

Title: PSI 2018/2019 - Relativity - Lecture 3

Date: Sep 06, 2018 09:00 AM

URL: <http://pirsa.org/18090004>

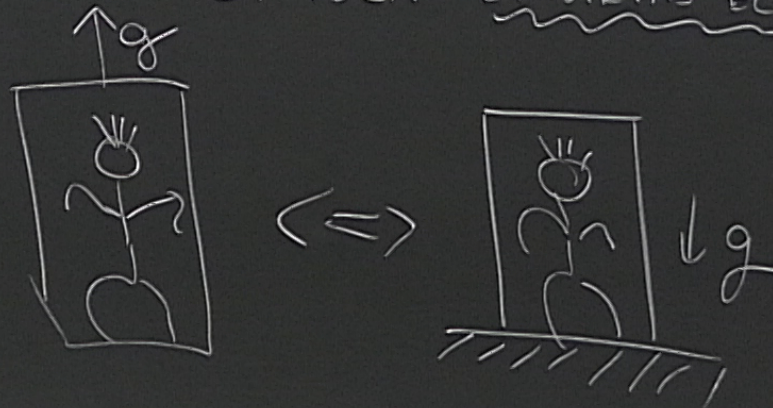
Abstract:

YESTERDAY. EQUIVALENCE PRINCIPLE

IMMEDIATE CO

- $m_I = mg \Rightarrow \boxed{a = g}$

- EINSTEIN'S KEY IDEA EINSTEIN'S ELEVATOR

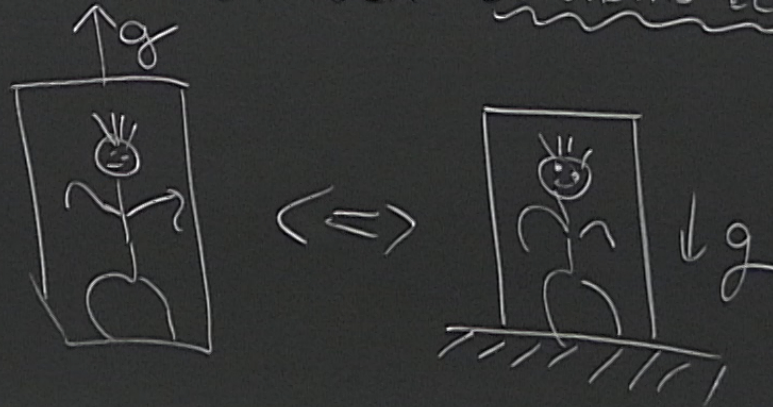


YESTERDAY. EQUIVALENCE PRINCIPLE

IMMEDIATE CO

- $m_I = m_g \Rightarrow \boxed{a = g}$

- EINSTEIN'S KEY IDEA EINSTEIN'S ELEVATOR



IMMEDIATE CONSEQUENCE: GRAVITATIONAL REDSHIFT

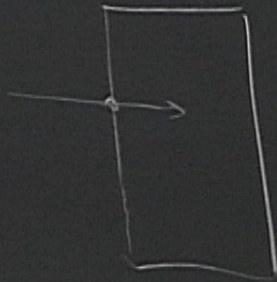
ω'
 $\downarrow g$
 ω

$\omega' < \omega$

IMMEDIATE CONSEQUENCE: GRAVITATIONAL REDSHIFT

ω'
 $\downarrow g$
 ω $\omega' < \omega$

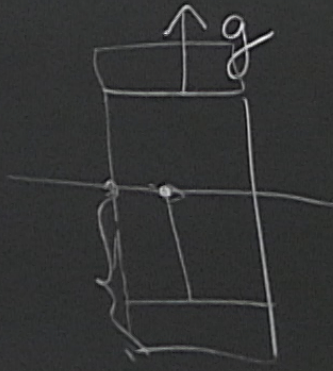
• BENDING OF LIGHT



IMMEDIATE CONSEQUENCE: GRAVITATIONAL REDSHIFT

ω'
 $\downarrow g$
 ω
 $\omega' < \omega$

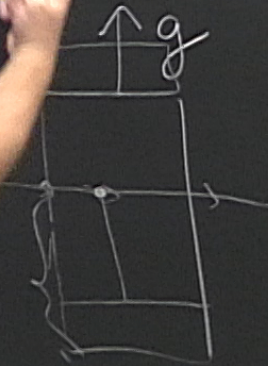
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IMMEDIATE CONSEQUENCE: GRAVITATIONAL REDSHIFT

ω'
 $\downarrow g$
 ω
 $\omega' < \omega$

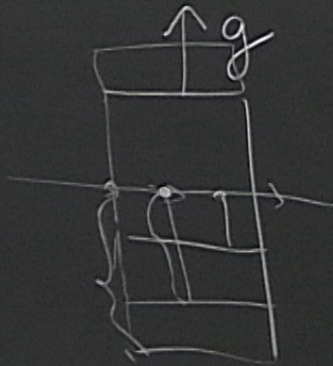
• BENDING OF LIGHT



IMMEDIATE CONSEQUENCE: GRAVITATIONAL REDSHIFT

$\uparrow g$ ω'
 $\downarrow g$ $\omega' < \omega$
 ω

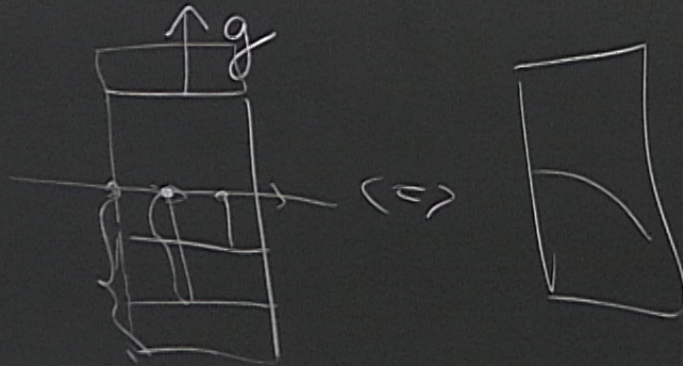
• BENDING OF LIGHT



IMMEDIATE CONSEQUENCE: GRAVITATIONAL REDSHIFT

ω'
 $\downarrow g$
 ω $\omega' < \omega$

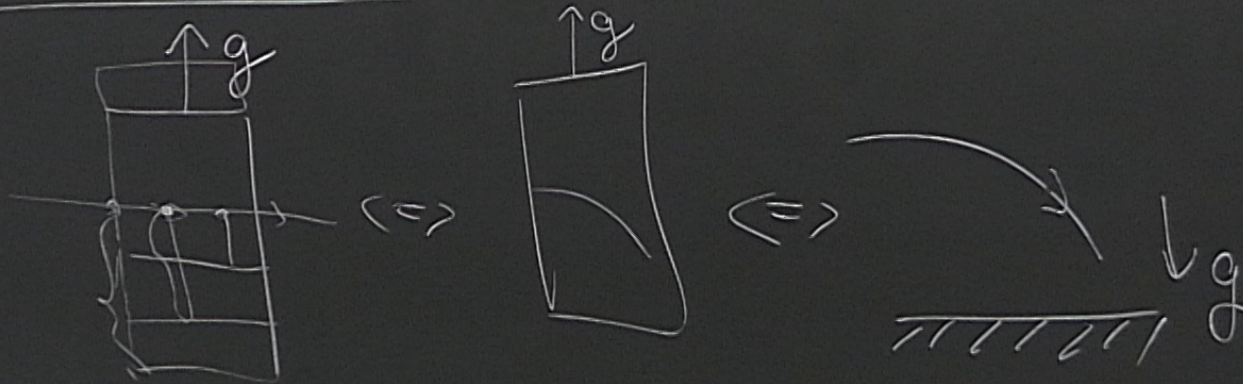
• BENDING OF LIGHT



IMMEDIATE CONSEQUENCE: GRAVITATIONAL REDSHIFT

ω'
 $\downarrow g$
 ω
 $\omega' < \omega$

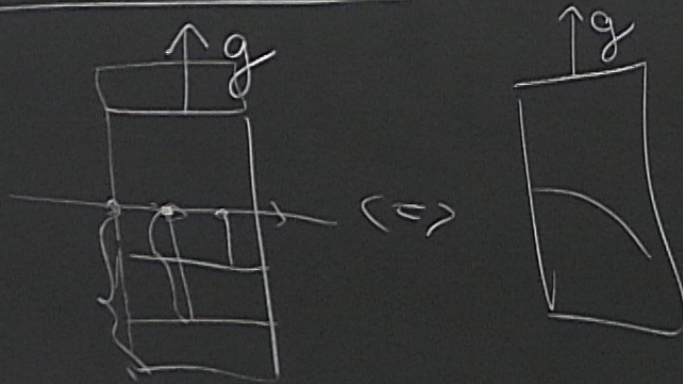
• BENDING OF LIGHT



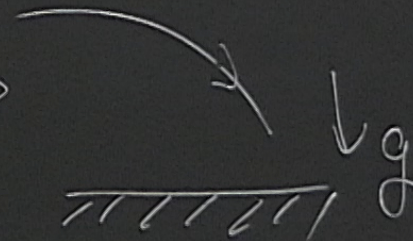
IMMEDIATE CONSEQUENCE: GRAVITATIONAL REDSHIFT

$\uparrow \omega'$
 $\downarrow g$ $\omega' < \omega$
 ω

• BENDING OF LIGHT



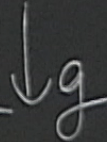
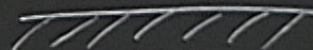
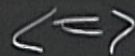
GRAVITY CHANGES WHAT IT MEANS
FOR A PATH TO BE
STRAIGHT.

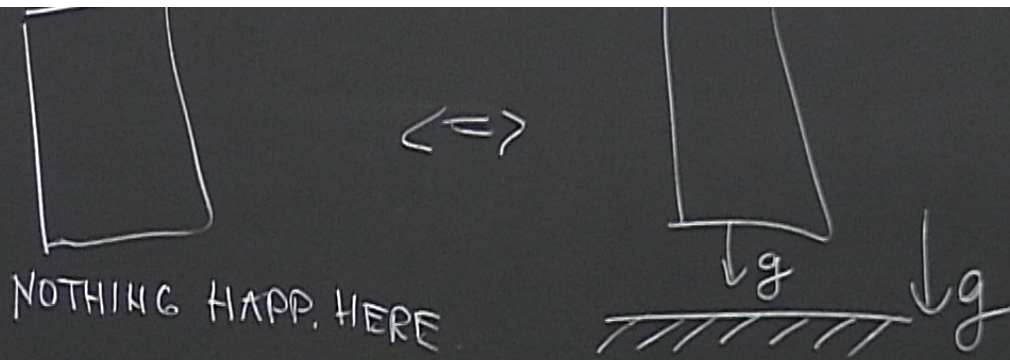


• LOCAL INERTIAL FRAME



NOTHING HAPP. HERE

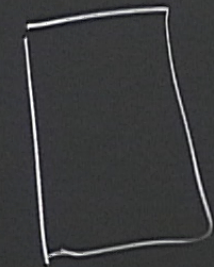




NOTHING HAPP. HERE

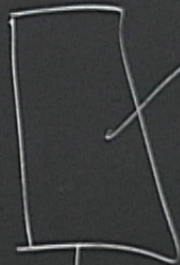
I CAN GET RID OF GRAVITY BY GOING
TO "FREELY FALLING FRAME"

• LOCAL INERTIAL FRAME

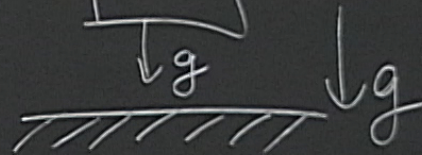


NOTHING HAPP. HERE

$\Leftrightarrow$



HERE SPECIAL RELATIVITY
IS VALID

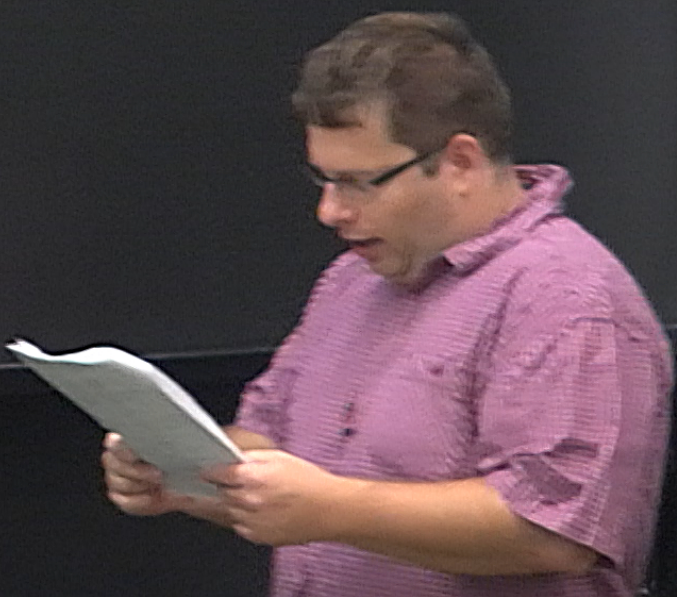


I CAN GET RID OF GRAVITY BY GOING
TO "FREELY FALLING FRAME"

• GEODESIC EQUATION

• LET ξ^μ ... BE LOCAL COORDINATES IN LOCAL INERTIAL FRAME.

