Title: PSI 2018/2019 - Relativity - Lecture 5

Date: Sep 07, 2018 09:00 AM

URL: http://pirsa.org/18080039

Abstract:

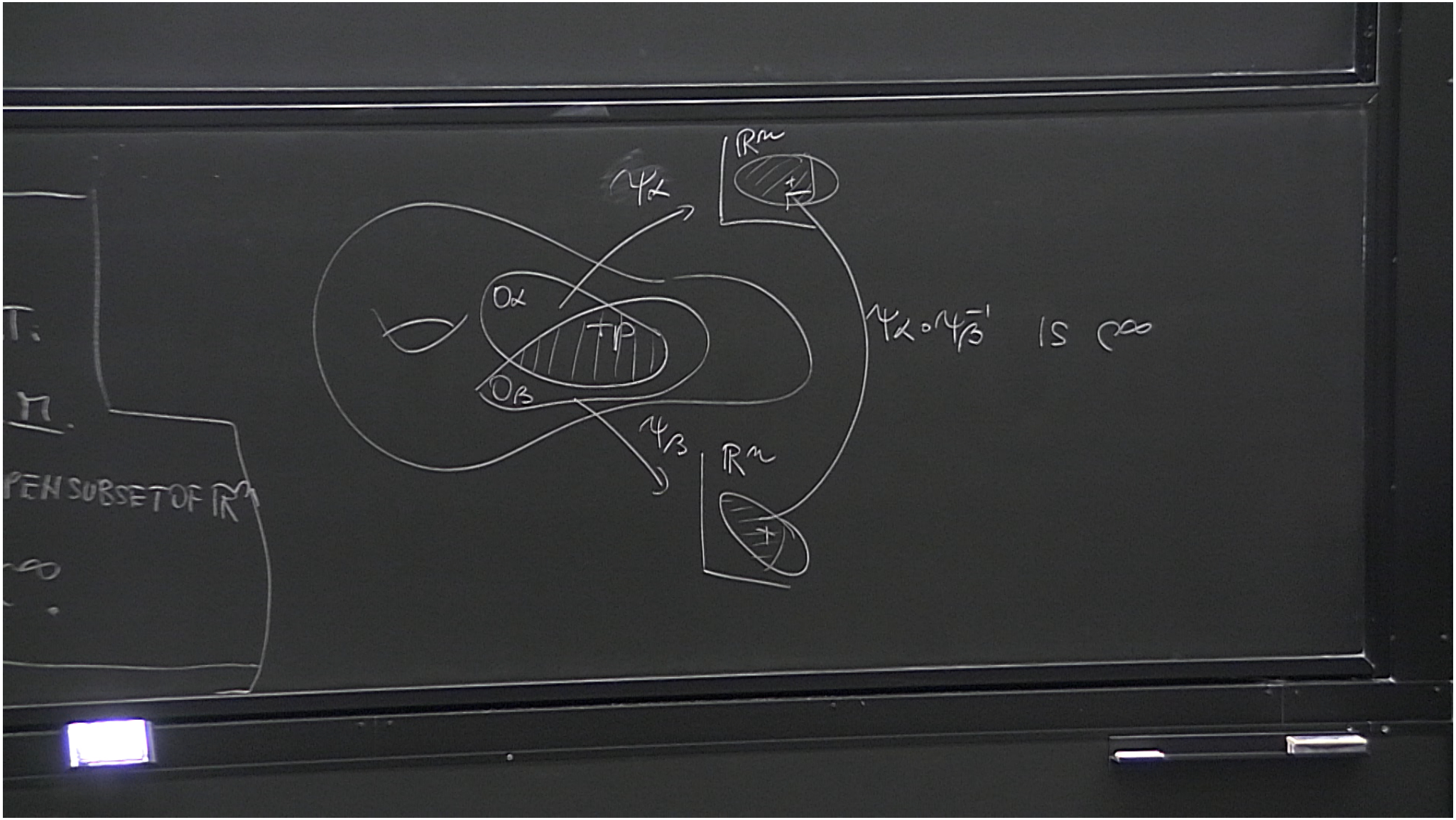
YESTERDAY:

DEF: AN n-DIM MANIFOLD M IS A SET OF POINTS TOGETHER WITH A COLLECTION OF SUBSETS $\{O_\alpha\}$, S.T.:

i) EACH $p \in M$ LIES IN AT LEAST ONE O_α, $\{O_\alpha\}$ COVER M.

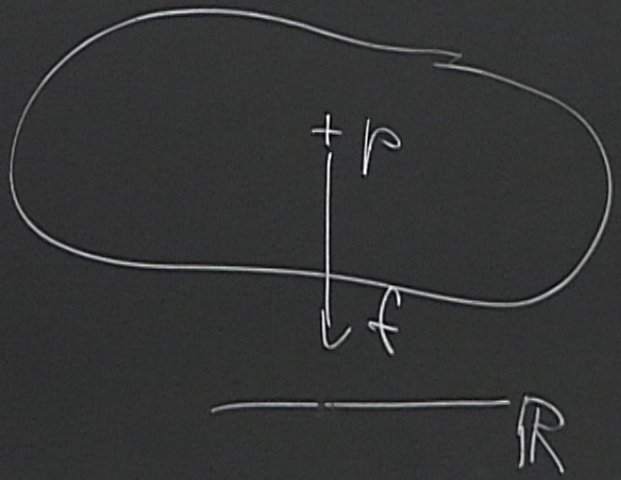
ii) FOR EACH α, THERE IS 1-1, ONTO, MAP $\psi_\alpha : O_\alpha \to U_\alpha$ OPEN SUBSET OF $\mathbb{R}^n$

iii) IF ANY 2 SETS $O_\alpha \cap O_\beta \neq 0 \Rightarrow \psi_\beta \circ \psi_\alpha^{-1}$ IS C^∞.


$\mathbb{R}^m$
ψ_α
U_α
p
U_β
ψ_β
$\mathbb{R}^m$
$\psi_\alpha \circ \psi_\beta^{-1}$ IS C^∞

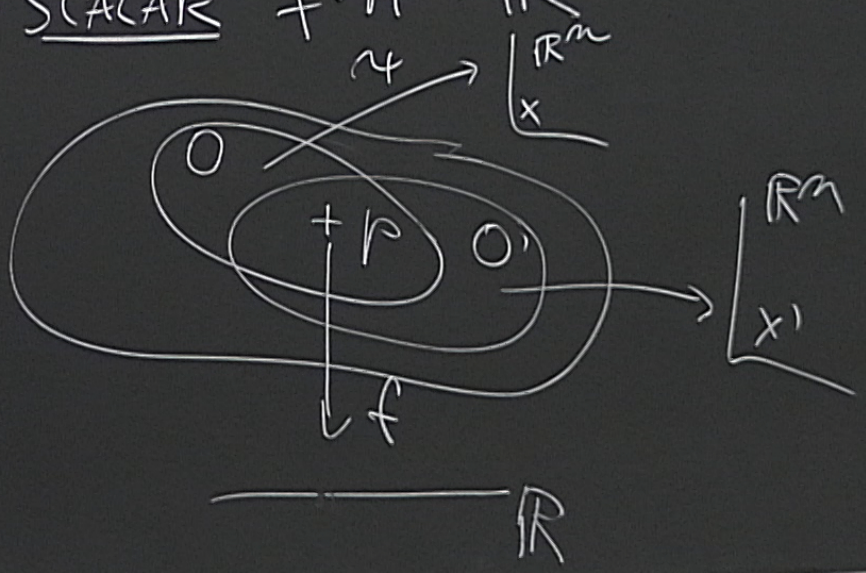
b) TENSORS

- SCALAR $f: M \to \mathbb{R}$



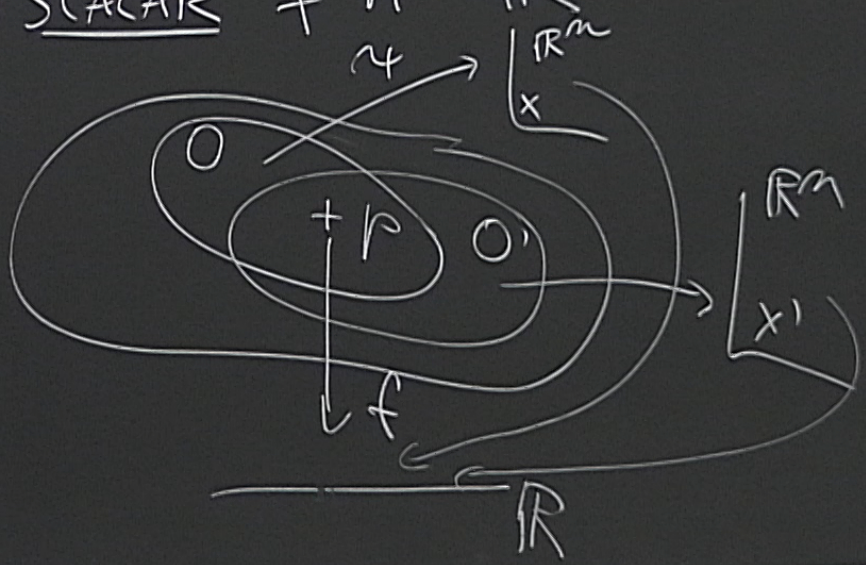
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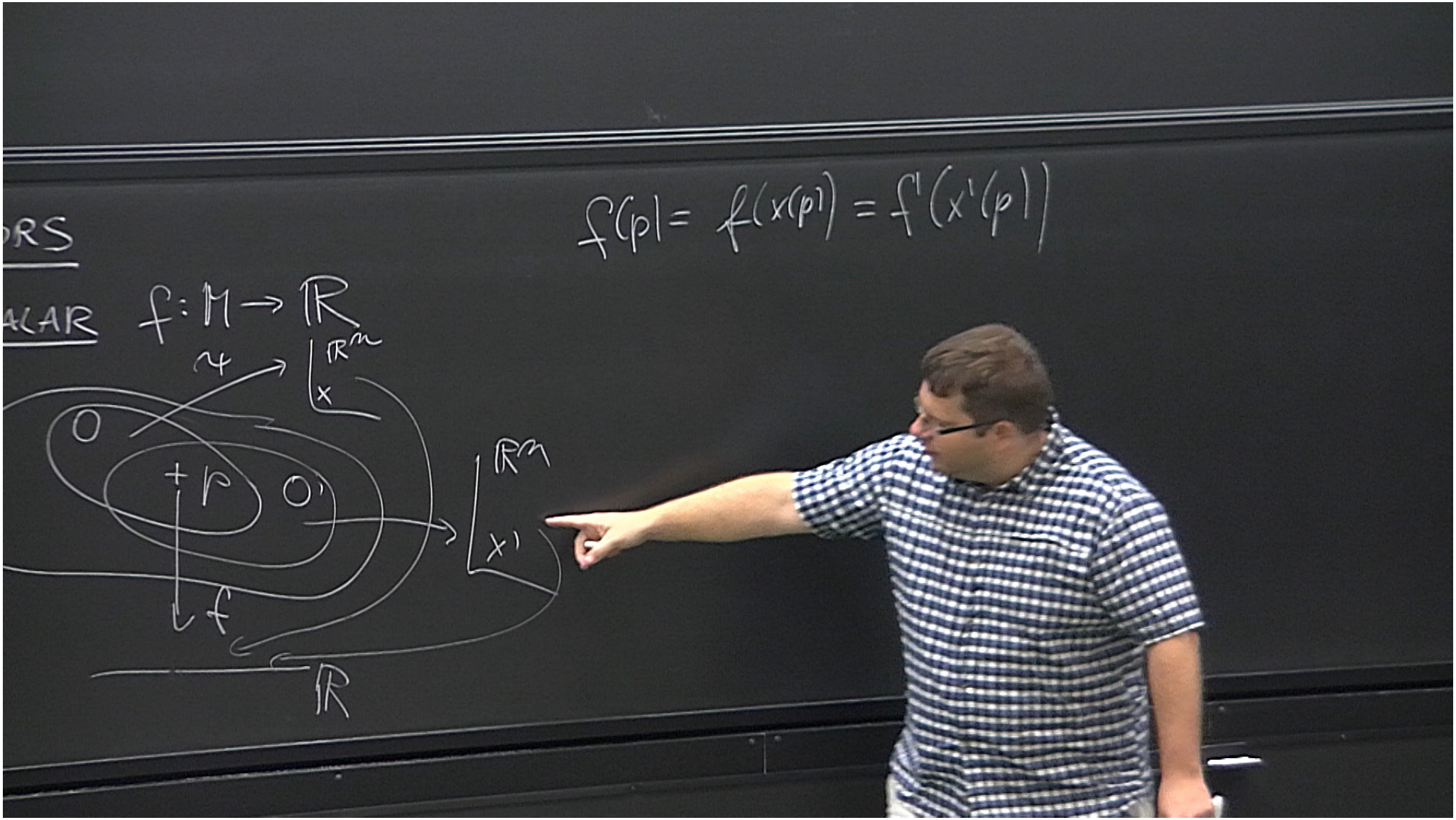
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b) TENSORS

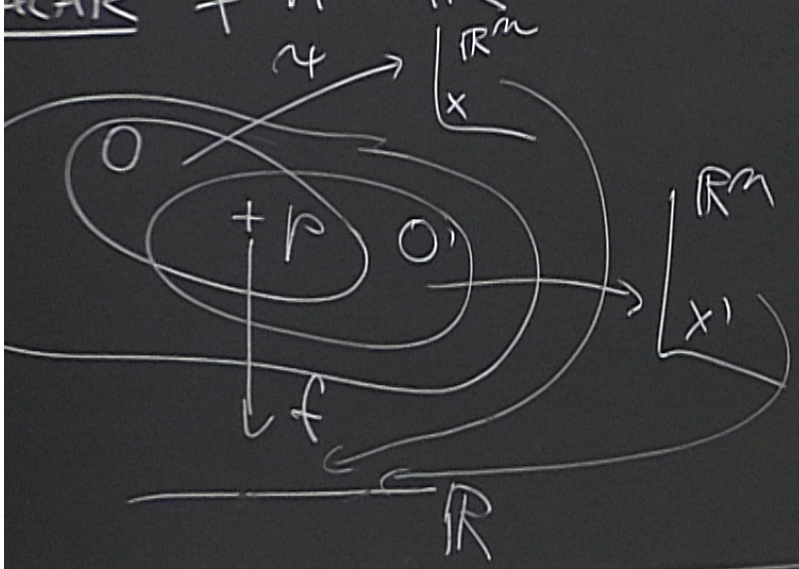
- SCALAR $f: M \to \mathbb{R}$




ORS
ALAR
f: M → ℝ
ℝᵐ
x
O
+p
O'
ℝᵐ
x'
f
ℝ
f(p) = f(x(p)) = f'(x'(p))

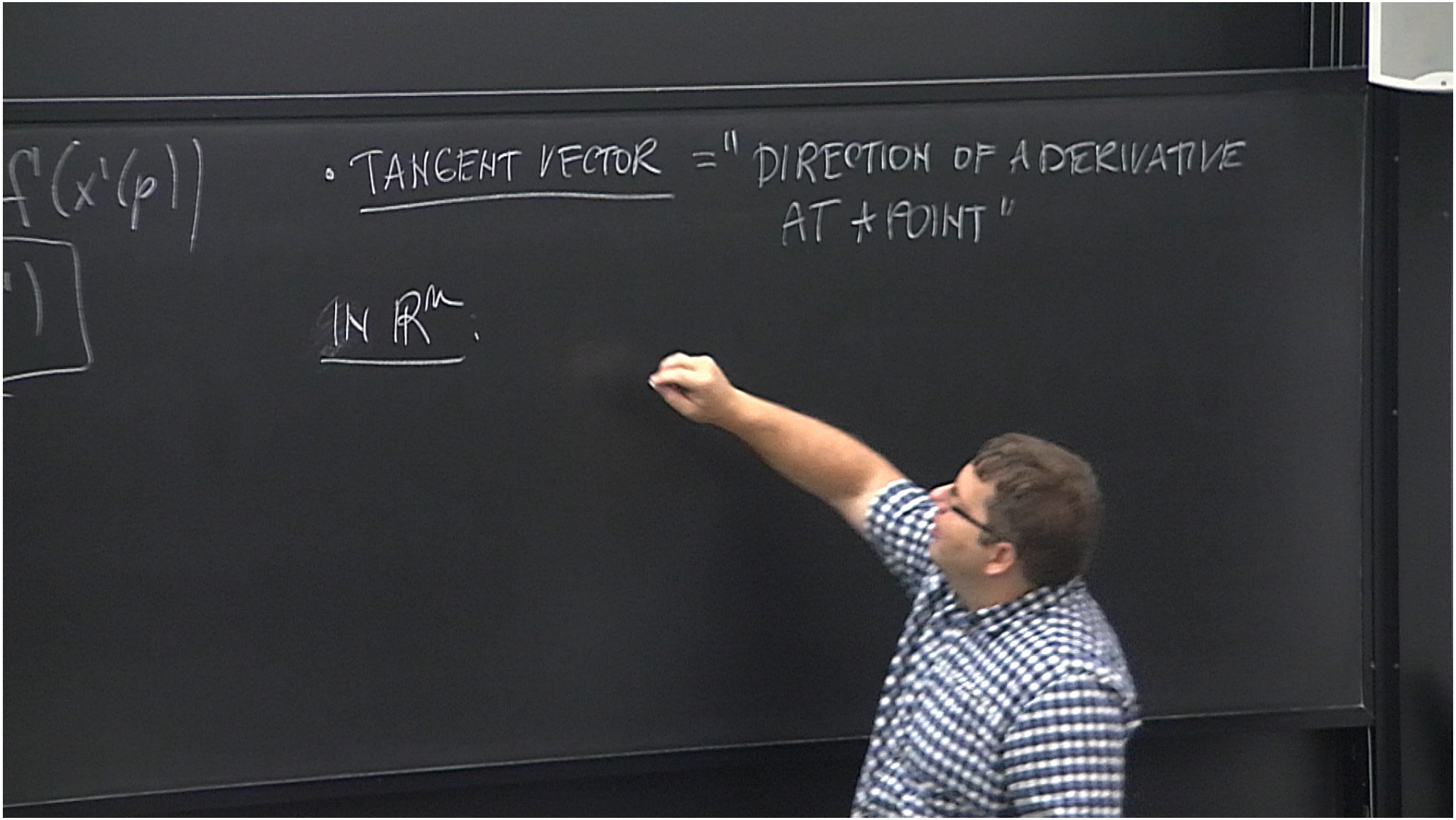
ORS

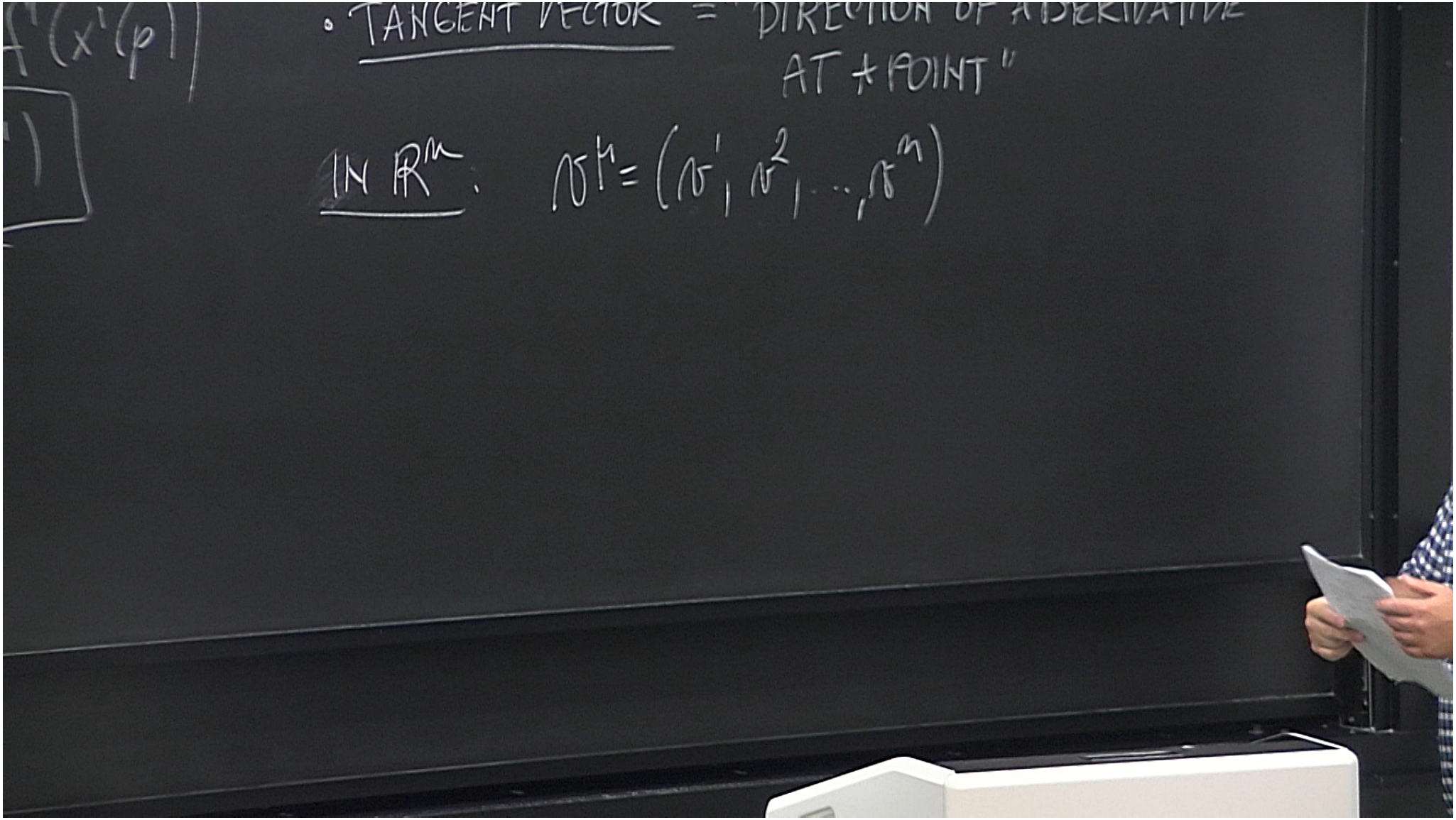
ALAR $f: M \to \mathbb{R}$

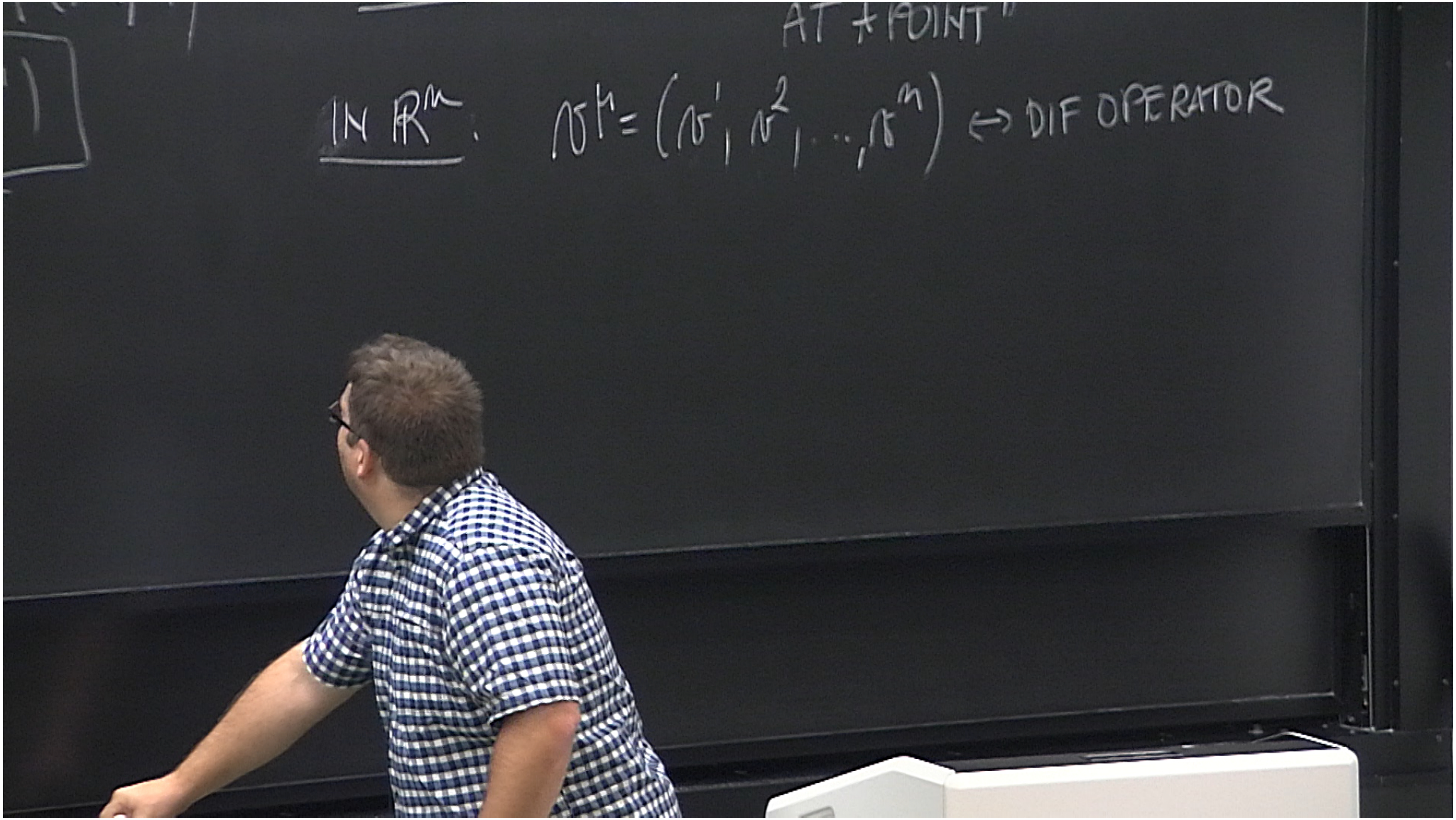


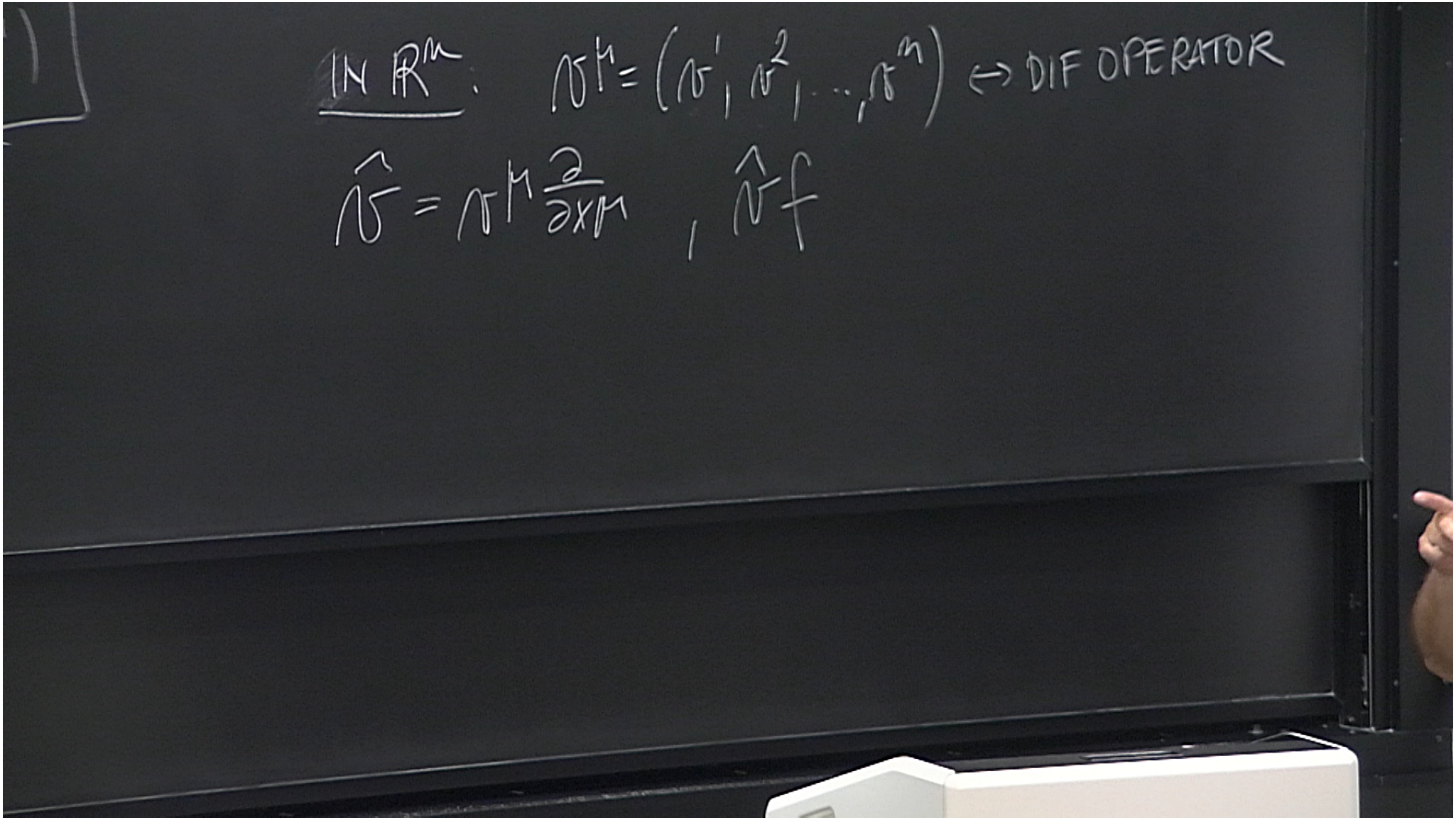
$$f(p) = f(x(p)) = f'(x'(p))$$

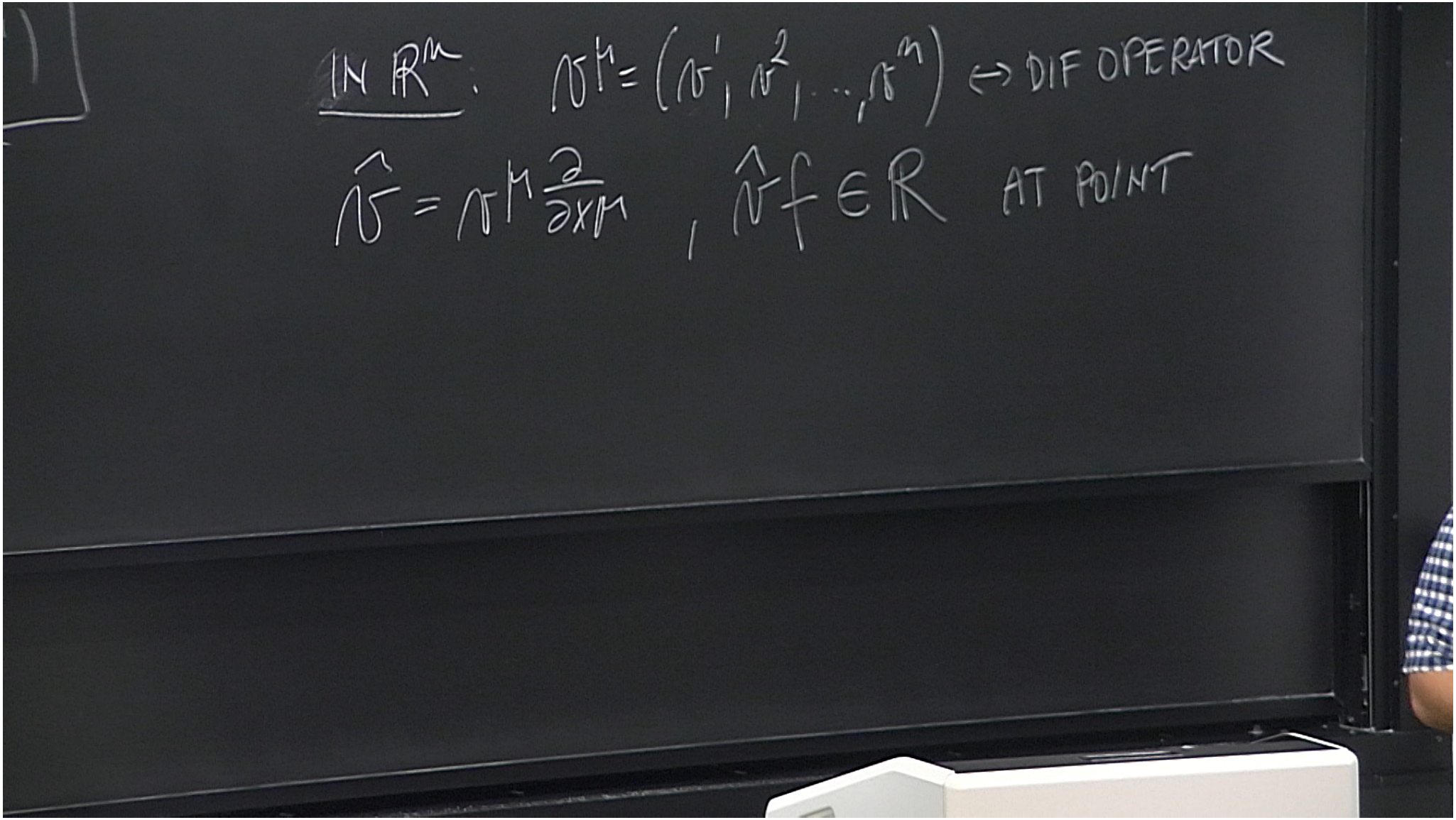
$$\boxed{f(x) = f'(x')}$$

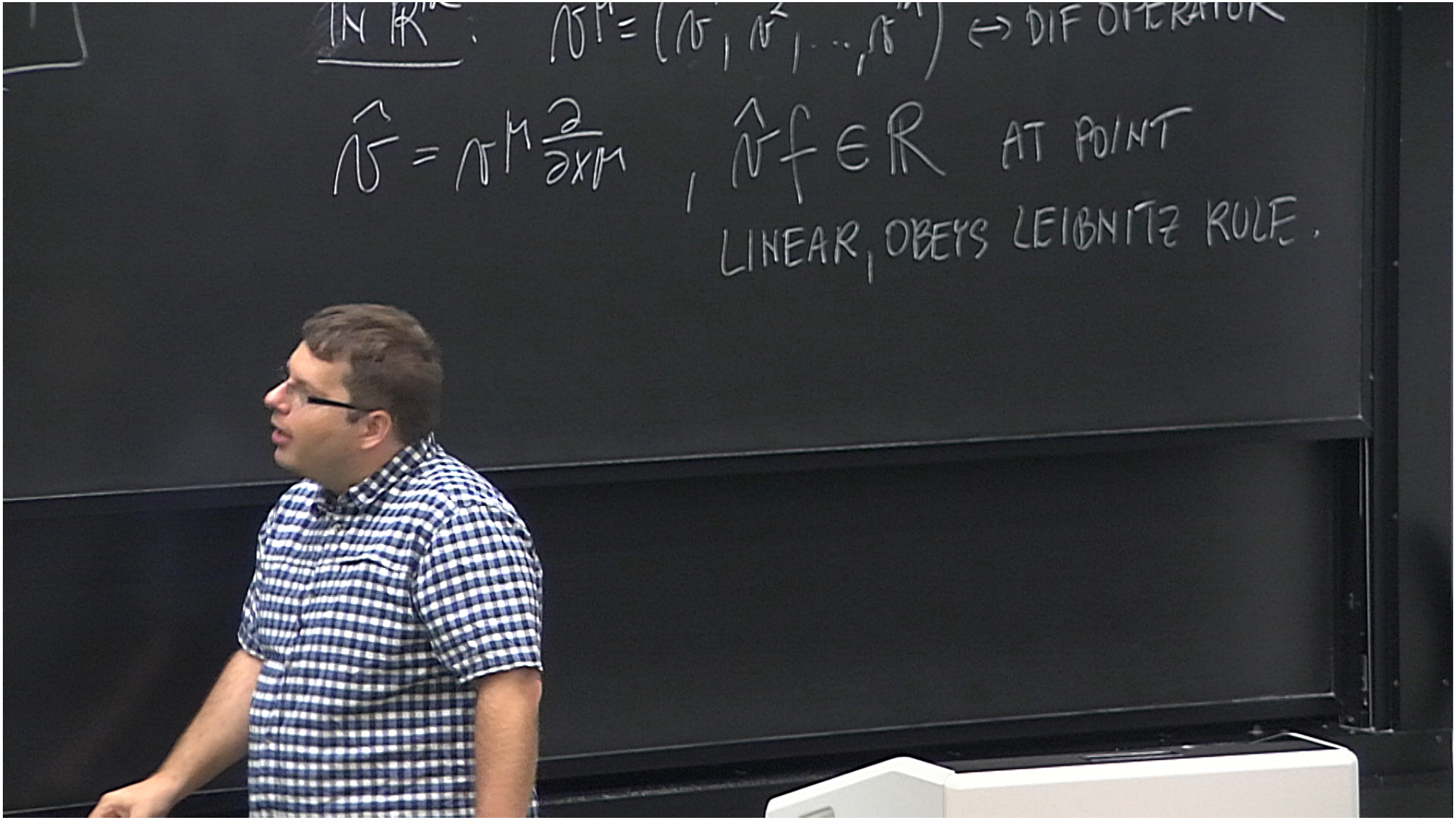

f'(x'(p))
• TANGENT VECTOR = " DIRECTION OF A DERIVATIVE AT A POINT "
IN $\mathbb{R}^m$:

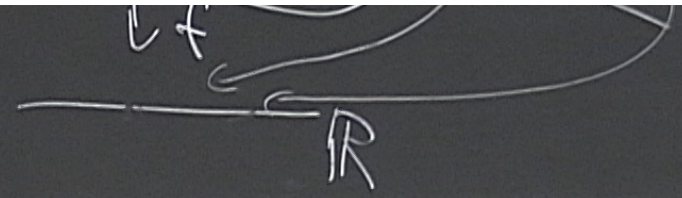

• TANGENT VECTOR = "DIRECTION OF A DERIVATIVE
AT A POINT"
IN $\mathbb{R}^n$:


AT A POINT
IN $\mathbb{R}^m$.
$\xi^\mu = (\xi^1, \xi^2, \ldots, \xi^m) \leftrightarrow$ DIF OPERATOR

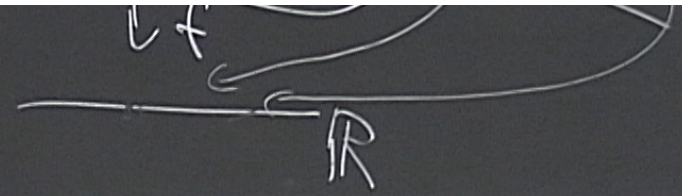

IN $\mathbb{R}^n$: $v^\mu = (v^1, v^2, \ldots, v^n) \leftrightarrow$ DIF OPERATOR
$\hat{v} = v^\mu \frac{\partial}{\partial x^\mu}$, $\hat{v} f$


IN $\mathbb{R}^n$: $v^\mu = (v^1, v^2, \ldots, v^n) \leftrightarrow$ DIF OPERATOR
$\hat{v} = v^\mu \frac{\partial}{\partial x^\mu}$, $\hat{v} f \in \mathbb{R}$ AT POINT


IN $\mathbb{R}^n$:
$\hat{v} = v^\mu \frac{\partial}{\partial x^\mu}$
$\hat{v} f \in \mathbb{R}$ AT POINT
LINEAR, OBEYS LEIBNITZ RULE.

