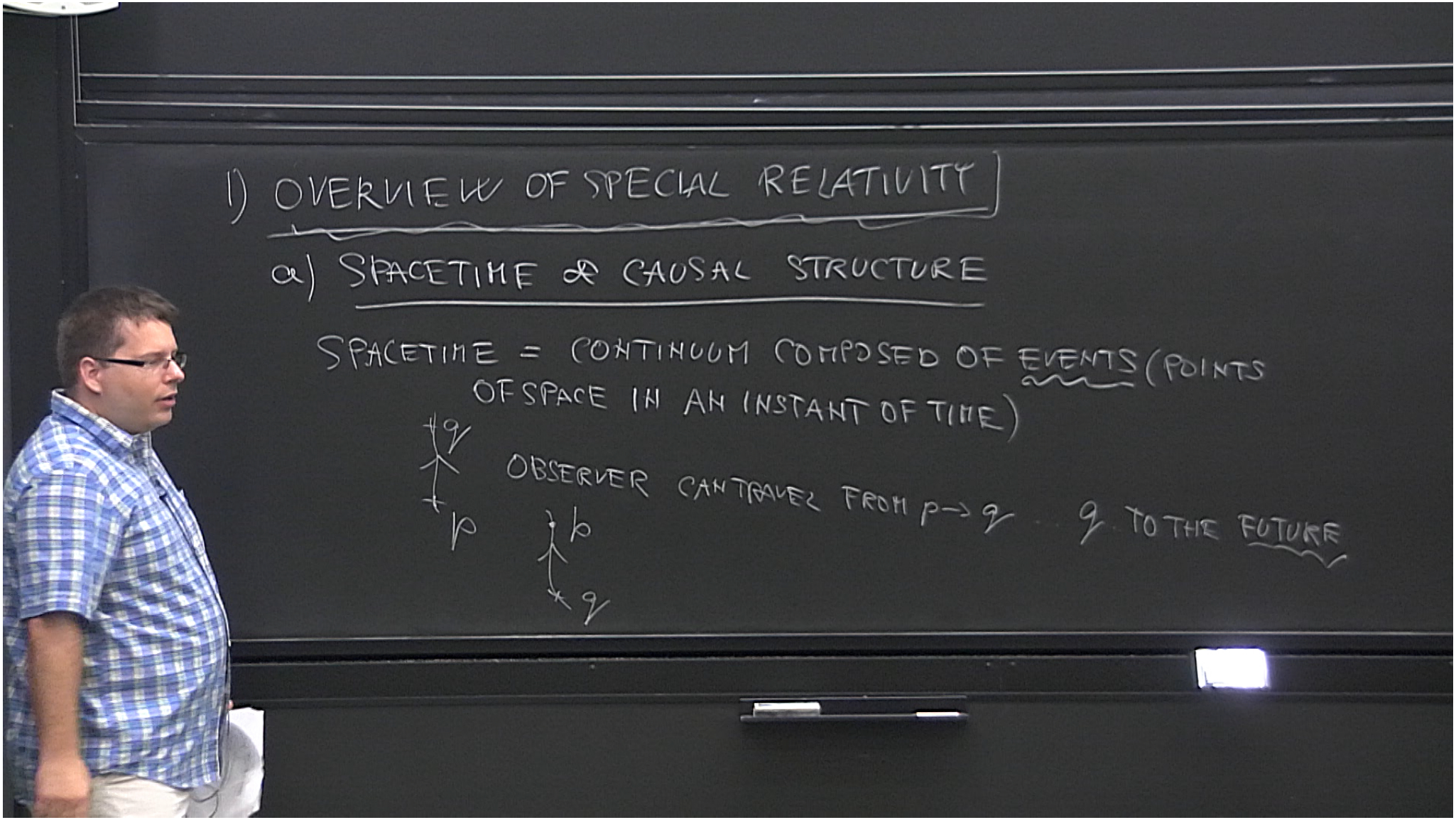
Title: PSI 2018/2019 - Relativity - Lecture 1

Date: Sep 04, 2018 09:00 AM

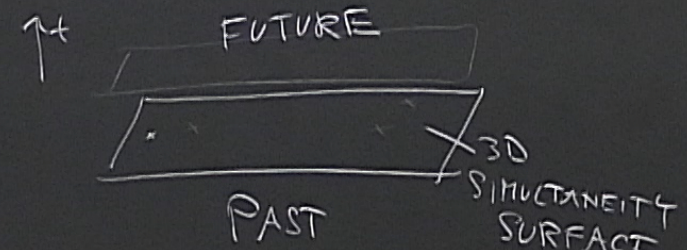
URL: http://pirsa.org/18090002

Abstract:


1) OVERVIEW OF SPECIAL RELATIVITY
a) SPACETIME & CAUSAL STRUCTURE
SPACETIME = CONTINUUM COMPOSED OF EVENTS (POINTS OF SPACE IN AN INSTANT OF TIME)
q
p
OBSERVER CAN TRAVEL FROM p → q
q TO THE FUTURE
p
q

3RD OPTION : OBSERVER CANNOT SEE BOTH p & q

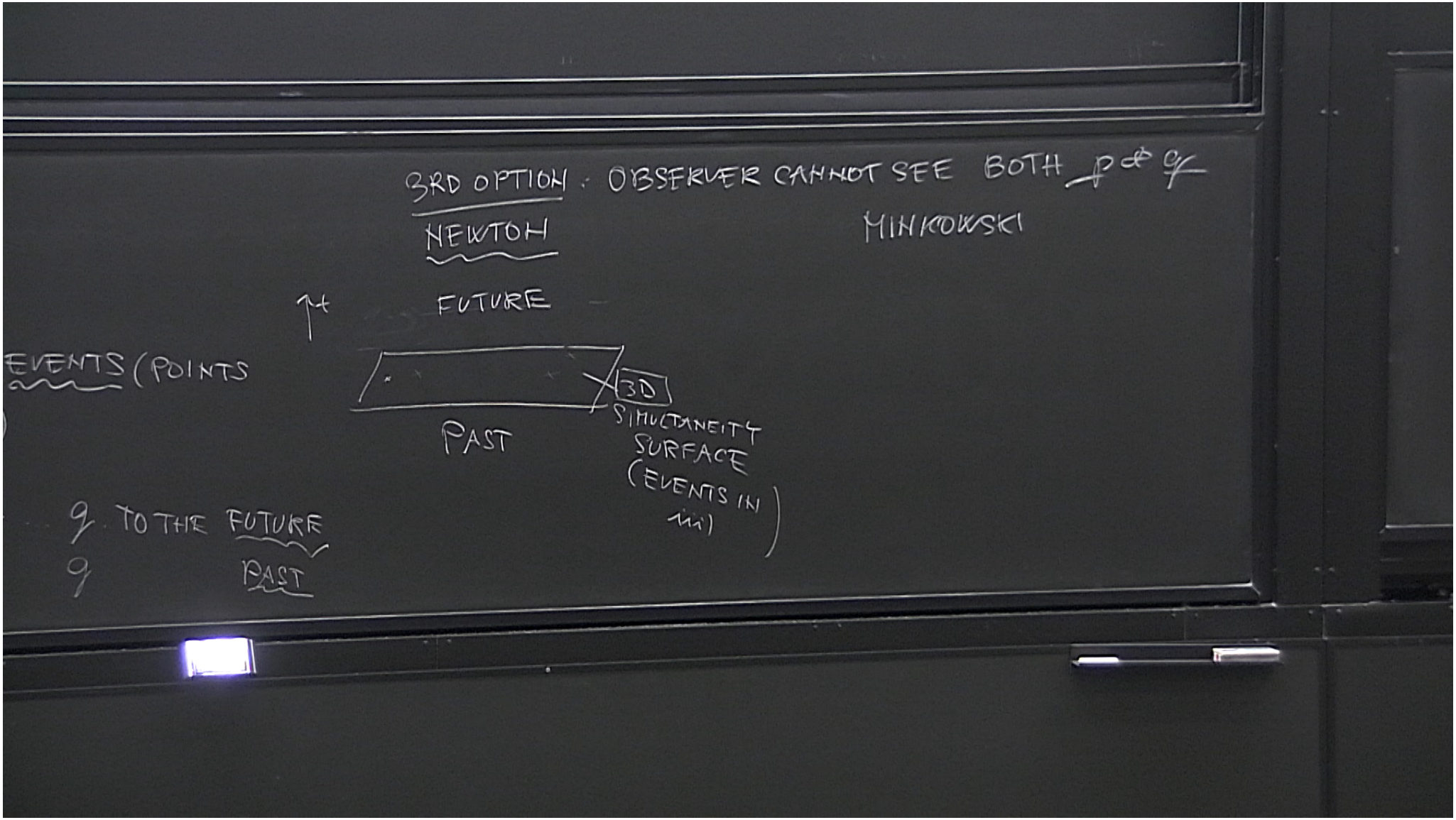
NEWTON



EVENTS (POINTS

q TO THE FUTURE

q PAST


3RD OPTION : OBSERVER CANNOT SEE BOTH p & q
NEWTON
MINKOWSKI
FUTURE
EVENTS (POINTS
3D
PAST
SIMULTANEITY SURFACE (EVENTS IN M)
q TO THE FUTURE
q PAST

ITT

URE

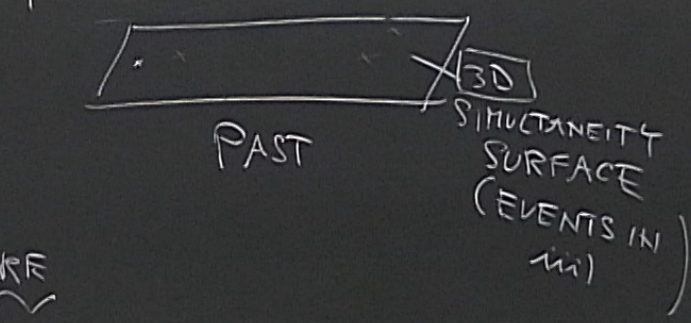
D OF EVENTS (POINTS
TIME)

