Title: G-equivariant factorization algebras

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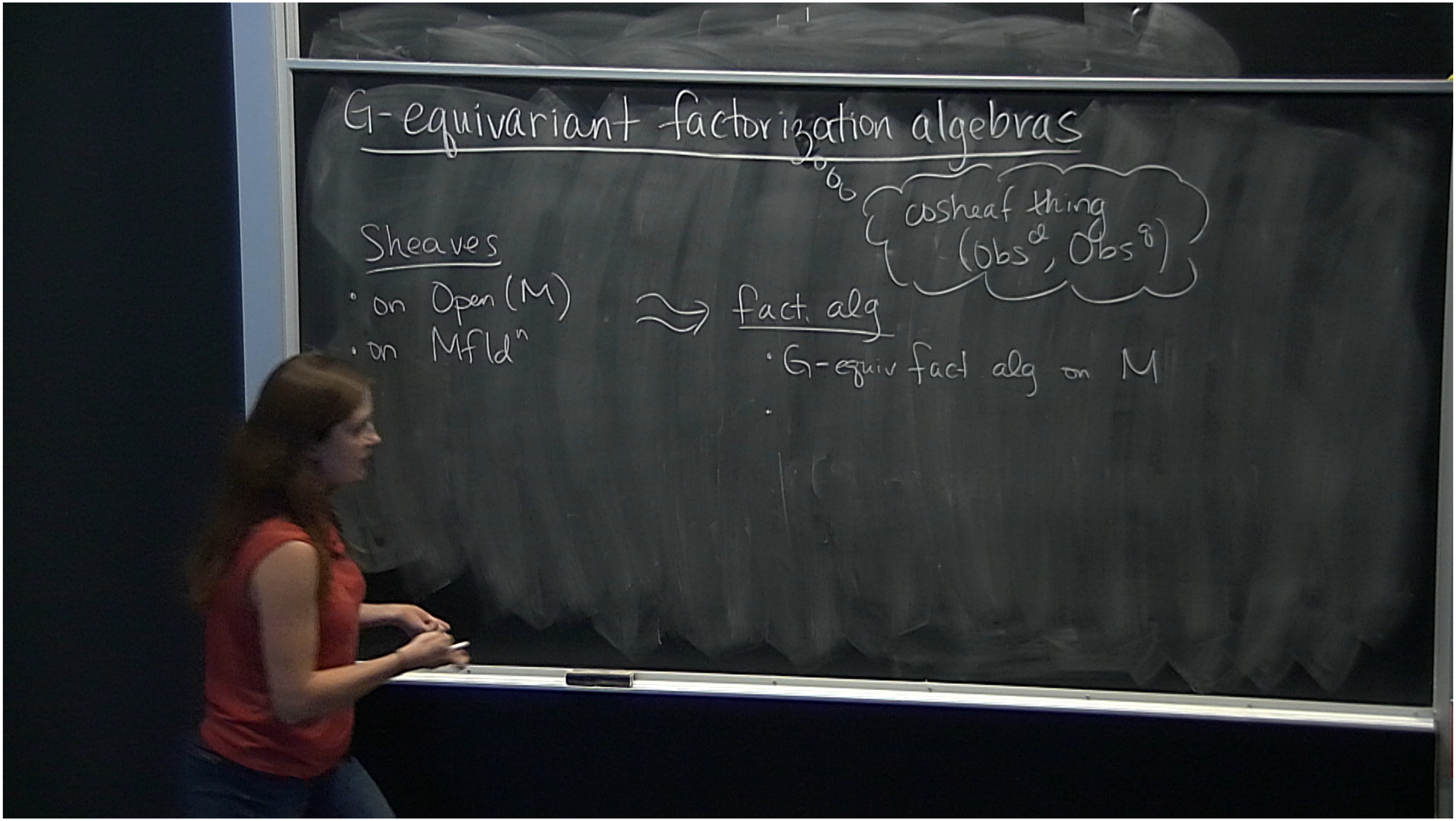
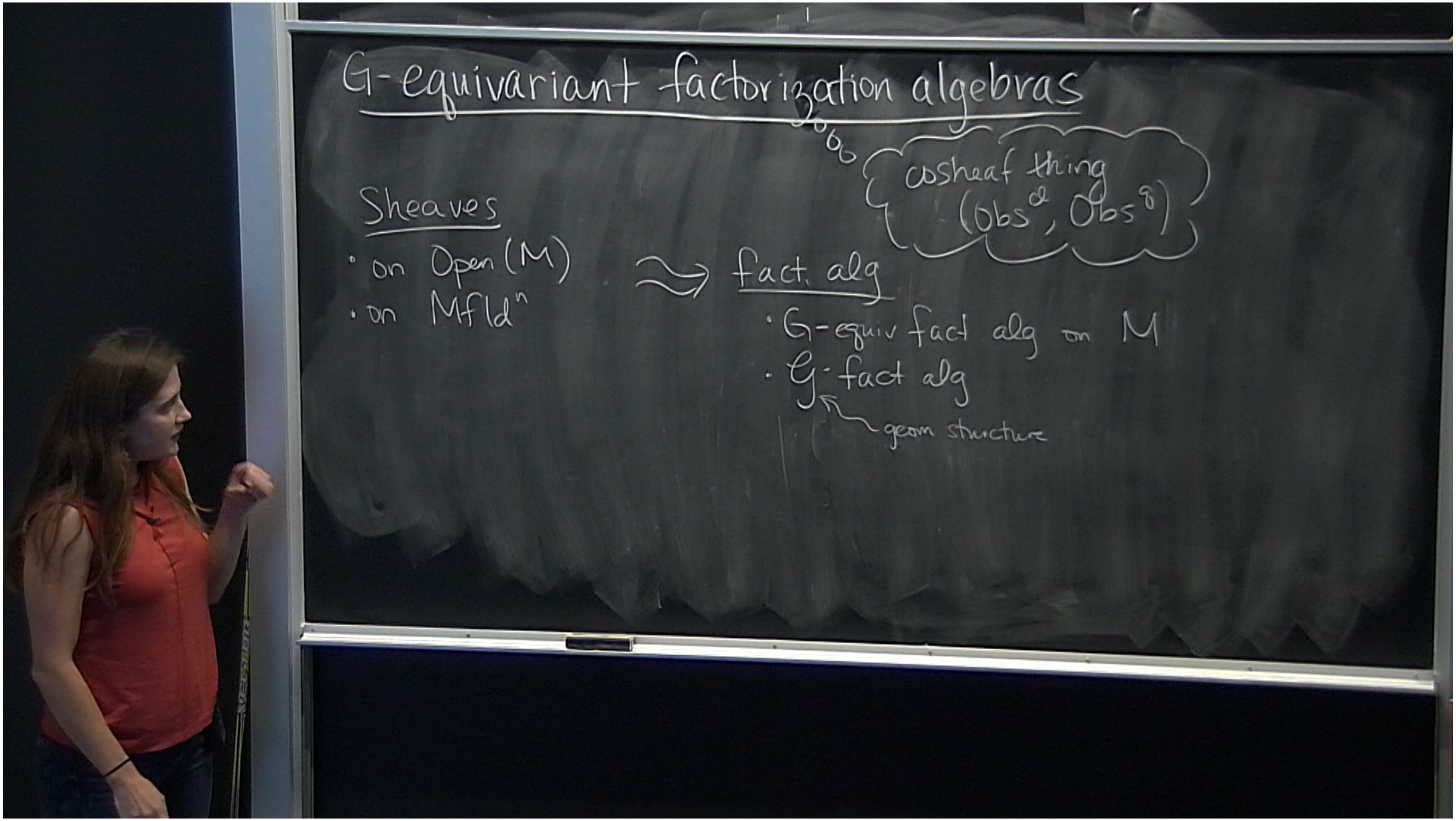
Abstract: There are various ways to define factorization algebras: one can define a factorization algebra that lives over the open subsets of some fixed manifold; or, alternatively, one can define a factorization algebra on the site of all manifolds of a given dimension (possibly with a specified geometric structure). In this talk, I will outline a comparison between G-equivariant factorization algebras on a fixed model space M to factorization algebras on the site of all manifolds equipped with a (M, G)-structure, given by an atlas with charts in M and transition maps given by elements of G. I will introduce the definitions of these two concepts and then sketch the proof that there is a quasi-equivalence between these dg-categories. This is work in progress