VITY



• LET $3x^i$... BE LOCAL COORDINATES IN LOCAL INERTIAL

$\Rightarrow \boxed{\frac{d^2 x^i}{d\tau^2} = 0} \dots$ STRAIGHT LINE.

$\Rightarrow \boxed{\frac{d^2 \mathbf{r}}{dt^2} = 0} \dots$ STRAIGHT LINE.

• LET $x^\alpha = x^\alpha(\tau)$ BE ANY OTHER (GLOBAL) COORDINATES
HOW DOES MY TRAJECTORY LOOK LIKE IN THIS
FRAME ?



TO 'FREELY FALLING FRAME'

$$0 = \frac{d^2 \xi^\mu}{d\tau^2} = \frac{d}{d\tau} \left(\frac{\partial \xi^\mu}{\partial x^\alpha} \frac{dx^\alpha}{d\tau} \right)$$

TO "FREELY FALLING FRAME"

$$0 = \frac{d^2 \xi^\mu}{d\tau^2} = \frac{d}{d\tau} \left(\frac{\partial \xi^\mu}{\partial x^\alpha} \frac{dx^\alpha}{d\tau} \right) = \frac{\partial \xi^\mu}{\partial x^\alpha} \frac{d^2 x^\alpha}{d\tau^2} + \frac{\partial^2 \xi^\mu}{\partial x^\alpha \partial x^\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau}$$

$$0 = \frac{d^2 \mathcal{L}}{d\tau^2} = \frac{d}{d\tau} \left(\frac{\partial \mathcal{L}}{\partial \dot{x}^\alpha} \frac{dx^\alpha}{d\tau} \right) = \frac{\partial \mathcal{L}}{\partial x^\alpha} \frac{d^2 x^\alpha}{d\tau^2} + \frac{\partial^2 \mathcal{L}}{\partial x^\alpha \partial x^\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau}$$

$$\frac{\partial^2 \mathcal{L}}{\partial x^\alpha \partial x^\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau}$$

$$\frac{\partial x^\nu}{\partial x^\alpha} = \delta_\alpha^\nu = \frac{\partial x^\nu}{\partial \mathcal{L}} \frac{\partial \mathcal{L}}{\partial x^\alpha}$$

$$0 = \frac{d^2 \xi^\mu}{d\tau^2} = \frac{d}{d\tau} \left(\frac{\partial \xi^\mu}{\partial x^\alpha} \frac{dx^\alpha}{d\tau} \right) = \frac{\partial \xi^\mu}{\partial x^\alpha} \frac{d^2 x^\alpha}{d\tau^2} + \frac{\partial^2 \xi^\mu}{\partial x^\alpha \partial x^\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau}$$

MULTIPLY BY $\frac{\partial x^\nu}{\partial \xi^\mu}$

$$\boxed{\frac{d^2 x^\nu}{d\tau^2} + \frac{\partial x^\nu}{\partial \xi^\mu} \frac{\partial^2 \xi^\mu}{\partial x^\alpha \partial x^\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = 0}$$

$$0 = \frac{d^2 \xi^\mu}{d\tau^2} = \frac{d}{d\tau} \left(\frac{\partial \xi^\mu}{\partial x^\alpha} \frac{dx^\alpha}{d\tau} \right) = \frac{\partial \xi^\mu}{\partial x^\alpha} \frac{d^2 x^\alpha}{d\tau^2} + \frac{\partial \xi^\mu}{\partial x^\alpha \partial x^\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau}$$

MULTIPLY BY $\frac{\partial x^\nu}{\partial \xi^\mu}$

$$\boxed{\frac{d^2 x^\nu}{d\tau^2} + \underbrace{\frac{\partial x^\nu}{\partial \xi^\mu} \frac{\partial^2 \xi^\mu}{\partial x^\alpha \partial x^\beta}}_{\substack{\Gamma^\nu \\ \alpha\beta}} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = 0}$$

GEODESIC EQUATION

$$\frac{dx^\mu}{d\tau} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = 0$$

$$\left(\frac{d^2 x^\nu}{d\tau^2} + \underbrace{\frac{\partial x^\nu}{\partial \xi^\mu} \frac{\partial^2 \xi^\mu}{\partial x^\alpha \partial x^\beta}}_{\text{CHRISTOFFEL SYMBOLS}} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = 0 \right)$$

$\underbrace{\quad}_{\alpha\beta} \dots$ CHRISTOFFEL SYMBOLS

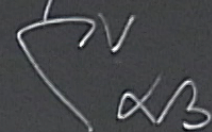
$\gamma^\mu_{\alpha\beta}$

CHRISTOFFEL SYMBOLS

• CURVED METRIC

FREELY FALLING FRAME

$$ds^2 = -d\tau^2 = \eta_{\mu\nu} d\xi^\mu d\xi^\nu$$



CHRISTOFFEL SYMBOLS

• CURVED METRIC

FREELY FALLING FRAME

$$\begin{aligned} ds^2 &= -d\tau^2 = \eta_{\mu\nu} d\xi^\mu d\xi^\nu \\ &= \eta_{\mu\nu} \frac{\partial \xi^\mu}{\partial x^\alpha} \frac{\partial \xi^\nu}{\partial x^\beta} dx^\alpha dx^\beta \end{aligned}$$

$\gamma^\nu_{\alpha\beta}$

CHRISTOFFEL SYMBOLS

• CURVED METRIC

FREELY FALLING FRAME

$$ds^2 = g_{\alpha\beta}$$

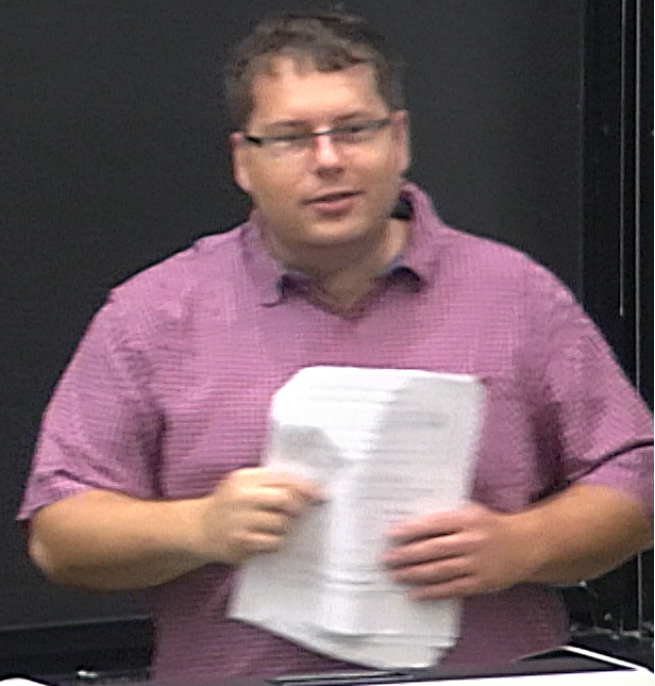
$$ds^2 = -d\tau^2 = \eta_{\mu\nu} d\xi^\mu d\xi^\nu$$

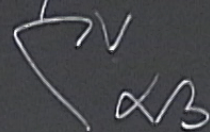
$$= \eta_{\mu\nu} \frac{\partial \xi^\mu}{\partial x^\alpha} \frac{\partial \xi^\nu}{\partial x^\beta} dx^\alpha dx^\beta$$

$$g_{\alpha\beta}$$

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta$$

↓ CURVED METRIC





CHRISTOFFEL SYMBOLS

FLAT METRIC

FREELY FALLING FRAME

$$ds^2 = -d\tau^2 = \eta_{\mu\nu} d\xi^\mu d\xi^\nu$$

$$= \eta_{\mu\nu} \frac{\partial \xi^\mu}{\partial x^\alpha} \frac{\partial \xi^\nu}{\partial x^\beta} dx^\alpha dx^\beta$$

$$g_{\alpha\beta}$$

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta$$

CURVED METRIC

$$g_{\alpha\beta} = g(\alpha\beta) = g_{\beta\alpha} \dots$$

↓ CURVED METRIC

$$g_{\alpha\beta} = g(\alpha/\beta) = g_{\beta\alpha} \dots$$

IN GENERAL... 10 COMPONENTS

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta = -dr^2$$

↓ CURVED METRIC

$$g_{\alpha\beta} = g(\alpha/\beta) = g_{\beta\alpha} \dots$$

IN GENERAL... 10 COMPONENTS

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta = -dr^2$$

↓ CURVED METRIC

$$g_{\alpha\beta} = g(\alpha/\beta) = g_{\beta\alpha} \dots$$

IN GENERAL... 10 COMPONENTS

$$u^2 = u^\alpha u_\alpha = u^\alpha u^\beta g_{\alpha\beta} = -1$$

STATEMENT.

$$\nabla^M \chi_B = \frac{1}{2} g^{M\sigma} (g_{\sigma\alpha, \beta} + g_{\sigma\beta, \alpha} - g_{\alpha\beta, \sigma})$$

$$\beta + g_{\beta, \alpha} - g_{\alpha, \beta})$$

$$g^{\alpha\beta} g_{\beta\gamma} = \delta^{\alpha}_{\gamma}$$

STATEMENT.

$$\nabla^M \alpha_B = \frac{1}{2} g^{M\alpha} (g_{\alpha B} + g_{B\alpha} - g_{\alpha B})$$

OR.

$$\nabla_{M\alpha B} = \frac{1}{2} (g_{M\alpha, B} + g_{M\beta, \alpha} - g_{\alpha B, M})$$

STATEMENT.

$$\nabla^M \underline{\alpha}_B = \frac{1}{2} g^{M\sigma} (g_{\sigma\alpha, B} + g_{\sigma B, \alpha} - g_{\alpha B, \sigma})$$

OR:

$$\nabla_{M\alpha B} = \frac{1}{2} (g_{M\alpha, B} + g_{MB, \alpha} - g_{\alpha B, M})$$

STATEMENT.

$$\nabla^M \underline{\alpha}_B = \frac{1}{2} g^{M\alpha} (g_{\alpha B} + g_{B\alpha} - g_{\alpha B})$$

OR.

$$\nabla^M \underline{\alpha}_B = \frac{1}{2} (g_{M\alpha, B} + g_{MB, \alpha} - g_{\alpha B, M})$$

STATEMENT:

$$\nabla_{\alpha}^M \underline{\chi}_B = \frac{1}{2} g^{M\sigma} (g_{\sigma\alpha, B} + g_{\sigma B, \alpha} - g_{\alpha B, \sigma})$$

OR:

$$\nabla_{\mu}^{\uparrow} \underline{\chi}_B = \frac{1}{2} (g_{\mu\alpha, B} + g_{\mu B, \alpha} - g_{\alpha B, \mu})$$

PROOF: PAIN:

$$g_{\mu\alpha, B} = \frac{\partial}{\partial x^B} \left(\eta_{\alpha\sigma} \frac{\partial x^{\sigma}}{\partial x^{\mu}} \frac{\partial x^{\sigma}}{\partial x^{\alpha}} \right)$$

$$\frac{1}{2} g^{\mu\sigma} (g_{\sigma\alpha, \beta} + g_{\sigma\beta, \alpha} - g_{\alpha\beta, \sigma})$$

$$g^{\alpha\beta} g_{\beta\gamma} = \delta^{\alpha}_{\gamma}$$

$$\frac{1}{2} (g_{\mu\alpha, \beta} + g_{\mu\beta, \alpha} - g_{\alpha\beta, \mu})$$

$$\frac{\partial}{\partial x^{\beta}} \left(\eta_{\sigma\delta} \frac{\partial x^{\sigma}}{\partial x^{\mu}} \frac{\partial x^{\delta}}{\partial x^{\alpha}} \right) = g_{\mu\alpha} \Gamma^{\sigma}_{\beta\sigma} + g_{\mu\sigma} \Gamma^{\sigma}_{\alpha\beta}$$

STATEMENT:

$$\nabla_{\alpha}^M \underline{\alpha}_B = \frac{1}{2} g^{M\sigma} (g_{\sigma\alpha, B} + g_{\sigma B, \alpha} - g_{\alpha B, \sigma})$$

OR:

$$\nabla_{\alpha}^M \underline{\alpha}_B = \frac{1}{2} (g_{M\alpha, B} + g_{MB, \alpha} - g_{\alpha B, M})$$

PROOF: PAIN!

$$g_{M\alpha, B} = \frac{\partial}{\partial x^B} \left(g_{\alpha\sigma} \frac{\partial x^{\sigma}}{\partial x^M} \right) = g_{\alpha\sigma} \frac{\partial x^{\sigma}}{\partial x^B} + 2 \text{ OTHER TERMS} \rightarrow \square$$

- EXAMPLE OF "FAKE GRAVITY" - RINDLER (ACCELERATED) FRAME
START IN FLAT SPACE OR CONSIDER AN ACCELERATED OBSERVER
WITH $a = \sqrt{a_\mu a^\mu} = \text{CONST}$

- EXAMPLE OF "FAKE GRAVITY" - RINDLER (ACCELERATED) FRAME
START IN FLAT SPACE OR CONSIDER AN ACCELERATED OBSERVER
WITH $a = \sqrt{a_\mu a^\mu} = \text{CONST}$ UNIFORM ACCELER.

