Title: Perturbative Anomalies of the Massless Free Fermion and Formal Moduli Problems

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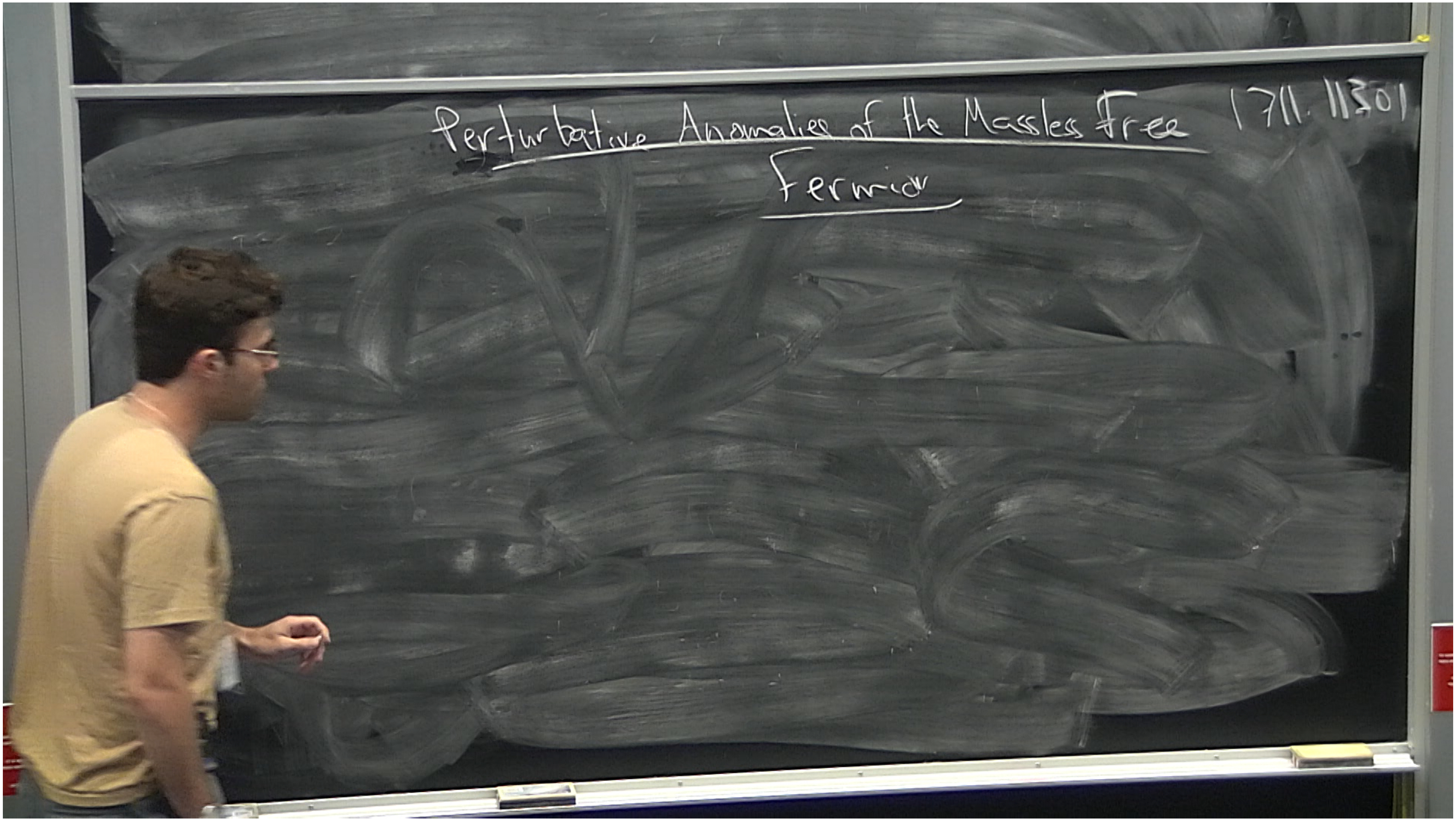
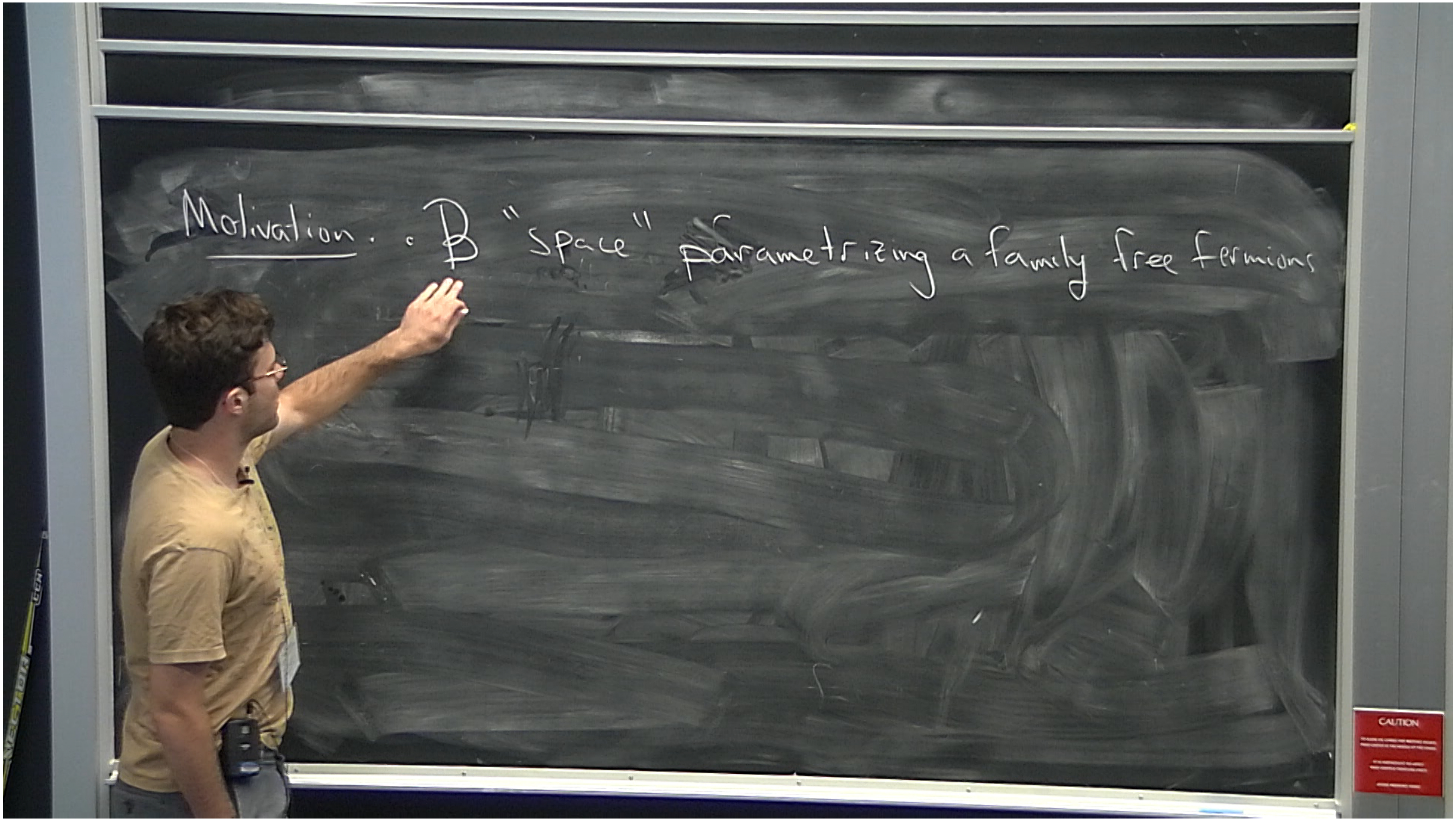
Abstract: It is conventional wisdom among physicists that anomalies of fermionic theories measure an obstruction to the existence of a well-defined (gauge-invariant) partition function. The aim of this talk is to use the formalism of Costello and Gwilliam to show how this wisdom is instantiated for perturbative anomalies of the massless free fermion. We will show how an action of a dg Lie algebra L on the massless free fermion theory gives rise to a line bundle over the formal moduli problem corresponding to L; the anomaly is precisely the failure of this line bundle to be trivial. Our running example will be the axial symmetry of the massless free fermion.


