Title: Higher operations in supersymmetric ï¬•eld theory

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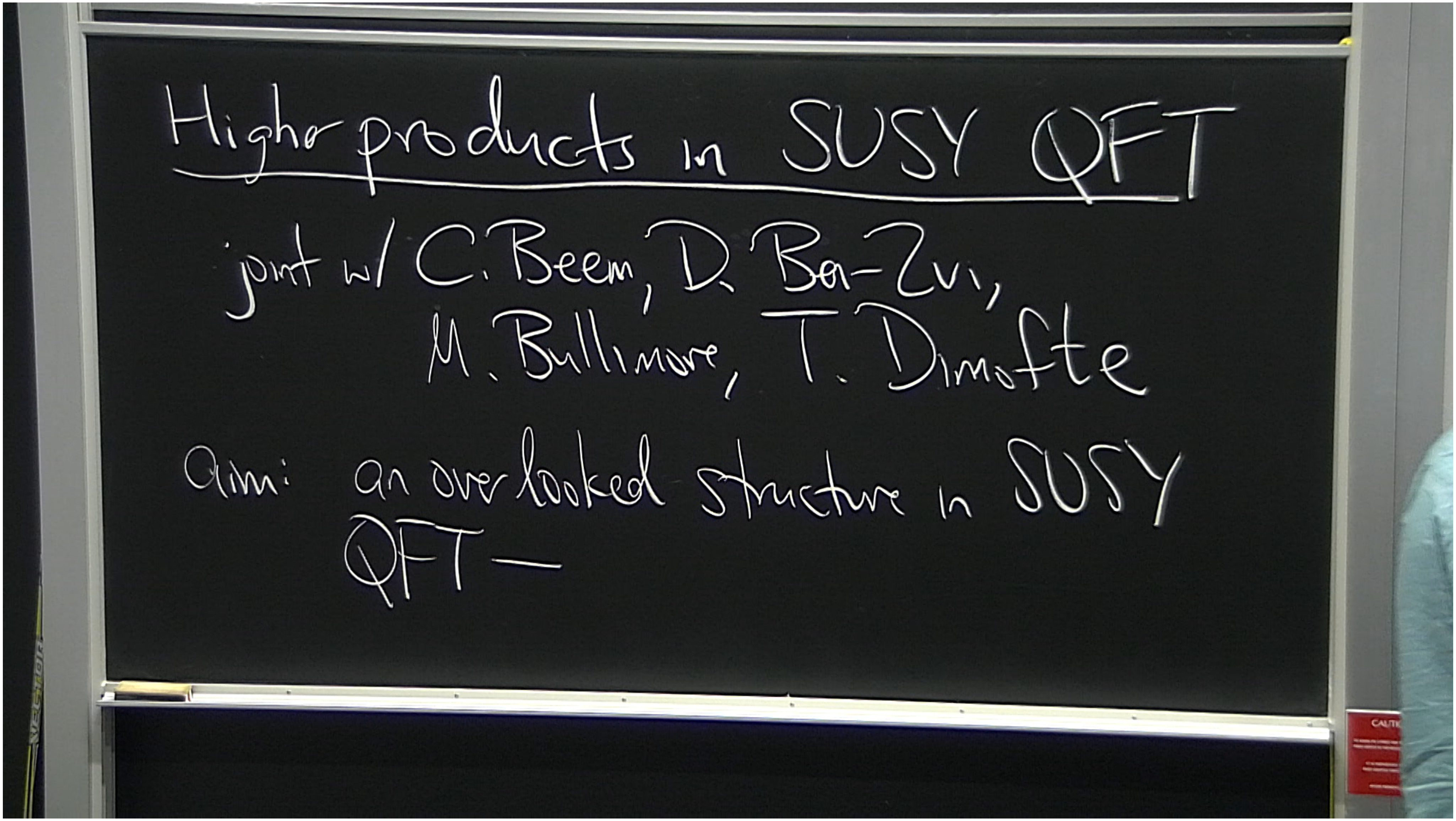
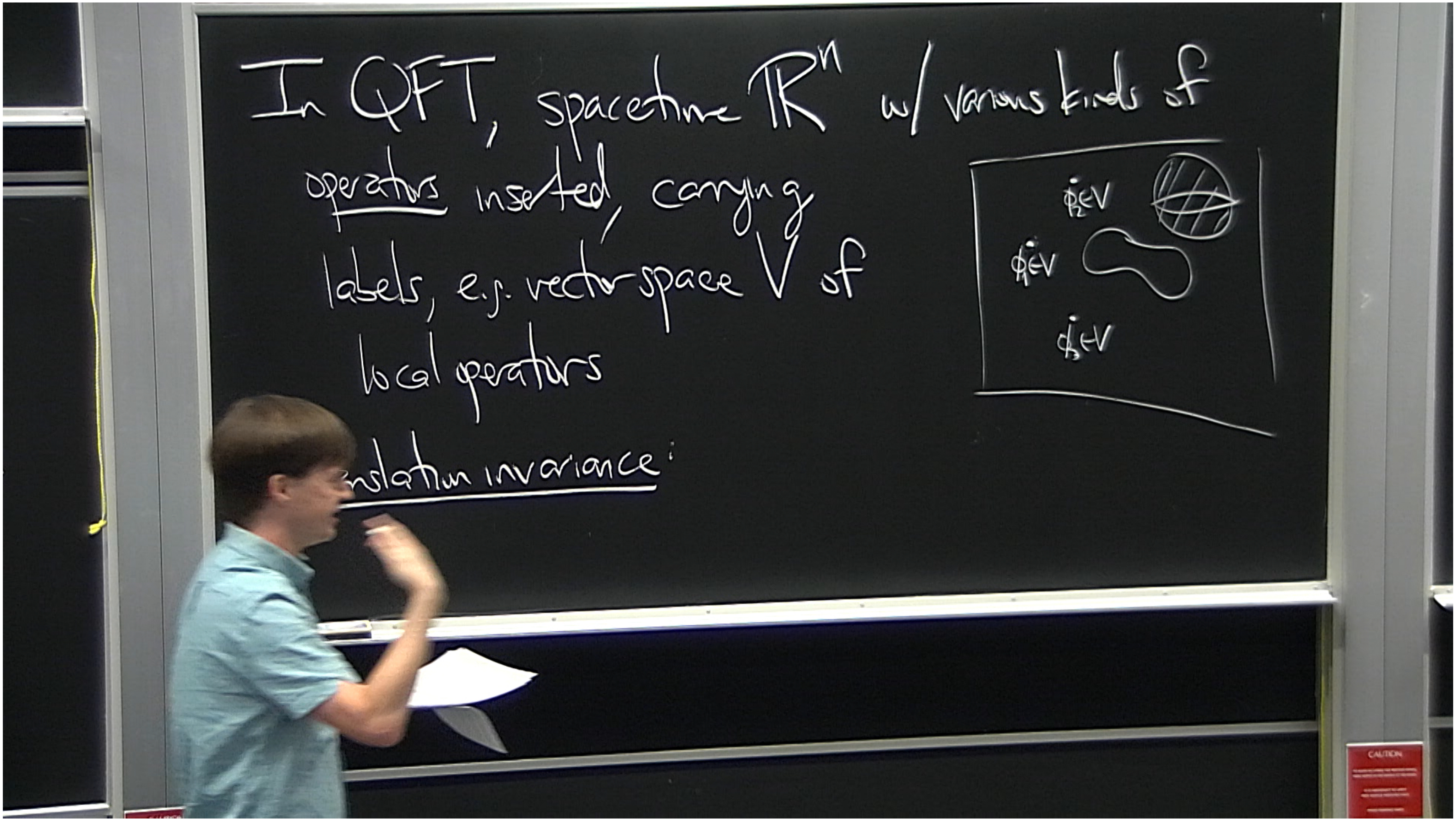
Abstract: I will review the construction of â€œhigher operationsâ€• on local and extended operators in topological ï¬•eld theory, and some applications of this construction in supersymmetric ï¬•eld theory. In particular, the higher operation on supersymmetric local operators in a 3d N=4 theory turns out to be induced by the holomorphic Poisson structure on the moduli space of the theory. This leads to a new way of establishing the non-renormalization properties of this Poisson structure, and also to a simple topological reason for the appearance of its deformation quantization when the theory is placed in Omega-background. This is an account of joint work with Christopher Beem, DavidB en-Zvi, Mathew Bullimore, and Tudor Dimofte.