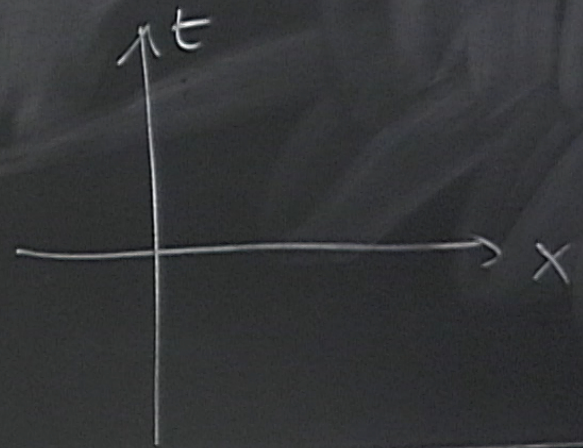
• EXAMPLE OF "FAKE GRAVITY" - RINDLER (ACCELERATED) FRAME

START IN FLAT SPACE OR CONSIDER AN ACCELERATED OBSERVER

WITH $a = \sqrt{a_\mu a^\mu} = \text{CONST}$

UNIFORM ACCELER.

$t =$



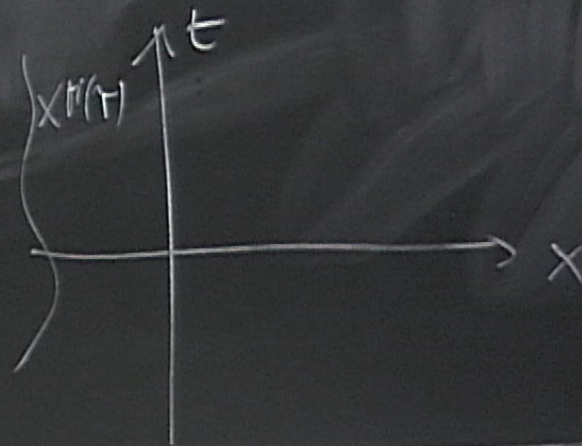
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START IN FLAT SPACE OR CONSIDER AN ACCELERATED OBSERVER

WITH $a = \sqrt{a_\mu a^\mu} = \text{CONST}$

UNIFORM ACCELER.

$t =$



• EXAMPLE OF "FAKE GRAVITY" - RINDLER (ACCELERATED) FRAME

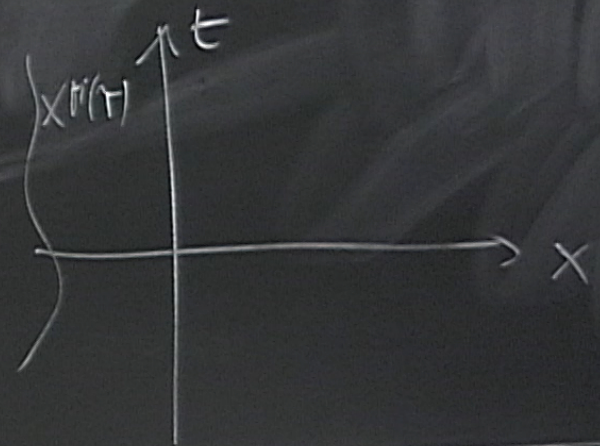
START IN FLAT SPACE OR CONSIDER AN ACCELERATED OBSERVER

WITH $a = \sqrt{a_\mu a^\mu} = \text{CONST}$

UNIFORM ACCELER.

$$t = \frac{1}{a} \sinh(a\tau)$$

$$x = \frac{1}{a} \cosh(a\tau)$$



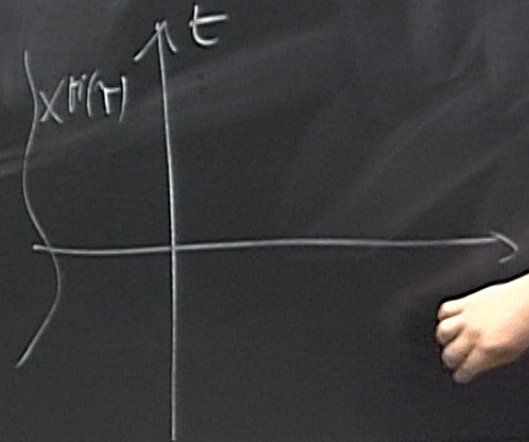
• EXAMPLE OF "FAKE GRAVITY" - RINDLER (ACCELERATED) FRAME

START IN FLAT SPACE OR CONSIDER AN ACCELERATED OBSERVER

WITH $a = \sqrt{a_\mu a^\mu} = \text{CONST}$

UNIFORM ACCELER.

$$\begin{aligned} t &= \frac{1}{a} \sinh(a\tau) \\ x &= \frac{1}{a} \cosh(a\tau) \end{aligned}$$

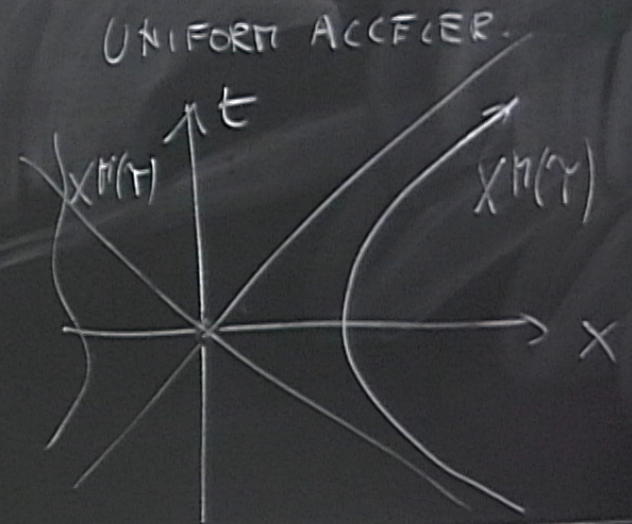


• EXAMPLE OF "FAKE GRAVITY" - RINDLER (ACCELERATED) FRAME

START IN FLAT SPACE OR CONSIDER AN ACCELERATED OBSERVER

WITH $a = \sqrt{a_\mu a^\mu} = \text{CONST}$

$$\begin{aligned} t &= \frac{1}{a} \sinh(a\tau) \\ x &= \frac{1}{a} \cosh(a\tau) \end{aligned}$$



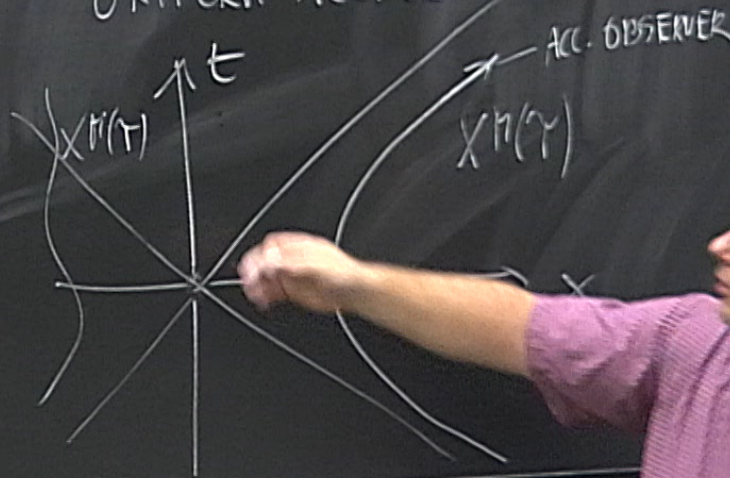
"KE GRAVITY" - RINDLER (ACCELERATED) FRAME

SPACE & CONSIDER AN ACCELERATED OBSERVER

$$a = \sqrt{a_\mu a^\mu} = \text{CONST}$$

UNIFORM ACCELER.

$$\begin{cases} \sinh(a\tau) \\ \cosh(a\tau) \end{cases}$$



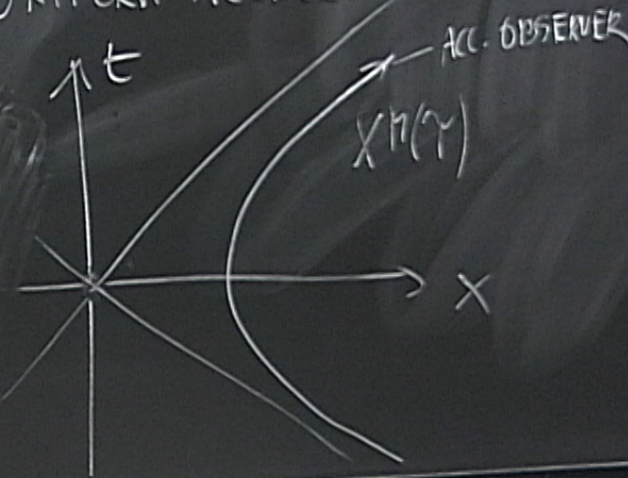
"LIKE GRAVITY" - RINDLER (ACCELERATED) FRAME

IN SPACE OR CONSIDER AN ACCELERATED OBSERVER

$$a = \sqrt{a_\mu a^\mu} = \text{CONST}$$

UNIFORM ACCELER.

$$\begin{cases} \sinh(a\tau) \\ \cosh(a\tau) \end{cases}$$



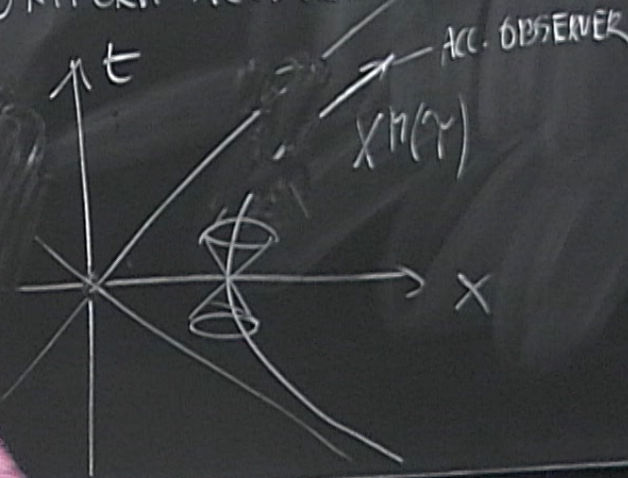
"KE GRAVITY" - RINDLER (ACCELERATED) FRAME

IN SPACE OR CONSIDER AN ACCELERATED OBSERVER

$$a = \sqrt{a_\mu a^\mu} = \text{CONST}$$

UNIFORM ACCELER.

$$\begin{cases} \sinh(a\tau) \\ \cosh(a\tau) \end{cases}$$



"KE GRAVITY" - RINDLER (ACCELERATED) FRAME

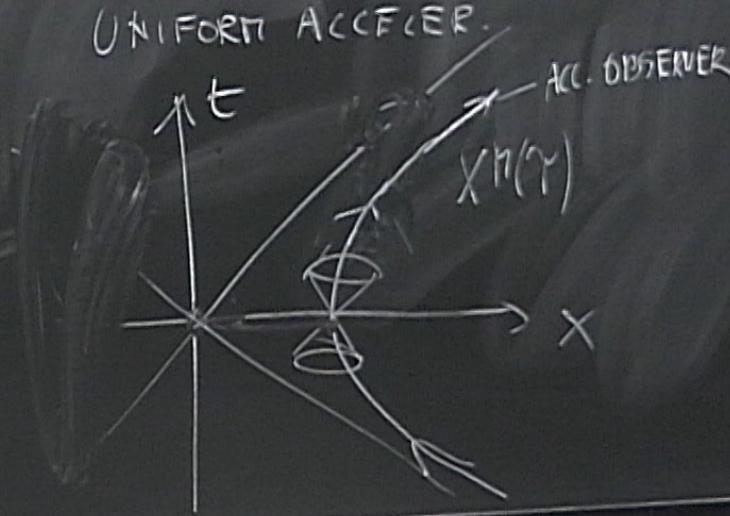
SPACE OR CONSIDER AN ACCELERATED OBSERVER

$$a = \sqrt{a_\mu a^\mu} = \text{CONST}$$

UNIFORM ACCELER.

