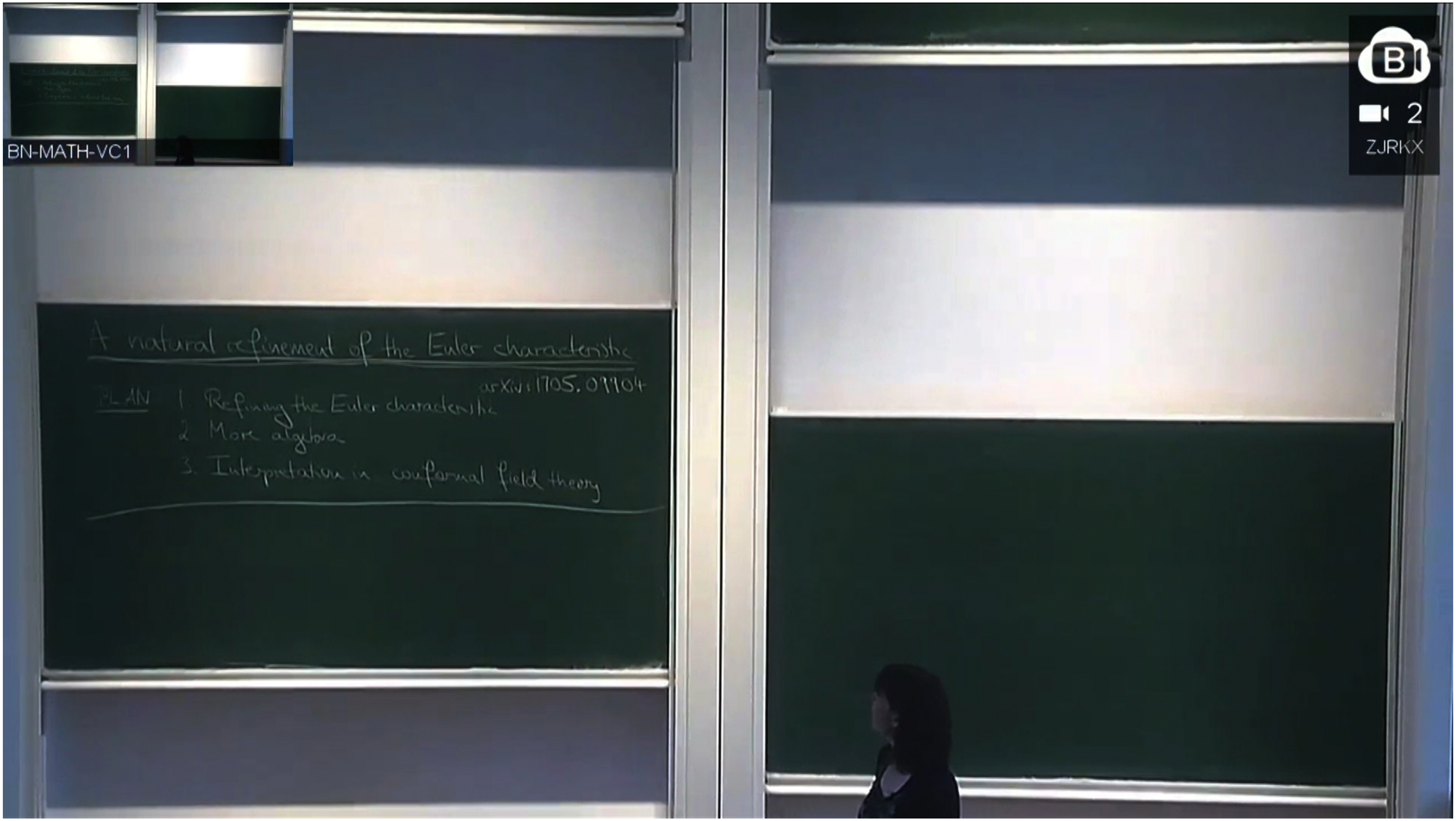
Title: A natural reï¬•nement of the Euler characteristic

Date: Aug 17, 2018 11:00 AM

URL: http://pirsa.org/18080079

Abstract: The Euler characteristic of a compact complex manifold M is a classical cohomological invariant. Depending on the viewpoint, it is most natural to interpret it as an index of an elliptic diï¬€erential operator on M, or as a supersymmetric index in superconformal ï¬•eld theories â€œon Mâ€•. Reï¬•ning the Euler characteristic but keeping with both index theoretic interpretations, one arrives at the notion of complex elliptic genera. We argue that superconformal ï¬•eld theory motivates further reï¬•nements of these elliptic genera which result in a choice of several new invariants, all of which have lost their interpretation in terms of index theory. However, at least if M is a K3 surface, then superconformal ï¬•eld theory and higher algebra select the same new invariant as a natural reï¬•nement of the complex elliptic genus.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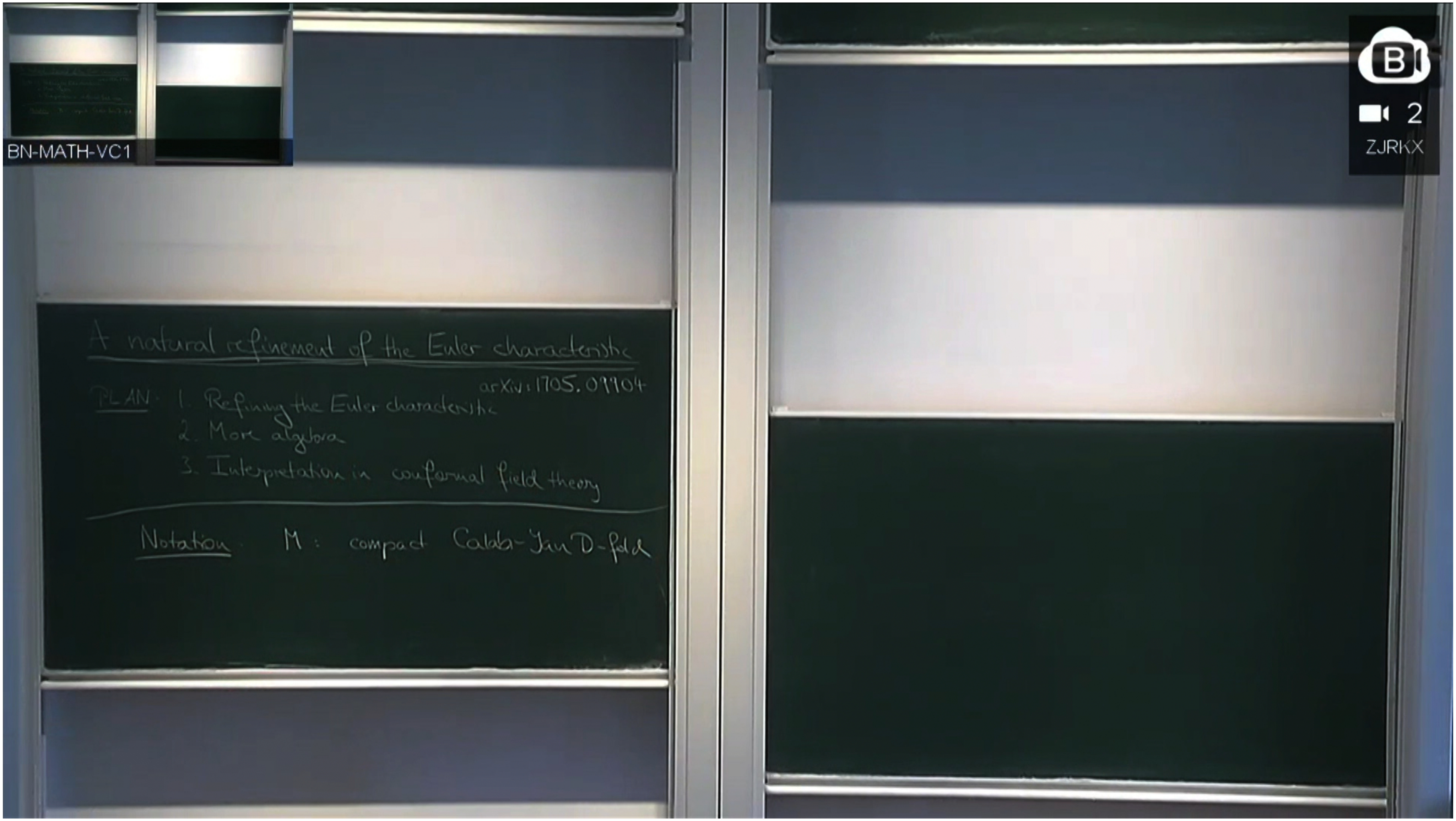
A natural refinement of the Euler characteristic
arXiv:1705.09904
PLAN
1. Refining the Euler characteristic
2. More algebra
3. Interpretation in conformal field theory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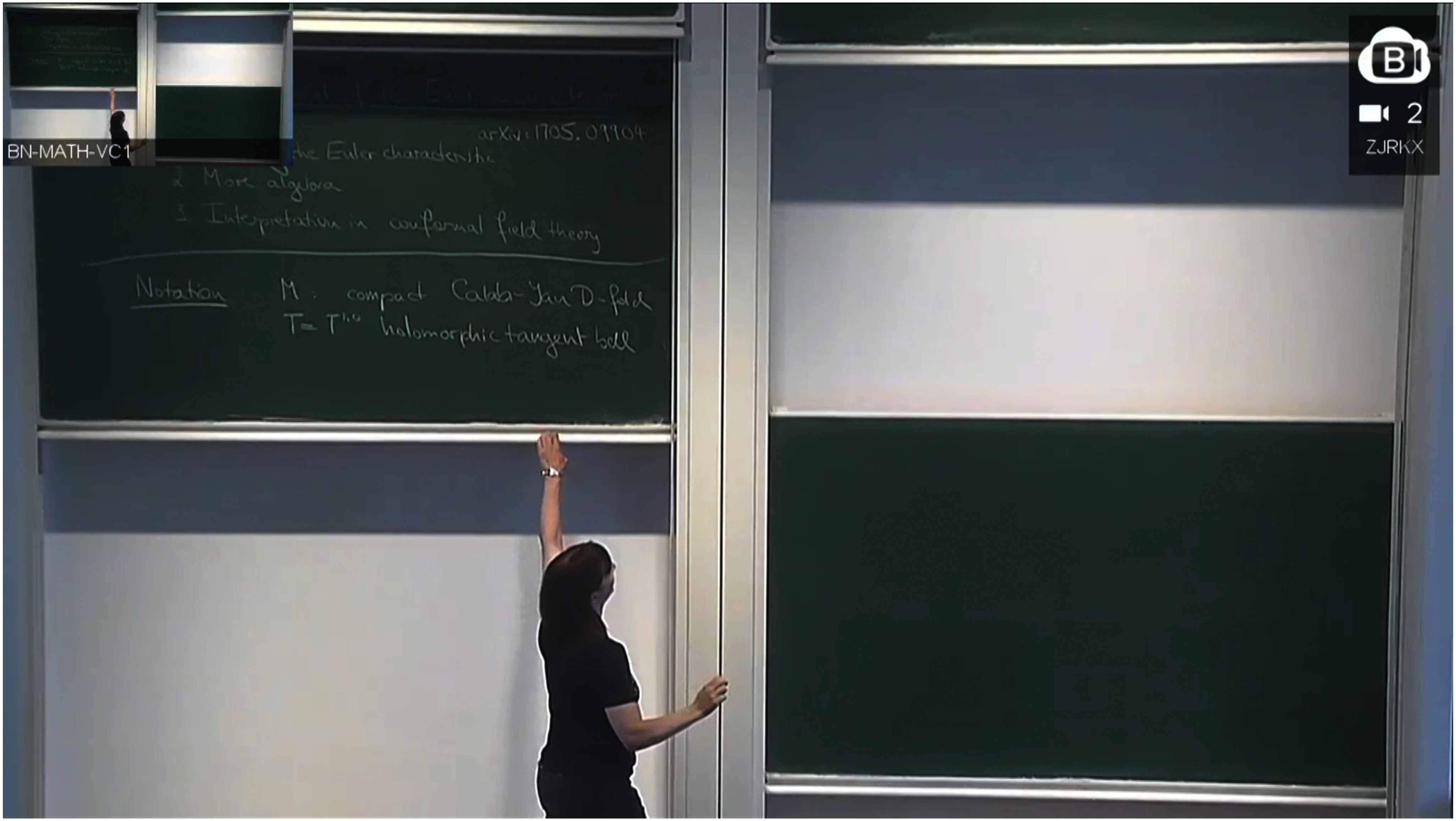
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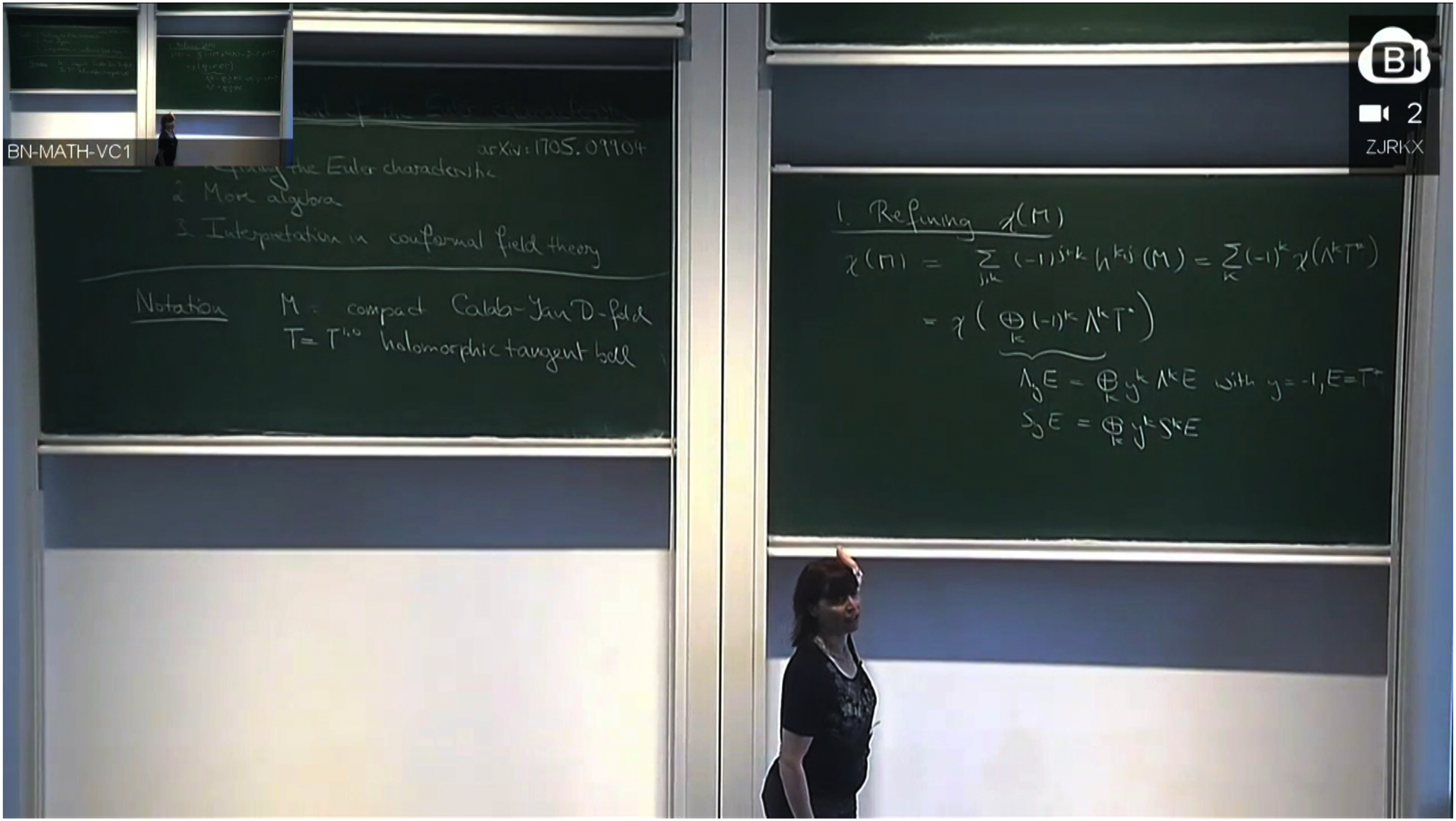
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χ(M) = Σ_{j,k} (-1)^{j+k} h^{k,j}(M) = Σ_k (-1)^k χ(Λ^k T*)
= χ(⊕_k (-1)^k Λ^k T*)
Λ_y E = ⊕_k y^k Λ^k E with y = -1, E = T*
S_y E = ⊕_k y^k S^k E
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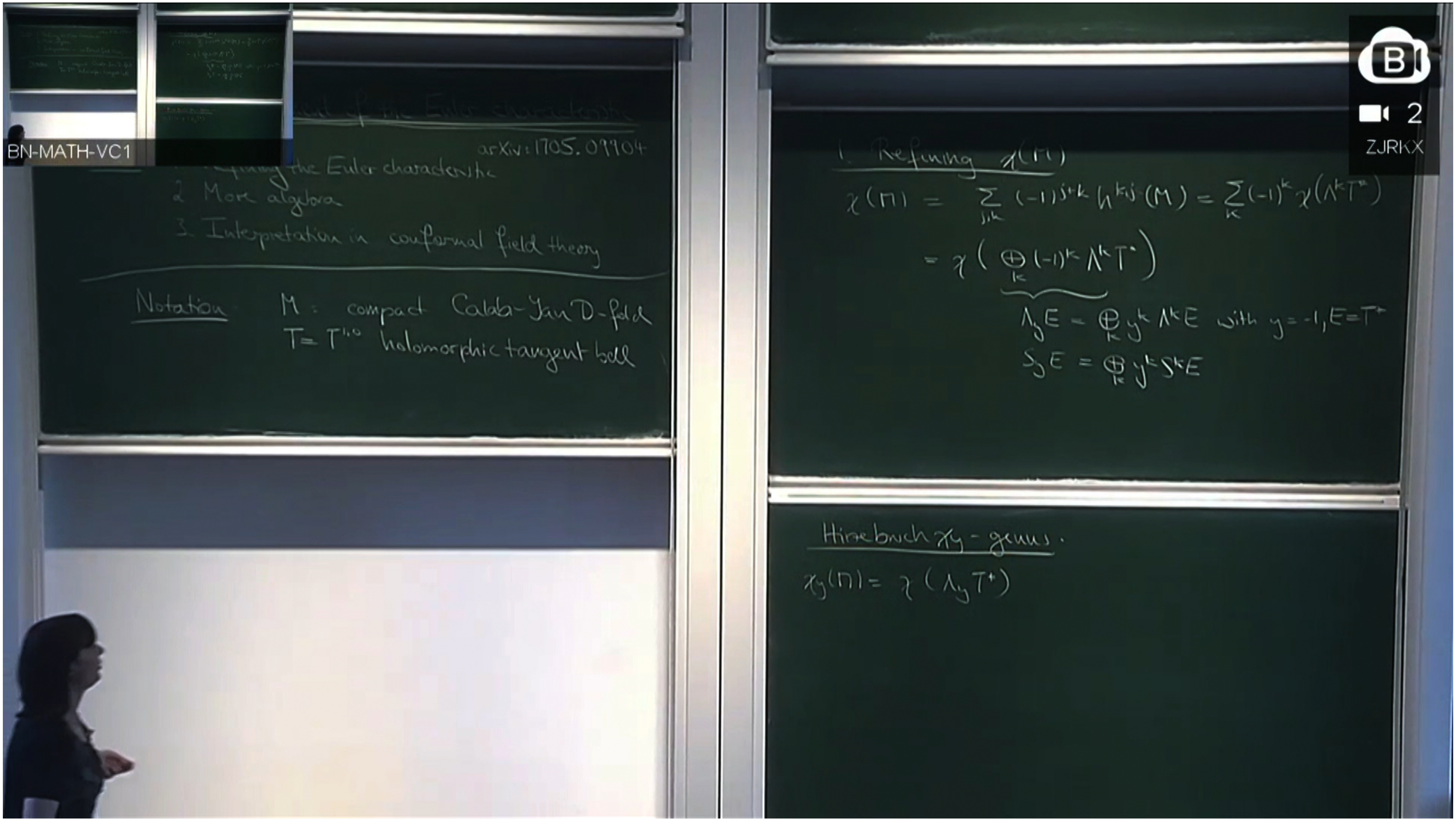
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χ_y(M) = χ(Λ_y T*)
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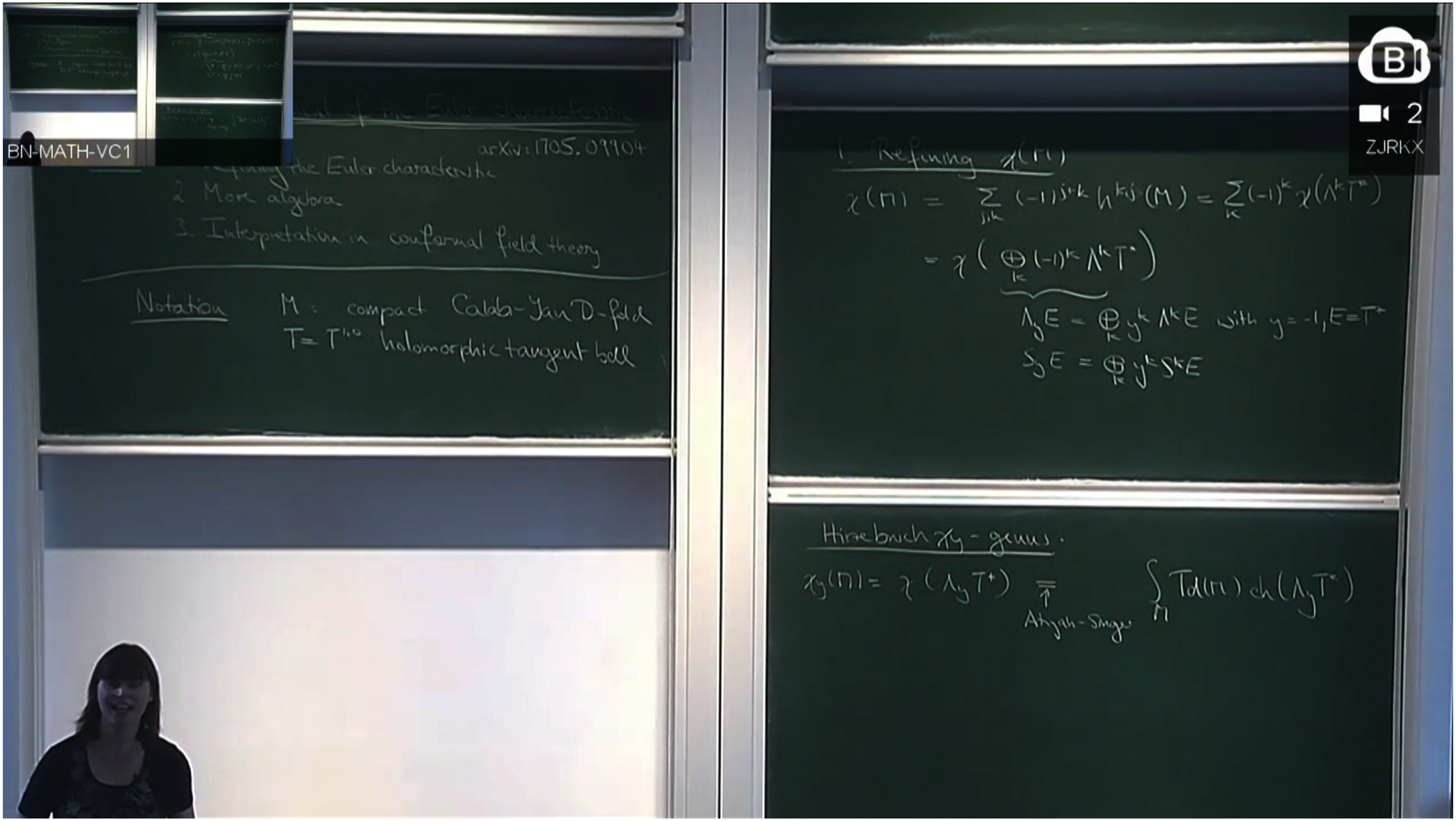
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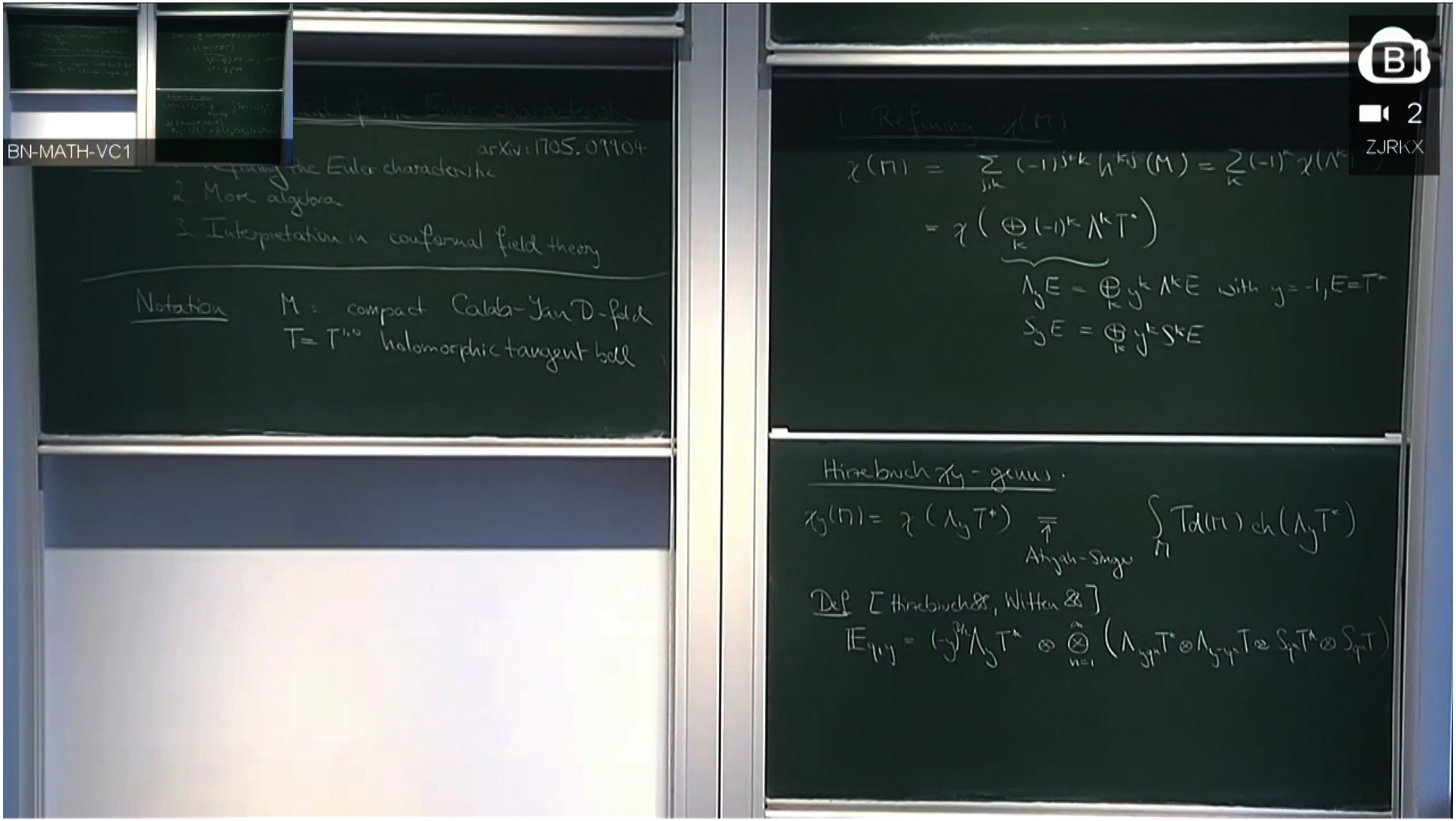
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$= \chi\left(\bigoplus_k (-1)^k \Lambda^k T^*\right)$
$\Lambda_y E = \bigoplus_k y^k \Lambda^k E$ with $y=-1, E=T^*$
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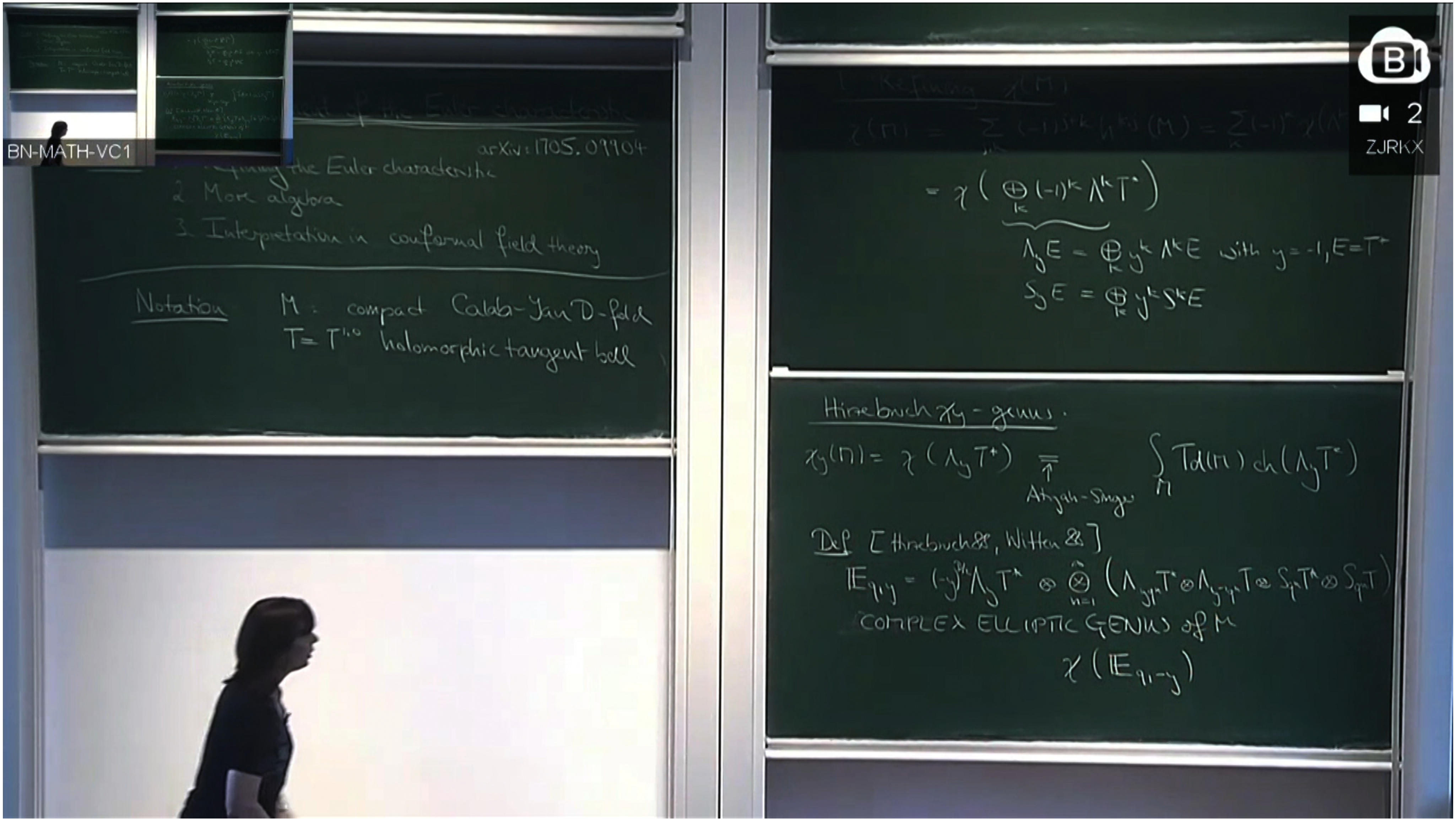
Hirzebruch χ_y-genus:
$\chi_y(M) = \chi(\Lambda_y T^*) = \int_M Td(M)\, ch(\Lambda_y T^*)$
Atiyah-Singer
Def [Hirzebruch88, Witten88]
$\mathbb{E}_{q,y} = (-y)^{D/2} \Lambda_y T^* \otimes \bigotimes_{n=1}^{\infty} \left(\Lambda_{yq^n} T^* \otimes \Lambda_{y^{-1}q^n} T \otimes S_{q^n} T^* \otimes S_{q^n} T\right)$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Def [Hirzebruch 88, Witten 88]
COMPLEX ELLIPTIC GENUS of M
χ (E_{q,-y})
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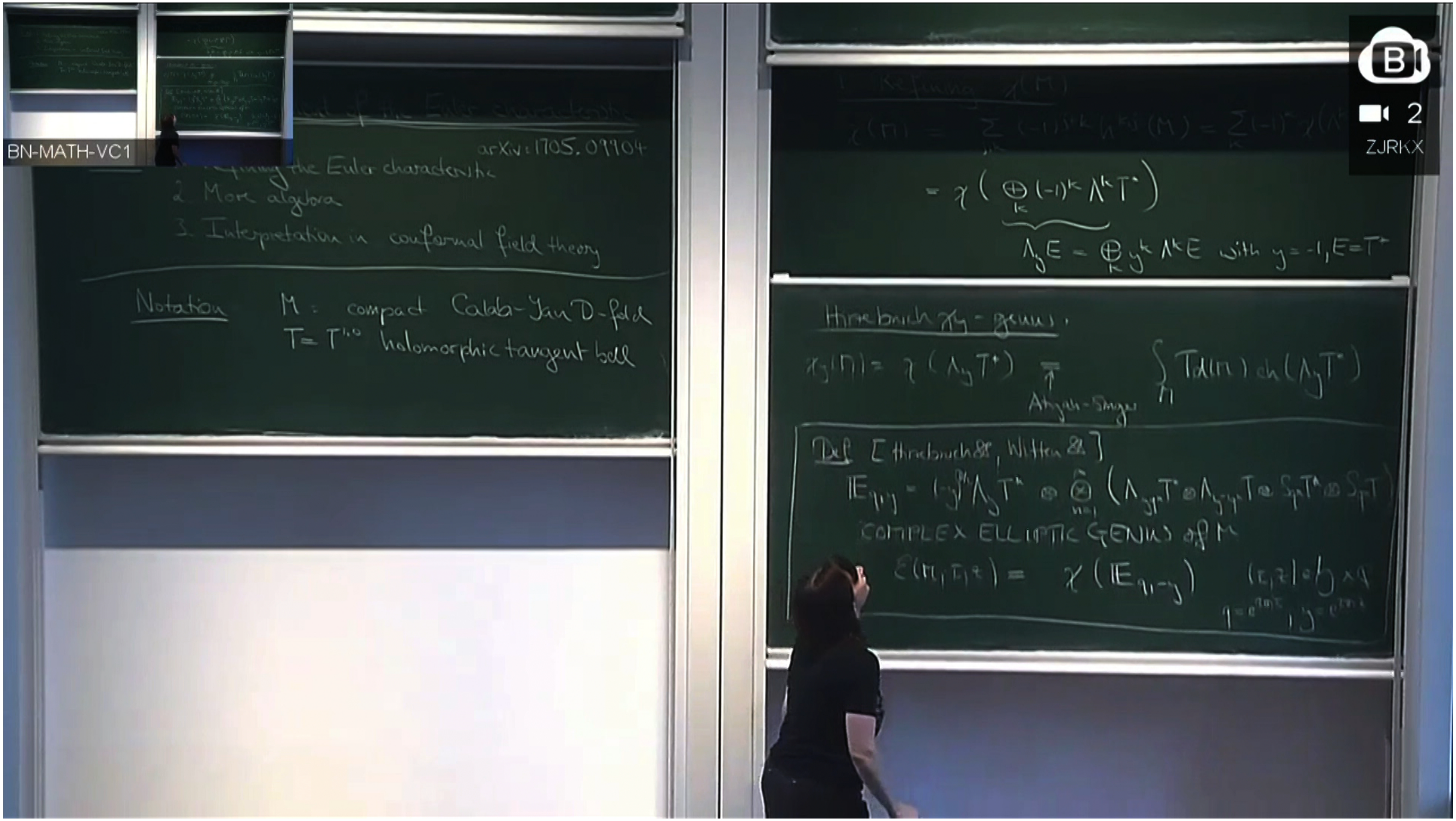
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$T = T^{1,0}$ holomorphic tangent bdl

