Title: Higher length-twist coordinates and applications - effective superpotentials from the geometry of opers

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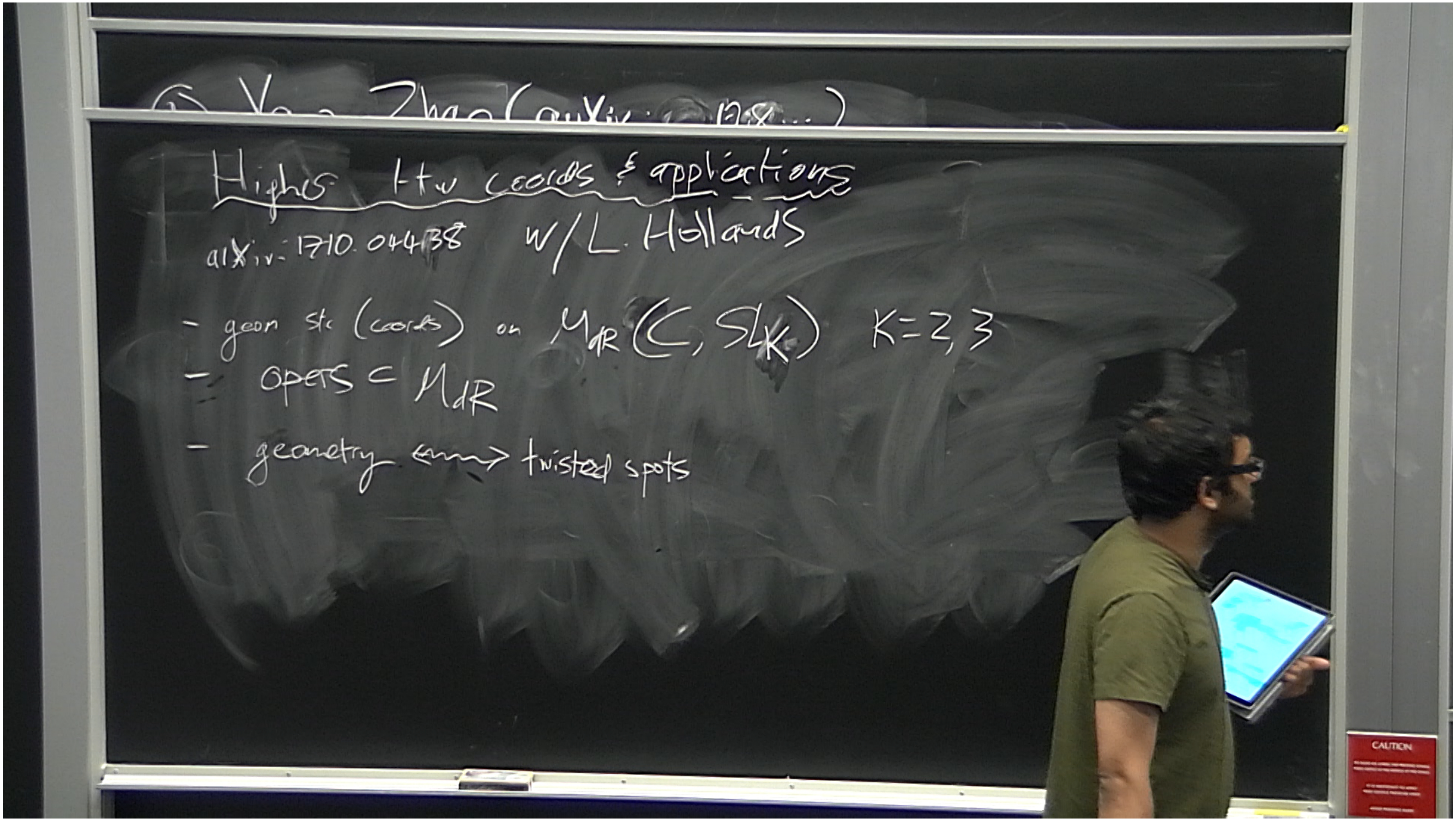
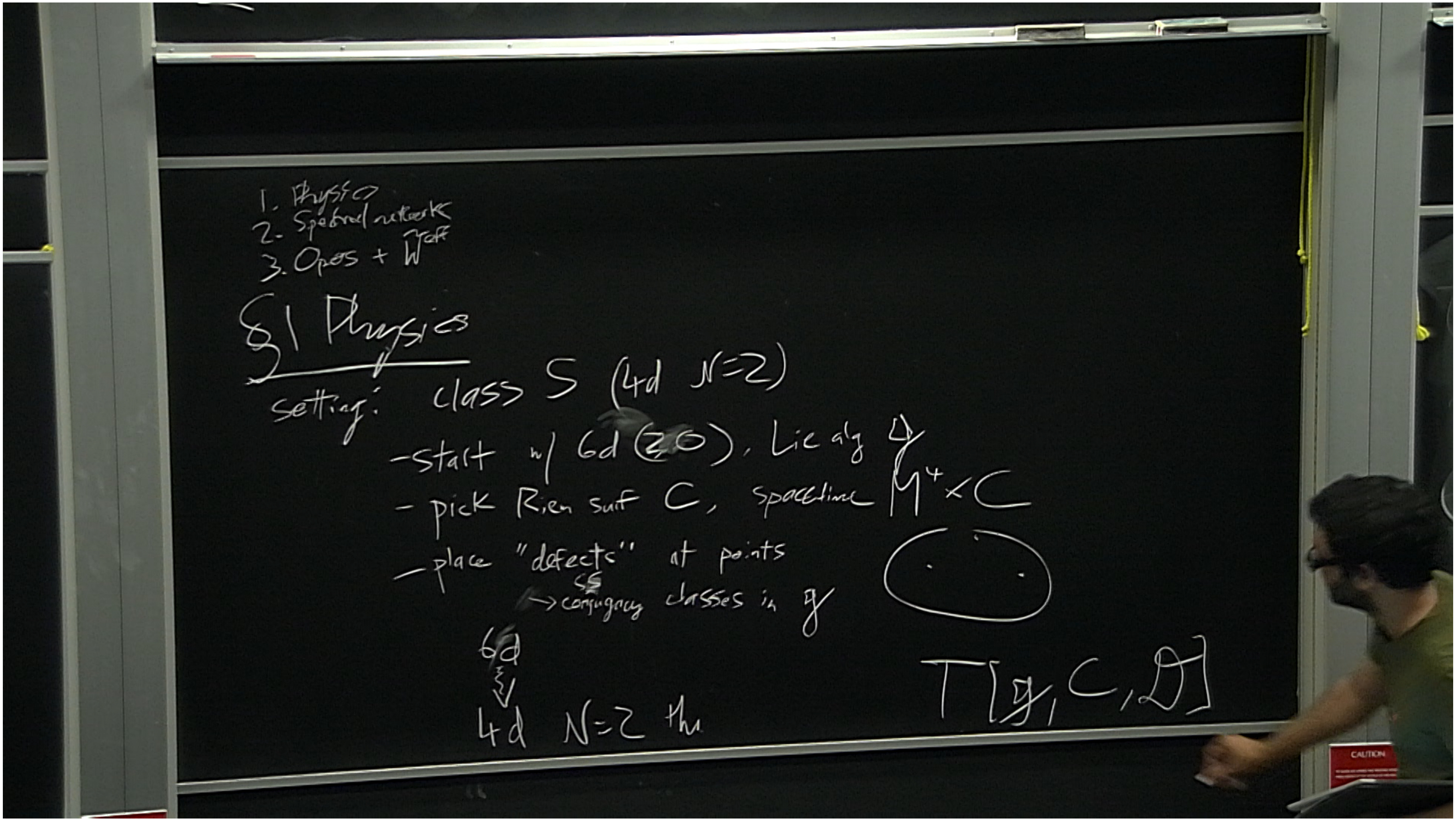
Abstract: We describe joint work with L. Hollands on the geometry of the moduli space of flat connections over a Riemann surface. On the one hand, we generalize and compute certain "complexified Fenchel-Nielsen" coordinates for SL(2)-connections to higher rank using the spectral network "abelianization" approach of Gaiotto-Moore-Neitzke. We then use these coordinates to compute superpotentials, following a conjecture of Nekrasov-Rosly-Shatashvili which roughly states the following: a certain low energy effective twisted superpotential arising from compactifying a theory of class S is equal to the generating function (in the sense of symplectic geometry), in some special coordinates, of the Lagrangian submanifold of opers in the associated moduli space of flat connections.