G-equivariant factorization algebras
cosheaf thing
$(\mathrm{Obs}^{cl}, \mathrm{Obs}^{q})$
Sheaves
· on Open(M)
· on Mfld^n
fact. alg
· G-equiv fact alg on M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on Mfld · G-equiv fact. alg on M

· $\mathcal{G}$-fact alg

geom structure

Why care?

- related to functional f.t.'s
  - Scheimbauer: loc. const. fact alg $\rightsquigarrow$ fully ext'd TFT
  - Dwyer-Stolz-Teichner: $\mathcal{G}$-fact alg $\rightsquigarrow$ twisted $\mathcal{G}$-field theory

I. $\mathcal{G}$-fact alg

G-equivariant factorization algebras

cosheaf thing
(Obs^{cl}, Obs^{q})

Sheaves

- on Open(M) $\rightsquigarrow$ fact. alg
- on $Mfld^n$

$\left\{ \begin{array}{l} \cdot\ G\text{-equiv fact alg on } M \\ \cdot\ \mathcal{G}\text{-fact alg} \end{array} \right.$

$\mathcal{G}$ ← geom structure

Why care?

- related to functorial f.t.'s
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	M	G
Eucl.	$\mathbb{R}^n$	$\mathbb{R}^n \rtimes O(n)$
spin	$\mathbb{R}^n$	$\mathbb{R}^n \rtimes \mathrm{Spin}(n)$
conformal	$\mathbb{R}^n$	$\mathbb{R}^n \rtimes (SO(n) \times \mathbb{R}_+)$

Defn a $\mathcal{G}$-fact. alg is lax sym mon. functor

$$\mathcal{F} : \mathcal{G}\mathrm{Man} \longrightarrow \mathrm{Ch}$$

sat. (i) mult. axiom $X_1, \dots, X_n \in \mathcal{G}\mathrm{Man}$

$$\mathcal{F}(X_1) \otimes \cdots \otimes \mathcal{F}(X_n) \xrightarrow[\text{w.e}]{\sim} \mathcal{F}(X_1 \sqcup \cdots \sqcup X_n)$$

(ii) descent axiom. for any Weiss cover $\mathcal{U} = \{U_a\}_{a \in A}$ of X

$$\mathcal{F}(X) \xleftarrow[\text{w.e}]{\sim} \mathrm{hocolim}\left(\bigoplus_{a_0} \mathcal{F}(U_{a_0}) \leftleftarrows \bigoplus_{a_0, a_1} \mathcal{F}(U_{a_0} \times_X U_{a_1}) \leftleftarrows \cdots \right)$$

<u>Defn</u> a <u>Weiss cover</u> of X is a covering $\mathcal{U} = \{ U_\alpha \xhookrightarrow{f_\alpha} X \}$ s.t. for any finite set $S \subset X$, $\exists\ \underset{\underset{\mathcal{U}}{\cap}}{U_\alpha} \xhookrightarrow{f_\alpha} X$ s.t. $S \subseteq f_\alpha(U_\alpha)$

Defn a $\mathcal{G}$-fact. alg is lax sym mon. functor

$$\mathcal{F} : \mathcal{G}\mathrm{Man} \longrightarrow \mathrm{Ch} \longleftarrow \text{(co)chain complexes}$$

