Title: N=1 supersymmetric vertex algebras of small index

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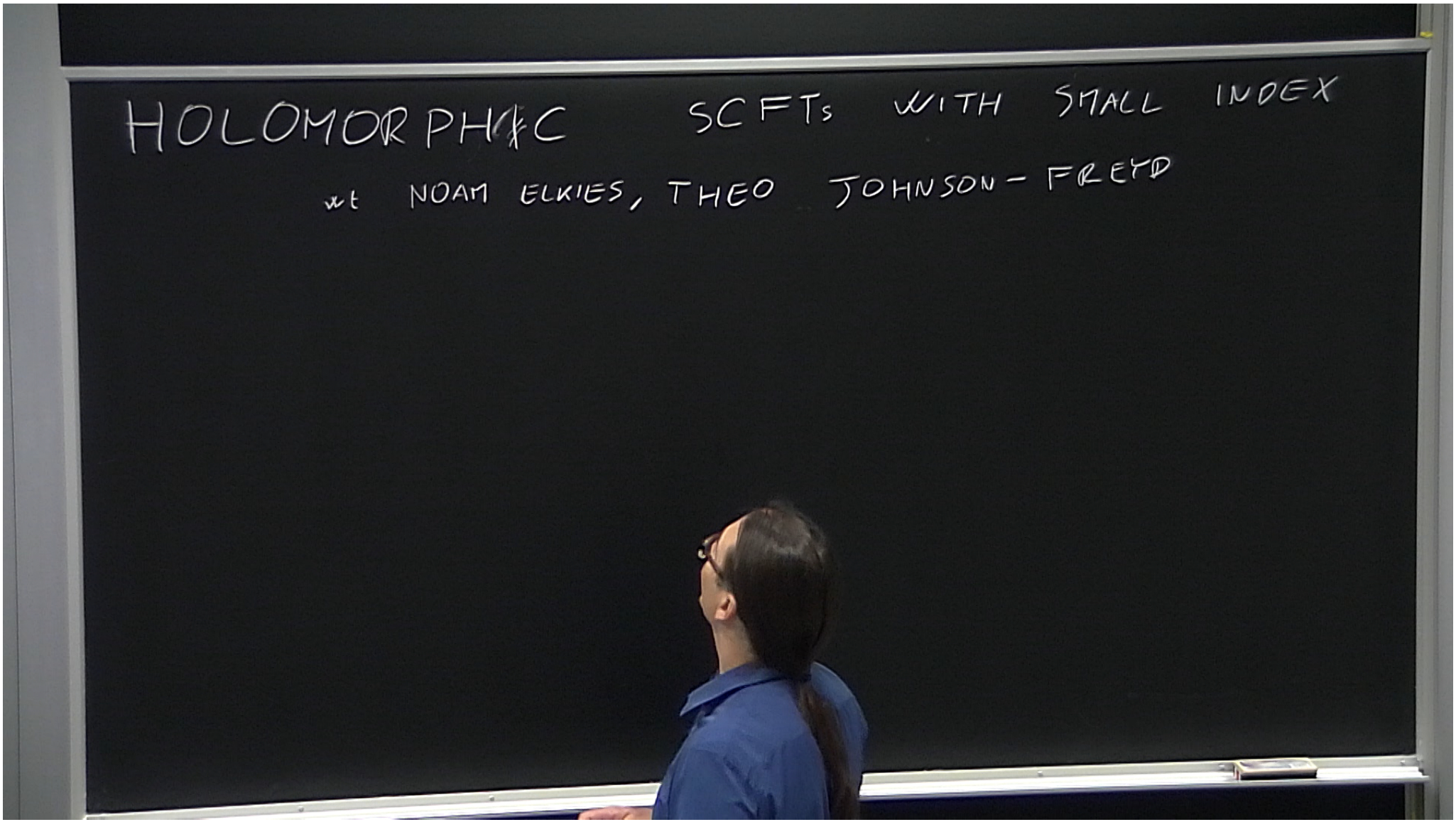
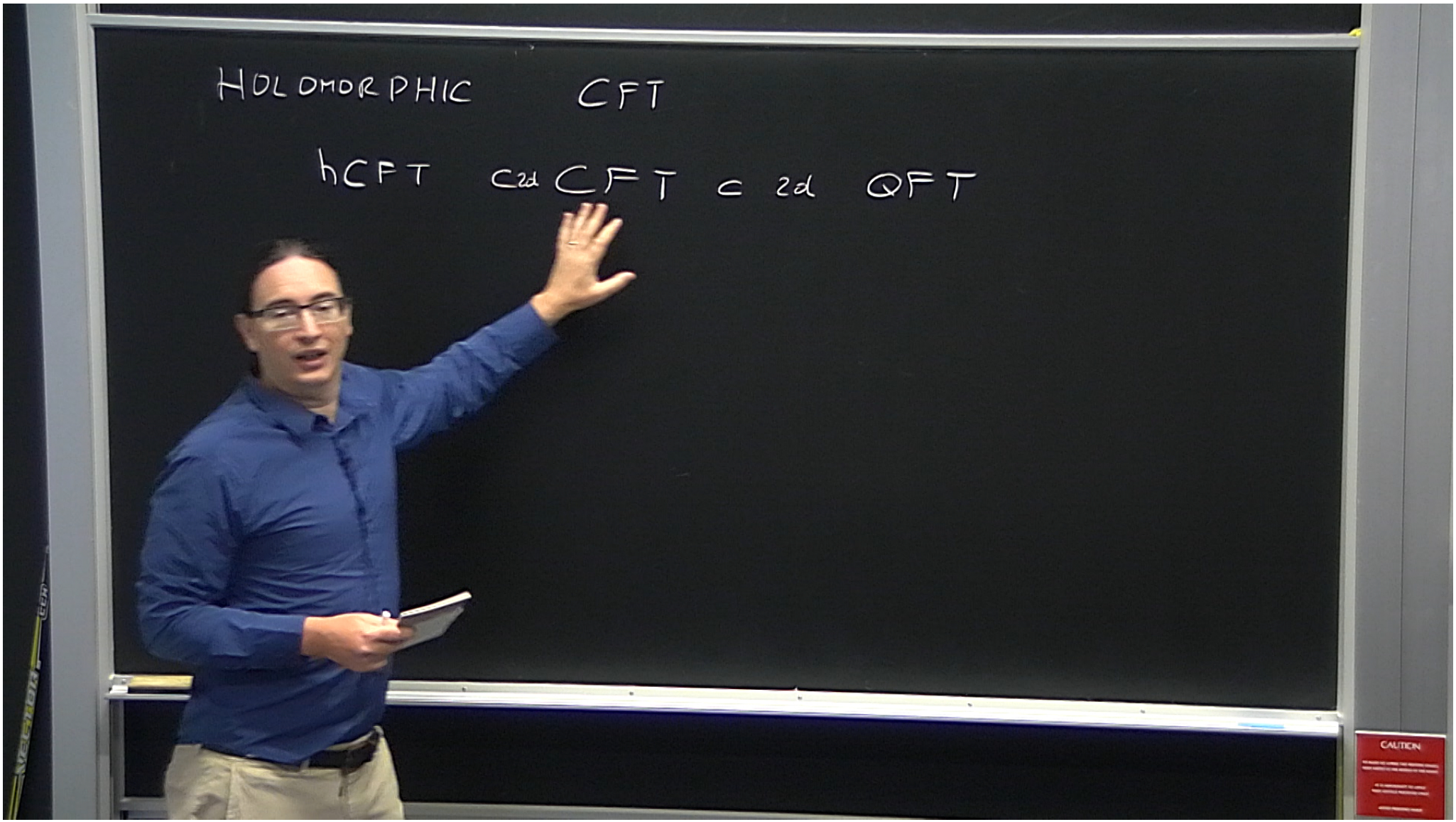
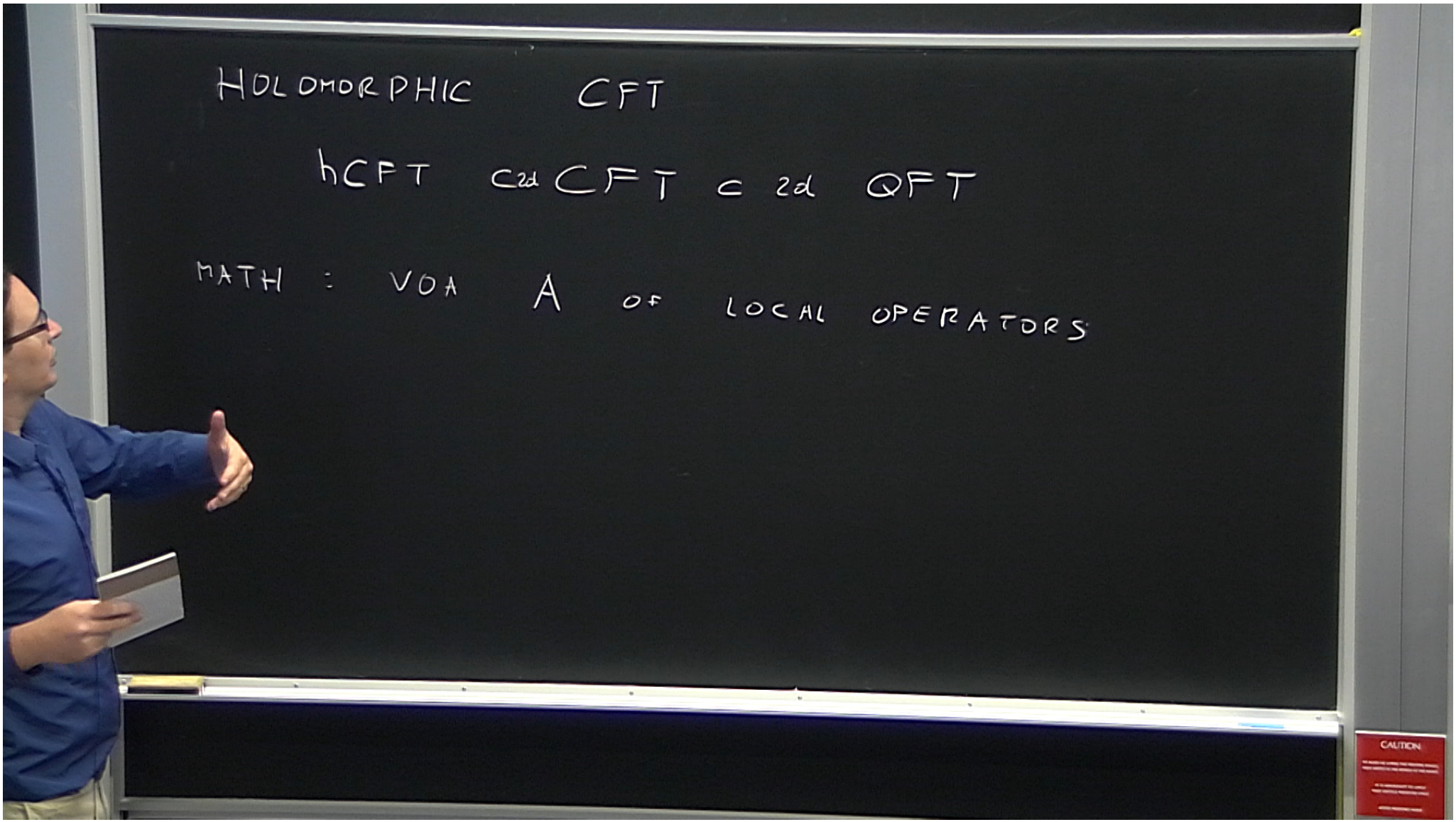
Abstract: I will describe examples of holomorphic N=1 super-symmetric vertex algebras with small (non-zero) values of the elliptic genus. I will speculate on a relation to certain patterns in the theory of topological modular forms.


HOLOMORPHIC SCFTs WITH SMALL INDEX
w/ NOAM ELKIES, THEO JOHNSON-FREYD


HOLOMORPHIC CFT
hCFT ⊂ 2d CFT ⊂ 2d QFT


HOLOMORPHIC CFT
hCFT ⊂ 2d CFT ⊂ 2d QFT
MATH : VOA A OF LOCAL OPERATORS

HOLOMORPHIC CFT

hCFT $\subset$ 2d CFT $\subset$ 2d QFT

MATH : VOA A OF LOCAL OPERATORS

A ONLY IRREDUCIBLE MODULE

HOLOMORPHIC CFT

hCFT $\subset$ 2d CFT $\subset$ 2d QFT

MATH : VOA A OF LOCAL OPERATORS

A ONLY IRREDUCIBLE MODULE

$A \ni T(z)$ $\quad T(z)\,T(0) \sim \frac{c}{z^4} + \frac{2T}{z^2} + \frac{\partial T}{z}$

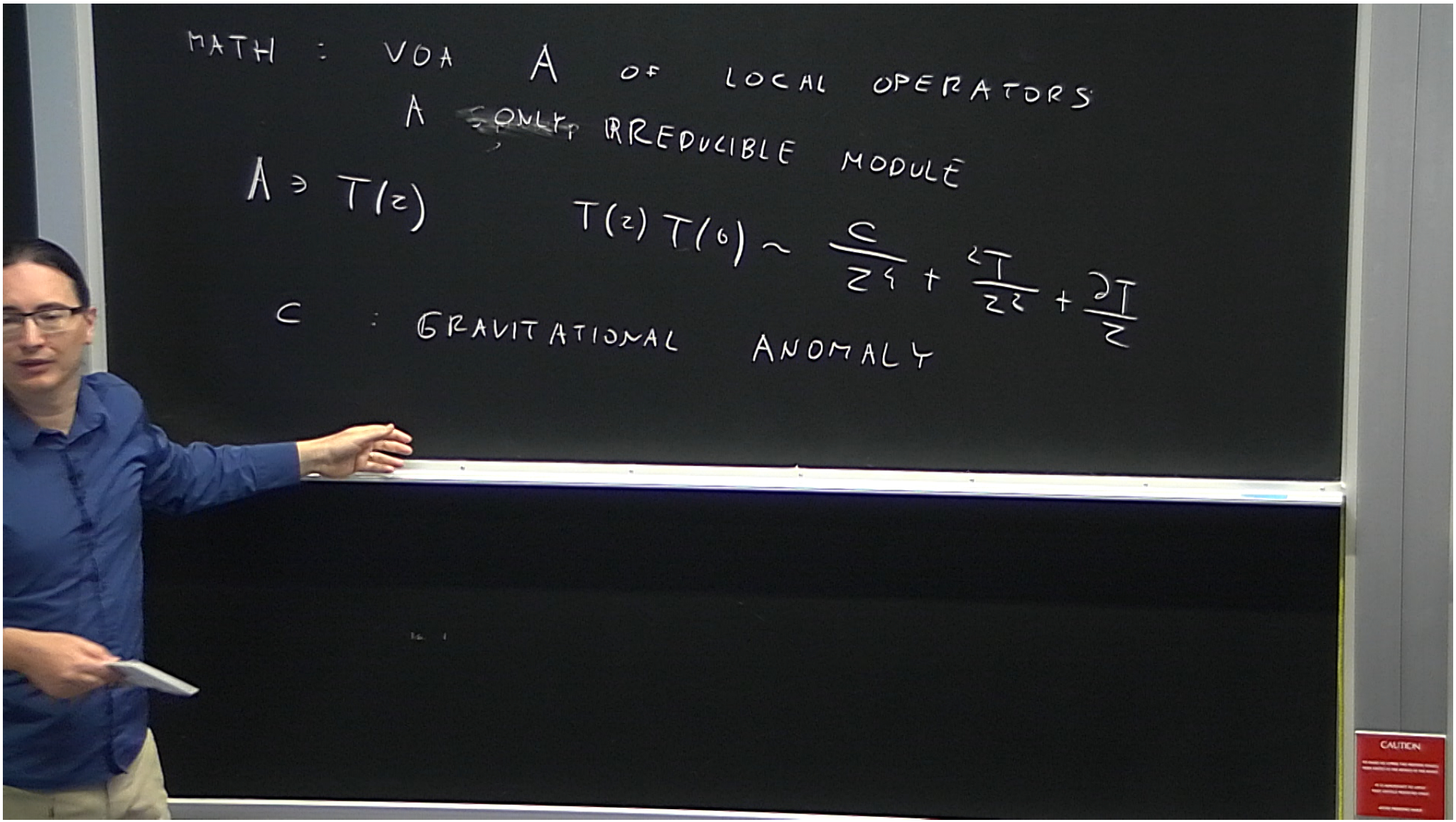
HOLOMORPHIC CFT (UNITARY)