$p \to q$ q TO THE FUTURE

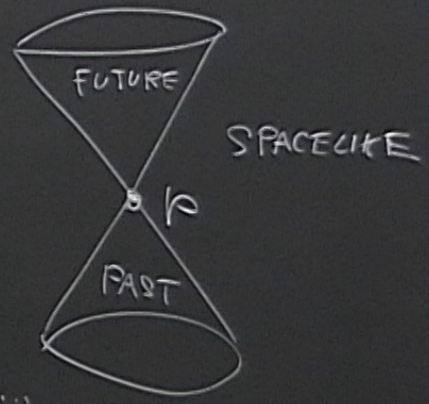
$q \to p$ q PAST

3RD OPTION : OBSERVER CANNOT SEE BOTH p & q

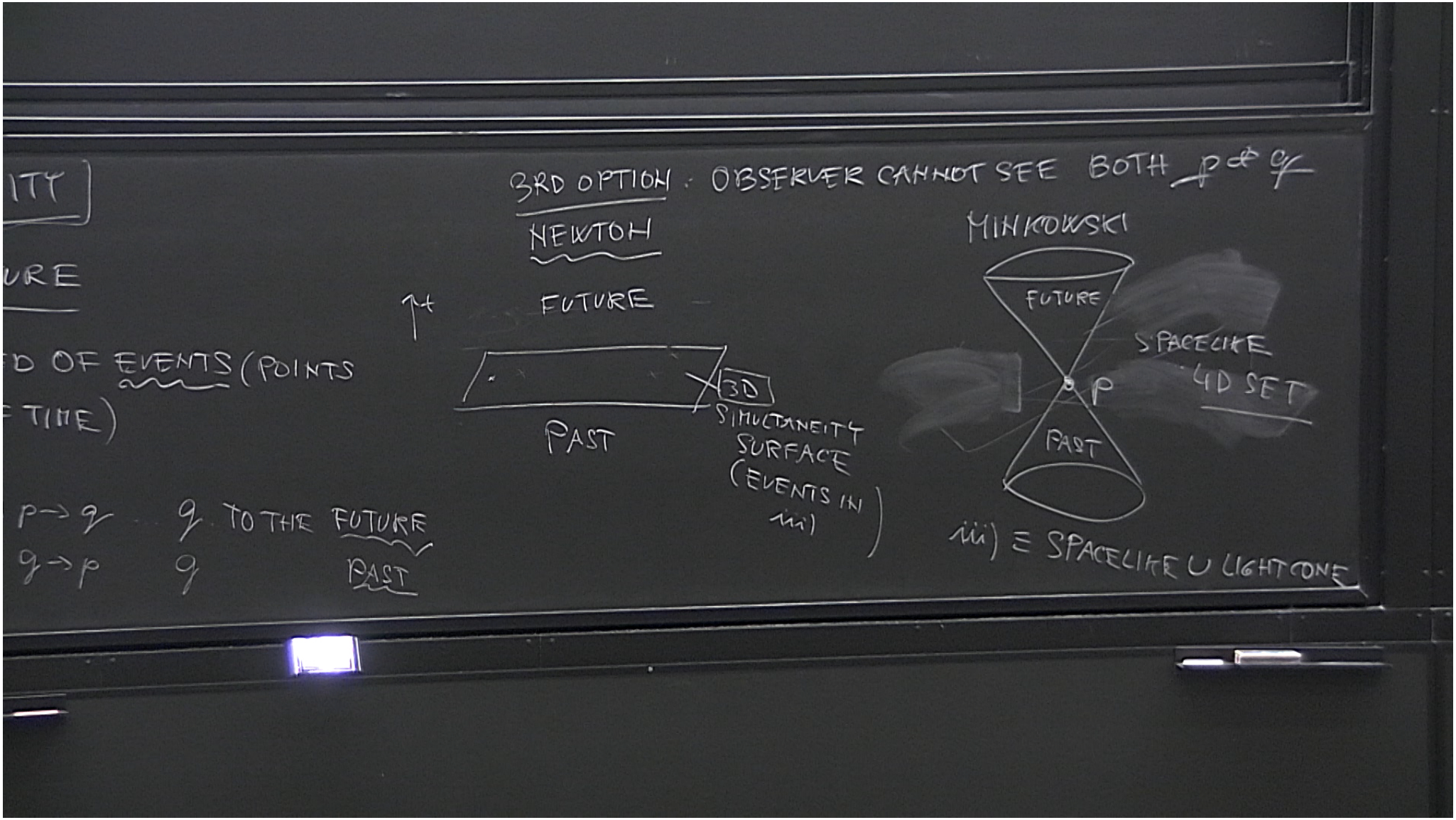
NEWTON



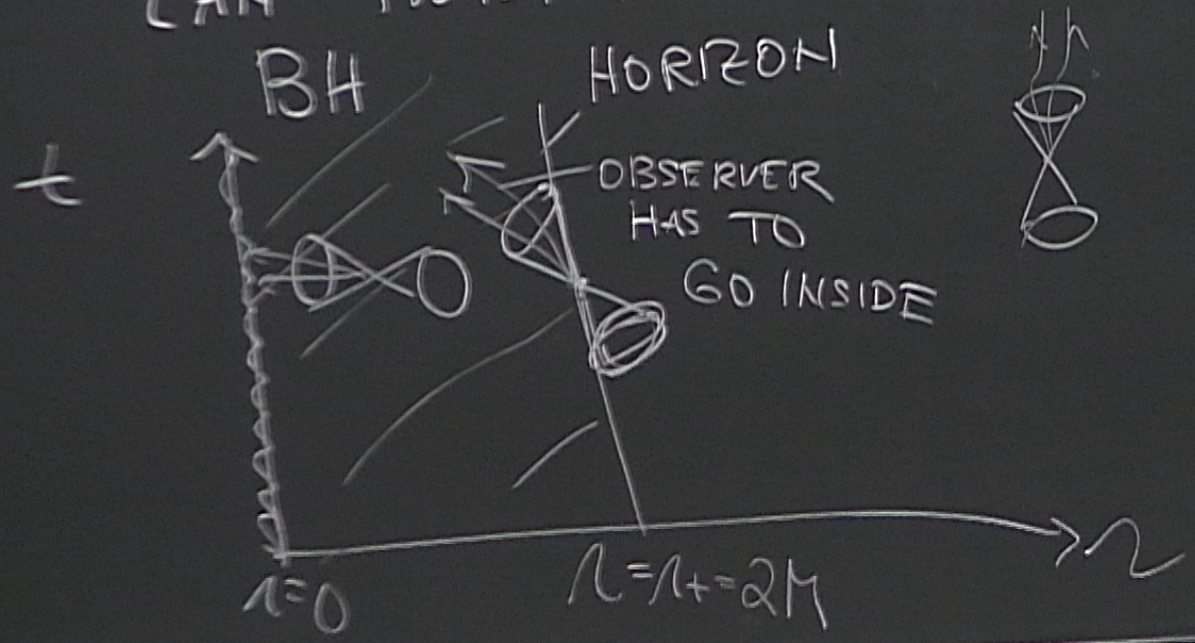
MINKOWSKI



iii) ≡ SPACELIKE ∪ LIGHTCONE


ITT
URE
D OF EVENTS (POINTS
E TIME)
p → q
q → p
q TO THE FUTURE
q PAST
3RD OPTION : OBSERVER CANNOT SEE BOTH p & q
NEWTON
FUTURE
PAST
3D
SIMULTANEITY SURFACE (EVENTS IN iii)
MINKOWSKI
FUTURE
P
PAST
SPACELIKE 4D SET
iii) ≡ SPACELIKE ∪ LIGHTCONE

REMARK. LIGHTCONE STRUCTURE ALSO IMPORTANT IN GENERAL RELATIVITY LOCALLY (LIGHTCONES CAN "TWIST AROUND"



- INERTIAL OBSERVER PLAYS IMPORTANT ROLE IN BOTH NEWTON & SR.
  - IN FACT THERE IS ∞-# OF THEM.
  - CONNECTED BY GALILEAN & POINCARE TRANSFORMATIONS.
    (BOOSTS & ROTATIONS & TRANSLATIONS)
    "ROTATIONS"

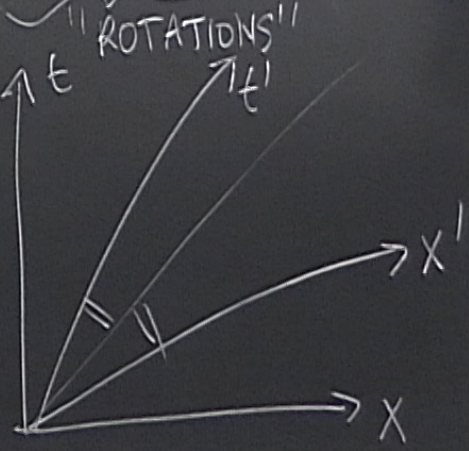
EX: BOOST

$$t' = \gamma(t - vx/c^2)$$
$$x' = \gamma(x - vt)$$
$$\gamma = \frac{1}{\sqrt{1-\beta^2}}, \quad \beta = \frac{v}{c}$$

- INERTIAL OBSERVER PLAYS IMPORTANT ROLE IN BOTH NEWTON & SR.
  - IN FACT THERE IS ∞-# OF THEM.
  - CONNECTED BY GALILEAN & POINCARE TRANSFORMATIONS.
    (BOOSTS & ROTATIONS & TRANSLATIONS)

EX: BOOST

$$t' = \gamma(t - vx/c^2)$$
$$x' = \gamma(x - vt)$$
$$\gamma = \frac{1}{\sqrt{1-\beta^2}}, \quad \beta = \frac{v}{c}$$



OBSERVER DEPENDENT SIMULTANEITY SURFACE

- ABSOLUTE QUANTITIES (OBSERVER INDEP.)

NEWTON: Δt (ABSOLUTE TIME)

