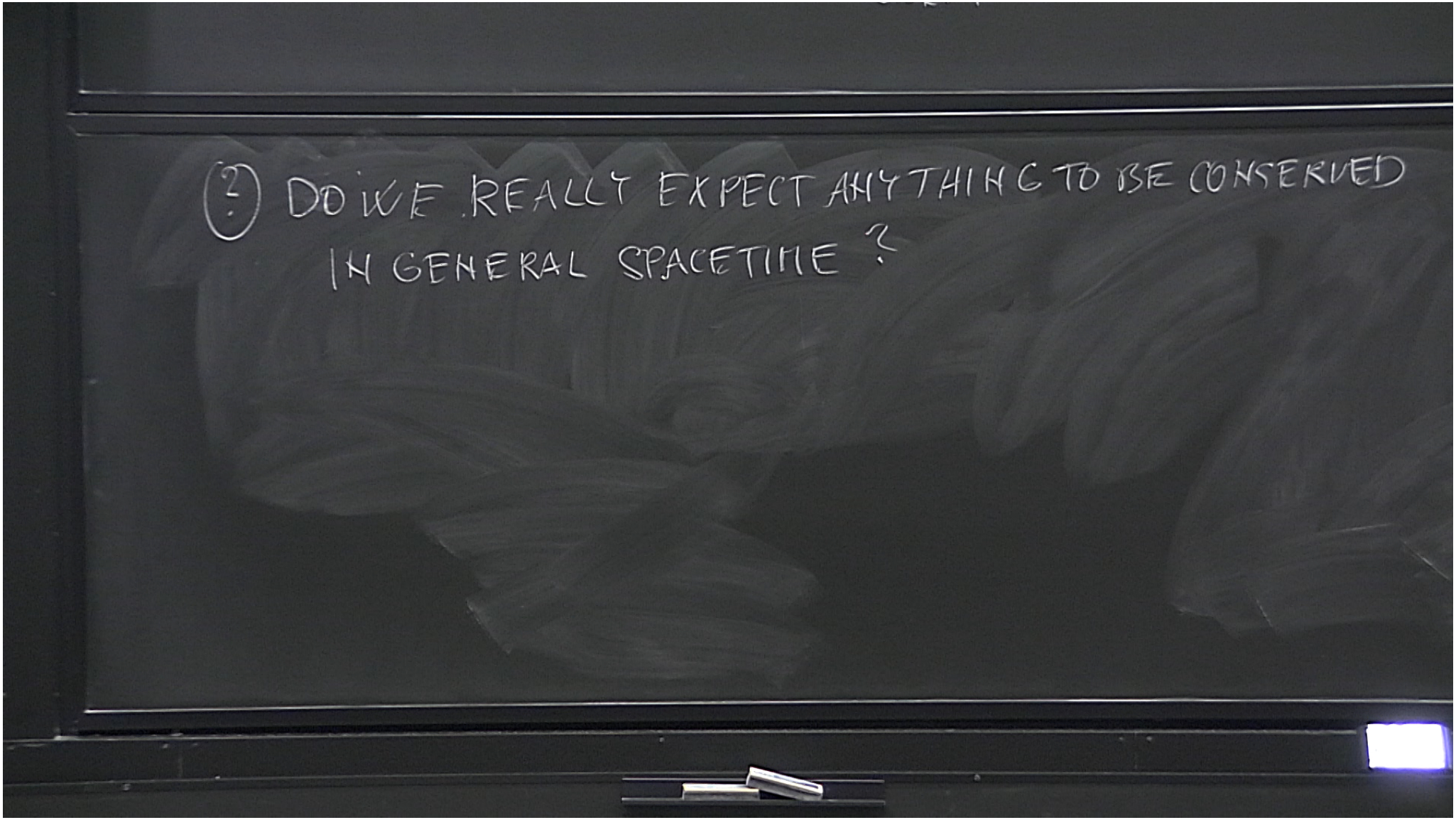

PROBLEMATIC : ANALOGY WITH MAXWELL
MAXWELL
GRAVITY
FIELD
ϕ ... EL STAT. POT
g

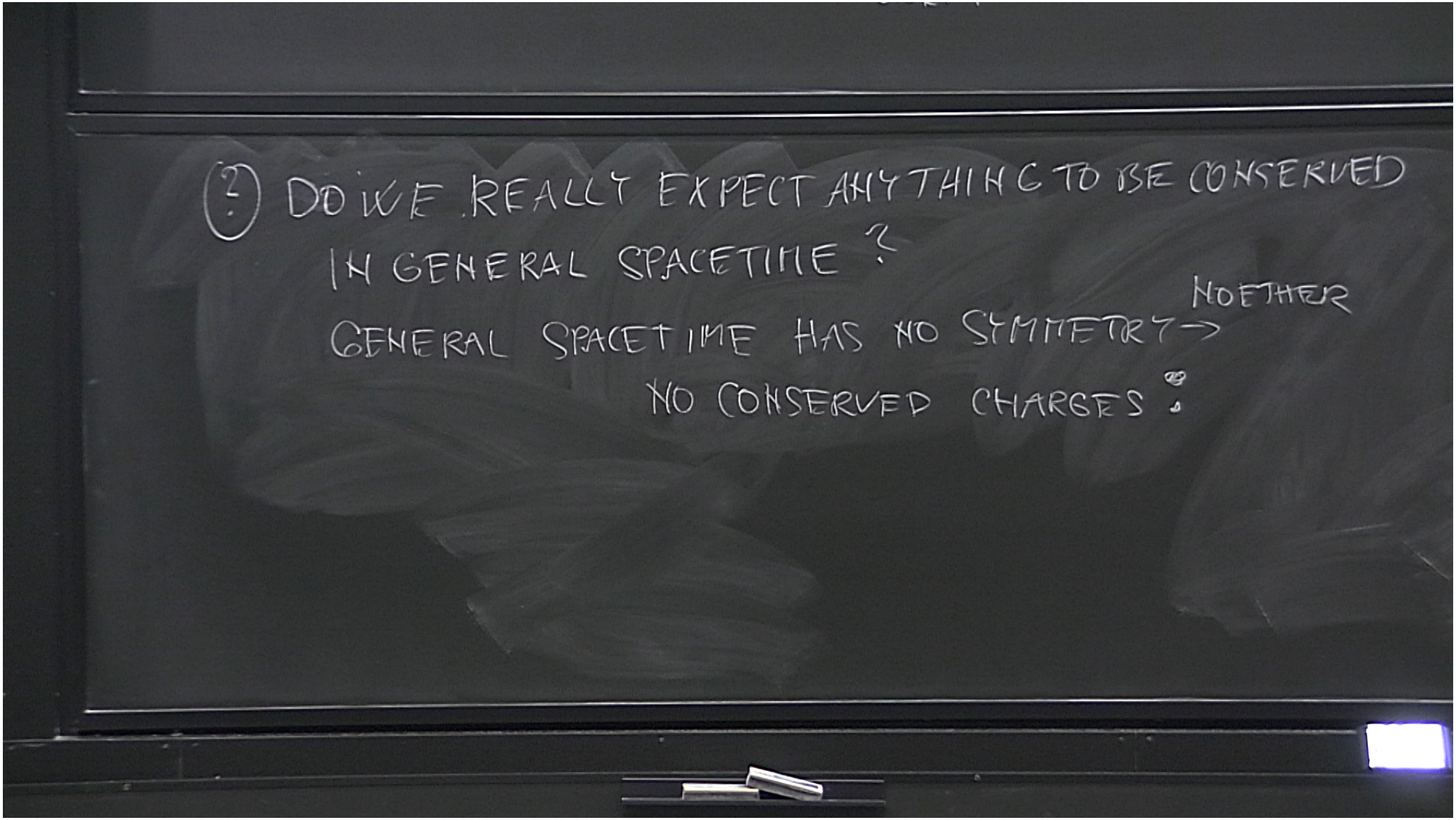

MAXWELL
GRAVITY
FIELD
ϕ ... EL STAT. POT
g
"STRENGTH"
$E \sim D\phi$

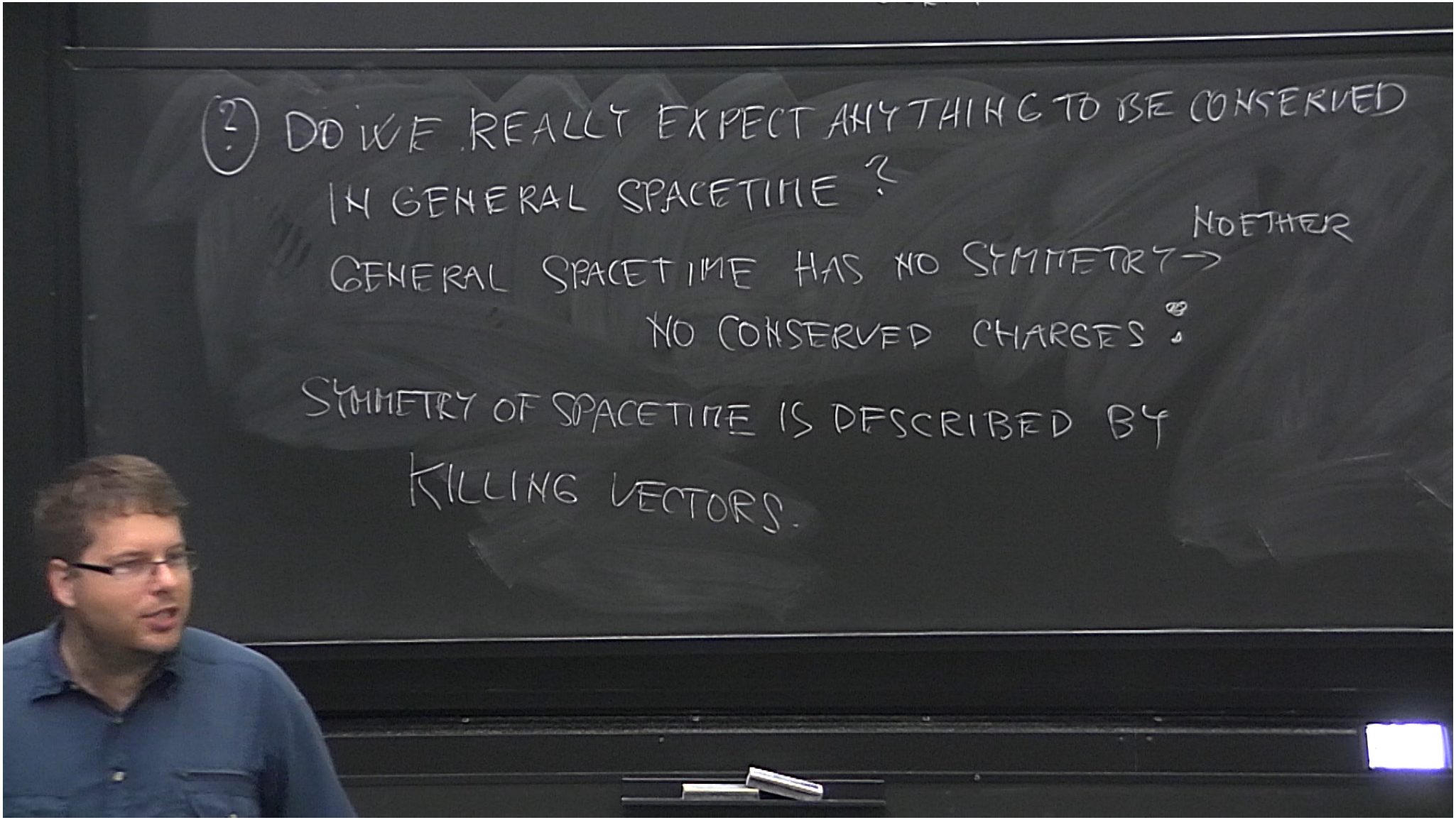

MAXWELL
FIELD
ϕ ... EL STAT. POT
g
"STRENGTH"
$E \sim D\phi$
$\Gamma \sim \partial g$

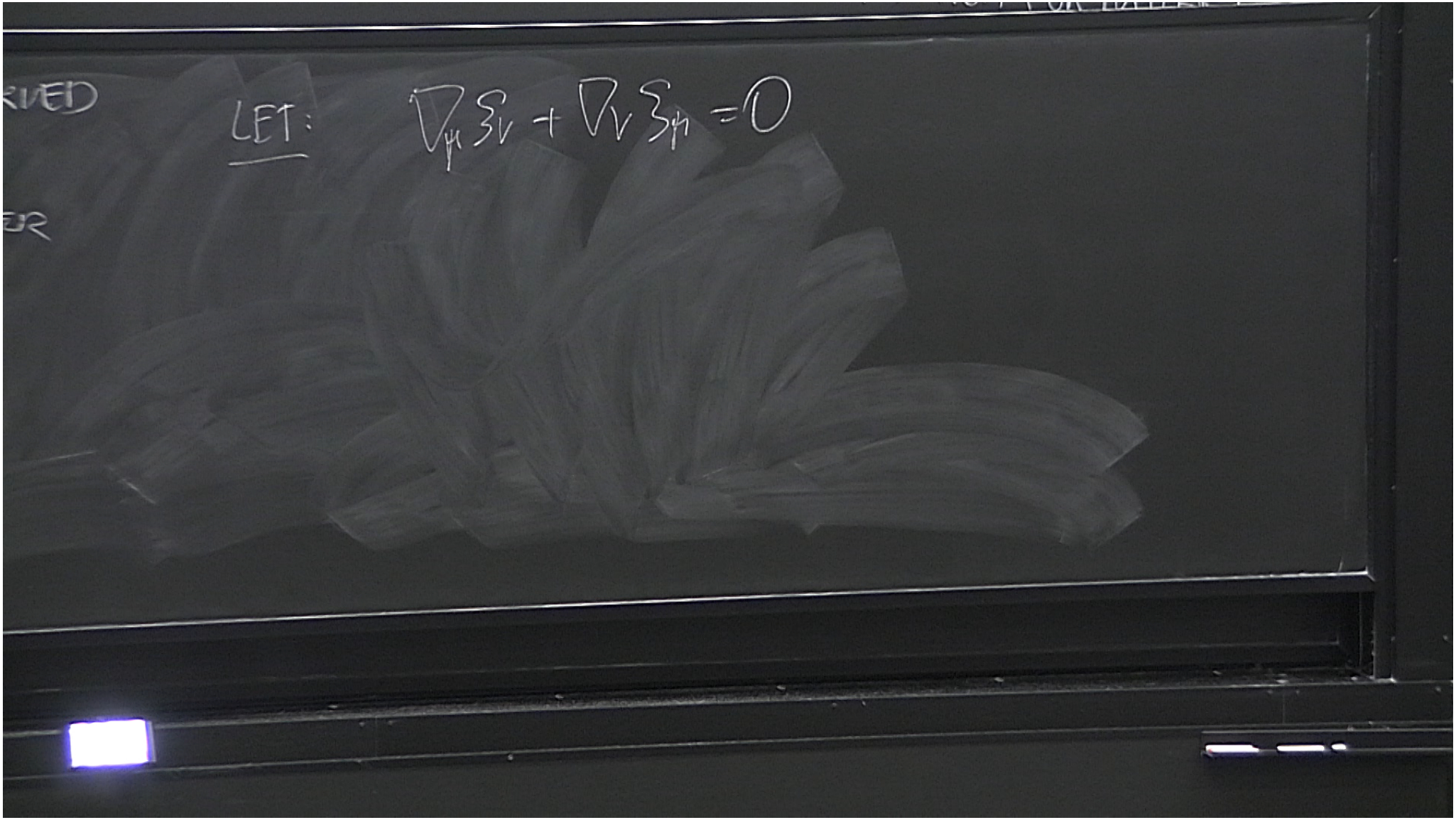

FIELD
Φ ... ELSTAT. POT
"STRENGTH"
E ~ Dϕ
ENERGY
ENERGY ~ E² ~ (Dϕ)²
ENERGY ~ ρ²

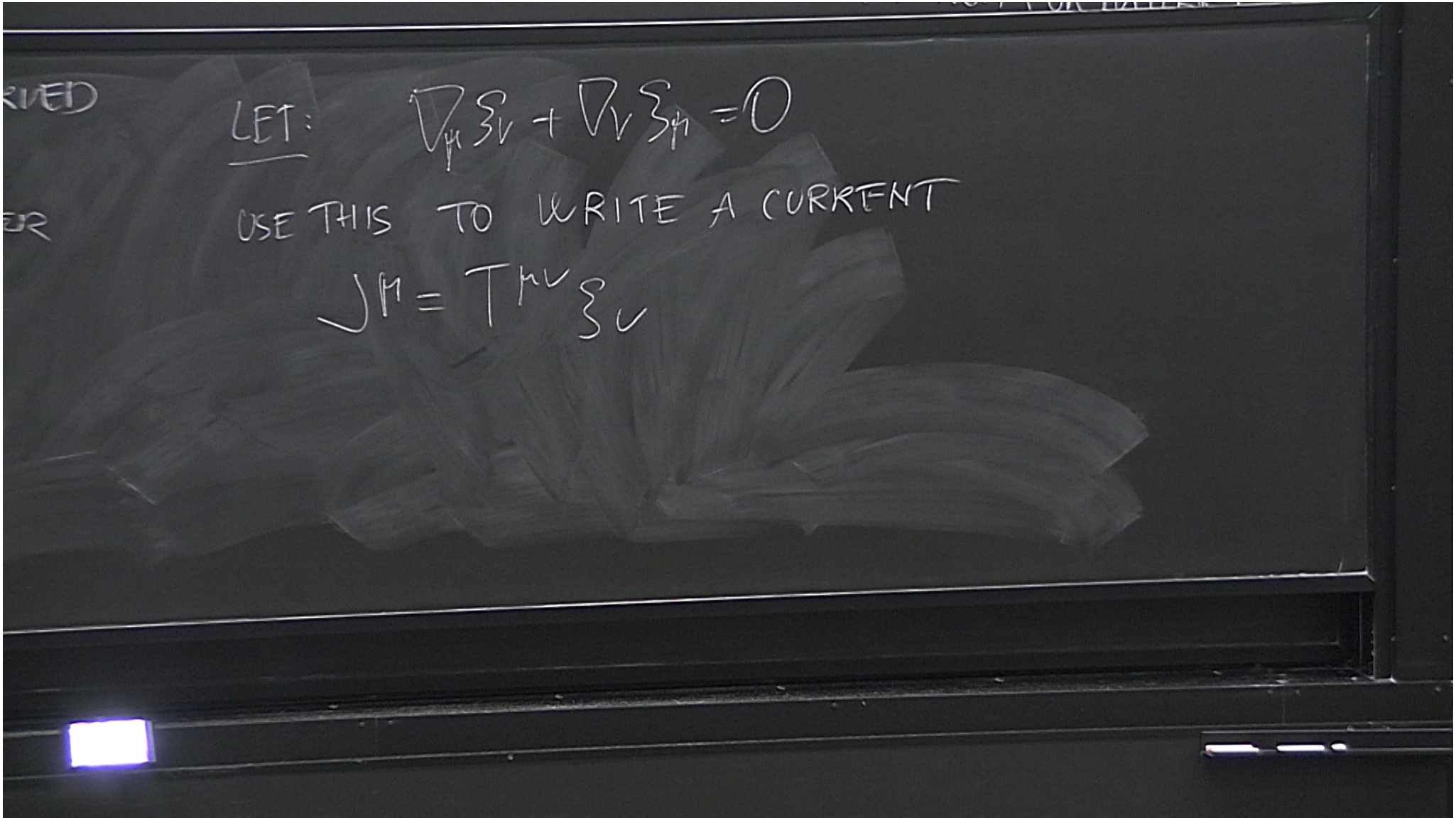

E ~ Dϕ
Γ ~ ∂g
ENERGY ~ E² ~ (Dϕ)²
ENERGY ~ Γ² ... NOT TENSOR!
HINT: GRAVITY IS HOLOGRAPHIC

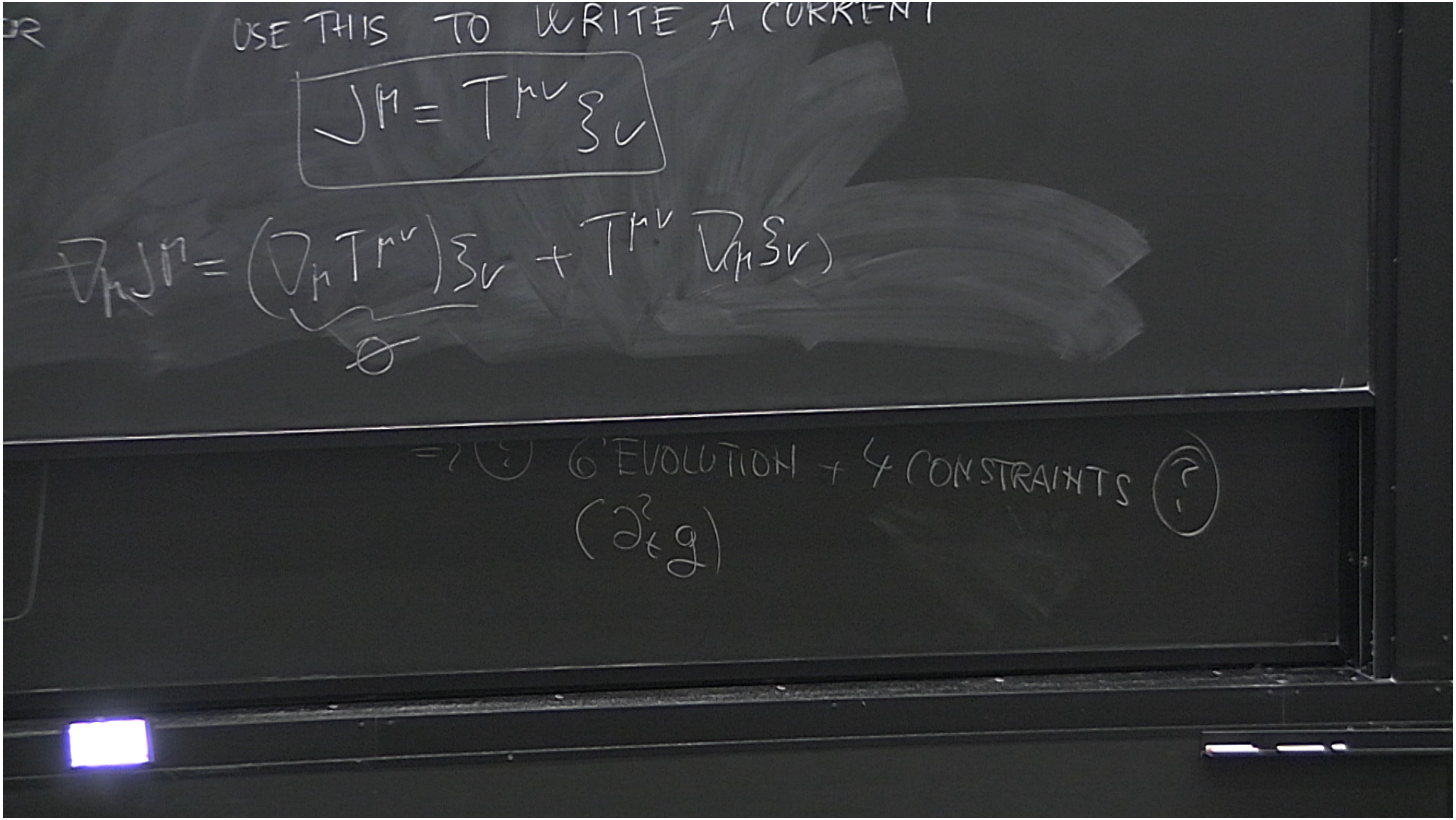

(?) DO WE REALLY EXPECT ANYTHING TO BE CONSERVED
IN GENERAL SPACETIME ?

