Title: A categorii¬•ed Doldâ€“Kan correspondence

Date: Aug 16, 2018 09:00 AM

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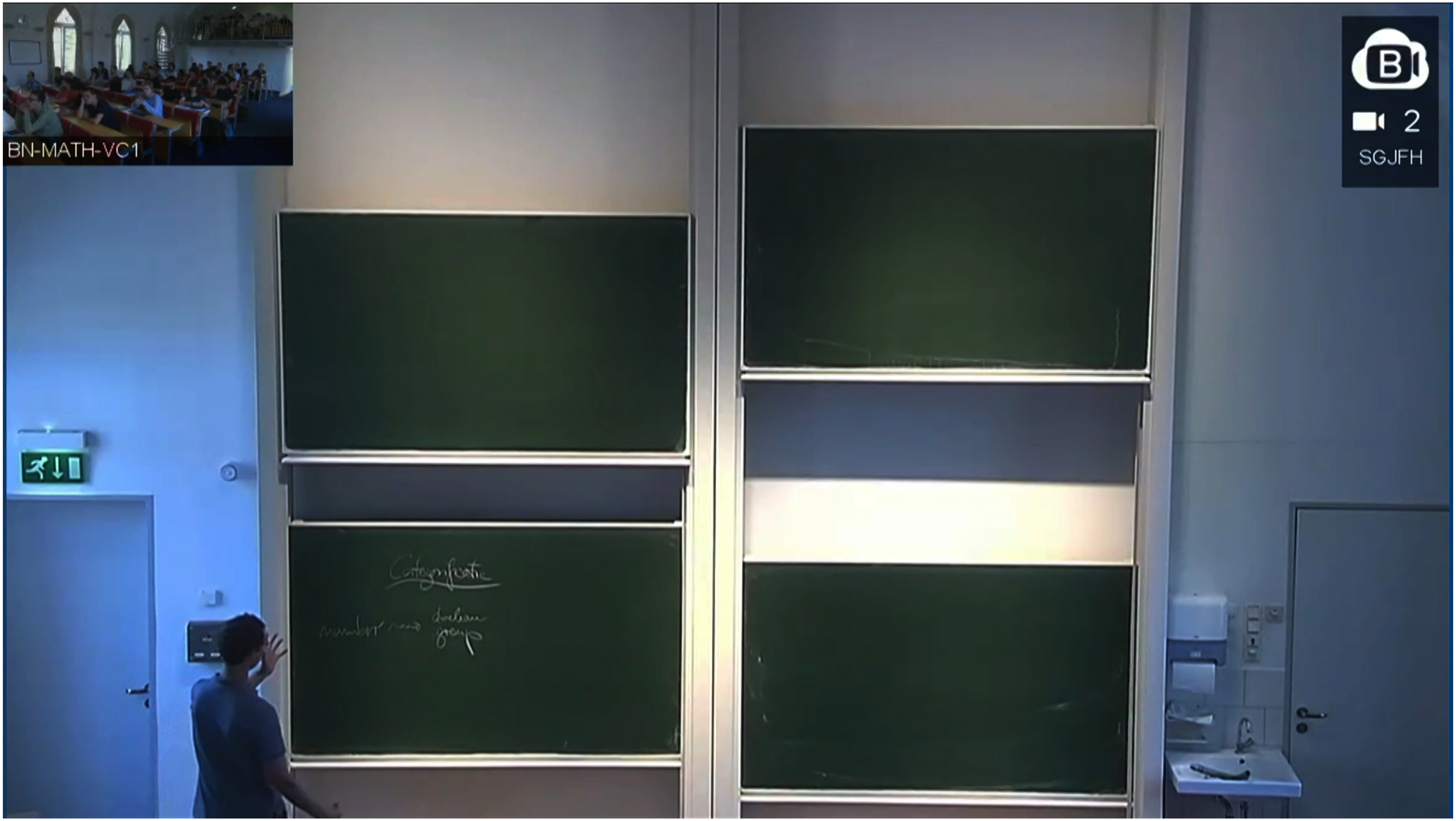
Abstract: Various recent developments, in particular in the context of topological Fukaya categories, seem to be glimpses of an emerging theory of categorii¬•ed homotopical and homological algebra. The increasing number of meaningful examples and constructions make it desirable to develop such a theory systematically. In this talk, we discuss a step towards this goal: a categorii¬•cation of the classical Doldâ€“Kan correspondence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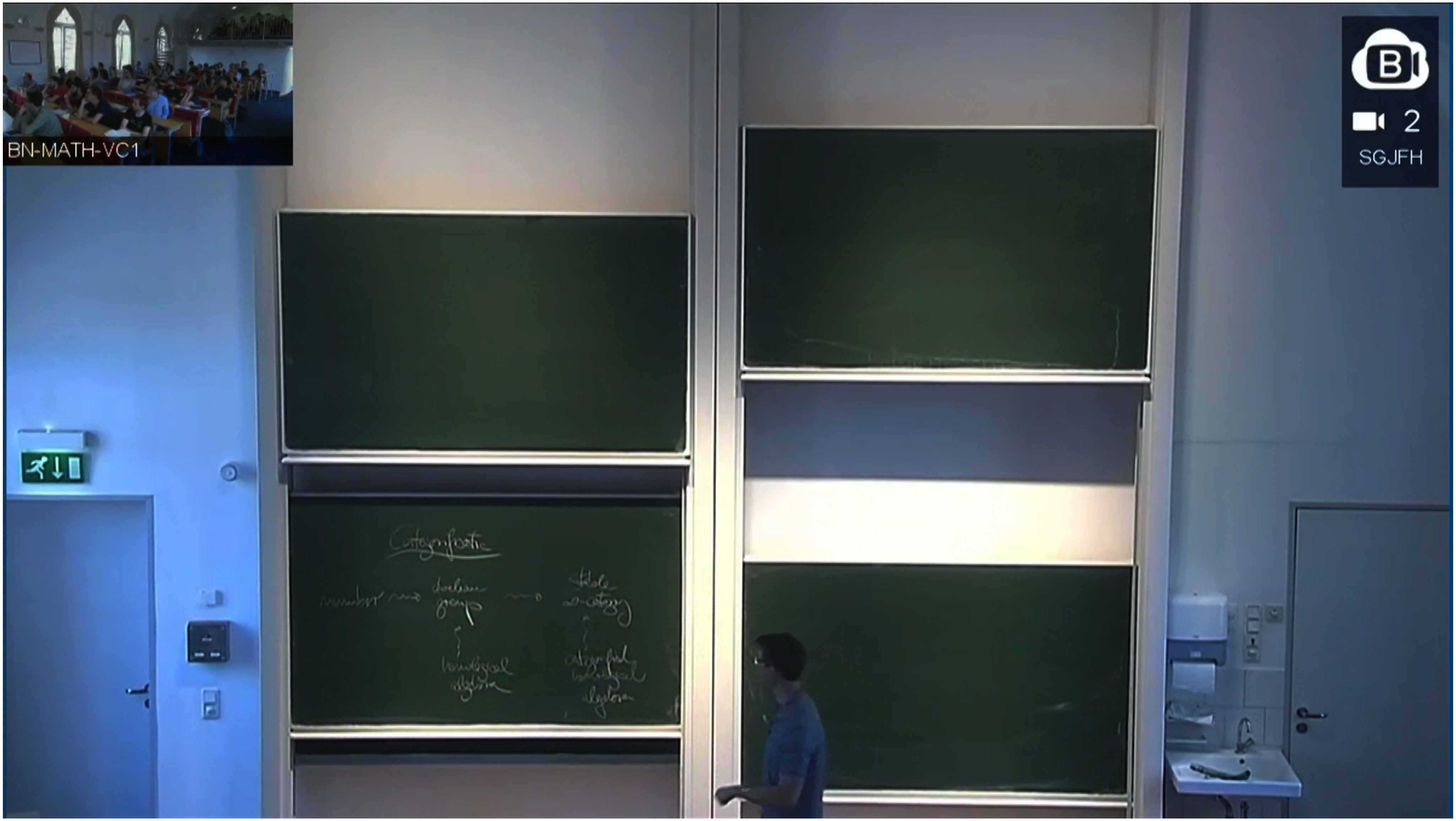
Categorification
number ⇝ abelian group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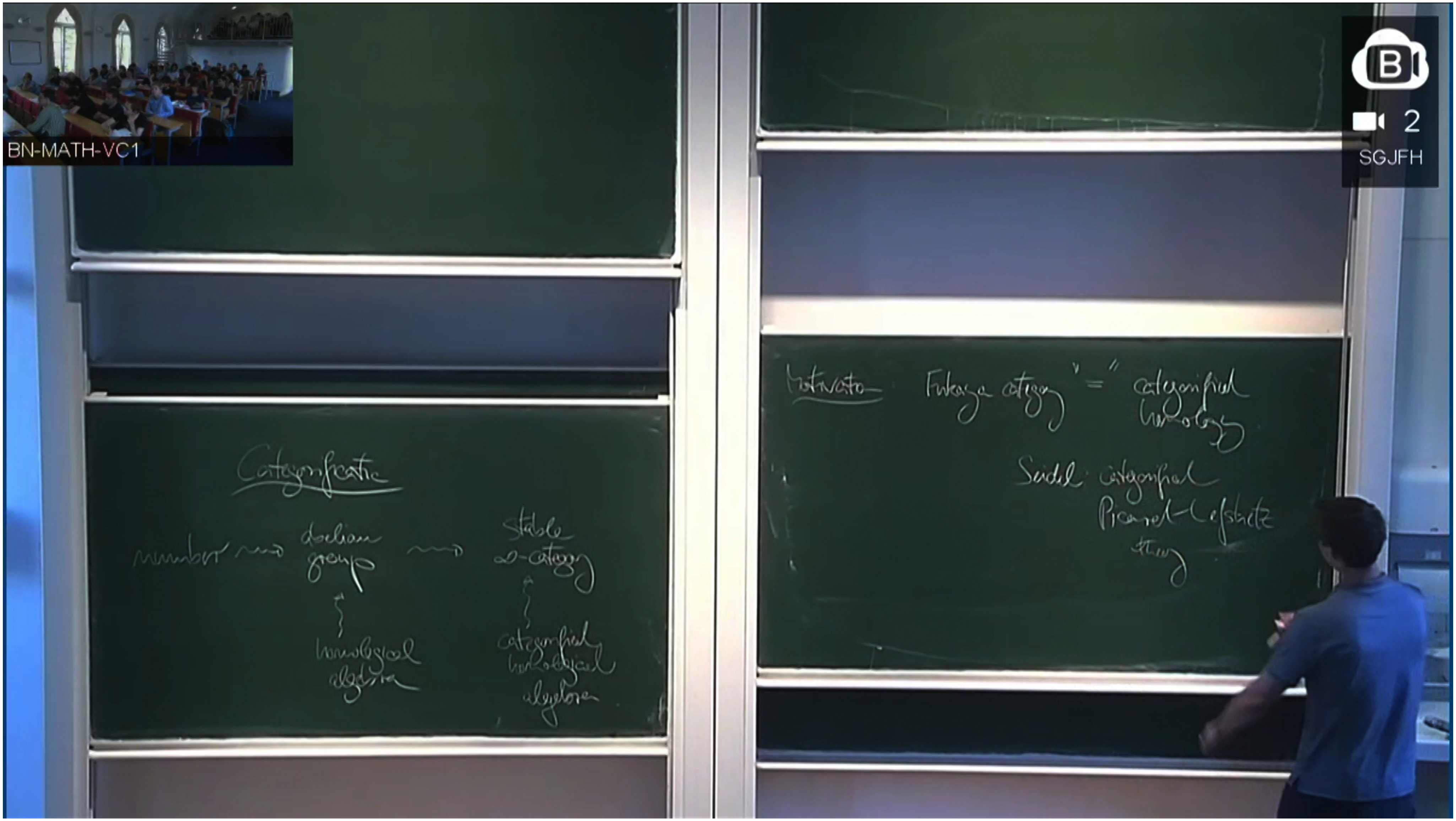