Higher Htw coords & applications
arXiv: 1710.04438 w/ L. Hollands
- geom str (coords) on $\mathcal{M}_{dR}(C, SL_K)$ K=2,3
- opers ⊂ $\mathcal{M}_{dR}$
- geometry ⟷ twisted spots
CAUTION


1. Physics
2. Spectral networks
3. Opers + W^{aff}
§1 Physics
setting: class S (4d $\mathcal{N}=2$)
- start w/ 6d (2,0), Lie alg $\mathfrak{g}$
- pick Riem surf C, spacetime $M^4 \times C$
- place "defects" at points
→ conjugacy classes in $\mathfrak{g}$
6d
4d $\mathcal{N}=2$ thy
$T[\mathfrak{g}, C, D]$

(2) Why?: concrete geometric str. $\mathcal{M}_H(C, SL_K)$

SW system $\longleftrightarrow$ $\mathcal{M}_H$

SW curve $\longleftrightarrow$ spectral curve

Coulomb branch $\longleftrightarrow$ $B = \bigoplus_{i=2}^{K} H^0(C, K_C^{\otimes i}(D))$

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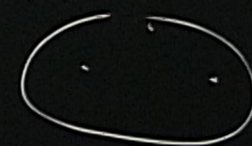


$$T[\underset{=A_{K-1}}{\mathfrak{g}}, C, D]$$

6d
$\rightsquigarrow$
4d $\mathcal{N}=2$ thy

Related: subject to Ω-def. on M^4 $(U(1)\times U(1))$

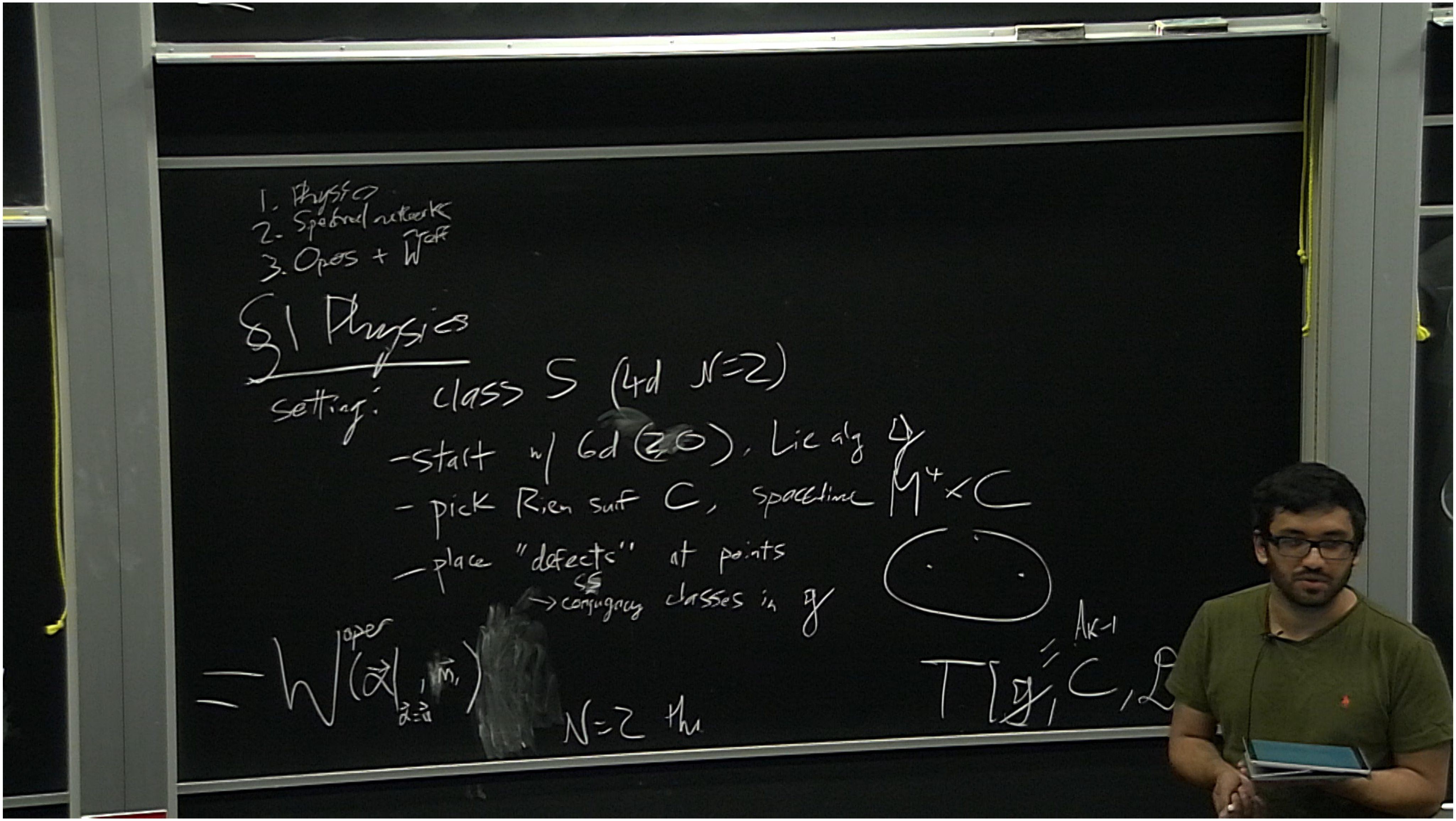
$$M^4 = (\text{cigar}_{\varepsilon}) \times S^1 \times \mathbb{R} \qquad \varepsilon$$

