Title: The low-energy TQFT of the generalized double semion model

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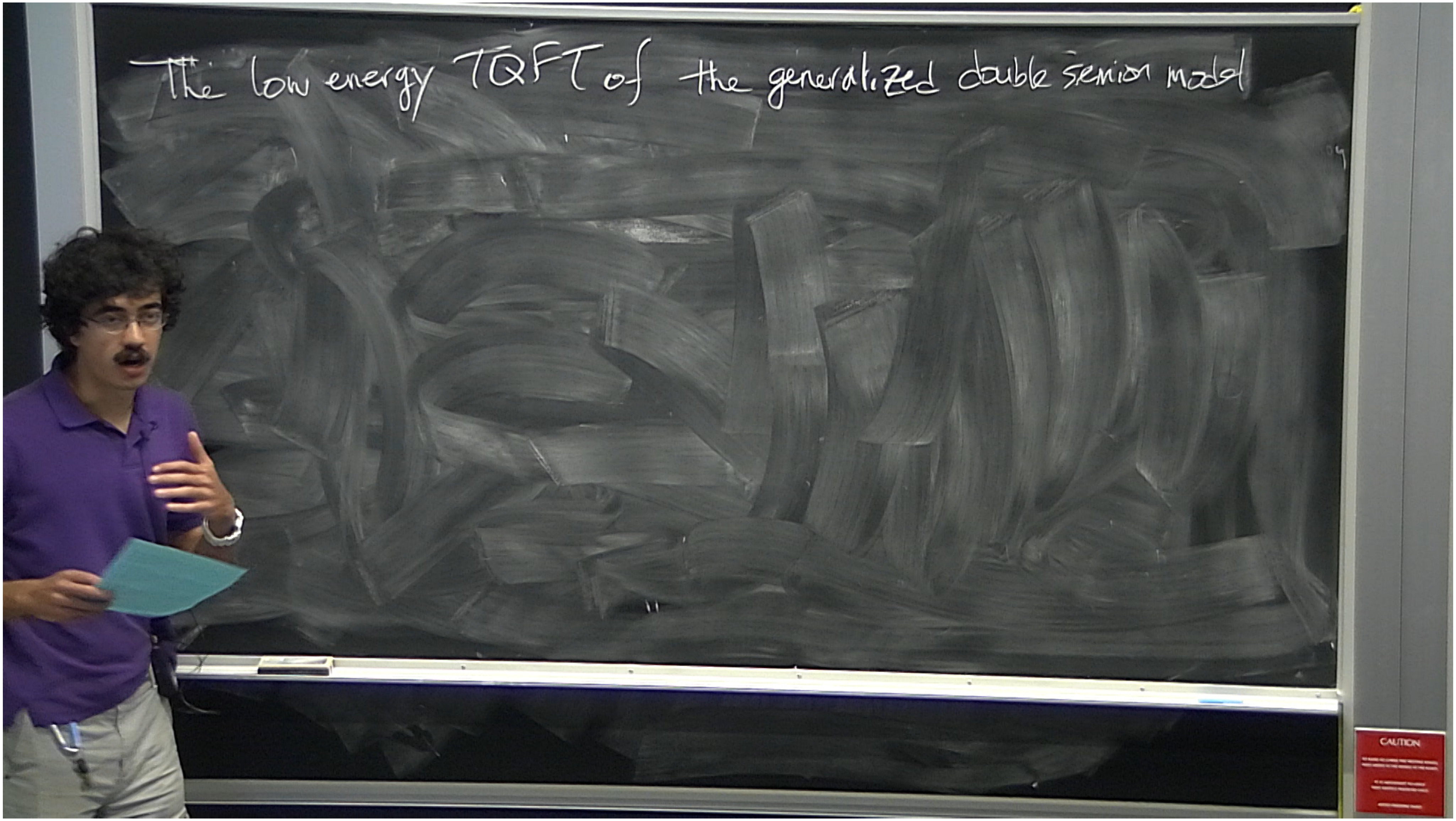
Abstract: The generalized double semion model, introduced by Freedman and Hastings, is a lattice field theory similar to the toric code, with a gapped Hamiltonian whose