Properties:

- $\mathcal{E}$ is a regularized $U(1)$-equivariant index of a Dirac operator on loop space

Atiyah-Singer M

Def [Hirzebruch88, Witten 88]

$$\mathbb{E}_{q,y} = (-y)^{-D/2} \Lambda_y T^* \otimes \hat{\bigotimes_{n=1}^{\infty}} \left(\Lambda_{yq^n} T^* \otimes \Lambda_{y^{-1}q^n} T \otimes S_{q^n} T^* \otimes \cdots \right)$$

COMPLEX ELLIPTIC GENUS of M

$$\mathcal{E}(M;\tau,z) = \chi\left(\mathbb{E}_{q,-y}\right) \qquad (\tau,z)\in\mathfrak{h}\times\mathbb{C}$$

$q = e^{2\pi i\tau}$, $y = e^{2\pi i z}$

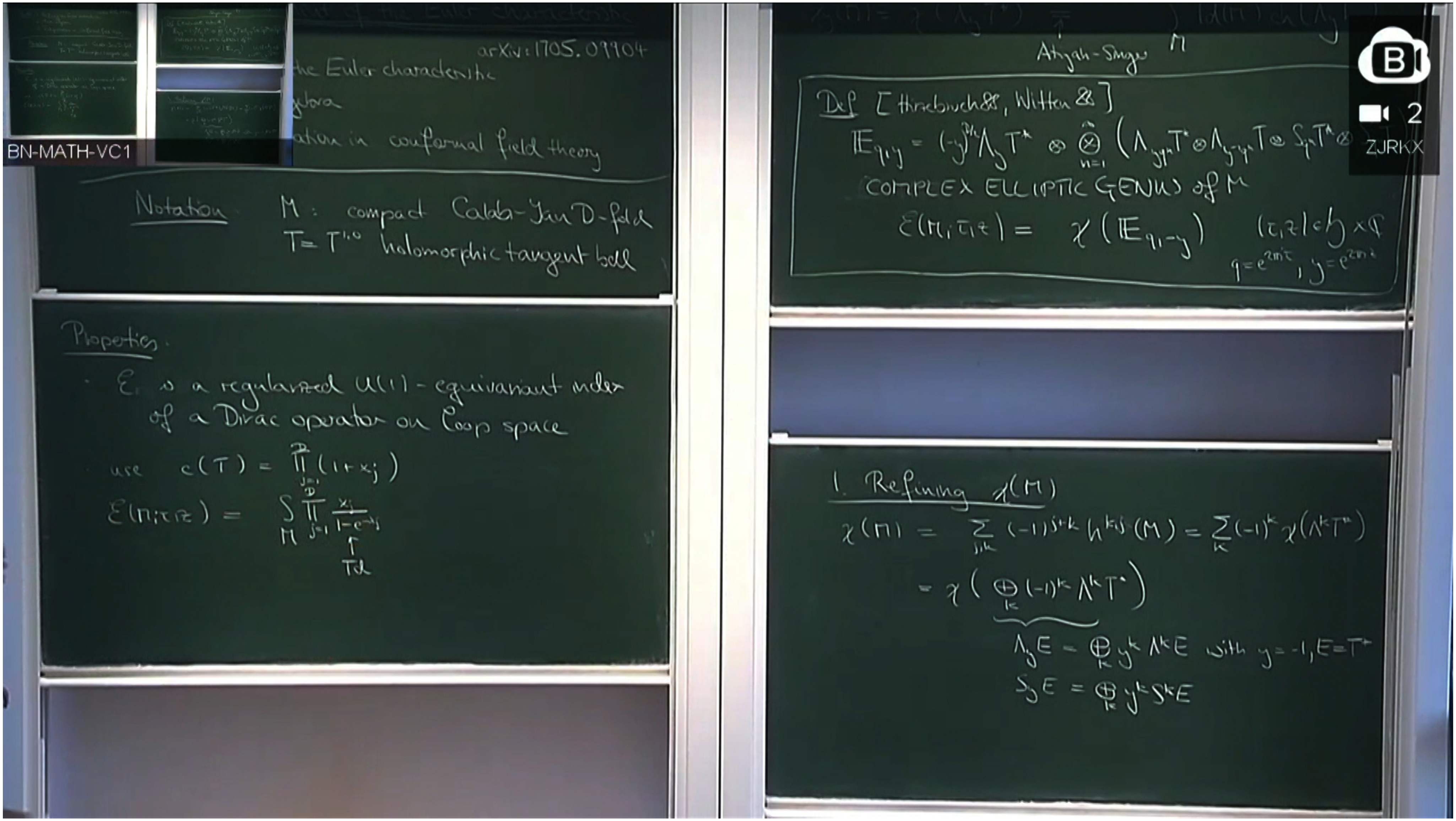
1. Refining $\chi(M)$

$$\chi(M) = \sum_{j,k} (-1)^{j+k} h^{k,j}(M) = \sum_k (-1)^k \chi(\Lambda^k T^*)$$

