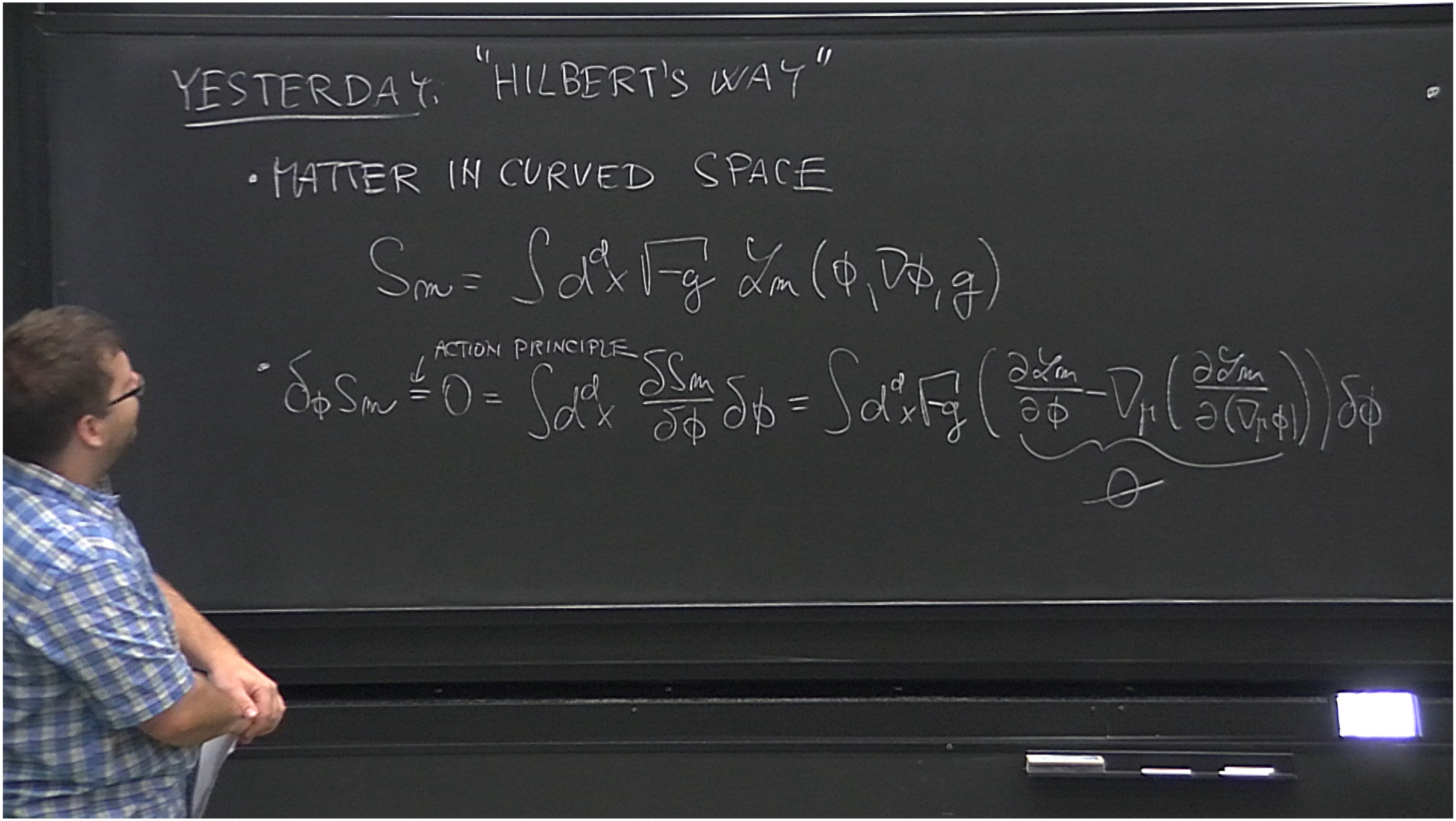
Title: PSI 2018/2019 - Relativity - Lecture 9

Date: Sep 13, 2018 09:00 AM

URL: http://pirsa.org/18090008

Abstract:


YESTERDAY: "HILBERT'S WAY"
• MATTER IN CURVED SPACE
$S_m = \int d^dx \sqrt{-g}\, \mathcal{L}_m(\phi, \nabla\phi, g)$
ACTION PRINCIPLE
• $\delta_\phi S_m \overset{!}{=} 0 = \int d^dx \frac{\delta S_m}{\delta\phi}\delta\phi = \int d^dx\sqrt{-g}\left(\underbrace{\frac{\partial \mathcal{L}_m}{\partial\phi} - \nabla_\mu\left(\frac{\partial\mathcal{L}_m}{\partial(\nabla_\mu\phi)}\right)}_{0}\right)\delta\phi$

YESTERDAY: "HILBERT'S WAY"

- MATTER IN CURVED SPACE $\quad (\eta_{\mu\nu} \to g_{\mu\nu},\ \partial_\mu \to \nabla_\mu)$

$$S_m = \int d^dx \sqrt{-g}\, \mathcal{L}_m(\phi, \nabla\phi, g)$$

- $$\delta_\phi S_m \overset{\text{ACTION PRINCIPLE}}{=} 0 = \int d^dx \frac{\delta S_m}{\delta \phi}\delta\phi = \int d^dx \sqrt{-g}\left(\underbrace{\frac{\partial \mathcal{L}_m}{\partial \phi} - \nabla_\mu\left(\frac{\partial \mathcal{L}_m}{\partial(\nabla_\mu \phi)}\right)}_{0}\right)\delta\phi$$

• $\delta_g S_m = -\frac{1}{2}\int d^dx \sqrt{-g}\, T_{\mu\nu}\, \delta g^{\mu\nu}$

ENERGY-MOMENTUM TENSOR (SYMMETRIC)

IF $\frac{\delta S_m}{\delta \phi} = 0 \quad \Rightarrow \quad \boxed{\nabla_\mu T^{\mu\nu} = 0}$

CONSERVATION OF $T^{\mu\nu}$ (MAYBE NOT ?)

• $\delta_g S_m = -\frac{1}{2}\int d^dx\, \sqrt{-g}\, T_{\mu\nu}\, \delta g^{\mu\nu}$

ENERGY-MOMENTUM TENSOR (SYMMETRIC)

IF $\frac{\delta S_m}{\delta \phi} = 0 \Rightarrow \boxed{\nabla_\mu T^{\mu\nu} = 0}$

CONSERVATION OF $T^{\mu\nu}$ (MAYBE NOT ?)

$\boxed{\partial_\mu T^{\mu\nu} = 0}$

$$\bullet\ \delta_g S_m = -\frac{1}{2}\int d^dx \sqrt{-g}\, T_{\mu\nu}\, \delta g^{\mu\nu}$$

ENERGY-MOMENTUM TENSOR (SYMMETRIC)

IF $\frac{\delta S_m}{\delta \phi} = 0 \Rightarrow \boxed{\nabla_\mu T^{\mu\nu} = 0}$

CONSERVATION OF $T^{\mu\nu}$ (MAYBE NOT ?)

$\boxed{\partial_\mu T^{\mu\nu} = 0} \rightarrow$ CONSERVATION OF ENERGY & MOMENTUM

• $\delta_g S_m = -\frac{1}{2}\int d^dx \sqrt{-g}\, T_{\mu\nu}\, \delta g^{\mu\nu}$

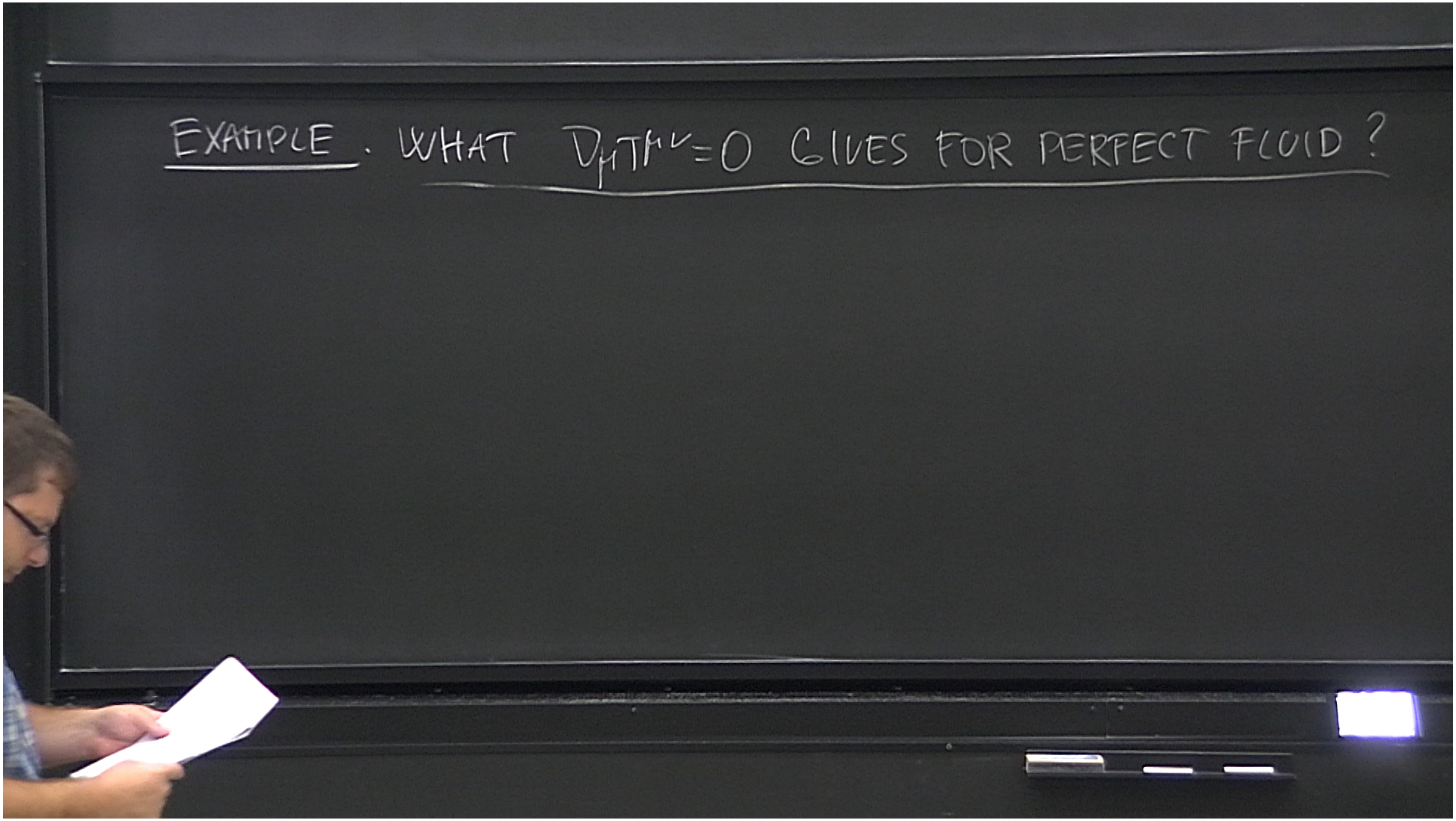
ENERGY-MOMENTUM TENSOR (SYMMETRIC)

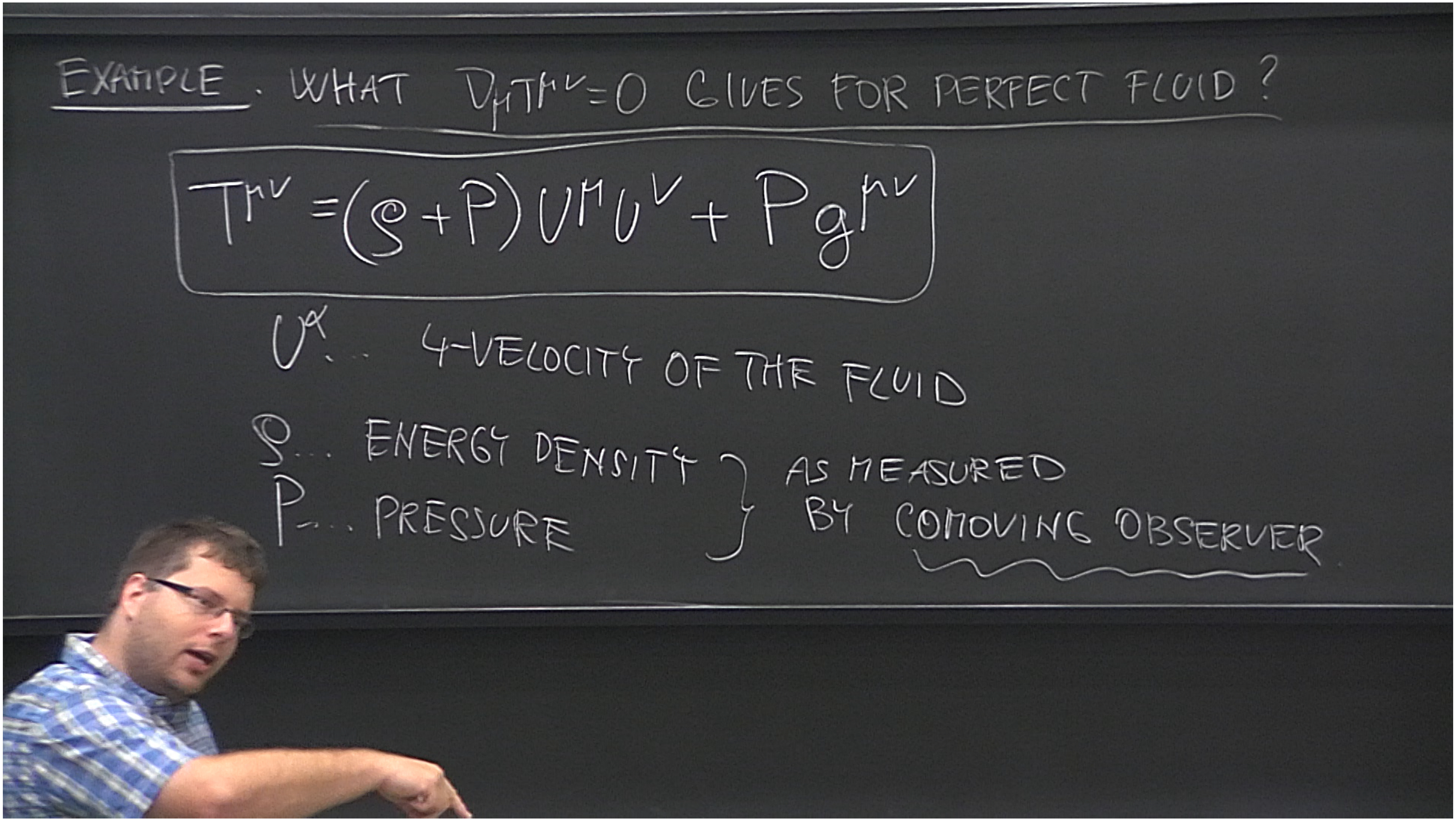
IF $\frac{\delta S_m}{\delta \phi} = 0 \Rightarrow \boxed{\nabla_\mu T^{\mu\nu} = 0}$

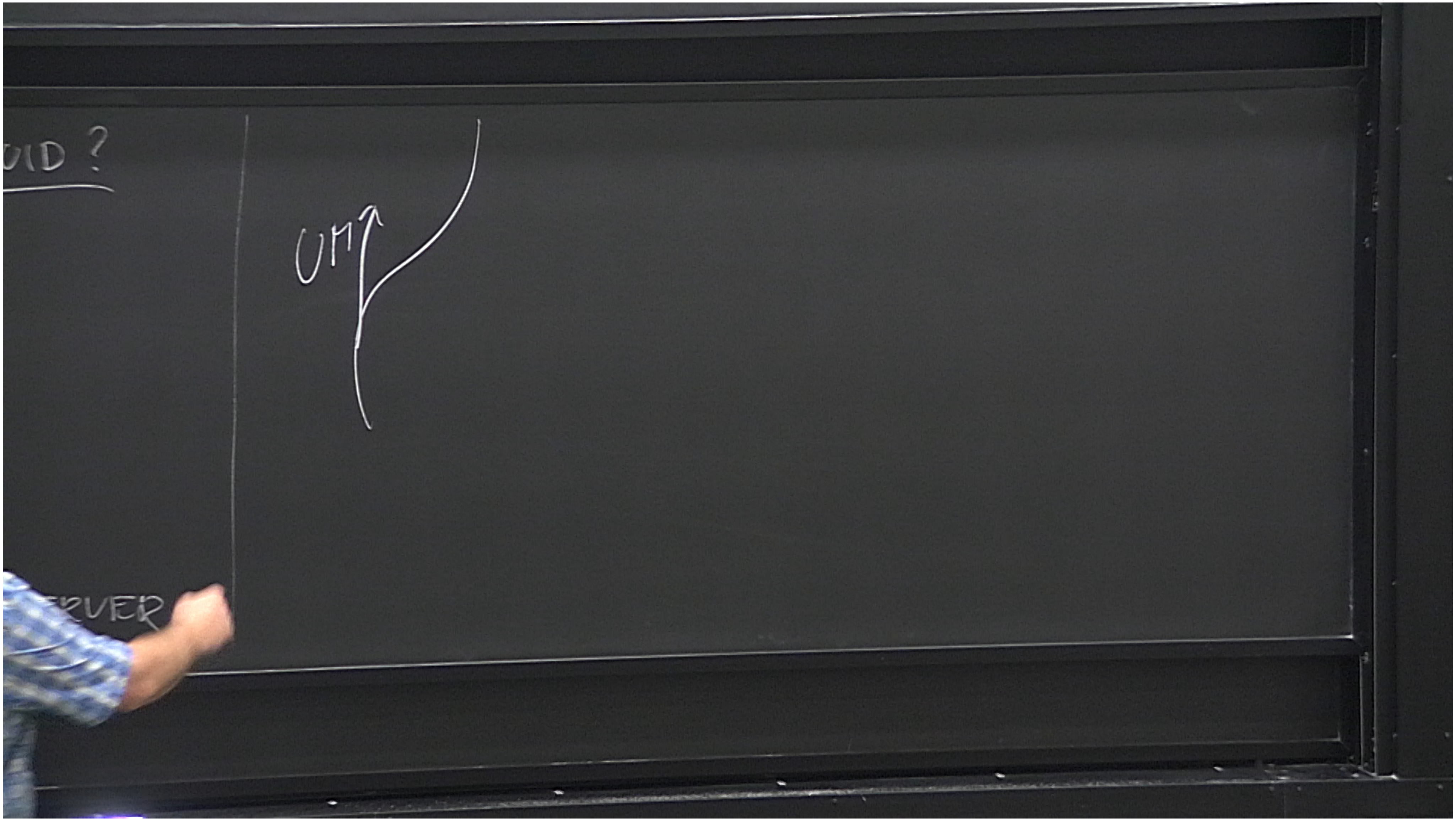
VERY DIFFERENT

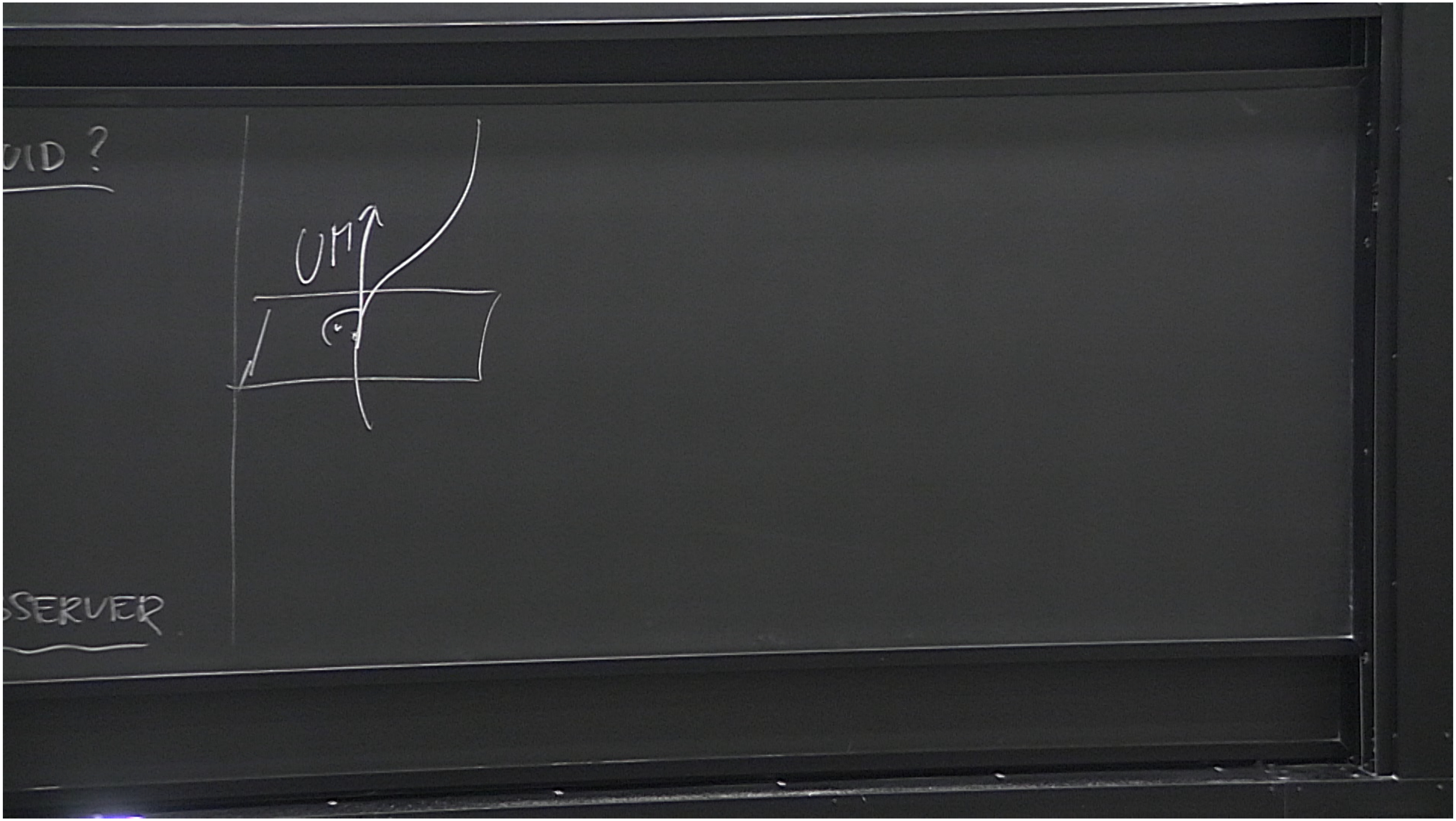
CONSERVATION OF $T^{\mu\nu}$ (MAYBE NOT ?)

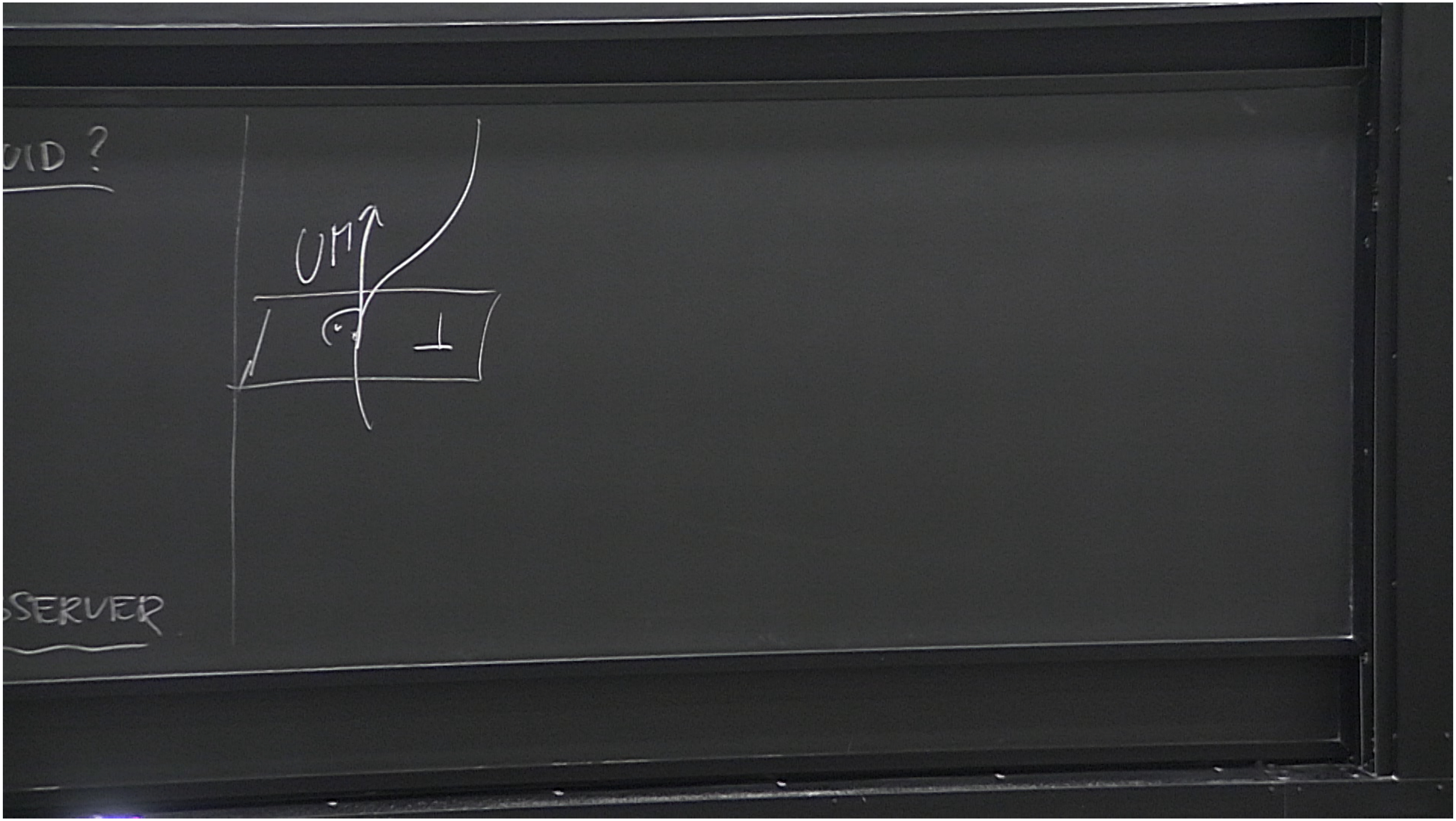
$\boxed{\partial_\mu T^{\mu\nu} = 0} \rightarrow$ CONSERVATION OF ENERGY & MOMENTUM

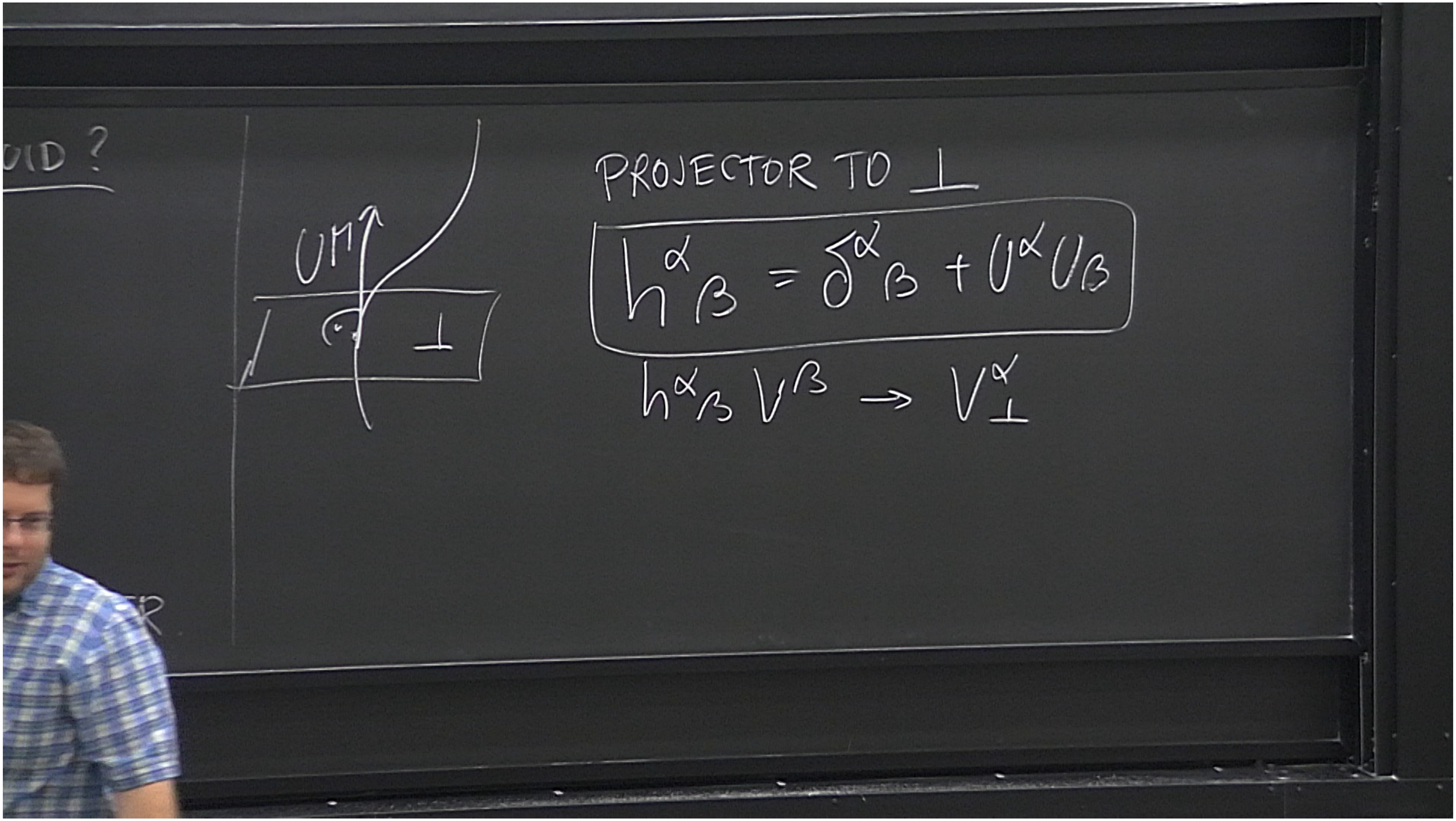

EXAMPLE. WHAT $D_\mu T^{\mu\nu}=0$ GIVES FOR PERFECT FLUID ?


EXAMPLE. WHAT $\nabla_\mu T^{\mu\nu}=0$ GIVES FOR PERFECT FLUID?
$T^{\mu\nu}=(\rho+P)U^\mu U^\nu+Pg^{\mu\nu}$
U^α... 4-VELOCITY OF THE FLUID
ρ... ENERGY DENSITY
P... PRESSURE
AS MEASURED BY COMOVING OBSERVER

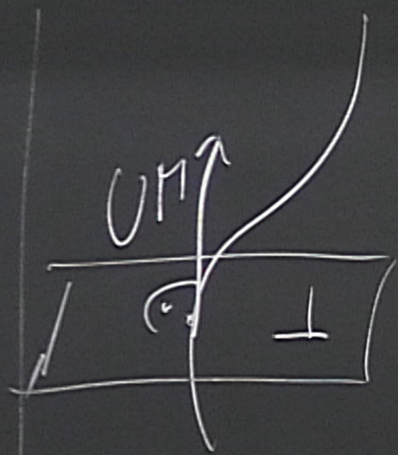



UID ?
UM
SSERVER


UID ?
UM
SSERVER


UID ?
PROJECTOR TO $\perp$
U^μ
$\perp$
$h^\alpha{}_\beta = \delta^\alpha{}_\beta + U^\alpha U_\beta$
$h^\alpha{}_\beta V^\beta \to V^\alpha_\perp$

UID ?



PROJECTOR TO $\perp$

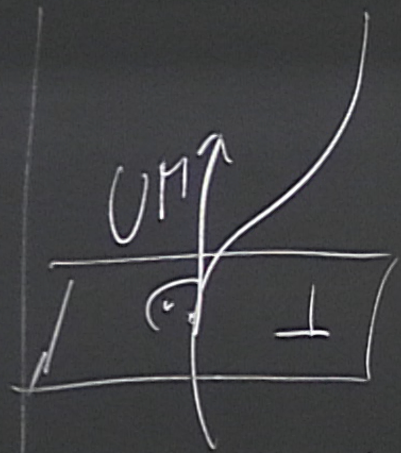
$$\boxed{h^\alpha{}_\beta = \delta^\alpha{}_\beta + U^\alpha U_\beta}$$

$$h^\alpha{}_\beta V^\beta \rightarrow V^\alpha_\perp$$

$$h^\alpha{}_\beta U^\beta = U^\alpha + U^\alpha U_\beta U^\beta$$

SERVER

UID ?



PROJECTOR TO $\perp$

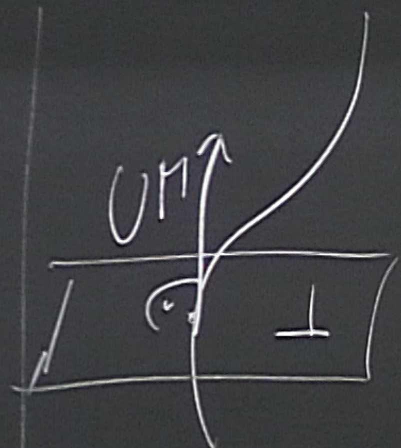
$$\boxed{h^\alpha{}_\beta = \delta^\alpha{}_\beta + U^\alpha U_\beta}$$

$$h^\alpha{}_\beta V^\beta \rightarrow V^\alpha_\perp$$

$$h^\alpha{}_\beta U^\beta = U^\alpha + U^\alpha U_\beta U^\beta = \underline{0}$$

SERVER

UID ?