space of ground states depends on the topology of the ambient manifold. In this talk, Iâ€™ll explain how to calculate its low-energy limit, which forms part of a topological field theory, in terms of characteristic classes of the ambient manifold.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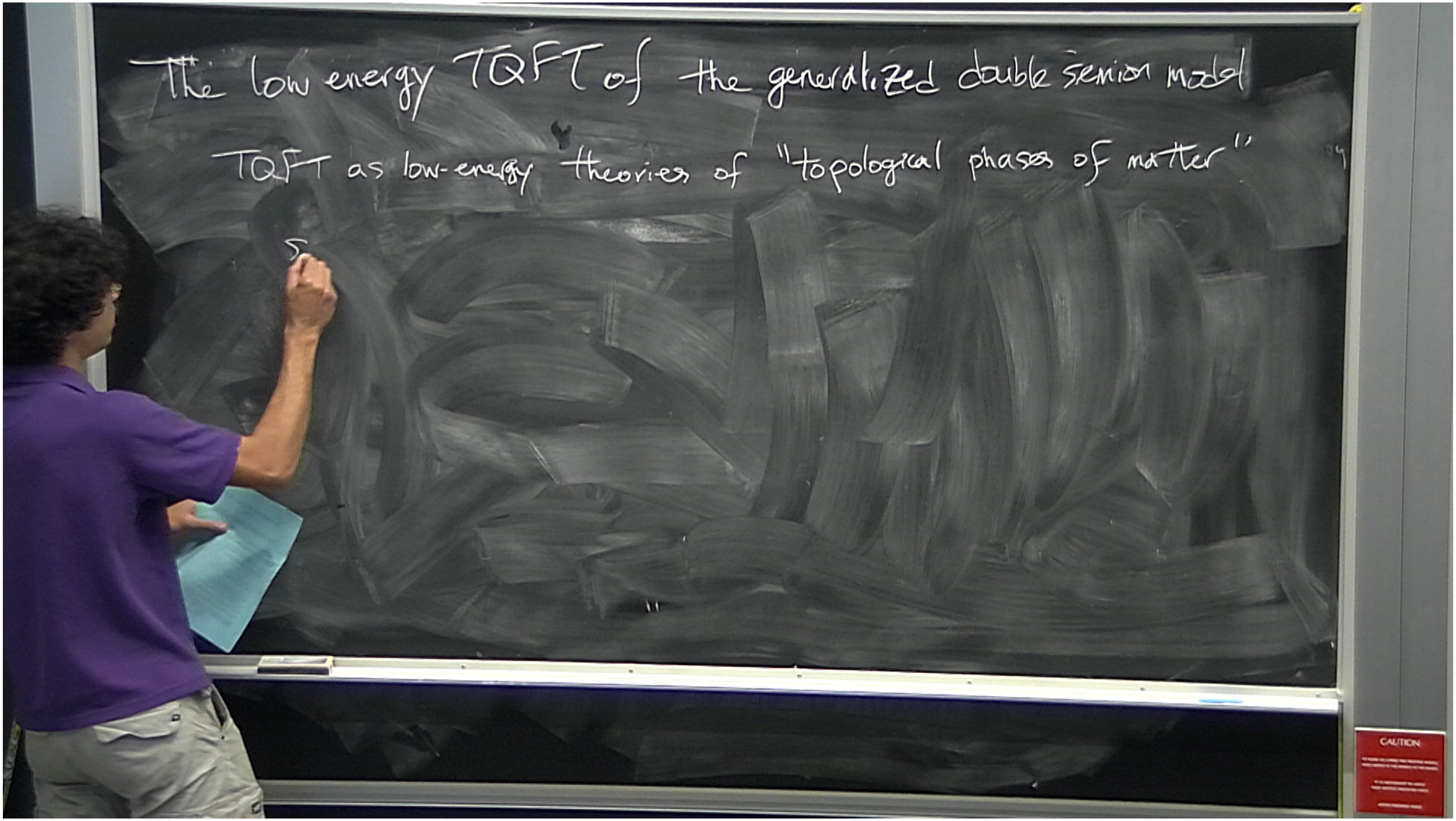