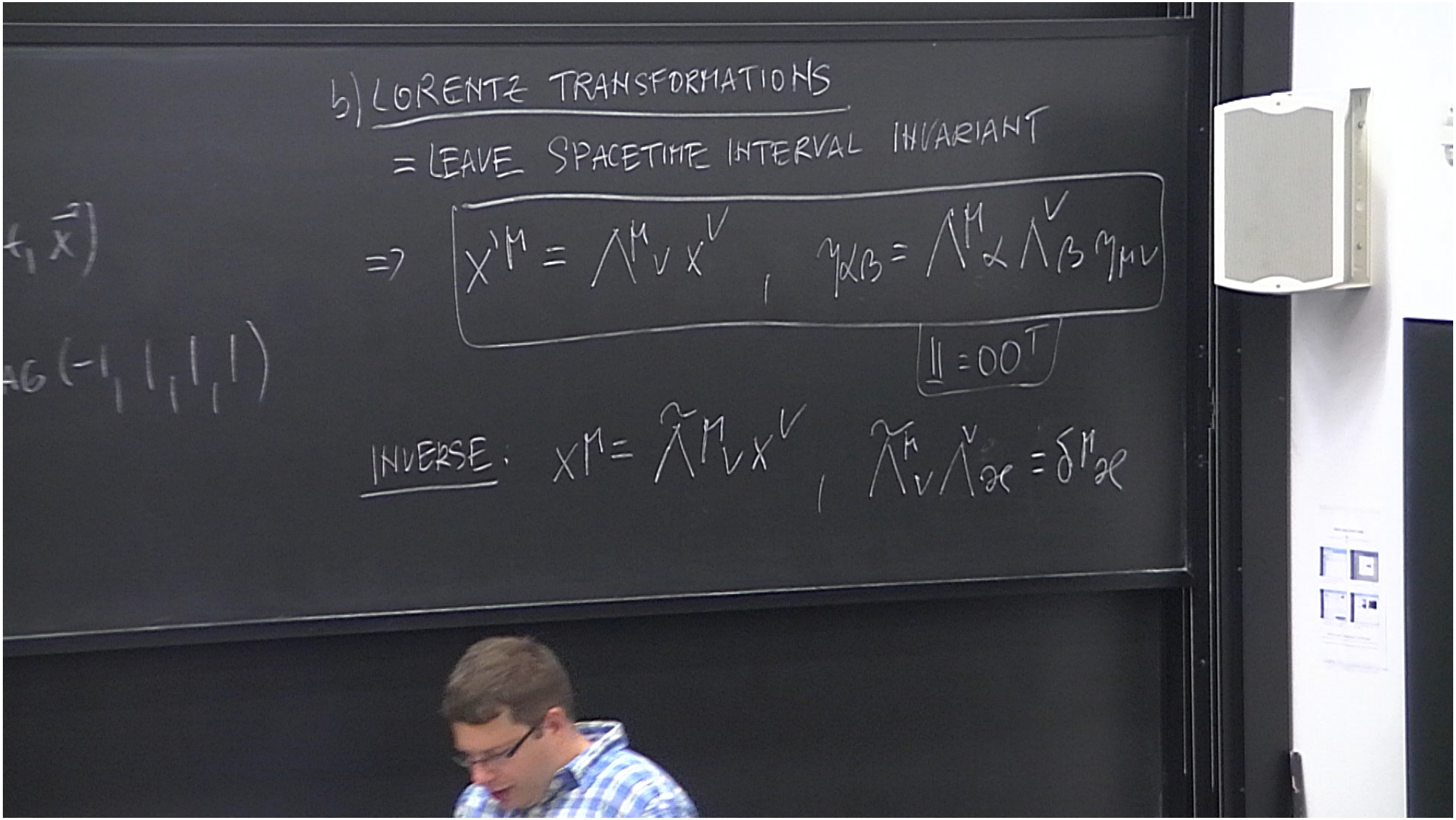
$|\vec{\Delta x}|$

$X^\mu = (ct, \vec{x})$

$\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$

SPECIAL REL. SPACETIME INTERVAL

$$\Delta s^2 = \eta_{\mu\nu} \Delta X^\mu \Delta X^\nu$$


b) LORENTZ TRANSFORMATIONS
= LEAVE SPACETIME INTERVAL INVARIANT
$\Rightarrow$ $x'^\mu = \Lambda^\mu{}_\nu x^\nu$, $\eta_{\alpha\beta} = \Lambda^\mu{}_\alpha \Lambda^\nu{}_\beta \eta_{\mu\nu}$
$\Lambda = \Theta\Theta^T$
INVERSE: $x^\mu = \tilde{\Lambda}^\mu{}_\nu x'^\nu$, $\tilde{\Lambda}^\mu{}_\nu \Lambda^\nu{}_\rho = \delta^\mu{}_\rho$

$\Delta s^2 = \eta_{\mu\nu} \Delta x^\mu \Delta x^\nu$

VECTOR: $V'^\mu = \Lambda^\mu{}_\nu V^\nu$

CO-VECTOR: $\omega'_\mu = \tilde{\Lambda}^\nu{}_\mu \omega_\nu$

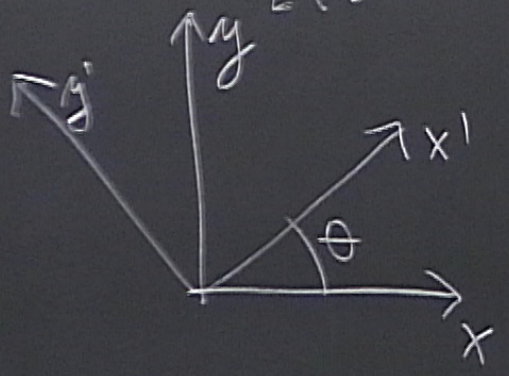
TENSOR = "PRODUCT OF VECTORS & CO-VECTORS"

SCALAR = $\varphi(x) = \varphi(x')$

EXS:

ROTATION (IN X-Y PLANE)

$$\Lambda^{\alpha}{}_{\beta} = \begin{array}{c} \\ t \\ x \\ y \\ z \end{array} \begin{array}{c} \begin{array}{cccc} t & x & y & z \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$



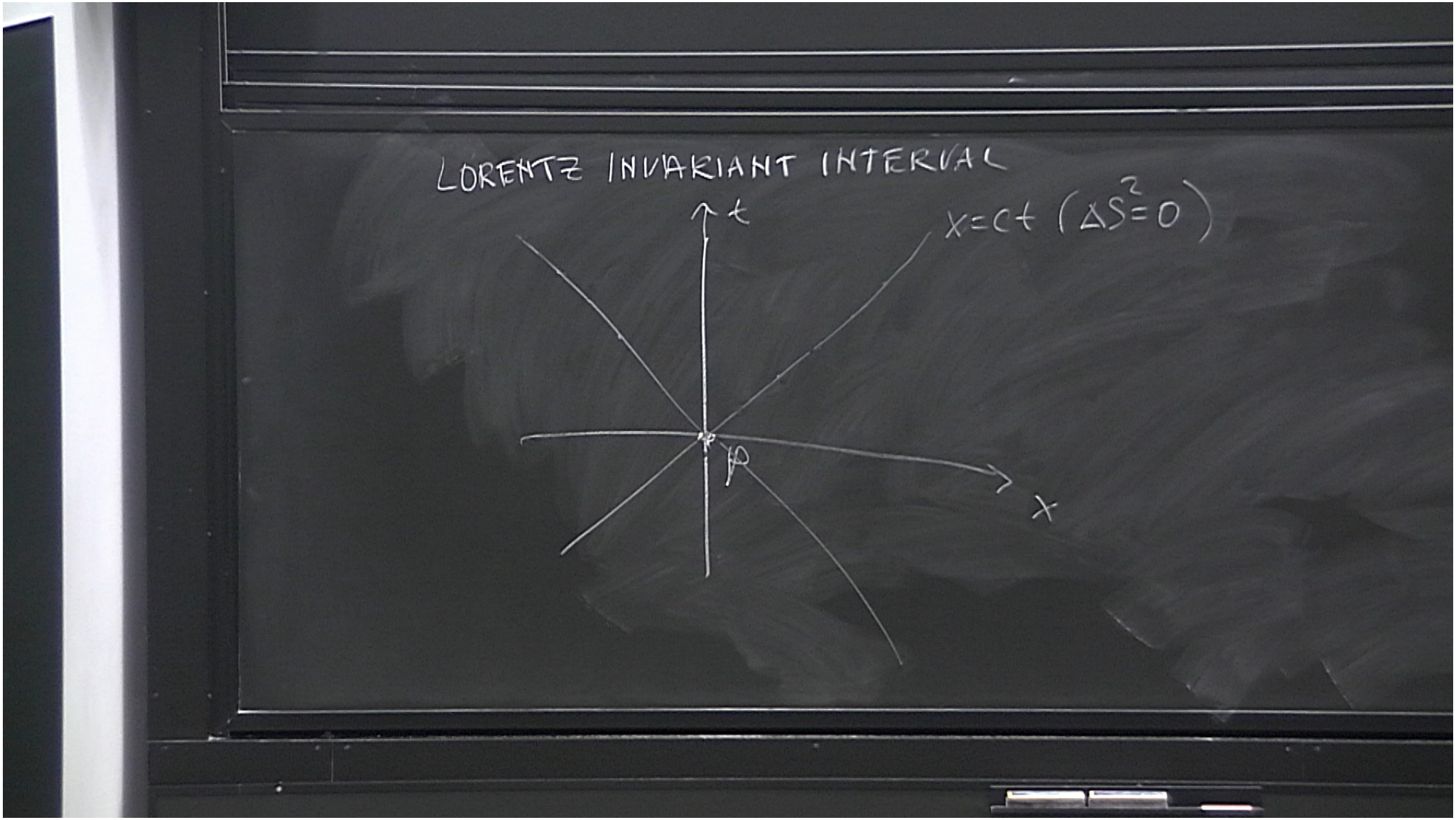
BOOST (X-DIR)

$$\Lambda^{\alpha}{}_{\beta} = \begin{array}{c} \begin{array}{cccc} t & x & y & z \end{array} \\ \begin{pmatrix} \cosh\theta & -\sinh\theta & 0 & 0 \\ -\sinh\theta & \cosh\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