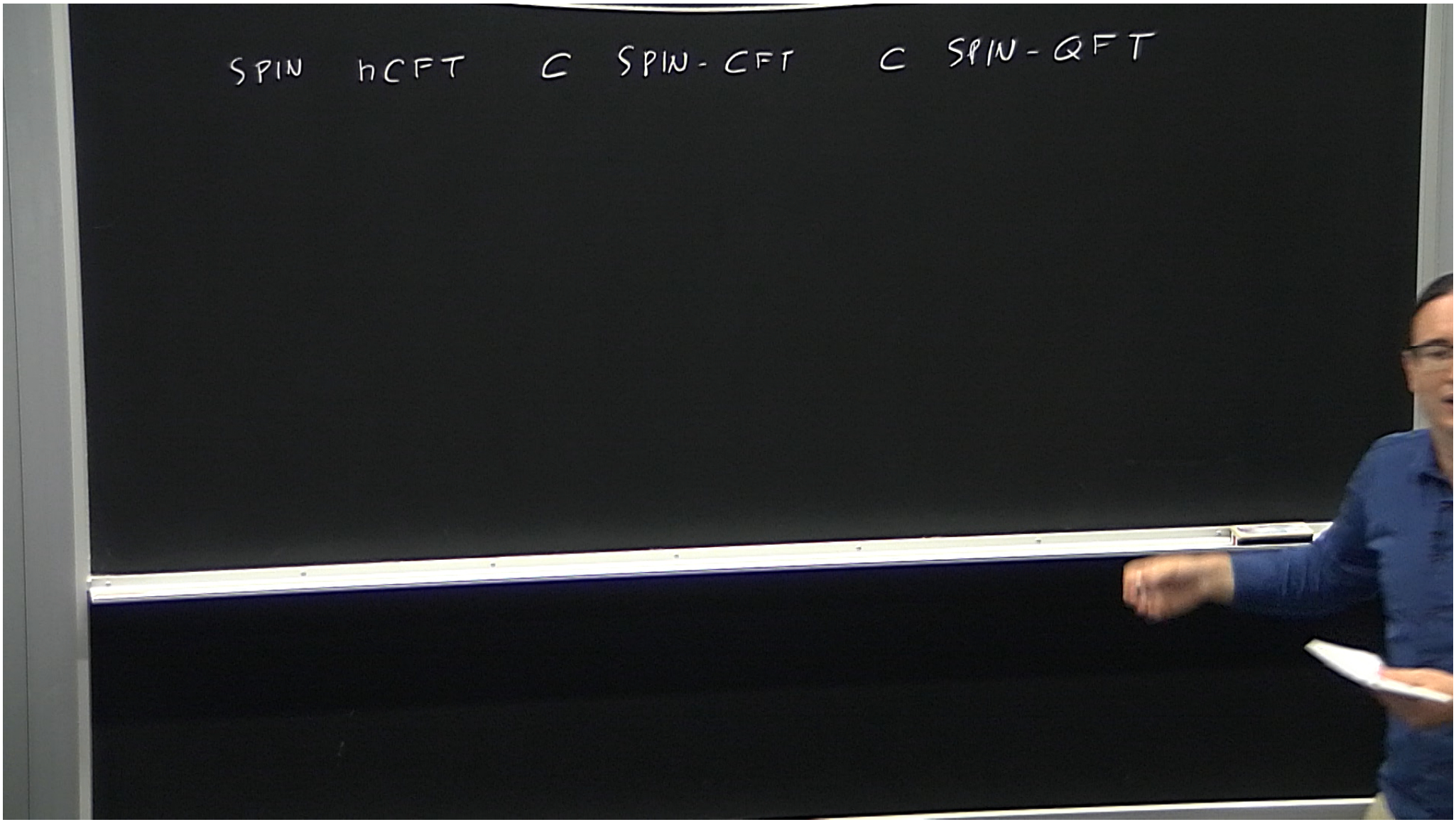
hCFT $\subset$ 2d CFT $\subset$ 2d QFT

MATH : VOA A OF LOCAL OPERATORS

A ONLY IRREDUCIBLE MODULE

$A \ni T(z)$ $\quad T(z)\,T(0) \sim \frac{c}{z^4} + \frac{2T}{z^2} + \frac{\partial T}{z}$


MATH : VOA A OF LOCAL OPERATORS
A ONLY IRREDUCIBLE MODULE
A ∋ T(z)
T(z) T(0) ~ c/z⁴ + 2T/z² + ∂T/z
c : GRAVITATIONAL ANOMALY


SPIN hCFT ⊂ SPIN-CFT ⊂ SPIN-QFT

$\mathcal{A}_e$ VOA OF BOSONIC LOCAL OPS

$\mathcal{A}_o$ MODULE OF FERMIONIC " "

$\mathcal{A}_e \oplus \mathcal{A}_o$

$\mathcal{A}_e$ VOA OF BOSONIC LOCAL OPS

$\mathcal{A}_o$ MODULE OF FERMIONIC " "

$\mathcal{A}_e \oplus \mathcal{A}_o = Z(S^1_b)$

SPIN hCFT $\subset$ SPIN-CFT $\subset$ SPIN-QFT

$\mathcal{A}_e$ VOA OF BOSONIC LOCAL OPS

$\mathcal{A}_o$ MODULE OF FERMIONIC " "

$\mathcal{A}_e \oplus \mathcal{A}_o = Z(S^1_b)$

$\mathcal{A}_R = Z(S^1_v)$

$N=1$ hSCFT $\subset$ $N=(1,0)$ SCFT $\subset$ $N=(1,0)$ SQFT

$N=1$ h SCFT $\subset$ $N=(1,0)$ SCFT $\subset$ $N=(1,0)$ SCFT

$A_e \ni T(z)$ $A_o \ni G(z)$ $h_G = \frac{3}{2}$

$G(z)G(0) \sim \frac{\hat{c}}{z^3} + \frac{T}{z}$

$N=1$ h SCFT $\subset$ $N=(1,0)$ SCFT $\subset$ $N=(1,0)$ SCFT

$A_e \ni T(z)$ $A_o \ni G(z)$ $h_G = \frac{3}{2}$

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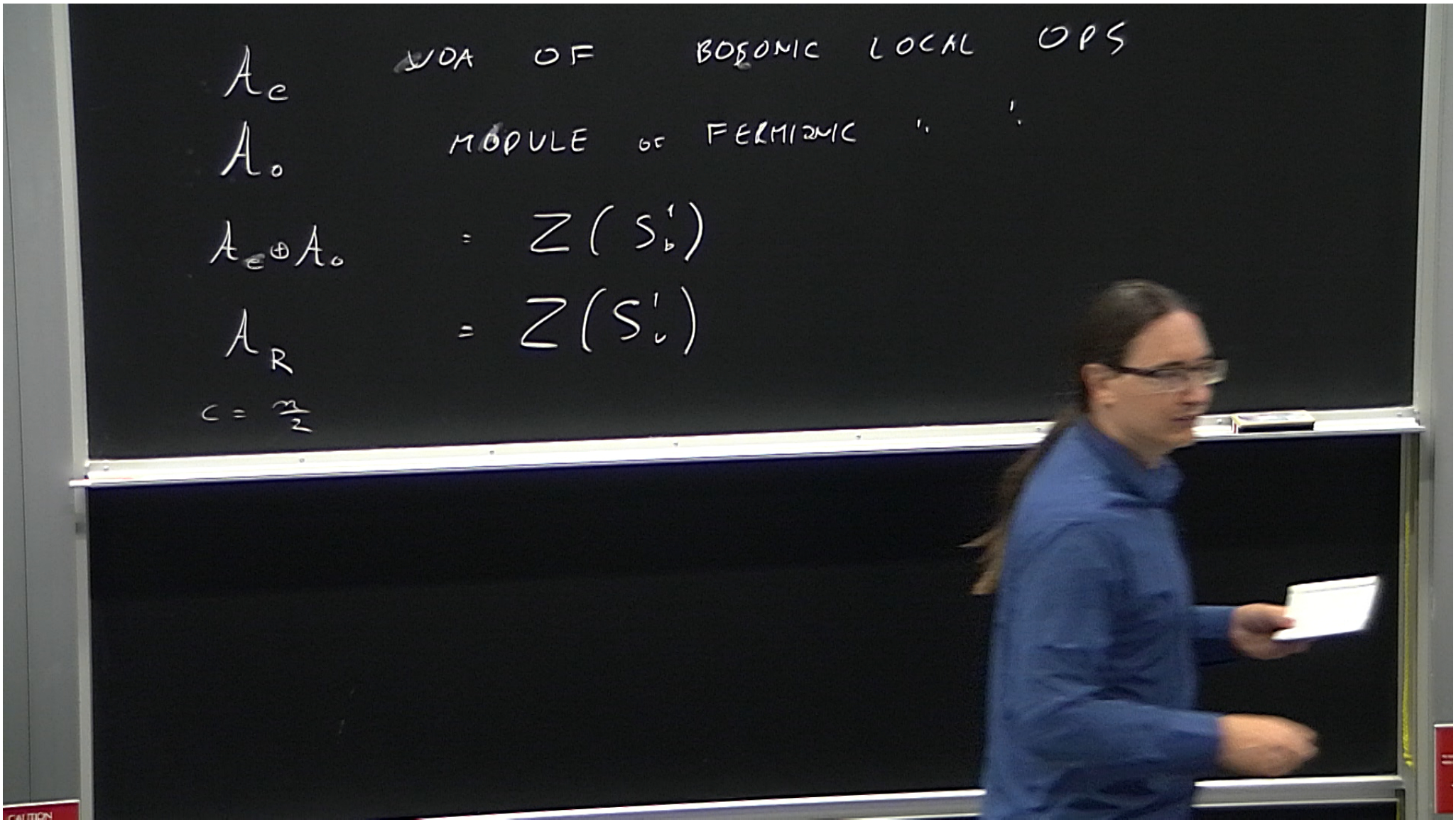
MATH : VOA $\mathcal{A}$ OF LOCAL OPERATORS

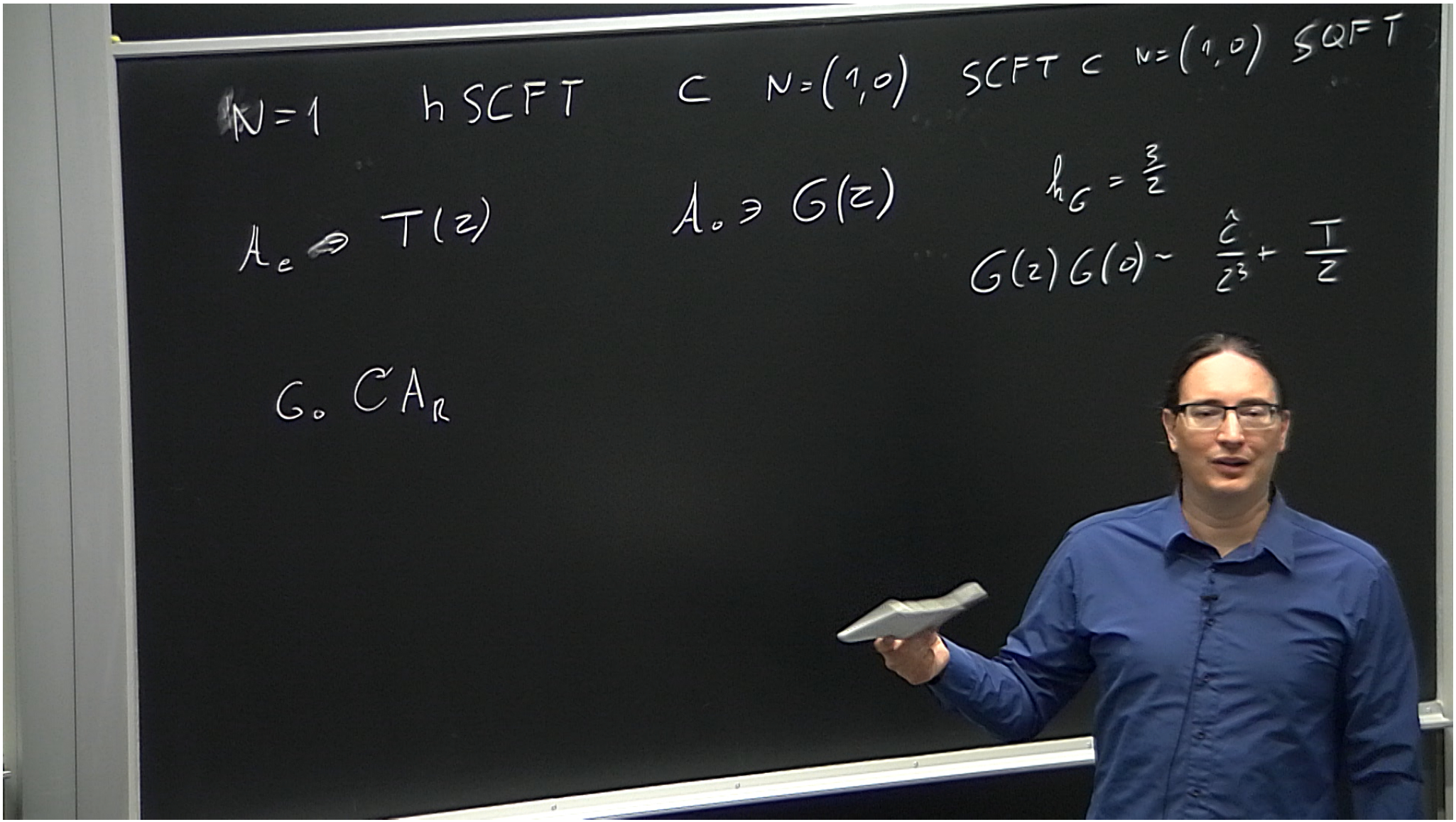
$\mathcal{A}$ ONLY IRREDUCIBLE MODULE

$\mathcal{A} \ni T(z)$ $\qquad T(z)\,T(0) \sim \frac{c}{z^4} + \frac{2T}{z^2} + \frac{\partial T}{z}$

c : GRAVITATIONAL ANOMALY

$c = 8n$


$\mathcal{A}_e$ VOA OF BOSONIC LOCAL OPS
$\mathcal{A}_o$ MODULE OF FERMIONIC " "
$\mathcal{A}_e \oplus \mathcal{A}_o = Z(S^1_b)$
$\mathcal{A}_R = Z(S^1_v)$
$c = \frac{n}{2}$


N=1 hSCFT ⊂ N=(1,0) SCFT ⊂ N=(1,0) SQFT
A_e ∋ T(z)
A_o ∋ G(z)
h_G = 3/2
G(z)G(0) ~ ĉ/z^3 + T/z
G_0 ⊂ A_R

$N=1$ h SCFT $\subset$ $N=(1,0)$ SCFT $\subset$ $N=(1,0)$ SQFT

$A_e \ni T(z)$ $\qquad$ $A_o \ni G(z)$ $\qquad$ $h_G = \frac{3}{2}$

$$G(z)G(0) \sim \frac{\hat{c}}{z^3} + \frac{T}{z}$$

$G_0 \subset A_R$ $\qquad$ $G_0^2 = L_0 - \frac{c}{24}$

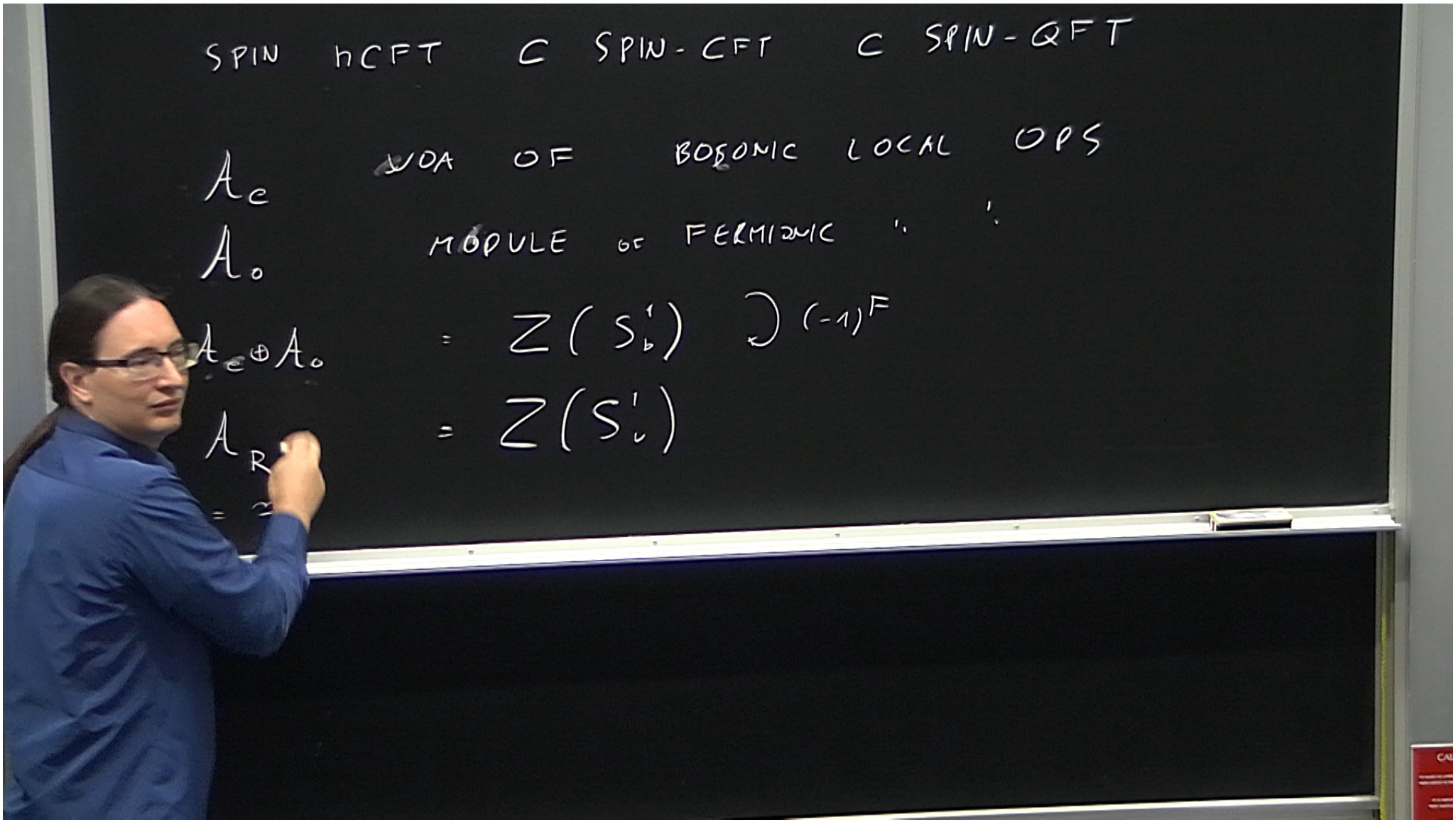
$N=1$ h SCFT $\subset$ $N=(1,0)$ SCFT $\subset$ $N=(1,0)$ ...