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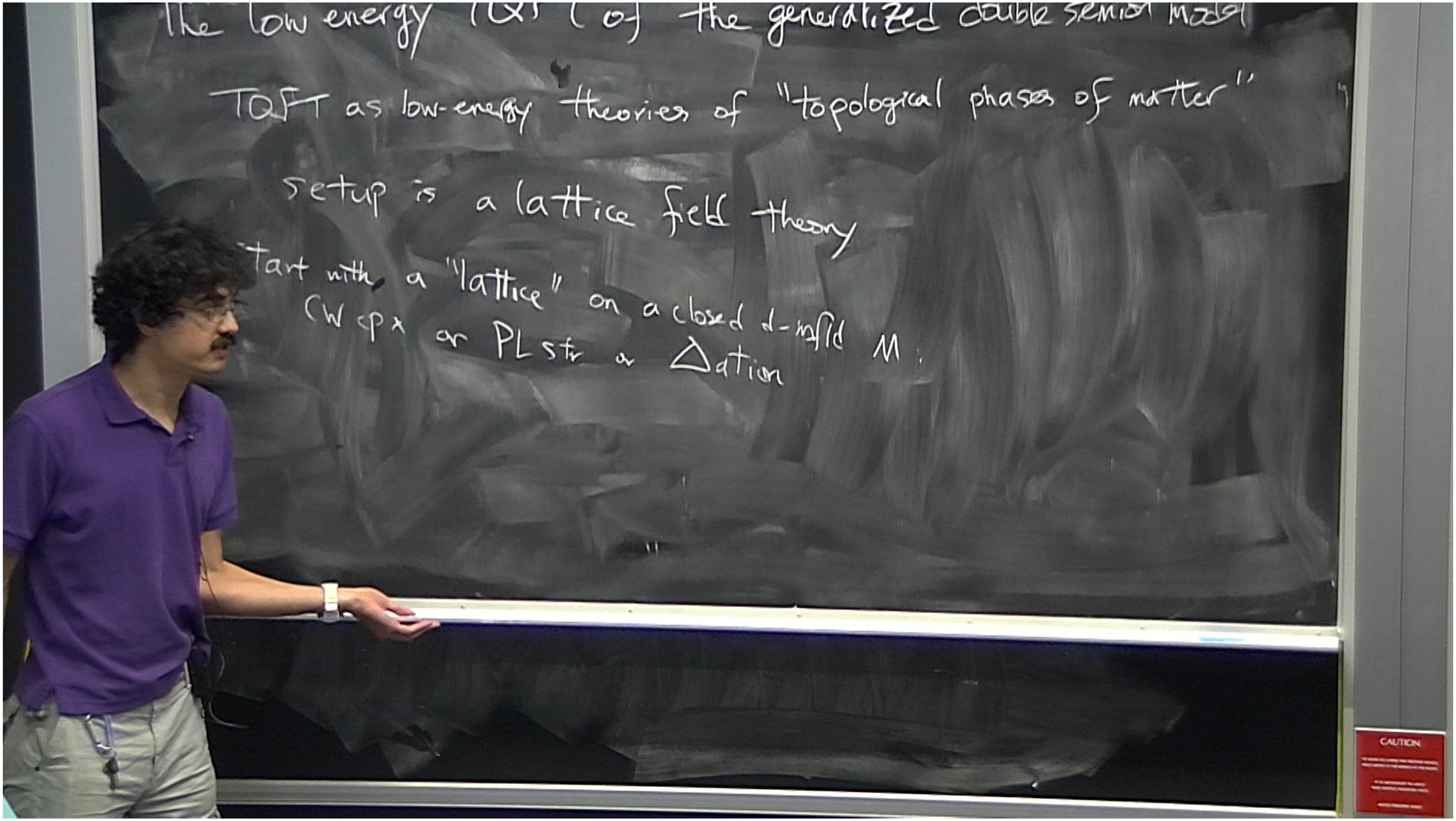
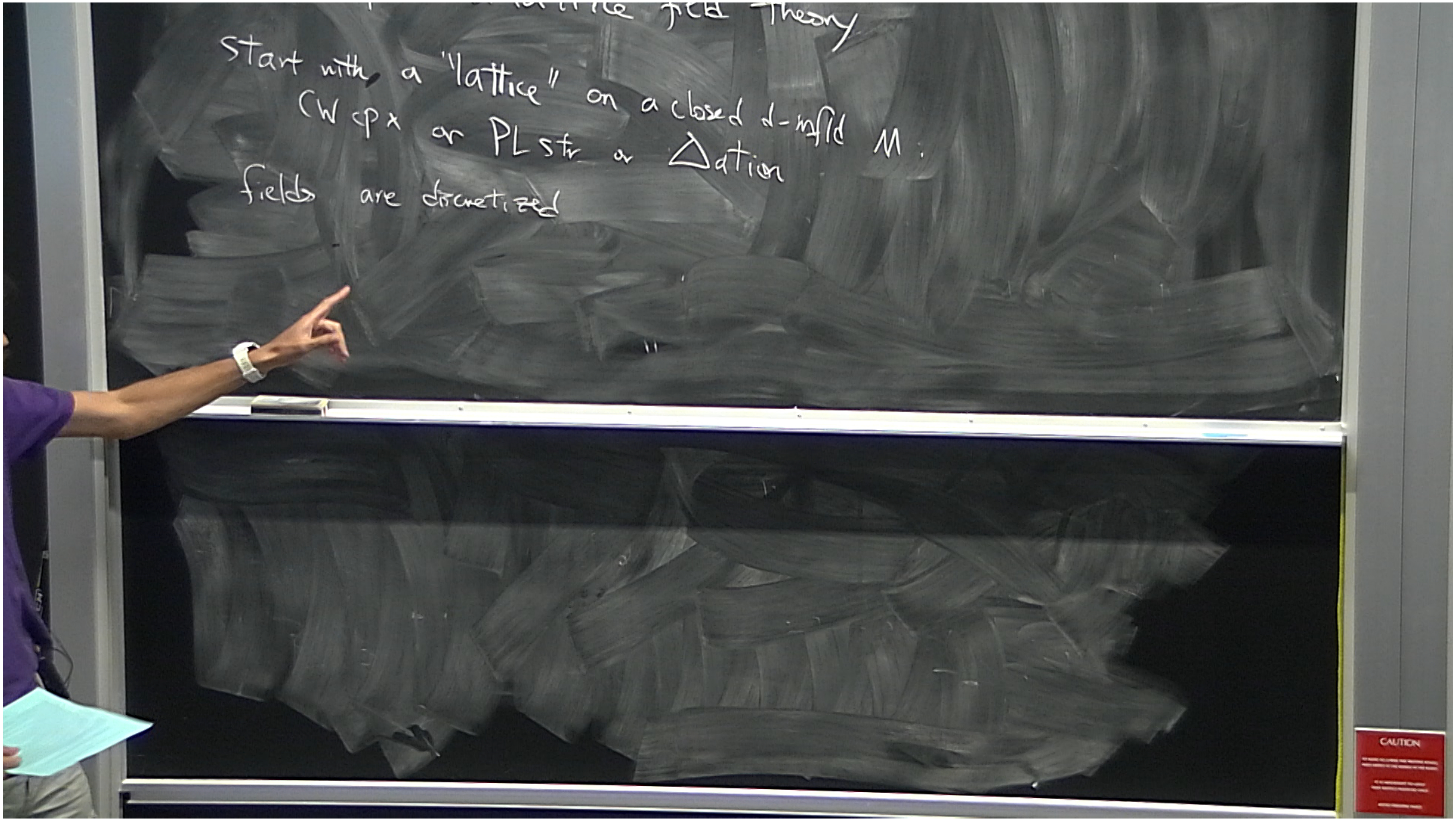
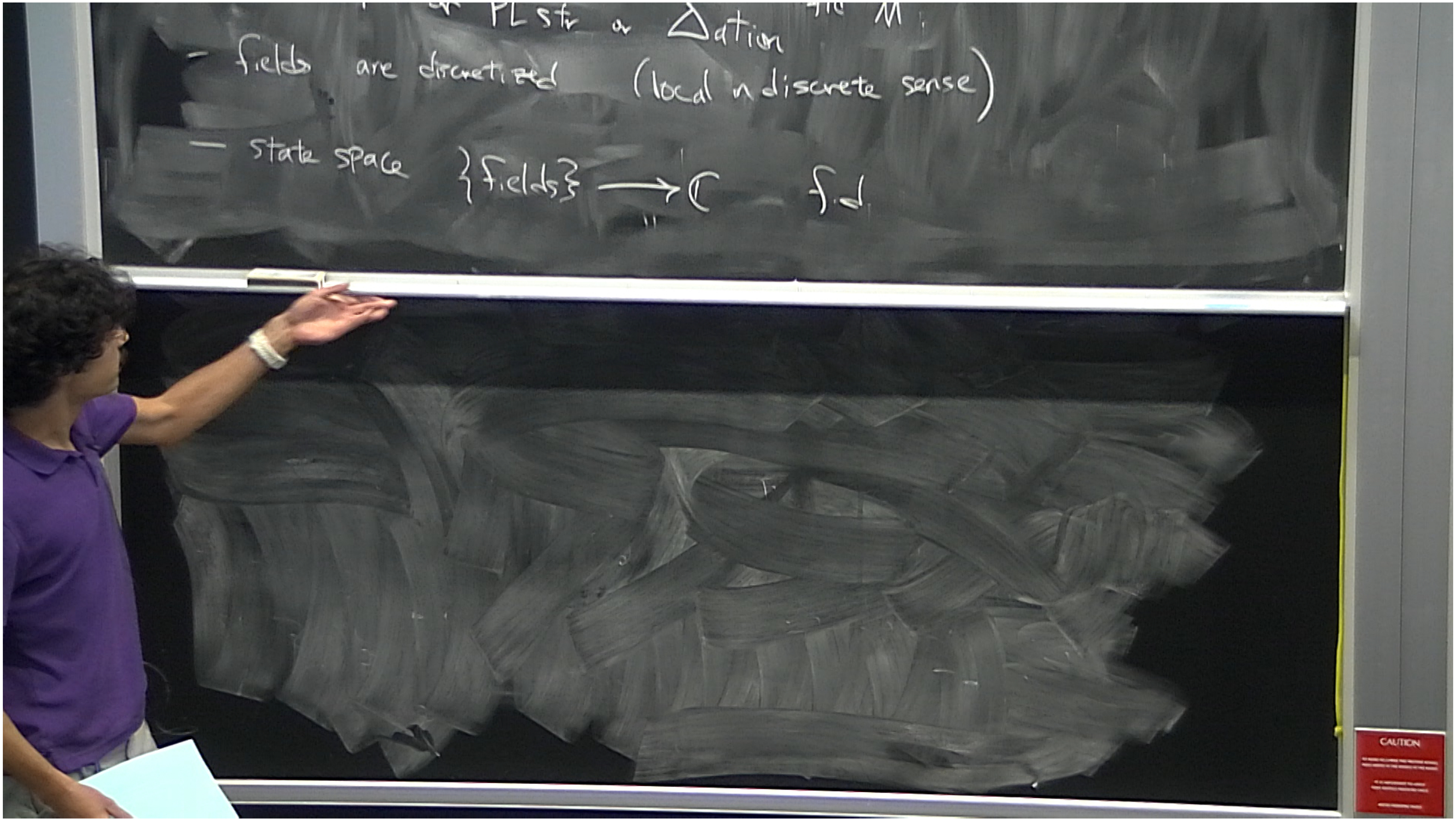
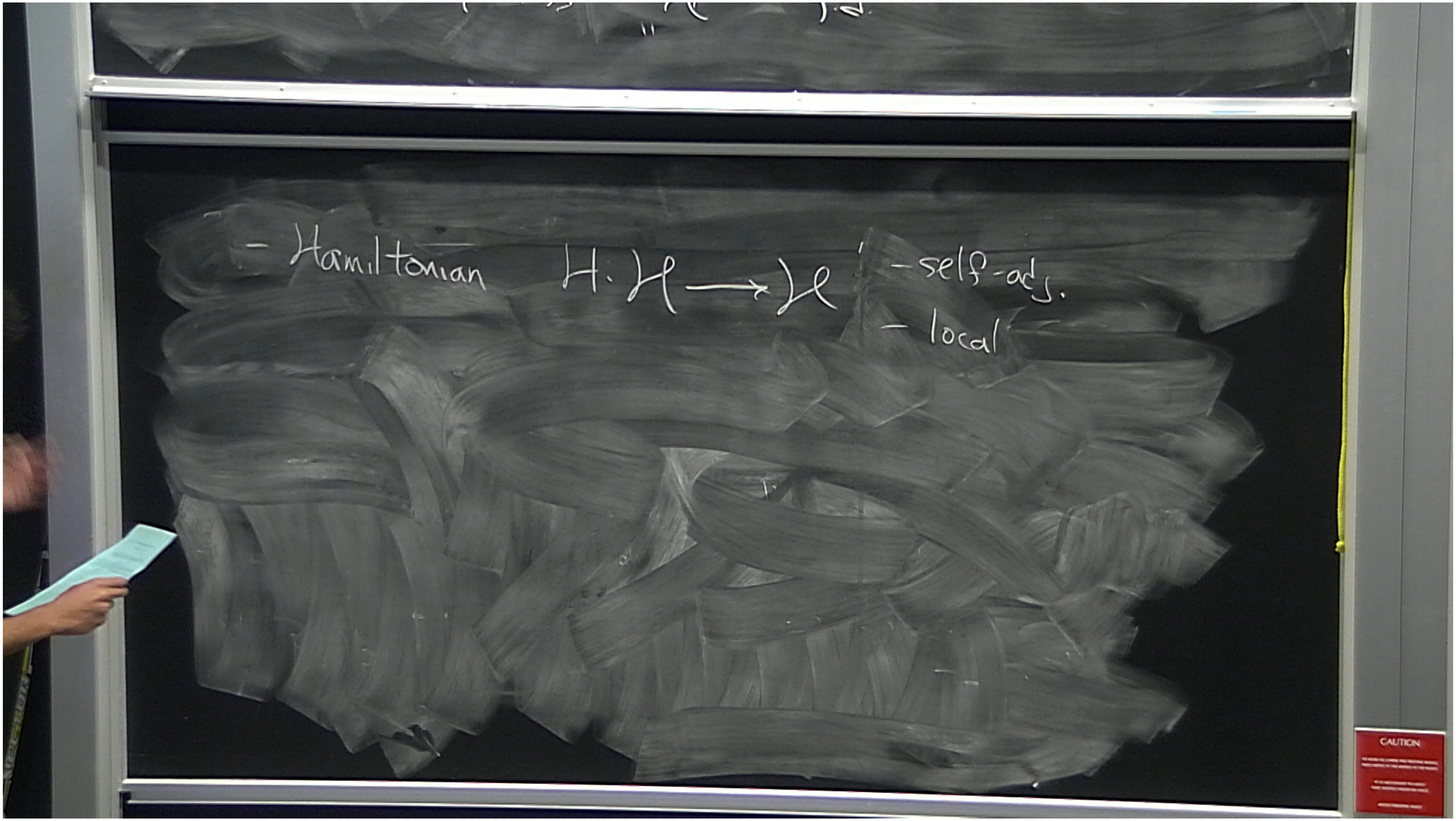
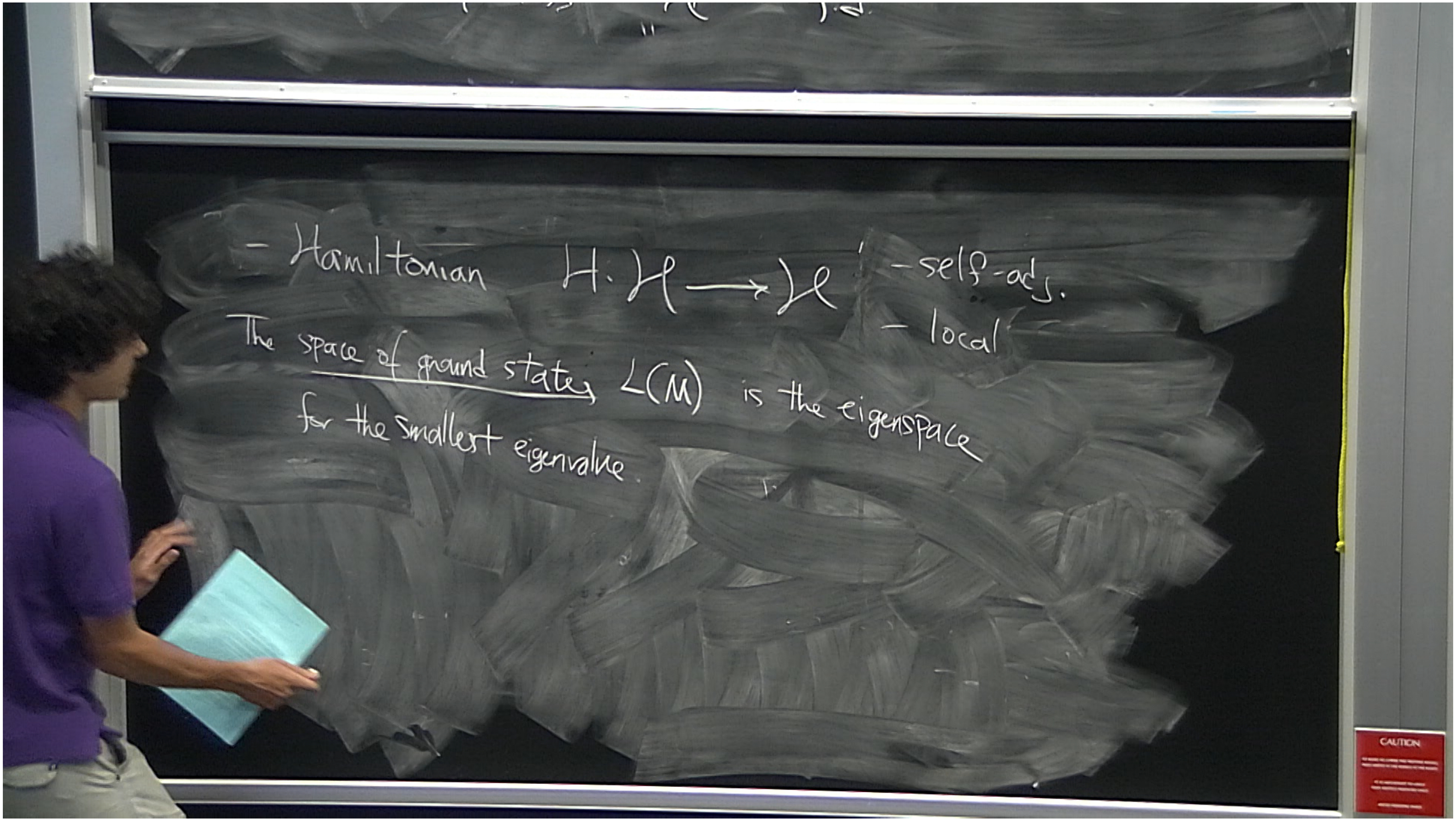
TQFT as low-energy theories of "topological phases of matter"


