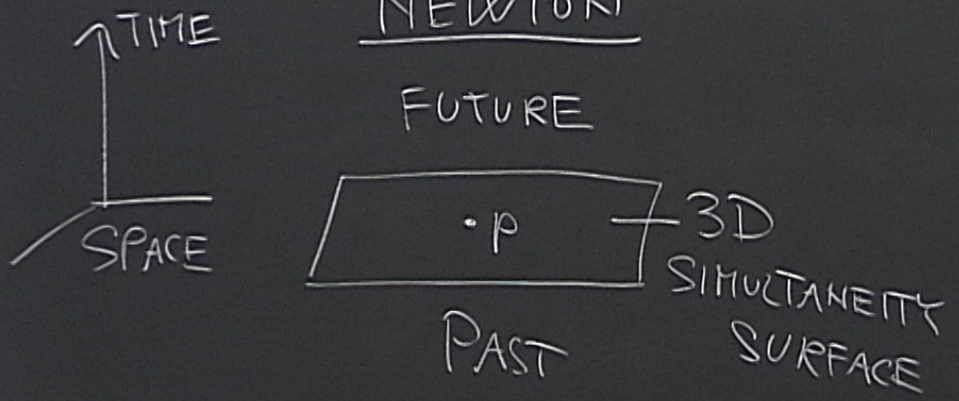
Title: PSI 2018/2019 - Relativity - Lecture 2

Date: Sep 05, 2018 09:00 AM

URL: http://pirsa.org/18090003

Abstract:

YESTERDAY. CAUSAL STRUCTURE OF SPACETIMES



NEWTON

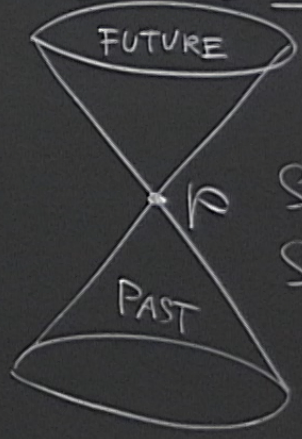
FUTURE

3D SIMULTANEITY SURFACE

PAST

SPECIAL RELAT

LIGHTCONE - GLOBAL



SPACELIKE SEPARATED

RELAT

ONE - GLOBAL

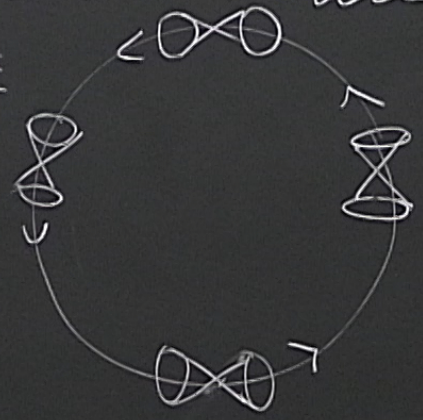
RE

SPACELIKE SEPARATED

GENERAL RELATIVITY

LIGHTCONES ARE LOCAL! (CAN TWIST AROUND)

E.G.:

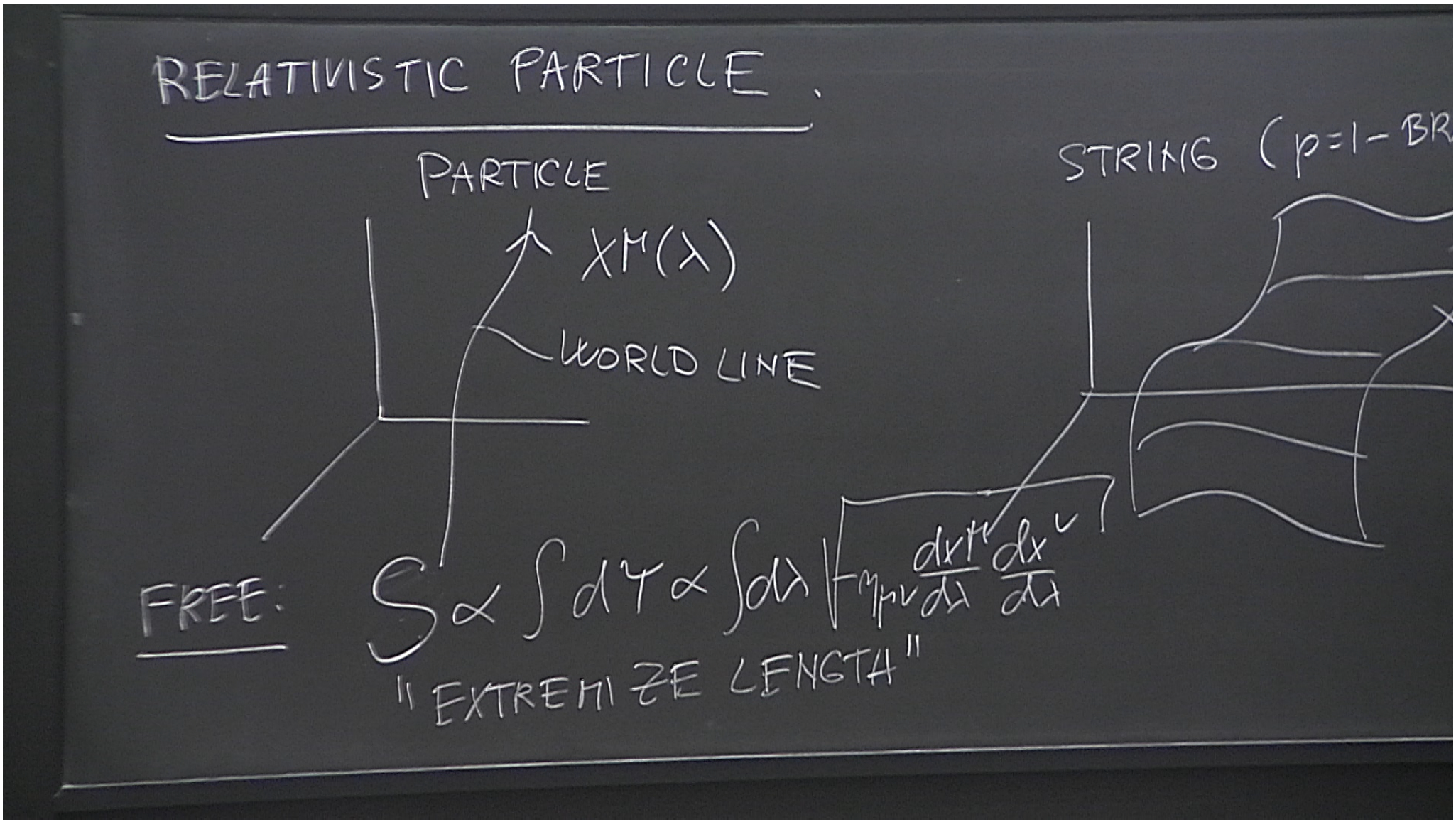


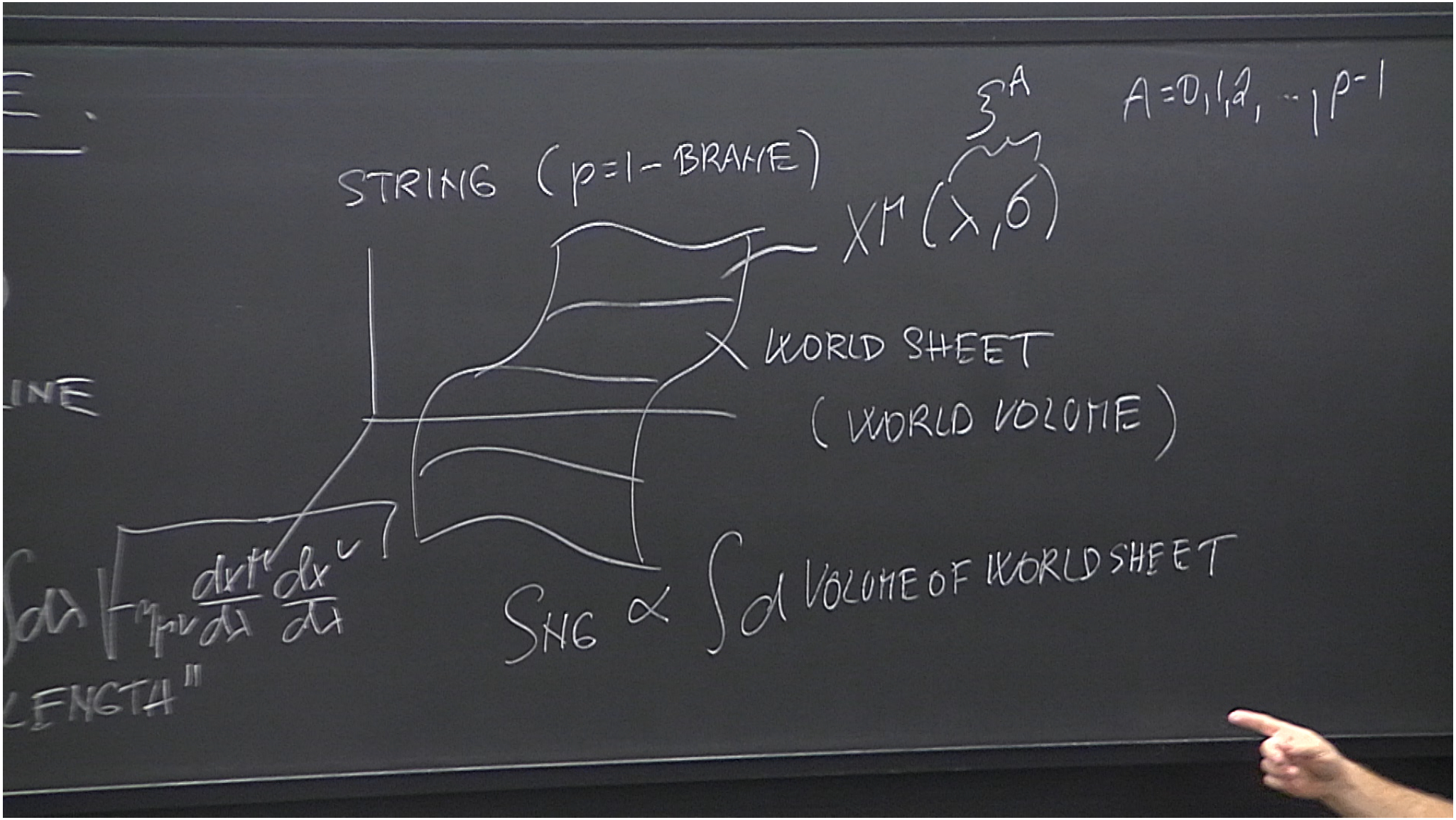
CLOSED TIMELIKE CURVES

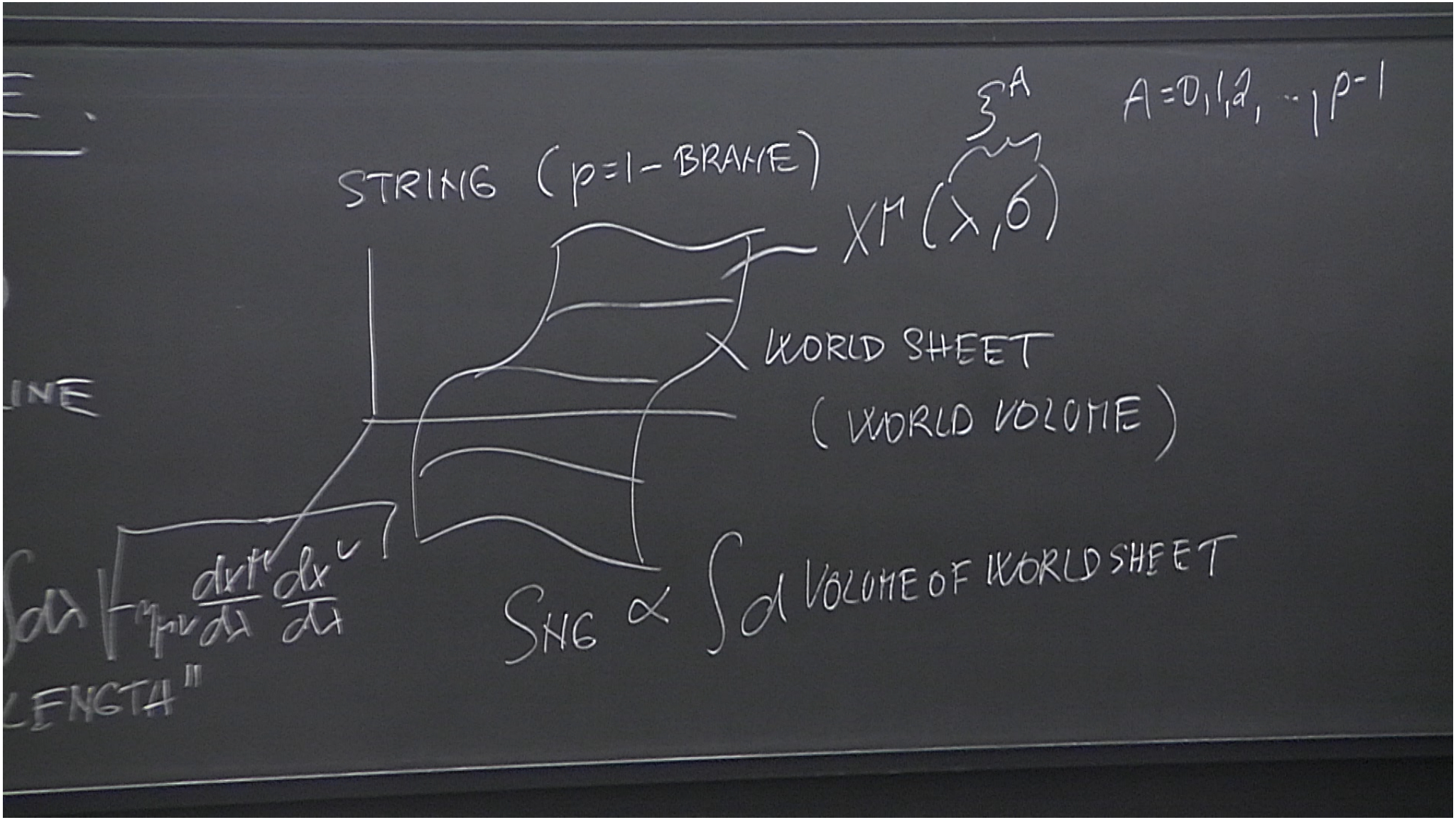
ARE POSSIBLE (KERR, TAUB-NUT)