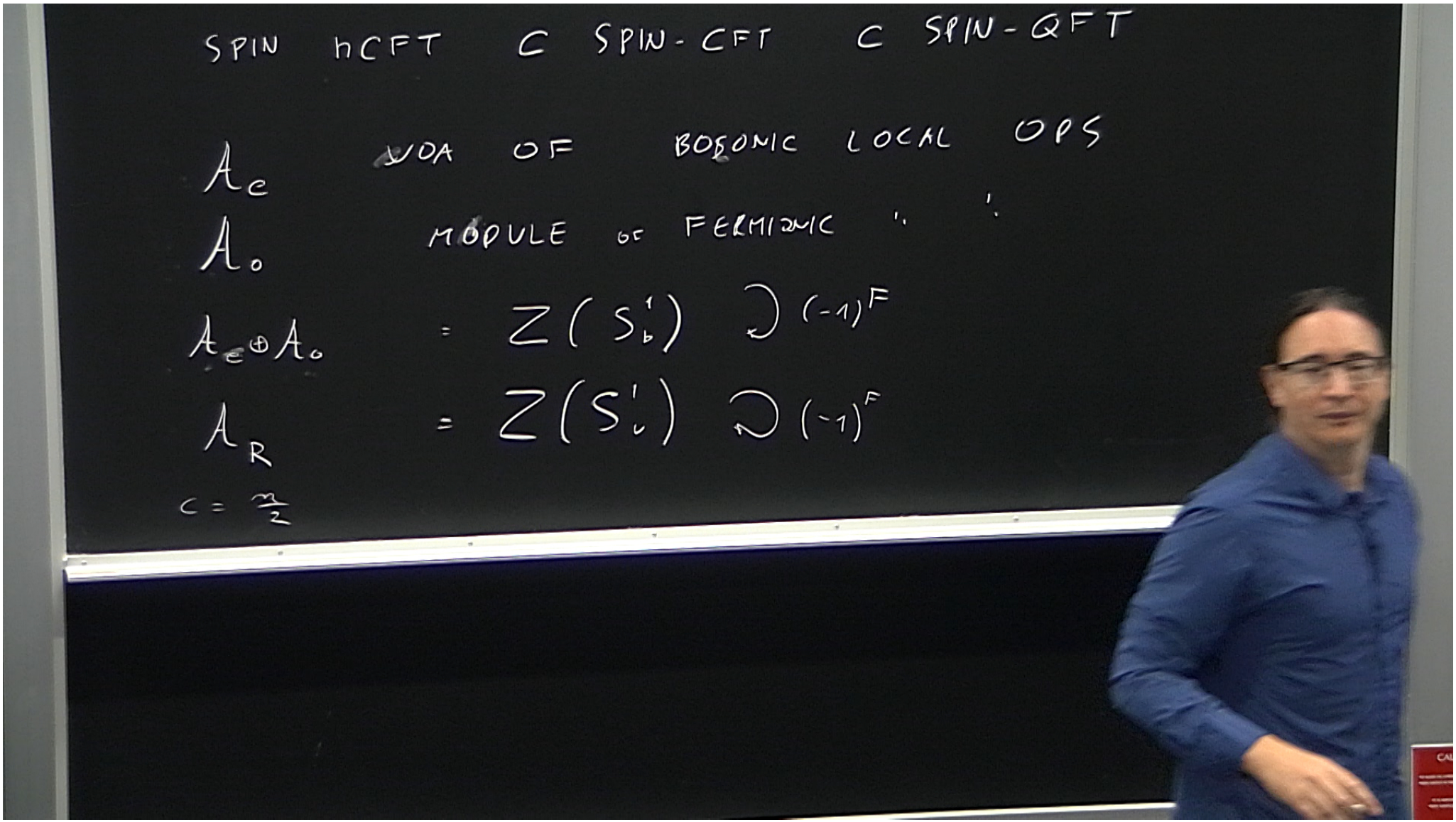
$\mathcal{A}_e \ni T(z)$ $\qquad$ $\mathcal{A}_o \ni G(z)$ $\qquad$ $h_G = \frac{3}{2}$

$$G(z)G(0) \sim \frac{\hat{c}}{z^3} + \frac{T}{z}$$

$G_0 \subset \mathcal{A}_R$ $\qquad$ $G_0^2 = L_0 - \frac{c}{24}$

$$\chi = \mathrm{Tr}_{\mathcal{A}_R} (-1)^F q^{L_0 - \frac{c}{24}}$$

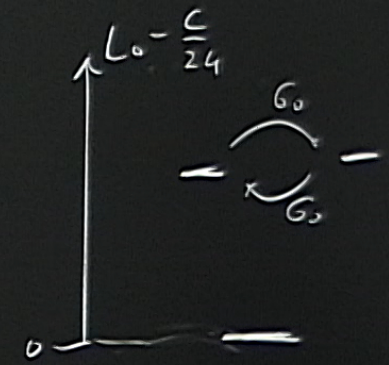

SPIN hCFT ⊂ SPIN-CFT ⊂ SPIN-QFT
$\mathcal{A}_e$ VOA OF BOSONIC LOCAL OPS
$\mathcal{A}_o$ MODULE OF FERMIONIC " "
$\mathcal{A}_e \oplus \mathcal{A}_o$ = $Z(S^1_b) \circlearrowleft (-1)^F$
$\mathcal{A}_R$ = $Z(S^1_v)$


SPIN hCFT ⊂ SPIN-CFT ⊂ SPIN-QFT
$\mathcal{A}_e$ VOA OF BOSONIC LOCAL OPS
$\mathcal{A}_o$ MODULE OF FERMIONIC " "
$\mathcal{A}_e \oplus \mathcal{A}_o = Z(S^1_b) \circlearrowleft (-1)^F$
$\mathcal{A}_R = Z(S^1_v) \circlearrowleft (-1)^F$
$c = \frac{n}{2}$

$A_e \ni T(z)$ $\qquad$ $A_o \ni G(z)$

$$G(z)G(0) \sim \frac{\hat{c}}{z^3} + \frac{T}{z}$$

$G_0 \subset A_R$ $\qquad$ $G_0^2 = L_0 - \frac{c}{24}$



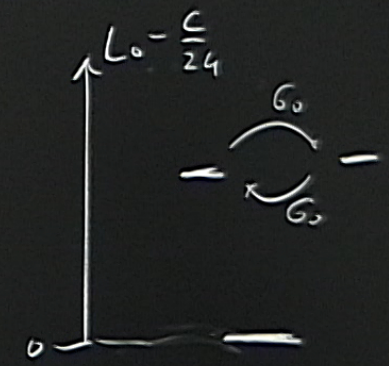
$$\chi = \mathrm{Tr}_{A_R} (-1)^F q^{L_0 - \frac{c}{24}} = \text{INTEGER}$$

$N=1$ hSCFT $\subset$ $N=(1,0)$ SCFT $\subset$ $N=(1,0)$ SQFT

$\mathcal{A}_e \ni T(z)$ $\qquad$ $\mathcal{A}_o \ni G(z)$ $\qquad$ $h_G = \frac{3}{2}$

$$G(z)G(0) \sim \frac{\hat{c}}{z^3} + \frac{T}{z}$$

$G_0 \subset \mathcal{A}_R$ $\qquad$ $G_0^2 = L_0 - \frac{c}{24}$



$$\chi = \mathrm{Tr}_{\mathcal{A}_R} (-1)^F q^{L_0 - \frac{c}{24}} = \text{INTEGER}$$

$$\chi = Z_{T^2}, \text{ ODD SPIN STRUCTURE} \quad (\bar{q})$$

FINP $\quad N=1 \quad$ hSCFT $\quad c = 12k$

$$\chi = \frac{24}{\gcd(24, k)}$$

FINP $\quad N=1 \quad$ hSCFT $\quad c = 12k$

$$\chi = \frac{24}{\gcd(24,k)}$$

$k = 1, 2, 3, 4, 5$

$c = 8n$

LATTICE hCFT

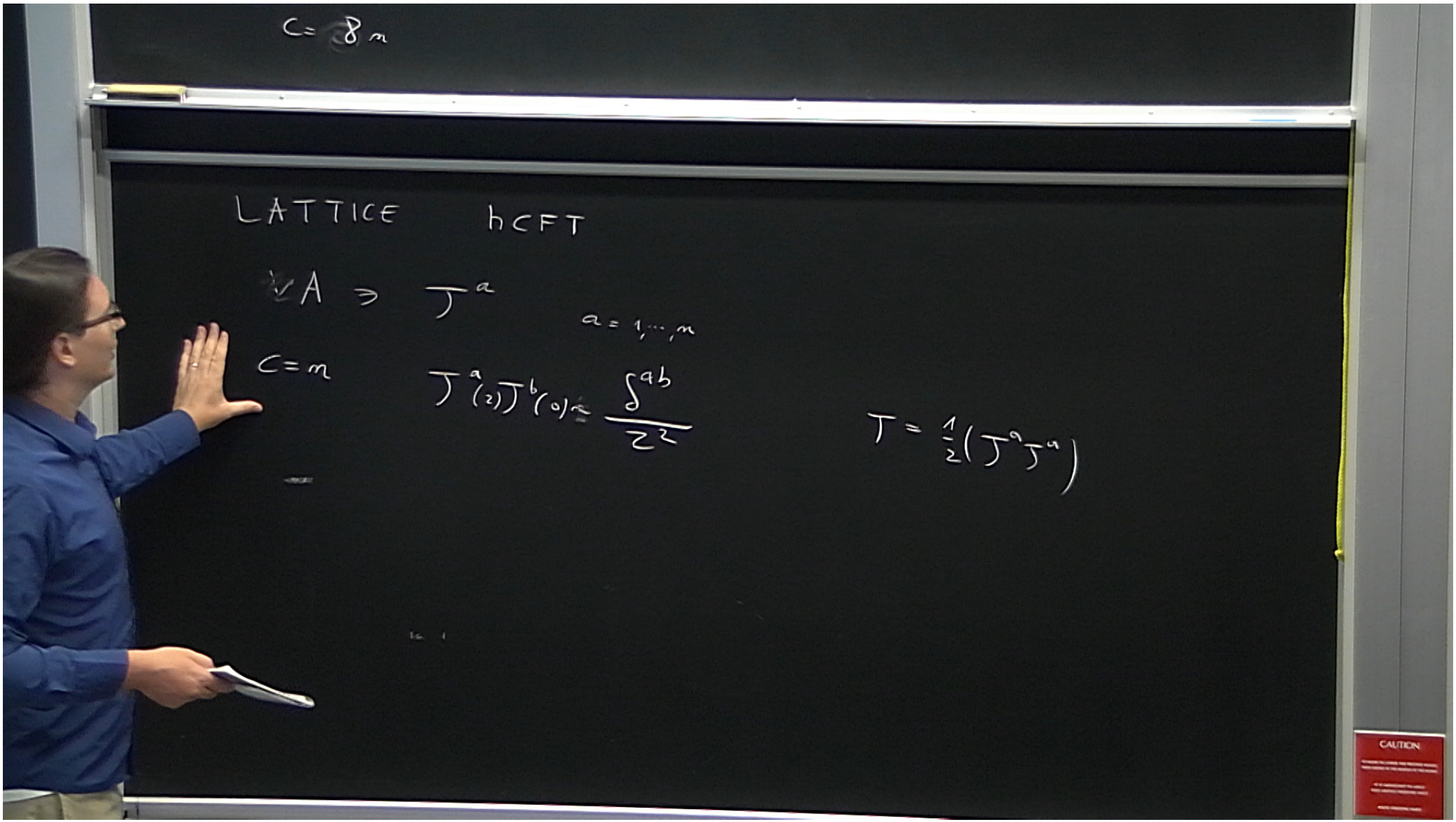
LATTICE hCFT

$A \Rightarrow J^a$

$a = 1, \cdots, n$

$c = n$

$$J^a(z) J^b(0) \sim \frac{\delta^{ab}}{z^2}$$


C= 8n
LATTICE hCFT
A ⇒ J^a
a = 1,⋯,n
C=n
J^a(z)J^b(0) ~ δ^ab / z^2
T = 1/2 (J^a J^a)

$c = 8n$

$c = n$

$$J^a(z) J^b(0) \sim \frac{\delta^{ab}}{z^2}$$

$$T = \frac{1}{2}(J^a J^a)$$

$$A = \sum_{\lambda \in L} M_\lambda \qquad \lambda \in R^n \qquad h_\lambda = \frac{\lambda^2}{2}$$