The low energy TQFT of the generalized double semion model
TQFT as low-energy theories of "topological phases of matter"
setup is a lattice field theory
start with a "lattice" on a closed d-mfld M:
CW cpx or PL str or △ation


field theory
Start with a "lattice" on a closed d-mfld M :
CW cpx or PL str or Δation
fields are discretized


PL str or Δation
- fields are discretized (local n discrete sense)
- state space {fields} → ℂ f.d.


- Hamiltonian $H: \mathcal{H} \to \mathcal{H}$
- self-adj.
- local



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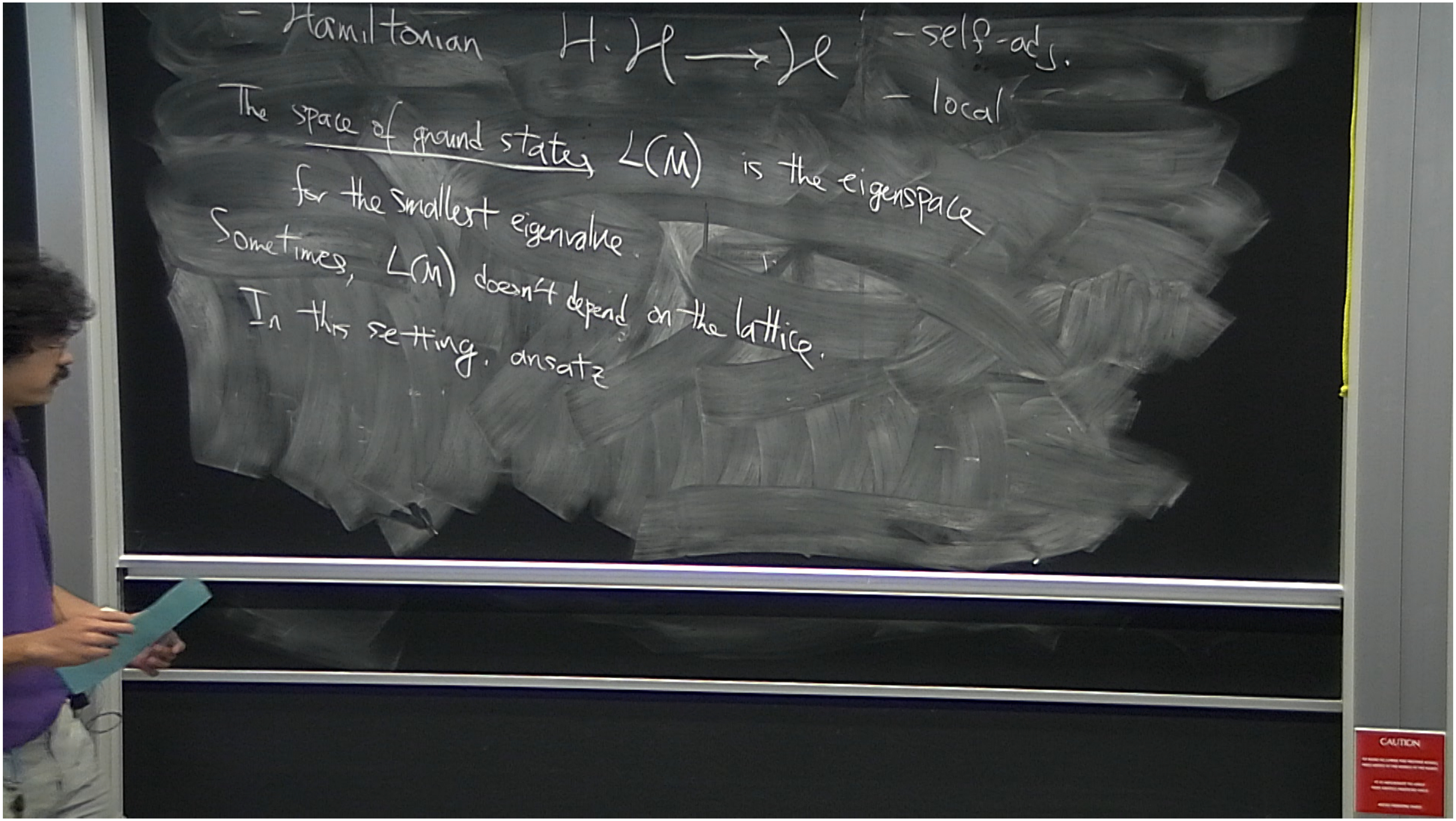
The space of ground states L(M) is the eigenspace
for the smallest eigenvalue.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Sometimes, L(M) doesn't depend on the lattice.
In this setting, ansatz

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The space of ground states $L(M)$ is the eigenspace for the smallest eigenvalue.