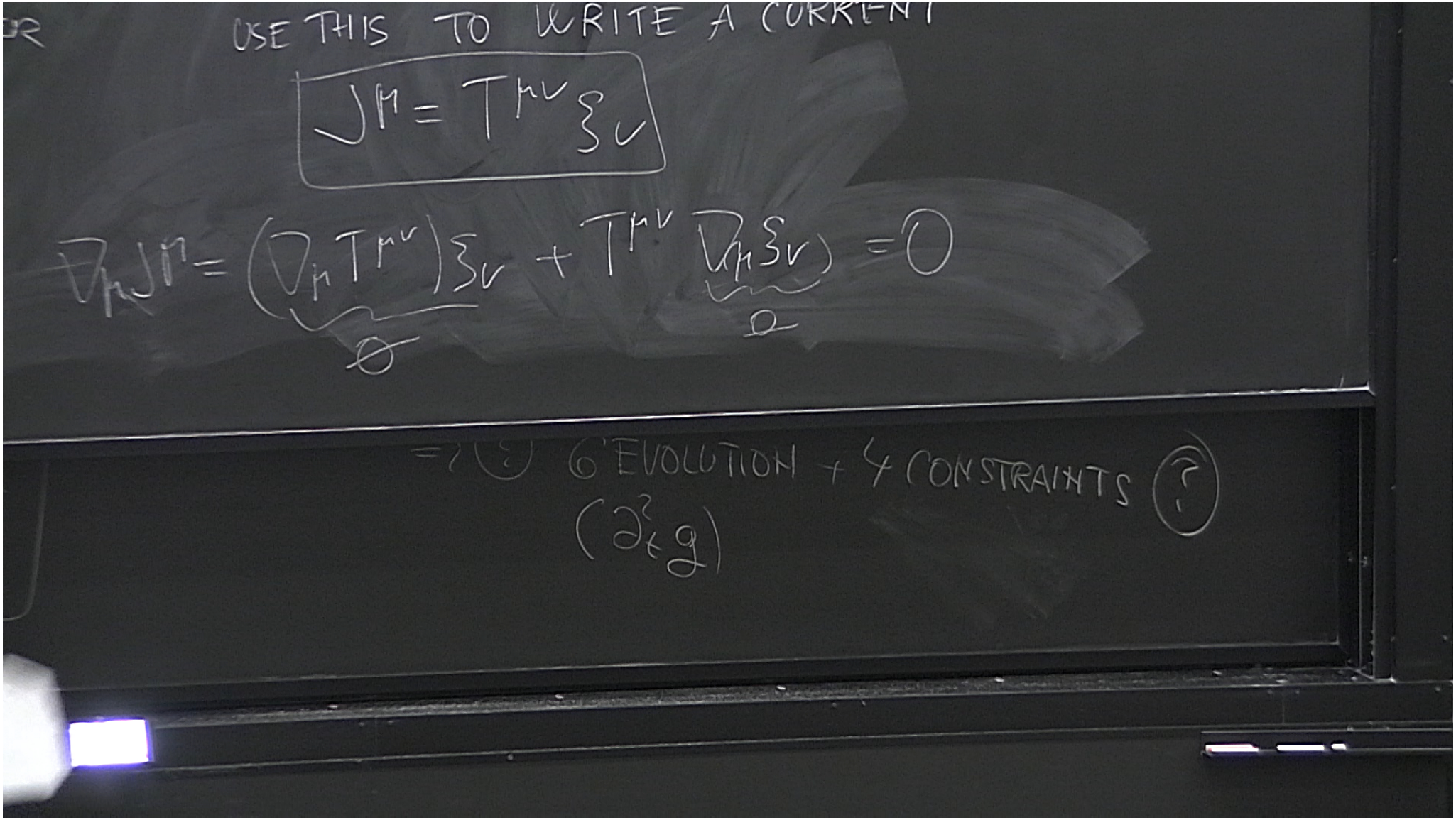

(?) DO WE REALLY EXPECT ANYTHING TO BE CONSERVED
IN GENERAL SPACETIME ?
GENERAL SPACETIME HAS NO SYMMETRY → NOETHER
NO CONSERVED CHARGES !

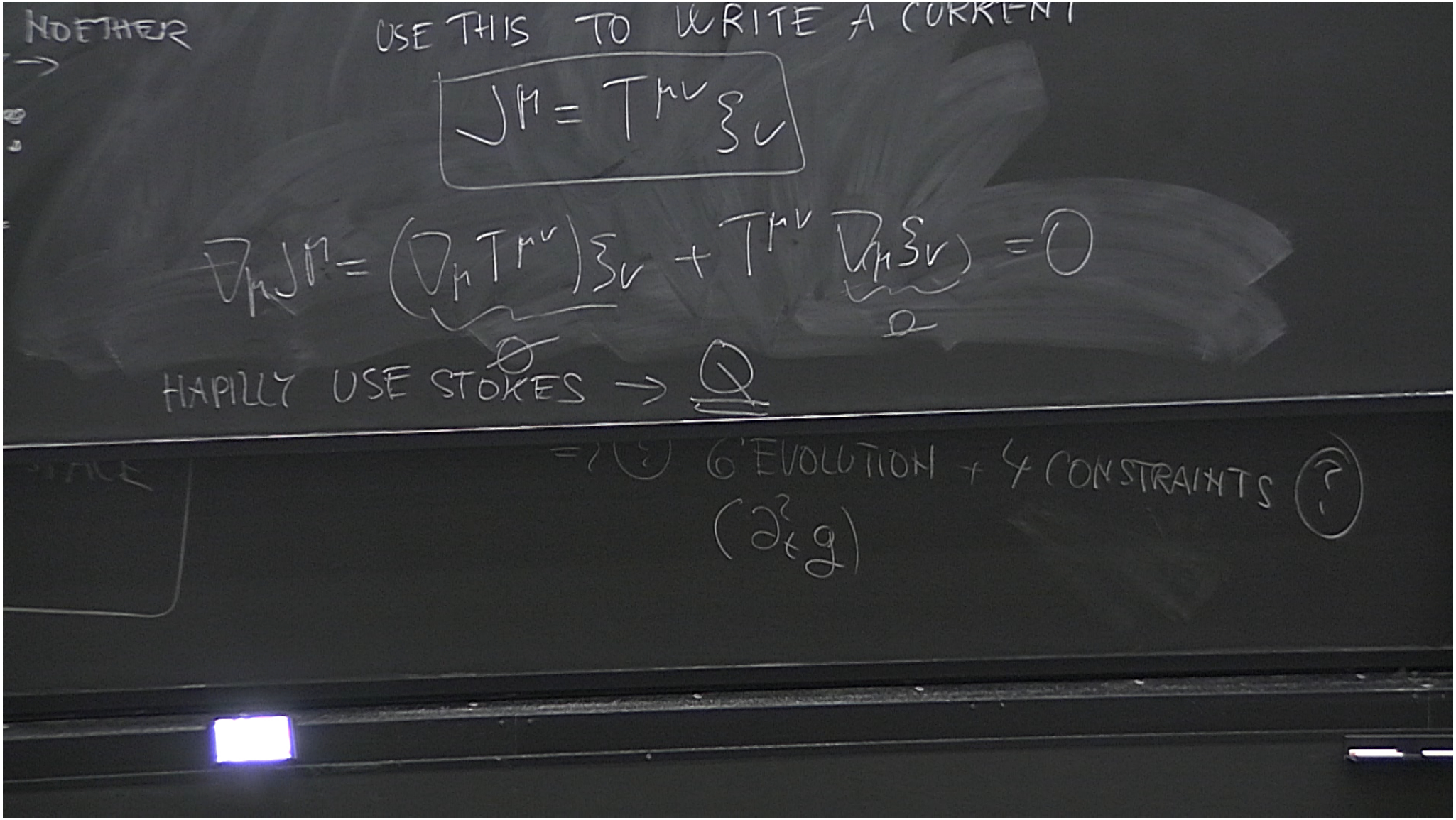

(?) DO WE REALLY EXPECT ANYTHING TO BE CONSERVED IN GENERAL SPACETIME ?
GENERAL SPACETIME HAS NO SYMMETRY → NOETHER
NO CONSERVED CHARGES !
SYMMETRY OF SPACETIME IS DESCRIBED BY
KILLING VECTORS.

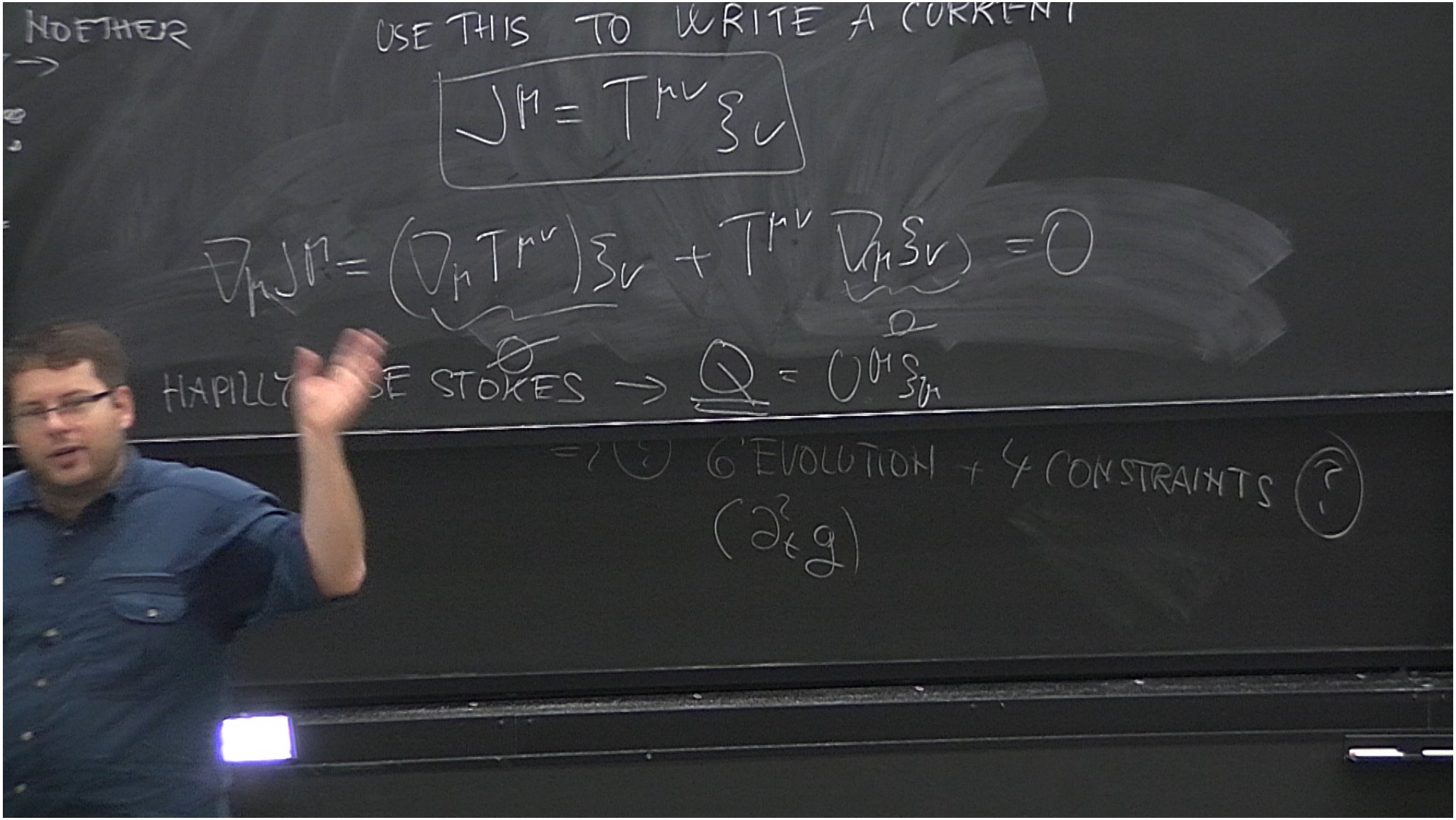

LET: $\nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = 0$

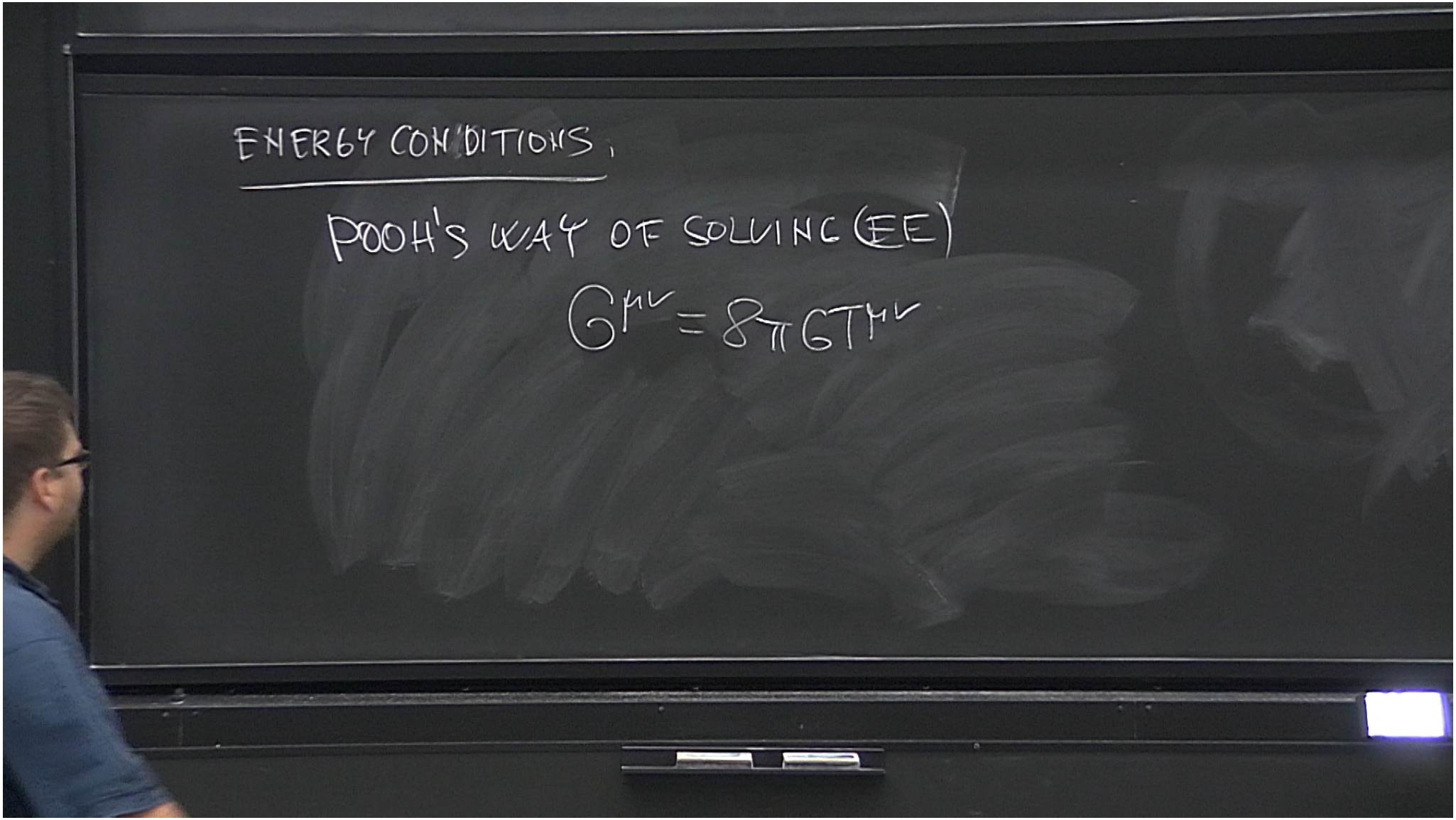

LET: $\nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = 0$
USE THIS TO WRITE A CURRENT
$J^\mu = T^{\mu\nu} \xi_\nu$

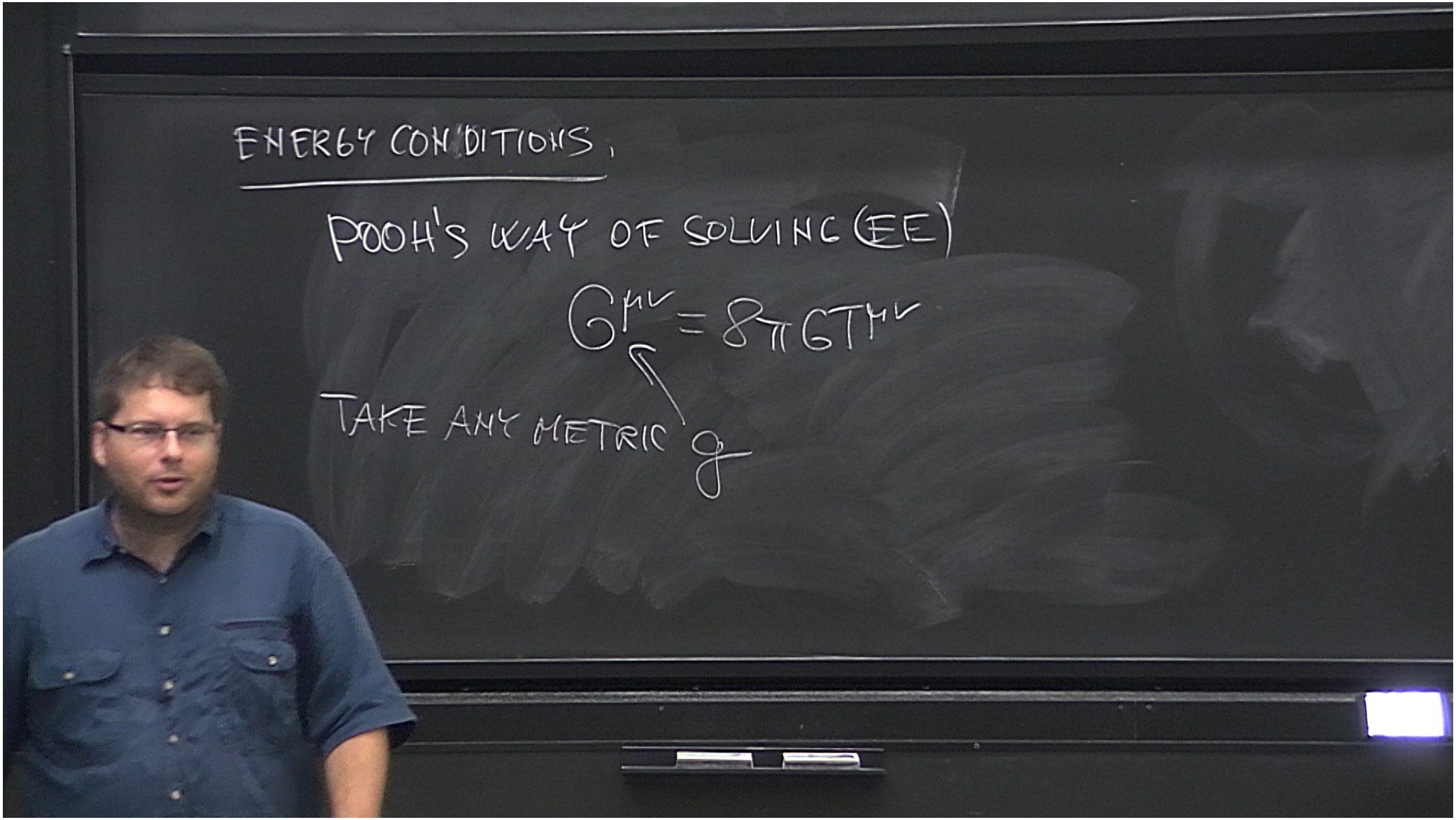

USE THIS TO WRITE A CURRENT
$J^\mu = T^{\mu\nu}\xi_\nu$
$\nabla_\mu J^\mu = (\nabla_\mu T^{\mu\nu})\xi_\nu + T^{\mu\nu}\nabla_{(\mu}\xi_{\nu)}$
$\Rightarrow$ 6 EVOLUTION + 4 CONSTRAINTS
$(\partial_t^2 g)$

