Title: An algebraic locality principle to renormalise higher zeta functions

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Abstract: According to the principle of locality in physics, events taking place at diï¬€erent locations should behave independently of each other, a feature expected to be reï¬‚ected in the measurements. We propose an algebraic locality framework to keep track of the independence, where sets are equipped with a binary symmetric relation we call a locality relation on the set, this giving rise to a locality set category. In this algebraic locality setup, we implement a multivariate regularisation, which gives rise to multivariate meromorphic functions. In this case, independence of events is reï¬‚ected in the fact that the multivariate meromorphic functions involve independent sets of variables. A minimal subtraction scheme deï¬•ned in terms of a projection map onto the holomorphic part then yields renormalised values. This multivariate approach can be implemented to renormalise at poles, various higher multizeta functions such as conical zeta functions (discrete sums on convex cones) and branched zeta functions (discrete sums associated with rooted trees). This renormalisation scheme strongly relies on the fact that the maps we are renormalizing can be viewed as locality algebra morphisms. This talk is based on joint work with Pierre Clavier, Li Guo and Bin Zhang.

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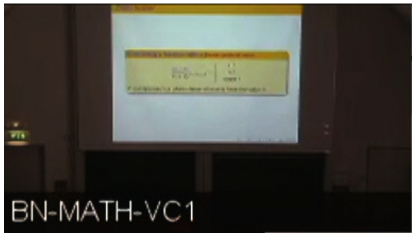
An algebraic locality principle to renormalise higher zeta functions

joint work with Pierre Clavier, Li Guo and Bin Zhang

Higher algebra and mathematical physics

Bonn, August 16th 2018



n teaser

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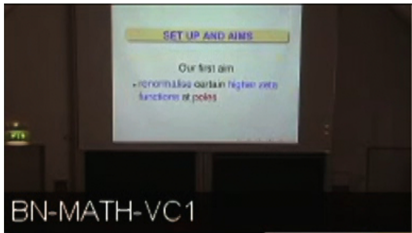
Evaluating a fraction with a linear pole at zero

$$\frac{z_1 - z_2}{z_1 + z_2}\Big|_{z_1=0, z_2=0} = \begin{cases} 1\,? \\ 0\,? \\ 10000\,? \end{cases}$$

In our approach, a given choice of locality fixes the value 0.



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SET UP AND AIMS

Our first aim

- renormalise certain higher zeta functions at poles

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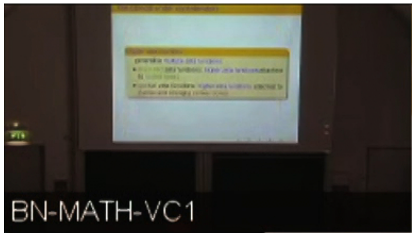
Multiple zeta functions as nested sums of symbols

On $\mathbb{R}_+$ consider the symbol map $\sigma_s : x \longmapsto x^{-s}$.



objects under consideration

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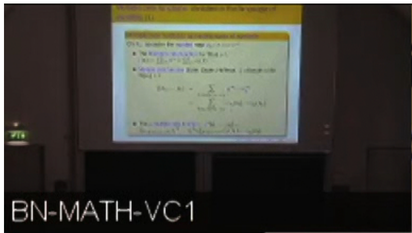
Higher zeta functions

generalise multiple zeta functions:

- branched zeta functions: higher zeta functions attached to rooted trees;
- conical zeta functions: higher zeta functions attached to (lattice and strongly) convex cones.



…iple zeta functions revisited in the language of …ools (1)

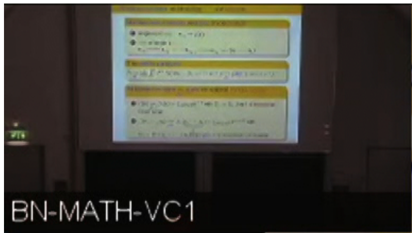
Multiple zeta functions as nested sums of symbols

On $\mathbb{R}_+$ consider the symbol map $\sigma_s : x \longmapsto x^{-s}$.