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In this setting, ansatz that there's a TQFT

$Z : \mathrm{Bord}_{d+1} \longrightarrow \mathrm{Vect}_{\mathbb{C}}$

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The space of ground states $L(M)$ is the eigenspace for the smallest eigenvalue.

Sometimes, $L(M)$ doesn't depend on the lattice.

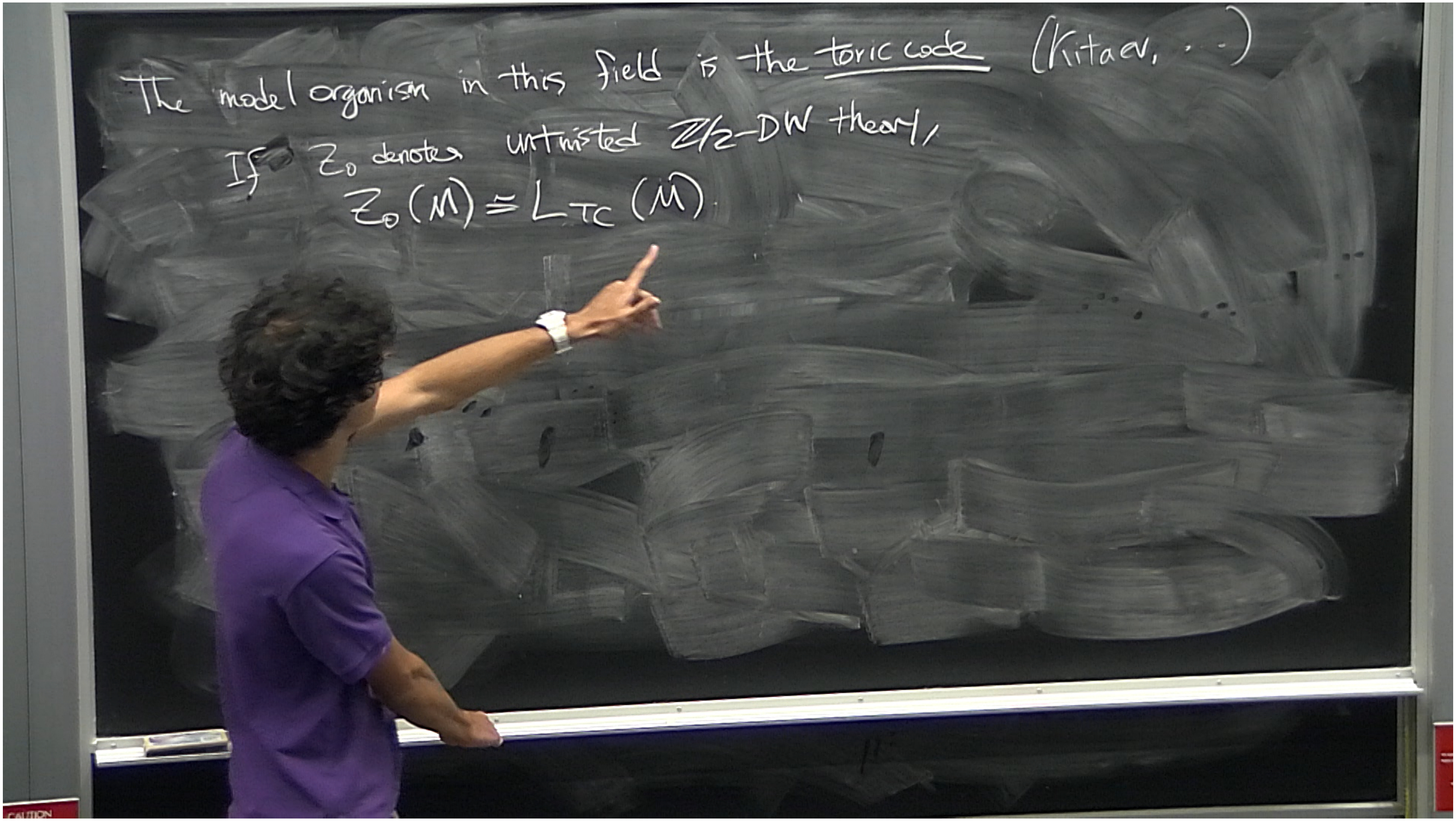
In this setting, ansatz that there's a TQFT $Z: \mathrm{Bord}_{d+1} \to \mathrm{Vect}_{\mathbb{C}}$

s.t. $Z(M) \cong L(M)$ $\forall M^d$.

In this case we say Z is the low energy TQFT of the model.

The model organism in this field is the toric code (Kitaev, . . .)

If Z_0 denotes untwisted $\mathbb{Z}/2$-DW theory,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$Z_0(M) \simeq L_{TC}(M)$

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$Z_{\alpha^3}(M) = Z_{DS}(M)$. (Freedman-Hastings) $\alpha^3 \in H^3(B\mathbb{Z}/2; \mathbb{R}/\mathbb{Z})$

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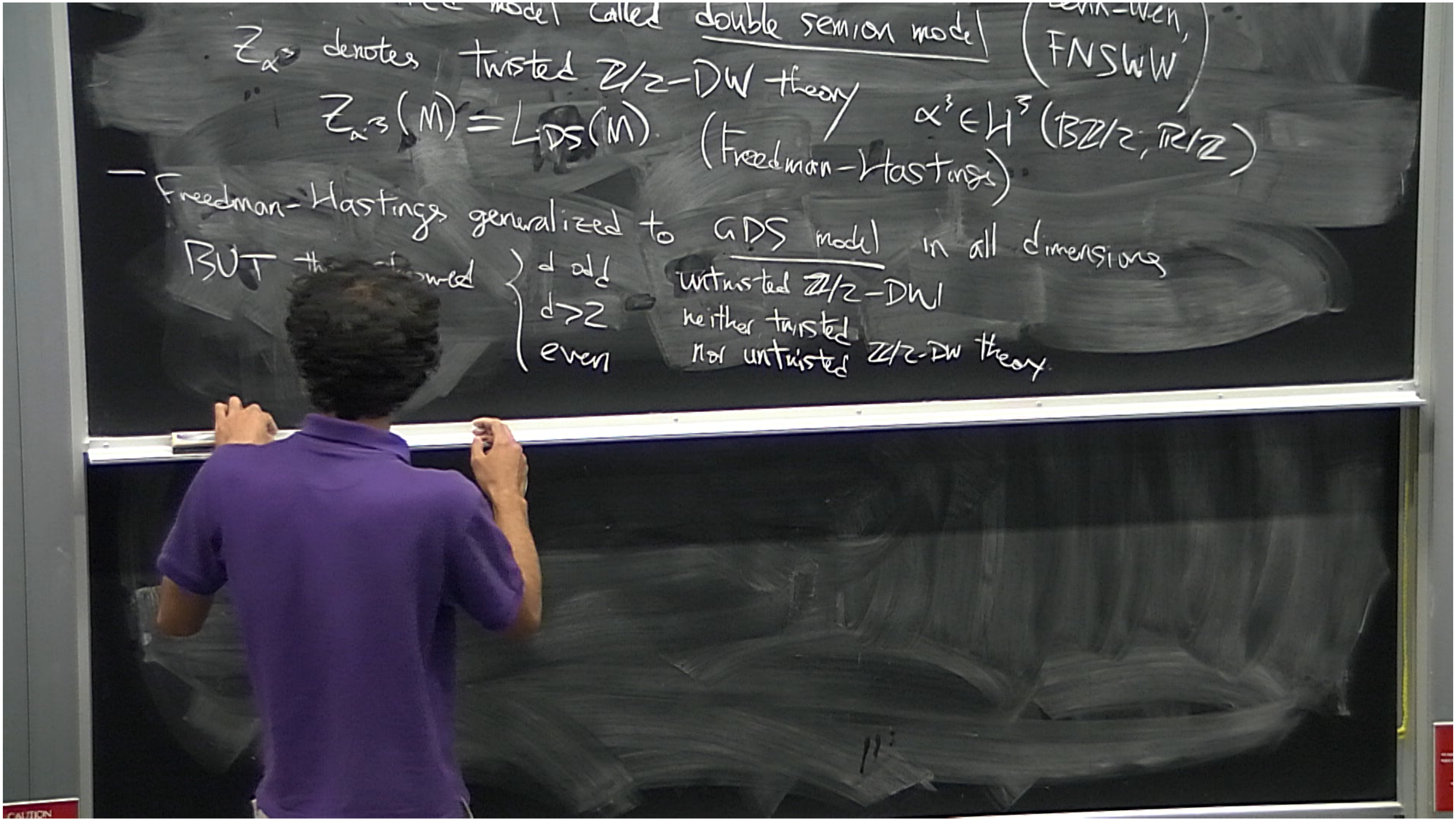
— Freedman-Hastings generalized to <u>GDS model</u> in all dimensions