$$= \chi\Big(\underbrace{\bigoplus_k (-1)^k \Lambda^k T^*}_{} \Big)$$

$\Lambda_y E = \bigoplus_k y^k \Lambda^k E$ with $y=-1$, $E = T^*$

$S_y E = \bigoplus_k y^k S^k E$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· E is a regularized U(1)-equivariant index of a Dirac operator on loop space
· use c(T) = ∏_{j=1}^{D} (1 + x_j)
E(M;τ,z) = ∫_M ∏_{j=1}^{D} x_j / (1 - e^{-x_j})
Td
Atiyah-Singer
M
Def [Hirzebruch 88, Witten 88]
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E(M;τ,z) = χ(E_{q,-y})
(τ,z) ∈ h × C
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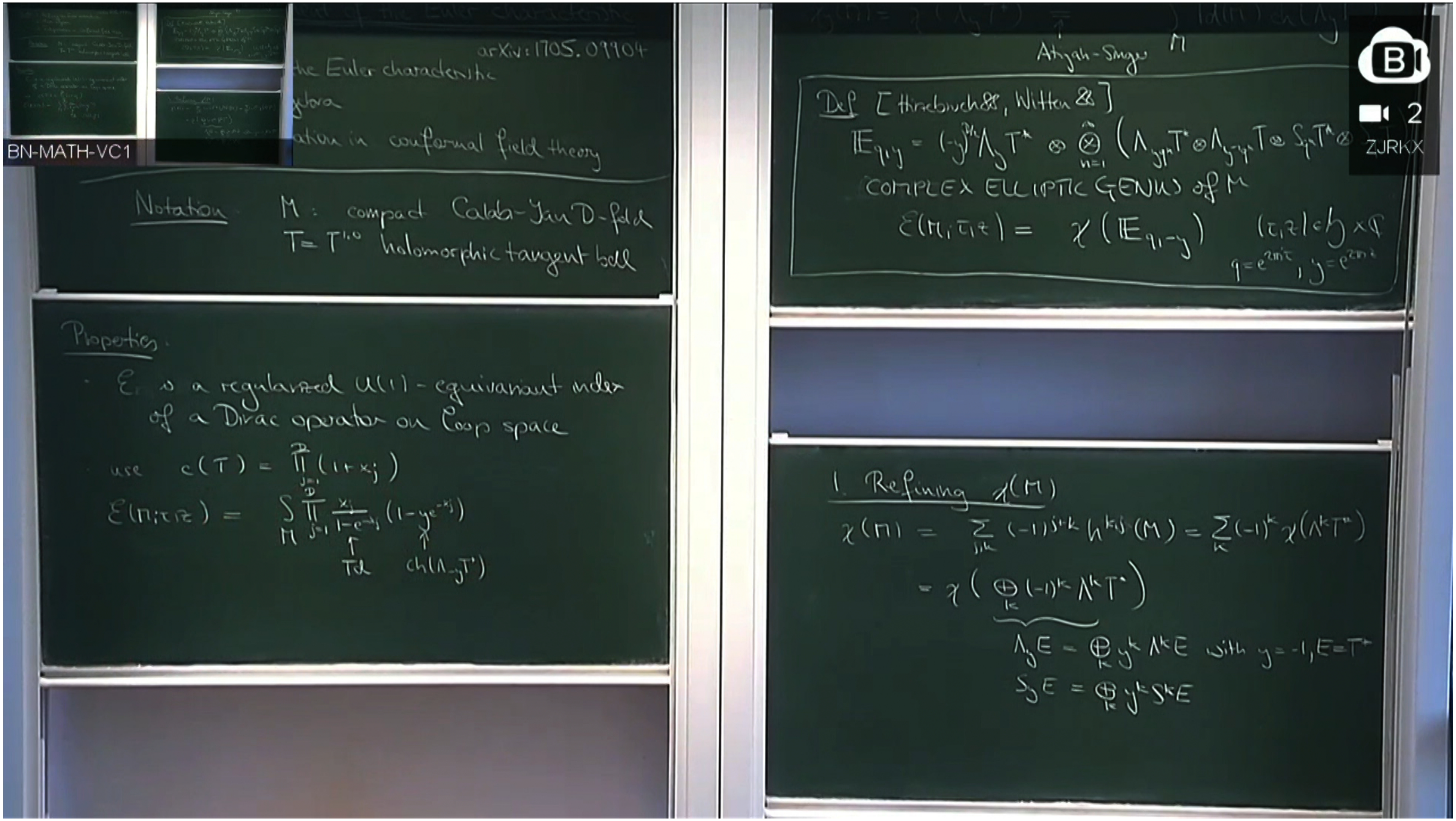
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E(M; τ, z) = ∫_M ∏_{j=1}^{D} x_j / (1 - e^{-x_j}) (1 - y e^{-x_j})
Td ch(Λ_{-y} T*)
Atiyah-Singer M
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E_{q,y} = (-y)^{-D/2} Λ_y T* ⊗ ⊗_{n=1}^{∞} (Λ_{yq^n} T* ⊗ Λ_{y^{-1}q^n} T ⊗ S_{q^n} T* ⊗
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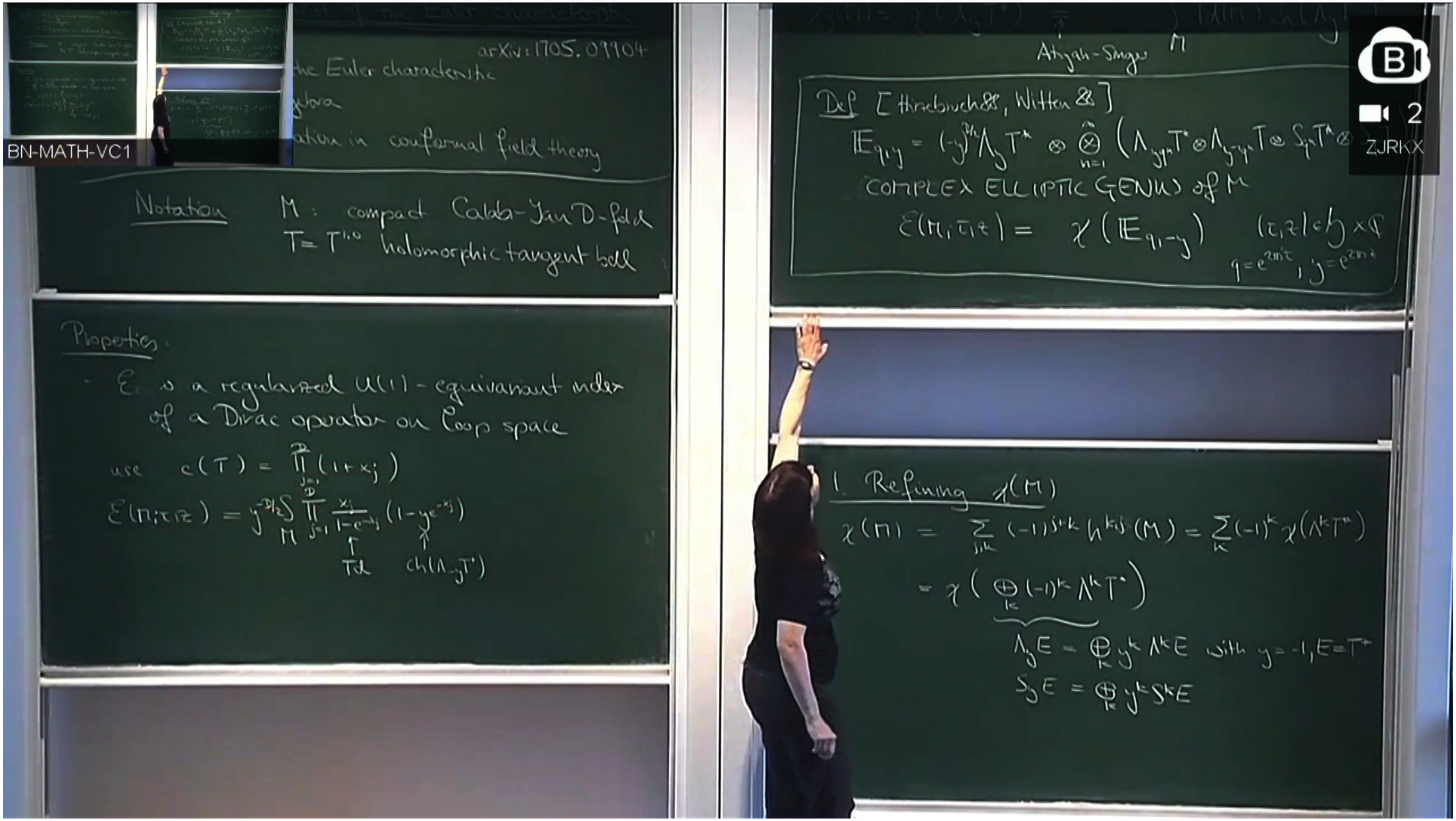
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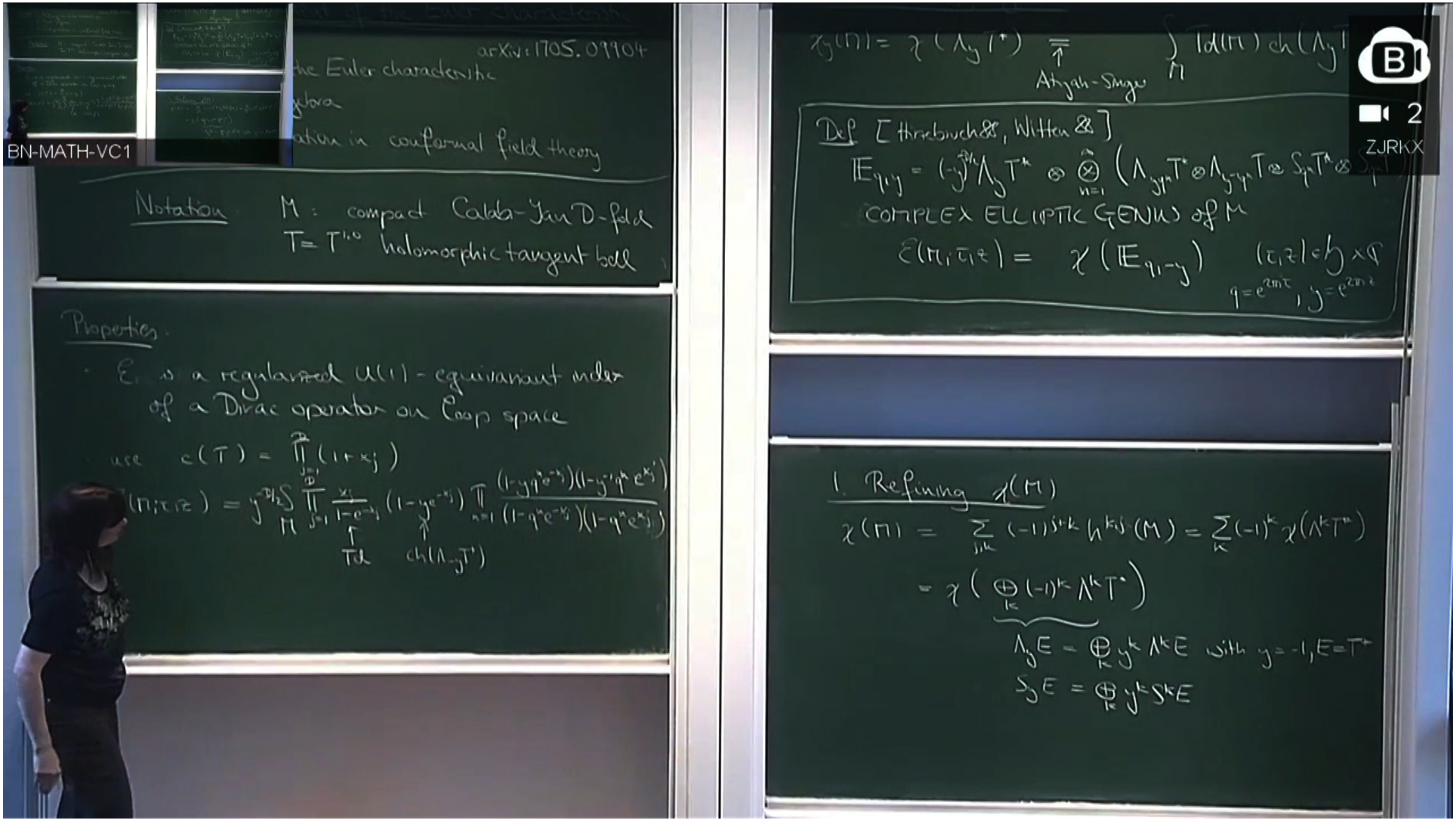
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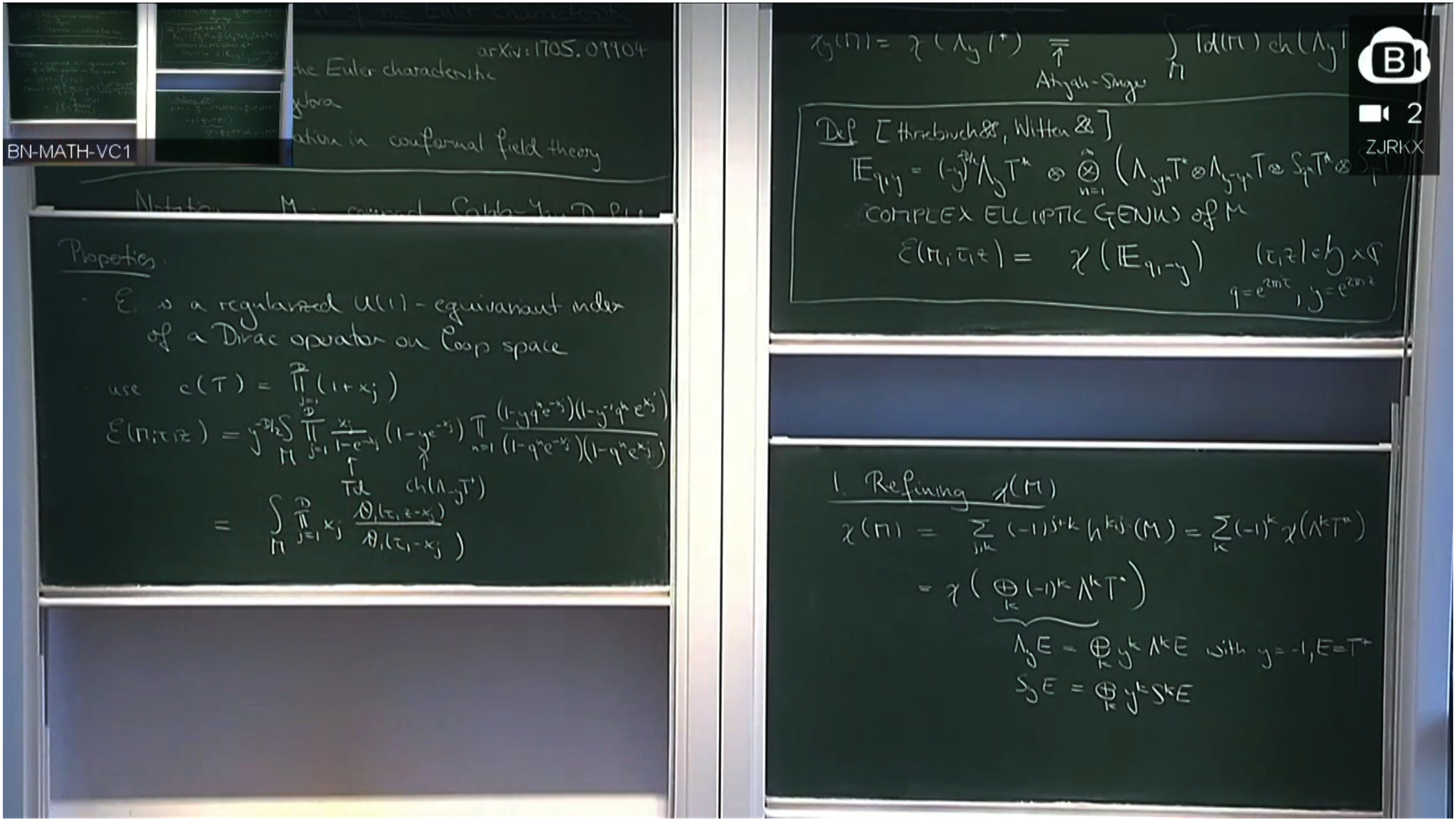
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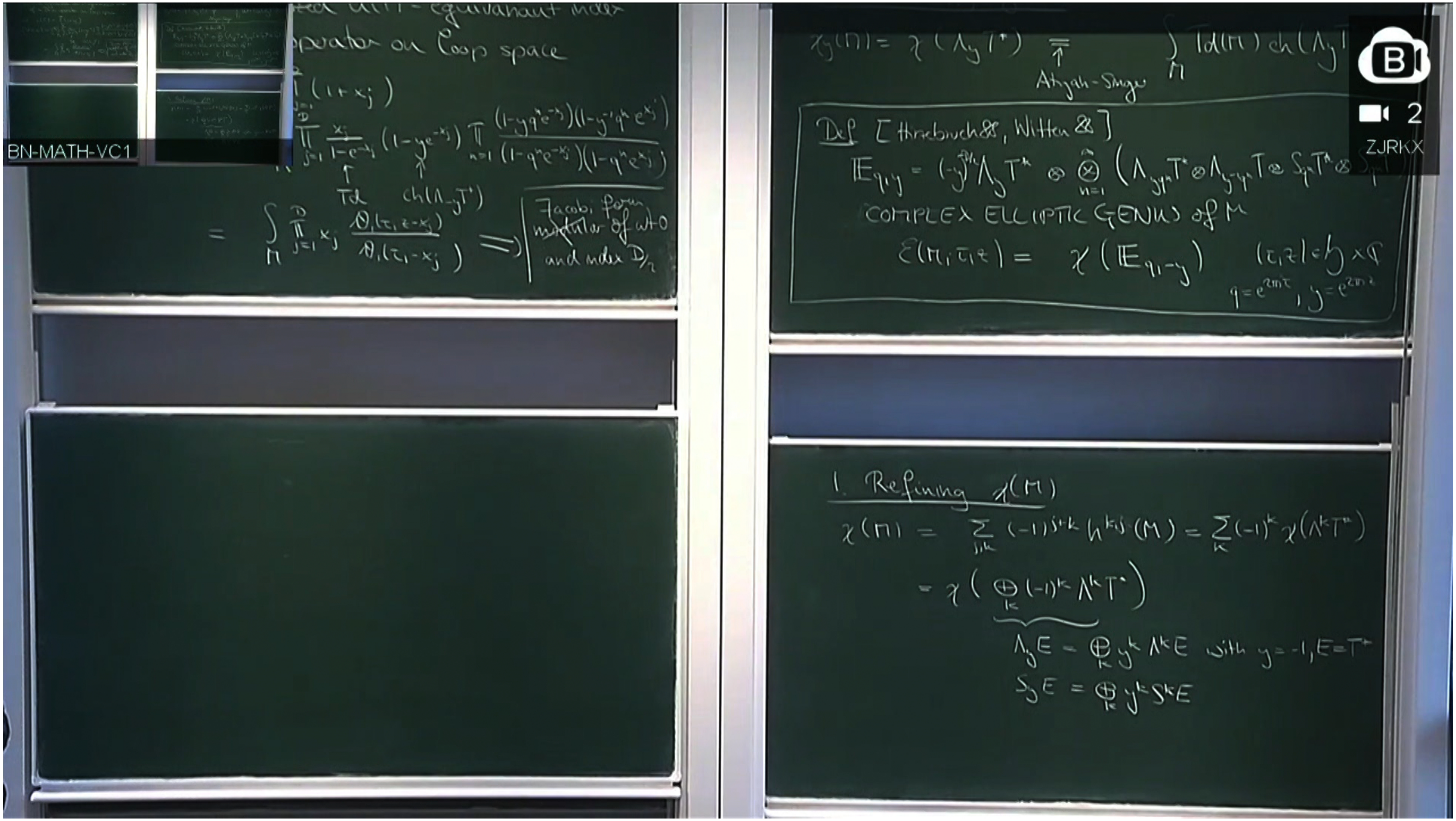
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operator on Loop space
$\prod (1+x_j)$
$\prod_{j=1}^{D} \frac{x_j}{1-e^{-x_j}} (1-y e^{-x_j}) \prod_{n=1} \frac{(1-yq^n e^{-x_j})(1-y^{-1}q^n e^{x_j})}{(1-q^n e^{-x_j})(1-q^n e^{x_j})}$
Td
$ch(\Lambda_{-y}T^*)$
$= \int_M \prod_{j=1}^{D} x_j \frac{\theta_1(\tau, z-x_j)}{\theta_1(\tau, -x_j)}$
Jacobi form modular of wt 0 and index $D/2$
$\chi_y(M) = \chi(\Lambda_y T^*) = \int_M Td(M)\, ch(\Lambda_y T^*)$
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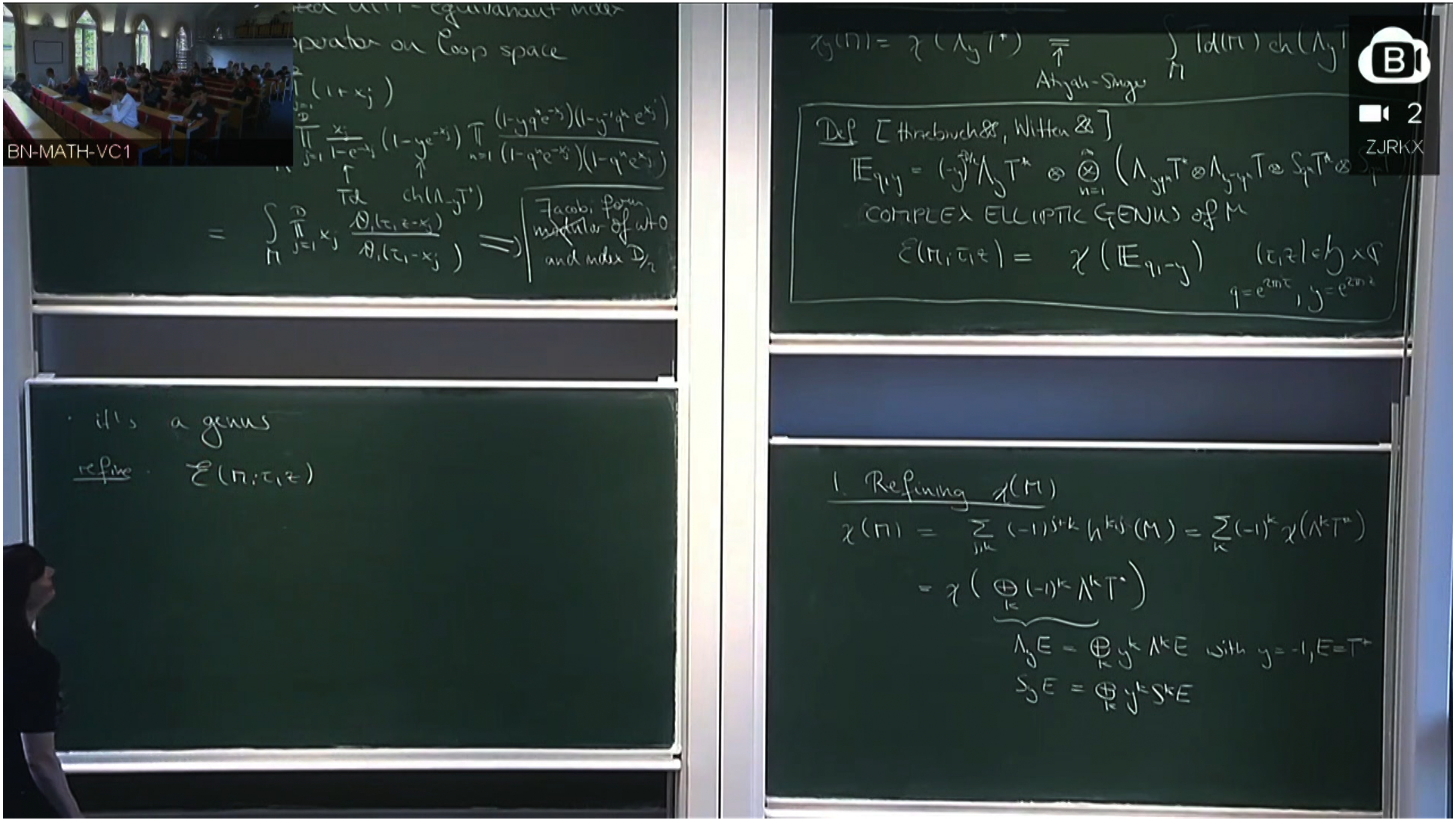
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operator on Loop space
= $\int_M \prod_{j=1}^{D} x_j \frac{\theta_1(\tau, z - x_j)}{\theta_1(\tau, -x_j)}$
Jacobi form modular of wt 0 and index D/2
· it's a genus
refine · $\mathcal{E}(M;\tau,z)$
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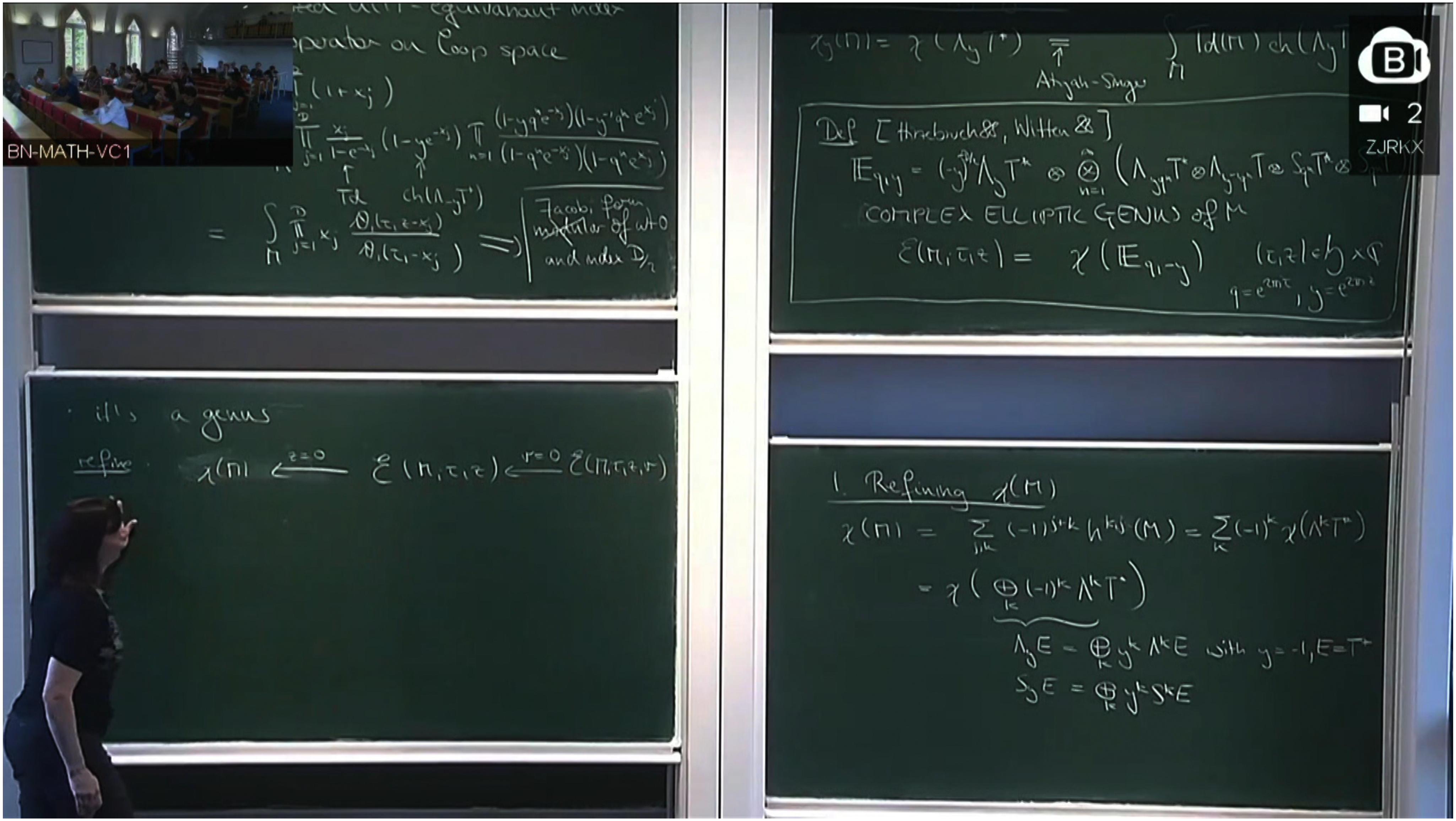
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operator on loop space
Atiyah-Singer
Def [Hirzebruch 88, Witten 88]
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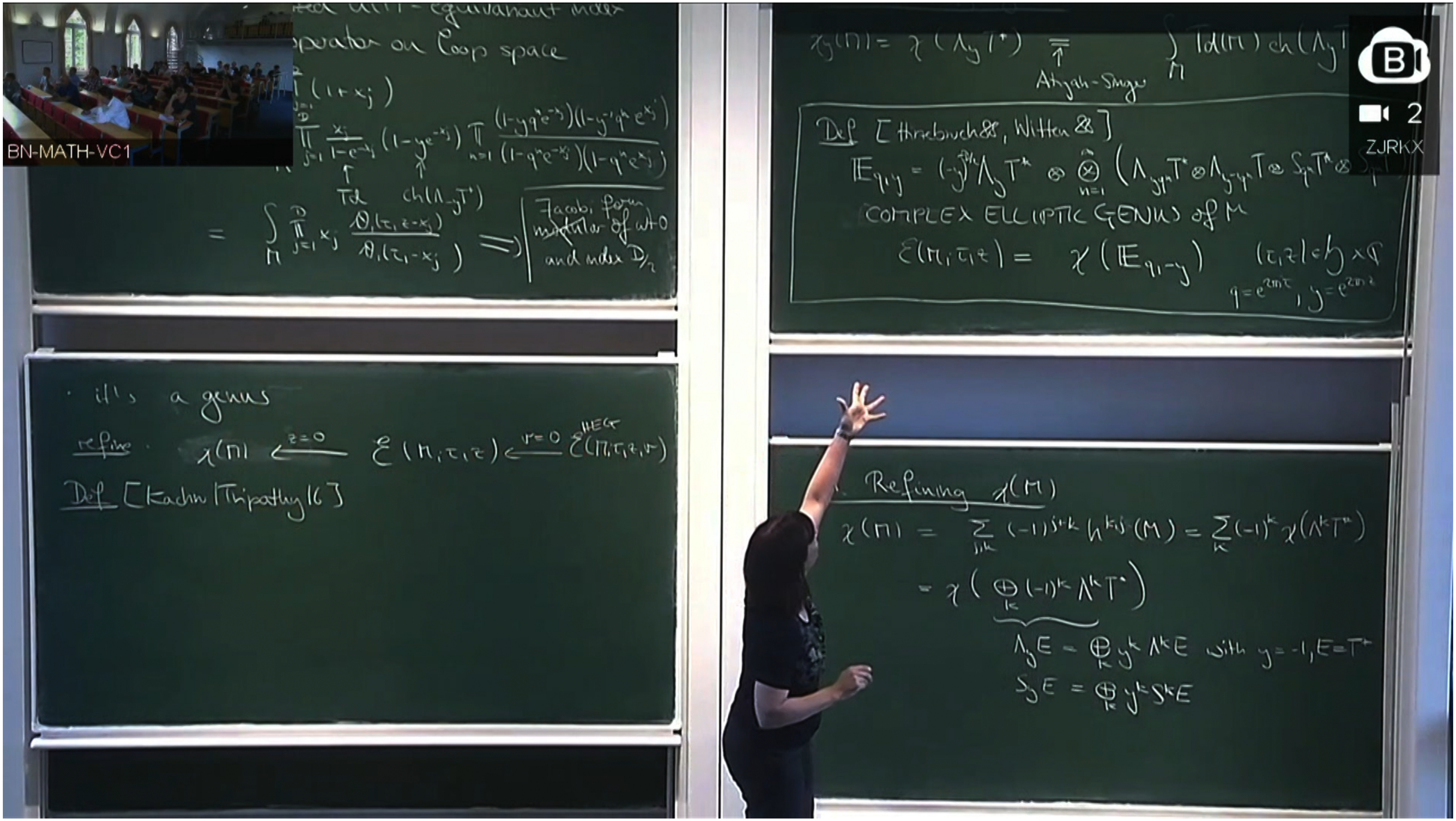
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Def [Kachru/Tripathy16]
Refining χ(M)
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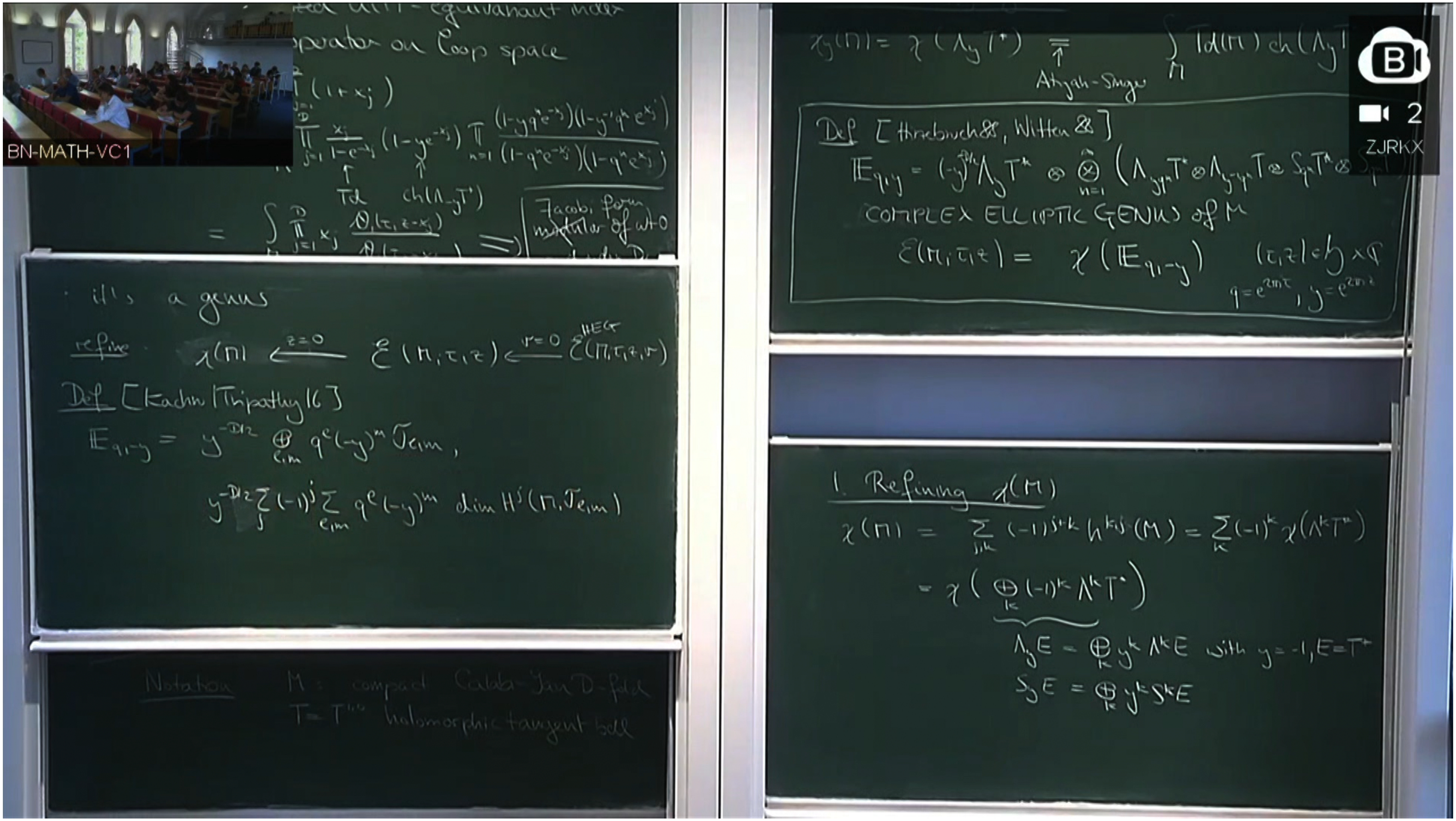
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operator on loop space
= ∫ Π x_j θ_1(τ, z − x_j)/θ_1(τ, −x_j) ⟹ Jacobi form, modular of wt 0
Td   ch(Λ_{−y}T*)
· it's a genus
refine   χ(M) ←(z=0)— ℰ(M;τ,z) ←(v=0)— ℰ(M,τ,z,v)
Def [Kachru|Tripathy 16]
E_{q,−y} = y^{−D/2} ⊕_{ℓ,m} q^ℓ(−y)^m J_{ℓ,m},
y^{−D/2} Σ_j (−1)^j Σ_{ℓ,m} q^ℓ(−y)^m dim H^j(M, J_{ℓ,m})
Notation   M: compact Calabi-Yau D-fold
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ℰ(M;τ,z) = χ(E_{q,−y})   (τ,z) ∈ ℍ × ℂ
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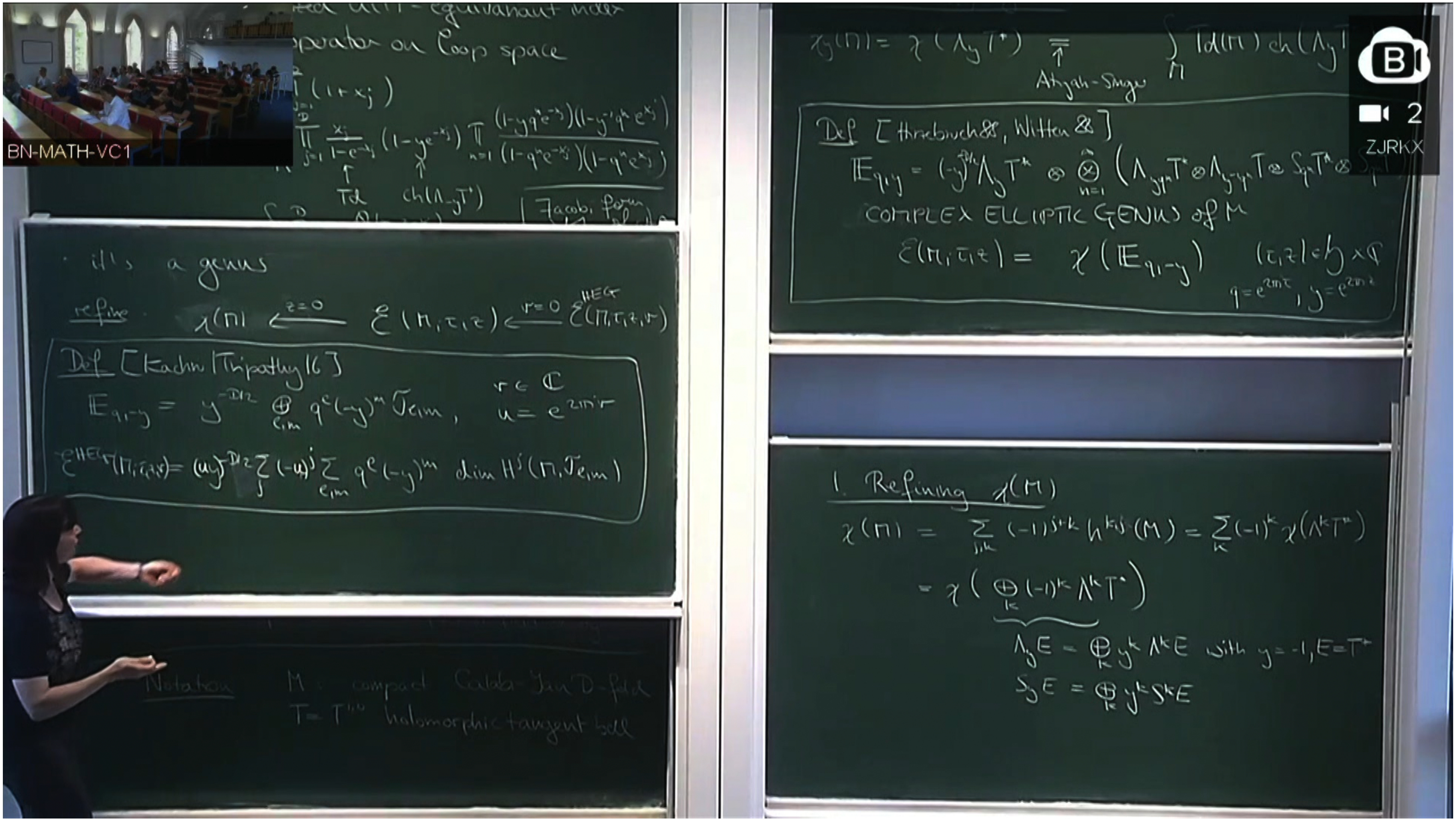
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operator on loop space
Jacobi form
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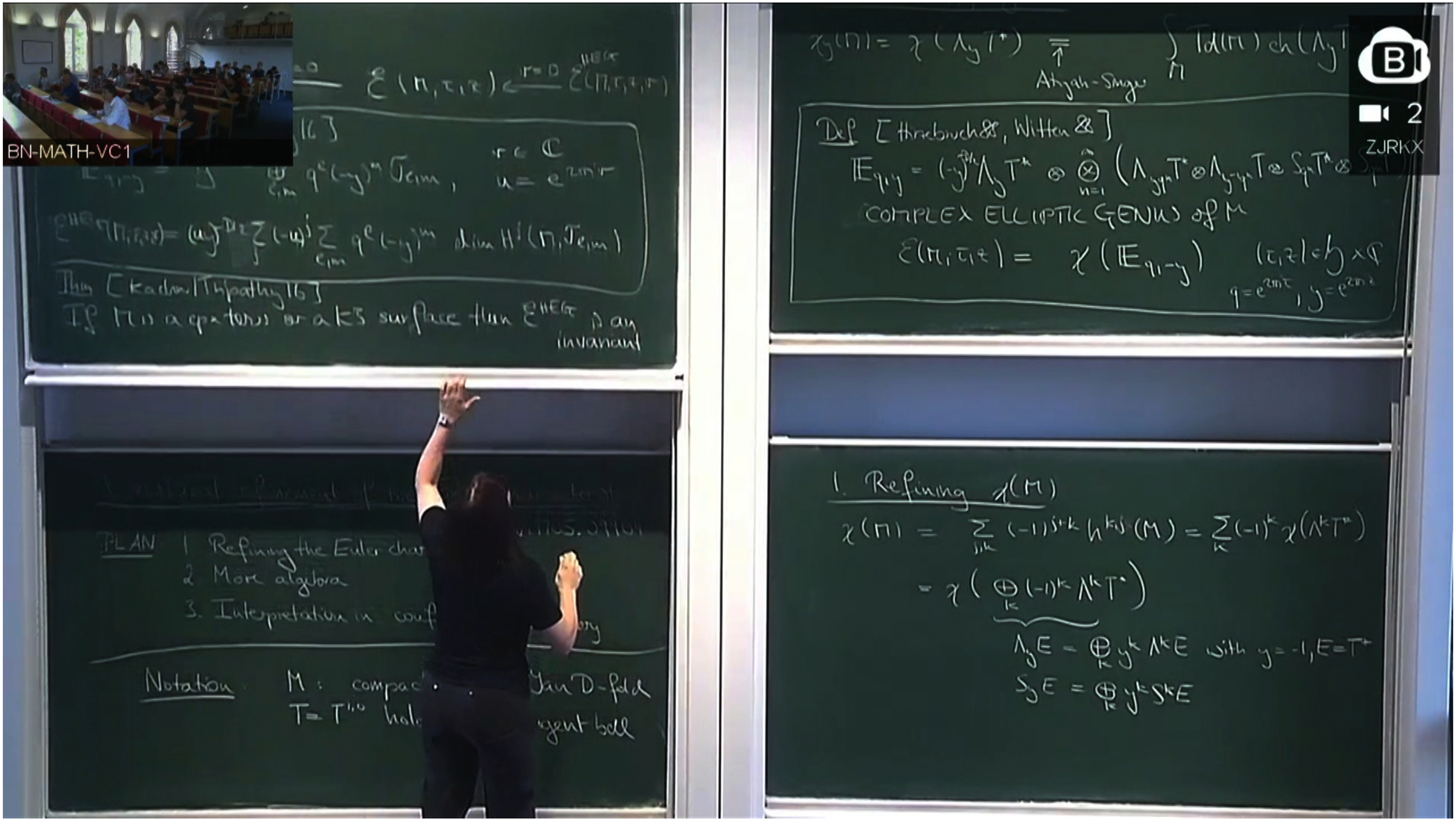
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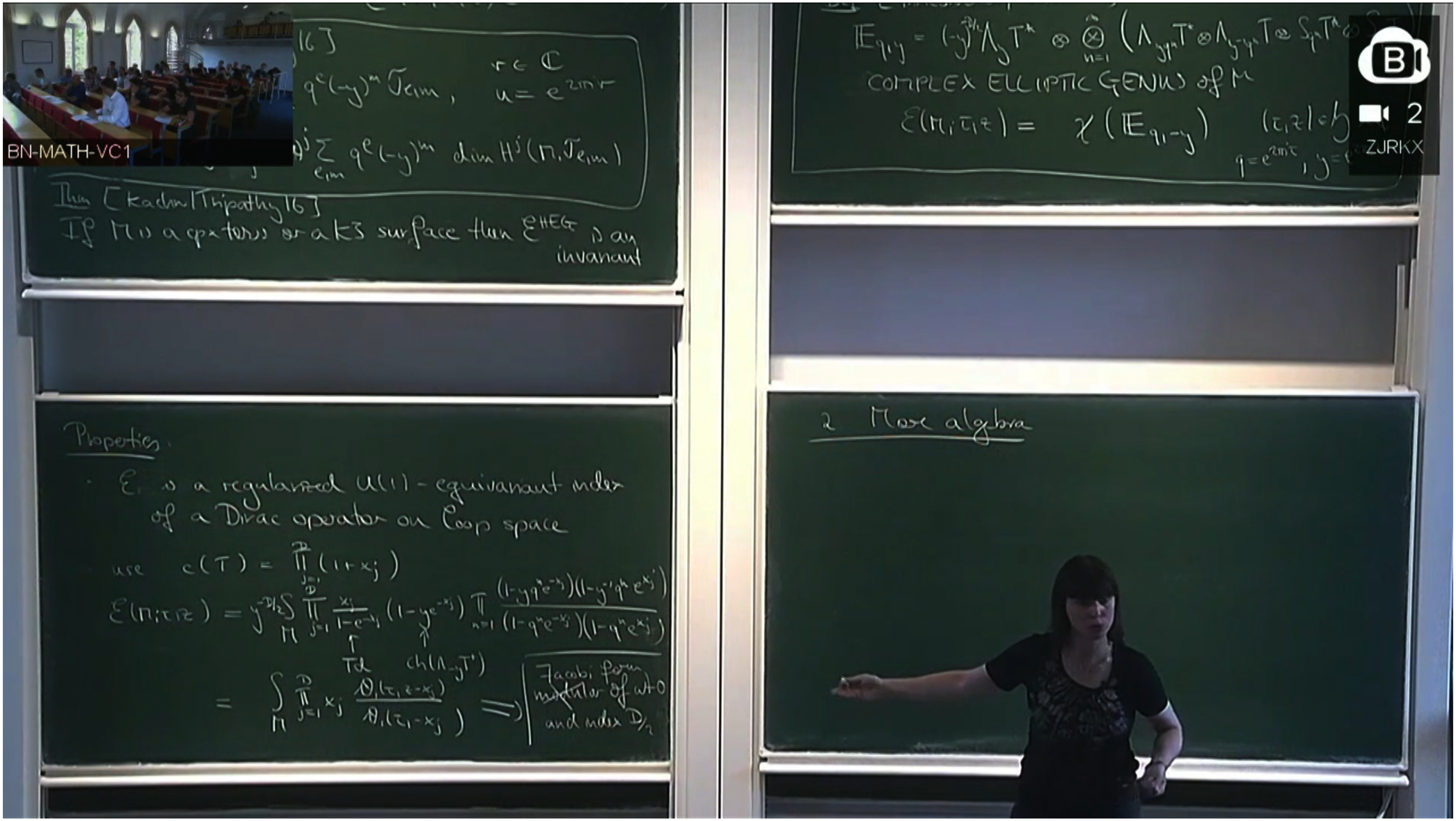
COMPLEX ELLIPTIC GENUS of M
Thm [Kachru/Tripathy 16]
If M is a complex torus or a K3 surface then E is an invariant
PLAN 1. Refining the Euler char
2. More algebra
3. Interpretation in conf
Notation M: compac
1. Refining χ(M)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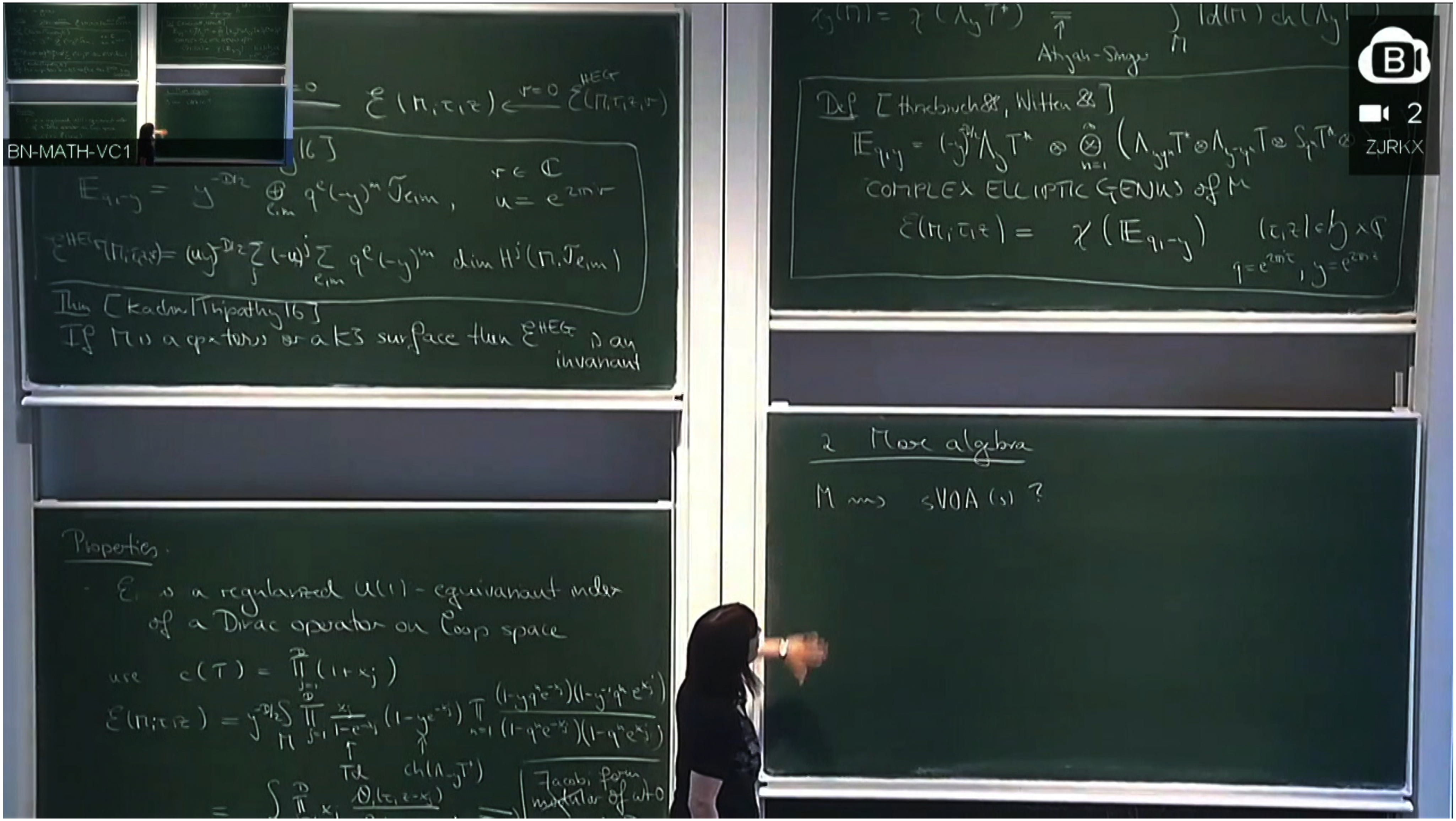
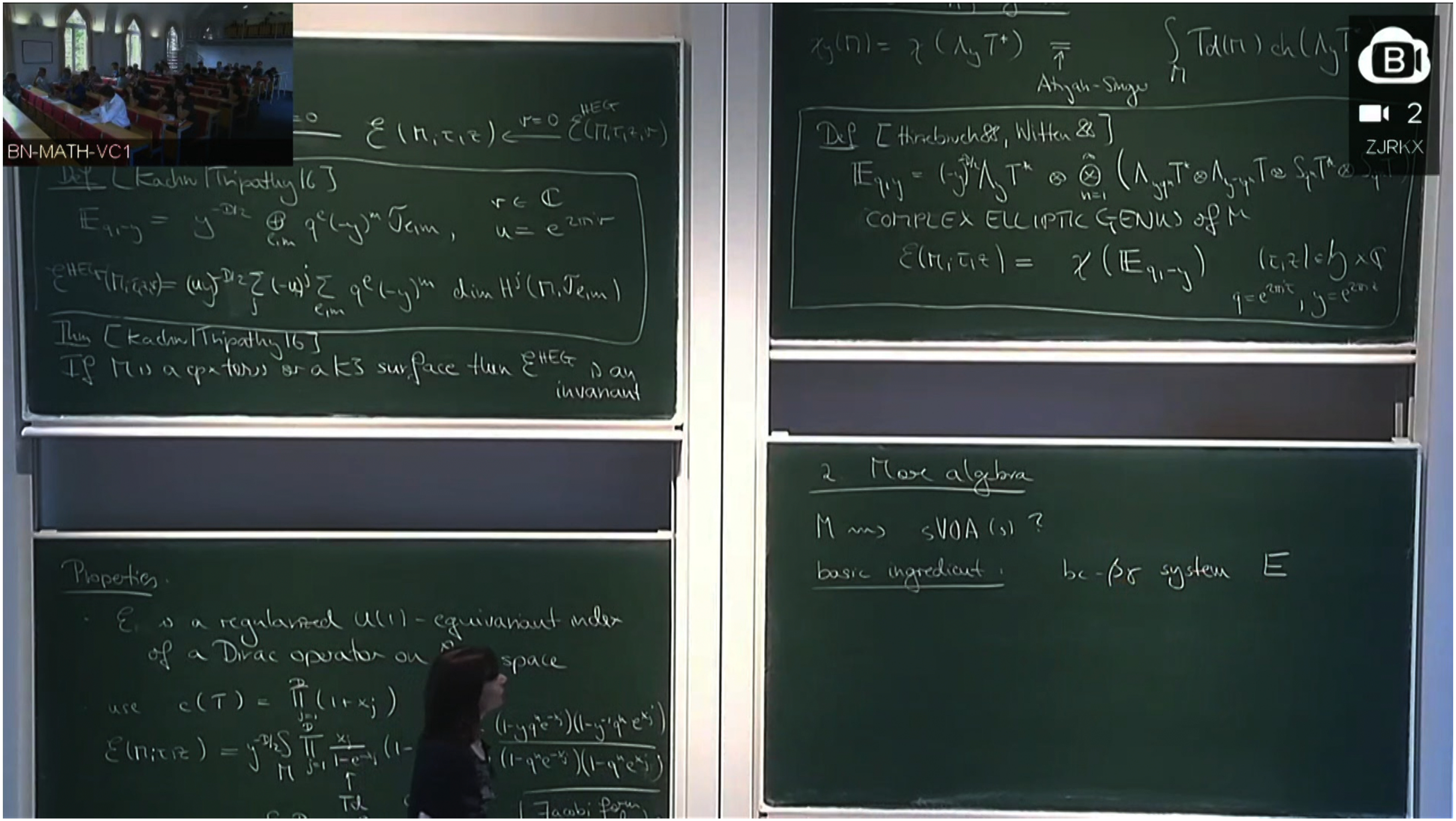
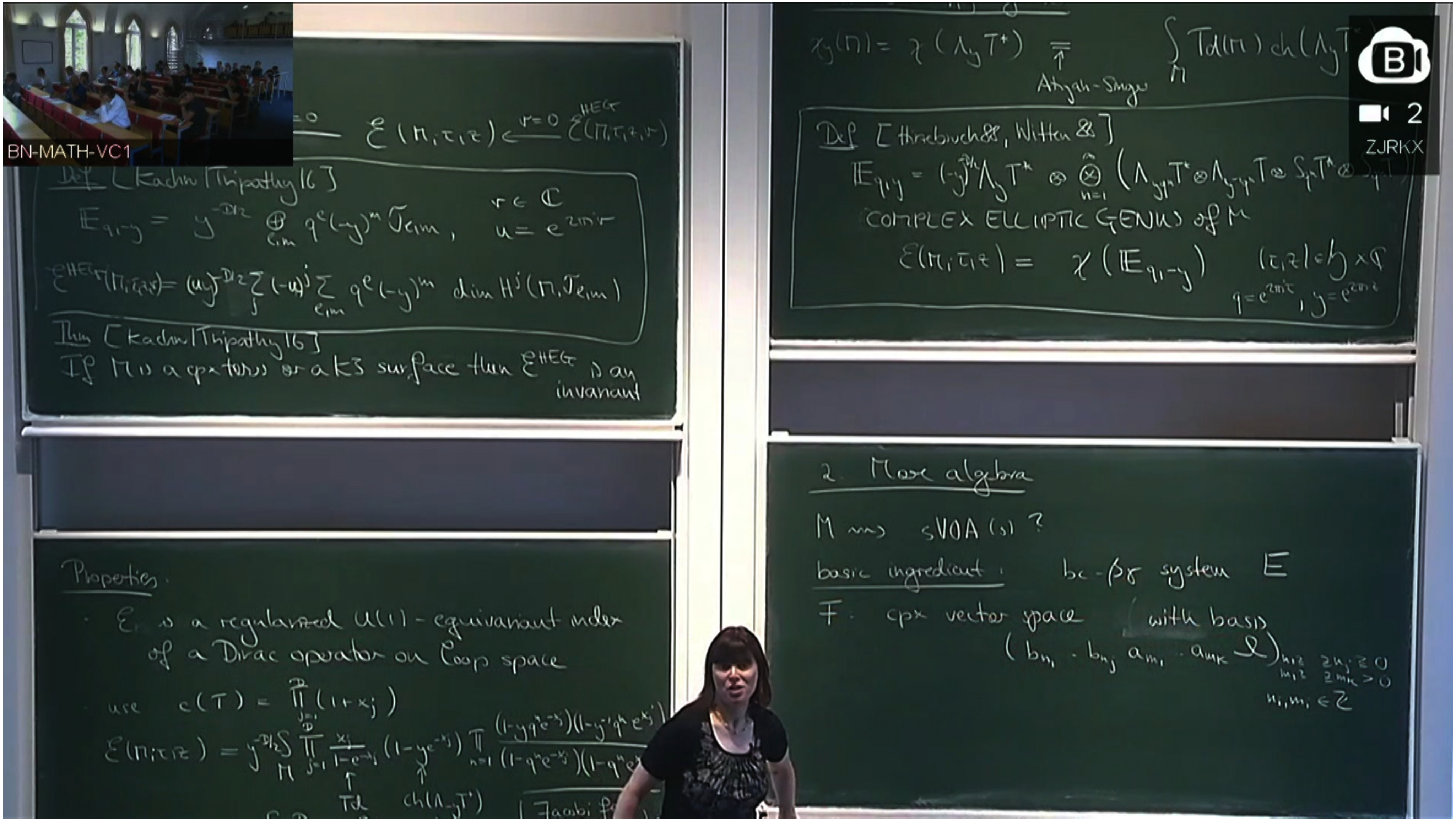
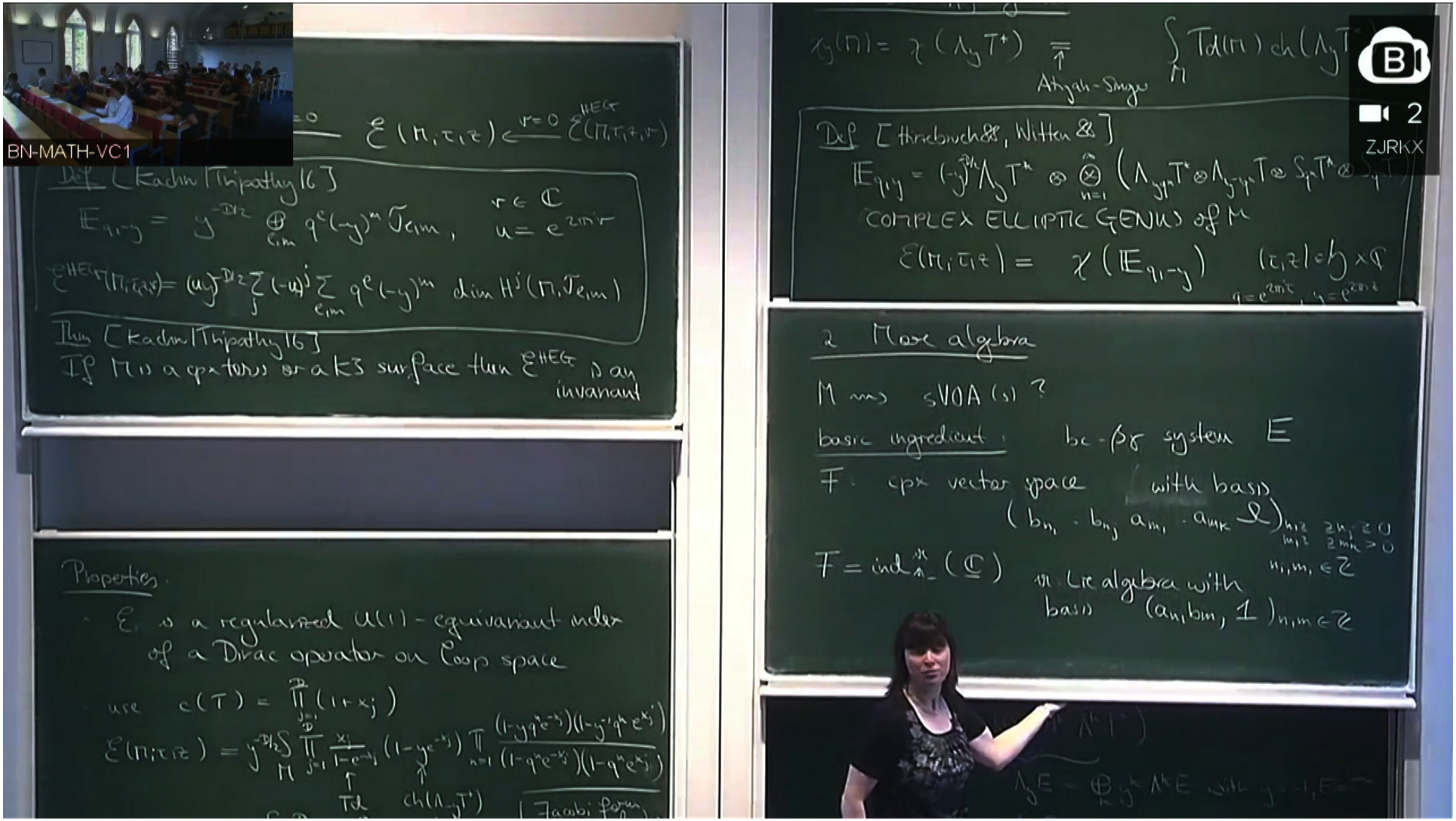
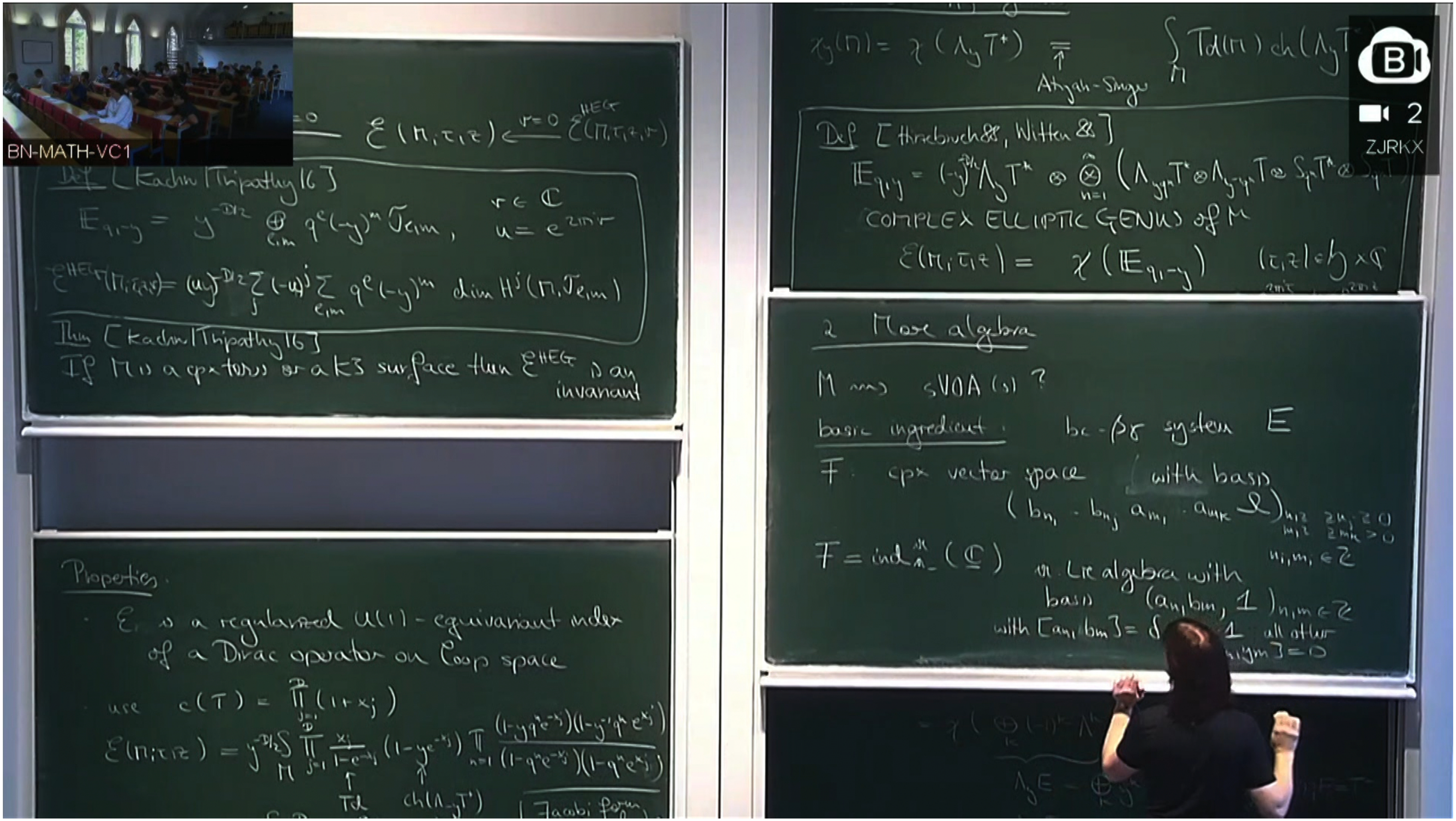
Thm [Kachru/Tripathy 16]
If M is a 4-torus or a K3 surface then $\mathcal{E}^{HEG}$ is an invariant
Properties
· $\mathcal{E}$ is a regularized U(1)-equivariant index of a Dirac operator on loop space
use $c(T) = \prod_{j=1}^{D} (1+x_j)$
Jacobi form, modular of wt 0 and index D/2
COMPLEX ELLIPTIC GENUS of M
$\mathcal{E}(M;\tau,z) = \chi(\mathbb{E}_{q,-y})$
ZJRKX
2 More algebra