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Categorification
number
abelian group
stable
∞-category
homological algebra
categorified homological algebra
Motivation
Fukaya category "=" categorified homology
Seidel: categorified Picard-Lefschetz theory
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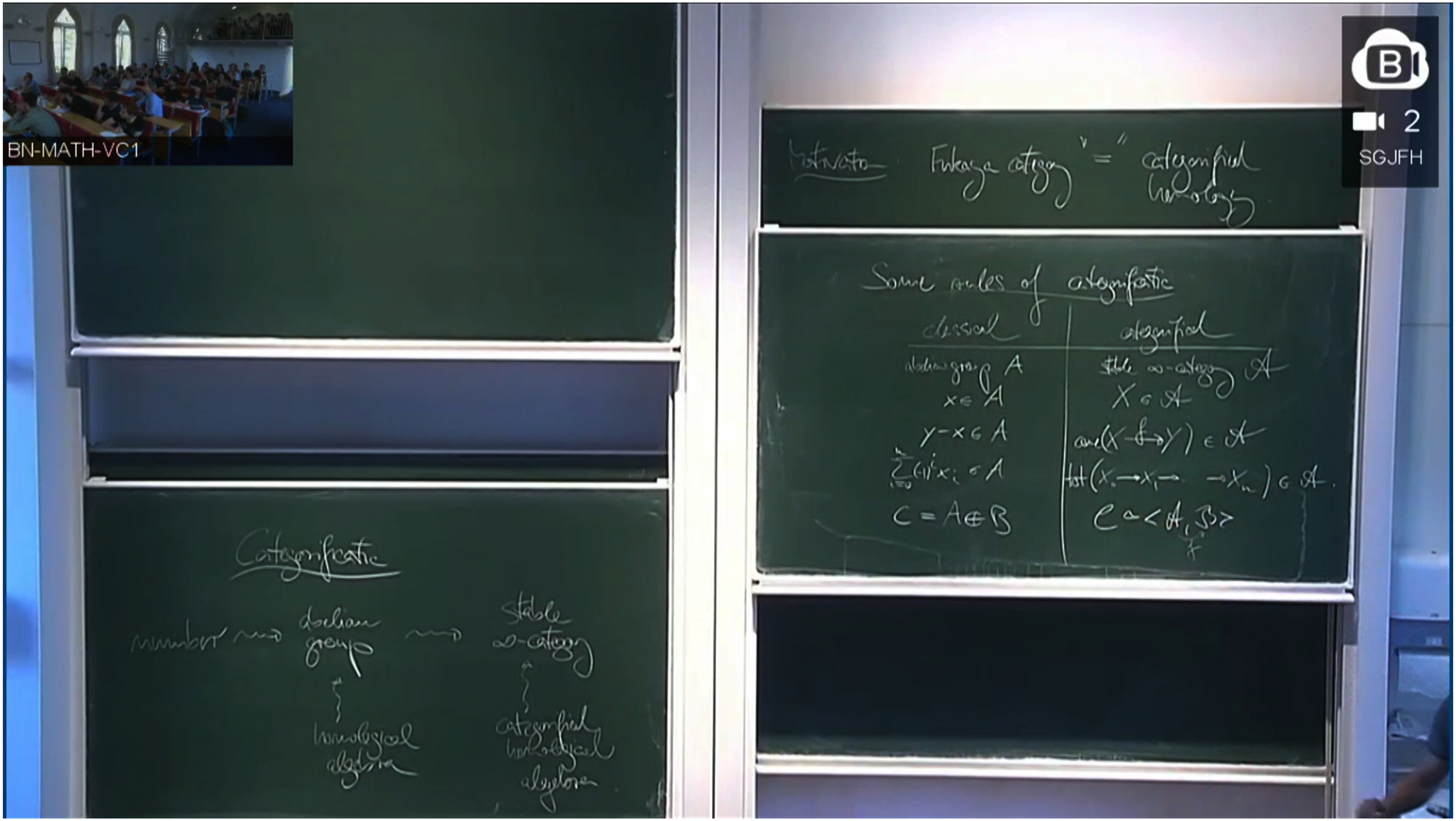
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Fukaya category "=" categorified homology
Some roles of categorification
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abelian group A
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Categorification
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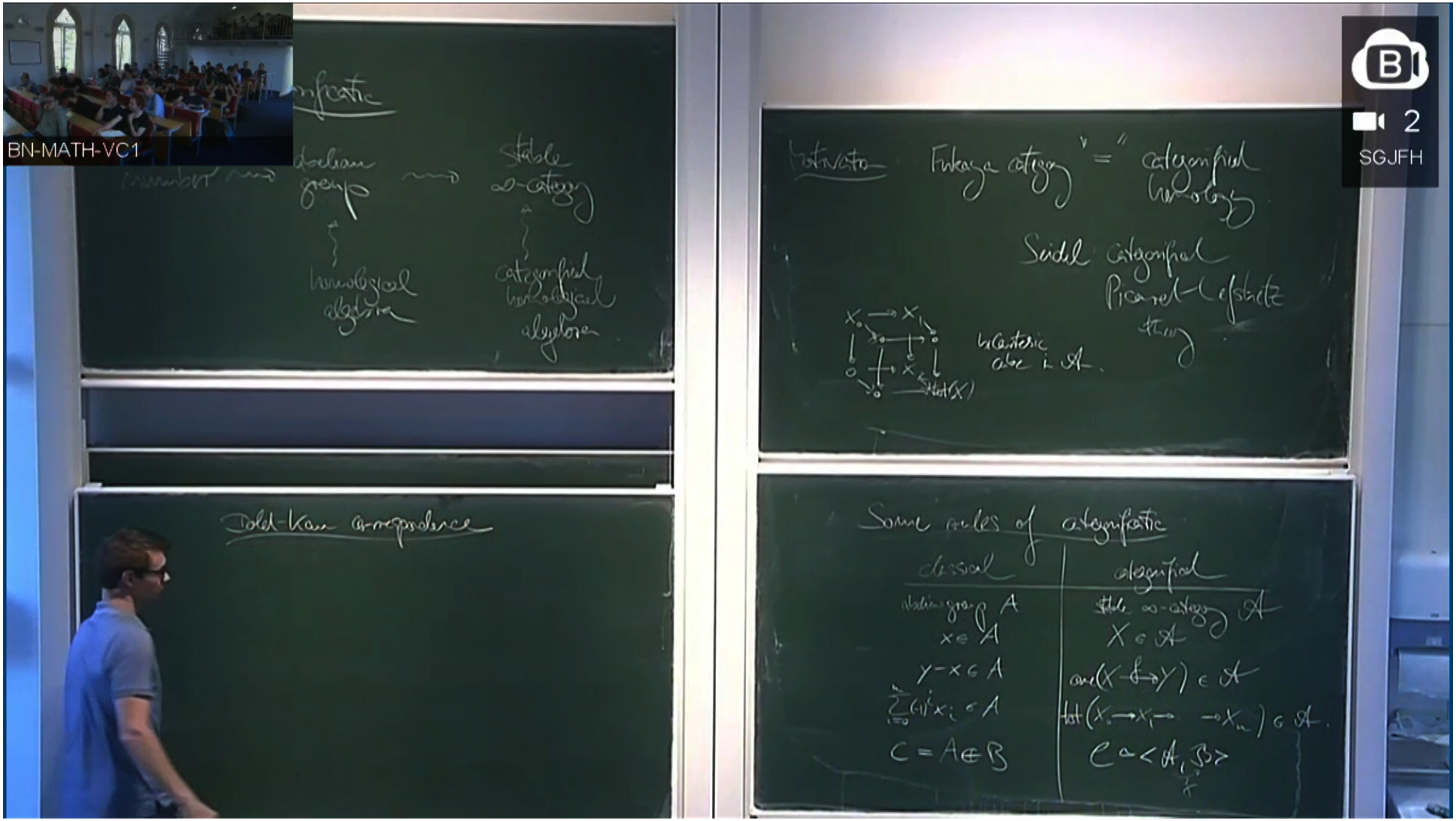
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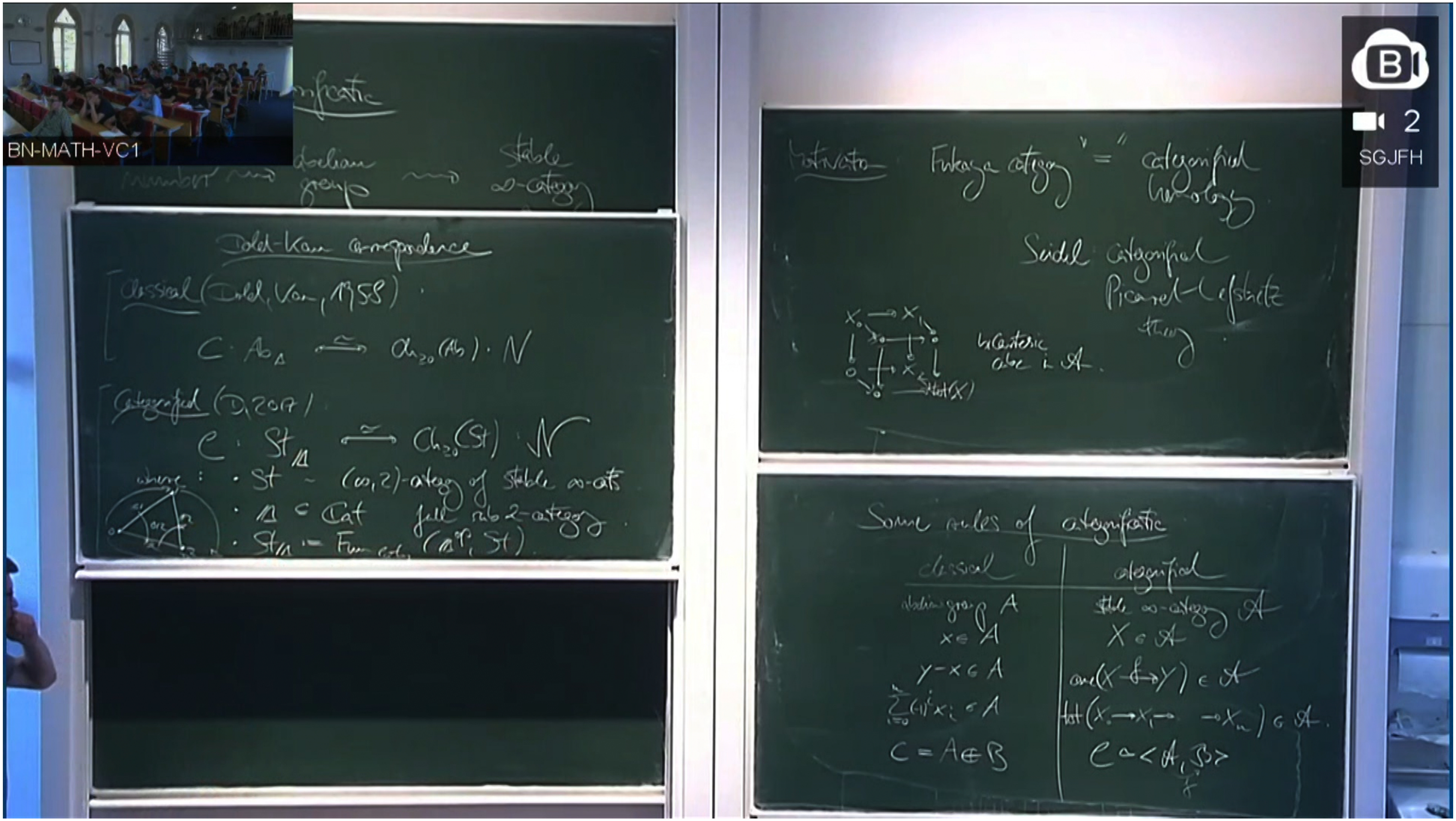
Dold-Kan correspondence