model called double semion model
Levin-Wen, FNSWW
Z_α3 denotes twisted Z/2-DW theory
Z_α3(M) = L_DS(M)
(Freedman-Hastings)
α³ ∈ H³(BZ/2; R/Z)
— Freedman-Hastings generalized to GDS model in all dimensions
BUT the ... showed
d odd: untwisted Z/2-DW
d>2 even: neither twisted nor untwisted Z/2-DW theory

$Z_{\alpha^3}(M) = L_{DS}(M)$ (Freedman-Hastings) $\alpha^3 \in H^3(B\mathbb{Z}/2;\, \mathbb{R}/\mathbb{Z})$

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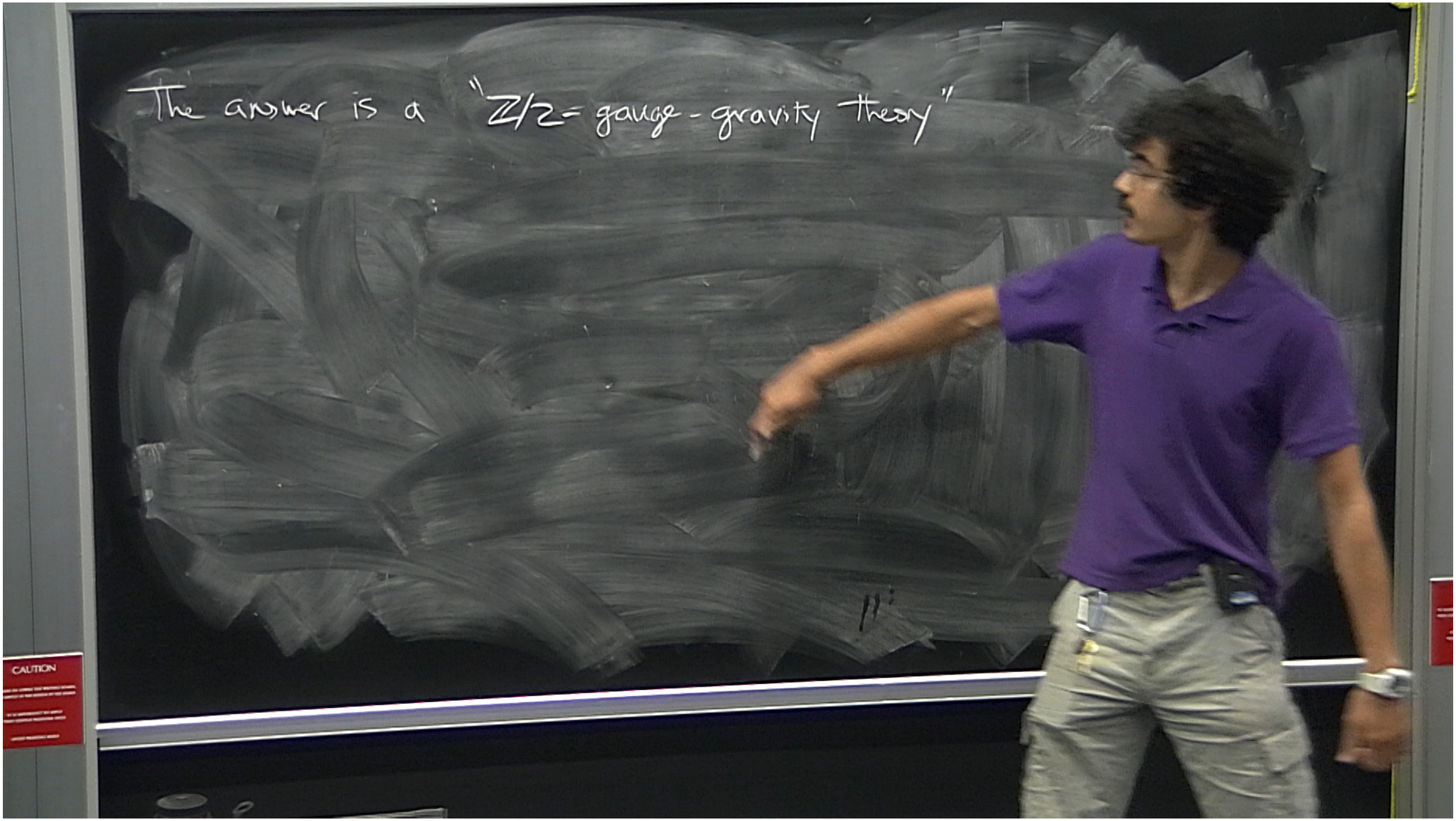
BUT they showed $\begin{cases} d \text{ odd} & \text{untwisted } \mathbb{Z}/2\text{-DW} \\ d>2 \text{ even} & \text{neither twisted nor untwisted } \mathbb{Z}/2\text{-DW theory} \end{cases}$


The answer is a "Z/2- gauge - gravity theory"
CAUTION

The answer is a "$\mathbb{Z}/2$-gauge-gravity theory"