PROJECTOR TO ⊥

$$\boxed{h^\alpha{}_\beta = \delta^\alpha{}_\beta + u^\alpha u_\beta}$$

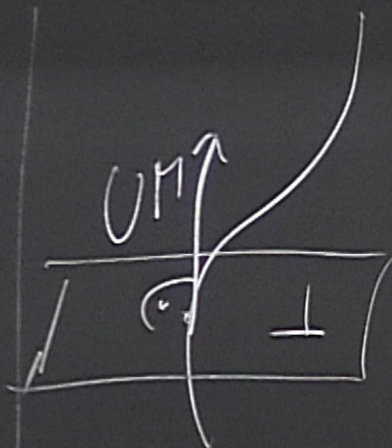
$$h^\alpha{}_\beta V^\beta \rightarrow V^\alpha_\perp$$

$$h^\alpha{}_\beta u^\beta = u^\alpha + u^\alpha u_\beta u^\beta = \underline{0}$$

$$h^\alpha{}_\beta h^\beta{}_\gamma = h^\alpha{}_\gamma$$

SSERVER

UID ?



PROJECTOR TO $\perp$

PROJECTOR TO $\parallel$

$$\boxed{h^\alpha{}_\beta = \delta^\alpha{}_\beta + U^\alpha U_\beta}$$

$$h^\alpha{}_\beta V^\beta \rightarrow V^\alpha_\perp$$

$$h^\alpha{}_\beta U^\beta = U^\alpha + U^\alpha U_\beta U^\beta = \underline{0}$$

$$h^\alpha{}_\beta h^\beta{}_\gamma = h^\alpha{}_\gamma$$

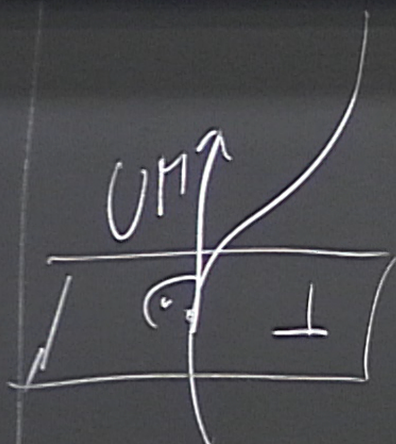
SERVER

$)/\partial\phi$

CONSERVATION OF $T^{\mu\nu}$ (MAYBE NOT ?)

$$\boxed{\partial_\mu T^{\mu\nu}=0} \to \text{CONSERVATION OF ENERGY \& MOMENTUM}$$

UID ?



PROJECTOR TO $\perp$

$$\boxed{h^\alpha{}_\beta = \delta^\alpha{}_\beta + U^\alpha U_\beta}$$

PROJECTOR TO $\parallel$

$$h^\alpha{}_\beta V^\beta \to V^\alpha_\perp$$

$$h^\alpha{}_\beta U^\beta = U^\alpha + U^\alpha \underbrace{U_\beta U^\beta}_{-1} = \underline{0}$$

$$h^\alpha{}_\beta h^\beta{}_\gamma = h^\alpha{}_\gamma$$

SERVER

ρ ... ENERGY DENSITY
P ... PRESSURE
} AS MEASURED BY COMOVING OBSERVER

$h^{\alpha}{}_{\beta} U^{\beta} = 0$

$h^{\alpha}{}_{\beta} h^{\beta}{}_{\gamma} = h^{\alpha}{}_{\gamma}$

$$\nabla_\mu T^{\mu\nu} = 0 = \nabla_\mu(\rho+P)\, U^\mu U^\nu + (\rho+P)(\nabla_\mu U^\mu) U^\nu + (\rho+P) U^\mu \nabla_\mu U^\nu + (\nabla_\mu P) g^{\mu\nu}$$

... OF THE FLUID

ρ ... ENERGY DENSITY } AS MEASURED
P ... PRESSURE } BY COMOVING OBSERVER

$$h^{\alpha}{}_{\beta} U^{\beta} = U^{\alpha} + U^{\alpha} \ldots$$

$$h^{\alpha}{}_{\beta} h^{\beta}{}_{\gamma} = h^{\alpha}{}_{\gamma}$$

$$\nabla_\mu T^{\mu\nu} = 0 = \nabla_\mu (\rho + P)\, U^\mu U^\nu + (\rho+P)(\nabla_\mu U^\mu)\, U^\nu + (\rho+P)\, U^\mu \nabla_\mu U^\nu + (\nabla_\mu P)\, g^{\mu\nu} + \emptyset$$

PROJECTION TO $\perp$.

... OF THE FLUID

ρ ... ENERGY DENSITY } AS MEASURED
P ... PRESSURE } BY COMOVING OBSERVER

$$h^{\alpha}{}_{\beta} U^{\beta} = U^{\alpha} + U^{\alpha} \ldots$$

$$h^{\alpha}{}_{\beta} h^{\beta}{}_{\gamma} = h^{\alpha}{}_{\gamma}$$

$$\nabla_\mu T^{\mu\nu} = 0 = \nabla_\mu(\rho+P)\, U^\mu U^\nu + (\rho+P)(\nabla_\mu U^\mu)\, U^\nu + (\rho+P)\, U^\mu \nabla_\mu U^\nu + (\nabla_\mu P)\, g^{\mu\nu} + 0$$

PROJECTION TO $\perp$. MULTIPLY BY $h^{\alpha}{}_{\nu}$

$$0 = 0 + 0 + (\rho+P)\, U^\mu h^{\alpha}{}_{\nu} \nabla_\mu U^\nu + h^{\alpha}{}_{\nu} (\nabla_\mu P)\, g^{\mu\nu}$$

... OF THE FLUID

ρ ... ENERGY DENSITY } AS MEASURED

P ... PRESSURE } BY COMOVING OBSERVER

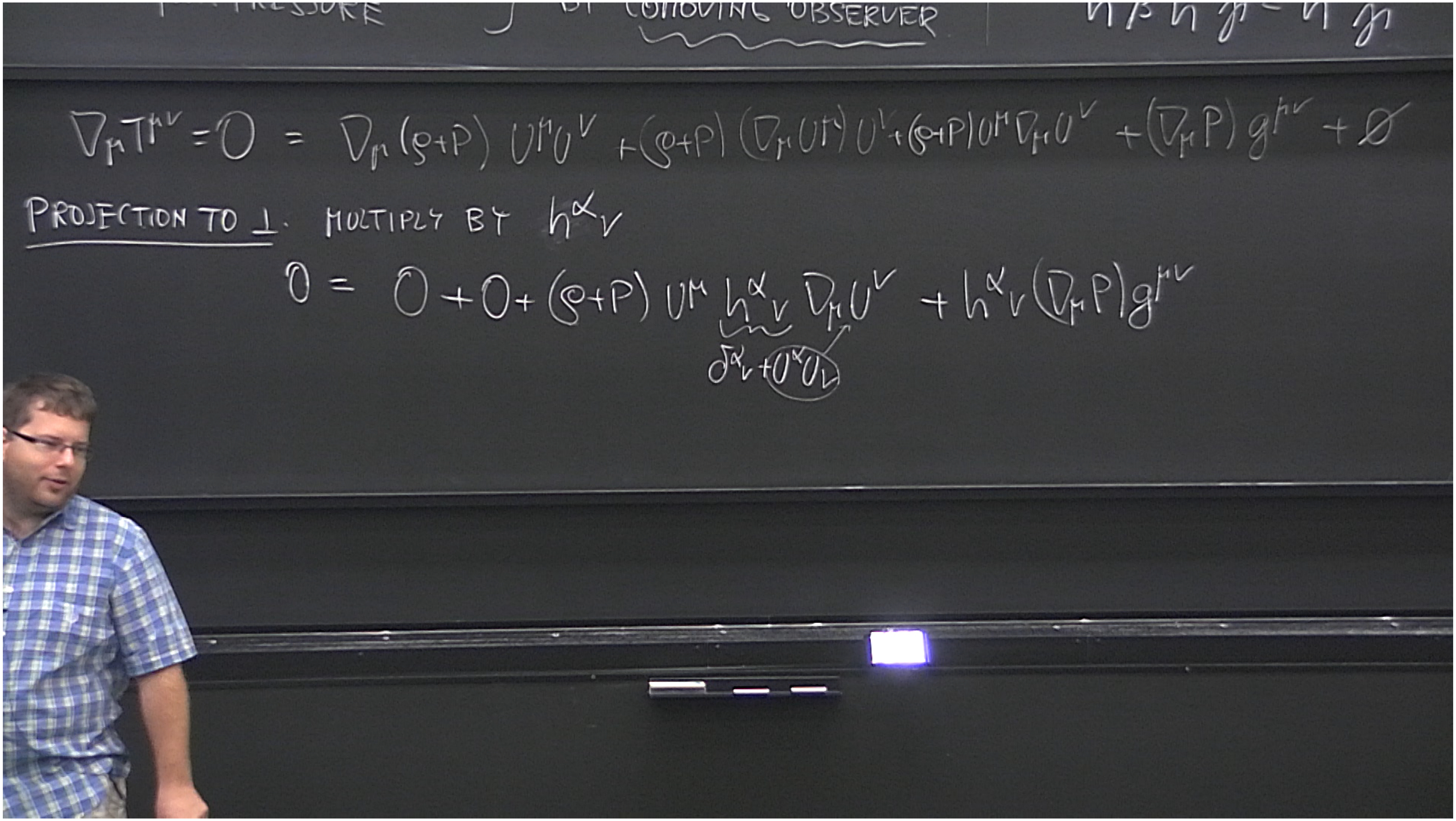
$$h^\alpha{}_\beta U^\beta = U^\alpha + U^\alpha$$

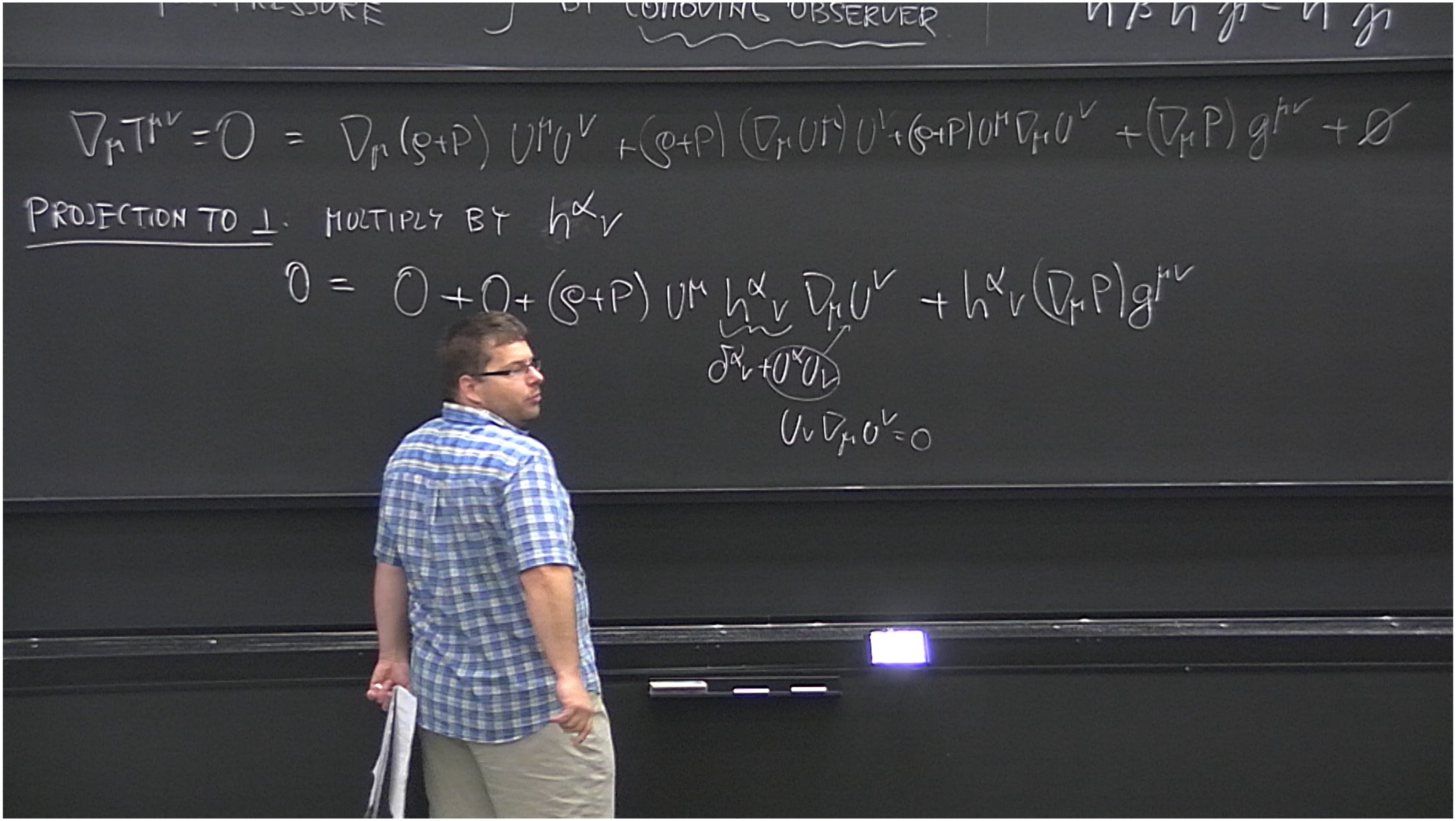
$$h^\alpha{}_\beta h^\beta{}_\gamma = h^\alpha{}_\gamma$$

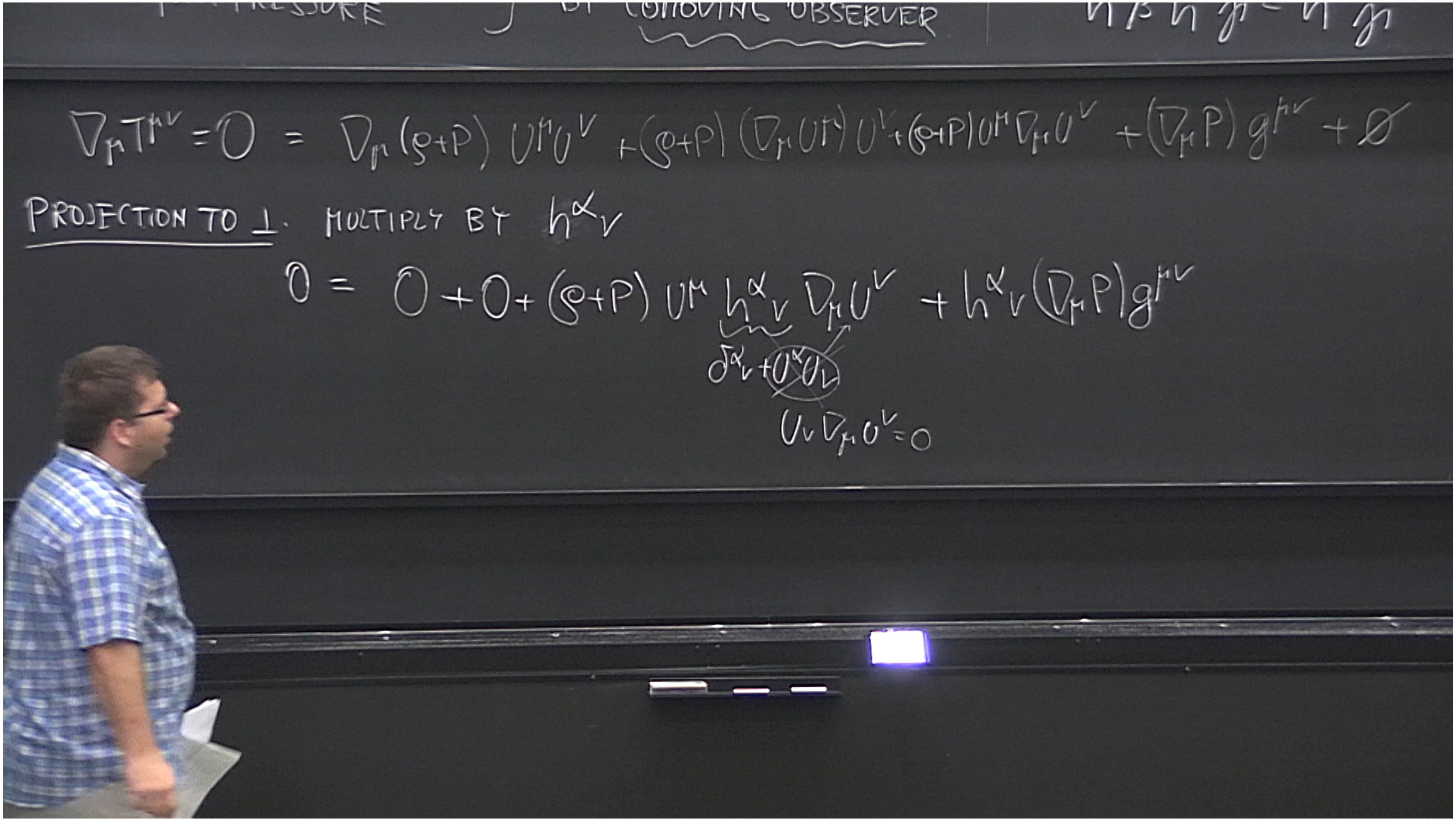
$$\nabla_\mu T^{\mu\nu} = 0 = \nabla_\mu(\rho+P)\,U^\mu U^\nu + (\rho+P)(\nabla_\mu U^\mu)U^\nu + (\rho+P)U^\mu \nabla_\mu U^\nu + (\nabla_\mu P)g^{\mu\nu} + \cancel{0}$$

PROJECTION TO $\perp$. MULTIPLY BY $h^\alpha{}_\nu$

$$0 = 0 + 0 + (\rho+P)\,U^\mu \underbrace{h^\alpha{}_\nu}_{\delta^\alpha_\nu + U^\alpha U_\nu} \nabla_\mu U^\nu + h^\alpha{}_\nu(\nabla_\mu P)g^{\mu\nu}$$


COMOVING OBSERVER
$\nabla_\mu T^{\mu\nu} = 0 = \nabla_\mu(\rho+P)\,U^\mu U^\nu + (\rho+P)(\nabla_\mu U^\mu)\,U^\nu + (\rho+P)\,U^\mu \nabla_\mu U^\nu + (\nabla_\mu P)\,g^{\mu\nu} + \cancel{0}$
PROJECTION TO $\perp$. MULTIPLY BY $h^\alpha{}_\nu$
$0 = 0 + 0 + (\rho+P)\,U^\mu \underbrace{h^\alpha{}_\nu}_{\delta^\alpha{}_\nu + U^\alpha U_\nu} \nabla_\mu U^\nu + h^\alpha{}_\nu(\nabla_\mu P)\,g^{\mu\nu}$


COMOVING OBSERVER
$\nabla_\mu T^{\mu\nu} = 0 = \nabla_\mu(\rho+P)\,U^\mu U^\nu + (\rho+P)(\nabla_\mu U^\mu)U^\nu + (\rho+P)U^\mu \nabla_\mu U^\nu + (\nabla_\mu P)\,g^{\mu\nu} + \cancel{0}$
PROJECTION TO $\perp$. MULTIPLY BY $h^\alpha{}_\nu$
$0 = 0 + 0 + (\rho+P)\,U^\mu h^\alpha{}_\nu \nabla_\mu U^\nu + h^\alpha{}_\nu(\nabla_\mu P)\,g^{\mu\nu}$
$\delta^\alpha{}_\nu + U^\alpha U_\nu$
$U_\nu \nabla_\mu U^\nu = 0$


COMOVING OBSERVER
$\nabla_\mu T^{\mu\nu} = 0 = \nabla_\mu(\rho+P)\,U^\mu U^\nu + (\rho+P)(\nabla_\mu U^\mu)U^\nu + (\rho+P)U^\mu \nabla_\mu U^\nu + (\nabla_\mu P)\,g^{\mu\nu} + 0$
PROJECTION TO $\perp$: MULTIPLY BY $h^\alpha{}_\nu$
$0 = 0 + 0 + (\rho+P)\,U^\mu h^\alpha{}_\nu \nabla_\mu U^\nu + h^\alpha{}_\nu (\nabla_\mu P)\,g^{\mu\nu}$
$\delta^\alpha{}_\nu + U^\alpha U_\nu$
$U_\nu \nabla_\mu U^\nu = 0$

... PRESSURE ... BY COMOVING OBSERVER

$$\nabla_\mu T^{\mu\nu} = 0 = \nabla_\mu(\rho+P)\,U^\mu U^\nu + (\rho+P)(\nabla_\mu U^\mu)U^\nu + (\rho+P)U^\mu \nabla_\mu U^\nu + (\nabla_\mu P)\,g^{\mu\nu} + \cancel{0}$$

PROJECTION TO $\perp$. MULTIPLY BY $h^\alpha{}_\nu$

$$0 = 0 + 0 + (\rho+P)\,U^\mu \underbrace{h^\alpha{}_\nu}_{\delta^\alpha{}_\nu + \cancel{U^\alpha U_\nu}} \nabla_\mu U^\nu + h^\alpha{}_\nu(\nabla_\mu P)\,g^{\mu\nu}$$

$$U_\nu \nabla_\mu U^\nu = 0$$

$$0 = (\rho+P)\,U^\mu \nabla_\mu U^\alpha$$

... PRESSURE ... BY COMOVING OBSERVER

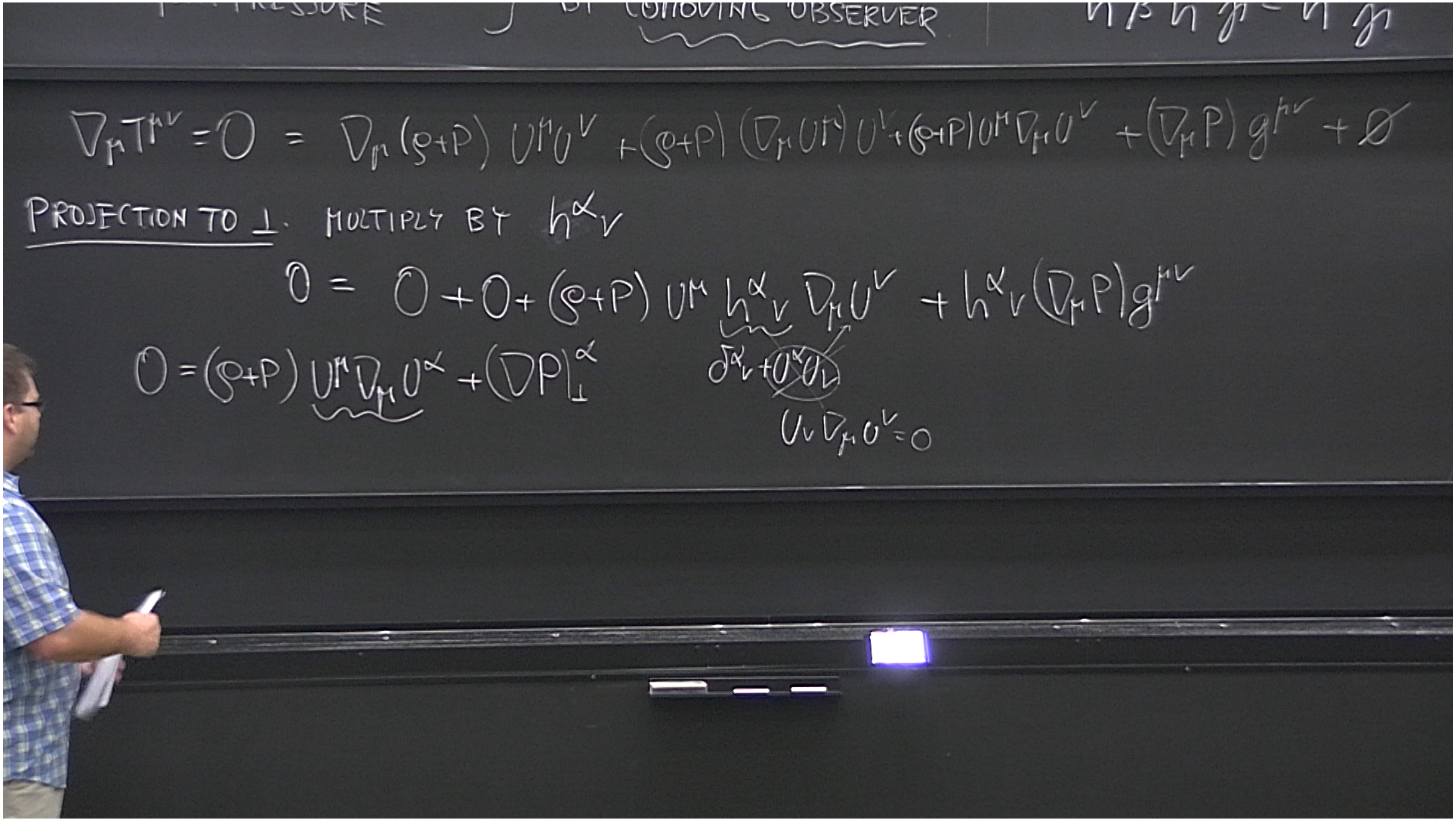
$$\nabla_\mu T^{\mu\nu} = 0 = \nabla_\mu(\rho+P)\,U^\mu U^\nu + (\rho+P)(\nabla_\mu U^\mu)U^\nu + (\rho+P)U^\mu\nabla_\mu U^\nu + (\nabla_\mu P)g^{\mu\nu} + \cancel{0}$$

PROJECTION TO $\perp$: MULTIPLY BY $h^\alpha{}_\nu$

$$0 = 0 + 0 + (\rho+P)\,U^\mu \underbrace{h^\alpha{}_\nu}_{\delta^\alpha{}_\nu + U^\alpha U_\nu} \nabla_\mu U^\nu + h^\alpha{}_\nu(\nabla_\mu P)g^{\mu\nu}$$

$$U_\nu \nabla_\mu U^\nu = 0$$

$$0 = (\rho+P)\,U^\mu\nabla_\mu U^\alpha + (\nabla P)^\alpha_\perp$$



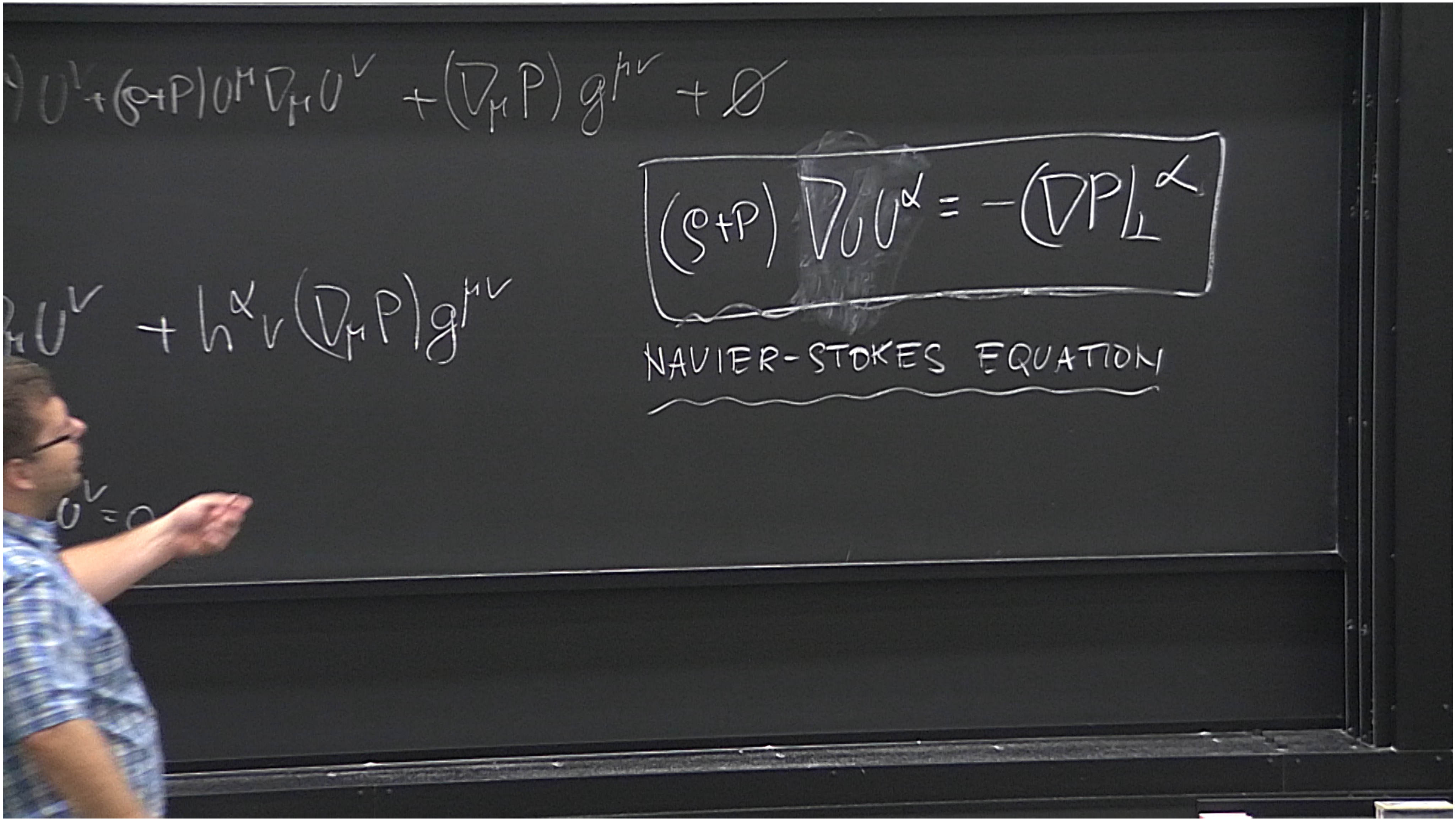
COMOVING OBSERVER
$\nabla_\mu T^{\mu\nu} = 0 = \nabla_\mu(\rho+P)\, U^\mu U^\nu + (\rho+P)(\nabla_\mu U^\mu) U^\nu + (\rho+P) U^\mu \nabla_\mu U^\nu + (\nabla_\mu P) g^{\mu\nu} + \cancel{0}$
PROJECTION TO $\perp$: MULTIPLY BY $h^\alpha{}_\nu$
$0 = 0 + 0 + (\rho+P) U^\mu \underbrace{h^\alpha{}_\nu}_{\delta^\alpha{}_\nu + U^\alpha U_\nu} \nabla_\mu U^\nu + h^\alpha{}_\nu (\nabla_\mu P) g^{\mu\nu}$
$U_\nu \nabla_\mu U^\nu = 0$
$0 = (\rho+P) U^\mu \nabla_\mu U^\alpha + (\nabla P)^\alpha_\perp$

$$) U^{\nu} + (\rho + P) U^{\mu} \nabla_{\mu} U^{\nu} + (\nabla_{\mu} P) g^{\mu\nu} + \cancel{0}$$

$$\boxed{(\rho + P) \nabla_U U^{\alpha} = -(\nabla P)_{\perp}^{\alpha}}$$

$$\nabla_{\mu} U^{\nu} + h^{\alpha}{}_{\nu} (\nabla_{\mu} P) g^{\mu\nu}$$

$$\nabla_{\mu} U^{\nu} = 0$$


NAVIER-STOKES EQUATION

$$) U^\nu + (\rho + P) U^\mu \nabla_\mu U^\nu + (\nabla_\mu P) g^{\mu\nu} + \cancel{0}$$

$$\boxed{\left(\rho + \frac{P}{c^2}\right) \nabla_U U^\alpha = -(\nabla P)_\perp^\alpha}$$

NAVIER-STOKES EQUATION

$$_\mu U^\nu + h^\alpha{}_\nu (\nabla_\mu P) g^{\mu\nu}$$

$$\nabla_\mu U^\nu = 0$$

$$) U^{\nu} + (\rho + P) U^{\mu} \nabla_{\mu} U^{\nu} + (\nabla_{\mu} P) g^{\mu\nu} + \cancel{0}$$

$$\boxed{\left(\rho + \frac{P}{c^2}\right) \nabla_U U^{\alpha} = -(\nabla P)_{\perp}^{\alpha}}$$

NAVIER-STOKES EQUATION

$$\nabla_{\mu} U^{\nu} + h^{\alpha}{}_{\nu} (\nabla_{\mu} P) g^{\mu\nu}$$

GR CORRECTION

$$\nabla_{\mu} U^{\nu} = 0$$

$$U^{\nu} + (\rho + P) U^{\mu} \nabla_{\mu} U^{\nu} + (\nabla_{\mu} P) g^{\mu\nu} + \emptyset$$

$$U^{\nu} + h^{\alpha}{}_{\nu} (\nabla_{\mu} P) g^{\mu\nu}$$

$$\boxed{\left(\rho + \frac{P}{c^2}\right) \nabla_U U^{\alpha} = -(\nabla P)_{\perp}^{\alpha}}$$