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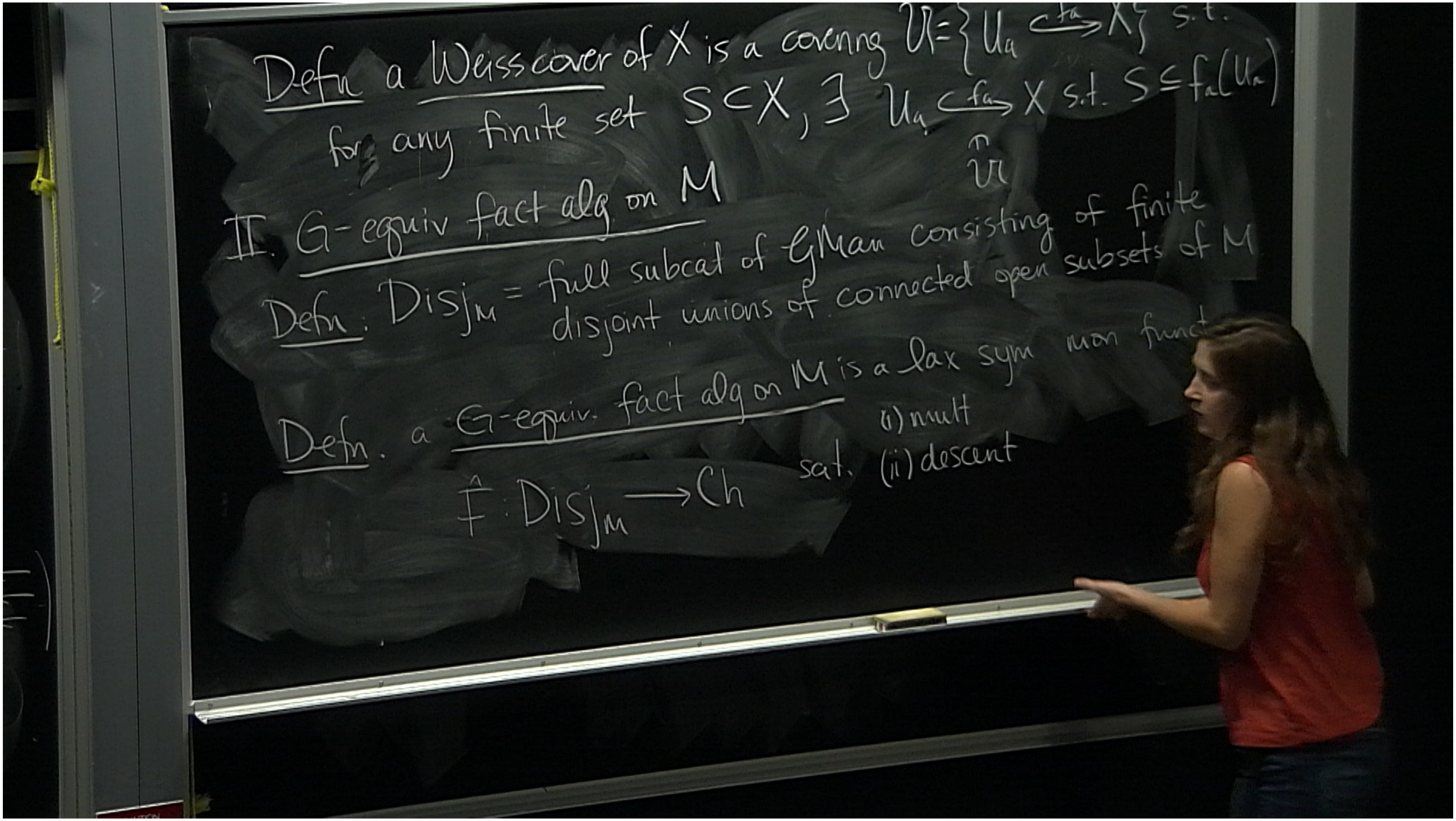
II. **G-equiv fact alg on M**

Defn: Disj_M = full subcat of $G\mathrm{Man}$ consisting of finite disjoint unions of connected open subsets of M

Defn.