- The Riemann zeta function: for $\Re(s) > 1$, $\zeta(s) := \sum_{n=1}^{\infty} n^{-s} = \sum_{n=1}^{\infty} \sigma_s(n)$;
- Multiple zeta function [Euler, Zagier, Hoffman, ..] of length k: for $\Re(s_1) > 1$

$$\zeta(s_1, \cdots, s_k) = \sum_{0<n_k<n_{k-1}<\cdots<n_1} n_1^{-s_1} \cdots n_k^{-s_k} = \sum_{0<n_k<n_{k-1}<\cdots<n_1} \sigma_{s_1}(n_1) \cdots \sigma_{s_k}(n_k).$$

- The $\star$-multiple zeta function $\zeta^*(s_1, \cdots, s_k) = \sum_{0<n_k\leq n_{k-1}\leq\cdots\leq n_1} n_1^{-s_1} \cdots n_k^{-s_k} = \sum_{0<n_k\leq\cdots\leq n_1} \sigma_{s_1}(n_1) \cdots \sigma_{s_k}(n_k)$.



ble zeta functions revisited using trees and cones (2)

ultiple zeta functions as sums on decorated ladder trees

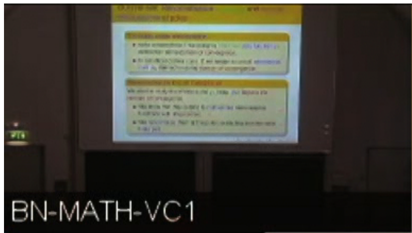
1. single root tree: $\bullet_{\sigma_s} \rightsquigarrow \zeta(s)$;
2. tree of length k:
   $\bullet_{\sigma_{s_1}} \text{———} \bullet_{\sigma_{s_2}} \cdots\cdots \bullet_{\sigma_{s_{k-1}}} \text{———} \bullet_{\sigma_{s_k}} \rightsquigarrow \zeta(s_1, \cdots, s_k)$.

The Mellin transform

$f \underset{\mathcal{M}}{\longmapsto} \frac{1}{\Gamma(\bullet)} \int_0^\infty \epsilon^{\bullet-1} f(\epsilon)\, d\epsilon; \quad (f_x : \epsilon \longmapsto e^{-\epsilon x}) \underset{\mathcal{M}}{\longmapsto} (\mathcal{M}(f_x) : s \longmapsto x^{-s})$

Multizeta functions $\underset{\mathcal{M}^{-1}}{\rightsquigarrow}$ sums on rational convex cones

1. $\zeta(s) \underset{\mathcal{M}^{-1}}{\rightsquigarrow} \mathcal{S}_1(\epsilon) = \sum_{n \in \mathbf{C}_1 \cap \mathbb{Z}} e^{-\epsilon n}$ with $\mathbf{C}_1 := \mathbb{R}_+$, the 1-dimensional Chen cone;
2. $\zeta(s_1, \cdots, s_k) \underset{\mathcal{M}^{-1}}{\rightsquigarrow} \mathcal{S}_k(\underbrace{\epsilon_1, \cdots, \epsilon_k}_{\vec{\epsilon}_k}) = \sum_{\vec{n} \in \mathbf{C}_k \cap \mathbb{Z}^k} e^{-\langle \vec{\epsilon}_k, \vec{n} \rangle}$ with
   $\mathbf{C}_k = \{0 < x_1 < \cdots < x_k\}$ the open k-dimensional Chen cone.



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first aim: Renormalisation branched and conical functions at poles

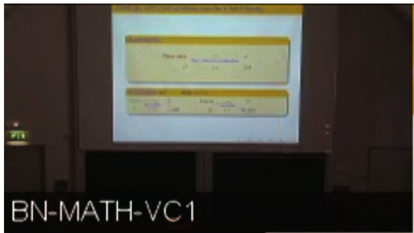
The maps under consideration

- so to a rooted tree **T** we assign a branched zeta function ζ_T defined on some domain of convergence;
- to a (lattice) convex cone **C** we assign a conical exponential sum $\mathcal{S}_C$ defined on some domain of convergence.

Renormalisation to cure divergences

We want to study the infinite sums ζ_T (resp. $\mathcal{S}_C$) beyond the domain of convergence.