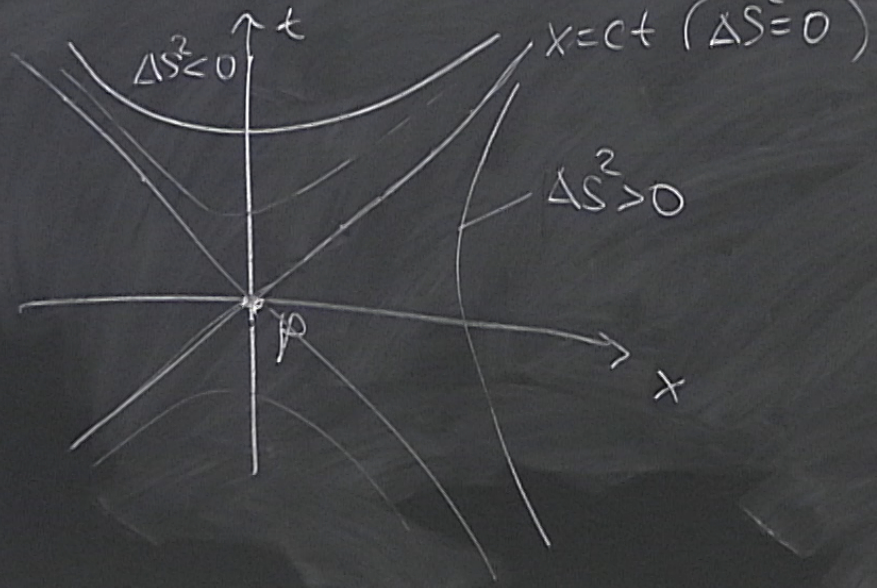
$$\beta = \frac{v}{c} = -\tanh\theta$$

$$\cosh\theta = \gamma$$

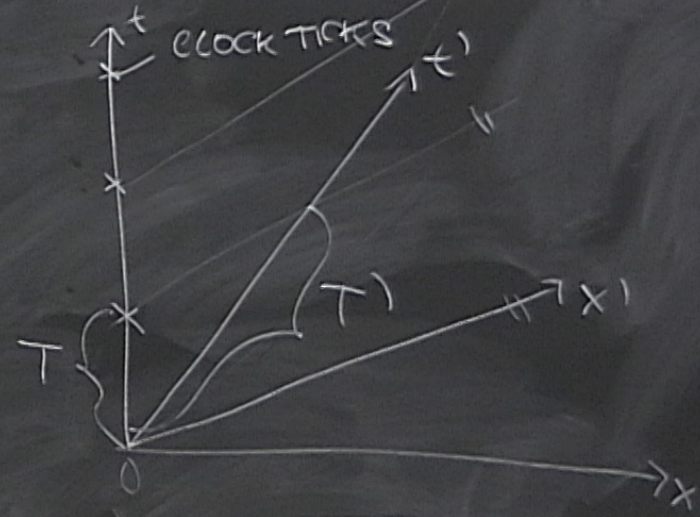
$$\sinh\theta = \beta\gamma$$


LORENTZ INVARIANT INTERVAL
t
x=ct (ΔS²=0)
P
x

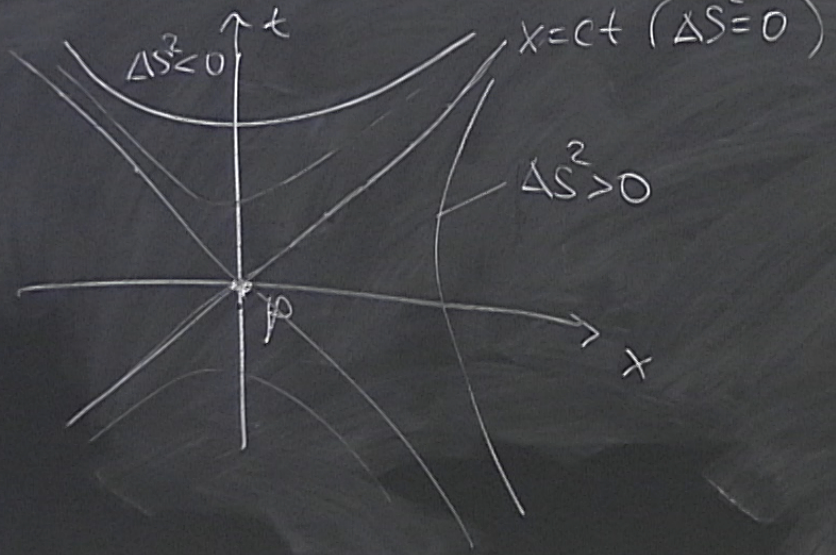
LORENTZ INVARIANT INTERVAL



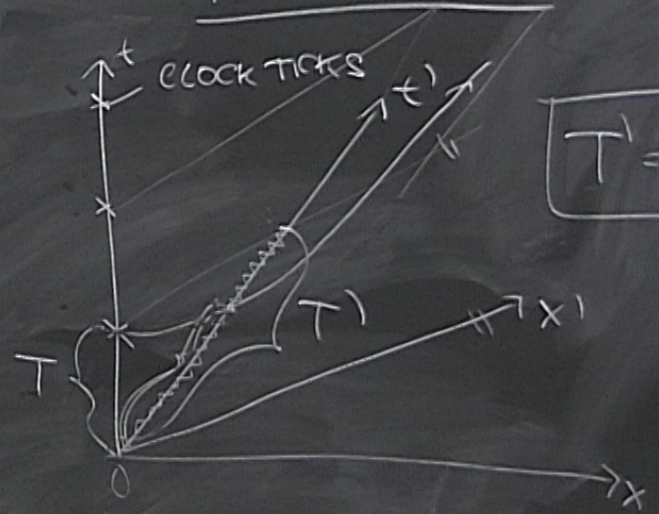
TIME DILATATION



LORENTZ INVARIANT INTERVAL

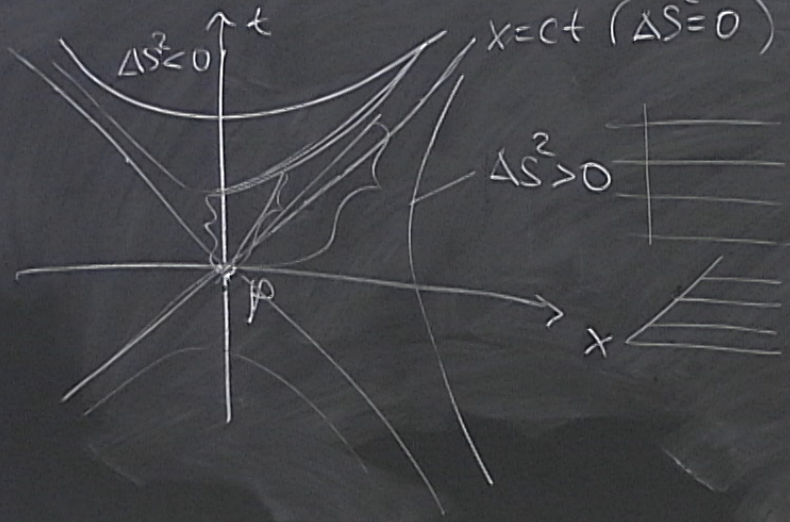


TIME DILATATION



$T' = \gamma T > T$

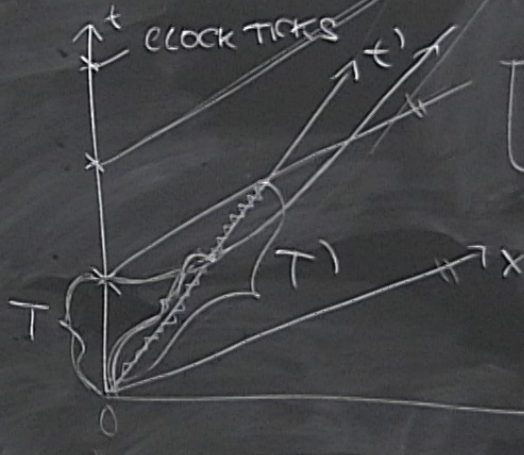
LORENTZ INVARIANT INTERVAL



$x = ct \ (\Delta S^2 = 0)$

$\Delta S^2 > 0$

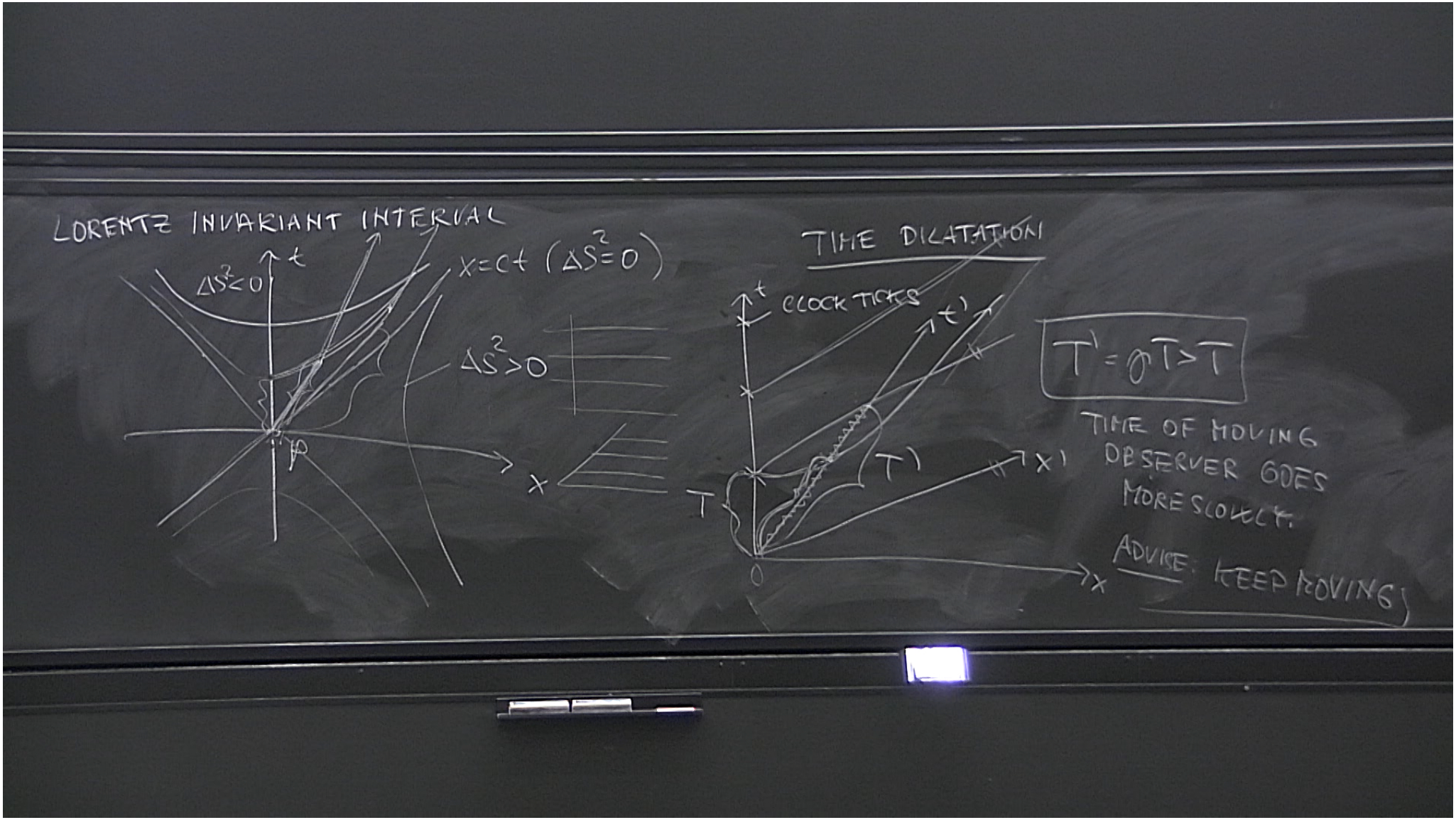
TIME DILATATION

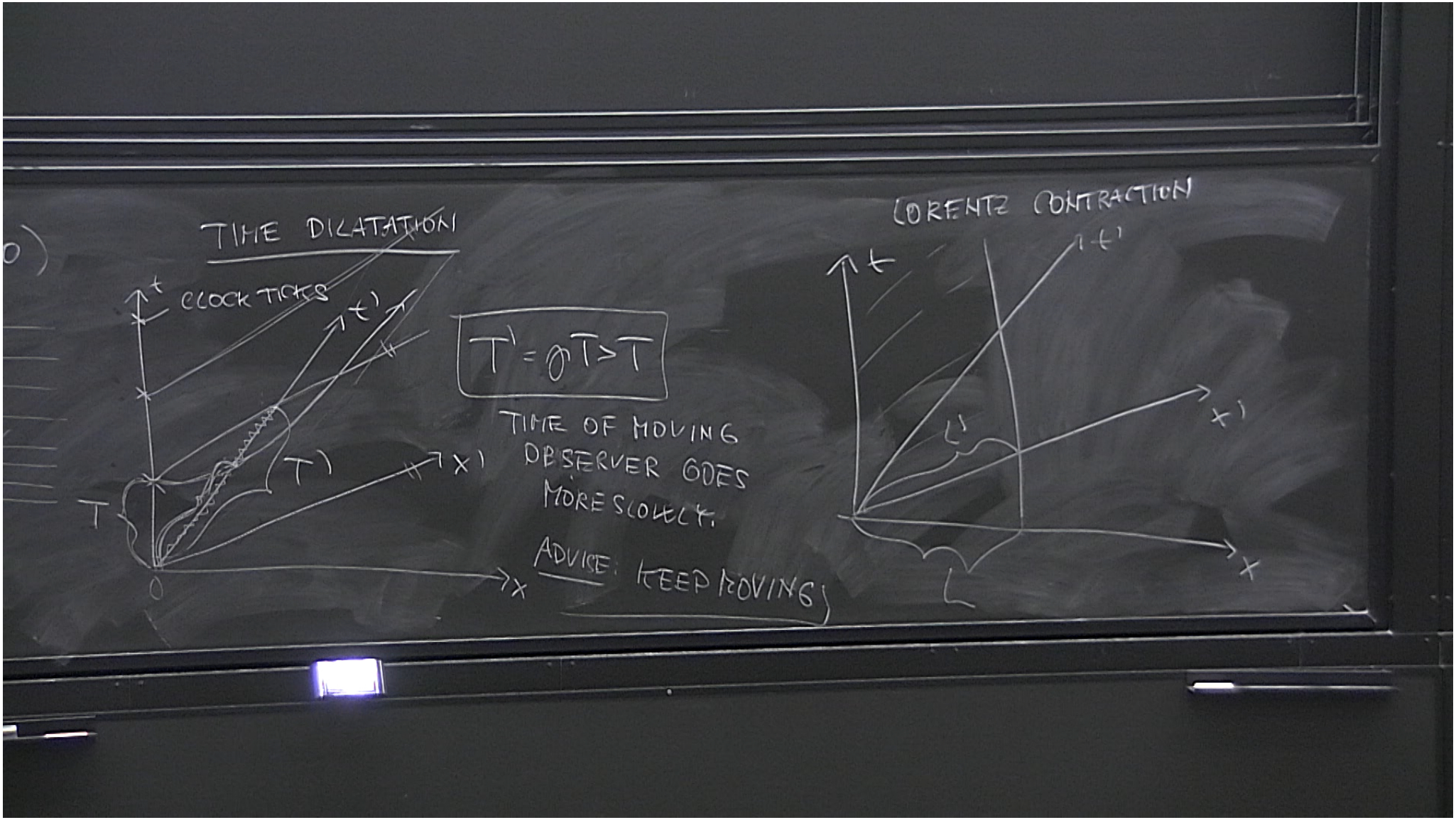


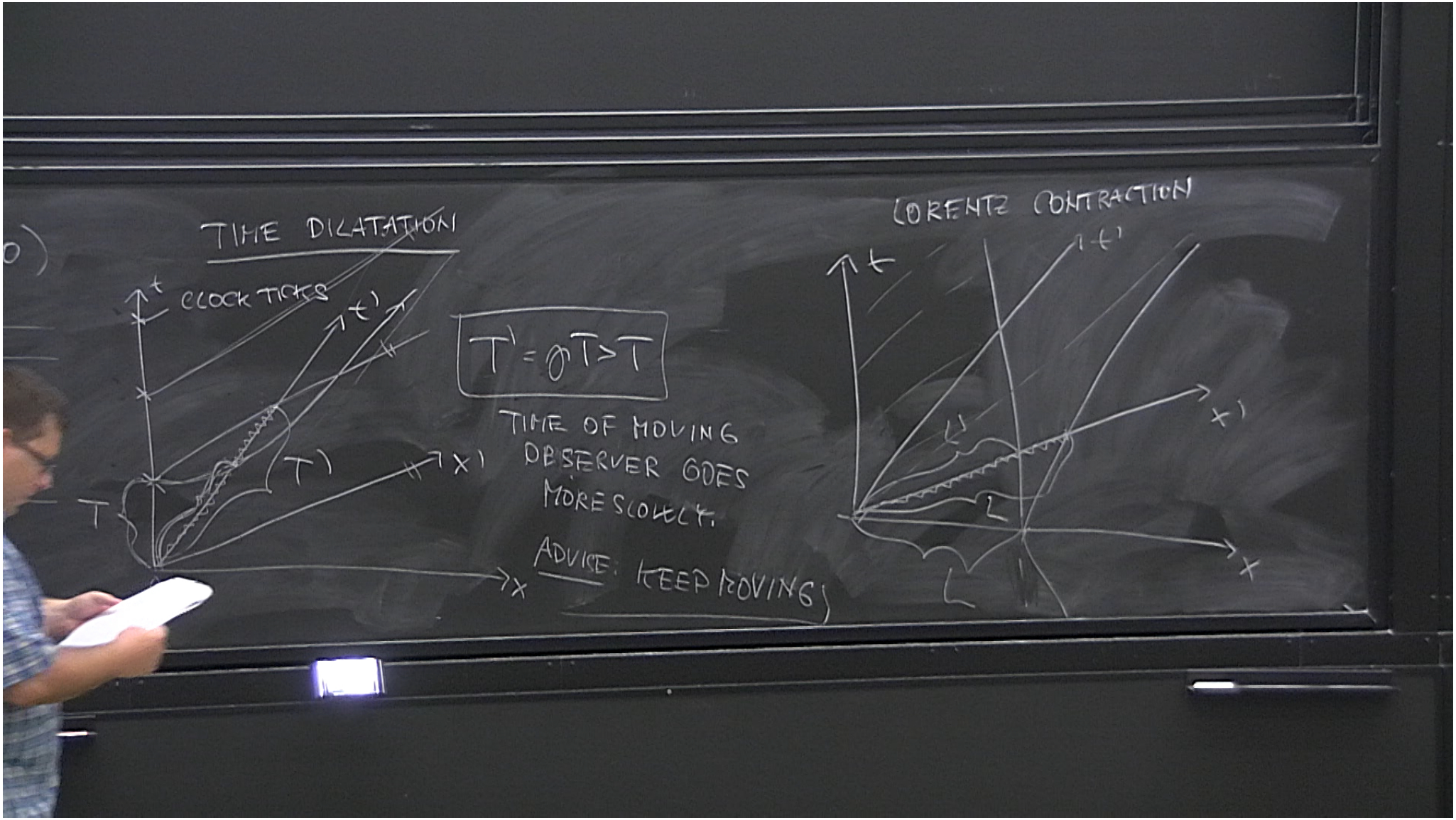
$T' = \gamma T > T$