L. EVEN SELF-DUAL LATTICE

LATTICE SPIN-LCFT

L : SELF-DUAL LATTICE

$A_e : L_e$ $A_o : L_o$ $A_R : L + \frac{w}{2}$

$w \cdot v = v^2 \mod 2$

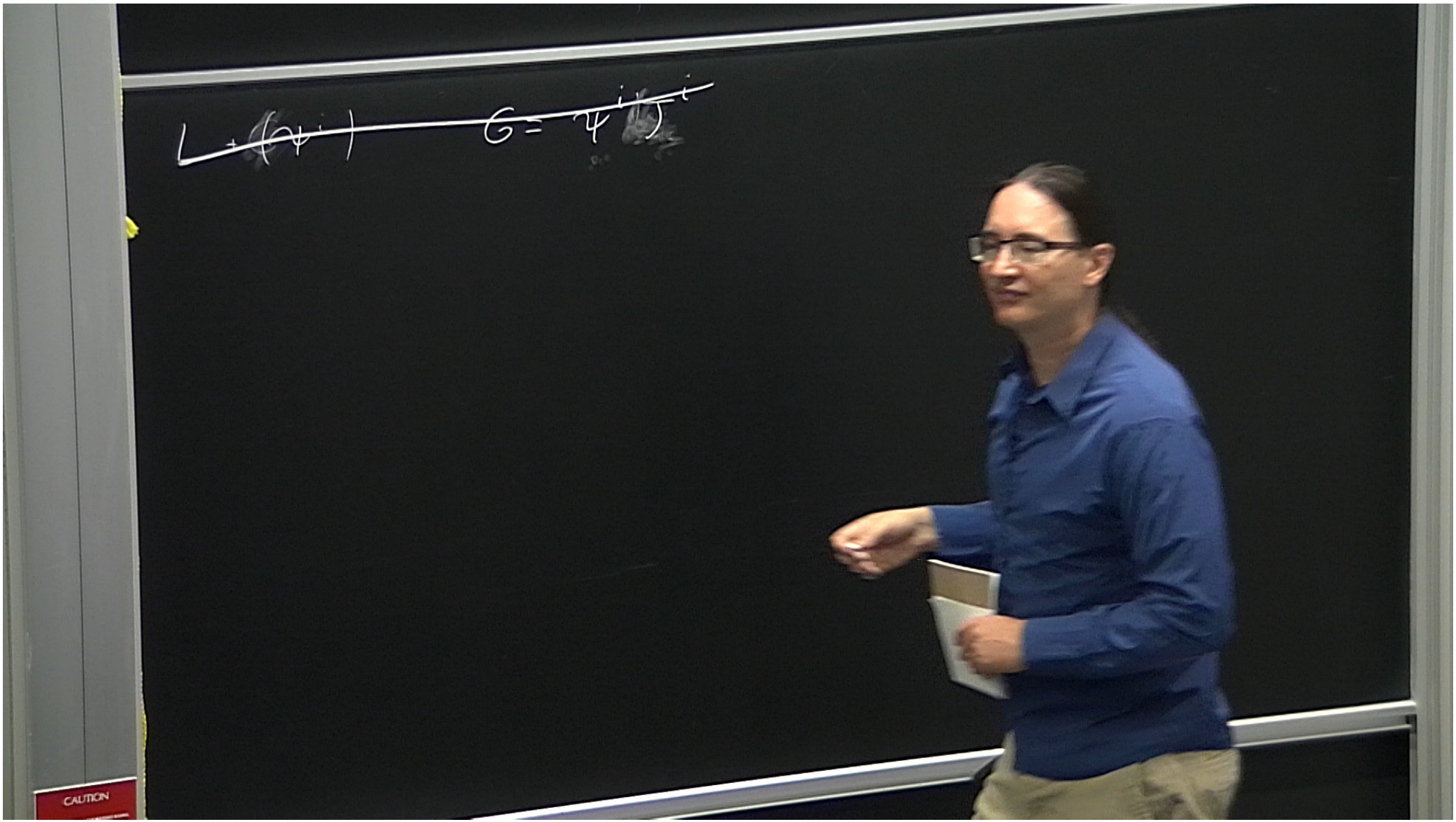
LATTICE SPIN-LCFT

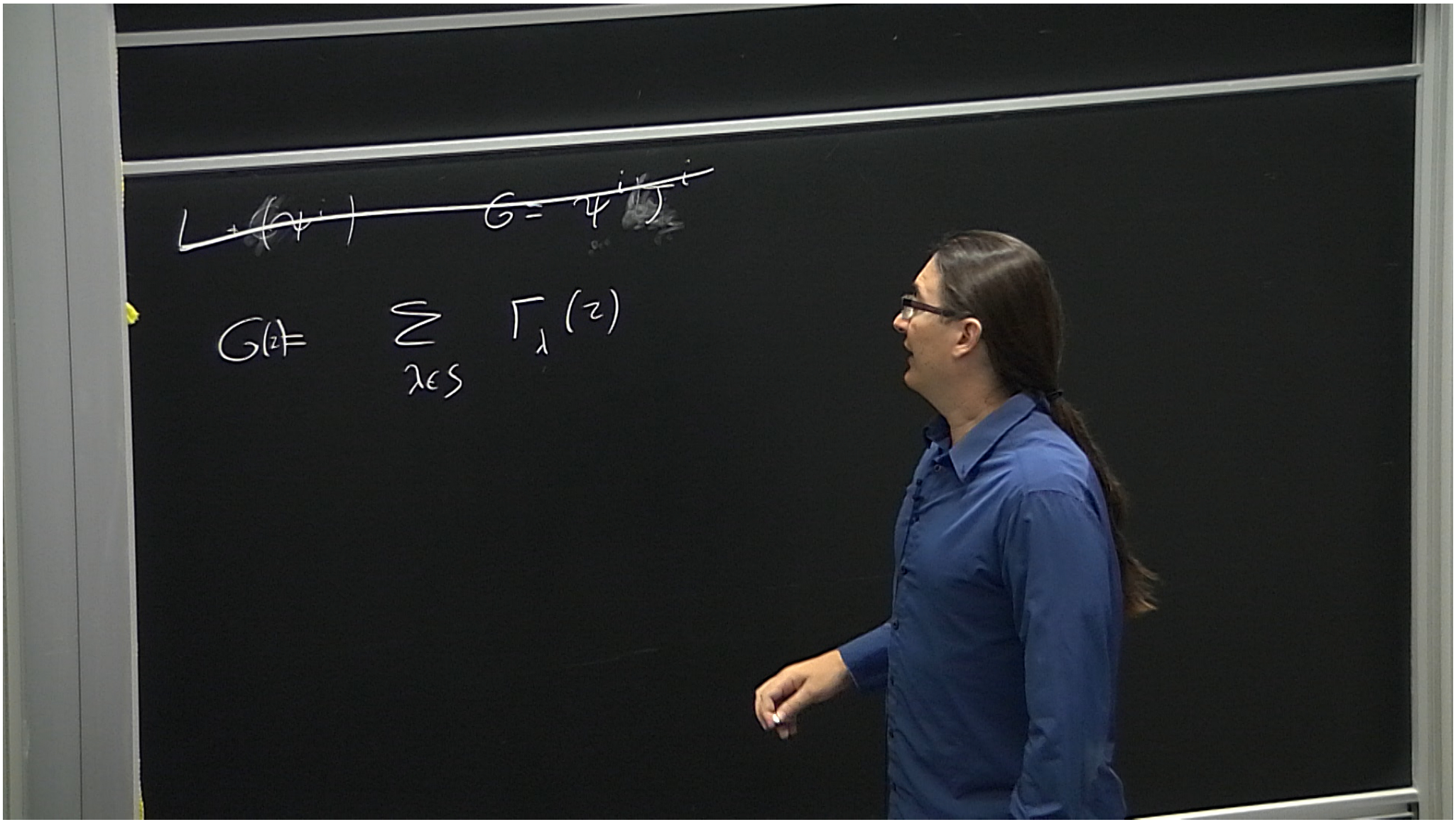
L : SELF-DUAL LATTICE

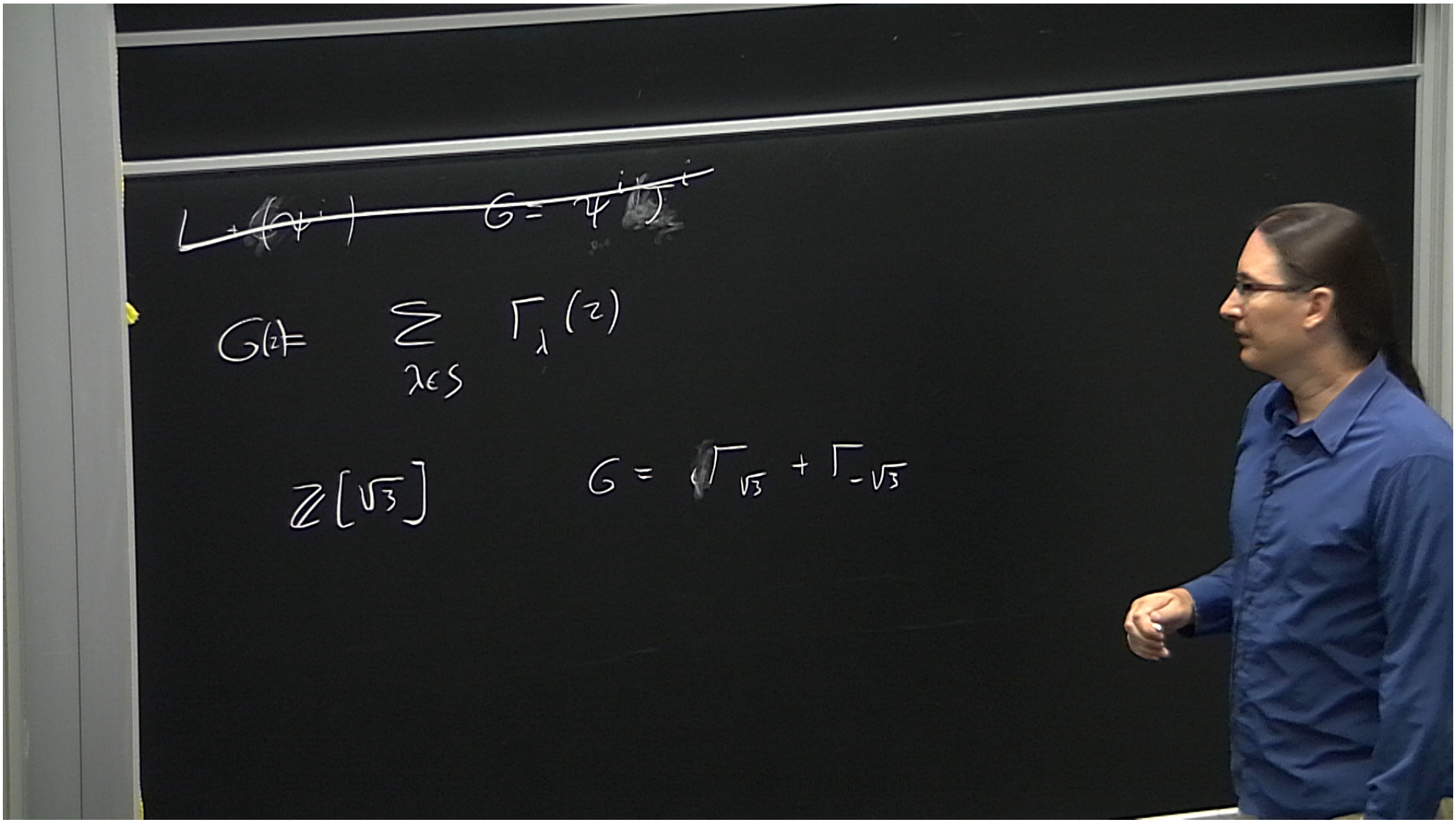
$A_e : L_e$ $\quad A_o : L_o$ $\quad A_R : L + \frac{w}{2}$

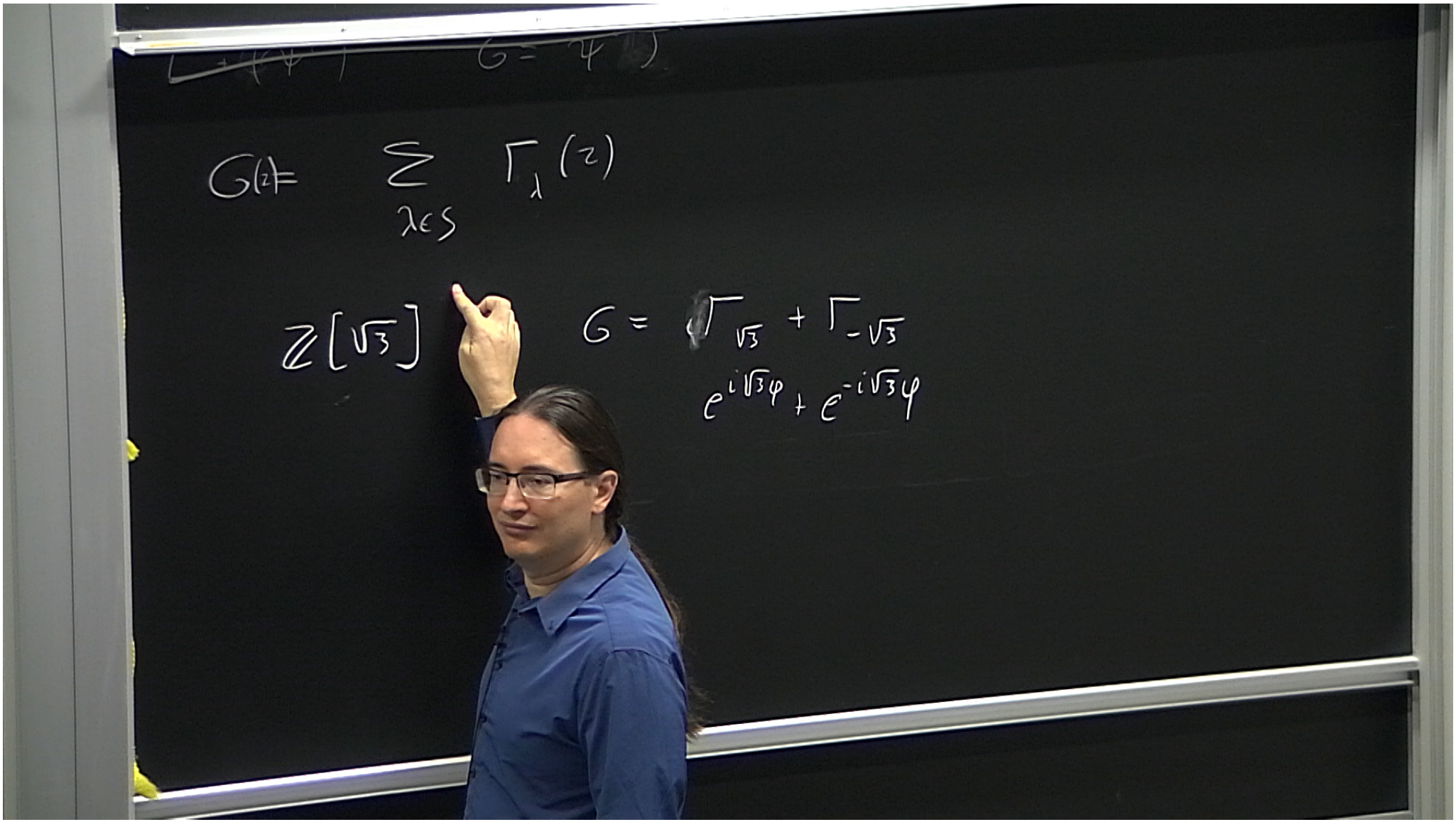
$w \cdot v = v^2 \mod 2$

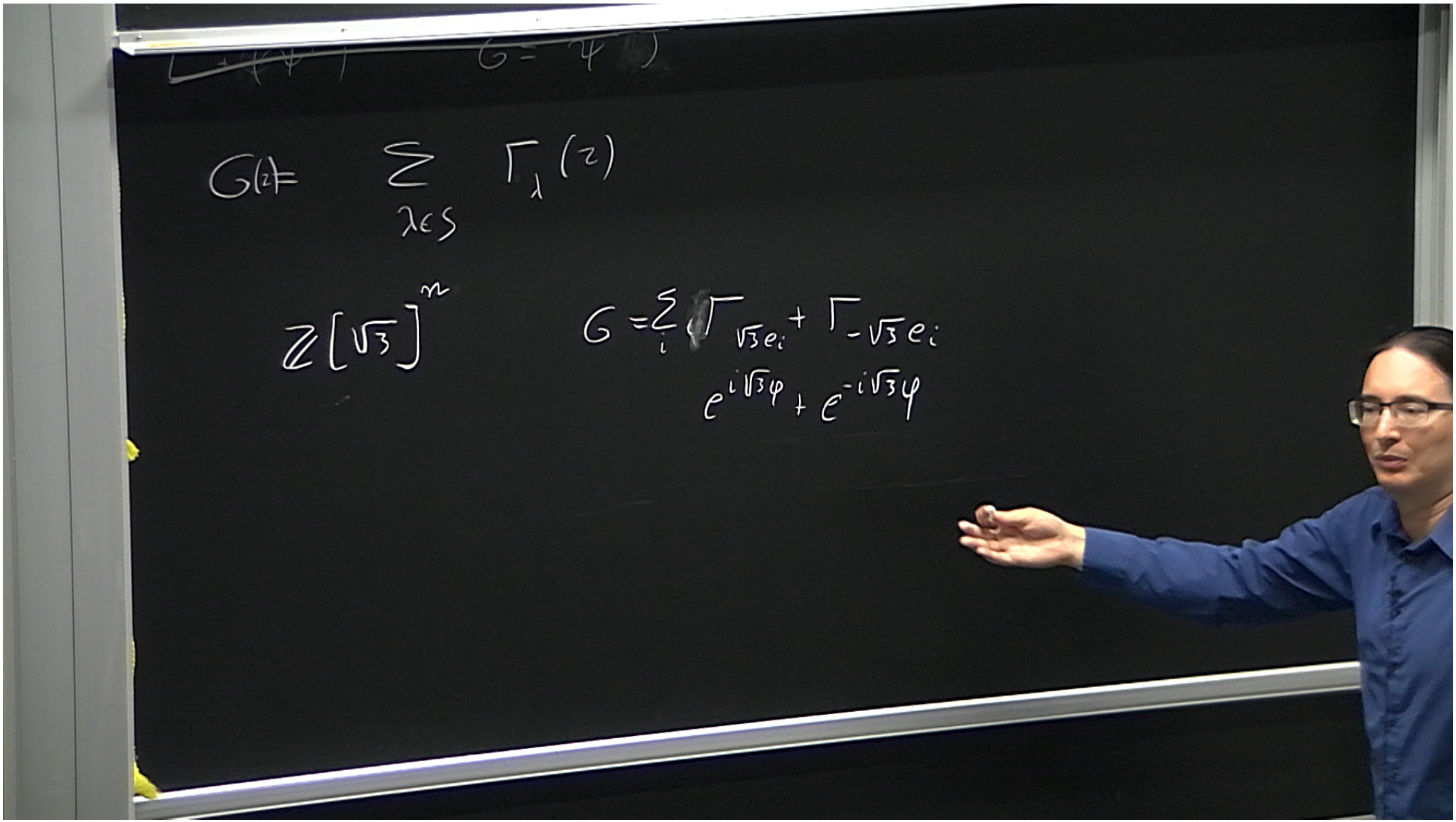
FERMIONS $\quad \psi^i(z)\,\psi^j(0) \sim \frac{\delta^{ij}}{z}$

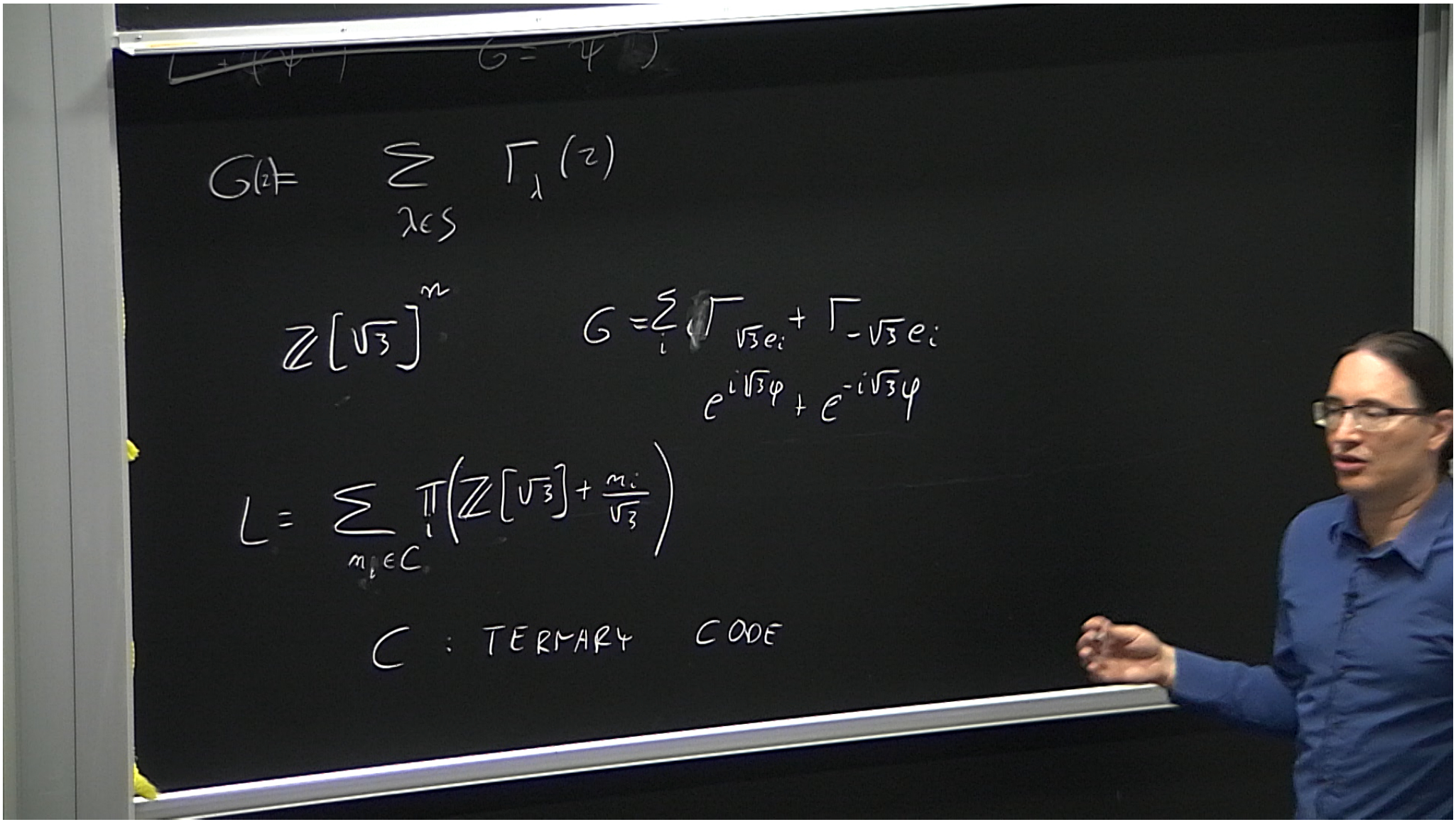

CAUTION


$G(z) = \sum_{\lambda \in S} \Gamma_\lambda(z)$


$G(z) = \sum_{\lambda \in S} \Gamma_\lambda(z)$
$\mathbb{Z}[\sqrt{3}]$
$G = \Gamma_{\sqrt{3}} + \Gamma_{-\sqrt{3}}$