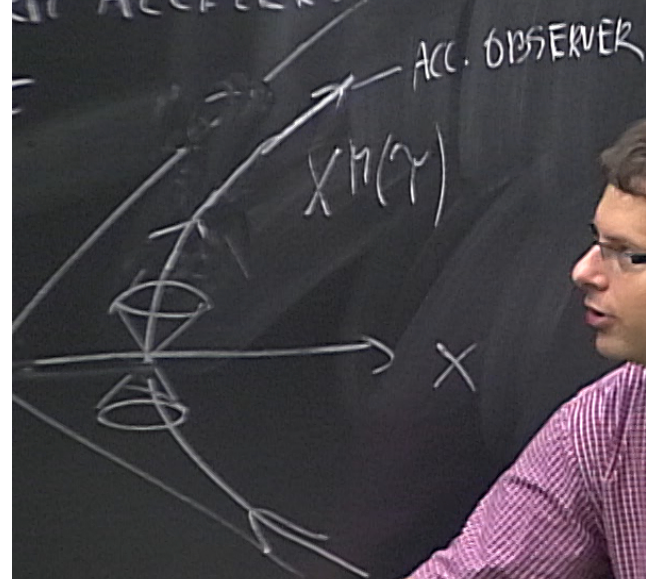
$$\sinh(a\tau)$$

$$\cosh(a\tau)$$



RATE
OBSERVER

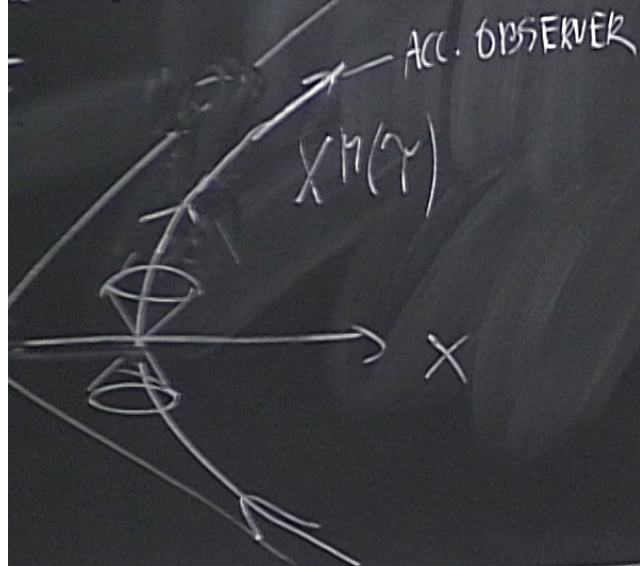
AT ACCELER.



RINDLER COORDINATES (NICE COORDINATES
FOR AN OBSERVER SITTING ON THE
ACCELERATED OBJECT)

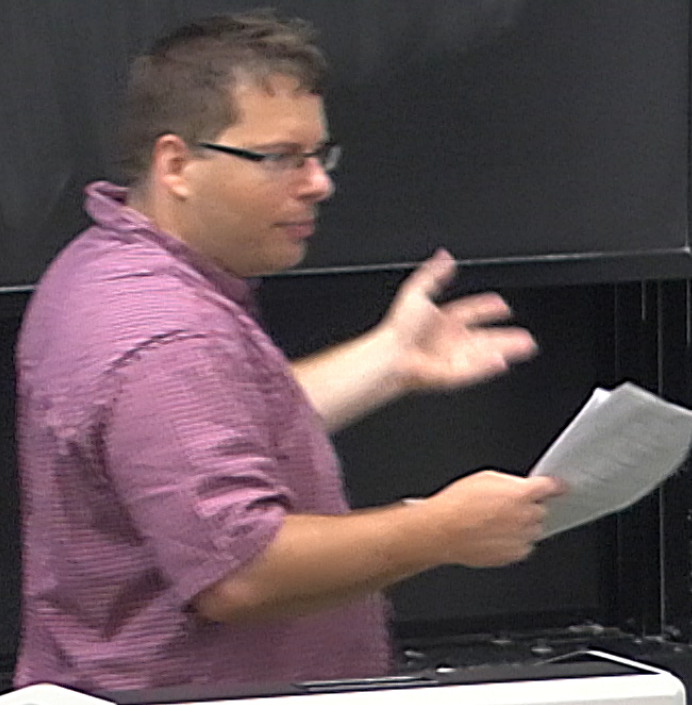
OBSERVER

IN ACCELER.



FOR AN OBSERVER SITTING ON THE
ACCELERATED OBJECT)

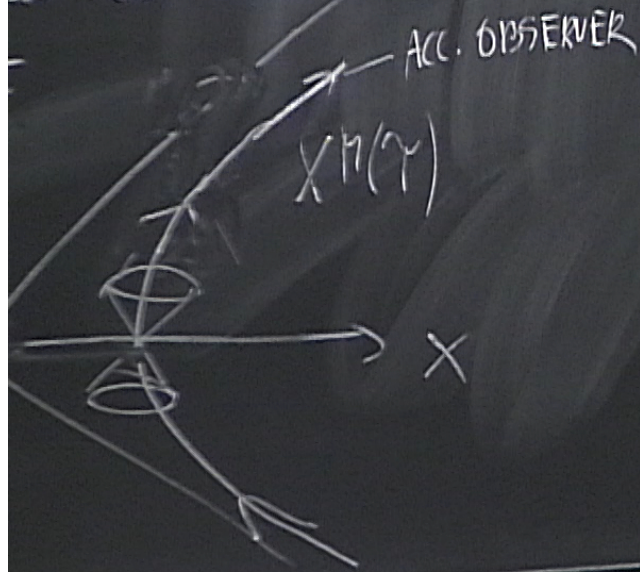
$$t = \left(\frac{1}{a} + X \right)$$



RATE

OBSERVER

IN ACCELER.



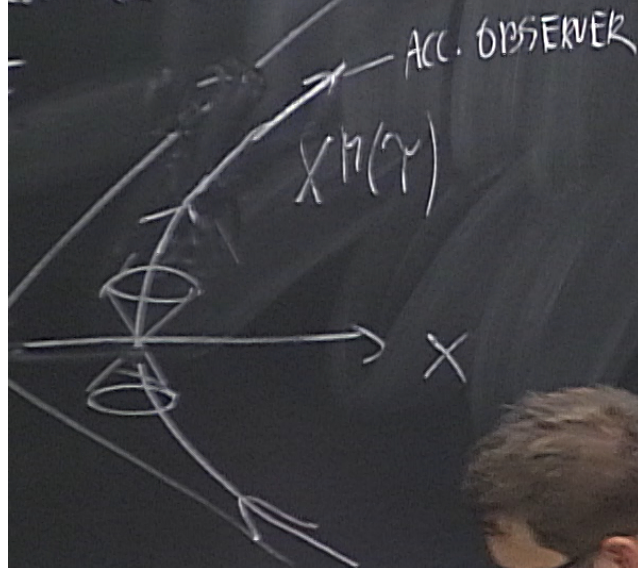
RINDLER COORDINATES (NICE COORDINATES
FOR AN OBSERVER SITTING ON THE
ACCELERATED OBJECT) (T, X)

$$t = \left(\frac{1}{a} + X \right)$$

RATE

OBSERVER

IN ACCELER.



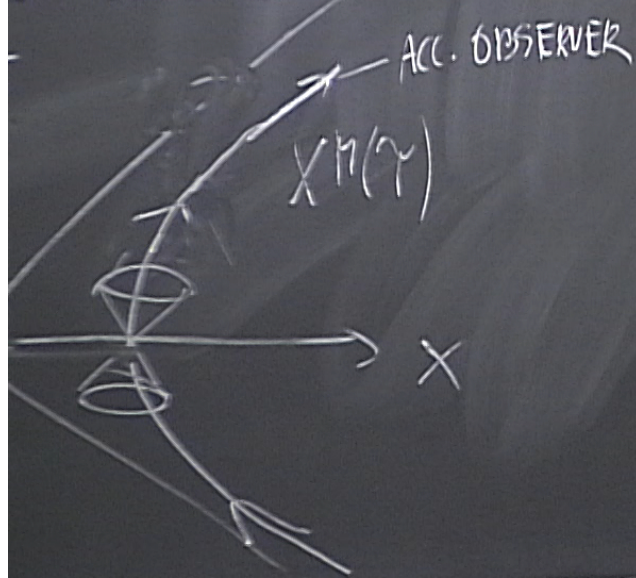
RINDLER COORDINATES (NICE COORDINATES
FOR AN OBSERVER SITTING ON THE
ACCELERATED OBJECT) (T, X)

$$\begin{aligned} t &= \left(\frac{1}{a} + X \right) \sinh(aT) \\ X &= \left(\frac{1}{a} + X \right) \cosh(aT) \end{aligned}$$

RATE

OBSERVER

IN ACCELER.



RINDLER COORDINATES (NICE COORDINATES
FOR AN OBSERVER SITTING ON THE
ACCELERATED OBJECT) (T, X)

$$t = \left(\frac{1}{a} + X \right) \sinh(aT)$$

$$x = \left(\frac{1}{a} + X \right) \cosh(aT)$$

$$X=0, T=\tau$$

$\gamma_{\alpha\beta}$

$$ds^2 = -dt^2 + dx^2 = -\frac{(1 - ax)^2 dT^2 + dx^2}{1 - 2ax + a^2 x^2}$$

$$dt = \frac{\partial t}{\partial T} dT + \frac{\partial t}{\partial x} dx$$

$$ds^2 = -dt^2 + dx^2 = -\frac{(1+ax)^2 dT^2 + dx^2}{1}$$

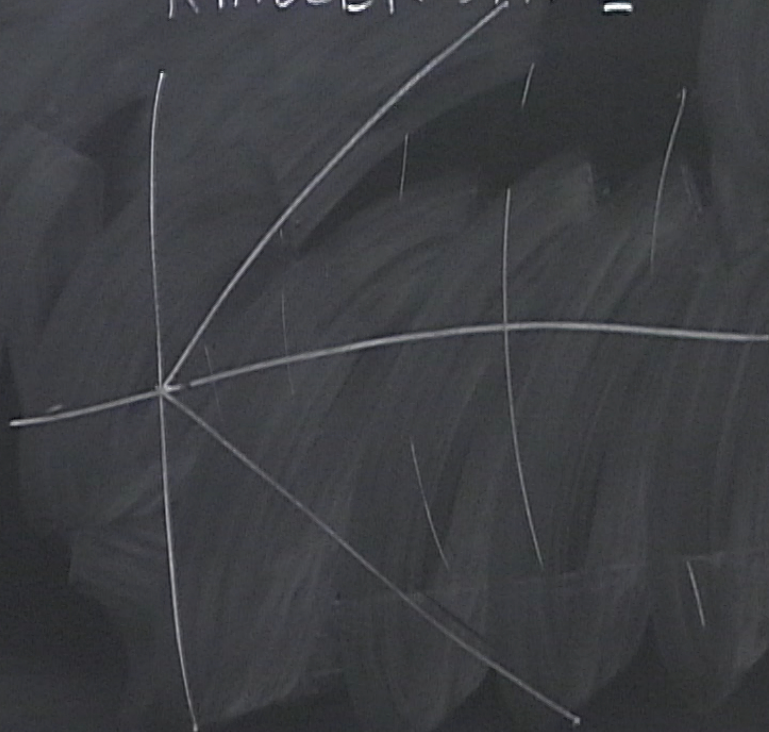
$$dt = \frac{\partial t}{\partial T} dT + \frac{\partial t}{\partial x} dx$$