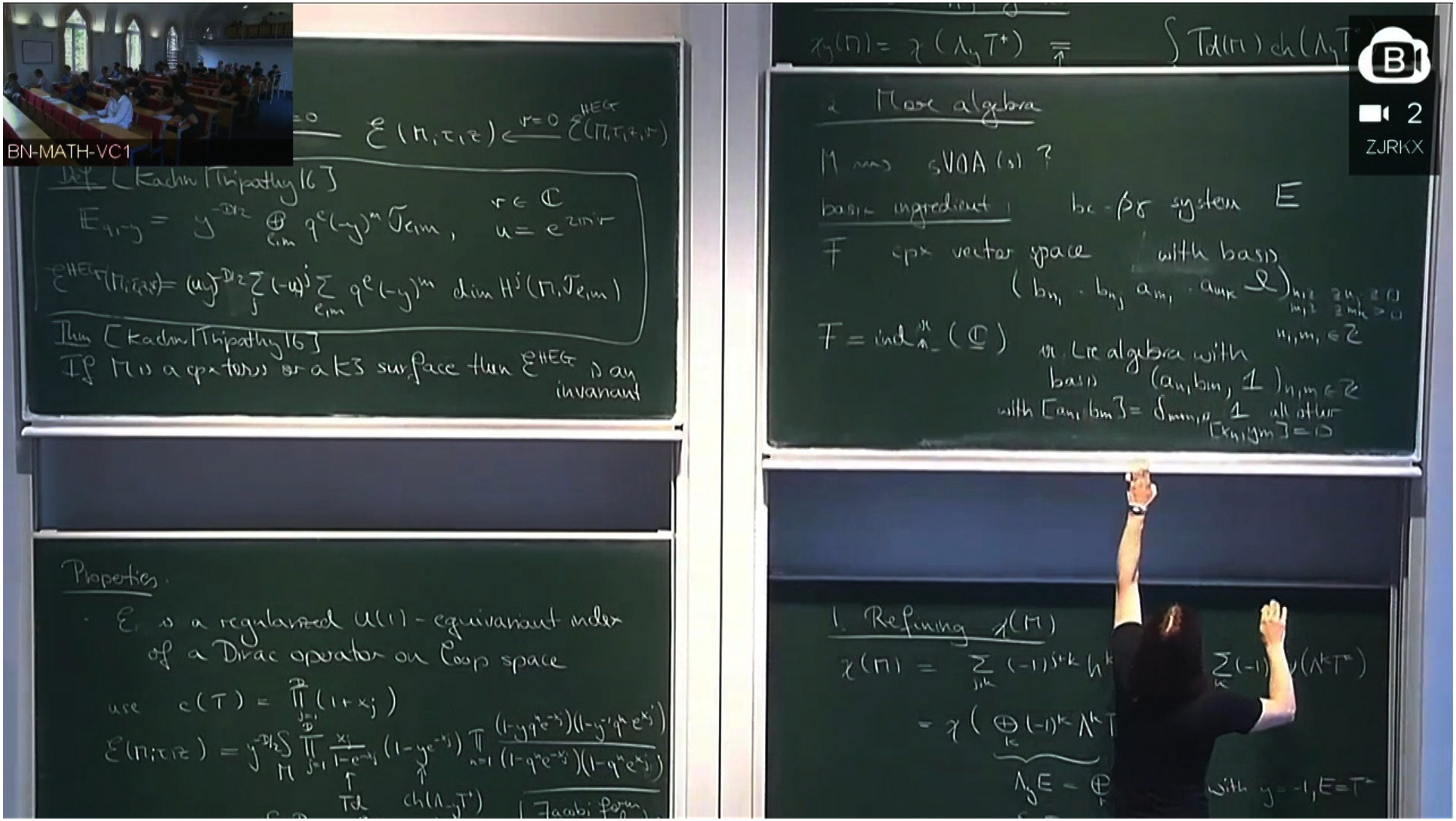

BN-MATH-VC1
ZJRKX
Atiyah-Singer
Def [Hirzebruch 88, Witten 88]
COMPLEX ELLIPTIC GENUS of M
Thm [Kachru/Tripathy 16]
If M is a complex torus or a K3 surface then E^HEG is an invariant
2 More algebra
M ⇝ sVOA(s) ?
Properties.
E is a regularized U(1) - equivariant index of a Dirac operator on loop space
Jacobi form modular of wt 0

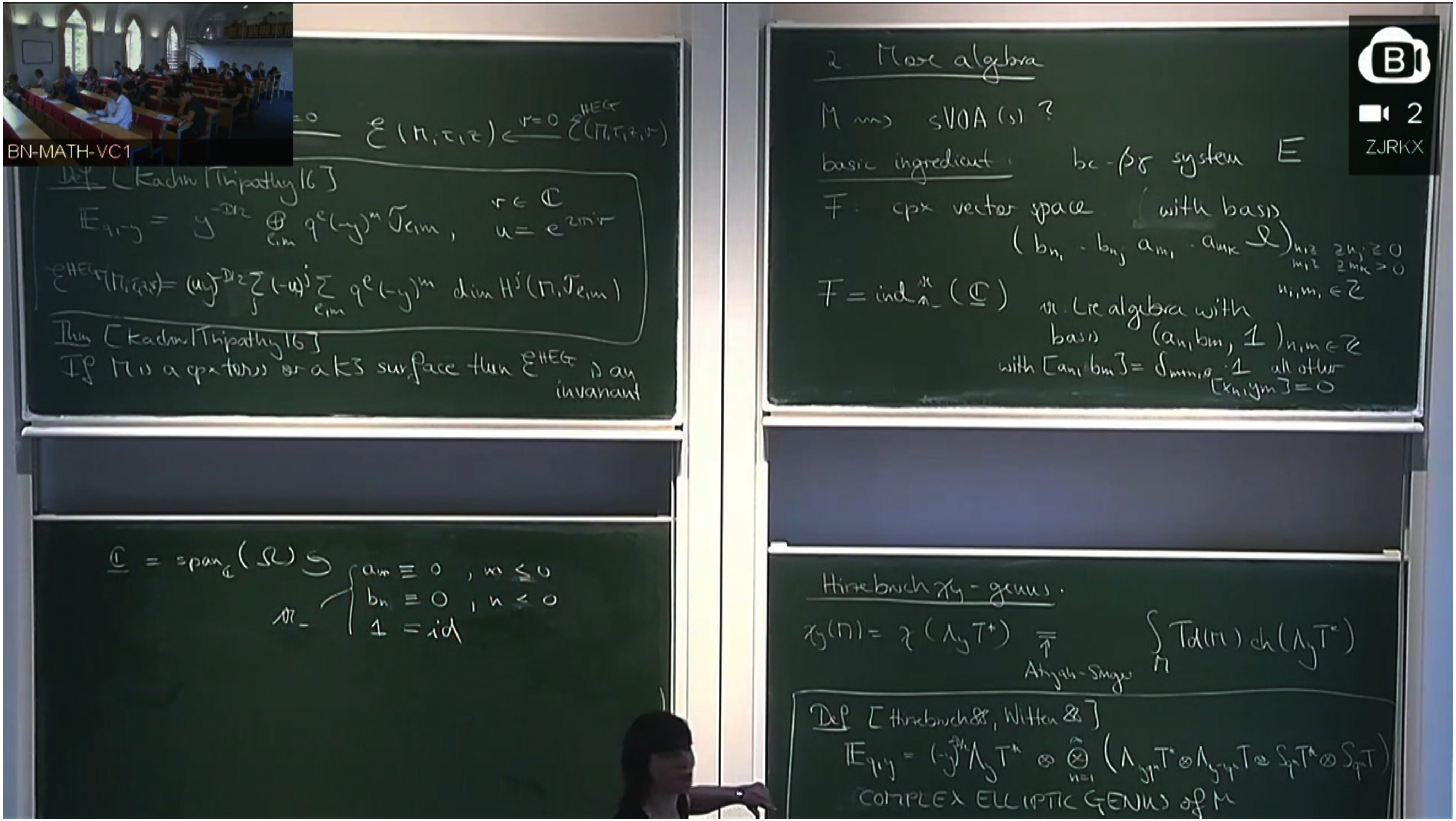

BN-MATH-VC1
Def [Kachru / Tripathy 16]
Thm [Kachru / Tripathy 16]
If M is a cpx torus or a K3 surface then E^HEG is an invariant
Properties.
E is a regularized U(1) - equivariant index of a Dirac operator on space
Jacobi form
Def [Hirzebruch 88, Witten 88]
COMPLEX ELLIPTIC GENUS of M
Atiyah-Singer
2 More algebra
M ⇝ sVOA(s) ?
basic ingredient: bc - βγ system E
ZJRKX

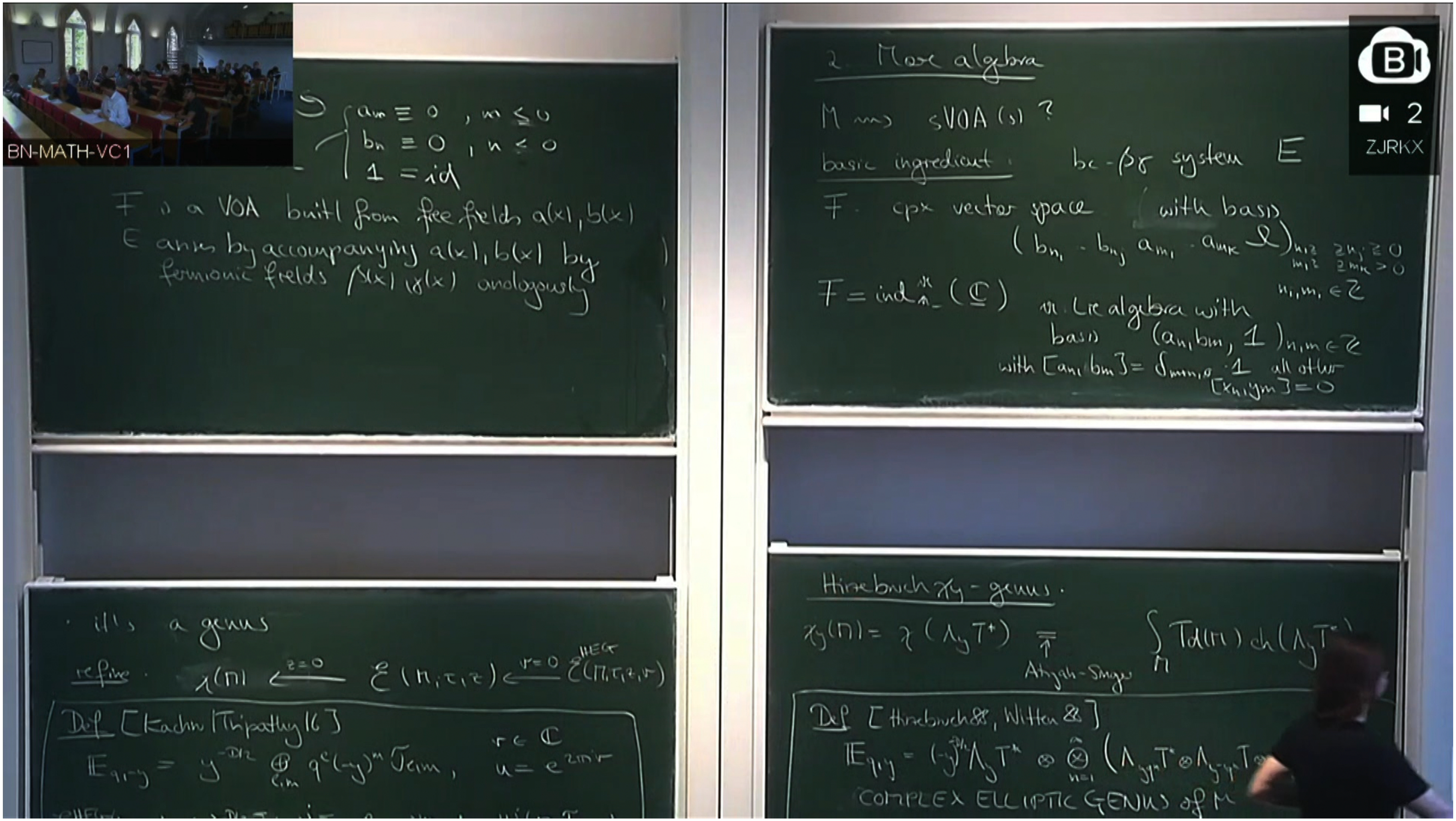

BN-MATH-VC1
2
ZJRKX
Def [Kachn/Tripathy 16]
Thm [Kachn/Tripathy 16]
If M is a cpx torus or a K3 surface then E^HEG is an invariant
Atiyah-Singer
Def [Hirzebruch 88, Witten 88]
COMPLEX ELLIPTIC GENUS of M
Properties
E is a regularized U(1)-equivariant index of a Dirac operator on loop space
Td
Jacobi
2. More algebra
M ~> sVOA(s)?
basic ingredient: bc-βγ system E
F: cpx vector space with basis