$G(z) = \sum_{\lambda \in S} \Gamma_\lambda(z)$
$\mathbb{Z}[\sqrt{3}]$
$G = \Gamma_{\sqrt{3}} + \Gamma_{-\sqrt{3}}$
$e^{i\sqrt{3}\varphi} + e^{-i\sqrt{3}\varphi}$


$G(z) = \sum_{\lambda \in S} \Gamma_\lambda(z)$
$\mathbb{Z}[\sqrt{3}]^n$
$G = \sum_i \Gamma_{\sqrt{3}e_i} + \Gamma_{-\sqrt{3}e_i}$
$e^{i\sqrt{3}\varphi} + e^{-i\sqrt{3}\varphi}$


$G(z) = \sum_{\lambda \in S} \Gamma_\lambda(z)$
$\mathbb{Z}[\sqrt{3}]^n$
$G = \sum_i \Gamma_{\sqrt{3}e_i} + \Gamma_{-\sqrt{3}e_i}$
$e^{i\sqrt{3}\varphi} + e^{-i\sqrt{3}\varphi}$
$L = \sum_{m_i \in C} \prod_i \left(\mathbb{Z}[\sqrt{3}] + \frac{m_i}{\sqrt{3}}\right)$
C : TERNARY CODE

$$G(z) = \sum_{\lambda \in S} \Gamma_\lambda(z)$$

$$\mathbb{Z}[\sqrt{3}]^n \qquad G = \sum_i \Gamma_{\sqrt{3}e_i} + \Gamma_{-\sqrt{3}e_i}$$

$$e^{i\sqrt{3}\varphi} + e^{-i\sqrt{3}\varphi}$$

$$L = \sum_{m_i \in C} \prod_i \left(\mathbb{Z}[\sqrt{3}] + \frac{m_i}{\sqrt{3}} \right)$$

χ : MULTIPLE OF 24

C : TERNARY CODE

$$G(z) = \sum_{\lambda \in S} \Gamma_\lambda(z)$$

$$\left(\frac{n_1}{\sqrt{3}}, \frac{n_2}{\sqrt{3}}, \frac{n_3}{\sqrt{3}} \cdots \right)$$

$$\mathbb{Z}[\sqrt{3}]^n$$

$$G = \sum_i \Gamma_{\sqrt{3}e_i} + \Gamma_{-\sqrt{3}e_i}$$

$$e^{i\sqrt{3}\varphi} + e^{-i\sqrt{3}\varphi}$$

$$L = \sum_{m_i \in C} \prod_i \left(\mathbb{Z}[\sqrt{3}] + \frac{m_i}{\sqrt{3}} \right)$$

χ : MULTIPLE OF 24

C : TERNARY CODE

$$n = 12k$$

$$G(z) = \sum_{\lambda \in S} \Gamma_\lambda(z)$$

$$\left(\frac{k_1}{\sqrt{3}}, \frac{k_2}{\sqrt{3}}, \frac{k_3}{\sqrt{3}} \cdots \right)$$

$$\mathbb{Z}[\sqrt{3}]^n$$

$$G = \sum_i \Gamma_{\sqrt{3}e_i} + \Gamma_{-\sqrt{3}e_i}$$

$$e^{i\sqrt{3}\varphi} + e^{-i\sqrt{3}\varphi}$$

$$L = \sum_{m_i \in C} \prod_i \left(\mathbb{Z}[\sqrt{3}] + \frac{m_i}{\sqrt{3}} \right)$$

χ : MULTIPLE OF 24

C : TERNARY CODE

LATTICE SPIN-LCFT

L : SELF-DUAL LATTICE $\lambda^2 = K$

$A_e : L_e$ $A_o : L_o$ $A_R : L + \frac{w}{2}$

$w \cdot v = v^2$ MOD 2

FERMIONS $\psi^i(z)\,\psi^{-j}(0) \sim \frac{\delta^{ij}}{z}$

LATTICE SPIN-LCFT

L : SELF-DUAL LATTICE $\lambda^2 = K$ $(-1)^{v \cdot w}$

$A_e : L_e$ $A_o : L_o$ $A_R : L + \frac{w}{2}$

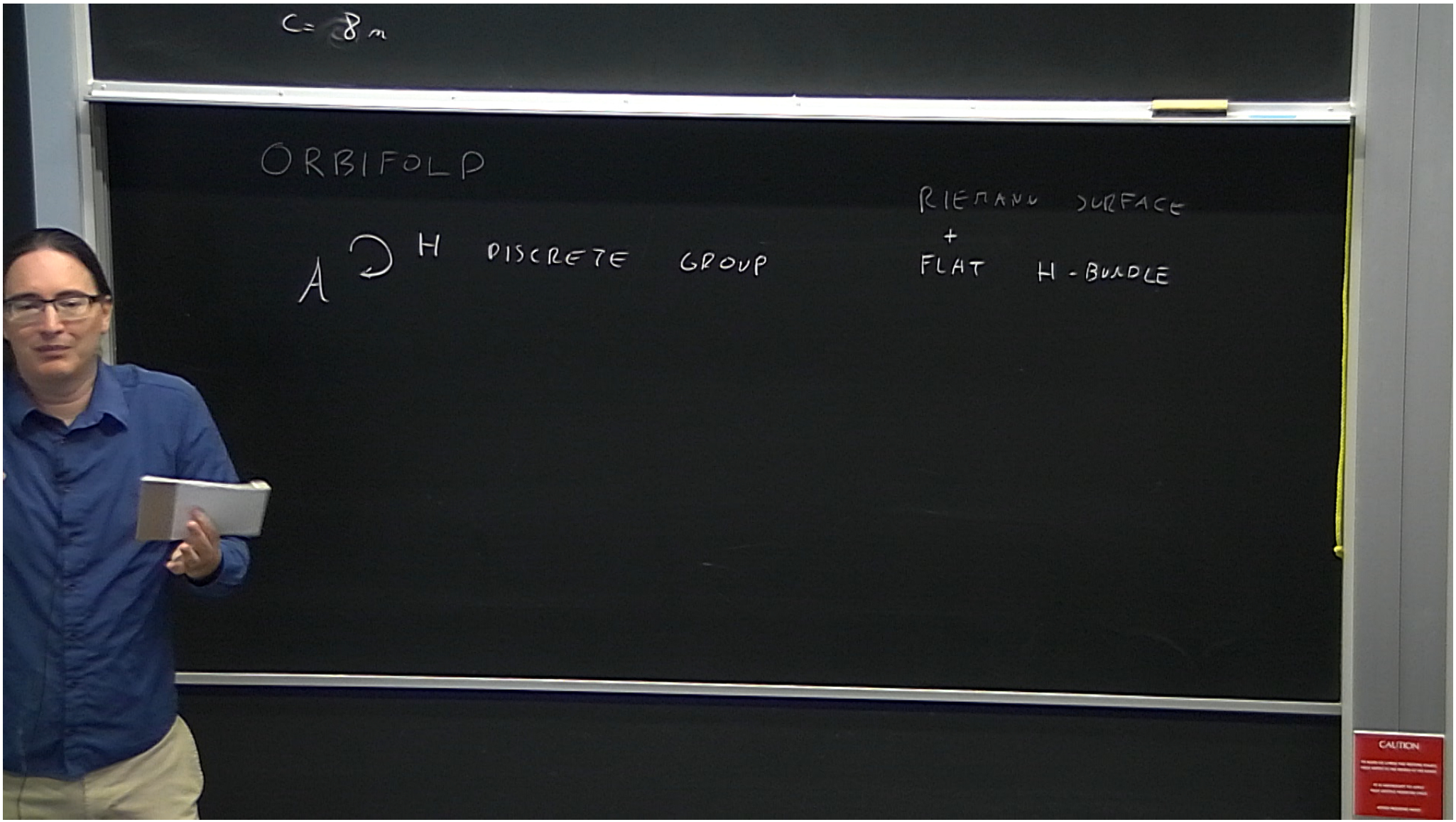
$w \cdot v \equiv v^2$ MOD 2

FERMIONS $\psi^i(z)\,\psi^j(0) \sim \frac{\delta^{ij}}{z}$

$c = 8n$

ORBIFOLD

$A \circlearrowleft H$ DISCRETE GROUP


C = 8 m
ORBIFOLD
A ↻ H DISCRETE GROUP
RIEMANN SURFACE
+
FLAT H-BUNDLE
CAUTION

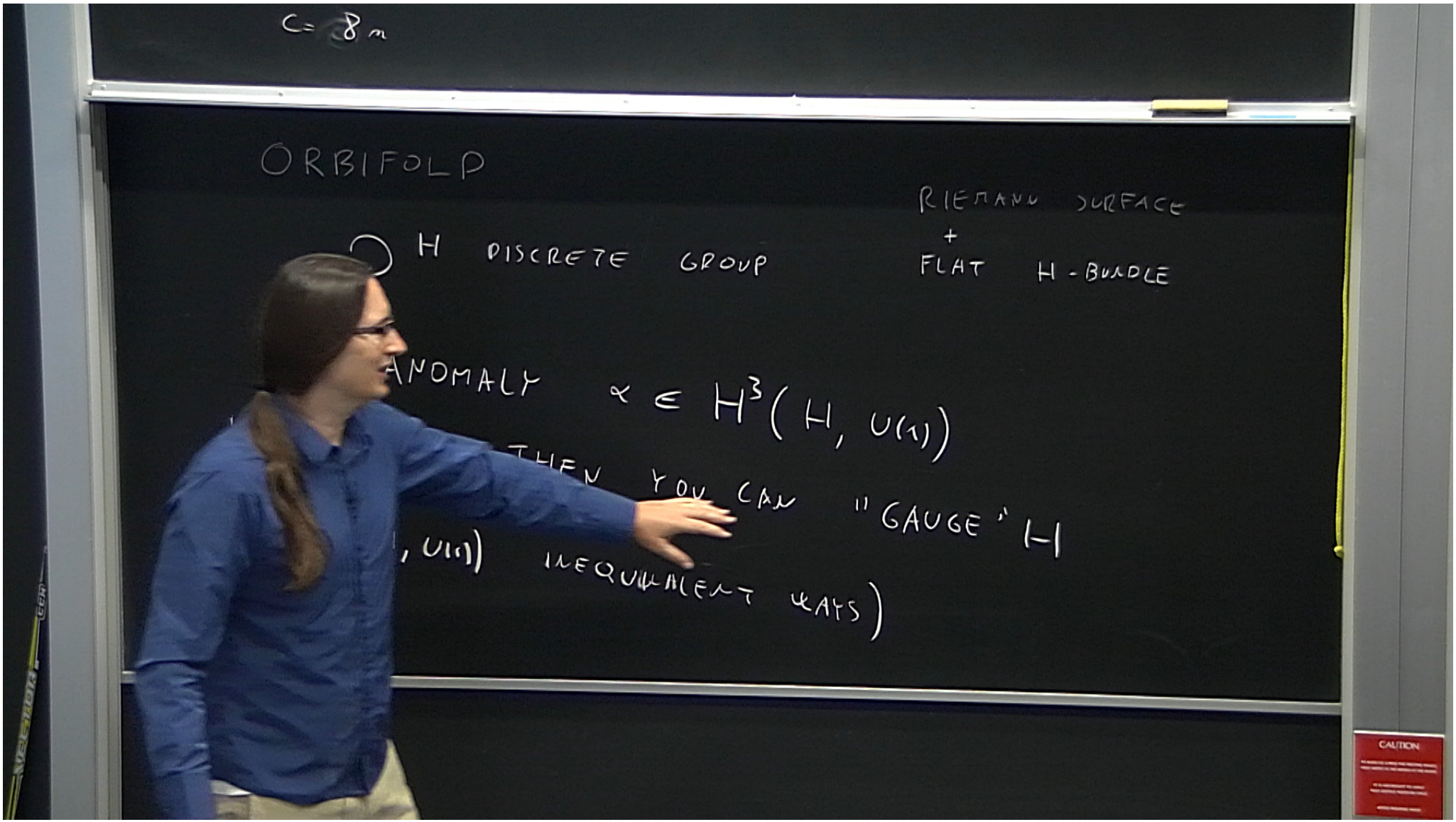
$c = 8n$

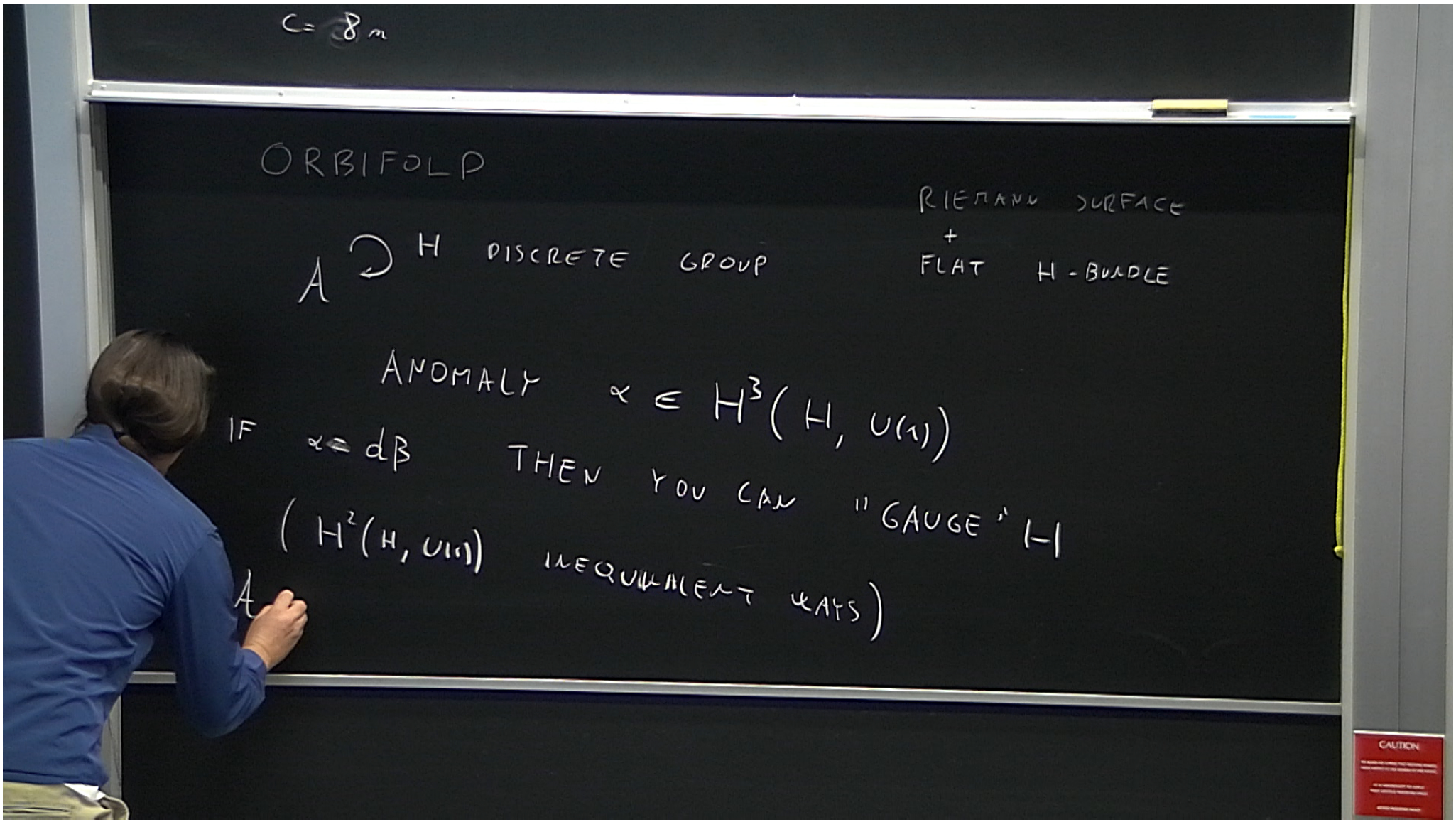
ORBIFOLD

$A \circlearrowleft H$ DISCRETE GROUP

RIEMANN SURFACE
+
FLAT H-BUNDLE

ANOMALY $\alpha \in H^3(H, U(1))$


C= 8 n
ORBIFOLD
H DISCRETE GROUP
RIEMANN SURFACE
+
FLAT H-BUNDLE
ANOMALY α ∈ H³(H, U(1))
THEN YOU CAN "GAUGE" H
, U(1)) INEQUIVALENT WAYS)


C = 8 n
ORBIFOLD
A ↻ H DISCRETE GROUP
RIEMANN SURFACE
+
FLAT H-BUNDLE
ANOMALY α ∈ H³(H, U(1))
IF α = dβ THEN YOU CAN "GAUGE" H
(H²(H, U(1)) INEQUIVALENT WAYS)
A