TIME OF MOVING OBSERVER GOES MORE SLOWLY.

ADVICE: KEEP MOVING!


LORENTZ INVARIANT INTERVAL
$\Delta S^2 < 0$
t
x
$x = ct$ $(\Delta S^2 = 0)$
$\Delta S^2 > 0$
TIME DILATATION
CLOCK TICKS
t'
T
T'
x'
$T' = \gamma T > T$
TIME OF MOVING OBSERVER GOES MORE SLOWLY.
ADVISE: KEEP MOVING!


TIME DILATATION
t
CLOCK TICKS
t'
T'
X'
T
0
X
T' = γT > T
TIME OF MOVING OBSERVER GOES MORE SLOWLY.
ADVICE: KEEP MOVING!
LORENTZ CONTRACTION
t
t'
x'
L'
L
x

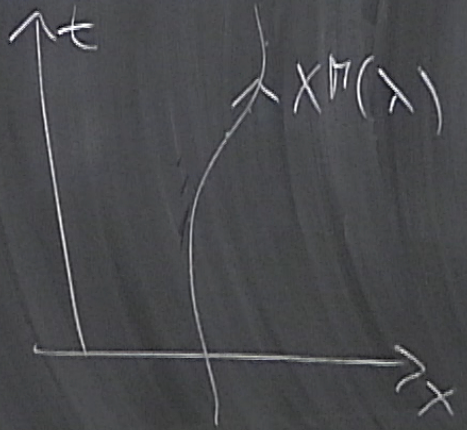

TIME DILATATION
CLOCK TICKS
t
t'
T
T'
x'
x
T' = γT > T
TIME OF MOVING OBSERVER GOES MORE SLOWLY.
ADVICE: KEEP MOVING!
LORENTZ CONTRACTION
t
t'
x'
x
1
2

SPECIAL REL. SPACETIME INTERVAL

$$\Delta s^2 = \eta_{\mu\nu}\Delta x^\mu \Delta x^\nu = -c^2\Delta t^2 + \Delta x^i \Delta x^i$$