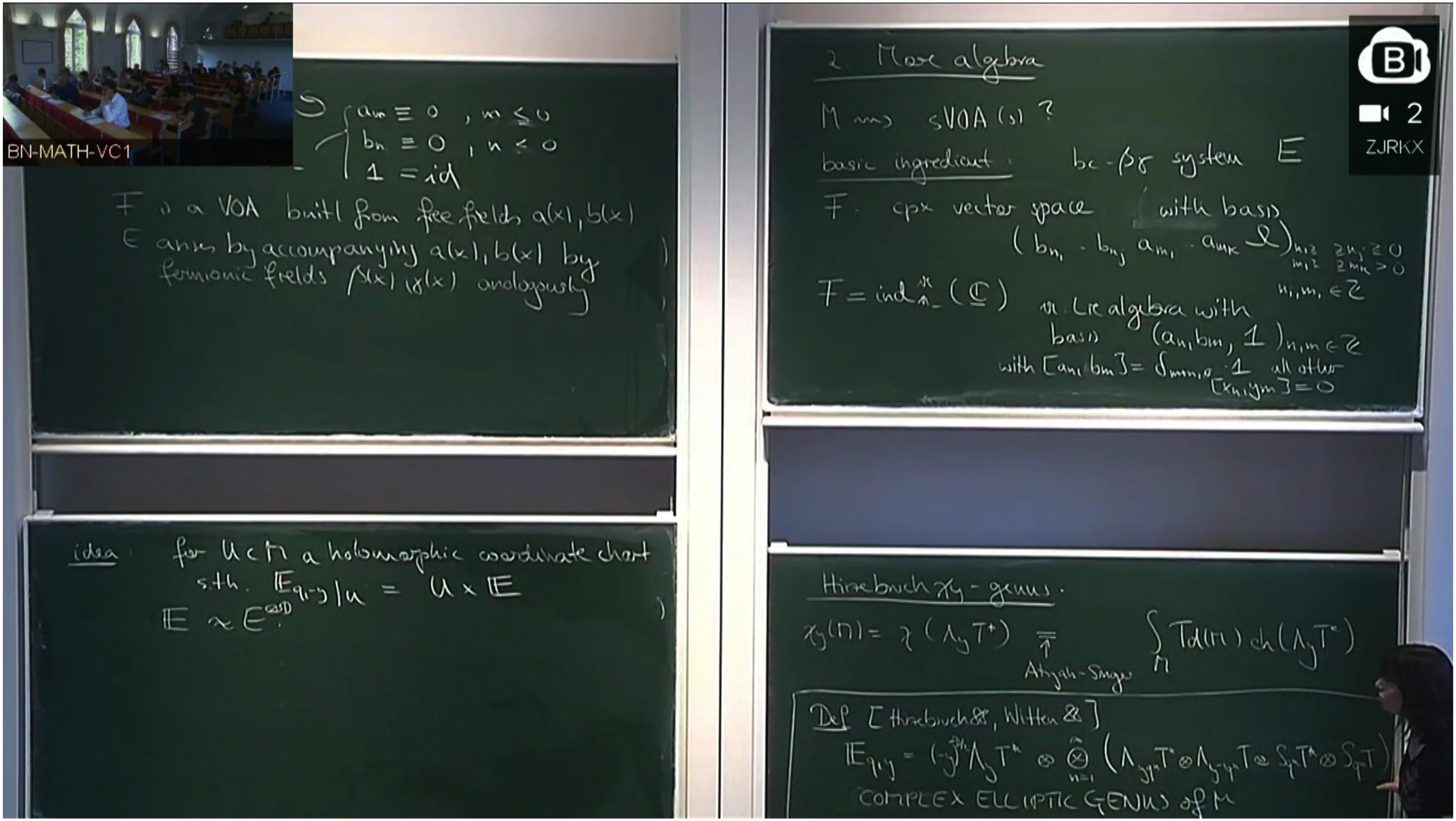

BN-MATH-VC1
Def [Kachru / Tripathy 16]
Thm [Kachru / Tripathy 16]
If M is a cpx torus or a K3 surface then E^HEG is an invariant
Properties
E is a regularized U(1)-equivariant index of a Dirac operator on loop space
use c(T) = ∏ (1+x_j)
Td   ch(Λ_{-y}T*)
Jacobi form
Atiyah-Singer
Def [Hirzebruch 88, Witten 88]
COMPLEX ELLIPTIC GENUS of M
2 More algebra
M ⇝ sVOA(s) ?
basic ingredient: bc-βγ system E
F: cpx vector space with basis
F = ind(ℂ)
Lie algebra with basis
ZJRKX

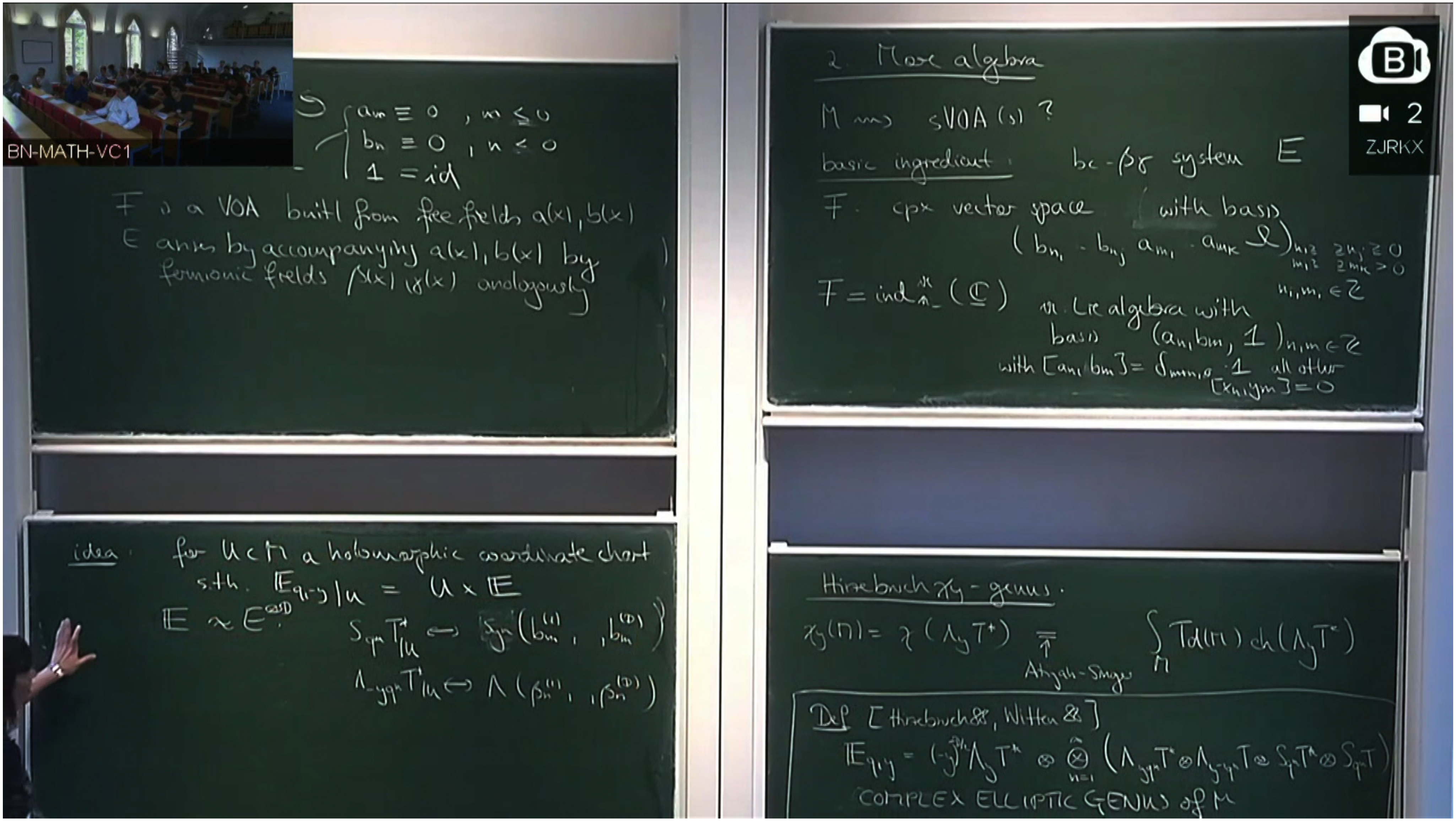

BN-MATH-VC1
ZJRKX
Def [Kachru/Tripathy 16]
$\mathbb{E}_{q,-y} = y^{-D/2} \bigoplus_{\ell,m} q^\ell (-y)^m \mathcal{J}_{\ell,m}$, $r \in \mathbb{C}$, $u = e^{2\pi i r}$
Thm [Kachru/Tripathy 16]
If M is a cpx torus or a K3 surface then $\mathcal{E}^{HEG}$ is an invariant
Properties
$\mathcal{E}$ is a regularized U(1)-equivariant index of a Dirac operator on loop space
use $c(T) = \prod_{j=1}^{D} (1+x_j)$
Jacobi form
$\chi_y(M) = \chi(\Lambda_y T^*)$
Atiyah-Singer
Def [Hirzebruch 88, Witten 88]
COMPLEX ELLIPTIC GENUS of M
$\mathcal{E}(M;\tau,z) = \chi(\mathbb{E}_{q,-y})$
2 More algebra
M ⇝ sVOA(s) ?
basic ingredient : bc-βγ system E
F : cpx vector space with basis
$F = \mathrm{ind}_{\mathfrak{a}_-}^{\mathfrak{a}}(\mathbb{C})$
$\mathfrak{a}$ Lie algebra with basis $(a_n, b_m, 1)_{n,m \in \mathbb{Z}}$
all other

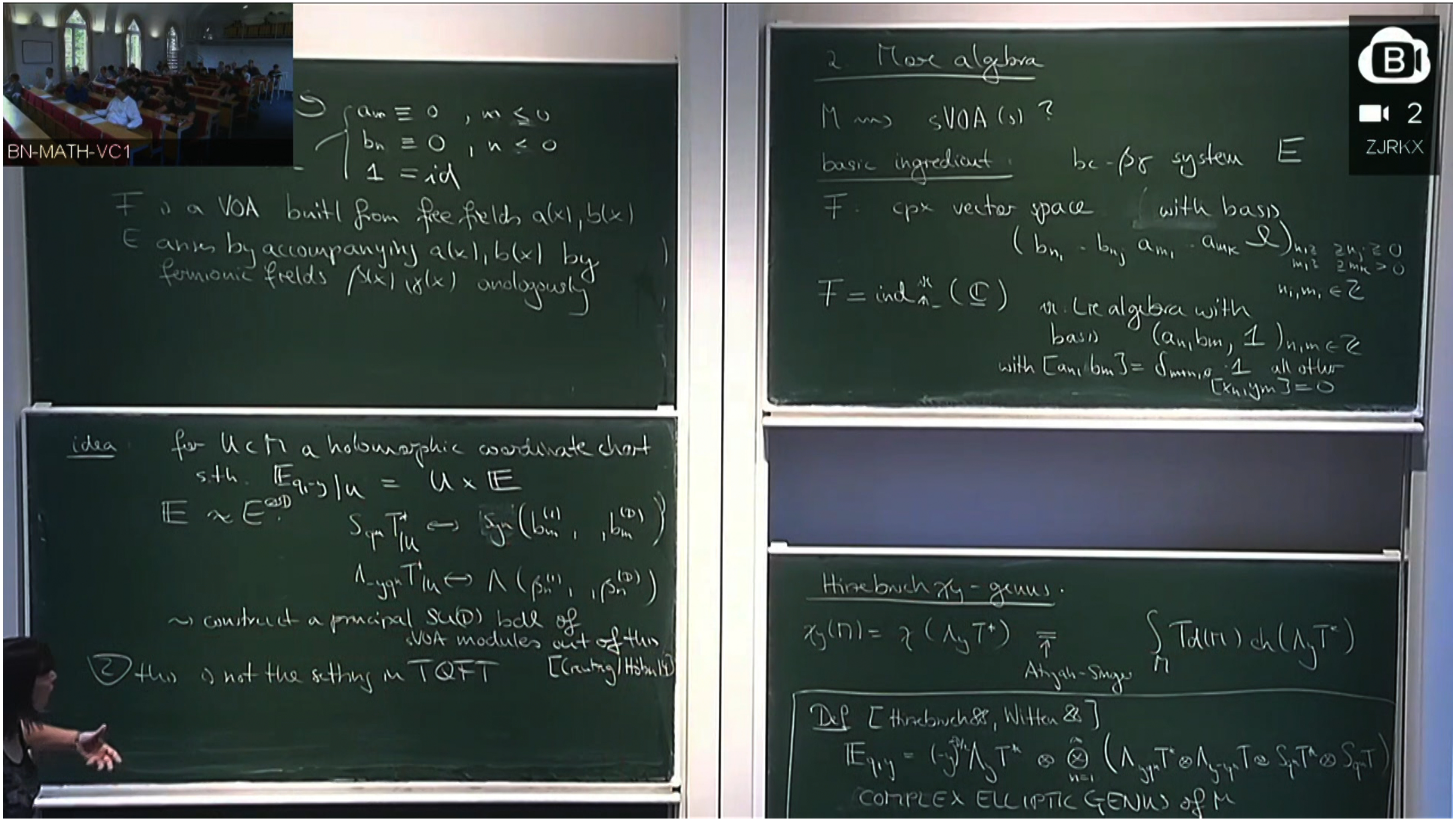

BN-MATH-VC1
2
ZJRKX
Def [Kachru/Tripathy 16]
Thm [Kachru/Tripathy 16]
If M is a cpx torus or a K3 surface then E^HEG is an invariant
Properties.
E is a regularized U(1)-equivariant index of a Dirac operator on loop space
use c(T) = ∏(1+x_j)
Jacobi form
2. More algebra
M ~~> sVOA(s) ?
basic ingredient: bc-βγ system E
F cpx vector space with basis
F = ind (ℂ)
𝔫 Lie algebra with basis (a_n, b_m, 1)
all other [x_n, y_m] = 0
1. Refining χ(M)


BN-MATH-VC1
ZJRKX
2
Def [Kachru/Tripathy 16]
Thm [Kachru/Tripathy 16]
If M is a cpx torus or a K3 surface then E^HEG is an invariant
C = span_C(Ω)
a_m ≡ 0 , m ≤ 0
b_n ≡ 0 , n < 0
1 = id
2. More algebra
M ⇝ sVOA(s) ?
basic ingredient: bc-βγ system E
F: cpx vector space with basis
F = ind^n_{n_-}(C)
n: Lie algebra with basis (a_n, b_m, 1)_{n,m ∈ Z}
with [a_n, b_m] = δ_{n+m,0} · 1, all other [x_n, y_m] = 0
Hirzebruch χ_y-genus:
Atiyah-Singer
Def [Hirzebruch 88, Witten 88]
COMPLEX ELLIPTIC GENUS of M


BN-MATH-VC1
ZJRKX
2
F is a VOA built from free fields a(x), b(x)
E arises by accompanying a(x), b(x) by fermionic fields β(x), γ(x) analogously
it's a genus
refine
Def [Kachru / Tripathy 16]
2. More algebra
M ⇝ sVOA(s) ?
basic ingredient: bc-βγ system E
F: cpx vector space with basis
F = ind (ℂ)
Lie algebra with basis
all other
Hirzebruch χy-genus.
Atiyah-Singer
Def [Hirzebruch 88, Witten 88]
COMPLEX ELLIPTIC GENUS of M


BN-MATH-VC1
ZJRKX
$a_m \equiv 0$, $m \leq 0$
$b_n \equiv 0$, $n \leq 0$
$1 = id$
F is a VOA built from free fields a(x), b(x)
E arises by accompanying a(x), b(x) by fermionic fields β(x), γ(x) analogously
idea: for U ⊂ M a holomorphic coordinate chart s.th. $\mathbb{E}_{q,-y}|_U = U \times \mathbb{E}$
$\mathbb{E} \sim E^{\otimes D}$
2. More algebra
M ⇝ sVOA(s) ?
basic ingredient: bc-βγ system E
F: cpx vector space with basis
$(b_{n_1} \cdots b_{n_j} a_{m_1} \cdots a_{m_k} \mathbb{1})$
$F = \mathrm{ind}_{\mathfrak{n}_-}^{\mathfrak{n}}(\mathbb{C})$
n: Lie algebra with basis $(a_n, b_m, 1)_{n,m \in \mathbb{Z}}$
with $[a_n, b_m] = \delta_{n+m,0} \cdot 1$ all other $[x_n, y_m] = 0$
Hirzebruch χ_y-genus:
$\chi_y(M) = \chi(\Lambda_y T^*) = \int_M Td(M)\, ch(\Lambda_y T^*)$
Atiyah-Singer
Def [Hirzebruch 88, Witten 88]
$\mathbb{E}_{q,y} = (-y)^{-\frac{D}{2}} \Lambda_y T^* \otimes \bigotimes_{n=1}^{\infty} (\Lambda_{yq^n} T^* \otimes \Lambda_{y^{-1}q^n} T \otimes S_{q^n} T^* \otimes S_{q^n} T)$
COMPLEX ELLIPTIC GENUS of M


BN-MATH-VC1
ZJRKX
2
a_m ≡ 0 , m ≤ 0
b_n ≡ 0 , n < 0
1 = id
F is a VOA built from free fields a(x), b(x)
E arises by accompanying a(x), b(x) by fermionic fields β(x), γ(x) analogously
idea: for U ⊂ M a holomorphic coordinate chart s.th. E_{q,-y}|_U = U × E
E ≃ E^{⊗D}
2. More algebra
M ⇝ sVOA(s) ?
basic ingredient: bc-βγ system E
F: cpx vector space with basis
F = ind (C)
Lie algebra with basis (a_n, b_m, 1)_{n,m ∈ Z}
with [a_n, b_m] = δ_{n+m,0} · 1 all other [x_n, y_m] = 0
Hirzebruch χ_y-genus:
χ_y(M) = χ(Λ_y T*) = ∫_M Td(M) ch(Λ_y T*)
Atiyah-Singer
Def [Hirzebruch 88, Witten 88]
COMPLEX ELLIPTIC GENUS of M


BN-MATH-VC1
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ZJRKX
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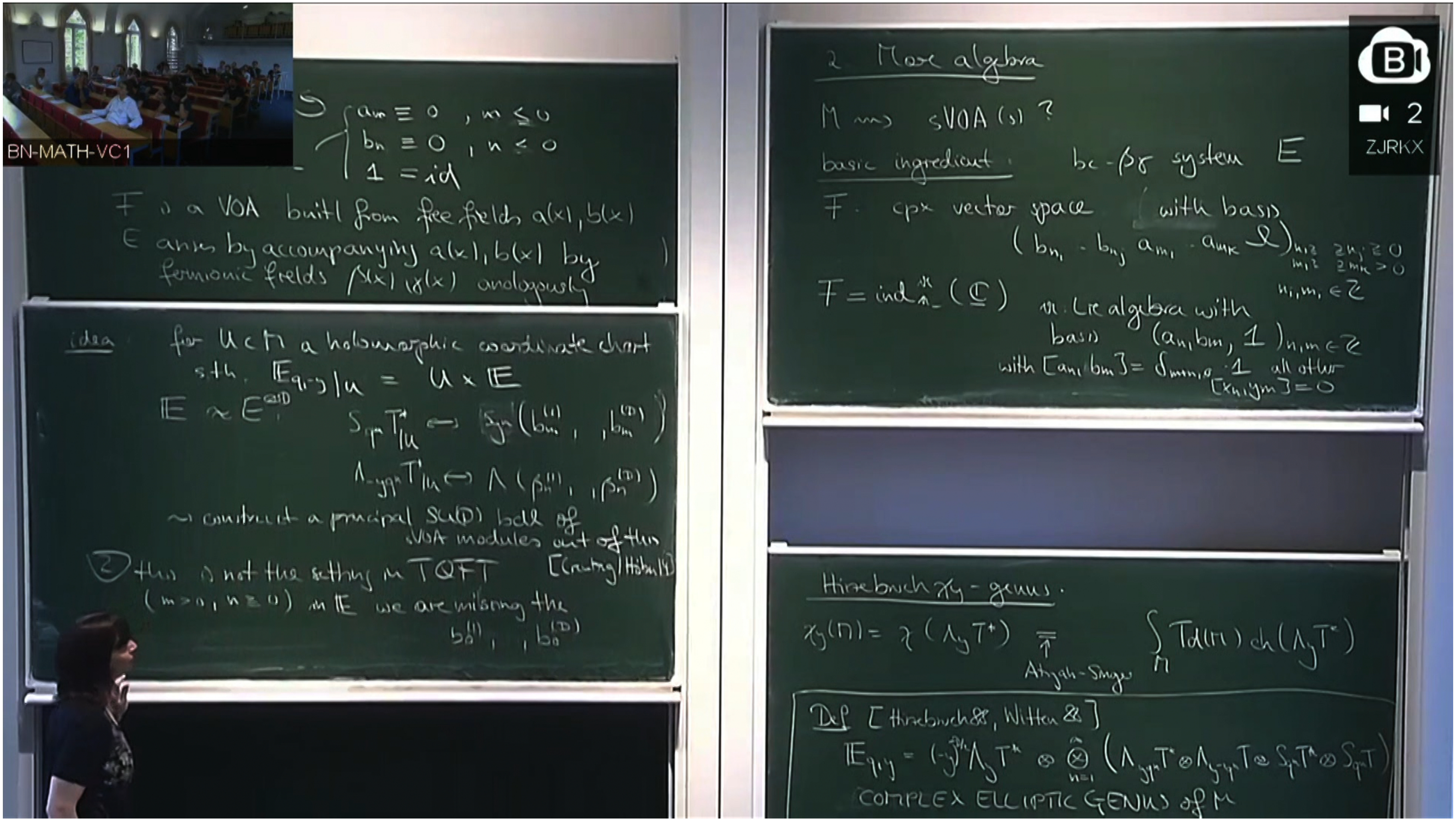
idea: for U ⊂ M a holomorphic coordinate chart s.th. E_{q,-y}|_U = U × E
E ≈ E^{⊗D}
∼ construct a principal SL(D) bdl of VOA modules out of this
this is not the setting in TQFT
2. More algebra
M ⇝ sVOA(s) ?
basic ingredient: bc-βγ system E
F: cpx vector space with basis
F = ind^n_{n_-}(ℂ)
n: Lie algebra with basis (a_n, b_m, 1)_{n,m ∈ ℤ}
with [a_n, b_m] = δ_{m+n,0} · 1, all other [x_n, y_m] = 0
Hirzebruch χ_y-genus:
χ_y(M) = χ(Λ_y T*) = ∫_M Td(M) ch(Λ_y T*)
Atiyah-Singer
Def [Hirzebruch 88, Witten 88]
COMPLEX ELLIPTIC GENUS of M


BN-MATH-VC1
a_m ≡ 0 , m ≤ 0
b_n ≡ 0 , n ≤ 0
1 = id
F is a VOA built from free fields a(x), b(x)
E arises by accompanying a(x), b(x) by fermionic fields β(x), γ(x) analogously
idea: for U ⊂ M a holomorphic coordinate chart s.th. E_{q,y}|U = U × E
E ≅ E^{⊗D}
S_{q^n} T*|U ⟷ Sym(b_m^{(1)}, ..., b_m^{(D)})
Λ_{-yq^n} T*|U ⟷ Λ(β_n^{(1)}, ..., β_n^{(D)})
~ construct a principal SU(D) bdl of sVOA modules out of this
② this is not the setting in TQFT [Costello, Hilburn 14]
(m > 0, n ≥ 0) in E we are missing the b_0^{(1)}, ..., b_0^{(D)}
2 More algebra
M ⇝ sVOA(s) ?
basic ingredient: bc-βγ system E
F: cpx vector space with basis (b_{n_1} ... b_{n_j} a_{m_1} ... a_{m_k} ℓ) n_i ≥ 0, m_i > 0, n_i, m_i ∈ ℤ
F = ind^{𝔫}_{𝔫_-}(ℂ)
𝔫: Lie algebra with basis (a_n, b_m, 1)_{n,m ∈ ℤ}
with [a_n, b_m] = δ_{n+m,0} · 1, all other [x_n, y_m] = 0
Hirzebruch χ_y-genus:
χ_y(M) = χ(Λ_y T*) = ∫_M Td(M) ch(Λ_y T*)
Atiyah-Singer
Def [Hirzebruch 88, Witten 88]
E_{q,y} = (-y)^{-D/2} Λ_y T* ⊗ ⊗_{n=1}^∞ (Λ_{yq^n} T* ⊗ Λ_{y^{-1}q^n} T ⊗ S_{q^n} T* ⊗ S_{q^n} T)
COMPLEX ELLIPTIC GENUS of M

a holomorphic coordinate chart

$\ldots|_U = U \times \mathbb{E}$

$S_{q^n} T^*|_U \leftrightarrow \mathrm{Sym}(b_m^{(1)}, \ldots, b_m^{(D)})$

$\Lambda_{-yq^n} T^*|_U \leftrightarrow \Lambda(\beta_n^{(1)}, \ldots, \beta_n^{(D)})$

$\rightsquigarrow$ construct a principal $SU(D)$ bdl of sVOA modules out of this

② this is not the setting in TQFT [Costello/Hohnhold 14]

$(m>0, n \geq 0)$ in $\mathbb{E}$ we are missing the $b_0^{(1)}, \ldots, b_0^{(D)}$

Thm [Malikov/Schechtman/Vaintrob 99]

2. More algebra

$M \rightsquigarrow$ sVOA(s) ?

basic ingredient: bc-$\beta\gamma$ system E

F: cpx vector space with basis $(b_{n_1} \ldots b_{n_j} a_{m_1} \ldots a_{m_k} \mathbb{1})_{n_i \geq 0,\, m_i > 0}$, $n_i, m_i \in \mathbb{Z}$

$F = \mathrm{ind}_{\mathfrak{n}_-}^{\mathfrak{n}}(\mathbb{C})$ $\quad$ $\mathfrak{n}$: Lie algebra with basis $(a_n, b_m, \mathbb{1})_{n,m \in \mathbb{Z}}$ with $[a_n, b_m] = \delta_{m+n,0} \mathbb{1}$, all other $[x_n, y_m] = 0$

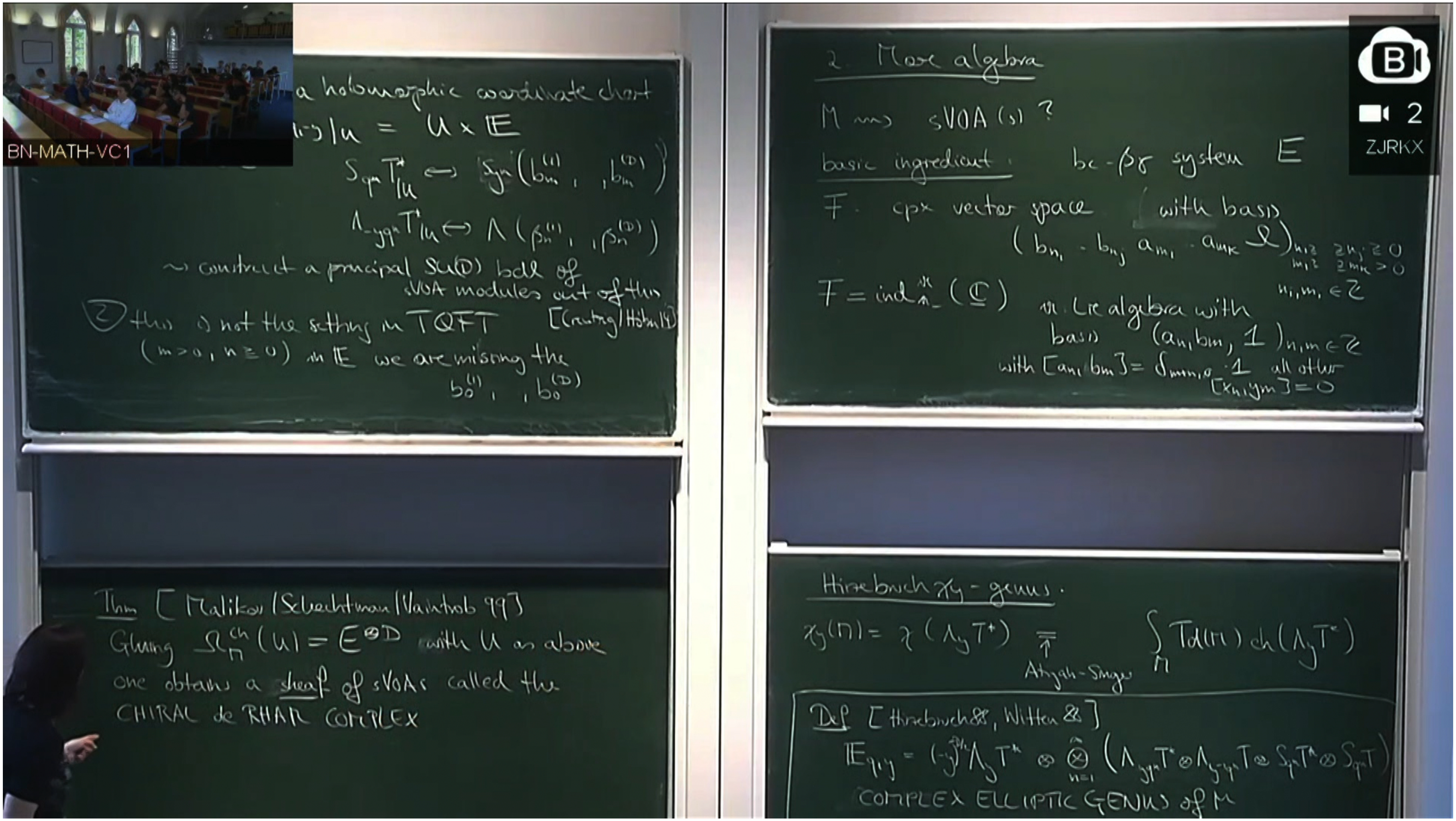
Hirzebruch χ_y-genus:

$$\chi_y(M) = \chi(\Lambda_y T^*) \underset{\text{Atiyah-Singer}}{=} \int_M Td(M)\, ch(\Lambda_y T^*)$$