Some rules of categorification
classical | categorified
abelian group A | stable ∞-category 𝒜
x ∈ A | X ∈ 𝒜
y − x ∈ A | cone(X → Y) ∈ 𝒜
C = A ⊕ B | 𝒞 ≃ <𝒜, ℬ>


BN-MATH-VC1
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Dold-Kan correspondence
Classical (Dold, Kan, 1958)
Categorified (D, 2017)
where :
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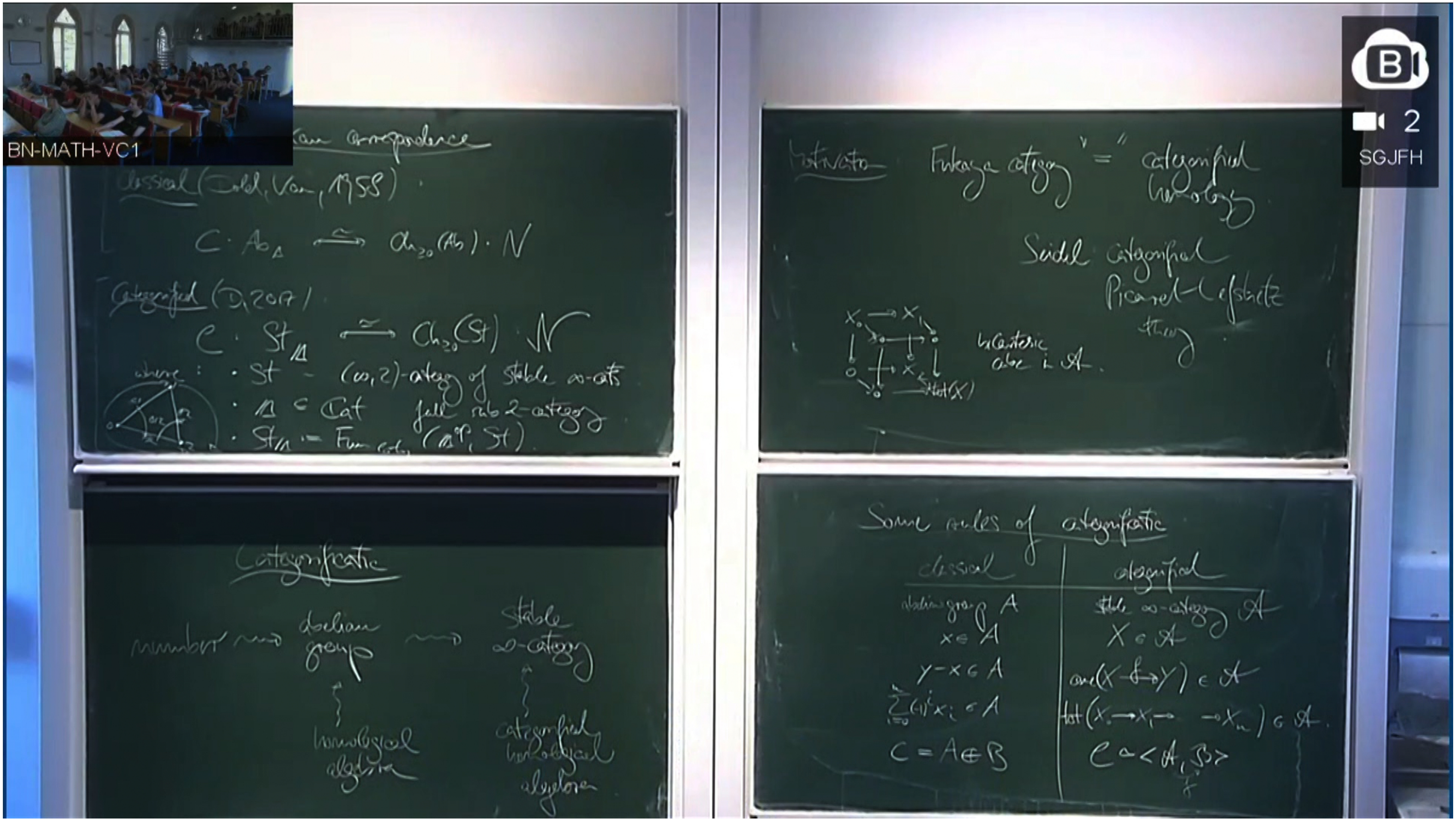
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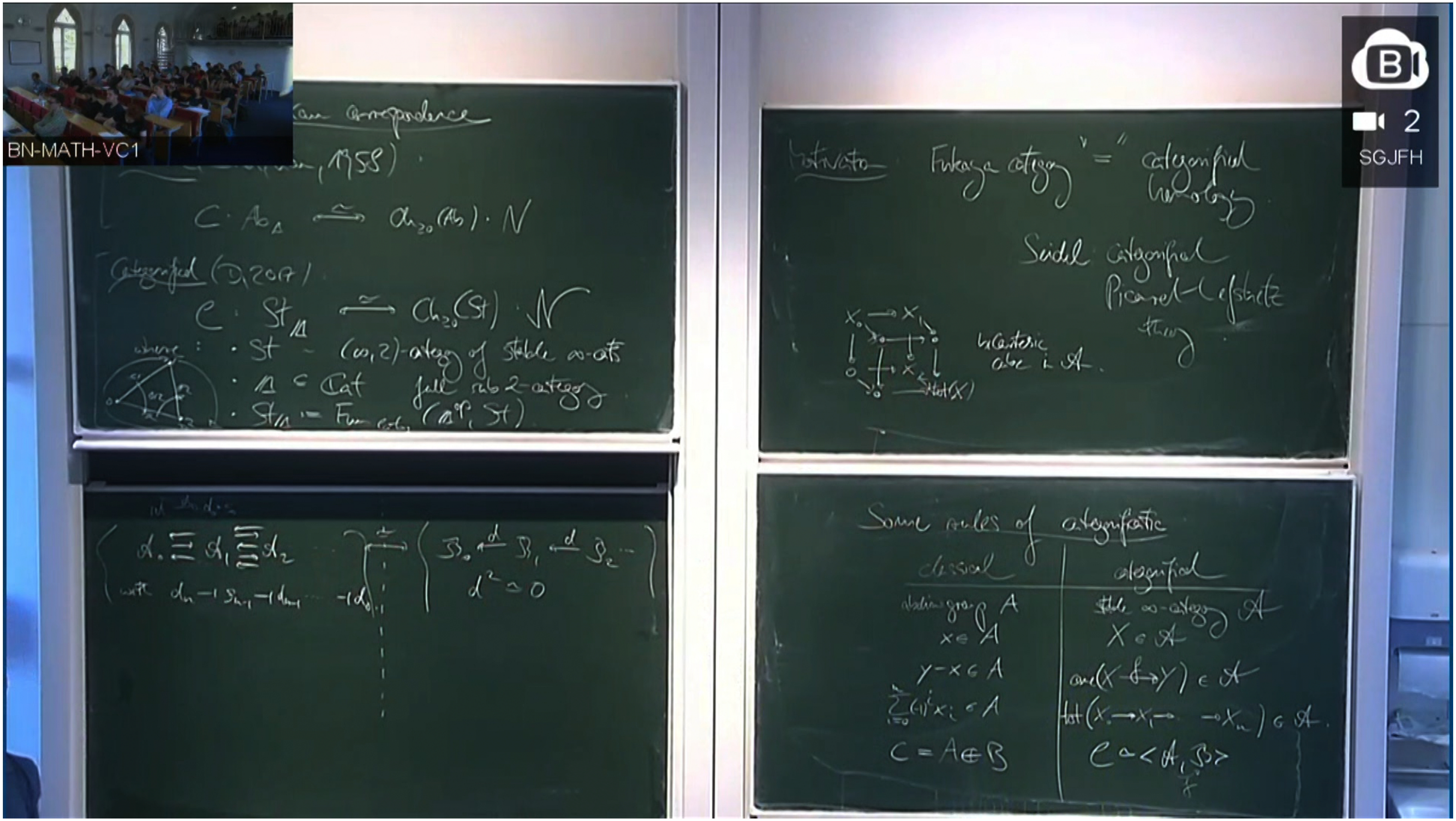
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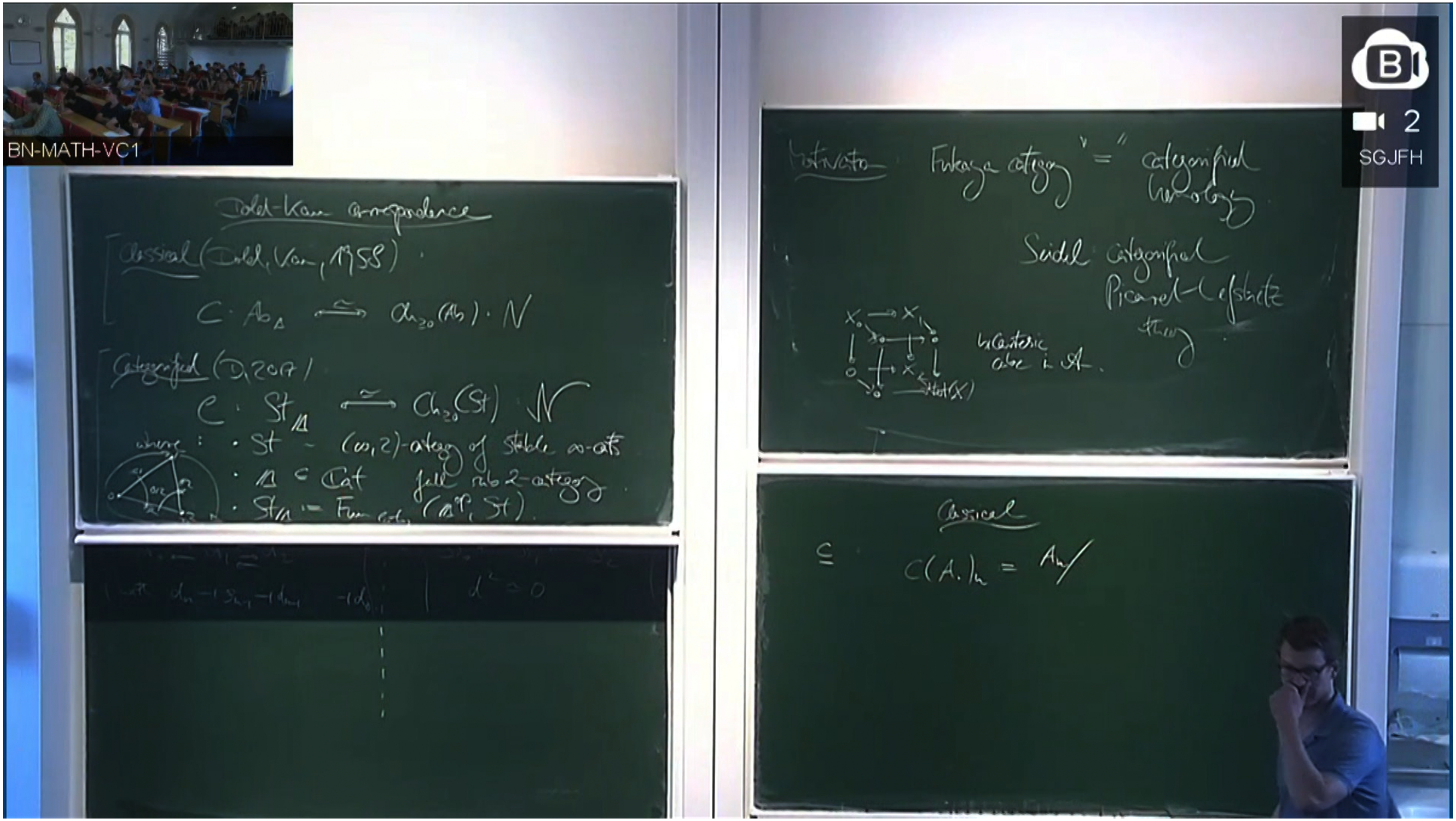
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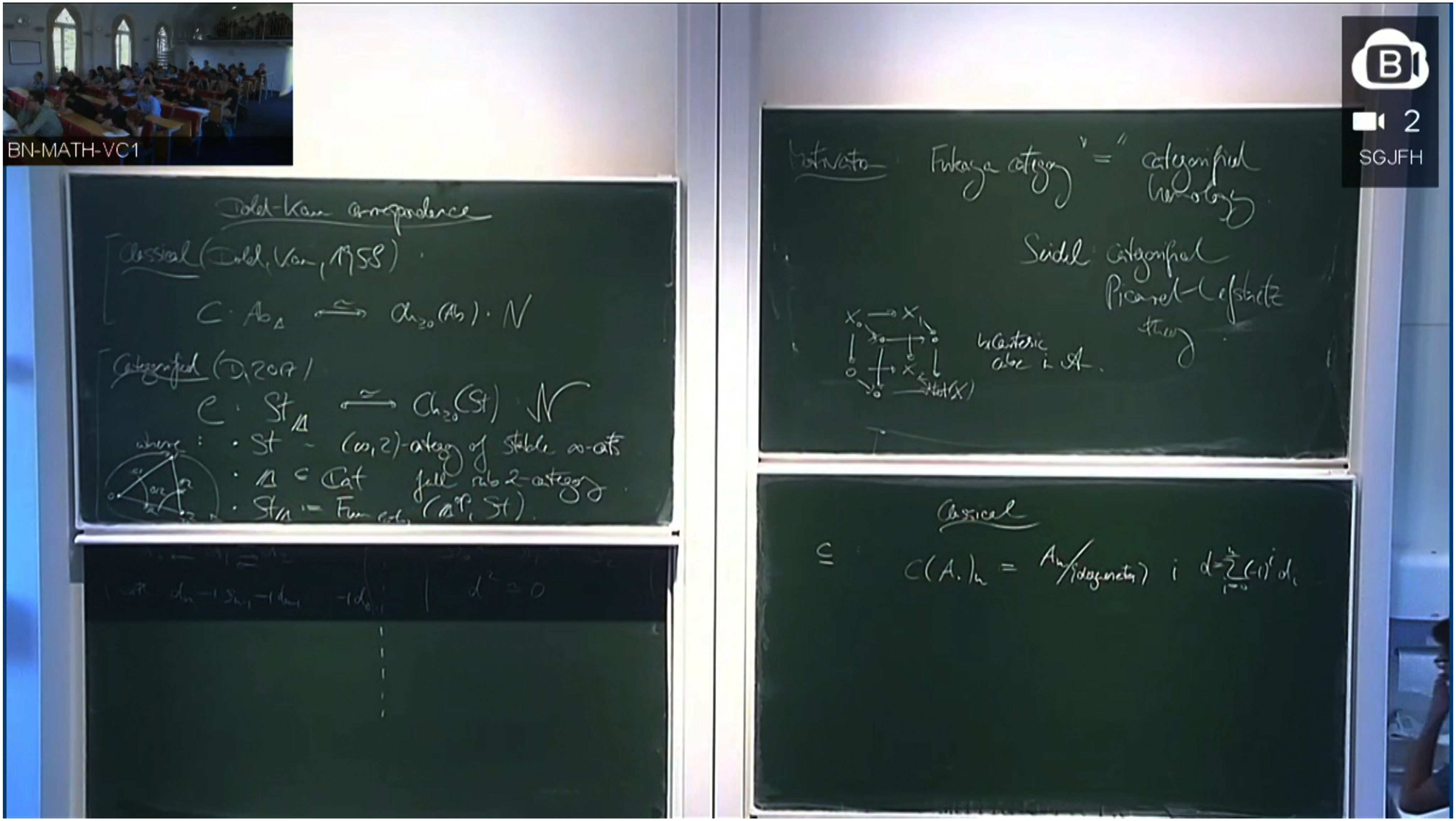
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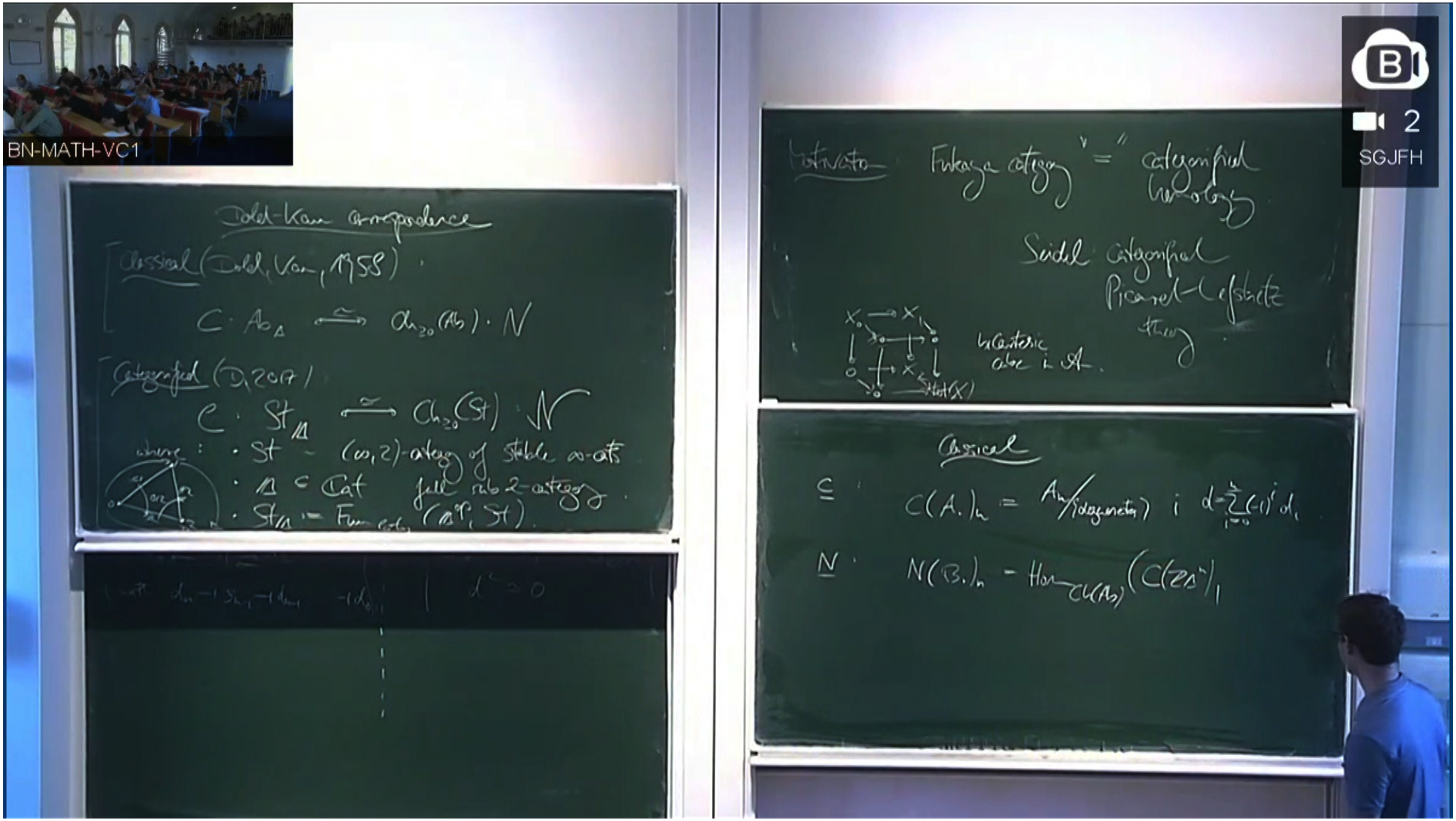
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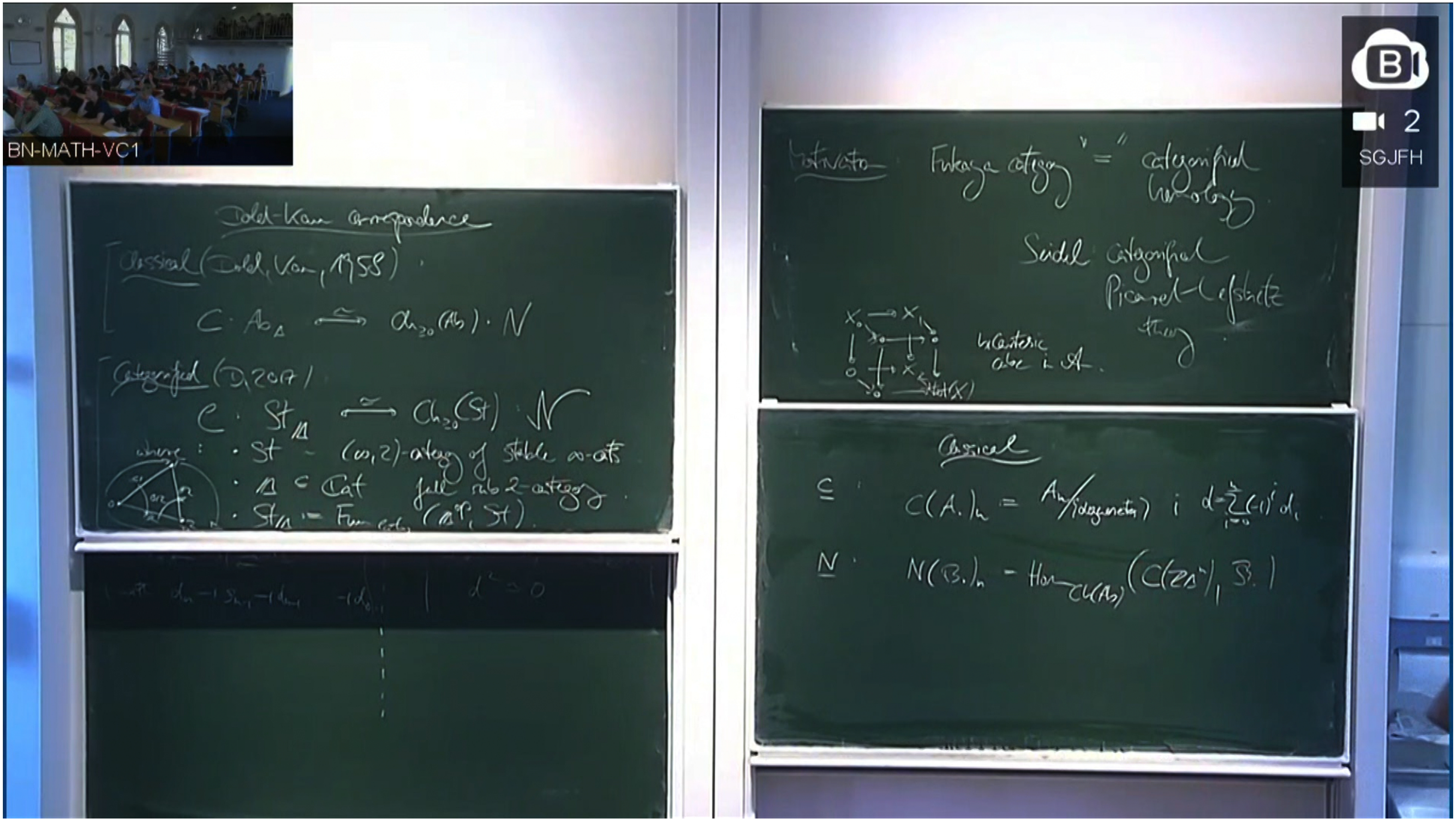