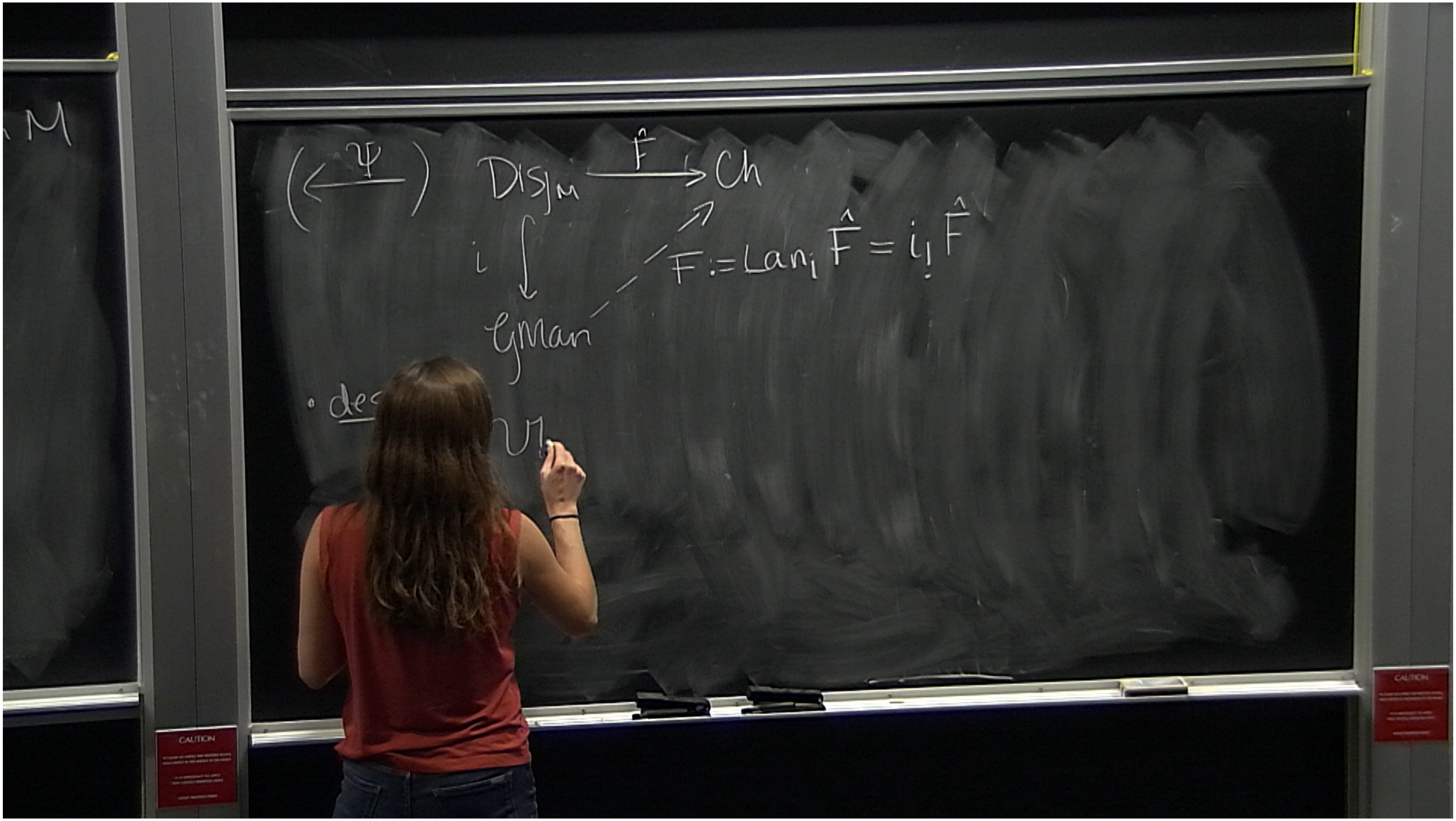

Defn a Weiss cover of X is a covering U = { U_a ↪ X } s.t.
for any finite set S ⊂ X, ∃ U_a ↪ X s.t. S ⊆ f_a(U_a)
II. G-equiv fact alg on M
Defn: Disj_M = full subcat of GMan consisting of finite
disjoint unions of connected open subsets of M
Defn. a G-equiv. fact alg on M is a lax sym mon funct
(i) mult
F̂ : Disj_M ⟶ Ch sat. (ii) descent