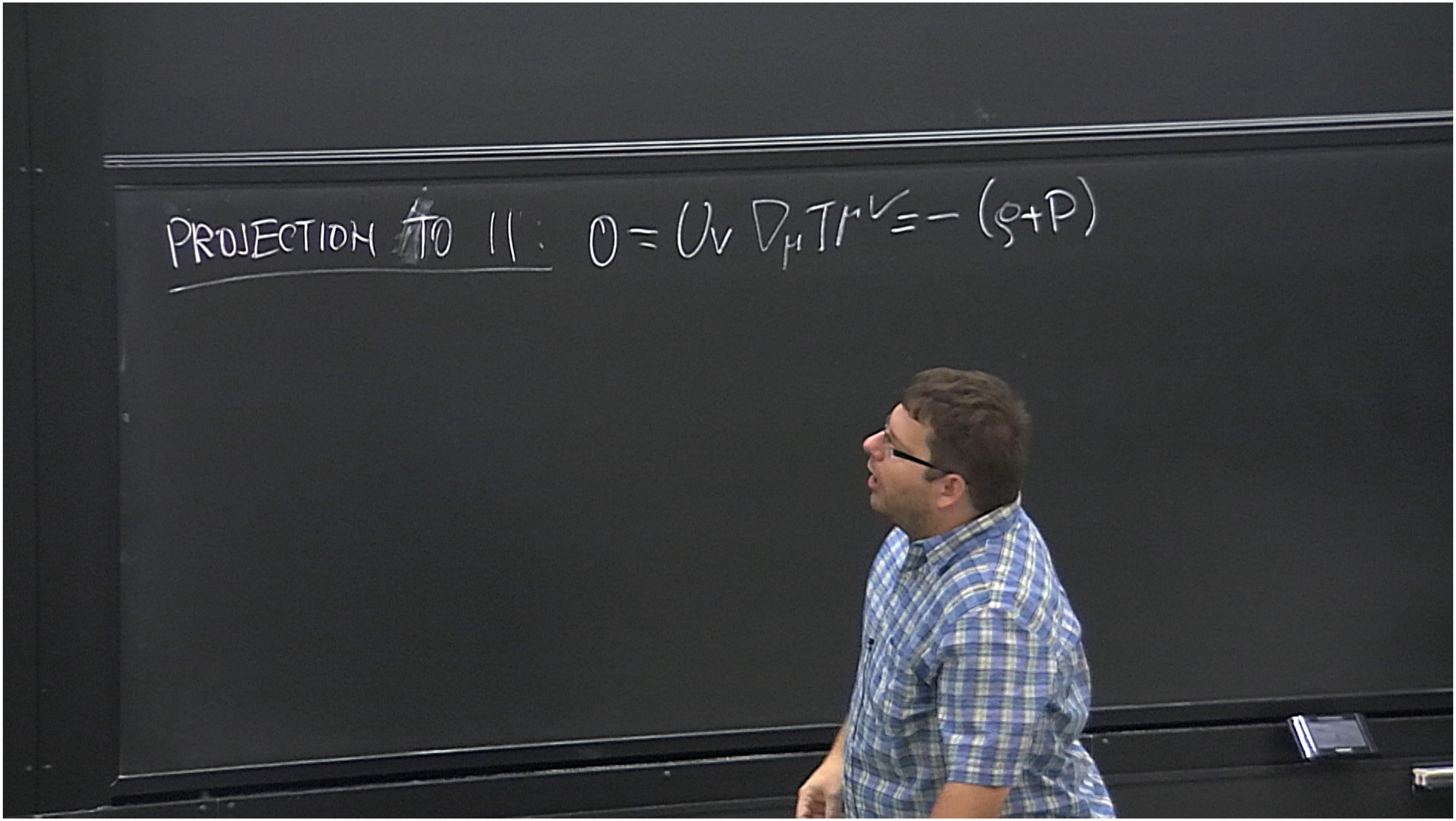
NAVIER-STOKES EQUATION

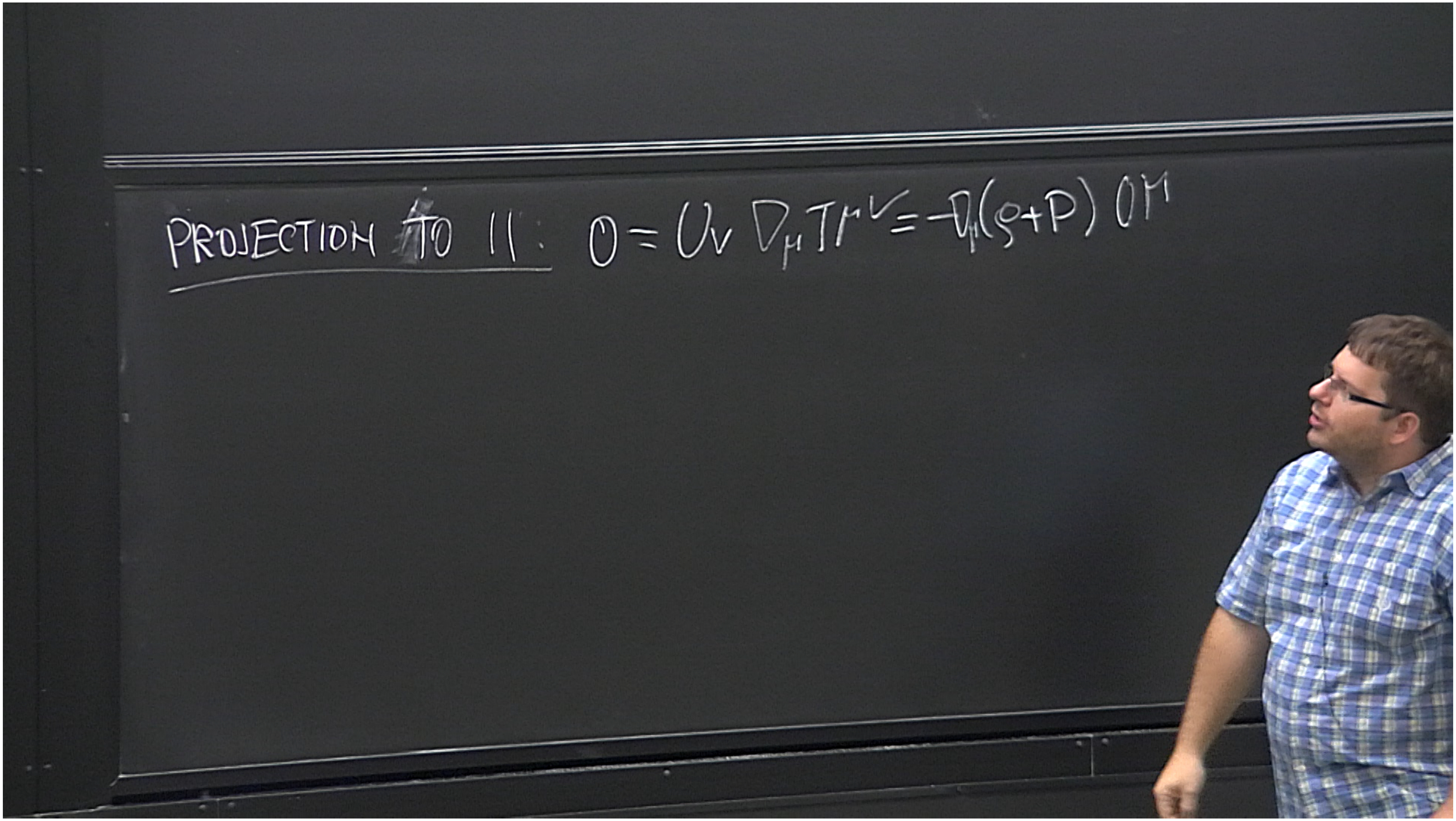
GR CORRECTION

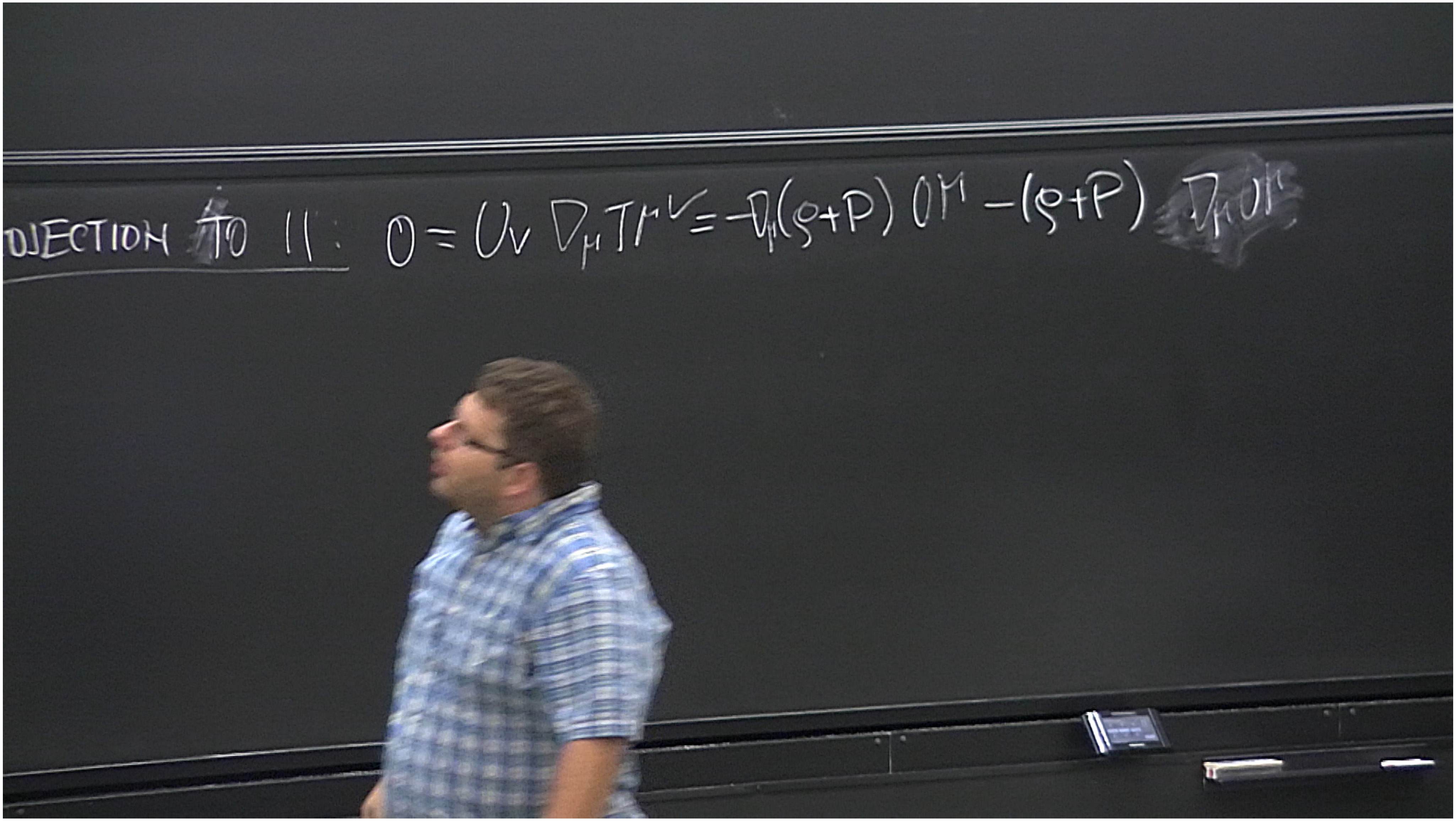
(EQ. OF MOTION FOR FLUID)

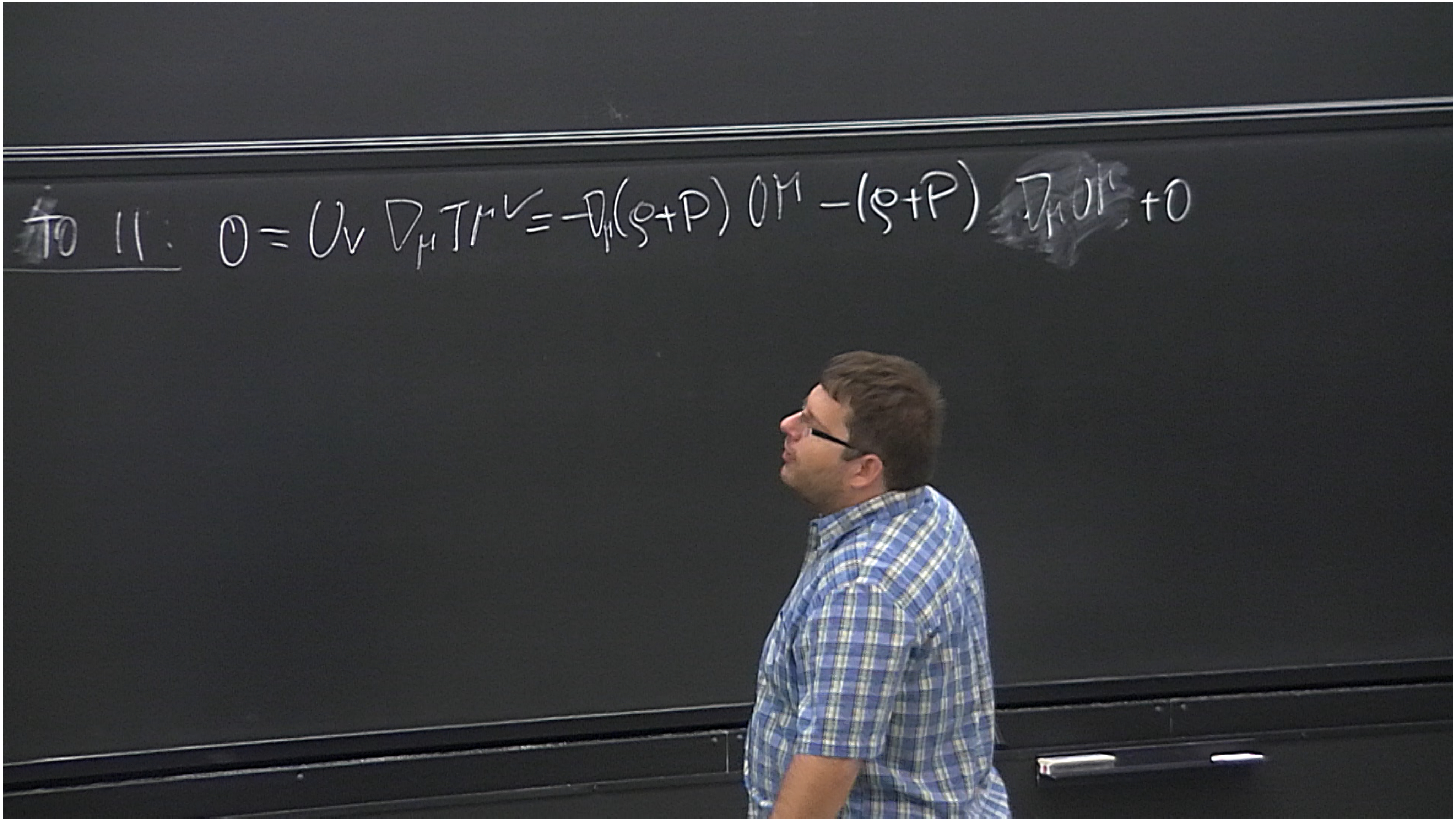
$$_{\mu} U^{\nu} = 0$$

PROJECTION TO $\parallel$: $0 = U_\nu D_\mu T^{\mu\nu}$

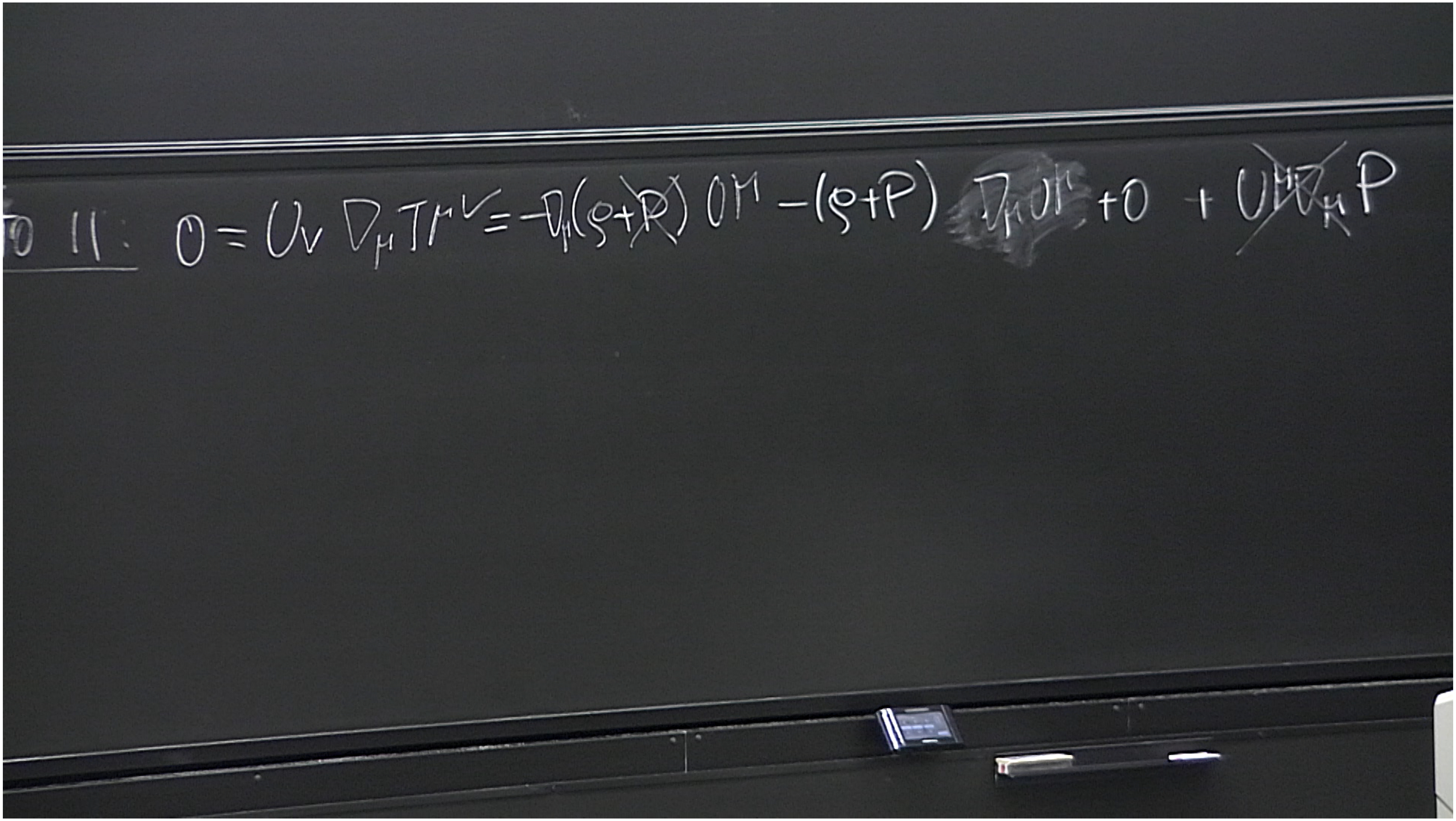

PROJECTION TO ||: $0 = U_\nu D_\mu T^{\mu\nu} = -(\rho+P)$


PROJECTION TO ||:
$0 = U_\nu D_\mu T^{\mu\nu} = -D_\mu(\rho+P)\, U^\mu$


OJECTION TO ||:
$0 = U_\nu D_\mu T^{\mu\nu} = -D_\mu(\rho+P)\,U^\mu - (\rho+P)\,D_\mu U^\mu$


TO II: $0 = U_\nu D_\mu T^{\mu\nu} = -D_\mu(\rho+P)\,U^\mu - (\rho+P)\,D_\mu U^\mu + 0$

o II: $0 = U_\nu \nabla_\mu T^{\mu\nu} = -\nabla_\mu(\rho+P)\, U^\mu - (\rho+P)\, \nabla_\mu U^\mu + 0 + U^\mu \nabla_\mu P$


o 11:
$0 = U_\nu D_\mu T^{\mu\nu} = -D_\mu(\rho + \cancel{P}) U^\mu - (\rho + P) D_\mu U^\mu + 0 + \cancel{U^\mu D_\mu} P$

o 11: $0 = U_\nu D_\mu T^{\mu\nu} = -D_\mu(\rho+\cancel{P})\, U^\mu - (\rho+P)\, D_\mu U^\mu + 0 + \cancel{U^\mu D_\mu} P$

$$\boxed{U^\mu D_\mu \rho + (\rho+P)\, D_\mu U^\mu = 0}$$

o 11: $0 = U_\nu D_\mu T^{\mu\nu} = -D_\mu(\rho+\cancel{P})\,U^\mu - (\rho+P)\,D_\mu U^\mu + 0 + \cancel{U^\mu D_\mu} P$

$$\boxed{U^\mu D_\mu \rho + (\rho+P)\,D_\mu U^\mu = 0}$$

CONTINUITY EQ.

PROJECTION TO $\parallel$: $0 = U_\nu D_\mu T^{\mu\nu} = -D_\mu(\rho+\cancel{P})\,U^\mu - (\rho+P)\,D_\mu U$

$$\boxed{U^\mu D_\mu \rho + (\rho+P)\,D_\mu U^\mu = 0}$$

CONTINUITY EQ.

$D_\mu T^{\mu\nu} = 0$ "IMPLIES EQ OF MOTION FOR MATTER"

PROJECTION TO $\parallel$: $0 = U_\nu D_\mu T^{\mu\nu} = -D_\mu(\rho + \cancel{P})\, U^\mu - (\rho + P)\, D_\mu U$

$$\boxed{U^\mu D_\mu \rho + (\rho + P)\, D_\mu U^\mu = 0}$$

CONTINUITY EQ.

$D_\mu T^{\mu\nu} = 0$ "IMPLIES EQ OF MOTION FOR MATTER"

INTERMEZZO: ACTION FOR ELECTROMAGNETISM

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- WANT A SCALAR(S) $f = f(A, \partial A)$

INTERMEZZO: ACTION FOR ELECTROMAGNETISM

WANT A SCALAR(S) $f = f(A, \partial A)$

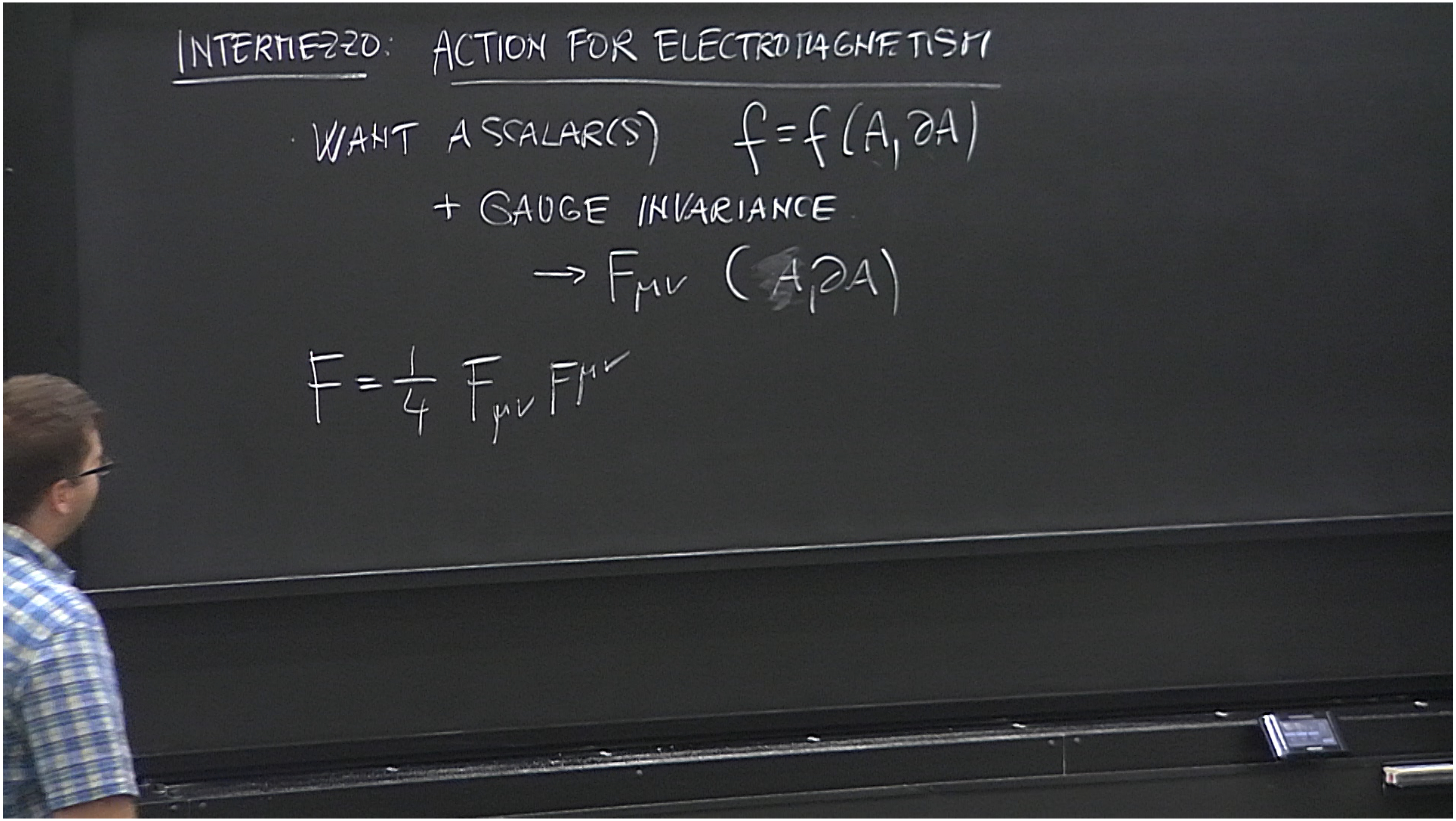
+ GAUGE INVARIANCE

INTERMEZZO: ACTION FOR ELECTROMAGNETISM

· WANT A SCALAR(S) $f = f(A, \partial A)$

+ GAUGE INVARIANCE

$\rightarrow F_{\mu\nu}(A, \partial A)$


INTERMEZZO: ACTION FOR ELECTROMAGNETISM
WANT A SCALAR(S) $f = f(A, \partial A)$
+ GAUGE INVARIANCE
$\rightarrow F_{\mu\nu}(A, \partial A)$
$F = \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$

INTERMEZZO: ACTION FOR ELECTROMAGNETISM

WANT A SCALAR(S) $f = f(A, \partial A)$

+ GAUGE INVARIANCE

$\rightarrow F_{\mu\nu}(A, \partial A)$

$$F = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \, , \quad \cancel{F_{\mu\nu}\eta^{\mu\nu}}$$

INTERMEZZO: ACTION FOR ELECTROMAGNETISM

WANT A SCALAR(S) $f = f(A, \partial A)$

\+ GAUGE INVARIANCE

$\rightarrow F_{\mu\nu}(A, \partial A)$

$$F = \frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \qquad \cancel{F_{\mu\nu}\eta^{\mu\nu}}$$

$$G = \frac{1}{4} F_{\mu\nu} {}^{*}F^{\mu\nu}$$

INTERMEZZO: ACTION FOR ELECTROMAGNETISM

WANT A SCALAR(S) $f = f(A, \partial A)$

\+ GAUGE INVARIANCE

$\to F_{\mu\nu}(A, \partial A)$

$$\boxed{\begin{aligned} F &= \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\ G &= \frac{1}{4} F_{\mu\nu} {}^{*}F^{\mu\nu} \end{aligned}}$$

$\cancel{F_{\mu\nu}\eta^{\mu\nu}}$

ANY $\mathcal{L} = \mathcal{L}(F, G)$ ... 2ND-ORDER "MAXWELL EQS."

ANY $\mathcal{L} = \mathcal{L}(F, G^2)$ ... 2ND-ORDER "MAXWELL EQS."

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- MAXWELL (LINEAR) $\boxed{\mathcal{L} = F}$

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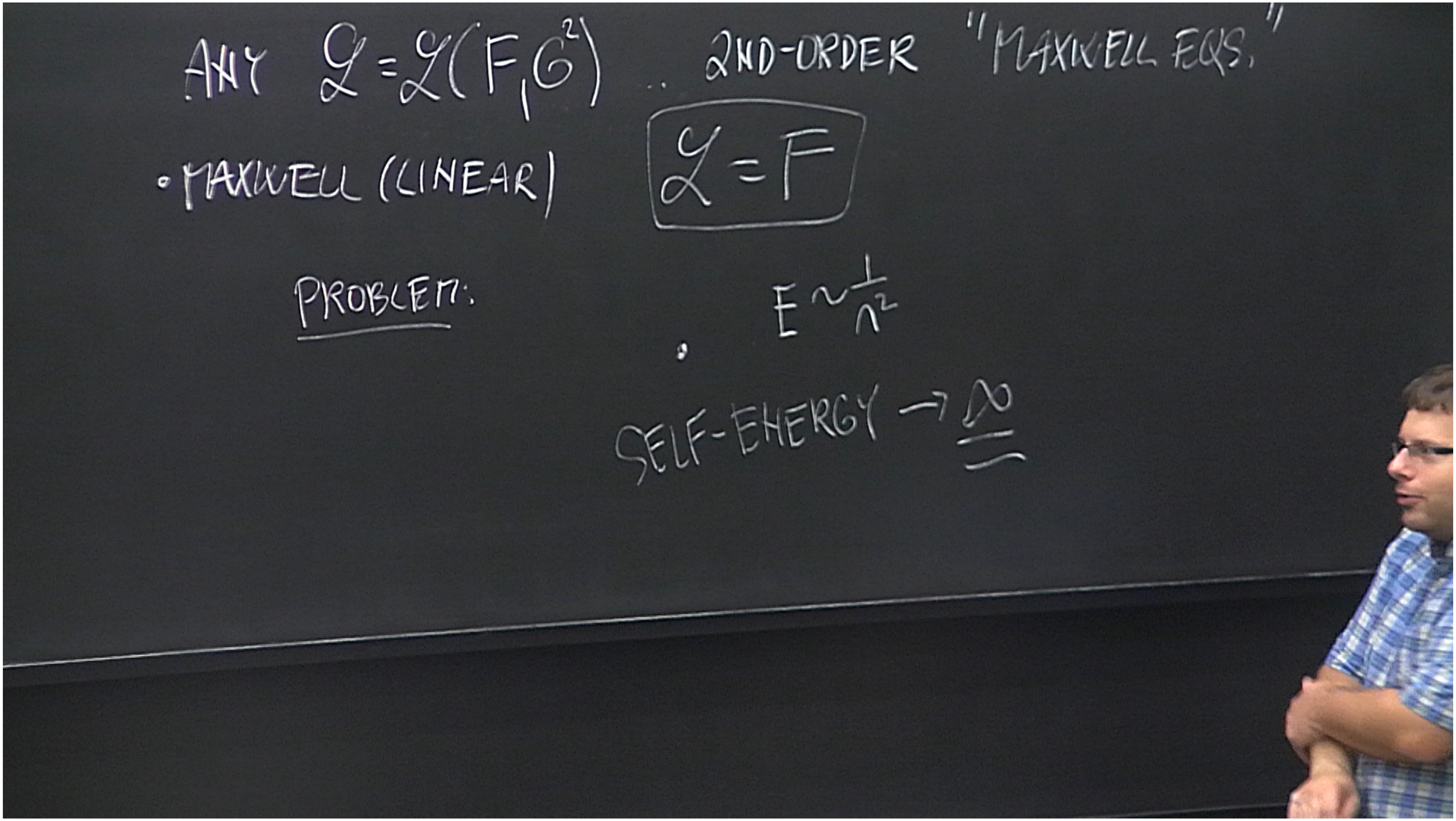
<u>PROBLEM</u>:

- $E \sim \frac{1}{r^2}$

SELF-ENERGY


ANY $\mathcal{L} = \mathcal{L}(F, G^2)$ .. 2ND-ORDER "MAXWELL EQS."
• MAXWELL (LINEAR)
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PROBLEM:
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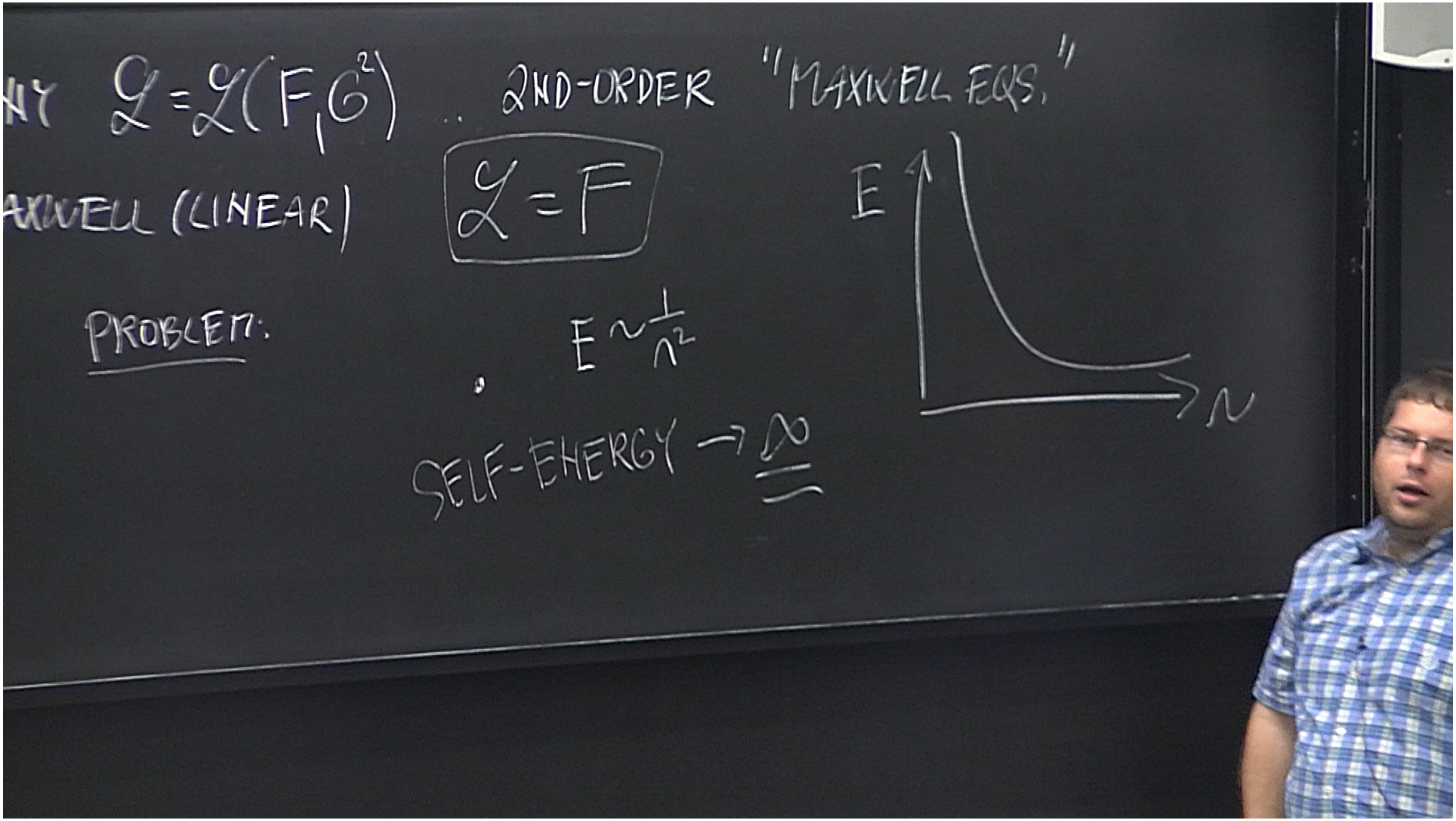
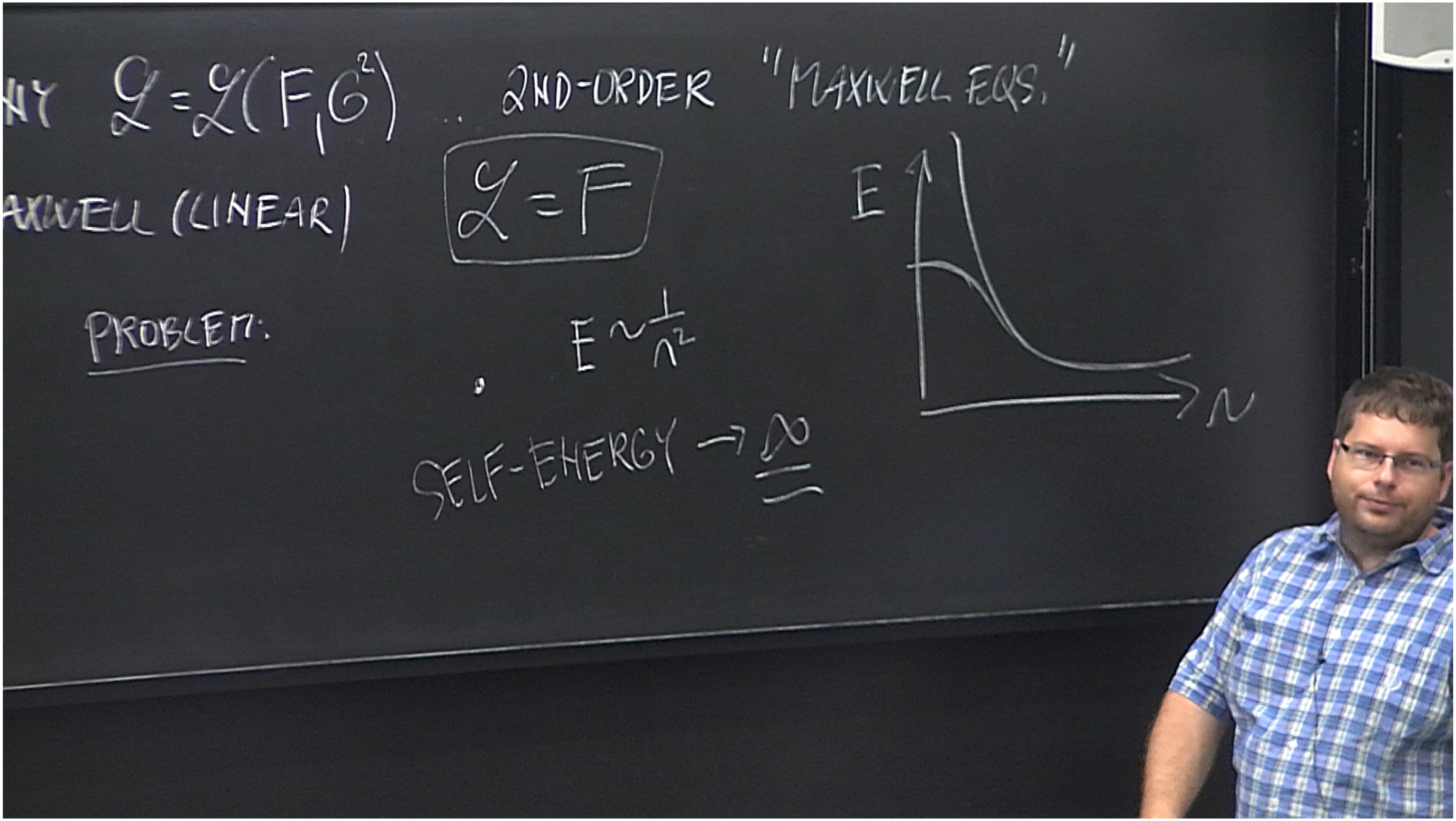


$\mathcal{L} = \mathcal{L}(F, G^2)$
2ND-ORDER "MAXWELL EQS."
AXWELL (LINEAR)
$\mathcal{L} = F$
PROBLEM:
$E \sim \frac{1}{r^2}$
SELF-ENERGY → ∞
E
r



2ND-ORDER "MAXWELL EQS."
AXWELL (LINEAR)
$\mathcal{L} = \mathcal{L}(F, G^2)$
$\mathcal{L} = F$
PROBLEM:
$E \sim \frac{1}{r^2}$
SELF-ENERGY $\to \infty$
E
r

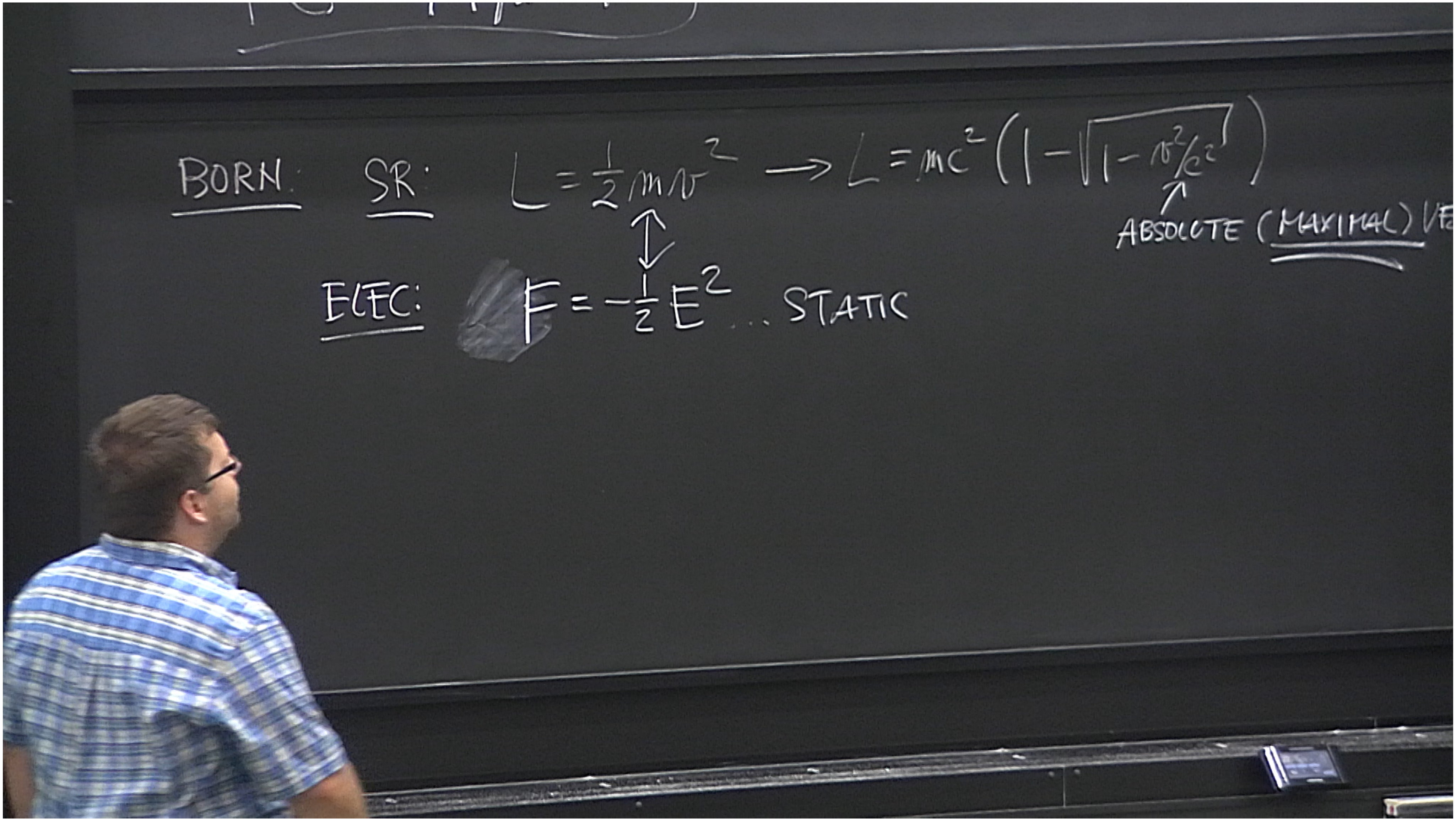
$$G = \frac{1}{4} F_{\mu\nu} {}^{*}F^{\mu\nu}$$

<u>BORN</u>: <u>SR</u>: $L = \frac{1}{2} m v^2$

$$F_{\mu\nu} \, {}^{*}F^{\mu\nu}$$

$$L = \frac{1}{2} m v^2 \longrightarrow L = mc^2\left(1 - \sqrt{1 - v^2/c^2}\right)$$

ABSOLUTE (MAXIMAL) VELOCITY

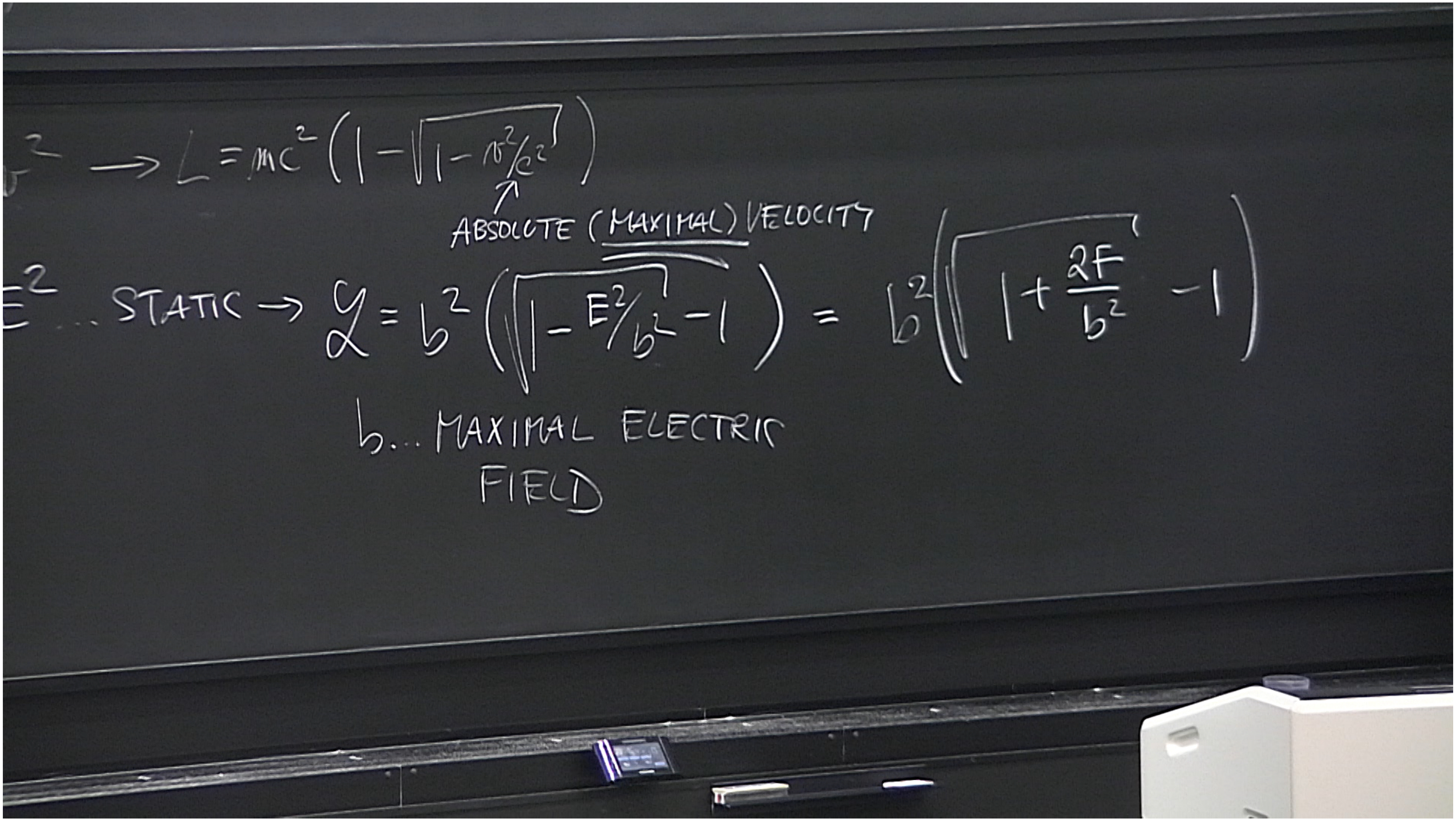

BORN:
SR:
$L = \frac{1}{2} m v^2 \longrightarrow L = mc^2 \left(1 - \sqrt{1 - v^2/c^2}\right)$
ABSOLUTE (MAXIMAL)
ELEC:
$F = -\frac{1}{2} E^2$ ... STATIC

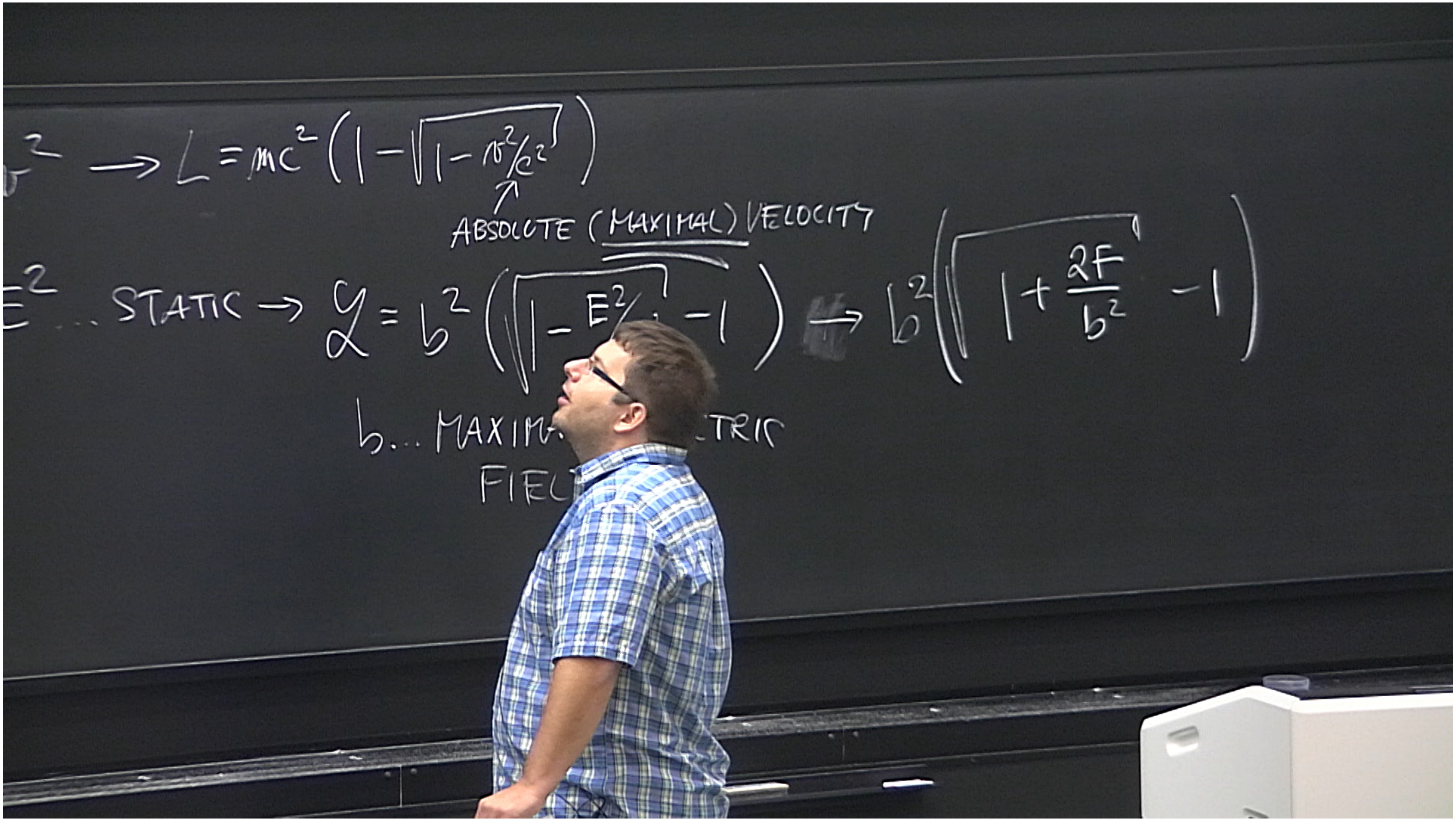
ORN: SR: $L = \frac{1}{2} m v^2 \longrightarrow L = mc^2\left(1-\sqrt{1-v^2/c^2}\right)$

ABSOLUTE (MAXIMAL) VELOCITY

ELEC: $F = -\frac{1}{2}E^2$ ... STATIC $\rightarrow$ $\mathcal{L} = b^2\left(\sqrt{1-E^2/b^2}-1\right)$

b... MAXIMAL ELECTRIC FIELD


$v^2 \longrightarrow L = mc^2\left(1-\sqrt{1-v^2/c^2}\right)$
ABSOLUTE (MAXIMAL) VELOCITY
E^2 ... STATIC $\longrightarrow \mathcal{L} = b^2\left(\sqrt{1-E^2/b^2}-1\right) = b^2\left(\sqrt{1+\frac{2F}{b^2}}-1\right)$
b... MAXIMAL ELECTRIC FIELD



$\longrightarrow L = mc^2\left(1-\sqrt{1-v^2/c^2}\right)$
ABSOLUTE (MAXIMAL) VELOCITY
STATIC $\to \mathcal{L} = b^2\left(\sqrt{1-E^2/\ldots}-1\right) \to b^2\left(\sqrt{1+\frac{2F}{b^2}}-1\right)$
b... MAXIM
TRIC
FIEL

...TE (MAXIMAL) VELOCITY

$$\sqrt{\;\ldots - E^2/b^2} - 1 \Big) \longrightarrow b^2\left(\sqrt{1+\frac{2F}{b^2}}-1\right)$$

...AL ELECTRIC

...ELD)

1908 ?

BORN-INFELD (30's)

$$\sqrt{-\det\left(g_{\mu\nu}+F_{\mu\nu}\right)}$$

NFELD (30's)

$$\sqrt{-\det\left(g_{\mu\nu}+F_{\mu\nu}\right)}-\sqrt{\det g_{\mu\nu}}=\sqrt{-g}\left(\sqrt{1+F-G^2}-1\right)$$

BORN-INFELD (30's)

$$\sqrt{-\det\left(g_{\mu\nu}+F_{\mu\nu}\right)} - \sqrt{\det g_{\mu\nu}} = \sqrt{-g}\left(\sqrt{1+\ldots}\right.$$

$$\boxed{\mathcal{L}_{BI} = b^2\left(\sqrt{1+\frac{2F}{b^2}-\frac{G^2}{b^4}}-1\right)}$$

BORN-INFELD (30's)

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PHOENIX FROM ASHES 60's

ELEC: $F = -\frac{1}{2}E^2$ ... STATIC $\rightarrow$ $\mathcal{L} = b^2\left(\sqrt{1 - E^2/b^2} - 1\right)$

60's ... RELATIVISTS

80's ... STRING

b ... MAXIMAL ELECTRIC FIELD

$$\boxed{\mathcal{L}_{BI} = b^2\left(\sqrt{1 + \frac{2F}{b^2} - \frac{G^2}{b^4}} - 1\right)}$$

PHOENIX 60'S

FROM ASHES

ELEC: $F = -\frac{1}{2}E^2$ ... STATIC $\rightarrow$ $\mathcal{L} = b^2\left(\sqrt{1 - E^2/b^2} - 1\right)$

60's ... RELATIVISTS

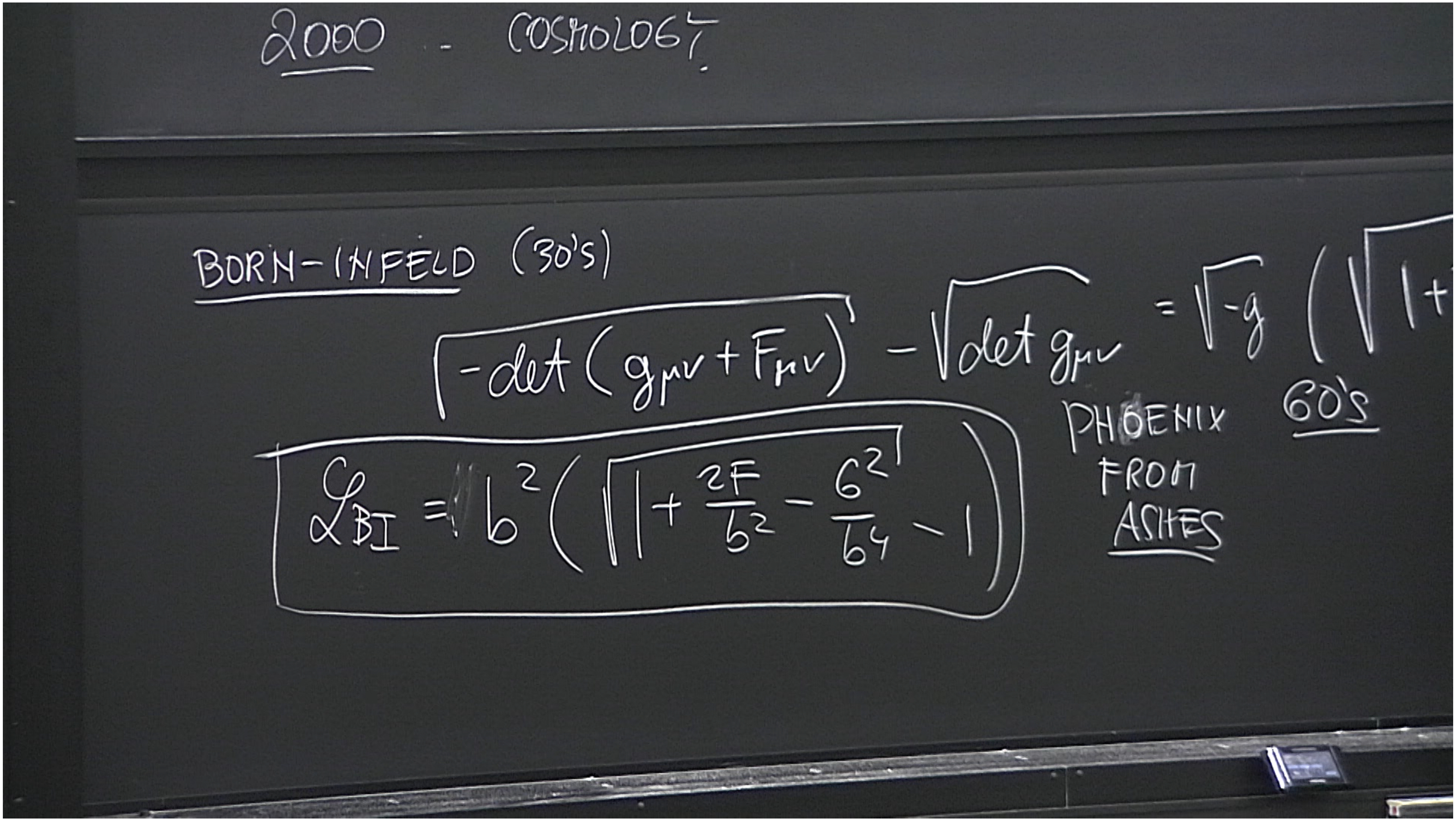
80's ... STRINGS & BRANES

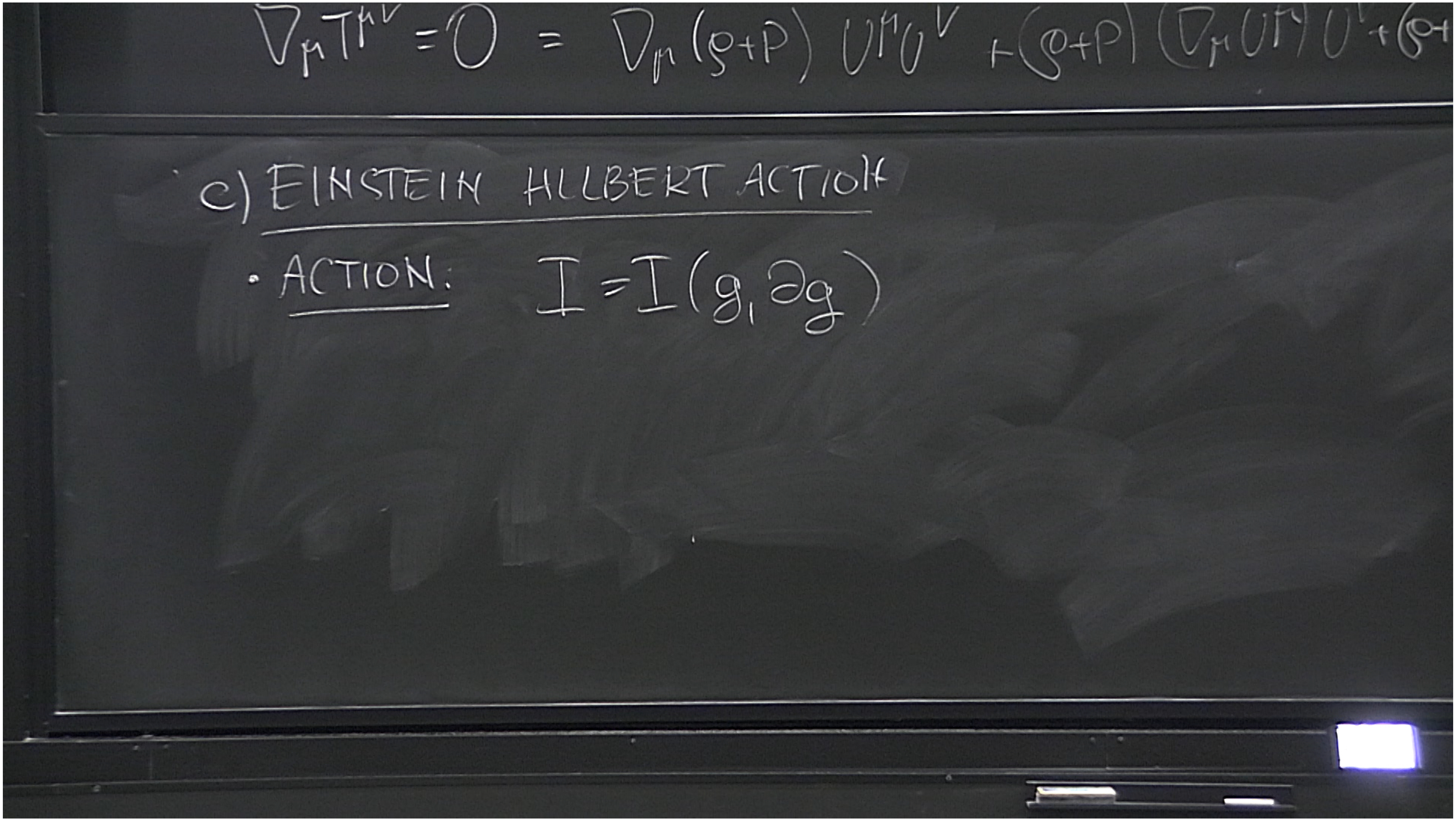
2000 .. COSMOLOGY

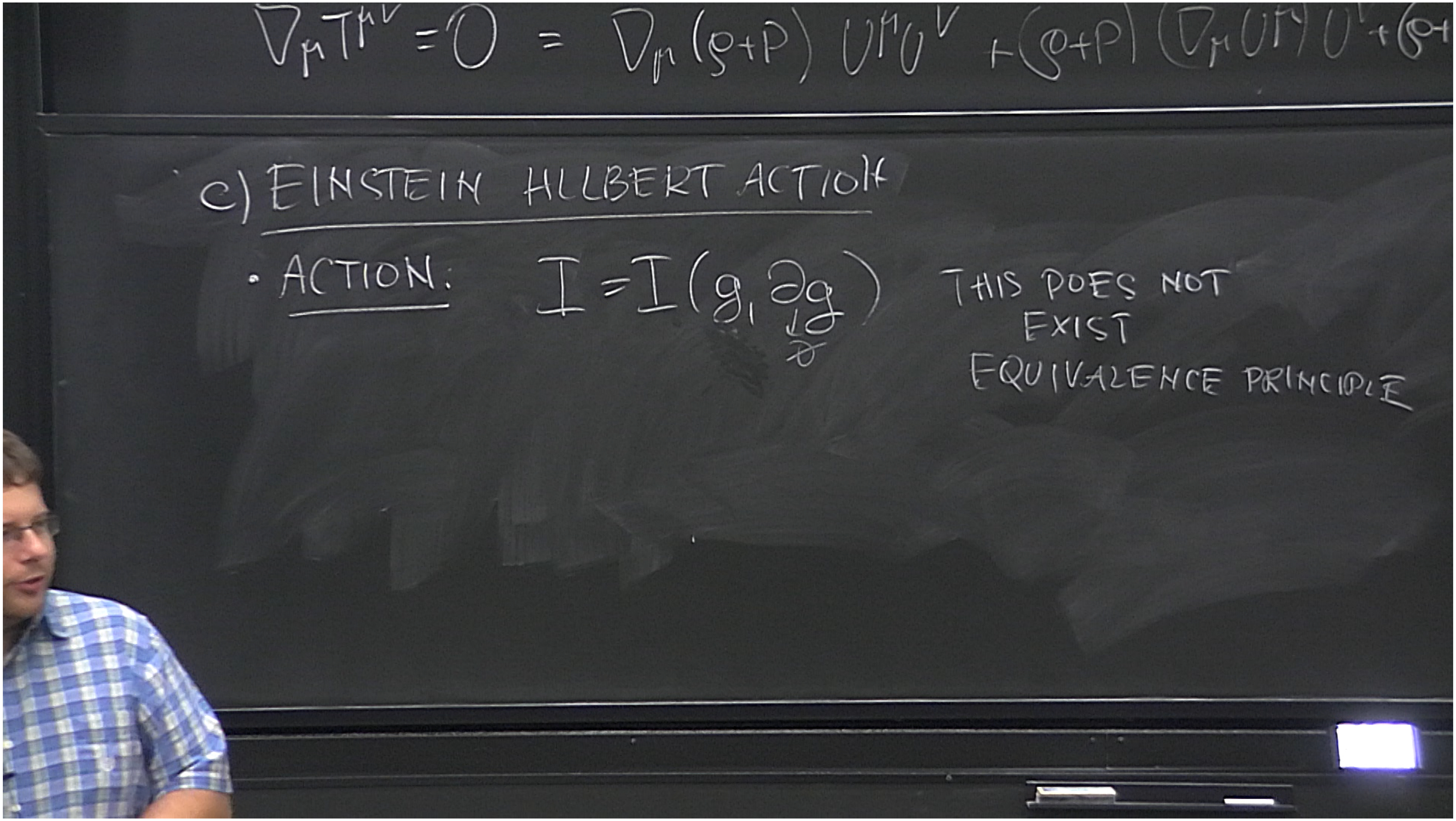
b ... MAXIMAL ELECTRIC FIELD

$$\boxed{\mathcal{L}_{BI} = b^2\left(\sqrt{1 + \frac{2F}{b^2} - \frac{G^2}{b^4}} - 1\right)}$$