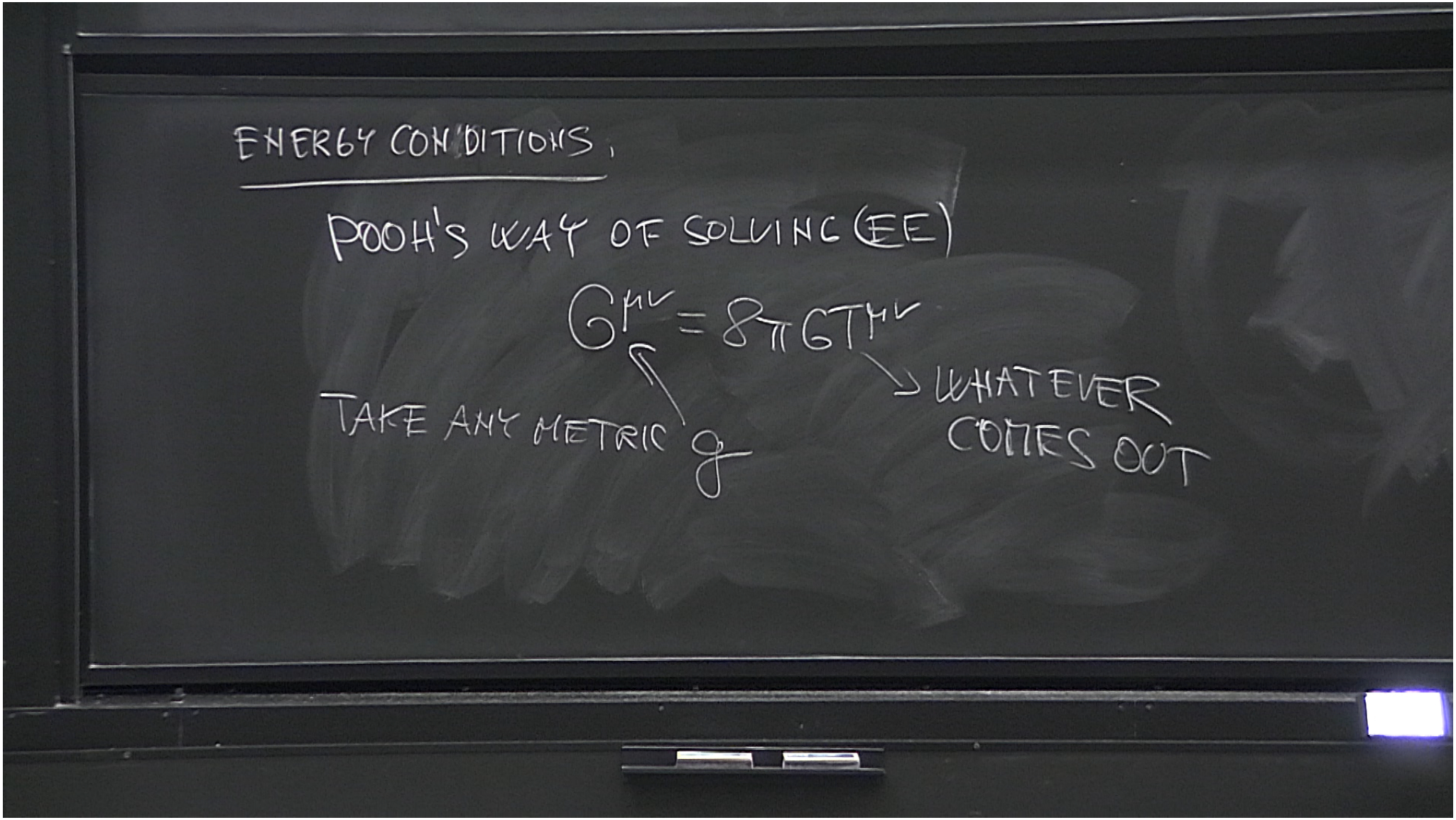

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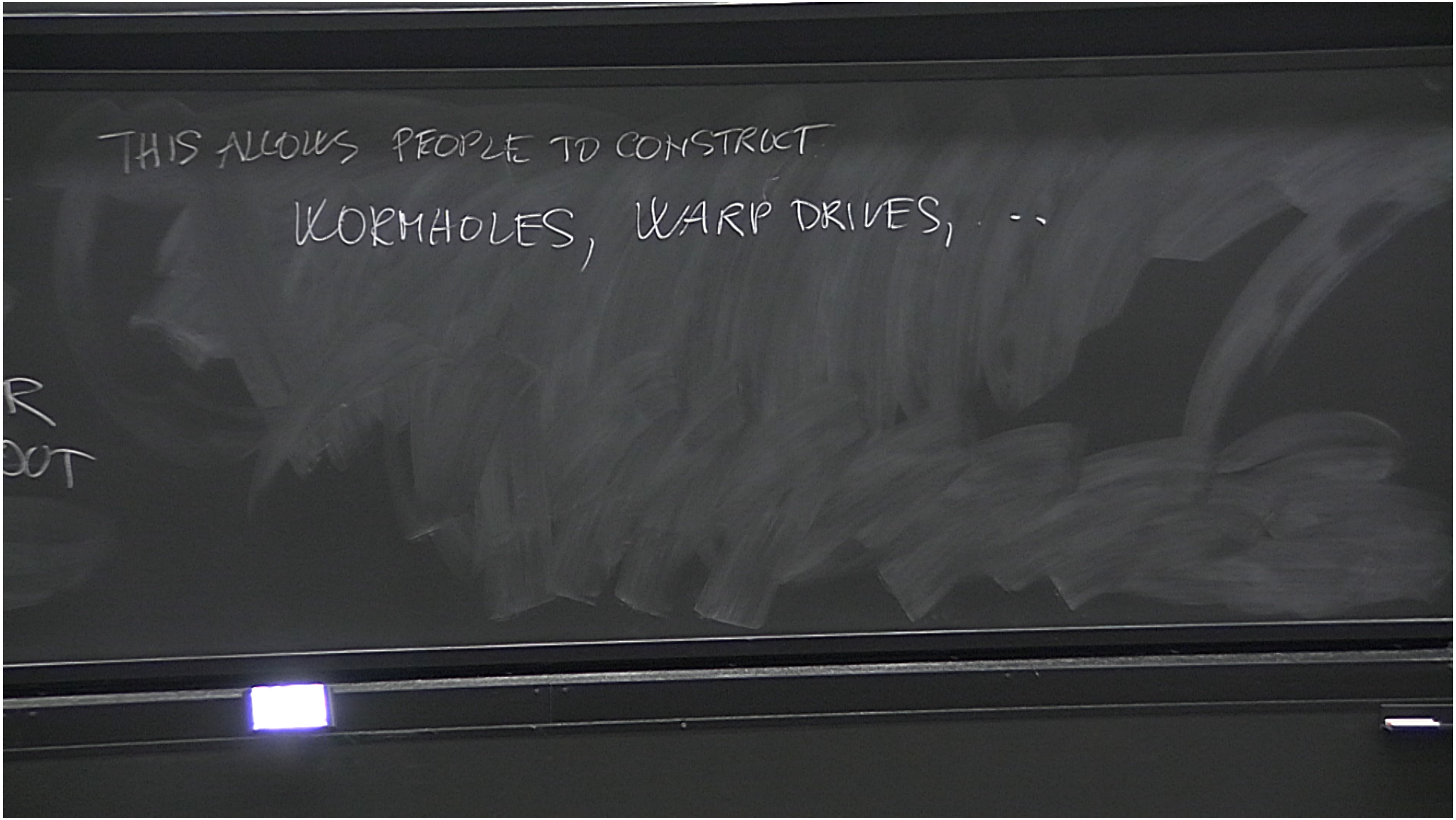

NOETHER
USE THIS TO WRITE A CURRENT
$J^\mu = T^{\mu\nu}\xi_\nu$
$\nabla_\mu J^\mu = (\nabla_\mu T^{\mu\nu})\xi_\nu + T^{\mu\nu}\nabla_{(\mu}\xi_{\nu)} = 0$
0
0
HAPILY USE STOKES → Q
=> (?) 6 EVOLUTION + 4 CONSTRAINTS (?)
$(\partial_t^2 g)$

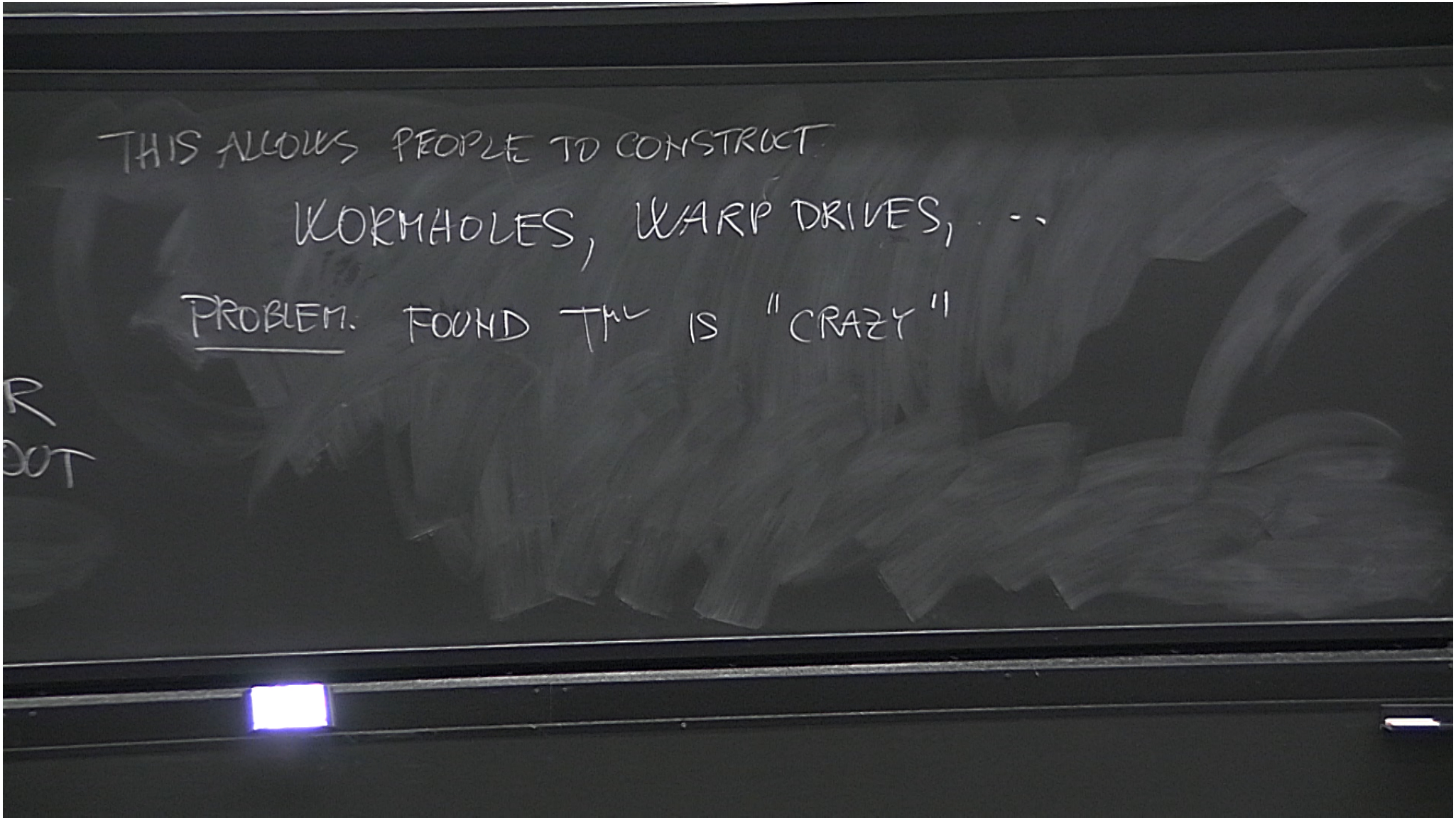

NOETHER
USE THIS TO WRITE A CURRENT
$J^\mu = T^{\mu\nu}\xi_\nu$
$\nabla_\mu J^\mu = (\nabla_\mu T^{\mu\nu})\xi_\nu + T^{\mu\nu}\nabla_{(\mu}\xi_{\nu)} = 0$
HAPILLY ... STOKES → $Q = U^\mu S_{3\mu}$
6 EVOLUTION + 4 CONSTRAINTS
$(\partial_t^2 g)$

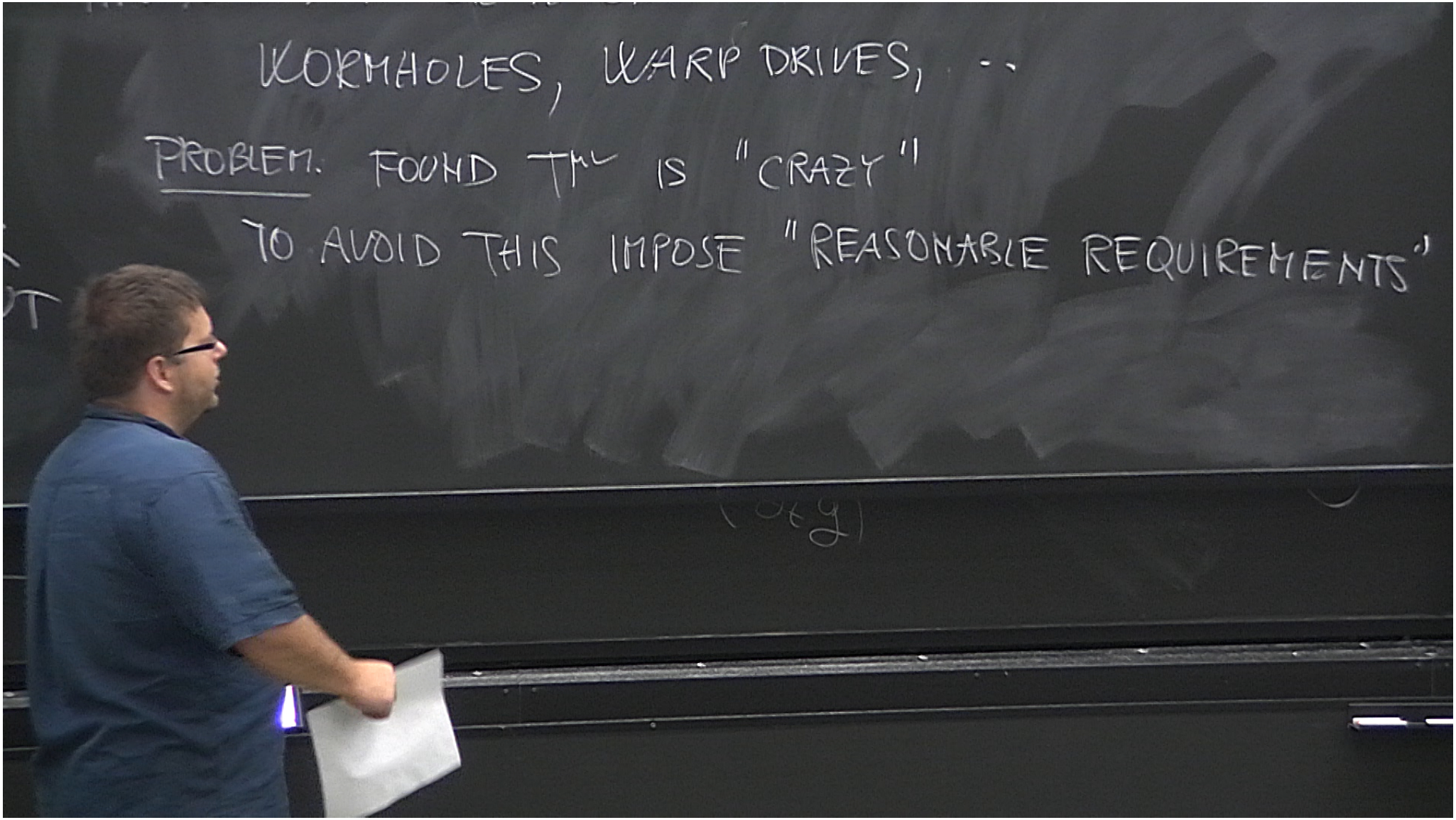

ENERGY CONDITIONS.
POOH'S WAY OF SOLVING (EE)
$G^{\mu\nu} = 8\pi G T^{\mu\nu}$


ENERGY CONDITIONS.
POOH'S WAY OF SOLVING (EE)
$G^{\mu\nu} = 8\pi G T^{\mu\nu}$
TAKE ANY METRIC g


ENERGY CONDITIONS.
POOH'S WAY OF SOLVING (EE)
$G^{\mu\nu} = 8\pi G T^{\mu\nu}$
TAKE ANY METRIC g
WHATEVER COMES OUT


THIS ALLOWS PEOPLE TO CONSTRUCT
WORMHOLES, WARP DRIVES, ..
R
OUT


THIS ALLOWS PEOPLE TO CONSTRUCT
WORMHOLES, WARP DRIVES, ..
PROBLEM. FOUND TH^NL IS "CRAZY"


WORMHOLES, WARP DRIVES, ..
PROBLEM. FOUND THM IS "CRAZY"
TO AVOID THIS IMPOSE "REASONABLE REQUIREMENTS"

HAPILY USE STOKES → $Q = \int_{S_{3+1}} \dots$

THIS ALLOWS PEOPLE TO CONSTRUCT

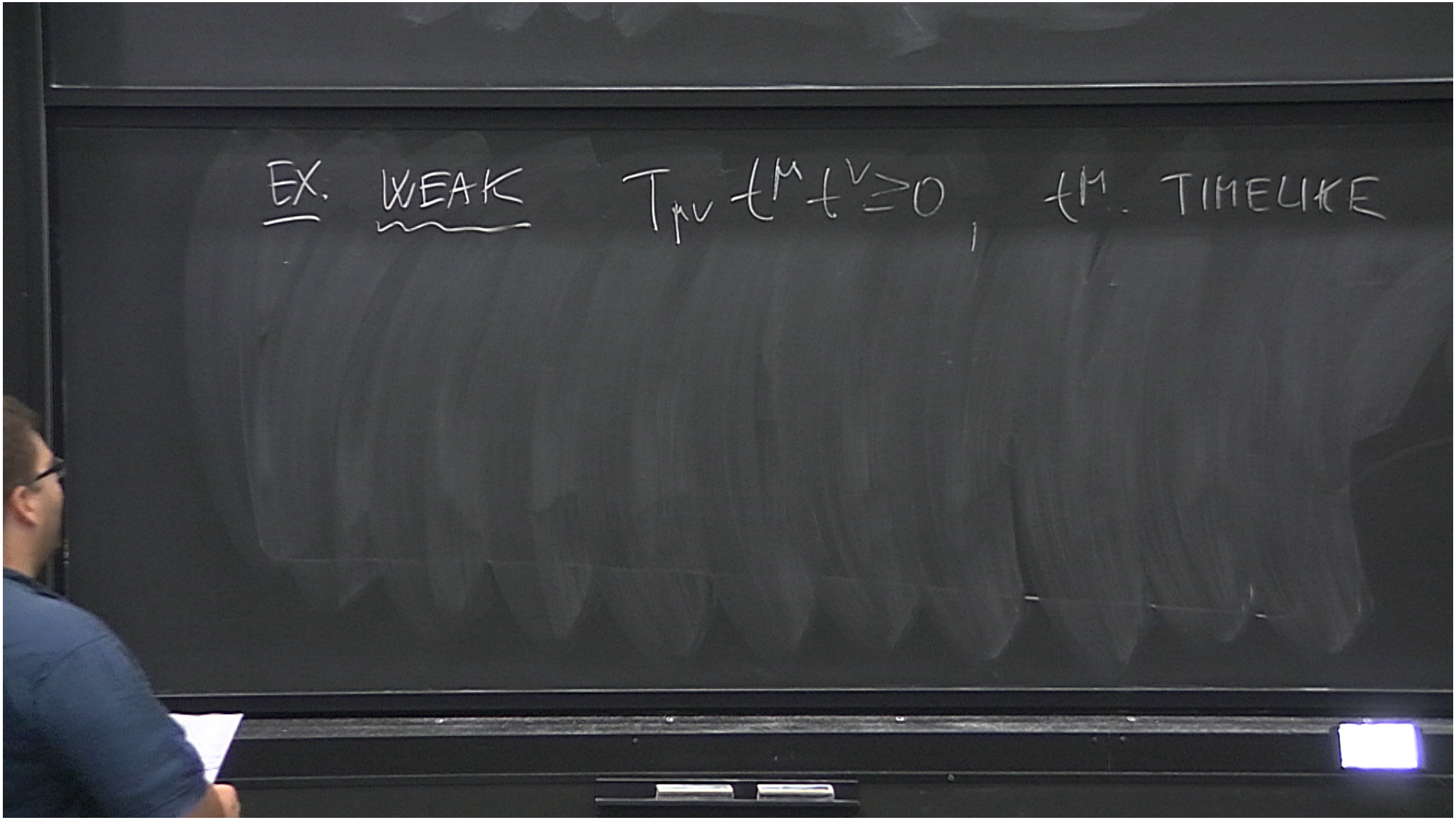
WORMHOLES, WARP DRIVES, ..

PROBLEM: FOUND $T^{\mu\nu}$ IS "CRAZY"

TO AVOID THIS IMPOSE "REASONABLE REQUIREMENTS"