Higher products in SUSY QFT
joint w/ C. Beem, D. Ben-Zvi,
M. Bullimore, T. Dimofte
aim: an overlooked structure in SUSY
QFT —
CAUTI

practitioners of (modern) TFT.

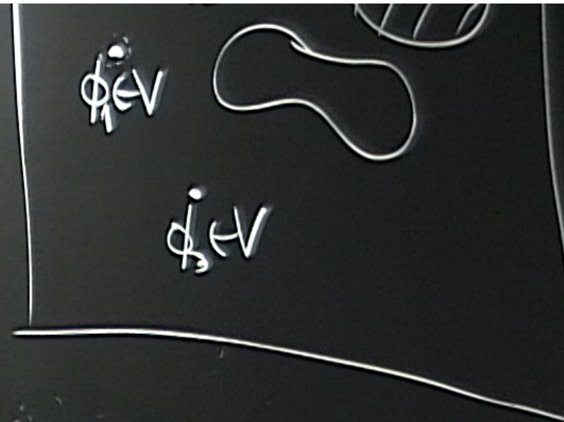
Companion to [Elliott–Safronov] on
E_n-algebra structures in SUSY QFT

I'll focus on homology, not chain level.



In QFT, spacetime $\mathbb{R}^n$ w/ various kinds of
operators inserted, carrying
labels, e.g. vector space V of
local operators
$\phi_2 \in V$
$\phi_1 \in V$
$\phi_3 \in V$
nslation invariance:

labels, e.g. vector space V of local operators



Translation invariance: $V \circlearrowleft \mathbb{R}^n$

by **momentum operators** P_μ $\quad \mu = 1, \dots, n$

$$\frac{\partial}{\partial x^\mu}\langle \phi(x) \cdots \rangle = \langle (P_\mu \phi)(x) \cdots \rangle$$

and P_μ is a <u>symmetry</u>:

$$\langle P_\mu(\cdots)\rangle = 0 \quad \text{e.g.} \quad \langle P_\mu\phi(x_1)\,\phi(x_2)\rangle + \langle \phi(x_1)\, P_\mu\phi(x_2)\rangle = 0$$

Now suppose the theory has ($\mathbb{Z}/_2$-odd) symmetry operators

Q, Q_μ with $Q^2 = 0$

$[Q, Q_\mu] = P_\mu$

(ex: Q is a "topological supercharge" in SUSY field theory)

If all $\underline{Q}\phi_i = 0$

then $\langle (Q\phi(x))\,\phi_1(x_1)\,\phi_2(x_2) \cdots \phi_n(x_n) \rangle$

$$= \langle Q\left(\phi(x)\,\phi_1(x_1) \cdots \phi_n(x_n)\right) \rangle = 0$$

$\Rightarrow \langle \prod \phi_i(x_i) \rangle$ depends only on $[\phi_i] \in H(Q)$!!

