Title: Quasi Many-Body Localization: Anyonic Self-induced Disorder Mechanism

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Abstract: <p>Many-body localization generalizes the concept of Anderson localization (i.e. single particle localization) to isolated interacting systems, where many-body eigenstates in the presence of sufficiently strong disorder can be localized in a region of Hilbert space even at nonzero temperature. This is an example of ergodicity breaking, which manifests failure of thermalization or more specifically the break down of eigenstate-thermalization hypothesis.</p>

<p>In this talk, I enquire into the quasi many-body localization in topologically ordered states of matter, revolving around the case of Kitaev toric code on the ladder geometry, where different types of anyonic defects carry different masses induced by environmental errors. Our study verifies that the presence of anyons generates a complex energy landscape solely through braiding statistics, which suffices to suppress the diffusion of defects in such clean, multicomponent anyonic liquid. This nonergodic dynamics suggests a promising scenario for investigation of quasi many-body localization. Our results unveil how self-generated disorder ameliorates the vulnerability of topological order away from equilibrium. This setting provides a new platform which paves the way toward impeding logical errors by self-localization of anyons in a generic, high energy state, originated exclusively in their exotic statistics.</p>

Quasi Many-Body Localization: Anyonic Self-induced Disorder Mechanism

Abdollah Langari
Department of Physics, Sharif University of Technology, Tehran

sharif.edu/~langari/

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Interplay of Thermalization & Localization

Topics:

- Eigenstate Thermalization Hypothesis (ETH)
- Ergodicity Breaking (Failure of ETH)
- Many Body Localization (MBL)
  I. MBL in a translationally invariant model
  II. Toric code on ladder geometry
  III. Glassy dynamics due to anyon statistics

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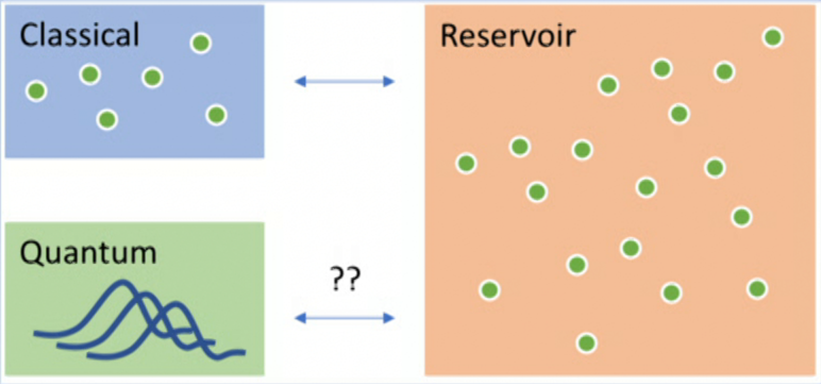
Diffusion

Diffusion is a process toward (thermal) equilibrium, which maximizes entropy.

3

Classical vs. Quantum Systems

- Classic Thermalization:
  - System in contact with reservoir, has energy (and particle) flow
  - can be described by few parameters after long time P, T, V, S, etc....



- Quantum systems
  - can use classical reservoir.
  - Is the concept of reservoir necessary?
  - **No!**

Closed Systems: Quantum Thermalization

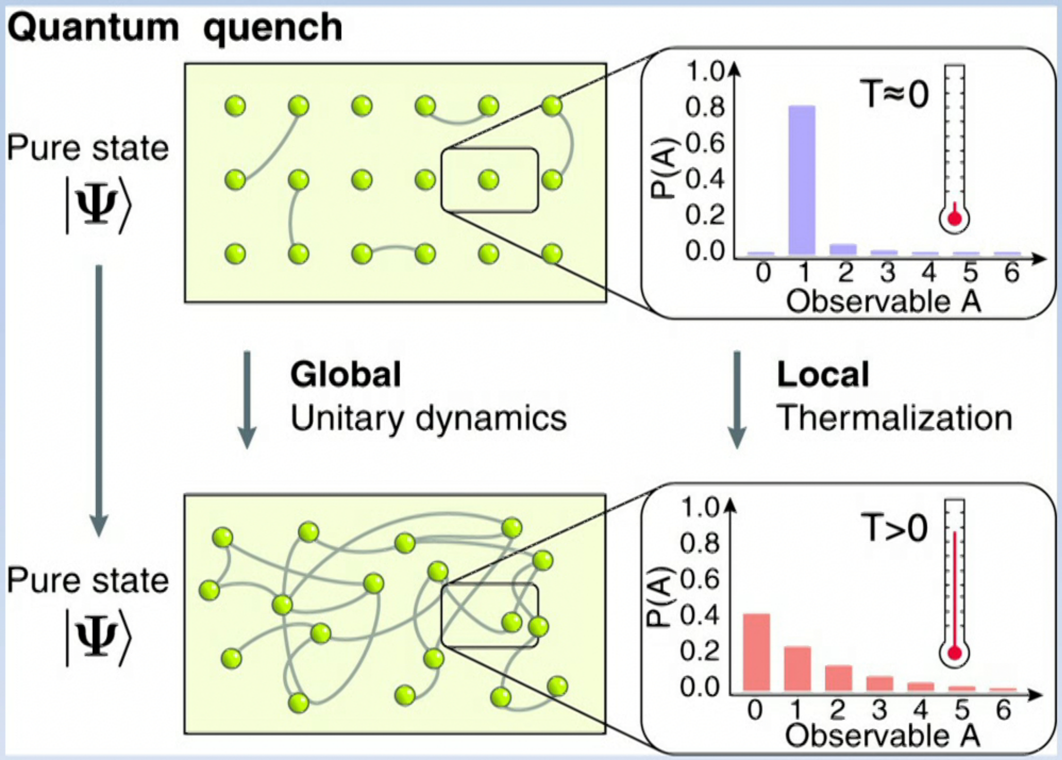
- In quantum system, initial state information perfectly preserved
  - Unitary evolution
  - System at time t depends on exact initial state – **no loss of information**
  - Obvious in eigenstate basis
  - Directly contradicts thermalization!
- But does **subsystem** look thermal at long times?
  - **Can it lose information about initial state to other subsystems!**

$$\rho(t) = e^{-iHt}\rho(0)e^{iHt}$$

$$\lim_{t\to\infty} \rho(t) = \rho(T)$$

$$\lim_{t\to\infty} \rho_S(t) \stackrel{?}{=} \rho_S(T)$$

Schematic of Thermalization Dynamics in Closed Systems



Adam M. Kaufman et al. Science 2016;353:794-800

Eigenstate Thermalization Hypothesis

- Quench protocol: Evolve initial state with a (many-body) Hamiltonian $|\Psi(t)\rangle = \exp(-iHt)|\Psi_0\rangle$

Q. : Does the system reach thermal equilibrium ?

- Expand $|\Psi_0\rangle = \sum_i a_n |n\rangle$ in eigenbasis of $H = \sum_n E_n |n\rangle\langle n|$
- Time-evolved observable (generic Hamiltonian)

$$\langle \mathcal{O}(t) \rangle = \sum_{n,n'} a^*_{n'} a_n e^{-i(E_{n'} - E_n)t} \mathcal{O}_{nn'} \xrightarrow{t\to\infty} \sum_n |a_n|^2 \mathcal{O}_{nn}$$

'Diagonal ensemble'

- Eigenstate thermalization hypothesis (ETH)

Deutsch, Srednicki, Rigol

$\langle n|\mathcal{O}|n\rangle \simeq \langle n'|\mathcal{O}|n'\rangle = \mathcal{O}(E)$ $\quad |n\rangle, |n'\rangle$ in the same energy shell

$\langle n|\mathcal{O}|n'\rangle$ vanish in the thermodynamic limit and for few-body observables

- ETH implies thermalization

$$\langle \mathcal{O}(t \to \infty) \rangle = \mathcal{O}(E) = \mathcal{O}(T) \qquad E = \langle \Psi_0 | H | \Psi_0 \rangle$$

$$E = \langle H \rangle_T$$

F. Alet

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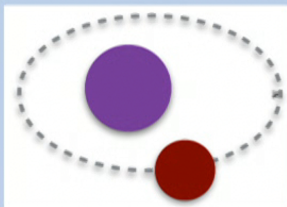
Luca D'Alessio, et. Al. *Advances in Physics*, 2016 Vol. 65, No. 3, 239–362

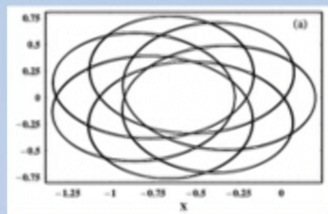
Break Down of ETH

1) Integrable systems

≡

Infinite number of "local" conserved quantities





2) Breaking of translational invariant symmetry in Space, Explicitly or even Spontaneously!

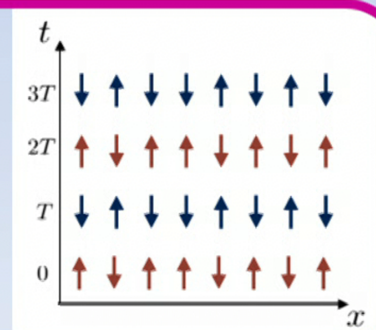
- ❑ Anderson (single particle) localization.
- ❑ Many-Body Localized (MBL) systems.