Def [Hirzebruch 88, Witten 88]

$$\mathbb{E}_{q,y} = (-y)^{-D/2} \Lambda_y T^* \otimes \hat{\bigotimes}_{n=1}^{\infty} \left(\Lambda_{yq^n} T^* \otimes \Lambda_{y^{-1}q^n} T \otimes S_{q^n} T^* \otimes S_{q^n} T\right)$$

COMPLEX ELLIPTIC GENUS of M


BN-MATH-VC1
a holomorphic coordinate chart
$|_U = U \times \mathbb{E}$
$S_{q^m} T^*|_U \leftrightarrow Sym(b_m^{(1)}, \ldots, b_m^{(D)})$
$\Lambda_{-yq^n} T^*|_U \leftrightarrow \Lambda(\beta_n^{(1)}, \ldots, \beta_n^{(D)})$
~ construct a principal SU(D) bdl of sVOA modules out of this
(2) this is not the setting in TQFT [Costello/Holstein]
(m > 0, n ≥ 0) in $\mathbb{E}$ we are missing the $b_0^{(1)}, \ldots, b_0^{(D)}$
Thm [Malikov/Schechtman/Vaintrob 99]
Gluing $\Omega^{ch}_M(U) = E^{\otimes D}$ with U as above one obtains a sheaf of sVOAs called the CHIRAL de RHAM COMPLEX
2. More algebra
M ⇝ sVOA(s) ?
basic ingredient: bc-βγ system E
F: cpx vector space with basis
$(b_{n_1} \ldots b_{n_j} a_{m_1} \ldots a_{m_k} \mathbb{1})$
$F = ind^{\mathfrak{n}}_{\mathfrak{n}_-}(\mathbb{C})$
n: Lie algebra with basis $(a_n, b_m, \mathbb{1})_{n,m \in \mathbb{Z}}$
with $[a_n, b_m] = \delta_{m+n,0} \mathbb{1}$ all other $[x_n, y_m] = 0$
Hirzebruch χ_y-genus:
$\chi_y(M) = \chi(\Lambda_y T^*) = \int_M Td(M)\, ch(\Lambda_y T^*)$
Atiyah-Singer
Def [Hirzebruch 88, Witten 88]
$\mathbb{E}_{q,y} = (-y)^{-D/2} \Lambda_y T^* \otimes \bigotimes_{n=1}^{\infty} (\Lambda_{yq^n} T^* \otimes \Lambda_{y^{-1}q^n} T \otimes S_{q^n} T^* \otimes S_{q^n} T)$
COMPLEX ELLIPTIC GENUS of M

a holomorphic coordinate chart

$\ldots|_U = U \times \mathbb{E}$

$S_{q^n} T^*|_U \leftrightarrow \mathrm{Sym}(b_m^{(1)}, \ldots, b_m^{(D)})$

$\Lambda_{-yq^n} T^*|_U \leftrightarrow \Lambda(\beta_n^{(1)}, \ldots, \beta_n^{(D)})$

$\rightsquigarrow$ construct a principal SU(D) bdl of sVOA modules out of this

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(m > 0, n ≥ 0) in $\mathbb{E}$ we are missing the $b_0^{(1)}, \ldots, b_0^{(D)}$

Thm [Malikov/Schechtman/Vaintrob 99]

Gluing $\mathcal{S}^{ch}_M(U) = E^{\otimes D}$ with U as above one obtains a sheaf of sVOAs called the CHIRAL de RHAM COMPLEX. It is filtered with associated graded $\mathbb{E}_{q,-y}$

2. More algebra

M $\rightsquigarrow$ sVOA(s)?

basic ingredient: bc-βγ system E

F: cpx vector space with basis

$(b_{n_1} \ldots b_{n_j}\, a_{m_1} \ldots a_{m_k}\, \Omega)_{n_1 \geq \ldots \geq n_j \geq 0,\; m_1 \geq \ldots \geq m_k > 0,\; n_i, m_i \in \mathbb{Z}}$

$F = \mathrm{ind}_{\mathfrak{n}_-}^{\mathfrak{n}}(\mathbb{C})$ $\mathfrak{n}$: Lie algebra with basis $(a_n, b_m, 1)_{n,m \in \mathbb{Z}}$ with $[a_n, b_m] = \delta_{m+n,0} \cdot 1$, all other $[x_n, y_m] = 0$

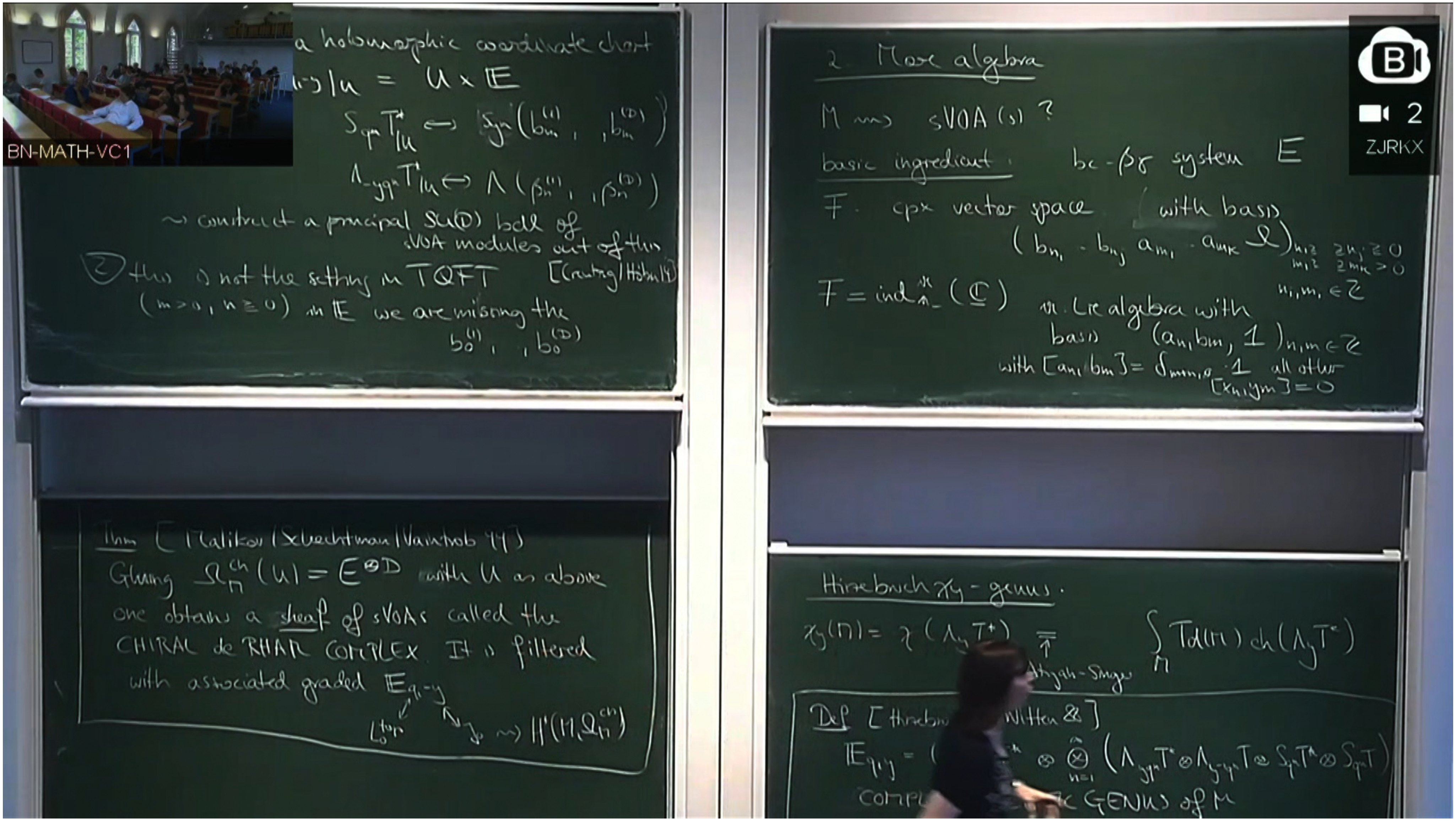
Hirzebruch χ_y-genus:

$$\chi_y(M) = \chi(\Lambda_y T^*) \underset{\text{Atiyah-Singer}}{=} \int_M \mathrm{Td}(M)\, \mathrm{ch}(\Lambda_y T^*)$$

Def [Hirzebruch 88, Witten 88]

$$\mathbb{E}_{q,y} = (-y)^{-D/2} \Lambda_y T^* \otimes \bigotimes_{n=1}^{\infty} \left(\Lambda_{yq^n} T^* \otimes \Lambda_{y^{-1}q^n} T \otimes S_{q^n} T^* \otimes S_{q^n} T\right)$$

COMPLEX ELLIPTIC GENUS of M


BN-MATH-VC1
2
ZJRKX
a holomorphic coordinate chart
~ construct a principal SU(D) bdl of sVOA modules out of this
(2) this is not the setting in TQFT
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Thm [Malikov/Schechtman/Vaintrob 99]
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2. More algebra
M ~> sVOA(s)?
basic ingredient: bc-βγ system
F: cpx vector space with basis
Lie algebra with basis
all other
Hirzebruch χy-genus:
Atiyah-Singer
Def [Hirzebruch, Witten 87]
GENUS of M

a holomorphic coordinate chart

$(\ldots)|_U = U \times \mathbb{E}$

$S_{q^m} T^*|_U \leftrightarrow S_{q^m}(b_m^{(1)}, \ldots, b_m^{(D)})$

$\Lambda_{-yq^n} T^*|_U \leftrightarrow \Lambda(\beta_n^{(1)}, \ldots, \beta_n^{(D)})$

⇝ construct a principal SU(D) bdl of sVOA modules out of this

② this is not the setting in TQFT [Costello/Höhn 14]
$(m>0,\ n\geq 0)$ in $\mathbb{E}$ we are missing the $b_0^{(1)}, \ldots, b_0^{(D)}$

Thm [Malikov/Schechtman/Vaintrob 99]
Gluing $\Omega^{ch}_M(U) = E^{\otimes D}$ with U as above one obtains a sheaf of sVOAs called the CHIRAL de RHAM COMPLEX. It is filtered with associated g… $\mathbb{E}_{q,-y}$

L_0^{top}, J_0 ⇝ $H^*(M, \Omega^{ch}_M)$

Hirzebruch χ_y-genus:

$$\chi_y(M) = \chi(\Lambda_y T^*) \underset{\text{Atiyah-Singer}}{=} \int_M Td(M)\, ch(\Lambda_y T^*)$$

Def [Hirzebruch 88, Witten 88]

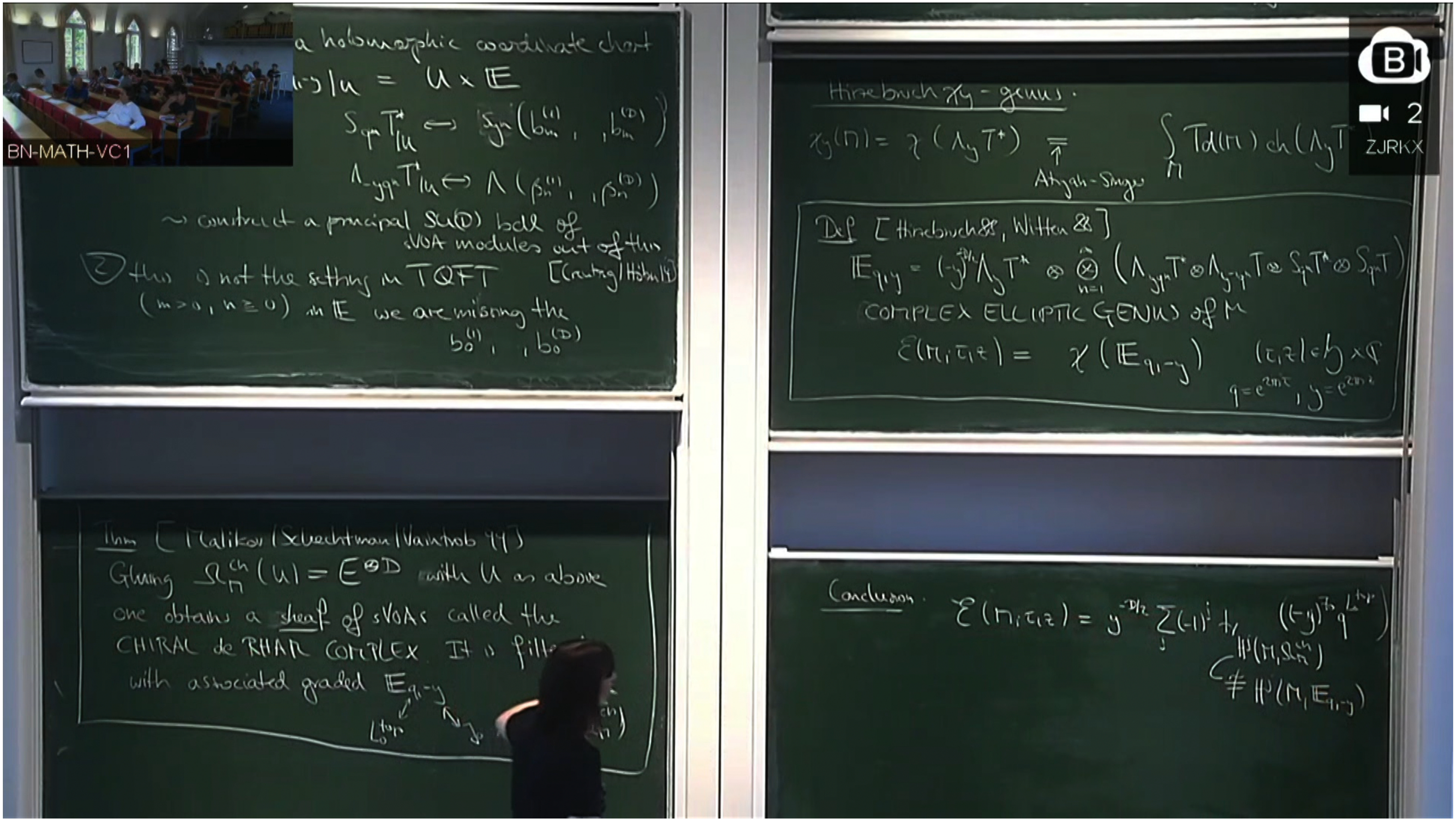
$$\mathbb{E}_{q,y} = (-y)^{-D/2} \Lambda_y T^* \otimes \bigotimes_{n=1}^{\infty} \left(\Lambda_{yq^n} T^* \otimes \Lambda_{y^{-1}q^n} T \otimes S_{q^n} T^* \otimes S_{q^n} T\right)$$

COMPLEX ELLIPTIC GENUS of M

$$\mathcal{E}(M;\tau,z) = \chi(\mathbb{E}_{q,-y}) \qquad (\tau,z)\in \mathfrak{h}\times\mathbb{C},\ q=e^{2\pi i\tau},\ y=e^{2\pi i z}$$

Conclusion.

$$\mathcal{E}(M;\tau,z) = y^{-D/2}\sum_j (-1)^j \operatorname{tr}_{H^j(M,\Omega^{ch}_M)} \left((-y)^{J_0} q^{L_0^{top}}\right)$$


BN-MATH-VC1
a holomorphic coordinate chart
U × E
~ construct a principal SU(D) bdl of sVOA modules out of this
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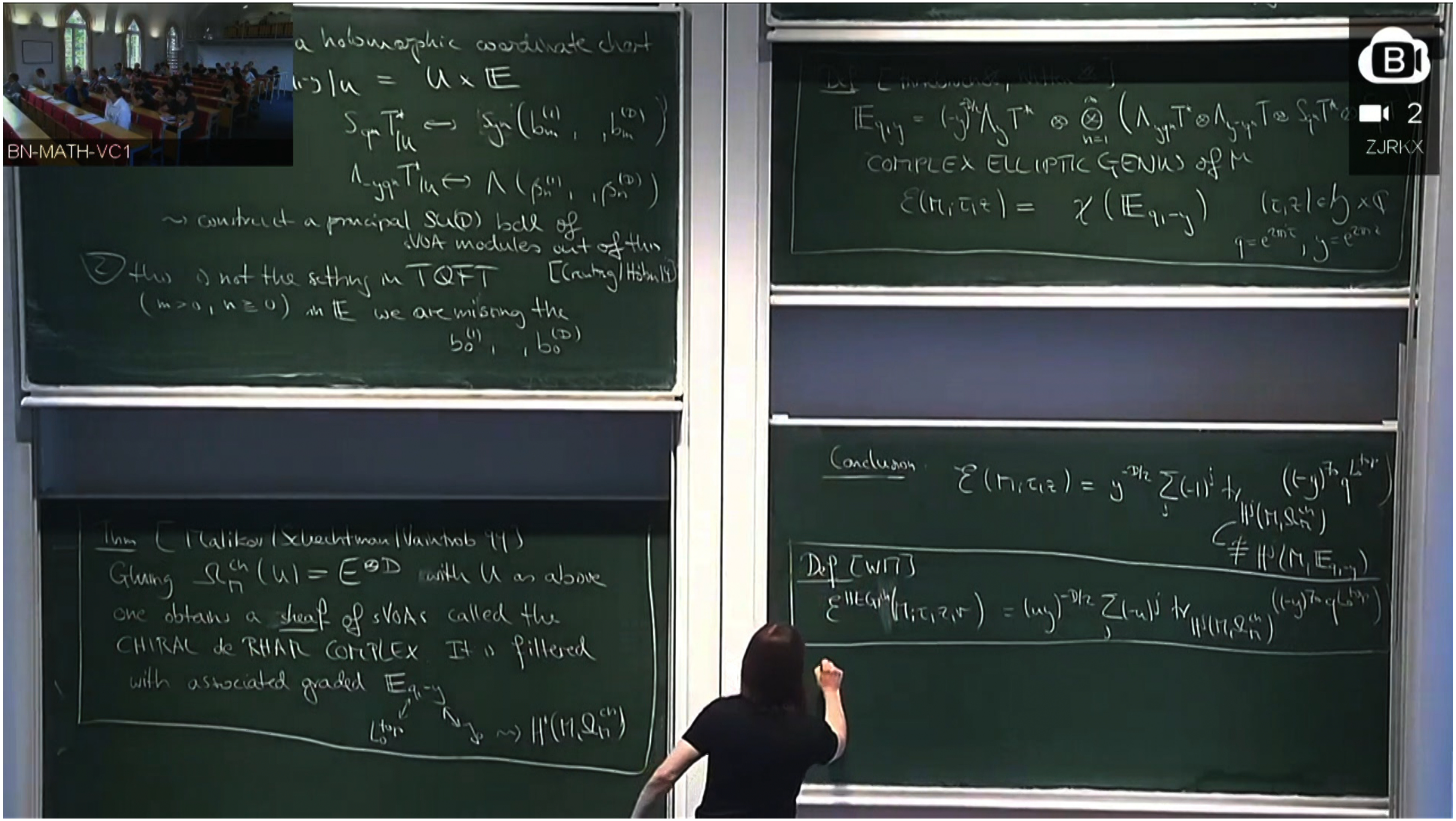
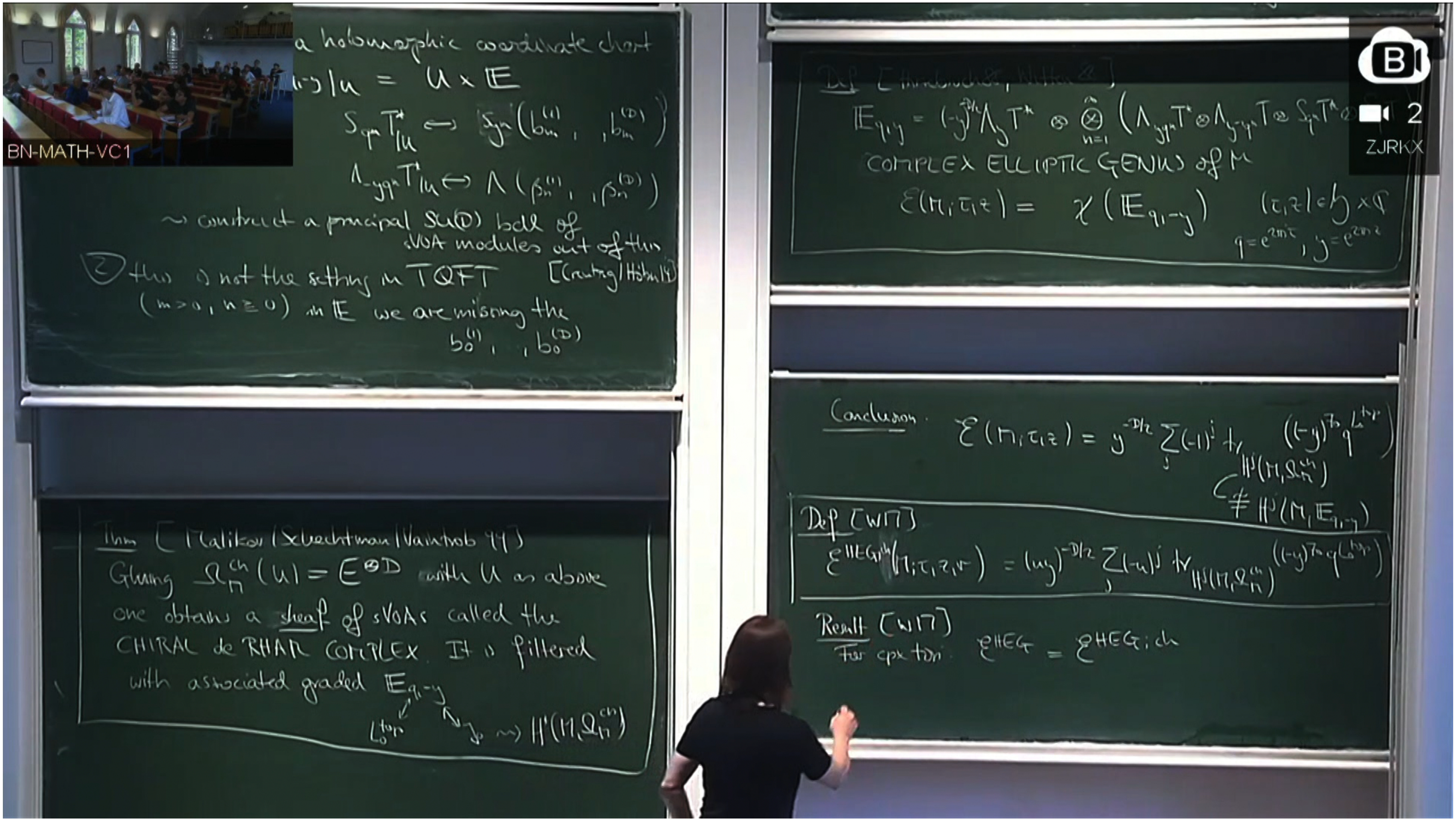
Hirzebruch χ_y - genus:
Atiyah-Singer
Def [Hirzebruch 88, Witten 88]
COMPLEX ELLIPTIC GENUS of M
Conclusion.
ZJRKX


BN-MATH-VC1
a holomorphic coordinate chart
~> construct a principal SU(D) bdl of
sVOA modules out of this
② this is not the setting in TQFT
we are missing the
Def
COMPLEX ELLIPTIC GENUS of M
Conclusion
Def [WIT]
Thm [Malikov/Schechtman/Vaintrob 99]
one obtains a sheaf of sVOAs called the
CHIRAL de RHAM COMPLEX. It is filtered
with associated graded
ZJRKX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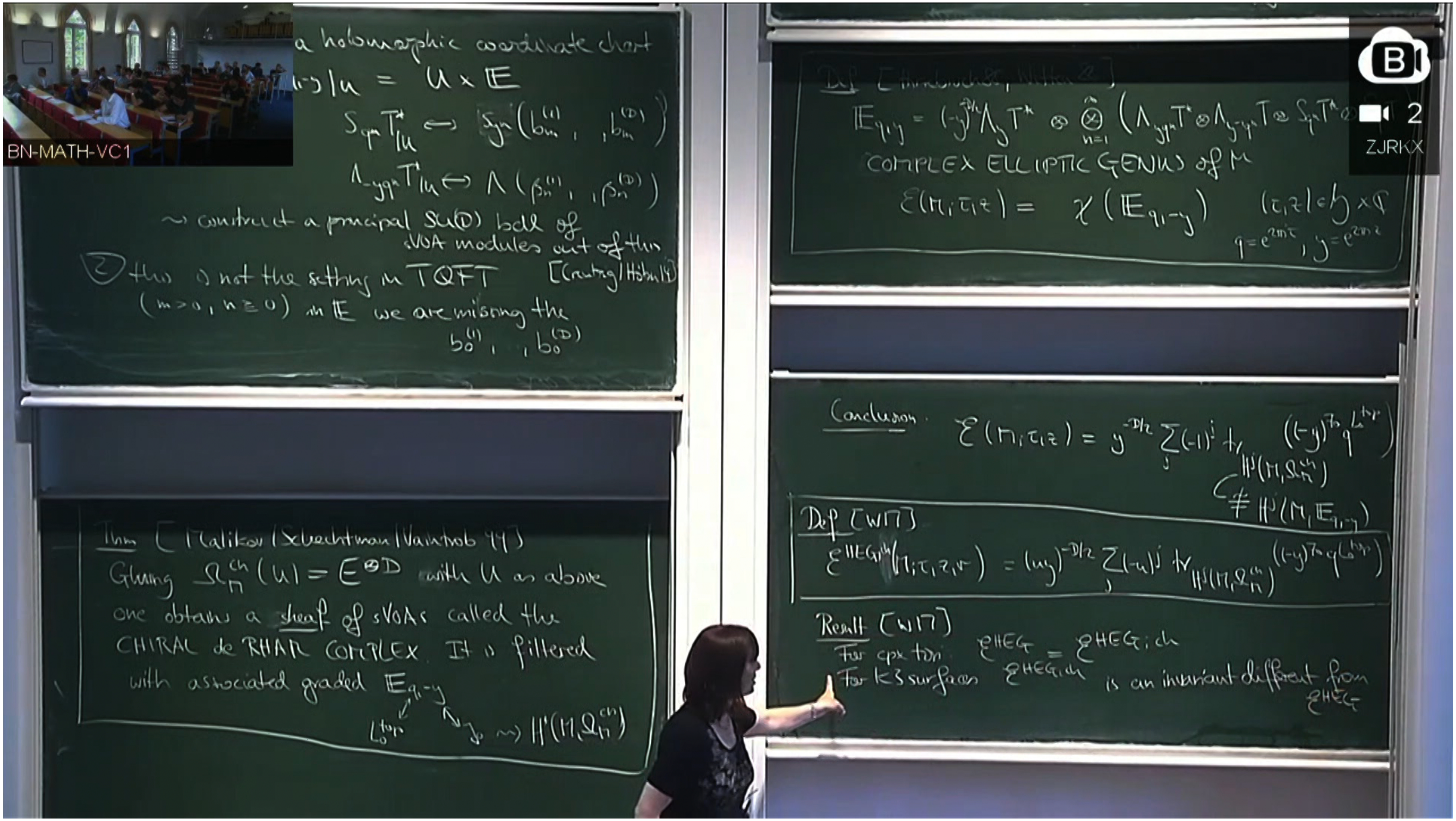
2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COMPLEX ELLIPTIC GENUS of M
Conclusion
Def [WIT]
Result [WIT]
For cpx tori
For K3 surfaces
is an invariant different from
2
ZJRKX

a holomorphic coordinate chart

$\mathbb{E}|_U = U \times \mathbb{E}$

$S_{q^m} T^*|_U \leftrightarrow \mathrm{Sym}(b_m^{(1)}, \ldots, b_m^{(D)})$

$\Lambda_{-yq^n} T^*|_U \leftrightarrow \Lambda(\beta_n^{(1)}, \ldots, \beta_n^{(D)})$

$\rightsquigarrow$ construct a principal $SU(D)$ bdl of sVOA modules out of this

(2) this is not the setting in TQFT [Creutzig/Höhn 14]

$(m > 0, n \geq 0)$ in $\mathbb{E}$ we are missing the $b_0^{(1)}, \ldots, b_0^{(D)}$

Thm [Malikov/Schechtman/Vaintrob 99]

Gluing $\Omega_M^{ch}(U) = E^{\otimes D}$ with U as above one obtains a sheaf of sVOAs called the CHIRAL de RHAM COMPLEX. It is filtered with associated graded ... $\rightsquigarrow H^\bullet(M, \Omega_M^{ch})$