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Dold-Kan correspondence
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C : Ab_Δ ⇄ Ch_≥0(Ab) : N
Categorified (D, 2017)
C : St_Δ ⇄ Ch_≥0(St) : N
where : · St – (∞,2)-category of stable ∞-cats
· Δ ⊂ Cat full sub 2-category
· St_Δ := Fun(Δ^op, St)
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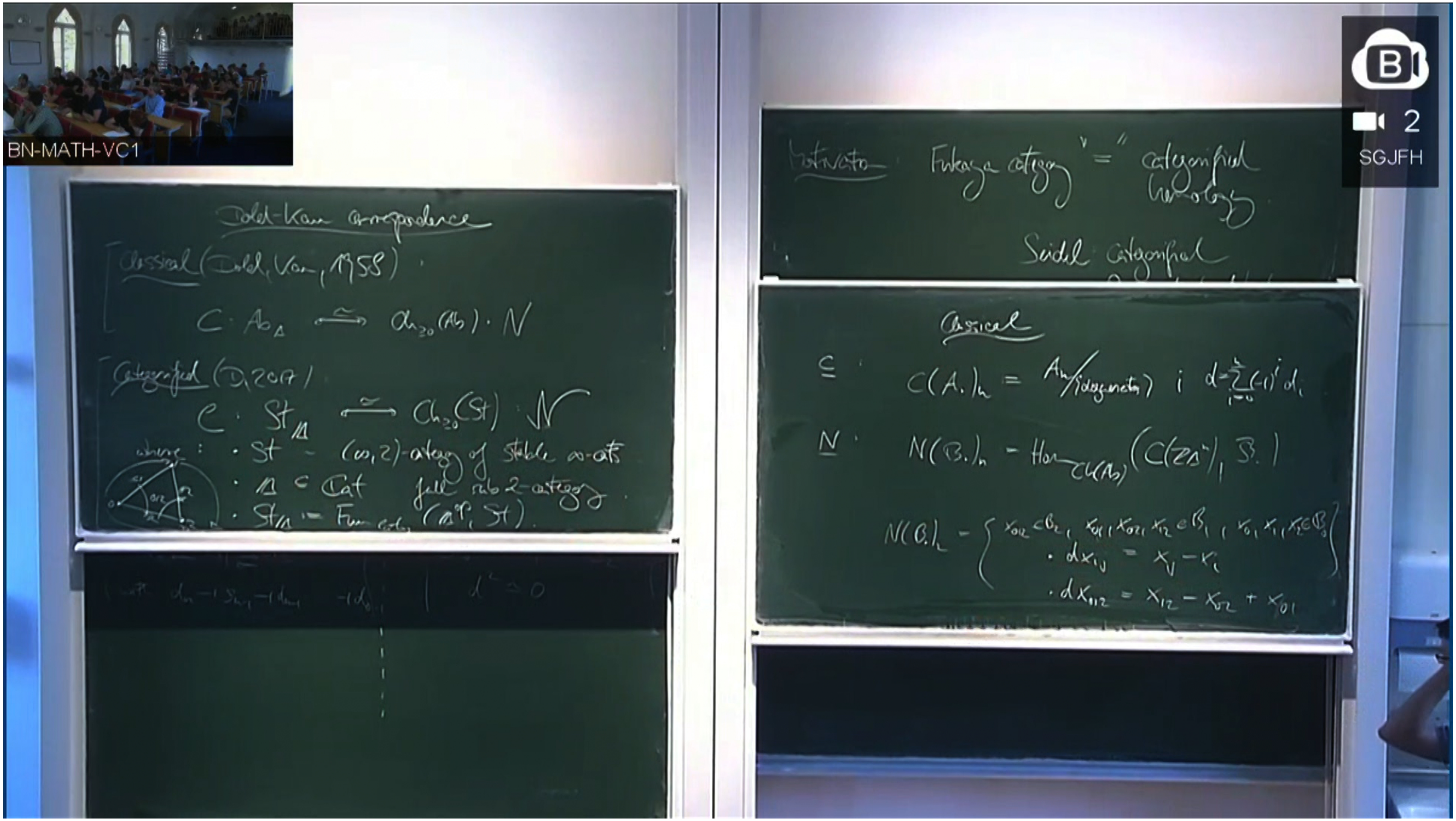
C : C(A.)_n = A_n/(degenerates) ; d = Σ(-1)^i d_i
N : N(B.)_n = Hom_Ch(Ab)(C(ZΔ^n), B.)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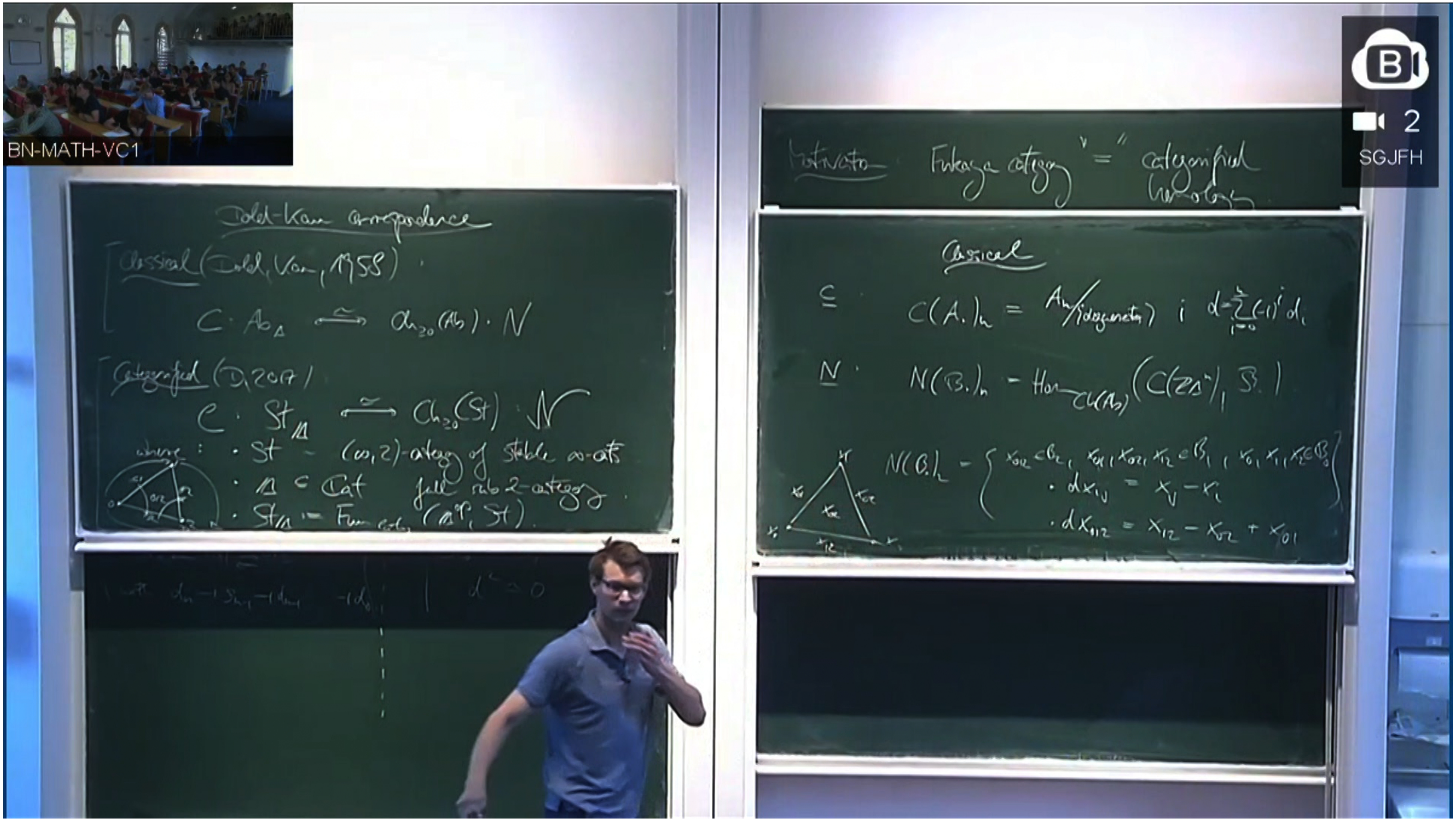

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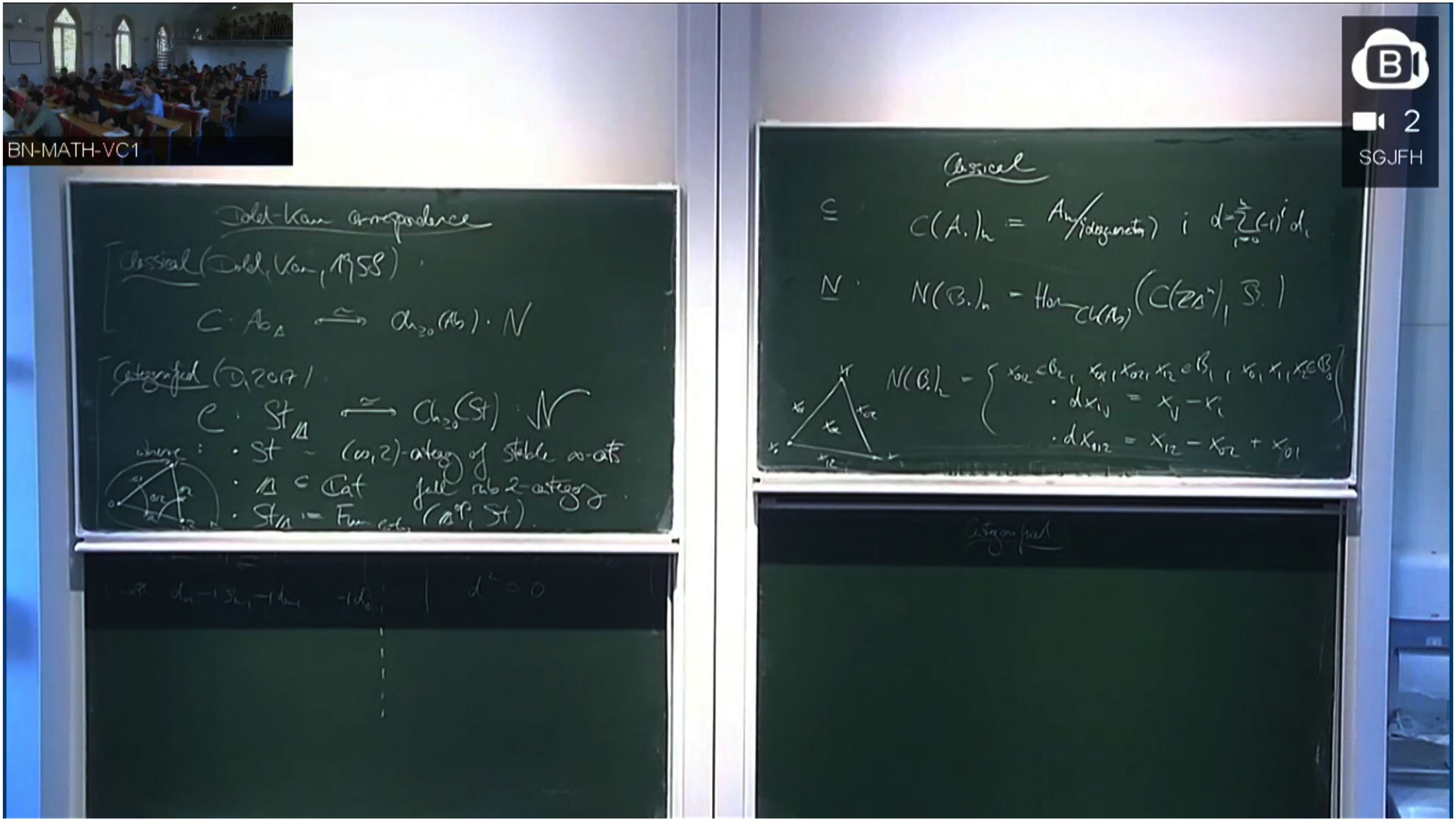
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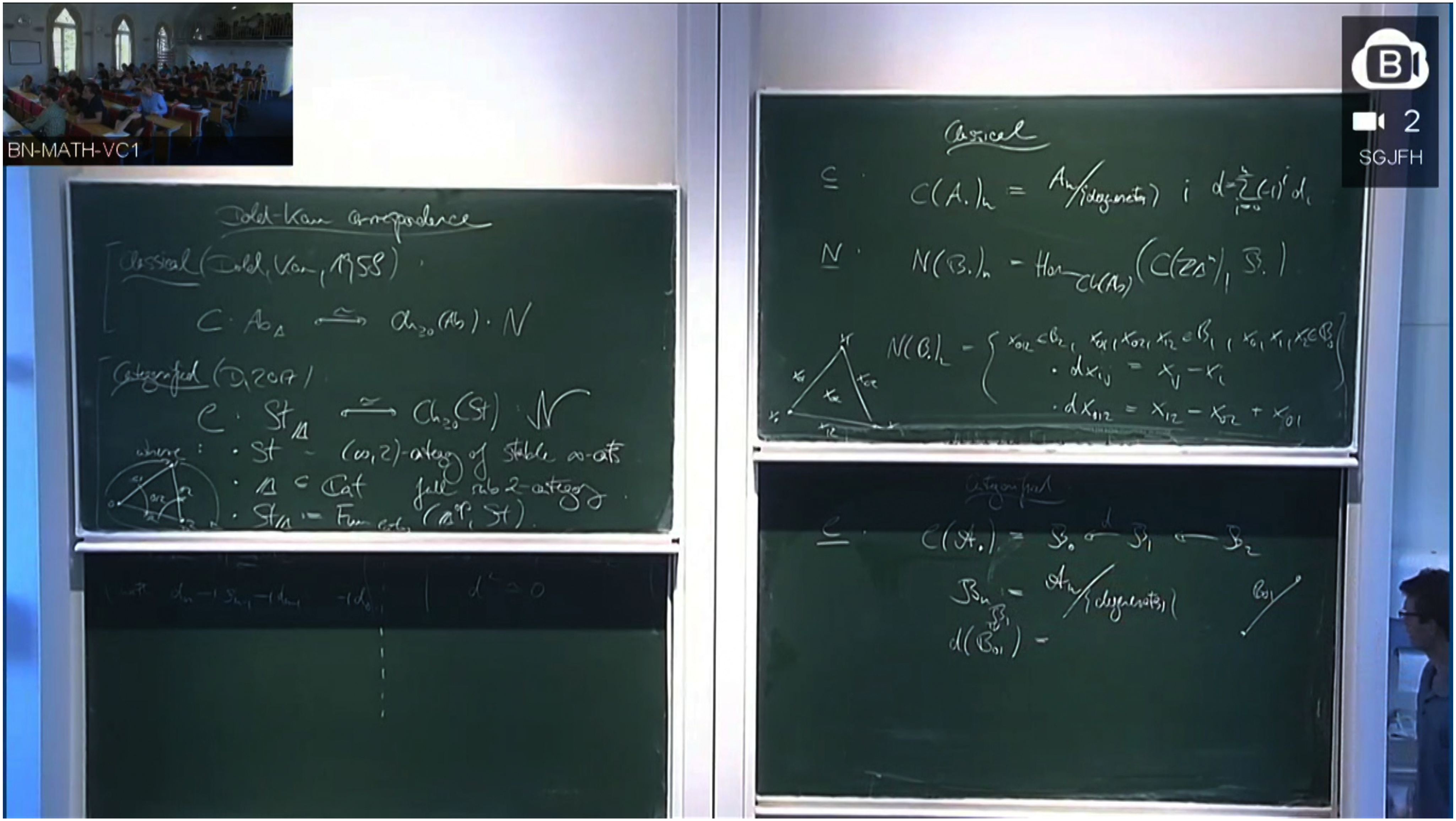