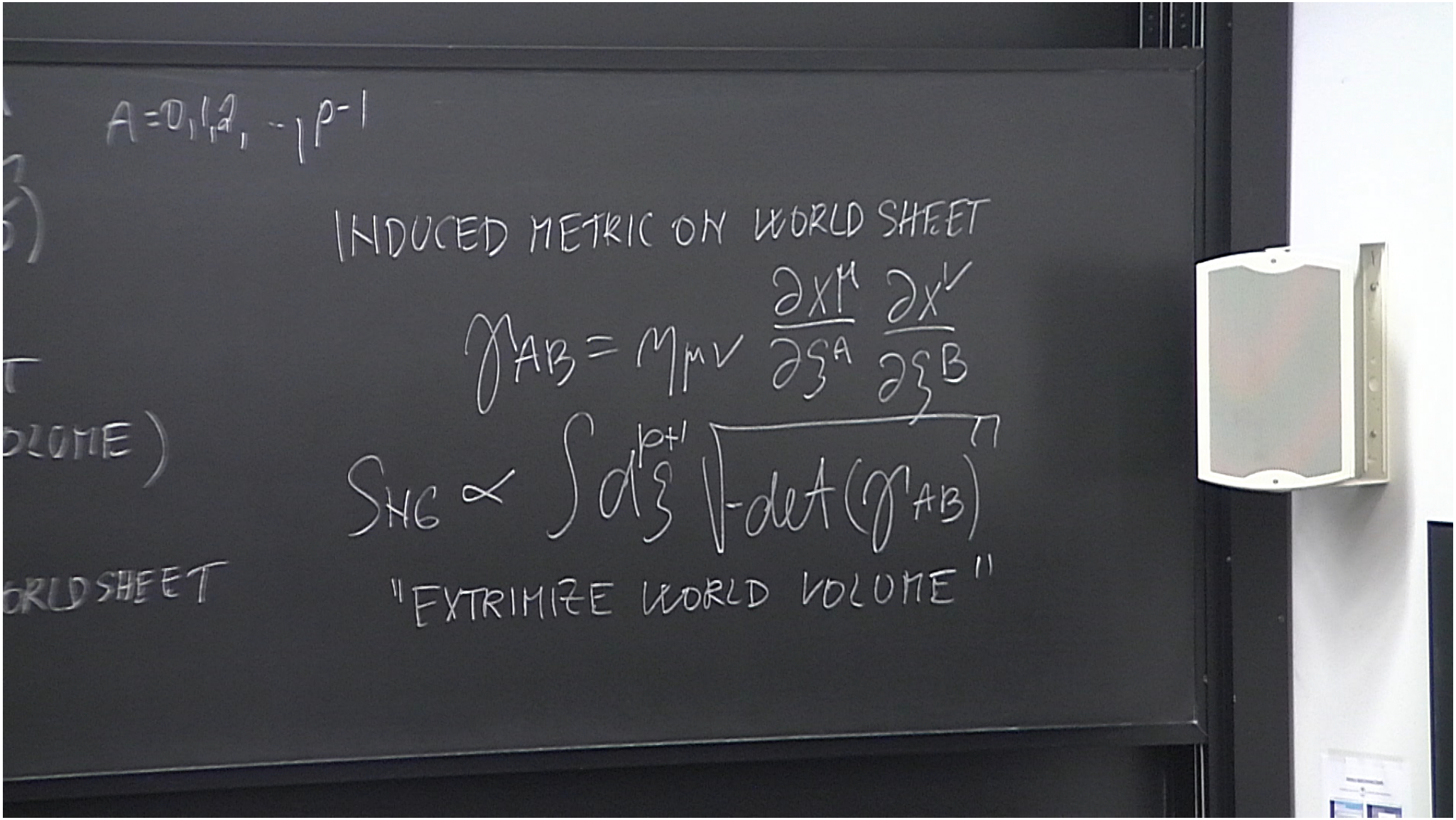
BUT ARE CONSIDERED PATHOLOGICAL

RELATIVISTIC PARTICLE.




STRING (p=1-BRANE)
Σ^A
$A = 0, 1, 2, \ldots, p-1$
$X^\mu(\lambda, \sigma)$
WORLD SHEET
(WORLD VOLUME)
$S_{NG} \propto \int d$ VOLUME OF WORLD SHEET
$\int d\lambda \sqrt{\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}}$
LENGTH"


STRING (p=1-BRANE)
ξ^A
$A=0,1,2,\ldots,p-1$
$X^\mu(\lambda,\sigma)$
WORLD SHEET
(WORLD VOLUME)
$\int d\lambda \sqrt{-\eta_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}}$
LENGTH"
$S_{NG} \propto \int d$ VOLUME OF WORLD SHEET


A = 0, 1, 2, ..., p-1
INDUCED METRIC ON WORLD SHEET
$\gamma_{AB} = \eta_{\mu\nu} \frac{\partial X^\mu}{\partial \xi^A} \frac{\partial X^\nu}{\partial \xi^B}$
$S_{NG} \propto \int d^{p+1}\xi \sqrt{-\det(\gamma_{AB})}$
"EXTRIMIZE WORLD VOLUME"

"EXTREMIZE LENGTH"

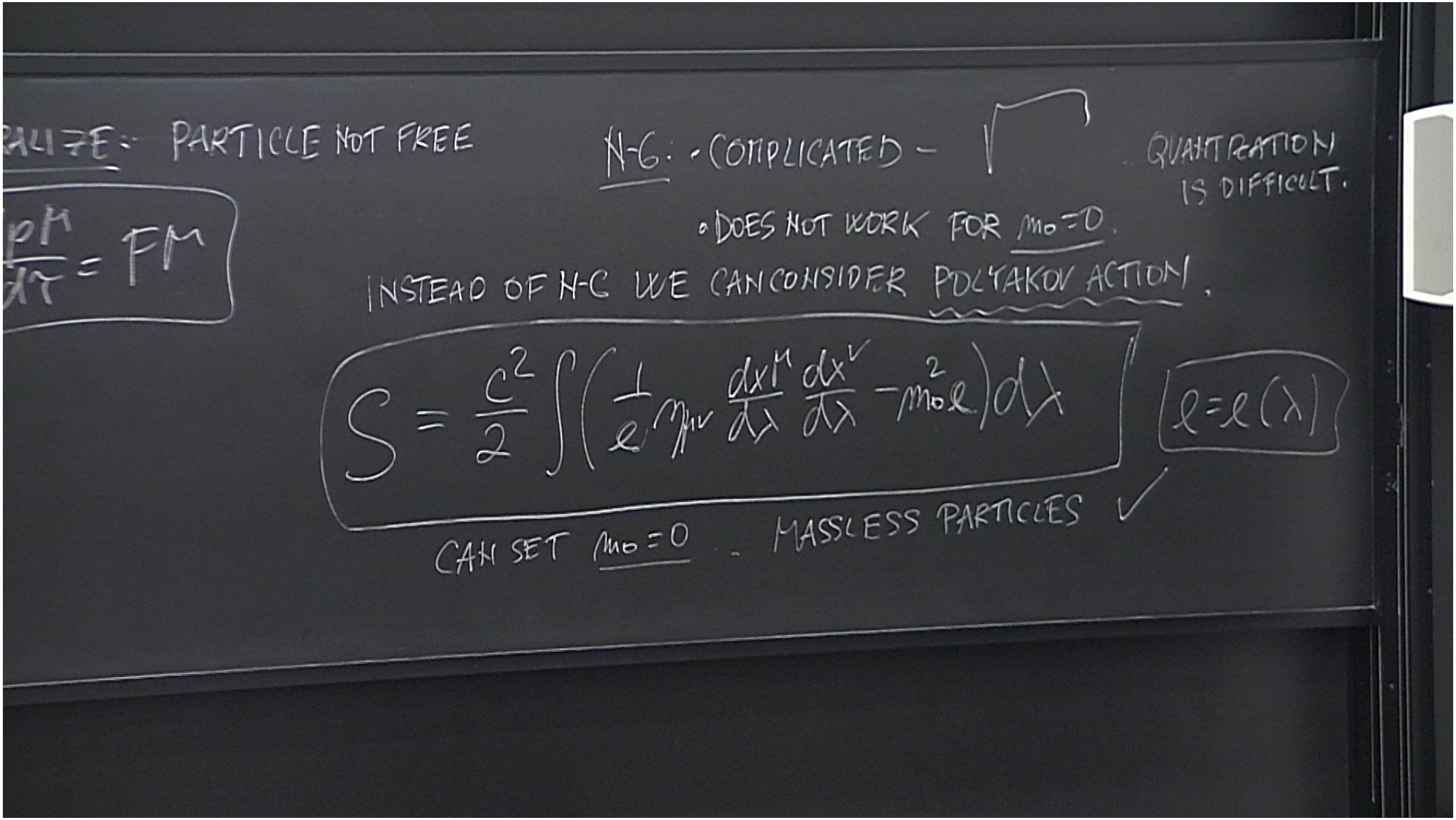
FOR PARTICLE:

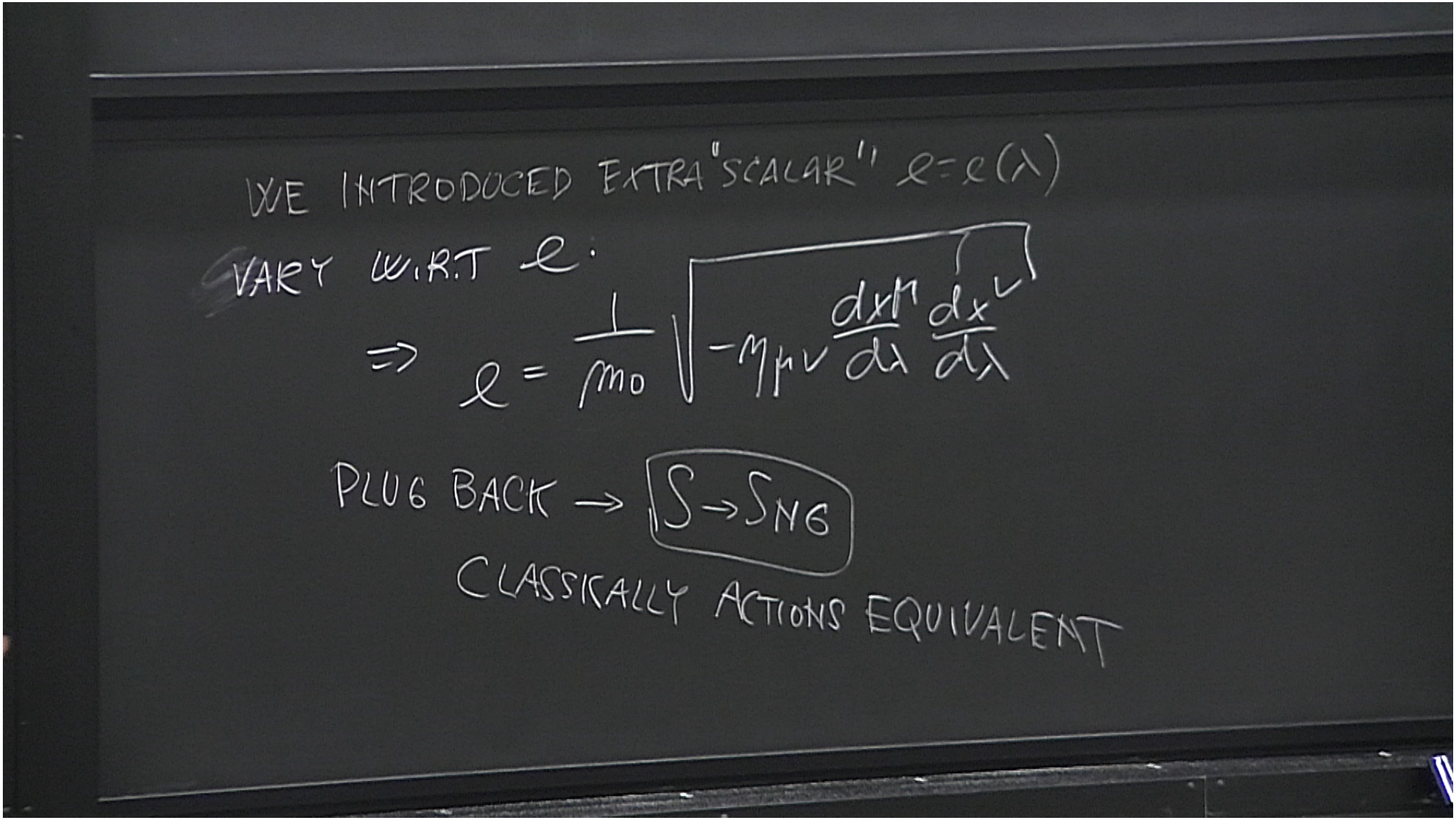
$$\frac{du^\mu}{d\tau} = 0 = \frac{dp^\mu}{d\tau}$$

$$\frac{d}{d\tau} = \frac{\partial x^\nu}{\partial \tau}\frac{\partial}{\partial x^\nu} = u^\nu \partial_\nu$$

(u^ν)

$$u^\nu \partial_\nu u^\mu = 0$$


RALIZE: PARTICLE NOT FREE
N-G: • COMPLICATED –
QUANTIZATION IS DIFFICULT.
$\frac{dp^\mu}{d\tau} = F^\mu$
• DOES NOT WORK FOR $m_0 = 0$.
INSTEAD OF N-G WE CAN CONSIDER POLYAKOV ACTION.
$S = \frac{c^2}{2}\int\left(\frac{1}{e}\eta_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda} - m_0^2 e\right)d\lambda$
$e = e(\lambda)$
CAN SET $m_0 = 0$. MASSLESS PARTICLES


WE INTRODUCED EXTRA "SCALAR" e=e(λ)
VARY W.R.T e:
⇒ e = 1/m0 √(−ημν dxμ/dλ dxν/dλ)
PLUG BACK → S→SNG
CLASSICALLY ACTIONS EQUIVALENT

FOR STRING.

$$S_{POL.} \propto \int d^2\xi \sqrt{-h}\, h^{AB}\, g_{AB}$$

MASSLESS STRING

h^{AB} ↑ INTERNAL METRIC

g_{AB} ↑ INDUCED METRIC

... 2D CFT

a) MAXWELL THEORY

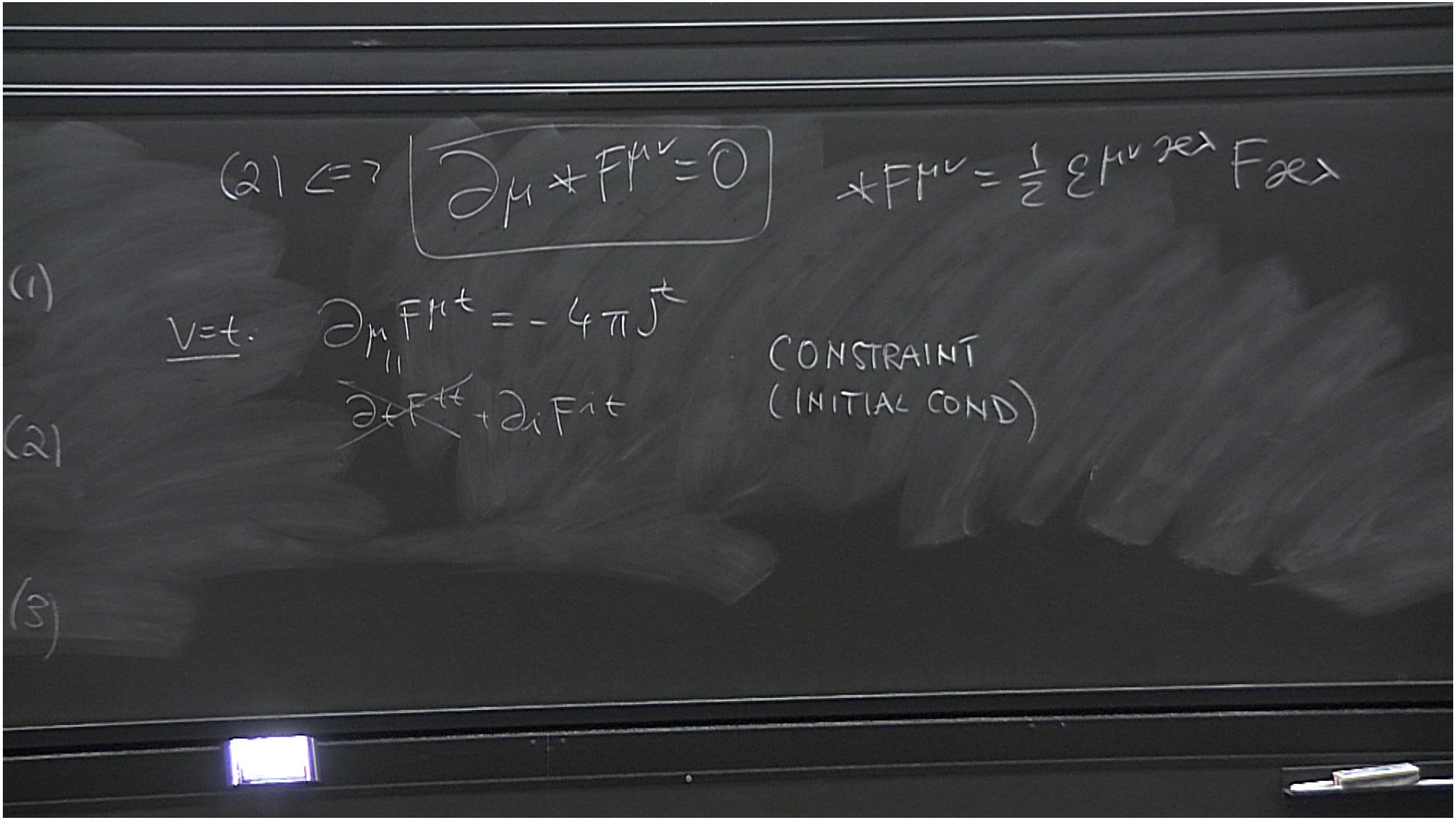
FIELD EQS.:

$$\partial_\mu F^{\mu\nu} = -4\pi J^\nu \qquad (1)$$

$$F_{[\mu\nu,\lambda]} = 0 \qquad (2)$$

EQS OF MOTION:

$$\frac{dp^\mu}{d\tau} = e F^{\mu\nu} u_\nu \qquad (3)$$

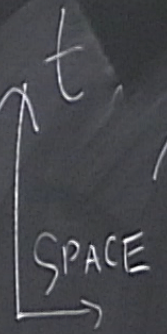

(2) ⟺ $\partial_\mu {}^*F^{\mu\nu} = 0$
${}^*F^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$
(1)
$\nu = t$: $\partial_\mu F^{\mu t} = -4\pi J^t$
$\partial_t F^{tt} + \partial_i F^{it}$
CONSTRAINT
(INITIAL COND)
(2)
(3)

VARY W.R.T e:

$\partial_\nu(1)$ $\quad \partial_\nu \partial_\mu F^{\mu\nu} = -4\pi \partial_\nu J^\nu$

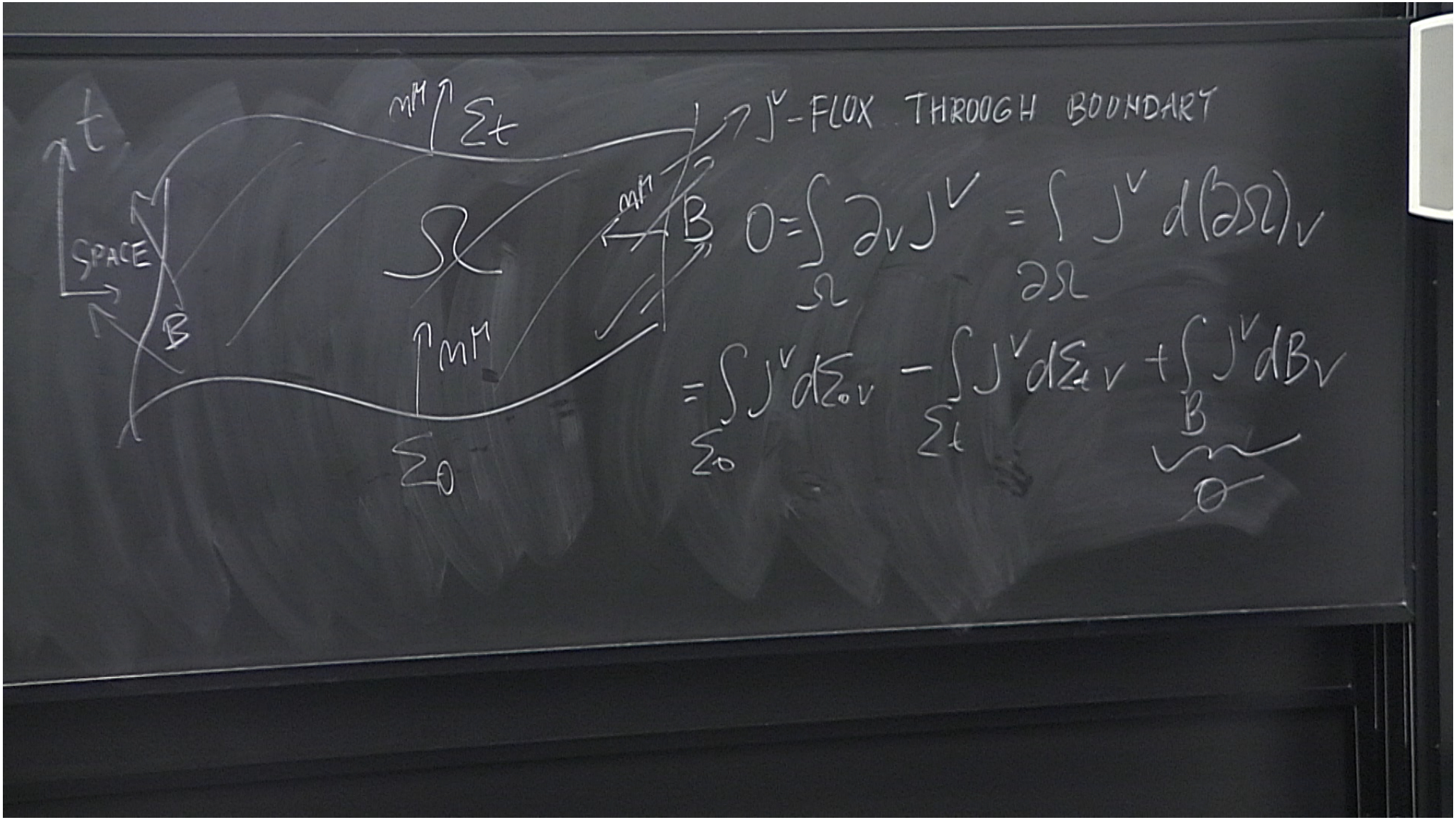
$\partial_\mu F^{\mu\nu} = -4\pi J^\nu$

$\boxed{0 = \partial_\nu J^\nu}$ .. CONTINUITY EQ.



$$Q = \int_{\Sigma} J^\nu \, d\Sigma_\nu$$

Σ SPATIAL SURFACE $\quad d\Sigma\, n_\nu$


t
SPACE
B
n^M
Σ_t
Ω
n^M
B
n^M
Σ_0
J^V-FLUX THROUGH BOUNDARY
$0 = \int_\Omega \partial_V J^V = \int_{\partial\Omega} J^V d(\partial\Omega)_V$
$= \int_{\Sigma_0} J^V d\Sigma_{0V} - \int_{\Sigma_t} J^V d\Sigma_{tV} + \underbrace{\int_B J^V dB_V}_{0}$

WE INTRODUCED EXTRA "SCALAR" $\varrho = \varrho(\lambda)$ FOR S

(2) AUTOMATICALLY SATISFIED $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

A^μ... 4-POTENTIAL

$\Rightarrow$ (1) SATISFIED

$$\partial_\mu A^\mu = 0$$

$$\Box A^\mu = -4\pi J^\mu$$

STATIC LIMIT: $A^\mu = (\phi, \vec{A}) = (\phi, 0)$, $\partial_t \phi = 0$

- LORENTZ GAUGE IS SATISFIED

$$\Delta\phi = -4\pi\rho \qquad J^\mu = (\rho, 0)$$

$$\frac{dp}{dt} = eE = -e\nabla\phi$$

ELECTROSTATICS.