Perturbative Anomalies of the Massless Free Fermion
1711.11301


Motivation. · B "space" parametrizing a family free fermions
CAUTION

Motivation.

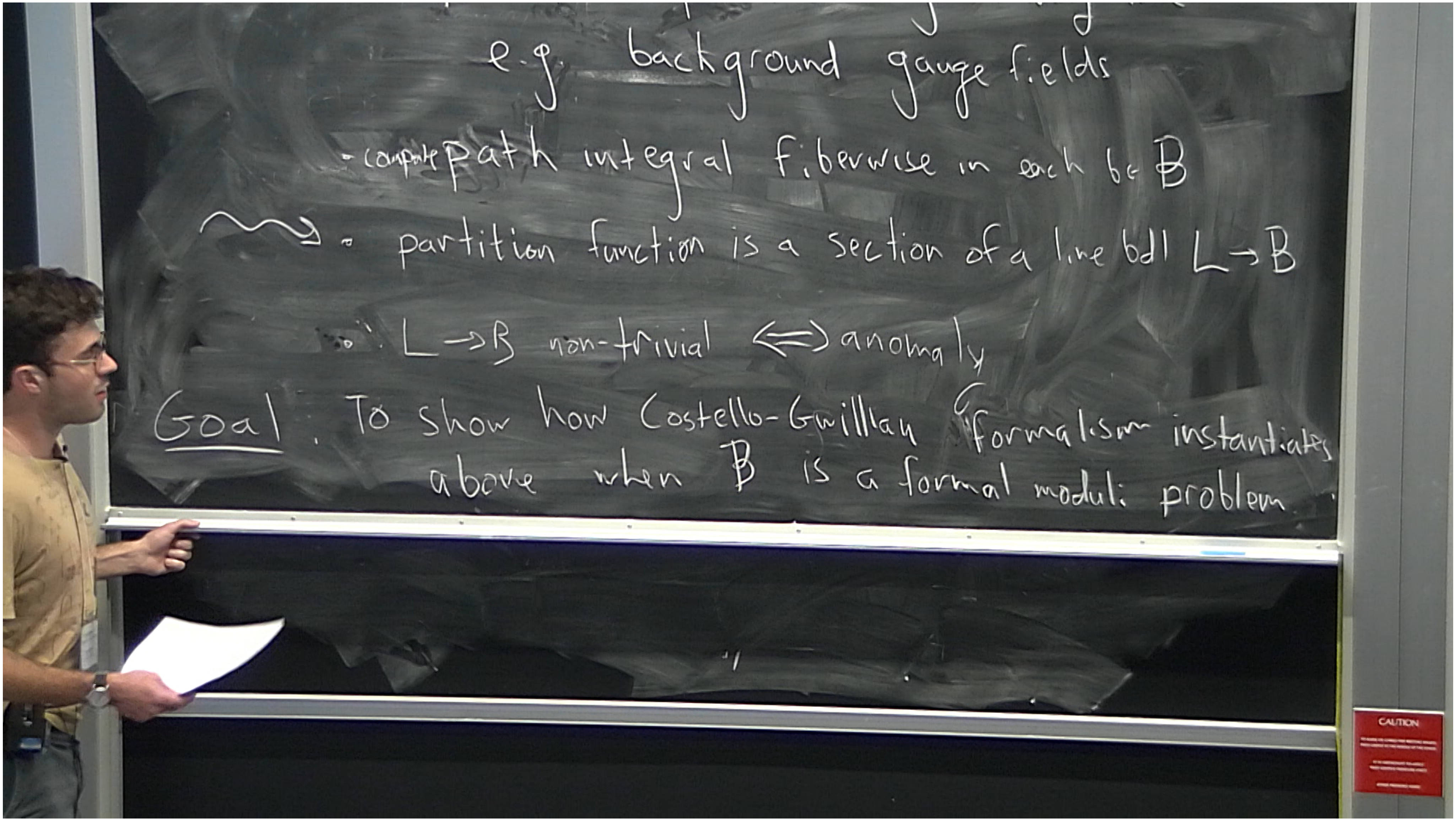
- $\mathcal{B}$ "space" parametrizing a family free fermions