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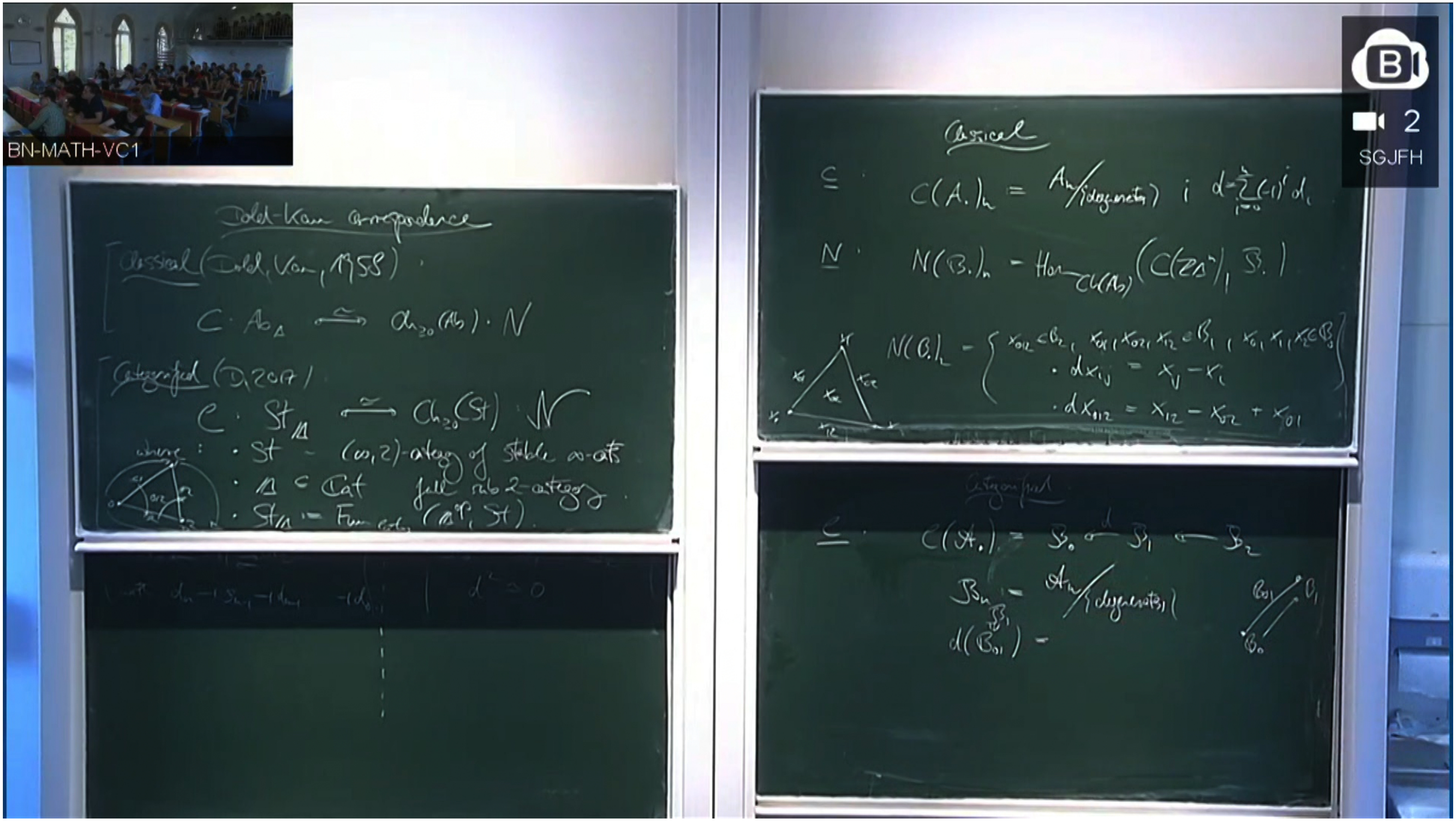
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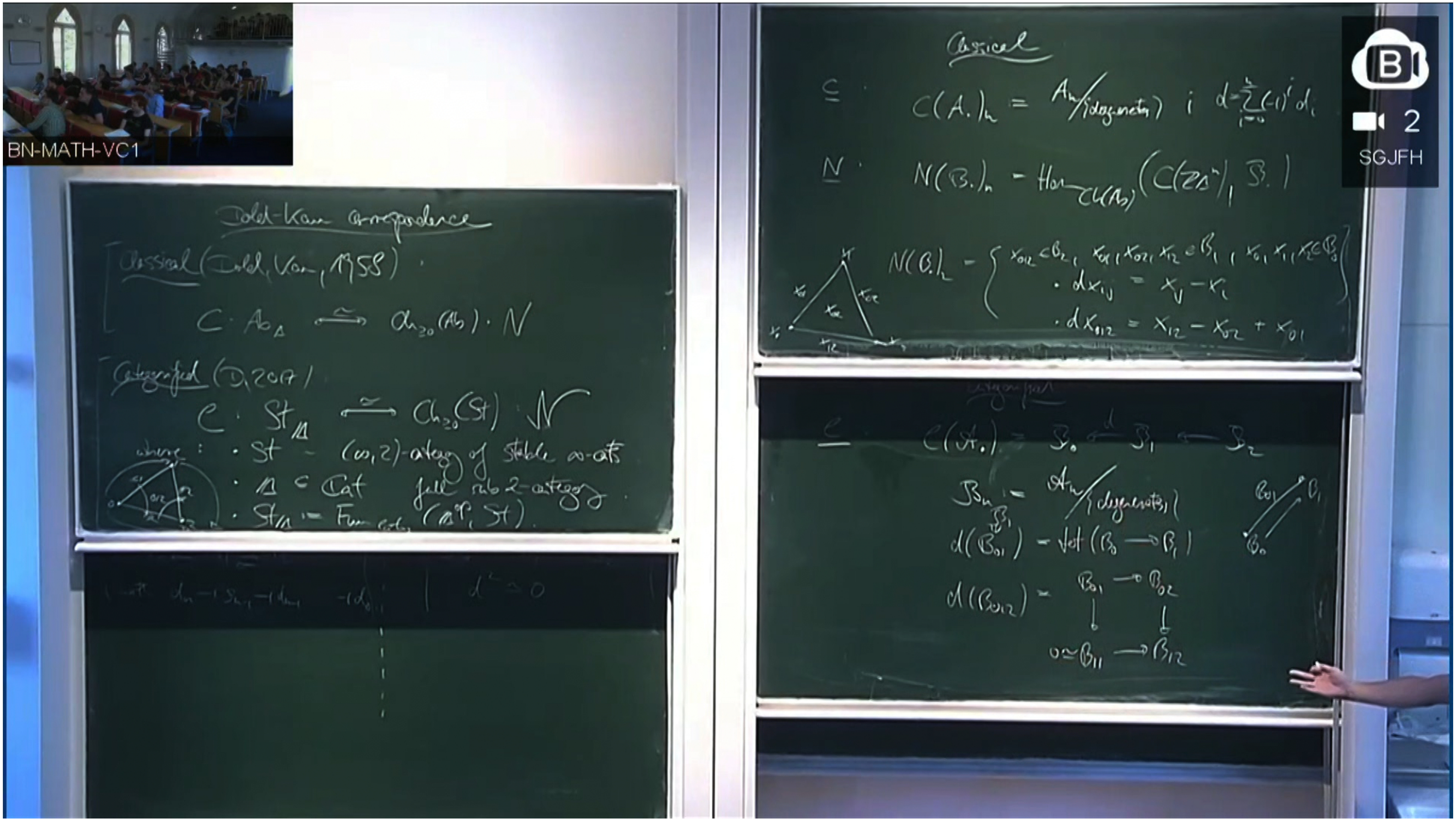
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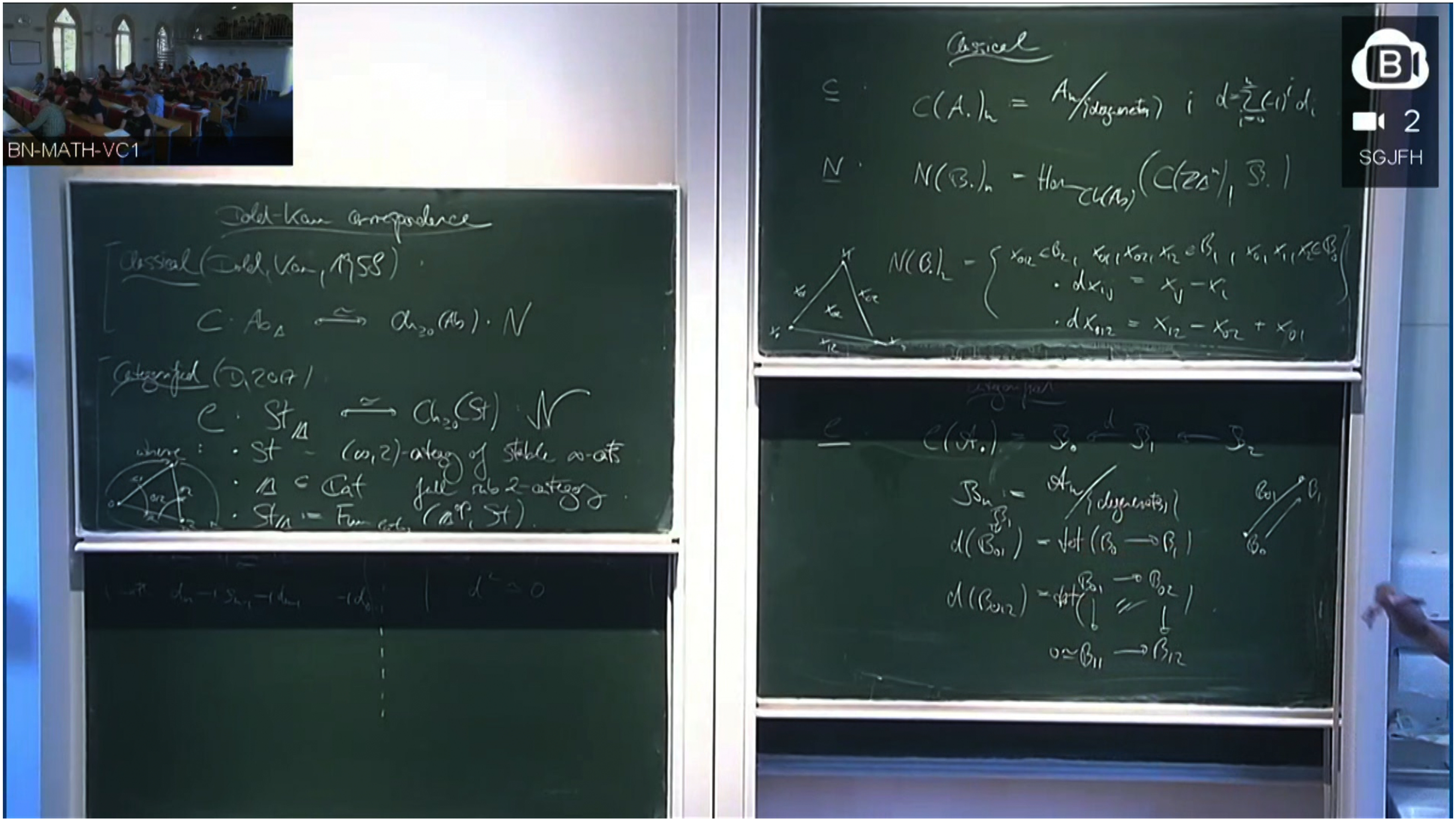
N(B.)_2 = { x_012 ∈ B_2, x_01, x_02, x_12 ∈ B_1, x_0, x_1, x_2 ∈ B_0 | d x_ij = x_j − x_i, d x_012 = x_12 − x_02 + x_01 }
Categorified
C: C(A.) = B_0 ← B_1 ← B_2
B_n = A_n/(degenerates)
d(B_01) = Vect(B_0 → B_1)
d(B_012) = B_01 → B_02, 0 ≃ B_11 → B_12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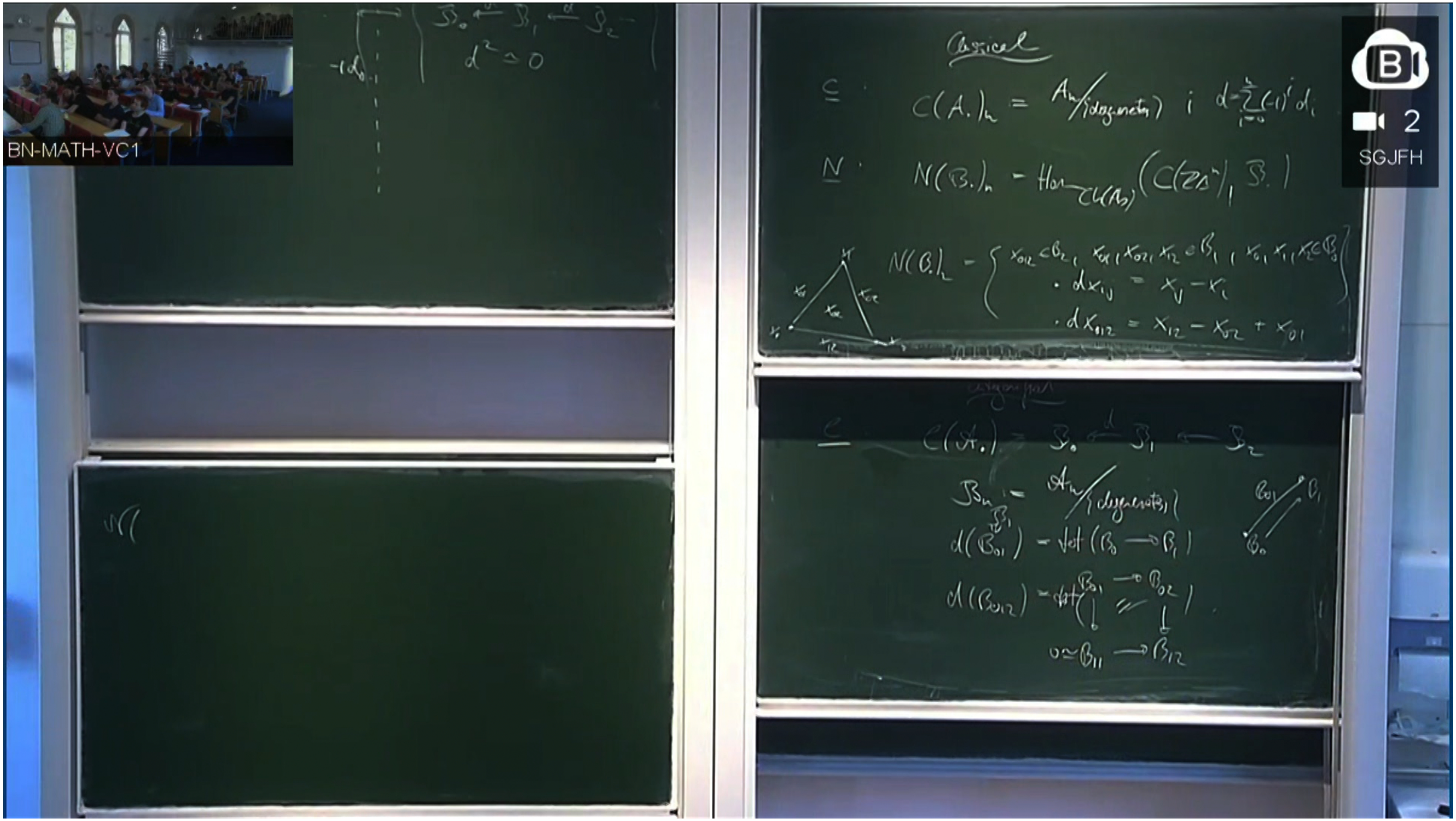
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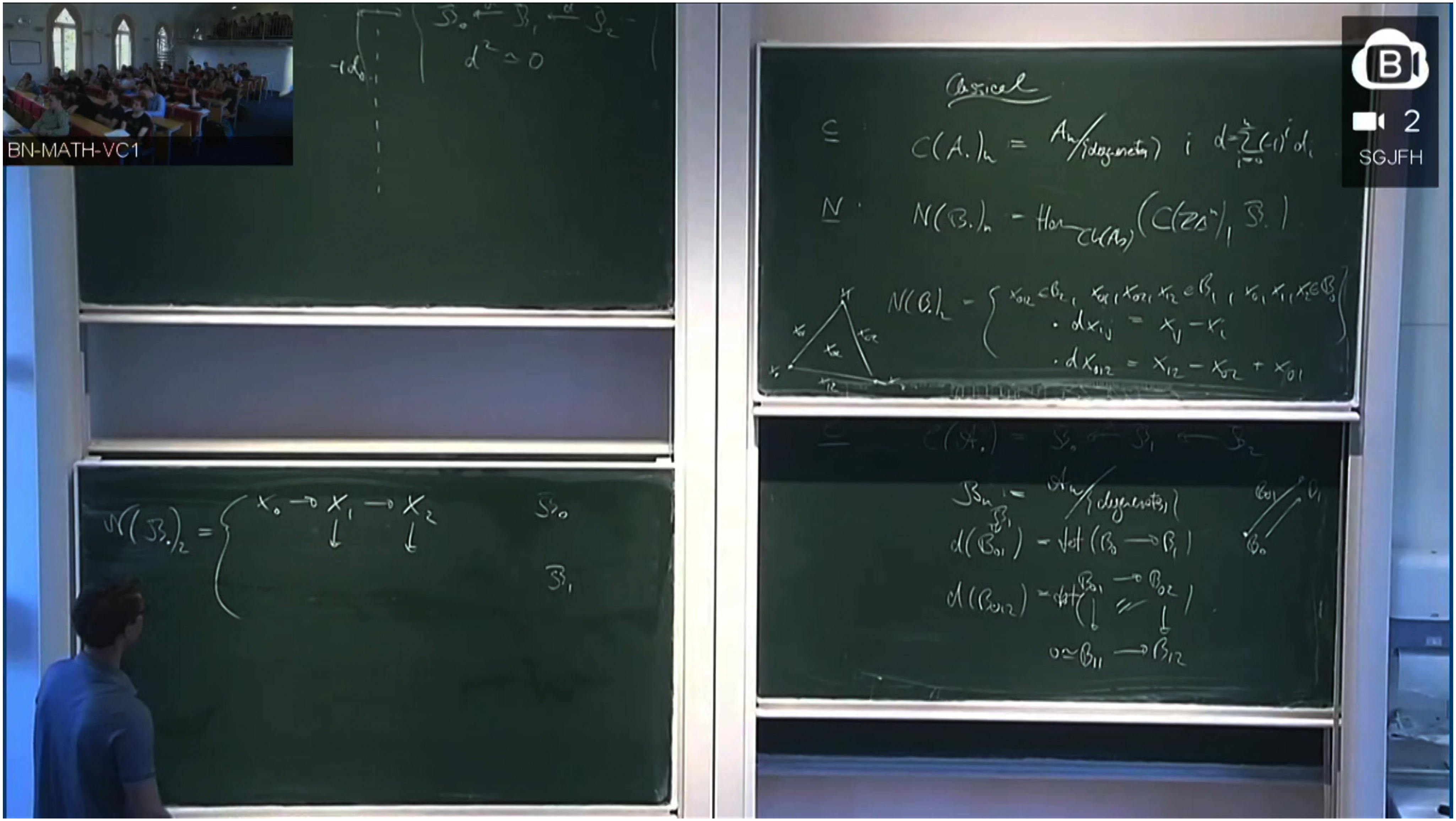
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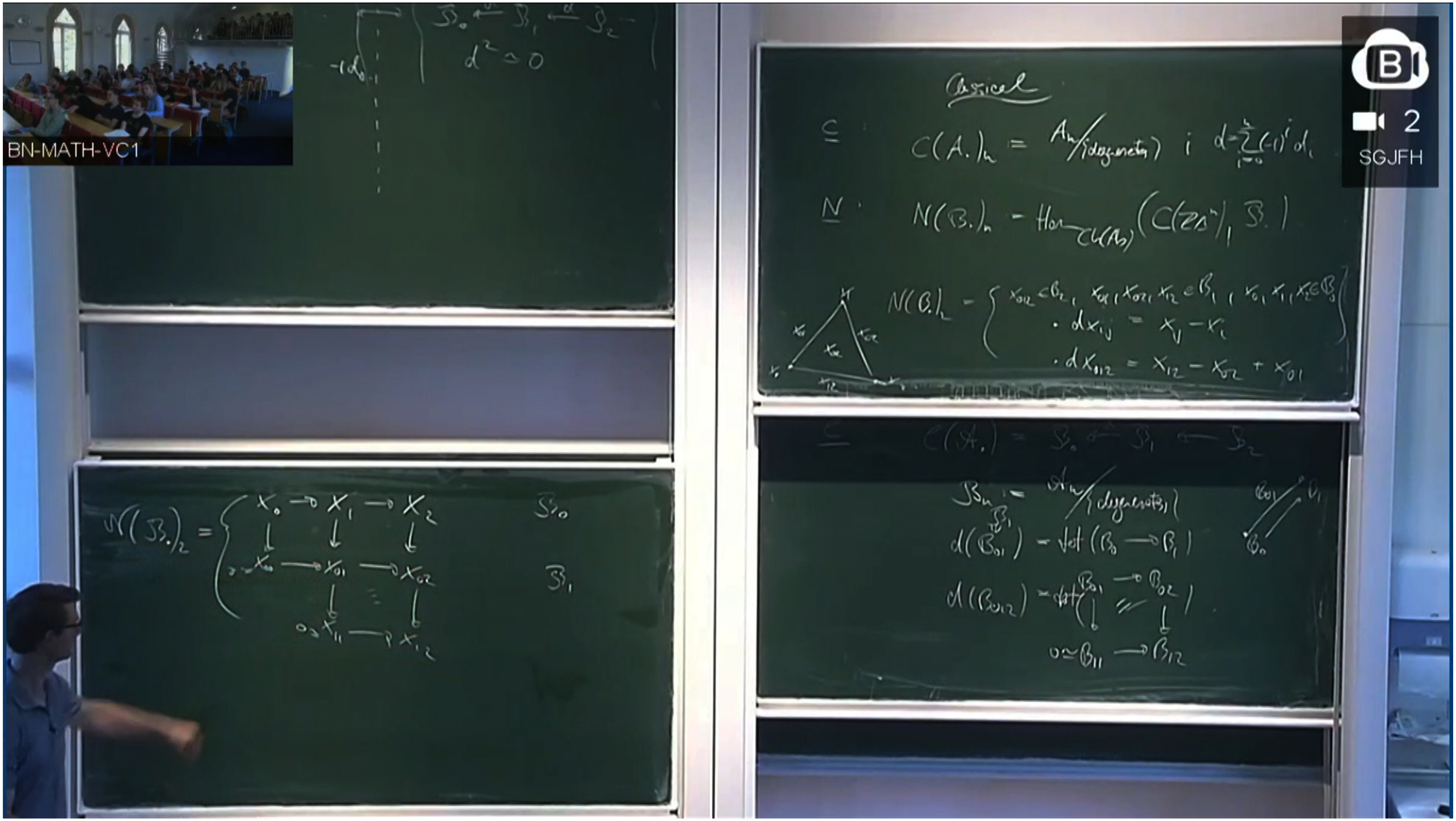
· dx_ij = x_j − x_i
· dx_012 = x_12 − x_02 + x_01