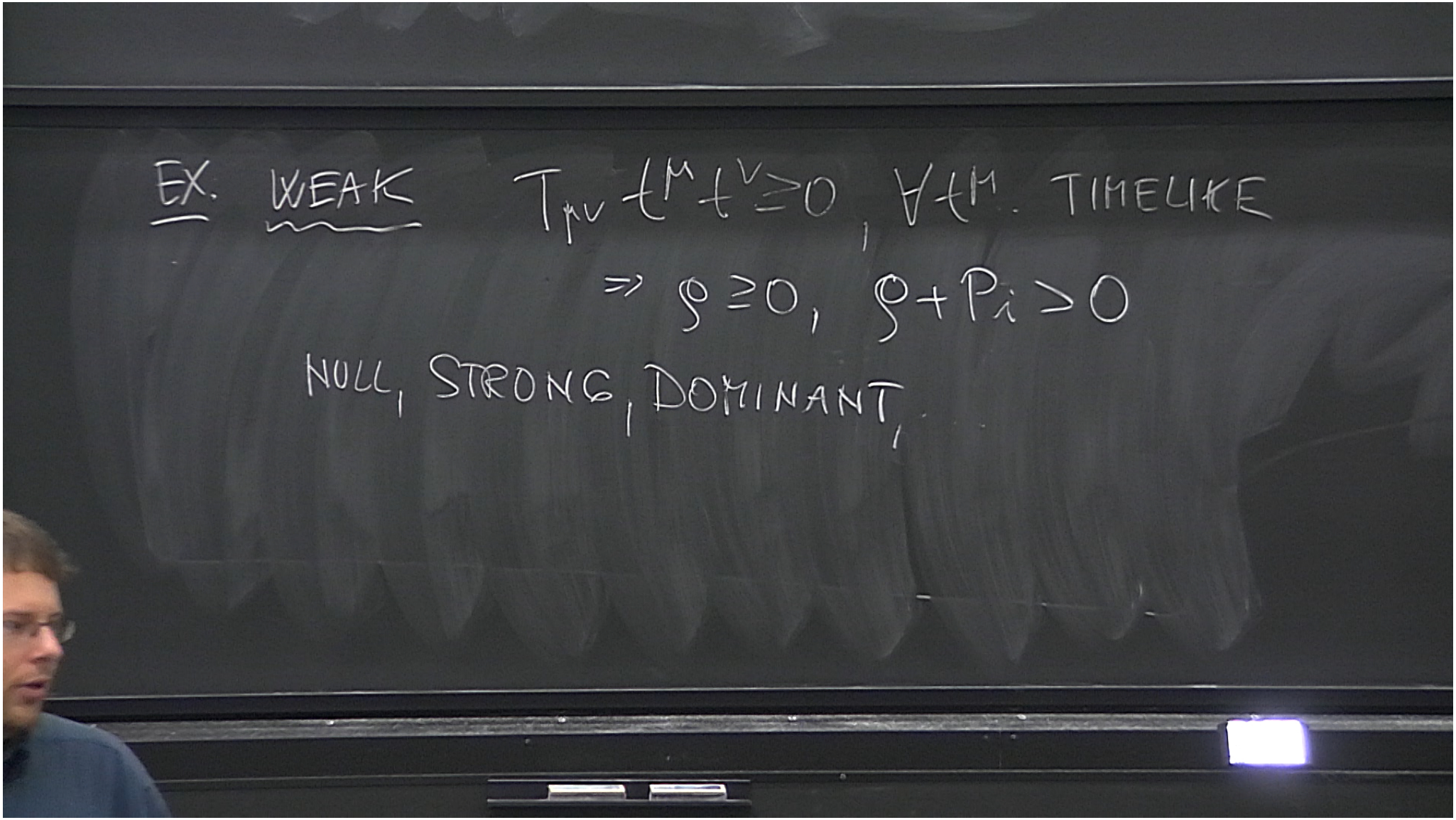
CALLED ENERGY CONDITIONS

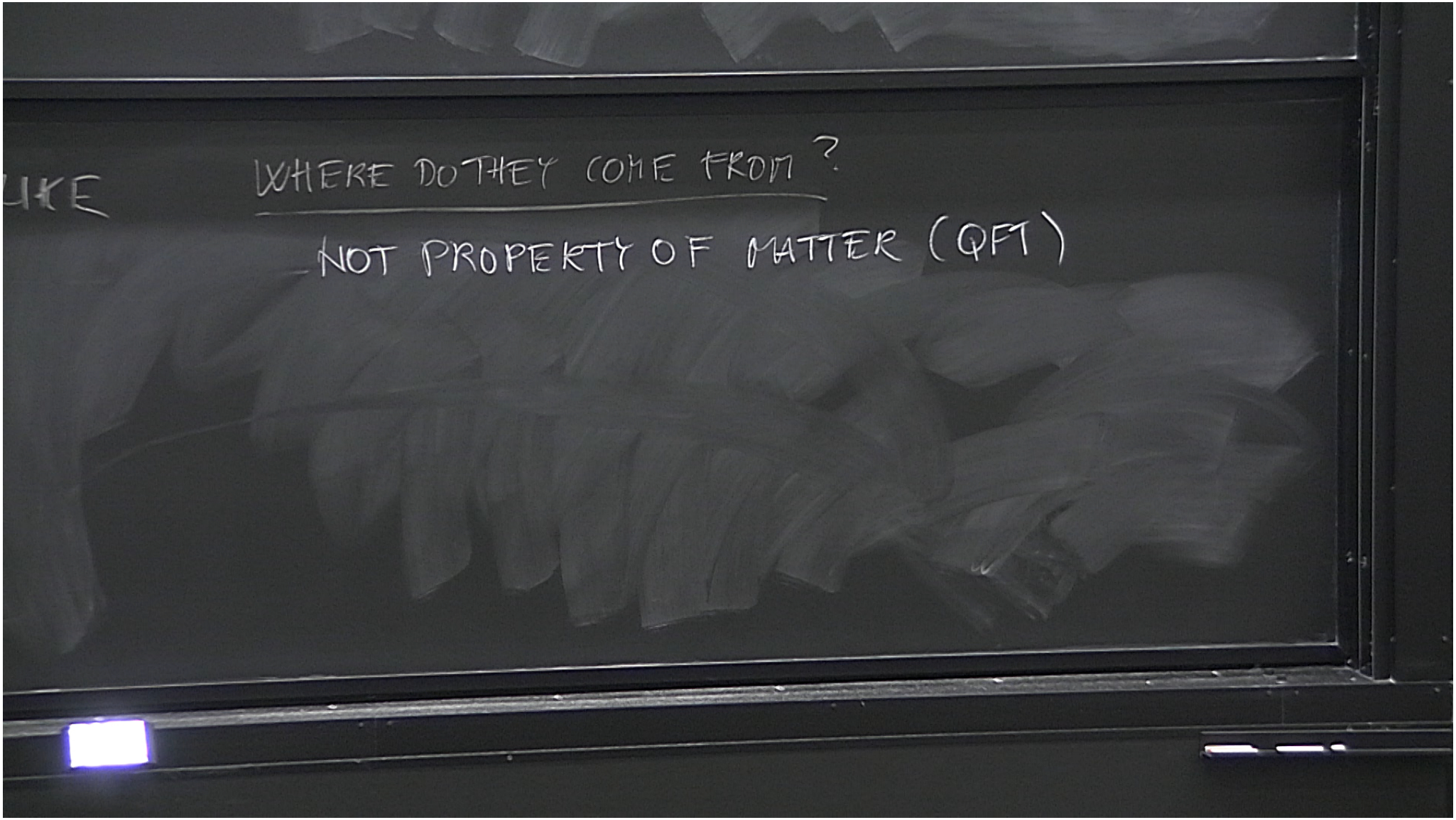
• HOW MANY EQS?

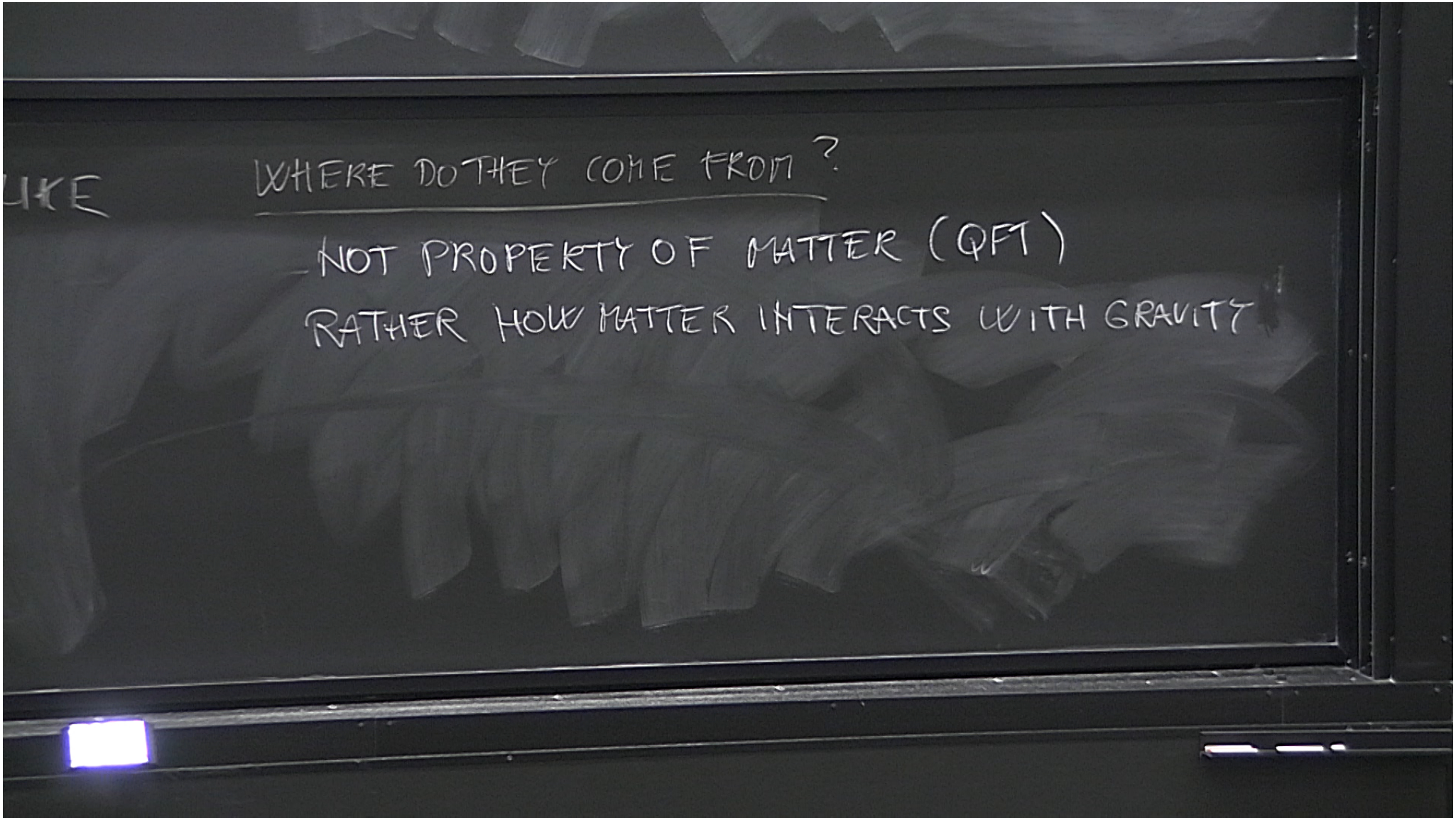

EX. WEAK $T_{\mu\nu} t^\mu t^\nu \geq 0$, t^μ TIMELIKE

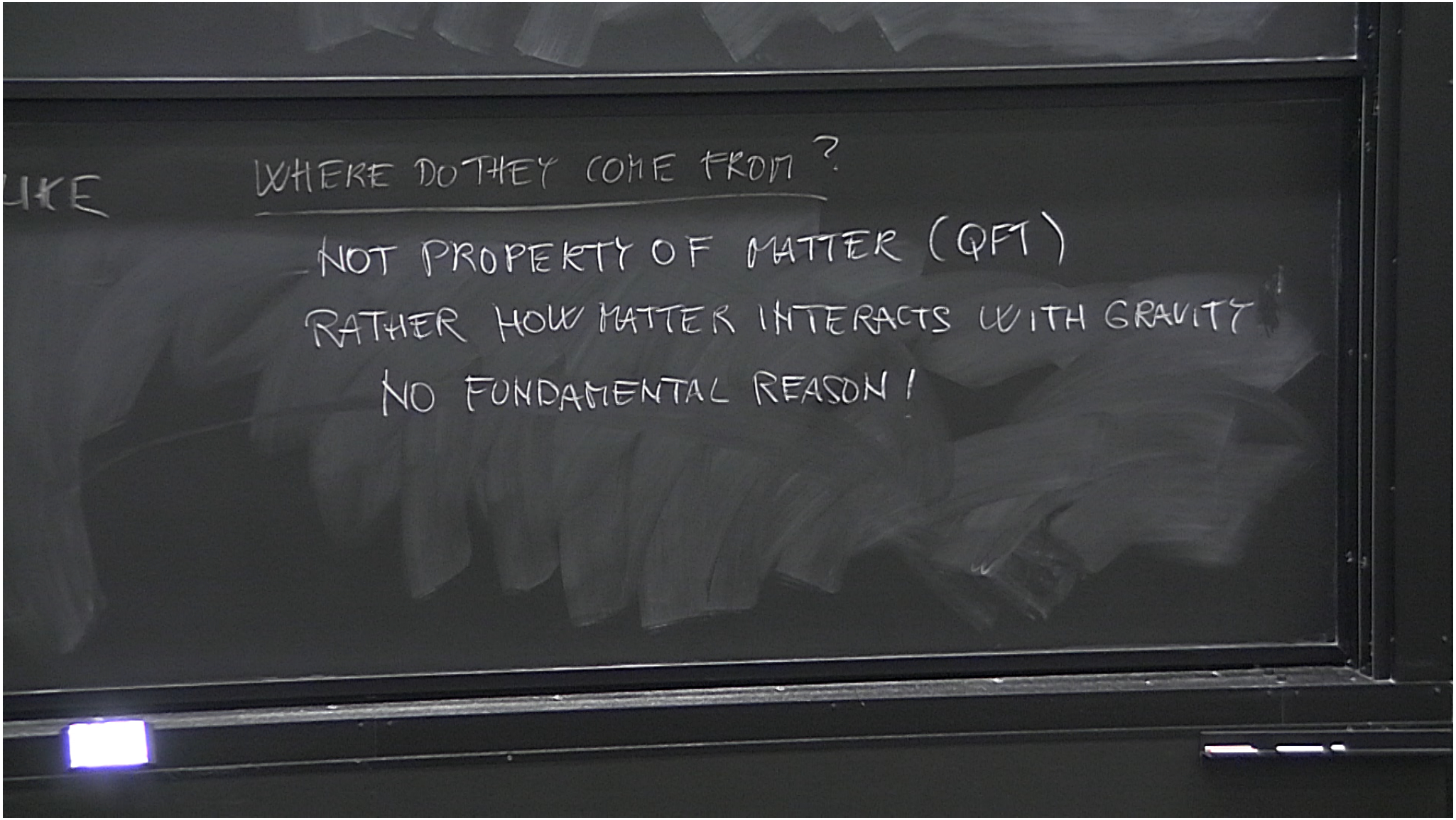
EX. WEAK $T_{\mu\nu} t^\mu t^\nu \geq 0$, $\forall t^\mu$ TIMELIKE

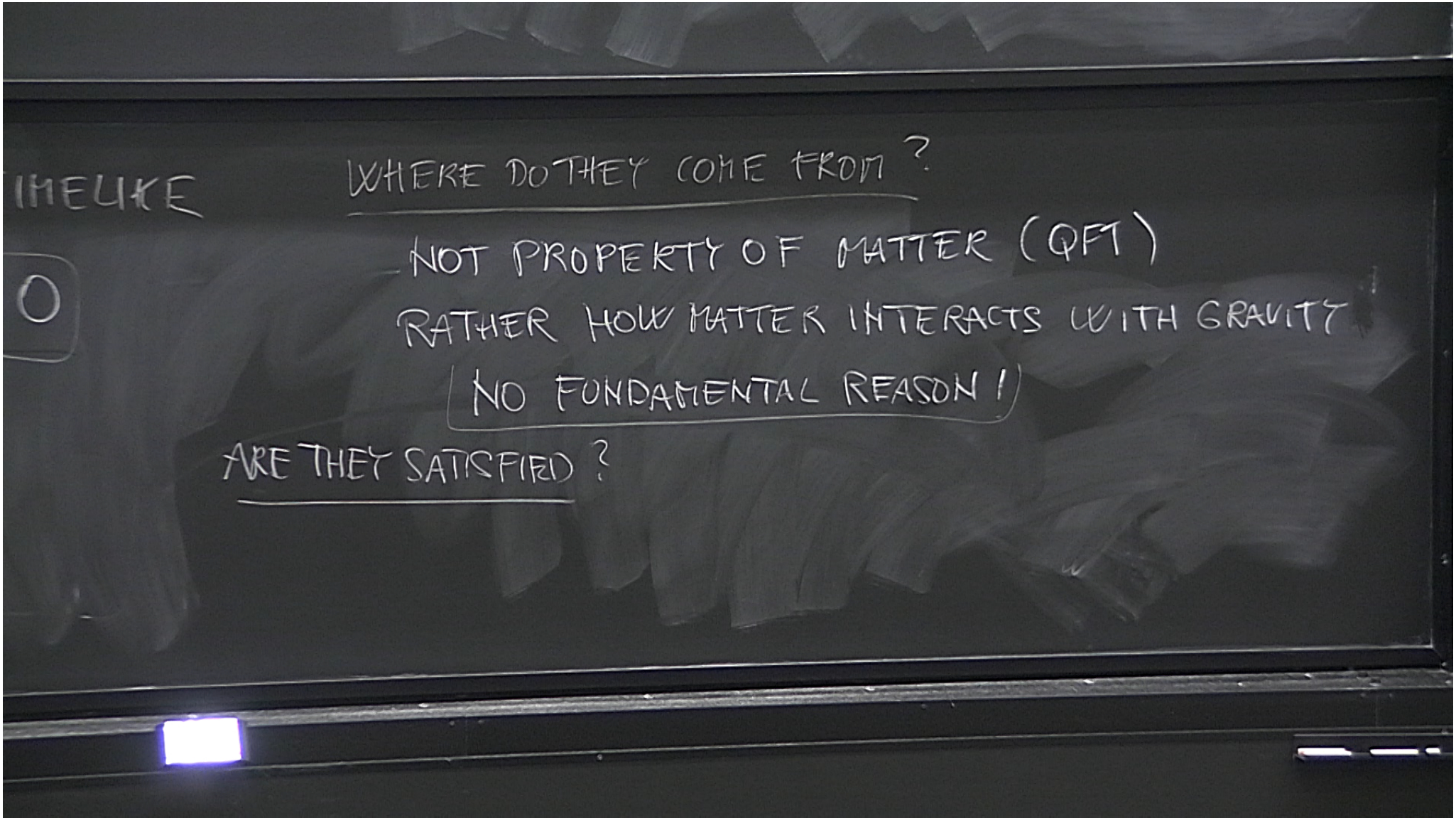
$\Rightarrow \rho \geq 0, \quad \rho + P_i > 0$

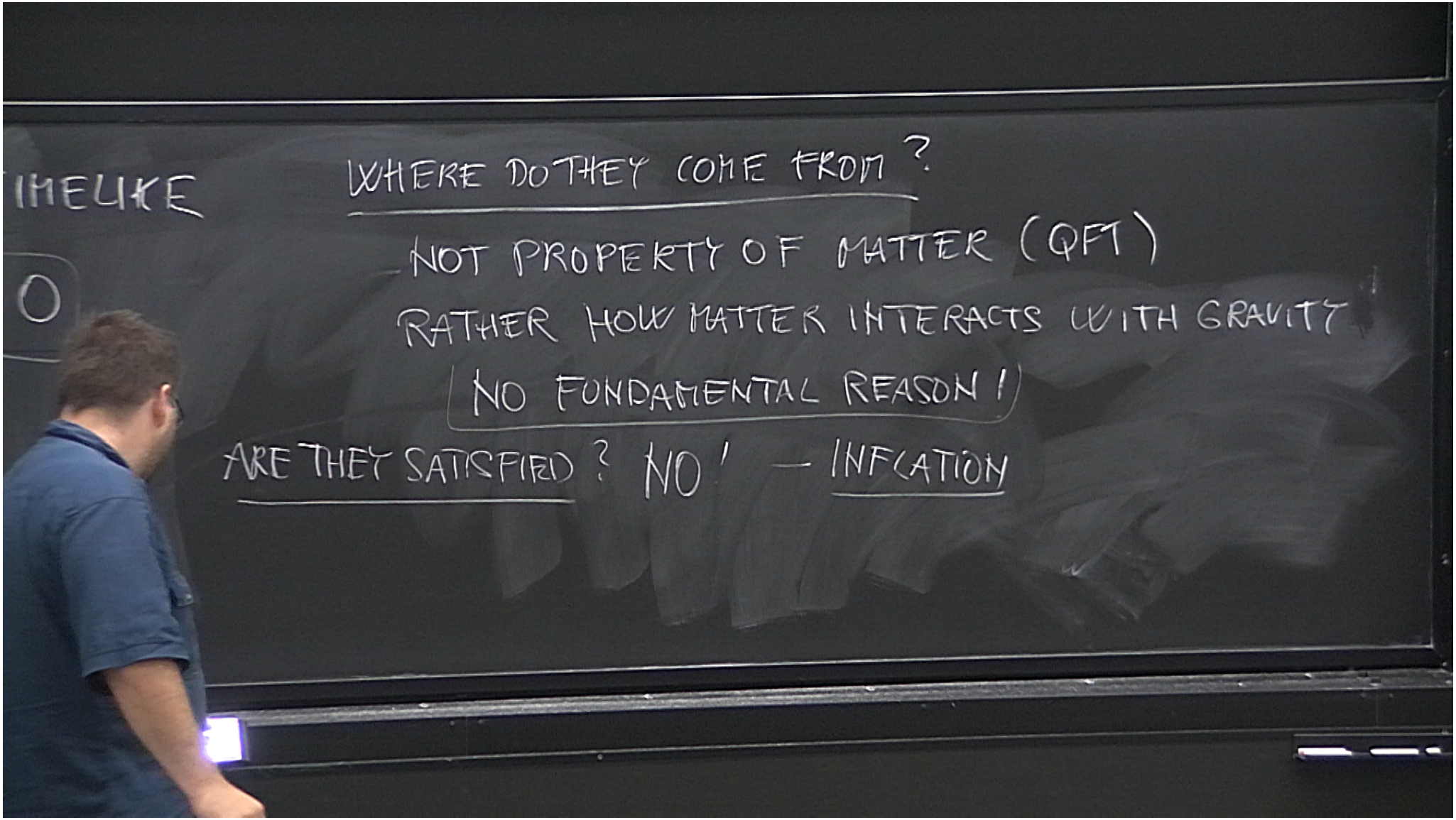

EX. WEAK $T_{\mu\nu} t^\mu t^\nu \geq 0$, $\forall t^\mu$ TIMELIKE
$\Rightarrow \rho \geq 0$, $\rho + P_i > 0$
NULL, STRONG, DOMINANT,

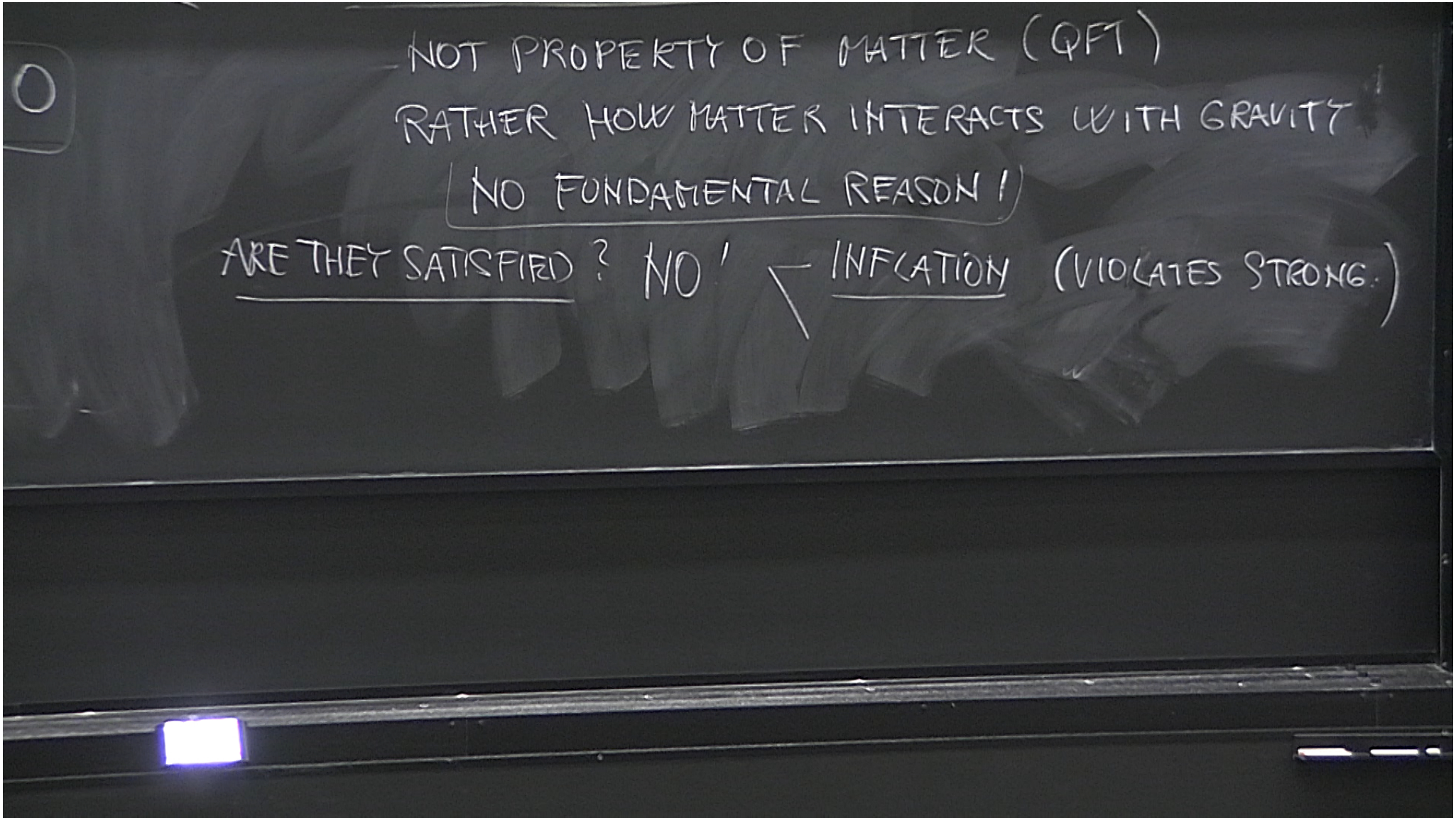

WHERE DO THEY COME FROM ?
NOT PROPERTY OF MATTER (QFT)