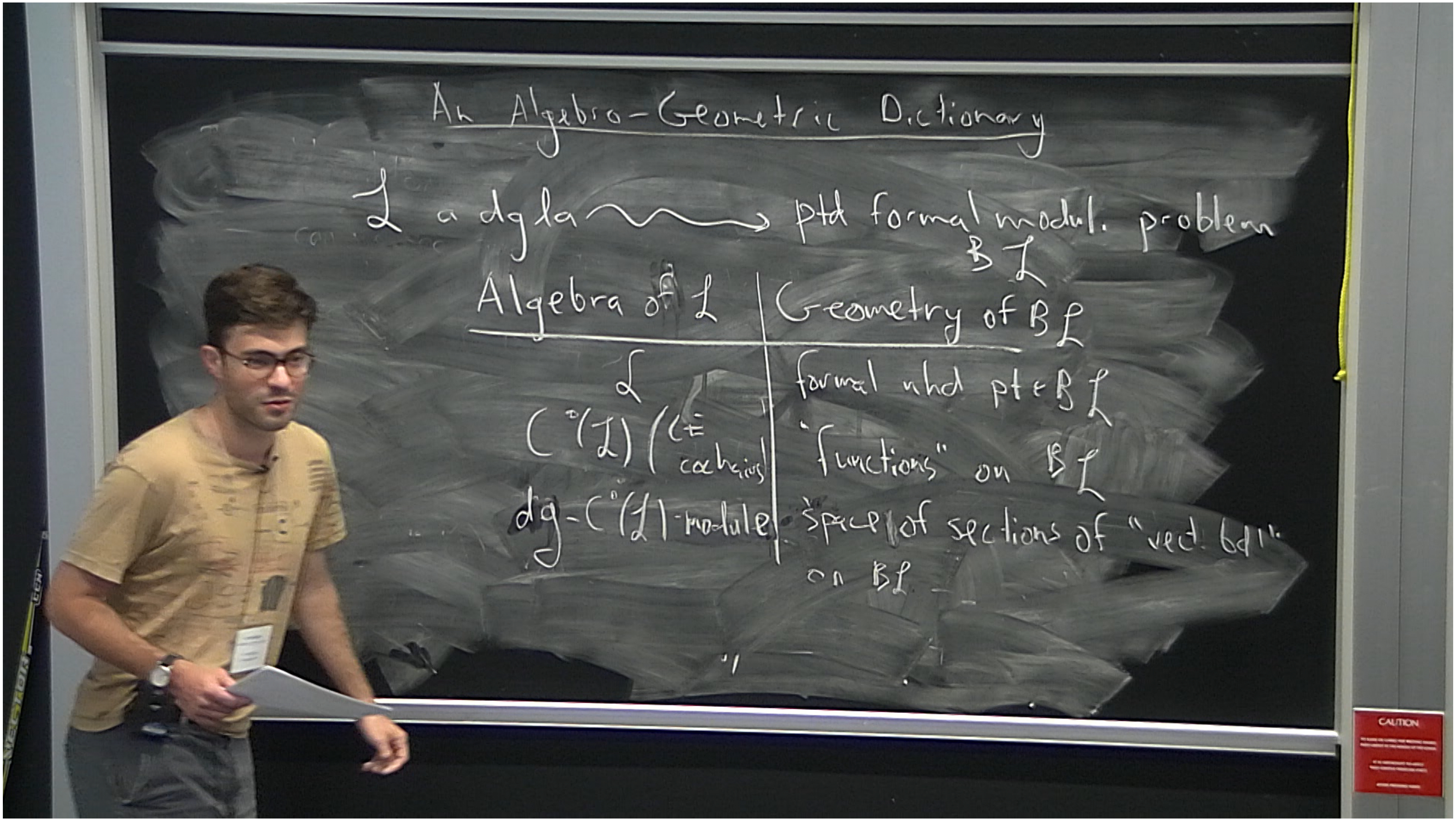
  e.g. background gauge fields

- compute path integral fiberwise in each $b \in \mathcal{B}$

$\rightsquigarrow$ • partition function is a section of a line bdl $L \to B$

- $L \to B$ non-trivial $\Longleftrightarrow$ anomaly


e.g. background gauge fields
· compute path integral fiberwise in each b ∈ B
· partition function is a section of a line bdl L → B
· ∴ L → B non-trivial ⟺ anomaly
Goal: To show how Costello-Gwilliam formalism instantiates above when B is a formal moduli problem.



An Algebro-Geometric Dictionary
$\mathcal{L}$ a dgla ⇝ ptd formal moduli problem $B\mathcal{L}$
Algebra of $\mathcal{L}$
Geometry of $B\mathcal{L}$
$\mathcal{L}$
formal nbd pt ∈ $B\mathcal{L}$
$C^*(\mathcal{L})$ (C^* = cochains)
"functions" on $B\mathcal{L}$
dg-$C^*(\mathcal{L})$-module
space of sections of "vect. bdl" on $B\mathcal{L}$

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Massless Free Fermion

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Def: A free BV thy is an elliptic cx $(\mathcal{F}, Q)$ together with a non-degen., skew-symm., deg (-1) pairing $\langle \, , \, \rangle$ (Q skew adj)

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$\rightsquigarrow Obs^{cl}, Obs^{q}$ cochain cxes non-degen., skew-symm., deg (-1) pairing $\langle \; , \; \rangle$ ...