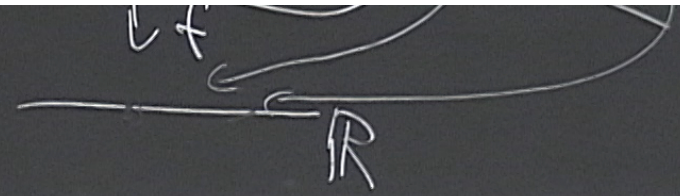


DEF: LET $\mathcal{F}$ BE A COLLECTION OF C^∞ SCALAR FUNCTIONS.
A TANGENT VECTOR V AT A POINT $p \in M$
IS A MAP $V : \mathcal{F} \to \mathbb{R}$



DEF: LET $\mathcal{F}$ BE A COLLECTION OF C^∞ SCALAR FUNCTIONS.
A TANGENT VECTOR V AT A POINT $p \in M$
IS A MAP $V: \mathcal{F} \to \mathbb{R}$ SO THAT

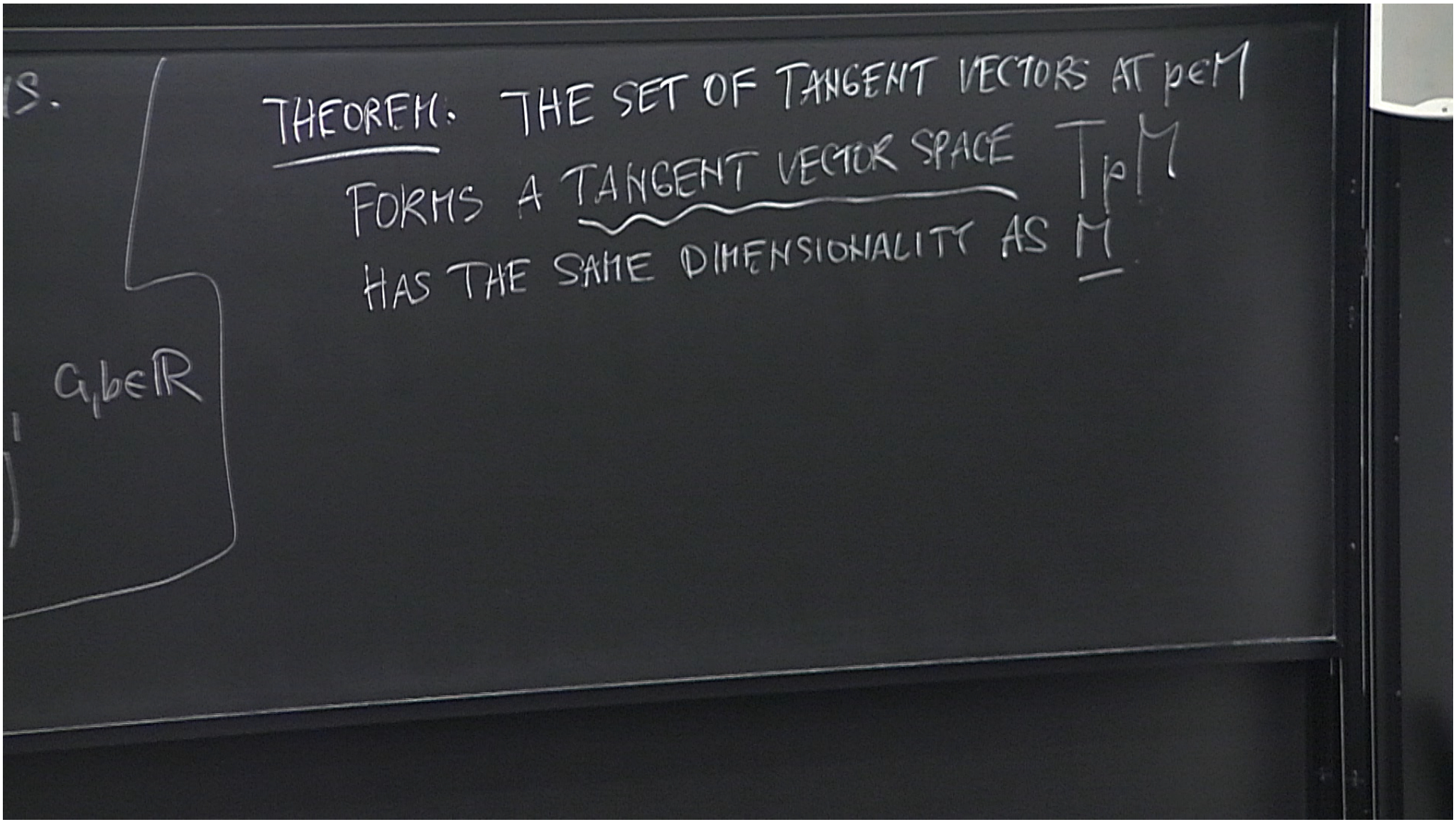
i) LINEAR $V(af+bg) = a\,V(f) + b\,V(g)$, $a,b \in \mathbb{R}$

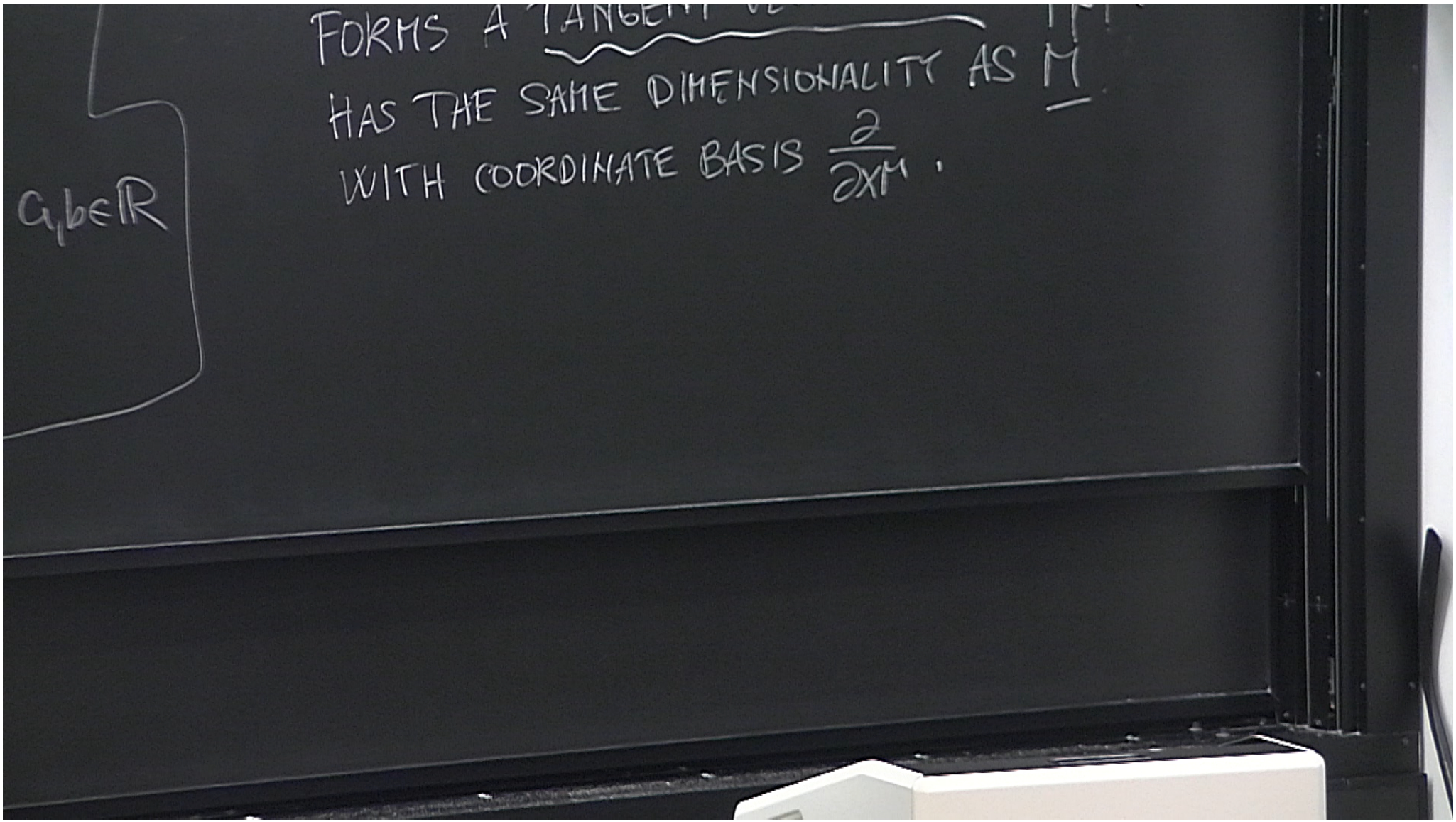


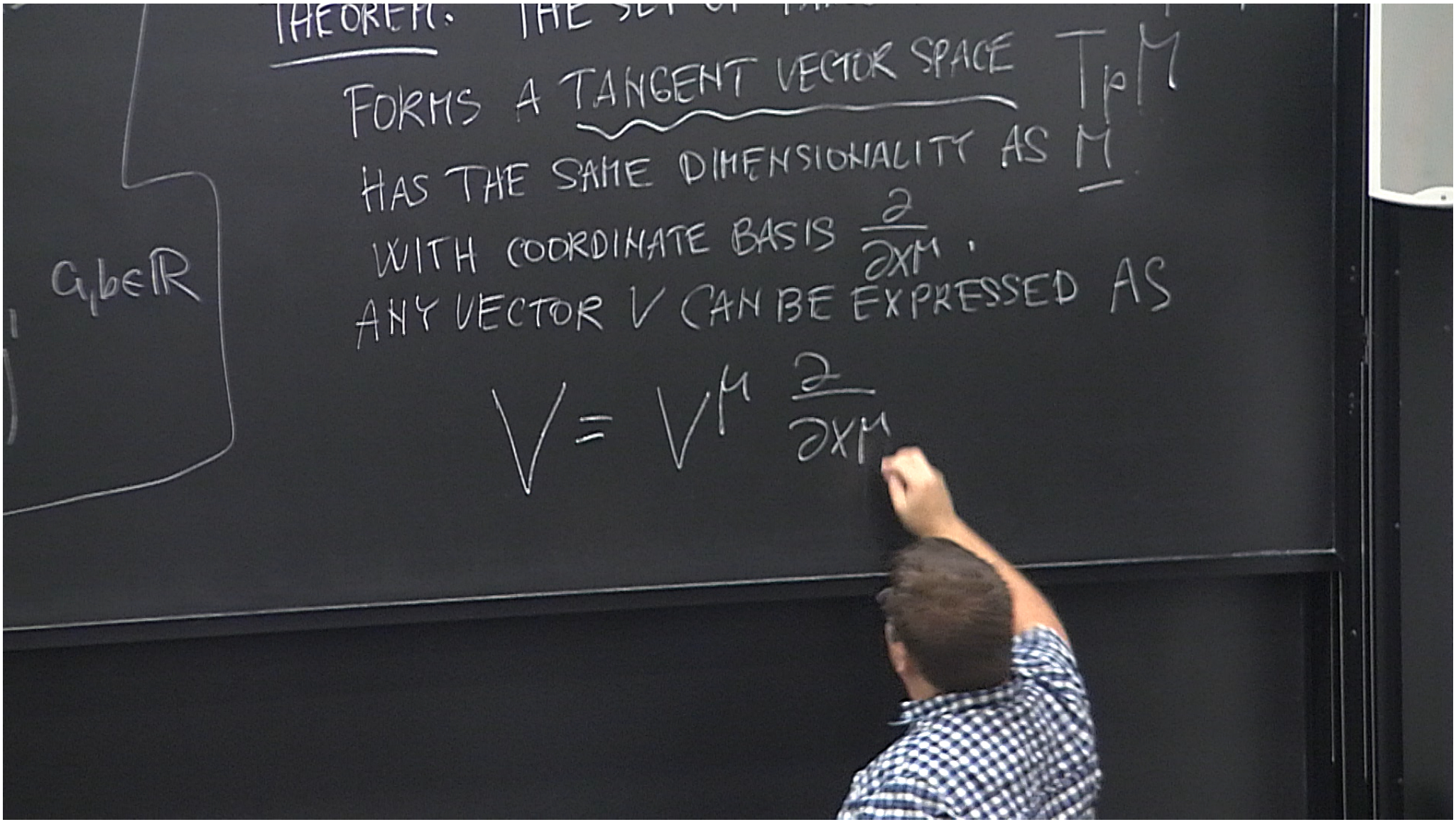
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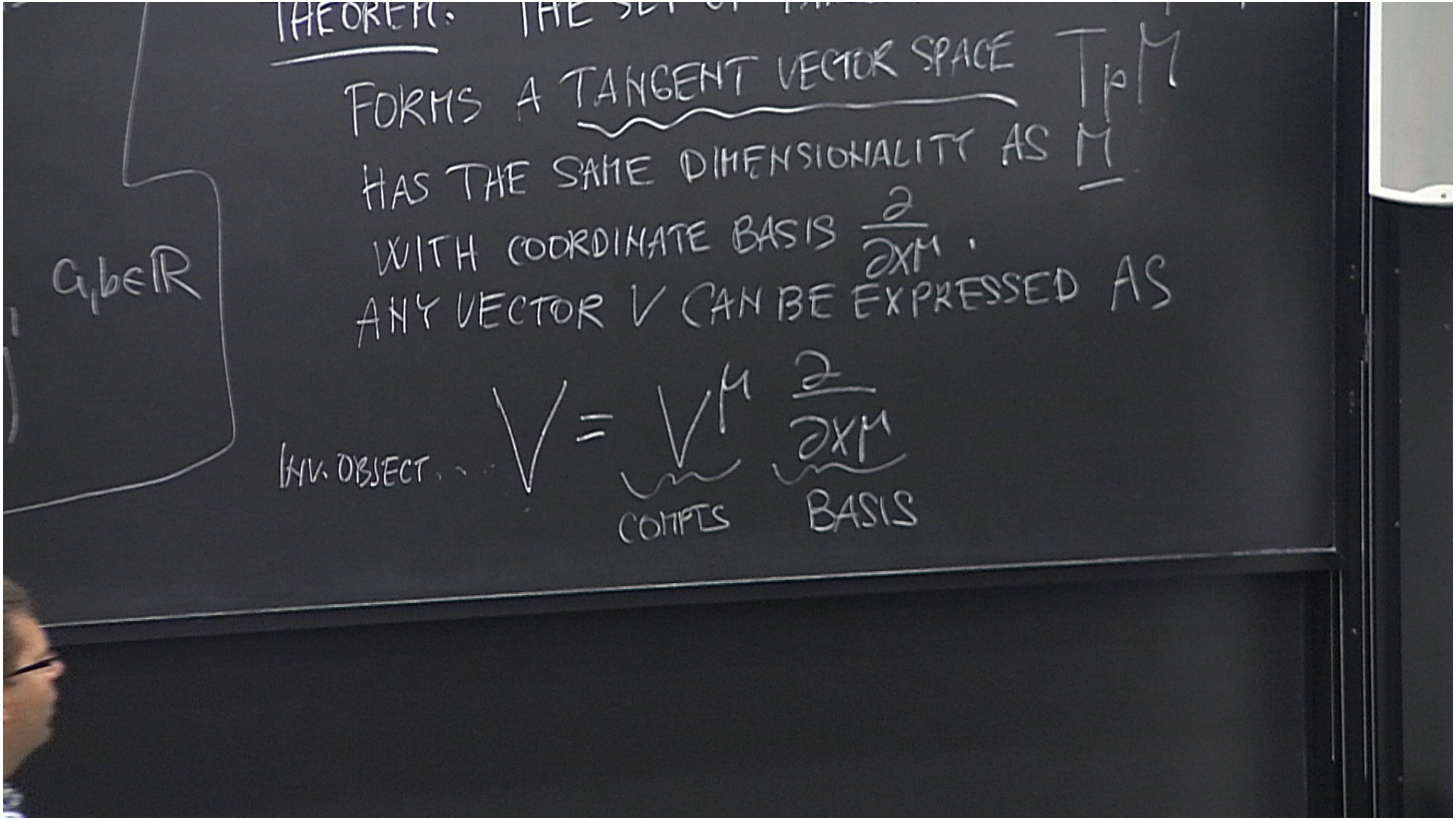
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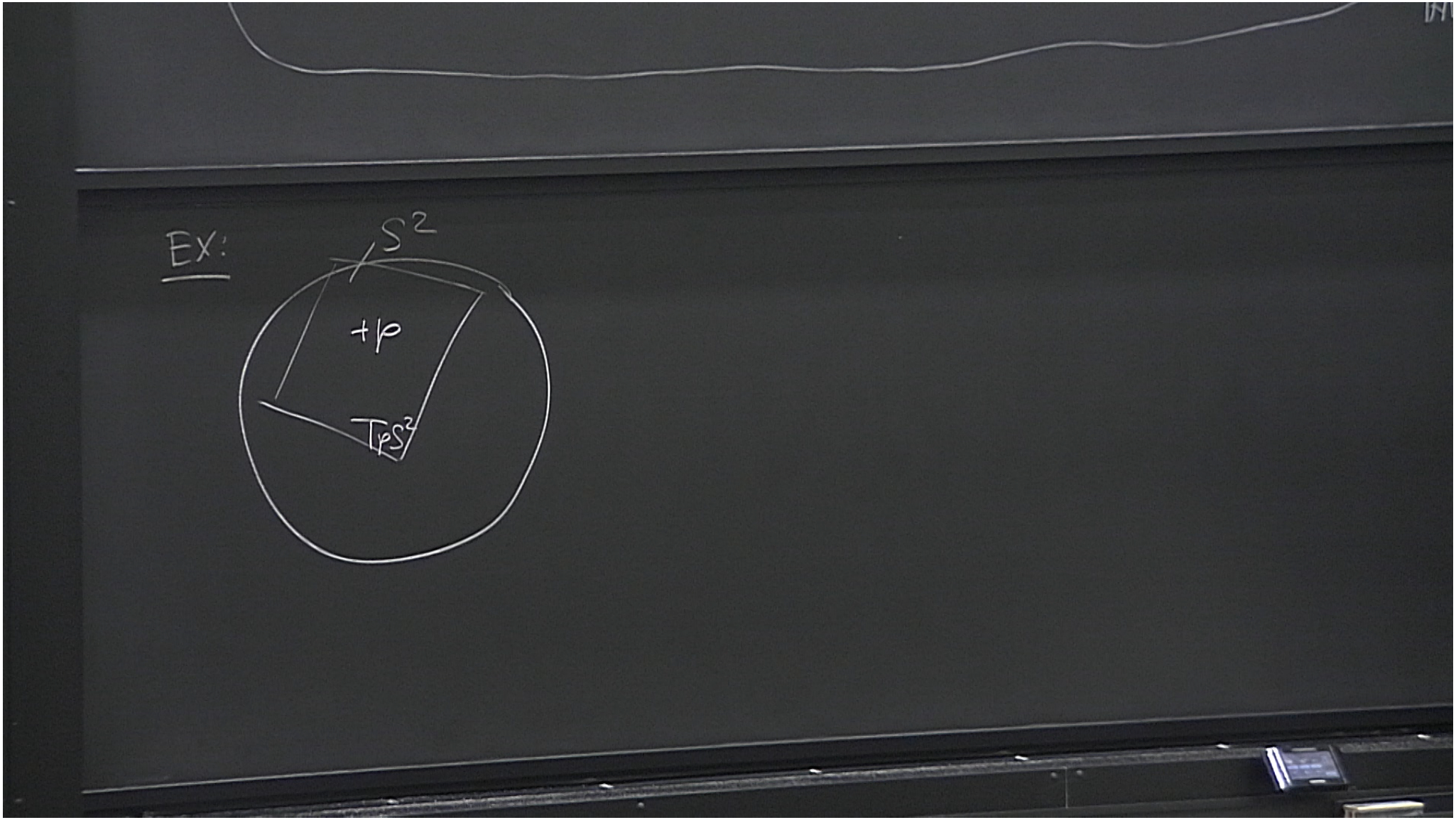
ii) LEIBNITZ: $V(fg) = f(p)\,V(g) + g(p)\,V(f)$


S.
THEOREM: THE SET OF TANGENT VECTORS AT p∈M
FORMS A TANGENT VECTOR SPACE T_pM
HAS THE SAME DIMENSIONALITY AS M
a,b∈R

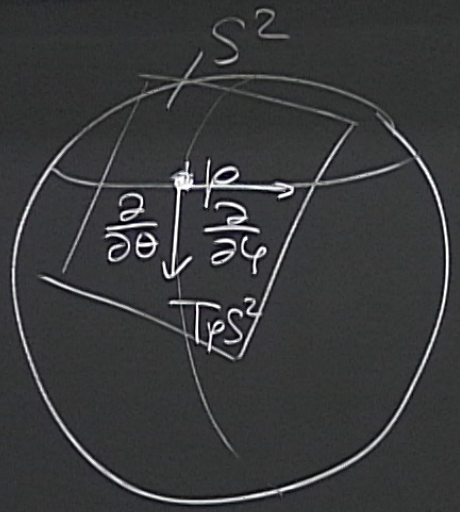

FORMS A TANGENT
HAS THE SAME DIMENSIONALITY AS M
WITH COORDINATE BASIS $\frac{\partial}{\partial x^\mu}$.
a,b∈R

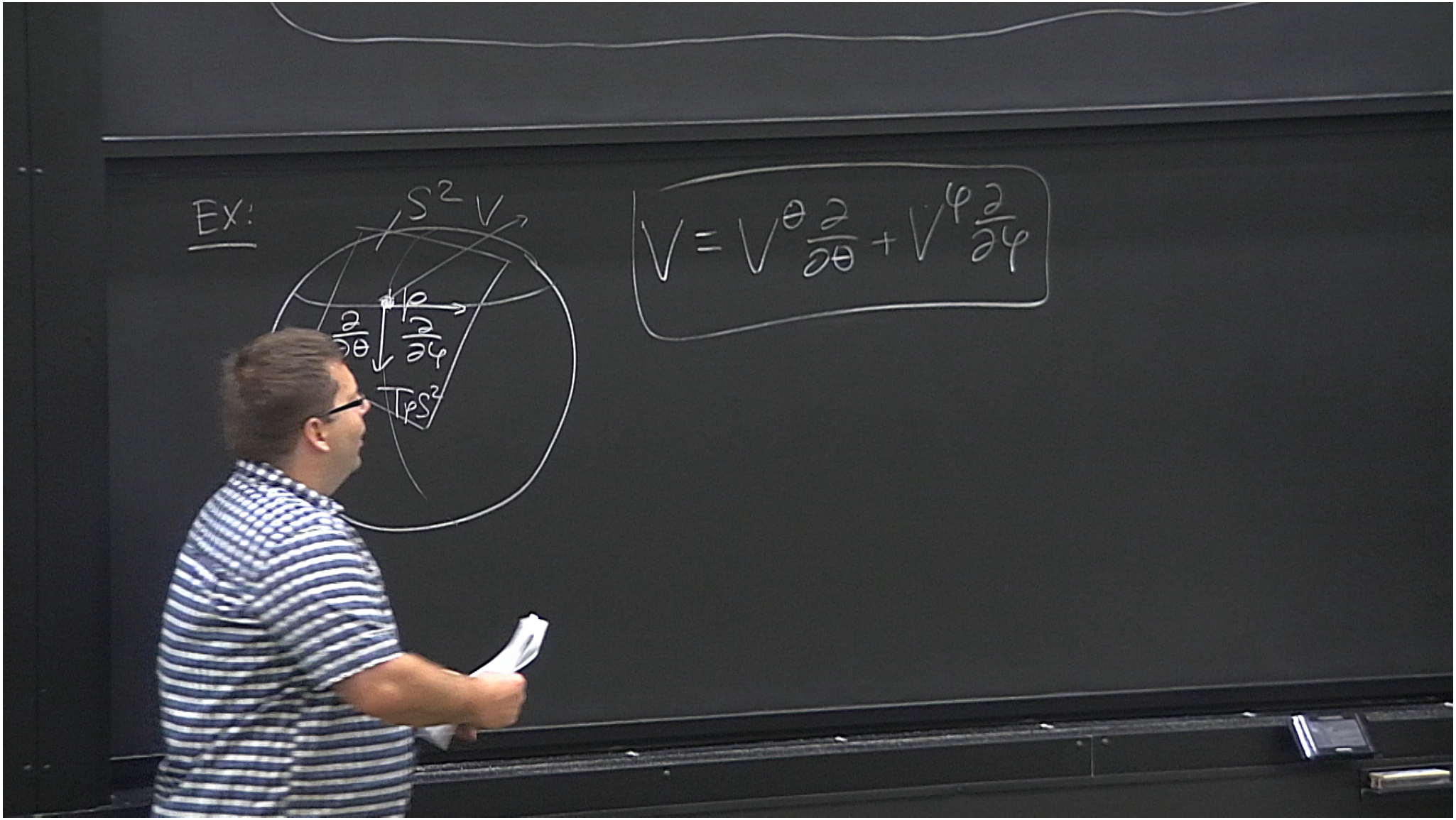

THEOREM. THE
FORMS A TANGENT VECTOR SPACE T_pM
HAS THE SAME DIMENSIONALITY AS M
WITH COORDINATE BASIS $\frac{\partial}{\partial x^\mu}$.
ANY VECTOR V CAN BE EXPRESSED AS
$V = V^\mu \frac{\partial}{\partial x^\mu}$
G,beR


THEOREM: THE
FORMS A TANGENT VECTOR SPACE T_pM
HAS THE SAME DIMENSIONALITY AS M
WITH COORDINATE BASIS $\frac{\partial}{\partial x^\mu}$.
ANY VECTOR V CAN BE EXPRESSED AS
INV. OBJECT
$V = V^\mu \frac{\partial}{\partial x^\mu}$
COMPTS
BASIS
G,beR

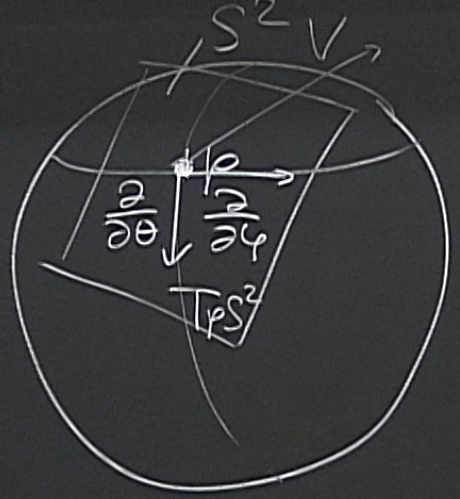

EX:
S^2
$+p$
T_pS^2

Ex:


S^2
$\frac{\partial}{\partial\theta}$
$\frac{\partial}{\partial\varphi}$
T_pS^2

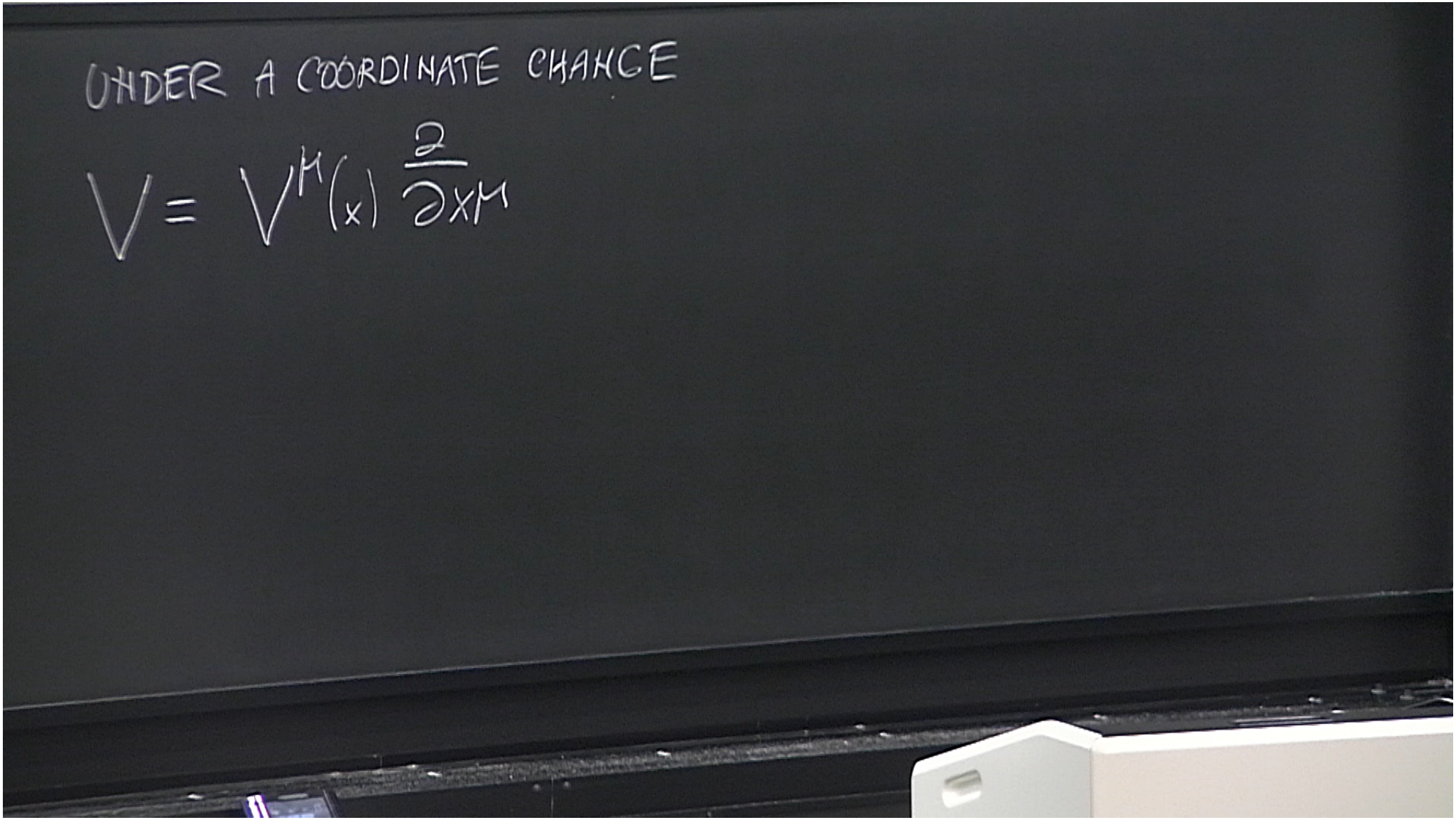

Ex:
S^2
V
$\frac{\partial}{\partial\theta}$
$\frac{\partial}{\partial\varphi}$
$T_p S^2$
$V = V^\theta \frac{\partial}{\partial\theta} + V^\varphi \frac{\partial}{\partial\varphi}$

Ex:



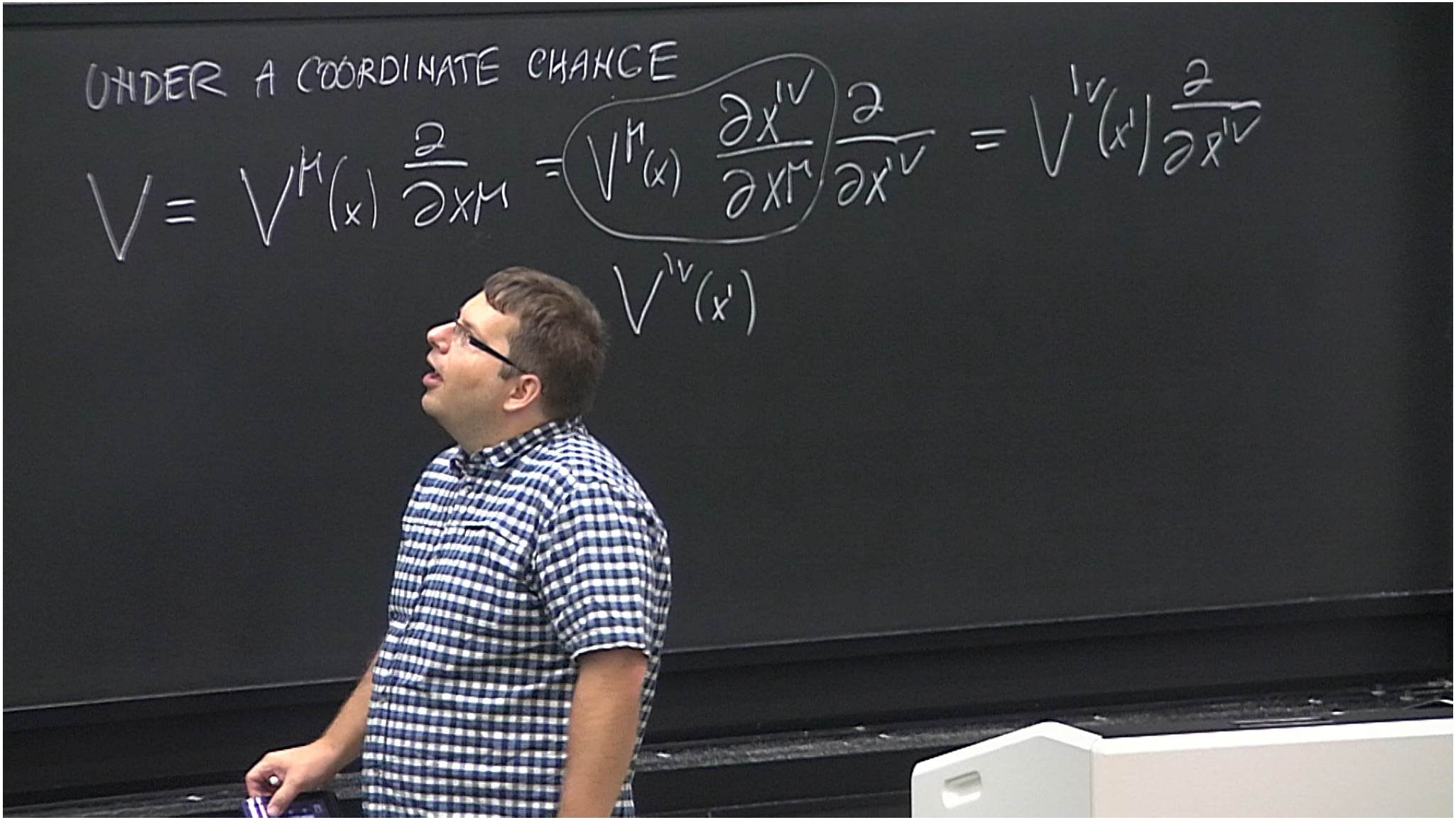
$$V = V^\theta \frac{\partial}{\partial\theta} + V^\varphi \frac{\partial}{\partial\varphi}$$

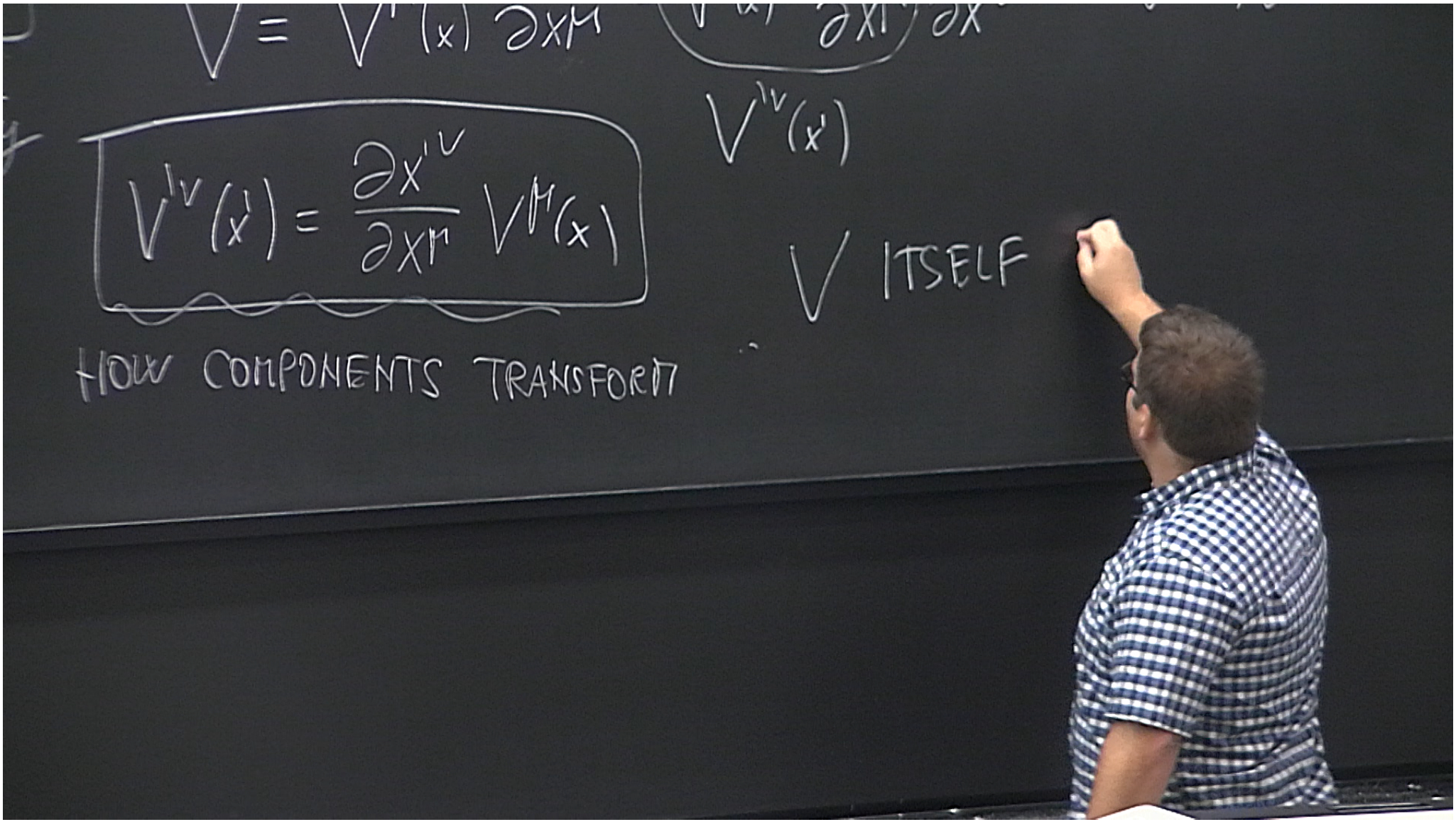
$$= V^x \frac{\partial}{\partial x} + V^y \frac{\partial}{\partial y}$$

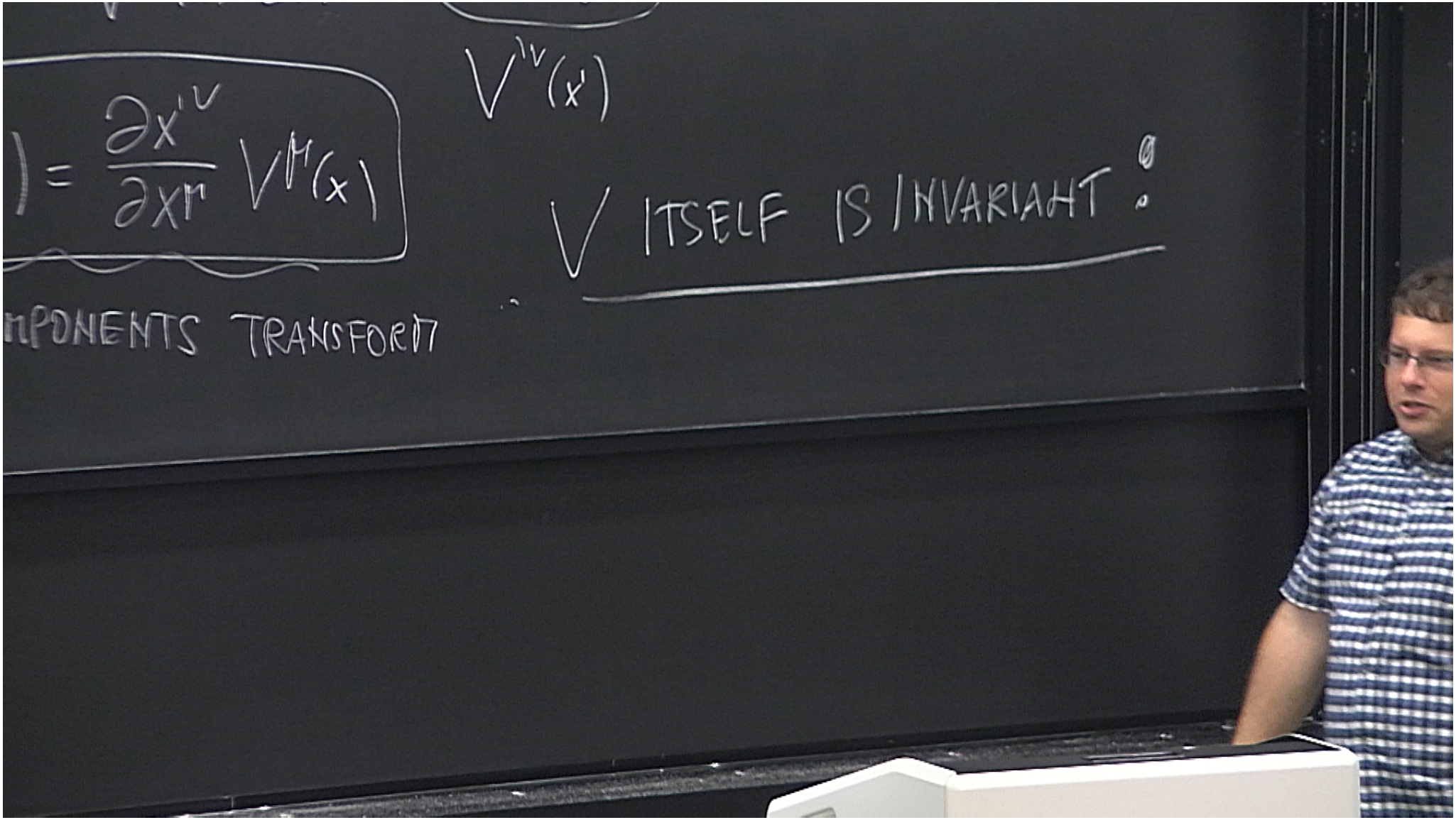

UNDER A COORDINATE CHANGE
$V = V^{\mu}(x) \frac{\partial}{\partial x^{\mu}}$

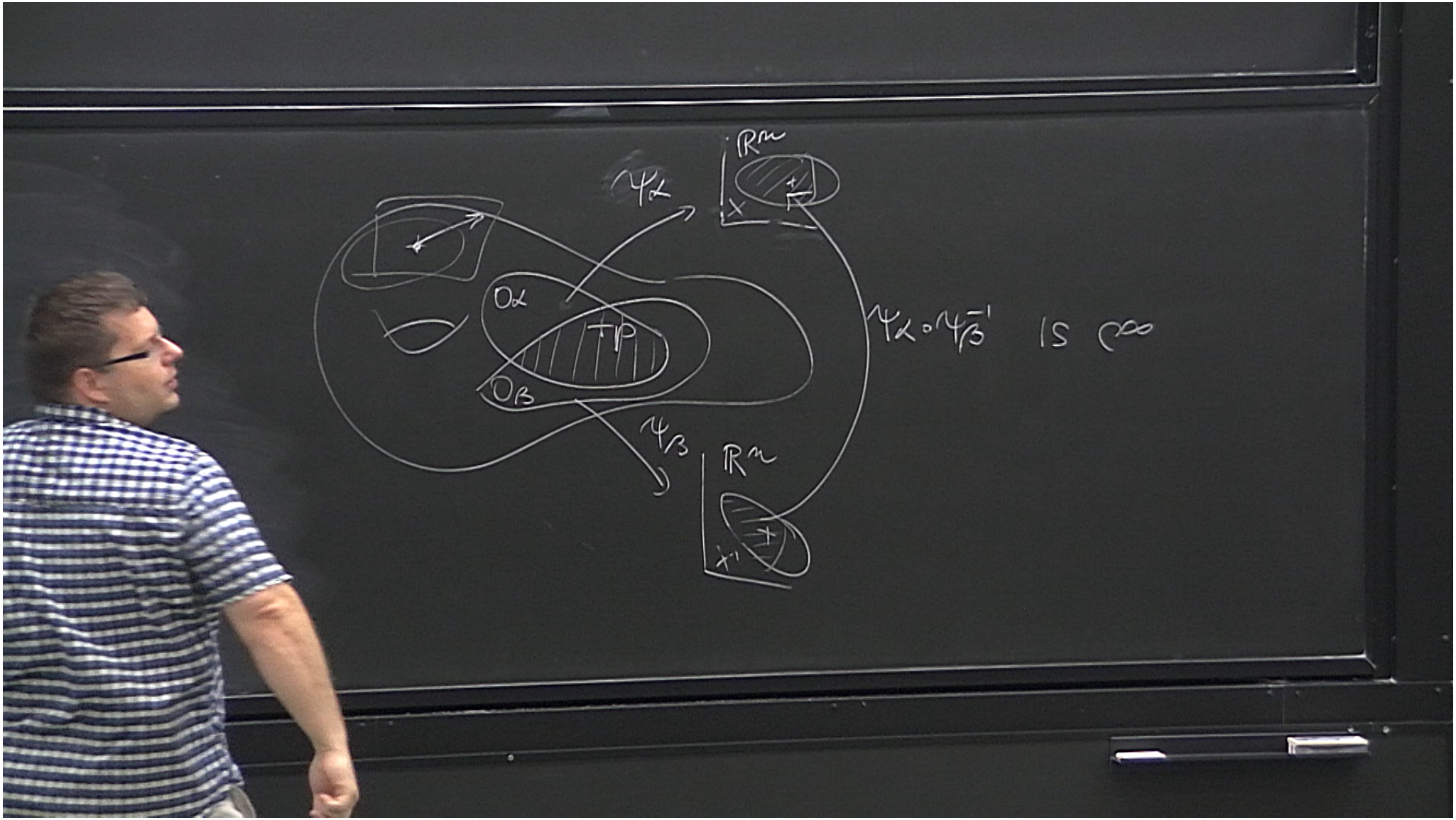
UNDER A COORDINATE CHANGE

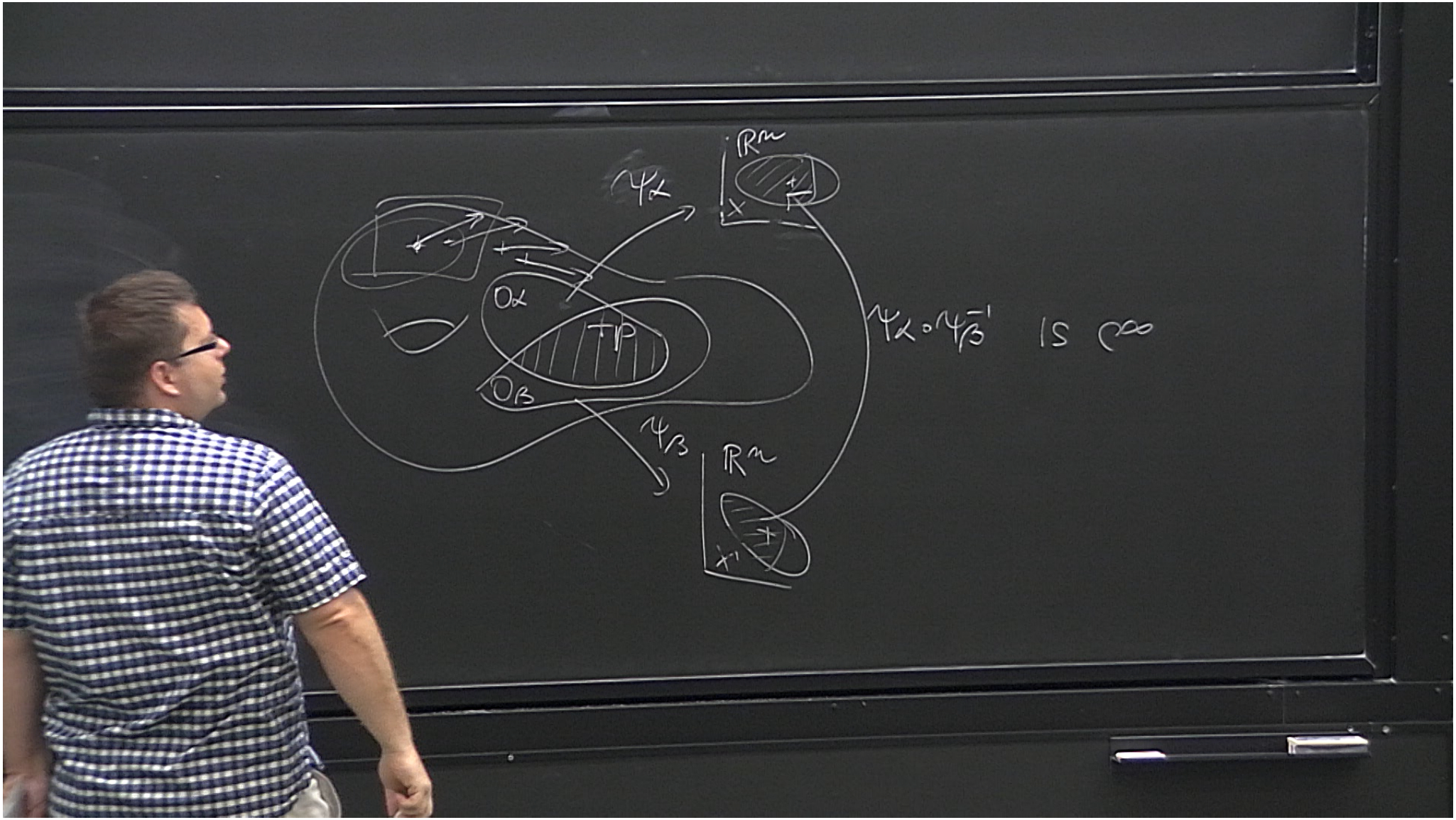
$$V = V^{\mu}(x) \frac{\partial}{\partial x^{\mu}} = V'^{\mu}(x) \frac{\partial x'^{\nu}}{\partial x^{\mu}} \frac{\partial}{\partial x'^{\nu}}$$

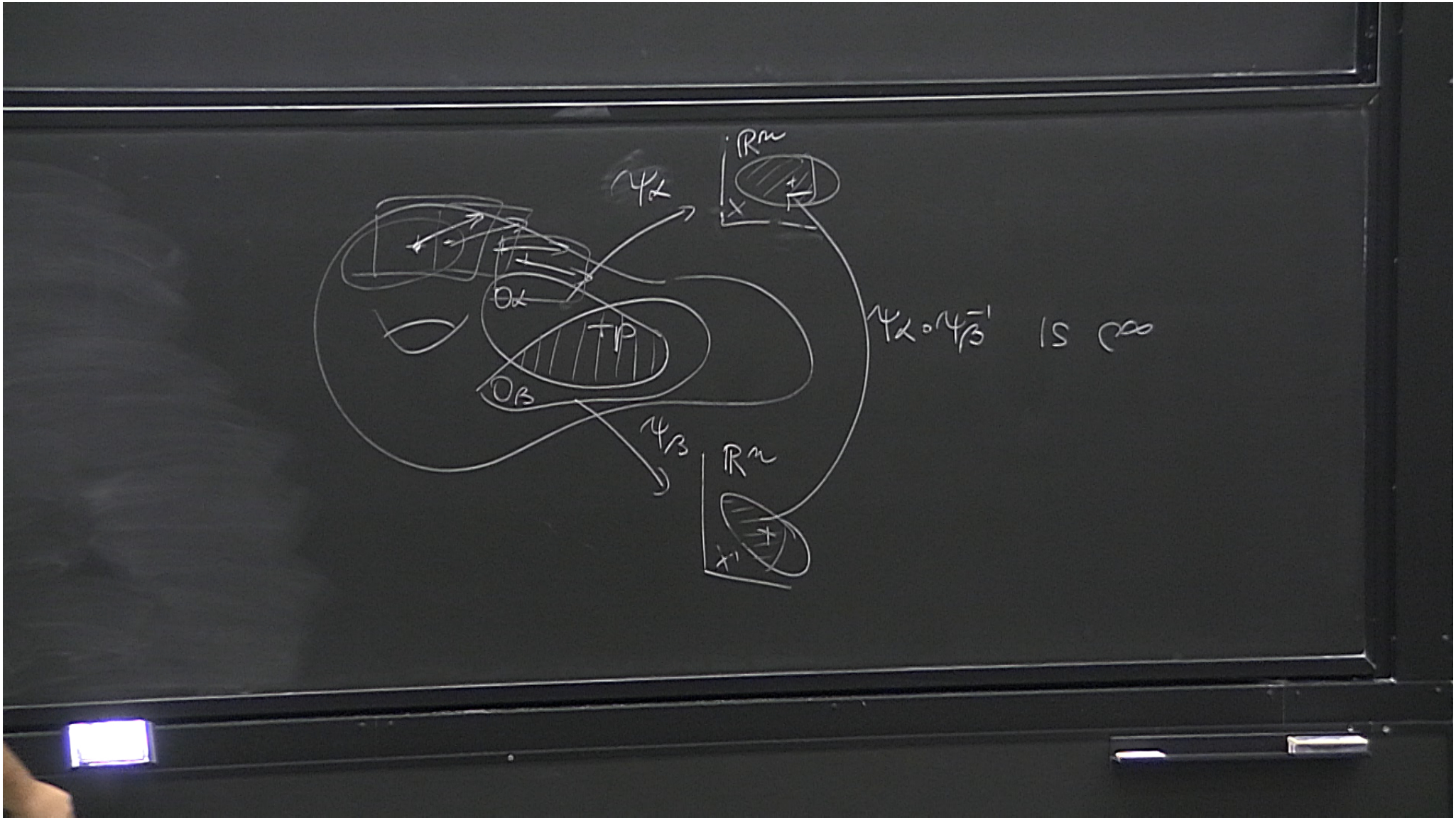

UNDER A COORDINATE CHANGE
$V = V^{\mu}(x) \frac{\partial}{\partial x^{\mu}} = V^{\mu}(x) \frac{\partial x'^{\nu}}{\partial x^{\mu}} \frac{\partial}{\partial x'^{\nu}} = V'^{\nu}(x') \frac{\partial}{\partial x'^{\nu}}$
$V'^{\nu}(x')$

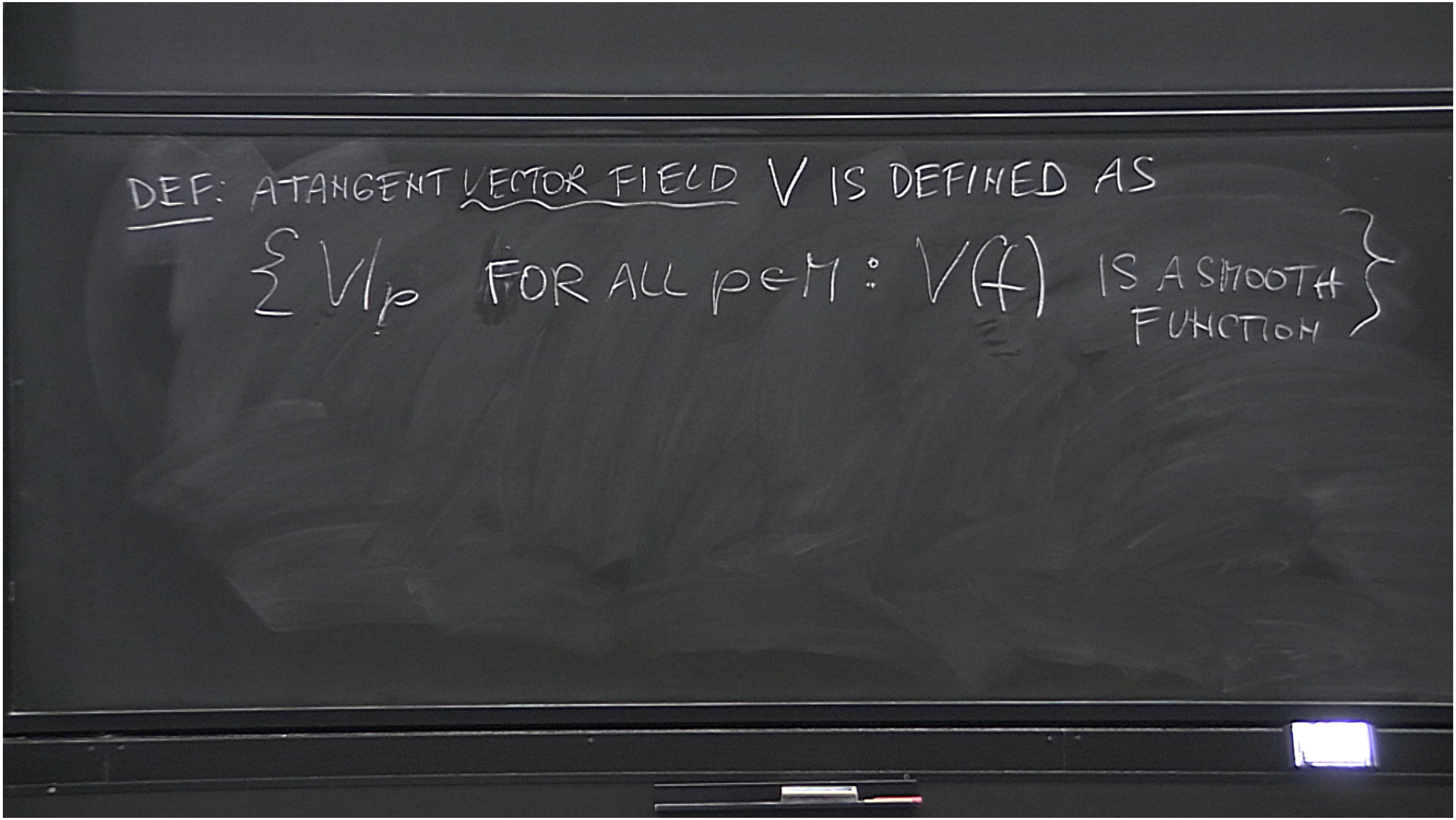

$V = V'(x)\,\partial x^{\mu}$
$V'^{\nu}(x')$
$V'^{\nu}(x') = \frac{\partial x'^{\nu}}{\partial x^{\mu}} V^{\mu}(x)$
V ITSELF
HOW COMPONENTS TRANSFORM

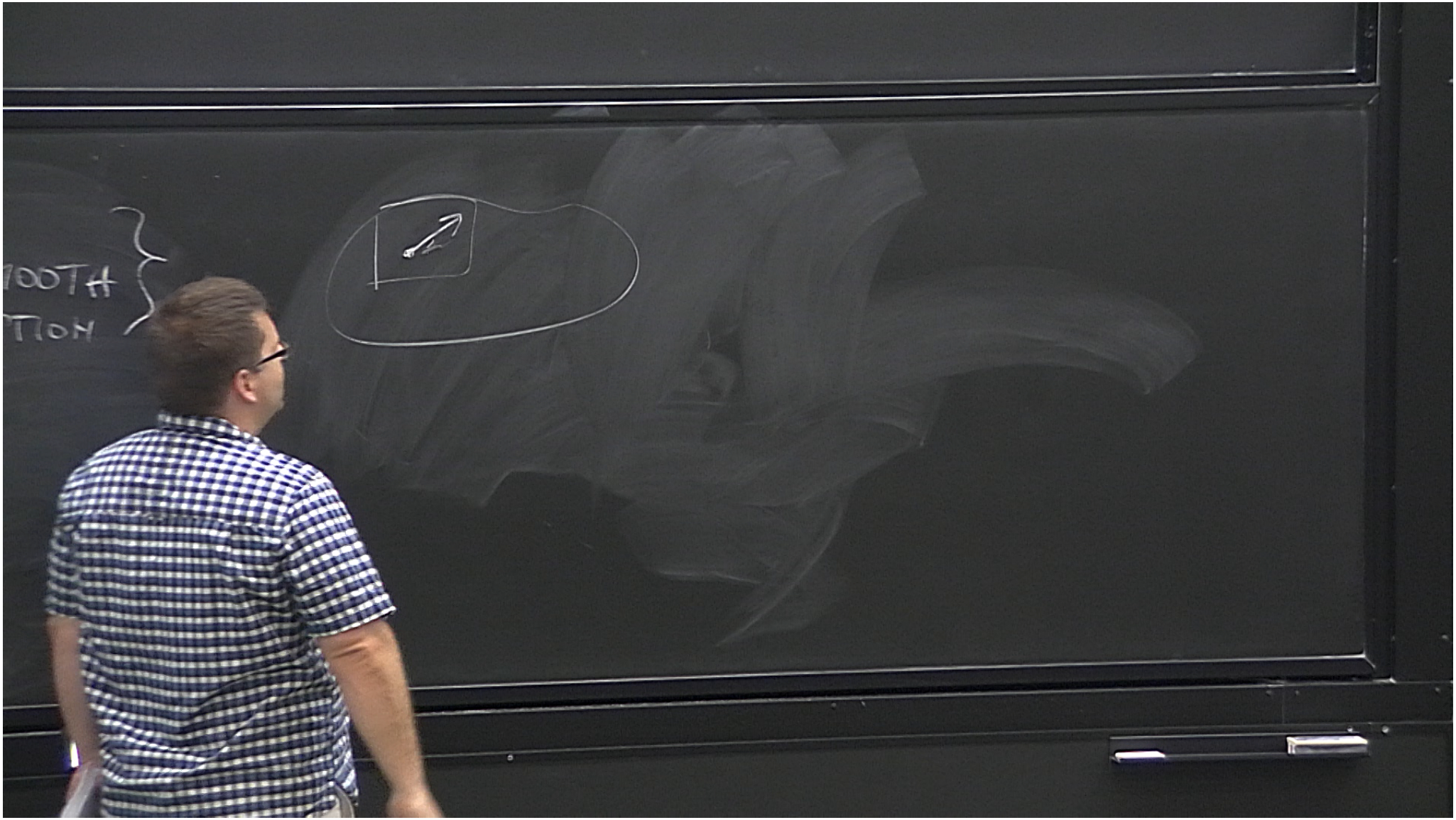

$V'^{\mu\nu}(x')$
$= \frac{\partial x'^{\mu}}{\partial x^{\rho}} V^{\rho\nu}(x)$
V ITSELF IS INVARIANT!
MPONENTS TRANSFORM

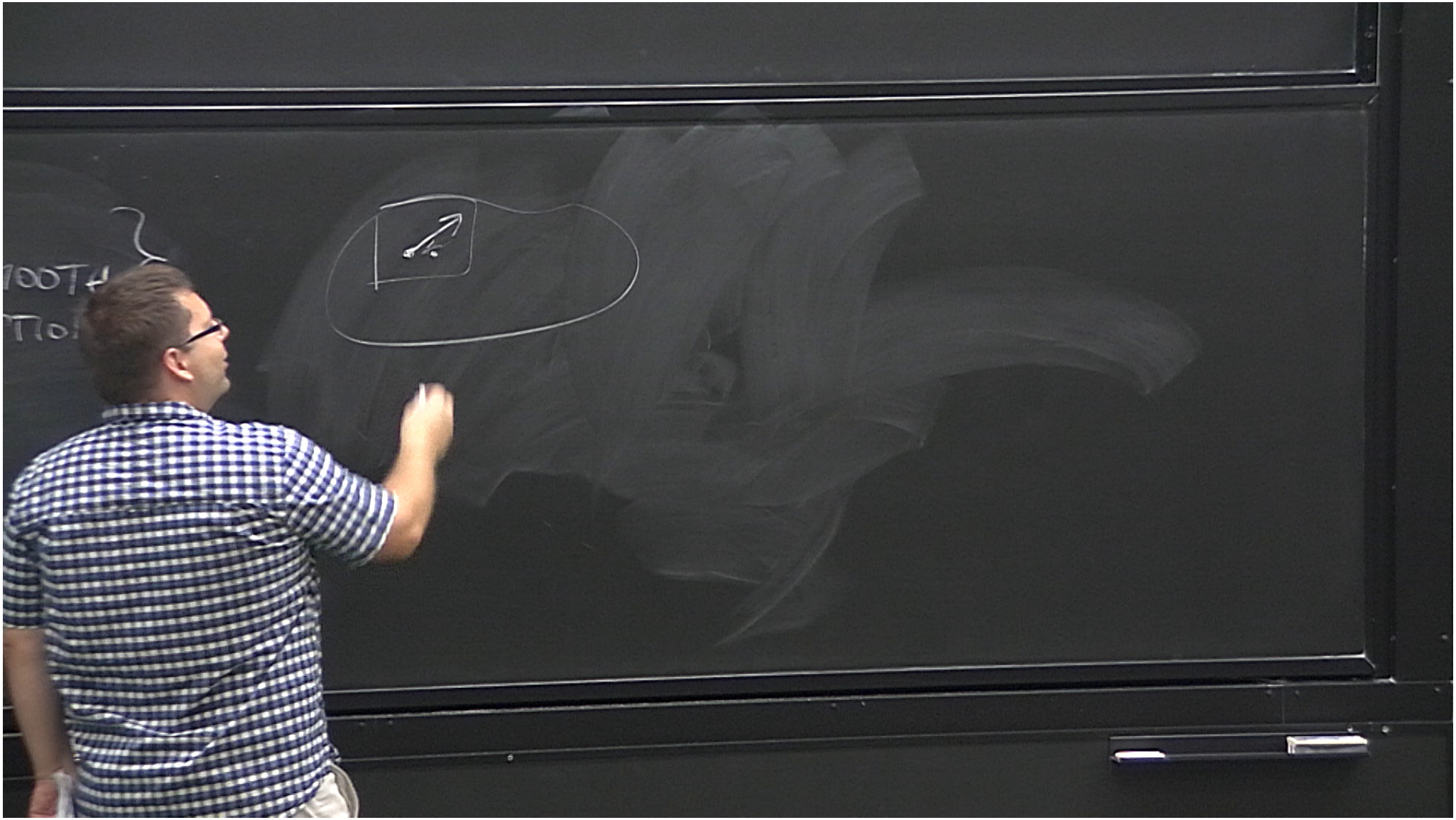

$\mathbb{R}^n$
ψ_α
U_α
p
U_β
ψ_β
$\mathbb{R}^n$
$\psi_\alpha \circ \psi_\beta^{-1}$ IS C^∞