- We show that they extend to multivariate meromorphic functions with linear poles;
- We renormalise them at the poles, extracting a reasonable finite part.



ogy with perturbative quantum field theory

Quantisation

$$\begin{array}{ccc} \text{Observable} & \xrightarrow[\text{Mean value over configurations}]{} & \mathbb{C} \\ \mathcal{O} & \longmapsto & \langle \mathcal{O} \rangle \end{array}$$

Summation on trees and cones

$$\begin{array}{ccc} \text{Trees} & \xrightarrow[\text{Summation}]{} & \mathbb{C} \\ \mathbf{T} & \longmapsto & \zeta_{\mathbf{T}}(p) \end{array} \qquad \begin{array}{ccc} \text{Cones} & \xrightarrow[\text{Summation}]{} & \mathbb{C} \\ \mathbf{C} & \longmapsto & \mathcal{S}_{\mathbf{C}}(p). \end{array}$$

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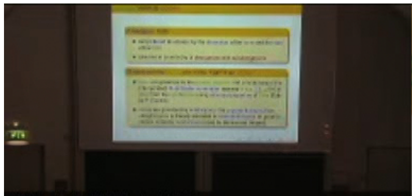

es versus cones

Analogies: both

- carry filtered structures: by the dimension of the cone and the size of the tree;
- give rise to a hierarchy of divergences with subdivergences;

Discrepancies: trees are more "rigid" than cones

- trees are governed by the grafting operator and concatenation: the (interpolated) Rota-Baxter summation operator $\sigma \longmapsto \sum_{n=1}^{\bullet} \sigma(n)$ is lifted from the root to trees using universal properties of trees [Talk by P. Clavier];
- cones are governed by subdivisions: the exponential sum S on smooth cones is linearly extended to exponential sums on general convex cones by subdivisions [used by Berline and Vergne].

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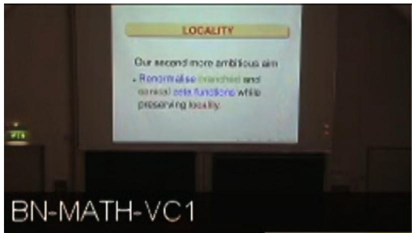

LOCALITY

Our second more ambitious aim

- Renormalise branched and conical zeta functions while preserving locality.

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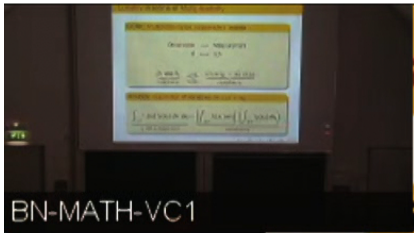

...ality in terms of Multiplicativity

...OAL: Multiplicativity on independent events

$$\text{Observable} \longrightarrow \text{Measurement}$$

$$O \longmapsto \langle O \rangle$$

$$\underbrace{O_1 \text{ and } O_2}_{\text{independent}} \underbrace{\Longrightarrow}_{\text{locality}} \underbrace{\langle O_1 \star O_2 \rangle = \langle O_1 \rangle \langle O_2 \rangle}_{\text{multiplicativity}}.$$

Analogy: separation of variables ($n = n_1 + n_2$)

$$\underbrace{\int_{\mathbb{R}^n} f_1(x_1)\, f_2(x_2)\, dx_1\, dx_2}_{x_1 \text{ and } x_2 \text{ independent}} = \underbrace{\left(\int_{\mathbb{R}^{n_1}} f_1(x_1)\, dx_1\right) \cdot \left(\int_{\mathbb{R}^{n_2}} f_2(x_2)\, dx_2\right)}_{\text{multiplicativity}}.$$

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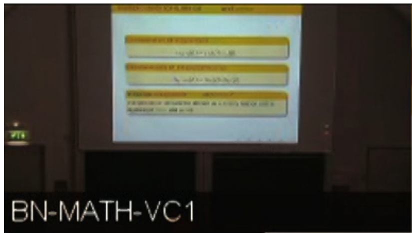

tiplicativity for sums on trees and cones

Concatenation of independent trees

$$\zeta_{T_1 \bullet T_2}(p) = \zeta_{T_1}(p) \cdot \zeta_{T_2}(p);$$

Minkowski sum of independent cones

$$S_{C_1 + C_2}(p) = S_{C_1}(p) \cdot S_{C_2}(p).$$

What are independent trees and cones?