PHOENIX 60'S FROM ASHES


2000 - COSMOLOGY
BORN-INFELD (30's)
$\sqrt{-\det(g_{\mu\nu}+F_{\mu\nu})} - \sqrt{\det g_{\mu\nu}} = \sqrt{-g}\left(\sqrt{1+\right.$
$\mathcal{L}_{BI} = b^2\left(\sqrt{1+\frac{2F}{b^2}-\frac{G^2}{b^4}}-1\right)$
PHOENIX FROM ASHES
60's


$\nabla_\mu T^{\mu\nu} = 0 = \nabla_\mu(\rho+P)\,U^\mu U^\nu + (\rho+P)(\nabla_\mu U^\mu)\,U^\nu + (\rho+$
c) EINSTEIN HILBERT ACTION
• ACTION: $I = I(g, \partial g)$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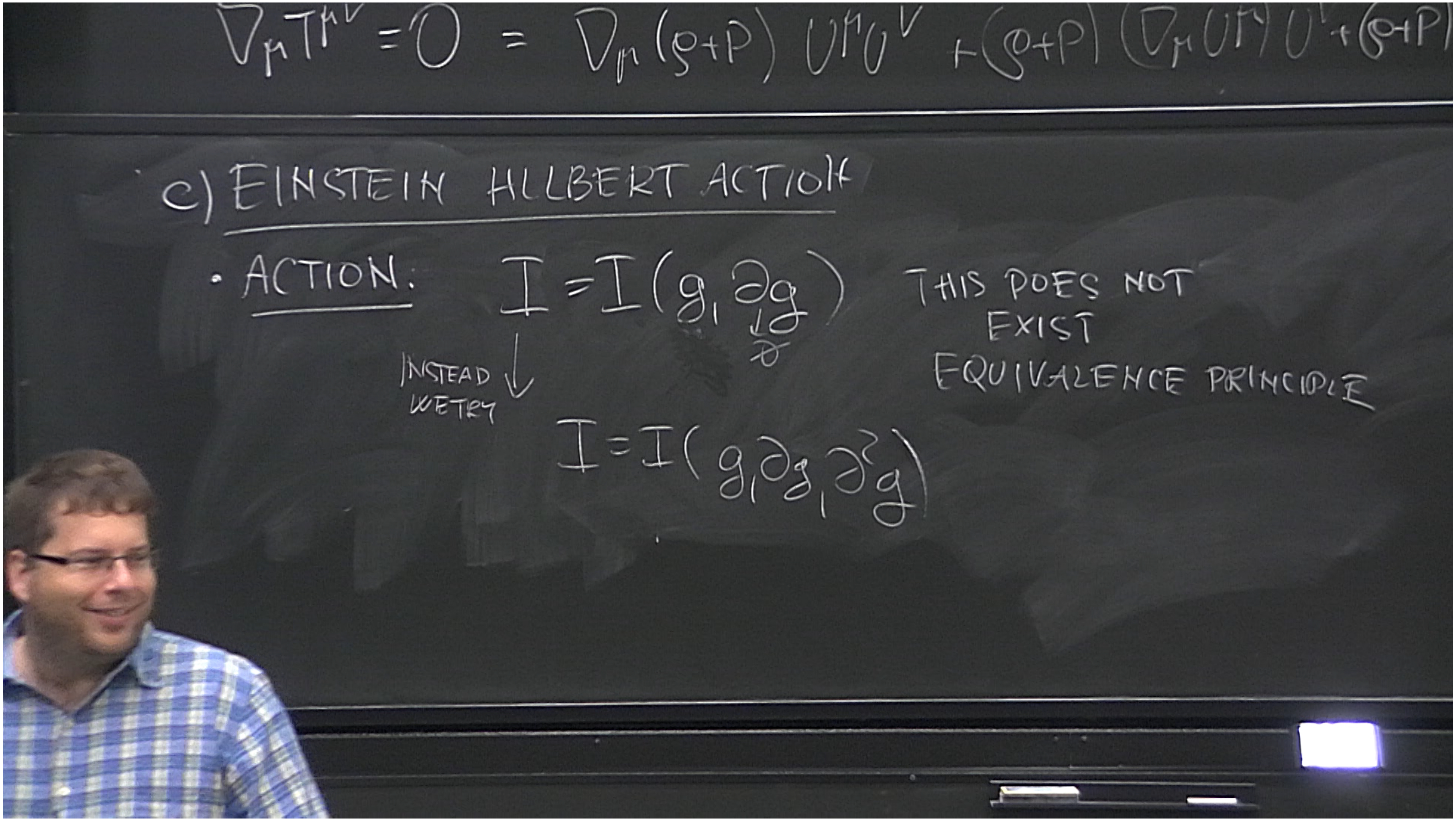
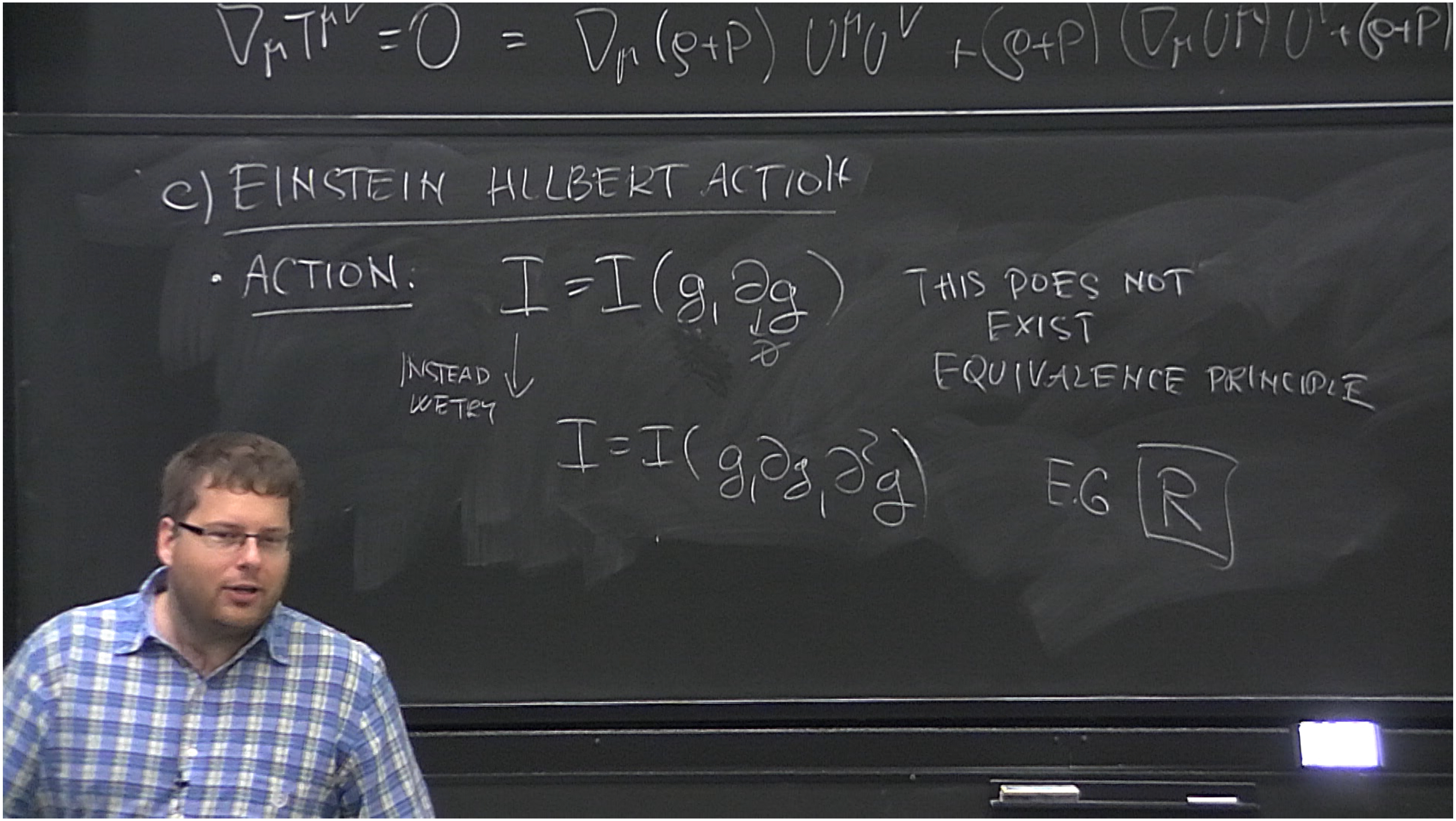
THIS DOES NOT EXIST
EQUIVALENCE PRINCIPLE


$\nabla_\mu T^{\mu\nu} = 0 = \nabla_\mu(\rho+P)\, U^\mu U^\nu + (\rho+P)(\nabla_\mu U^\mu) U^\nu + (\rho+P)$
c) EINSTEIN HILBERT ACTION
• ACTION: $I = I(g, \partial g)$
THIS DOES NOT EXIST
EQUIVALENCE PRINCIPLE
INSTEAD WE TRY
$I = I(g, \partial g, \partial^2 g)$


$\nabla_\mu T^{\mu\nu} = 0 = \nabla_\mu(\rho+P)U^\mu U^\nu + (\rho+P)(\nabla_\mu U^\mu)U^\nu + (\rho+P)$
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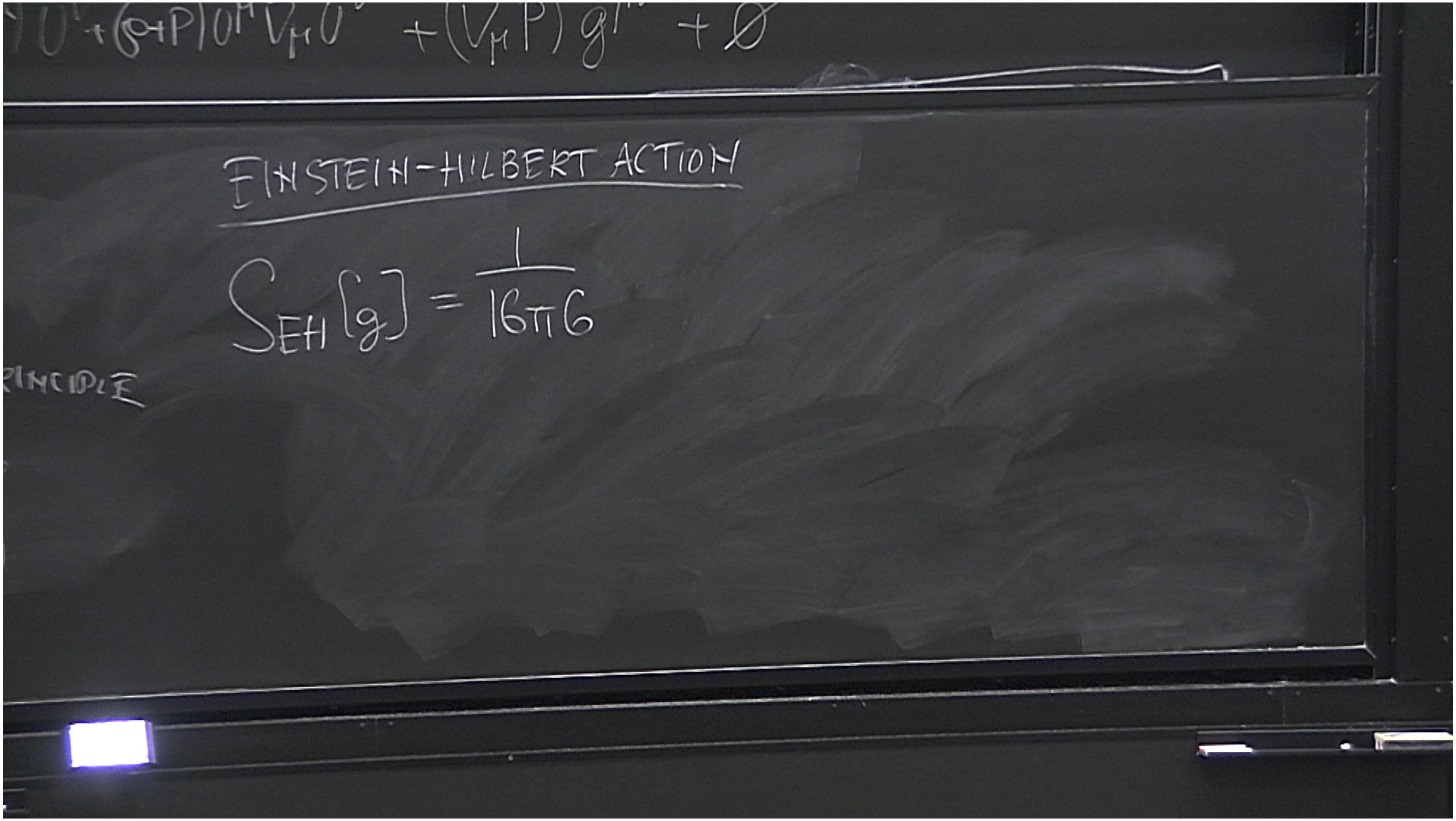
$I = I(g, \partial g, \partial^2 g)$
E.G $\boxed{R}$


$+(\rho+P)U^\mu \nabla_\mu U^\nu + (\nabla_\mu P) g^{\mu\nu} + \emptyset$
EINSTEIN-HILBERT ACTION
$S_{EH}[g] = \frac{1}{16\pi G}$
RINCIPLE

$$\ldots U^{\nu} + (\rho + P) U^{\mu} \nabla_{\mu} U^{\nu} + (\nabla_{\mu} P) g^{\mu\nu} + \emptyset$$

EINSTEIN-HILBERT ACTION

$$\boxed{S_{EH}[g] = \frac{1}{16\pi G} \int d^4x \sqrt{-g}\, R(g, \partial g, \partial^2 g)}$$

RINCIPLE

EINSTEIN-HILBERT ACTION

$$\boxed{S_{EH}[g] = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}\;R(g, \partial g, \partial^2 g)}$$

...INCIPLE

DIRTY TRICKS: • $R = g^{\mu\nu} R_{\mu\nu}$

$$\boxed{h^\alpha{}_\beta = \delta^\alpha{}_\beta + U^\alpha U_\beta}$$

$\parallel$ $\perp$

$$h^\alpha{}_\beta V^\beta \to V^\alpha_\perp$$

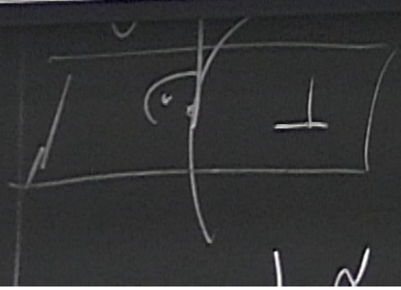
EINSTEIN-HILBERT ACTION

$$\boxed{S_{EH}[g] = \frac{1}{16\pi G}\int d^4x \sqrt{-g}\, R(g, \partial g, \partial^2 g)}$$

...INCIPLE

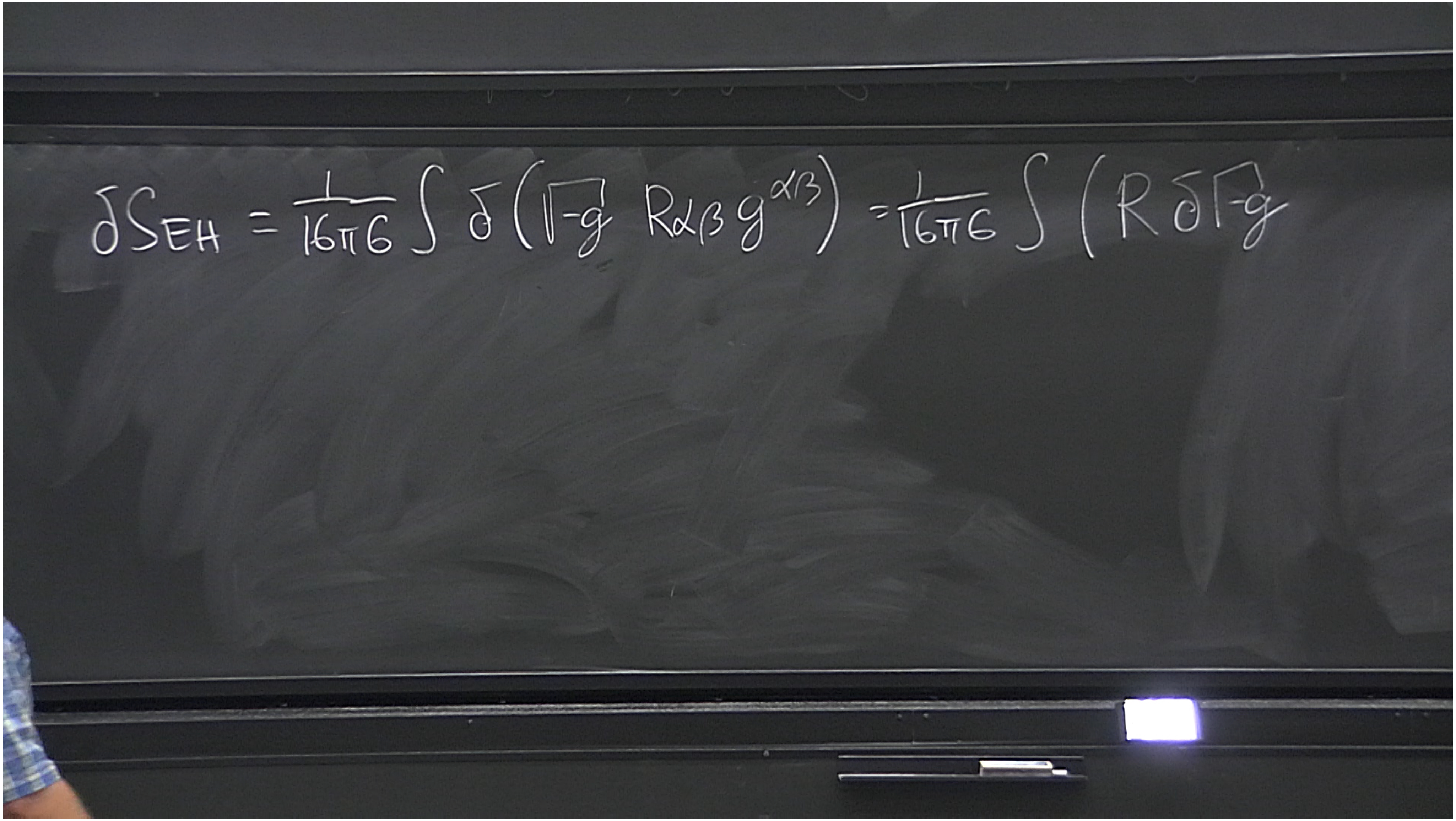
DIRTY TRICKS:

- $R = g^{\mu\nu} R_{\mu\nu}$
- $\delta\sqrt{-g} = -\frac{1}{2}\sqrt{-g}\, g_{\mu\nu}\,\delta g^{\mu\nu}$

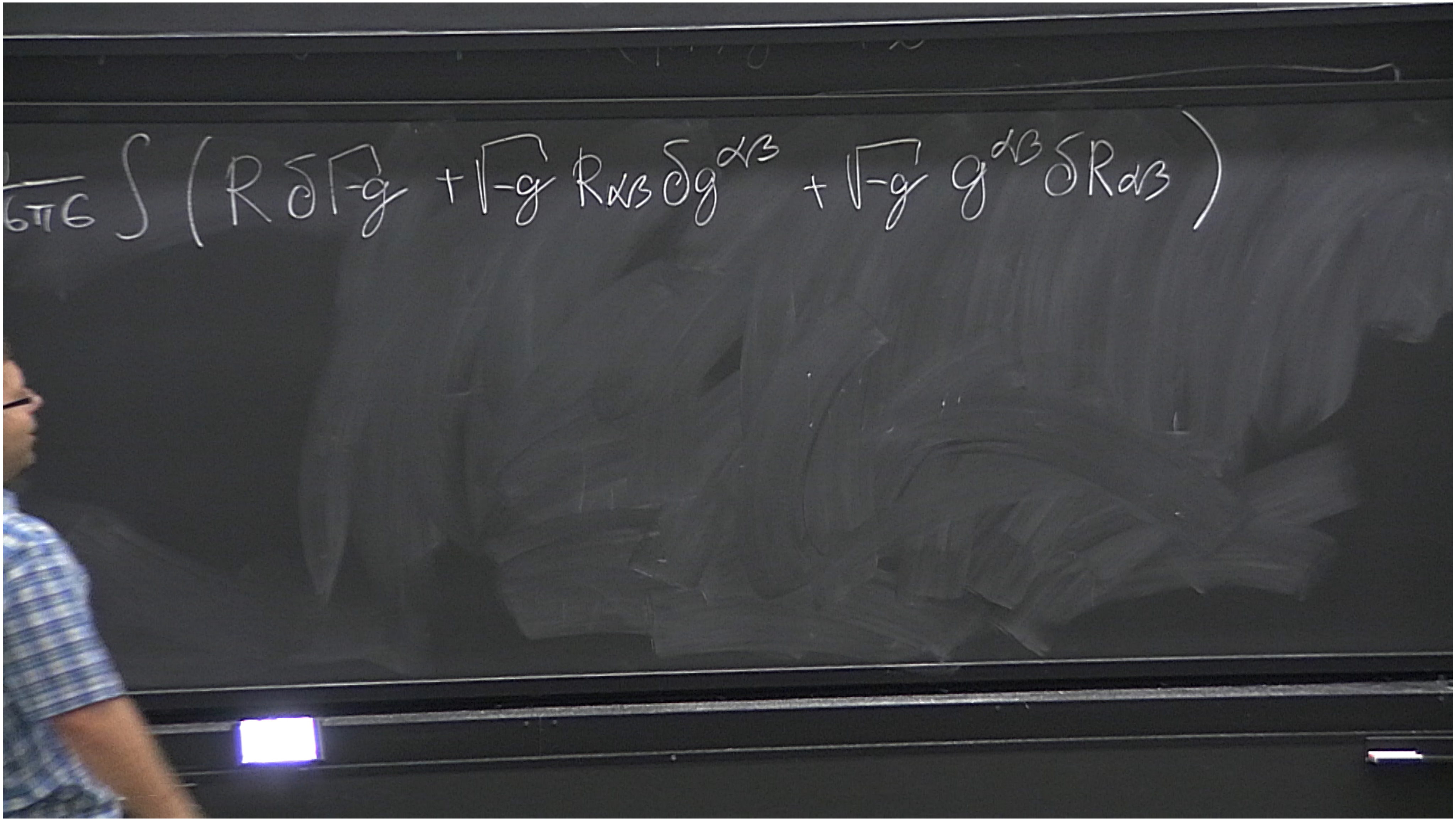


$$\boxed{h^\alpha{}_\beta = \delta^\alpha{}_\beta + U^\alpha U_\beta}$$

$$h^\alpha{}_\beta V^\beta \to V^\alpha_\perp$$


$\delta S_{EH} = \frac{1}{16\pi G} \int \delta \left(\sqrt{-g}\, R_{\alpha\beta}\, g^{\alpha\beta} \right) = \frac{1}{16\pi G} \int \left(R\, \delta \sqrt{-g} \right.$

$$\left(R\,\delta\sqrt{-g} + \sqrt{-g}\,R_{\alpha\beta}\,\delta g^{\alpha\beta}\right.$$


$\overline{6\pi G} \int \left(R\,\delta\sqrt{-g} + \sqrt{-g}\,R_{\alpha\beta}\,\delta g^{\alpha\beta} + \sqrt{-g}\,g^{\alpha\beta}\,\delta R_{\alpha\beta} \right)$

• ACTION: $I = I(g, \partial g)$ THIS DOES NOT EXIST EQUIVALENCE PRINCIPLE

INSTEAD WE TRY $I = I(g, \partial g, \partial^2 g)$ EG $\boxed{R}$

$$S_{EH}[g] = \frac{1}{16\pi G}\int d^4x \sqrt{-g}\, R(g, \partial g, \partial g)$$

DIRTY TRICKS: • $R = g^{\mu\nu} R_{\mu\nu}$

• $\delta\sqrt{-g} = -\frac{1}{2}\sqrt{-g}\, g_{\mu\nu}\delta g^{\mu\nu}$

$$\delta S_{EH} = \frac{1}{16\pi G}\int \delta\left(\sqrt{-g}\, R_{\alpha\beta} g^{\alpha\beta}\right) = \frac{1}{16\pi G}\int\left(R\,\delta\sqrt{-g} + \sqrt{-g}\, R_{\alpha\beta}\delta g^{\alpha\beta} + \sqrt{-g}\, g^{\alpha\beta}\delta R_{\alpha\beta}\right)$$

$$\alpha\beta) = \frac{1}{16\pi G}\int\Big(\underbrace{R\,\delta\sqrt{-g}}_{-\frac{1}{2}R\sqrt{-g}\,g_{\alpha\beta}\,\delta g^{\alpha\beta}} + \sqrt{-g}\,R_{\alpha\beta}\,\delta g^{\alpha\beta} + \sqrt{-g}\,g^{\alpha\beta}\,\delta R_{\alpha\beta}\Big)$$

$$\sqrt{-g}\,\Big(\underbrace{R_{\alpha\beta} - \frac{1}{2}R g_{\alpha\beta}}_{G_{\alpha\beta}}\Big)\,\delta g^{\alpha\beta}$$

$$\Big(\underbrace{R\,\delta\sqrt{-g}}_{-\frac{1}{2}R\sqrt{-g}\,g_{\alpha\beta}\,\delta g^{\alpha\beta}} + \sqrt{-g}\,R_{\alpha\beta}\,\delta g^{\alpha\beta} + \sqrt{-g}\,\underbrace{g^{\alpha\beta}\,\delta R_{\alpha\beta}}_{\nabla_\mu V^\mu} \Big)$$

$$V_\mu = \nabla^\beta(\delta g_{\mu\beta}) - g^{\alpha\beta}\nabla_\mu(\delta g_{\alpha\beta})$$

$$\sqrt{-g}\,\Big(\underbrace{R_{\alpha\beta} - \frac{1}{2}R\,g_{\alpha\beta}}_{G_{\alpha\beta}}\Big)\,\delta g^{\alpha\beta}$$

$$\Big(\underbrace{R\,\delta\sqrt{-g}}_{\tfrac{1}{2}R\sqrt{-g}\,g_{\alpha\beta}\,\delta g^{\alpha\beta}} + \sqrt{-g}\,R_{\alpha\beta}\,\delta g^{\alpha\beta} + \sqrt{-g}\,\underbrace{g^{\alpha\beta}\,\delta R_{\alpha\beta}}_{\nabla_\mu V^\mu}\Big)$$

✓

$\nabla_\mu V^\mu$ ... BOUNDARY TERM

$$V_\mu = \nabla^\beta(\delta g_{\mu\beta}) - g^{\alpha\beta}\nabla_\mu(\delta g_{\alpha\beta})$$

$$\sqrt{-g}\,\big(\underbrace{R_{\alpha\beta} - \tfrac{1}{2}R\,g_{\alpha\beta}}_{G_{\alpha\beta}}\big)\,\delta g^{\alpha\beta}$$

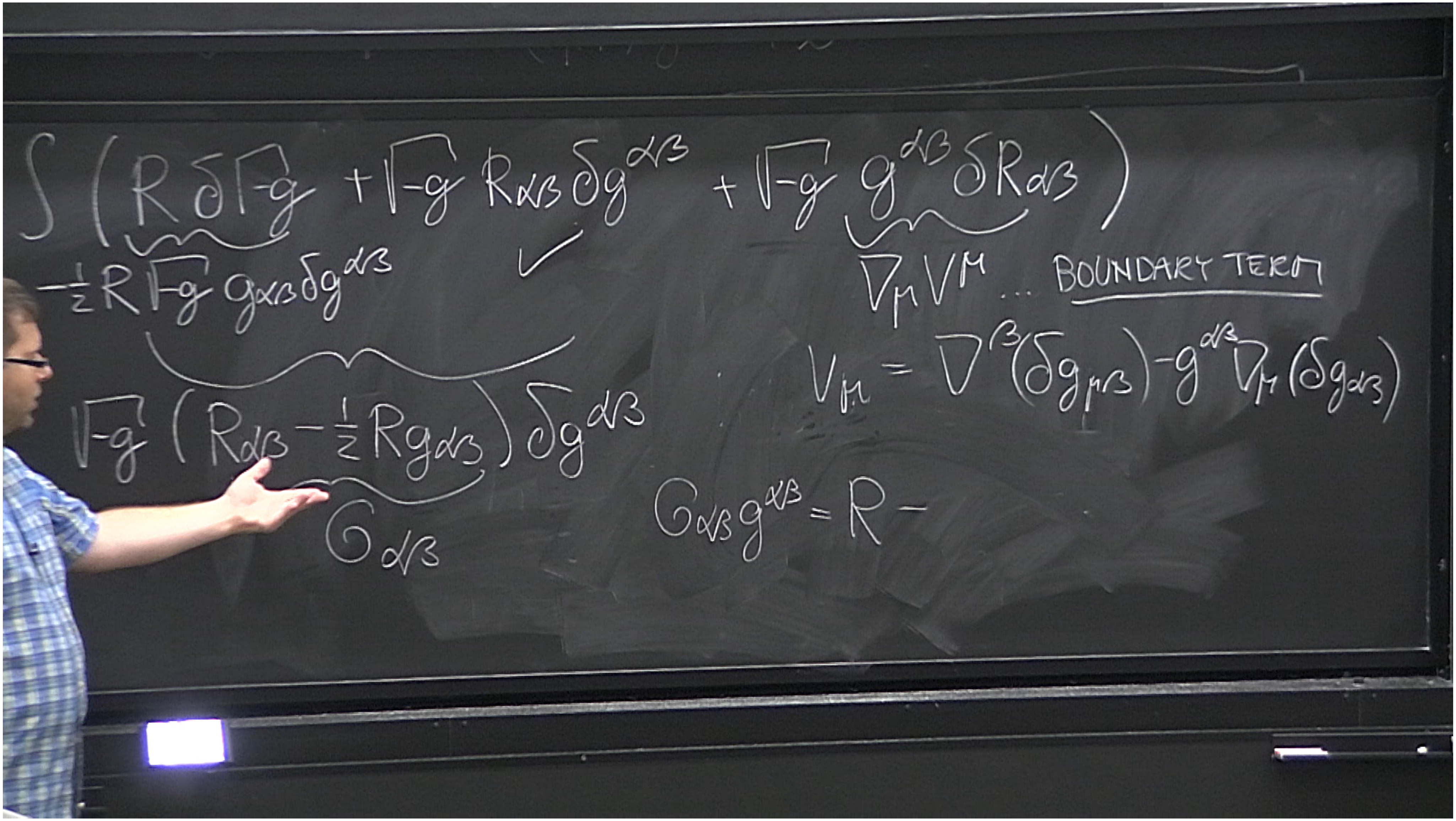
$$\delta S_{EH} = \frac{1}{16\pi G} \int \delta \left(\sqrt{-g}\, R_{\alpha\beta}\, g^{\alpha\beta} \right) = \frac{1}{16\pi G} \int \Big(R\, \delta \Gamma \ldots$$

$$-\tfrac{1}{2} R \sqrt{-g}\, g \ldots$$

$$\sqrt{-g} \, (R_{\alpha} \ldots$$

VACUUM EINSTEIN EQS:

$$\boxed{G_{\mu\nu} = 0}$$


$\int \left(\underbrace{R\,\delta\sqrt{-g}}_{-\frac{1}{2}R\sqrt{-g}\,g_{\alpha\beta}\delta g^{\alpha\beta}} + \sqrt{-g}\,R_{\alpha\beta}\,\delta g^{\alpha\beta} + \sqrt{-g}\,\underbrace{g^{\alpha\beta}\delta R_{\alpha\beta}}_{\nabla_\mu V^\mu} \right)$
$\nabla_\mu V^\mu$ ... BOUNDARY TERM
$V_\mu = \nabla^\beta(\delta g_{\mu\beta}) - g^{\alpha\beta}\nabla_\mu(\delta g_{\alpha\beta})$
$\sqrt{-g}\left(\underbrace{R_{\alpha\beta} - \frac{1}{2}R g_{\alpha\beta}}_{G_{\alpha\beta}}\right)\delta g^{\alpha\beta}$
$G_{\alpha\beta}\, g^{\alpha\beta} = R -$

$$\int \left(\underbrace{R\,\delta\sqrt{-g}}_{-\frac{1}{2}R\sqrt{-g}\,g_{\alpha\beta}\,\delta g^{\alpha\beta}} + \sqrt{-g}\,R_{\alpha\beta}\,\delta g^{\alpha\beta} + \sqrt{-g}\,\underbrace{g^{\alpha\beta}\,\delta R_{\alpha\beta}}_{\nabla_\mu V^\mu} \right)$$

$\nabla_\mu V^\mu$ ... BOUNDARY TERM

$$\sqrt{-g}\,\Big(\underbrace{R_{\alpha\beta} - \tfrac{1}{2} R\, g_{\alpha\beta}}_{G_{\alpha\beta}}\Big)\,\delta g^{\alpha\beta}$$

$$V_\mu = \nabla^\beta(\delta g_{\mu\beta}) - g^{\alpha\beta}\nabla_\mu(\delta g_{\alpha\beta})$$

$$G_{\alpha\beta}\,g^{\alpha\beta} = R - \tfrac{1}{2} R d = 0$$

$$\int \Big(\underbrace{R\,\delta\sqrt{-g}}_{-\frac{1}{2} R \sqrt{-g}\, g_{\alpha\beta}\,\delta g^{\alpha\beta}} + \sqrt{-g}\, R_{\alpha\beta}\,\delta g^{\alpha\beta} \checkmark + \sqrt{-g}\, \underbrace{g^{\alpha\beta}\,\delta R_{\alpha\beta}}_{\nabla_\mu V^\mu} \Big)$$