given $\beta \in H^{d+1}(B\mathbb{Z}/2 \times BO_n; \mathbb{Z}/2)$

can define a TQFT

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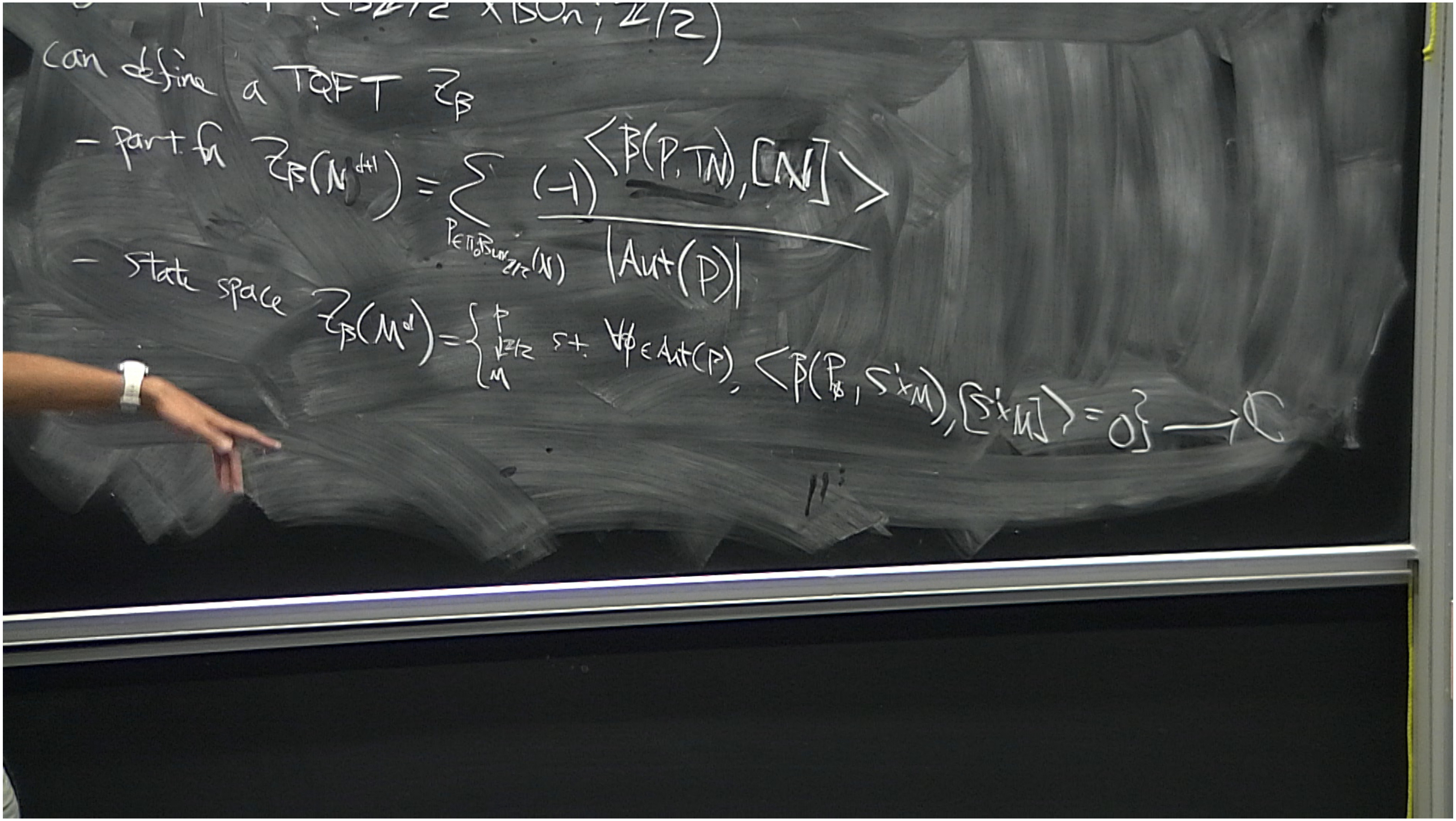
can define a TQFT Z_β

- part. fn $Z_\beta(N^{d+1}) = \sum_{P \in \pi_0 \mathrm{Bun}_{\mathbb{Z}/2}(N)} \frac{(-1)^{\langle \beta(P, TN), [N] \rangle}}{|\mathrm{Aut}(P)|}$



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- State space $Z_\beta(M^d) = \{ P \downarrow^{\mathbb{Z}/2} M$ s.t. $\forall \phi \in \mathrm{Aut}(P), \langle \beta(P_\phi, S^1 \times M), [S^1 \times M] \rangle = 0 \} \to \mathbb{C}$

The answer is a "$\mathbb{Z}/2$-gauge-gravity theory"

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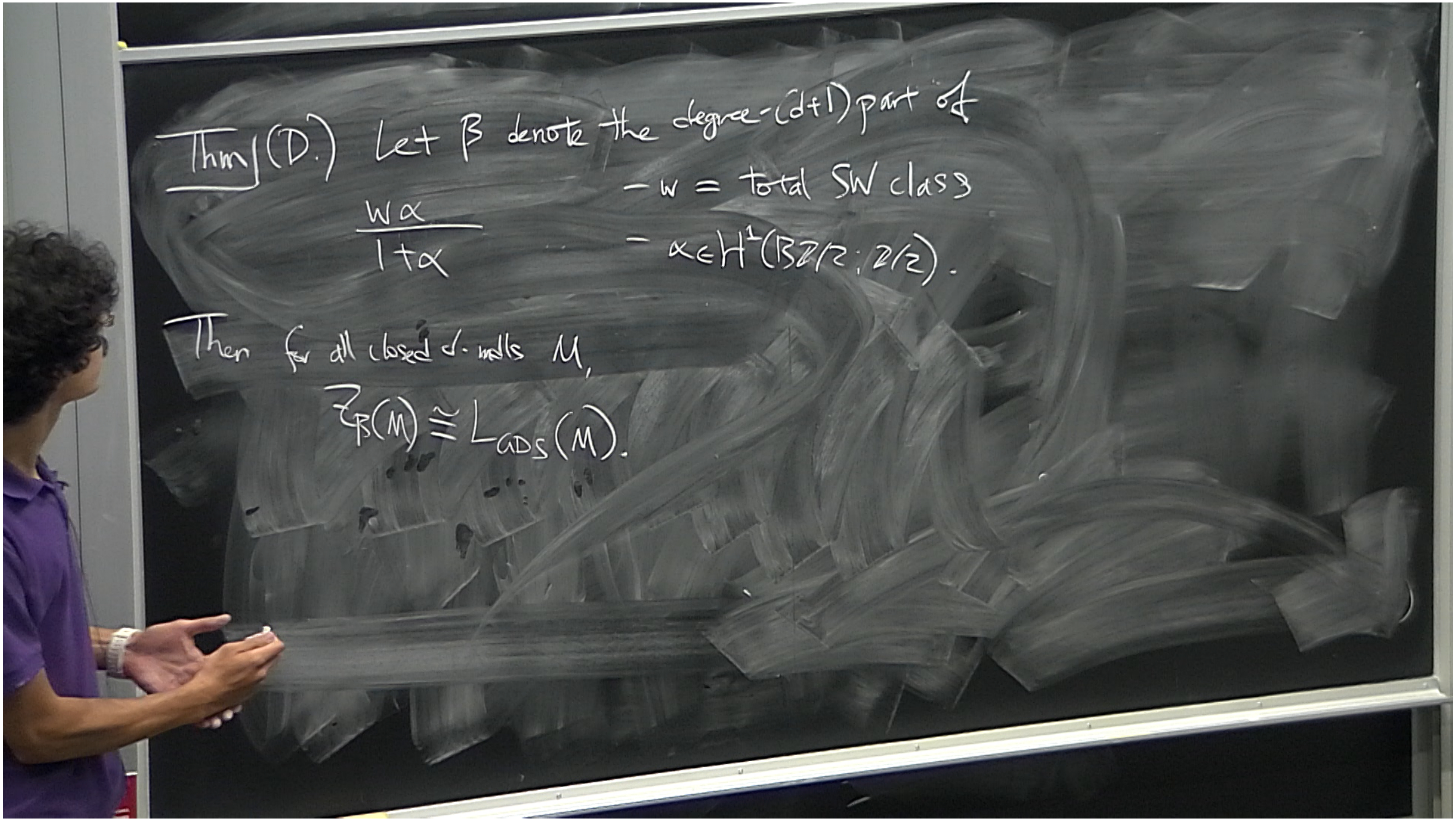
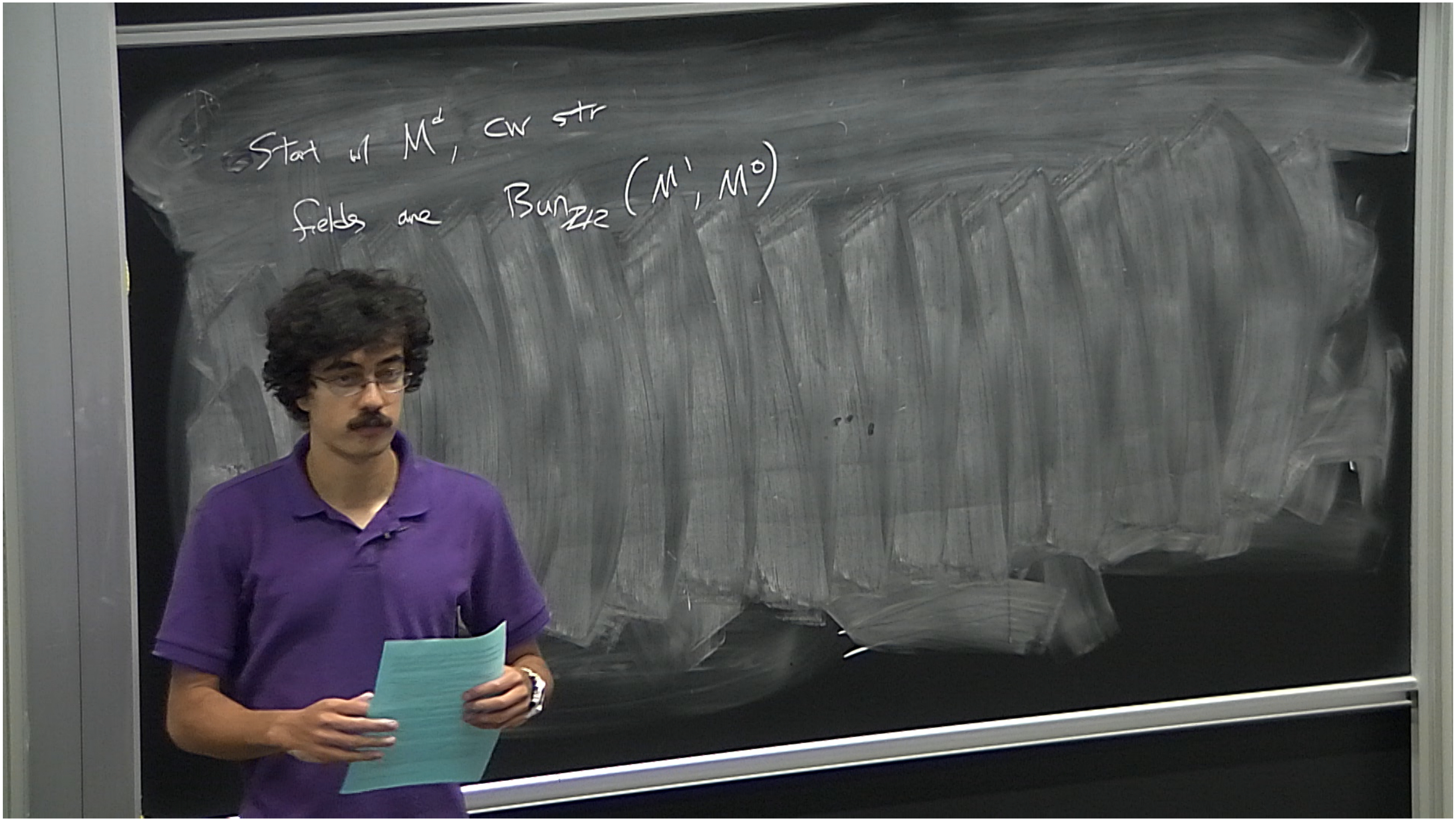
- state space $$Z_\beta(M^d) = \left\{ \begin{matrix} P \\ \downarrow \mathbb{Z}/2 \\ M \end{matrix} \text{ s.t. } \forall \phi \in \mathrm{Aut}(P),\ \langle \beta(P_\phi, S^1 \times M), [S^1 \times M] \rangle = 0 \right\} \longrightarrow \mathbb{C}$$