Independence will later be defined as a locality relation on the algebras of trees and cones.

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e generally: $\Phi \rightsquigarrow \Phi^{ren}$

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ata: $(\mathcal{A}, \mathcal{M}, \Phi)$

- a (commutative) algebra $(\mathcal{A}, \star)$ (e.g.: trees, cones),
- an algebra of meromorphic germs at zero to be defined $\mathcal{M}$,
- an algebra morphism $\Phi : (\mathcal{A}, \star) \longrightarrow (\mathcal{M}, \cdot)$ (e.g: $\zeta, \mathcal{S}$)

$$\Phi\,(a_1 \star a_2) = \Phi(a_1) \cdot \Phi(a_2). \qquad (1)$$

Our aim: $(\mathcal{A}, \mathbb{C}, \Phi^{ren})$

Build a map

$$\Phi^{\rm ren} : (\mathcal{A}, \star) \longrightarrow (\mathbb{C}, \cdot)$$

that satisfies a locality condition:

$$a_1 \text{ independent of } a_2 \Longrightarrow \Phi^{\rm ren}\,(a_1 \star a_2) = \Phi^{\rm ren}(a_1) \cdot \Phi^{\rm ren}(a_2). \qquad (2)$$

k to higher zeta functions

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The algebra $\mathcal{A}$

1. pointed convex cones **C** in $\mathbb{R}^\infty$ equipped with the Minkowski sum (L. Guo, S.-P., B. Zhang 2017);
2. rooted forests **F** equipped with the concatenation product (P.Clavier, L. Guo, S.-P., B. Zhang 2018)

- **Preliminary remark:** $s_i \longmapsto s_i + z_i \Longrightarrow$ poles at zero.

The map $\Phi : \mathcal{A} \longrightarrow \mathcal{M}(\mathbb{C}^\infty)$

1. Exponential sums on cones: $\mathbf{C} \longmapsto \mathcal{S}_{\mathbf{C}}$;
2. Branched zeta functions: $\mathbf{F} \longmapsto \zeta_{\mathbf{F}}$

THE ANALYTIC SET UP

at is $\mathcal{M}$? What are the poles of $f = \Phi(a)$?

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A first naive approach: one variable

- $\mathcal{M}(\mathbb{C})$ meromorphic germs in one variable: $f(z) = \sum_{j=1}^{k} a_j z^{-j} + h(z)$;
- Subtract the pole and evaluate the holomorphic part at the pole (here zero): $\mathrm{ev}_0^{\mathrm{reg}} f = \mathrm{ev}_0^{\mathrm{reg}} \left(\sum_{j=1}^{k} a_j z^{-j} + h(z) \right) := h(0)$.
- Obstacle: $\mathrm{ev}_0^{\mathrm{reg}}(f_1 \cdot f_2) \neq \mathrm{ev}_0^{\mathrm{reg}}(f_1) \cdot \mathrm{ev}_0^{\mathrm{reg}}(f_2)$ so multiplicativity is ruined: $1 = \mathrm{ev}_0^{\mathrm{reg}} \left(\frac{1}{z} \cdot z \right) \neq \mathrm{ev}_0^{\mathrm{reg}} \left(\frac{1}{z} \right) \cdot \mathrm{ev}_0^{\mathrm{reg}}(z) = 0$.