2) FIRST LOOK AT GR

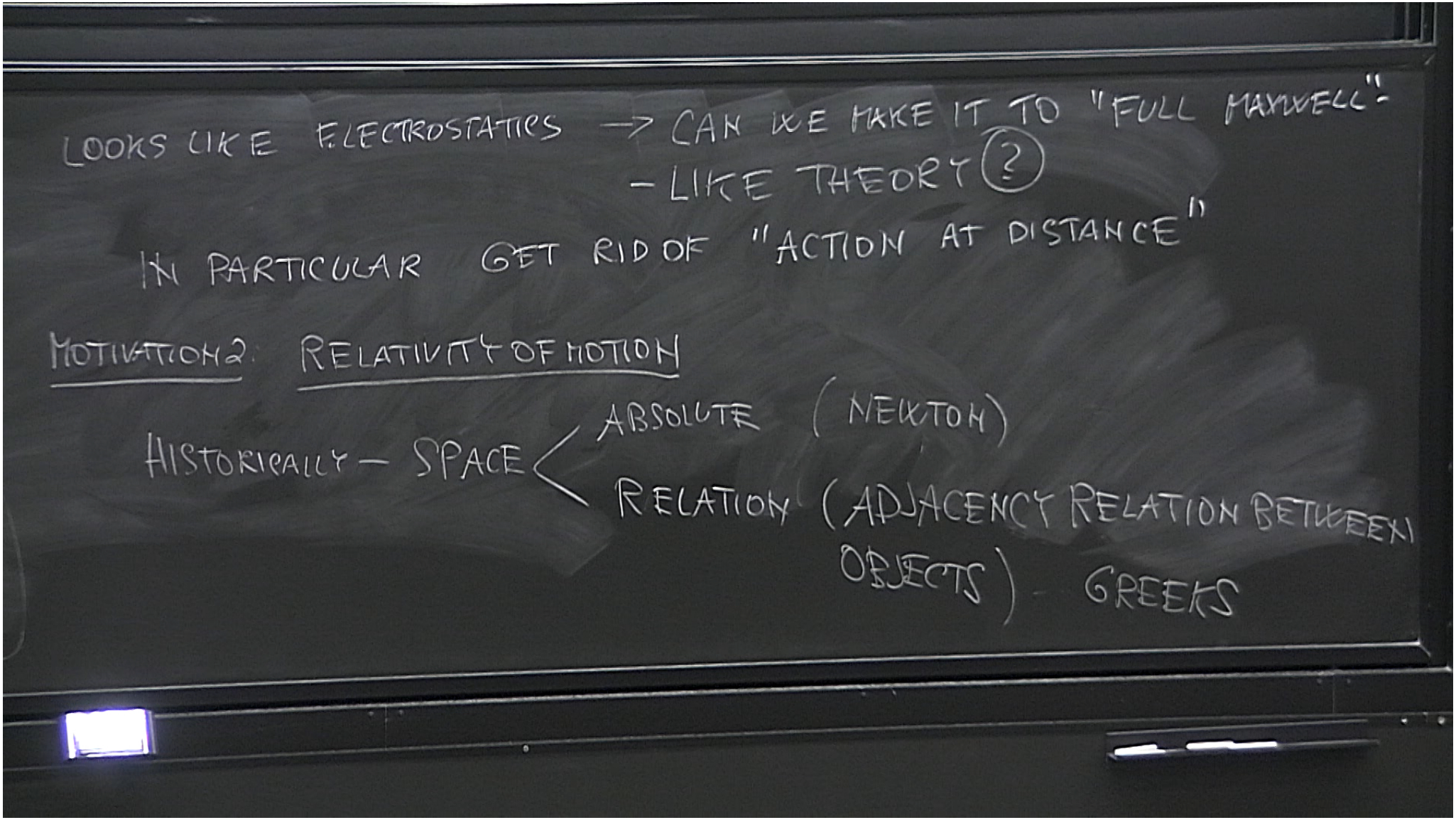
a) PATH TO THE THEORY

MOTIVATION 1.

NEWTON'S GRAVITY: ... GRAV. POTENTIAL ... ϕ_N

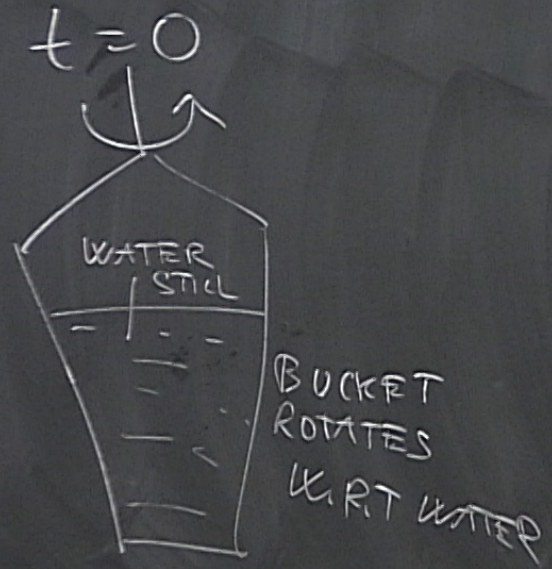
$$\Delta\phi_N = 4\pi G \rho_m \quad , \quad \frac{dp}{dt} = -m\nabla\phi$$

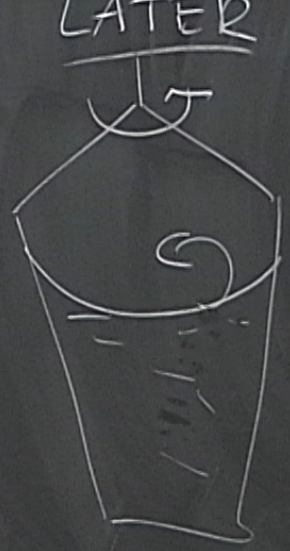
NEWTON'S GRAV. ↑


LOOKS LIKE ELECTROSTATICS → CAN WE MAKE IT TO "FULL MAXWELL"-
-LIKE THEORY ?
IN PARTICULAR GET RID OF "ACTION AT DISTANCE"
MOTIVATION 2: RELATIVITY OF MOTION
HISTORICALLY – SPACE
ABSOLUTE (NEWTON)
RELATION (ADJACENCY RELATION BETWEEN OBJECTS) - GREEKS