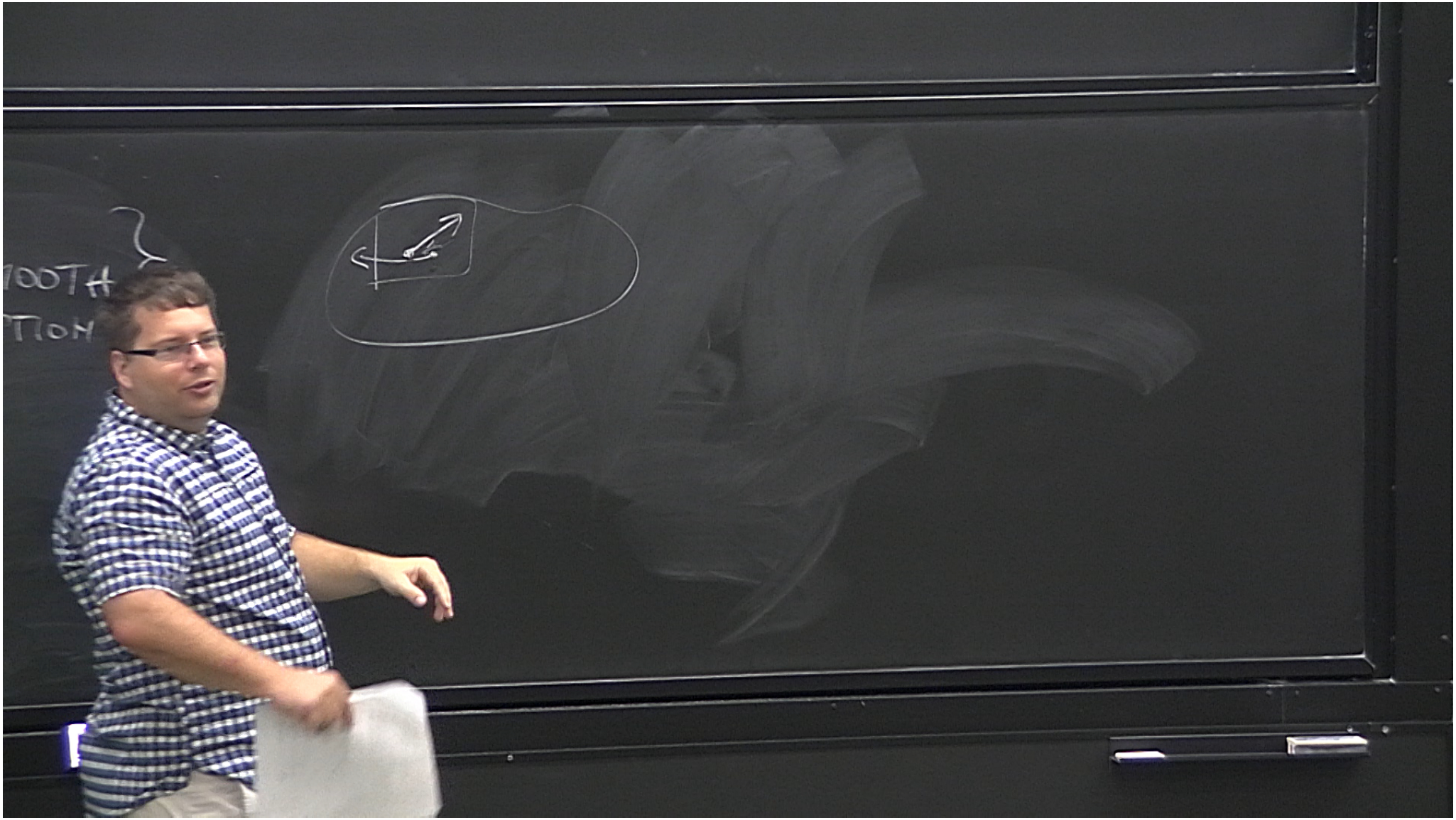

$\mathbb{R}^n$
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p
U_β
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$\psi_\alpha \circ \psi_\beta^{-1}$ IS C^∞

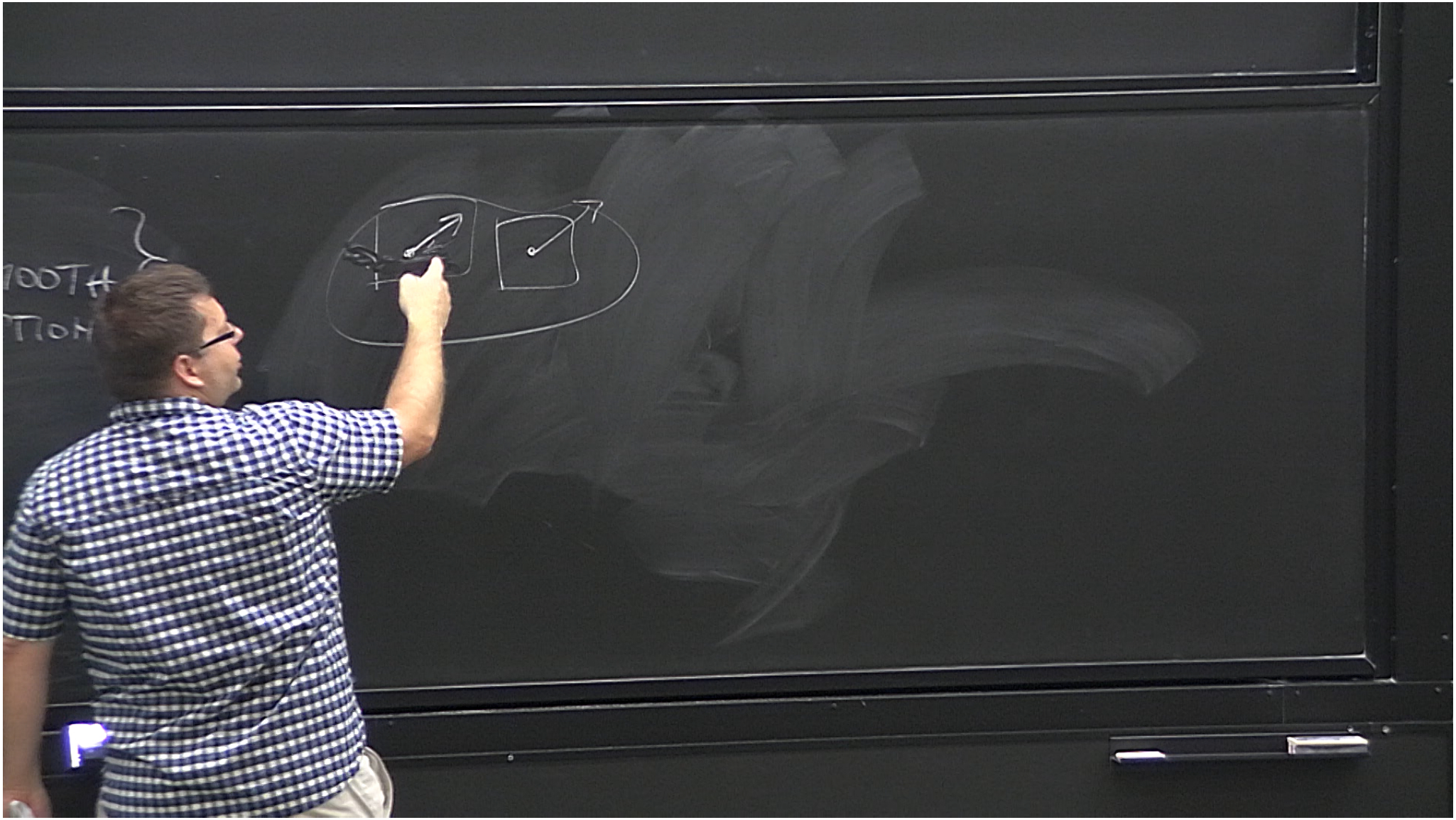

$\mathbb{R}^n$
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U_α
p
U_β
ψ_β
$\mathbb{R}^n$
$\psi_\alpha \circ \psi_\beta^{-1}$ IS C^∞

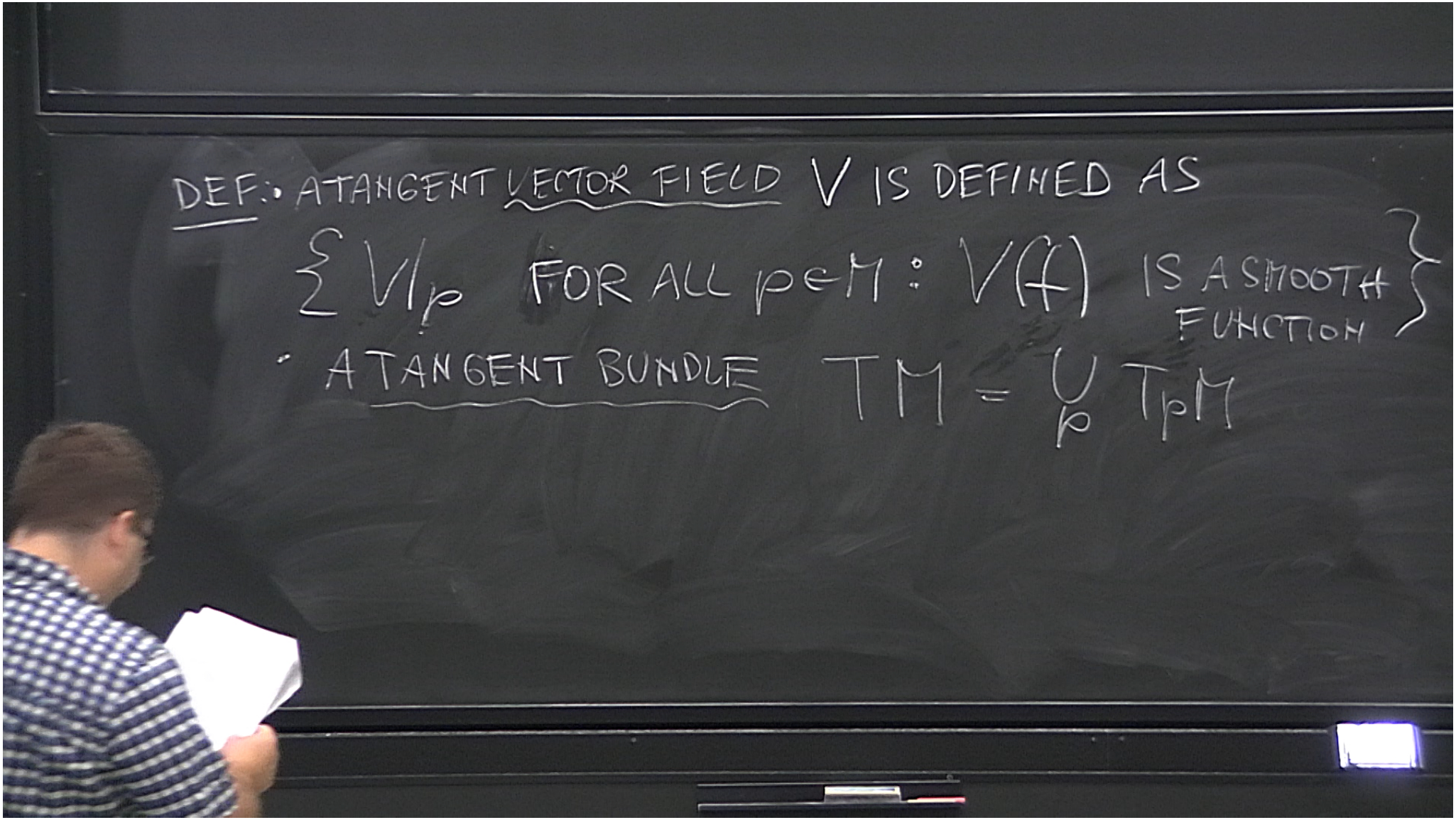

DEF: A TANGENT VECTOR FIELD V IS DEFINED AS
$\{ V|_p$ FOR ALL $p \in M$: $V(f)$ IS A SMOOTH FUNCTION $\}$

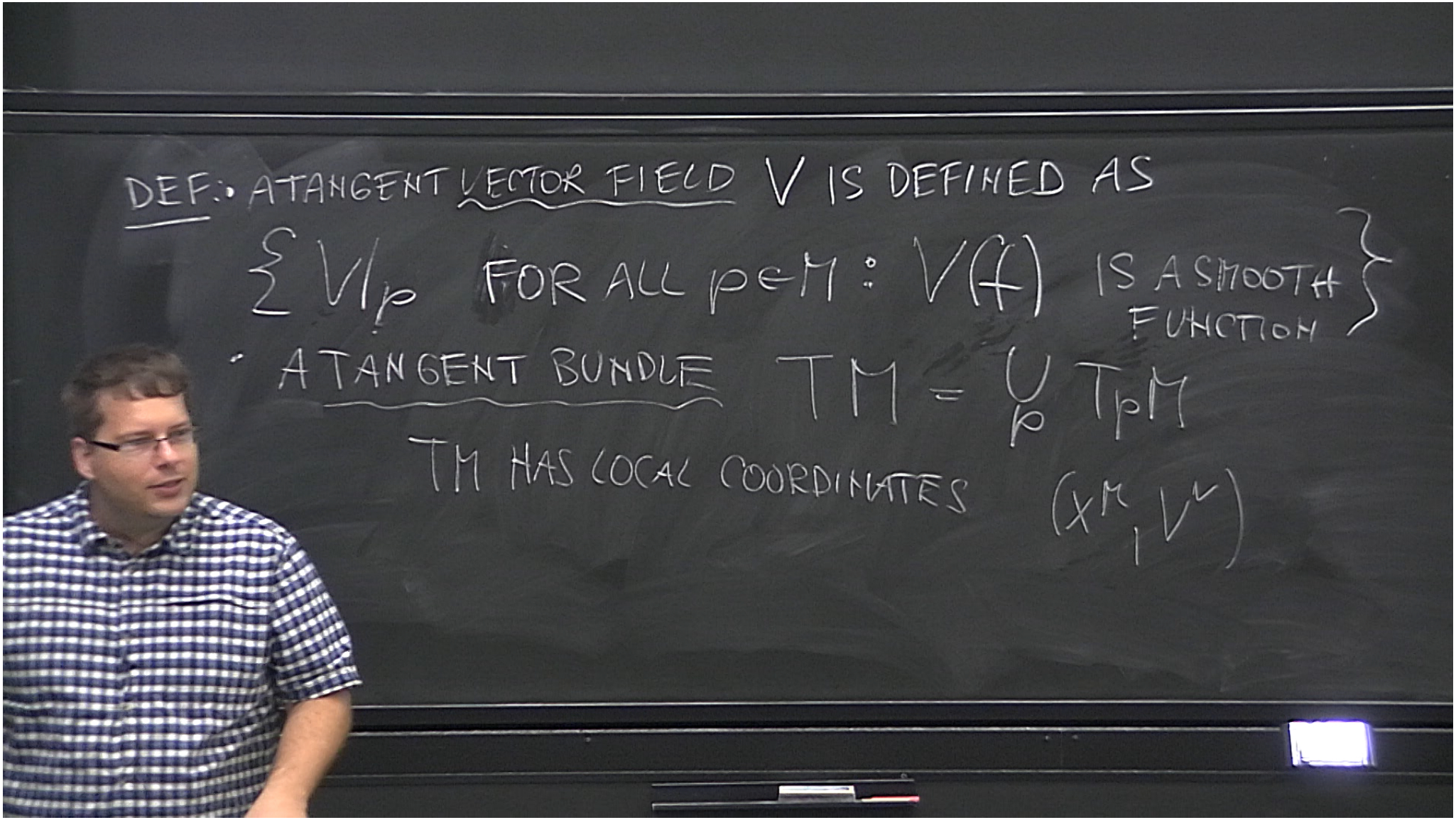

OOTH
TION

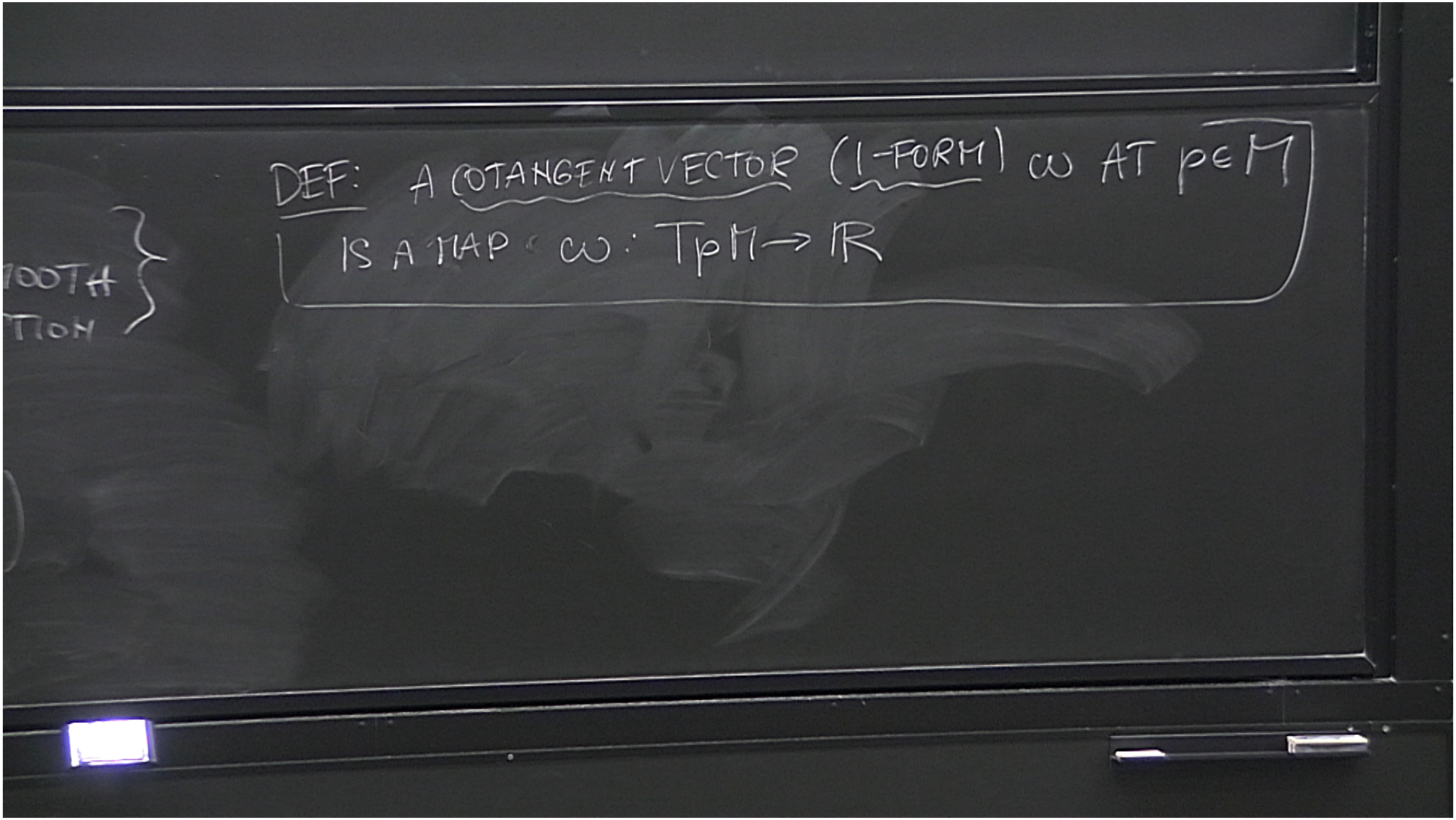


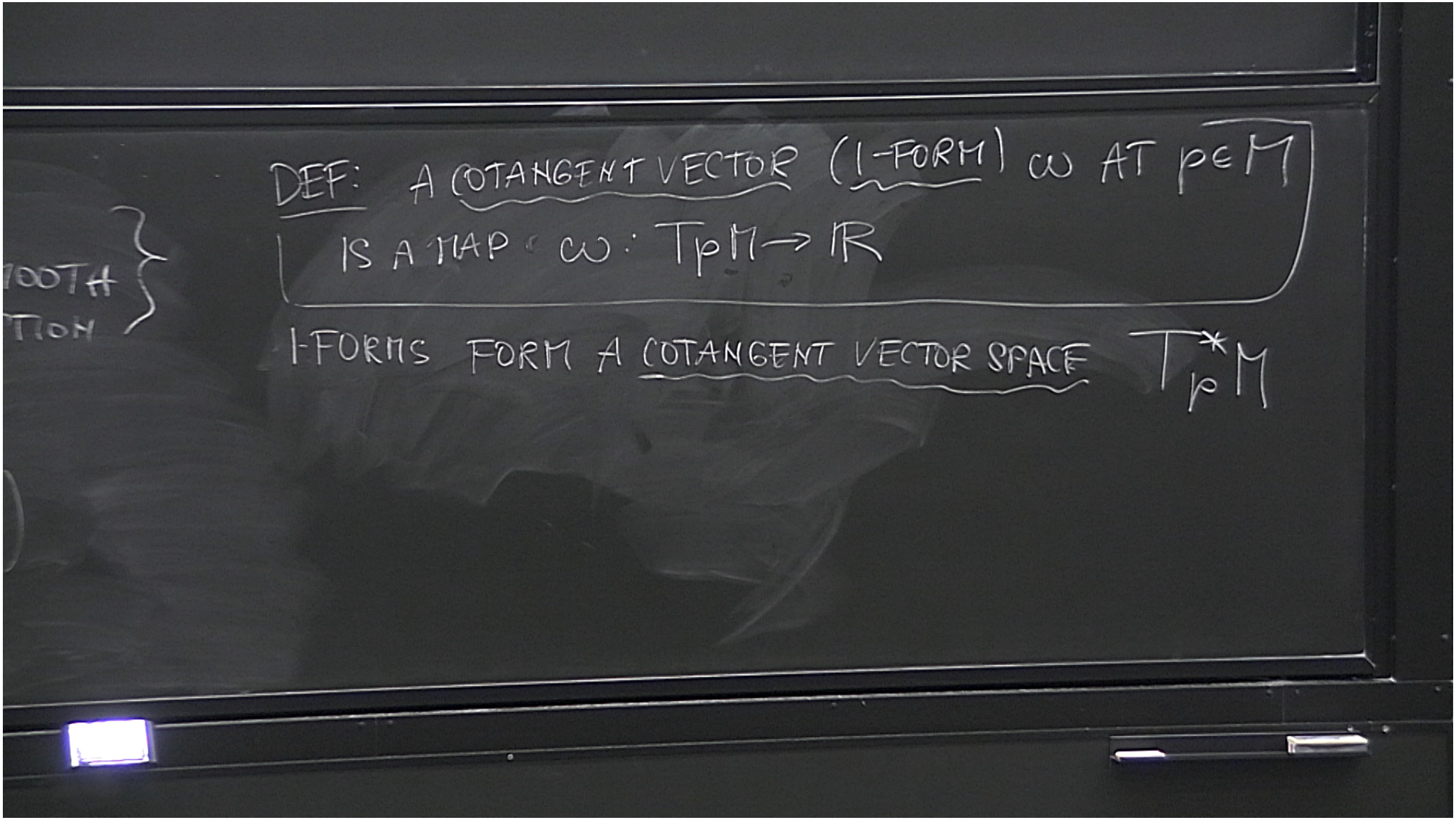



OOTH
TION


DEF: A TANGENT VECTOR FIELD V IS DEFINED AS
$\{ V|_p$ FOR ALL $p \in M$: $V(f)$ IS A SMOOTH FUNCTION $\}$
A TANGENT BUNDLE $TM = \bigcup_p T_pM$


DEF: A TANGENT VECTOR FIELD V IS DEFINED AS
$\{ V|_p$ FOR ALL $p \in M : V(f)$ IS A SMOOTH FUNCTION $\}$
A TANGENT BUNDLE $TM = \bigcup_p T_pM$
TM HAS LOCAL COORDINATES (x^μ, V^ν)


DEF: A COTANGENT VECTOR (1-FORM) ω AT p∈M
IS A MAP ω: TpM → ℝ

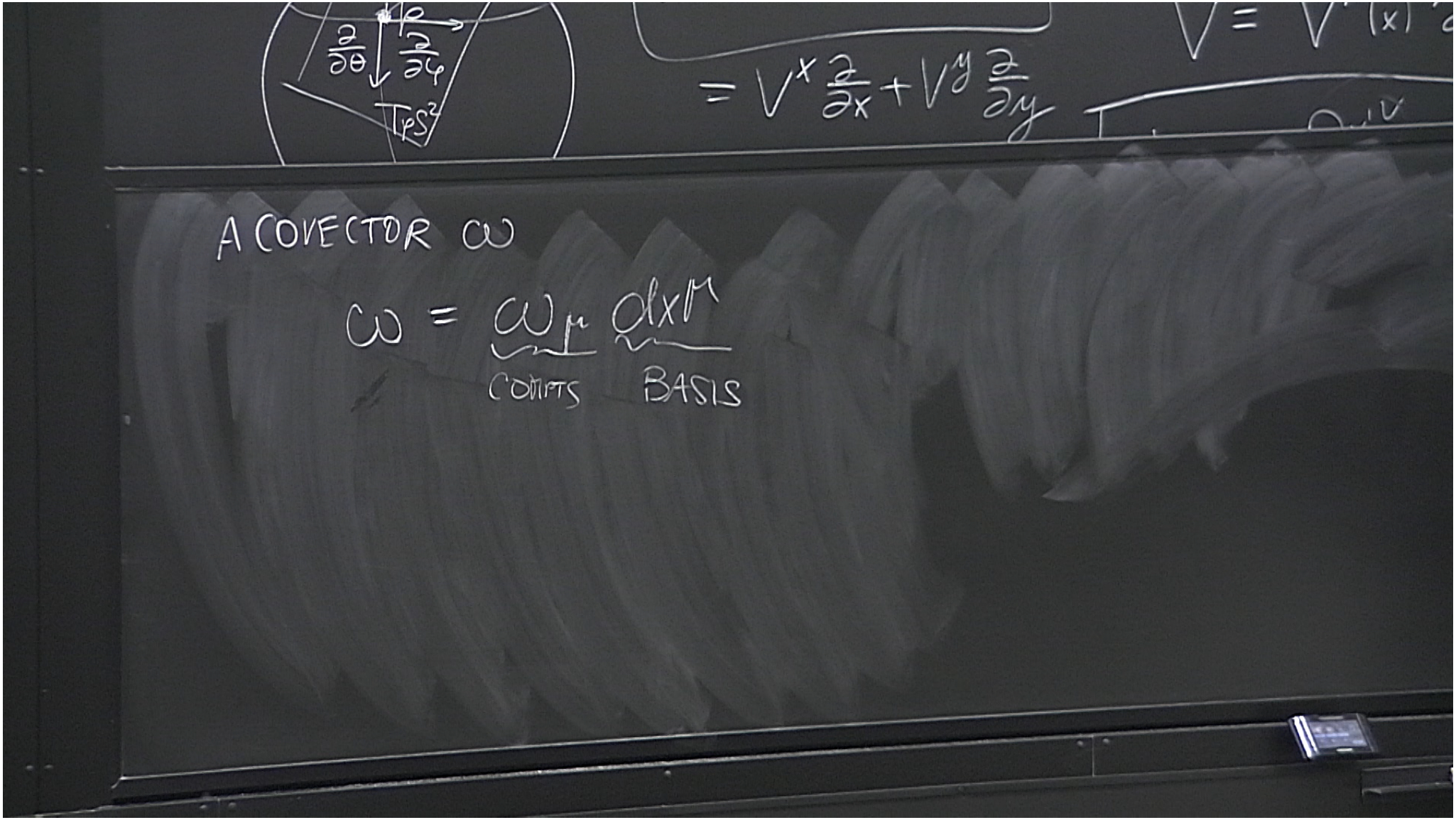

DEF: A COTANGENT VECTOR (1-FORM) ω AT p∈M
IS A MAP ω: TpM → ℝ
1-FORMS FORM A COTANGENT VECTOR SPACE T*pM

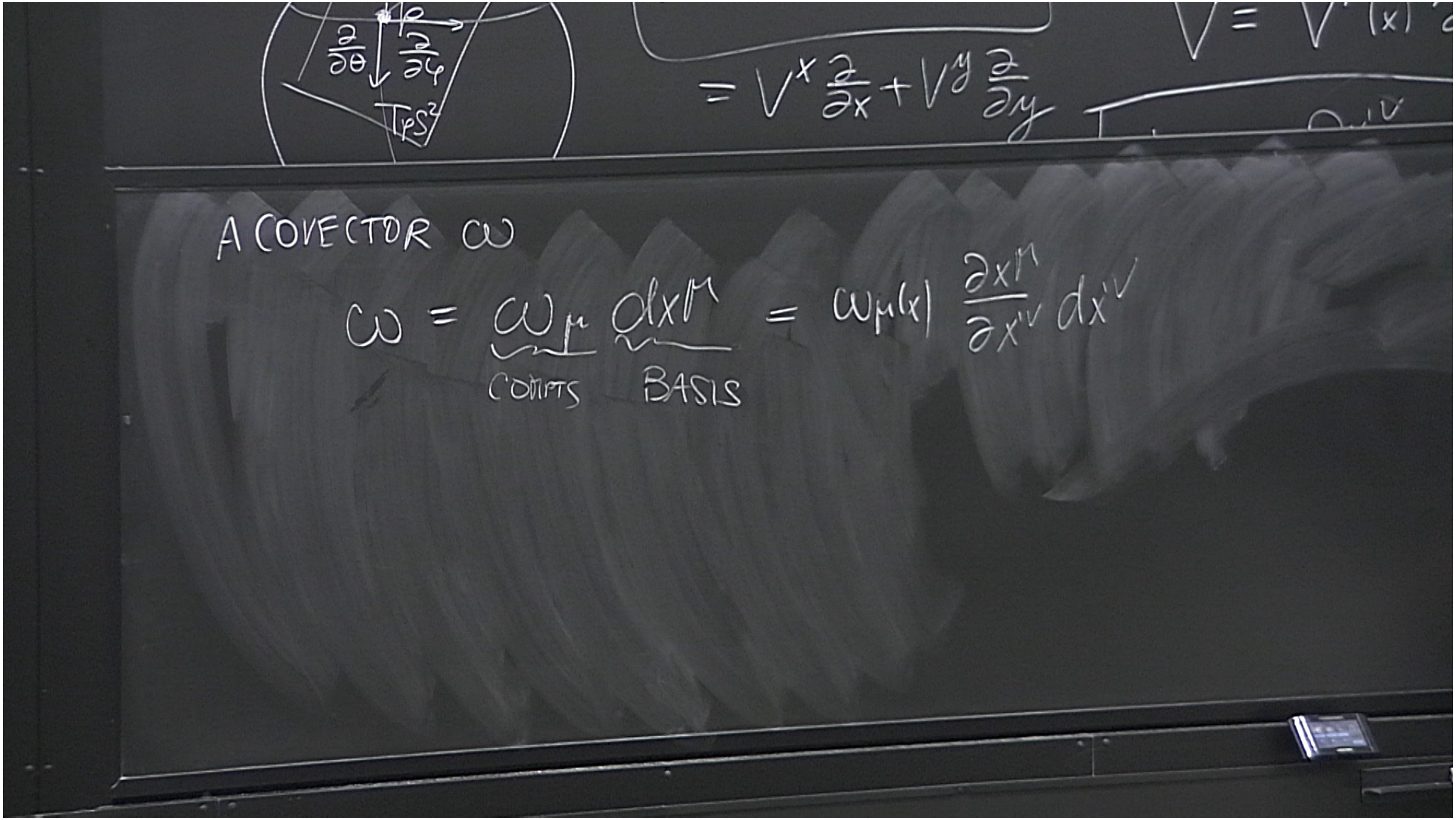
OOTH
TION

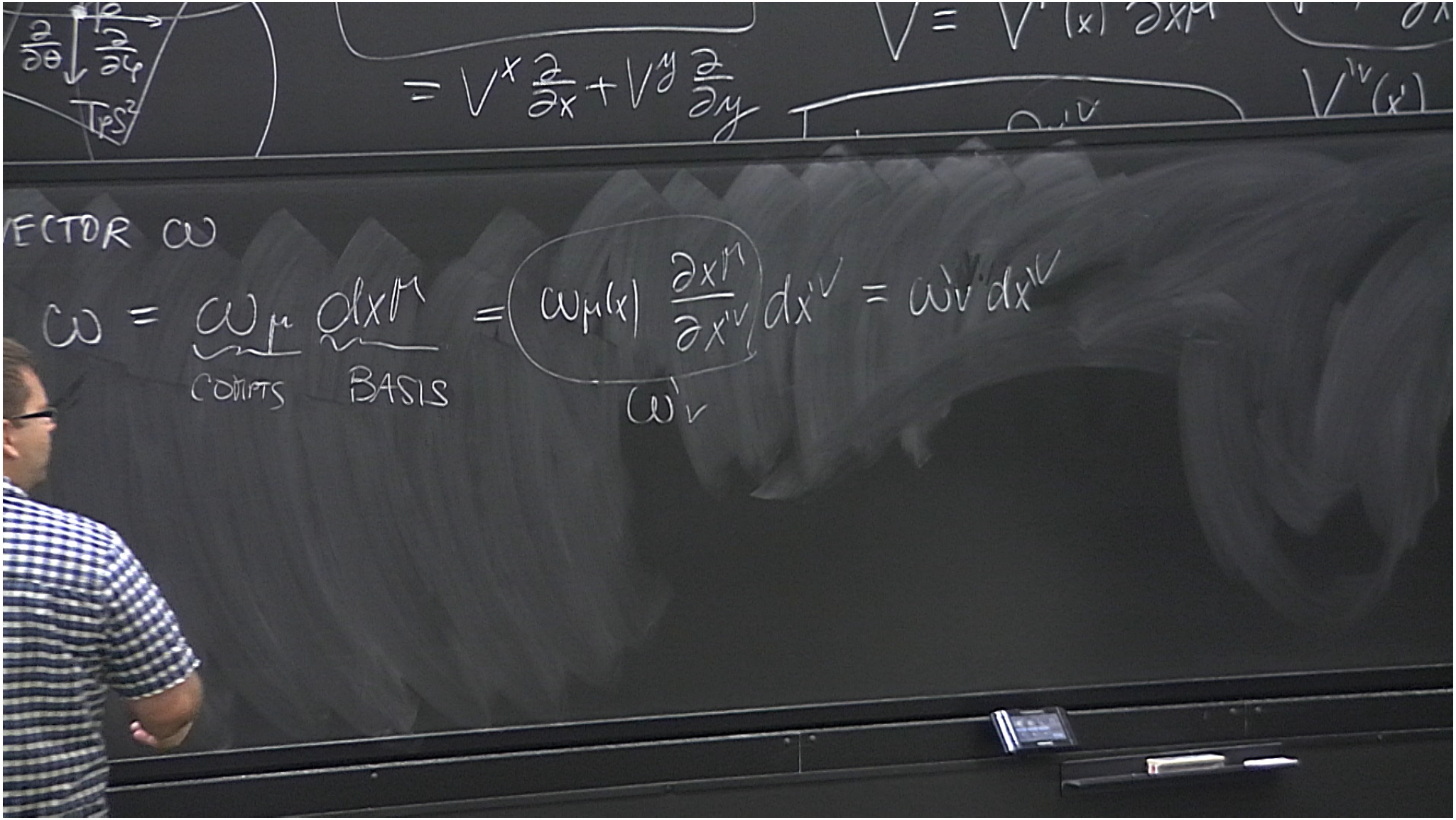
DEF: A COTANGENT VECTOR (1-FORM) ω AT $p \in M$ IS A MAP $\omega : T_pM \to \mathbb{R}$