III "Thm" $\mathcal{G}$-fact alg are equiv. to G-equiv. fact alg on M

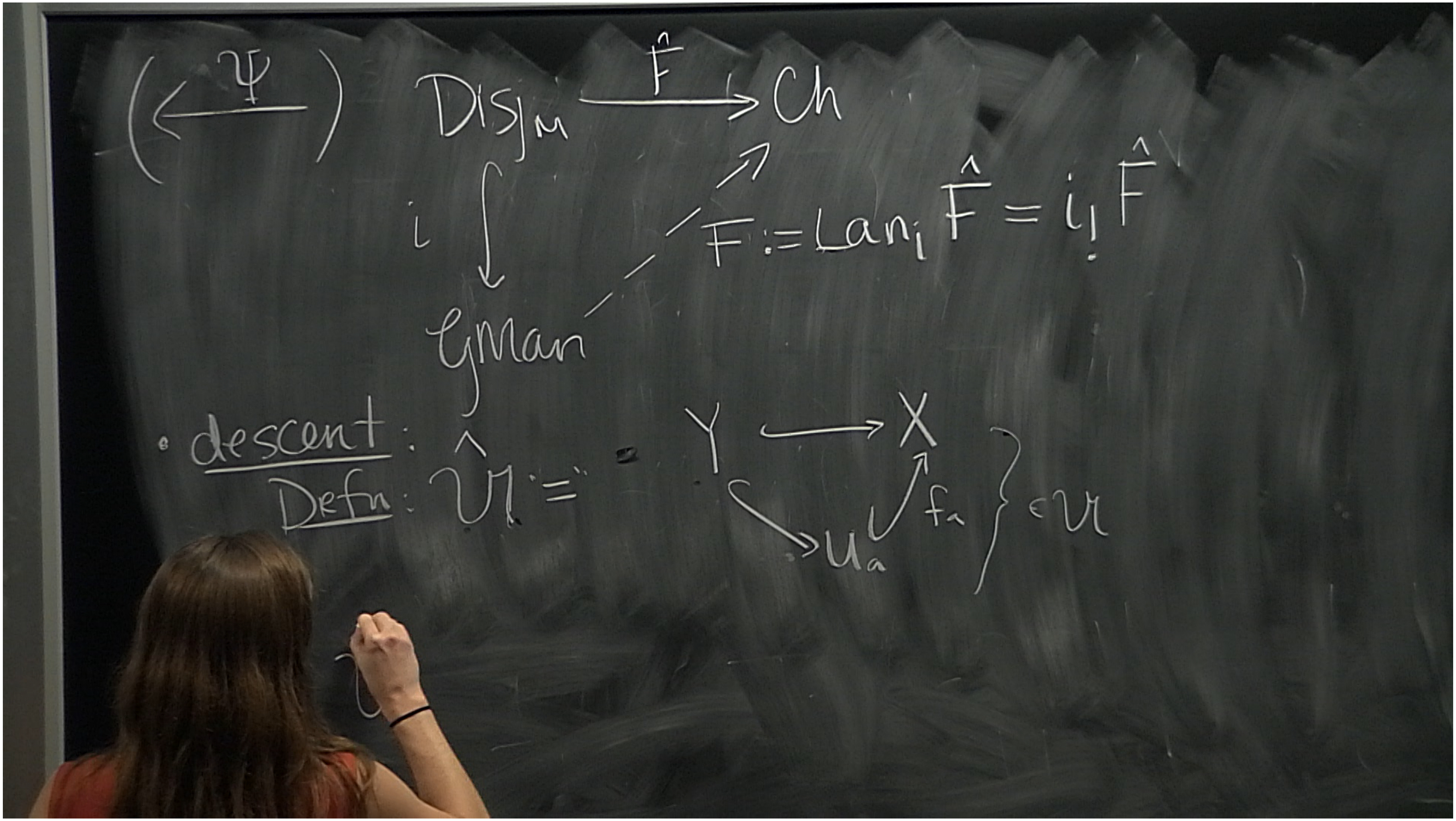
$$\begin{tikzcd} \mathcal{G}\mathrm{Fac} \arrow[r, bend left, "\Phi"] & \mathrm{Fac}_M^G \arrow[l, bend left, "\Psi"] \end{tikzcd}$$