RANGE OF COORDINATES (METRIC IS NON-DEGENERATE)

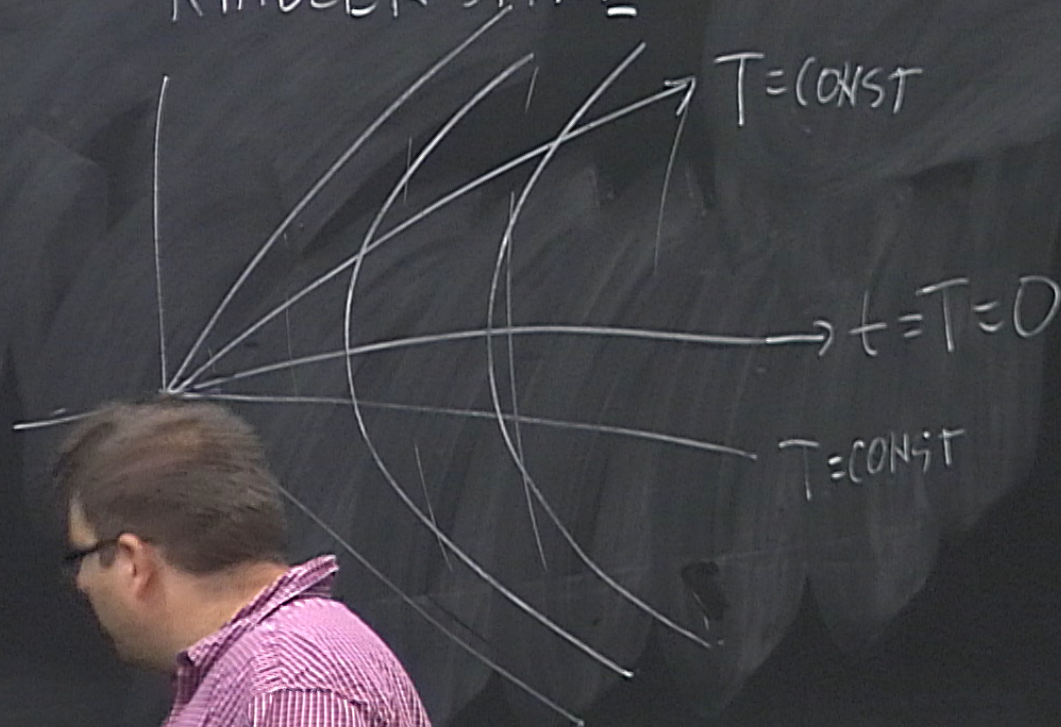
$$x \in \left(-\frac{1}{a}, \infty\right)$$

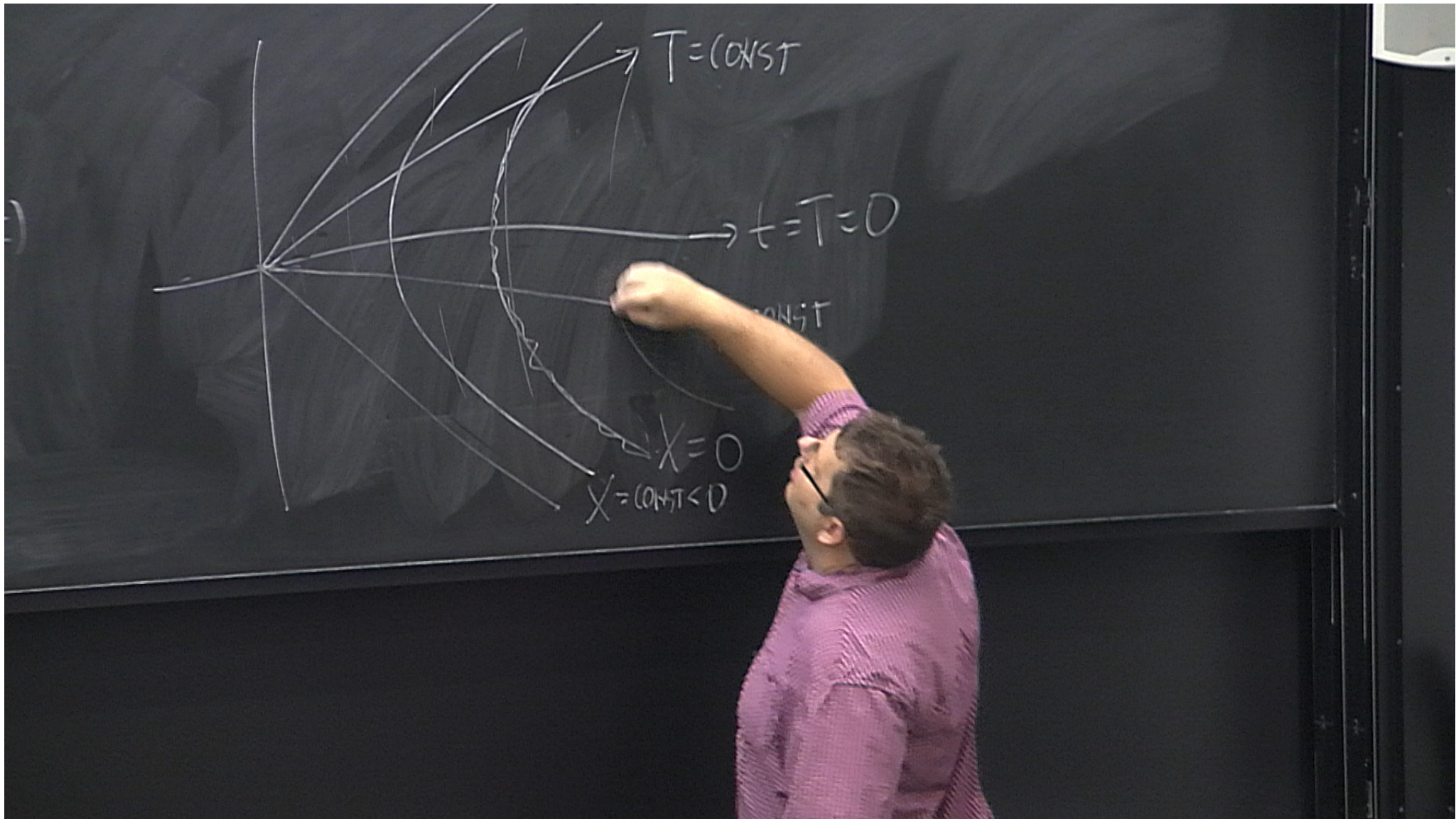
$$T \in (-\infty, \infty)$$

RINDLER SPACE

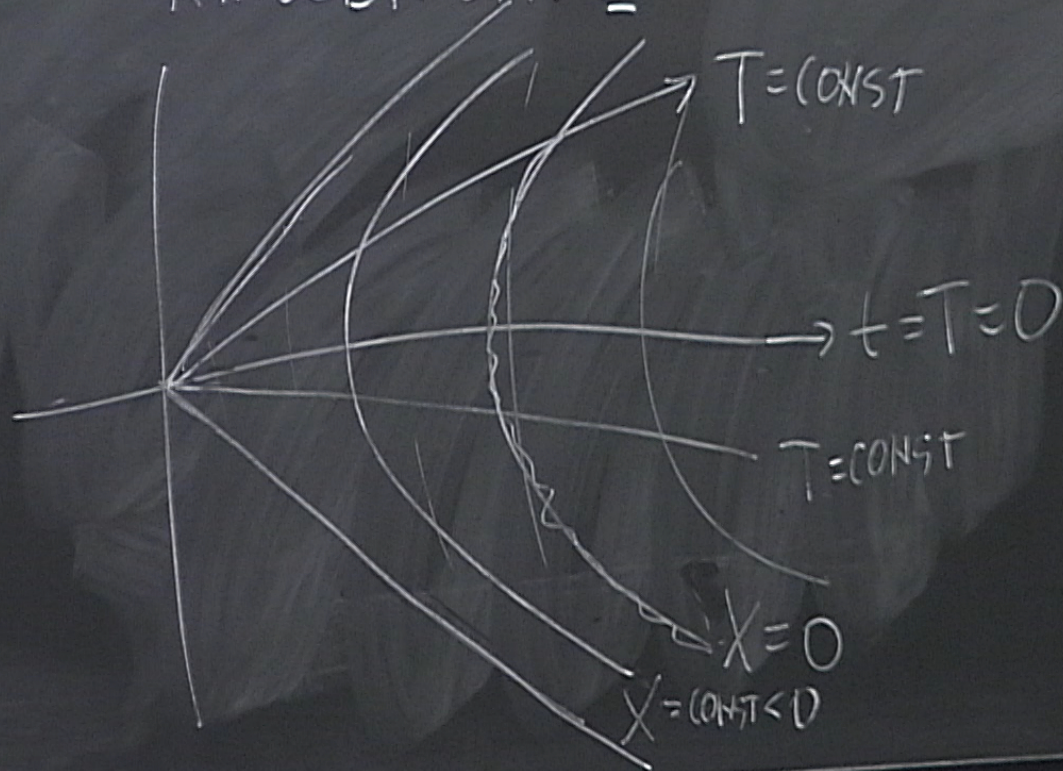


RINDLER SPACE





RINDLER SPACE



$$dt = \frac{\partial t}{\partial T} dT + \frac{\partial t}{\partial X} dX$$

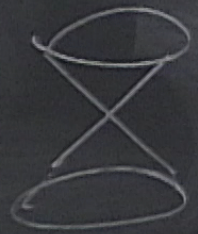
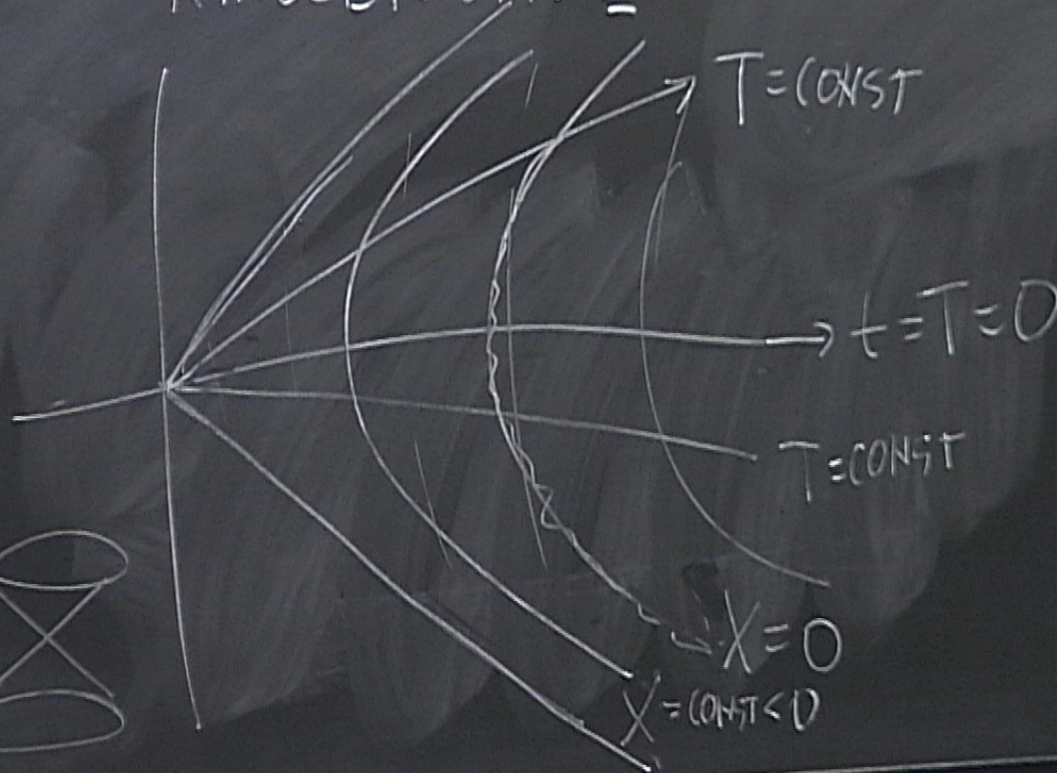
RANGE OF COORDINATES (METRIC IS NON-DEGENERATE)