If all $\underline{Q}\phi_i = 0$

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$$= \langle Q\big(\phi(x)\,\phi_1(x_1)\cdots\phi_n(x_n)\big)\rangle = 0$$

$\Longrightarrow \langle \prod \phi_i(x_i)\rangle$ depends only on $[\phi_i] \in H^i(Q)$

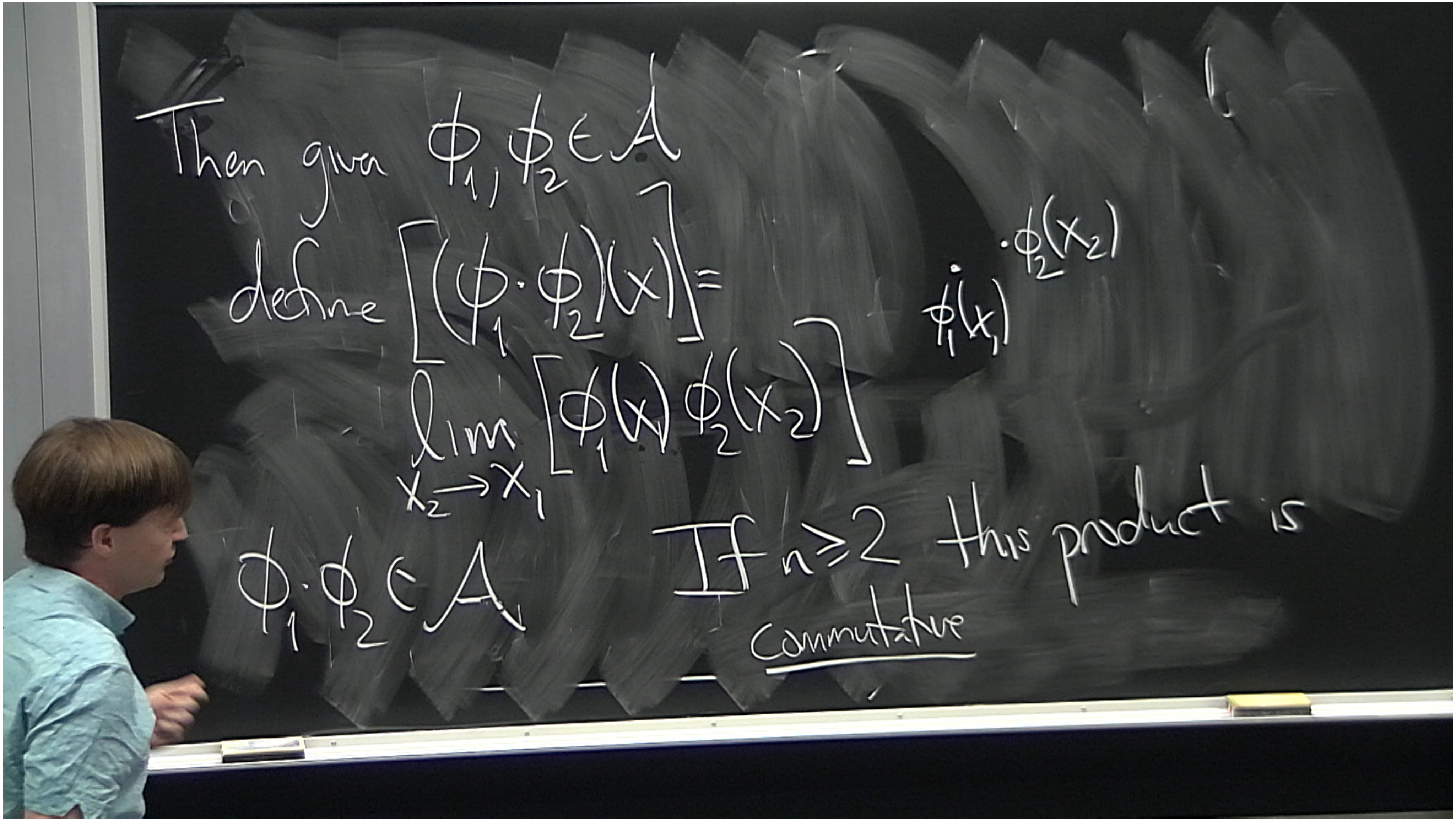
!!

A

$$Q\phi = 0 \Rightarrow P_\mu \phi = [Q, Q_\mu]\phi$$

$$= Q(Q_\mu \phi) \quad Q\text{-exact}$$

$$\Rightarrow \left\langle \prod \phi_i(x_i) \right\rangle \text{ is } \underline{\text{locally constant}} \text{ f}^{\text{n}} \text{ of } \{x_i\}$$