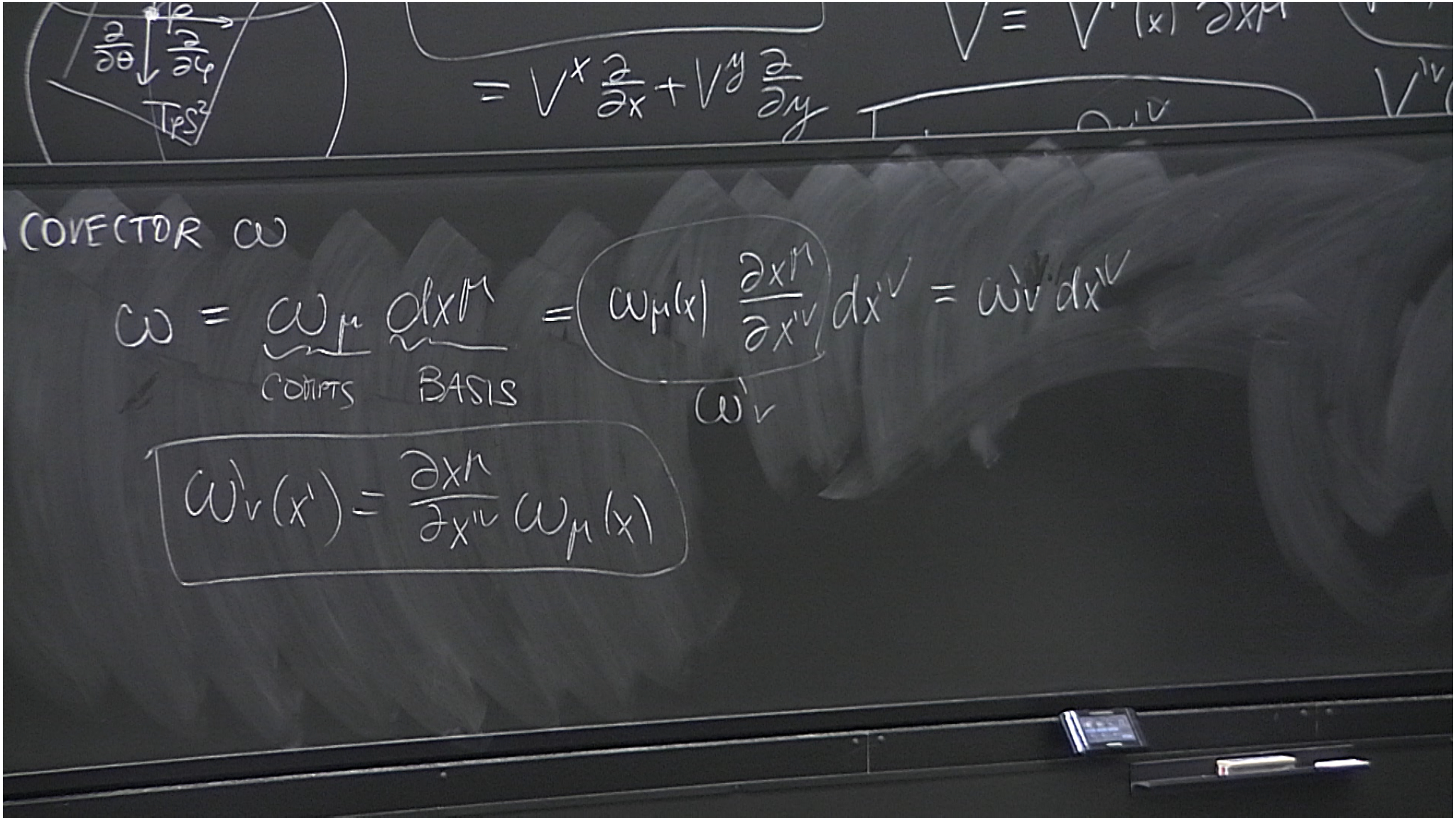
1-FORMS FORM A COTANGENT VECTOR SPACE T^*_pM WITH A COORDINATE BASIS dx^μ DEFINED AS

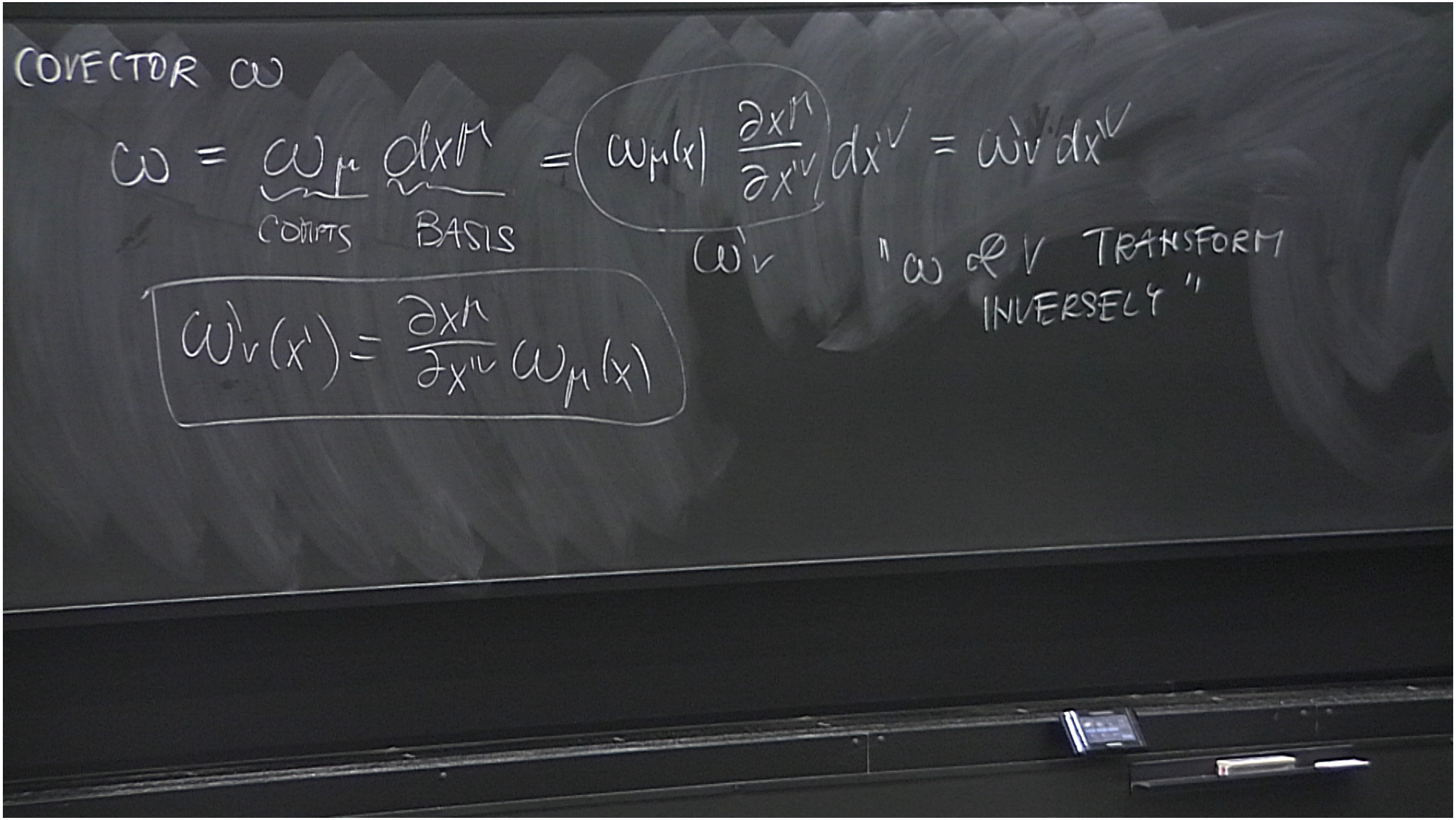
$$dx^\mu\left(\frac{\partial}{\partial x^\nu}\right) = \delta^\mu_\nu$$

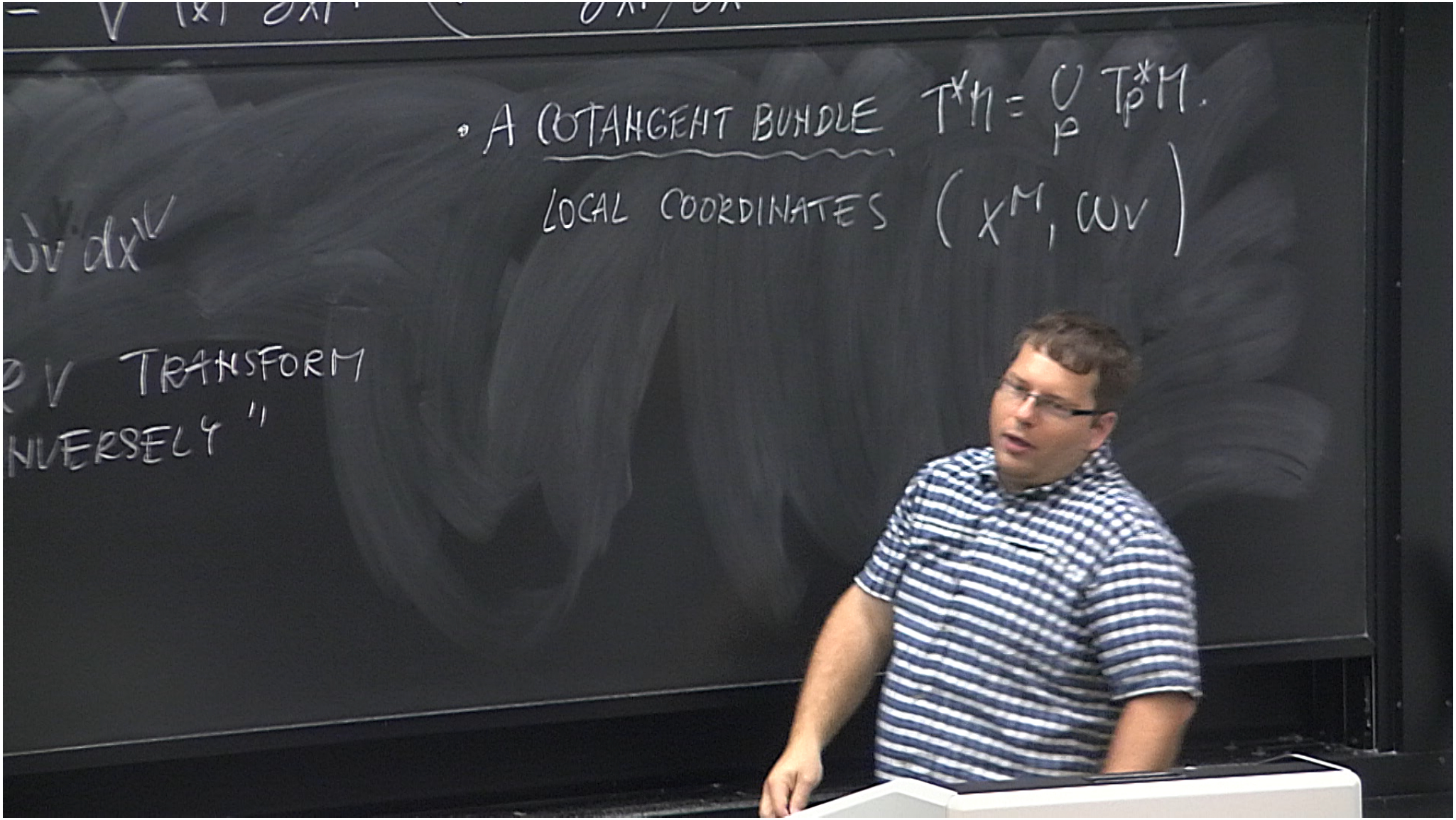

$\frac{\partial}{\partial\theta}$
$\frac{\partial}{\partial\varphi}$
T_pS^2
$= V^x \frac{\partial}{\partial x} + V^y \frac{\partial}{\partial y}$
A COVECTOR ω
$\omega = \omega_\mu dx^\mu$
COMPTS
BASIS

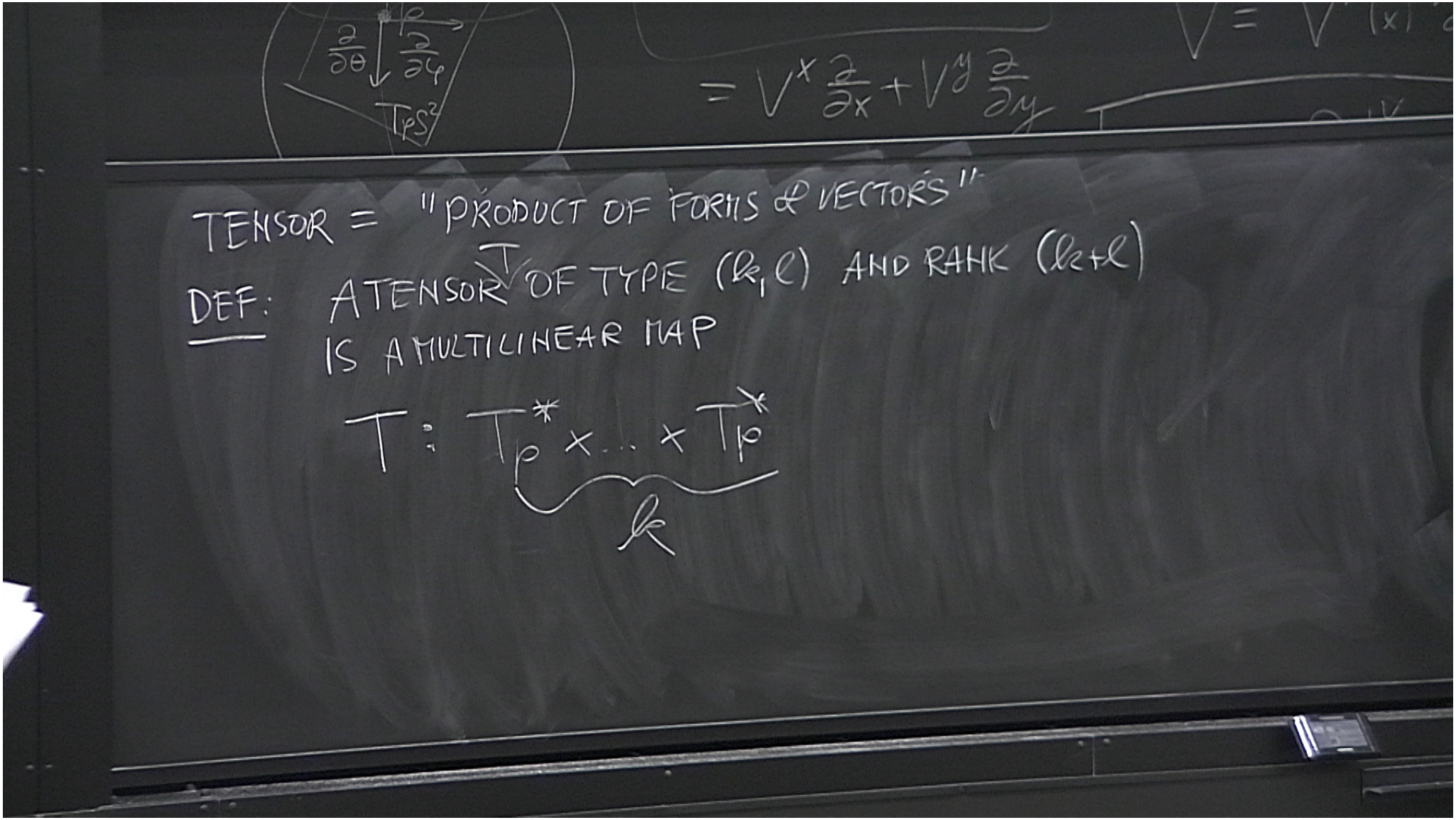

$\frac{\partial}{\partial\theta}$
$\frac{\partial}{\partial\varphi}$
$T_p S^2$
$= V^x \frac{\partial}{\partial x} + V^y \frac{\partial}{\partial y}$
A COVECTOR ω
$\omega = \underbrace{\omega_\mu}_{\text{COMPTS}} \underbrace{dx^\mu}_{\text{BASIS}} = \omega_\mu(x) \frac{\partial x^\mu}{\partial x'^\nu} dx'^\nu$

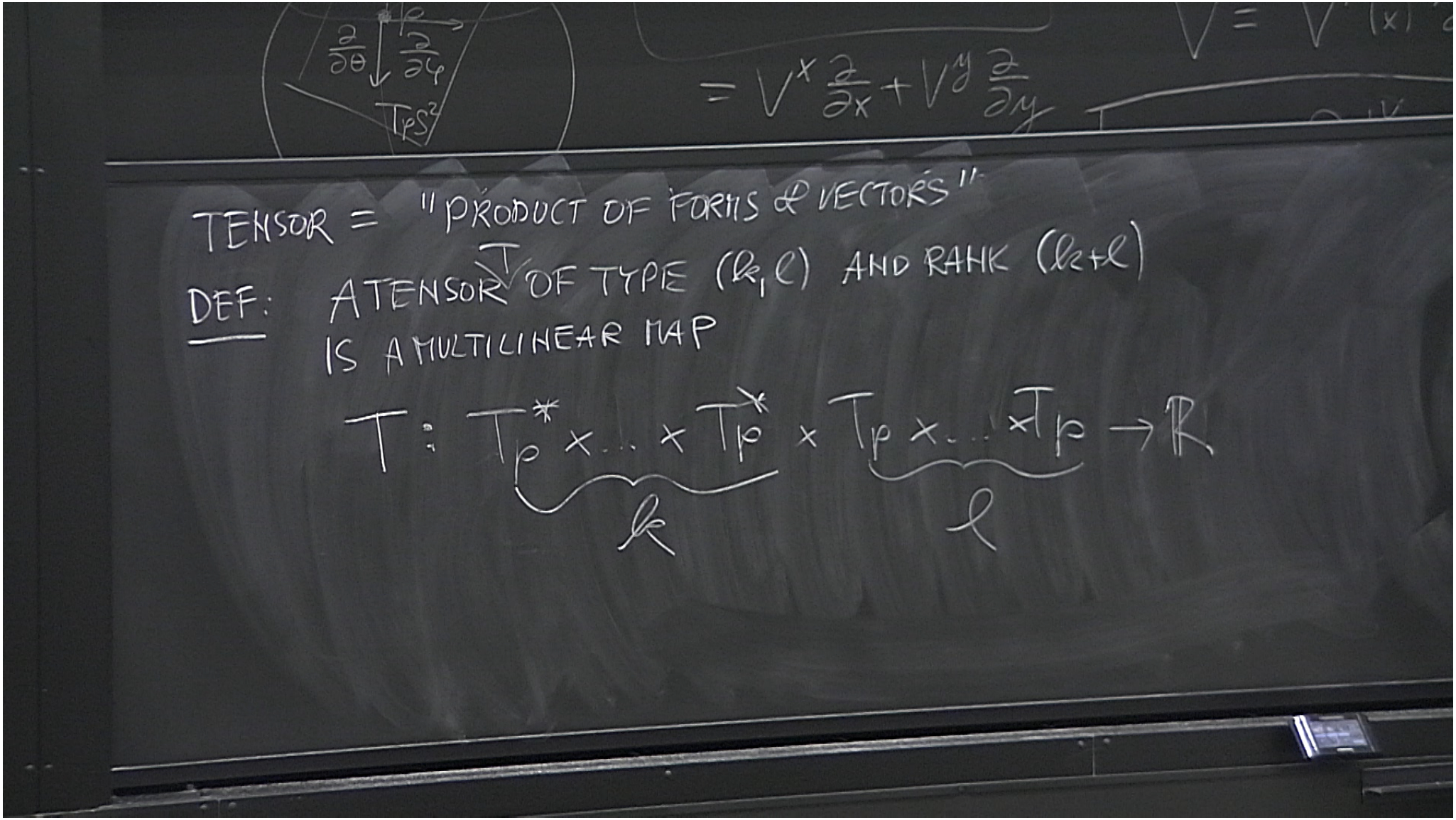

$= V^x \frac{\partial}{\partial x} + V^y \frac{\partial}{\partial y}$
$V = V'(x) \frac{\partial x'}{\partial x}$
$V'^{\nu}(x')$
ECTOR ω
$\omega = \underbrace{\omega_\mu}_{\text{COVECTS}} \underbrace{dx^\mu}_{\text{BASIS}} = \underbrace{\omega_\mu(x) \frac{\partial x^\mu}{\partial x'^\nu}}_{\omega'_\nu} dx'^\nu = \omega'_\nu dx'^\nu$

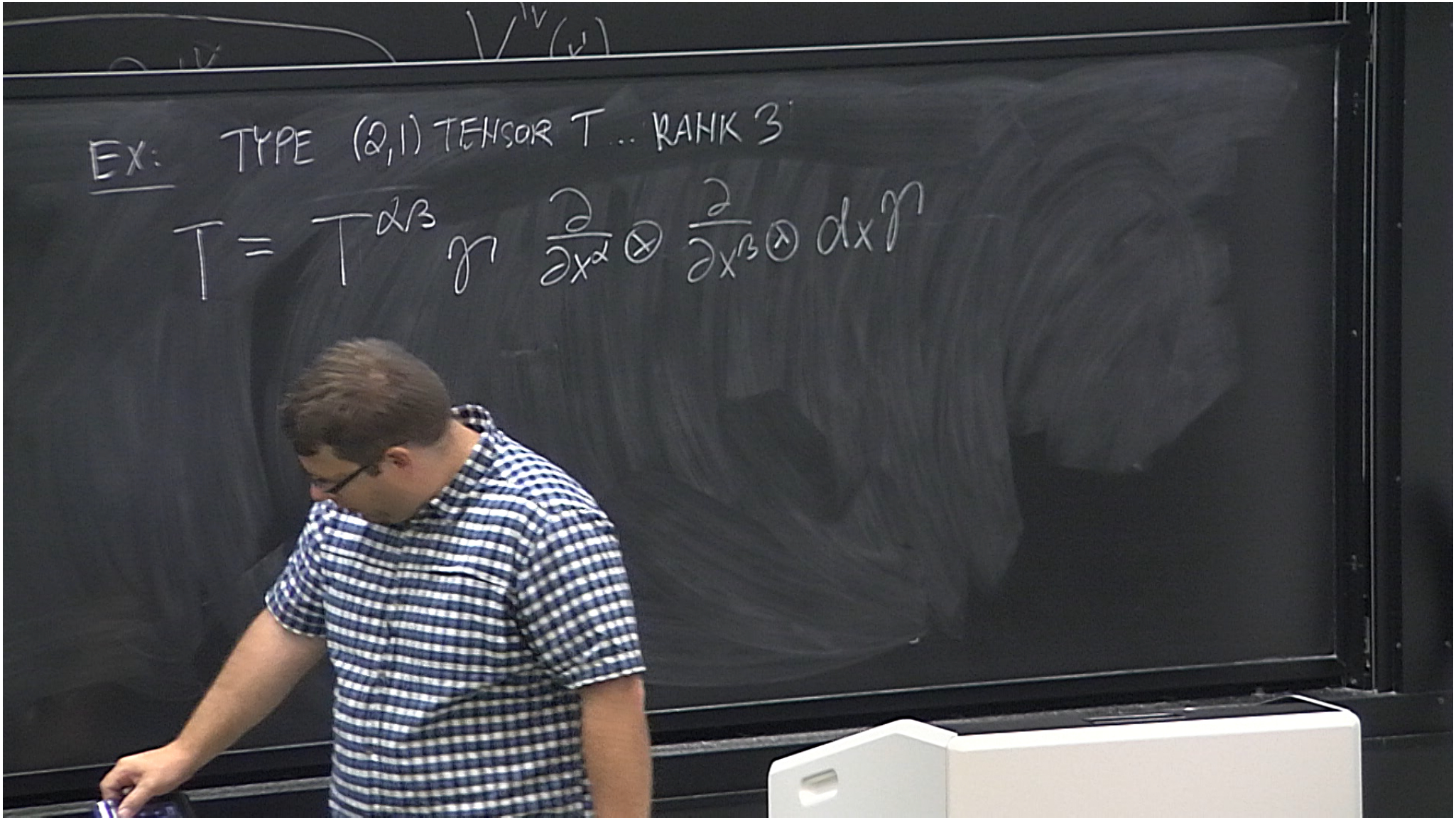

$\frac{\partial}{\partial\theta}$ $\frac{\partial}{\partial\varphi}$
T_pS^2
$= V^x \frac{\partial}{\partial x} + V^y \frac{\partial}{\partial y}$
$V = V'(x) \frac{\partial x'^\mu}{\partial x^\nu}$
COVECTOR ω
$\omega = \underbrace{\omega_\mu}_{\text{COMPTS}} \underbrace{dx^\mu}_{\text{BASIS}} = \omega_\mu(x) \frac{\partial x^\mu}{\partial x'^\nu} dx'^\nu = \omega'_\nu dx'^\nu$
ω'_ν
$\omega'_\nu(x') = \frac{\partial x^\mu}{\partial x'^\nu} \omega_\mu(x)$

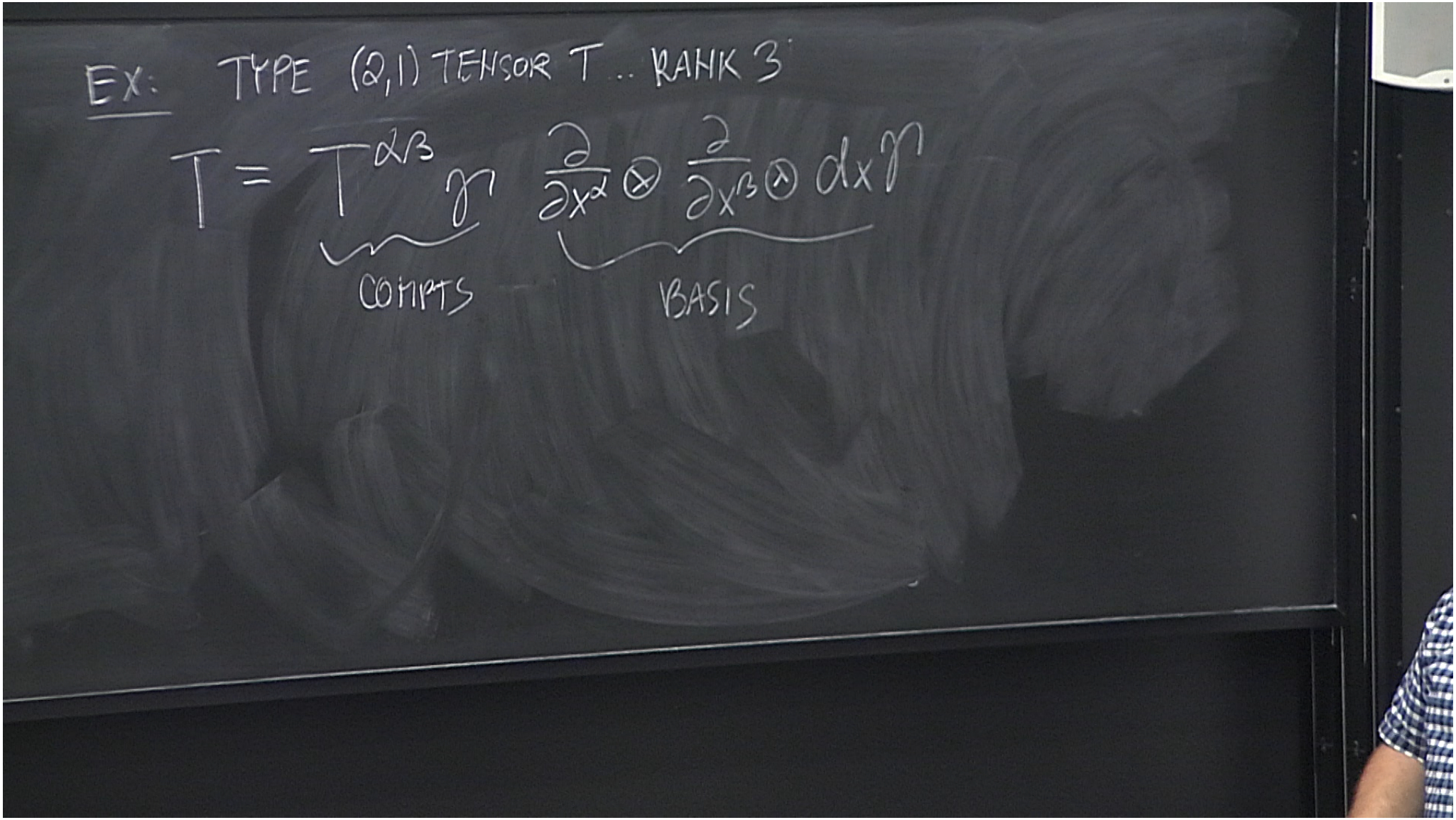

COVECTOR ω
ω = ω_μ dx^μ = ω_μ(x) ∂x^μ/∂x'^ν dx'^ν = ω'_ν dx'^ν
COMPTS
BASIS
ω'_ν
"ω & V TRANSFORM INVERSELY"
ω'_ν(x') = ∂x^μ/∂x'^ν ω_μ(x)

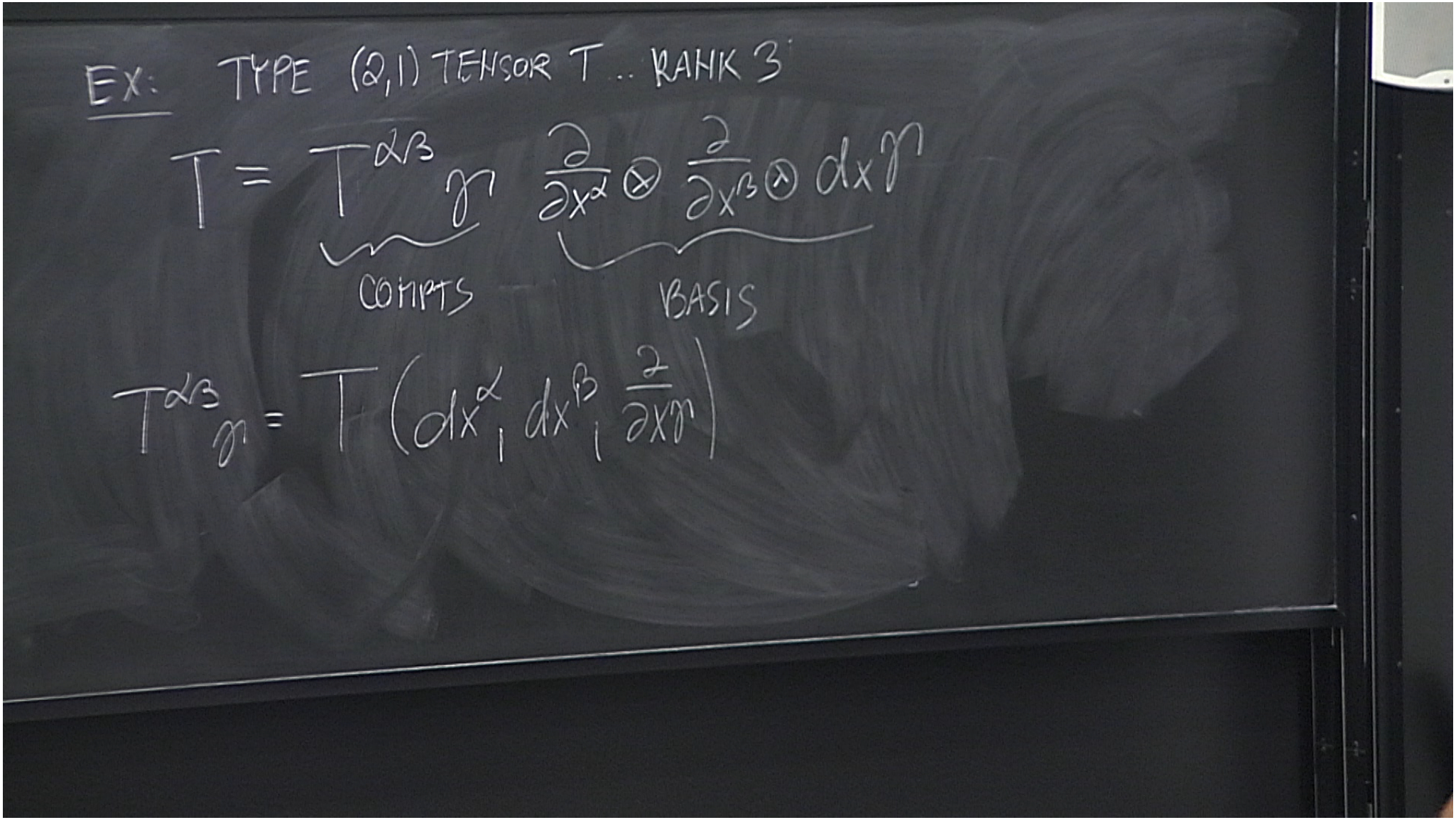

• A COTANGENT BUNDLE $T^*M = \bigcup_p T_p^*M$.
LOCAL COORDINATES (x^μ, ω_ν)
$\omega_\nu dx^\nu$
TRANSFORM
NIVERSELY "

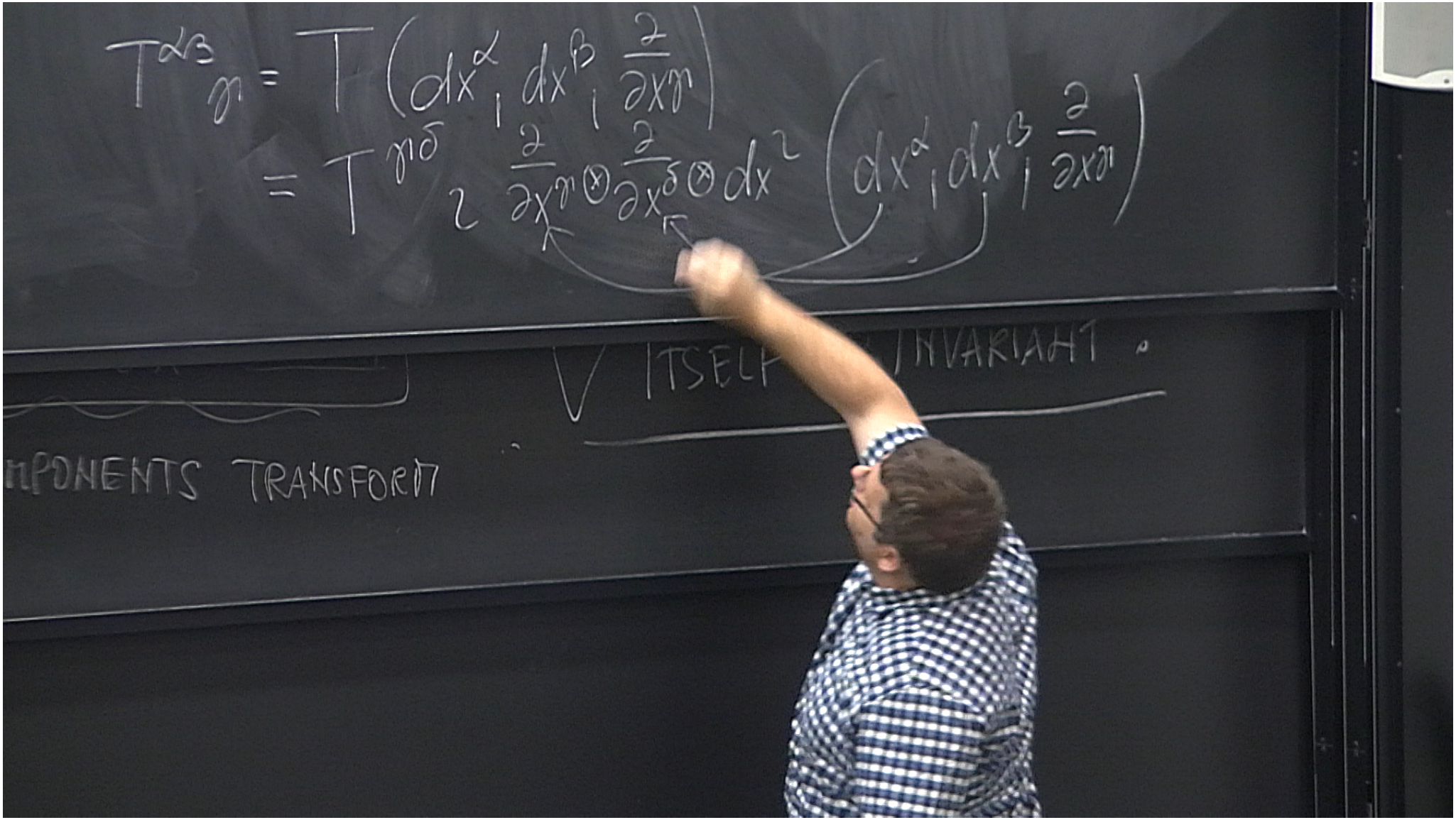

$\frac{\partial}{\partial\theta}$
$\frac{\partial}{\partial\varphi}$
T_pS^2
$= V^x \frac{\partial}{\partial x} + V^y \frac{\partial}{\partial y}$
TENSOR = "PRODUCT OF FORMS & VECTORS"
DEF: A TENSOR T OF TYPE (k,ℓ) AND RANK $(k+\ell)$
IS A MULTILINEAR MAP
$T : T_p^* \times \ldots \times T_p^*$
k