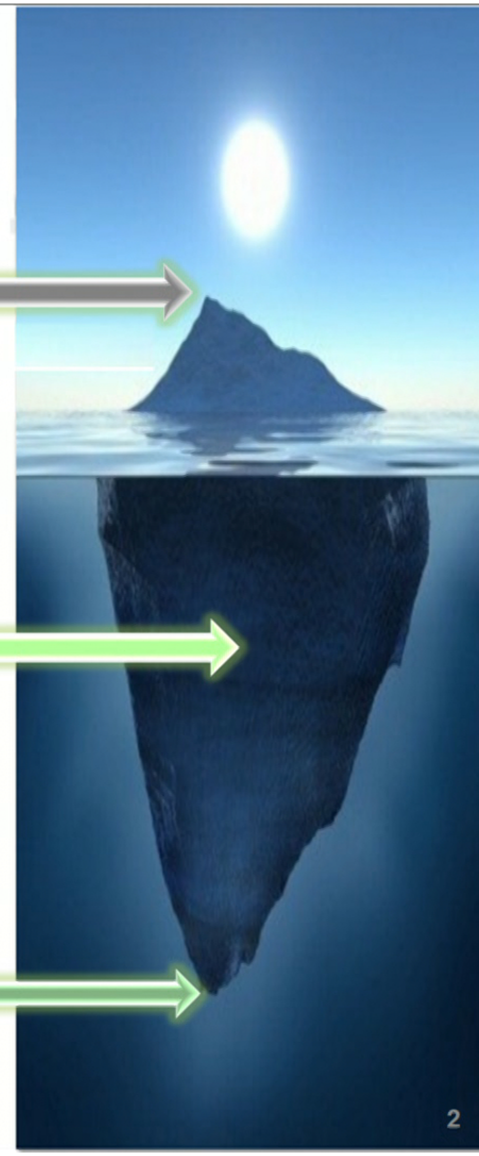
3) Time crystals.

$$H(t) = H(t+T)$$

BUT

$$\langle O(t) \rangle \neq \langle O(t+T) \rangle$$





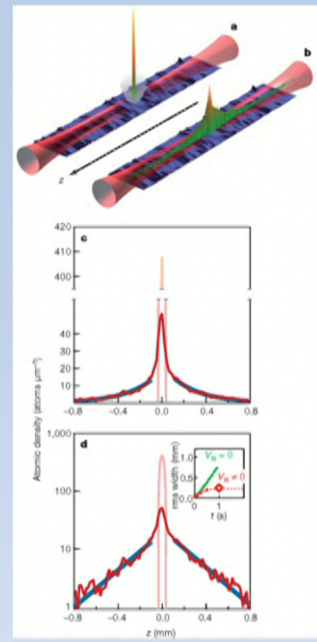
Anderson Localization: Single-Particle Localization

PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

Absence of Diffusion in Certain Random Lattices

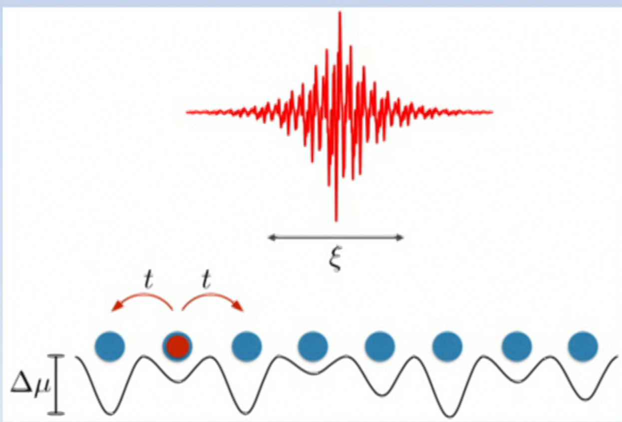
P. W. Anderson
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received October 10, 1957)

This paper presents a simple model for such processes as spin diffusion or conduction in the "impurity band." These processes involve transport in a lattice which is in some sense random, and in them diffusion is expected to take place via quantum jumps between localized sites. In this simple model the essential randomness is introduced by requiring the energy to vary randomly from site to site. It is shown that at low enough densities no diffusion at all can take place, and the criteria for transport to occur are given.



$$H = \sum_i \varepsilon_i |i\rangle\langle i| + \sum_{\langle i,j\rangle} t_{ij} |i\rangle\langle j| + h.c$$

$\Delta\mu = \varepsilon_i = [-W, +W]$

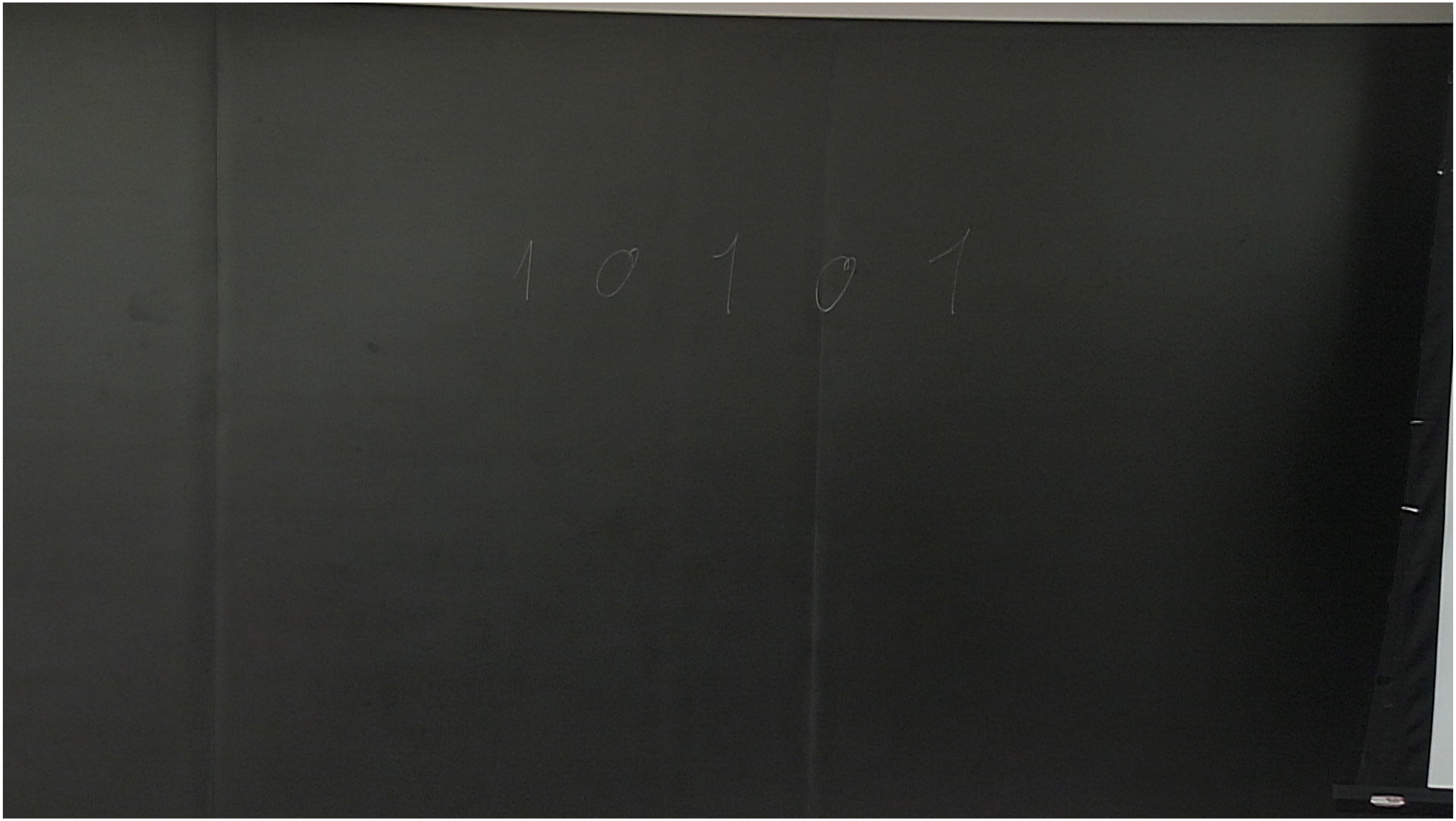


$W > t_c \rightarrow$ localization

Billy et al. "Direct observation of Anderson localizationof matter waves in a controlled disorder". Nature 453, 891-894 (2008).

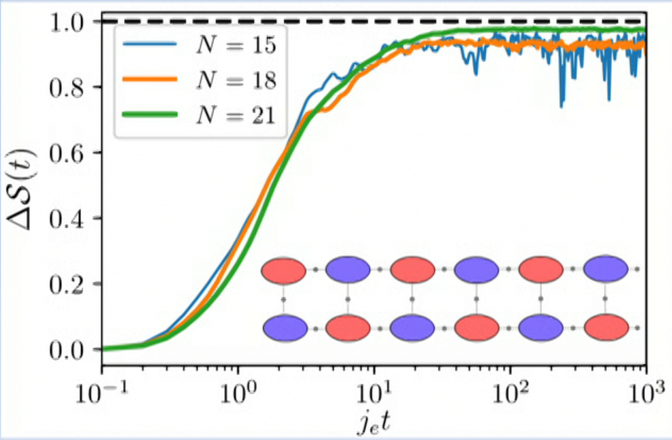
Roati et al. "Anderson localization of a non-interacting Bose-Einstein condensate". Nature 453, 895-898 (2008).