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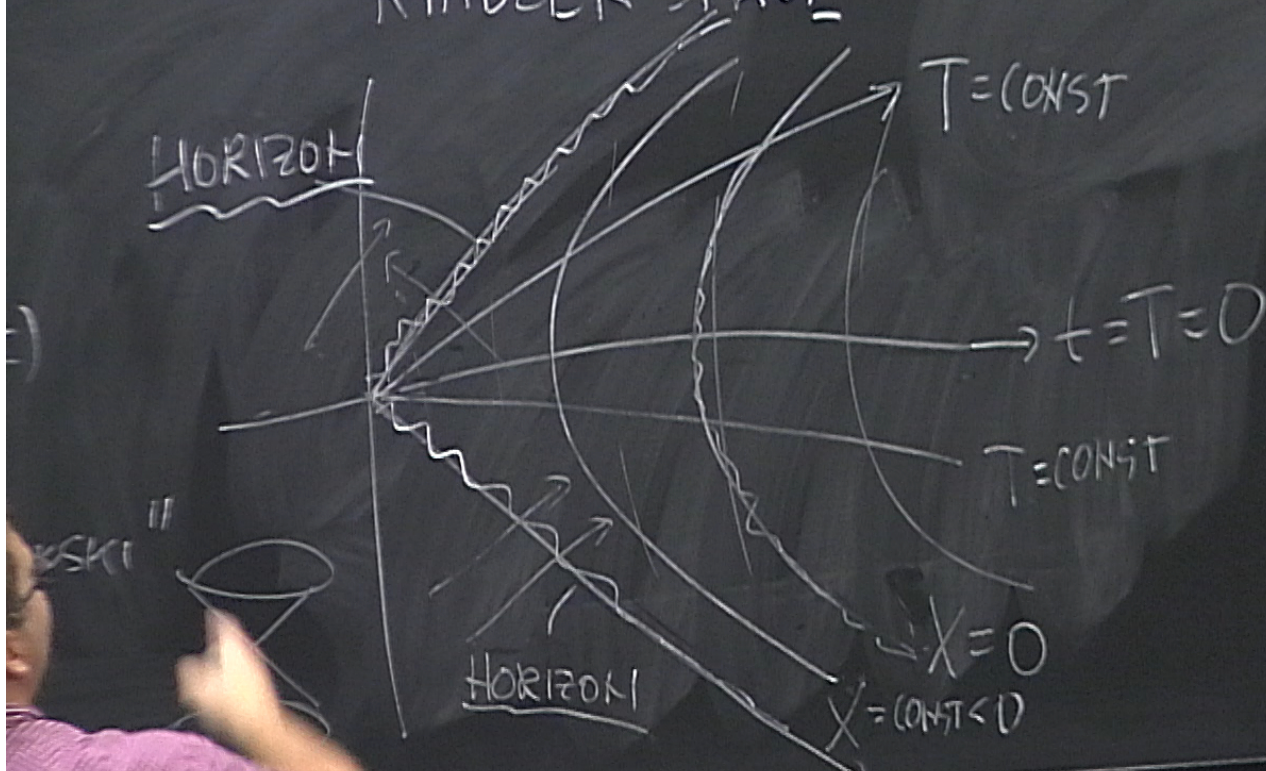
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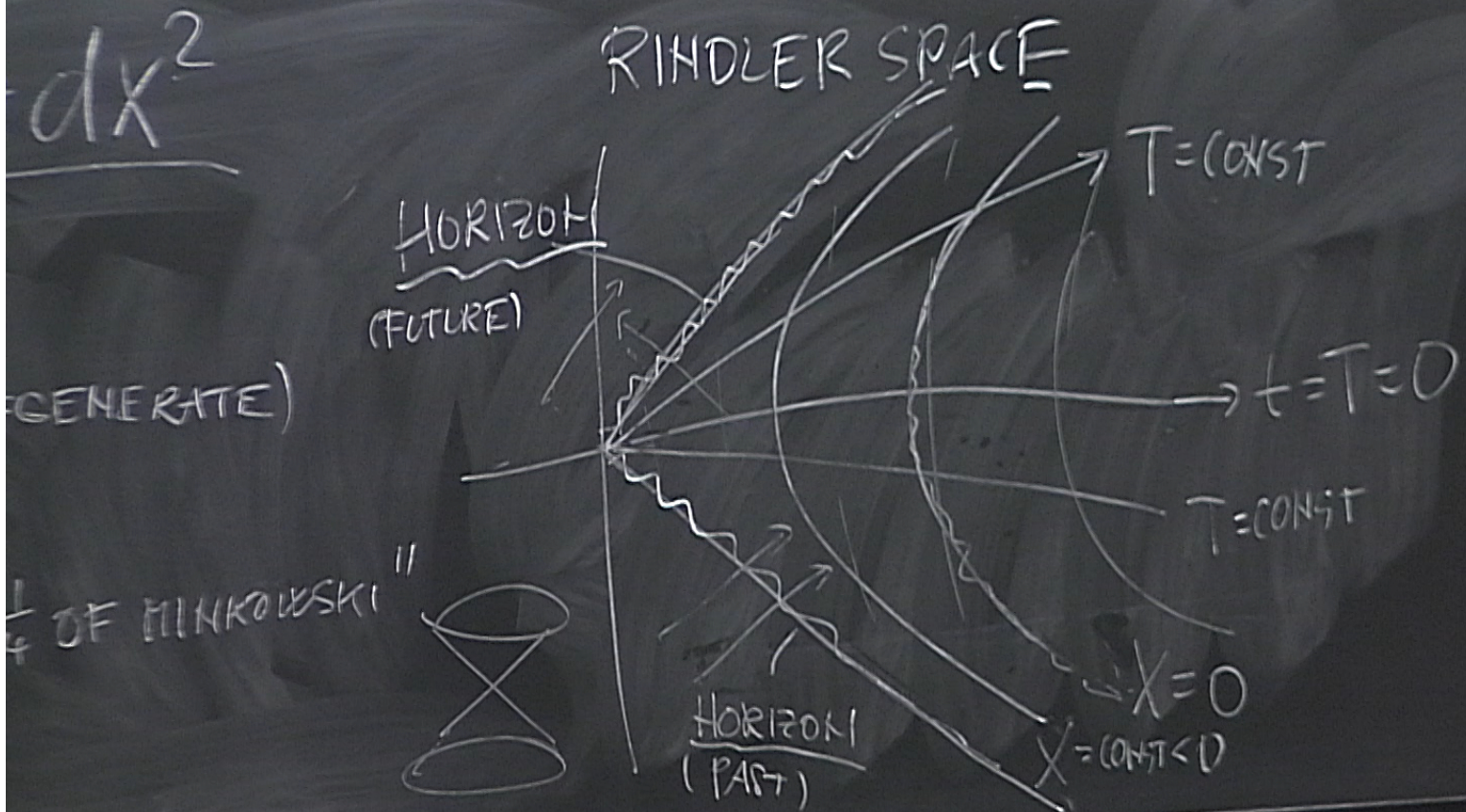
" $\frac{1}{4}$ OF MINKOWSKI"

RINDLER SPACE

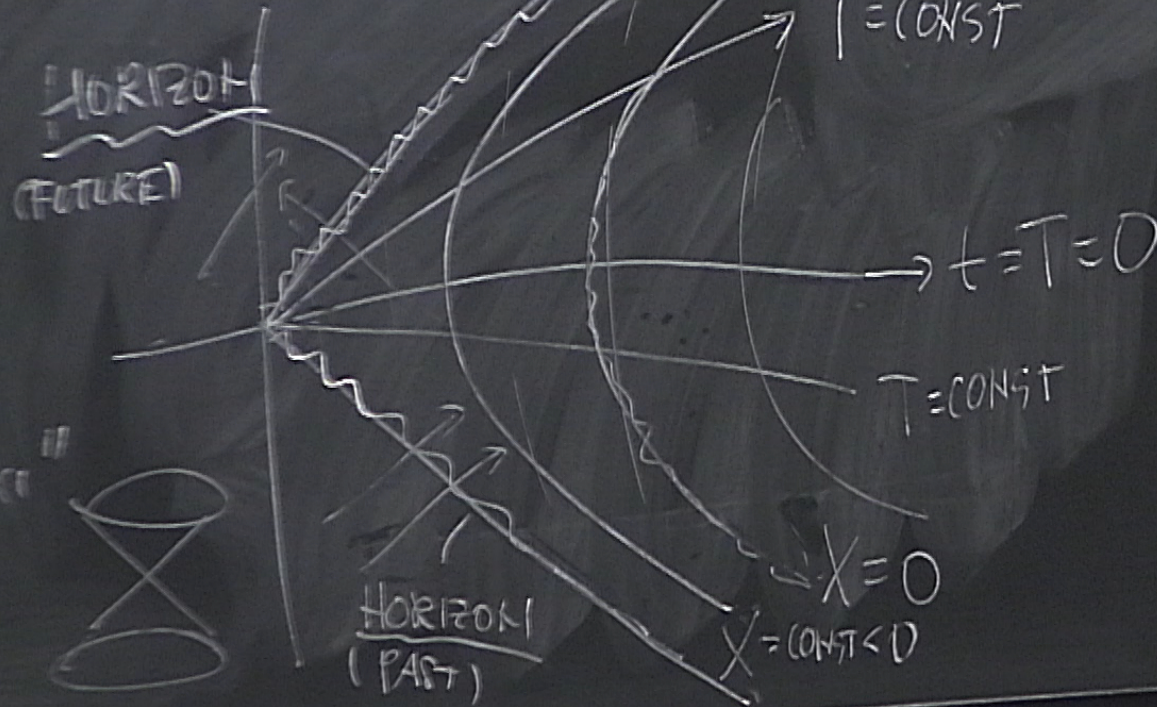


RINDLER SPACE





RINDLER SPACE

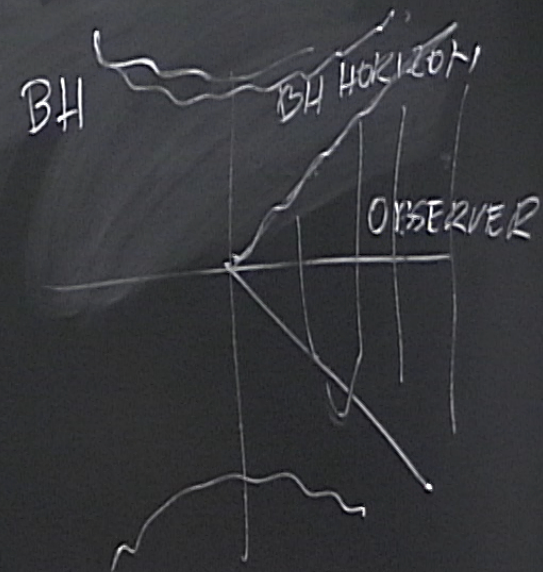
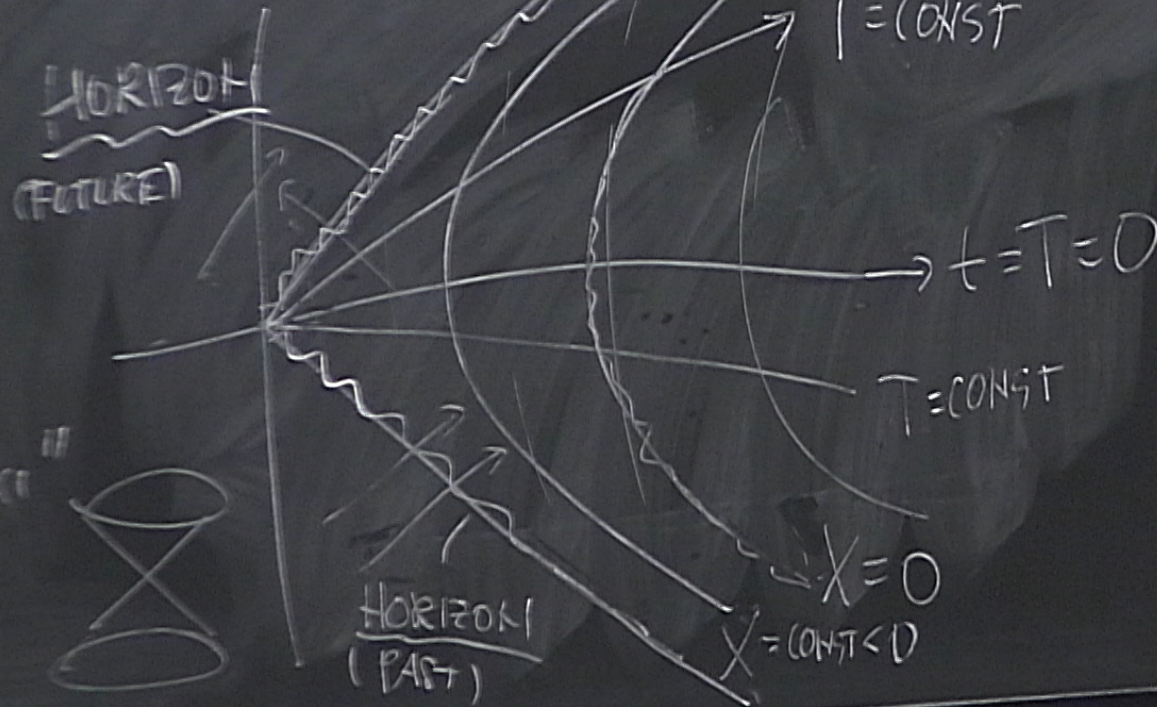