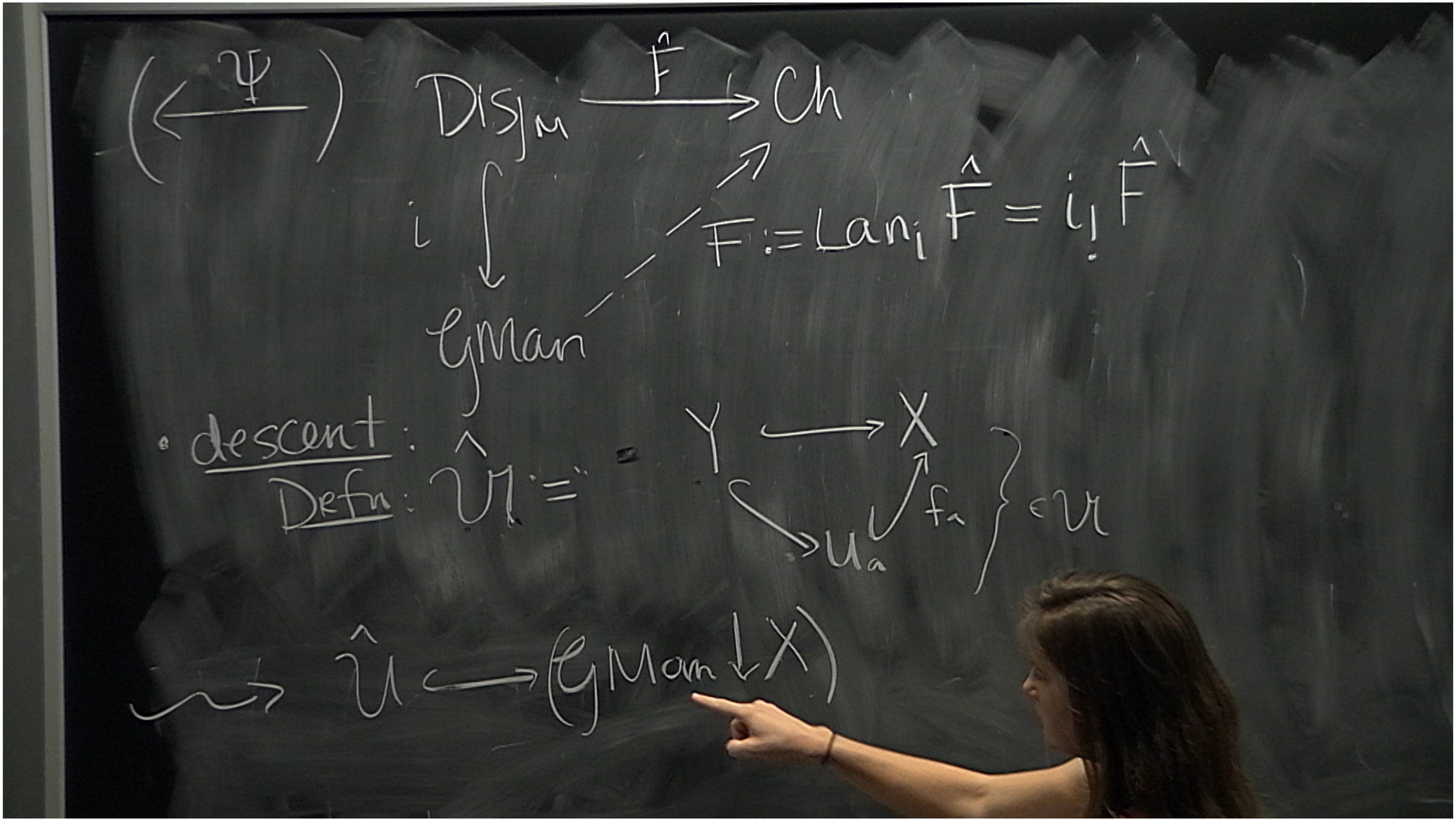
$\left(\xrightarrow{\Phi}\right)$

$$\begin{tikzcd} \mathrm{Disj}_M \arrow[r, dashed, "\hat{F} := F|_{\mathrm{Disj}_M}"] \arrow[d, "i"'] & \mathrm{Ch} \\ \mathcal{G}\mathrm{Man} \arrow[ur, "F"'] & \end{tikzcd}$$



$(\xleftarrow{\Psi})$ $\mathrm{Disj}_M \xrightarrow{\hat{F}} \mathrm{Ch}$
i
Gman
$F := \mathrm{Lan}_i \hat{F} = i_! \hat{F}$