F. Pollmann

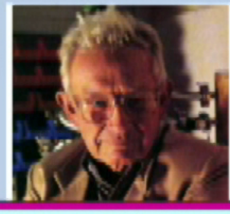

1 0 1 0 1

Resilience of the topological order following a quench

Heating procedure: $$\Delta\mathcal{S} = \frac{S_{ent}(t) - S_0}{S_{page} - S_0}$$



H. Yarloo, AL, A. Vaezi, Phys. Rev. B. 97, 054304 (2018)



Anderson was actually interested in MBL: problem of quantum spin diffusion.

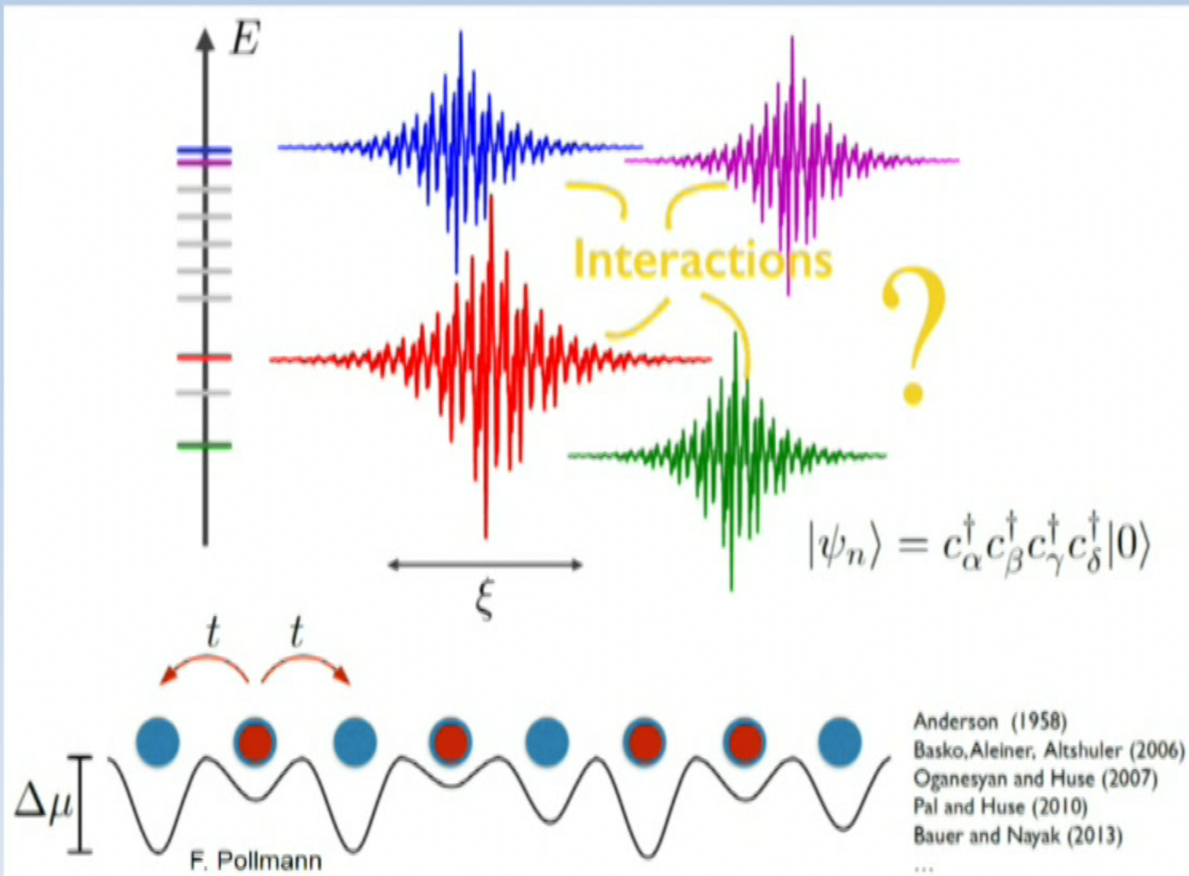
Anderson used a single particle model as an (over) simplification.

Absence of Diffusion in Certain Random Lattices

P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received October 10, 1957)

Such a theorem is of interest for a number of reasons: first, because it may apply directly to spin diffusion among donor electrons in Si, a situation in which Feher[3] has shown experimentally that spin diffusion is negligible; second, and probably more important, as an example of a real physical system with an infinite number of degrees of freedom, having no obvious oversimplification, in which the approach to equilibrium is simply impossible; and third, as the irreducible minimum from which a theory of this kind of transport, if it exists, must start. In particular, it re-emphasizes the caution with which we must treat ideas such as "the thermodynamic system of spin interactions" when there is no obvious contact with a real external heat bath.

Many-Body Localization (MBL)



Many-Body Localization (MBL)

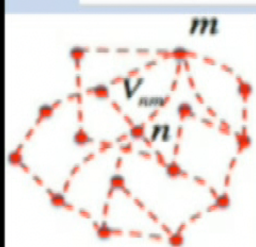
$$H = \sum_i \epsilon_i c_i^\dagger c_i + t \sum_{\langle i,j \rangle} c_i^\dagger c_j + V \sum_i n_i n_{i+1}$$

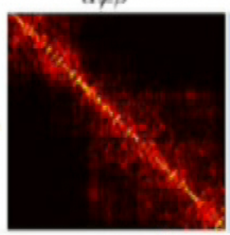
➢ Central assumption:

✓All single particle states are localized.

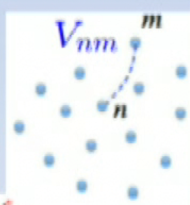
✓Absence of thermal Bath

$$H = \sum_\alpha \mu_\alpha |\alpha\rangle\langle\alpha| + \sum_{\alpha\neq\beta} V_{\beta\alpha} |\beta\rangle\langle\alpha| + h.c$$

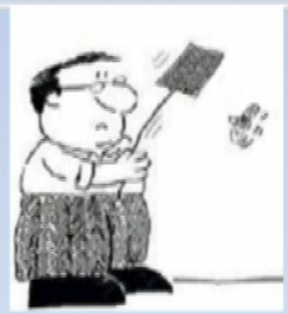


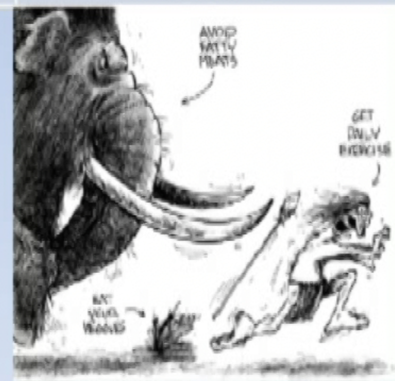


$H_{ij} \propto e^{-|i-j|/\xi}$



Many-body Loc.	Anderson Loc.
Hilbert space	Real space
Random Hartree-Fock energy	Random On-site energy
Interaction	Hopping





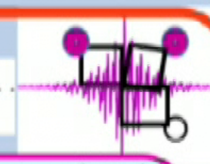
Rough definition of MBL: AL in Hilbert space

MBL and adiabatic continuity

$$H_{AL} = \sum_i \mathcal{J}_i \, \tilde{n}_i$$

Diagonal and non-interacting
just like
Free fermions

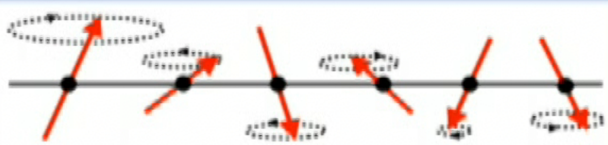
$$H^{eff}_{MBL} = \sum_i \mathcal{J}_i \, \tilde{n}_i + \sum_{i,j} \mathcal{J}_{ij} \, \tilde{n}_i \tilde{n}_j + \sum_{i,j,k} \mathcal{J}_{ijk} \, \tilde{n}_i \tilde{n}_j \tilde{n}_k + \dots$$



Diagonal But interacting
just like
Fermi-liquid theory

Anderson orbitals
only dressed by
interaction

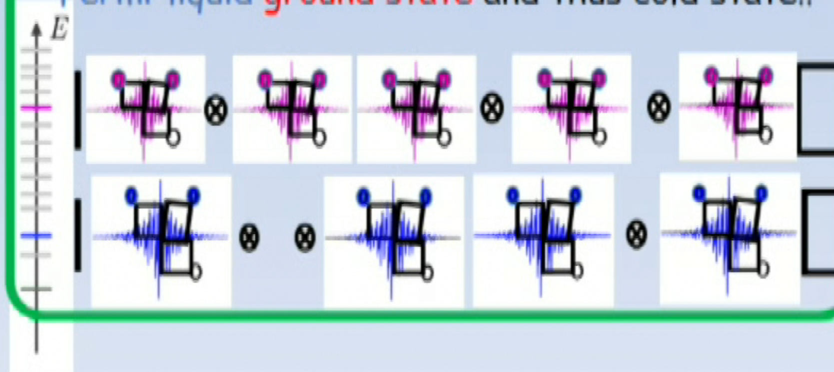
➢Mutually commuting quasi-local conserved quantities:



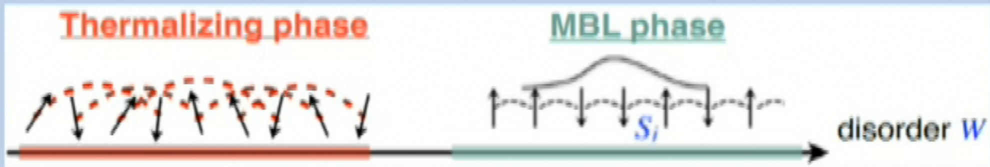
$\tilde{n}_i = n_i + \text{exponential tail}$

MBL: Emergent integribility in non-integreble systems.