Then reduce to 2d, pass to low energy.

$\longrightarrow$ characterised by $\widetilde{W}^{\text{eff}}(\vec{a},\vec{m},\varepsilon)$

Nekrasov-Rosly-Shatashvili: Conj: $\exists$ some holom. Darboux $\{\alpha_i,\beta_i\}=1$ on $M^{e}_{dR}(C, SL_K)$ s.t. $\widetilde{W}^{\text{eff}}(\vec{a},\vec{m},\varepsilon|_{\varepsilon=1}) =$

$e = \exp(2\pi i \vec{m})$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§1 Physics
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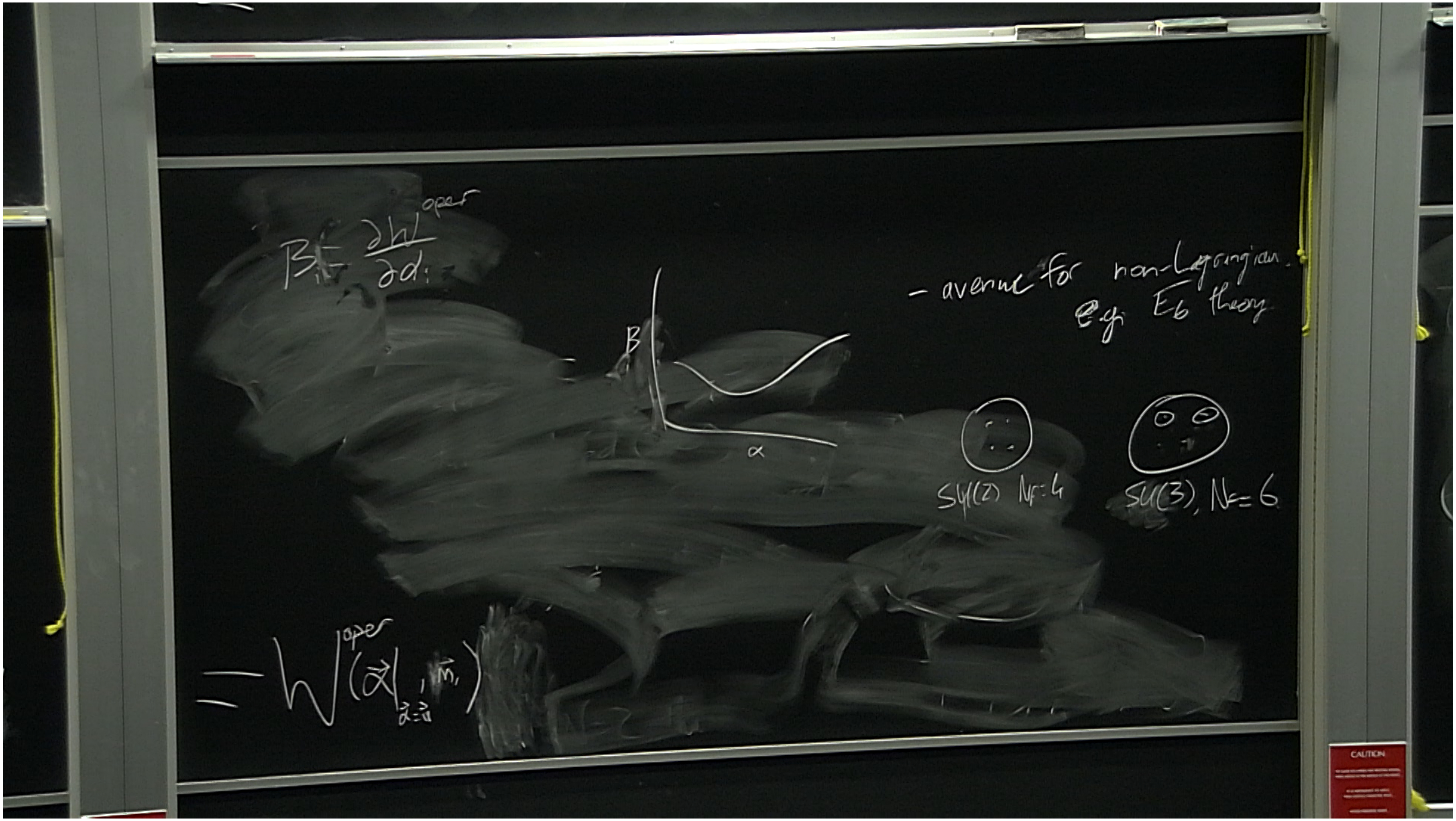
= W oper
N=2 th
T[g, C, D]
AK-1