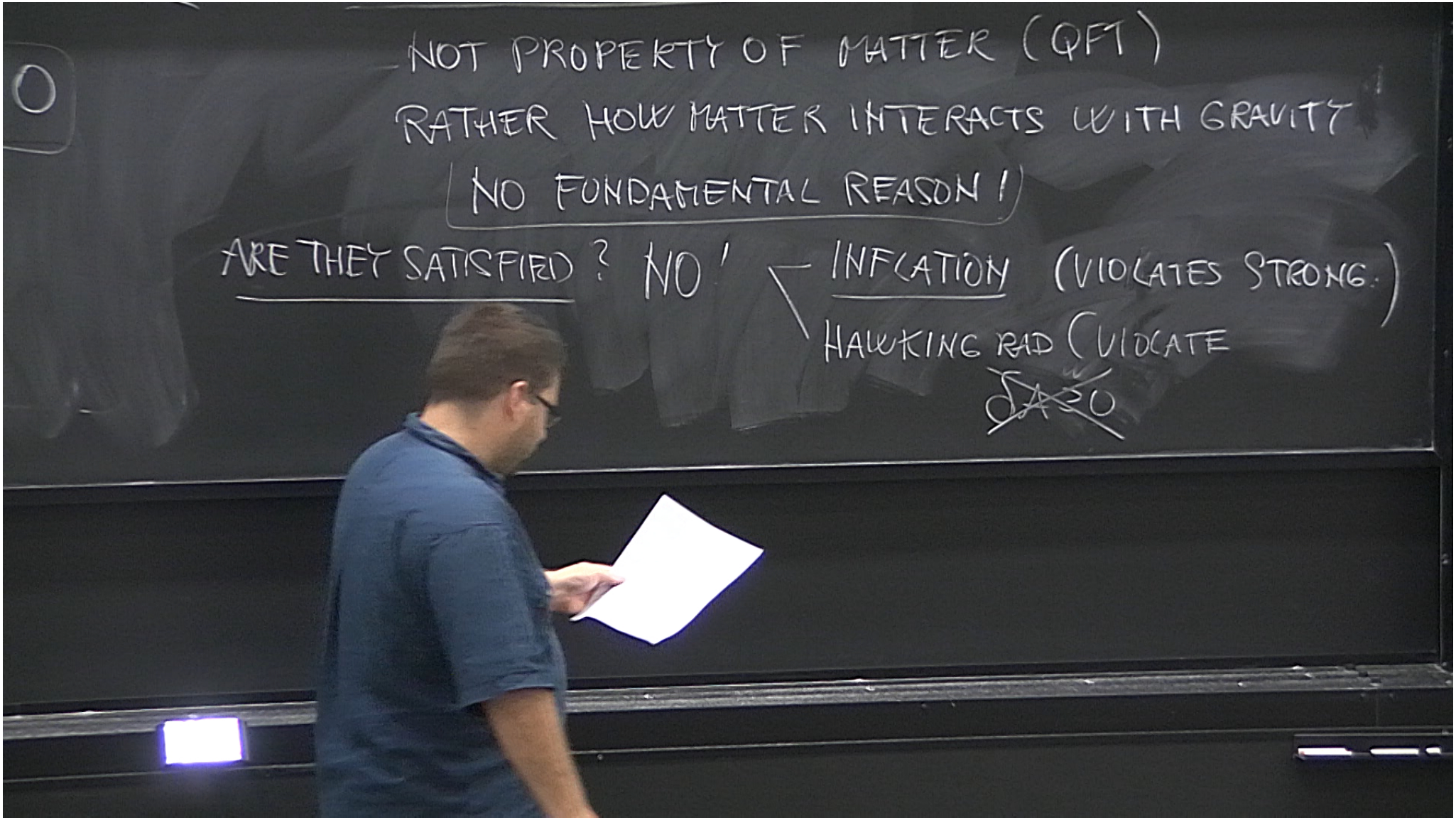

LIKE
WHERE DO THEY COME FROM ?
NOT PROPERTY OF MATTER (QFT)
RATHER HOW MATTER INTERACTS WITH GRAVITY

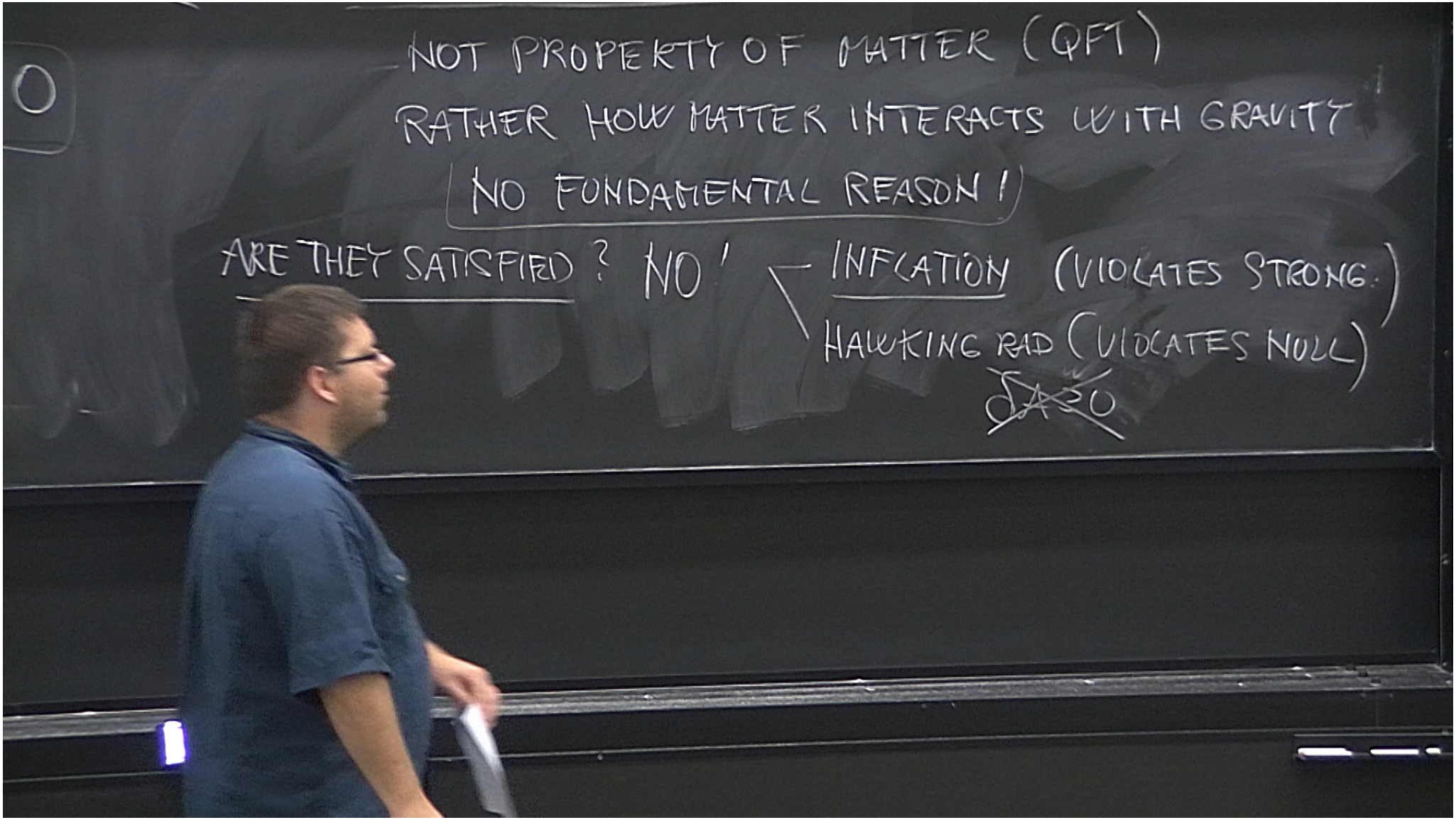

WHERE DO THEY COME FROM ?
NOT PROPERTY OF MATTER (QFT)
RATHER HOW MATTER INTERACTS WITH GRAVITY
NO FUNDAMENTAL REASON !

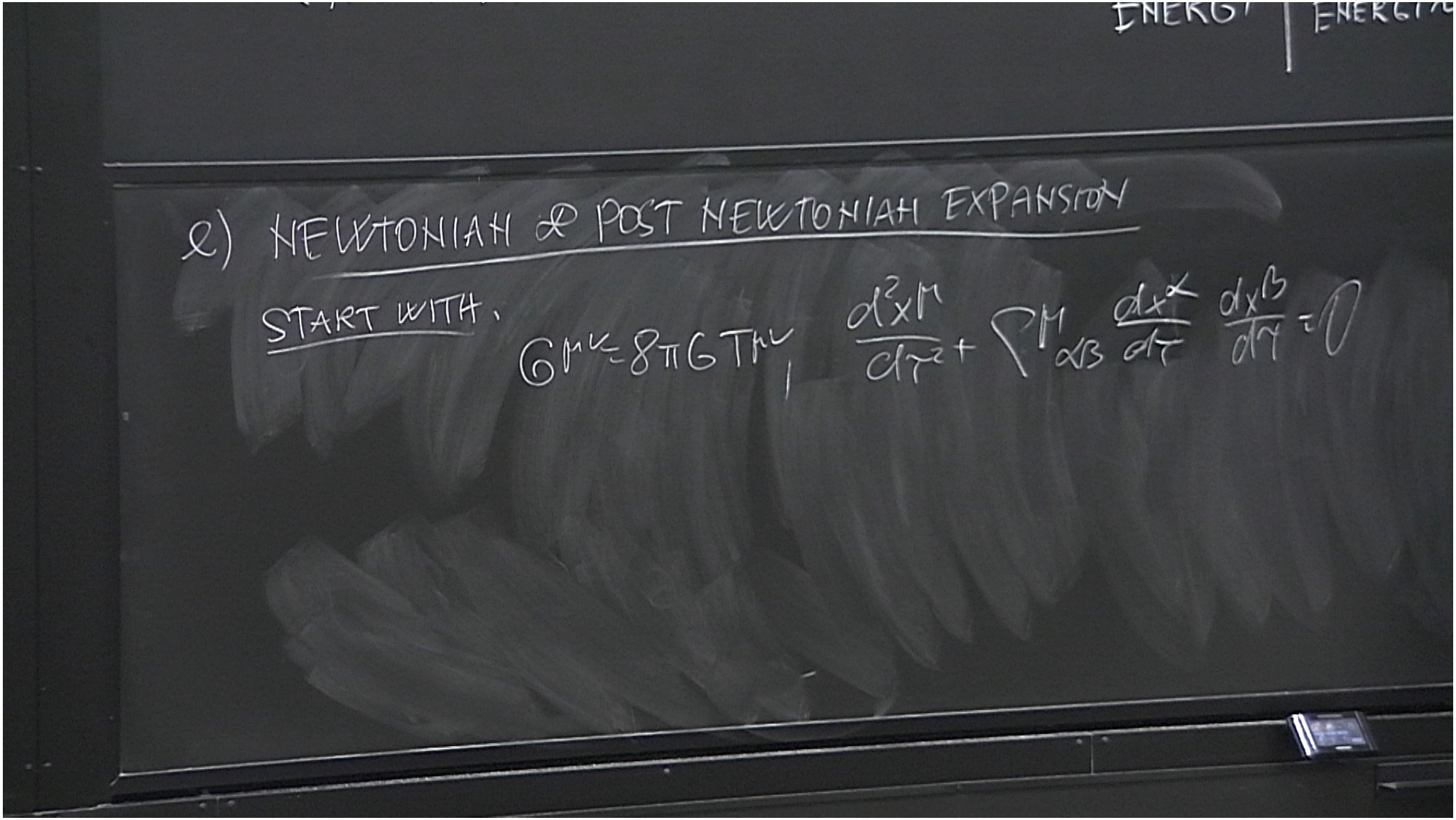

IMELIKE
O
WHERE DO THEY COME FROM ?
NOT PROPERTY OF MATTER (QFT)
RATHER HOW MATTER INTERACTS WITH GRAVITY
NO FUNDAMENTAL REASON !
ARE THEY SATISFIED ?

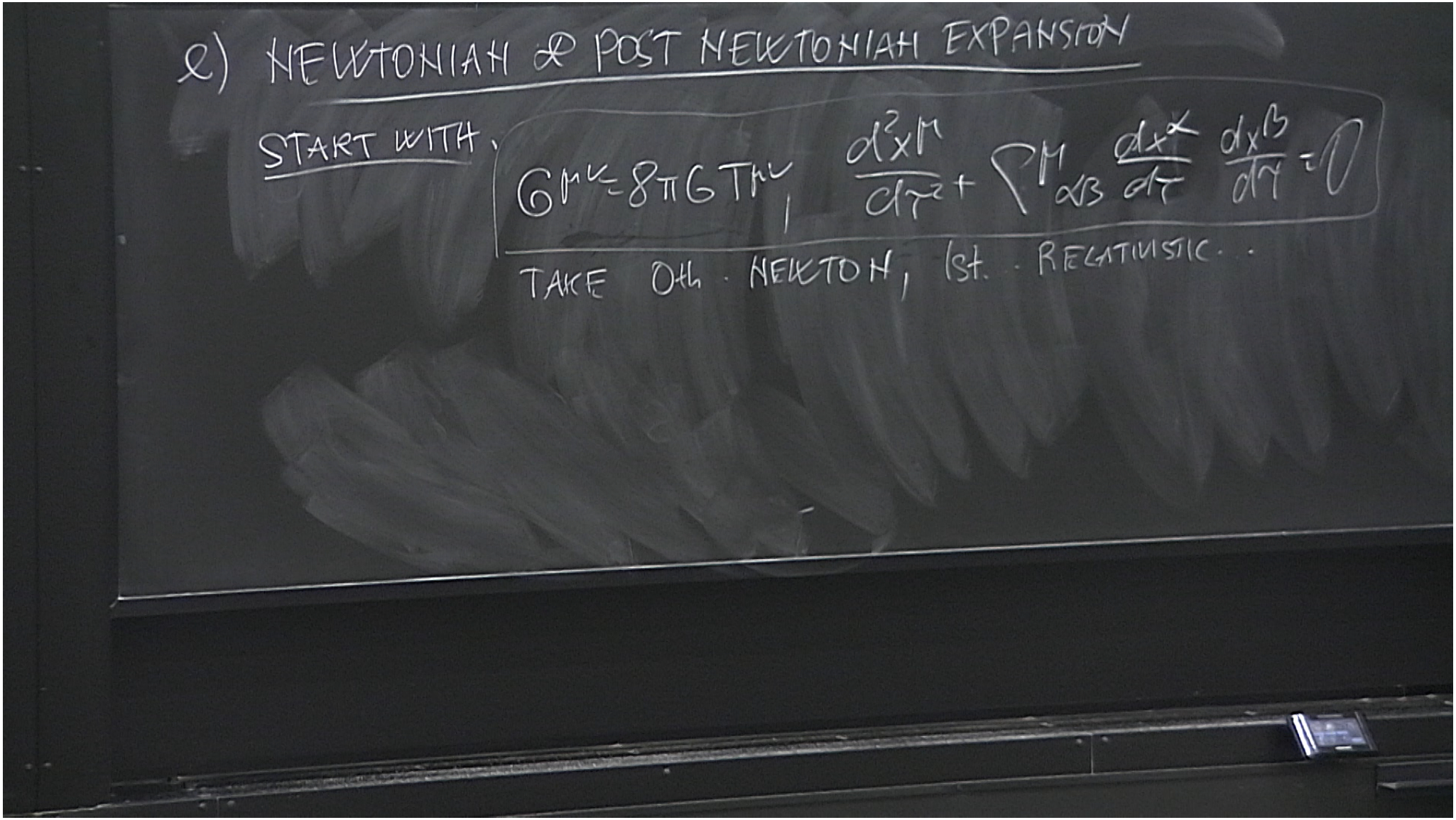

IMELIKE
O
WHERE DO THEY COME FROM ?
NOT PROPERTY OF MATTER (QFT)
RATHER HOW MATTER INTERACTS WITH GRAVITY
NO FUNDAMENTAL REASON !
ARE THEY SATISFIED ? NO ! — INFLATION

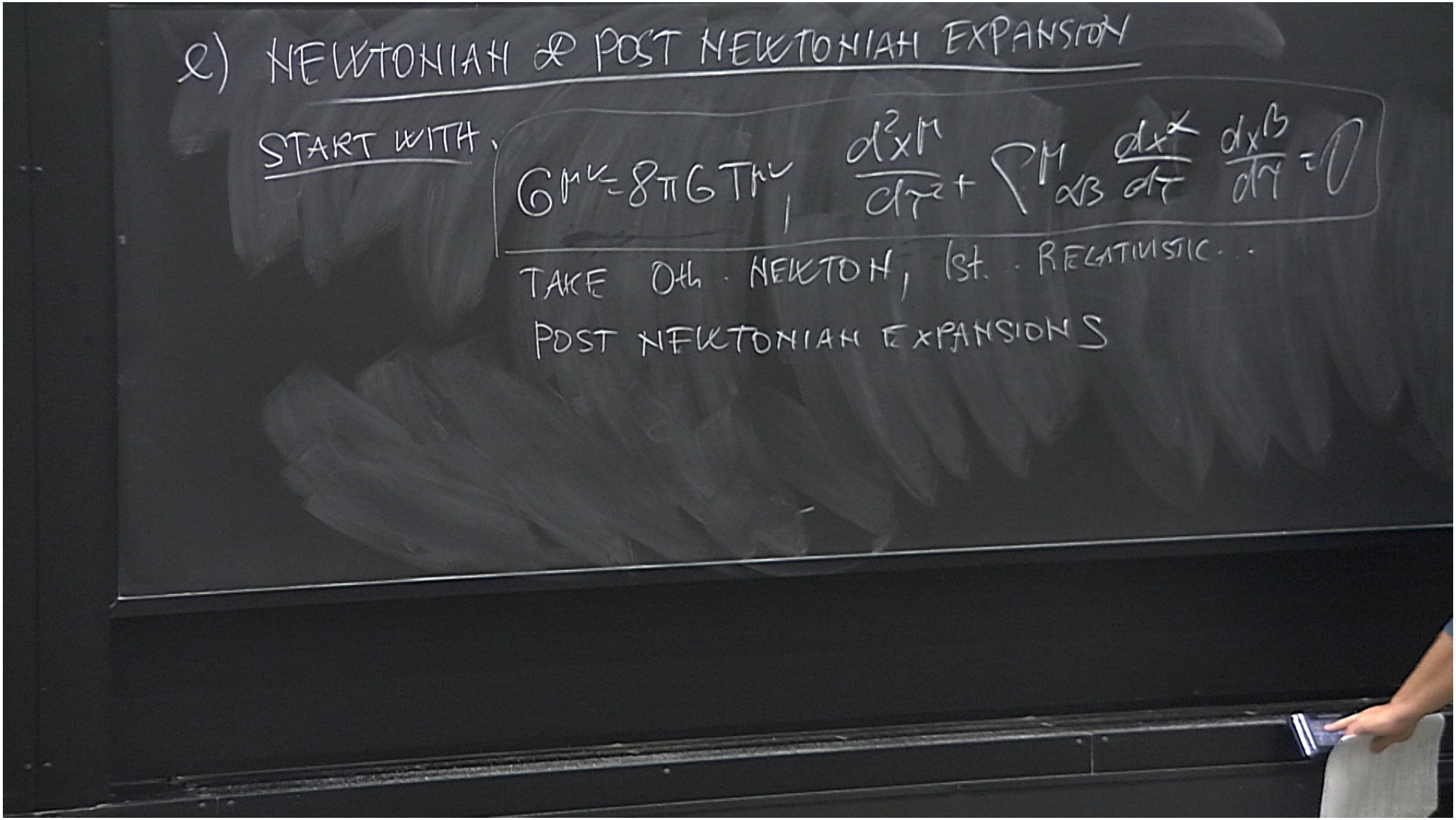

NOT PROPERTY OF MATTER (QFT)
RATHER HOW MATTER INTERACTS WITH GRAVITY
NO FUNDAMENTAL REASON!
ARE THEY SATISFIED? NO!
INFLATION (VIOLATES STRONG.)