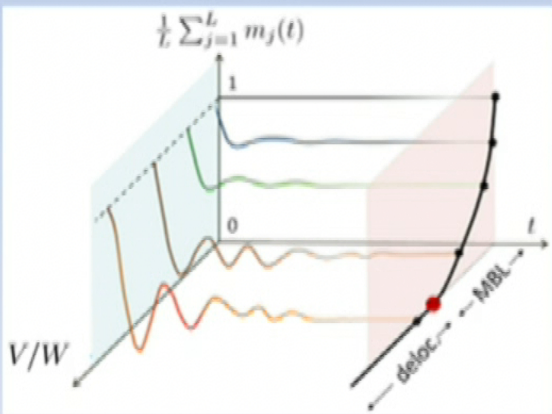
✓Every MBL eigenstates
is like
Fermi-liquid ground state and thus cold state!!



Universal characteristic properties of MBL

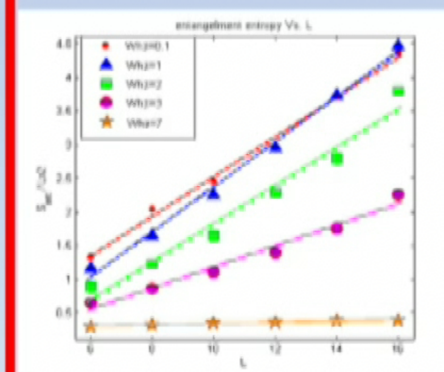


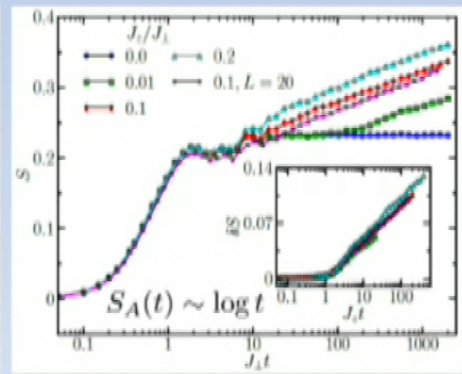
✓Initial stored information never decay even in infinite time



✓Area-law entanglement of highly excited states

✓Logarithmic growth of entanglement (surprisingly unbounded !)



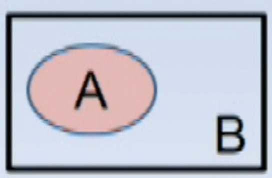


J. Bardarson, F. Pollmann, J. E. Moore, Phys. Rev. Lett. 109, 017202 (2012)

Characteristics of MBL phase

Thermal phase	Single-particle localized	Many-body localized
Memory of initial conditions hidden in global operators at long times	Some memory of local initial conditions preserved in local observables at long times	Some memory of local initial conditions preserved in local observables at long times
Eigenstate thermalization hypothesis (ETH) true	ETH false	ETH false
May have nonzero DC conductivity	Zero DC conductivity	Zero DC conductivity
Continuous local spectrum	Discrete local spectrum	Discrete local spectrum
Eigenstates with volume-law entanglement	Eigenstates with area-law entanglement	Eigenstates with area-law entanglement
Power-law spreading of entanglement from nonentangled initial condition	No spreading of entanglement	Logarithmic spreading of entanglement from nonentangled initial condition
Dephasing and dissipation	No dephasing, no dissipation	Dephasing but no dissipation

S ~ L^d　　　S ~ L^{d-1}　　　S ~ L^{d-1}



$\rho_A^\alpha = \mathrm{tr}_B |\alpha\rangle\langle\alpha|$　　$S_{\mathrm{ent}}^\alpha(A) = -\mathrm{tr}_A \left(\rho_A^\alpha \ln \rho_A^\alpha\right)$

D. A. Abanin and Z. Papic, Ann. Phys. (Berlin) 529, No. 7, 1700169 (2017)
R. Nandkishore and D. A. Huse, Annu. Rev. Condens. Matter Phys. 6:15–38 (2015)

MBL requirements:

- Interaction
- Disorder

Can MBL phase appear in a translationally invariant system? i.e. without disorder

MBL in translationally invariant systems (quasi-MBL) :

➢ Q1: Is Non-ergodic dynamics is possible in the absence of disorder ?

A1: yes !!

N. Y. Yao, C. R. Laumann, J. I. Cirac, M. D. Lukin, and J. E. Moore, Phys. Rev. Lett. **117**, 240601 (2016).

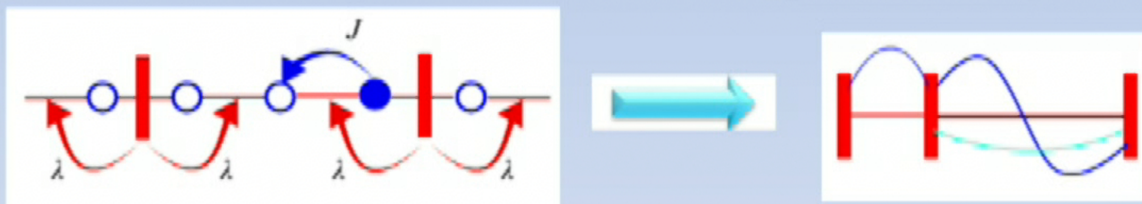
- 1D many body Falikov-Kimball Hamiltonian:

$$H = -J\sum_i \left(a^\dagger_{i+1}a_i + a^\dagger_i a_{i+1}\right)\left(1 - b^\dagger_i b_i\right) \;-\lambda \sum \left(b^\dagger_{i+1}b_i + b^\dagger_i b_{i+1}\right)$$

massive b particle

light a particle

$J \gg \lambda$:



M. Schiulaz, A. Silva, and M. Müller, Phys. Rev. B 91, 184202 (2015)

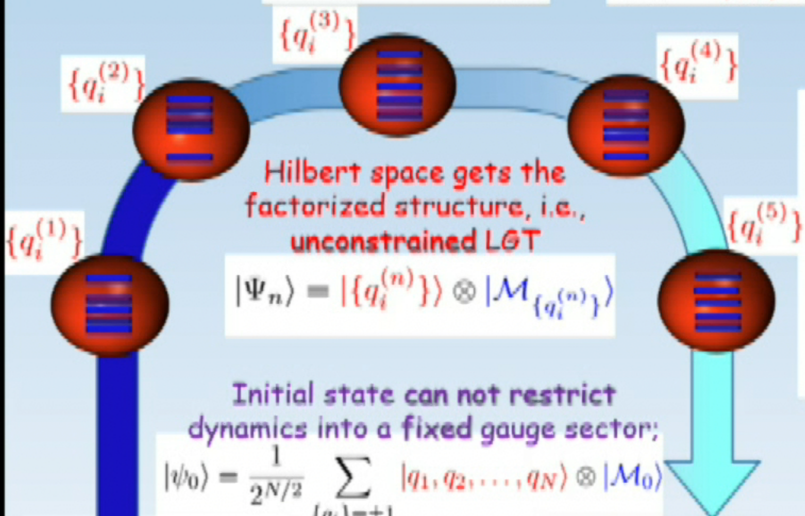
High entropic configuration ⟹ Higher Configurational disorder ⟹ Non-ergodic dynamics

Disorder-free localization due to super-selection rule:

$$\hat{H} = -J\sum_{\langle ij\rangle}\hat{\sigma}^z_{i,j}\hat{f}^\dagger_i\hat{f}_j - h\sum_i \hat{\sigma}^x_{i-1,i}\hat{\sigma}^x_{i,i+1}$$

$|\psi(t)\rangle = e^{-iHt}|\psi_0\rangle$

$|\psi_0\rangle = |\psi_s\rangle \otimes |\psi_f\rangle$ $\quad |\psi_s\rangle = |\uparrow,\uparrow,\ldots,\uparrow\rangle$ $\quad |\psi_f\rangle = |1,0,1,0,\ldots\rangle$