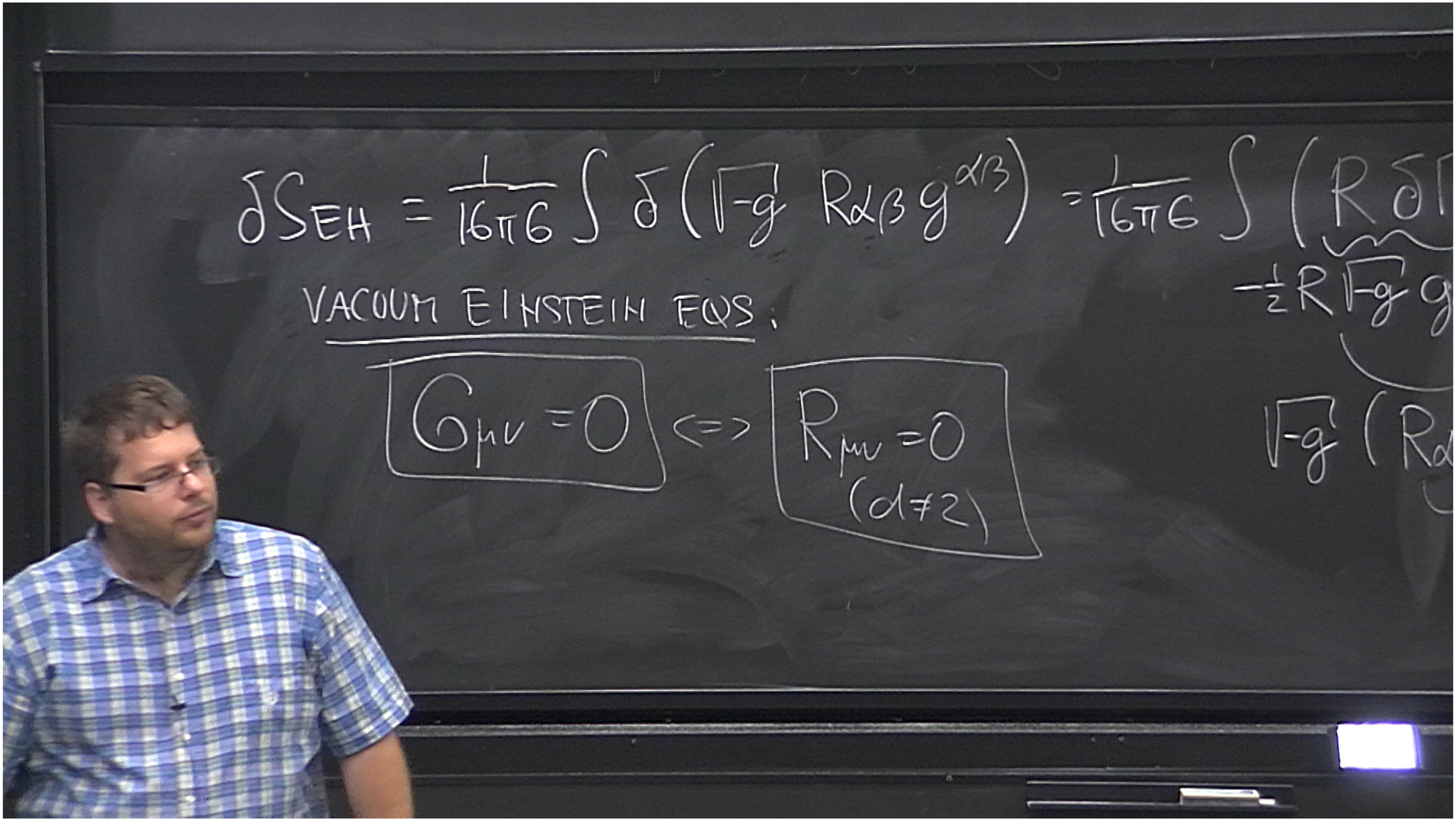
$\nabla_\mu V^\mu$ ... BOUNDARY TERM

$$\sqrt{-g}\,\Big(\underbrace{R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta}}_{G_{\alpha\beta}}\Big)\,\delta g^{\alpha\beta}$$

$$V_\mu = \nabla^\beta(\delta g_{\mu\beta}) - g^{\alpha\beta}\nabla_\mu(\delta g_{\alpha\beta})$$

$$G_{\alpha\beta}\, g^{\alpha\beta} = R - \frac{1}{2} R d = 0$$

$\boxed{R=0}$ $d \neq 2$



$\delta S_{EH} = \frac{1}{16\pi G}\int \delta\left(\sqrt{-g}\, R_{\alpha\beta}\, g^{\alpha\beta}\right) = \frac{1}{16\pi G}\int \Big(R\,\delta\Gamma$
$-\frac{1}{2} R \sqrt{-g}\, g$
VACUUM EINSTEIN EQS:
$\boxed{G_{\mu\nu} = 0} \Leftrightarrow \boxed{R_{\mu\nu} = 0}$
$(d \neq 2)$
$\sqrt{-g}\,\big(R_{\alpha}$

2 REMARKS:

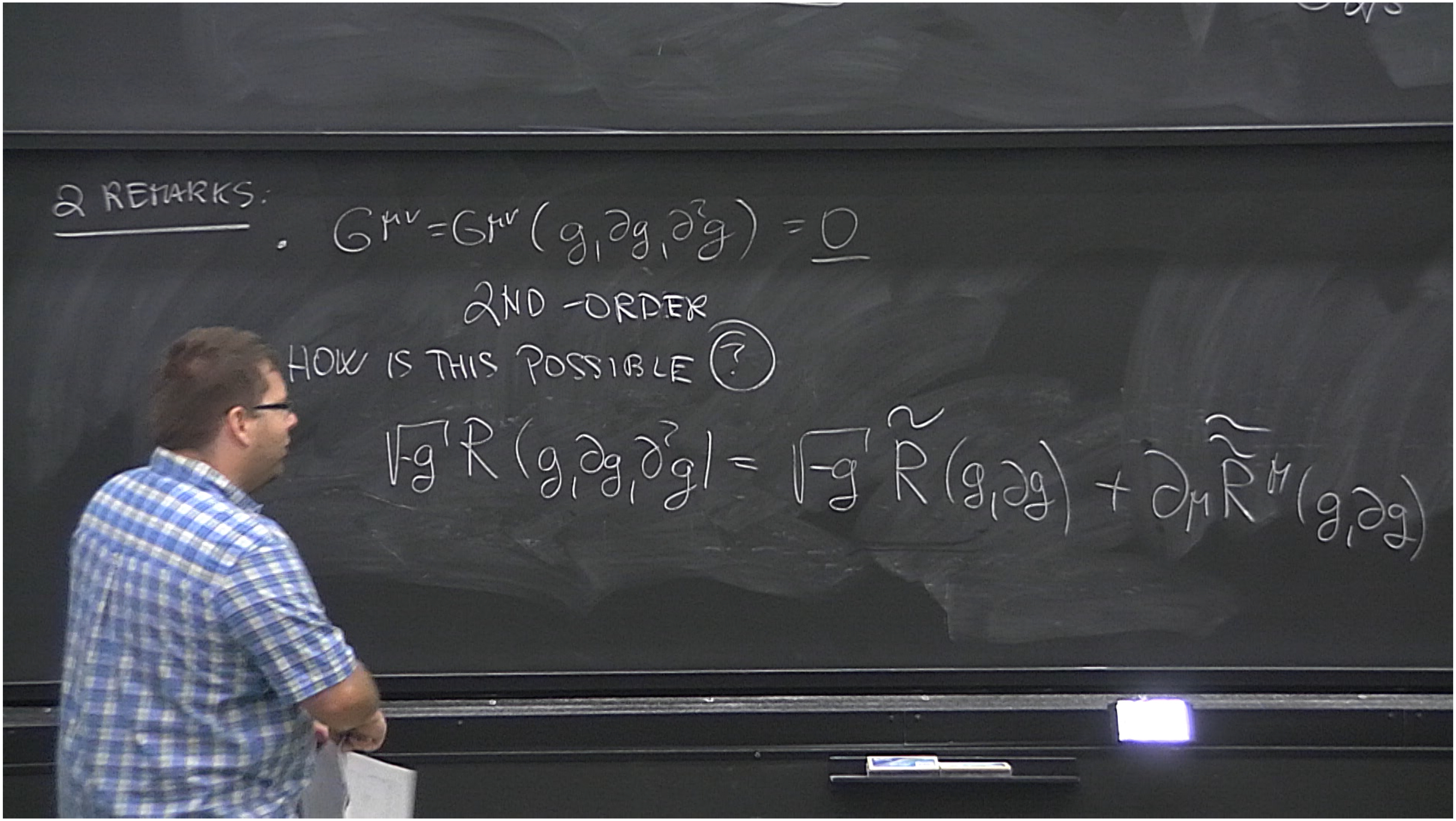
- $G^{\mu\nu} = G^{\mu\nu}(g, \partial g, \partial^2 g)$

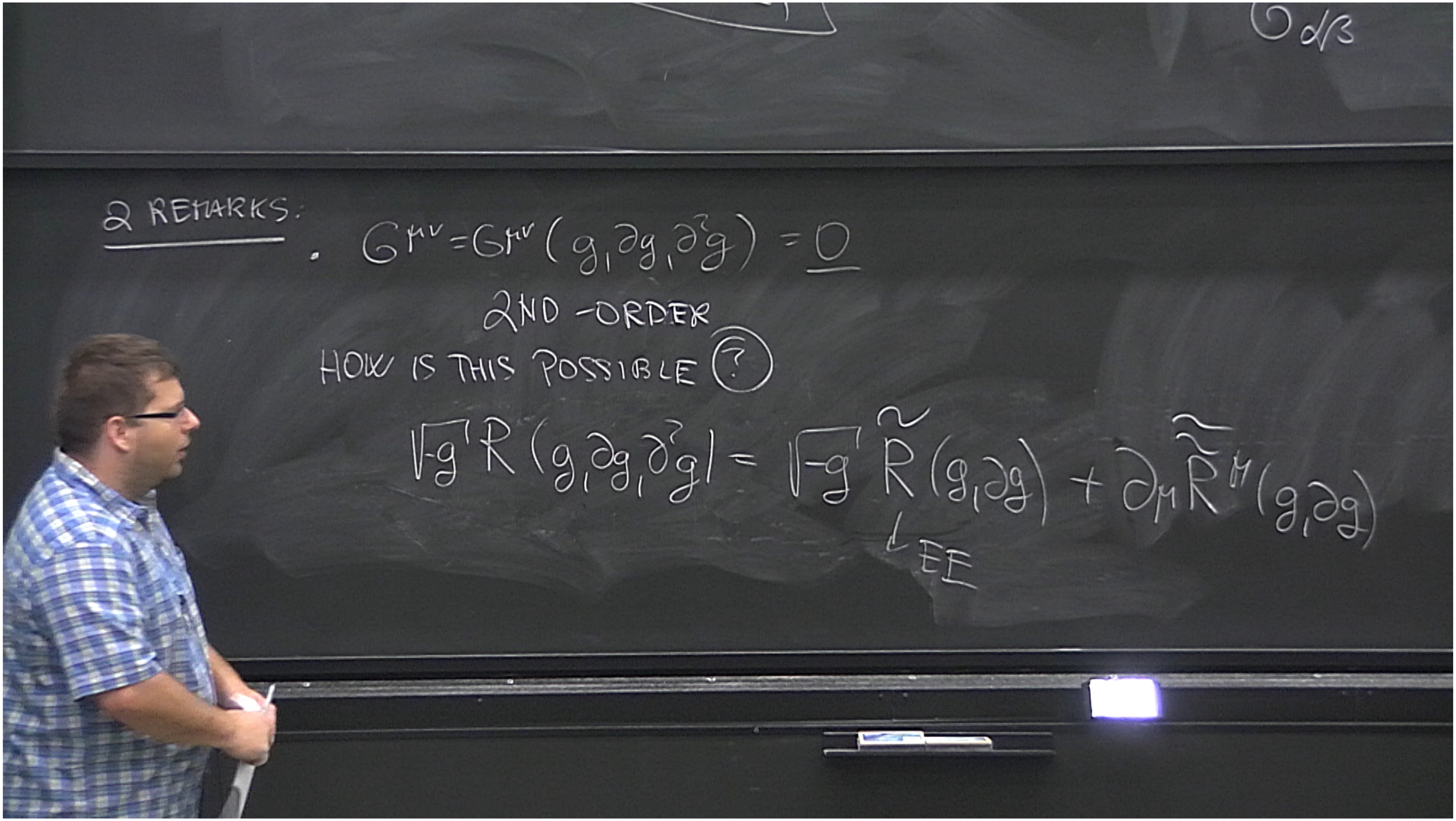
2 REMARKS:

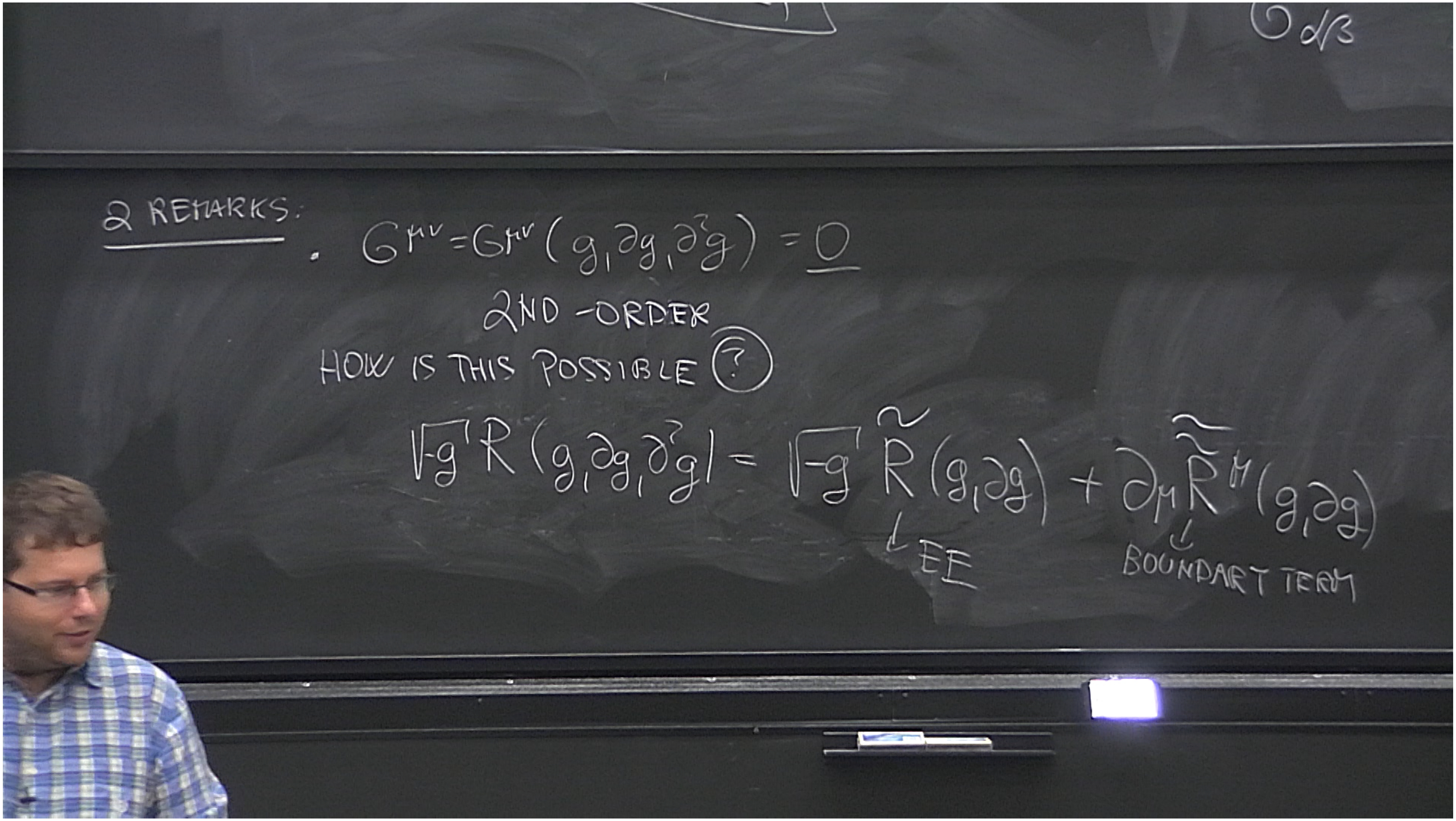
- $G^{\mu\nu} = G^{\mu\nu}(g, \partial g, \partial^2 g) = \underline{0}$

2ND-ORDER

HOW IS THIS POSSIBLE ?


2 REMARKS:
• $G^{\mu\nu} = G^{\mu\nu}(g, \partial g, \partial^2 g) = 0$
2ND-ORDER
HOW IS THIS POSSIBLE ?
$\sqrt{-g}\, R(g, \partial g, \partial^2 g) = \sqrt{-g}\, \tilde{R}(g, \partial g) + \partial_\mu \tilde{\tilde{R}}^\mu(g, \partial g)$


2 REMARKS:
• $G^{\mu\nu} = G^{\mu\nu}(g, \partial g, \partial^2 g) = 0$
2ND-ORDER
HOW IS THIS POSSIBLE ?
$\sqrt{-g}\, R(g, \partial g, \partial^2 g) = \sqrt{-g}\, \tilde{R}(g, \partial g) + \partial_\mu \tilde{\tilde{R}}^\mu (g, \partial g)$
EE


2 REMARKS:
• $G^{\mu\nu} = G^{\mu\nu}(g, \partial g, \partial^2 g) = 0$
2ND-ORDER
HOW IS THIS POSSIBLE ?
$\sqrt{-g}\,R(g,\partial g,\partial^2 g) = \sqrt{-g}\,\tilde{R}(g,\partial g) + \partial_\mu \tilde{\tilde{R}}^\mu(g,\partial g)$
EE
BOUNDARY TERM

$\boxed{R=0}$ $d \neq 2$

TO KILL THIS NEED TO PRESCRIBE $g, \partial g$ ON BOUNDARY.

$$) + \partial_M \tilde{\tilde{R}}^M(g, \partial g)$$

BOUNDARY TERM

$\boxed{R=0}$ $d\neq 2$

TO KILL THIS NEED TO PRESCRIBE $g, \partial g$ ON BOUNDARY.

$\Rightarrow$ NO SOLUTIONS

E-H ACTION IS ILL POSED AS VARIATIONAL PRINCIPLE

$$) + \partial_\mu \tilde{\tilde{R}}^\mu (g, \partial g)$$

BOUNDARY TERM

RKS:

• $G^{\mu\nu} = G^{\mu\nu}(g, \partial g, \partial^2 g) = \underline{0}$

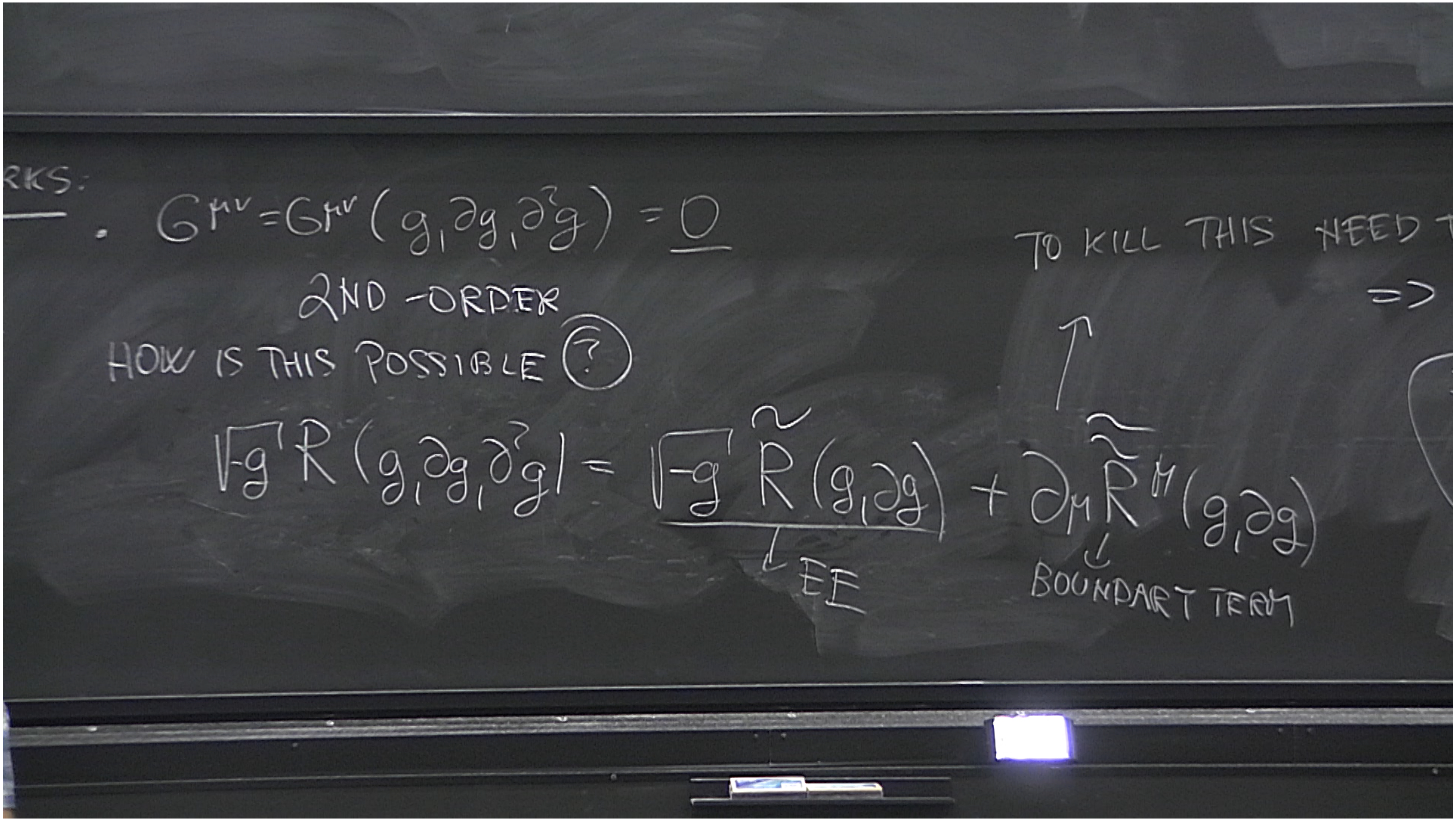
2ND-ORDER

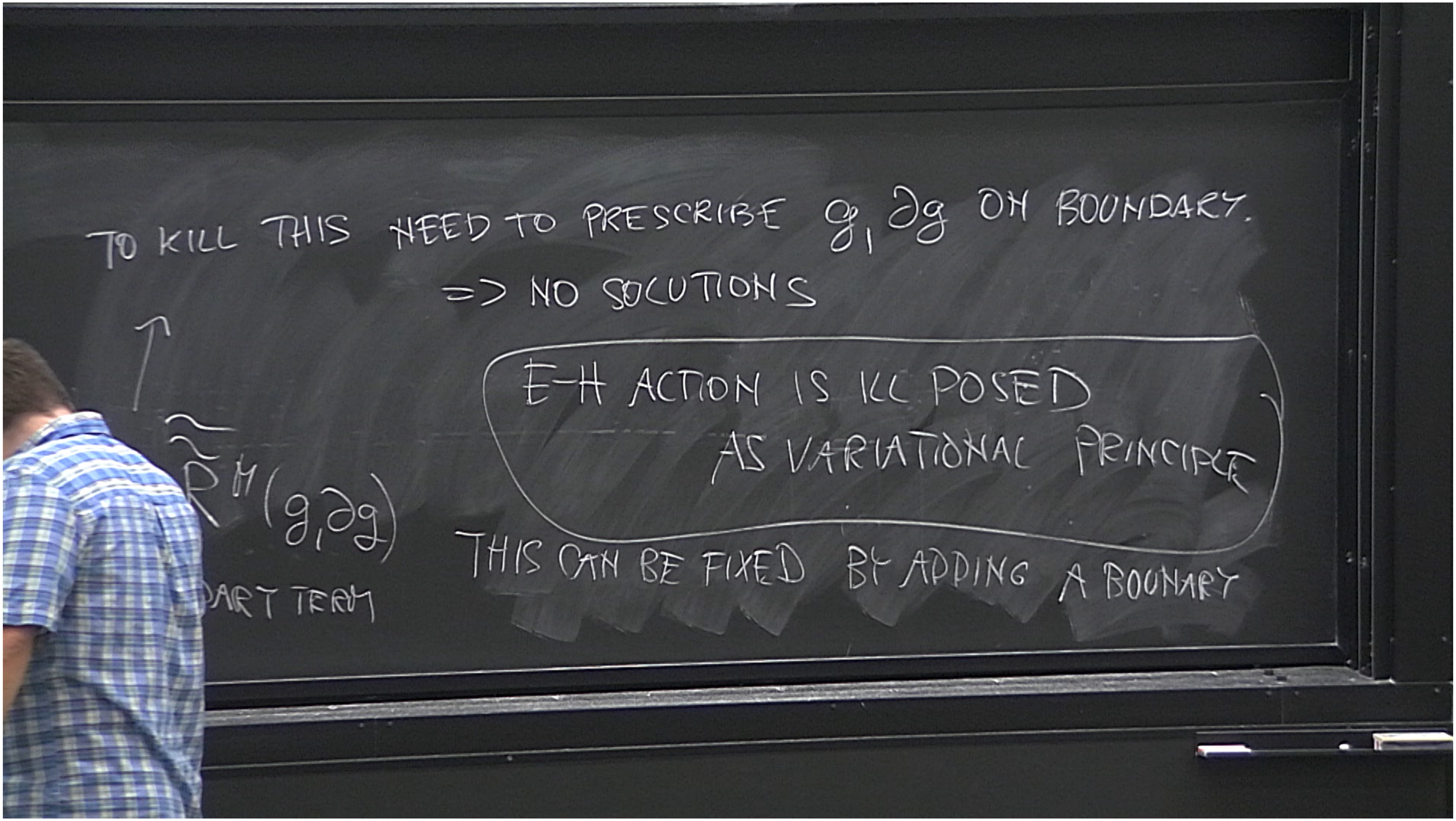
HOW IS THIS POSSIBLE ?

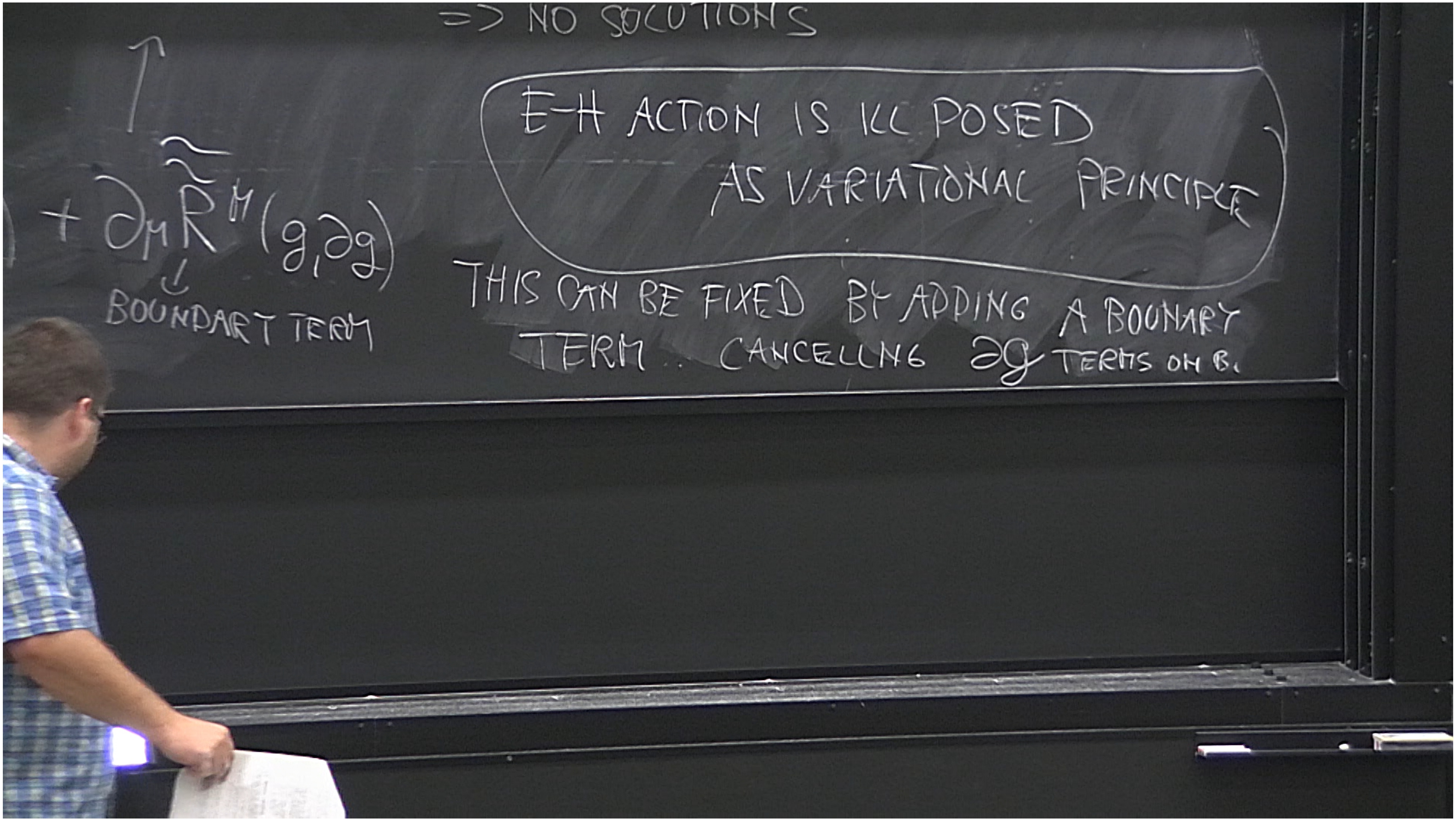
TO KILL THIS NEED T

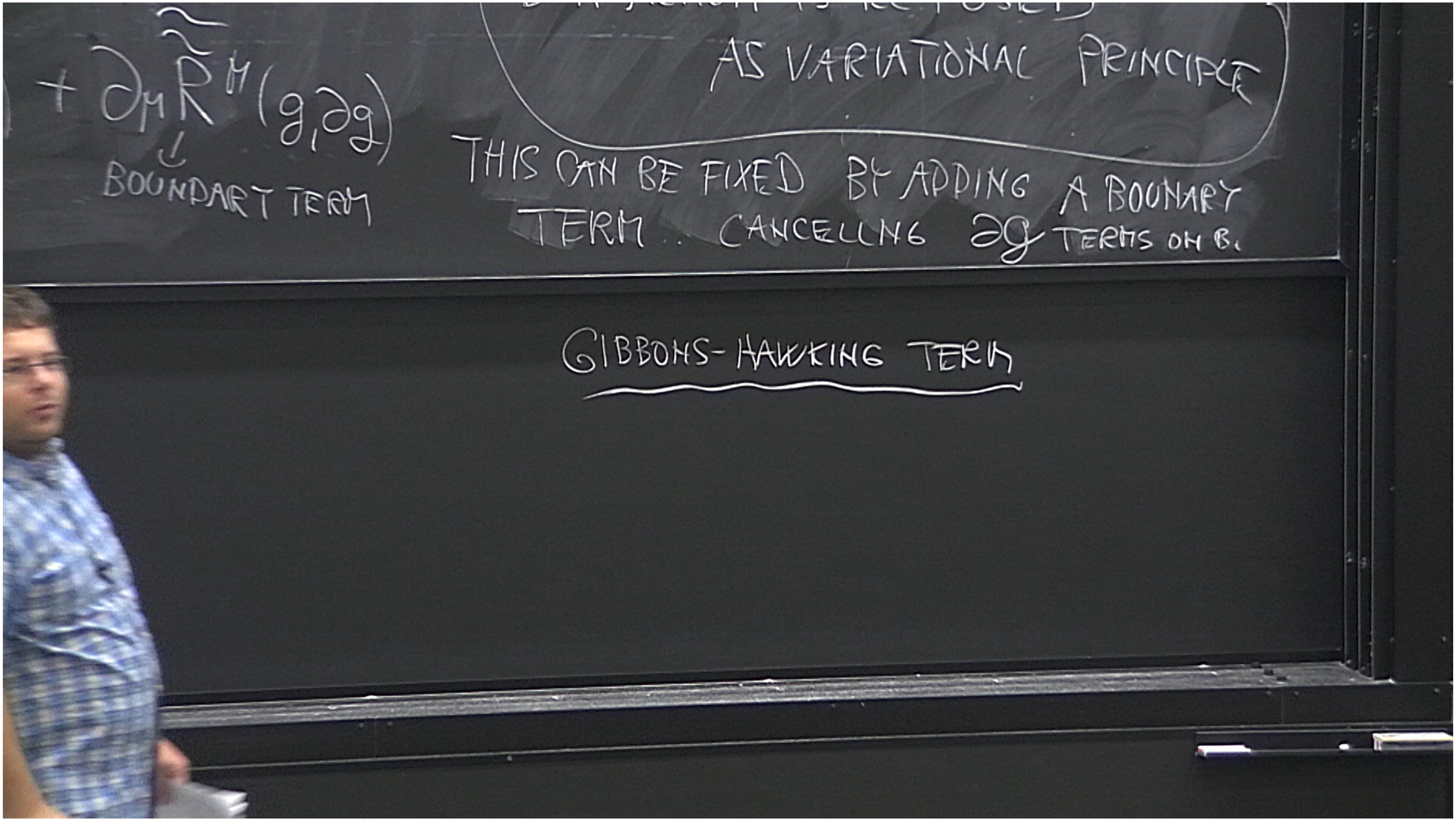
=>

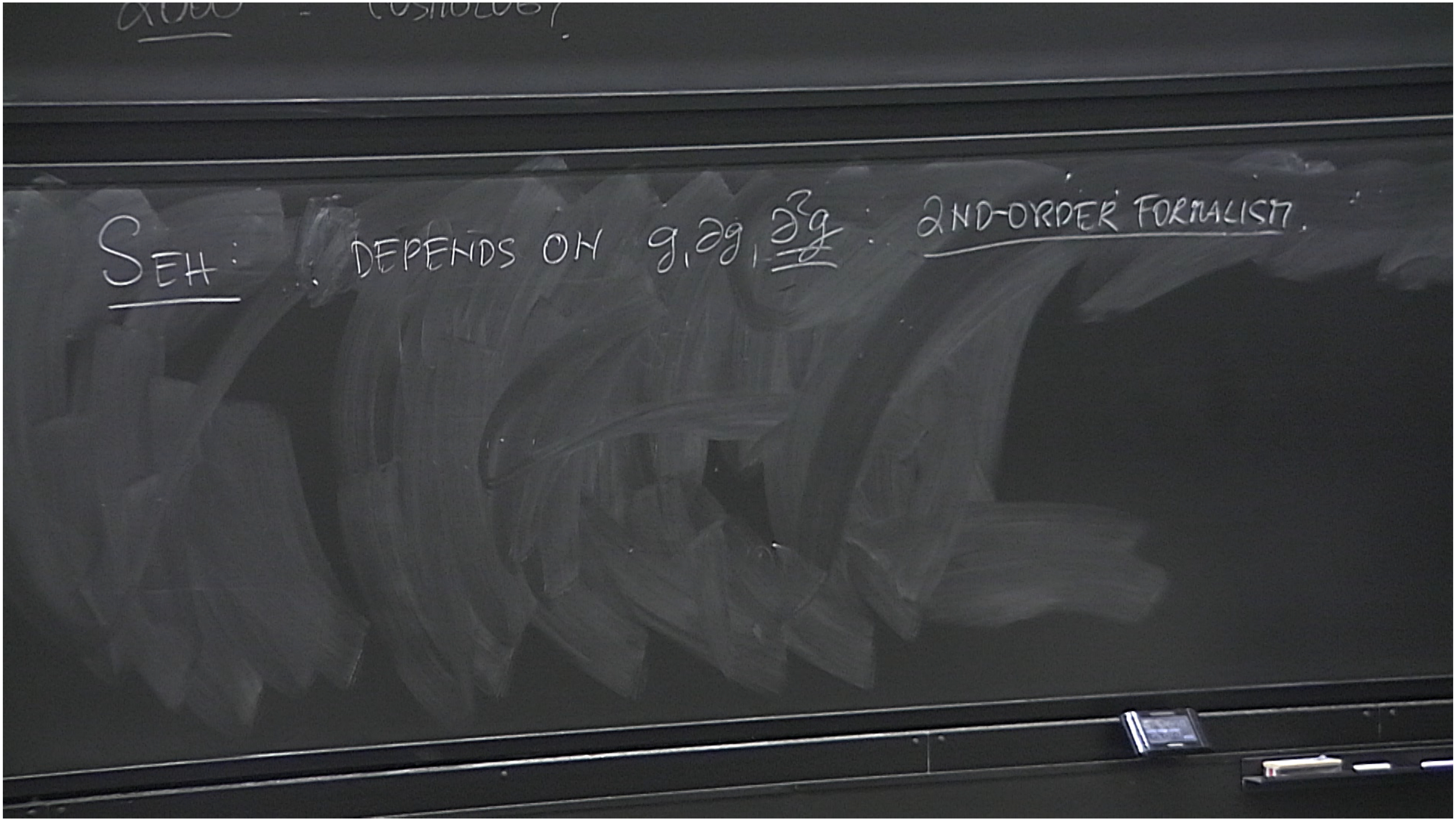
$$\sqrt{-g}\,R(g,\partial g,\partial^2 g) = \underbrace{\sqrt{-g}\,\tilde{R}(g,\partial g)}_{\text{EE}} + \underbrace{\partial_\mu \tilde{\tilde{R}}^{\mu}(g,\partial g)}_{\text{BOUNDARY TERM}}$$