Counterterms

- $\mathcal{M}(\mathbb{C}) := \mathbb{C}[z^{-1}, z]] \ni f$ is an algebra;
- $\mathcal{M}_+(\mathbb{C}) := \mathbb{C}[z]] \ni h$ is an algebra;
- Counterterms: $\mathcal{M}_-(\mathbb{C}) := z^{-1}\,\mathbb{C}[z^{-1}, z]] \ni f - h$ is not an ideal;

ivariate meromorphic germs

lternative approach: several variables

- multivariate meromorphic germs: e.g. $f(z_1, z_2) = \frac{z_1}{z_2}$; $f(z_1, z_2) = \frac{z_1 - z_2}{z_1 + z_2}$
- independence/ locality/ orthogonality relation: $\frac{1}{z_1} \perp z_2$; $\frac{1}{z_1+z_2} \perp z_1 - z_2$
- a (partial) product on independent germs: e.g. $\frac{z_1}{z_2} = z_1 \cdot \frac{1}{z_2}$, $\frac{z_1 - z_2}{z_1 + z_2} = (z_1 - z_2) \cdot \frac{1}{z_1 + z_2}$.

Multivariate meromorphic germs with linear poles

- $\mathcal{M}(\mathbb{C}^k) \ni f = \frac{h(\ell_1, \cdots, \ell_n)}{L_1^{s_1} \cdots L_n^{s_n}}$, h holomorphic germ, $s_i \in \mathbb{Z}_{\geq 0}$;
- $\ell_i : \mathbb{C}^k \to \mathbb{C}$ and $L_j : \mathbb{C}^k \to \mathbb{C}$ linear forms;
- Dependence set $\mathrm{Dep}(f) := \langle \ell_1, \cdots, \ell_m, L_1, \cdots, L_n \rangle$;

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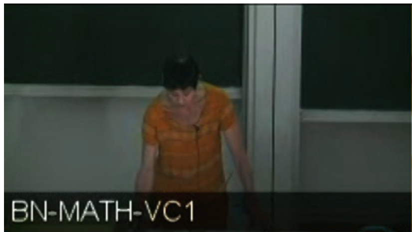

lity and multivariate regularisation

heorem (L. Guo, S.-P., B. Zhang/ N. Berline, M. Vergne 2015)

$\mathcal{M}(\mathbb{C}^k) = \mathcal{M}_-(\mathbb{C}^k) \oplus^\perp \mathcal{M}_+(\mathbb{C}^k)$, where $\mathcal{M}_-(\mathbb{C}^k) \ni \frac{h(\ell_1, \cdots, \ell_n)}{L_1^{s_1} \cdots L_n^{s_n}}$ with $\mathrm{Dep}(h) \perp \langle L_1, \cdots, L_n \rangle$ and $f_1 \perp f_2 \iff \mathrm{Dep}(f_1) \perp \mathrm{Dep}(f_2)$.

Locality ideal

- $\mathcal{M}(\mathbb{C}^k)$ is a locality algebra (to be defined);
- $\mathcal{M}_+(\mathbb{C}^k)$ is a locality algebra ;
- No counterterms: $\mathcal{M}_-(\mathbb{C}^k)$ is a locality ideal (to be defined);

Our main protagonists

- Orthogonal projection $\pi_+ : \mathcal{M}(\mathbb{C}^k) \longrightarrow \mathcal{M}_+(\mathbb{C}^k)$ is a locality morphism (to be defined) ;
- Generalised evaluator $ev_0^{\mathrm{ren}} := ev_0 \circ \pi_+ : \mathcal{M}(\mathbb{C}^k) \longrightarrow \mathbb{C}$.