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<u>Ex</u> (Massless Free Fermion): (M, g) Riem mfld, $V \to M$ $\mathbb{Z}/2$-graded

$V \to M$ w metric $(\cdot, \cdot)$, $\mathbb{D}$ a formally self.-adj't Dirac operator non-degen. skew-symm. , deg (-1) pairing ... ad

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Let Γ chirality involution. $\mathcal{F} := \underset{0}{(\mathcal{E}(M;V)} \xrightarrow{D\Gamma} \underset{1}{\mathcal{E}(M;V))}$

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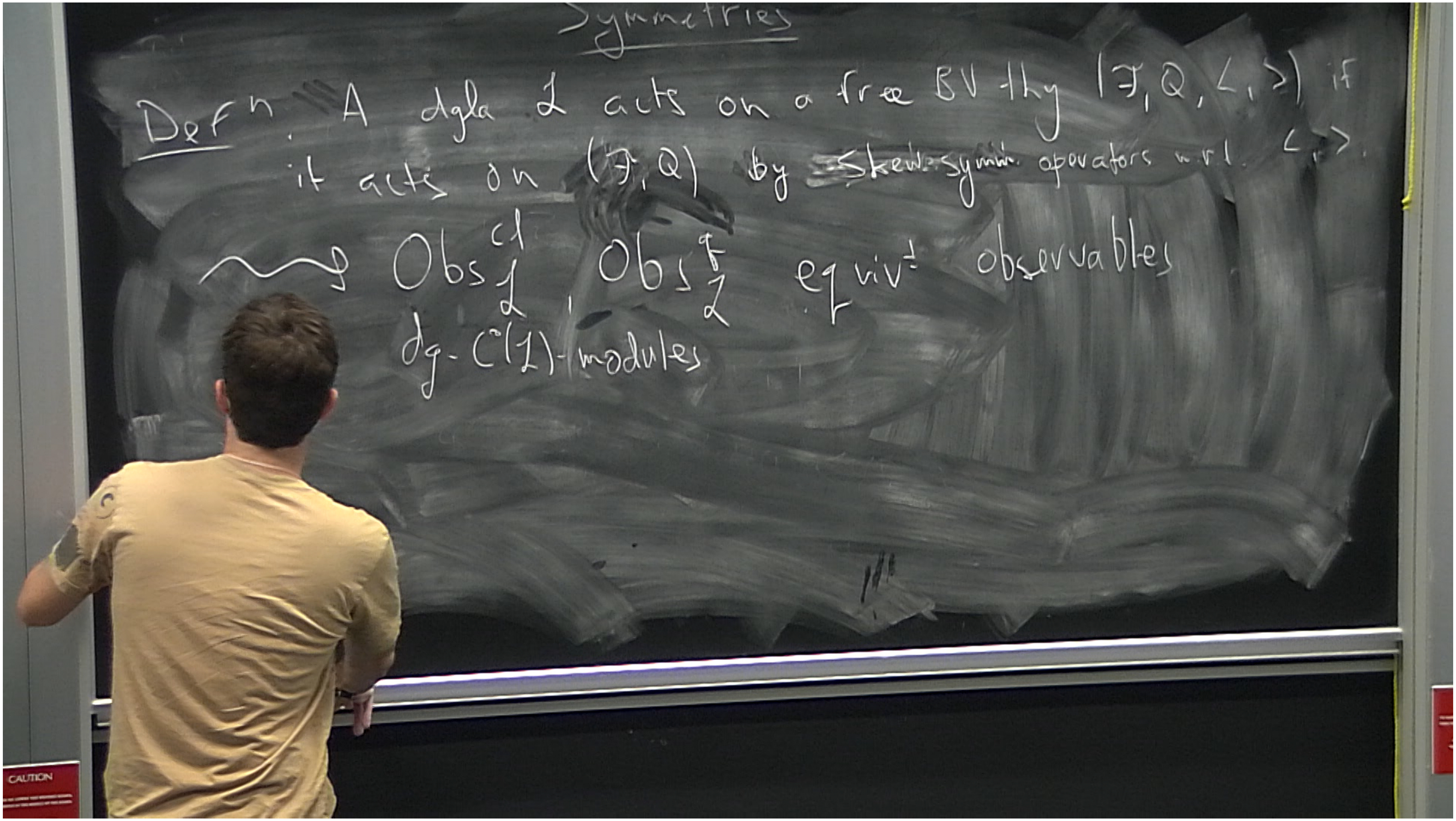
$\langle \varphi, \psi \rangle = \int (\varphi, \psi)\, d\mathrm{Vol}_g$