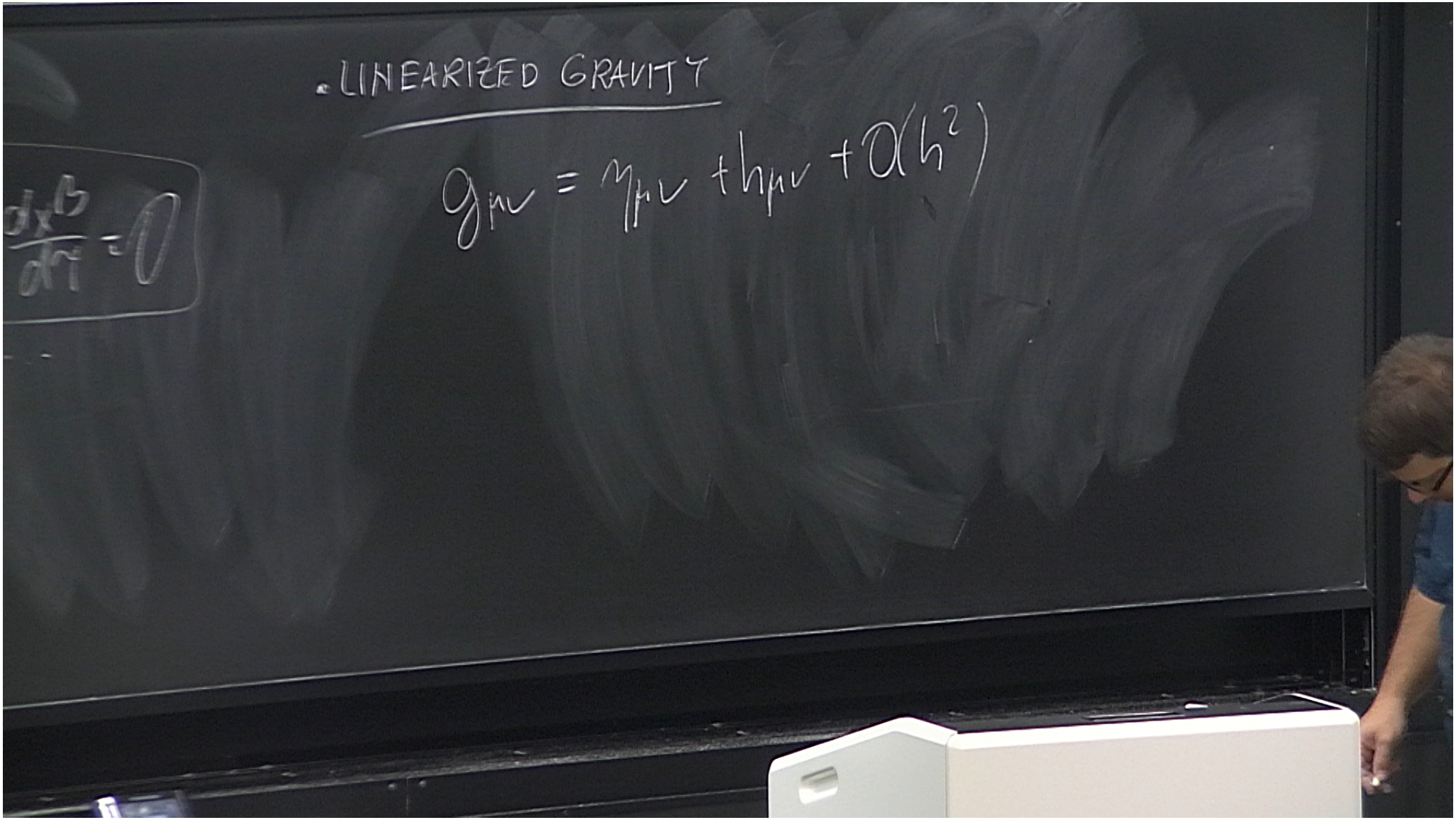

NOT PROPERTY OF MATTER (QFT)
RATHER HOW MATTER INTERACTS WITH GRAVITY
NO FUNDAMENTAL REASON!
ARE THEY SATISFIED? NO!
INFLATION (VIOLATES STRONG.
HAWKING RAD (VIOLATE

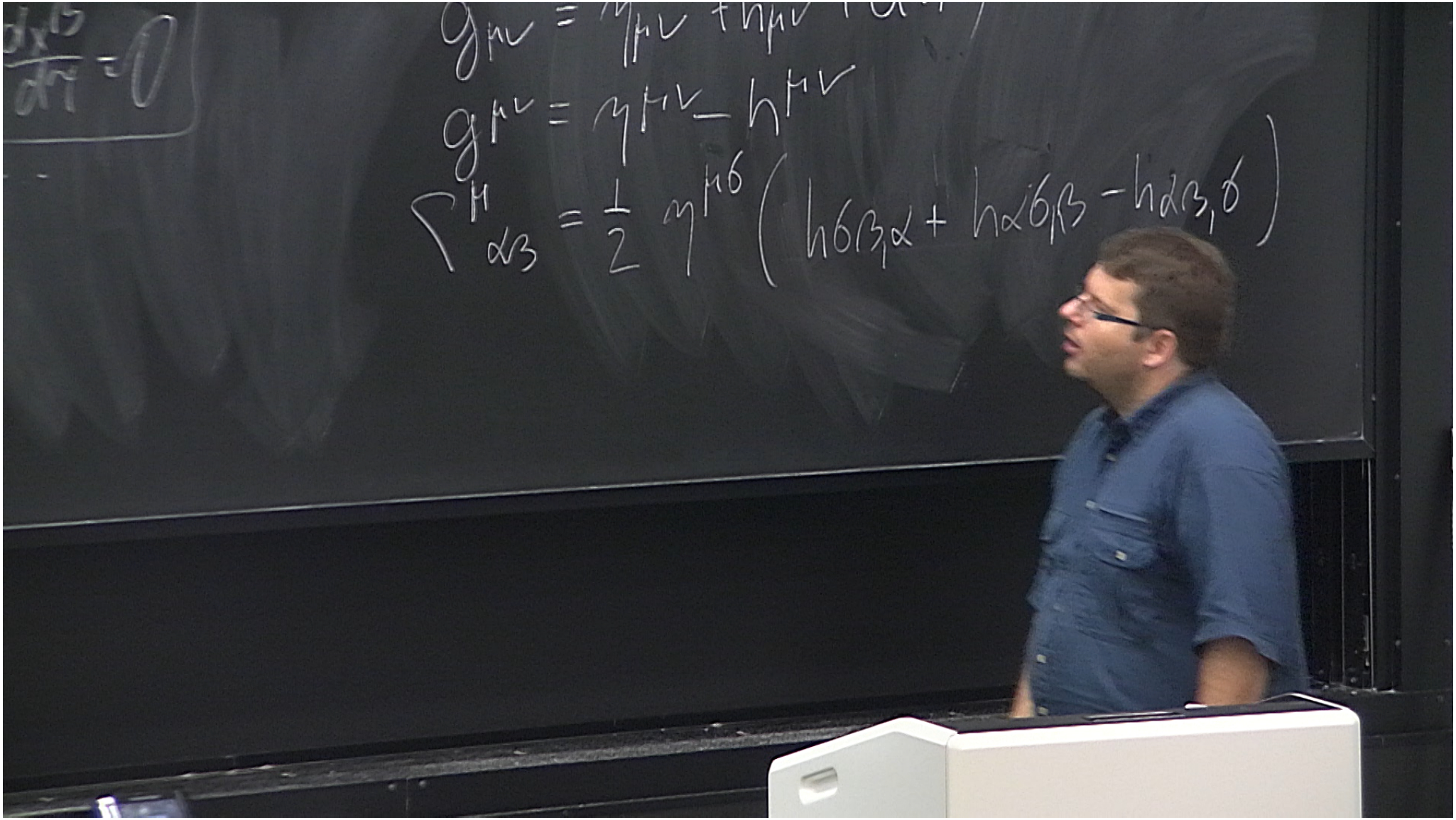

NOT PROPERTY OF MATTER (QFT)
RATHER HOW MATTER INTERACTS WITH GRAVITY
NO FUNDAMENTAL REASON!
ARE THEY SATISFIED? NO!
INFLATION (VIOLATES STRONG)
HAWKING RAD (VIOLATES NULL)

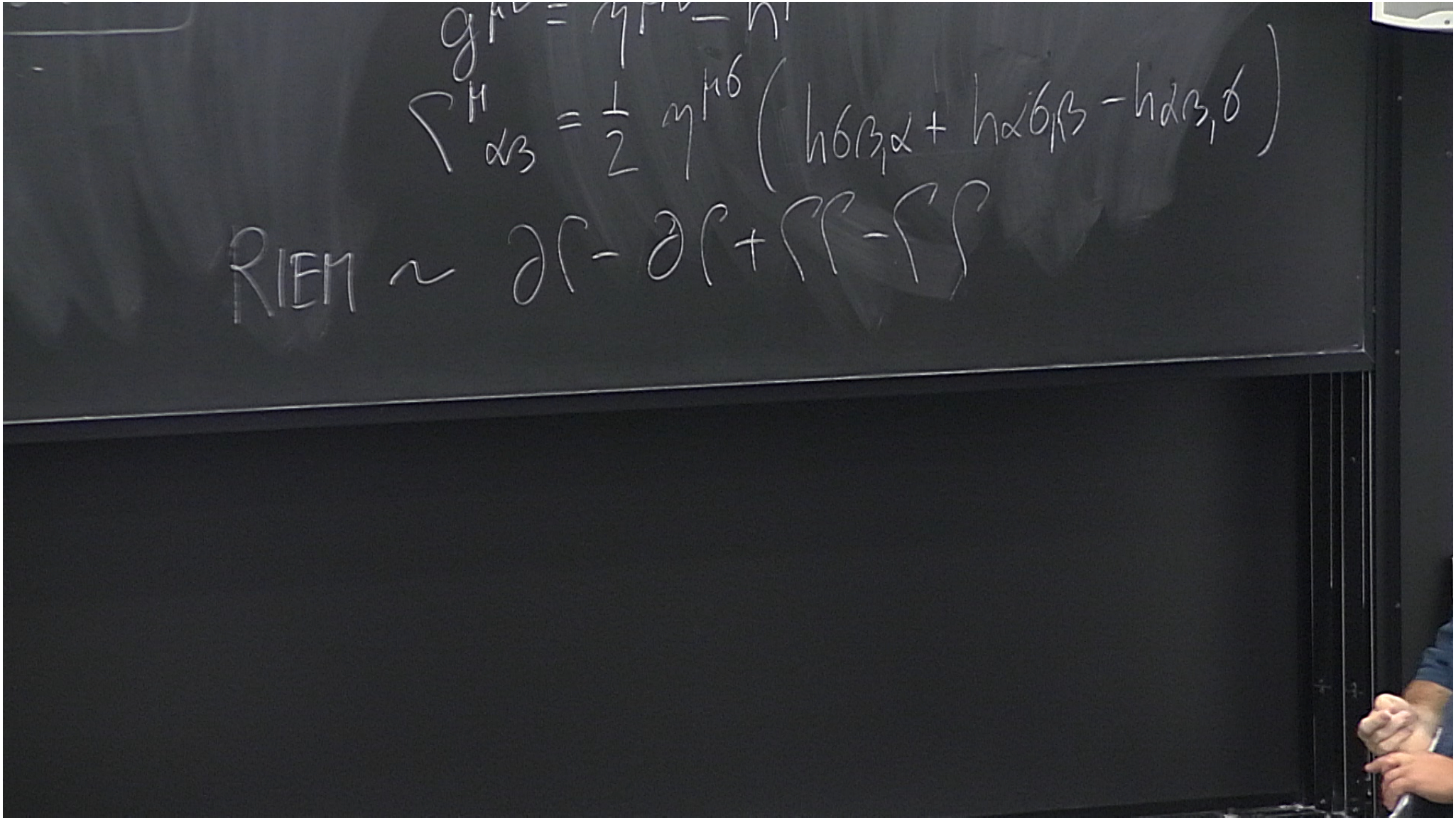

ENERGY
ENERGY
c) NEWTONIAN & POST NEWTONIAN EXPANSION
START WITH.
$G^{\mu\nu} = 8\pi G T^{\mu\nu}$, $\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = 0$

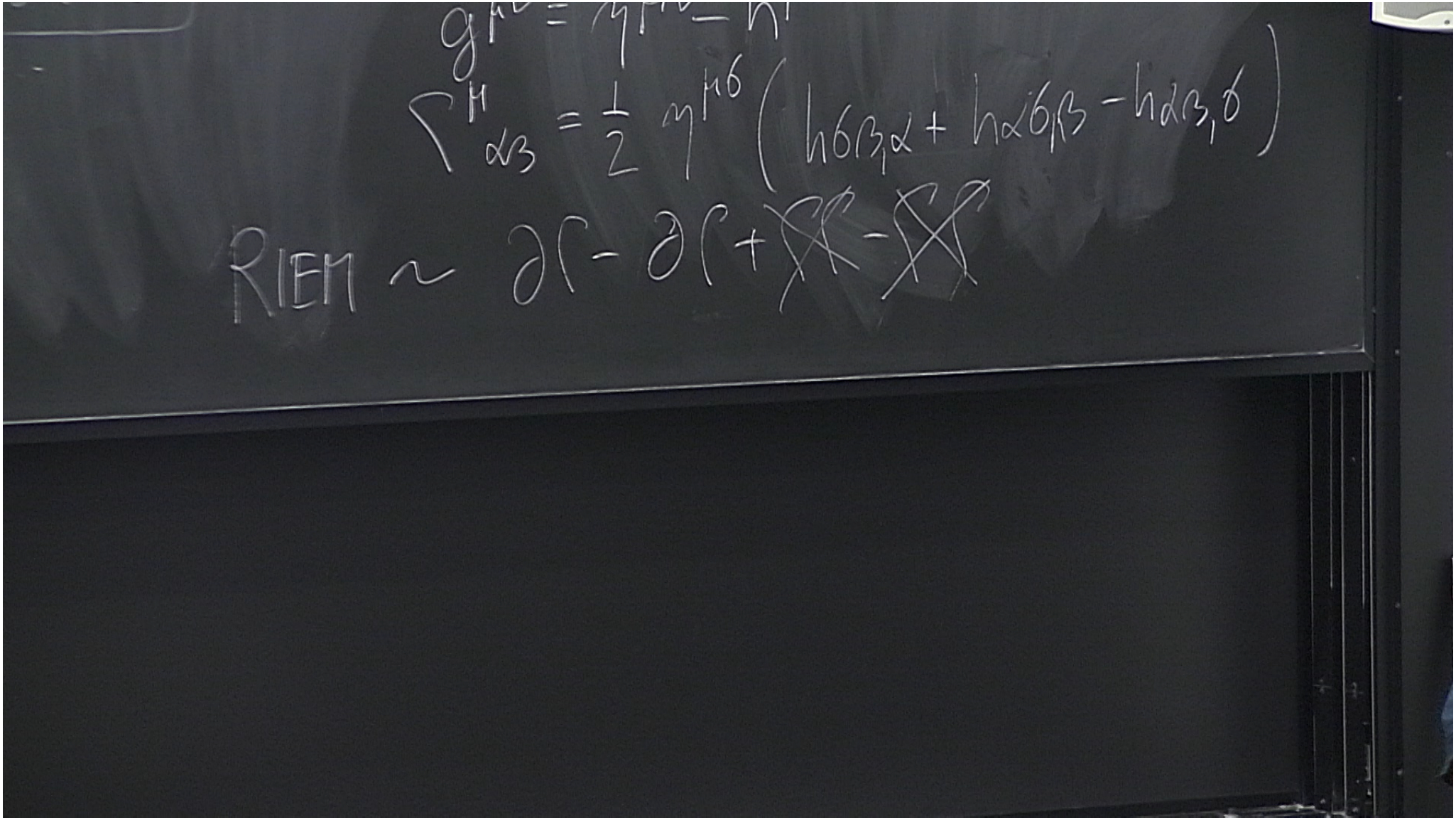

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TAKE 0th. NEWTON, 1st. RELATIVISTIC..

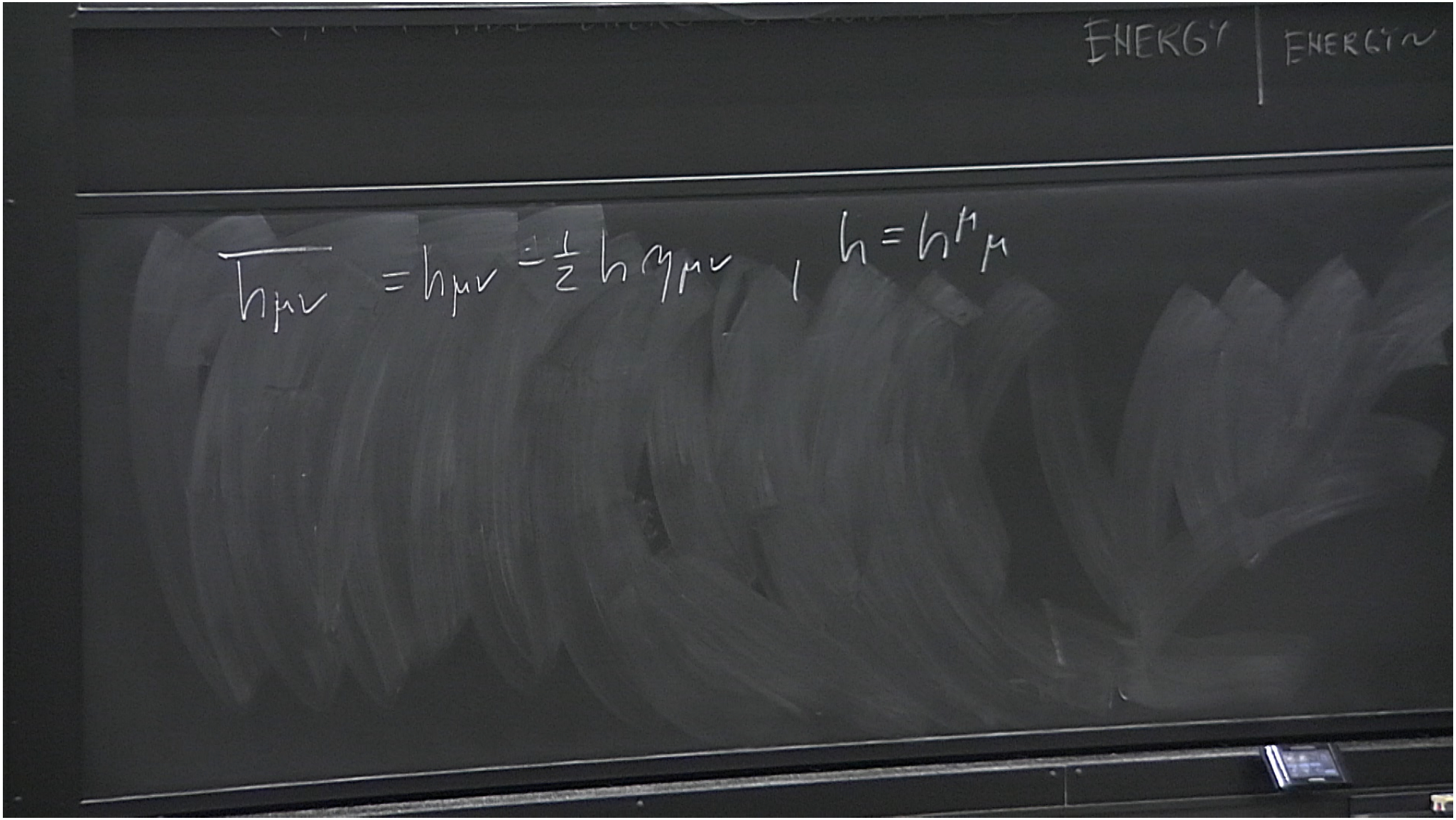

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TAKE 0th. NEWTON, 1st. RELATIVISTIC..
POST NEWTONIAN EXPANSIONS

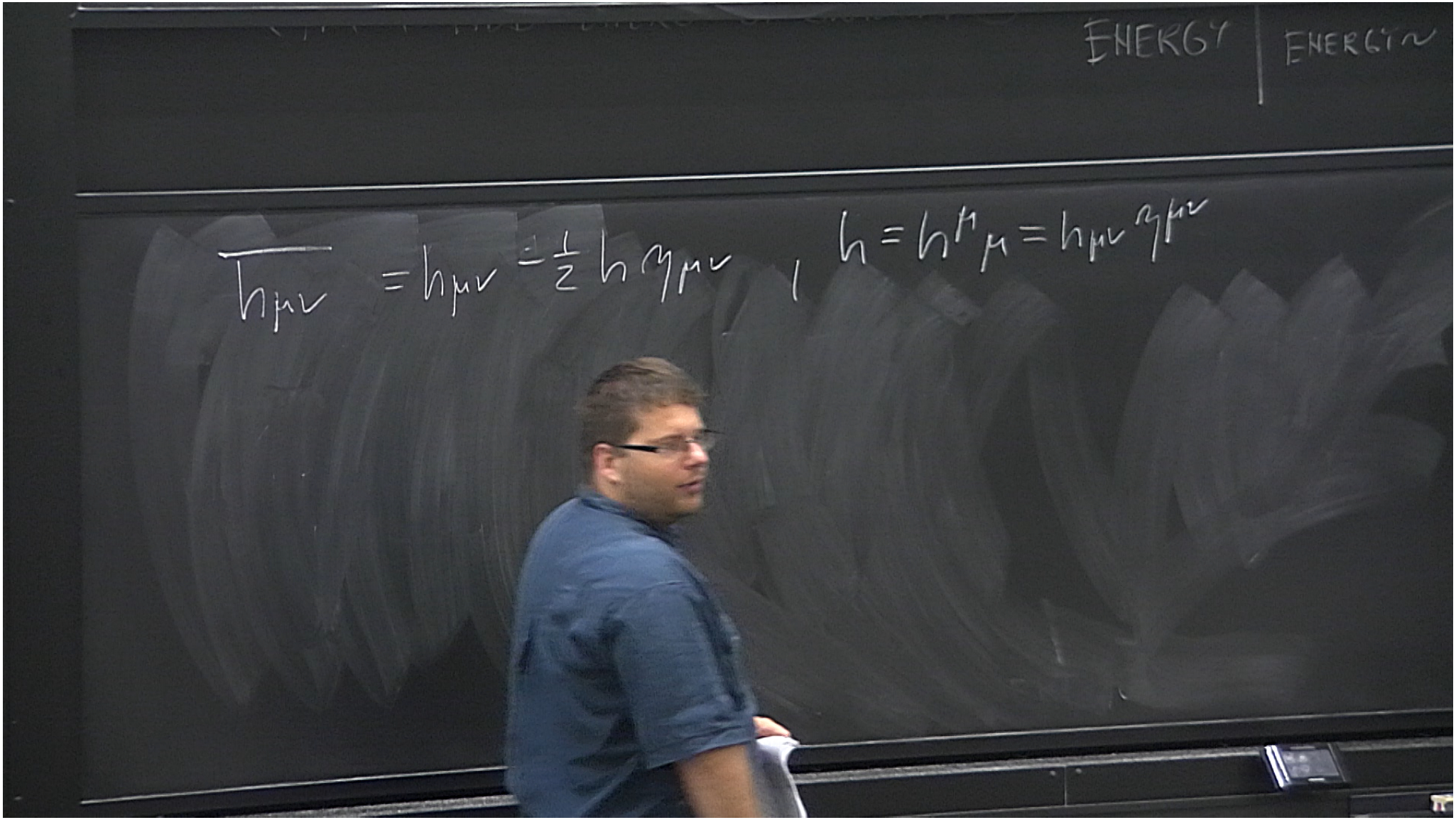

LINEARIZED GRAVITY
$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + O(h^2)$
$\frac{dx^b}{d\tau} = 0$

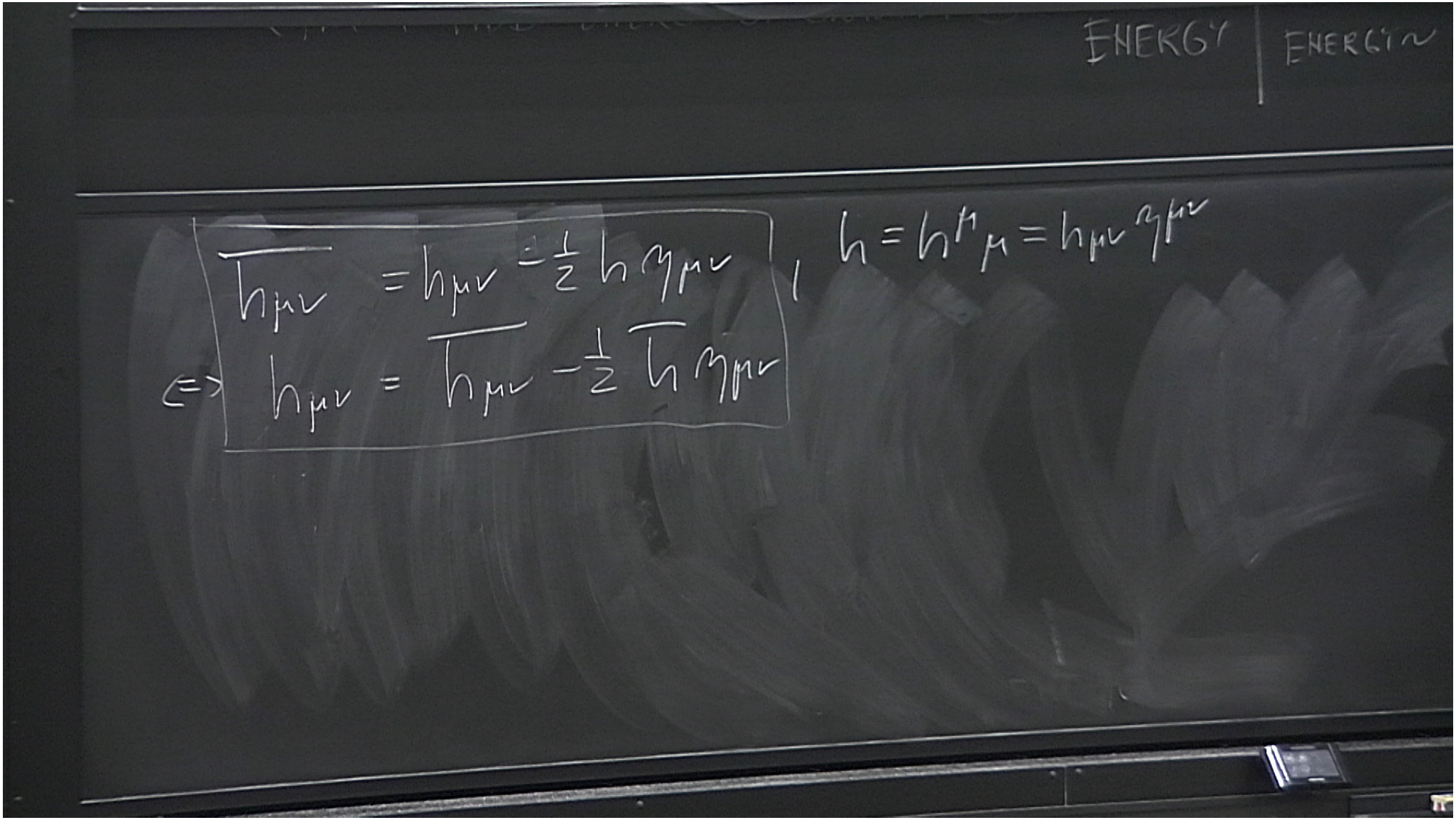

$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
$g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu}$
$\Gamma^{\mu}_{\alpha\beta} = \frac{1}{2}\eta^{\mu\sigma}\left(h_{\sigma\beta,\alpha} + h_{\alpha\sigma,\beta} - h_{\alpha\beta,\sigma}\right)$