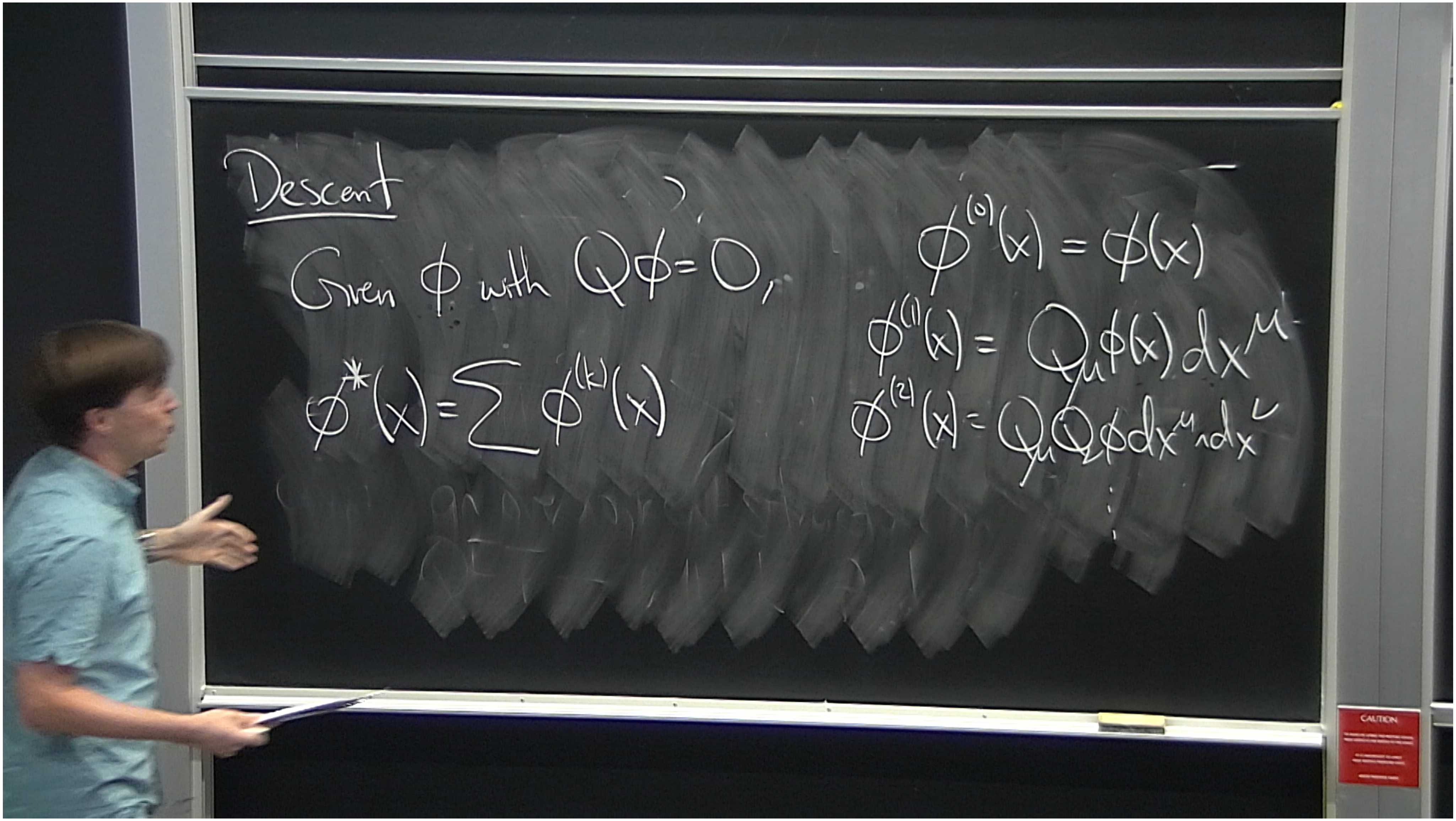

Then given $\phi_1, \phi_2 \in A$
define $[(\phi_1 \cdot \phi_2)(x)] = \lim_{x_2 \to x_1} [\phi_1(x_1)\phi_2(x_2)]$
$\phi_1(x_1) \cdot \phi_2(x_2)$
$\phi_1 \cdot \phi_2 \in A$
If $n \geq 2$ this product is commutative

If $n=1$ define $\phi_1 \cdot \phi_2(x) = \phi_1(x_1) \leftarrow \phi_2(x_2)$

$$\phi_2 \cdot \phi_1(x) = \phi_2(x_2) \leftarrow \phi_1(x_1)$$

This product is always associative —
in dim $n \geq 2$ that's because $\mathcal{C}_3(\text{ball})$ is connected

"topological ring"



Descent
Given ϕ with $Q\phi = 0$,
$\phi^{*}(x) = \sum \phi^{(k)}(x)$
$\phi^{(0)}(x) = \phi(x)$
$\phi^{(1)}(x) = Q_\mu \phi(x)\, dx^\mu$
$\phi^{(2)}(x) = Q_\mu Q_\nu \phi\, dx^\mu \wedge dx^\nu$
$\vdots$
CAUTION

Given ϕ with $Q\phi = 0$

$$\phi^{(0)}(x) = \phi(x)$$

$$\phi^{(1)}(x) = Q_\mu \phi(x)\, dx^\mu$$

$$\phi^{(2)}(x) = Q_\mu Q_\nu \phi\, dx^\mu \wedge dx^\nu$$

$$\vdots$$

$$\phi^{*}(x) = \sum \phi^{(k)}(x)$$

$$d\phi^{(0)}(x) = P_\mu \phi(x)\, dx^\mu = Q Q_\mu \phi(x)\, dx^\mu$$

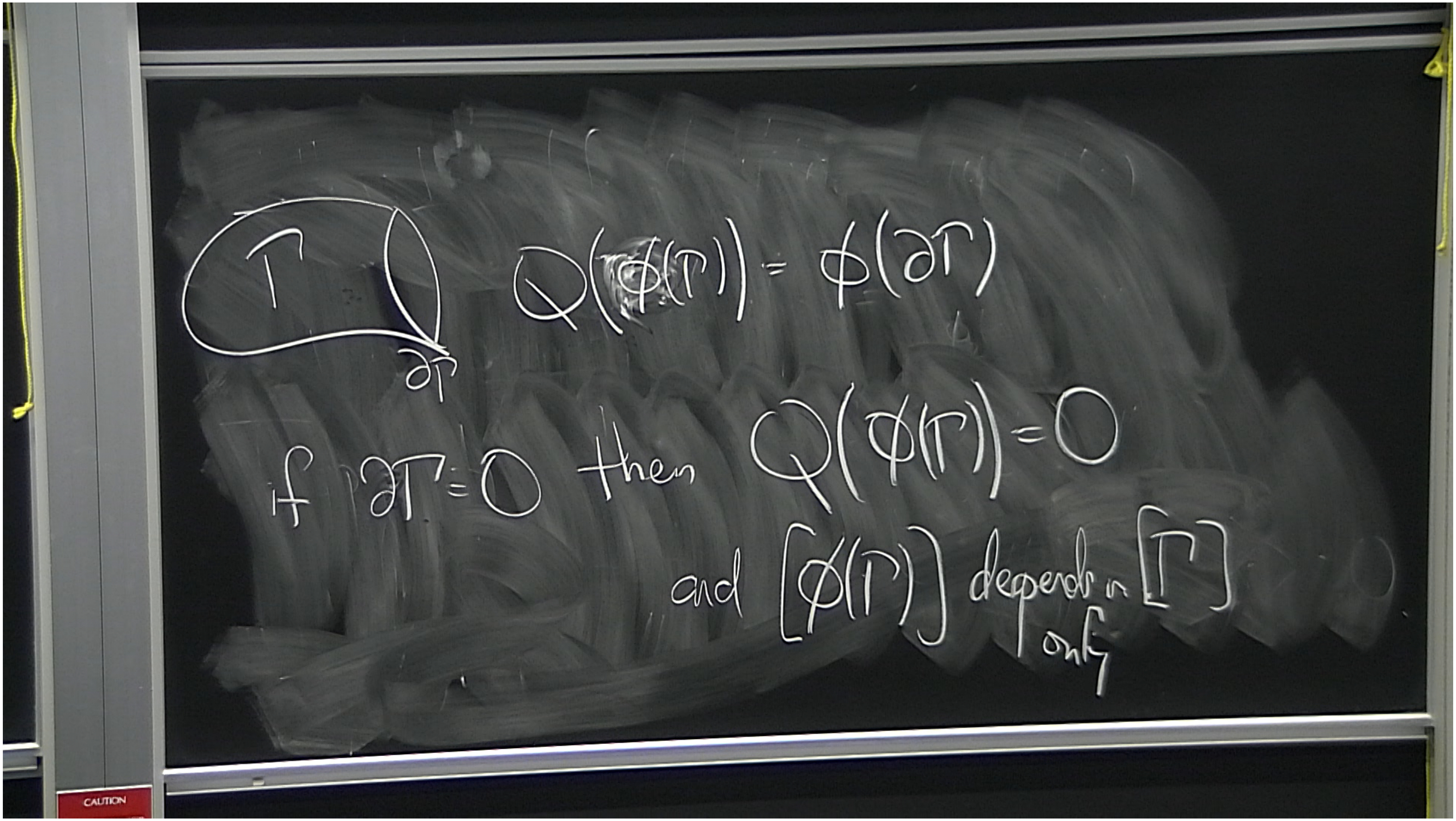
For any k-chain $\Gamma \subset \mathbb{R}^n$