Symmetries
Defn: A dgla $\mathfrak{L}$ acts on a free BV thy $(\mathcal{F}, Q, \langle , \rangle)$ if
it acts on $(\mathcal{F}, Q)$ by skew-symm. operators w.r.t. $\langle , \rangle$.
$\rightsquigarrow$ $Obs^{cl}_{\mathfrak{L}}$, $Obs^{q}_{\mathfrak{L}}$ equivt observables
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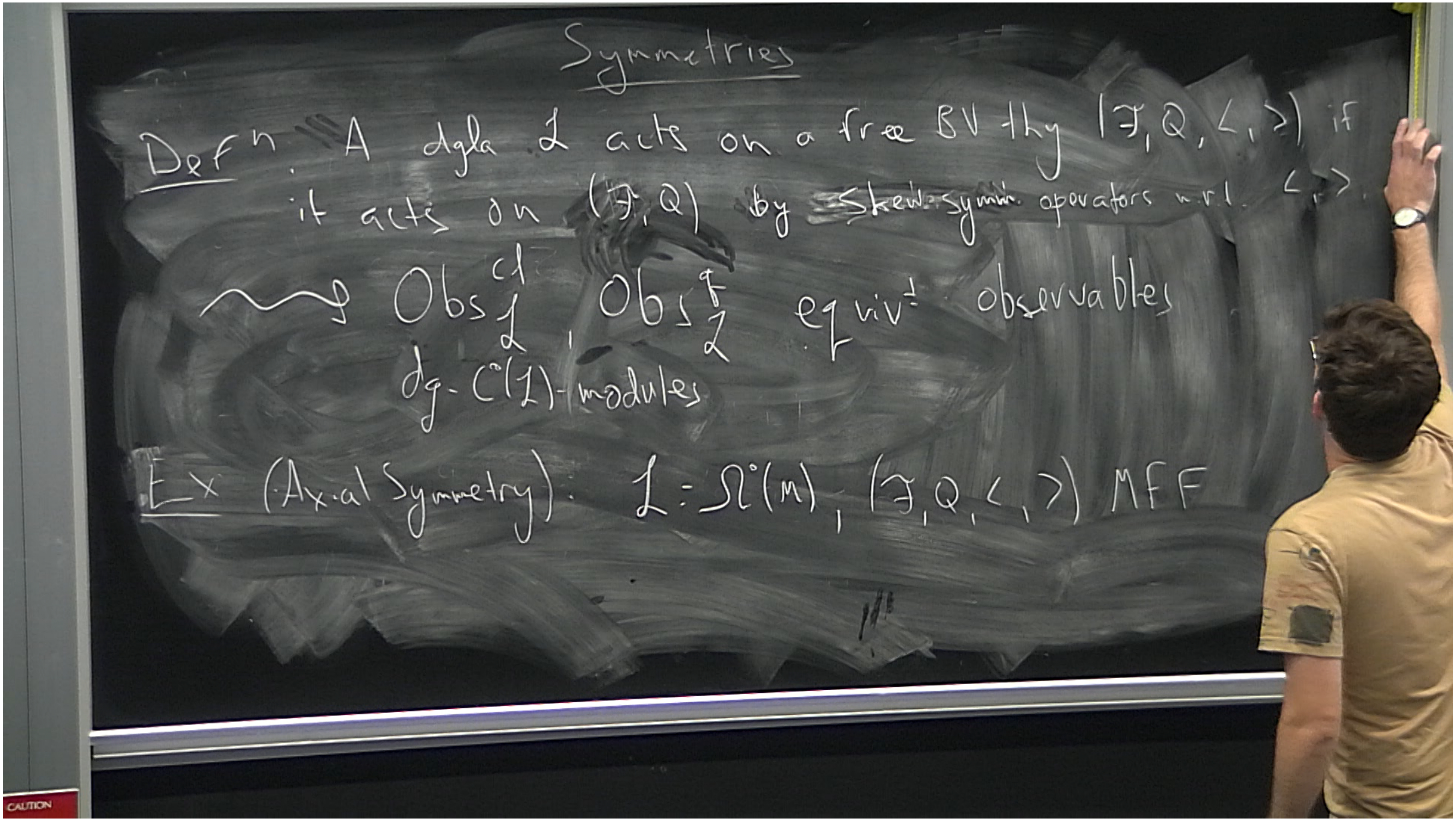
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dg-$C^\bullet(\mathcal{L})$-modules
Ex (Axial Symmetry): $\mathcal{L} = \Omega^\bullet(M)$, $(\mathcal{F}, Q, \langle , \rangle)$ MFF

Def: A free BV thy is an elliptic cx $(\mathcal{F}, Q)$ together with a non-degen., skew-symm., deg (-1) pairing $\langle\ ,\ \rangle$ (Q skew adj)

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$\langle \varphi, \psi \rangle = \int (\varphi, \psi)\, dVol_g$