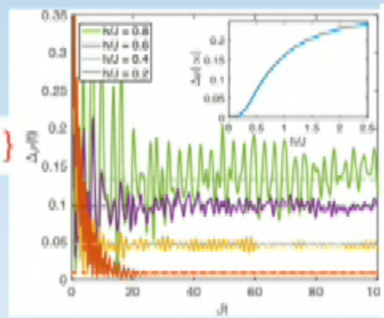
Hilbert space gets the factorized structure, i.e., unconstrained LGT

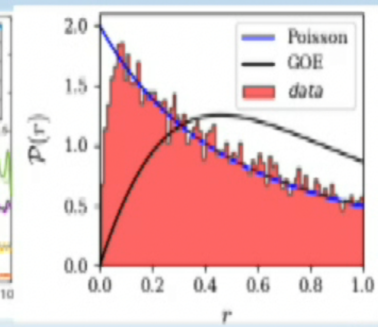
$$|\Psi_n\rangle = |\{q_i^{(n)}\}\rangle \otimes |\mathcal{M}_{\{q_i^{(n)}\}}\rangle$$

Initial state can not restrict dynamics into a fixed gauge sector;

$$|\psi_0\rangle = \frac{1}{2^{N/2}}\sum_{\{q_i\}=\pm 1}|q_1,q_2,\ldots,q_N\rangle \otimes |\mathcal{M}_0\rangle$$

$|\psi_s\rangle = |\uparrow,\uparrow,\ldots,\uparrow\rangle$ $\quad |\psi_{\mathcal{M}}(t)\rangle = e^{itH_{\{q_i\}}}|\mathcal{M}_0\rangle$





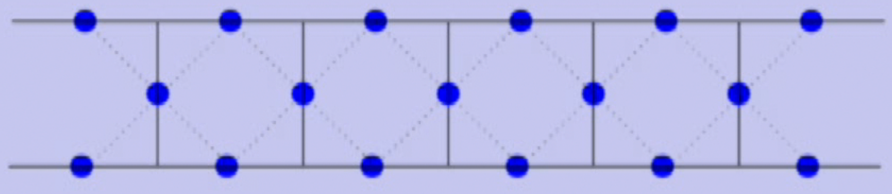
$$\hat{H}_{\{q_j\}} = -J\sum_{\langle ij\rangle}\hat{c}^\dagger_i\hat{c}_j + 2h\sum_j q_j(\hat{c}^\dagger_j\hat{c}_j - 1/2)$$

Emergent dynamical disorder

$$\mathcal{P}(q) = \frac{1}{2}\delta(q_i+1) + \frac{1}{2}\delta(q_i-1)$$

A. Smith, J. Knolle, D. L. Kovrizhin, and R. Moessner, Phys. Rev. Lett. 118, 266601 (2017).

New mechanism for disorder free MBL in: Kitaev-toric code on ladder geometry (a quasi-one dimensional model)



$$\mathcal{H}_K = -J_v \sum_{\perp,\top} A_v \quad - \quad J_p \sum_{\Box} B_p,$$

$$A_v \equiv \prod_i \sigma_i^x, \;\; i \in \perp \text{ or } \top\,; \;\; J_v > 0, \qquad B_p \equiv \prod_j \sigma_j^z, \;\; j \in \Box\,; \;\; J_p > 0,$$

Let $|\Omega\rangle \equiv \otimes_i |+\rangle_i$, $\sigma_i^x |+\rangle_i = |+\rangle_i$ the 2-fold degenerate ground states:

$$|\psi_K\rangle = \frac{1}{2^N} \prod_p (1 + B_p)|\Omega\rangle, \quad |\psi'_K\rangle = W_z|\psi_K\rangle, \quad W_z = \prod_{\ell \in one\ leg} \sigma_\ell^z$$

2D: A. Kitaev, Annals of Physics 303, 2 (2003); 2-legs ladder: V. Karimipour, Phys. Rev. B. **79**, 214435 (2009).

AL, A. Mohammad-Aghaei and R. Haghshenas, Phys. Rev. B. 91, 024415 (2015)

The main idea

$$H_0^{KL} = -j_m \sum_i B_p(i) - j_e \sum_i \left(A_s^r(i) + A_s^b(i)\right)$$

- $A_v|\psi\rangle = -|\psi\rangle$ implies an e anyon on v

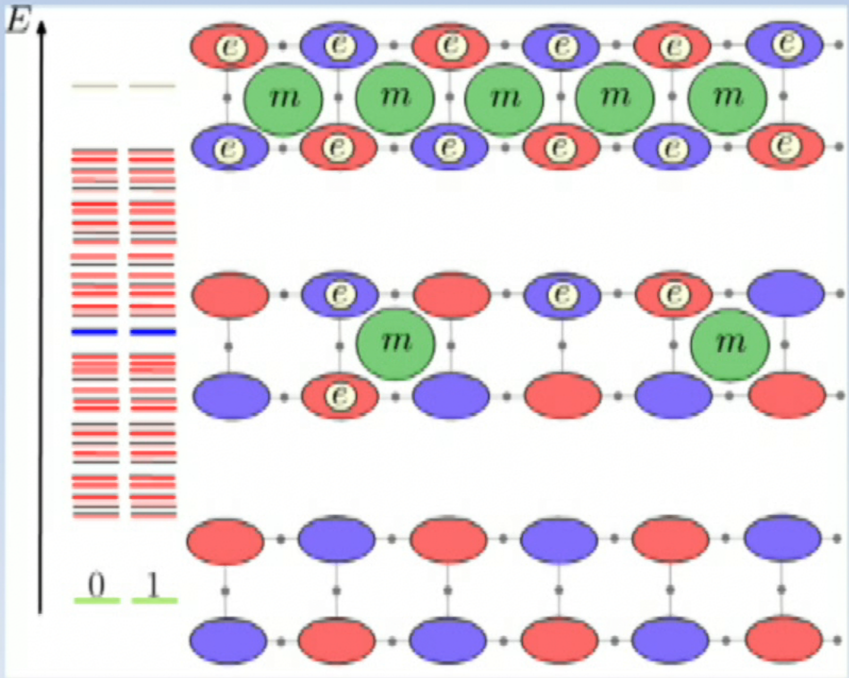
$$\hat{n}_s^e = (1 - \hat{A}_s)/2$$

- $B_p|\psi\rangle = -|\psi\rangle$ implies an m anyon on p

$$\hat{n}_p^m = (1 - \hat{B}_p)/2$$

The main idea

$$H_0^{KL} = -j_m \sum_i B_p(i) - j_e \sum_i \left(A_s^r(i) + A_s^b(i)\right)$$



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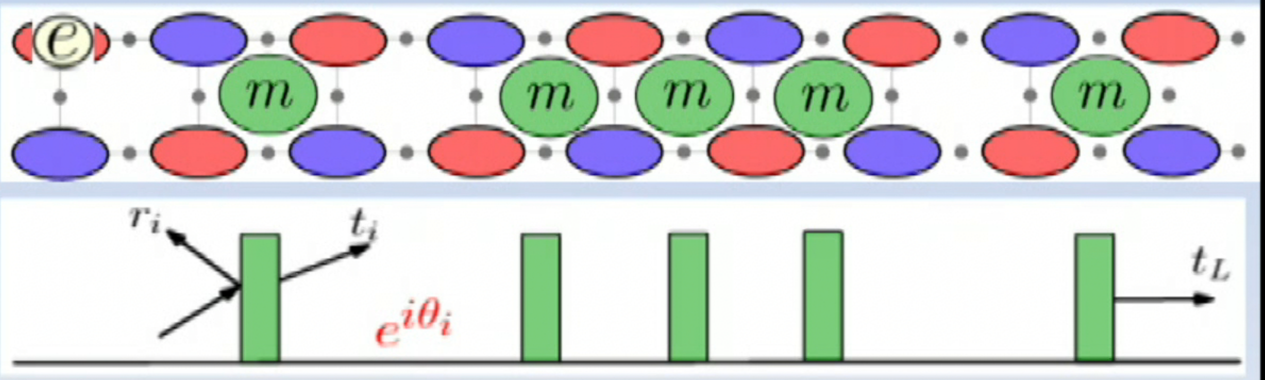
$$H_0^{KL} = -j_m \sum_i B_p(i) - j_e \sum_i \left(A_s^r(i) + A_s^b(i)\right)$$

Anyons feel a random phase while moving across the system just like to what happens in AL.