BH

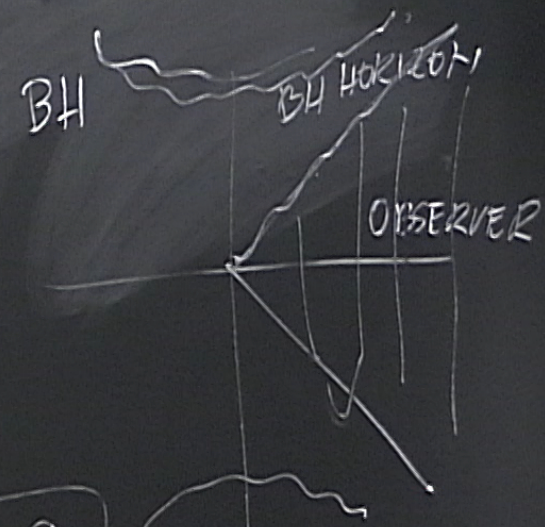
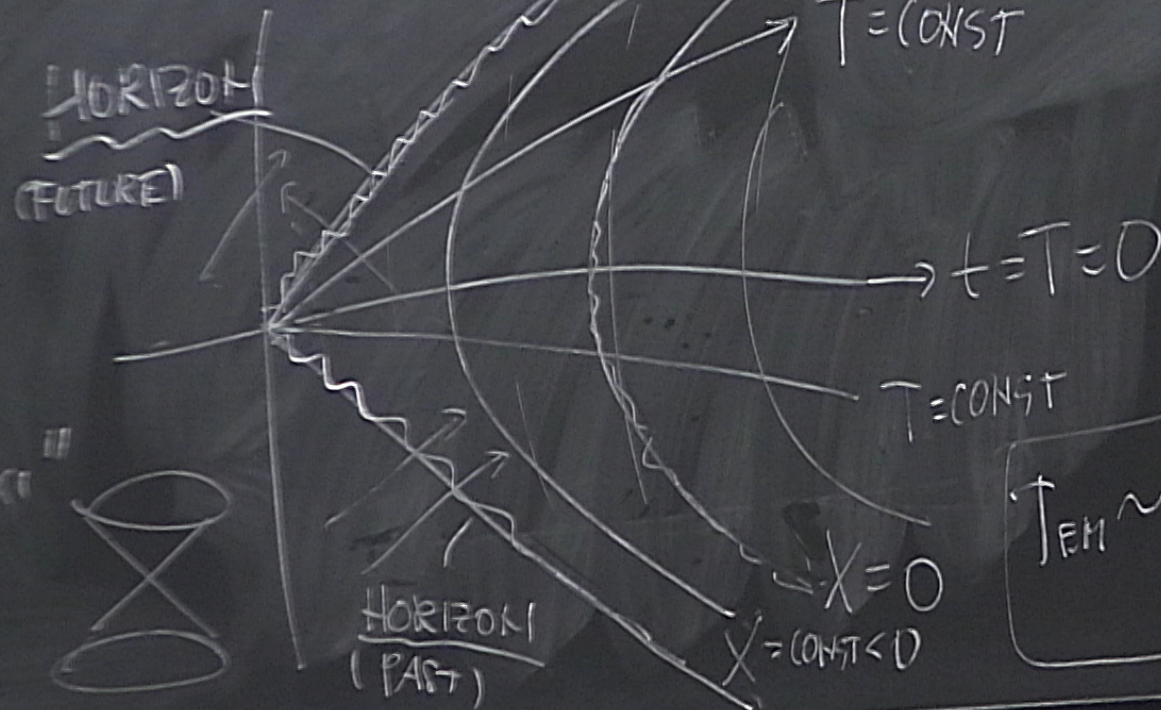
BH HORIZON

OBSERVER

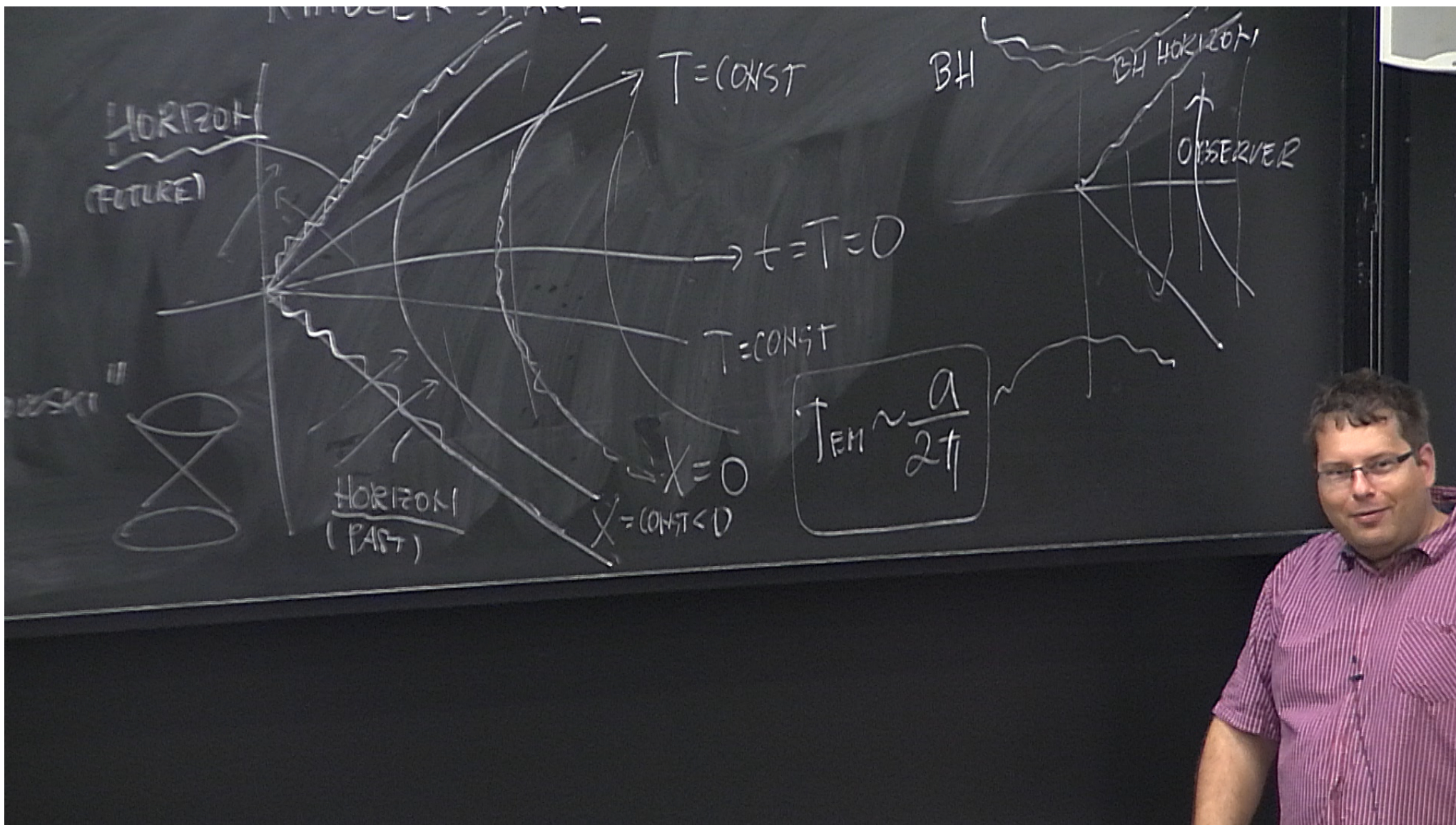
RINDLER SPACE

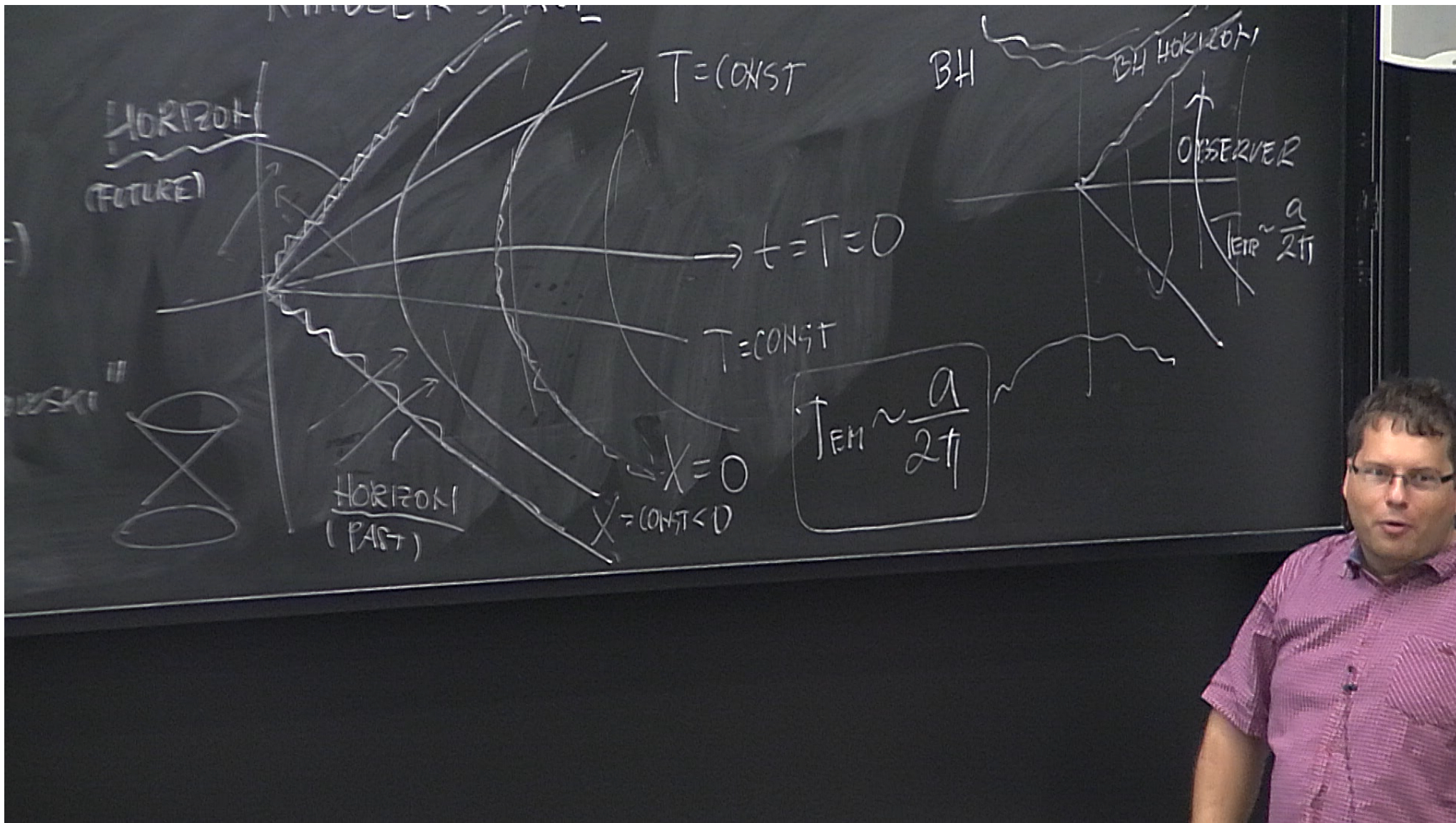


RINDLER SPACE



$$T_{EM} \sim \frac{a}{2\pi}$$





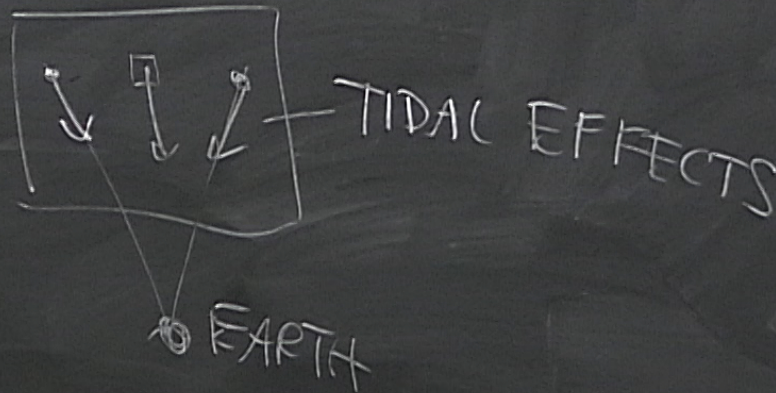
• GENERAL GRAVITATIONAL FIELD

CANNOT BE FULLY MIMICKED BY ACCELERATION
(EXCEPT AT A POINT)