Def [Hirzebruch, Witten 88]

$$\mathbb{E}_{q,y} = (-y)^{-D/2} \Lambda_{y} T^* \otimes \bigotimes_{n=1}^{\infty} \left(\Lambda_{yq^n} T^* \otimes \Lambda_{y^{-1}q^n} T \otimes S_{q^n} T^* \otimes S_{q^n} T \right)$$

COMPLEX ELLIPTIC GENUS of M

$$\mathcal{E}(M; \tau, z) = \chi(\mathbb{E}_{q,-y}) \qquad (\tau, z) \in \mathfrak{h} \times \mathbb{C}$$

$q = e^{2\pi i \tau}, \ y = e^{2\pi i z}$

Conclusion

$$\mathcal{E}(M; \tau, z) = y^{-D/2} \sum_j (-1)^j \, \mathrm{tr}_{H^j(M, \Omega_M^{ch})} \left((-y)^{J_0} q^{L_0} \right)$$

$\neq H^j(M, \mathbb{E}_{q,-y})$

Def [WM]

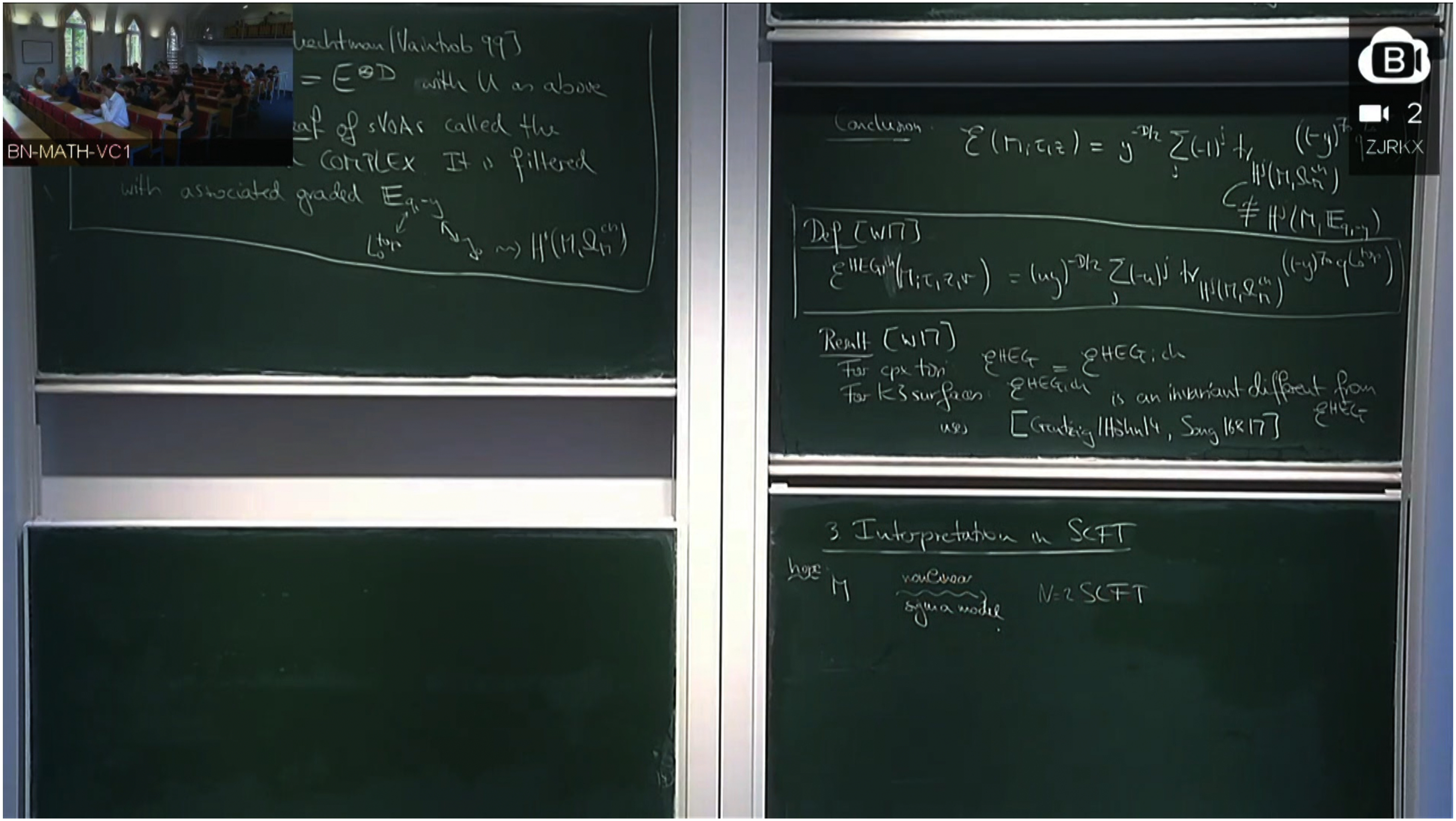
$$\mathcal{E}^{HEG,ch}(M; \tau, z, u) = (uy)^{-D/2} \sum_j (-u)^j \, \mathrm{tr}_{H^j(M, \Omega_M^{ch})} \left((-y)^{J_0} q^{L_0} \right)$$

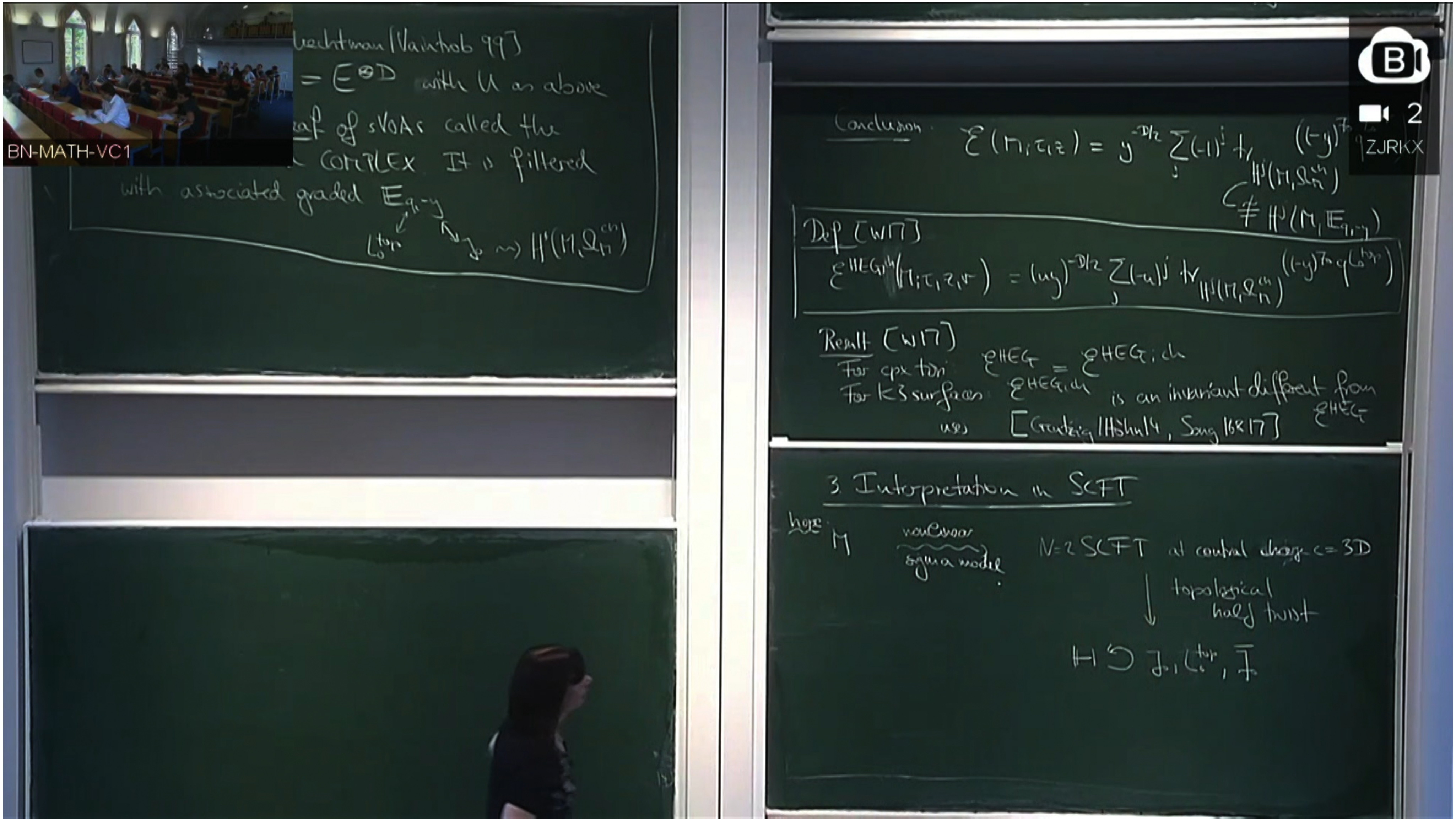
Result [WM]

For cpx tori $\mathcal{E}^{HEG} = \mathcal{E}^{HEG,ch}$

For K3 surfaces $\mathcal{E}^{HEG,ch}$ is an invariant different from $\mathcal{E}^{HEG}$

uses [Creutzig/Höhn 14, Song 16&17]


BN-MATH-VC1
2
ZJRKX
echtman [Vaintrob 99]
= E⊗D with U as above
of sVOAs called the
COMPLEX It is filtered
with associated graded E_{q,-y}
Conclusion: E(M; τ, z) = y^{-D/2} Σ_j (-1)^j tr
Def [W17]
Result [W17]
For cpx tori E^HEG = E^HEG;ch
For K3 surfaces E^HEG;ch is an invariant different from E^HEG
uses [Creutzig/Höhn 14, Song 16&17]
3 Interpretation in SCFT
here: M
nonlinear sigma model
N=2 SCFT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Def [W17]
Result [W17]
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3 Interpretation in SCFT
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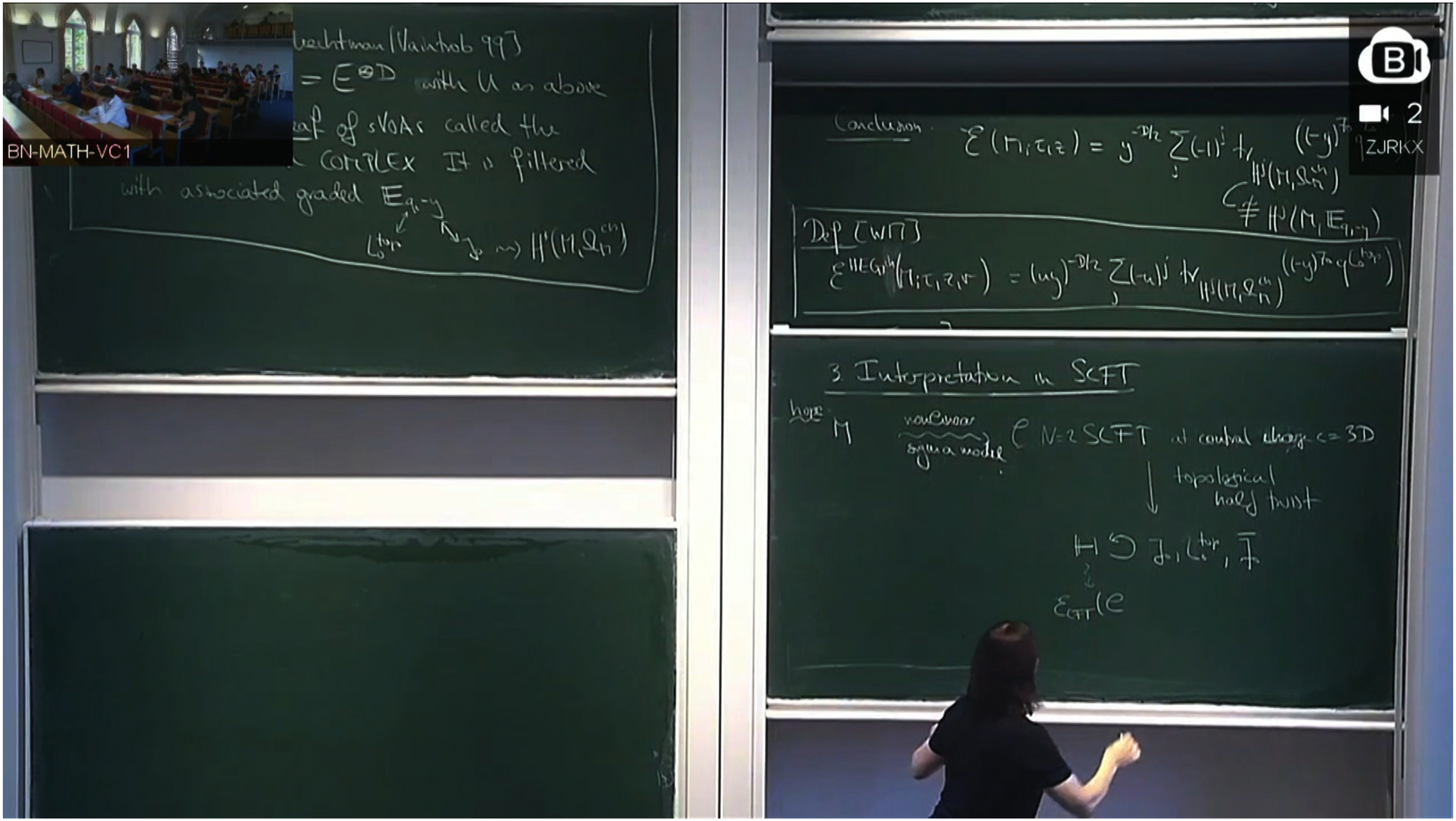
sigma model
N=2 SCFT at central charge c=3D
topological
half twist


BN-MATH-VC1
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Def [WM]
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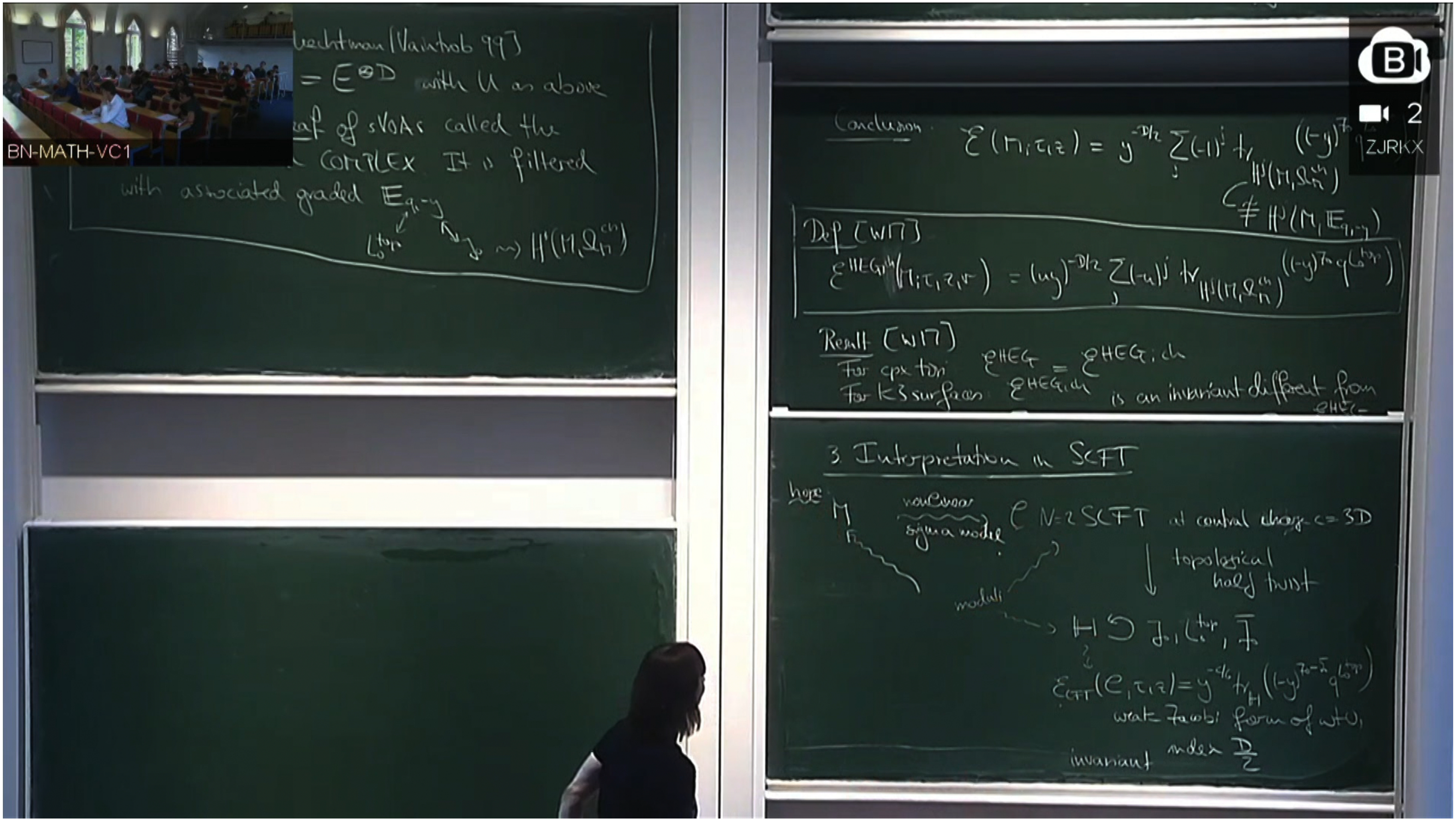
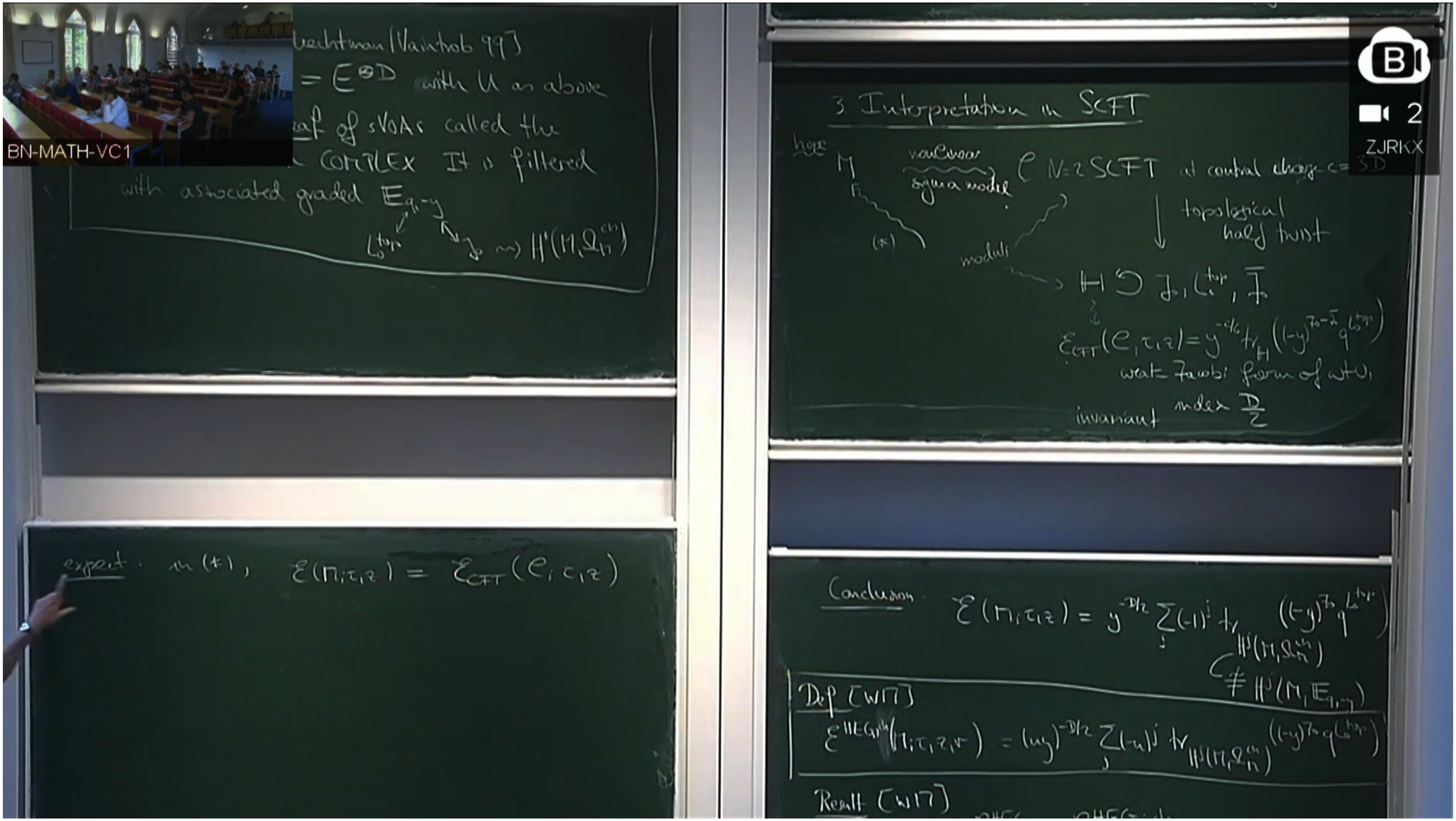
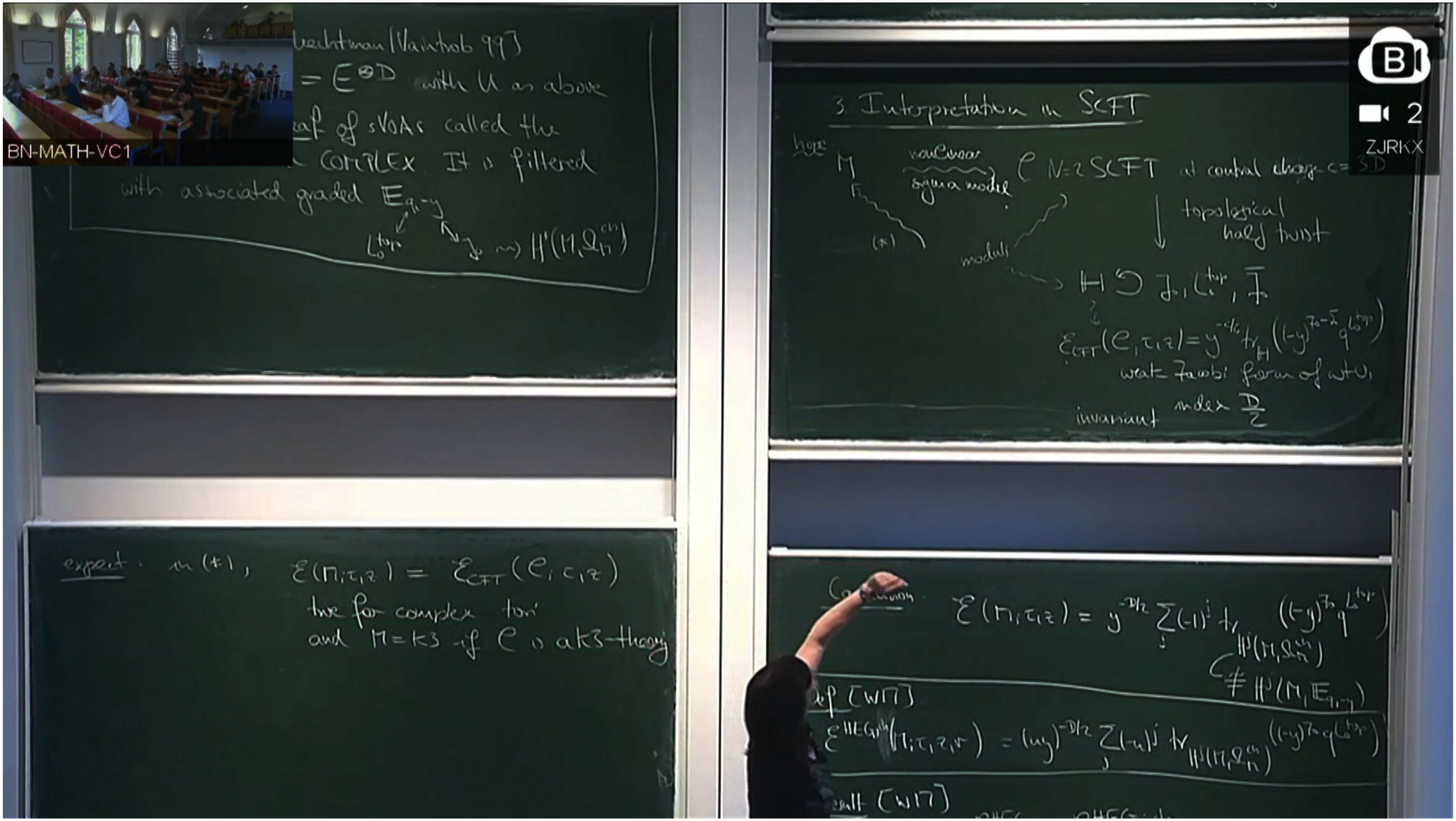
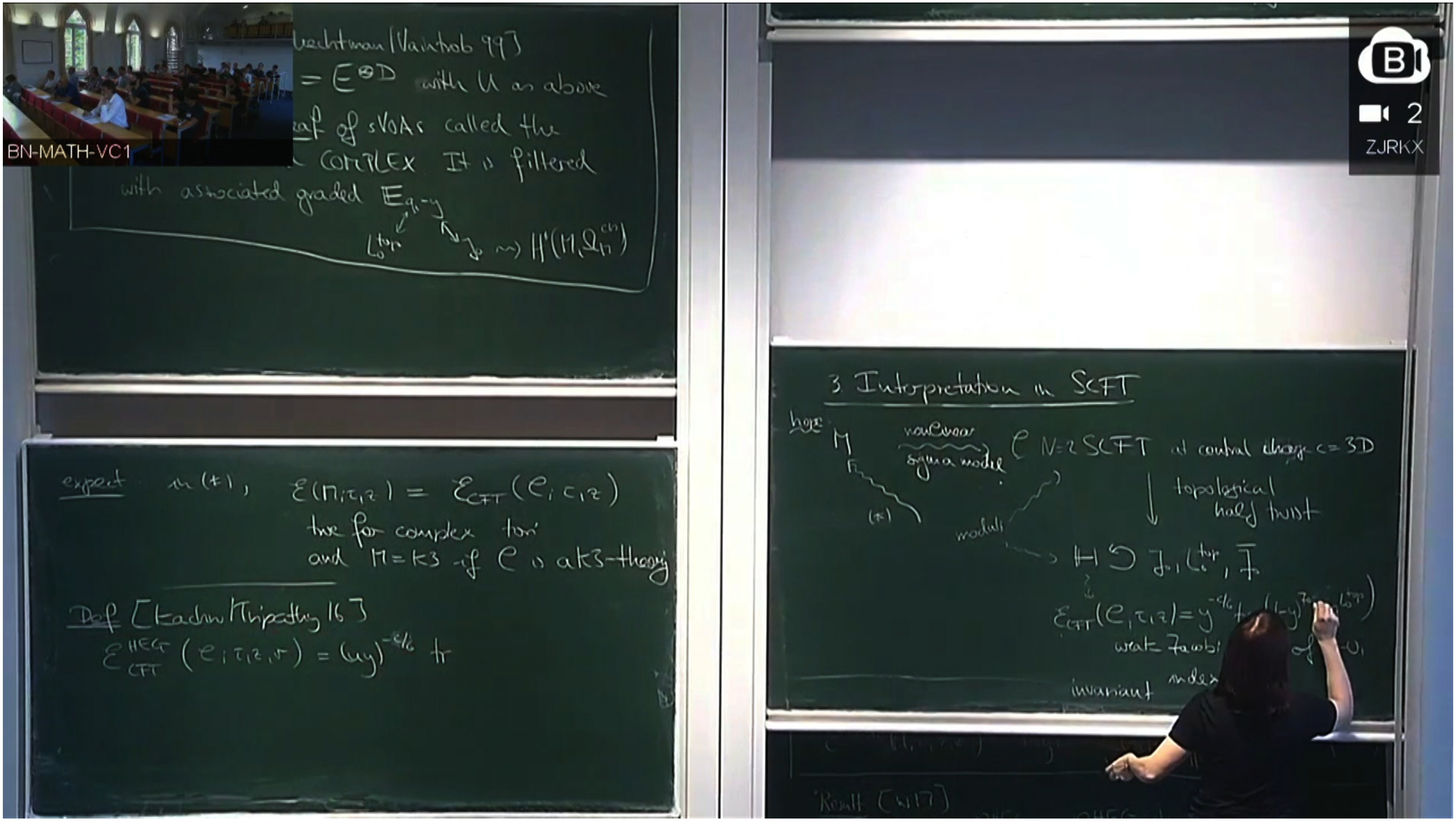

BN-MATH-VC1
2
ZJRKX
with associated graded
Conclusion
Def [WT]
Result [WT]
For cpx tori
For K3 surfaces
is an invariant different from
3. Interpretation in SCFT
here
non-linear sigma model
N=2 SCFT at central charge c = 3D
topological half twist
moduli
weak Jacobi form of wt 0, index D/2
invariant


BN-MATH-VC1
ZJRKX
2
with associated graded $\mathbb{E}_{q,-y}$
3. Interpretation in SCFT
nonlinear sigma model
$N=2$ SCFT at central charge $c = 3D$
topological half twist
moduli
weak Jacobi form of wt 0, index $\frac{D}{2}$
invariant
expect: in (*), $\mathcal{E}(M;\tau,z) = \mathcal{E}_{CFT}(\mathcal{C};\tau,z)$
Conclusion
Def [WIT]
Result [WIT]


BN-MATH-VC1
ZJRKX
3. Interpretation in SCFT
expect: in (*), $\mathcal{E}(M;\tau,z) = \mathcal{E}_{CFT}(\mathcal{C},\tau,z)$
true for complex tori and M = K3 if $\mathcal{C}$ is a K3-theory
weak Jacobi form of wt 0, index $\frac{D}{2}$
Def [WM]
Recall [WM]