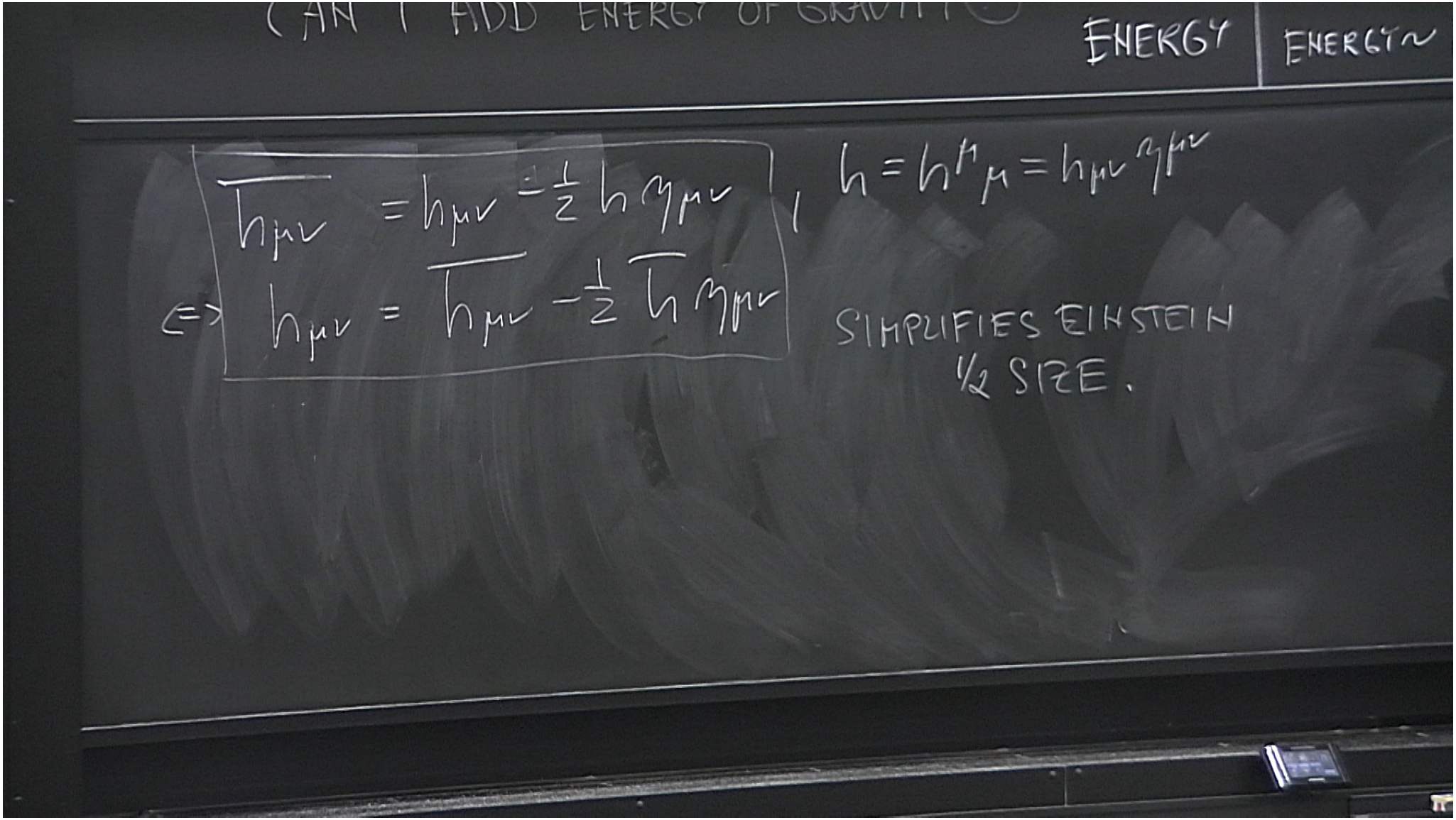

$g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu}$
$\Gamma^{\mu}_{\alpha\beta} = \frac{1}{2}\eta^{\mu\sigma}\left(h_{\sigma\beta,\alpha} + h_{\alpha\sigma,\beta} - h_{\alpha\beta,\sigma}\right)$
RIEM $\sim \partial\Gamma - \partial\Gamma + \Gamma\Gamma - \Gamma\Gamma$


$g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu}$
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RIEM $\sim \partial\Gamma - \partial\Gamma + \Gamma\Gamma - \Gamma\Gamma$


ENERGY
ENERGIA
$\overline{h_{\mu\nu}} = h_{\mu\nu} - \frac{1}{2} h \eta_{\mu\nu}$, $h = h^{\mu}{}_{\mu}$


ENERGY
ENERGIA
$\overline{h_{\mu\nu}} = h_{\mu\nu} - \frac{1}{2} h \eta_{\mu\nu}$, $h = h^{\mu}{}_{\mu} = h_{\mu\nu}\eta^{\mu\nu}$


ENERGY
ENERGIA
$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} h \eta_{\mu\nu}$
$\Leftrightarrow h_{\mu\nu} = \bar{h}_{\mu\nu} - \frac{1}{2} \bar{h} \eta_{\mu\nu}$
$h = h^{\mu}{}_{\mu} = h_{\mu\nu} \eta^{\mu\nu}$


CAN I ADD ENERGY OF GRAVITY?
ENERGY
ENERGY~
$\overline{h_{\mu\nu}} = h_{\mu\nu} - \frac{1}{2} h \eta_{\mu\nu}$
$h = h^{\mu}{}_{\mu} = h_{\mu\nu}\eta^{\mu\nu}$
$\Leftrightarrow h_{\mu\nu} = \overline{h}_{\mu\nu} - \frac{1}{2}\overline{h}\eta_{\mu\nu}$
SIMPLIFIES EINSTEIN
½ SIZE.

CAN I ADD ENERGY OF GRAVITY?

ENERGY | ENERGY

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} h \eta_{\mu\nu}$$

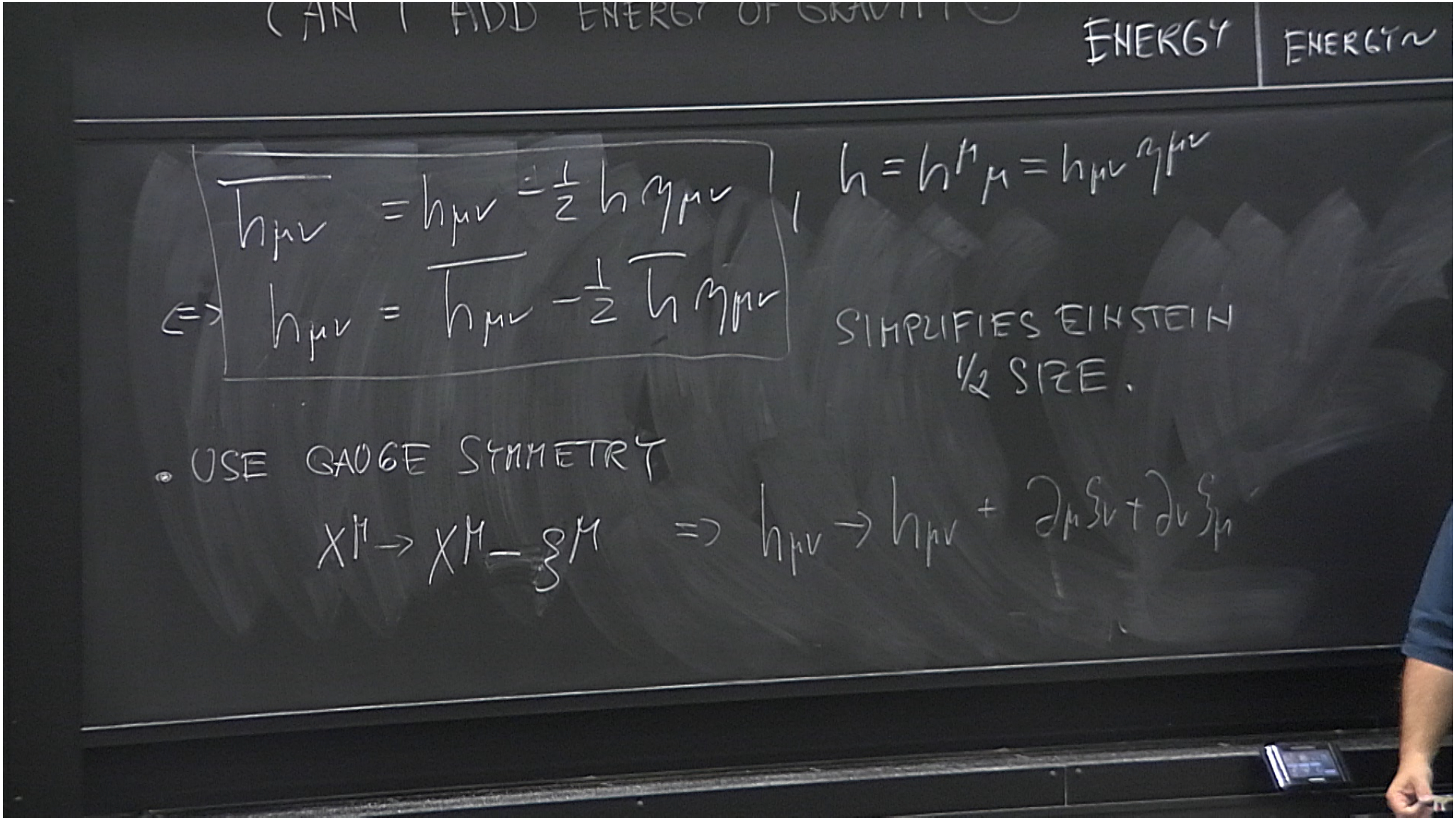
$$\Leftrightarrow \quad h_{\mu\nu} = \bar{h}_{\mu\nu} - \frac{1}{2} \bar{h} \eta_{\mu\nu}$$

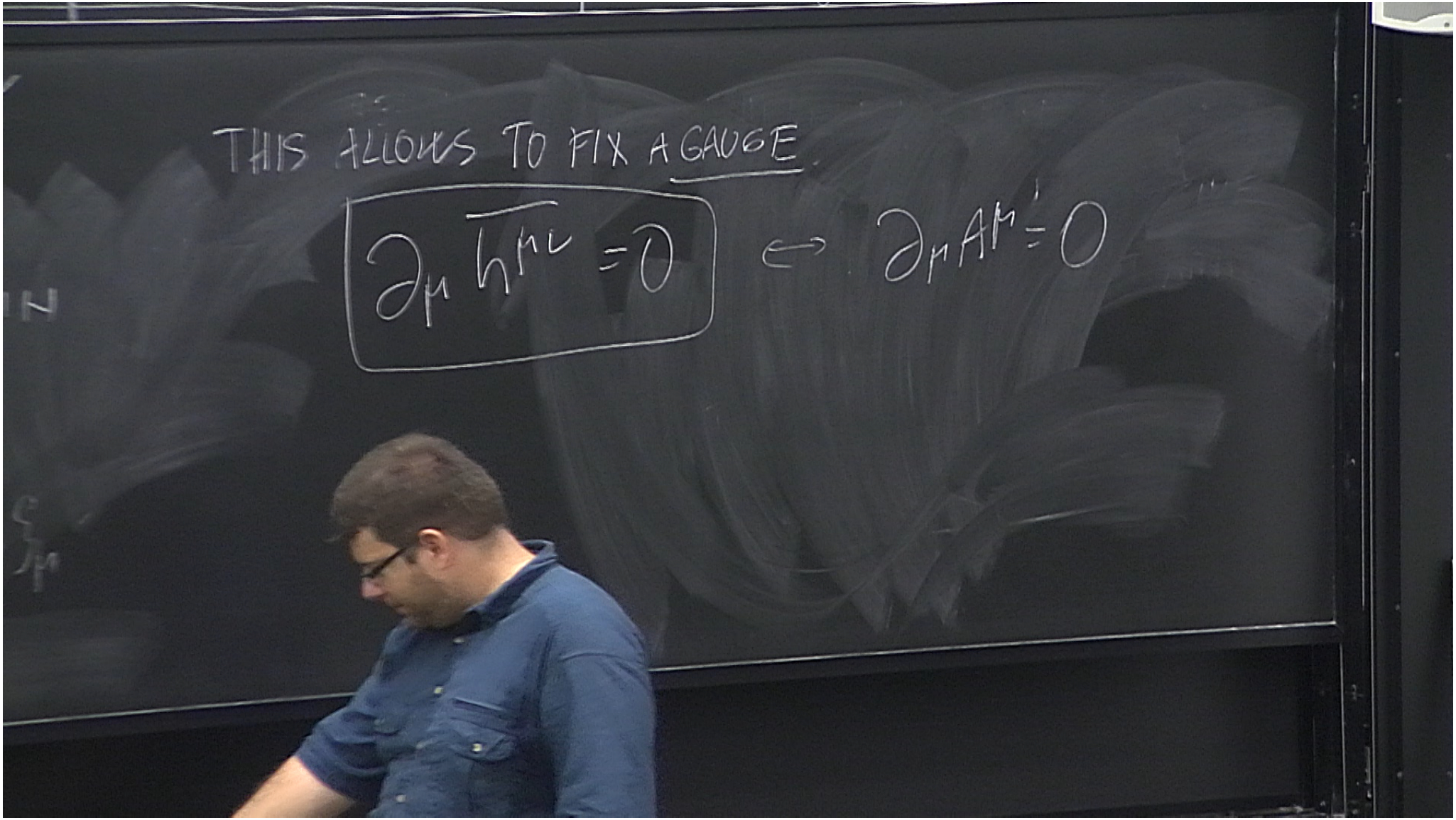
, $h = h^{\mu}{}_{\mu} = h_{\mu\nu} \eta^{\mu\nu}$

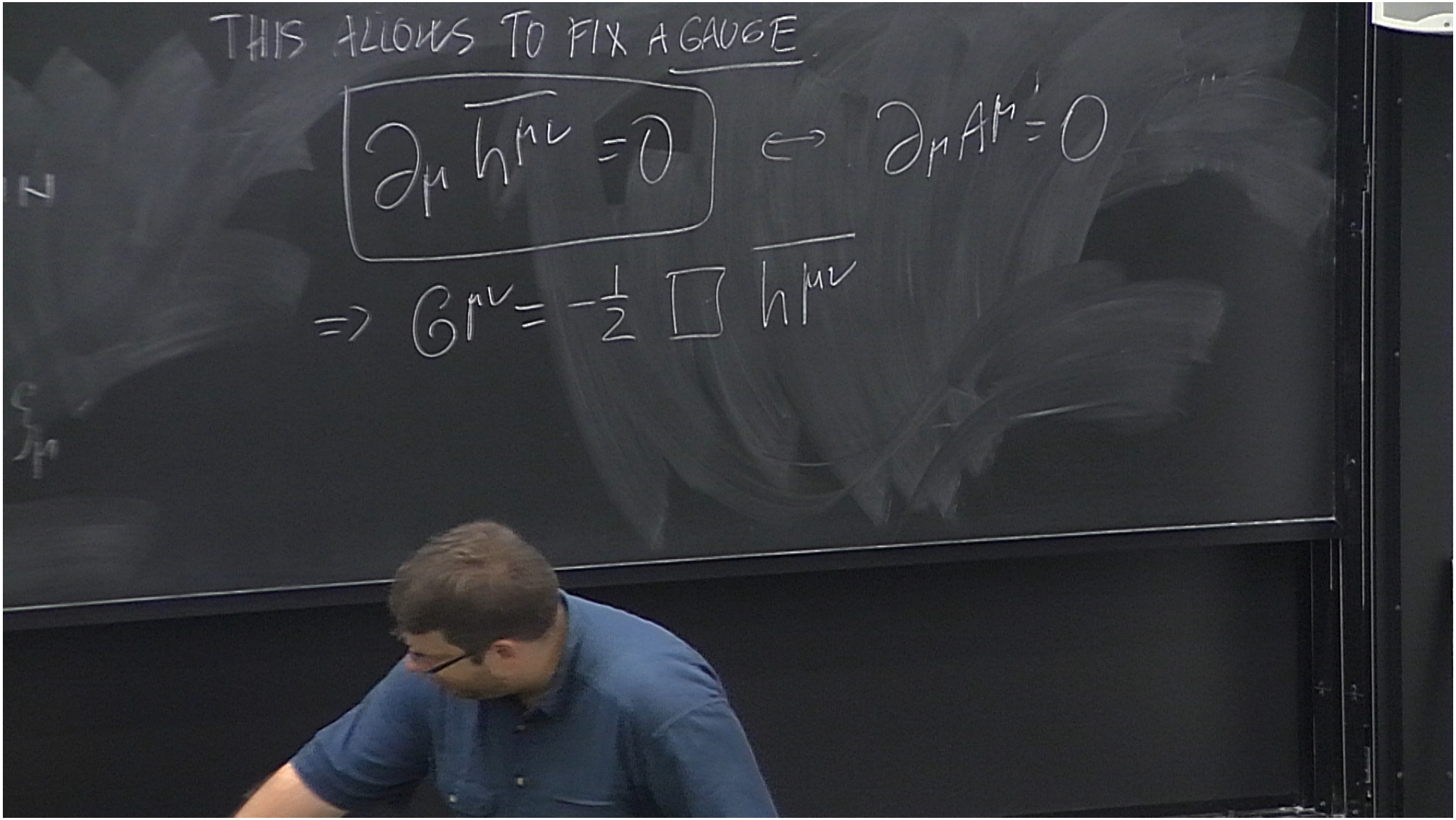
SIMPLIFIES EINSTEIN ½ SIZE.