Ψ
Disj_M
F̂
Ch
i
Gman
F := Lan_i F̂ = i_! F̂
descent:
Defn: Û :=
Y
X
U_a
f_a
∈ U



$(\xleftarrow{\Psi})$
$\mathrm{Disj}_M \xrightarrow{\hat{F}} \mathrm{Ch}$
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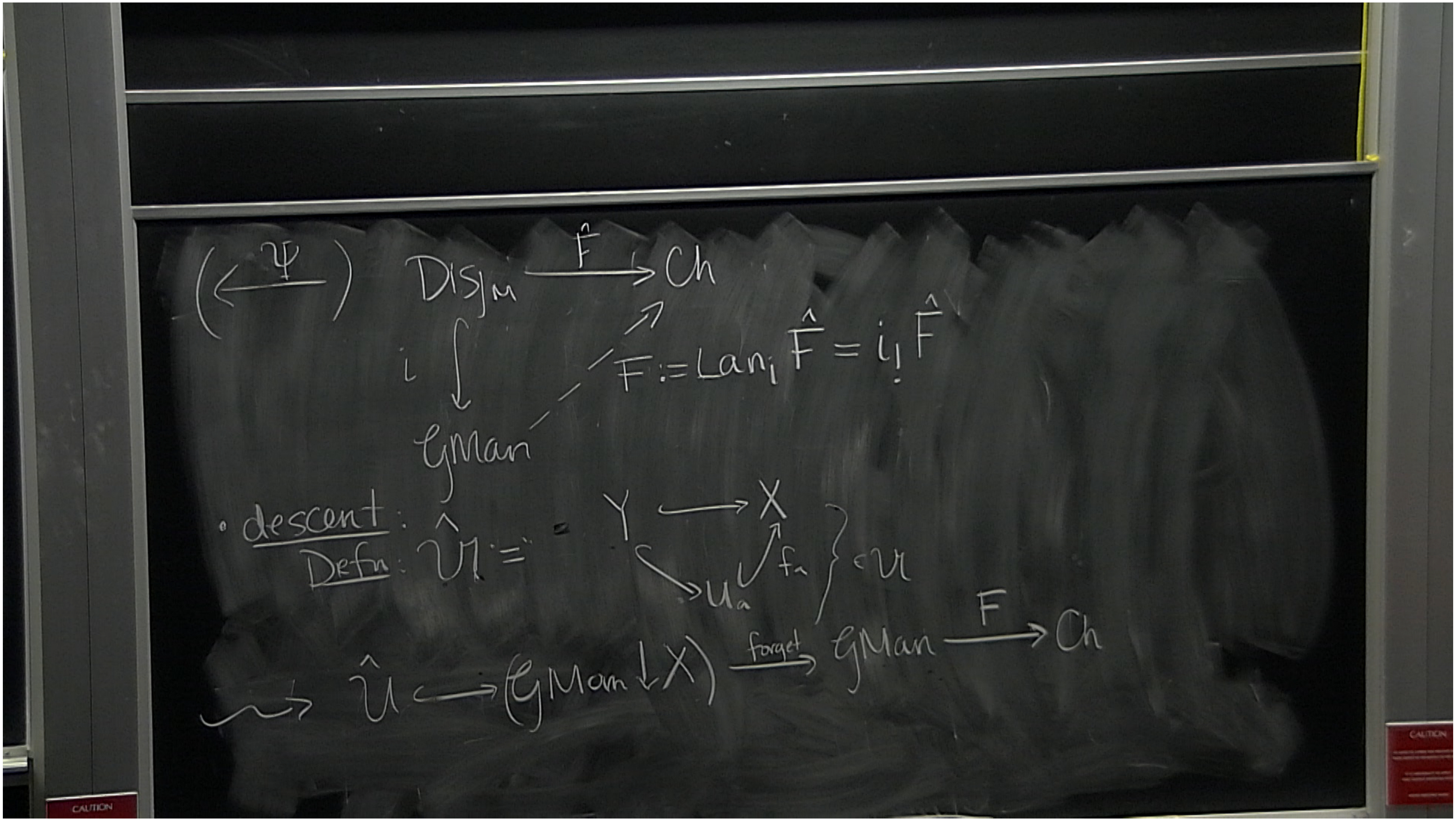
$F := \mathrm{Lan}_i \hat{F} = i_! \hat{F}$
GMan
descent:
Defn: $\hat{U} :=$ "
$Y \to X$
U_a
f_a
$\in U$
$\rightsquigarrow \hat{U} \hookrightarrow (\mathrm{GMan} \downarrow X)$



$(\xleftarrow{\Psi})$ $\mathrm{Disj}_M \xrightarrow{\hat{F}} \mathrm{Ch}$
i
GMan
$F := \mathrm{Lan}_i \hat{F} = i_! \hat{F}$
• descent:
Defn: $\hat{\mathcal{U}} :=$
$Y \longrightarrow X$
U_a
f_a
$\in \mathcal{U}$
$\rightsquigarrow \hat{\mathcal{U}} \hookrightarrow (\mathrm{GMan} \downarrow X) \xrightarrow{\text{forget}} \mathrm{GMan} \xrightarrow{F} \mathrm{Ch}$