B_i = ∂W^oper / ∂a_i
- avenue for non-Lagrangian
e.g. E_6 theory
B
α
SU(2) N_f=4
SU(3), N_f=6
= W^oper(α, m)

§ Spectral networks & abelianization

- Gaiotto-Moore-Neitzke, Berk-Nevins-Roberts, Aoki-Kawai-Takei "Stokes graph"
- "WKB spectral network":
  - pick $(\varphi_2, \dots \varphi_K) \in \mathrm{Hitch}(T)$, angle ϑ

$$\Sigma = \{\lambda^K + \lambda^{K-2}\varphi_2 + \dots \varphi_K = 0\} \subset T^*C$$

branched covering

§ Spectral networks

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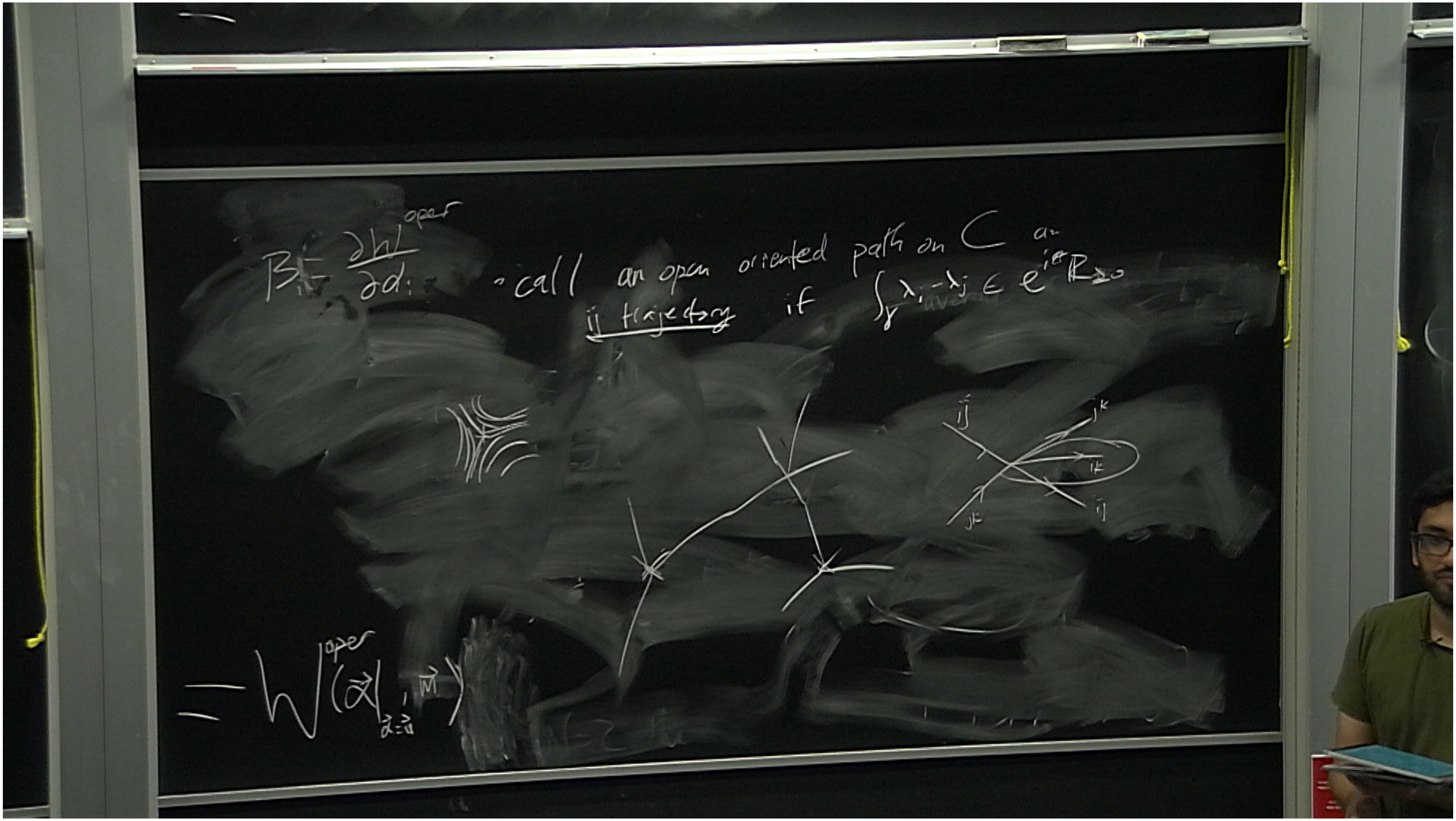
branched covering

· canonically produce a "SN"


B_i = ∂W^oper / ∂α_i
call an open oriented path on C an
ij trajectory if ∫_γ λ_i − λ_j ∈ e^{iθ} ℝ_{≥0}
ij
jk
ik
ij
jk
= W^oper(α, m)

Abelianization $\quad SL_K\ (\mathcal{E},\nabla) \longleftrightarrow GL_1\ (\mathcal{L}',\nabla^{ab})$

$\mathcal{E} \downarrow C \qquad \mathcal{L}' \downarrow \Sigma' = \Sigma \setminus \mathrm{ram}(\pi)$

a W-pair is $(\mathcal{E},\nabla,\iota,\mathcal{L}',\nabla^{ab})$

tendle iso $\iota : \mathcal{E}|_{C\setminus W} \longrightarrow \pi_*\mathcal{L}'|_{C\setminus W}$

s.t. a) ι takes ∇ to $\pi_*\nabla^{ab}$ $\qquad e_w : \mathcal{L}_i \to \mathcal{L}_j$

b) jumps at ij wall by $S_w = 1 + e_w \in \mathrm{End}(\pi_*\mathcal{L})$,

$e = \exp(2\pi i m)$ $\qquad$ on $\mathcal{M}^{e}_{dR}(C, SL_K)$ s.t. $\widetilde{W}^{eff}(\vec{a},\vec{m},\varepsilon|_{\varepsilon=1})$