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GET RID OF GRAVITY

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$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

GRAVITY = GEOMETRY

$$dt = \frac{\partial t}{\partial T} dT + \frac{\partial t}{\partial x} dx$$

HORIZON
(FUTURE)

2) NEWTONIAN LIMIT

• SLOW MOTION $\left| \frac{dx^i}{dt} \right| \ll \frac{dt}{dt} \quad (v \ll c)$

$$dt = \frac{\partial t}{\partial T} dT + \frac{\partial t}{\partial x} dx$$

HORIZONTAL
(FUTURE)

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$$\frac{d^2 x^\mu}{dt^2} + \underbrace{\sum_{\alpha, \beta} \Gamma_{\alpha\beta}^\mu \frac{dx^\alpha}{dt} \frac{dx^\beta}{dt}} = 0$$

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• STATIC FIELD $g_{\mu\nu,t} = 0$

$$\nabla^M_{+t} = \frac{1}{2} g^{16} (g_{6t,t} + g_{6t,t} - g_{tt,6})$$

• STATIC FIELD $g_{\mu\nu,t} = 0$

$$\nabla M_{tt} = \frac{1}{2} g^{ab} (g_{at,t} + g_{bt,t} - g_{tt,b})$$

• STATIC FIELD $g_{\mu\nu,t} = 0$

$$\nabla^M_{+t} = \frac{1}{2} g^{\mu 6} (g_{6t,t} + g_{6t,t} - g_{tt,6}) = -g^{\mu 6} g_{tt,6}$$

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• WEAK FIELD

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu},$$

$h_{\mu\nu}$... "SMALL"

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• WEAK FIELD

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu},$$

$$g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu} \quad \checkmark$$

$h_{\mu\nu}$ is "SMALL"

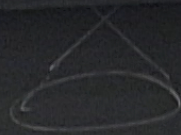
WEAK FIELD

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu} \quad \checkmark$$

$$\Gamma^{\mu}_{tt} = -\frac{1}{2} \eta^{\mu\sigma} h_{tt,\sigma}$$

$h_{\mu\nu}$: SPACE



HORIZON
(PAST)

$X=0$
 $X=(0 < T < 1)$

IER 21



$$\bullet \quad \underline{\mu = t} \quad \frac{d^2 t}{dr^2} + \frac{2}{t} \left(\frac{dt}{dr} \right)^2 = 0$$

$$\bullet \quad \underline{\mu = t} \quad \frac{d^2 t}{dr^2} + \underbrace{\frac{t}{r}}_{\neq 0} \left(\frac{dt}{dr} \right)^2 = 0$$

$$\boxed{\frac{dt}{dr} = c = \text{CONST.}}$$

- $\mu = t$ $\frac{d^2 t}{d\tau^2} + \underbrace{\int_t^t}_{\emptyset} \left(\frac{dt}{d\tau} \right)^2 = 0$

$$\boxed{\frac{dt}{d\tau} = c = \text{CONST.}}$$

- $\mu = i$ $\frac{d^2 x^i}{d\tau^2} + \int_i^i \left(\frac{dt}{d\tau} \right)^2 = 0$

- $\mu = t$ $\frac{d^2 t}{d\tau^2} + \underbrace{\left(\frac{dt}{d\tau} \right)^2}_{\neq 0} = 0$

$$\boxed{\frac{dt}{d\tau} = c = \text{CONST.}}$$

- $\mu = i$ $\frac{d^2 x^i}{d\tau^2} + \underbrace{\left(\frac{dt}{d\tau} \right)^2}_{c^2} = 0$
 $\frac{d^2 x^i}{d\tau^2} = -c^2$

- $\mu = t$

$$\frac{d^2 t}{d\tau^2} + \underbrace{\Gamma^t_{tt}}_{\neq 0} \left(\frac{dt}{d\tau} \right)^2 = 0$$

$$\frac{d^2 x^i}{dt^2} = -\Gamma^i_{tt}$$

$$\boxed{\frac{dt}{d\tau} = c = \text{CONST.}}$$

- $\mu = r$

$$\underbrace{\frac{d^2 x^i}{d\tau^2}}_{\frac{d^2 x^i}{dt^2} c^2} + \underbrace{\Gamma^r_{tt}}_{c^2} \left(\frac{dt}{d\tau} \right)^2 = 0$$

$$\frac{dx^i}{d\tau^2} + \underbrace{\Gamma^i_{tt}}_0 \left(\frac{dt}{d\tau} \right)^2 = 0$$

$$\boxed{\frac{dt}{d\tau} = c = \text{CONST}}$$

$$\frac{dx^i}{d\tau^2} + \underbrace{\Gamma^i_{tt}}_{c^2} \left(\frac{dt}{d\tau} \right)^2 = 0$$

$$\frac{d^2 x^i}{dt^2} = - \underbrace{\Gamma^i_{tt}}_{= \frac{1}{2} h_{tt,i}^{'}}$$

$$\frac{d^2 t}{d\tau^2} + \underbrace{\gamma_{tt} \left(\frac{dt}{d\tau} \right)^2}_{\neq 0} = 0$$

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NEWTON: $\frac{d^2 x^i}{d\tau^2} = -\nabla^i \phi$

$$\frac{d^2 t}{d\tau^2} + \underbrace{\gamma_{tt}}_{\neq 0} \left(\frac{dt}{d\tau} \right)^2 = 0$$

$$\frac{dt}{d\tau} = c = \text{CONST.}$$

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NEWTON. $\frac{d^2 x^i}{d\tau^2} = -\nabla^i \phi$

$$\phi = -\frac{1}{2} h_{tt}$$

g

$$h_{tt} = \frac{1}{2} h_{tt, \hat{t}}$$

$$g_{tt} = - \left(1 + \frac{\phi}{c^2} \right)$$

$$\frac{1}{2} \nabla^i \phi$$

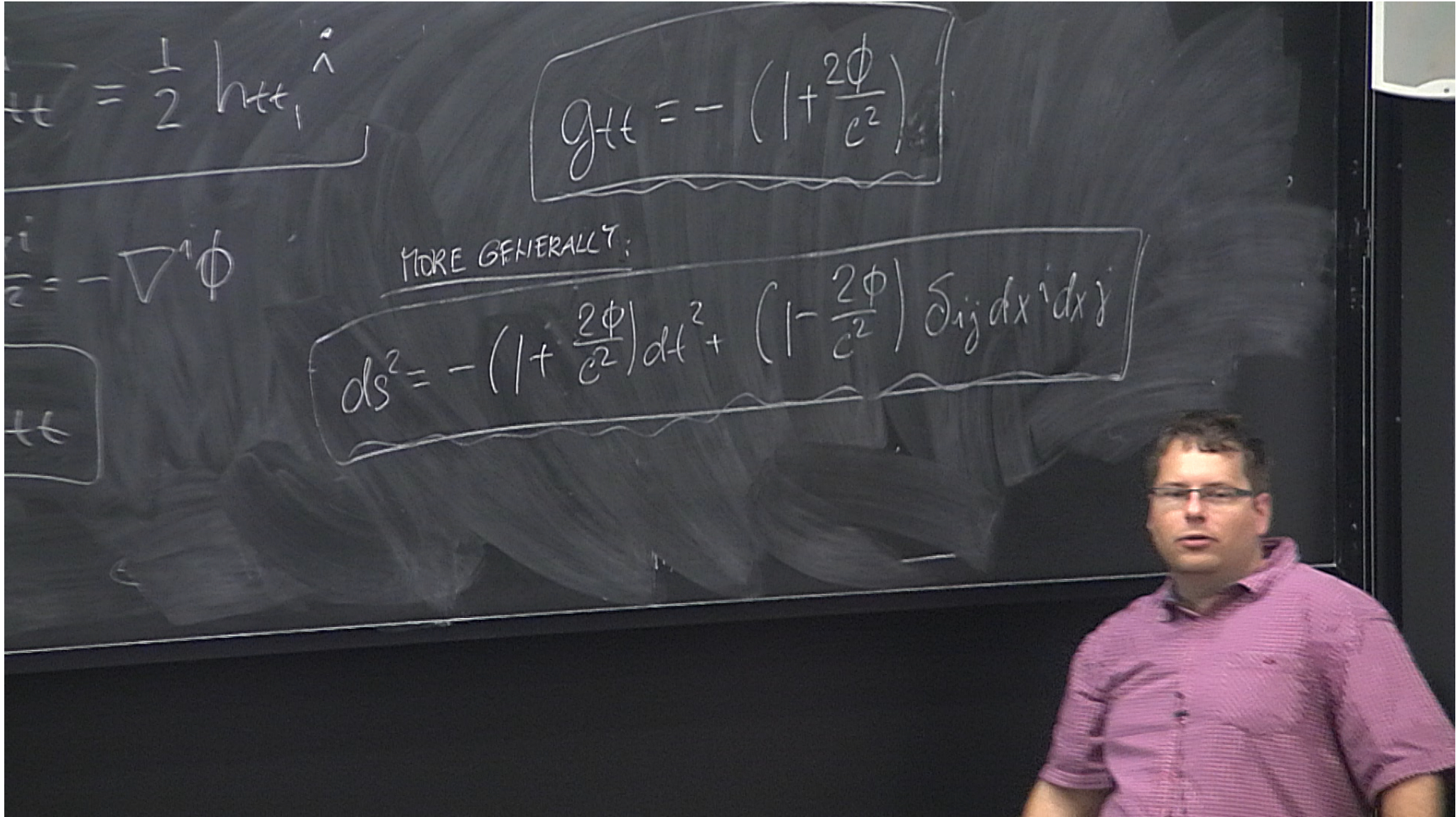
$$h_{tt}$$

$$h_{tt} = \frac{1}{2} h_{tt, \hat{r}}$$

$$g_{tt} = - \left(1 + \frac{2\phi}{c^2} \right)$$

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$$h_{tt}$$



$$h_{tt} = \frac{1}{2} h_{tt, \hat{r}}$$

$$g_{tt} = - \left(1 + \frac{2\phi}{c^2} \right)$$

$$\vec{e}_i = - \nabla^i \phi$$

MORE GENERALLY:

$$ds^2 = - \left(1 + \frac{2\phi}{c^2} \right) dt^2 + \left(1 - \frac{2\phi}{c^2} \right) \delta_{ij} dx^i dx^j$$

EARTH $\frac{\phi}{c^2} \sim 10^{-9}$

$$g_{tt} = -(1 + \frac{2\phi}{c^2})$$

MORE GENERALLY:

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EARTH $\frac{\phi}{c^2} \sim 10^{-9}$ SUN: 10^{-6}



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