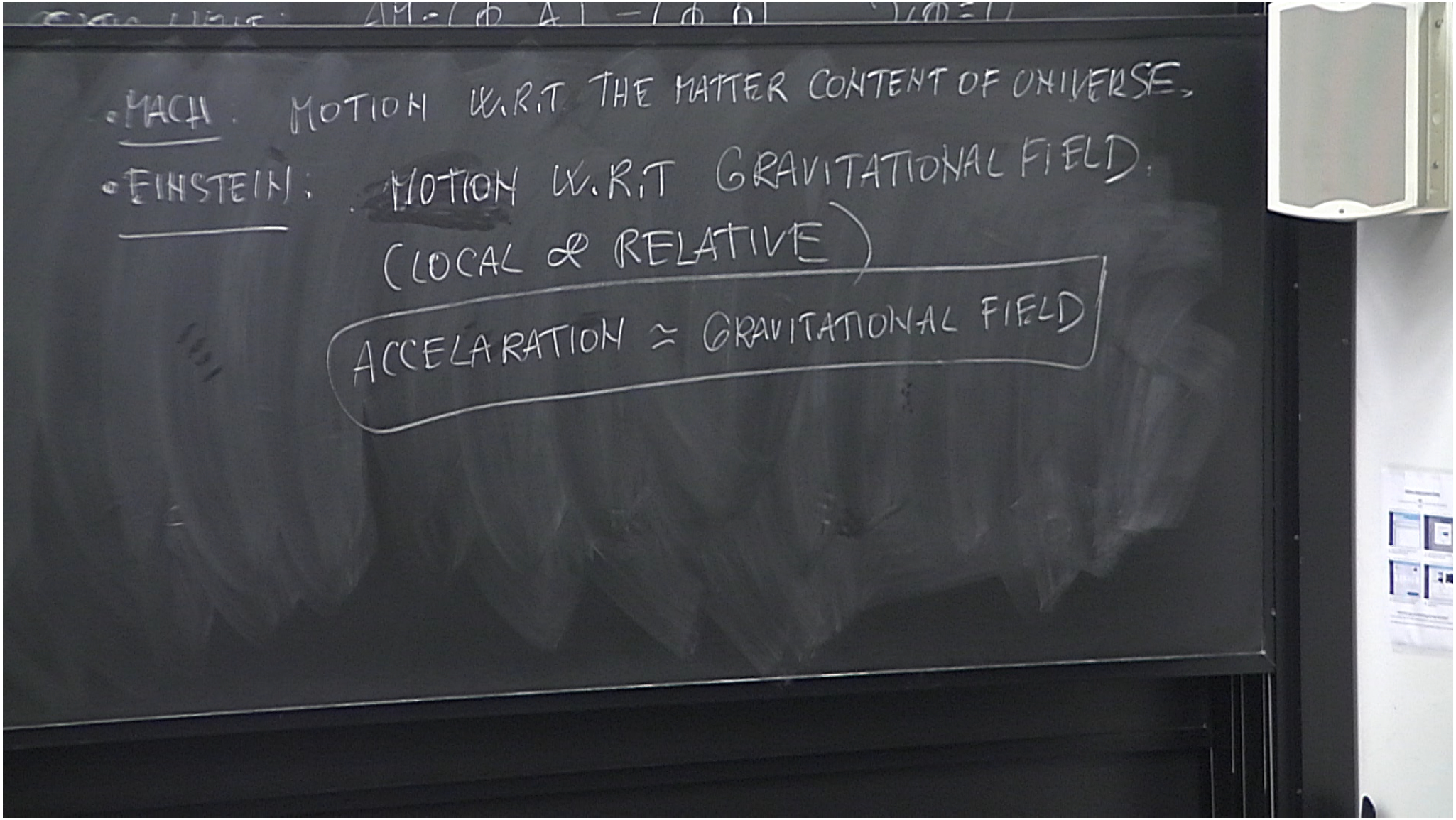
$\Box A = -4\pi J^{M}$

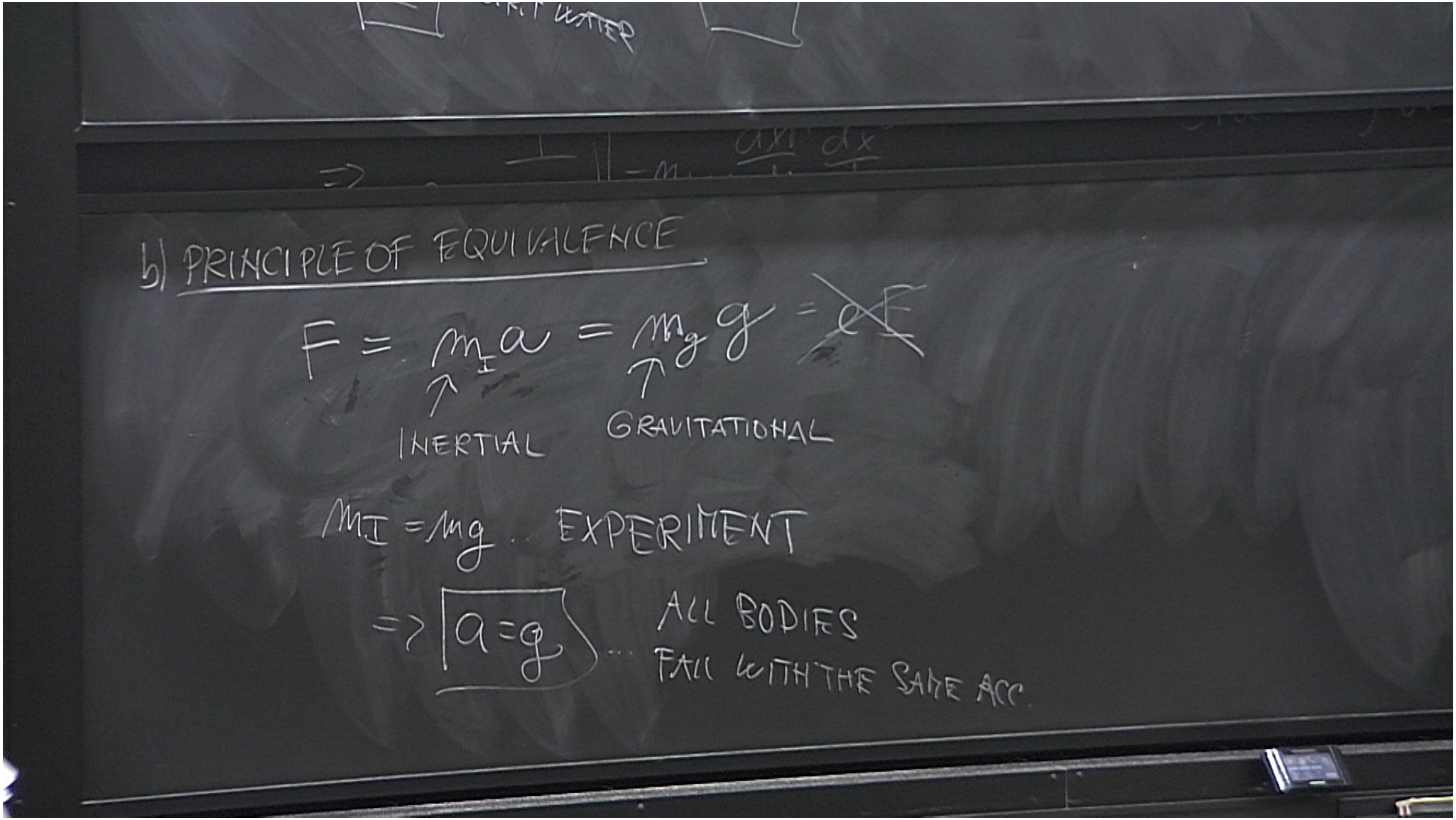
NEWTON INTRODUCED ABSOLUTE SPACE BASED ON THE FOLLOWING "EXPERIMENT" ... NEWTON'S BUCKET



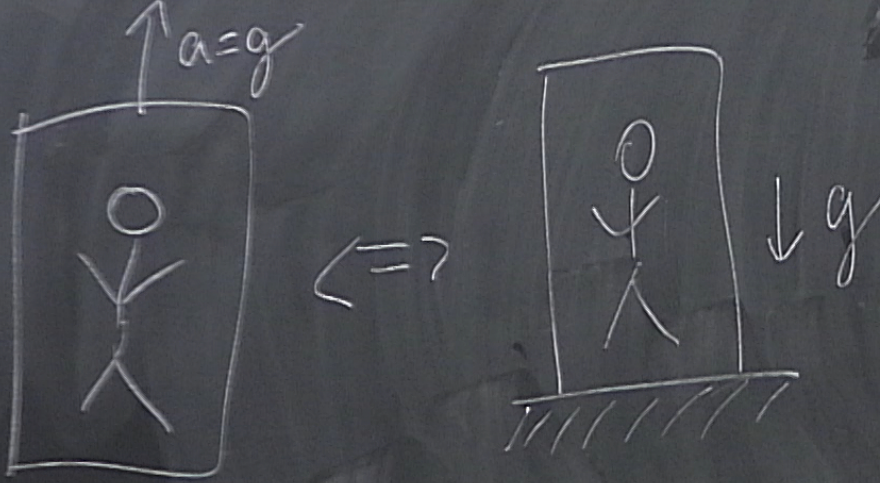


=> EXISTENCE OF ABSOLUTE SPACE


MACH. MOTION W.R.T THE MATTER CONTENT OF UNIVERSE.
EINSTEIN: MOTION W.R.T GRAVITATIONAL FIELD.
(LOCAL & RELATIVE)
ACCELARATION ≃ GRAVITATIONAL FIELD


b) PRINCIPLE OF EQUIVALENCE
$F = m_I a = m_g g = \cancel{eF}$
INERTIAL
GRAVITATIONAL
$m_I = m_g$ .. EXPERIMENT
$\Rightarrow \boxed{a = g}$ ... ALL BODIES FALL WITH THE SAME ACC.