d² = 0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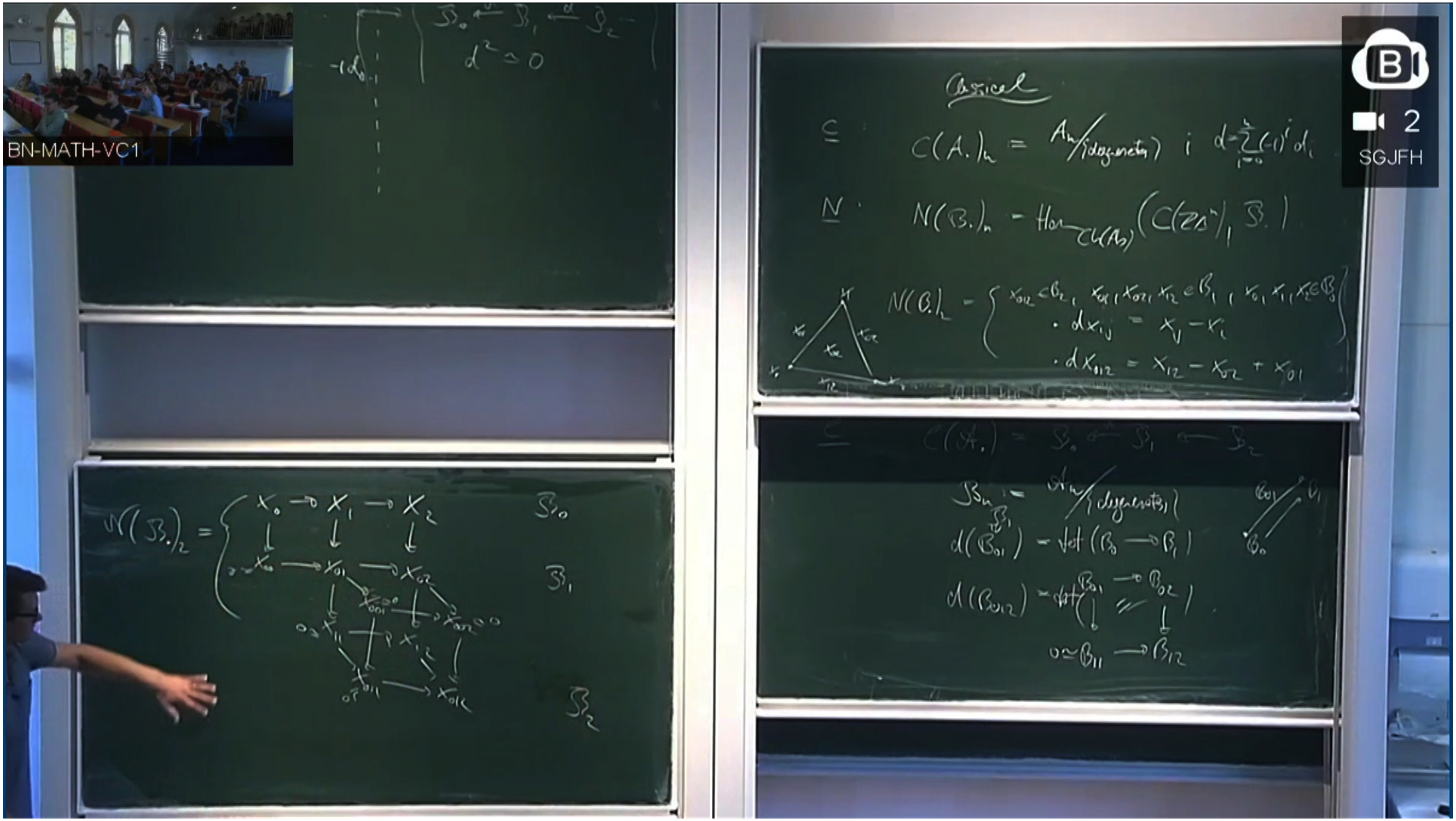
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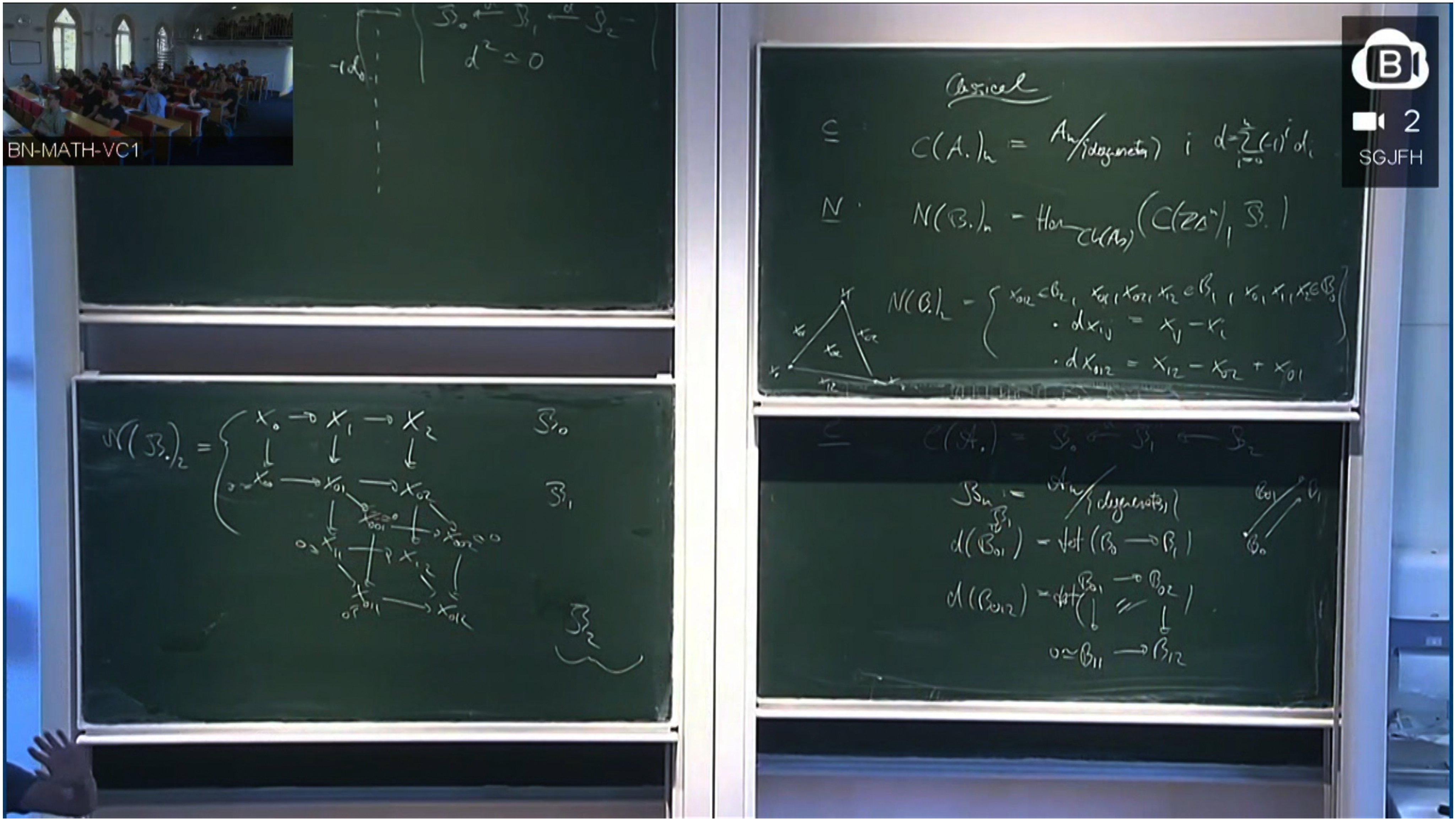
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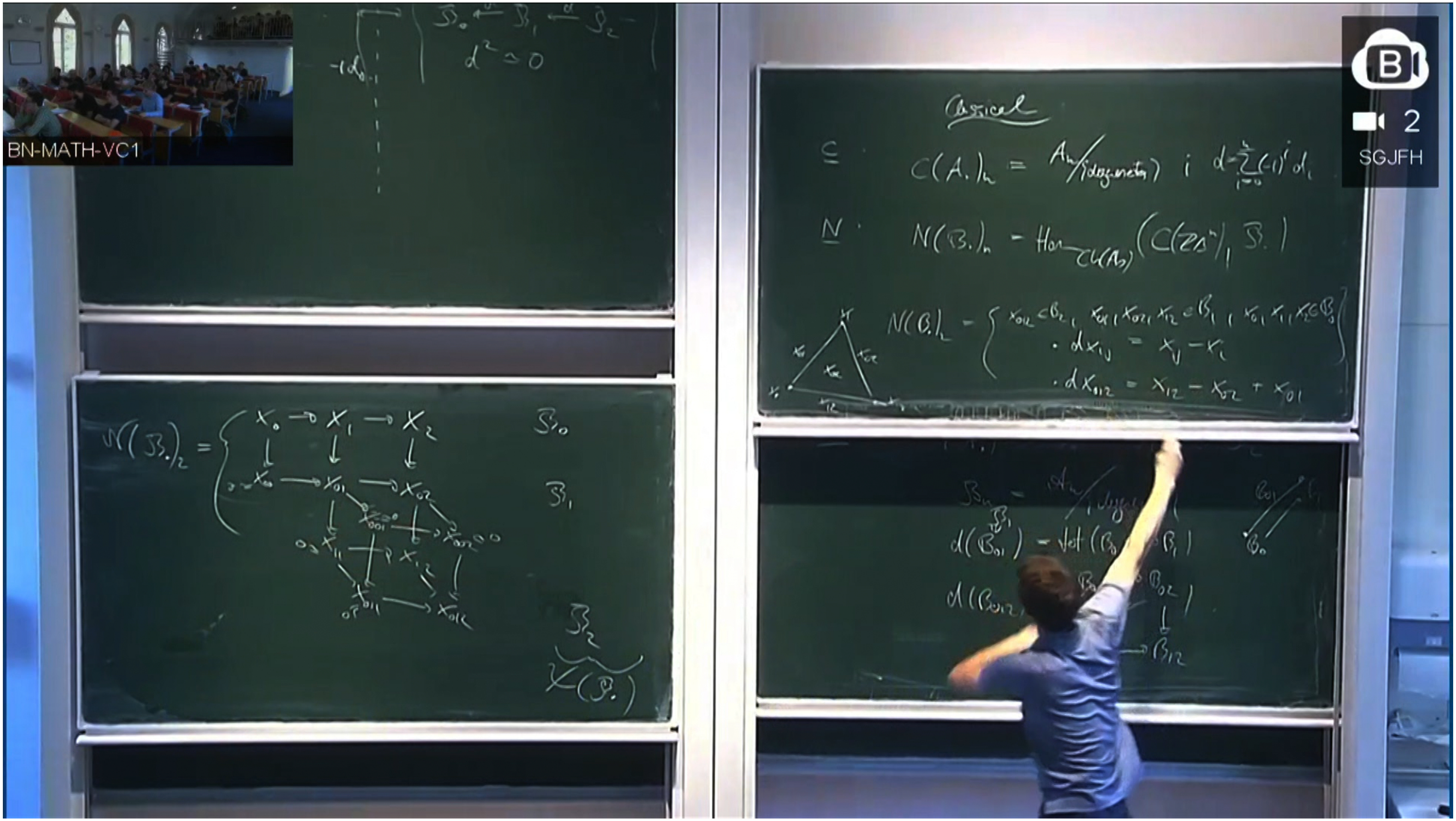
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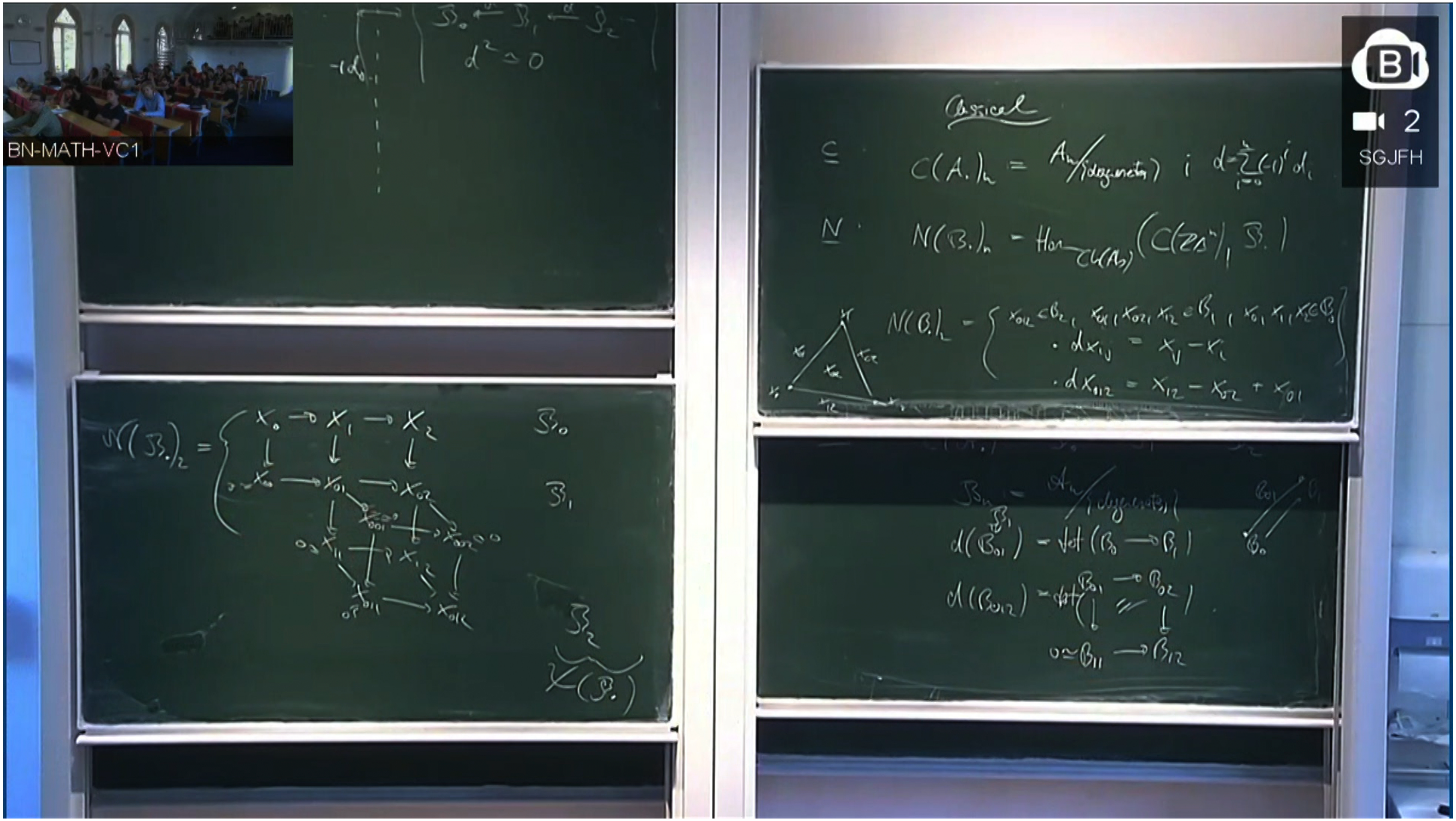
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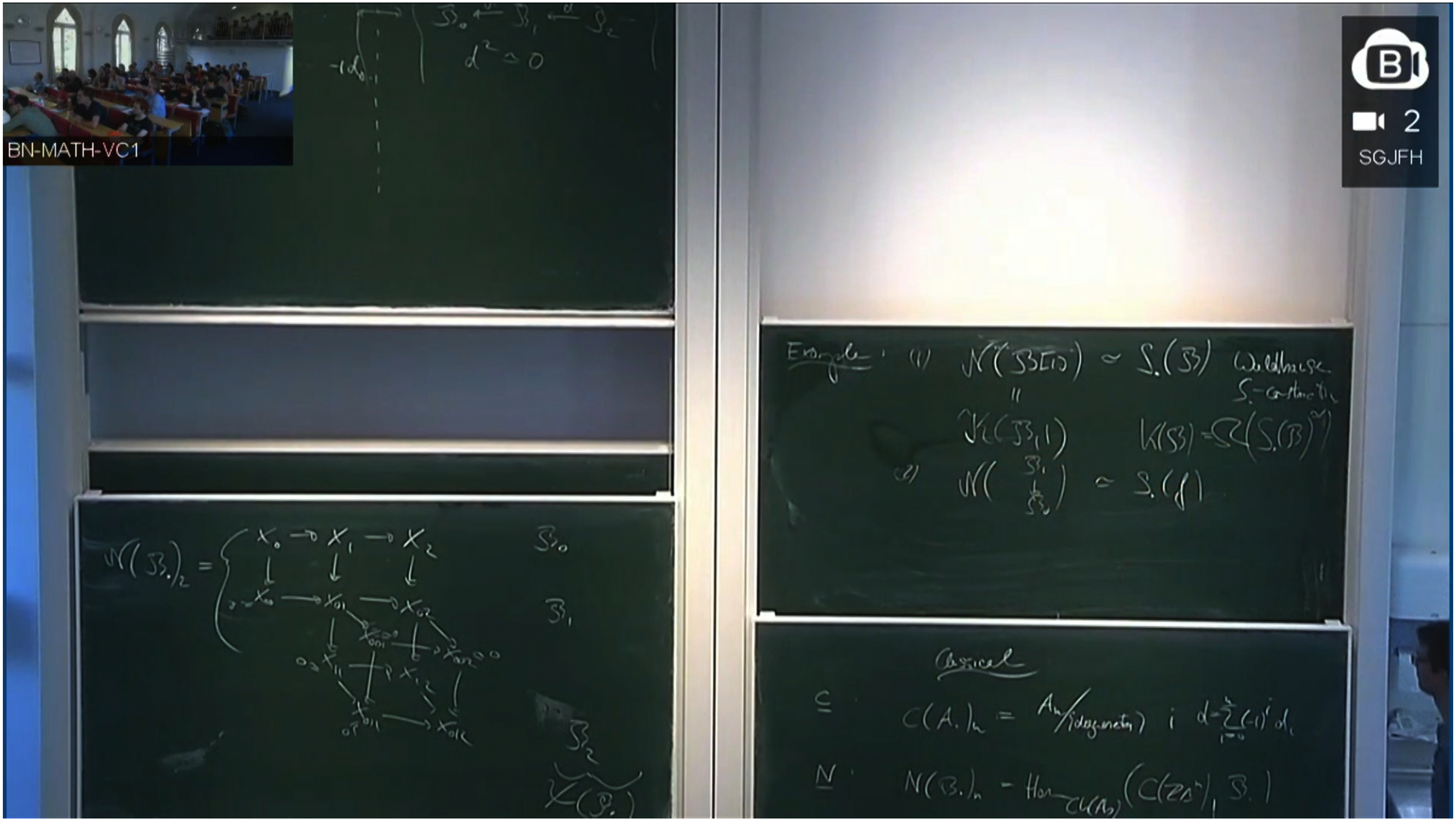
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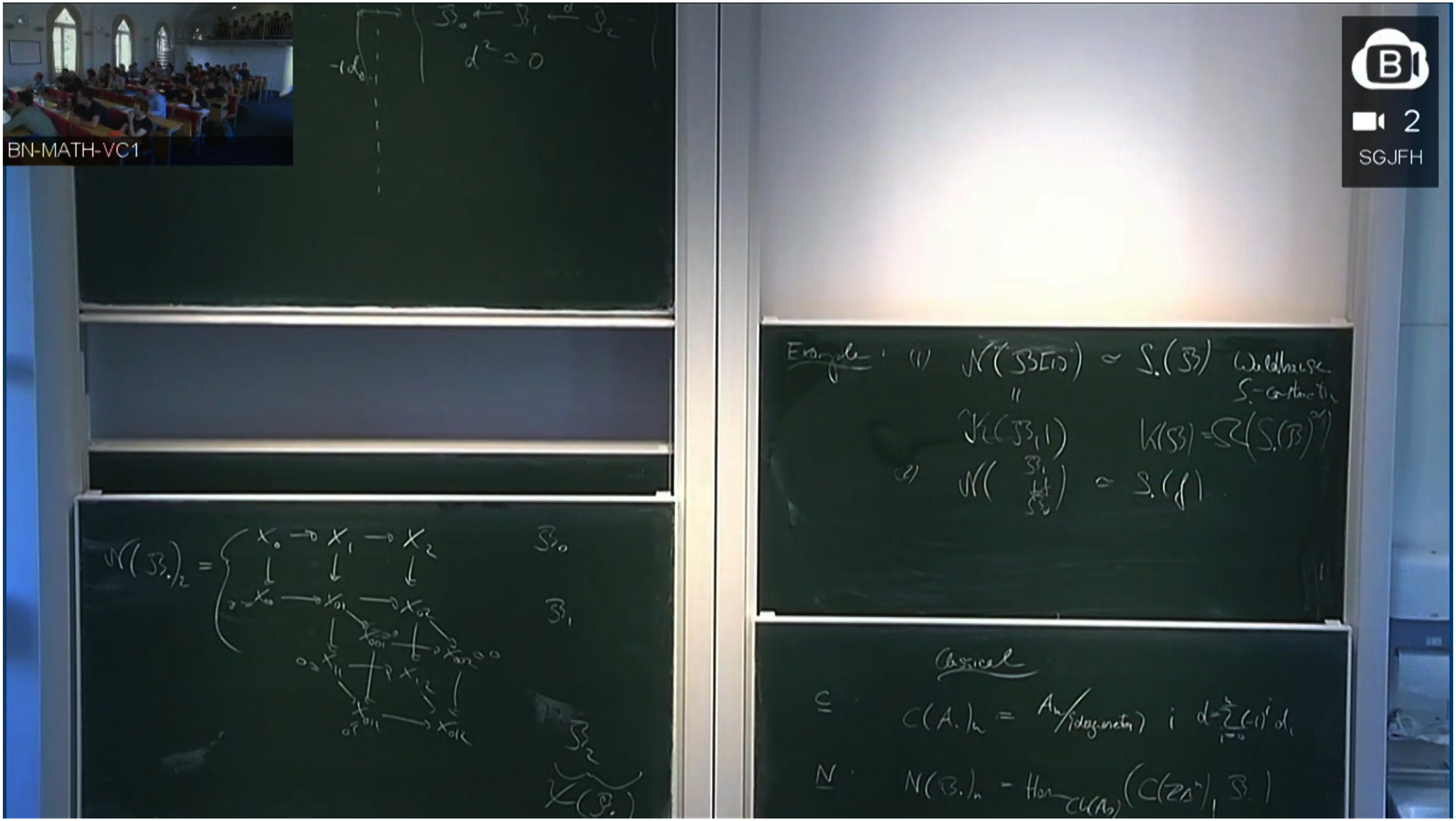
SGJFH
Example
Waldhause
S.-construction
Classical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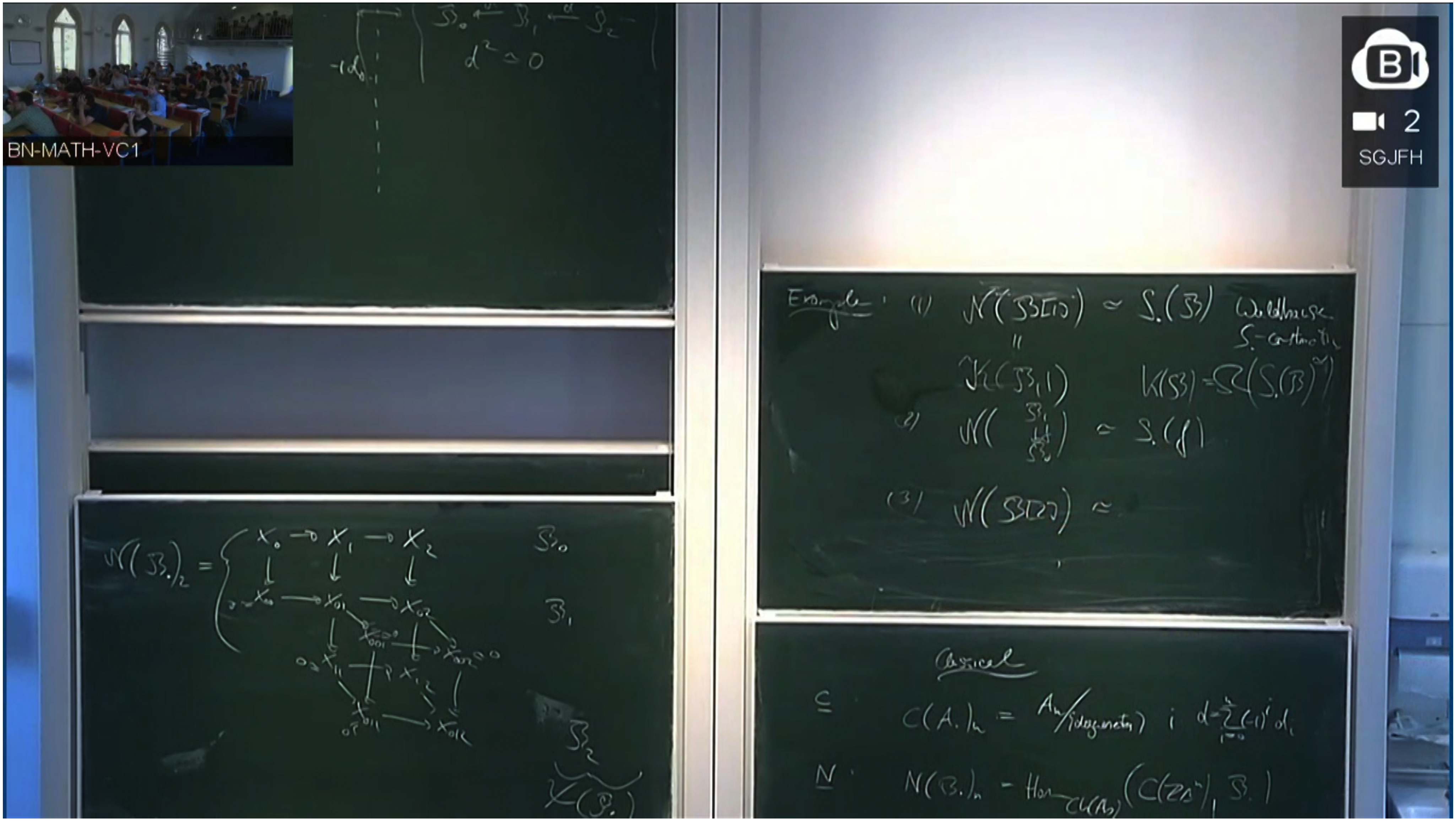
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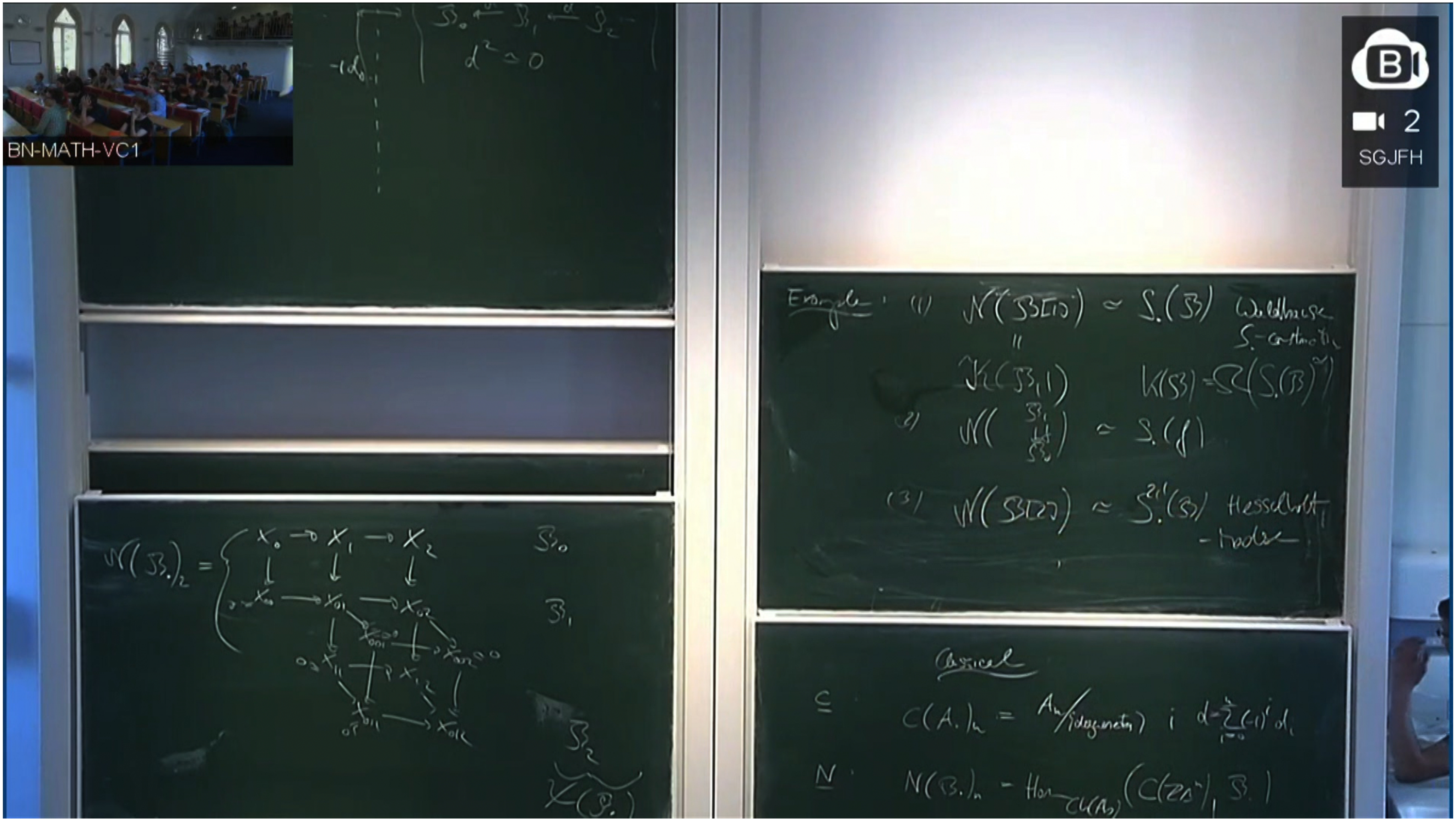