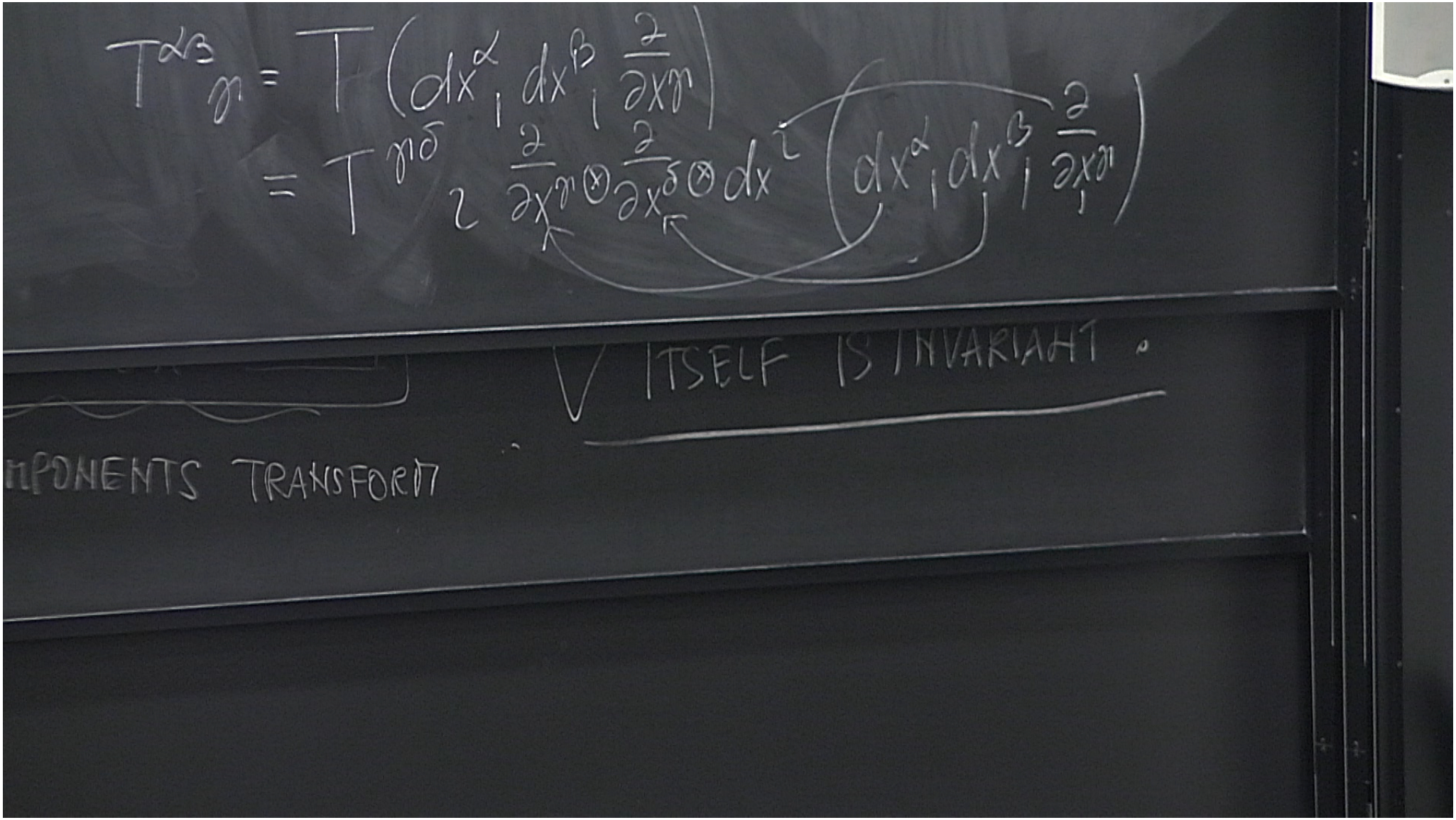

$\frac{\partial}{\partial\theta}$
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TENSOR = "PRODUCT OF FORMS & VECTORS"
DEF: A TENSOR T OF TYPE (k,ℓ) AND RANK $(k+\ell)$
IS A MULTILINEAR MAP
$T : \underbrace{T_p^* \times \dots \times T_p^*}_{k} \times \underbrace{T_p \times \dots \times T_p}_{\ell} \to \mathbb{R}$

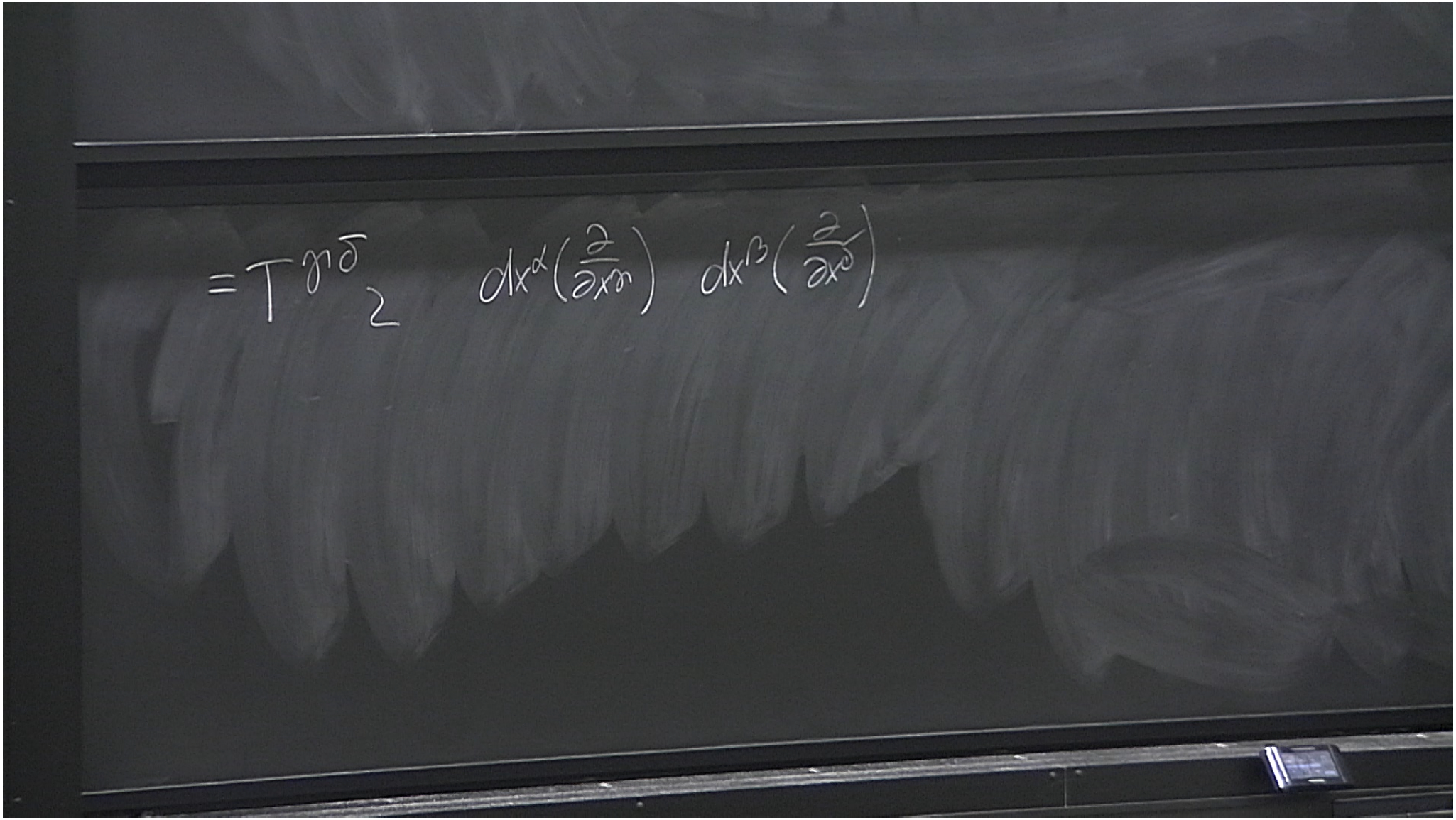

EX: TYPE (2,1) TENSOR T .. RANK 3
$T = T^{\alpha\beta}{}_{\gamma} \frac{\partial}{\partial x^\alpha} \otimes \frac{\partial}{\partial x^\beta} \otimes dx^\gamma$

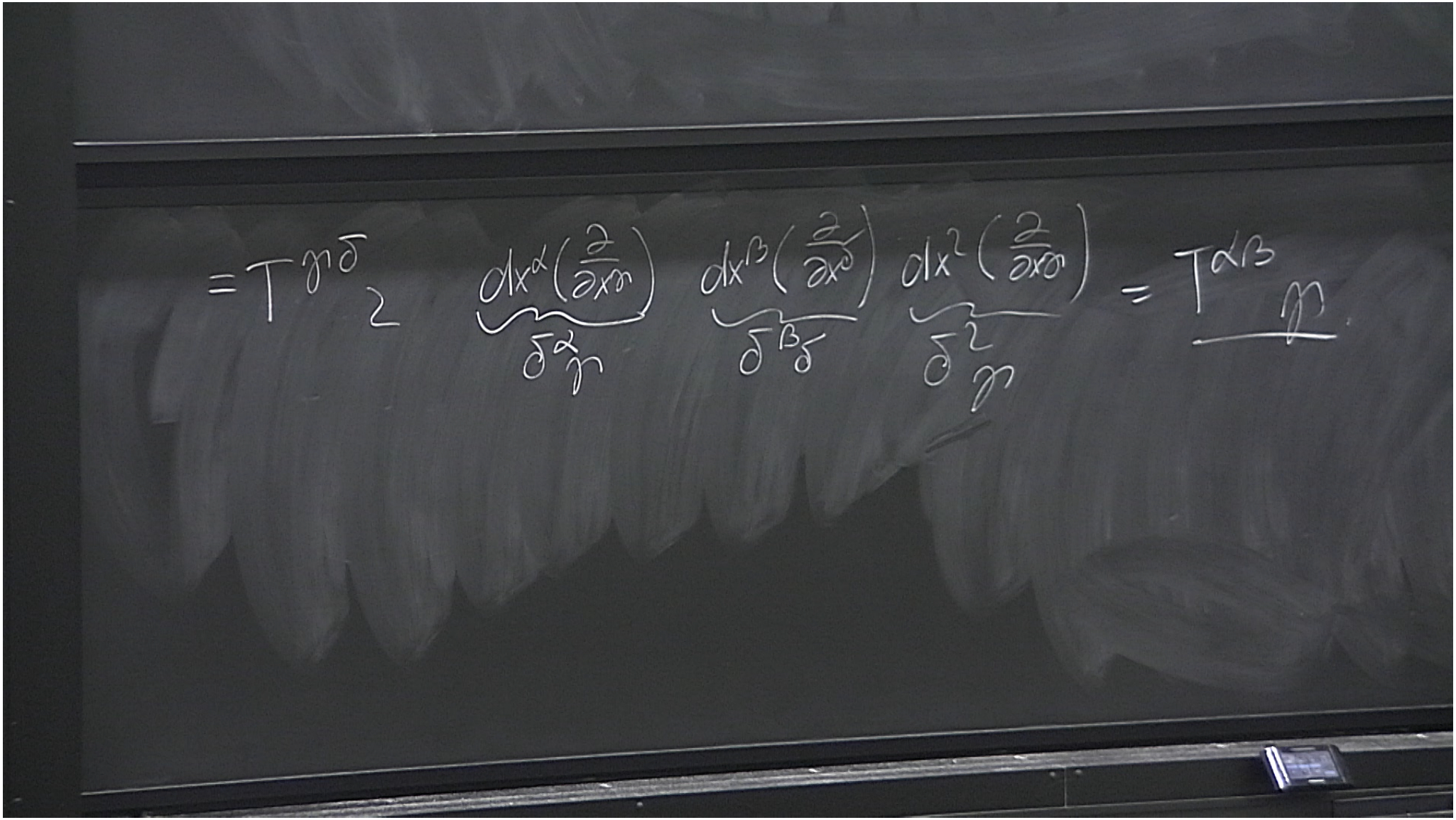

EX: TYPE (2,1) TENSOR T .. RANK 3
$T = T^{\alpha\beta}{}_{\gamma} \frac{\partial}{\partial x^\alpha} \otimes \frac{\partial}{\partial x^\beta} \otimes dx^\gamma$
COMPTS
BASIS

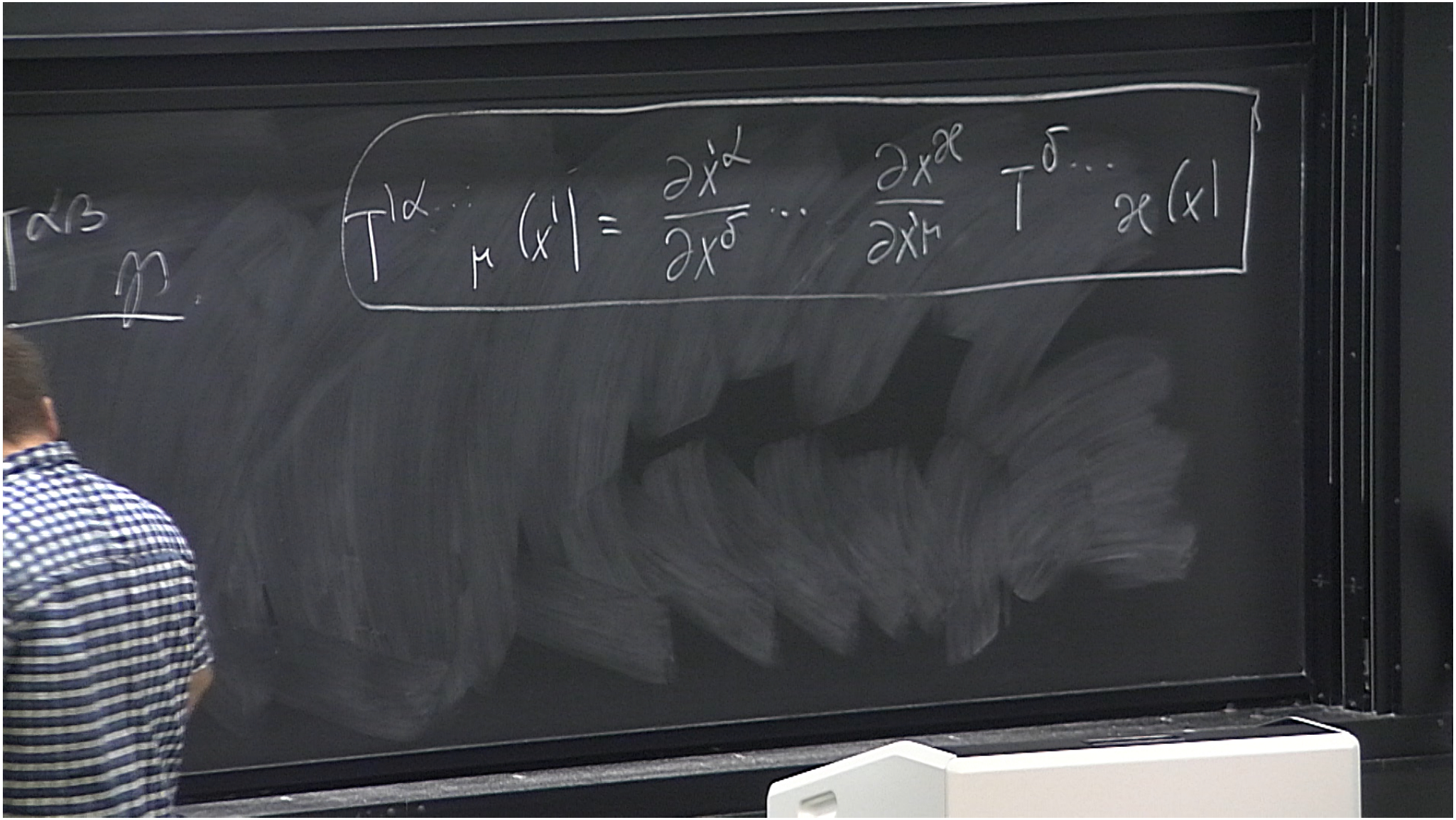

EX: TYPE (2,1) TENSOR T .. RANK 3
$T = T^{\alpha\beta}{}_{\gamma}\ \frac{\partial}{\partial x^\alpha} \otimes \frac{\partial}{\partial x^\beta} \otimes dx^\gamma$
COMPTS
BASIS
$T^{\alpha\beta}{}_{\gamma} = T\left(dx^\alpha, dx^\beta, \frac{\partial}{\partial x^\gamma}\right)$


$T^{\alpha\beta}{}_{\gamma} = T\left(dx^{\alpha}, dx^{\beta}, \frac{\partial}{\partial x^{\gamma}}\right)$
$= T^{\gamma\delta}{}_{\lambda} \frac{\partial}{\partial x^{\gamma}} \otimes \frac{\partial}{\partial x^{\delta}} \otimes dx^{\lambda}\left(dx^{\alpha}, dx^{\beta}, \frac{\partial}{\partial x^{\gamma}}\right)$
ITSELF
NVARIANT
MPONENTS TRANSFORM


$T^{\alpha\beta}{}_{\gamma} = T\left(dx^{\alpha}, dx^{\beta}, \frac{\partial}{\partial x^{\gamma}}\right)$
$= T^{\gamma\delta}{}_{\zeta} \frac{\partial}{\partial x^{\gamma}} \otimes \frac{\partial}{\partial x^{\delta}} \otimes dx^{\zeta}\left(dx^{\alpha}, dx^{\beta}, \frac{\partial}{\partial x^{\gamma}}\right)$
ITSELF IS INVARIANT.
MPONENTS TRANSFORM


$= T^{\gamma\sigma}{}_{2} \quad dx^{\alpha}\left(\frac{\partial}{\partial x^{m}}\right) \quad dx^{\beta}\left(\frac{\partial}{\partial x^{0}}\right)$


$= T^{\gamma\sigma}{}_{2} \quad \underbrace{dx^{\alpha}\left(\frac{\partial}{\partial x^{m}}\right)}_{\partial^{\alpha}_{m}} \quad \underbrace{dx^{\beta}\left(\frac{\partial}{\partial x^{\sigma}}\right)}_{\partial^{\beta}{}_{\sigma}} \quad \underbrace{dx^{2}\left(\frac{\partial}{\partial x^{n}}\right)}_{\partial^{2}_{m}} = T^{\alpha\beta}{}_{m}$


$T^{\alpha\beta}$
$T'^{\alpha\ldots}{}_{\mu\ldots}(x') = \frac{\partial x'^{\alpha}}{\partial x^{\sigma}} \cdots \frac{\partial x^{\rho}}{\partial x'^{\mu}} T^{\sigma\ldots}{}_{\rho\ldots}(x)$

$$= T^{\gamma\sigma}{}_{\lambda} \underbrace{dx^{\alpha}\left(\frac{\partial}{\partial x^{\gamma}}\right)}_{\delta^{\alpha}_{\gamma}} \underbrace{dx^{\beta}\left(\frac{\partial}{\partial x^{\sigma}}\right)}_{\delta^{\beta}_{\sigma}} \underbrace{dx^{\lambda}\left(\frac{\partial}{\partial x^{\mu}}\right)}_{\delta^{\lambda}_{\mu}} = T^{\alpha\beta}{}_{\mu}.$$

COOK BOOK:

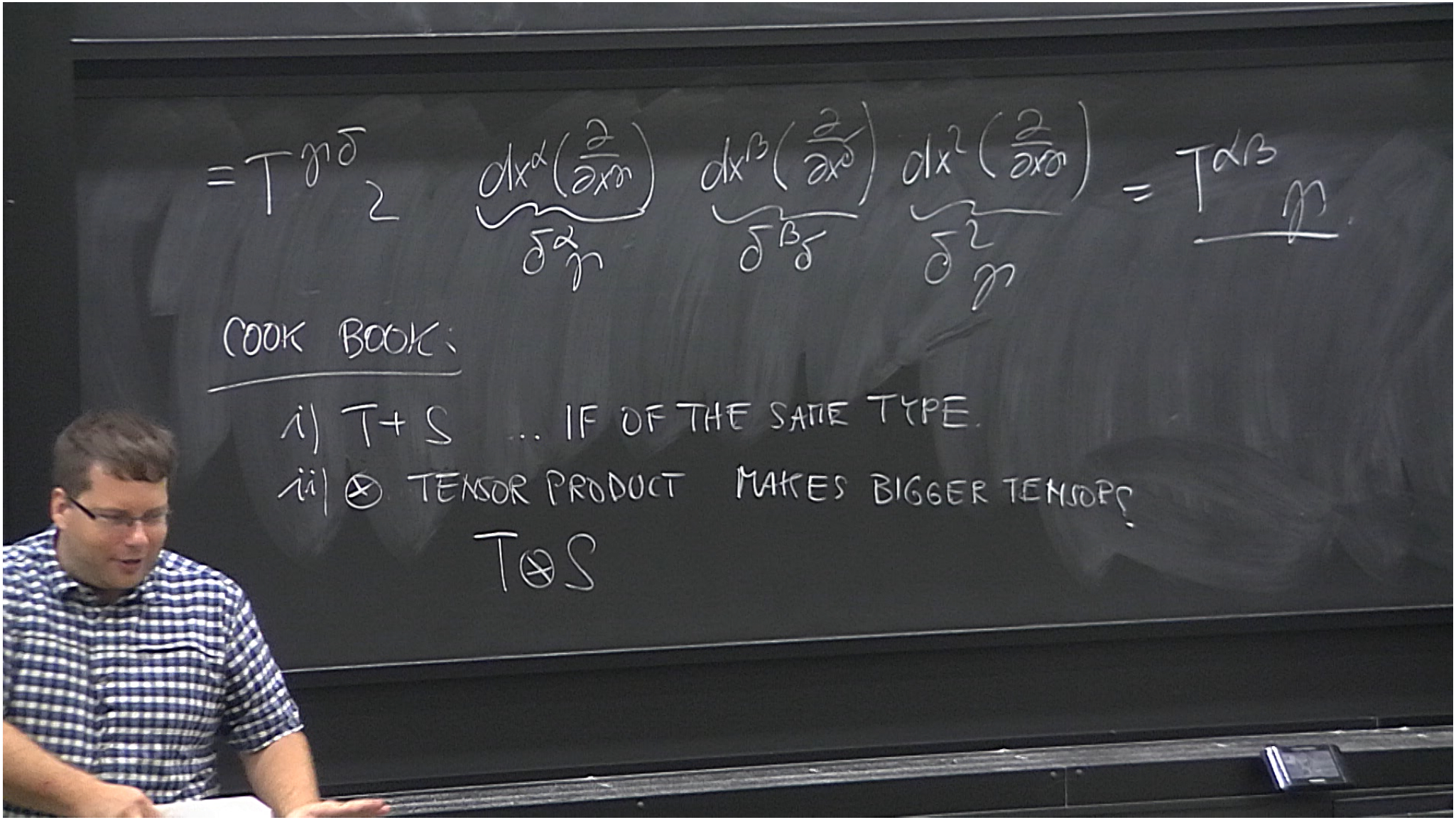
i) $T + S$ ... IF OF THE SAME TYPE.

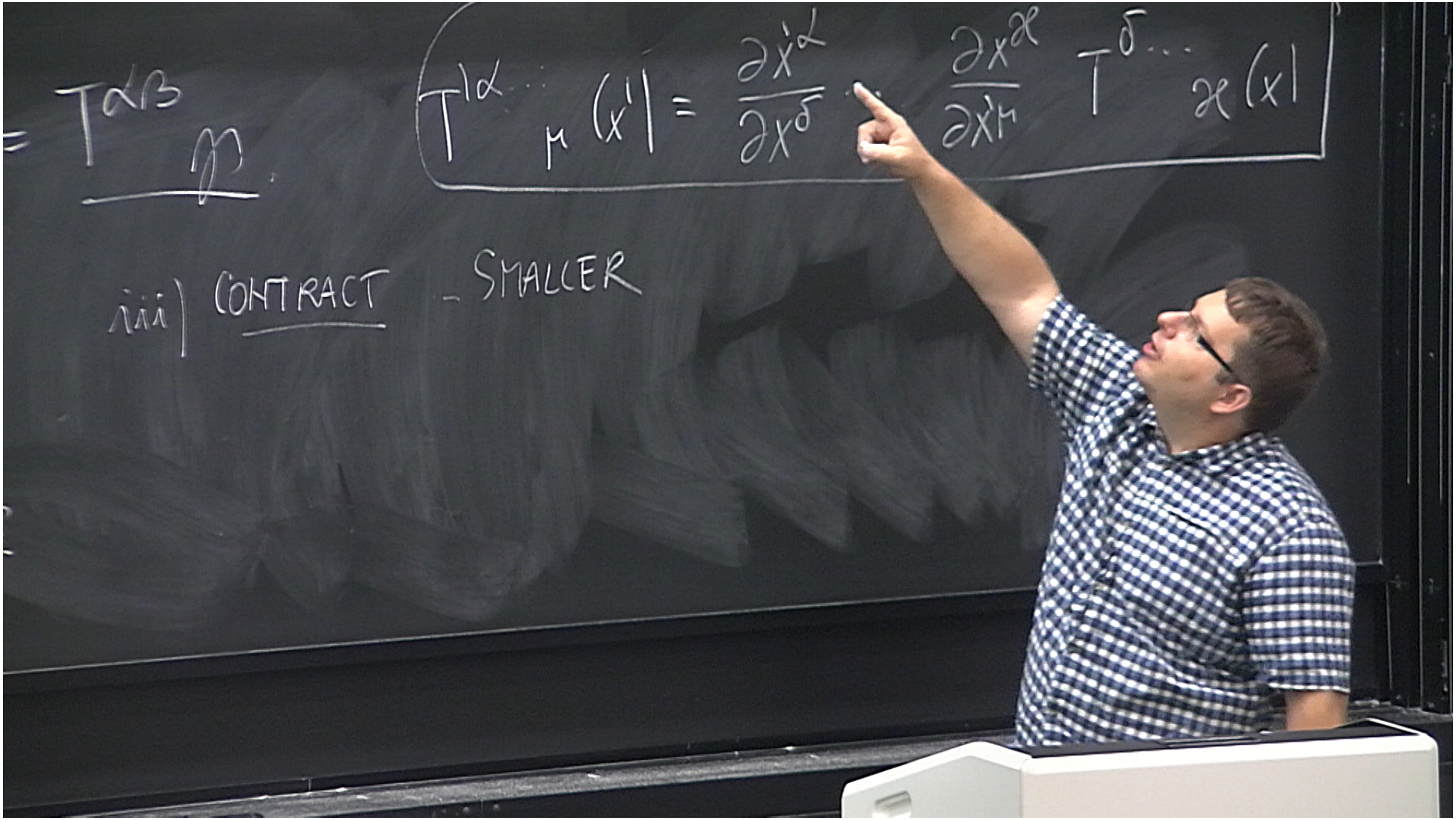
$$= T^{\gamma\delta}{}_{2} \underbrace{dx^{\alpha}\left(\frac{\partial}{\partial x^{\gamma}}\right)}_{\delta^{\alpha}_{\gamma}} \underbrace{dx^{\beta}\left(\frac{\partial}{\partial x^{\delta}}\right)}_{\delta^{\beta}_{\delta}} \underbrace{dx^{2}\left(\frac{\partial}{\partial x^{0}}\right)}_{\delta^{2}_{0}} = T^{\alpha\beta}{}_{0}$$

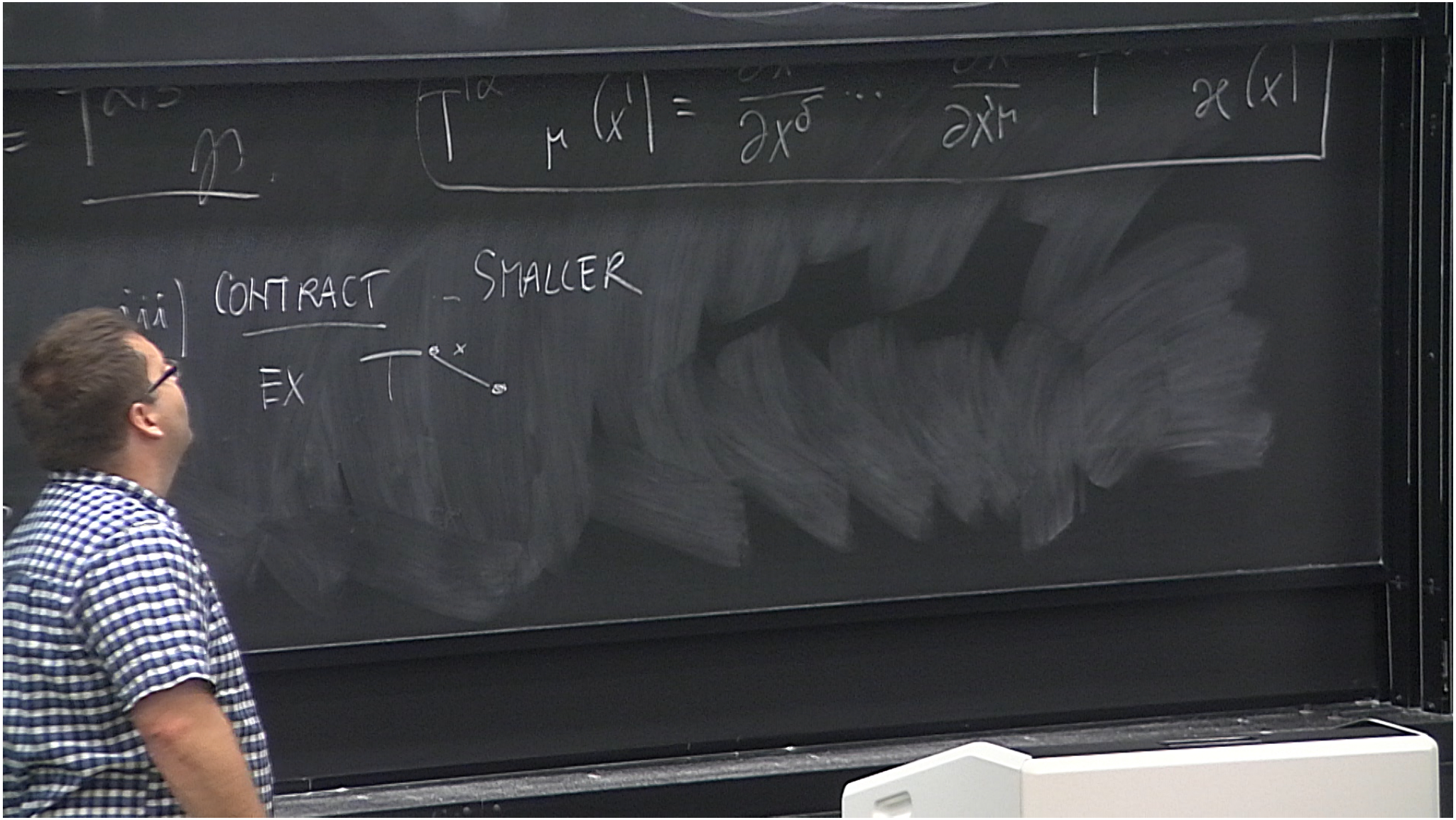
COOK BOOK:

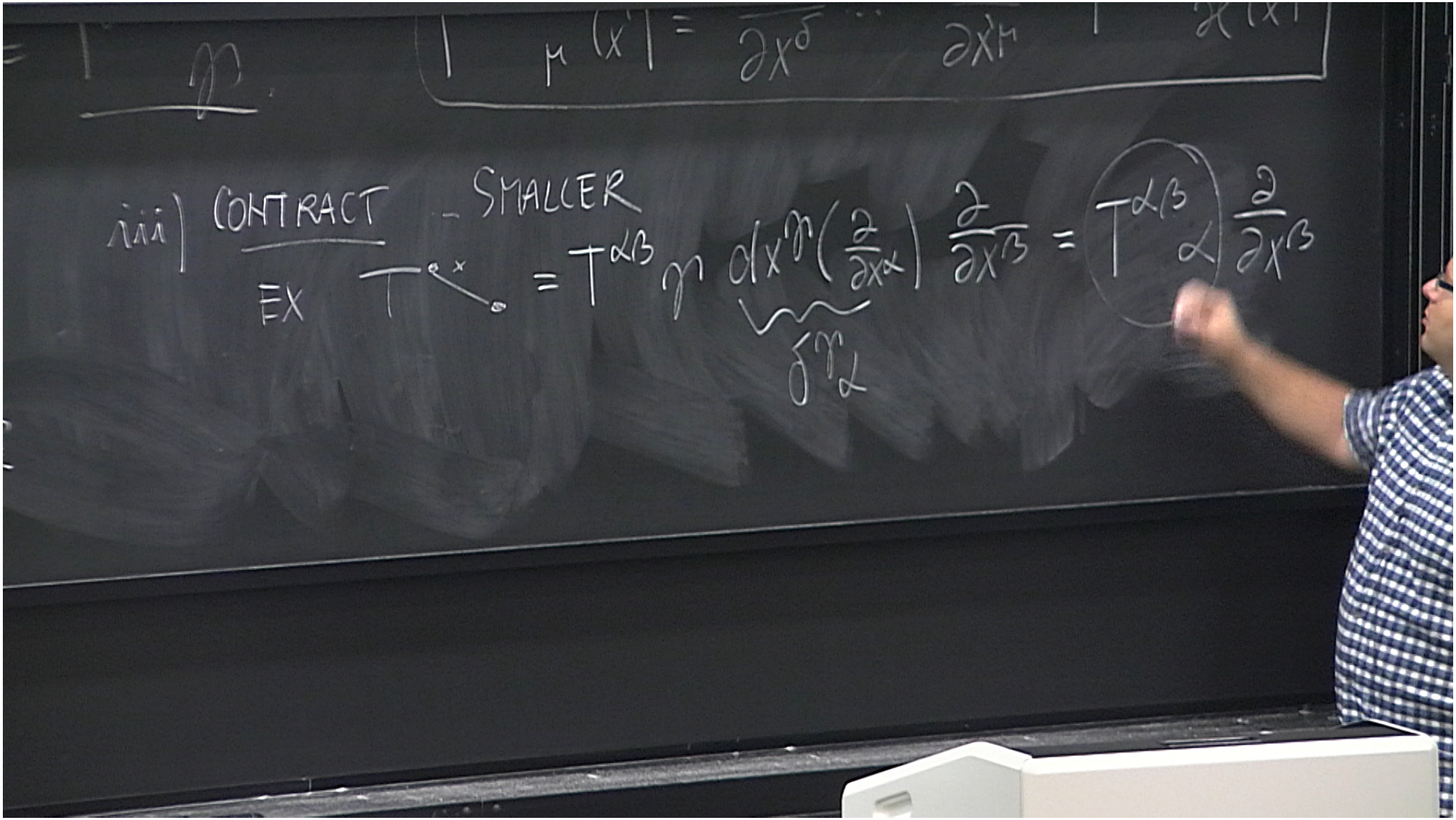
i) $T + S$ .. IF OF THE SAME TYPE.