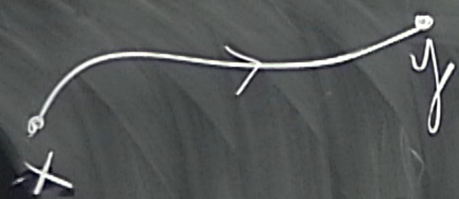
define $\phi(\Gamma) = \int_\Gamma \phi^{(k)}$

extended operator

$$d\phi^{(0)} = Q\phi^{(1)}$$
$$d\phi^{(1)} = Q\phi^{(2)}$$
$$d\phi^{*} = Q\phi^{*}$$

$$Q\phi(\Gamma) = \int_\Gamma Q\phi^{(*)} = \int_\Gamma d\phi^{(*)} = \int_{\partial\Gamma} \phi^{(*)} = \phi(\partial\Gamma)$$


Γ
∂Γ
Q(φ(Γ)) = φ(∂Γ)
if ∂Γ = 0 then Q(φ(Γ)) = 0
and [φ(Γ)] depends on [Γ] only



$[\phi(x)] = [\phi(y)]$

What to do with $\phi(\Gamma)$?

One application: [Witten]

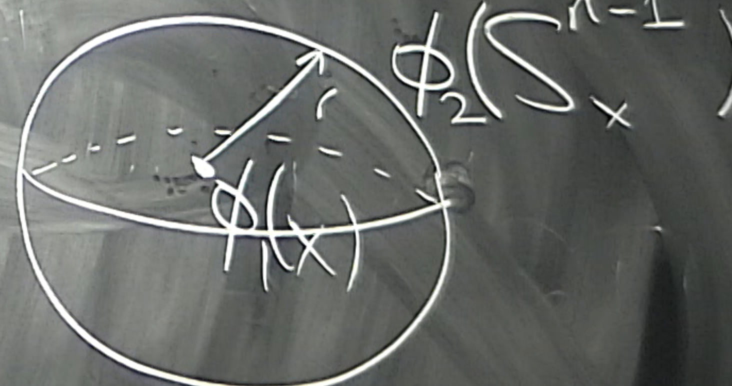
build twisted version of SUSY theory on n-manifold X

then insert $\phi(\Gamma)$ on nontriv. cycle in X.

Secondary product

Say $Q\phi_1 = Q\phi_2 = 0$

$$\{\phi_1, \phi_2\}(x) = \lim_{r \to 0} \phi_1(x)\, \phi_2(S_x^{n-1})$$



$$A \otimes A \longrightarrow A$$

Then $\{\cdot,\cdot\}$ gives a (degree $n-1$) Poisson bracket on A.

$\mathbb{R}^n$

To prove $\{\cdot,\cdot\}$ is a Poisson bracket:

(Skew) Symmetry: extend $\phi_1(x_1)\phi_2(x_2)$ to form-valued op $(\phi_1\phi_2)^*$ on $\mathcal{C}_2(\text{ball})$