$c = 8n$

ORBIFOLD

RIEMANN SURFACE
+
FLAT H-BUNDLE

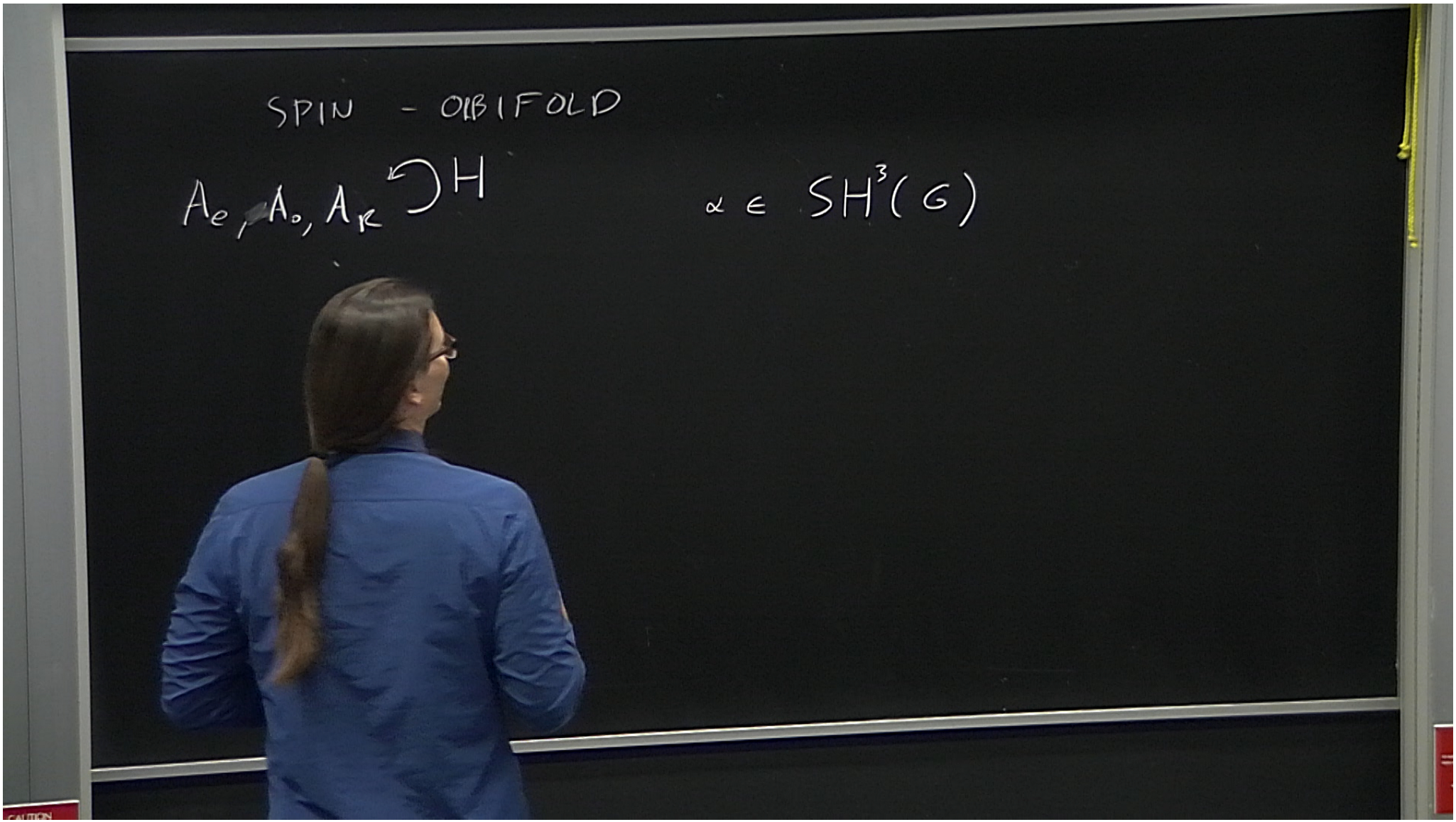
$A \circlearrowleft H$ DISCRETE GROUP

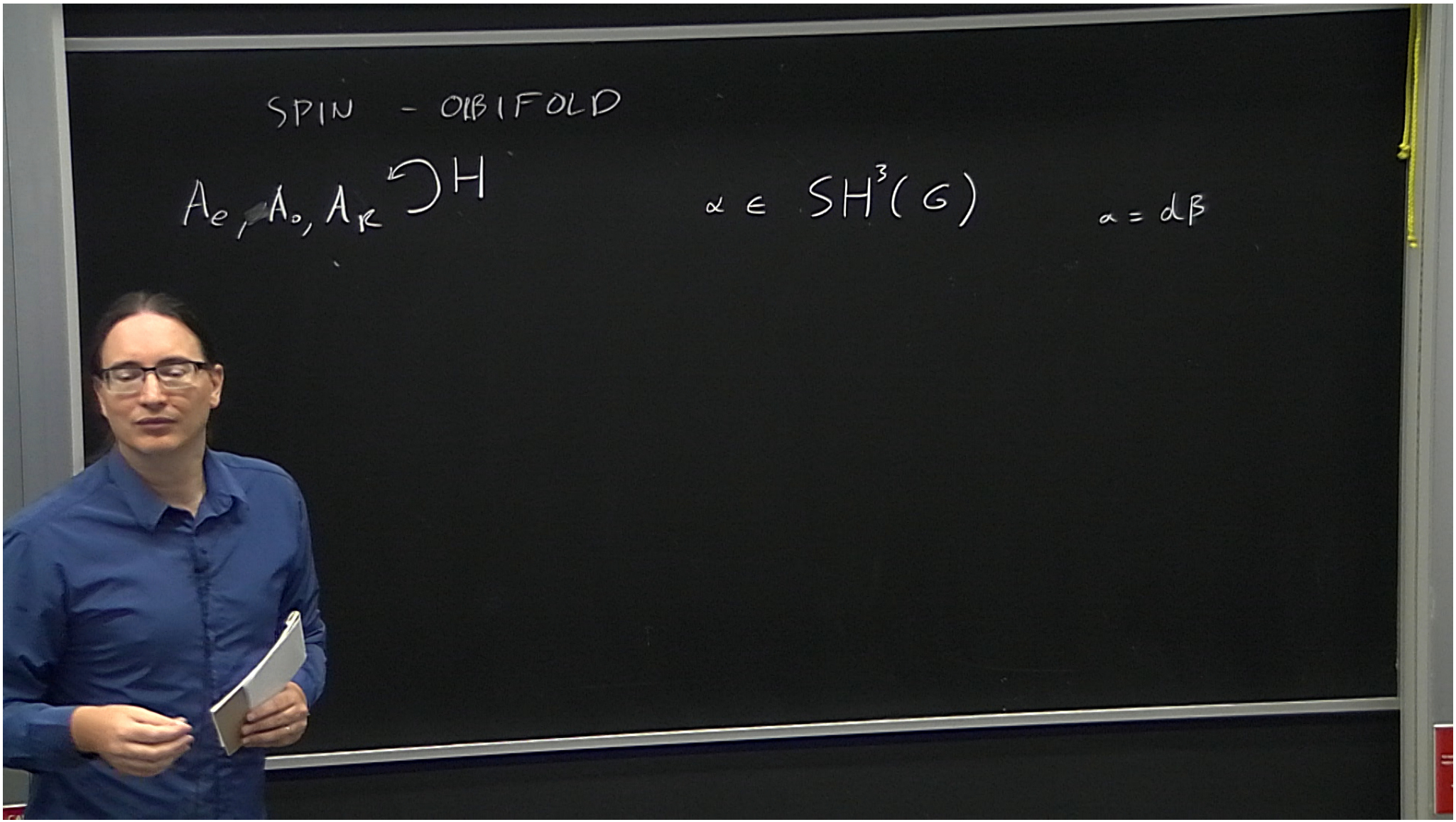
ANOMALY $\alpha \in H^3(H, U(1))$

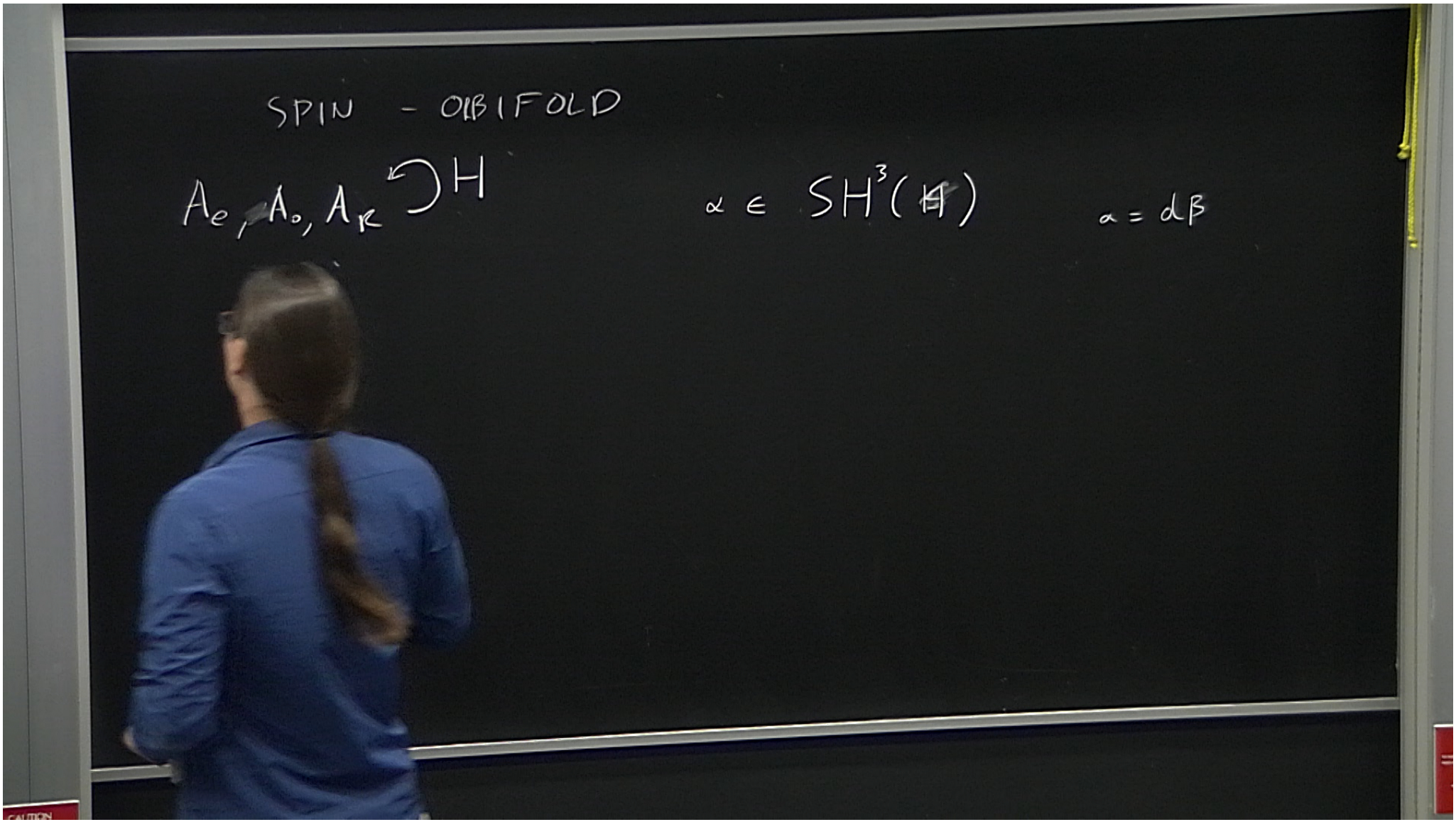
IF $\alpha = d\beta$ THEN YOU CAN "GAUGE" $|-|$

($H^2(H, U(1))$ INEQUIVALENT WAYS)

$A /\!/ H$


SPIN - ORBIFOLD
$A_e, A_o, A_R \circlearrowleft H$
$\alpha \in SH^3(G)$

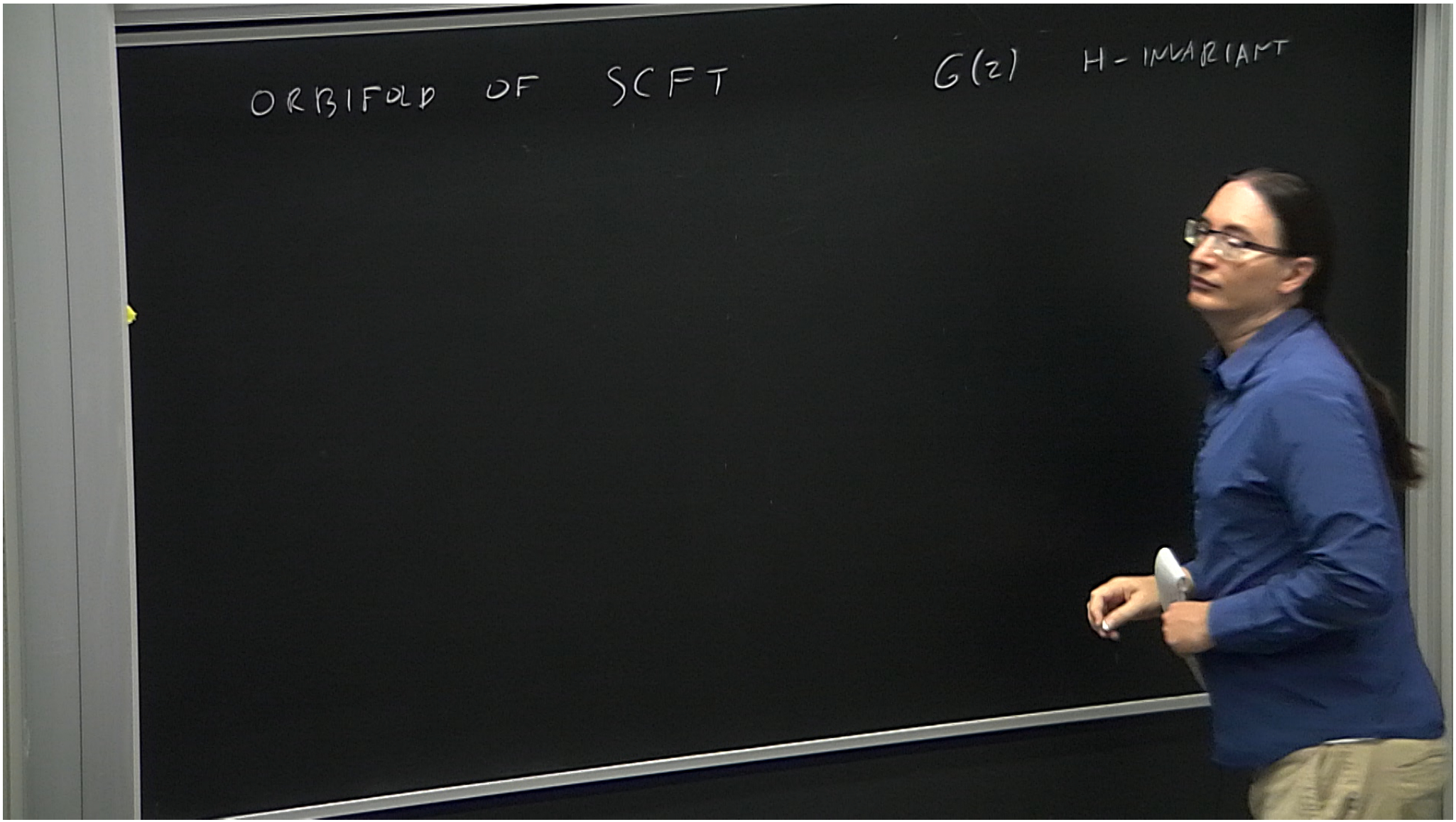

SPIN - ORBIFOLD
$A_e, A_o, A_R \circlearrowleft H$
$\alpha \in SH^3(G)$
$\alpha = d\beta$

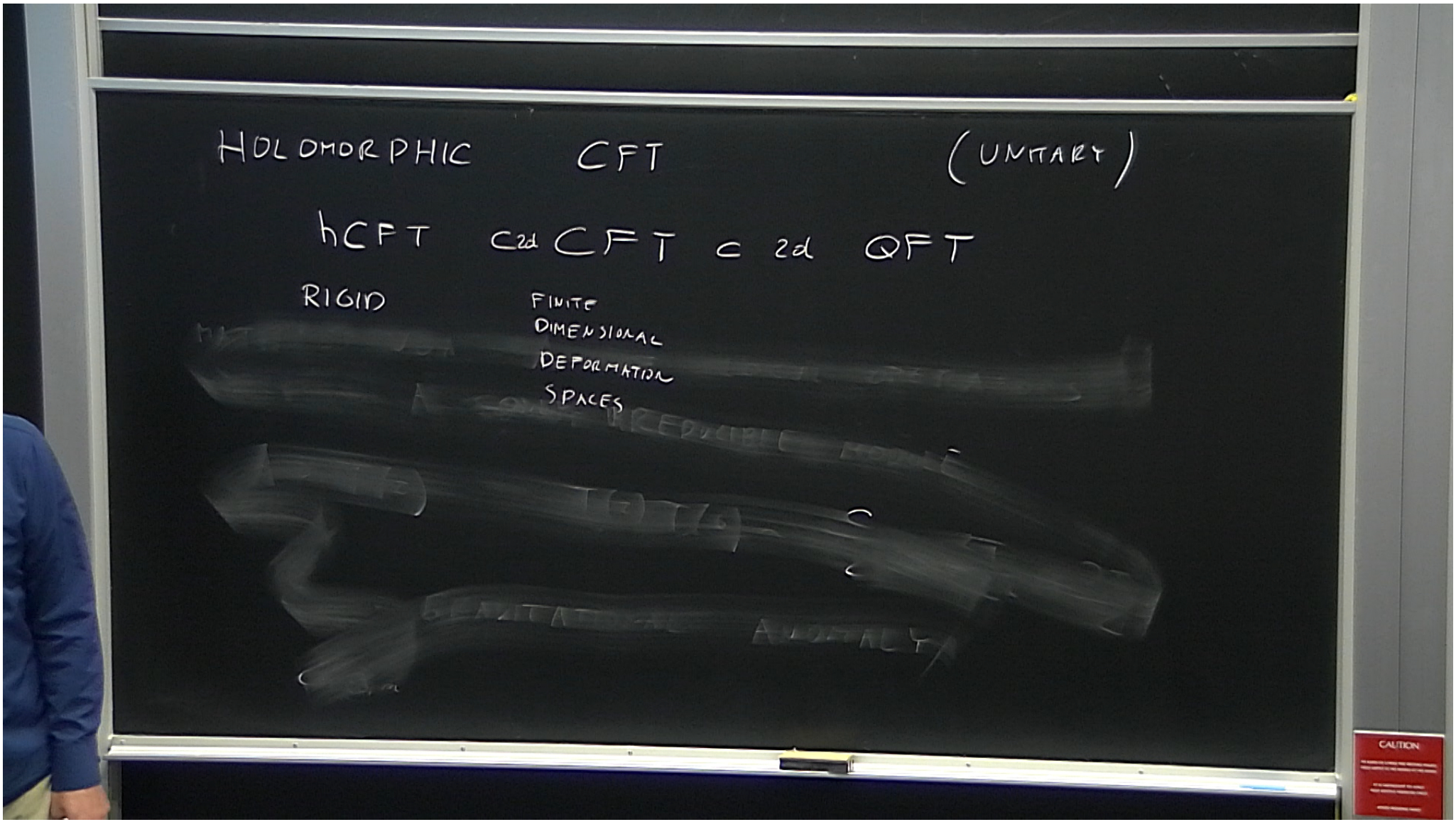

SPIN - ORBIFOLD
$A_e, A_o, A_R \circlearrowleft H$
$\alpha \in SH^3(H)$
$\alpha = d\beta$