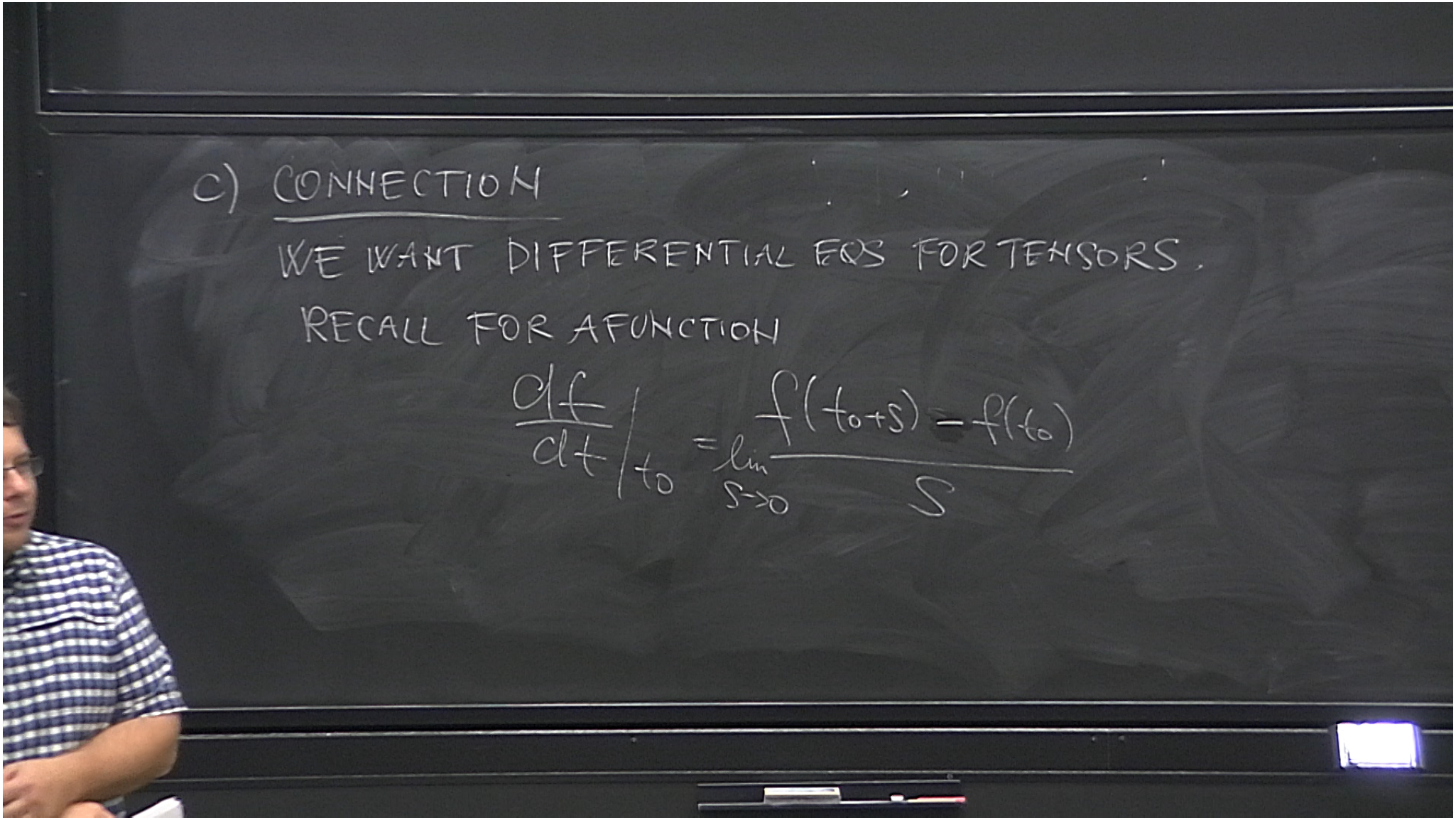
ii) $\otimes$ TENSOR PRODUCT MAKES BIGGER TENSORS

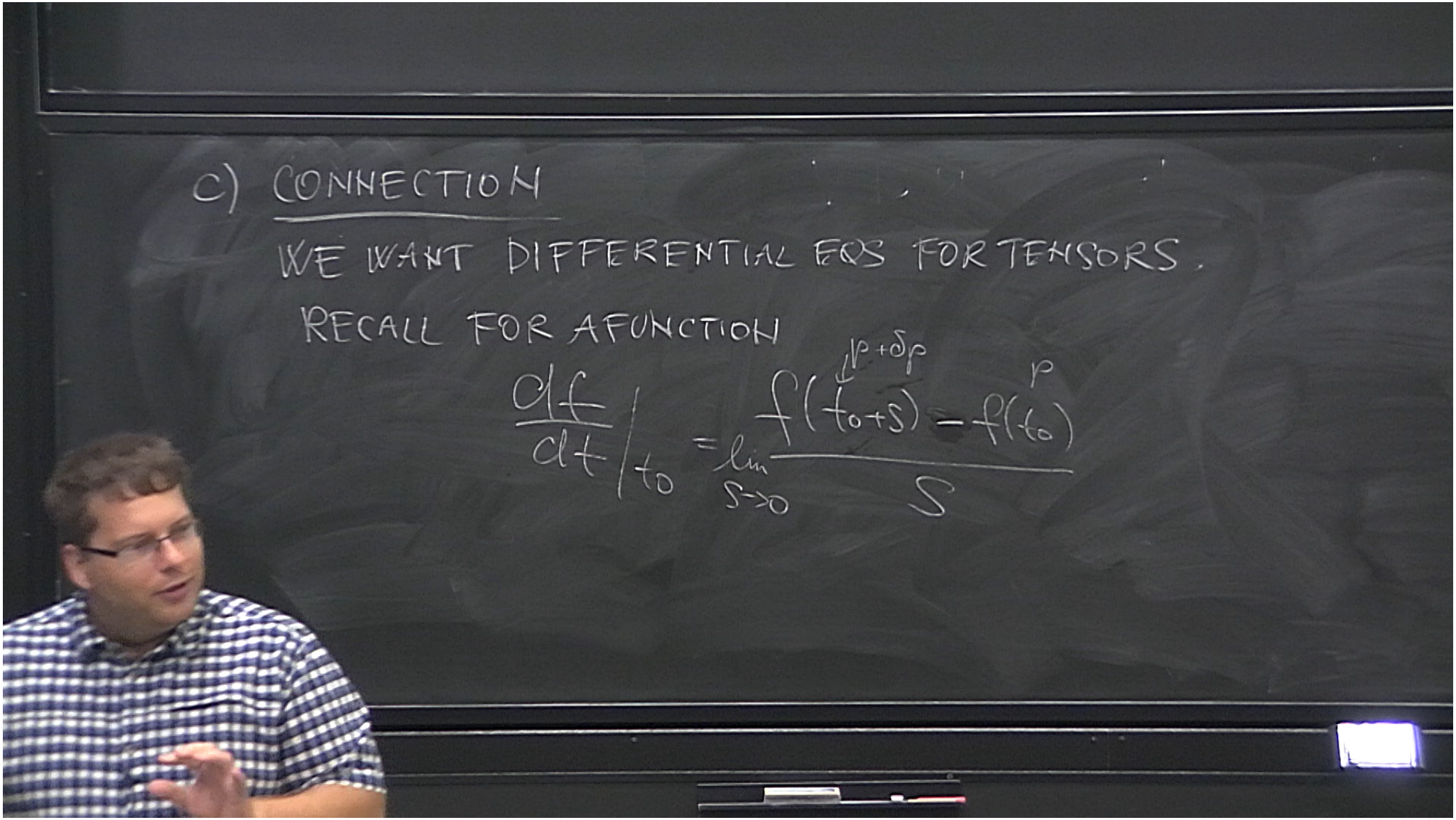

$= T^{\gamma\delta}{}_{2} \frac{dx^{\alpha}(\frac{\partial}{\partial x^{\gamma}})}{\delta^{\alpha}_{\gamma}} \frac{dx^{\beta}(\frac{\partial}{\partial x^{\delta}})}{\delta^{\beta}_{\delta}} \frac{dx^{2}(\frac{\partial}{\partial x^{\sigma}})}{\delta^{2}_{\gamma}} = T^{\alpha\beta}{}_{\gamma}$
COOK BOOK:
i) T+S .. IF OF THE SAME TYPE.
ii) ⊗ TENSOR PRODUCT MAKES BIGGER TENSORS
T⊗S

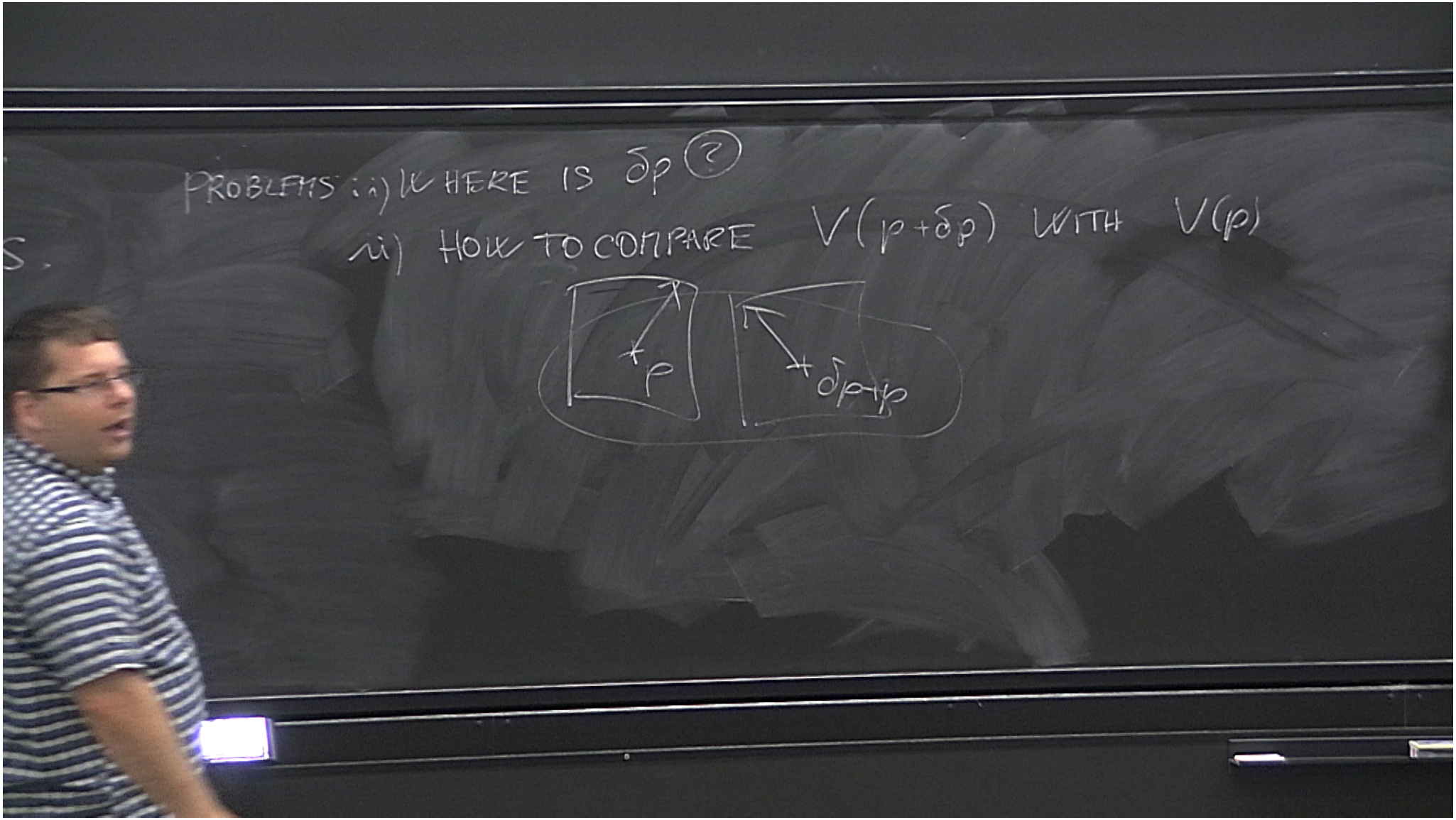

$T^{\alpha\beta}{}_{\mu}$
$T'^{\alpha\ldots}{}_{\mu}(x') = \frac{\partial x'^{\alpha}}{\partial x^{\sigma}} \ldots \frac{\partial x^{\rho}}{\partial x'^{\mu}} T^{\sigma\ldots}{}_{\rho\ldots}(x)$
iii) CONTRACT - SMALLER


$T^{\alpha\beta}$
$T^{\alpha}{}_{\mu}(x') = \frac{\partial x^{\alpha}}{\partial x^{\sigma}} \cdots \frac{\partial x^{\lambda}}{\partial x^{\mu}} T \quad \text{rest}(x)$
iii) CONTRACT - SMALLER
EX T


iii) CONTRACT - SMALLER
EX
$= T^{\alpha\beta}{}_{\alpha}\frac{\partial}{\partial x^\beta}$

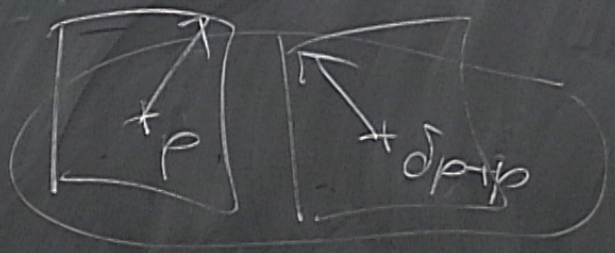

c) CONNECTION
WE WANT DIFFERENTIAL EQS FOR TENSORS.
RECALL FOR A FUNCTION
$\frac{df}{dt}\Big|_{t_0} = \lim_{s\to 0} \frac{f(t_0+s) - f(t_0)}{s}$


c) CONNECTION
WE WANT DIFFERENTIAL EQS FOR TENSORS.
RECALL FOR A FUNCTION
$\frac{df}{dt}\Big|_{t_0} = \lim_{s\to 0} \frac{f(t_0+s) - f(t_0)}{s}$
$p+\delta p$
p


PROBLEMS: i) WHERE IS δρ ?
ii) HOW TO COMPARE V(ρ+δρ) WITH V(ρ)
ρ
δρ+ρ

PROBLEMS: i) WHERE IS $\delta\rho$?

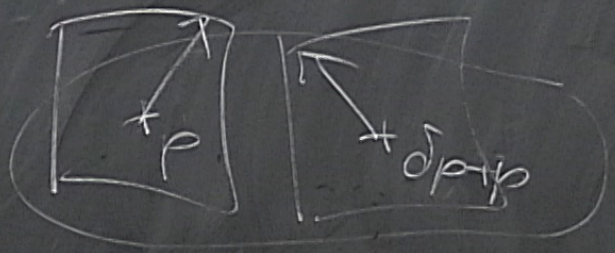
ii) HOW TO COMPARE $V(\rho+\delta\rho)$ WITH $V(\rho)$



CANNOT BE DONE IN GENERAL.

PROBLEMS: i) WHERE IS δp ?

ii) HOW TO COMPARE $V(p+\delta p)$ WITH $V(p)$



CANNOT BE DONE IN GENERAL, UNLESS EXTRA STRUCTURE EXISTS IN MANIFOLD

$\delta^2 g$ $\delta^B \delta$ $\delta^2 g$

COOK BOOK:

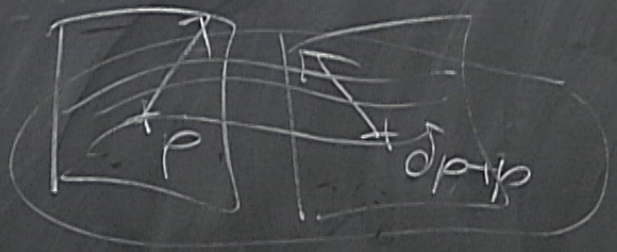
iii) CONTRAC

3 STANDARD POSSIBILITIES:

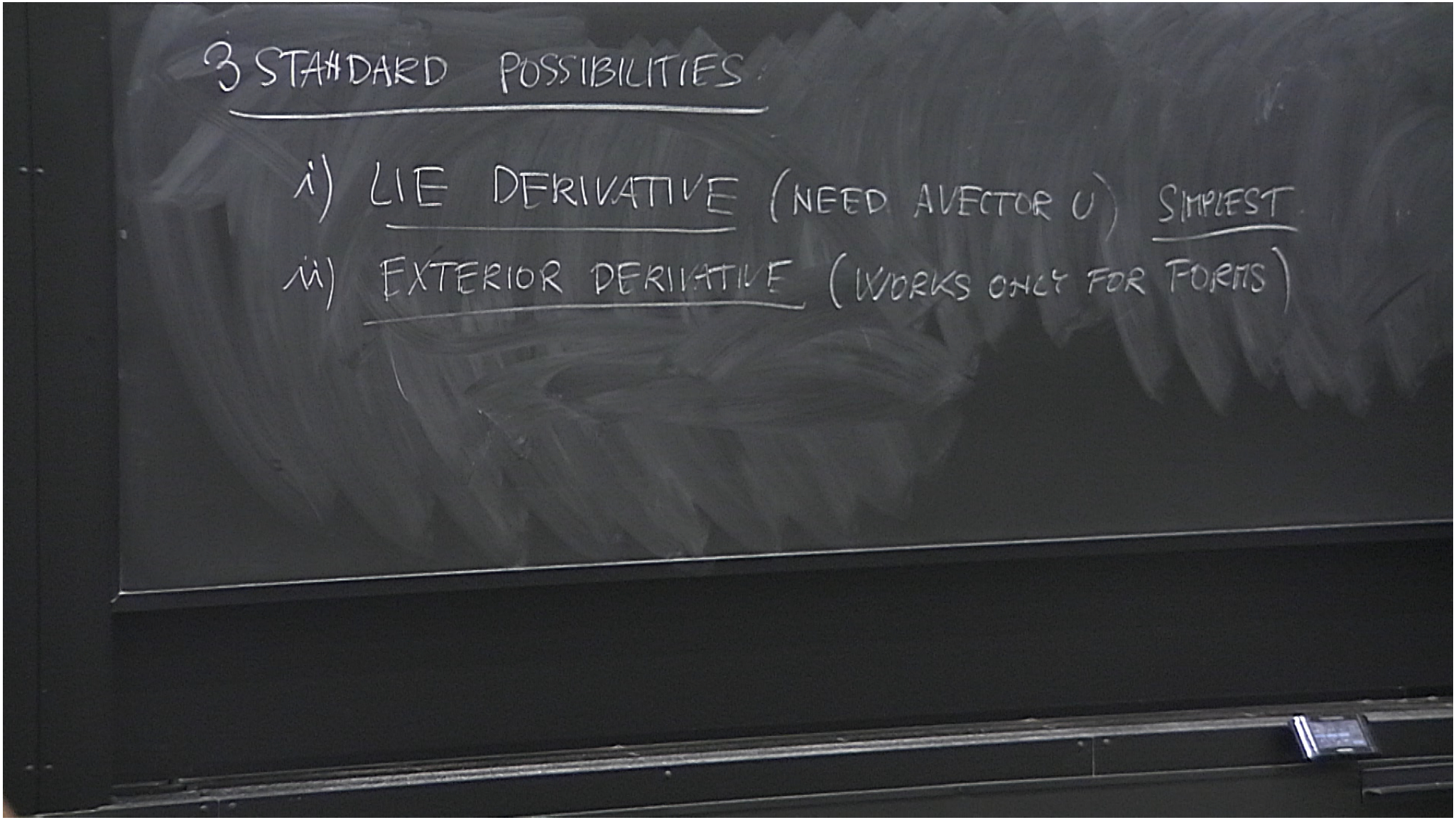
i) LIE DERIVATIVE (NEED A VECTOR U)

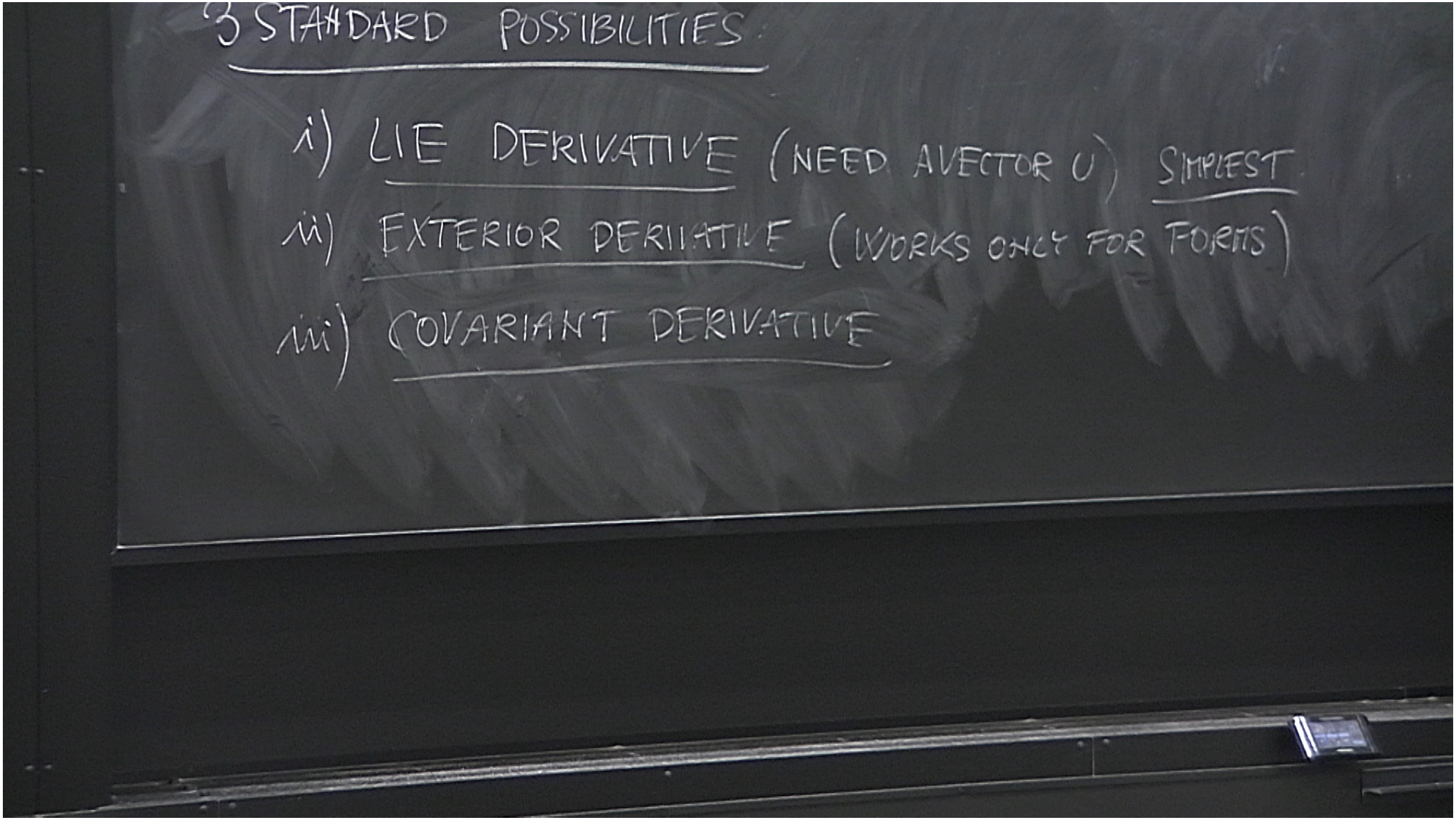
PROBLEMS: i) WHERE IS δp (?)

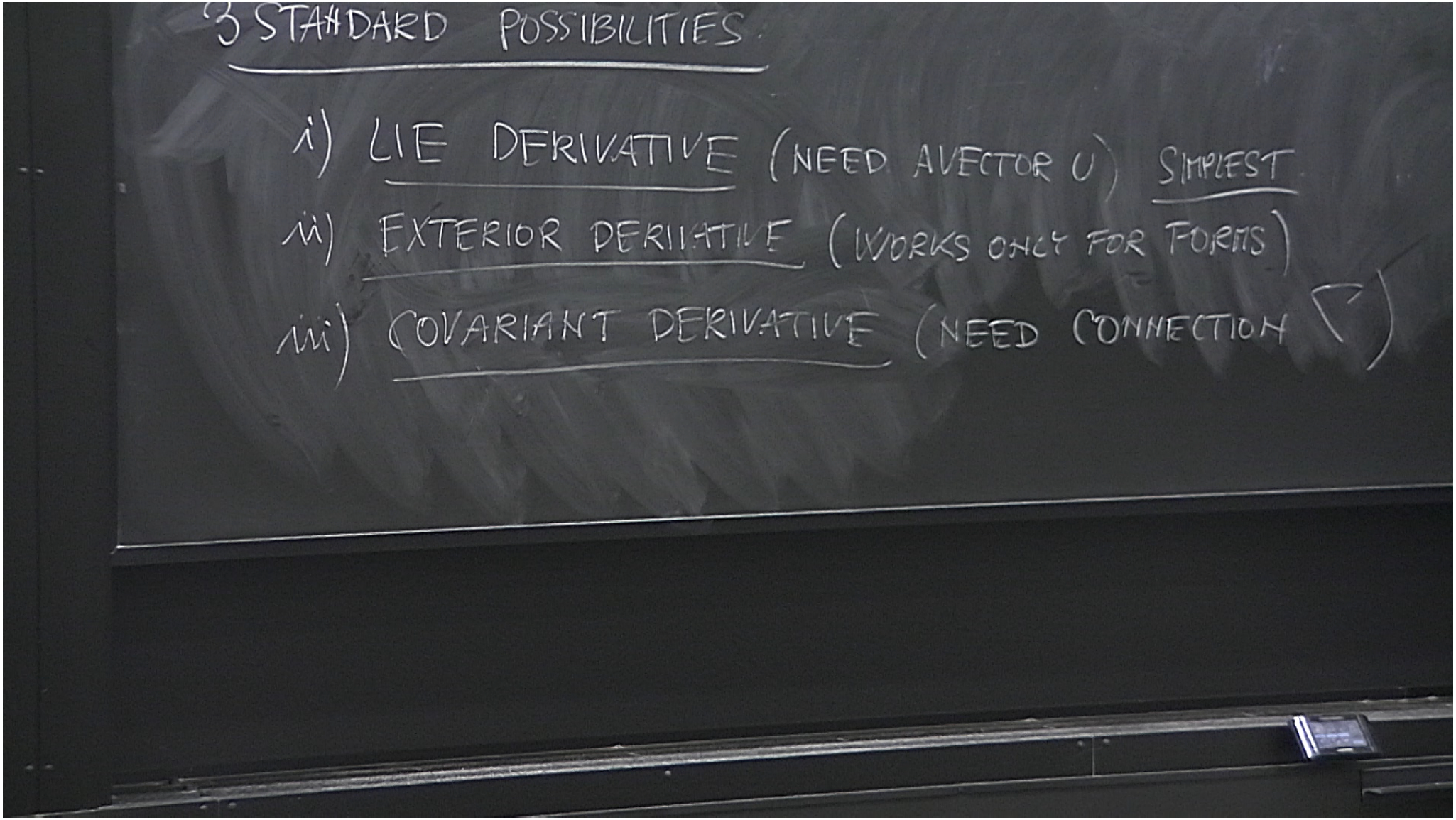
ii) HOW TO COMPARE $V(p+\delta p)$ WITH $V(p)$