$\Longrightarrow$ every class $\alpha \in H_*(\mathcal{C}_2(\text{ball}))$

gives a map $*_\alpha : A \otimes A \longrightarrow A$

and $H_{n-1}(\mathcal{C}_2(\text{ball})) \simeq \mathbb{Z}$

$\mathcal{C}_2(\text{ball}) \simeq S^{n-1}$

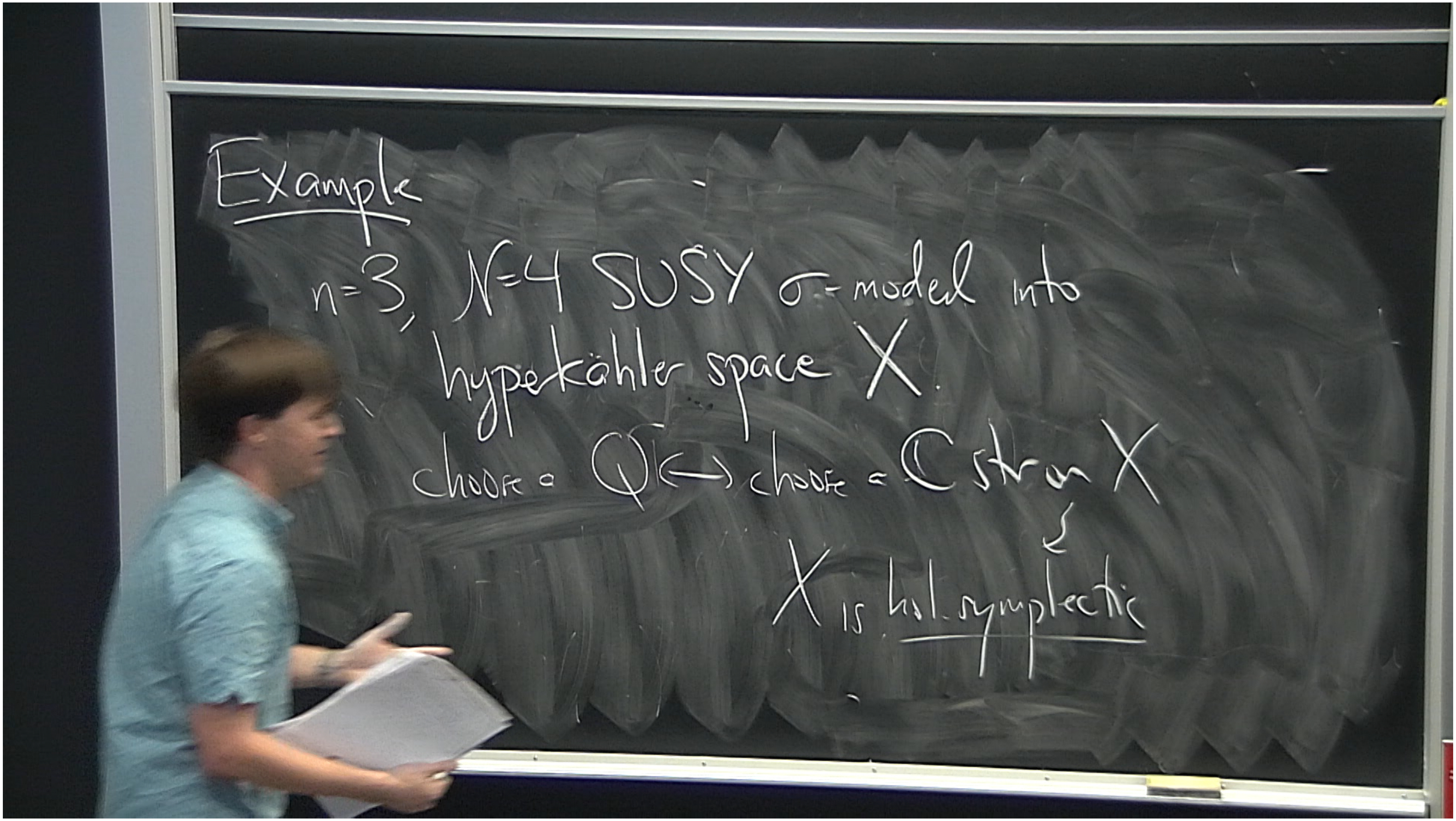
Jacobi comes from rel'n in $H_{2n-2}(\mathcal{C}_3(\text{ball}))$

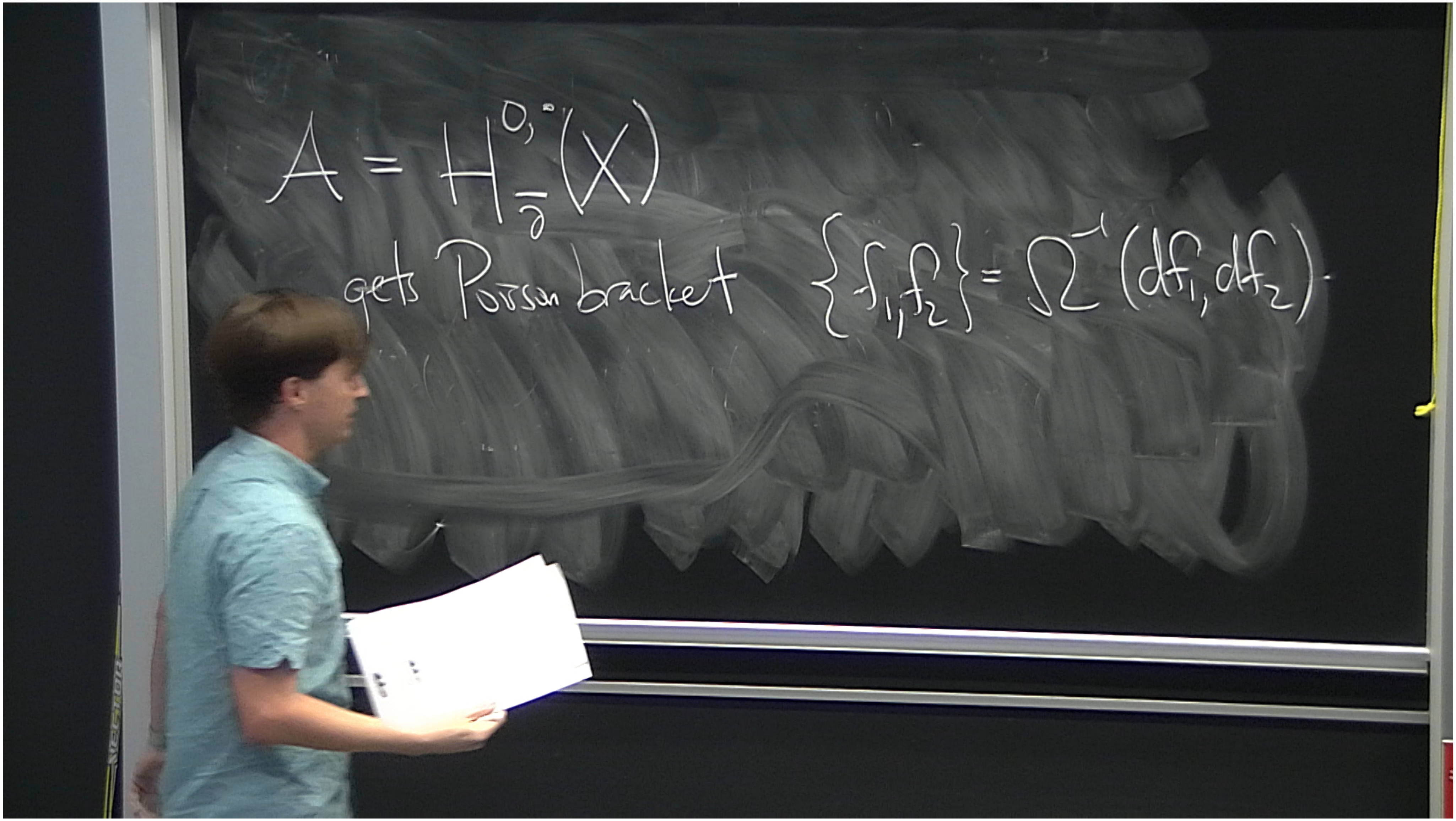
Rk All operations we get in this way are built from $\cdot$ and $\{ , \}$