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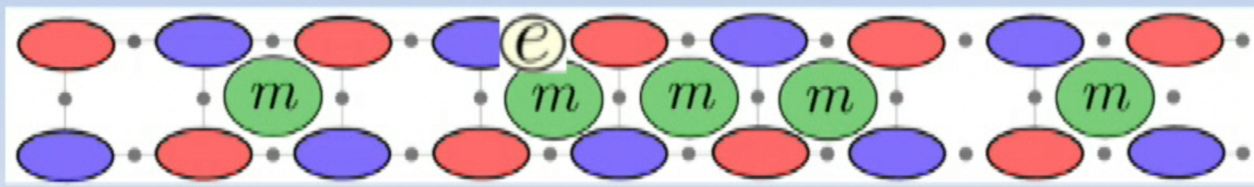

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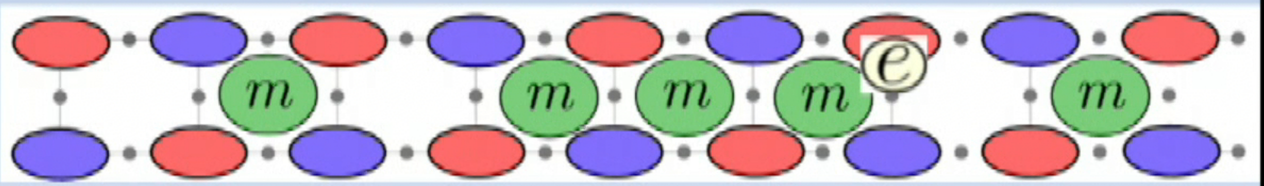

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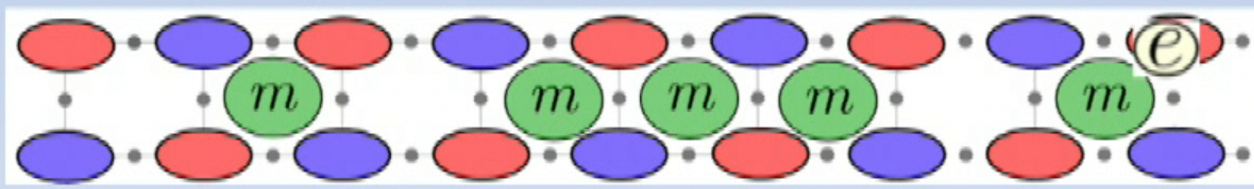
Anyons feel a random phase while moving across the system just like to what happens in AL.



The main idea

$$H_0^{KL} = -j_m \sum_i B_p(i) - j_e \sum_i \left(A_s^r(i) + A_s^b(i)\right)$$

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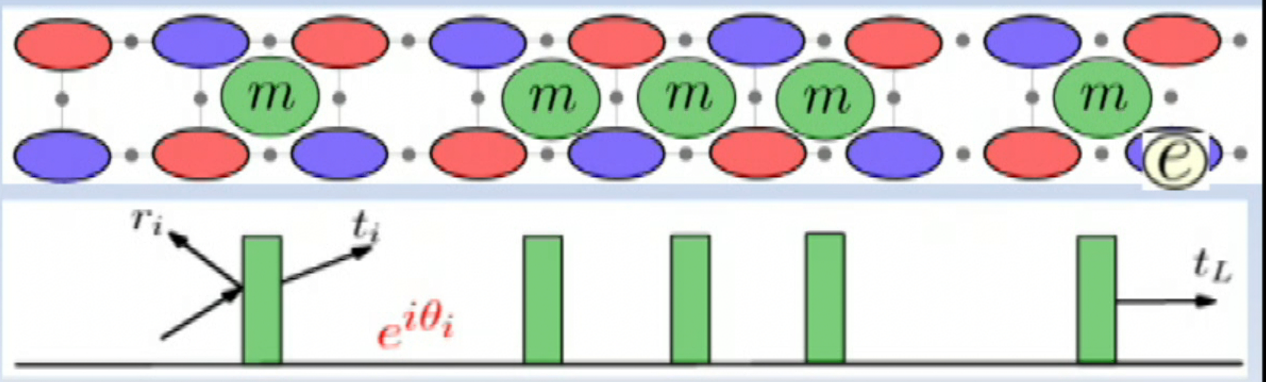
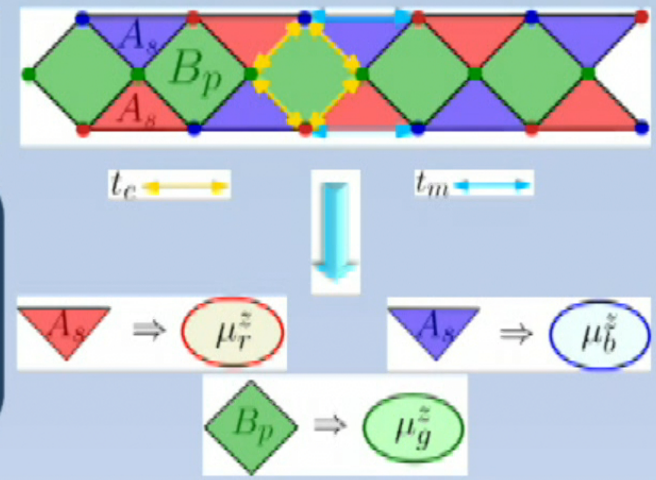
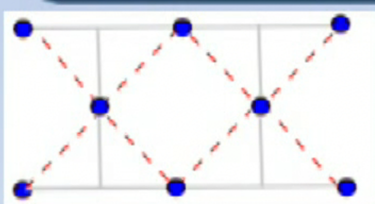


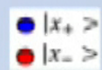
Disorder-free localization in Kitaev Ladder system

$$H_{KLI} = -j_m \sum_{p=1}^{L} \hat{B}_p - j_e \sum_{s=1}^{2L} \hat{A}_s^{\perp,\top} - t_e \sum_{\langle i,j \rangle \in p} Z_i Z_j - t_m \sum_{\langle i,j \rangle \in leg} X_i X_j$$

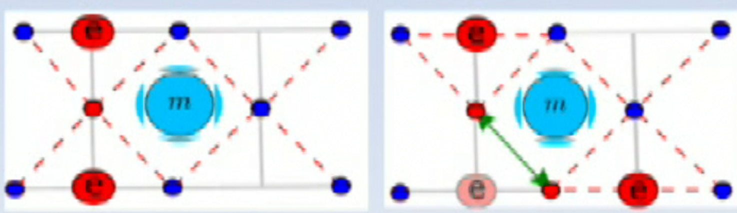


- ✓ The presence of one type of anyons hinders the kinetic term of the other kind.
- ✓ The e-charges are confined between static m-charges.
- ✓ Anyons feel a random phase while moving across the system that purely rooted in nontrivial anyonic statistics.





$$H_{\rm dual}^{KL} = -\sum_i \left(j_m \mu_{g,i}^z + t_m \mu_{g,i}^x \mu_{g,i+1}^x (1 + \mu_{r,i+1}^z \mu_{b,i+1}^z)\right)$$
$$-\sum_i \left(j_e \mu_{b,i}^z + t_e \mu_{b,i}^x \mu_{b,i+1}^x (1 + \mu_{g,i}^z)\right)$$
$$-\sum_i \left(j_e \mu_{r,i}^z + t_e \mu_{r,i}^x \mu_{r,i+1}^x (1 + \mu_{g,i}^z)\right)$$



H. Yarloo, AL, A. Vaezi, Phys. Rev. B. 97, 054304 (2018)

The main idea

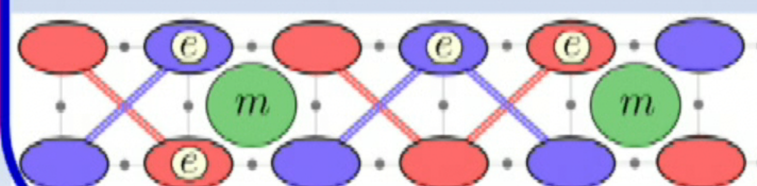
$$H^{KL} = H_0^{KL} - t_e \sum_{\langle i,j\rangle \in \partial p} Z_i Z_j - t_m \sum_{i \in legs} X_i X_{i+1}$$

✓ Light particles (charges) see Heavy ones (fluxes) as static barriers.

✓ After non-local dual mapping:

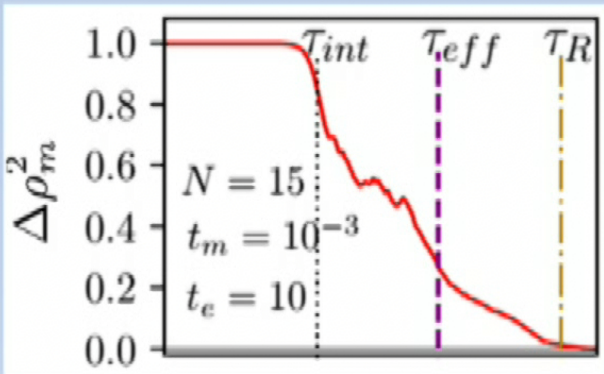
$$t_e'(i) = 2t_e(1 - n_i^m)$$

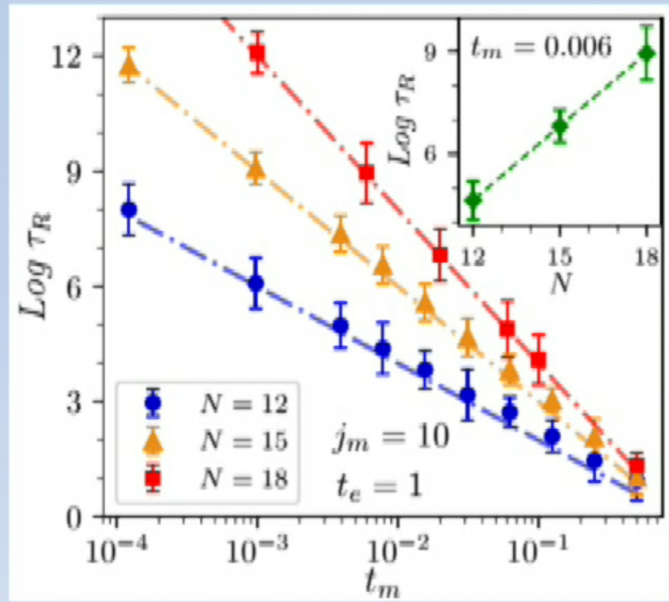
✓ The e-charges are confined between static m-charges.