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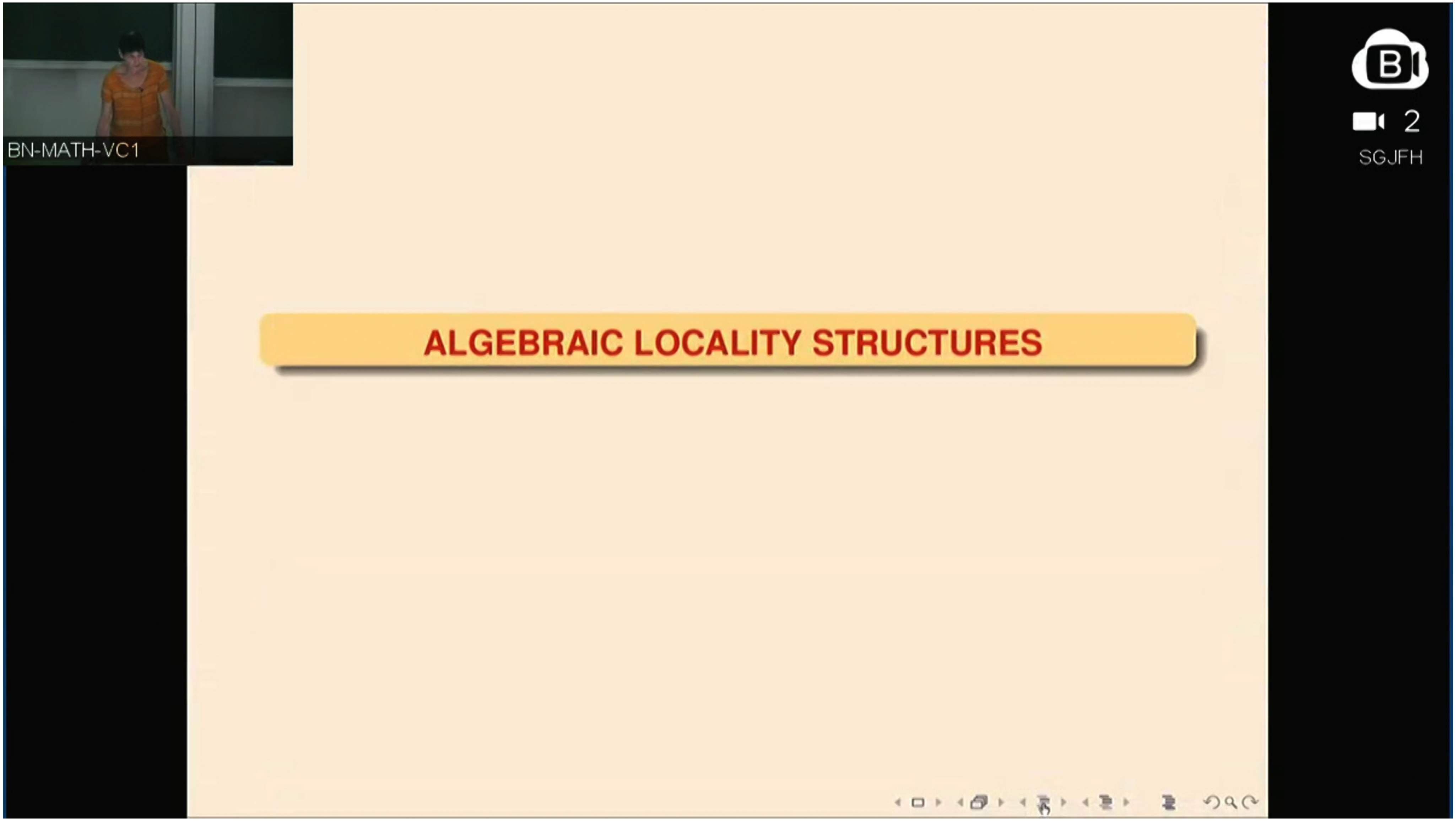
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ALGEBRAIC LOCALITY STRUCTURES
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ebraic locality

Definition of locality

A **locality set** is a couple $(X, \top)$ where X is a set and $\top \subseteq X \times X$ is a symmetric relation on X, called **locality relation** (or **independence relation**) of the locality set.

$$x_1 \top x_2 \Longleftrightarrow (x_1, x_2) \in \top, \quad \forall x_1, x_2 \in X.$$

Two basic yet important examples of locality

- $X \top Y \Longleftrightarrow X \cap Y = \emptyset$ on subsets X, Y of a set Z.
- $X \top Y \Longleftrightarrow X \perp Y$ on subsets X, Y of an euclidean vector space V.

Separation of variables

On $\mathcal{M}(\mathbb{C}^\infty)$, $f_1 \perp f_2 \Longleftrightarrow \mathrm{Dep}(f_1) \perp \mathrm{Dep}(f_2)$.