RKS:
• $G^{\mu\nu} = G^{\mu\nu}(g, \partial g, \partial^2 g) = \underline{0}$
2ND-ORDER
HOW IS THIS POSSIBLE ?
$\sqrt{-g}\, R(g, \partial g, \partial^2 g) = \underbrace{\sqrt{-g}\, \tilde{R}(g, \partial g)}_{\text{EE}} + \partial_\mu \tilde{\tilde{R}}^\mu (g, \partial g)$
BOUNDARY TERM
TO KILL THIS NEED
=>


TO KILL THIS NEED TO PRESCRIBE $g, \partial g$ ON BOUNDARY.
=> NO SOLUTIONS
E-H ACTION IS ILL POSED
AS VARIATIONAL PRINCIPLE
$\tilde{R}^{M}(g, \partial g)$
ARY TERM
THIS CAN BE FIXED BY ADDING A BOUNARY


=> NO SOLUTIONS
$+ \partial_\mu \tilde{\tilde{R}}^\mu(g, \partial g)$
BOUNDARY TERM
E-H ACTION IS ILL POSED
AS VARIATIONAL PRINCIPLE
THIS CAN BE FIXED BY ADDING A BOUNDARY
TERM CANCELLING ∂g TERMS ON B.


AS VARIATIONAL PRINCIPLE
$+ \partial_M \tilde{\tilde{R}}^M(g,\partial g)$
BOUNDARY TERM
THIS CAN BE FIXED BY ADDING A BOUNARY
TERM CANCELLING ∂g TERMS ON B.
GIBBONS-HAWKING TERM


S_{EH} : DEPENDS ON $g, \partial g, \partial^2 g$: 2ND-ORDER FORMALISM.

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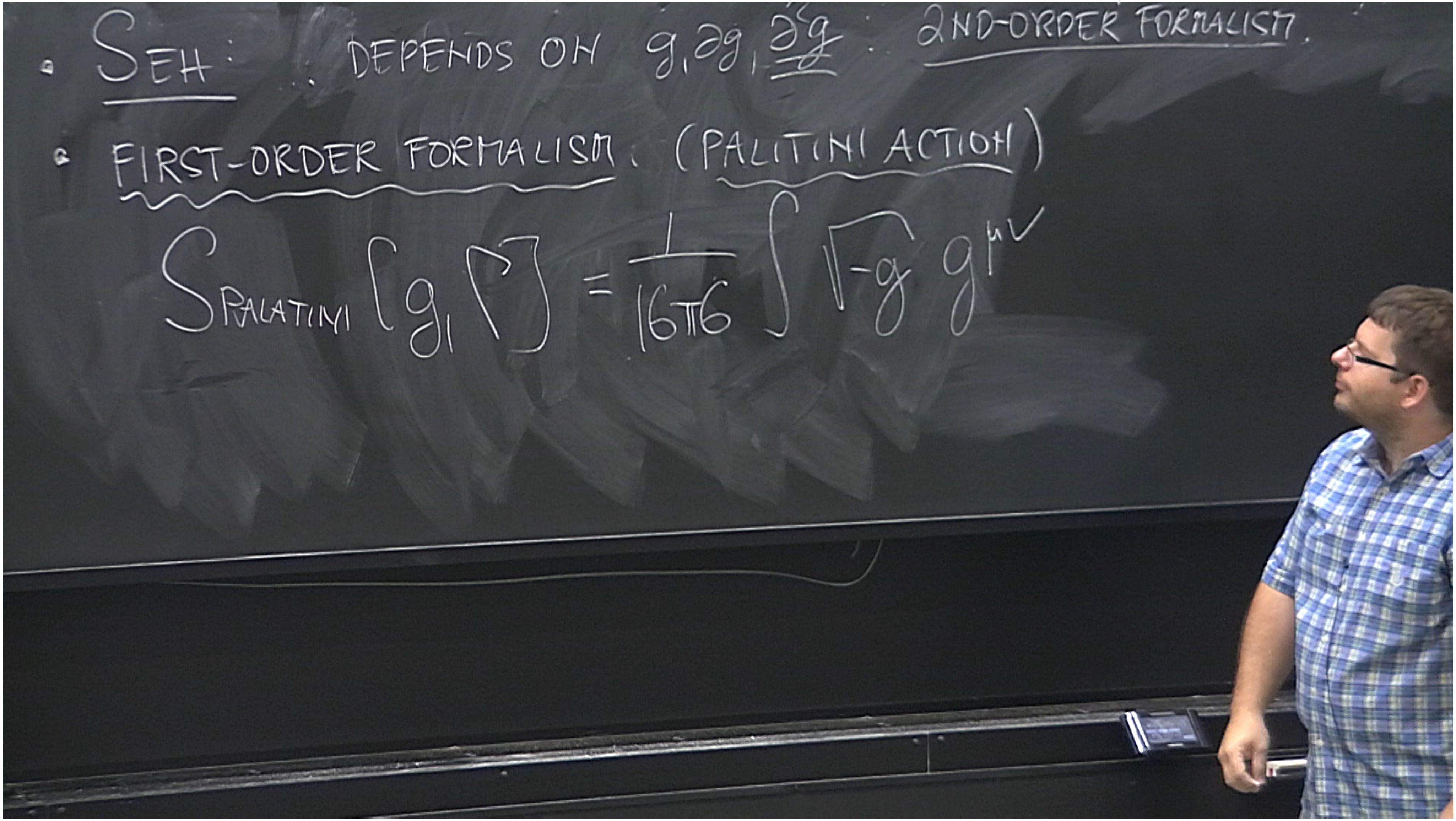
$$S_{PALATINI}[g, \Gamma]$$



S_{EH} : DEPENDS ON $g, \partial g, \partial^2 g$: 2ND-ORDER FORMALISM.
FIRST-ORDER FORMALISM: (PALITINI ACTION)
$S_{PALATINI}[g, \Gamma] = \frac{1}{16\pi G} \int \sqrt{-g}\, g^{\mu\nu}$

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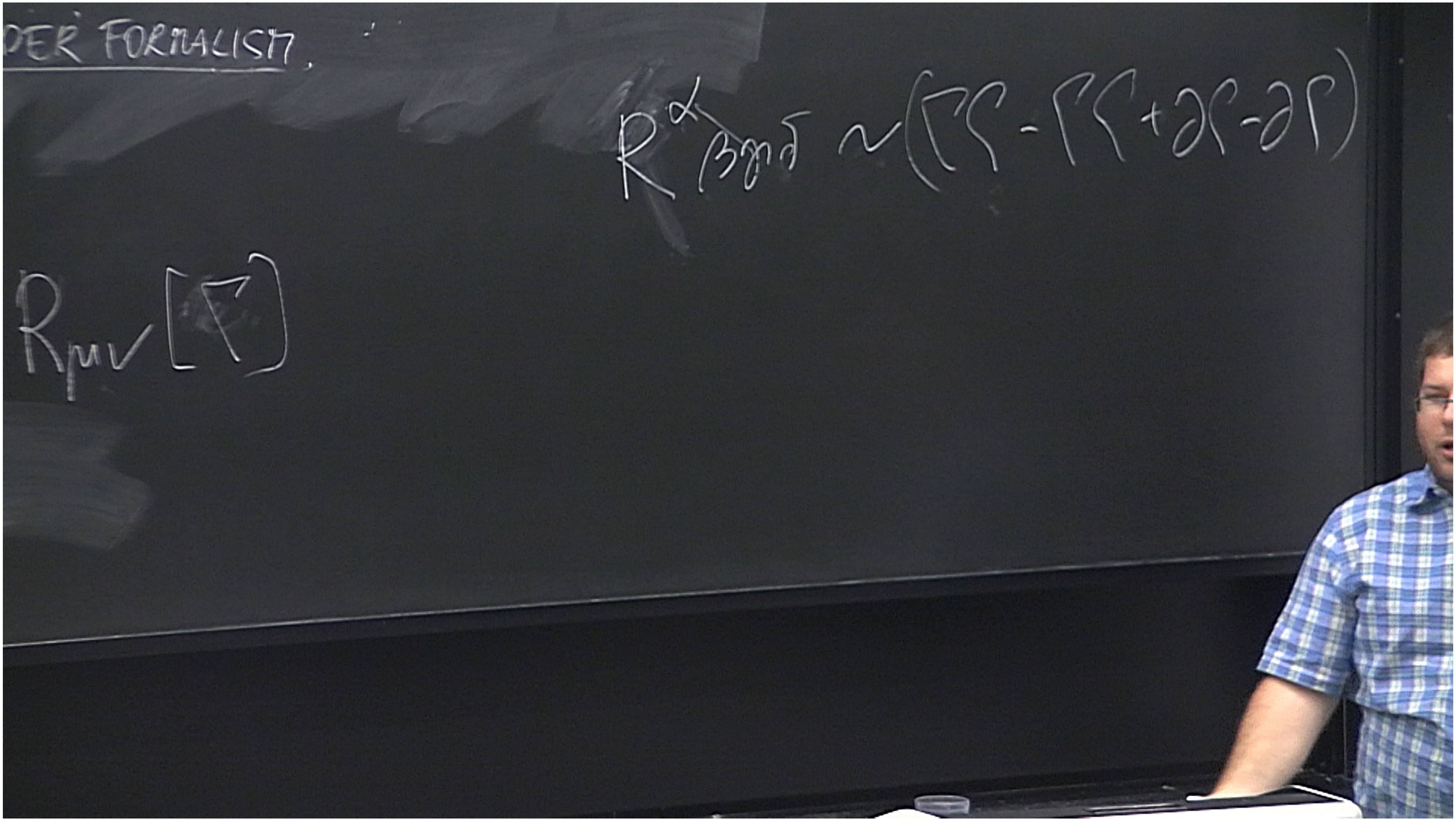
- S_{EH} : ∴ DEPENDS ON $g, \partial g, \underline{\partial^2 g}$ ∴ 2ND-ORDER FORMALISM.
- FIRST-ORDER FORMALISM: (PALITINI ACTION)

$$S_{PALATINI}[g, \Gamma] = \frac{1}{16\pi G} \int \sqrt{-g}\, g^{\mu\nu} R_{\mu\nu}[\Gamma]$$

DER FORMALISM.

$$R^{\alpha}{}_{\beta\gamma\delta} \sim (\Gamma\Gamma - \Gamma\Gamma + \partial\Gamma - \partial\Gamma)$$

$$R_{\mu\nu}[\Gamma]$$


DER FORMALISM.
$R^{\alpha}{}_{\beta\gamma\delta} \sim (\Gamma\Gamma - \Gamma\Gamma + \partial\Gamma - \partial\Gamma)$
$R_{\mu\nu}[\Gamma]$

DER FORMALISM.

$R_{\mu\nu}(\Gamma)$

$$R^{\alpha}{}_{\beta\gamma\delta} \sim (\Gamma\Gamma - \Gamma\Gamma + \partial\Gamma - \partial\Gamma)$$

$$R_{\alpha\beta} = R_{\alpha\beta}(\Gamma)$$

- S_{EH} : DEPENDS ON $g, \partial g, \underline{\partial^2 g}$: 2ND-ORDER FORMALISM

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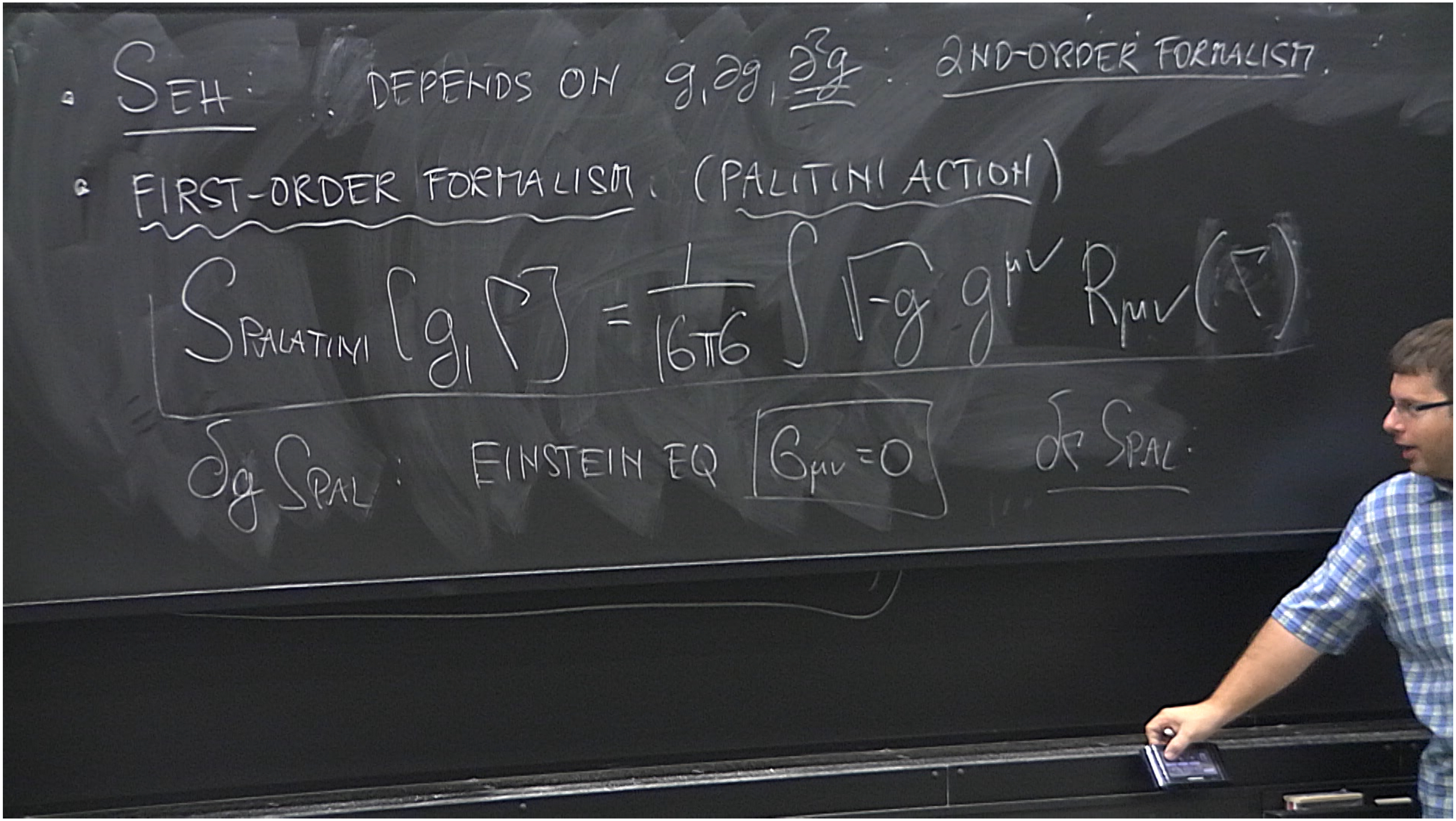
$$\boxed{S_{PALATINI}[g, \Gamma] = \frac{1}{16\pi G} \int \sqrt{-g}\, g^{\mu\nu} R_{\mu\nu}(\Gamma)}$$

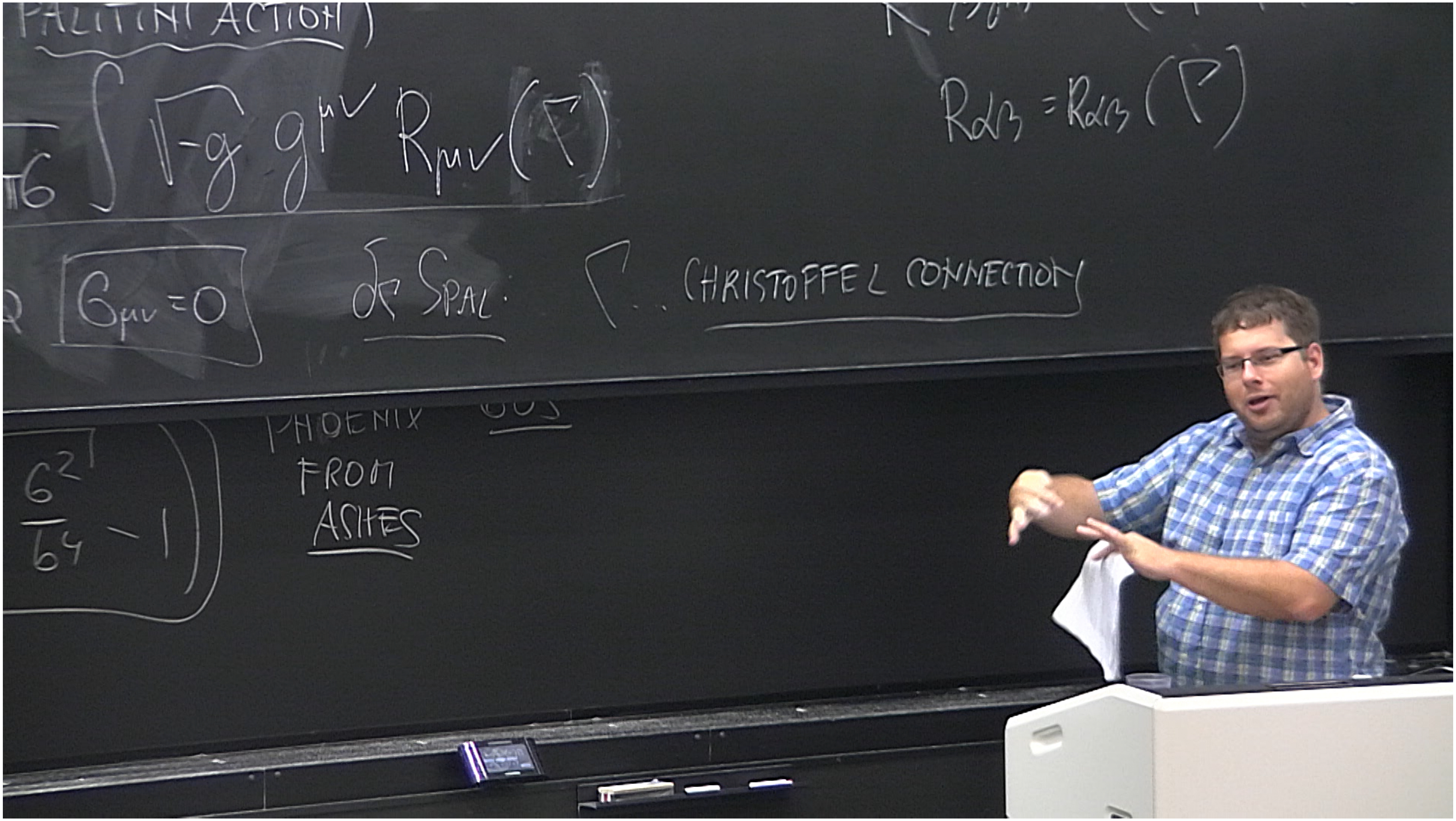
$\delta_g S_{PAL}$:

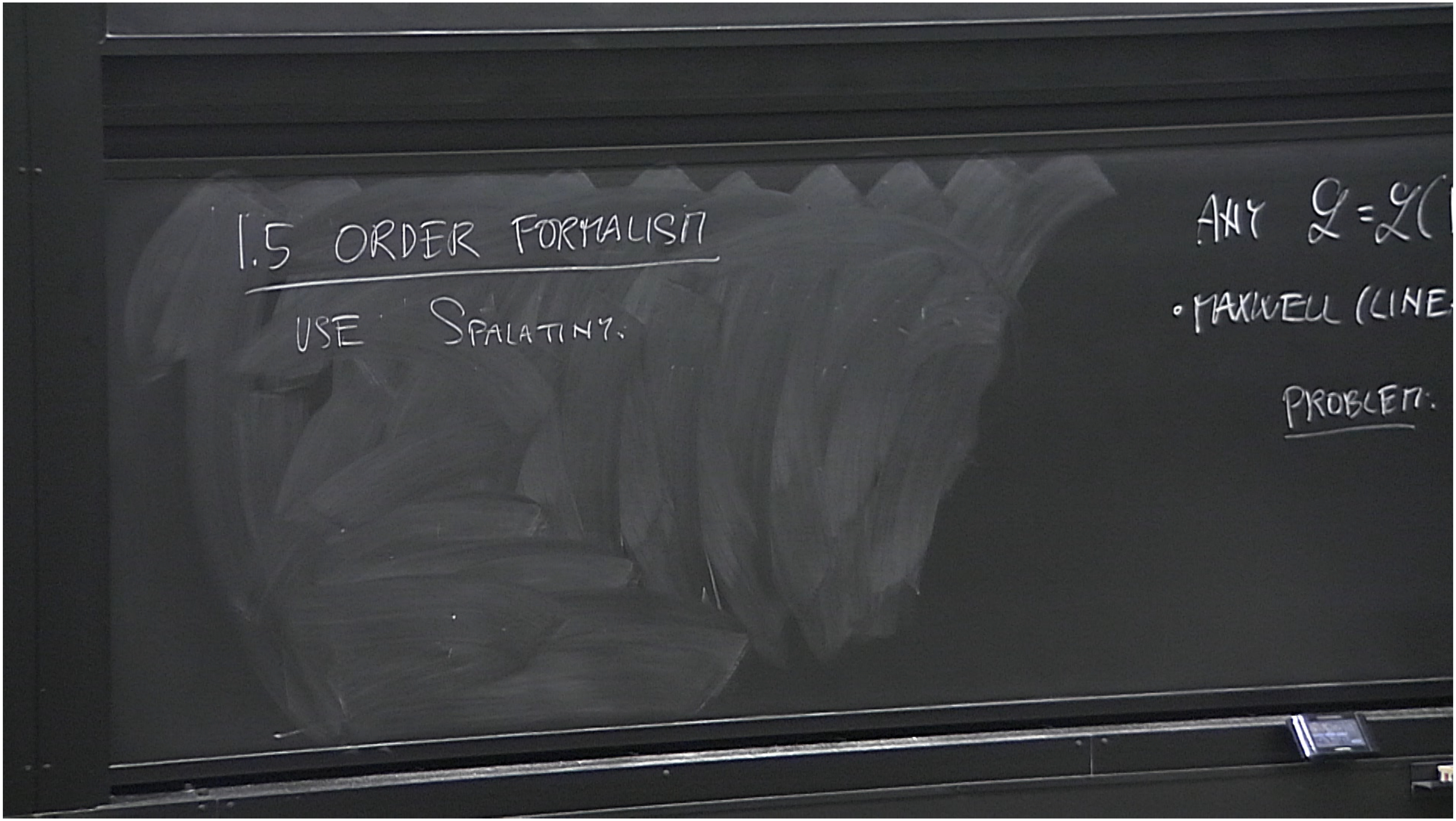
- S_{EH} : DEPENDS ON $g, \partial g, \underline{\partial^2 g}$: <u>2ND-ORDER FORMALISM</u>.
- <u>FIRST-ORDER FORMALISM</u>: (<u>PALITINI ACTION</u>)

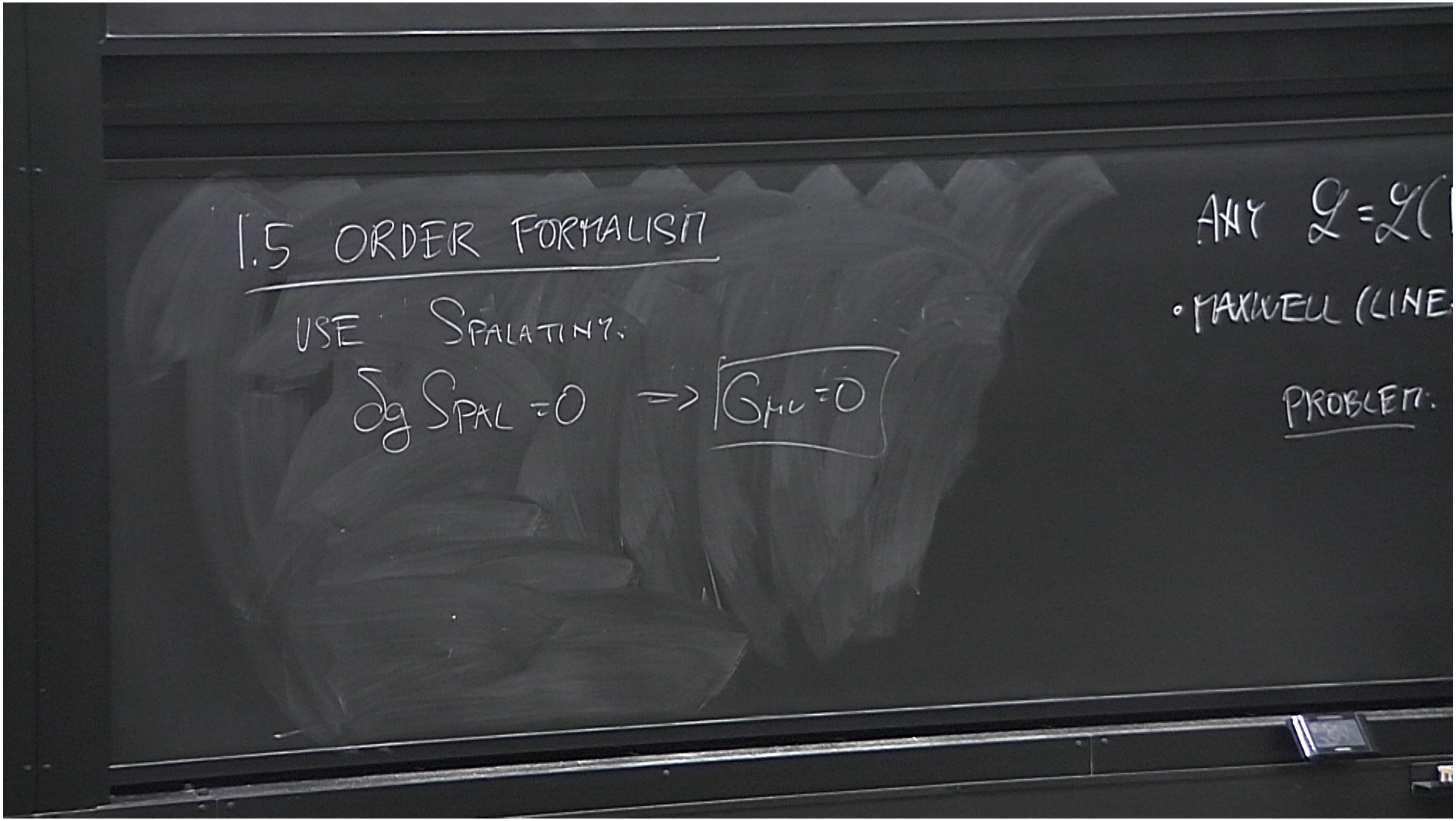
$$\boxed{S_{PALATINI}[g, \Gamma] = \frac{1}{16\pi G}\int \sqrt{-g}\, g^{\mu\nu} R_{\mu\nu}(\Gamma)}$$

$\delta_g S_{PAL}$: EINSTEIN EQ $\boxed{G_{\mu\nu} = 0}$


S_{EH} : DEPENDS ON $g, \partial g, \partial^2 g$: 2ND-ORDER FORMALISM.
FIRST-ORDER FORMALISM : (PALITINI ACTION)
$S_{PALATINI}[g, \Gamma] = \frac{1}{16\pi G} \int \sqrt{-g}\, g^{\mu\nu} R_{\mu\nu}(\Gamma)$
$\delta_g S_{PAL}$: EINSTEIN EQ $G_{\mu\nu} = 0$
$\delta_\Gamma S_{PAL}$:


PALITINI ACTION
$\int \sqrt{-g}\, g^{\mu\nu} R_{\mu\nu}(\Gamma)$
$R_{\alpha\beta} = R_{\alpha\beta}(\Gamma)$
$\boxed{G_{\mu\nu} = 0}$
$\delta_\Gamma S_{PAL}$
Γ ... CHRISTOFFEL CONNECTION
PHOENIX
FROM
ASHES
$\frac{6^2}{6^4} - 1$


1.5 ORDER FORMALISM
USE SPALATINY.
ANY $\mathcal{L} = \mathcal{L}($
• MAXWELL (LINE
PROBLEM:


1.5 ORDER FORMALISM
USE SPALATINY:
$\delta_g S_{PAL} = 0 \Rightarrow \boxed{G_{\mu\nu} = 0}$
ANY $\mathcal{L} = \mathcal{L}($
• MAXWELL (LINE
PROBLEM:

1.5 ORDER FORMALISM

USE SPALATINY:

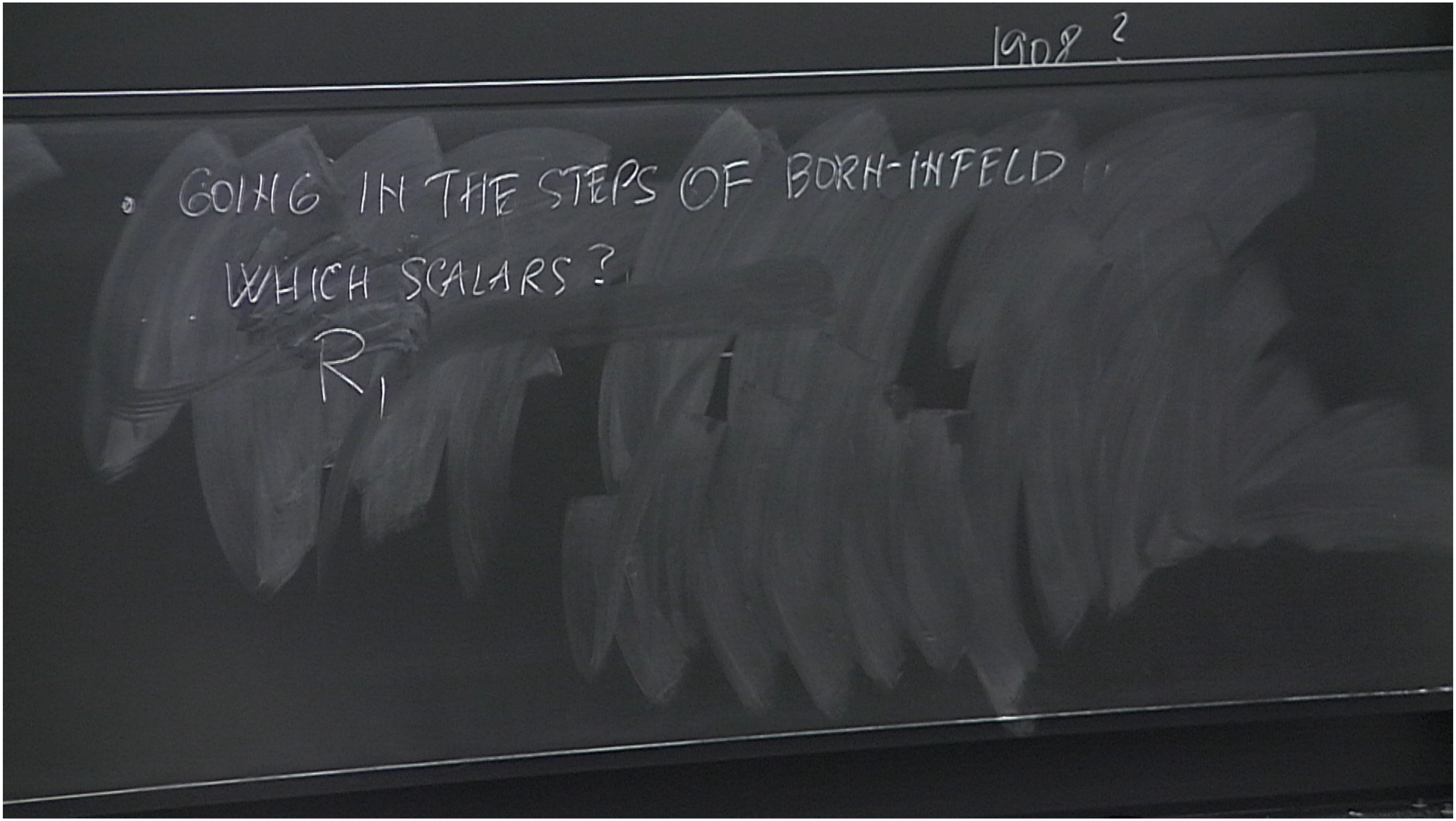
$$\delta_g S_{PAL} = 0 \implies \boxed{G_{\mu\nu} = 0}$$

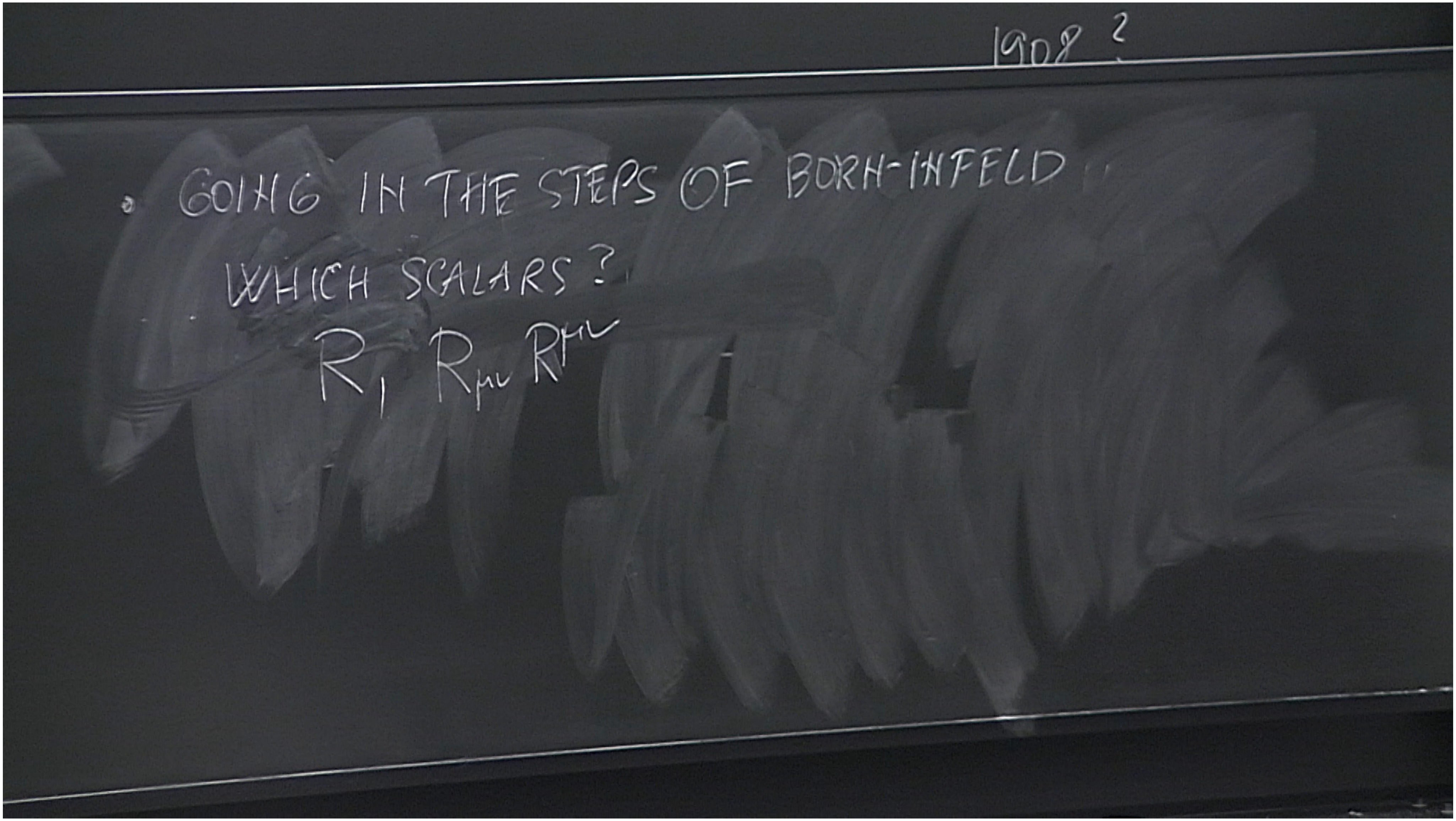
+ USE CHRISTOFFEL CONNECTION

ANY $\mathcal{L} = \mathcal{L}($

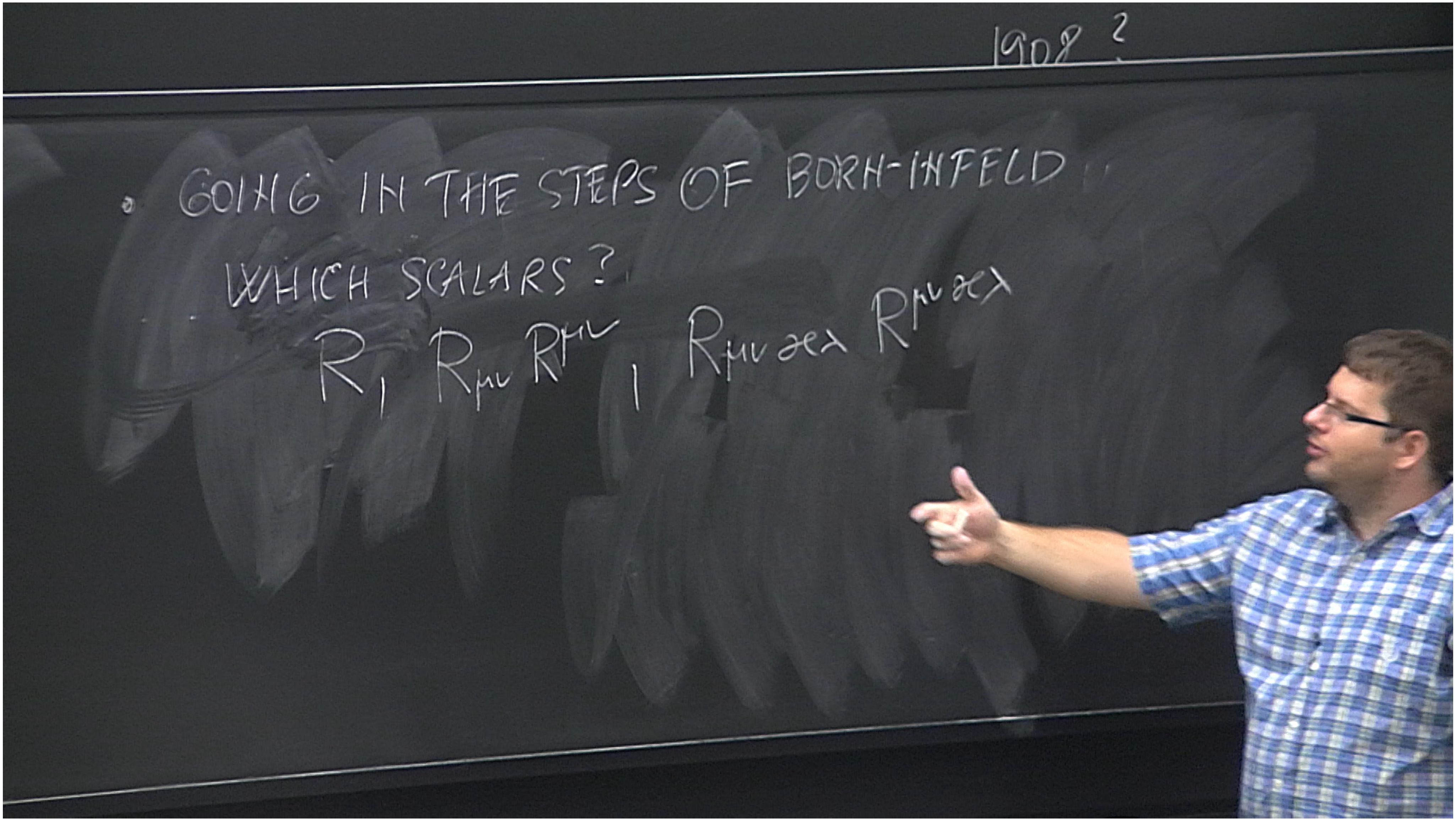
• MAXWELL (LINE

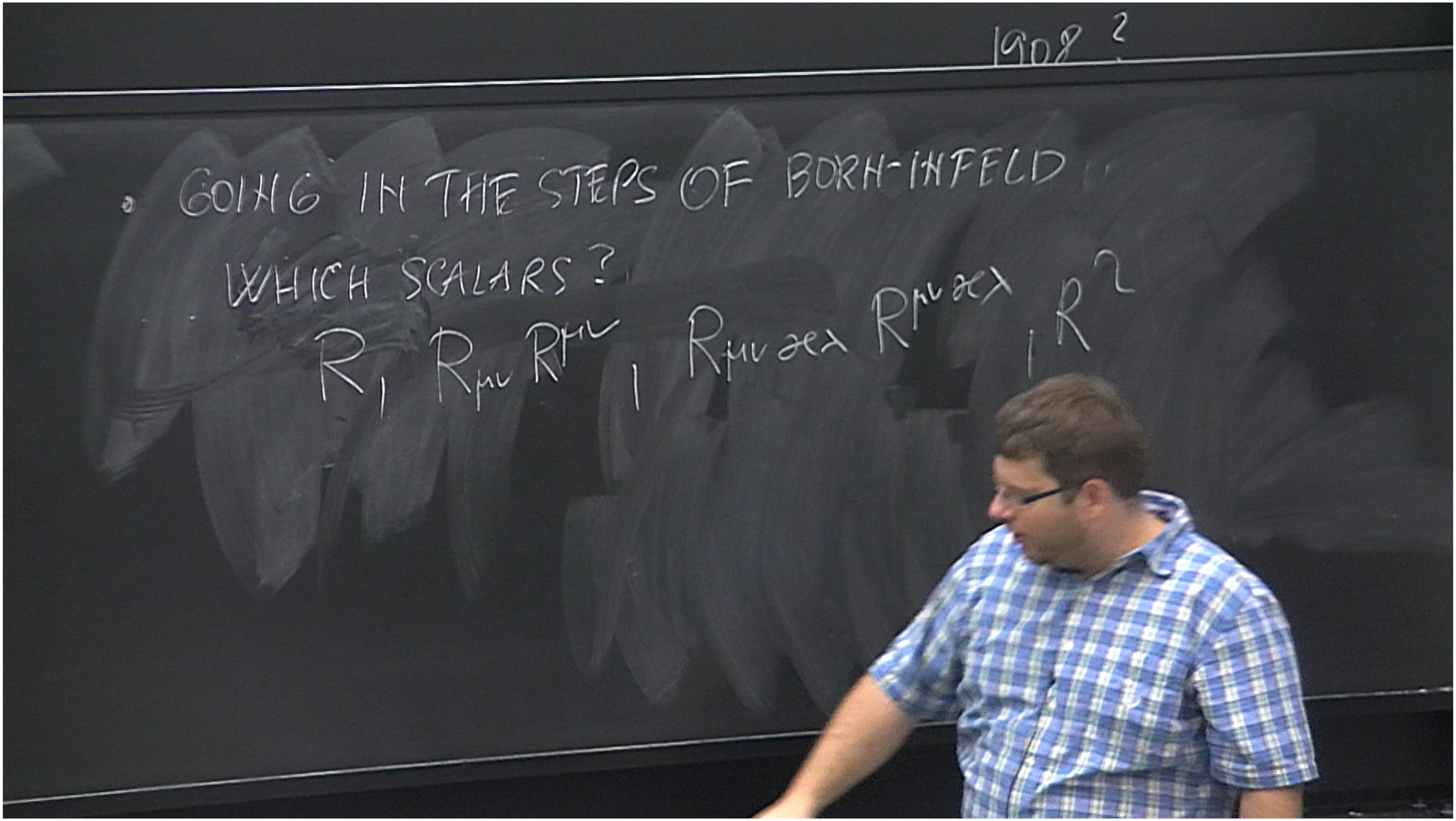
PROBLEM:

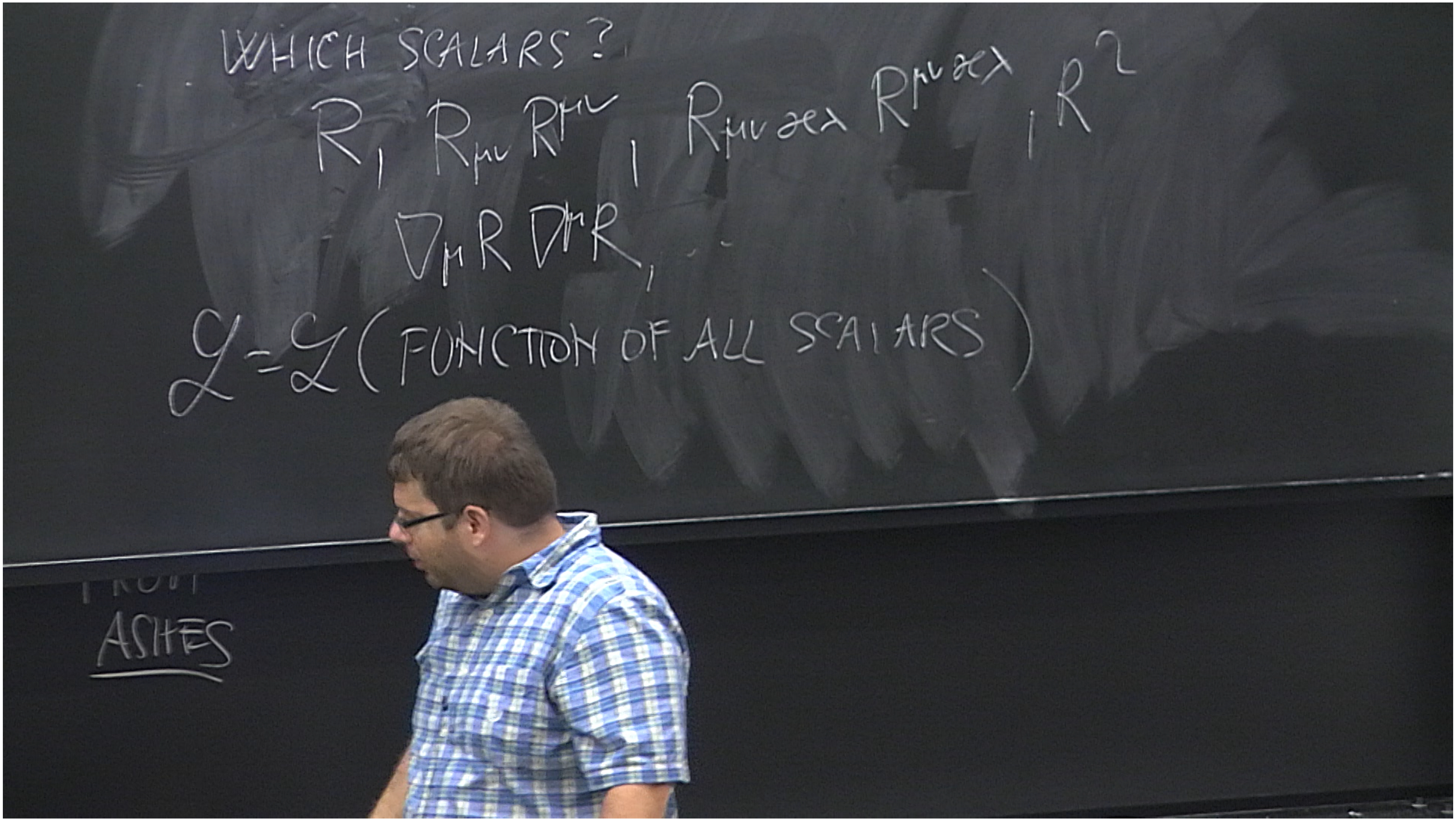

1908 ?
GOING IN THE STEPS OF BORN-INFELD
WHICH SCALARS?
R,



1908 ?
GOING IN THE STEPS OF BORN-INFELD
WHICH SCALARS ?
R, $R_{\mu\nu} R^{\mu\nu}$


1908 ?
GOING IN THE STEPS OF BORN-INFELD
WHICH SCALARS ?
R, $R_{\mu\nu}R^{\mu\nu}$, $R_{\mu\nu\sigma\lambda}R^{\mu\nu\sigma\lambda}$


1908 ?
GOING IN THE STEPS OF BORN-INFELD
WHICH SCALARS?
R, $R_{\mu\nu} R^{\mu\nu}$, $R_{\mu\nu\sigma\lambda} R^{\mu\nu\sigma\lambda}$, R^2


WHICH SCALARS?
R, $R_{\mu\nu}R^{\mu\nu}$, $R_{\mu\nu\sigma\lambda}R^{\mu\nu\sigma\lambda}$, R^2
$\nabla_\mu R \nabla^\mu R$, ...
$\mathcal{L} = \mathcal{L}$ (FUNCTION OF ALL SCALARS)
ASHES

WHICH SCALARS?

$$R, \; R_{\mu\nu}R^{\mu\nu}, \; R_{\mu\nu\sigma\lambda}R^{\mu\nu\sigma\lambda}, \; R^2$$

$$\nabla_\mu R \, \nabla^\mu R, \; \ldots$$

$$\mathcal{L} = \mathcal{L}(\text{FUNCTION OF ALL SCALARS})$$

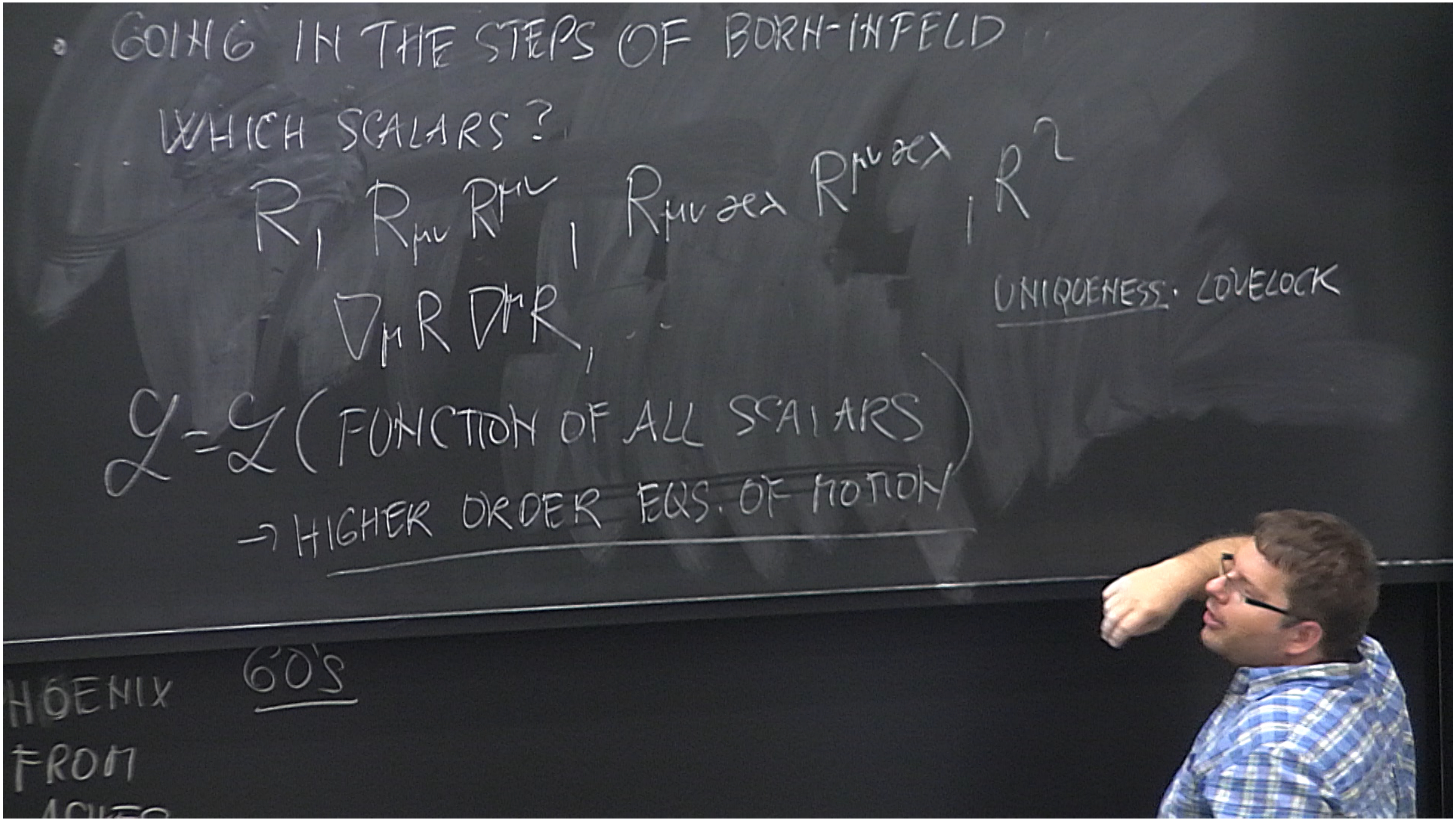
$\rightarrow$ <u>HIGHER ORDER EQS. OF MOTION</u>

ON

$g_{\mu\nu}$

PHOENIX <u>60's</u>

FROM


GOING IN THE STEPS OF BORN-INFELD
WHICH SCALARS?
R, $R_{\mu\nu}R^{\mu\nu}$, $R_{\mu\nu\sigma\lambda}R^{\mu\nu\sigma\lambda}$, R^2
$\nabla_\mu R \nabla^\mu R$, ...
UNIQUENESS: LOVELOCK
$\mathcal{L}=\mathcal{L}$(FUNCTION OF ALL SCALARS)
→ HIGHER ORDER EQS. OF MOTION
HOENIX
60's
FROM

- GOING IN THE STEPS OF BORN-INFELD

WHICH SCALARS?

$$R, \; R_{\mu\nu} R^{\mu\nu}, \; R_{\mu\nu\sigma\lambda} R^{\mu\nu\sigma\lambda}, \; R^2$$

$$\nabla_\mu R \, \nabla^\mu R, \; \ldots$$

$$\mathcal{L} = \mathcal{L}(\text{FUNCTION OF ALL SCALARS})$$

$\rightarrow$ HIGHER ORDER EQS. OF MOTION

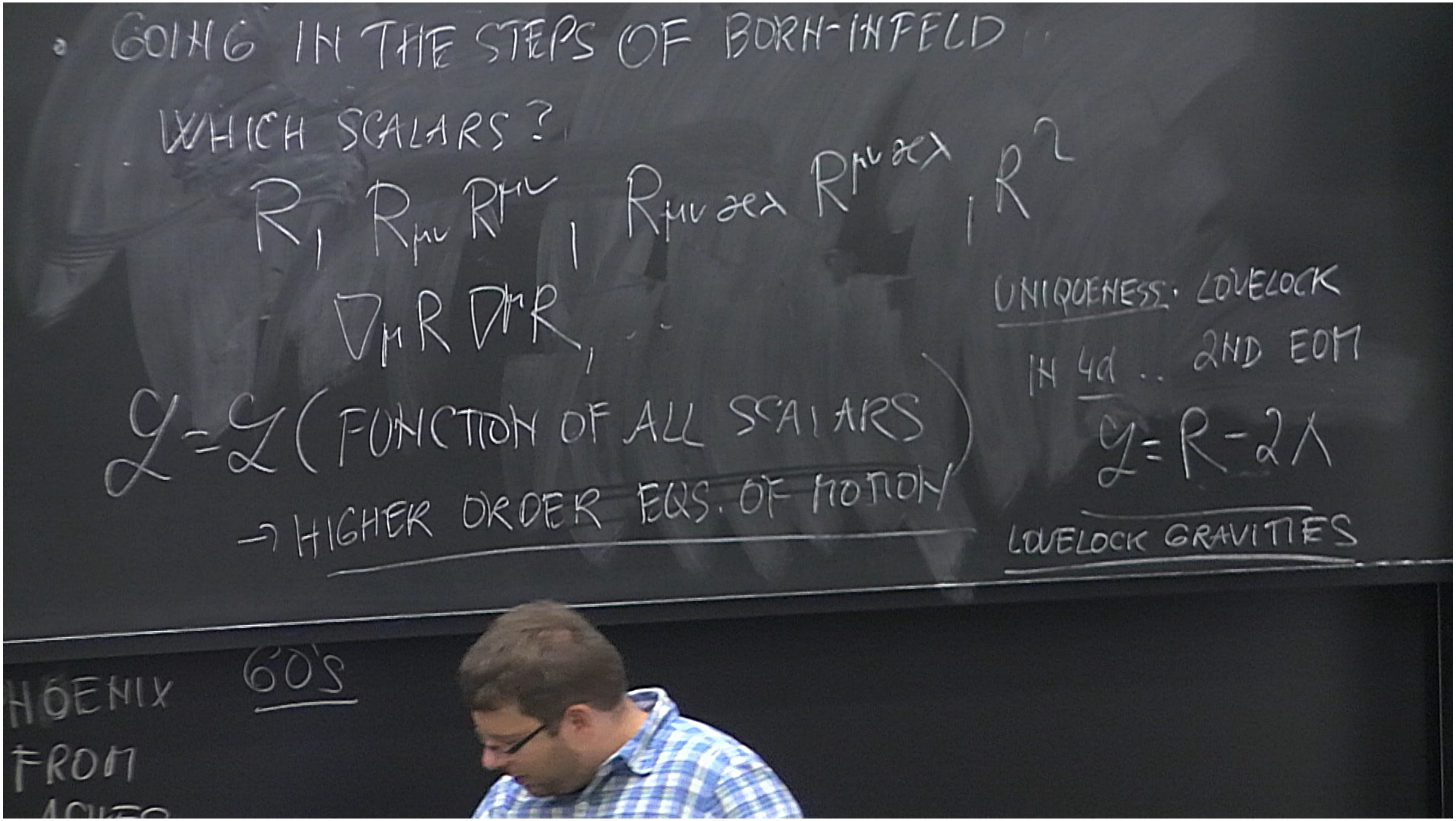
UNIQUENESS: LOVELOCK

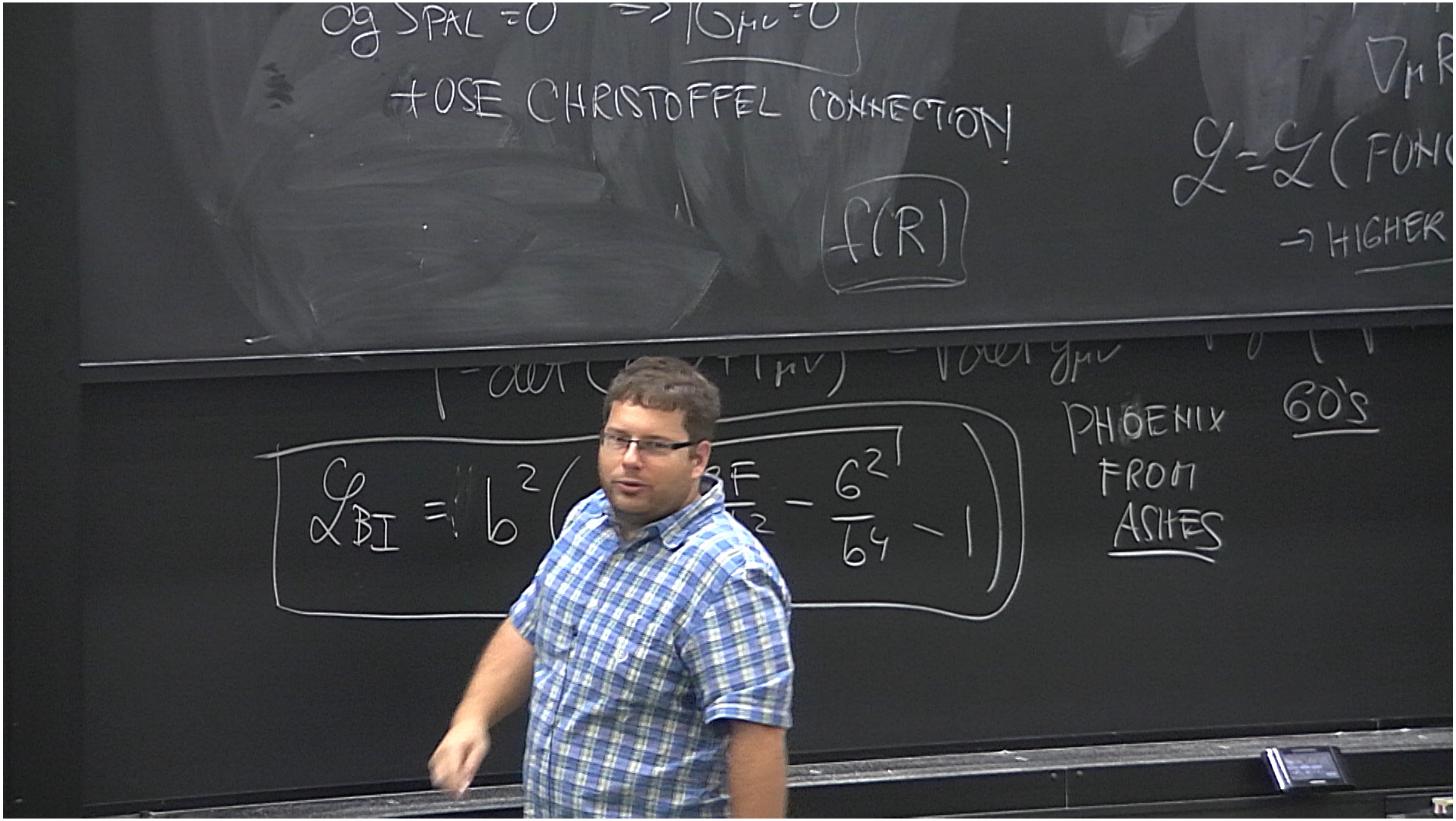
IN 4d .. 2ND EOM

$$\mathcal{L} = R - 2\Lambda$$

HOENIX 60's

FROM


GOING IN THE STEPS OF BORN-INFELD
WHICH SCALARS?
R, $R_{\mu\nu}R^{\mu\nu}$, $R_{\mu\nu\sigma\lambda}R^{\mu\nu\sigma\lambda}$, R^2
$\nabla_\mu R \nabla^\mu R$, ...
$\mathcal{L} = \mathcal{L}$(FUNCTION OF ALL SCALARS)
→ HIGHER ORDER EQS. OF MOTION
UNIQUENESS: LOVELOCK
IN 4d .. 2ND EOM
$\mathcal{L} = R - 2\Lambda$
LOVELOCK GRAVITIES
60's
FROM


+ USE CHRISTOFFEL CONNECTION!
f(R)
→ HIGHER
PHOENIX FROM ASHES
60's