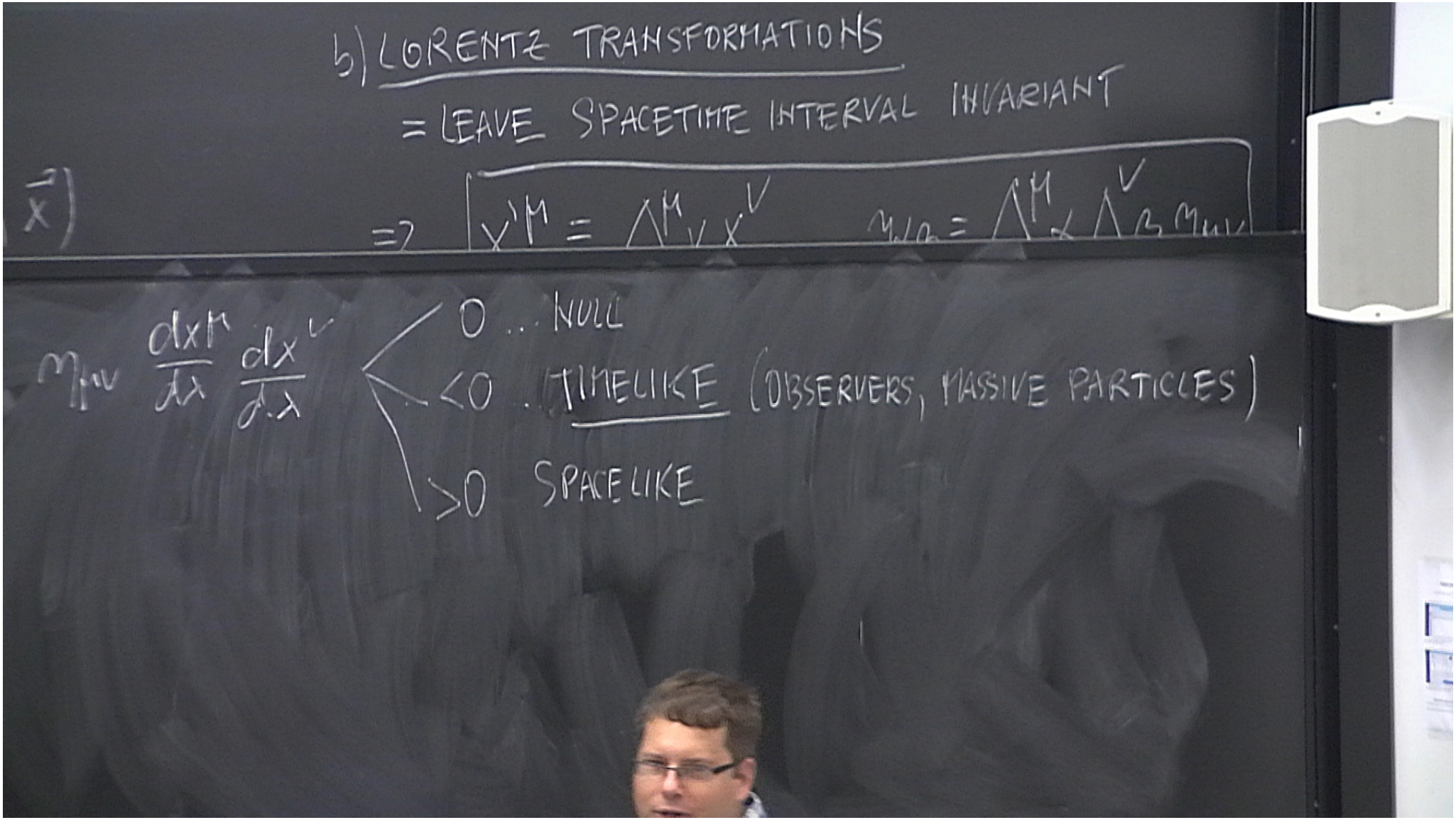
c) RELATIVISTIC PARTICLE

A POINT LIKE OBJECT SWEEPS A TRAJECTORY IN SPACETIME



VELOCITY W.R.T. λ

$$\frac{dx^\mu}{d\lambda}$$


b) LORENTZ TRANSFORMATIONS
= LEAVE SPACETIME INTERVAL INVARIANT
$\vec{x}$)
$\Rightarrow$ $x'^\mu = \Lambda^\mu{}_\nu x^\nu$ $\eta_{\mu\nu} = \Lambda^\alpha{}_\mu \Lambda^\beta{}_\nu \eta_{\alpha\beta}$
$\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}$
0 ... NULL
<0 ... TIMELIKE (OBSERVERS, MASSIVE PARTICLES)
>0 SPACELIKE

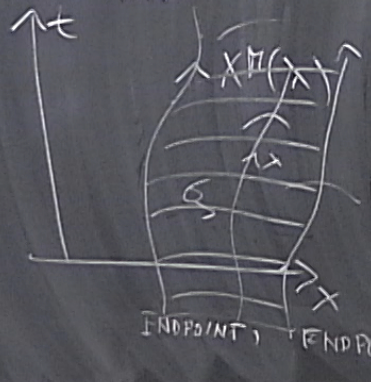
OBSERVER DEPENDENT SIMULTANEITY SURFACE.

b) LORENTZ TRANSFORMATION

c) RELATIVISTIC PARTICLE

A POINT LIKE OBJECT SWEEPS A TRAJECTORY

IN SPACETIME



VELOCITY W.R.T. λ

$$\frac{dx^\mu}{d\lambda}$$

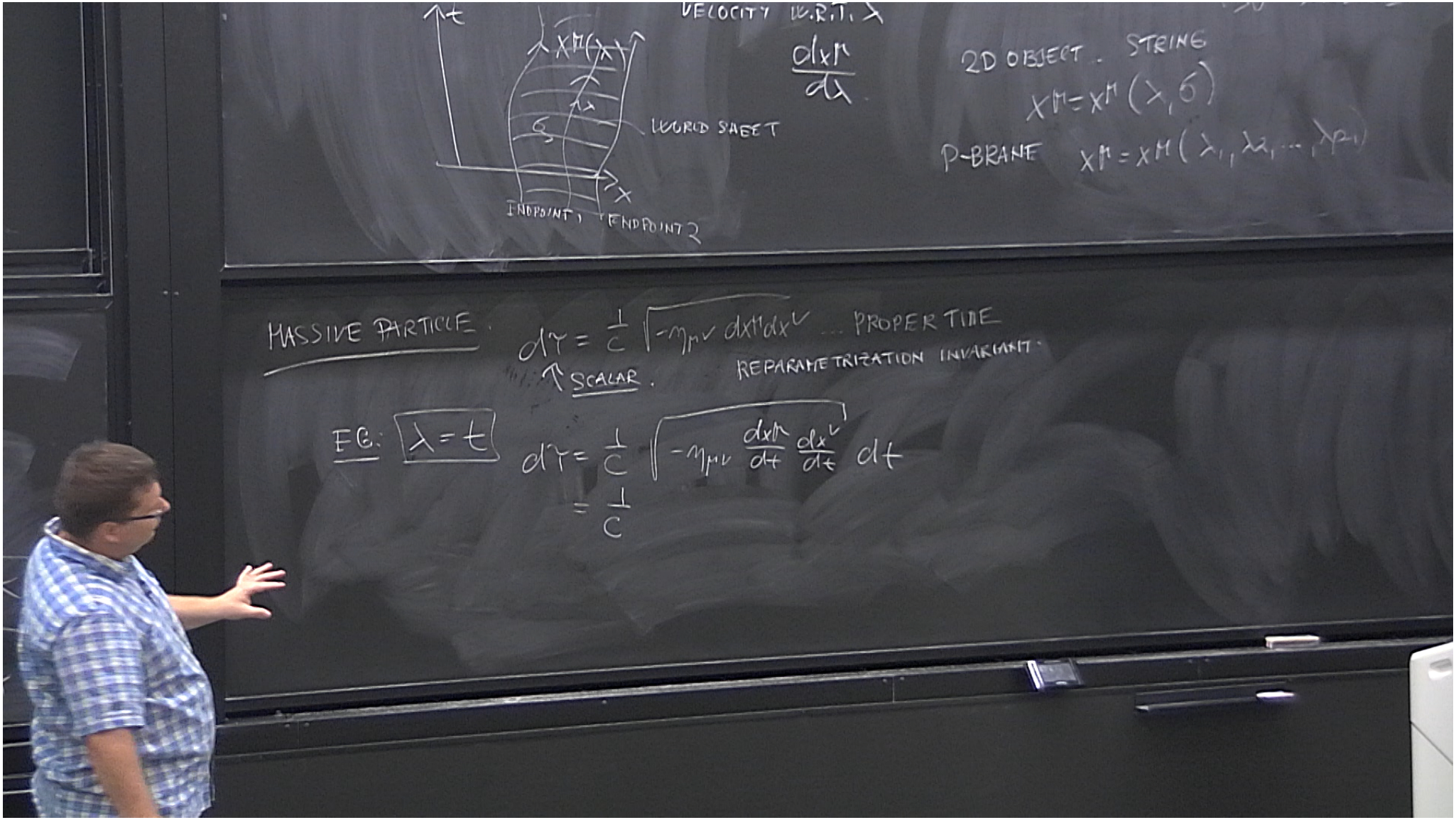
$$\eta_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}$$

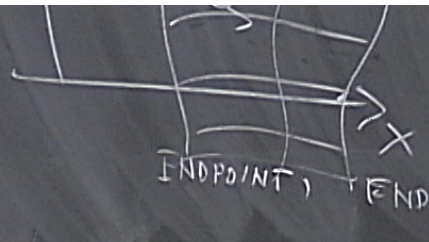
- $=0$ .. NULL
- <0 . TIMELIKE (OBSER
- >0 SPACELIKE

2D OBJECT . STRING

$$X^\mu = X^\mu(\lambda, \sigma)$$

P-BRANE $X^\mu = X^\mu(\lambda_1, \lambda_2, \ldots, \lambda_{p+1})$


t
$X^\mu(\lambda)$
λ_2
λ_1
WORLD SHEET
x
ENDPOINT 1
ENDPOINT 2
VELOCITY W.R.T. λ
$\frac{dx^\mu}{d\lambda}$
2D OBJECT. STRING
$X^\mu = X^\mu(\lambda, \sigma)$
P-BRANE $X^\mu = X^\mu(\lambda_1, \lambda_2, \ldots, \lambda_{p-1})$
MASSIVE PARTICLE.
$d\tau = \frac{1}{c}\sqrt{-\eta_{\mu\nu}\, dx^\mu dx^\nu}$ ... PROPER TIME
↑ SCALAR. REPARAMETRIZATION INVARIANT.
EG: $\lambda = t$
$d\tau = \frac{1}{c}\sqrt{-\eta_{\mu\nu}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}}\, dt$
$= \frac{1}{c}$



P-BRANE $X^\mu = X$

MASSIVE PARTICLE.