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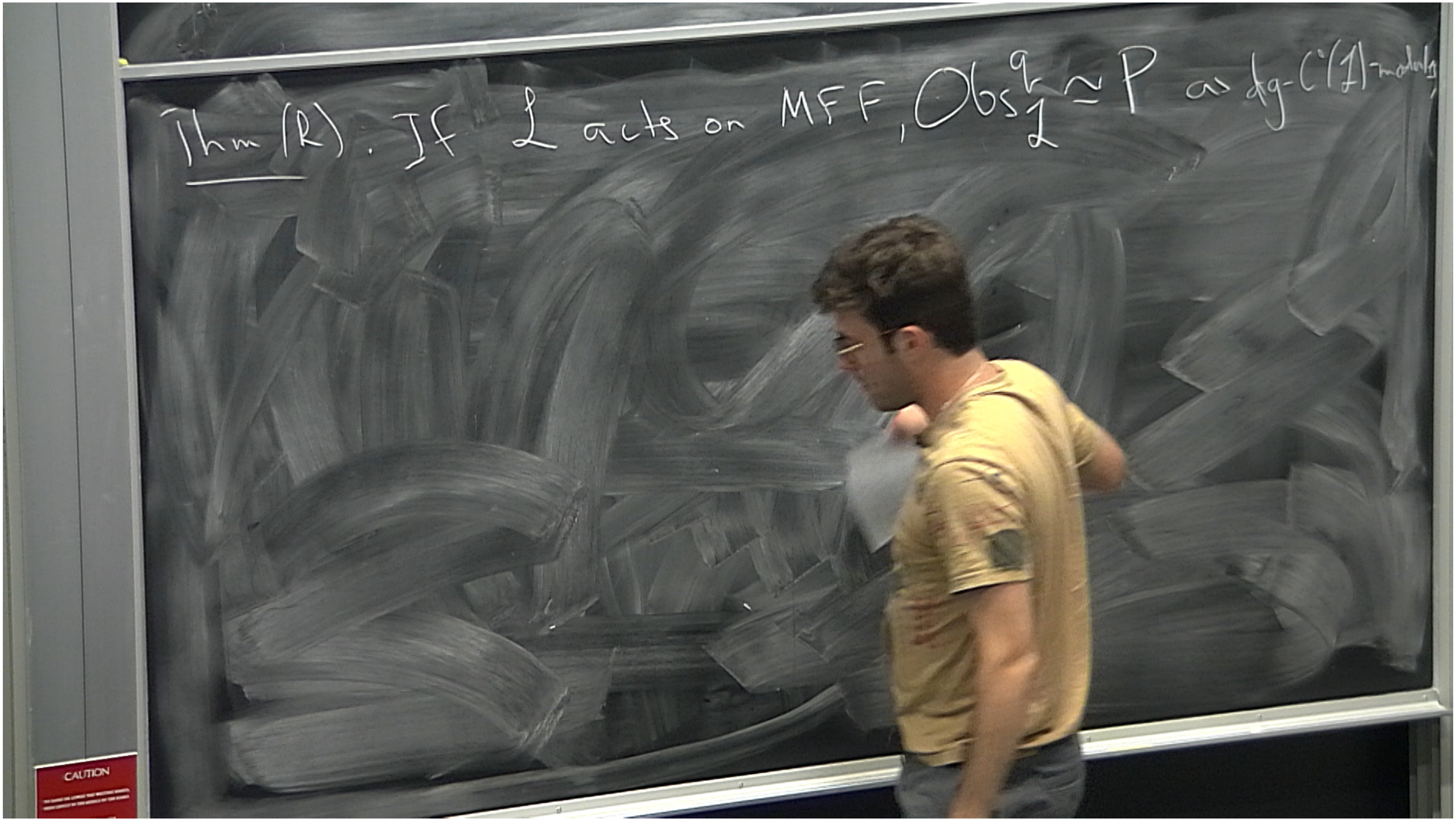
Ex (Massless Free Fermion): (M, g) Riem mfld, $V \to M$ $\mathbb{Z}/2$-graded $V \to M$ w metric $(\cdot,\cdot)$, D a formally self.-adj.t Dirac operator