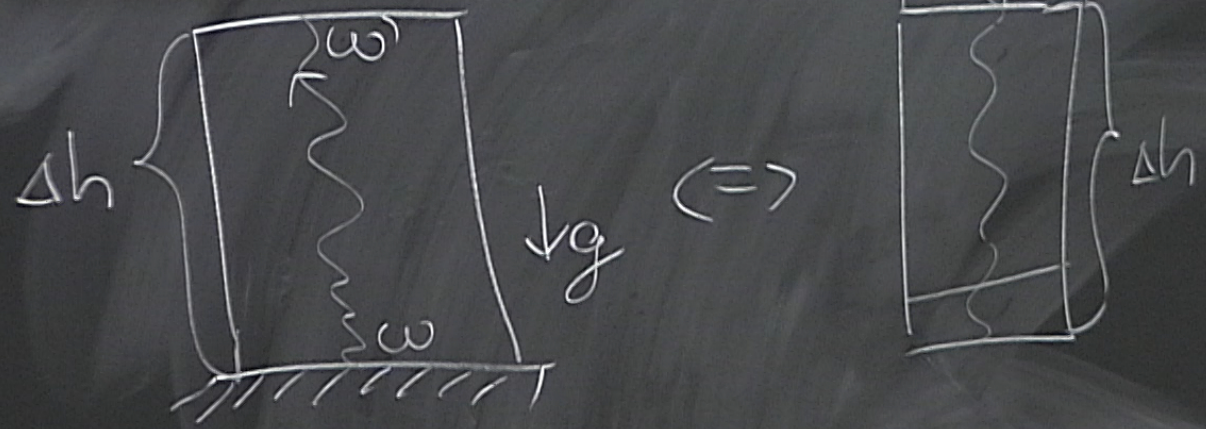
EINSTEIN'S ELEVATOR



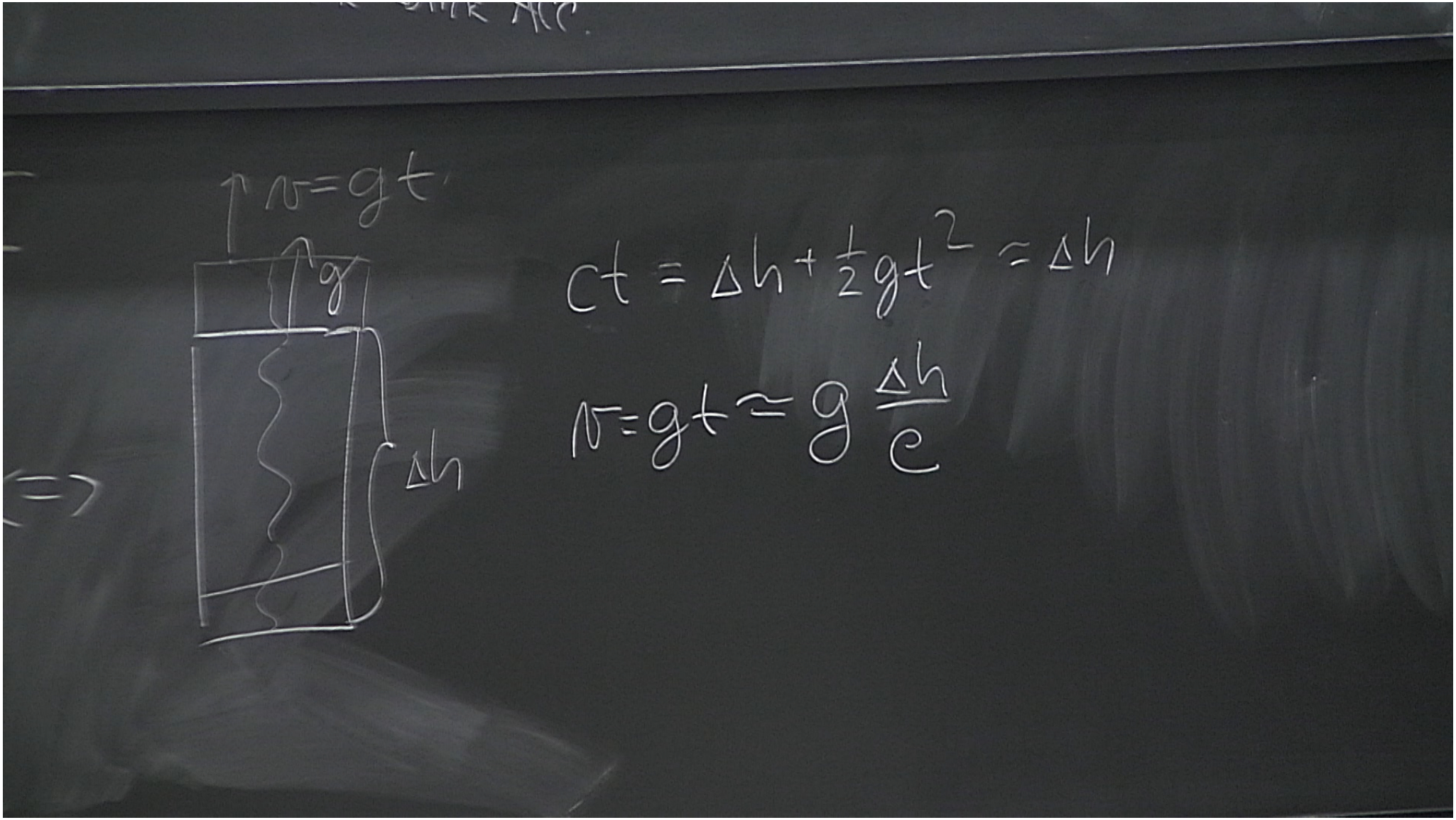
WE CAN MIMIC GRAVITATIONAL FIELD BY GOING TO THE ACCELERATED FRAME.

$m_1 = g$ ... FALL WITH THE SAME ACC.

GRAVITATIONAL REDSHIFT



$$ct = \Delta h + \frac{1}{2} g t^2$$


v = gt
g
Δh
ct = Δh + ½gt² ≈ Δh
v = gt ≈ g Δh/c

$= \Delta h$

BOTTOM: $k^\mu = \frac{\omega}{c}(1,0,0,1)$ (z)

UP: $k'^\mu = \frac{\omega'}{c}(1,0,0,1) = \gamma \frac{\omega}{c}(1-\beta)(1,0,0,1)$

$= \Delta h + \frac{1}{2} g t^2 \simeq \Delta h$

$gt \simeq g \frac{\Delta h}{c}$

BOTTOM: $k^\mu = \frac{\omega}{c}(1,0,0,1)$ (z)

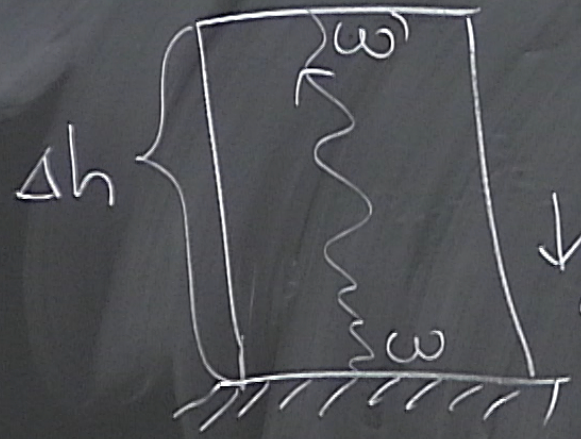
UP: $k'^\mu = \frac{\omega'}{c}(1,0,0,1) = \gamma \frac{\omega}{c}(1-\beta)(1,0,0,1)$

$\omega' = \gamma\omega(1-\beta) \simeq \omega(1-\beta) = \omega\left(1 - \frac{g\Delta h}{c^2}\right)$ DOPPLER

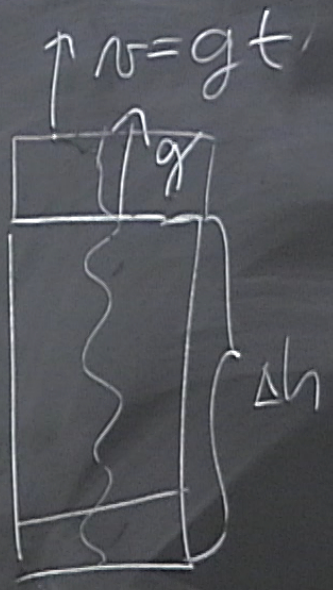
$$z = \frac{\omega' - \omega}{\omega} = -\frac{g\Delta h}{c^2}$$

POUND-REBKA (1960)

GRAVITATIONAL REDSHIFT



$\Leftrightarrow$



$ct = \Delta h$

$v = gt \simeq$

PROPER TIME $\sim \frac{1}{\omega}$

IN "STRONG GRAV FIELD" PROPER TIME GOES MORE SLOWLY.