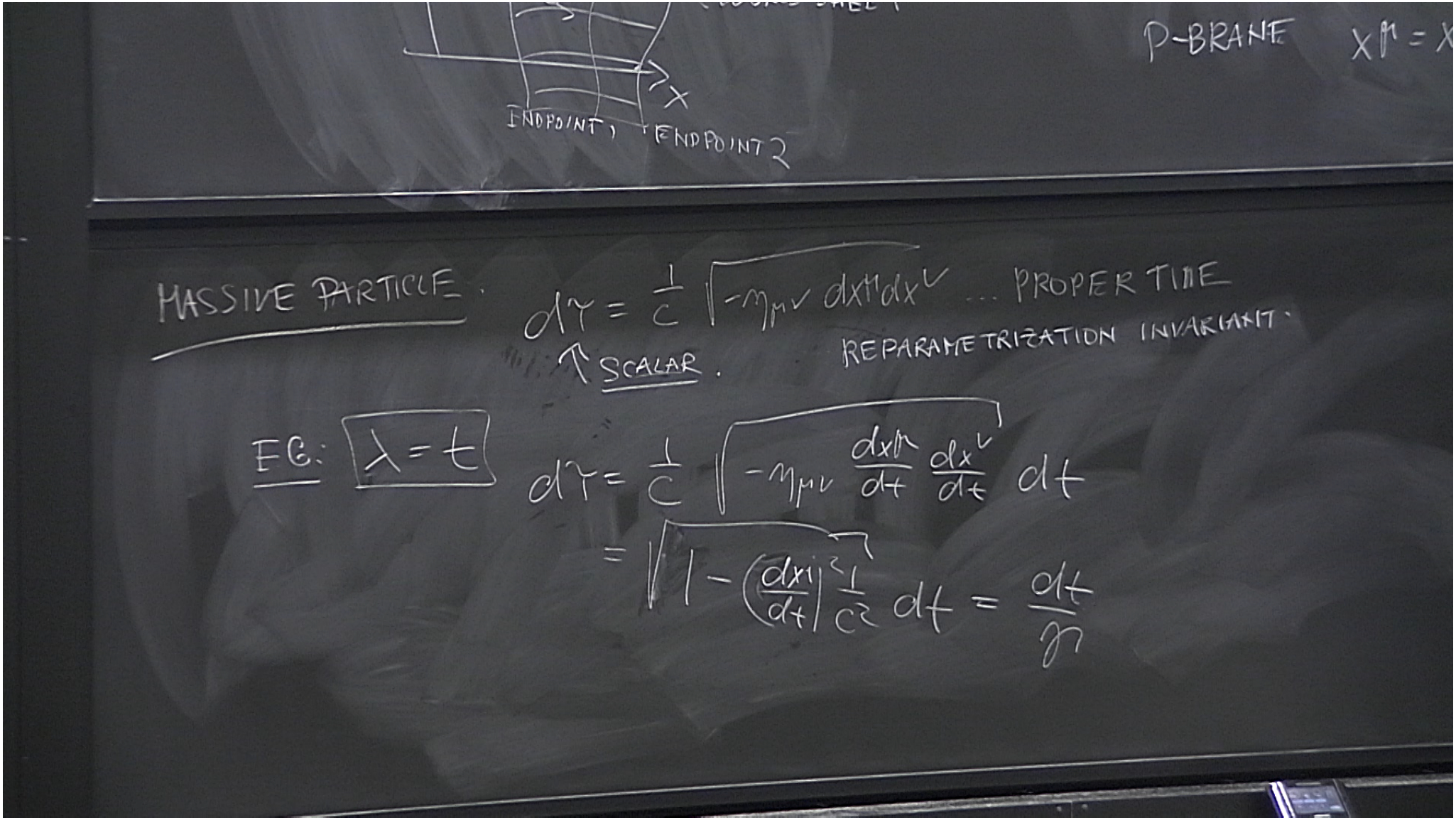
$$d\tau = \frac{1}{c}\sqrt{-\eta_{\mu\nu}\,dx^\mu dx^\nu}$$ ... PROPER TIME

↑ SCALAR. REPARAMETRIZATION INVARIANT.

EG: $\boxed{\lambda = t}$

$$d\tau = \frac{1}{c}\sqrt{-\eta_{\mu\nu}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}}\,dt$$

$$= \frac{1}{c}$$


ENDPOINT 1
ENDPOINT 2
X
P-BRANE
$X^\mu = X$
MASSIVE PARTICLE.
$d\tau = \frac{1}{c}\sqrt{-\eta_{\mu\nu}\,dx^\mu dx^\nu}$ ... PROPER TIME
↑ SCALAR.
REPARAMETRIZATION INVARIANT.
EG: $\lambda = t$
$d\tau = \frac{1}{c}\sqrt{-\eta_{\mu\nu}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}}\,dt$
$= \sqrt{1-\left(\frac{dx_i}{dt}\right)^2\frac{1}{c^2}}\,dt = \frac{dt}{\gamma}$

- 4-VELOCITY

$$u^\mu = \frac{dx^\mu}{d\tau} = \frac{dx^\mu}{dt}\left(\frac{dt}{d\tau}\right)_\gamma = \gamma(c, \vec{v})$$

$$u^2 = -1$$

- 4-MOMENTUM

$$p^\mu = m_0 u^\mu = (\gamma c m_0, \gamma m_0 \vec{v})$$

m_0 ↑ REST MASS

m

$\vec{p}$

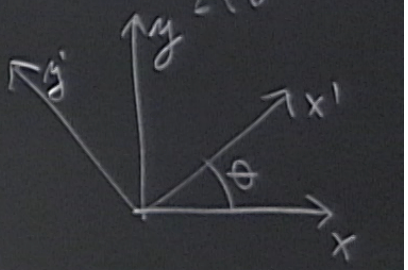
$m = m_0 \gamma$

$p^\mu = (c m, \vec{p})$

EX:

ROTATION

$\Lambda^\alpha_\beta =$



EXS

$m = m_0 \gamma$ $\quad E = mc^2$ (TUTORIAL)

$p^\mu = (cm, \vec{p}) = (E/c, \vec{p})$

$p^2 = p_\mu p^\mu = -E^2/c^2 + \vec{p}^2$



EXS:

$m = m_0 \gamma$ $\quad E = mc^2$ (TUTORIAL)

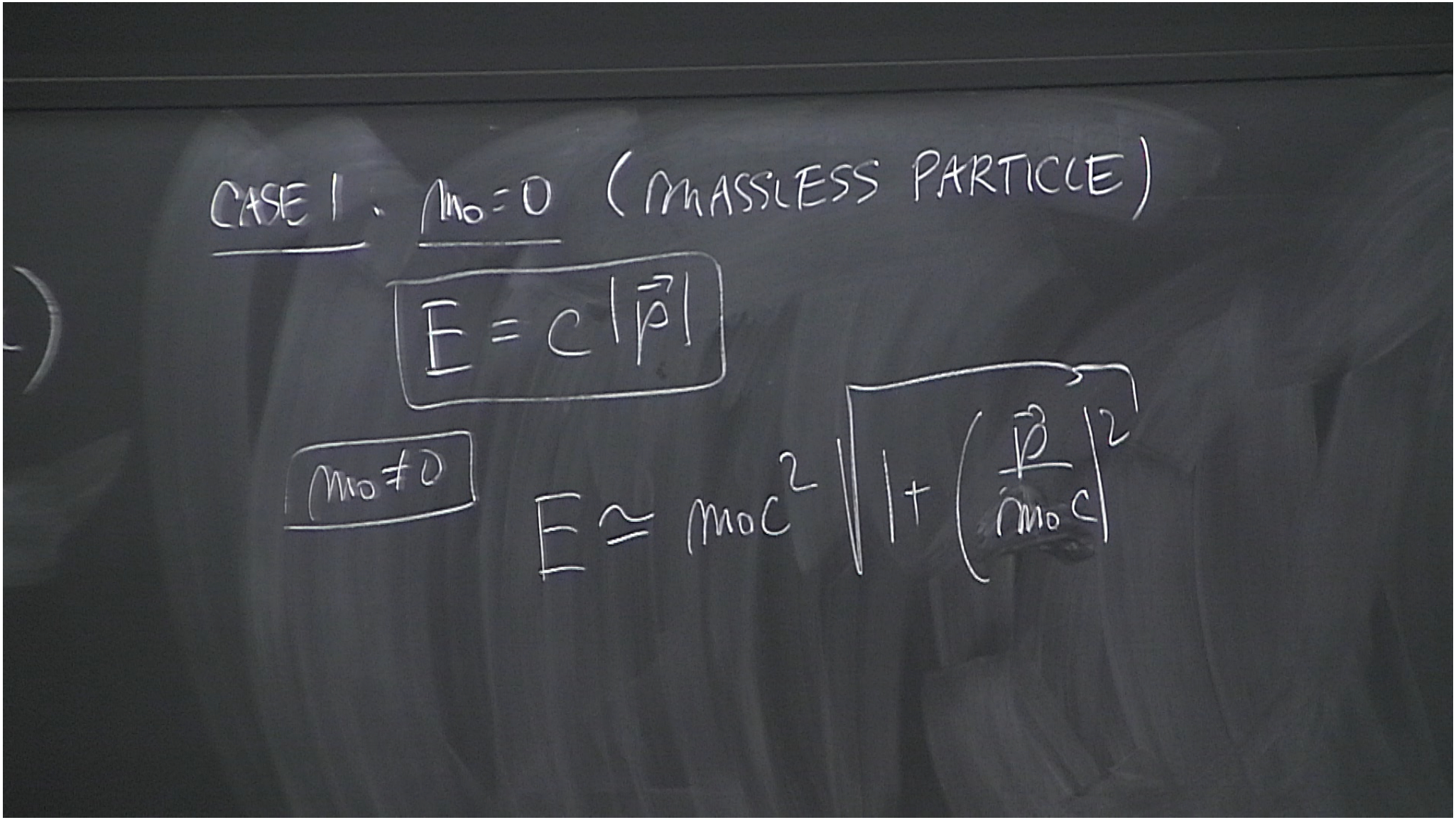
$p^\mu = (cm, \vec{p}) = (E/c, \vec{p})$

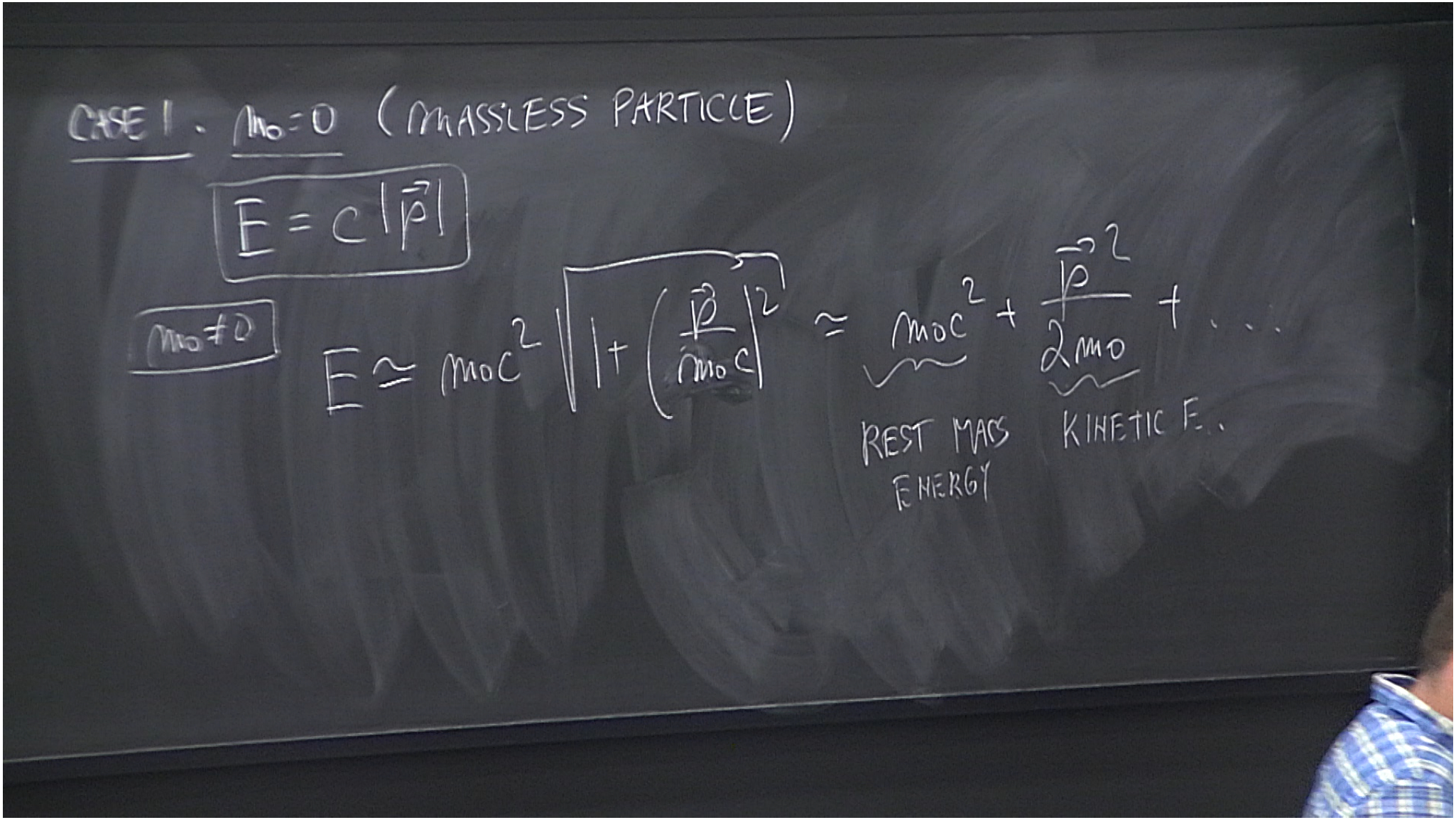
$p^2 = p_\mu p^\mu = -E^2/c^2 + \vec{p}^{\,2} = -m_0^2 c^2$