Thm (D.) Let β denote the degree-$(d+1)$ part of
$\frac{w\alpha}{1+\alpha}$
$-\ w$ = total SW class
$-\ \alpha \in H^1(B\mathbb{Z}/2; \mathbb{Z}/2)$.
Then for all closed d-mflds M,
$Z_\beta(M) \cong L_{ADS}(M)$.


Start w/ M^d, cw str
fields are $\mathrm{Bun}_{\mathbb{Z}/2}(M^1, M^0)$

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- Hamiltonian $H = \sum_v H_v + \sum_f H_f$

$H_v = \frac{1}{2}(1 - A_v)$ — A_v: switching the triv. at v

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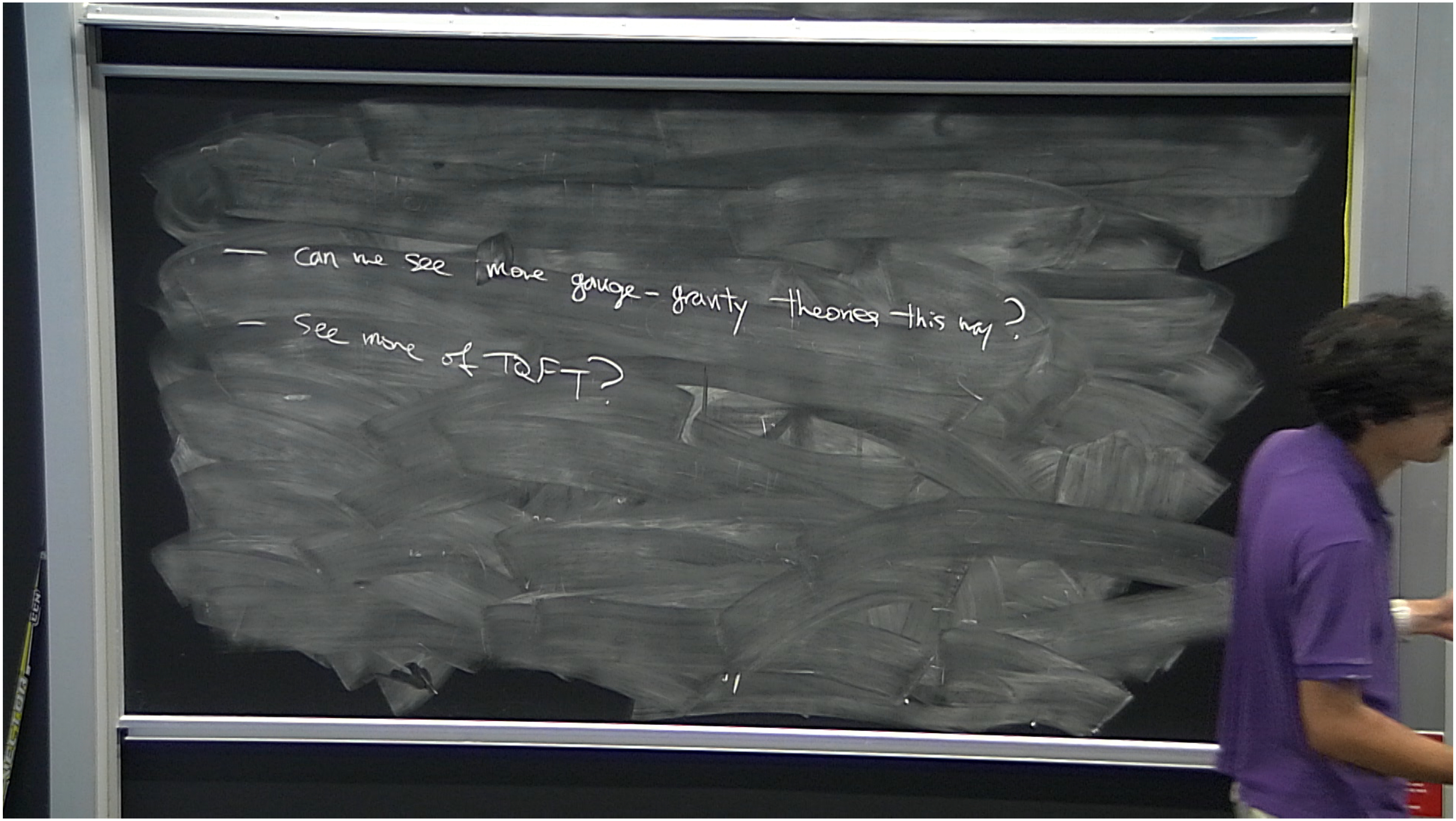
$H_f = \frac{1}{2}(1 - B_f)$ B_f: multiplies by $(-1)^{\mathrm{hol}}$

Space of ground states: $\mathbb{C}[\mathrm{Bun}_{\mathbb{Z}/2}(M)] = Z_0(M)$


— Can we see more gauge-gravity theories this way?
— See more of TQFT?