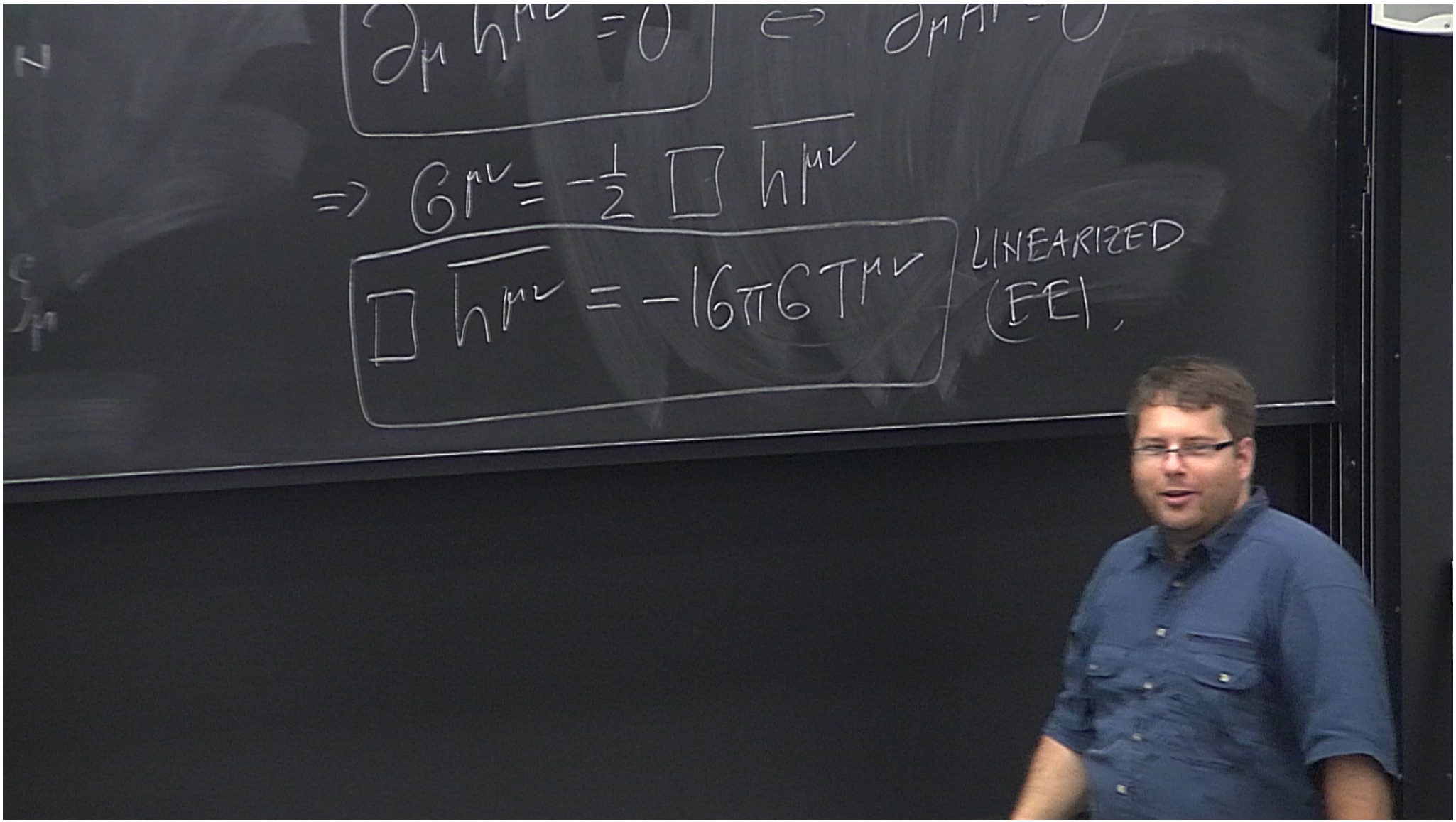
- USE GAUGE SYMMETRY

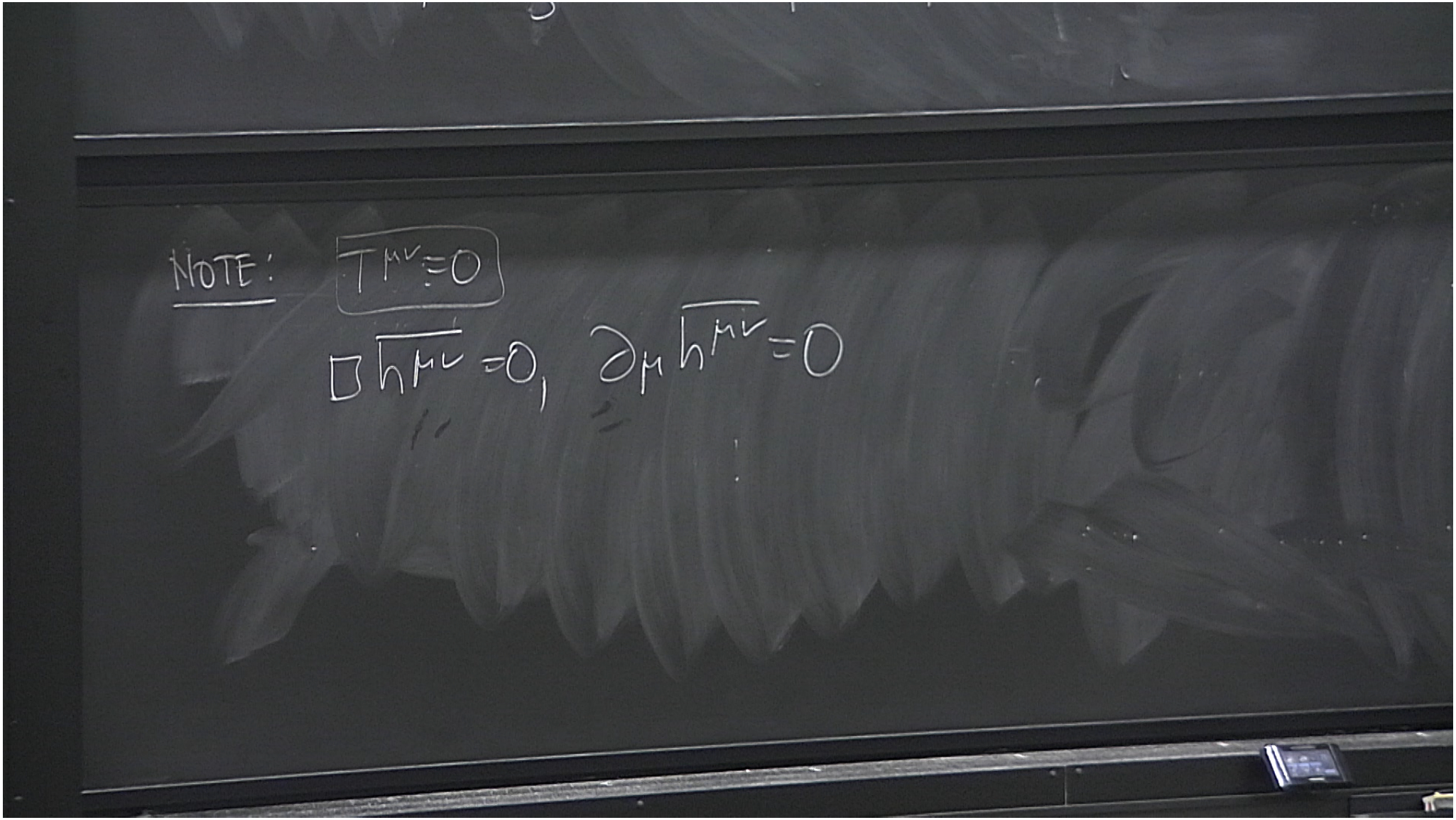
$$X^{\mu} \to X^{\mu} - \xi^{\mu}$$


ENERGY
ENERGY
$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} h \eta_{\mu\nu}$
$h = h^{\mu}{}_{\mu} = h_{\mu\nu}\eta^{\mu\nu}$
$\Leftrightarrow h_{\mu\nu} = \bar{h}_{\mu\nu} - \frac{1}{2}\bar{h}\eta_{\mu\nu}$
SIMPLIFIES EINSTEIN
½ SIZE.
• USE GAUGE SYMMETRY
$X^{\mu} \to X^{\mu} - \xi^{\mu} \Rightarrow h_{\mu\nu} \to h_{\mu\nu} + \partial_{\mu}\xi_{\nu} + \partial_{\nu}\xi_{\mu}$


THIS ALLOWS TO FIX A GAUGE
$\partial_\mu \bar{h}^{\mu\nu} = 0$
$\Leftrightarrow \partial_\mu A^\mu = 0$


THIS ALLOWS TO FIX A GAUGE
$\partial_\mu \bar{h}^{\mu\nu} = 0 \iff \partial_\mu A^\mu = 0$
$\Rightarrow G^{\mu\nu} = -\frac{1}{2} \Box \bar{h}^{\mu\nu}$

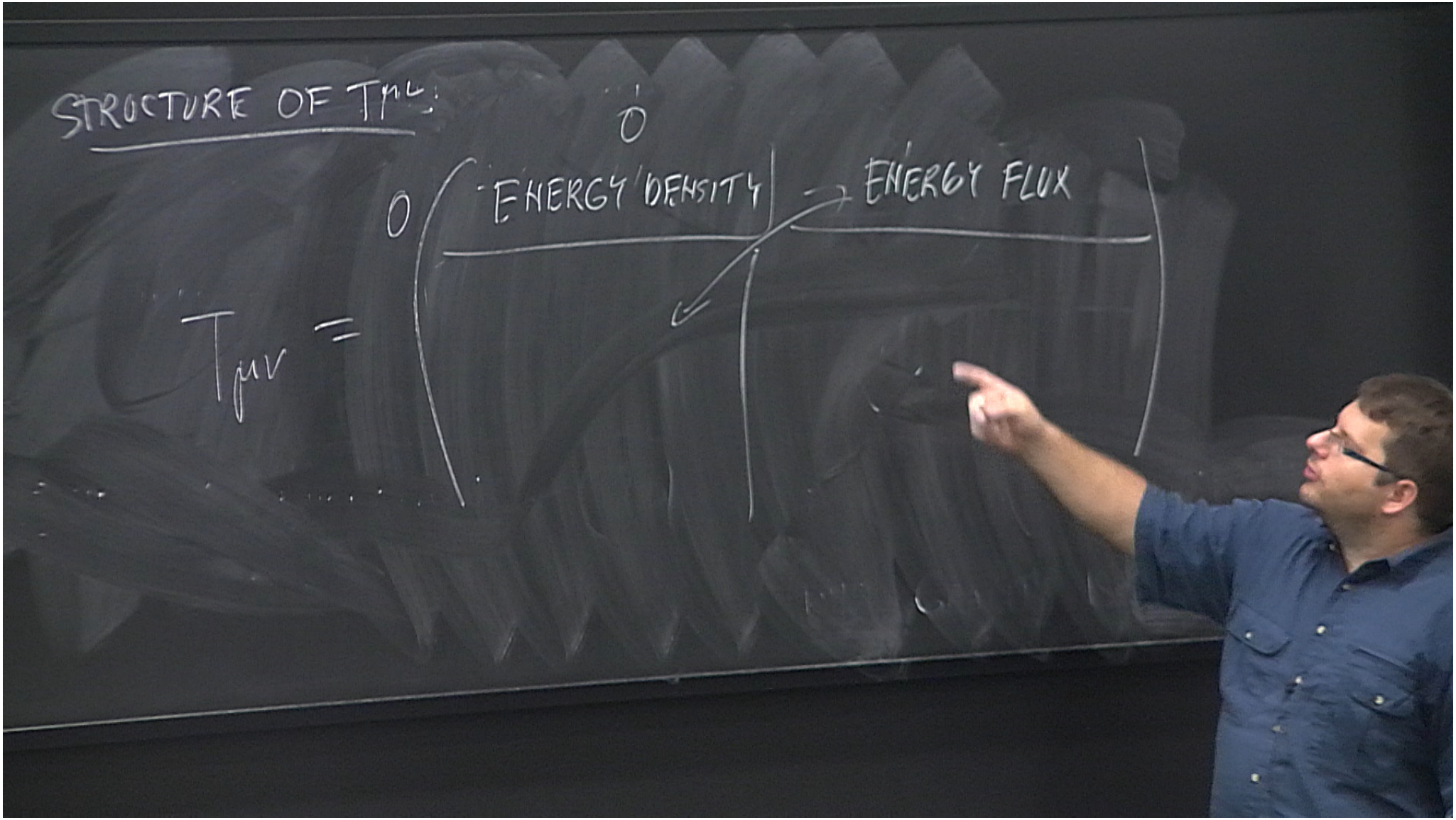

$\partial_\mu h^{\mu\nu} = 0$
$\Leftrightarrow$
$\Rightarrow G^{\mu\nu} = -\frac{1}{2} \Box \overline{h}^{\mu\nu}$
$\Box \overline{h}^{\mu\nu} = -16\pi G T^{\mu\nu}$
LINEARIZED
(EE).

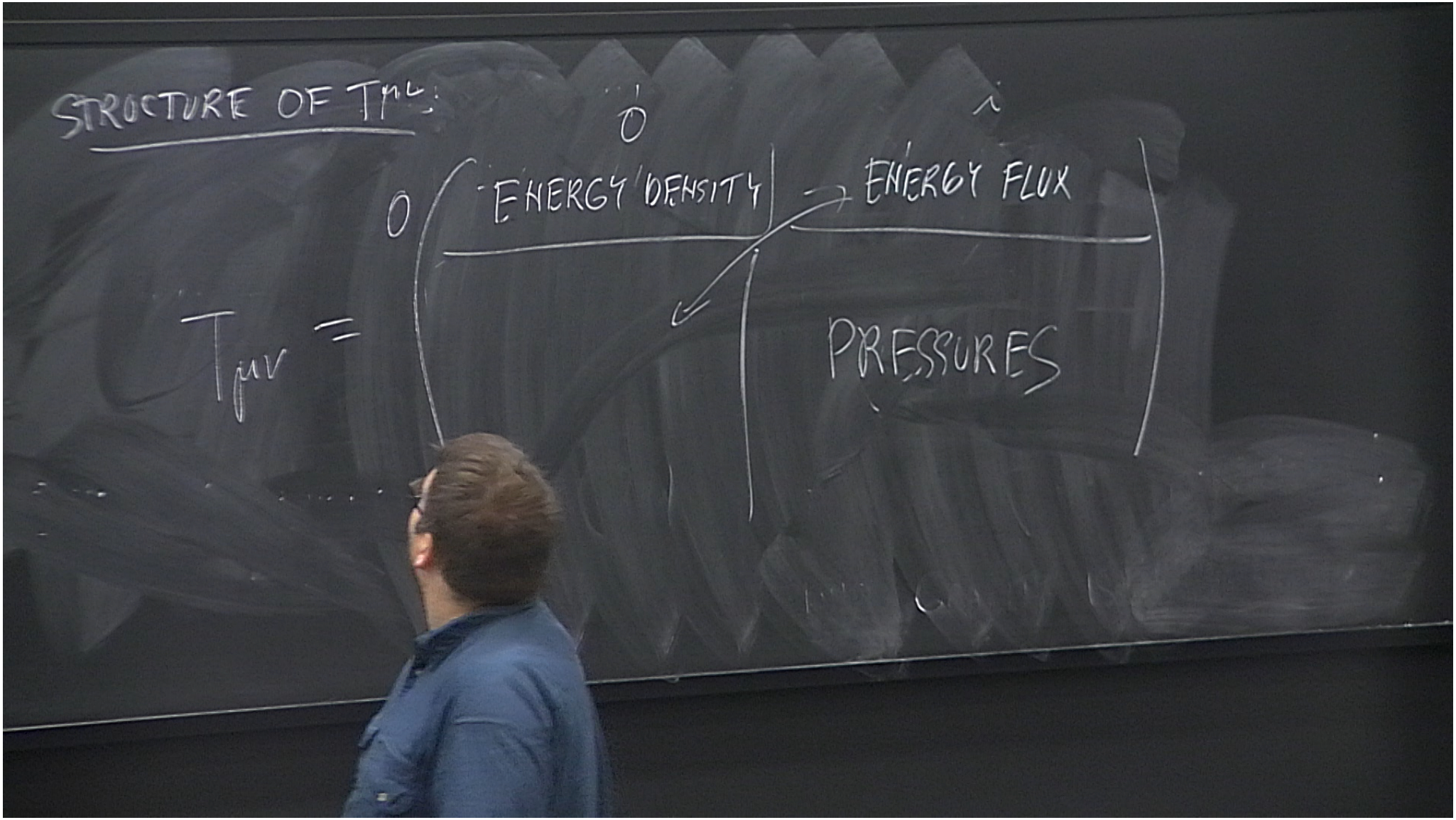

NOTE: $T^{\mu\nu}=0$
$\Box \bar{h}^{\mu\nu}=0, \quad \partial_\mu \bar{h}^{\mu\nu}=0$

NOTE: $T^{\mu\nu} = 0$

$$\Box \bar{h}^{\mu\nu} = 0, \quad \partial_\mu \bar{h}^{\mu\nu} = 0$$

FIELD EQS FOR SPIN-2 PARTICLE IN FLAT SPACE ... PAULI-FIERZ (39)


STRUCTURE OF $T^{\mu\nu}$:
0
0
ENERGY DENSITY
ENERGY FLUX
$T_{\mu\nu} =$


STRUCTURE OF $T^{\mu\nu}$:
0
i
0
ENERGY DENSITY
ENERGY FLUX
$T_{\mu\nu}$ =
PRESSURES

$\Rightarrow \boxed{\rho \geq 0, \quad \rho + P_i > 0}$

NEWTONIAN LIMIT:

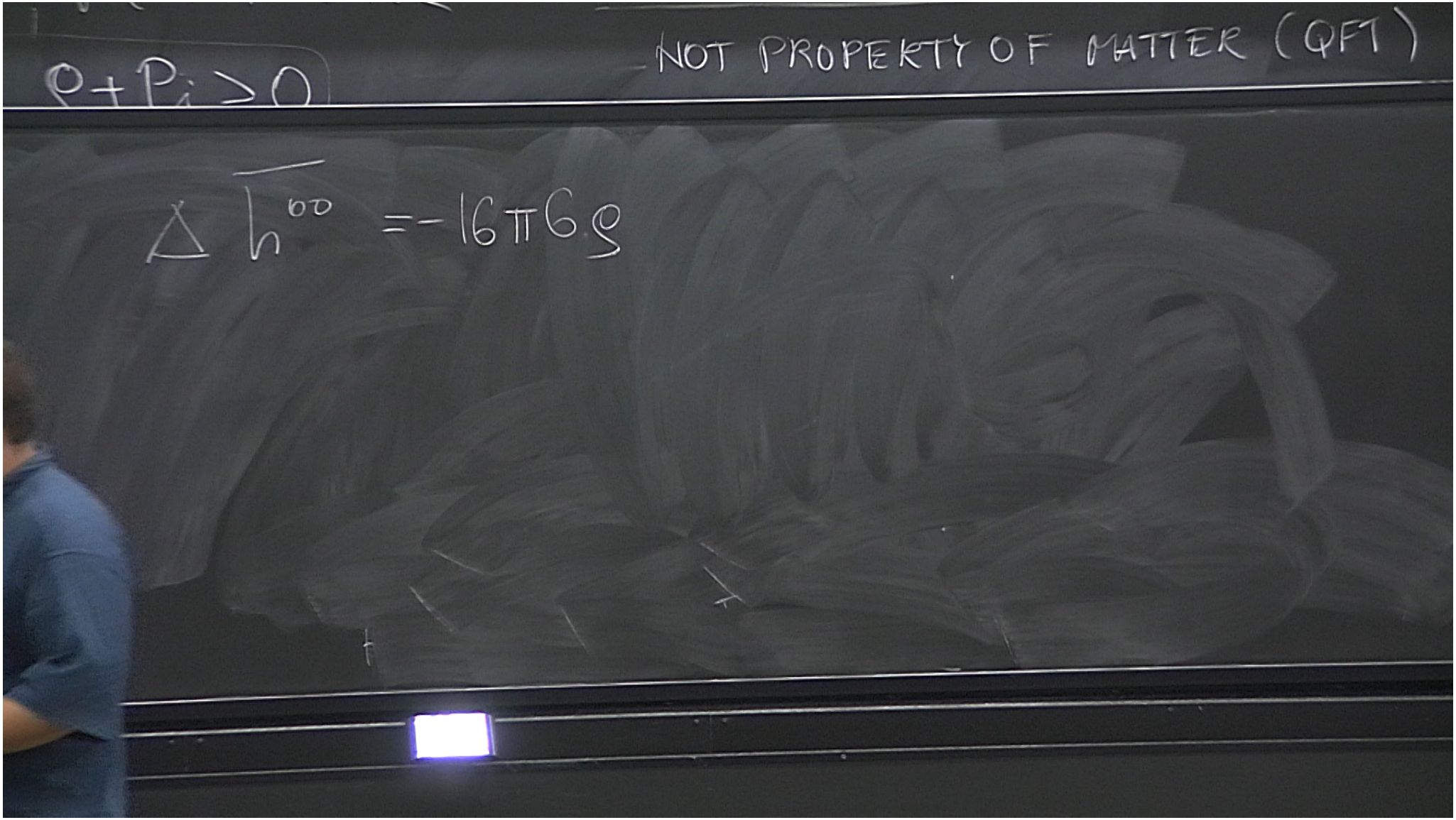
$$T_{\mu\nu} = \begin{pmatrix} \rho & 0 \\ 0 & 0 \end{pmatrix}$$

$\Rightarrow \rho \geq 0, \quad \rho + P_i > 0$

NEWTONIAN LIMIT:

$$T_{\mu\nu} = \begin{pmatrix} \rho & 0 \\ 0 & 0 \end{pmatrix}$$

FIELD IS STATIC $\quad T^{\mu\nu}{}_{,0} = 0$


NOT PROPERTY OF MATTER (QFT)
ρ+P_i>0
Δ h̄^00 = -16πGρ

$\rho + P_i > 0$

NOT PROPERTY OF MATTER (QFT)

$$\Delta \bar{h}^{00} = -16\pi G \rho \quad \leftrightarrow \quad \boxed{\Delta\phi = 4\pi G \rho}$$

$$\phi = -\frac{1}{4}\bar{h}_{00}$$

$$\boxed{g_{00} = -1 - 2\phi}$$

$\rho + P_i > 0$

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NOT PROPERTY OF MATTER (QFT)

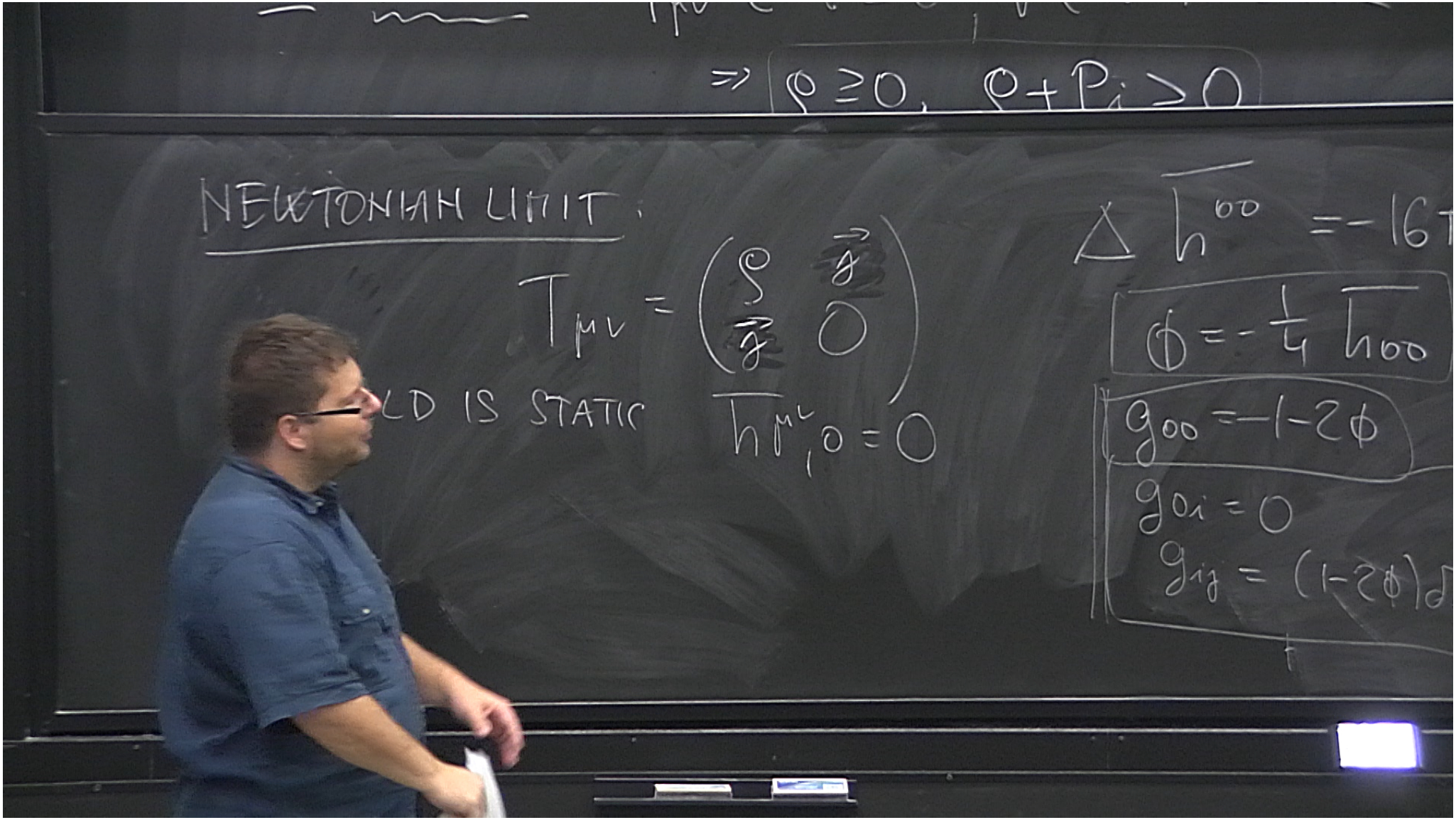
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$\Rightarrow \rho \geq 0, \quad \rho + P_i > 0$
NEWTONIAN LIMIT:
$T_{\mu\nu} = \begin{pmatrix} \rho & \vec{g} \\ \vec{g} & 0 \end{pmatrix}$
LD IS STATIC
$T^{\mu\nu}{}_{,0} = 0$
$\Delta \bar{h}^{00} = -16\pi$
$\phi = -\frac{1}{4} h_{00}$
$g_{00} = -1 - 2\phi$
$g_{0i} = 0$
$g_{ij} = (1 - 2\phi)\delta$