BN-MATH-VC1
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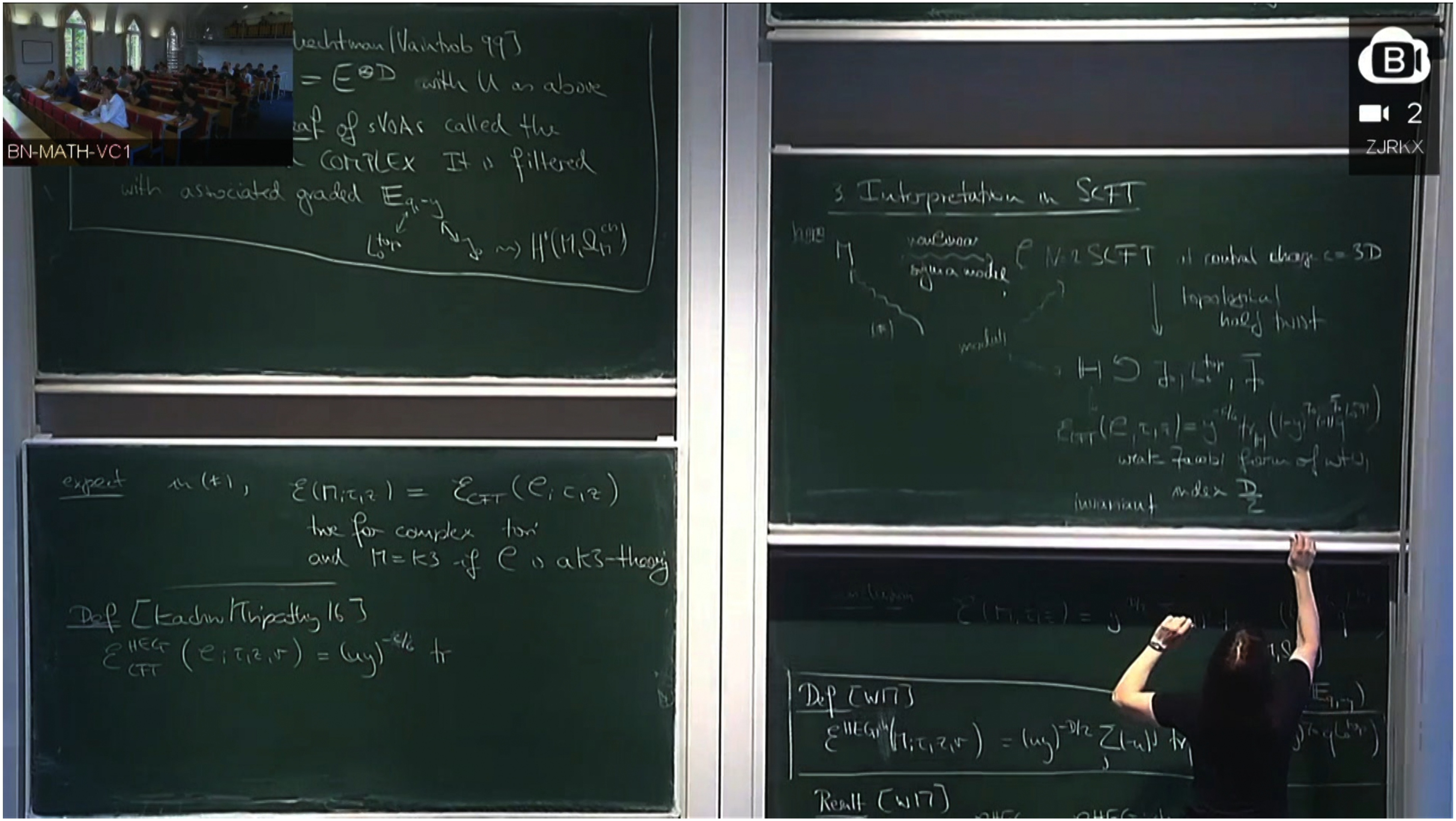
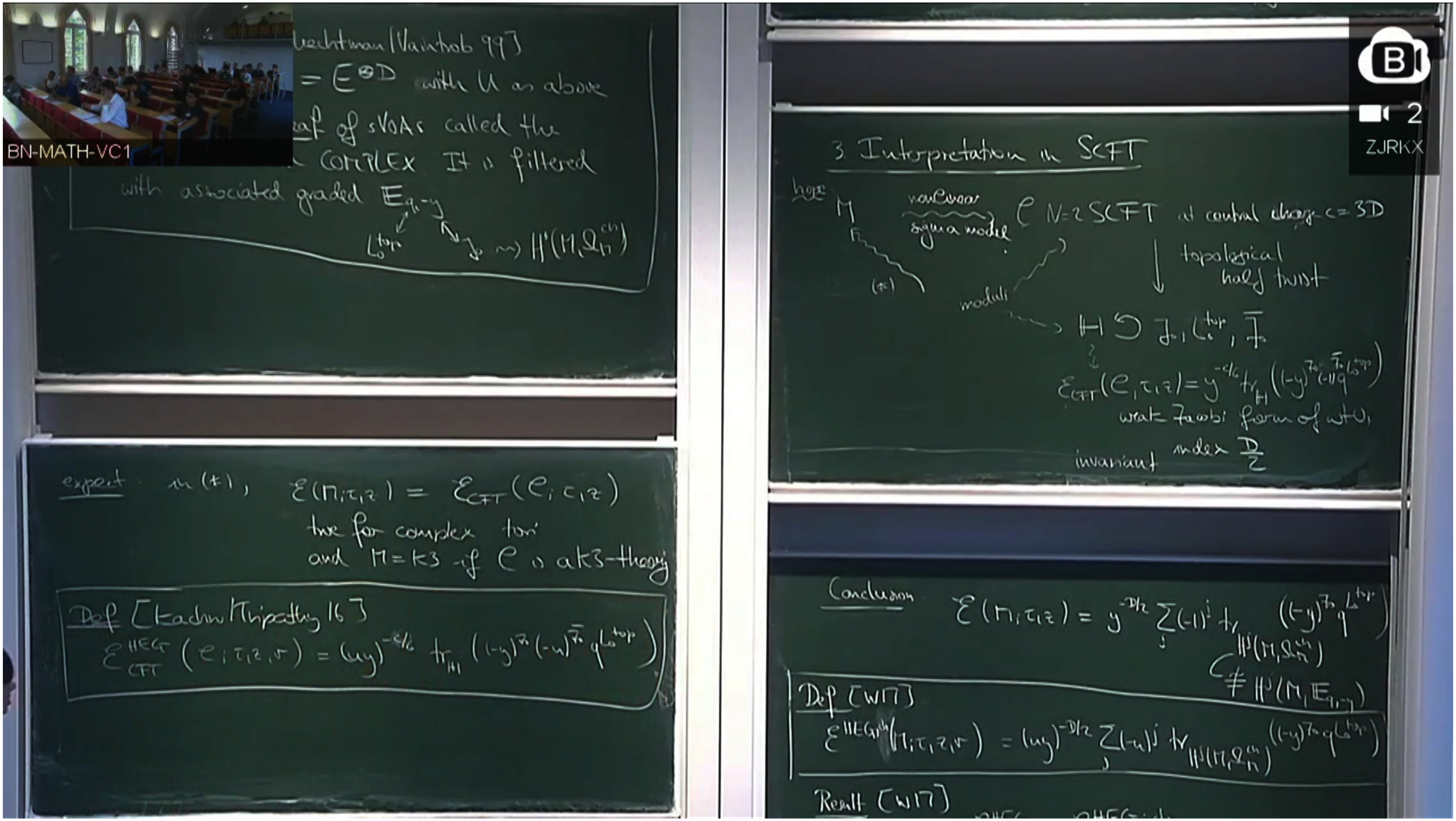
3. Interpretation in SCFT
nonlinear sigma model
N=2 SCFT at central charge c=3D
topological half twist
moduli
weak Jacobi
invariant
index
expect
true for complex tori
Def [Kachru Tripathy 16]
Result (W17)


BN-MATH-VC1
ZJRKX
2
with associated graded $\mathbb{E}_{q,-y}$
expect in (*), $\mathcal{E}(M;\tau,z) = \mathcal{E}_{CFT}(\mathcal{C};\tau,z)$
true for complex tori
and M = K3 if $\mathcal{C}$ is a K3-theory
3. Interpretation in SCFT
N=2 SCFT
topological half twist
weak Jacobi form of wt 0,
index $\frac{D}{2}$
Def [WM]
Result [WM]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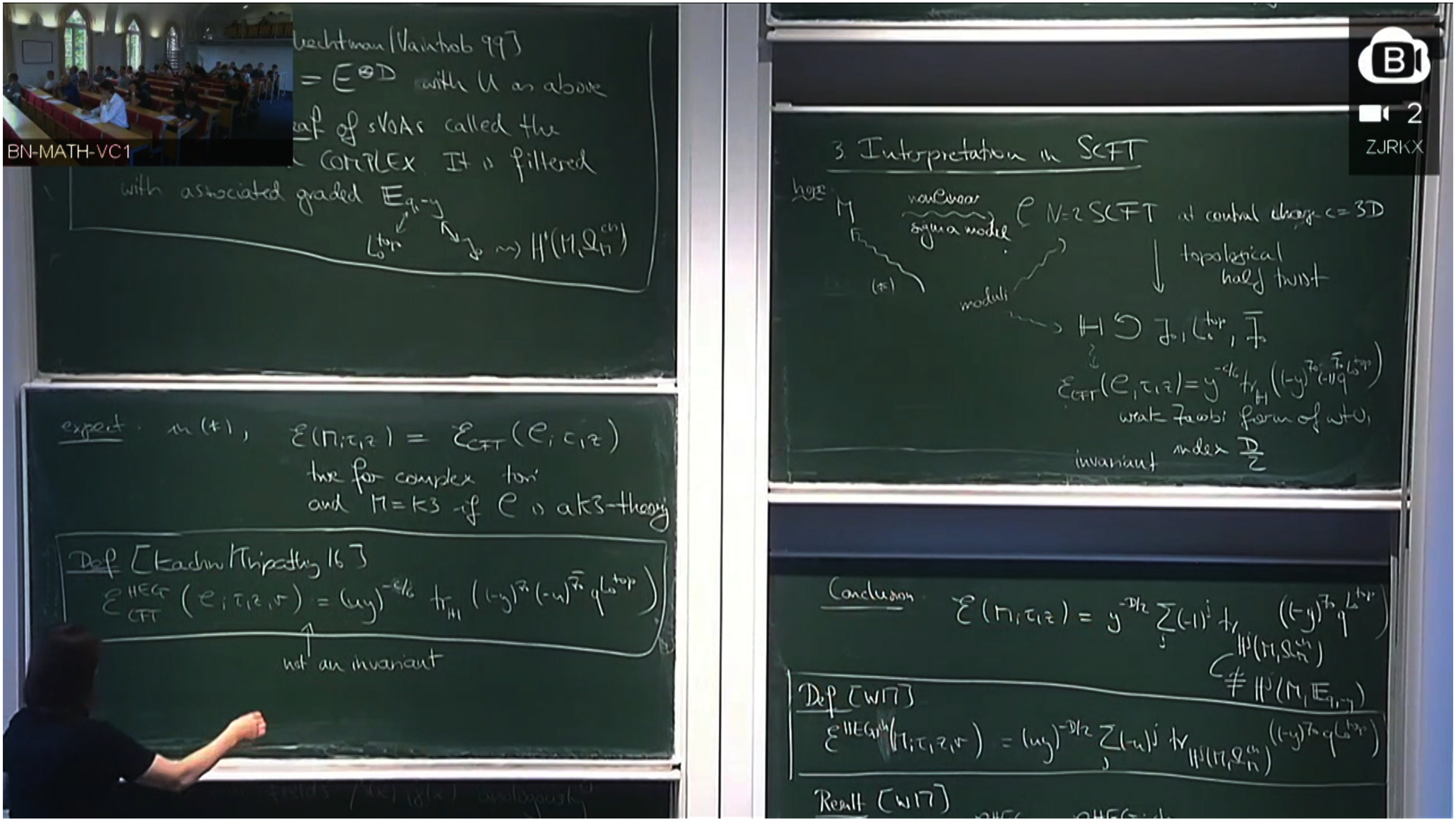
3. Interpretation in SCFT
hope
M
nonlinear sigma model
C N=2 SCFT at central charge c = 3D
topological half twist
moduli
H ⊃ J_0, L_0^top, J̄_0
weak Jacobi form of wt 0, index D/2
invariant
expect in (*), E(M;τ,z) = E_CFT(C;τ,z)
true for complex tori
and M = K3 if C is a K3-theory
Def [Kachru Tripathy 16]
Conclusion
Def [WM]
Result [WM]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expect
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Def [Kachru/Tripathy 16]
not an invariant
Conclusion
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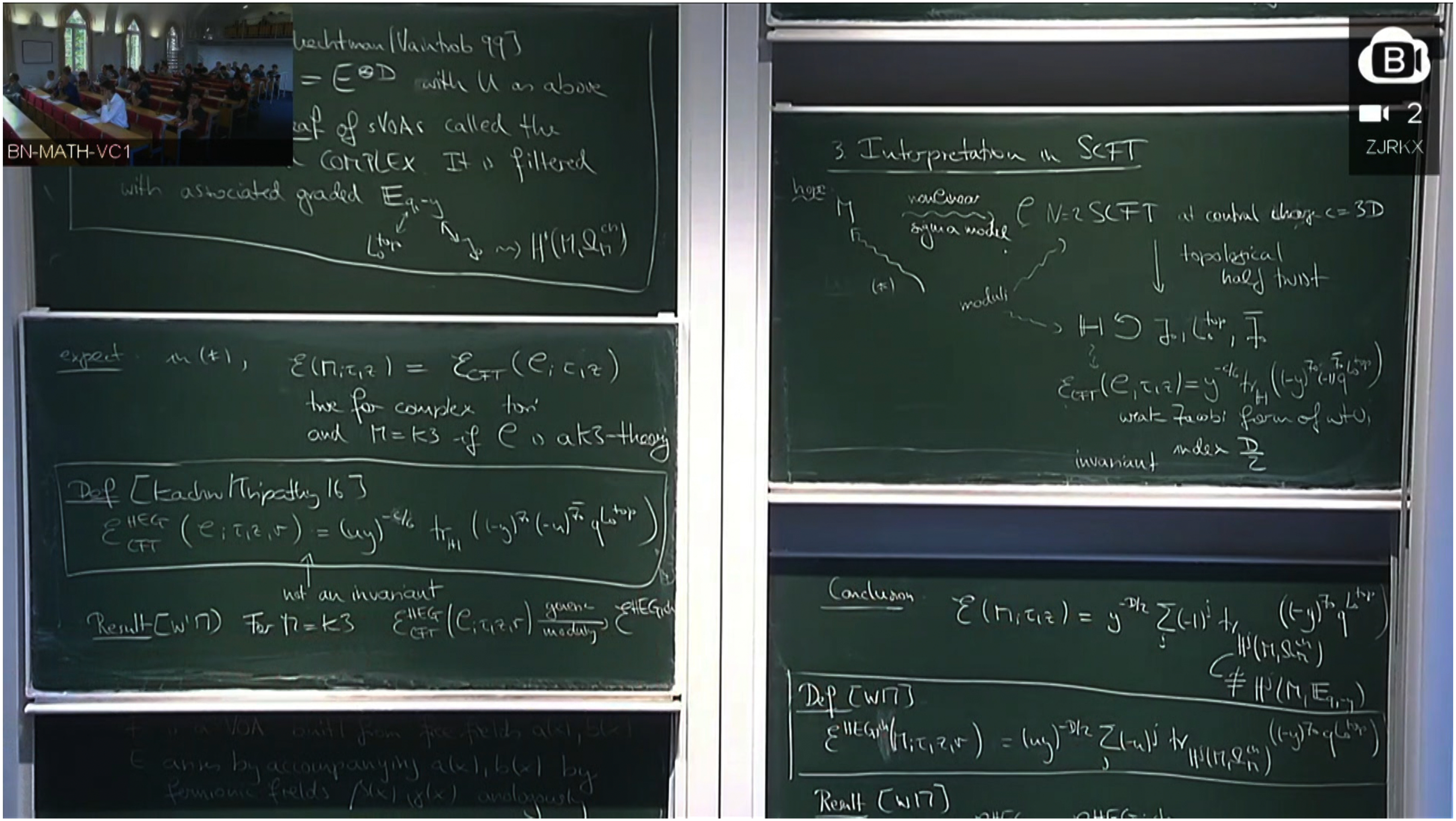
Result [WM]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with associated graded $\mathbb{E}_{q,-y}$
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Def [Kachru/Tripathy 16]
$\mathcal{E}^{HEG}_{CFT}(\mathcal{C};\tau,z,\nu) = (uy)^{-c/6}\, tr_{H}\left((-y)^{J_0}(-u)^{\bar{J}_0} q^{L_0^{top}}\right)$
not an invariant
Result [W'17] For M = K3
3. Interpretation in SCFT
hope
nonlinear sigma model
N=2 SCFT at central charge c = 3D
topological half twist
moduli
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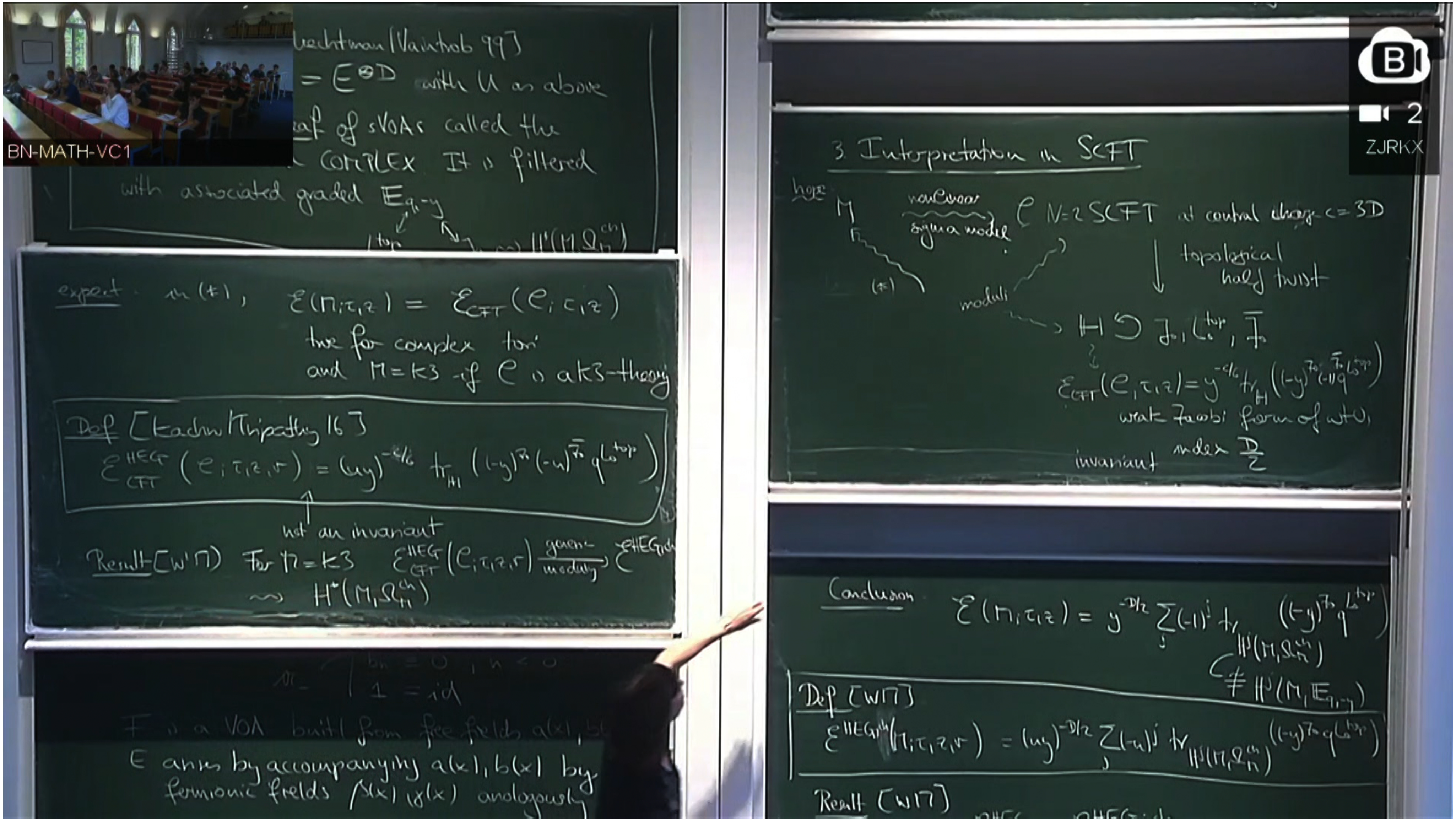
weak Jacobi form of wt 0, index D/2
invariant
Conclusion
$\mathcal{E}(M;\tau,z) = y^{-D/2}\sum_j (-1)^j\, tr\left((-y)^{J_0} q^{L_0^{top}}\right)$
Def [W17]
$\mathcal{E}^{HEG}(M;\tau,z,\nu) = (uy)^{-D/2}\sum_j (-u)^j\, tr_{H^j(M,\Omega^{ch}_M)}\left((-y)^{J_0} q^{L_0^{top}}\right)$
Result [W17]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3. Interpretation in SCFT
nonlinear sigma model
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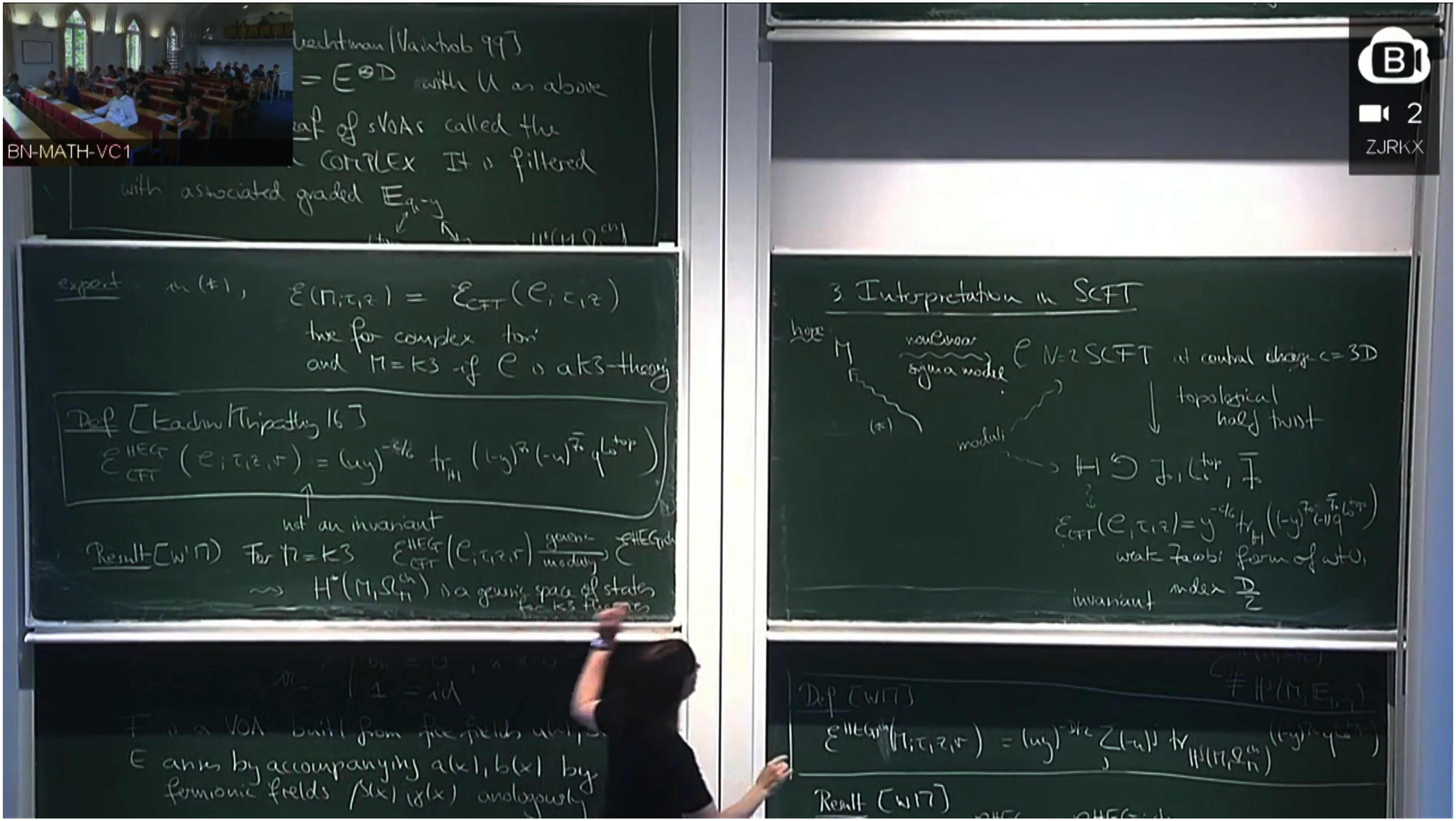
weak Jacobi form of wt 0, index D/2
Conclusion
Def [WM]
Result [WM]
E arises by accompanying a(x), b(x) by fermionic fields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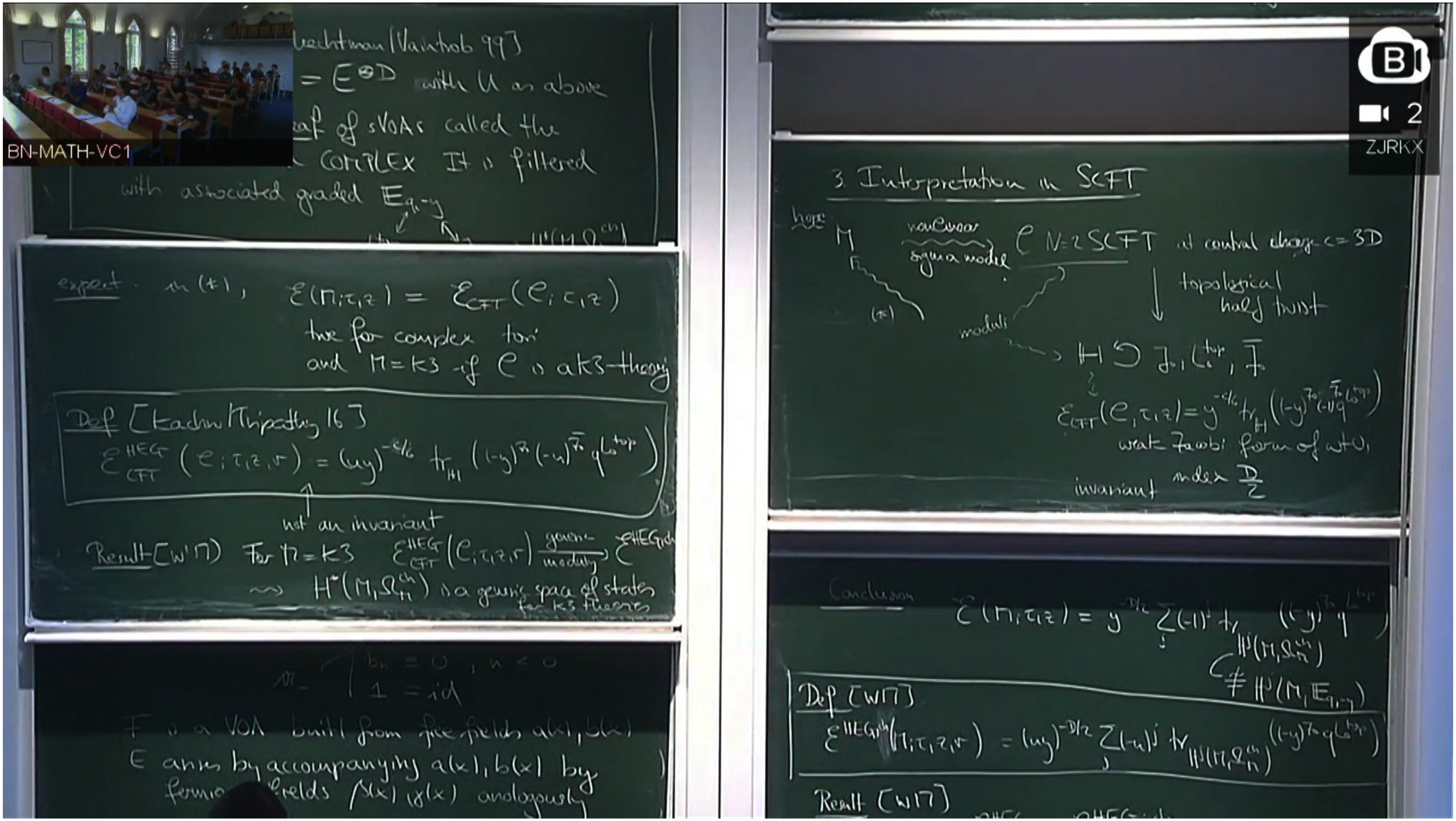
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and M = K3 if C is a K3-theory
Def [Kachru/Tripathy 16]
E^{HEG}_{CFT}(C;τ,z,ν) = (-y)^{-c/6} tr_H ((-y)^{J_0} (-u)^{J̄_0} q^{L_0^top})
not an invariant
Result [W'17] For M = K3 E^{HEG}_{CFT}(C;τ,z,ν) generic moduli E^{HEG}
H*(M,Ω^ch_M) is a generic space of states for K3 theories
F is a VOA built from free fields a(x), b(x)
E arises by accompanying a(x), b(x) by fermionic fields β(x), γ(x) analogously
3. Interpretation in SCFT
hope M nonlinear sigma model C N=2 SCFT at central charge c=3D
topological half twist
moduli
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E_CFT(C;τ,z) = y^{-c/6} tr_H ((-y)^{J_0} (-1)^{J̄_0} q^{L_0^top})
weak Jacobi form of wt 0, index D/2
invariant
Conclusion
E(M;τ,z) = y^{-D/2} Σ_j (-1)^j tr_{H^j(M,Ω^ch_M)} ((-y)^{J_0} q^{L_0^top})
≠ H^j(M, E_{q,-y})
Def [W17]
E^{HEG,ch}(M;τ,z,ν) = (-y)^{-D/2} Σ_j (-u)^j tr_{H^j(M,Ω^ch_M)} ((-y)^{J_0} q^{L_0^top})
Result [W17]