SPIN - ORBIFOLD

$A_e, A_o, A_R \circlearrowleft H$ $\qquad \alpha \in SH^3(H)$ $\qquad \alpha = d\beta$

$SH^2(H)$ INEQUIVALENT SPIN-ORBIFOLD


ORBIFOLD OF SCFT
G(z) H-INVARIANT


HOLOMORPHIC CFT (UNITARY)
hCFT ⊂ 2d CFT ⊂ 2d QFT
RIGID
FINITE
DIMENSIONAL
DEFORMATION
SPACES

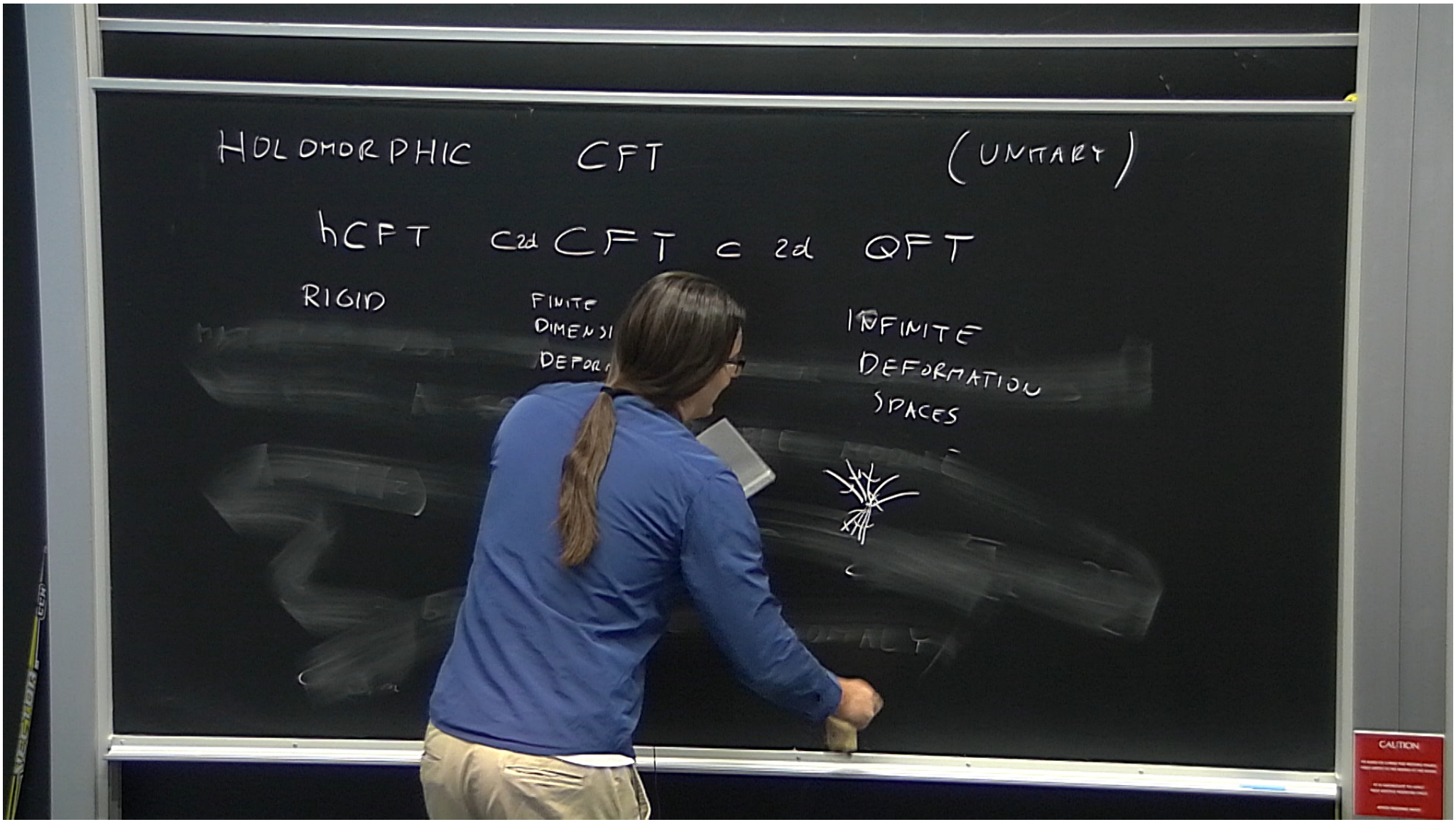
HOLOMORPHIC CFT (UNITARY)

hCFT $\subset$ 2d CFT $\subset$ 2d QFT

RIGID

FINITE DIMENSIONAL DEFORMATION SPACES

INFINITE DEFORMATION SPACES


HOLOMORPHIC CFT (UNITARY)
hCFT ⊂ 2d CFT ⊂ 2d QFT
RIGID
FINITE
INFINITE
DEFORMATION
SPACES

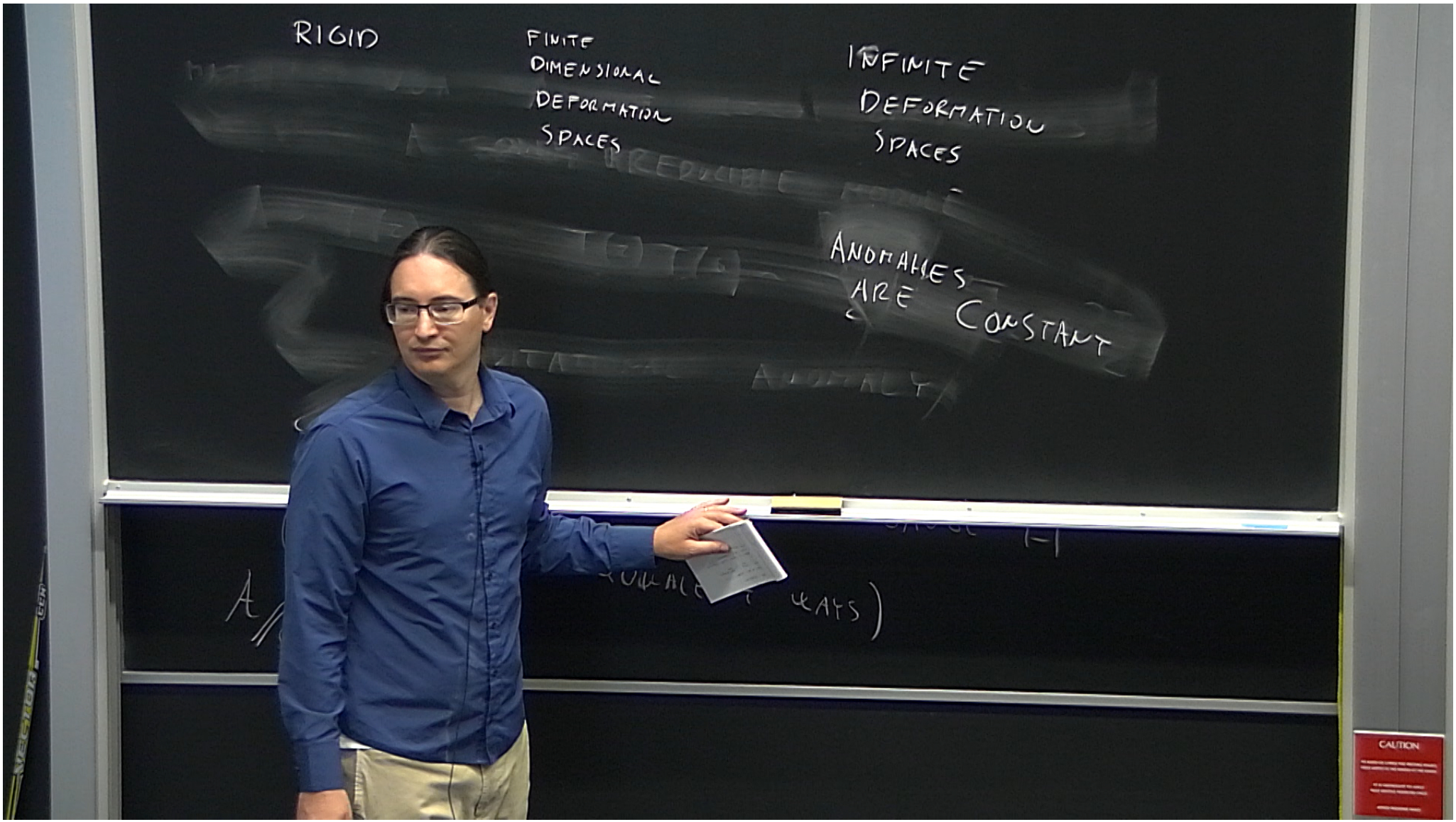
HOLOMORPHIC CFT (UNITARY)

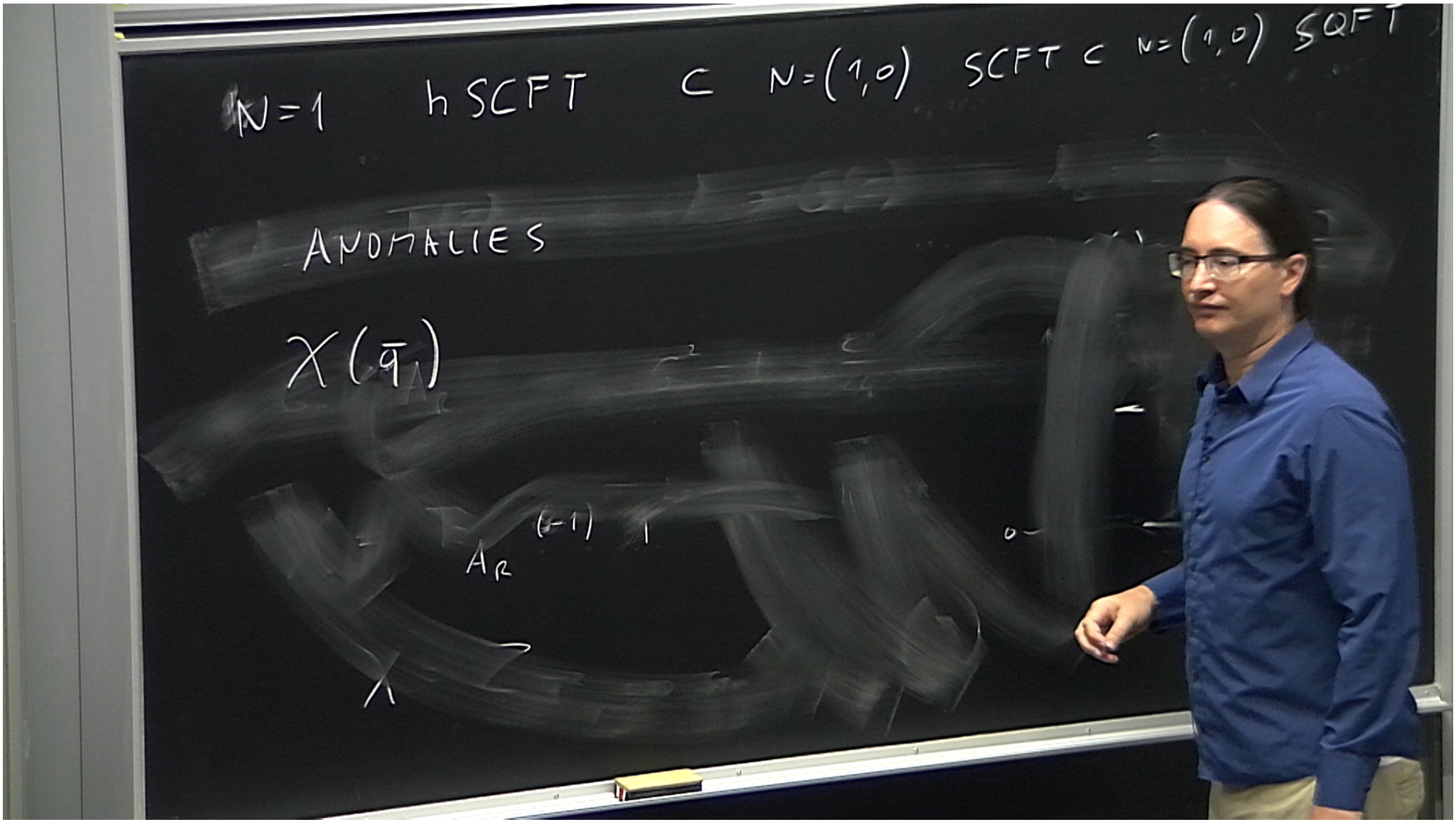
hCFT $\subset$ 2d CFT $\subset$ 2d QFT

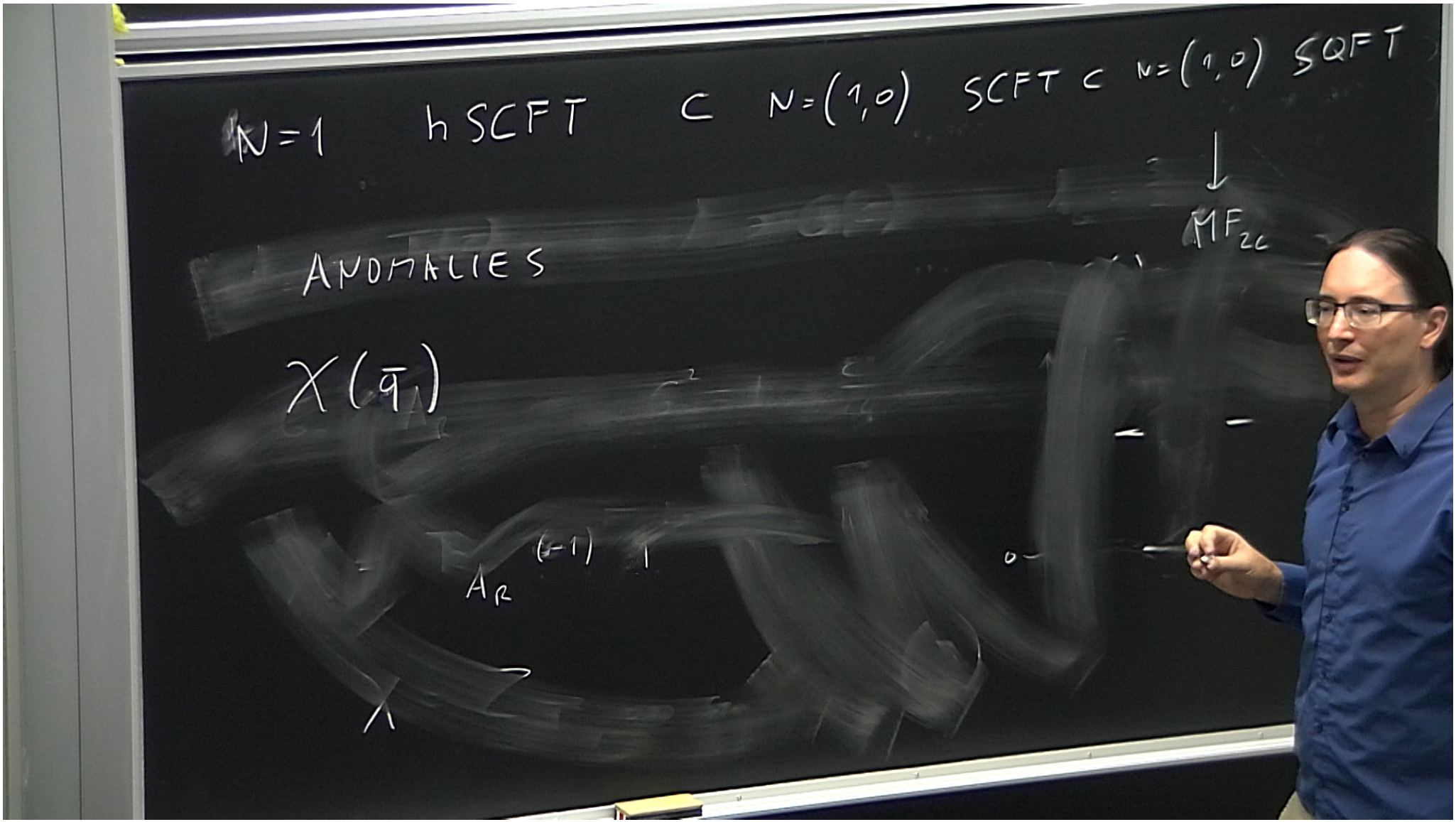
RIGID

FINITE DIMENSIONAL DEFORMATION SPACES

INFINITE DEFORMATION SPACES


RIGID
FINITE DIMENSIONAL DEFORMATION SPACES
INFINITE DEFORMATION SPACES
ANOMALIES ARE CONSTANT


N=1 hSCFT ⊂ N=(1,0) SCFT ⊂ N=(1,0) SQFT
ANOMALIES
χ(q̄)
A_R (-1)