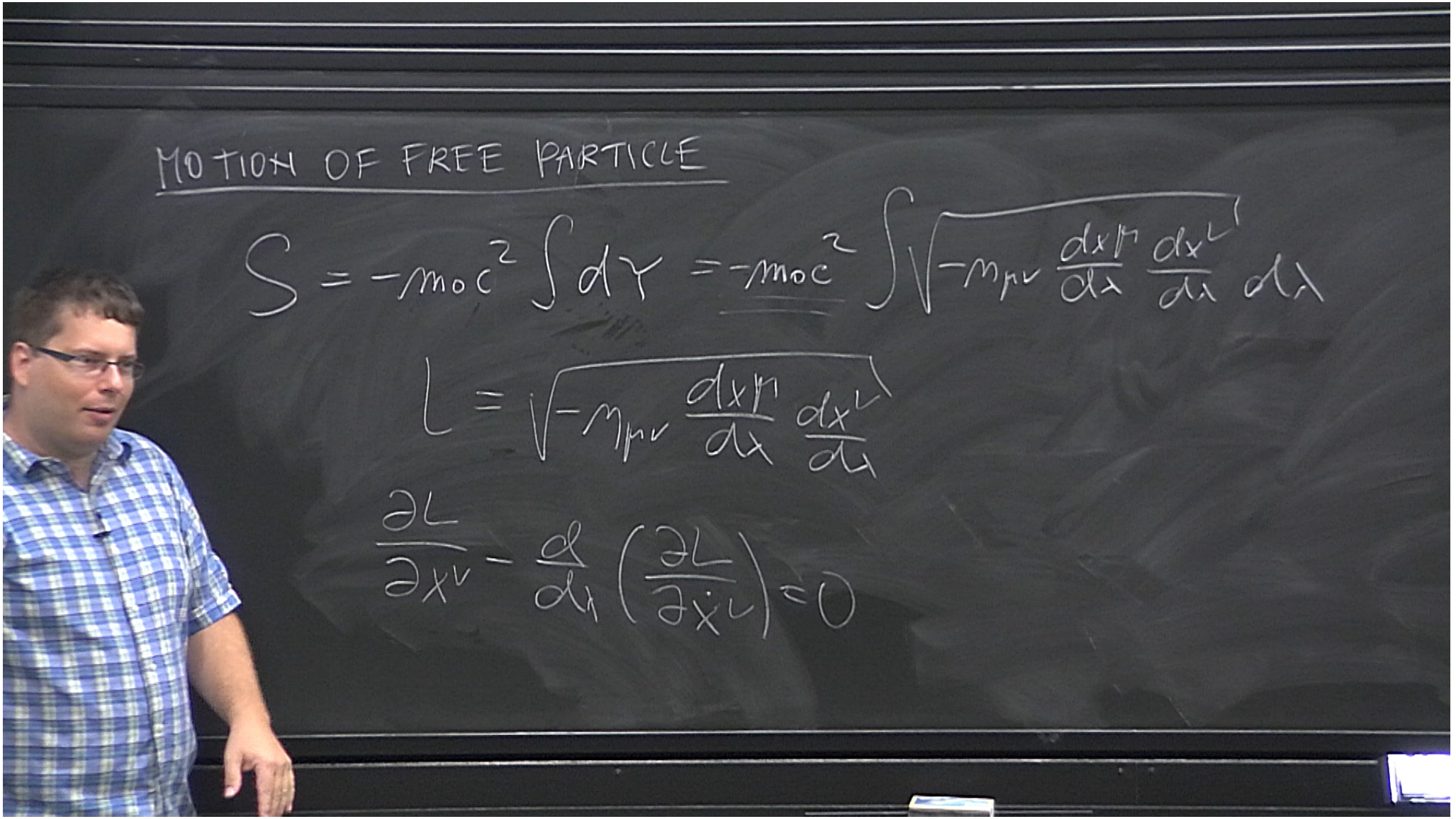
↑ REST FRAME

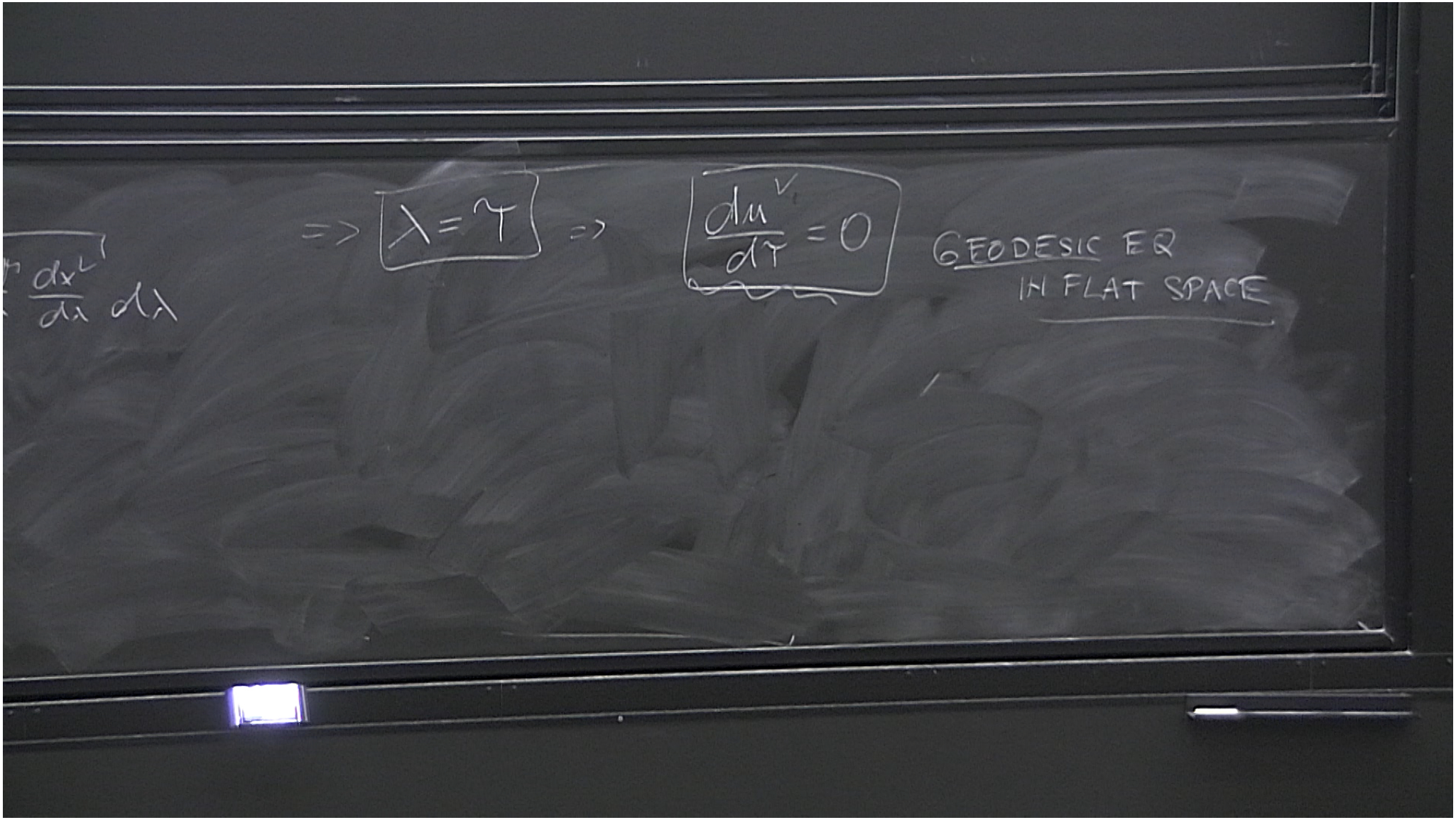
$E = c\sqrt{\vec{p}^{\,2} + m_0^2 c^2}$




CASE 1. $m_0 = 0$ (MASSLESS PARTICLE)
$E = c|\vec{p}|$
$m_0 \neq 0$
$E \simeq m_0 c^2 \sqrt{1 + \left(\frac{\vec{p}}{m_0 c}\right)^2}$


CASE 1. $m_0 = 0$ (MASSLESS PARTICLE)
$E = c|\vec{p}|$
$m_0 \neq 0$
$E \simeq m_0 c^2 \sqrt{1 + \left(\frac{\vec{p}}{m_0 c}\right)^2} \simeq m_0 c^2 + \frac{\vec{p}^2}{2m_0} + \ldots$
REST MASS ENERGY
KINETIC E.


MOTION OF FREE PARTICLE
$S = -m_0 c^2 \int d\tau = -m_0 c^2 \int \sqrt{-\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}}\, d\lambda$
$L = \sqrt{-\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}}$
$\frac{\partial L}{\partial x^\nu} - \frac{d}{d\lambda}\left(\frac{\partial L}{\partial \dot{x}^\nu}\right) = 0$


$\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}$
$\Rightarrow \boxed{\lambda = \tau} \Rightarrow \boxed{\frac{du^\nu}{d\tau} = 0}$
GEODESIC EQ
IN FLAT SPACE