§ Spectral networks

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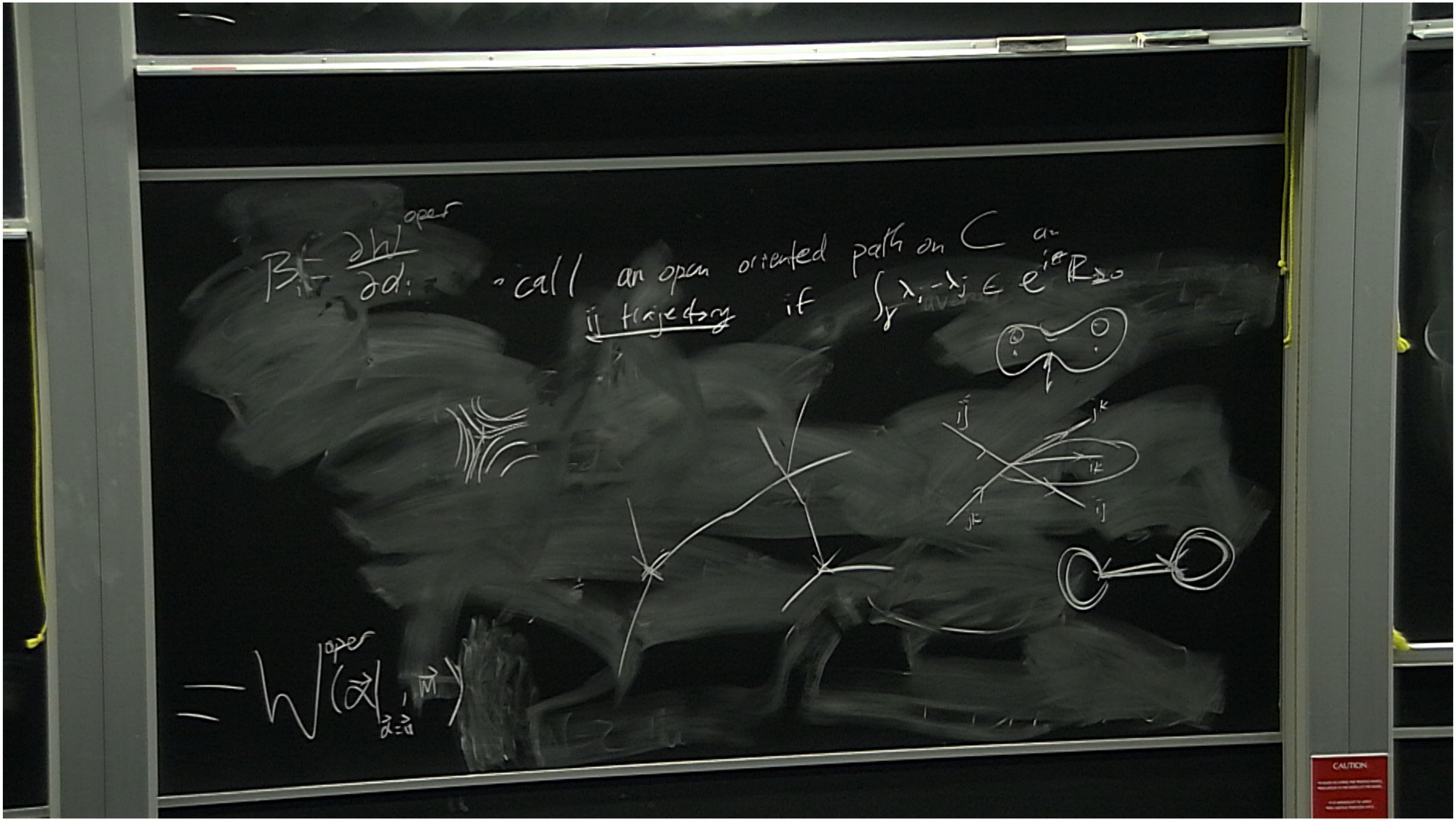
branched covering

· canonically produce a SN.

$$\nabla \mapsto \nabla^{ab} \qquad \mathrm{Hol}_\gamma(\nabla^{ab}) \in \mathbb{C}^*, \ \gamma \in H_1(\Sigma', \mathbb{Z})$$

use as coords.



$B_i = \frac{\partial W^{oper}}{\partial \alpha_i}$
call an open oriented path on C an ij trajectory if $\int_\gamma \lambda_i - \lambda_j \in e^{i\theta}\mathbb{R}_{\geq 0}$
ij
jk
ik
ij
jk
$= W^{oper}(\vec{\alpha}, \vec{m})|_{\vec{\alpha}=\vec{a}}$