...tial operations

...peration on the graph of a locality relation

- Locality set: $(X, \top)$,
- Graph: $\top = \{(a,b) \in X^2, \quad a \top b\}$,
- Partial operation:

$$\begin{aligned} \star : X \times X \supset \top & \longrightarrow X \\ (a,b) & \longmapsto a \star b. \end{aligned}$$

Partial product on meromorphic germs

The partial product on $\mathcal{M}(\mathbb{C}^\infty) = \bigcup_{k \in \mathbb{N}} \mathcal{M}(\mathbb{C}^k)$:

$$\begin{aligned} \mathcal{M}(\mathbb{C}^\infty) \times \mathcal{M}(\mathbb{C}^\infty) \supset \top & \longrightarrow \mathcal{M}(\mathbb{C}^\infty) \\ \left(f_1 = \frac{h_1(\vec{\ell}_1)}{\vec{L}_1^{\vec{s}_1}}, f_2 = \frac{\widetilde{h}_2(\vec{\ell}_2)}{\vec{L}_2^{\vec{s}_2}} \right) & \longmapsto f_1 \cdot f_2 = \frac{h_1(\vec{\ell}_1) \cdot h_2(\vec{\ell}_2)}{\vec{L}_1^{\vec{s}_1} \cdot \vec{L}_2^{\vec{s}_2}}. \end{aligned}$$

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braic locality structures

ocality structures

- A locality semigroup is a locality set $(A, \top_A)$ with a product law $m_A : A \times A \supset \top_A \longrightarrow A$ compatible with the locality relation: $\forall x, y, z \in A, \quad ((x \top_A y) \wedge (y \top_A z) \wedge (x \top_A z)) \Longrightarrow ((m_A(x, y) \top_A z) \wedge (x \top_A m_A(y, z)))$ and locally associative.
- A locality vector space is a locality set $(V, \top_V)$ with a linear structure such that for any subset X of V, the set $X^{\top_V}$ is a linear subspace of V.
- A locality (unital) algebra is a locality vector space $(A, \top_A)$ equipped with a bilinear map $m_A : \top_A \longrightarrow A$ such that $(A, \top_A, m_A)$ is a locality semi-group (monoid).

A counterexample relevant for applications

On $\mathbb{R}$ equipped with the relation $x \top y \Longleftrightarrow x + y \notin \mathbb{Z}$ the addition does not yield a locality semi-group: for $U = \{1/3\}$ we have $(1/3, 1/3) \in (U^\top \times U^\top) \cap \top$ but $1/3 + 1/3 = 2/3 \notin U^\top$.

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n locality semigroups to locality ideals

oncepts related to locality semigroups

1. (Non commutative) **partial semi-groups**
   R. H. **Schelp**, A partial semigroup approach to partially ordered sets (1972);
2. correspond to **Weinstein**'s **selective categories** with one object
   D. Li-Bland, A. Weinstein, Selective categories and linear canonical relations, Symmetry, Integrability and Geometry: Methods and Applications (2014).

$\mathcal{M}_-(\mathbb{C}^k)$ is not an ideal in $\mathcal{M}(\mathbb{C}^k)$ yet

$\mathcal{M}_-(\mathbb{C}^k)$ is a locality ideal in $\mathcal{M}(\mathbb{C}^k)$

$$\mathcal{M}_+(\mathbb{C}^k) \ni h' \perp \frac{h}{\vec{L}^{\vec{s}}} \in \mathcal{M}_-(\mathbb{C}^k) \Longrightarrow h' \cdot \frac{h}{\vec{L}^{\vec{s}}} \in \mathcal{M}_-(\mathbb{C}^k).$$

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Locality maps

$\Phi : (X, \top_X) \longmapsto (Y, \top_Y)$ is a locality map if $\Phi \otimes \Phi(\top_X) \subset \top_Y$

(almost)-Locality of distribution kernels

Let $U \subset \mathbb{R}^n$ be open. We compare two locality relations on $\mathcal{D}(U)$:

- $\phi \top^\epsilon \psi \Longleftrightarrow d\,(\mathrm{Supp}(\phi), \mathrm{Supp}(\psi)) > \epsilon$;
- $\phi \top^K \psi \Longleftrightarrow \int_U \phi(x)\, K(x, y)\, \psi(y)\, dx\, dy = 0$.

The kernel K is ϵ-local $\Longleftrightarrow$ $\mathrm{Id} : (\mathcal{D}(U), \top^\epsilon) \longrightarrow \left(\mathcal{D}(U), \top^K\right)$ is a locality map.