Disorder-free localization

$$\Delta\rho_m^2(t) \equiv \frac{1}{L}\sum_{p=1}^{L} |\langle\psi_{N_m}|(n_{p+1}^m(t) - n_p^m(t))|\psi_{N_m}\rangle|^2$$





- translational symmetry is *dynamically* broken for $t_m \ll t_e$.

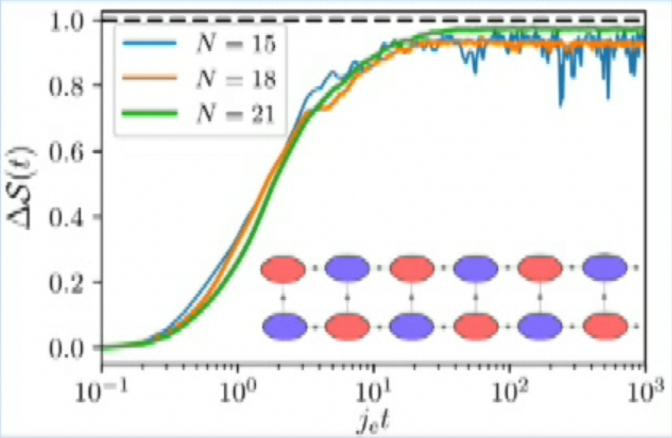
✓ Exponentially diverging behavior for complete relaxation time:
$\tau_R t_m \propto (t_e/t_m)^{N_m - 1}$

Translational symmetry is *dynamically* broken.

H. Yarloo, AL, A. Vaezi, Phys. Rev. B. 97, 054304 (2018)

Resilience of the topological order following a quench

Heating procedure: $\Delta \mathcal{S} = \dfrac{S_{ent}(t) - S_0}{S_{page} - S_0}$



H. Yarloo, AL, A. Vaezi, Phys. Rev. B. 97, 054304 (2018)

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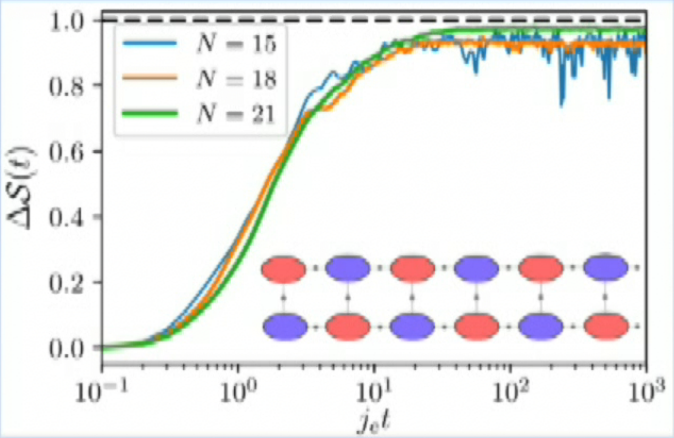

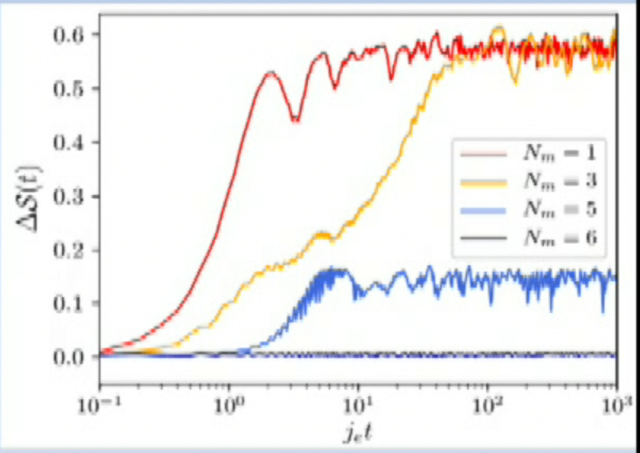

It is more favorable for the initial information to be encoded in subspaces with higher density of errors.

H. Yarloo, AL, A. Vaezi, Phys. Rev. B. 97, 054304 (2018)

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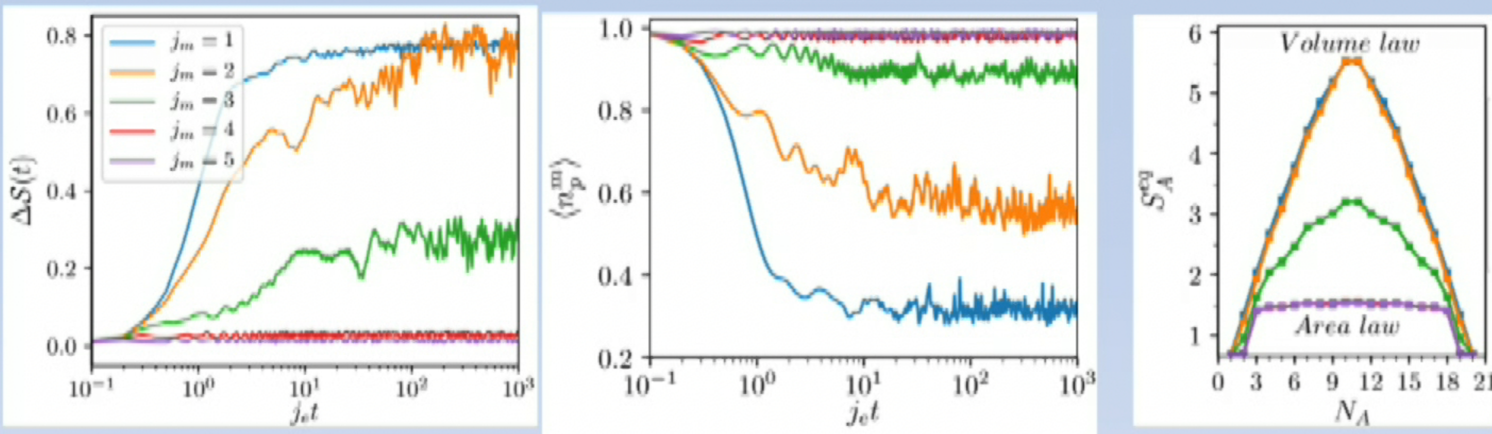