Thm the homology of the little n-discs operad is the degree $n-1$ Poisson operad

(Cohen)

(Dev Sinha)


Example
n=3, N=4 SUSY σ-model into
hyperkähler space X.
choose a Q ⟷ choose a ℂ str on X
X is hol. symplectic

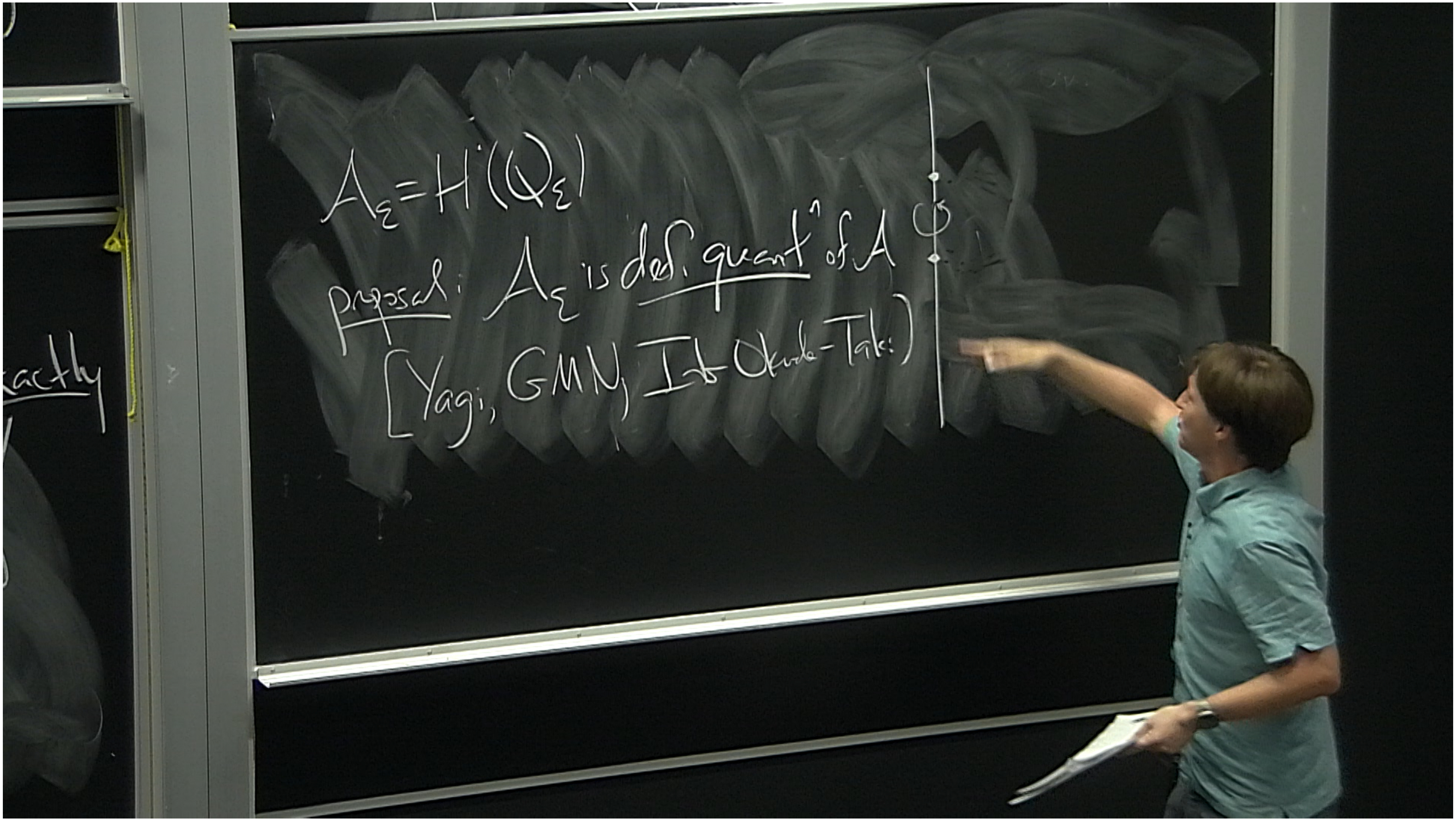

$A = H^{0,\bullet}_{\bar\partial}(X)$
gets Poisson bracket
$\{f_1, f_2\} = \Omega^{-1}(df_1, df_2)$