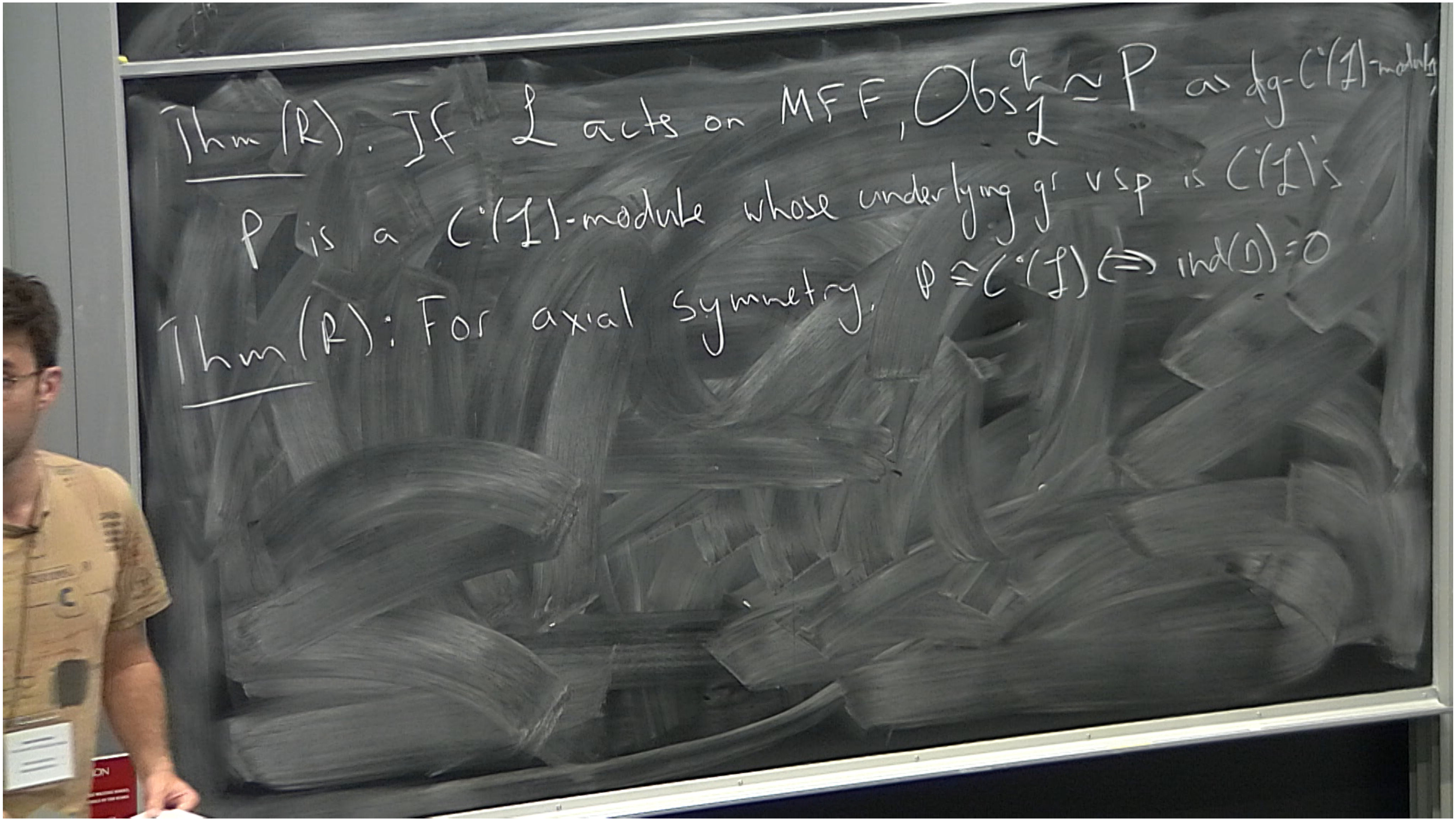
Let Γ chirality involution $\mathcal{F}$: $= \Gamma(M;V) \xrightarrow[c(\alpha)]{D\Gamma} \Gamma(M;V)$ $f\Gamma$ $f \in \Omega^0(M)$ $\alpha \in \Omega^1(M)$

$$\langle \varphi, \psi \rangle = \int (\varphi, \psi) \, dVol_g$$


Thm (R). If $\mathcal{L}$ acts on MFF, $\mathrm{Obs}^{q}_{\mathcal{L}} \simeq P$ as dg-$C^{\bullet}(\mathcal{L})$-modules
CAUTION

Thm (R). If $\mathcal{L}$ acts on MFF, $\mathrm{Obs}^{q}_{\mathcal{L}} \simeq P$ as dg-$C^{*}(\mathcal{L})$-modules,

P is a $C^{*}(\mathcal{L})$-module whose underlying gr vsp is $C^{*}(\mathcal{L})$'s


Thm (R). If $\mathcal{L}$ acts on MFF, $\mathrm{Obs}^q_{\mathcal{L}} \simeq P$ as dg-$C^\bullet(\mathcal{L})$-modules
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Thm (R): For axial symmetry, $P \cong C^\bullet(\mathcal{L}) \iff \mathrm{ind}(D)=0$