Locality morphisms of algebras

A locality map $\Phi : (A, \top_A, m_A) \longmapsto (B, \top_B, m_B)$ is a locality morphism of locality algebras if $a_1 \top_A a_2 \Longrightarrow \Phi(m_A(a_1, a_2)) = m_B(\Phi(a_1), \Phi(a_2))$.


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MULTIVARIATE REGULARISATION
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artially multiplicative renormalised map

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Back to our two main protagonists

- The orthogonal projection $\pi_+ : \left(\mathcal{M}(\mathbb{C}^k), \perp\right) \longrightarrow \left(\mathcal{M}_+(\mathbb{C}^k), \perp\right)$ is a locality morphism of locality algebras;
- The generalised evaluator $ev_0^{\mathrm{ren}} := ev_0 \circ \pi_+ : \left(\mathcal{M}_+(\mathbb{C}^k), \perp\right) \longrightarrow \mathbb{C}$ is a locality character.

Theorem (P. Clavier, L. Guo, S.-P., B. Zhang 2018)

A locality morphism $\Phi : (\mathcal{A}, \top) \longrightarrow \left(\mathcal{M}(\mathbb{C}^k), \perp\right)$ gives rise to a locality character

$$\Phi^{\mathrm{ren}} := ev_0^{\mathrm{ren}} \circ \Phi : (\mathcal{A}, \top) \longrightarrow \mathbb{C}.$$

clusions

artial multiplicativity preserved after renormalisation!

The renormalised map Φ^{ren} is partially multiplicative

$$a_1 \top_A a_2 \Longrightarrow \Phi^{\mathrm{ren}}(a_1 \star a_2) = \Phi^{\mathrm{ren}}(a_1) \cdot \Phi^{\mathrm{ren}}(a_2). \quad (3)$$

Multivariate minimal subtraction scheme

Provided $\Phi(\mathcal{A}) \subset \mathcal{M}(\mathbb{C}^\infty)$, we can renormalise while preserving partial multiplicativity using the locality projection π_+ to extract the finite part at zero.

Back to higher zeta functions: one can renormalise at poles

1. Exponential sums on rational convex cones equipped with an orthogonality locality relation (L. Guo, S.-P., B. Zhang 2017);
2. Branched zeta functions equipped with an orthogonality locality relation (P. Clavier, L. Guo, S.-P., B. Zhang 2018).

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perties of the renormalised sums at poles

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- The renormalised higher zeta values at poles are rational, due to the rationality of the Bernoulli numbers (arising in the Euler-Maclaurin formula), the rationality of the convex lattice cones and the fact that the underlying algebraic procedures are compatible with linearity;
- The renormalised higher zeta values restricted to ladder trees and Chen cones yield renormalised multiple zeta values;
- Stuffle relations generalise to compatibility with subdivisions for sums on cones;
- The branched sum on trees factorises through words via a "flatening operator".

k in progress and open question

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Work in progress

- Generalisation of the generalised evaluators $\mathrm{ev}_0^{\mathrm{ren},Q} := \mathrm{ev}_0 \circ \pi_+{}^Q$ to abstract generalised evaluators for an abstract locality relation $\top$;
- Study of the dependence on the underlying inner product Q on $\mathbb{R}^\infty$:The general linear group $\mathrm{GL}(\mathbb{R}^\infty) \ni L$ acts (transitively) on inner products $Q \longrightarrow Q_L := L^t\, Q\, L$ used to build the projection π_+^Q, and hence on the locality relation $\perp^Q$ on $\mathcal{M}(\mathbb{C}^\infty)$;
- Generalisation of the locality relation $\top^Q$ on $\mathcal{M}(\mathbb{C}^\infty)$ to an abstract locality relation $\top$, by means of a generalisation of the orthogonal complement map to an abstract complement map;

Open question

Identification and description of a (the renormalisation ?) subgroup of $\mathrm{GL}(\mathbb{R}^\infty)$ acting transitively on abstract generalised evaluators.


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THANK YOU FOR YOUR ATTENTION!