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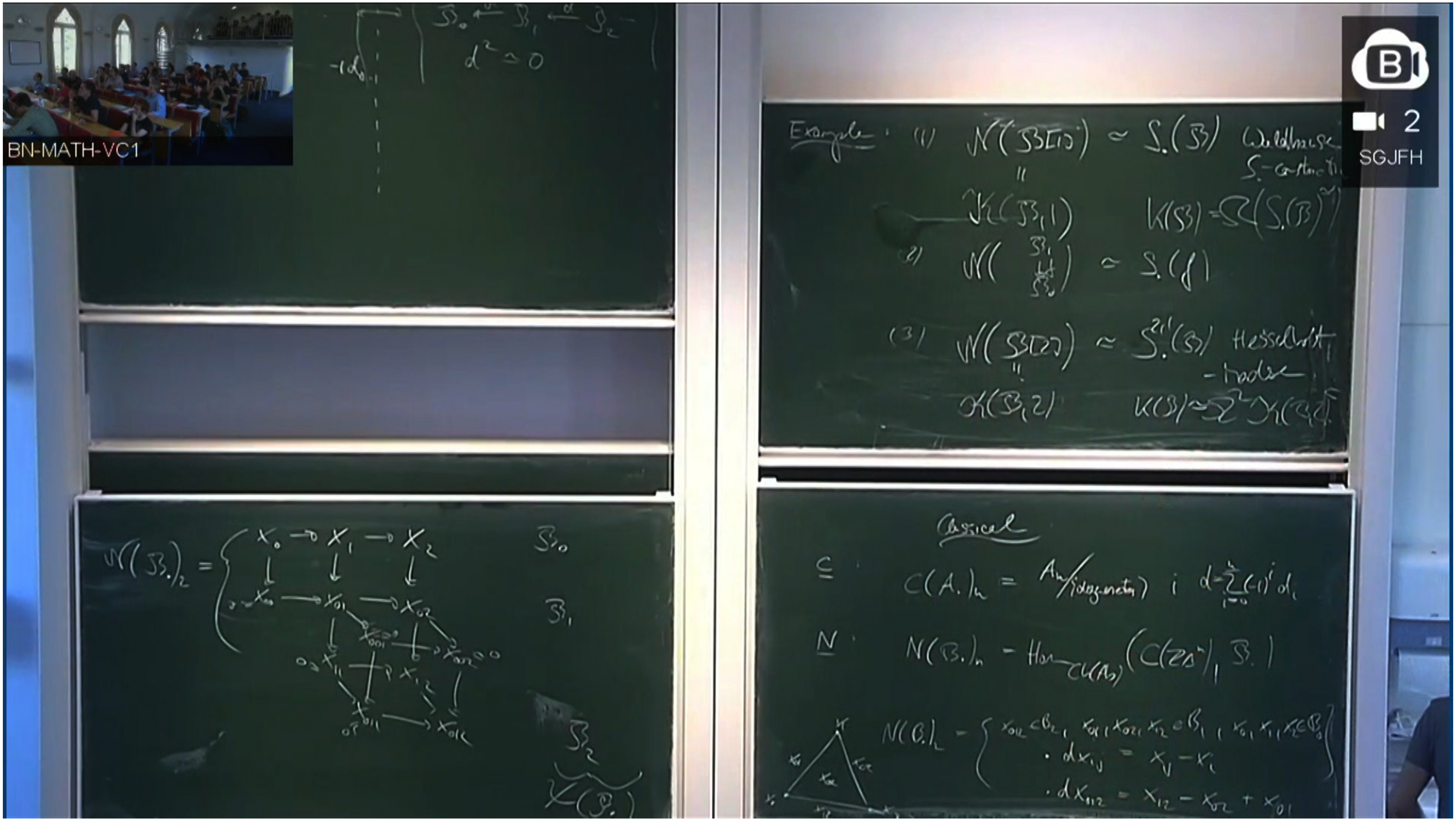
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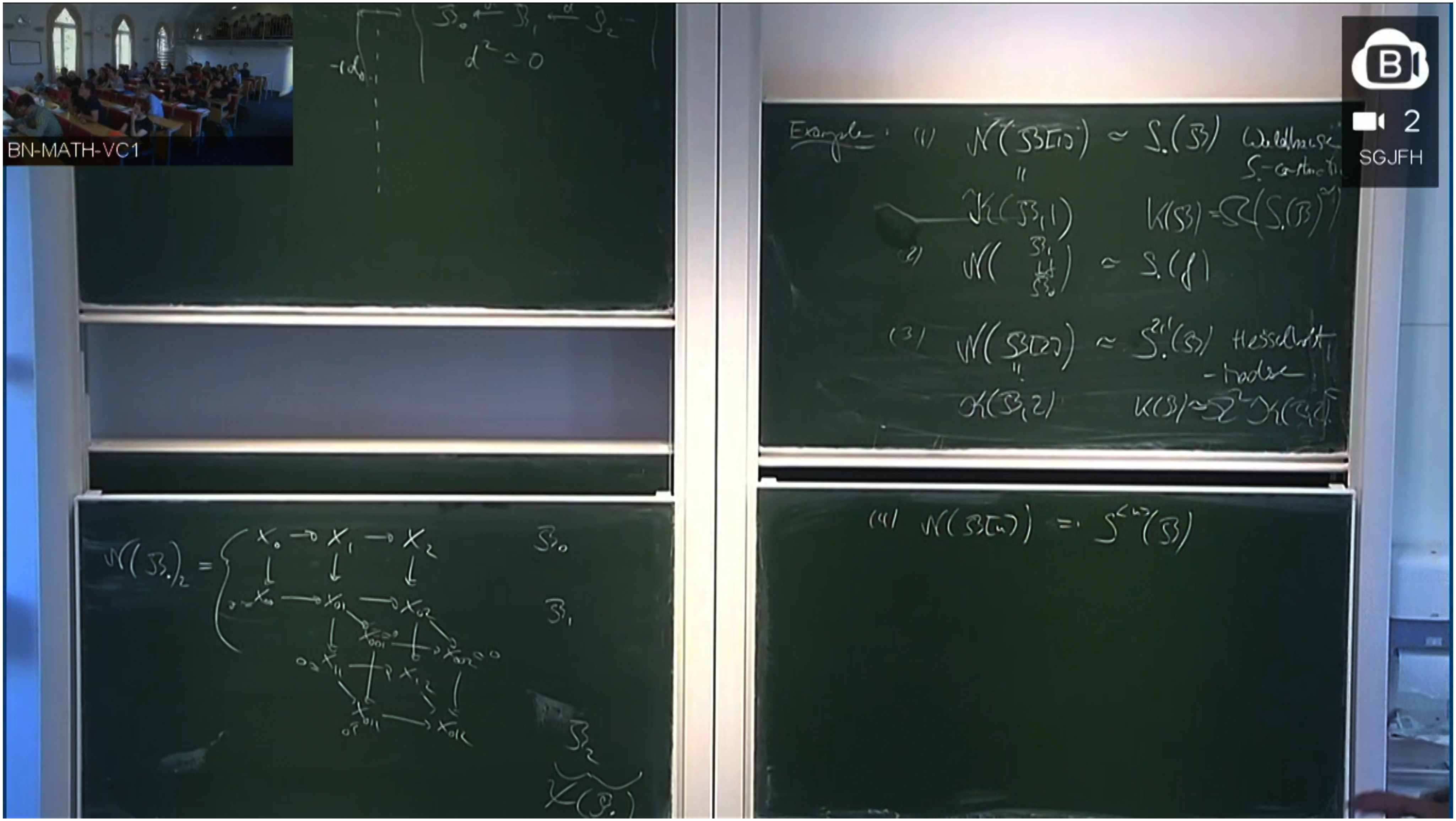
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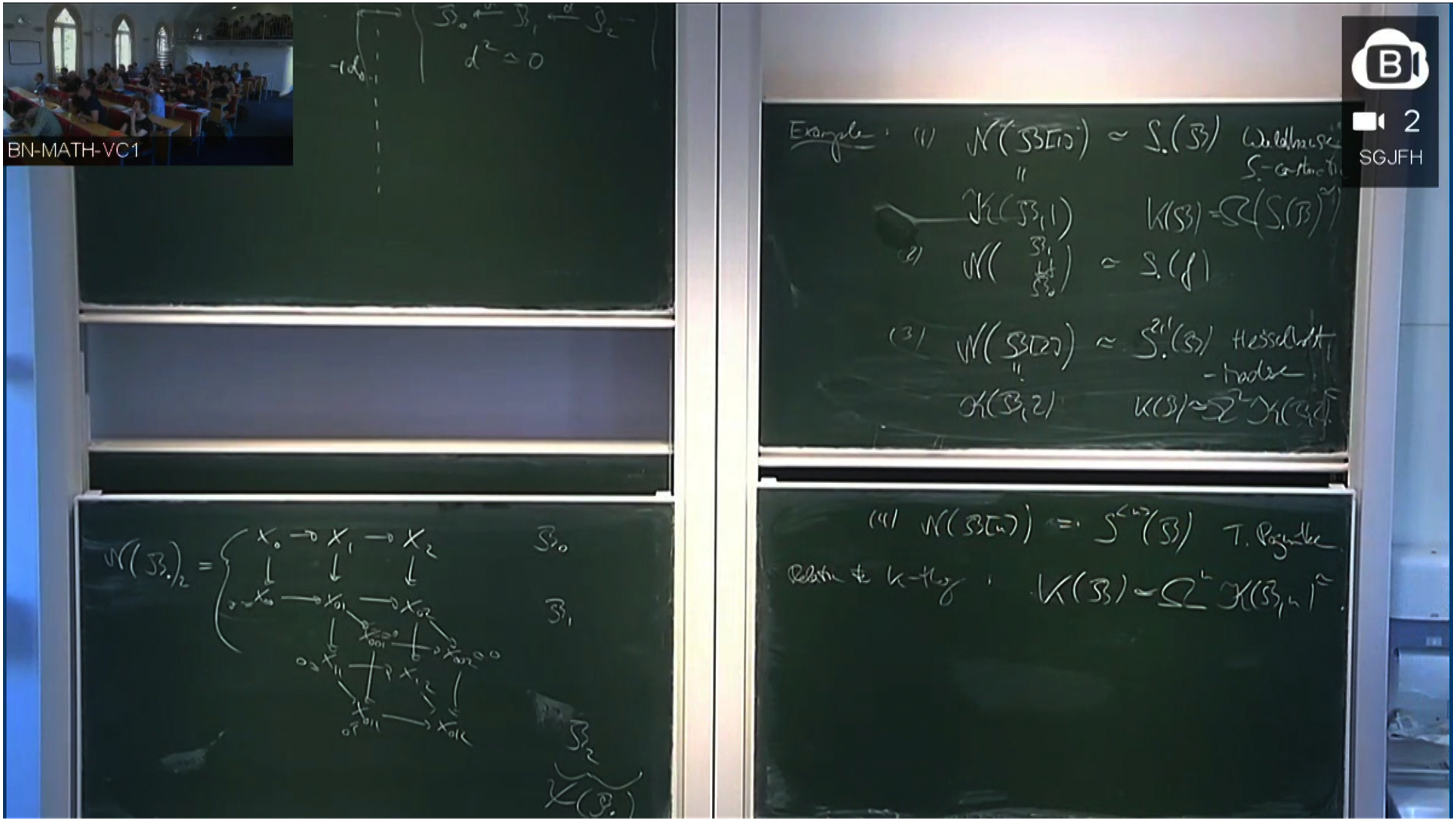
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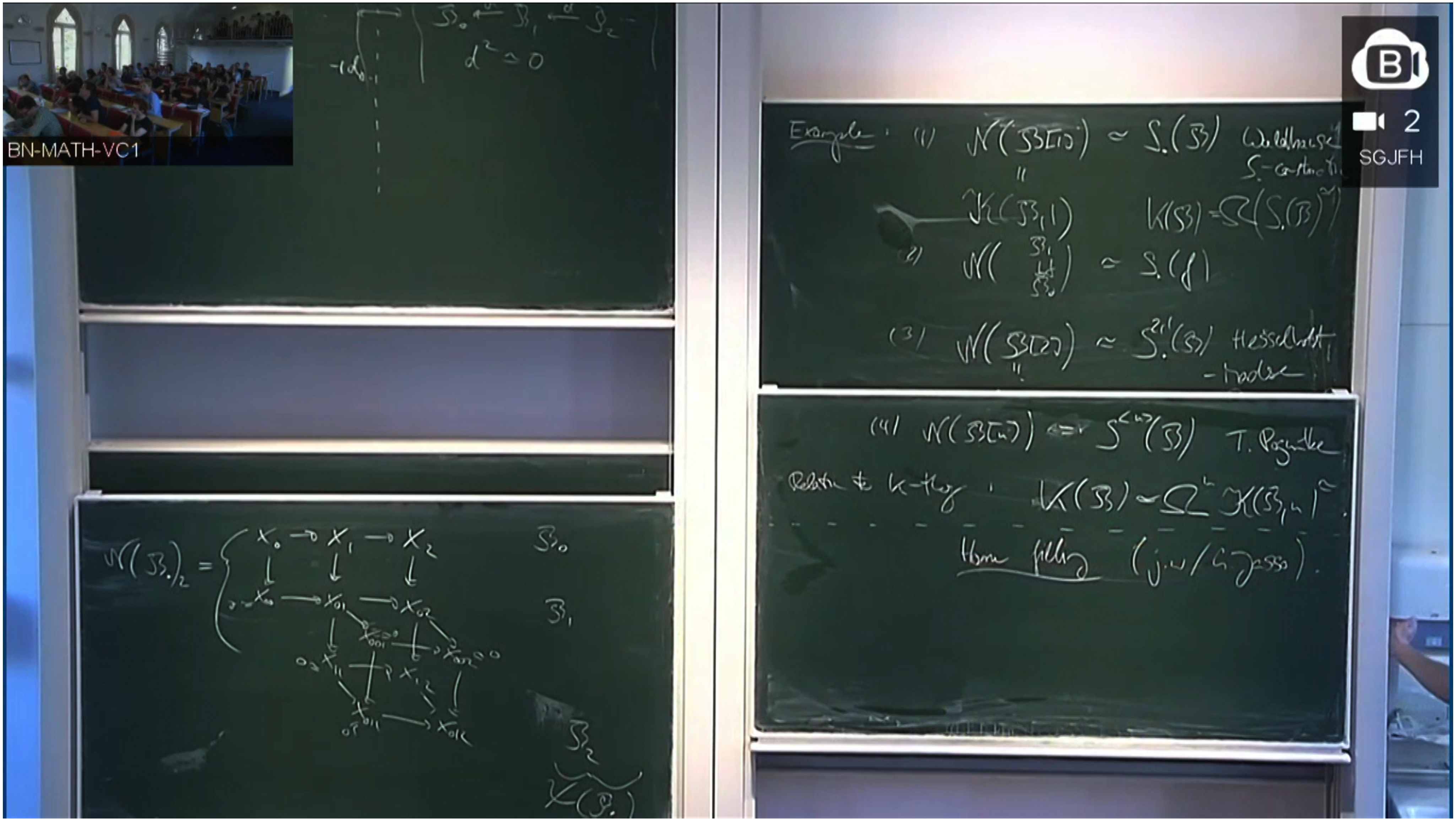
T. Poguntke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Example
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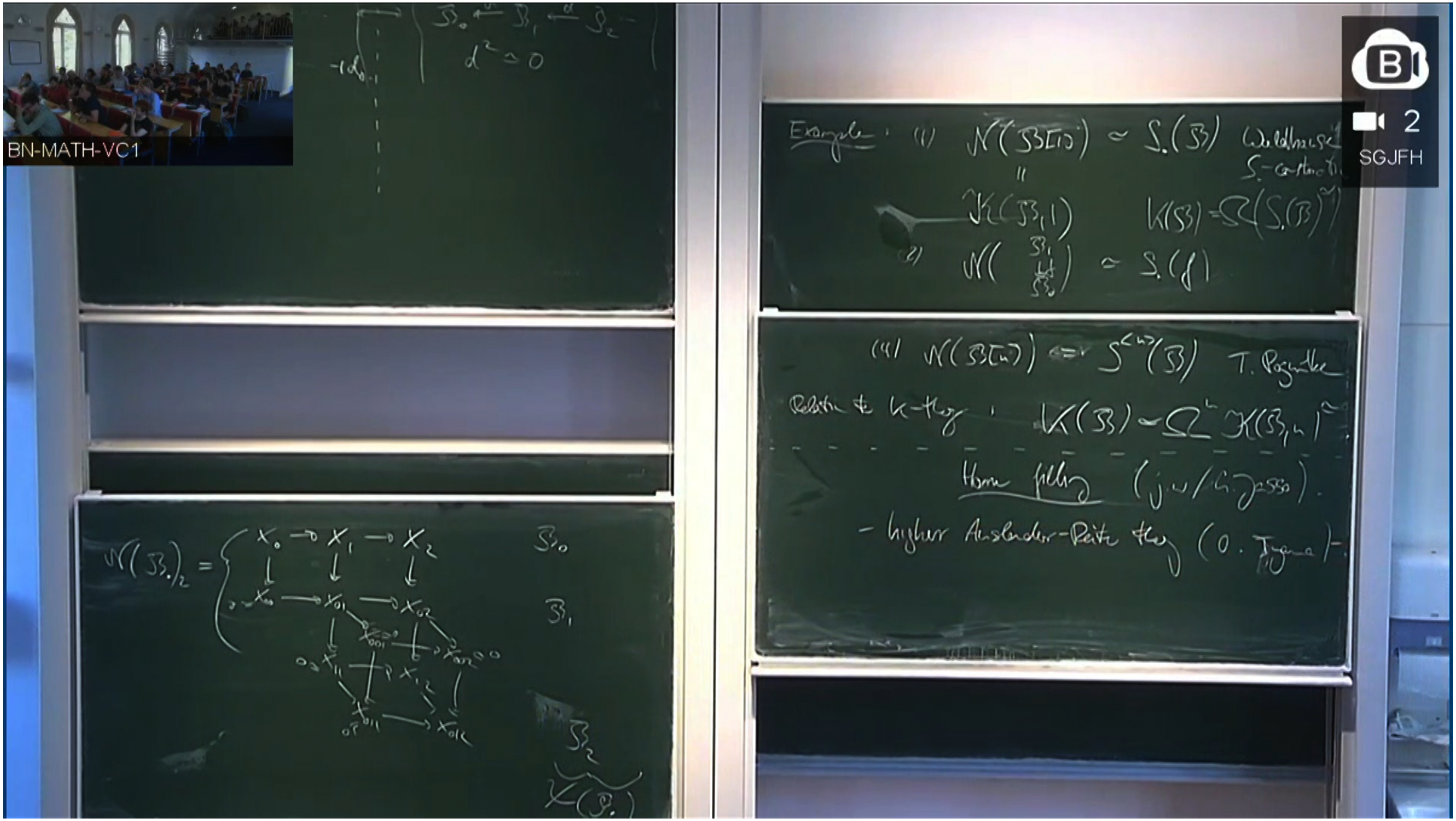
Relation to K-theory
Horn filling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Example:
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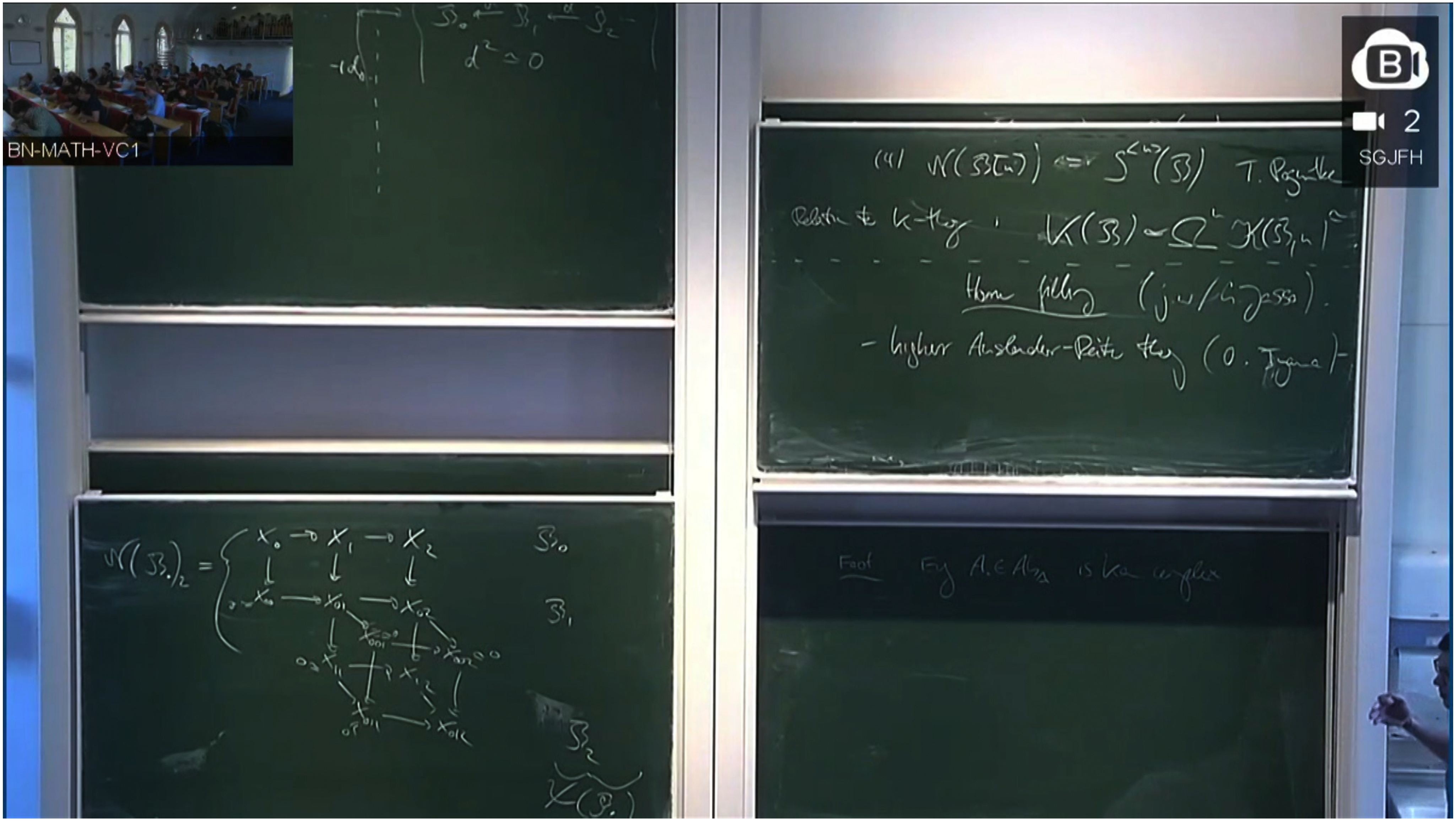
higher Auslander-Reiten theory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Horn filling