N=1 hSCFT ⊂ N=(1,0) SCFT ⊂ N=(1,0) SQFT
MF2c
ANOMALIES
χ(q)
A_R (-1)

$N=1$ hSCFT $\subset$ $N=(1,0)$ SCFT $\subset$ $N=(1,0)$ SQFT

$\downarrow$

MF_{2c}

ANOMALIES

$\chi(\bar{q})$

A_R (-1)

λ

$N=1$ hSCFT $\subset$ $N=(1,0)$ SCFT $\subset$ $N=(1,0)$ SQFT

$\downarrow$

MF_{2c}

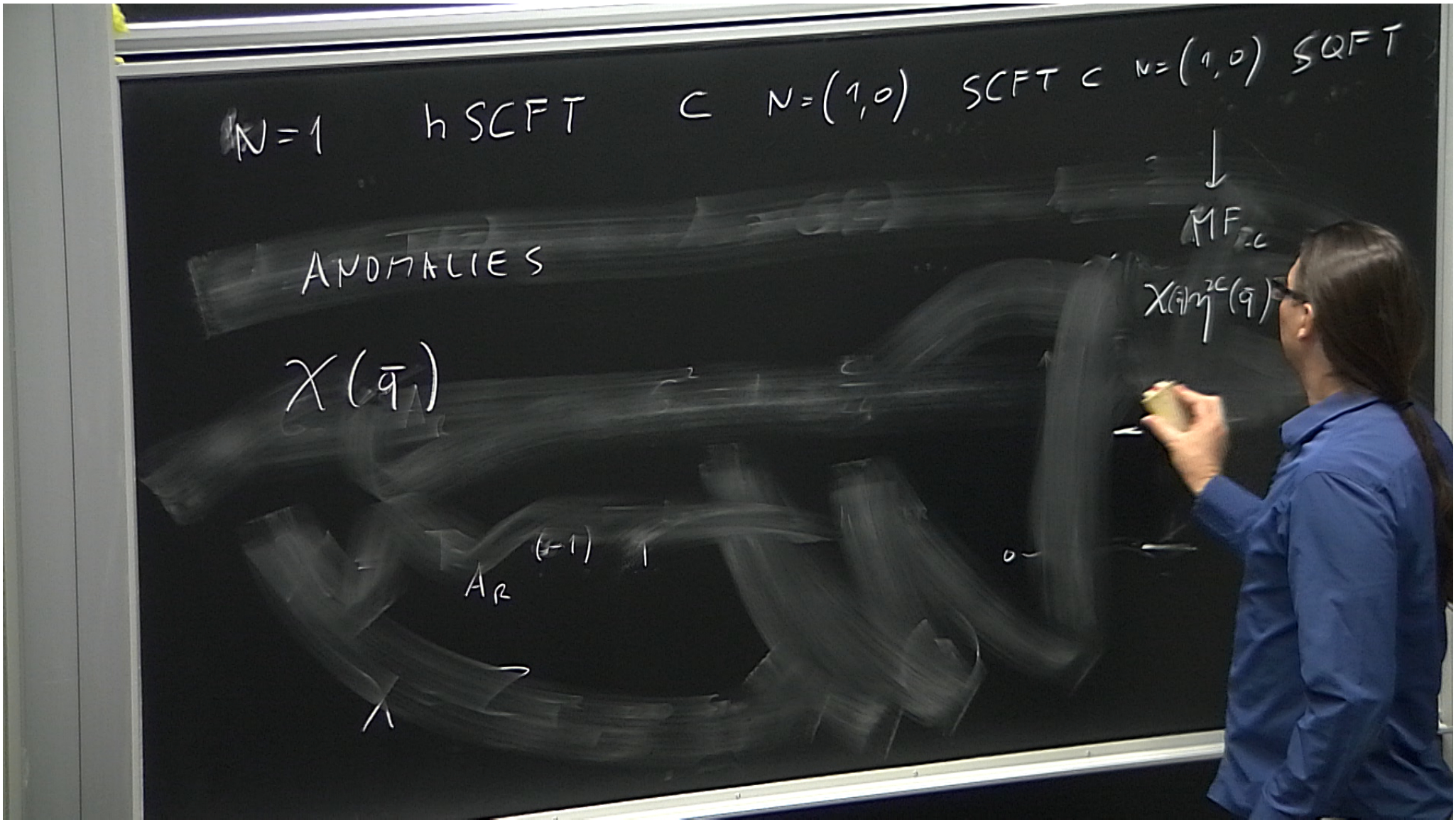
$\chi\, \eta^{2c}(\bar{q})$

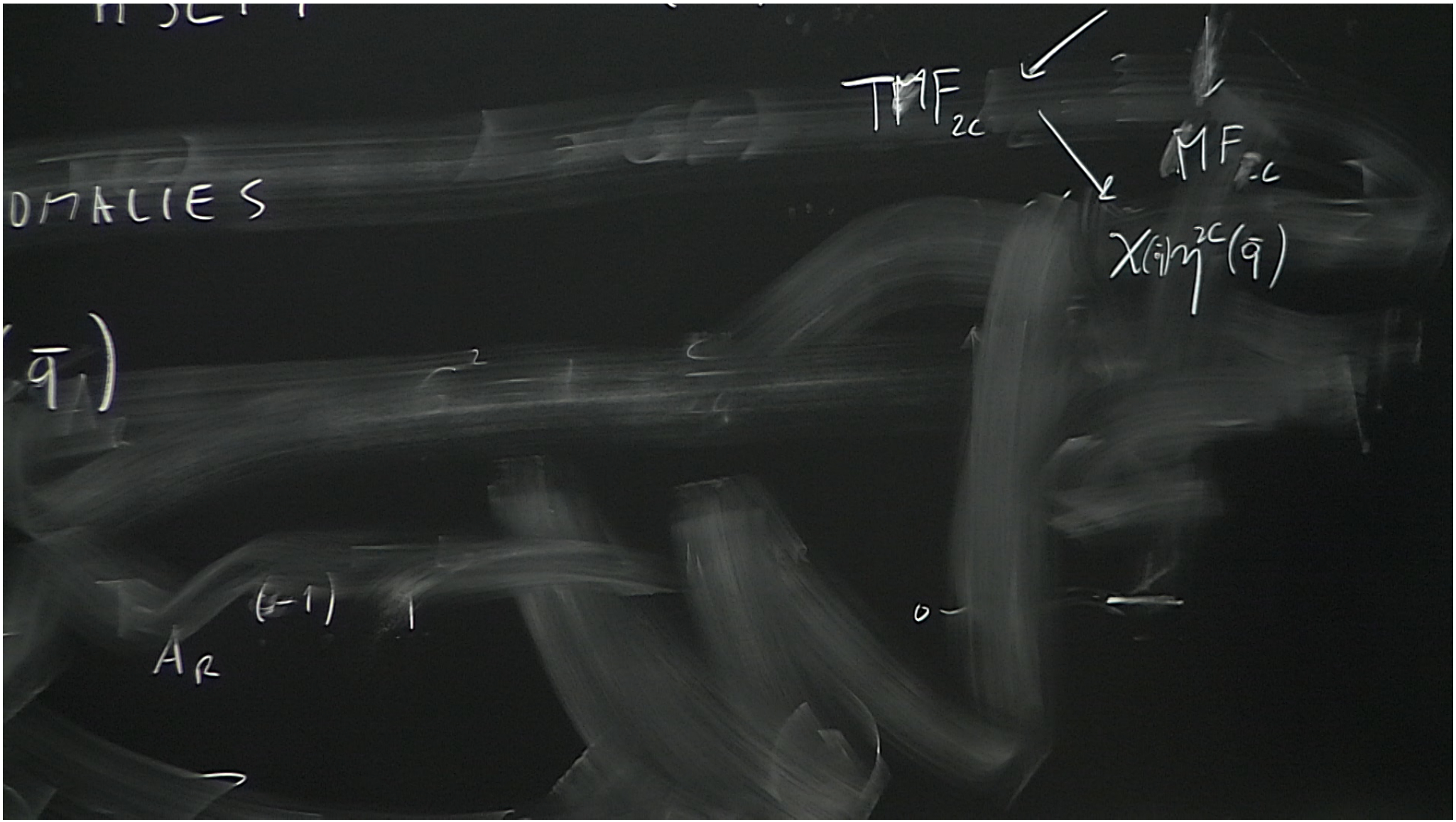
ANOMALIES

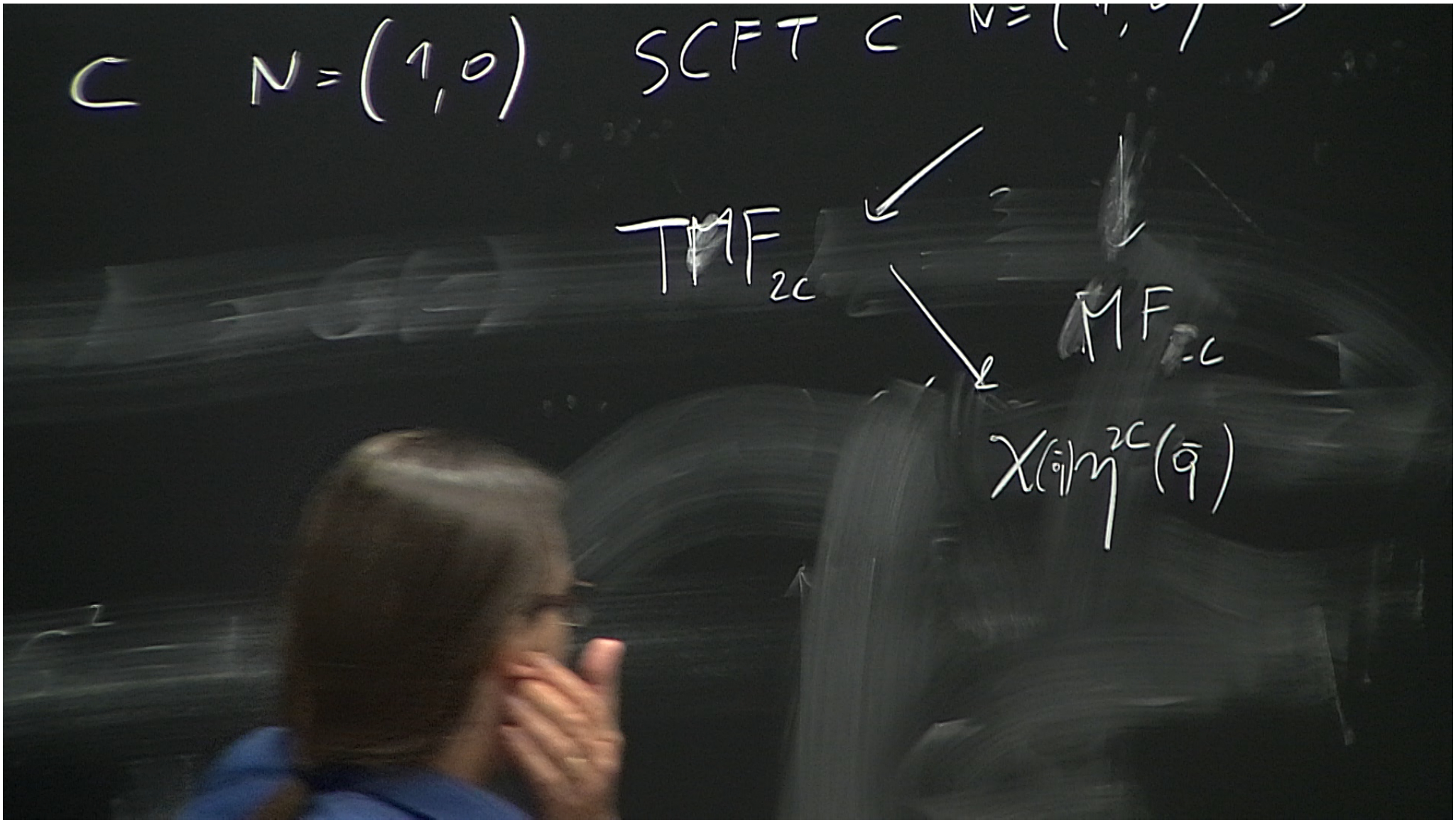
$\chi(\bar{q})$

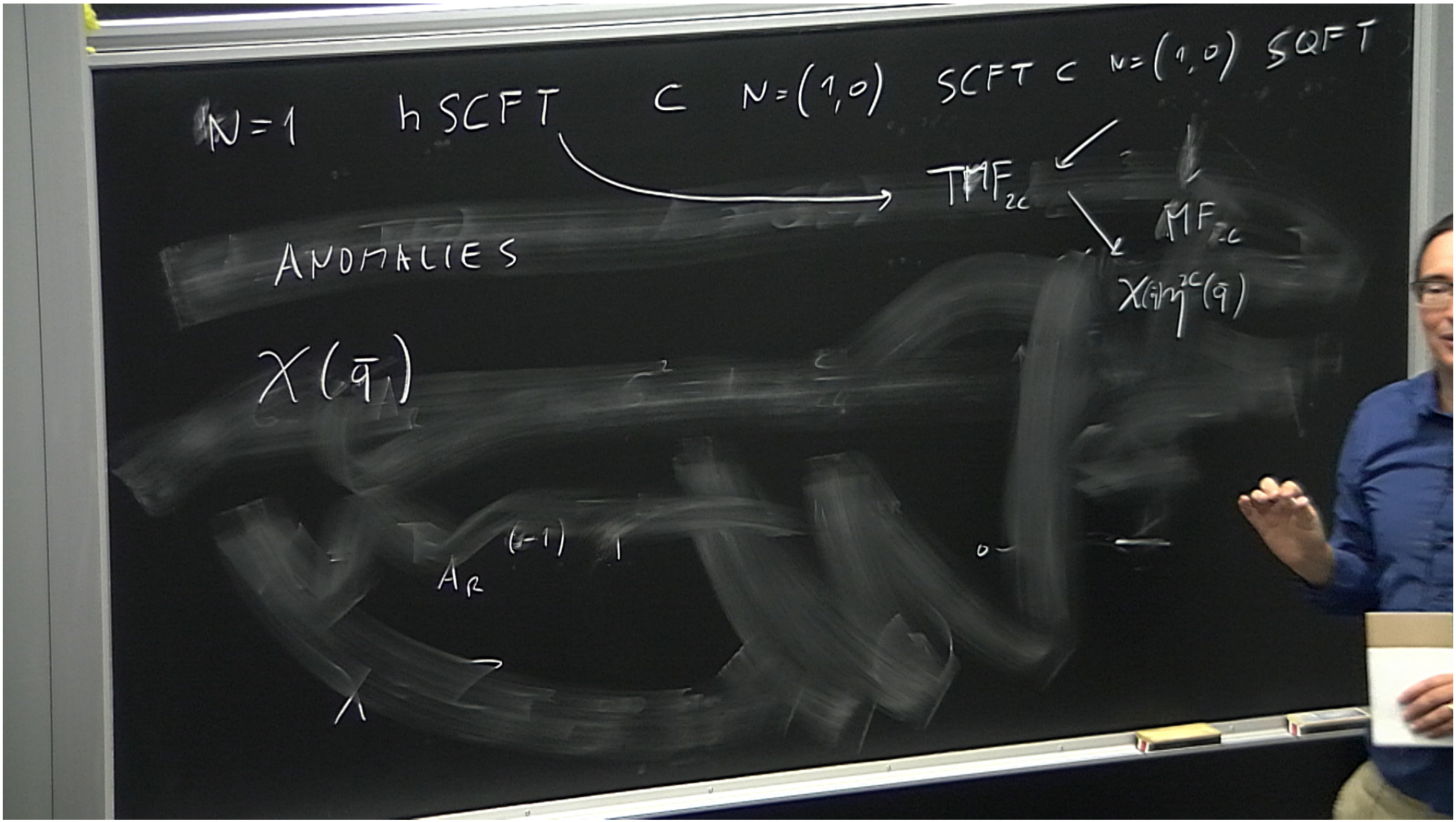
A_R (-1)

χ


N=1 hSCFT ⊂ N=(1,0) SCFT ⊂ N=(1,0) SQFT
MF
ANOMALIES
χ(q̄)
A_R
(-1)


TMF2c
MF2c
X(i)η2c(q̄)
OMALIES
(q̄)
(-1)
0
AR


C N=(1,0) SCFT C N=(1,0)
TMF2c
MF-c
χ(q̄)η2c(q̄)


N=1 hSCFT ⊂ N=(1,0) SCFT ⊂ N=(1,0) SQFT
TMF2c
MF2c
χ(q)η2c(q)
ANOMALIES
χ(q̄)
AR
(-1)

$N=1$ hSCFT $\subset$ $N=(1,0)$ SCFT $\subset$ $N=(1,0)$ SQFT

$\longrightarrow$ TMF_{2c} $\longrightarrow$ MF_{2c}

$\chi(\bar q)\eta^{2c}(\bar q)$

ANOMALIES

$\chi(\bar q)$

$\frac{24}{\gcd(24, \frac{c}{12})}\eta^{2c}(\bar q)$

A_R (-1) 1

0

χ