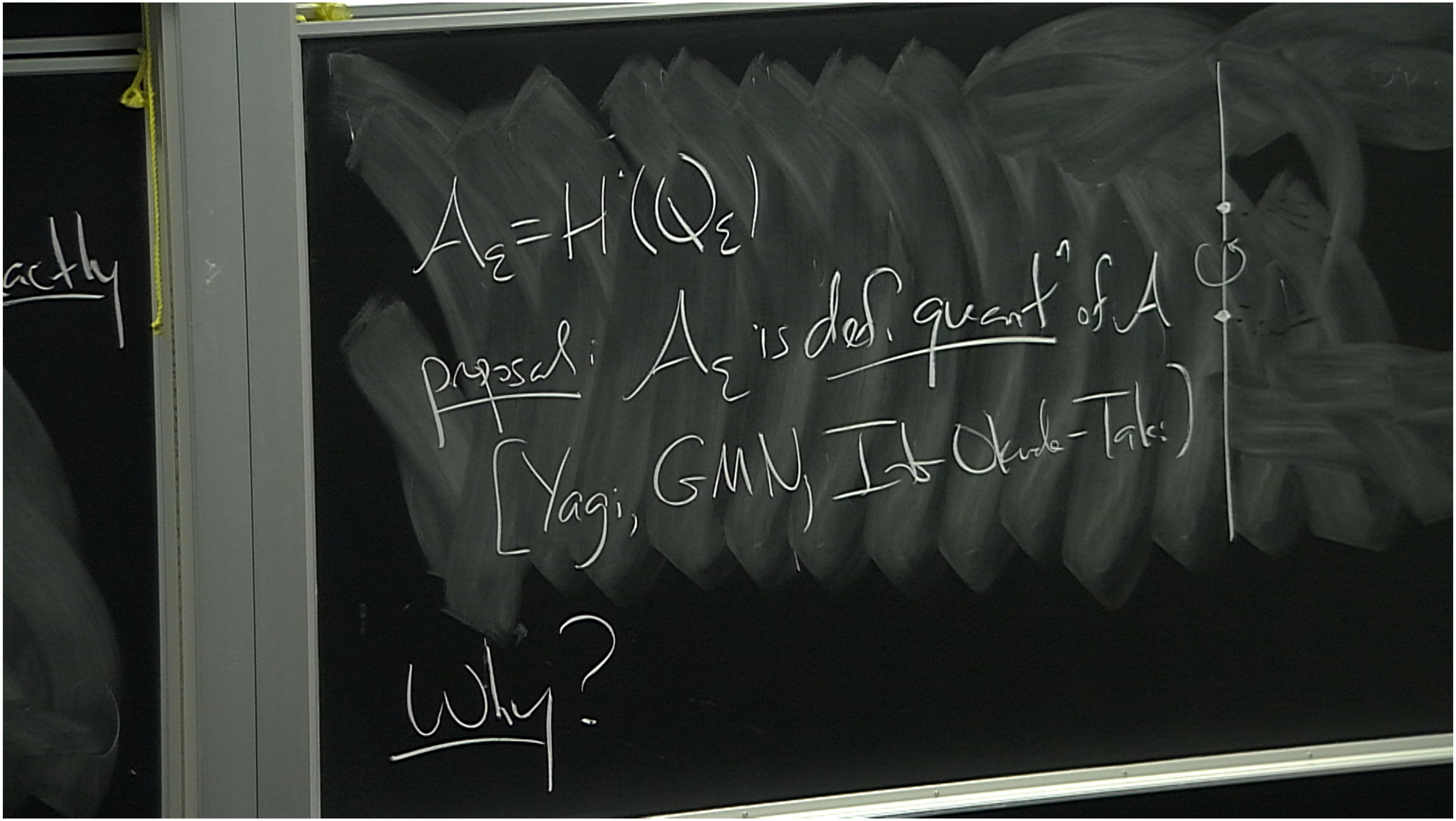
Applications

a) Non-renormalization: $\{\,,\}$ can be computed exactly either in the IR or the UV

[Bullimore-Dimofte-Gaiotto, Gaiotto-Moore-N]

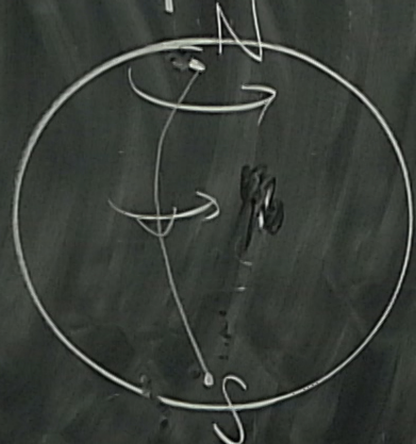
b) Ω-background


A_ε = H^·(Q_ε)
proposal: A_ε is def. quant^n of A
[Yagi, GMN, Ito-Okuda-Taki]


actly
A_ε = H'(Q_ε)
proposal: A_ε is def. quant'n of A
[Yagi, GMN, Ito-Okuda-Taki]
Why?

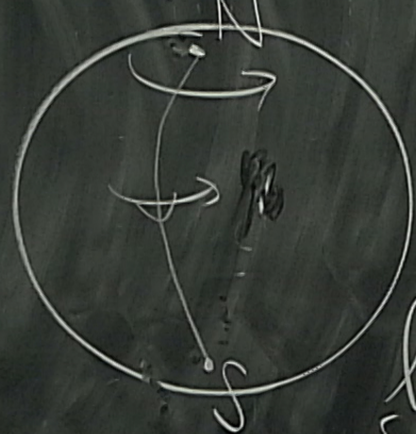
Operations on A_{ε} come from $\underline{U(1)\text{-equiv}}$ homology of $\mathcal{C}_n(\text{ball})$

$\mathcal{C}_2(\text{ball}) \sim$



$[N] - [S] = \varepsilon[S^2]$

Operations on A_ε come from $U(1)$-equiv homology of $\mathcal{C}_n(\text{ball})$

$\mathcal{C}_2(\text{ball}) \sim$ 

$$[N]_0 - [S]_0 = \varepsilon [S^2]_2$$

$$\lim_{\varepsilon \to 0} \frac{\phi_1 \phi_2 - \phi_2 \phi_1}{\varepsilon} = \{\phi_1, \phi_2\}$$