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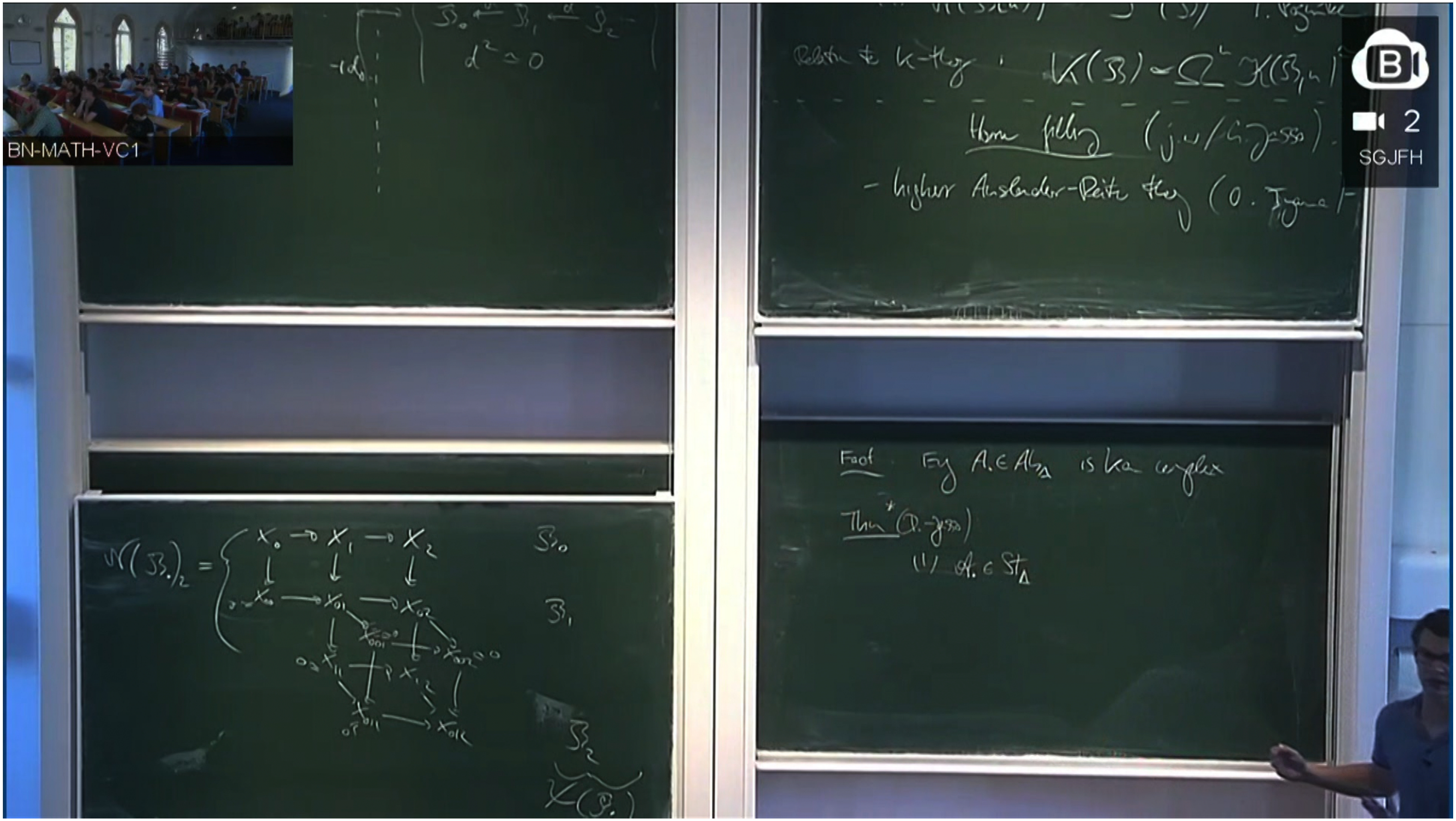
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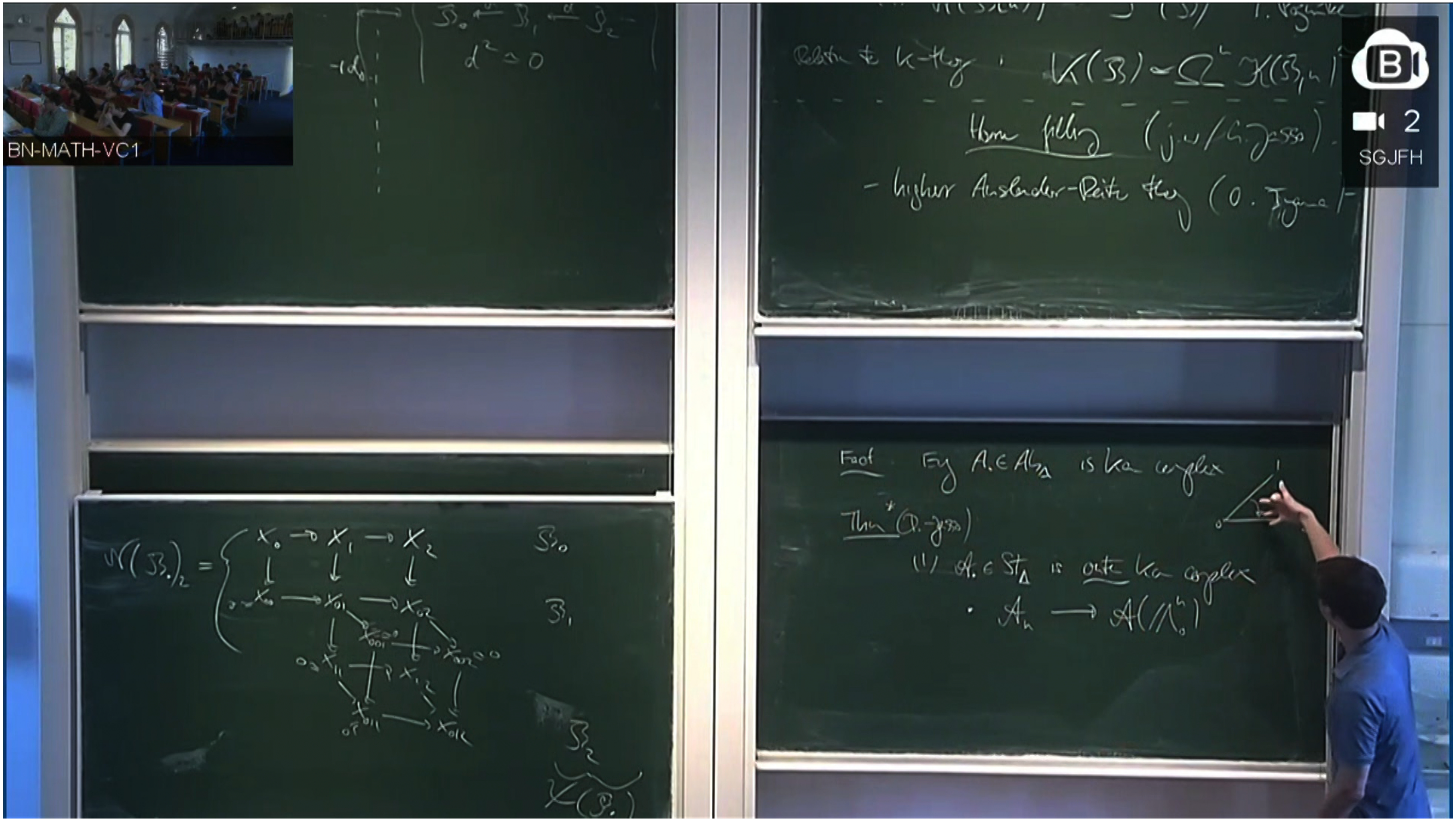
Fact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Hom filling
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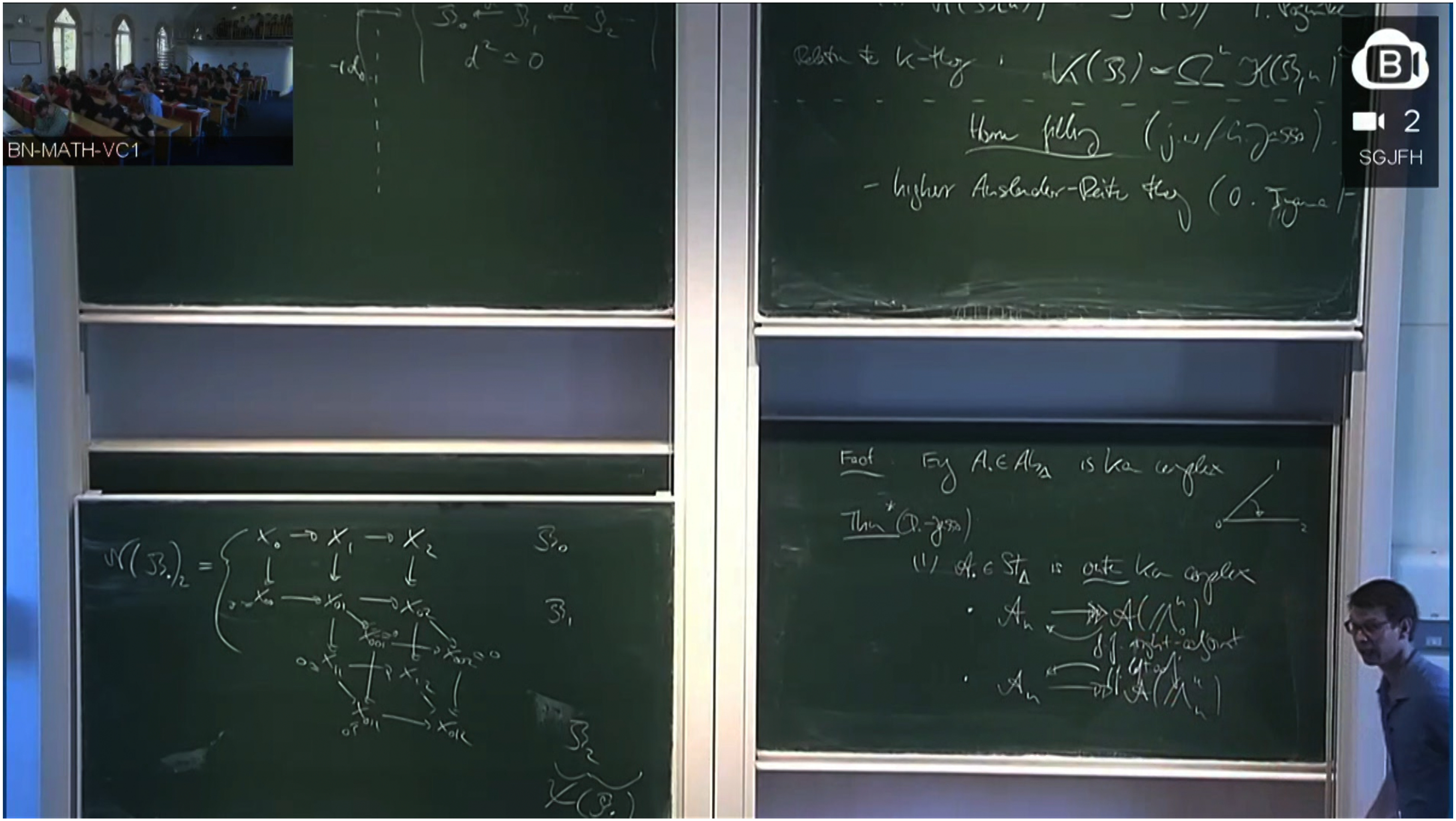
Fact
Thm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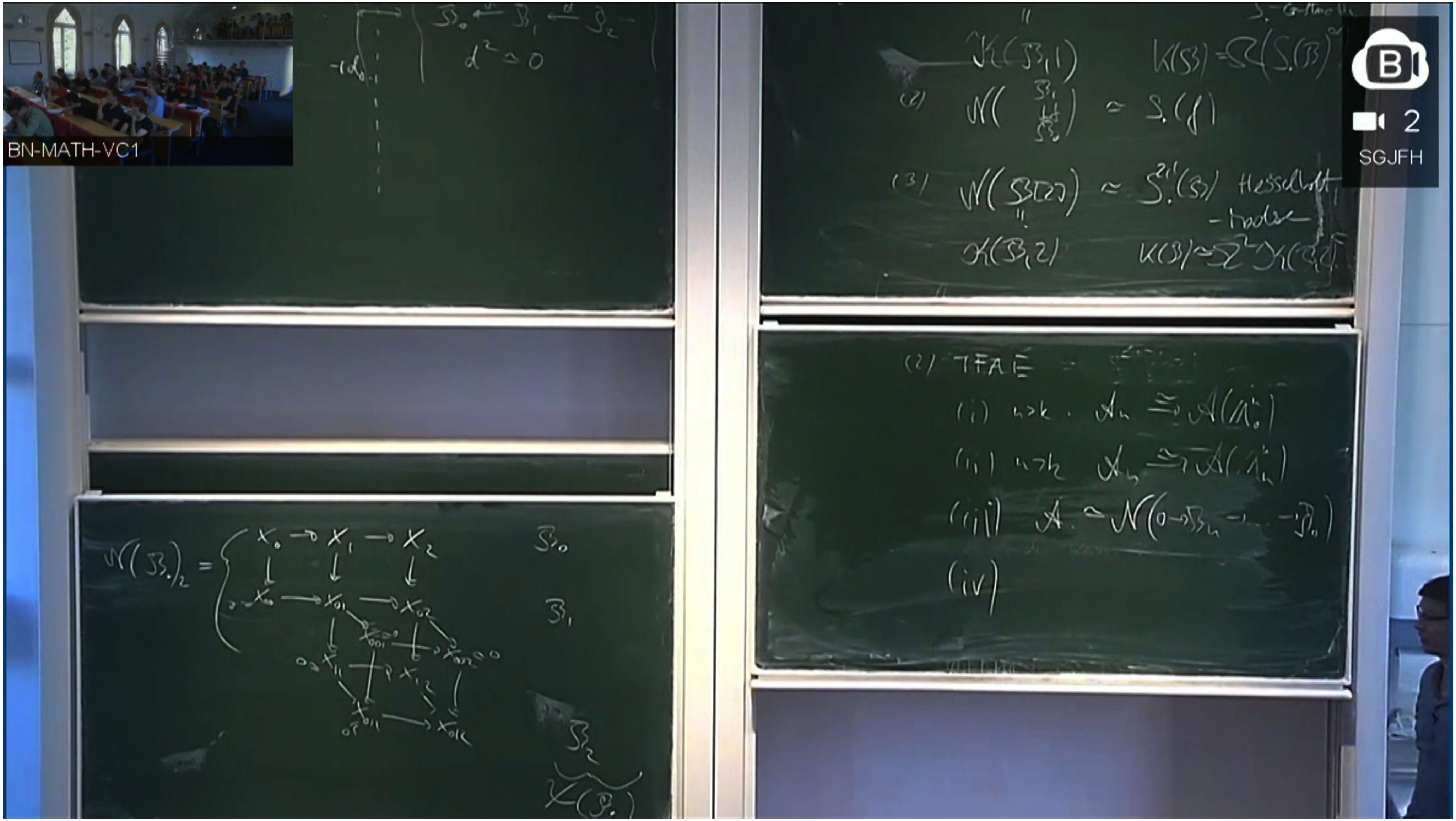
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(2) TFAE
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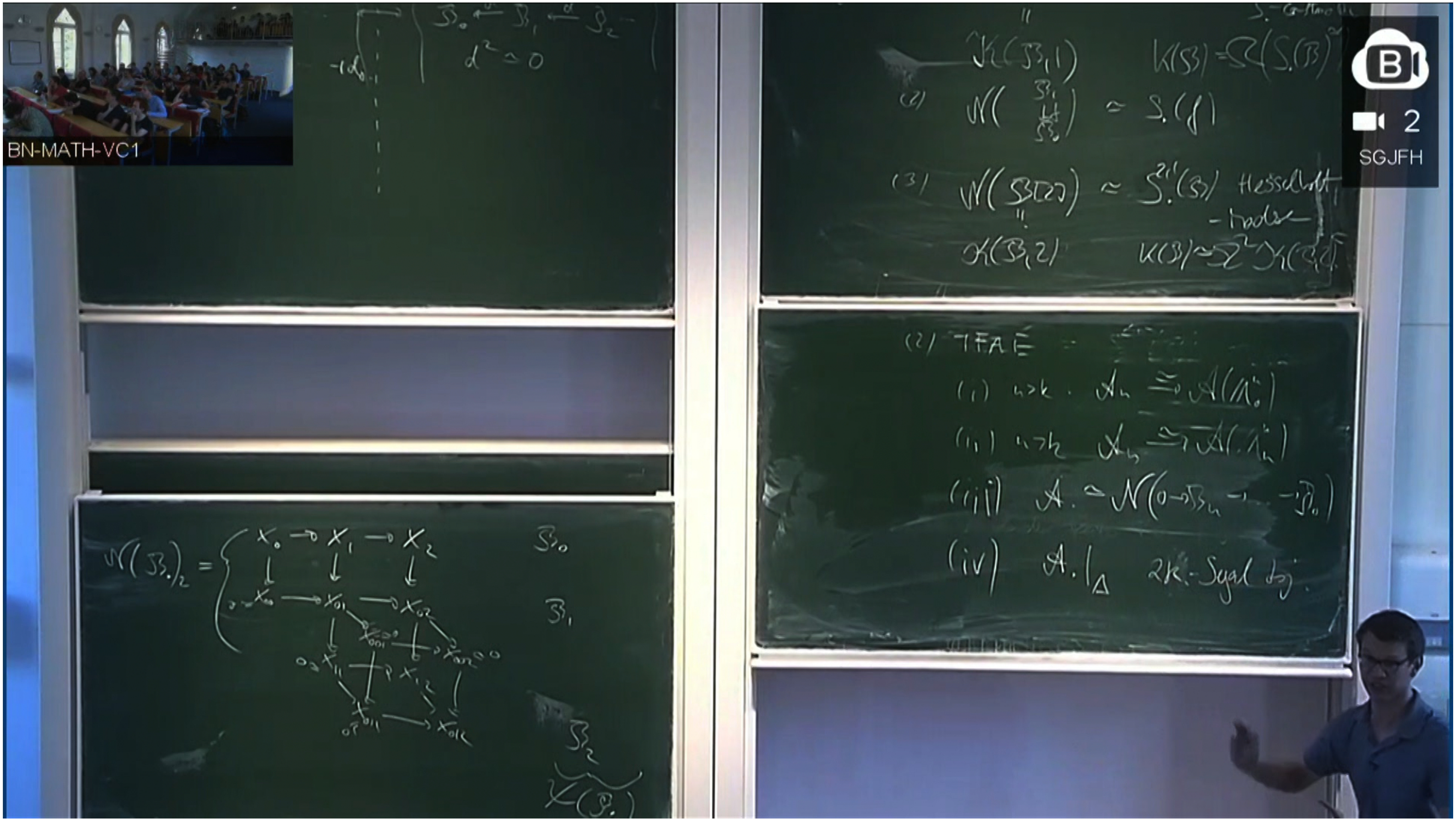
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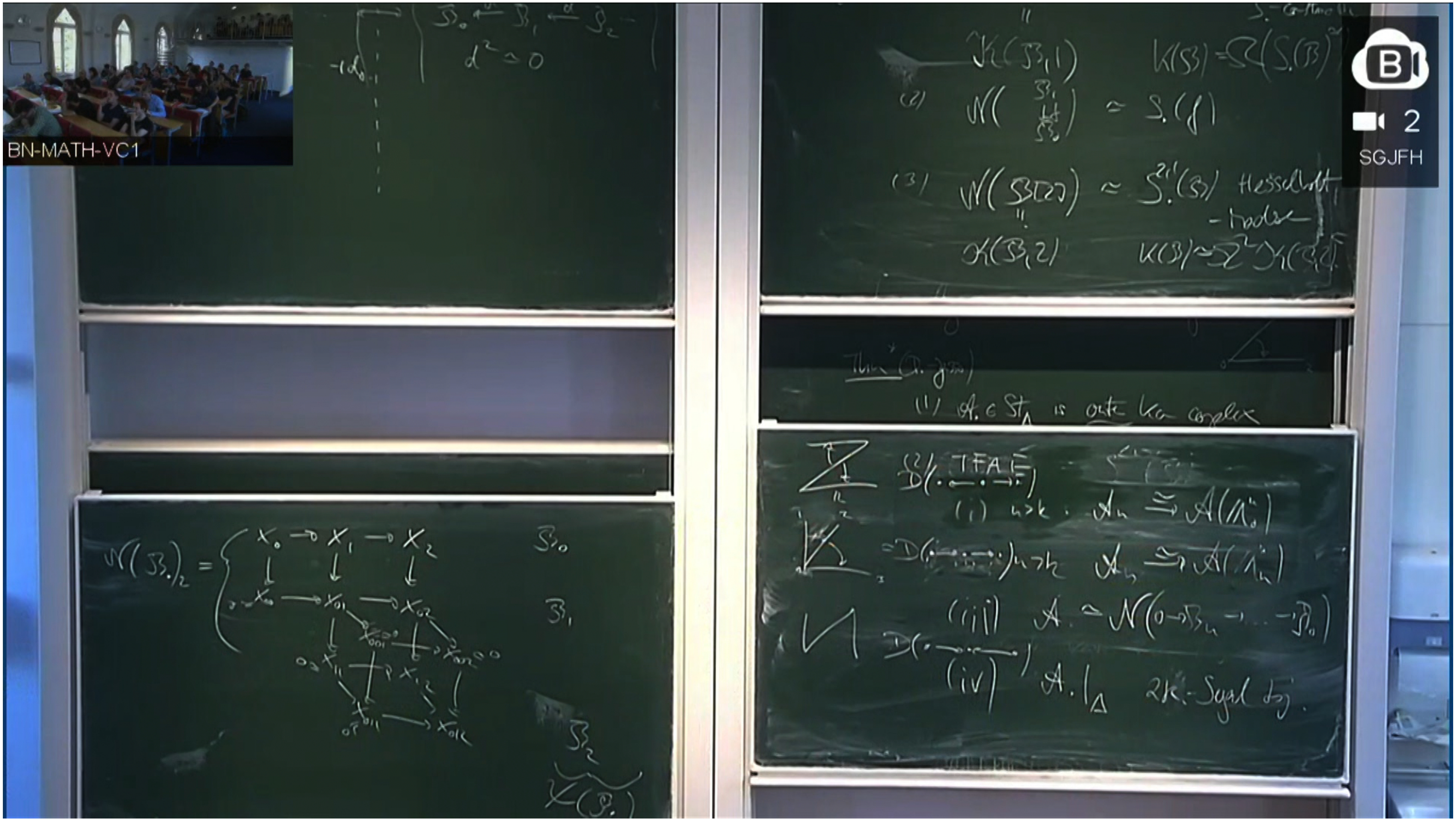
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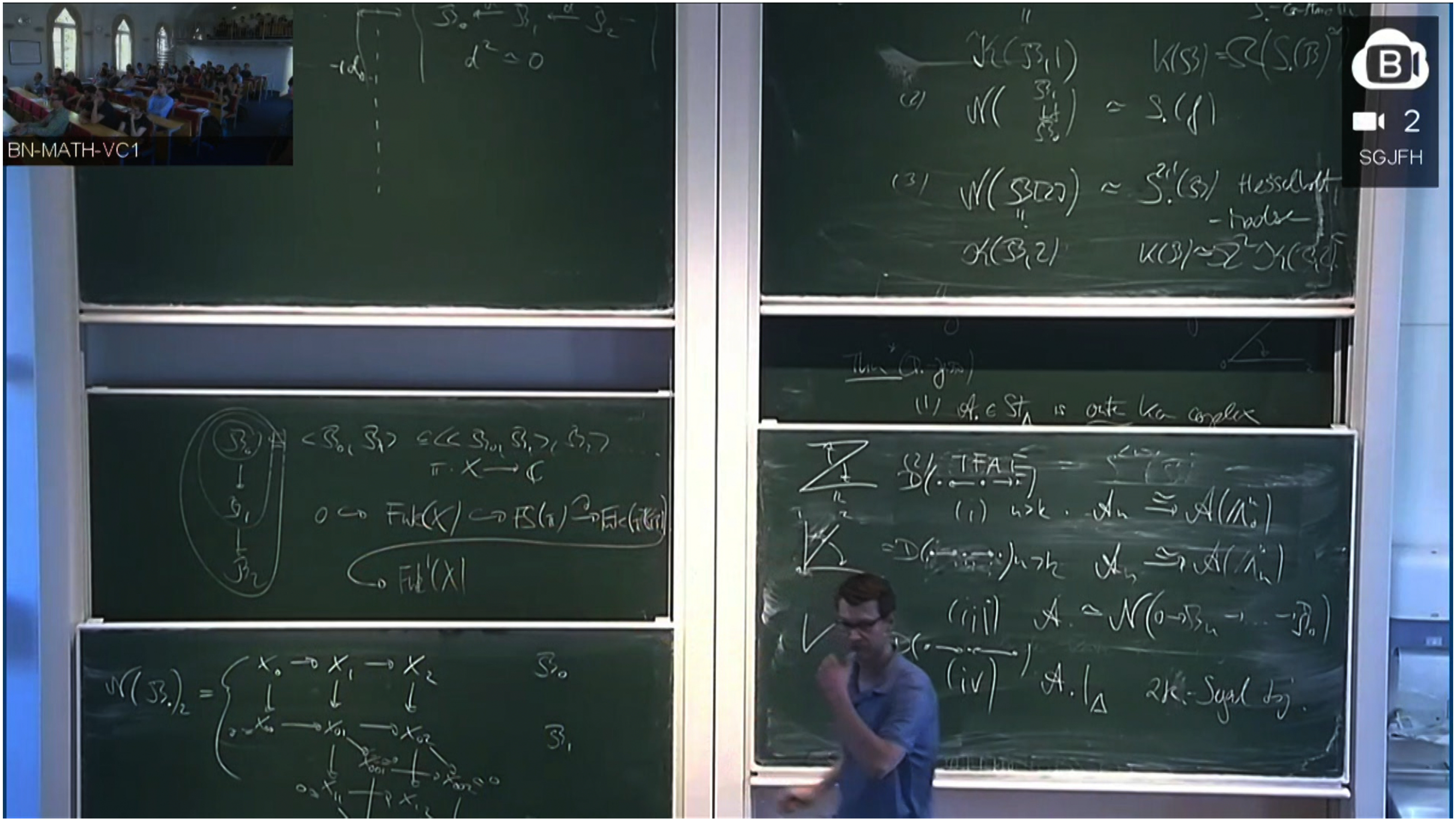
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