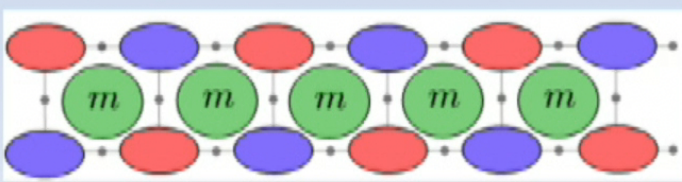

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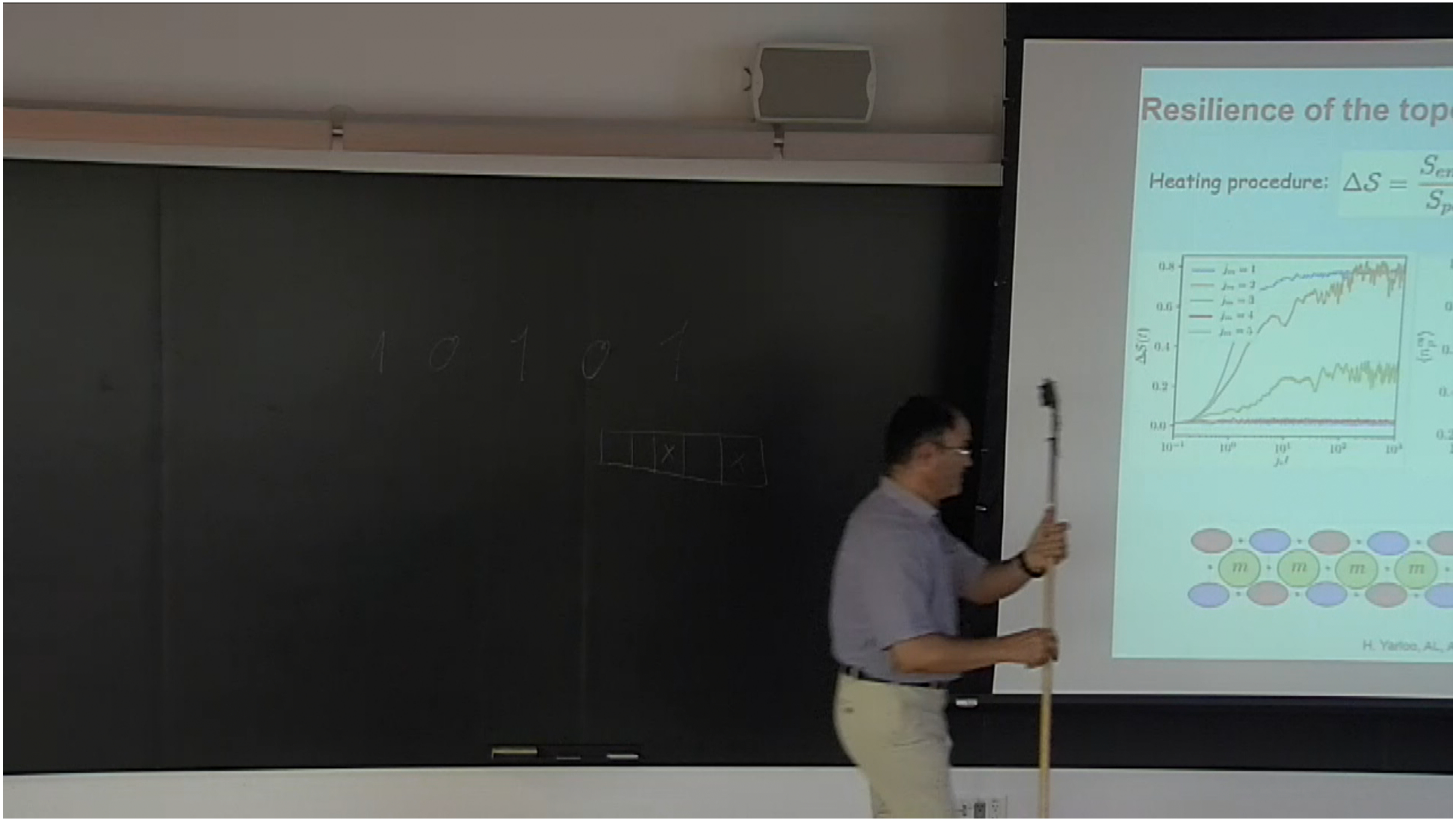
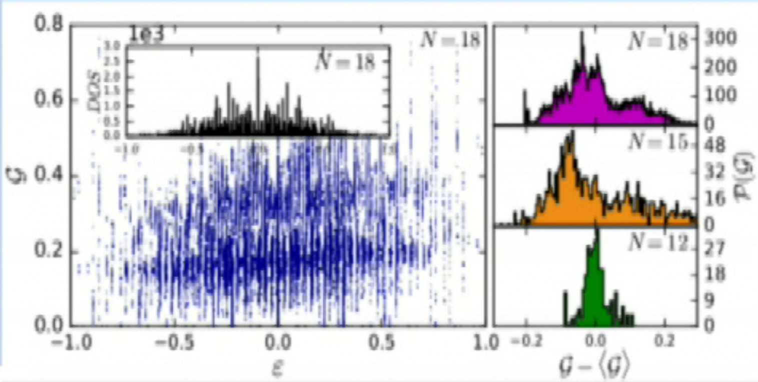
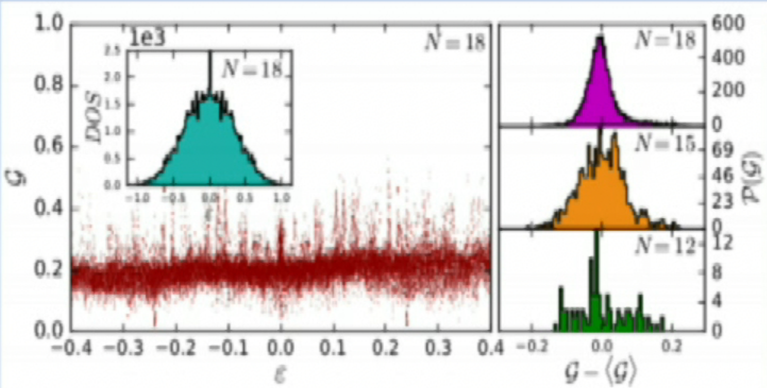


Resilience of the top
Heating procedure:
H. Yarloo, AL,

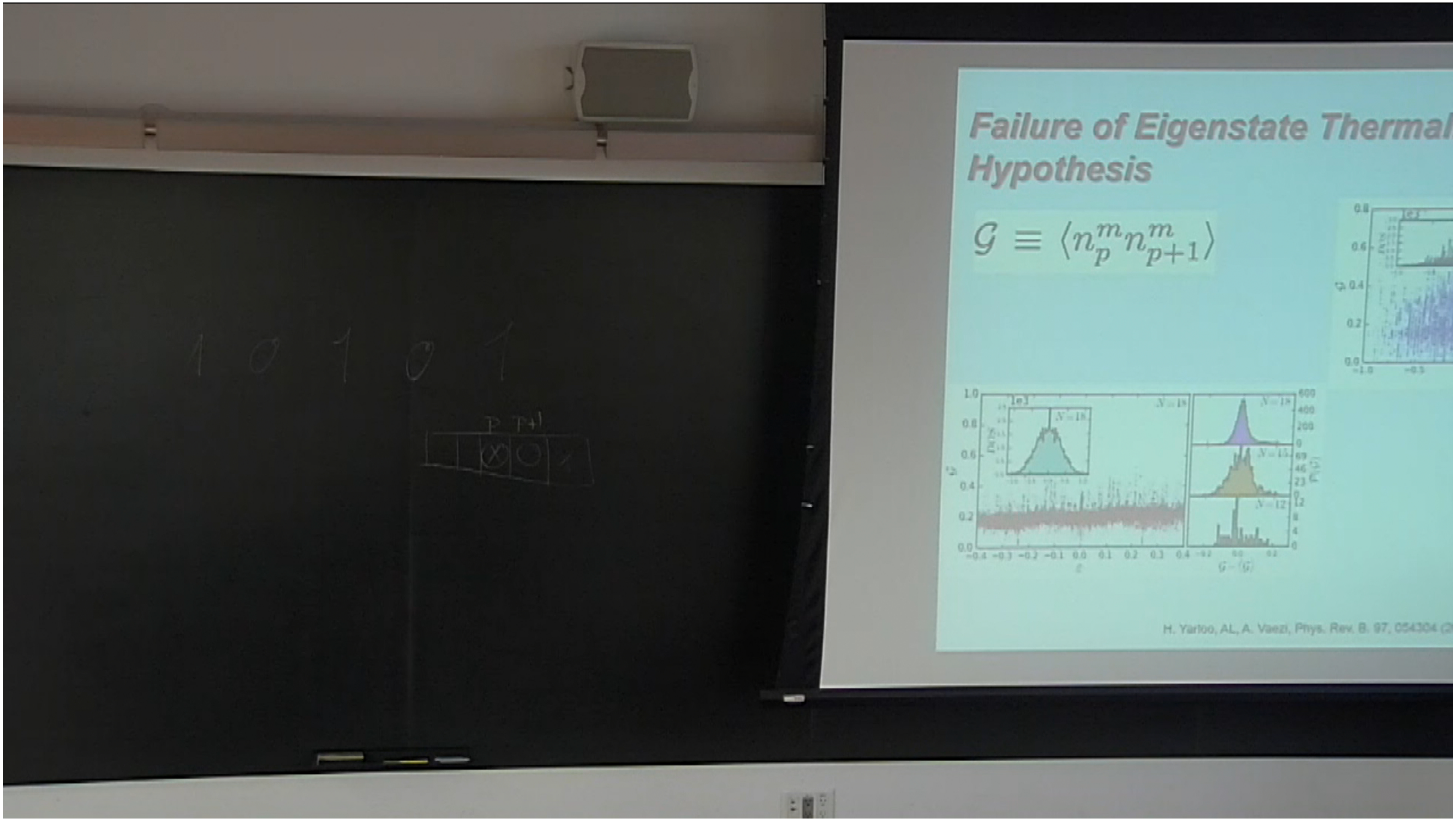
Failure of Eigenstate Thermalization Hypothesis

$$\mathcal{G} \equiv \langle n_p^m n_{p+1}^m \rangle$$





H. Yarloo, AL, A. Vaezi, Phys. Rev. B. 97, 054304 (2018)


Failure of Eigenstate Thermal
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$\mathcal{G} \equiv \langle n_p^m n_{p+1}^m \rangle$
H. Yarloo, AL, A. Vaezi, Phys. Rev. B. 97, 054304
1 0 1 0 1
p p+1

Summary:

- Random arrangement of heavy anyons in high energy eigenstates dynamically suppresses the diffusion of light ones.
- Self-induced disorder, is purely rooted in nontrivial anyonic statistics.
- This effect increases the robustness of topological order.
- A typical initial inhomogeneity gives birth to a glassy dynamics.
- Slow growth of entanglement entropy, with characteristic time scales bearing resemblance with those of inhomogeneity relaxation.
- impeding logical errors in highly excited states by self-localization of anyons.
- the stronger environmental perturbations the higher strength of self-disorder and boosting the tendency of anyons toward self-localization
- It is more favorable for the initial information to be encoded in subspaces with higher density of errors!

Collaborators:

Abolhassan Vaezi (Stanford University)

Hadi Yarloo (Sharif University of Technology)

Many thanks for your attention

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