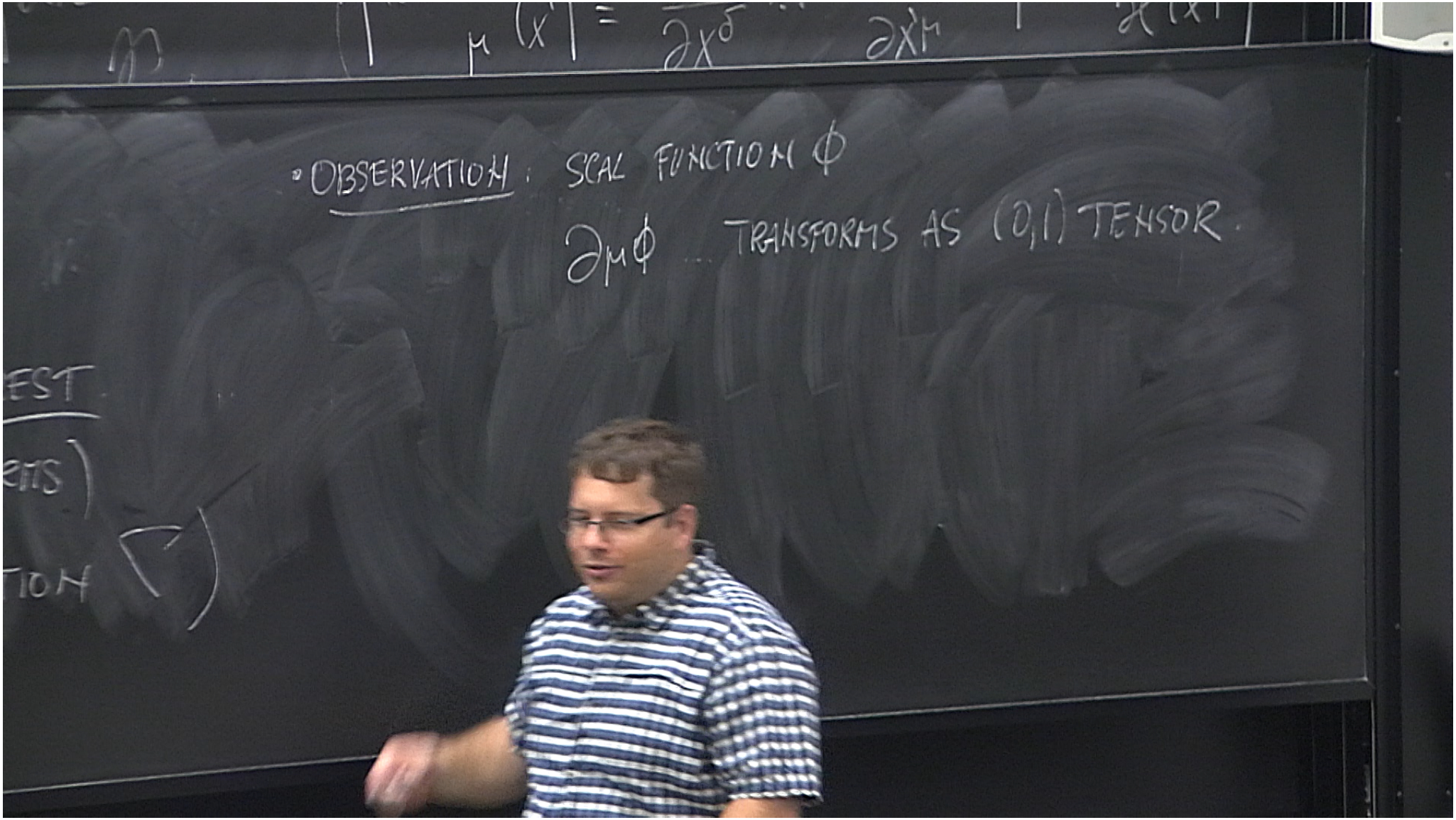


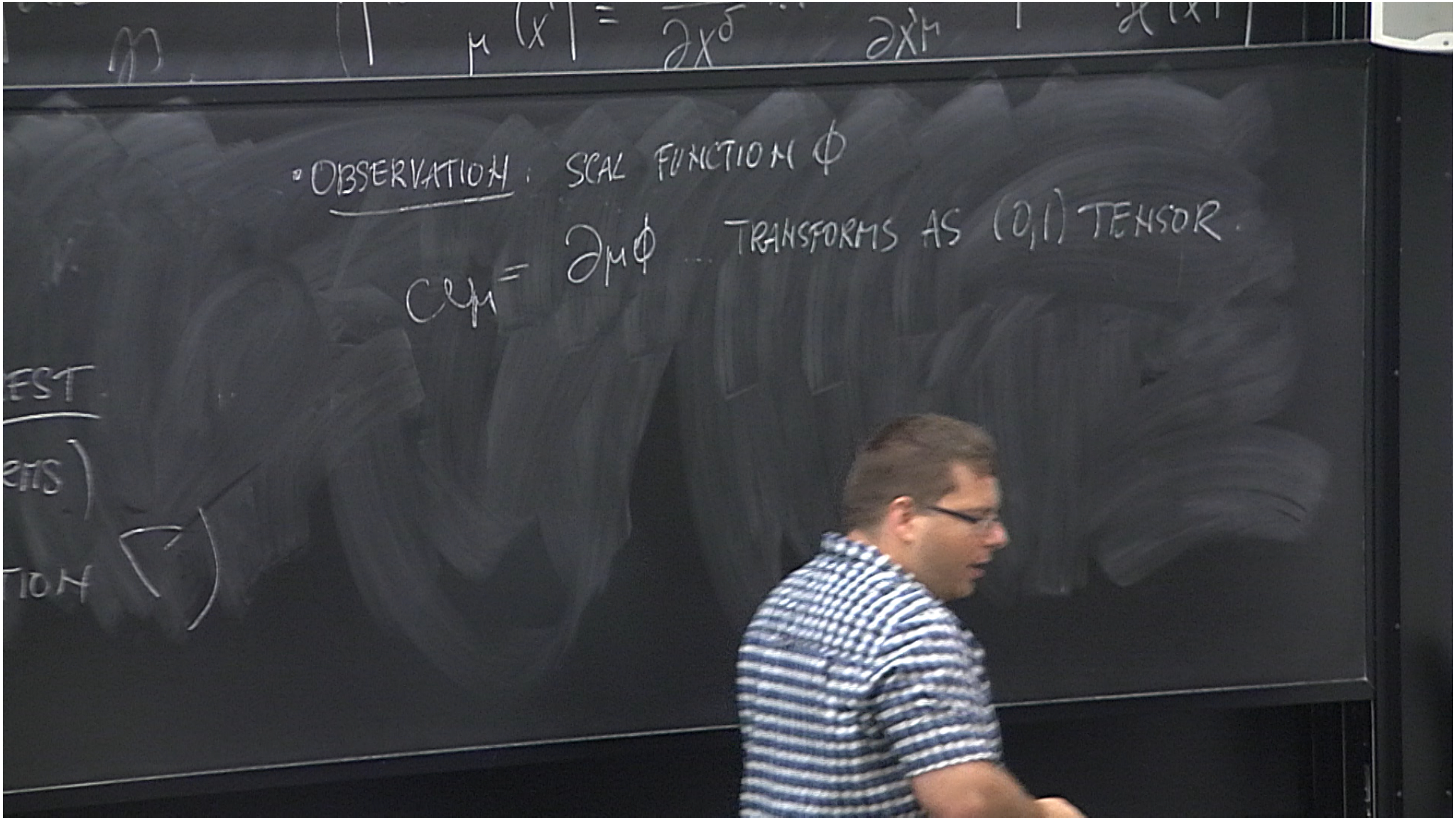
CANNOT BE DONE IN GENERAL, UNLESS EXTRA STRUCTURE EXISTS IN MANIFOLD

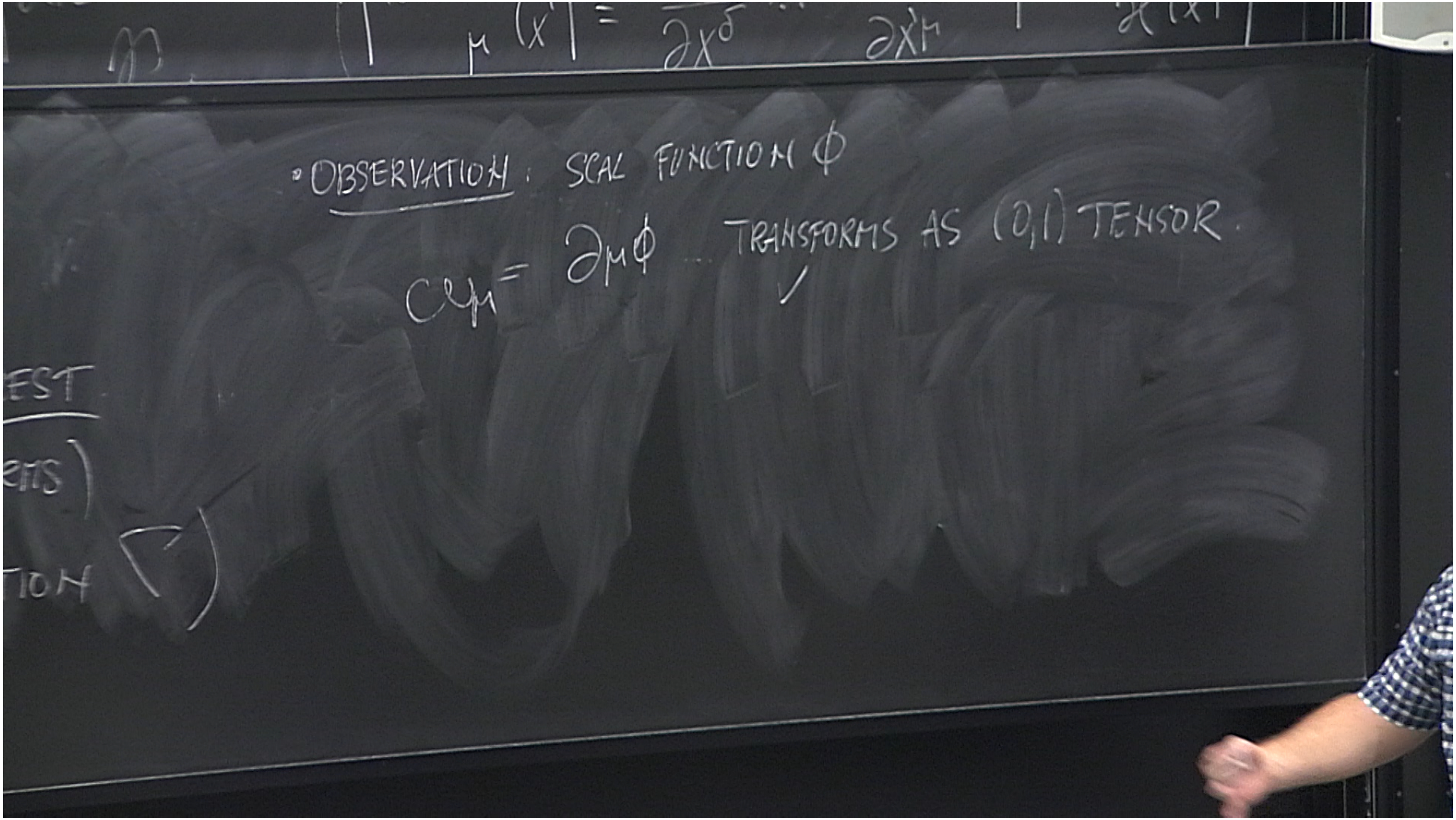

3 STANDARD POSSIBILITIES:
i) LIE DERIVATIVE (NEED A VECTOR U) SIMPLEST
ii) EXTERIOR DERIVATIVE (WORKS ONLY FOR FORMS)

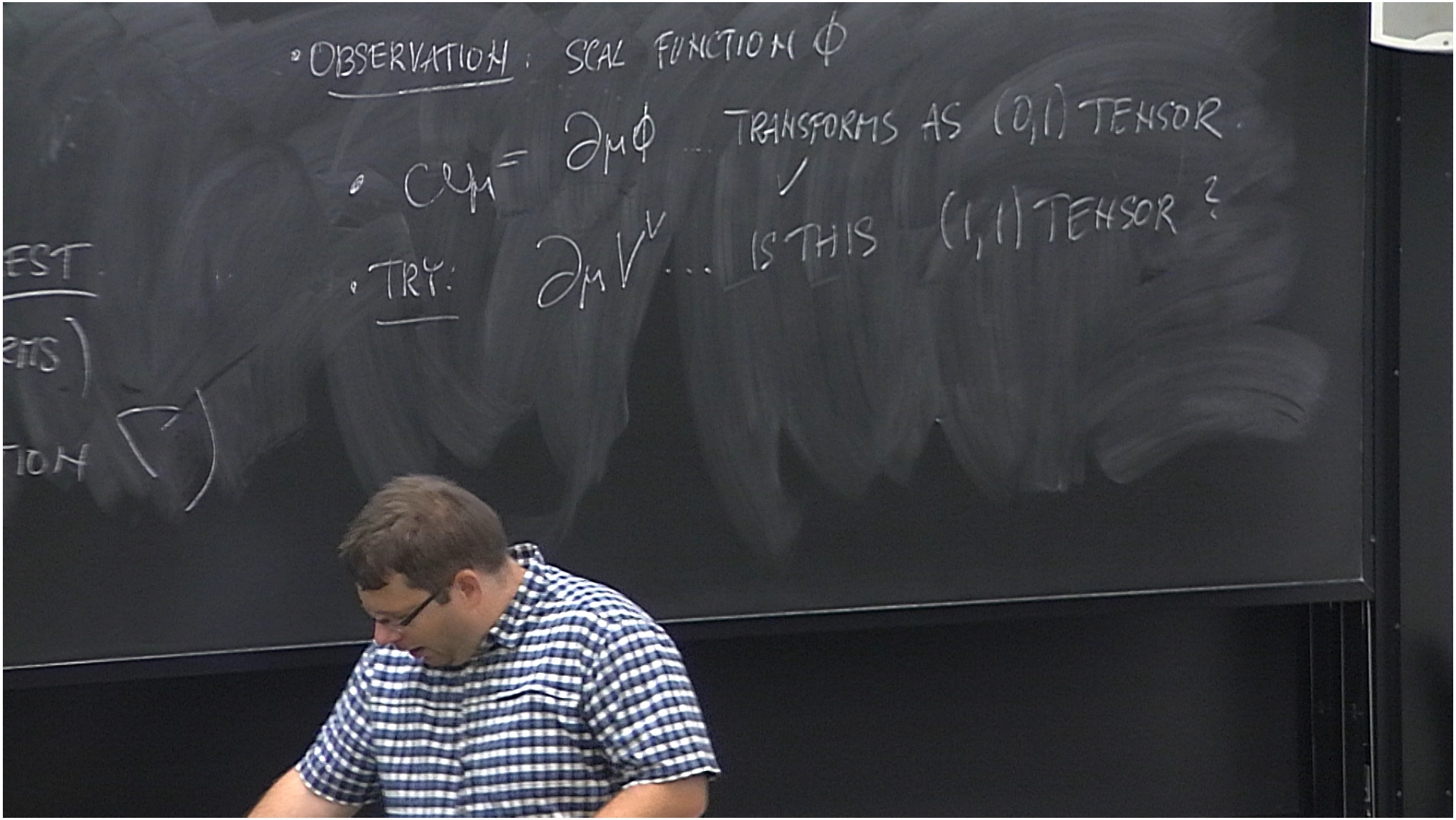

3 STANDARD POSSIBILITIES:
i) LIE DERIVATIVE (NEED A VECTOR U) SIMPLEST
ii) EXTERIOR DERIVATIVE (WORKS ONLY FOR FORMS)
iii) COVARIANT DERIVATIVE

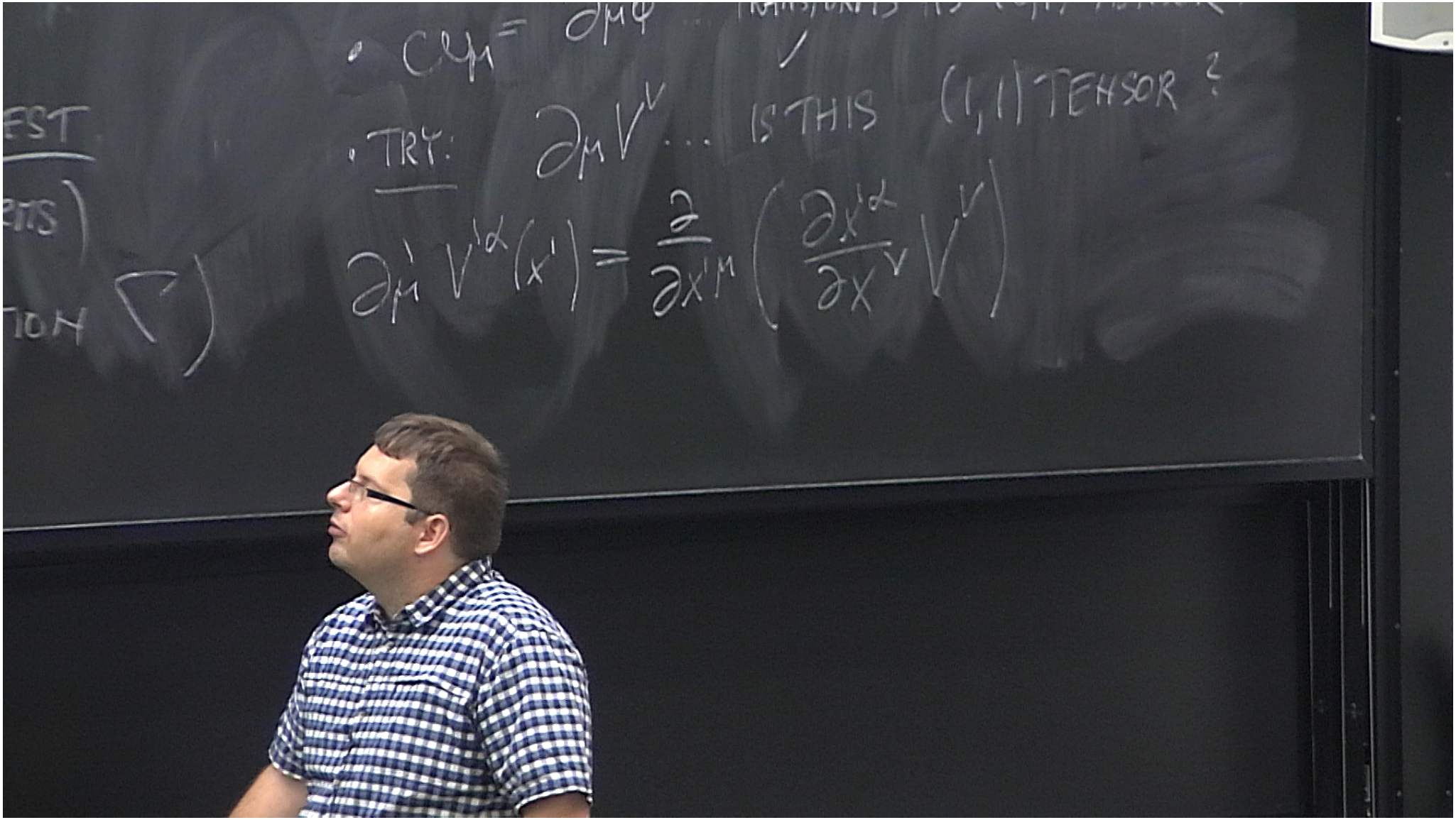

3 STANDARD POSSIBILITIES
i) LIE DERIVATIVE (NEED A VECTOR U) SIMPLEST
ii) EXTERIOR DERIVATIVE (WORKS ONLY FOR FORMS)
iii) COVARIANT DERIVATIVE (NEED CONNECTION ∇)

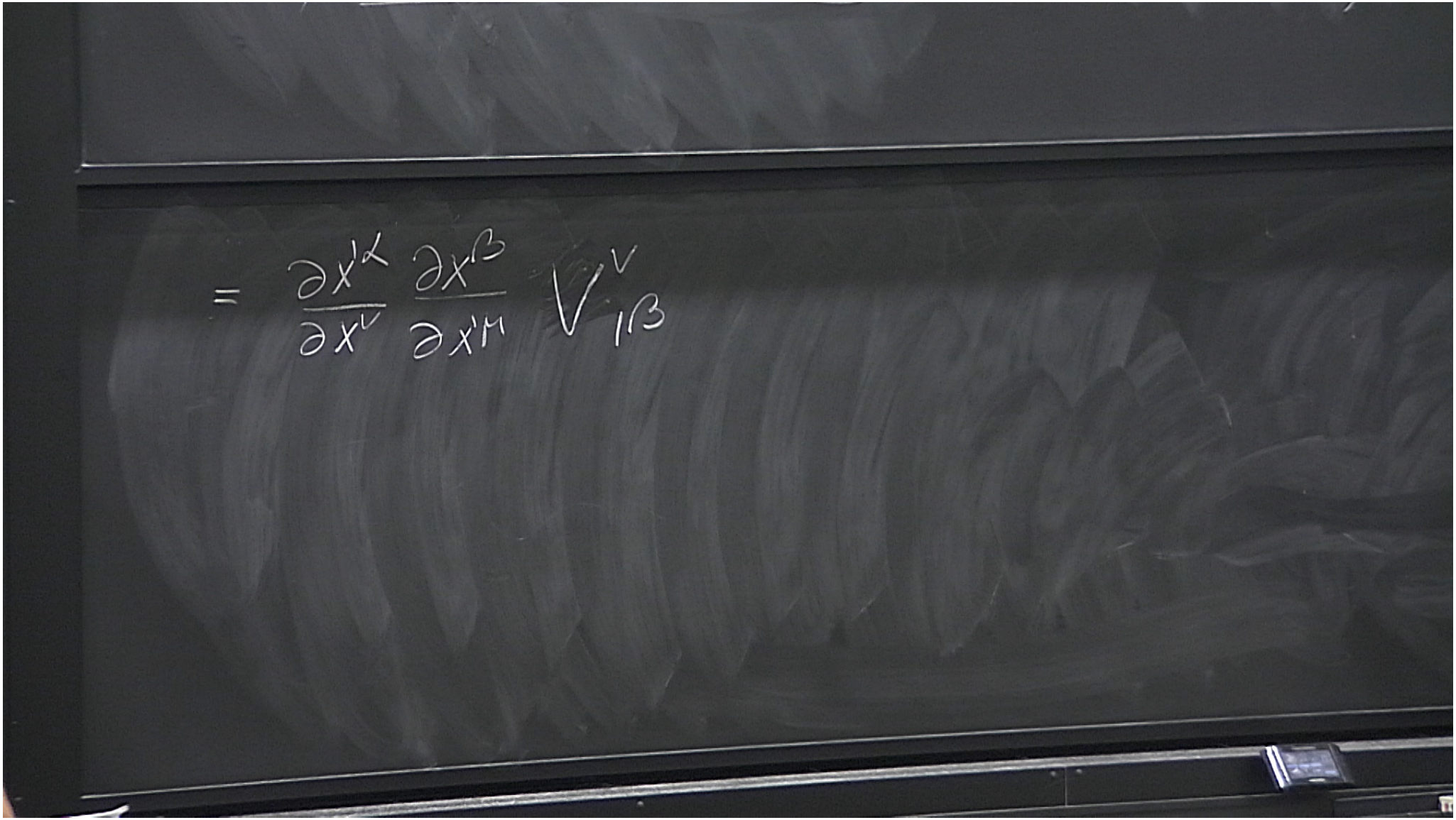

• OBSERVATION: SCAL FUNCTION ϕ
$\partial_\mu \phi$ ... TRANSFORMS AS (0,1) TENSOR.

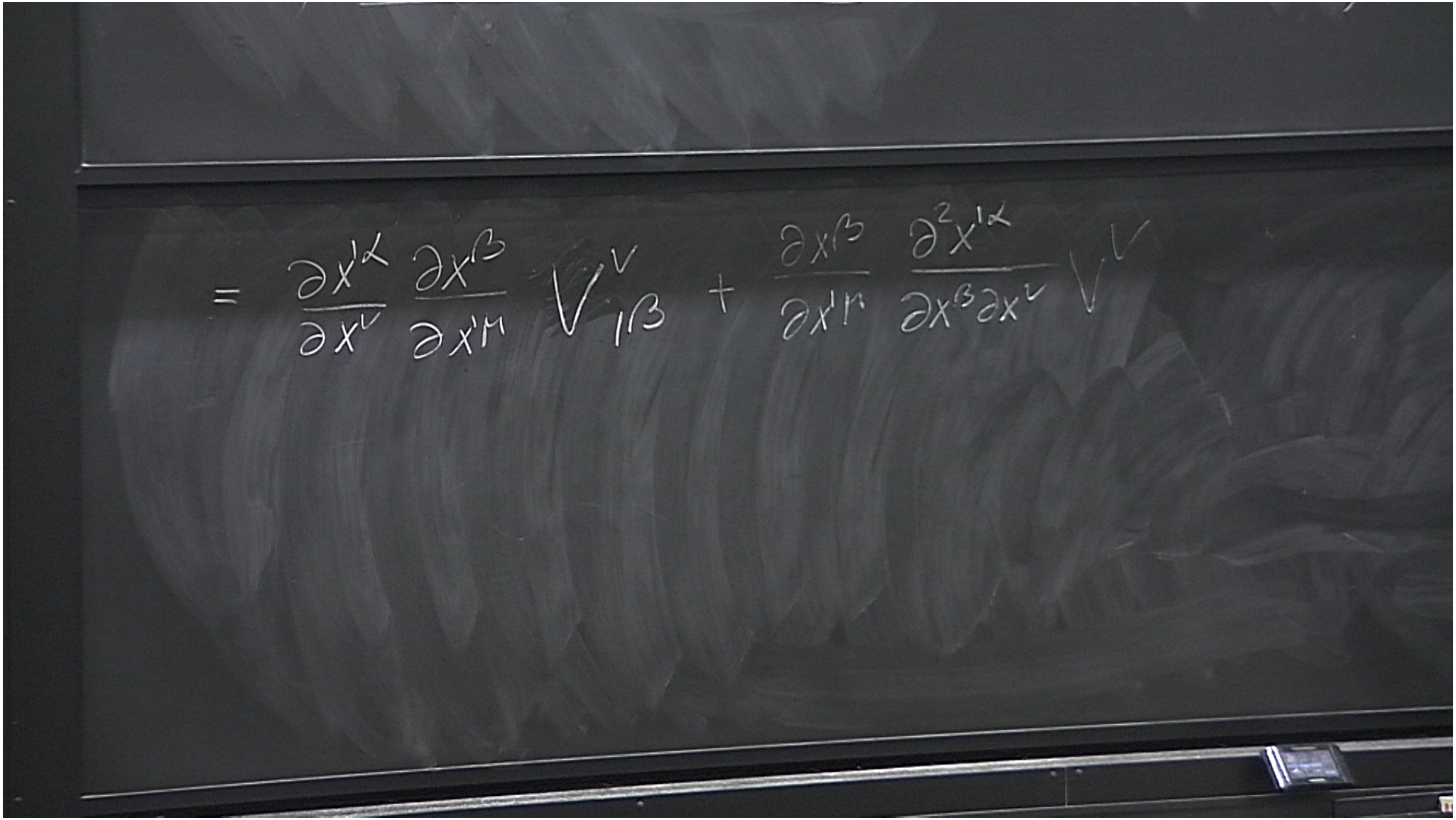

• OBSERVATION : SCAL FUNCTION ϕ
$\partial_\mu \phi$ ... TRANSFORMS AS (0,1) TENSOR.


• OBSERVATION: SCAL FUNCTION ϕ
$\partial_\mu \phi$ TRANSFORMS AS (0,1) TENSOR.


• OBSERVATION: SCAL FUNCTION φ
• $C\Psi_\mu = \partial_\mu \phi$ ... TRANSFORMS AS (0,1) TENSOR.
• TRY: $\partial_\mu V^\nu$ ... IS THIS (1,1) TENSOR?


• $C_{\mu\nu} = \partial_\mu \Phi$ ...
• TRY: $\partial_\mu V^\nu$ ... IS THIS (1,1) TENSOR ?
$\partial'_\mu V'^\alpha(x') = \frac{\partial}{\partial x'^\mu}\left(\frac{\partial x'^\alpha}{\partial x^\nu} V^\nu\right)$


$= \frac{\partial x'^{\alpha}}{\partial x^{\nu}} \frac{\partial x^{\beta}}{\partial x'^{\mu}} V^{\nu}_{\mu\beta}$


$= \frac{\partial x'^{\alpha}}{\partial x^{\nu}} \frac{\partial x^{\beta}}{\partial x'^{\mu}} V^{\nu}_{|\beta} + \frac{\partial x^{\beta}}{\partial x'^{\mu}} \frac{\partial^2 x'^{\alpha}}{\partial x^{\beta} \partial x^{\nu}} V^{\nu}$

$$= \underbrace{\frac{\partial x'^{\alpha}}{\partial x^{\nu}} \frac{\partial x^{\beta}}{\partial x'^{\mu}} V^{\nu}{}_{\beta}}_{\text{OK ... (1,1) TENSOR}} + \frac{\partial x^{\beta}}{\partial x'^{\mu}} \frac{\partial^2 x'^{\alpha}}{\partial x^{\beta} \partial x^{\nu}} V^{\nu}$$

$$= \frac{\partial x'^{\alpha}}{\partial x^{\nu}} \frac{\partial x^{\beta}}{\partial x'^{\mu}} V^{\nu}{}_{|\beta} + \frac{\partial x^{\beta}}{\partial x'^{\mu}} \frac{\partial^2 x'^{\alpha}}{\partial x^{\beta} \partial x^{\nu}} V^{\nu}$$

OK ... (1,1) TENSOR

PROBLEM

$$= \frac{\partial x'^{\alpha}}{\partial x^{\nu}} \frac{\partial x^{\beta}}{\partial x'^{\mu}} V^{\nu}{}_{|\beta} + \frac{\partial x^{\beta}}{\partial x'^{\mu}} \frac{\partial^2 x'^{\alpha}}{\partial x^{\beta} \partial x^{\nu}} V^{\nu}$$

OK ... (1,1) TENSOR

PROBLEM !

TO DEAL WITH THIS (RIEMANN 1830)

$$= \frac{\partial x'^{\alpha}}{\partial x^{\nu}} \frac{\partial x^{\beta}}{\partial x'^{\mu}} V^{\nu}_{\ ,\beta} + \frac{\partial x^{\beta}}{\partial x'^{\mu}} \frac{\partial^2 x'^{\alpha}}{\partial x^{\beta} \partial x^{\nu}} V^{\nu}$$

OK ... (1,1) TENSOR

PROBLEM !

- TO DEAL WITH THIS (RIEMANN 1830)

COVARIANT DERIVATIVE

$$\nabla_{\mu} V^{\nu} = \partial_{\mu} V^{\nu} + \Gamma^{\nu}_{\mu\alpha} V^{\alpha}$$

$$= \frac{\partial x'^{\alpha}}{\partial x^{\nu}} \frac{\partial x^{\beta}}{\partial x'^{\mu}} V^{\nu}_{\ ,\beta} + \frac{\partial x^{\beta}}{\partial x'^{\mu}} \frac{\partial^2 x'^{\alpha}}{\partial x^{\beta} \partial x^{\nu}} V^{\nu}$$

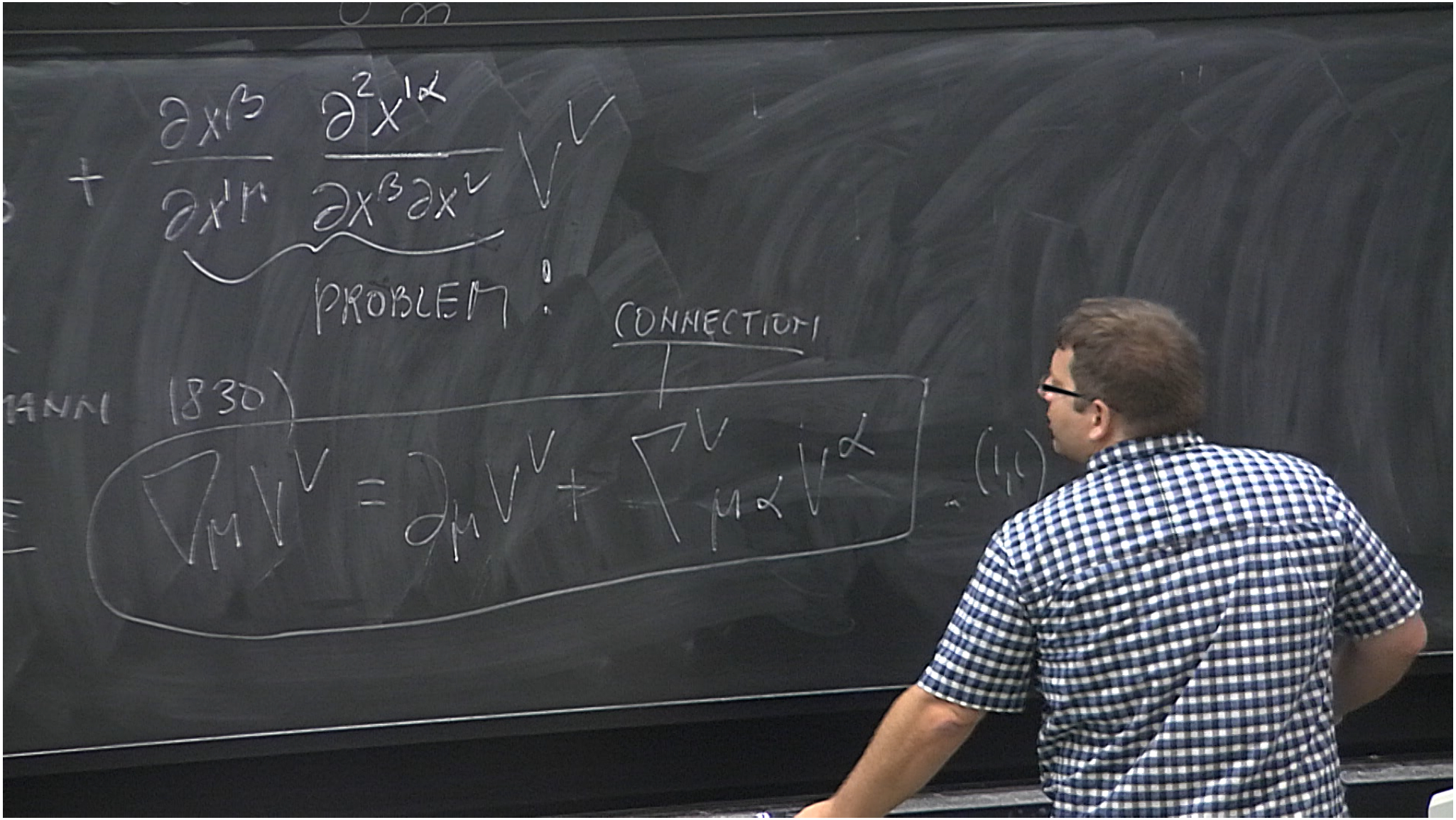
OK ... (1,1) TENSOR

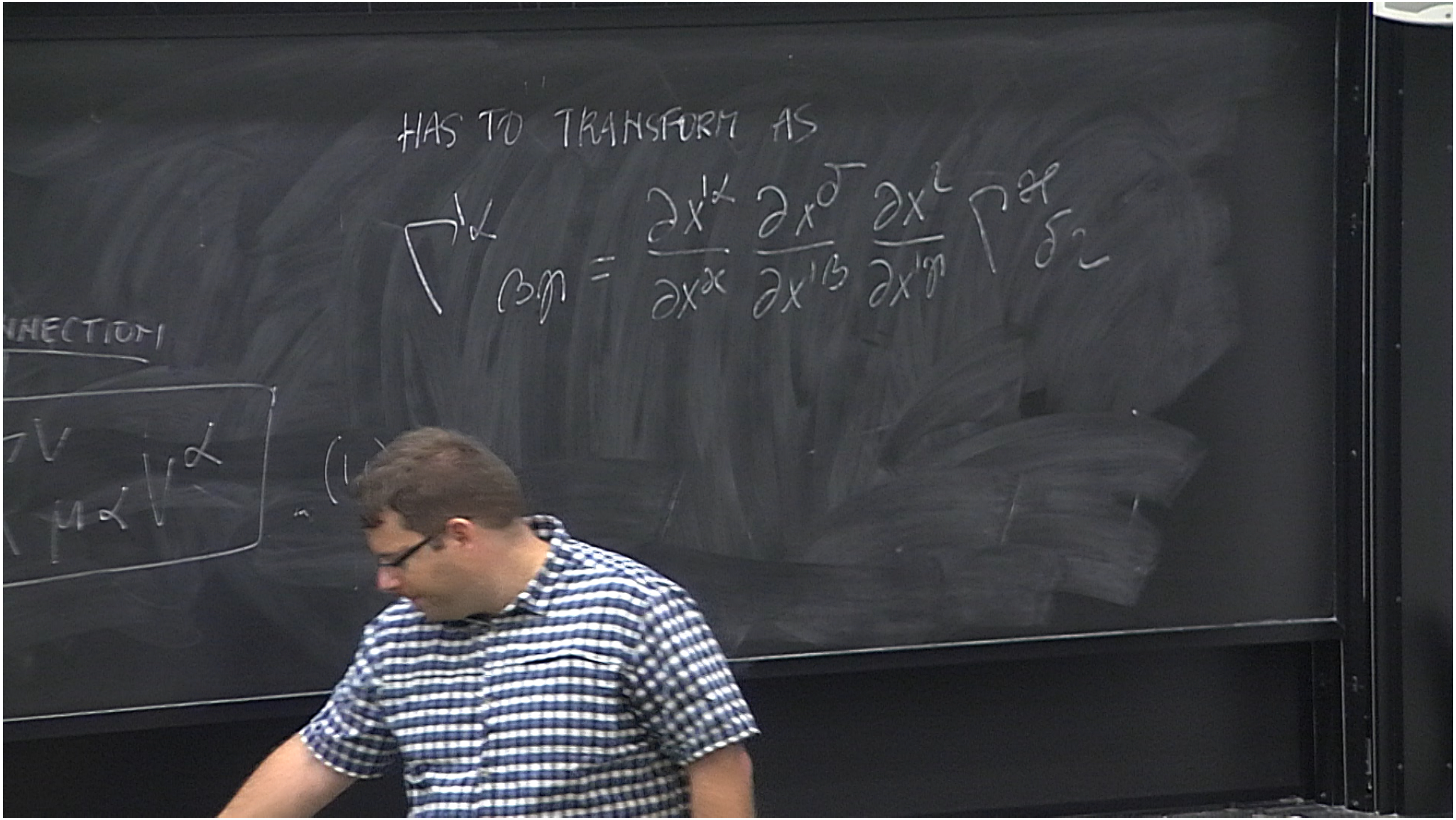
PROBLEM !

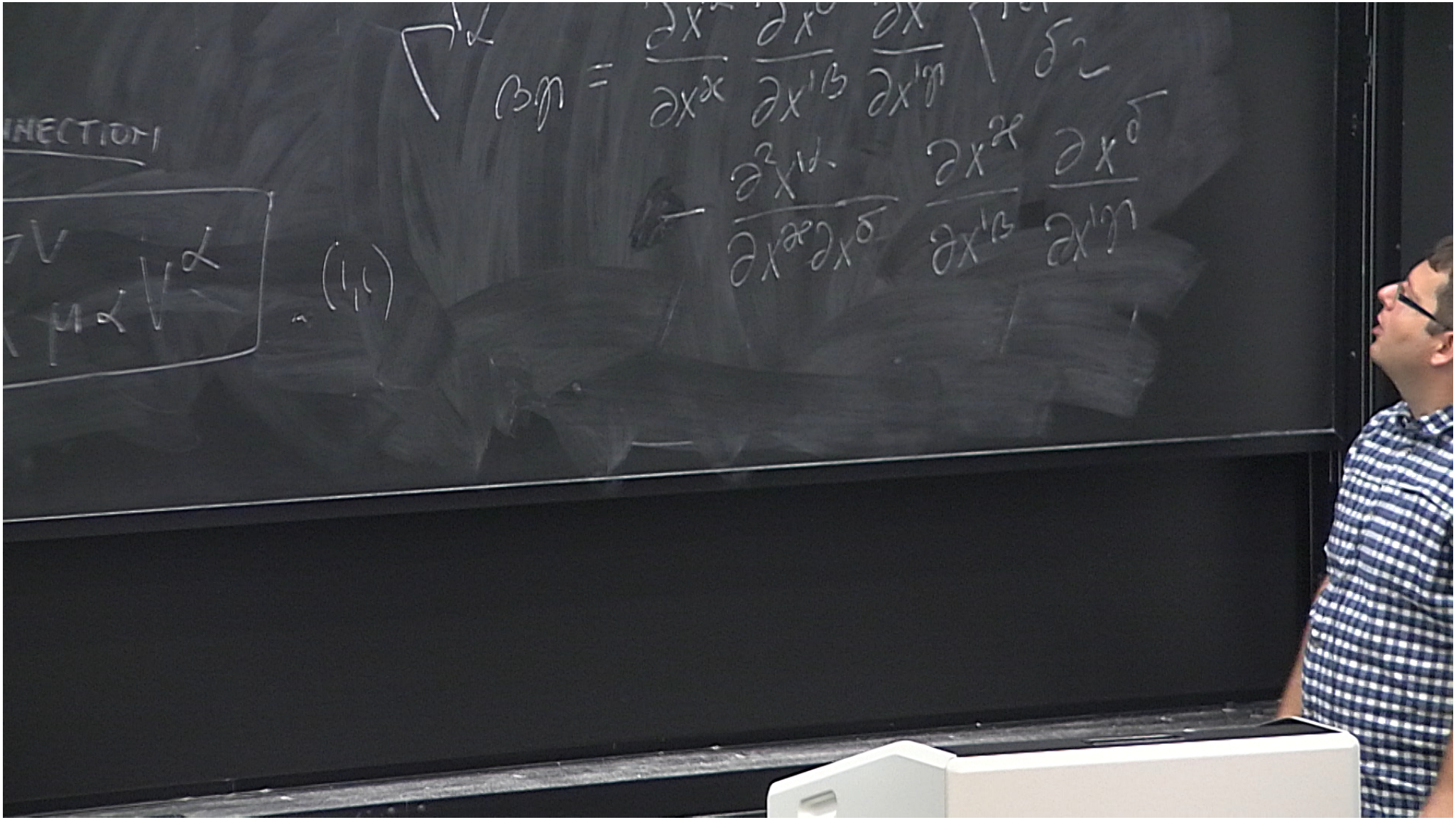
- TO DEAL WITH THIS (RIEMANN 1830)

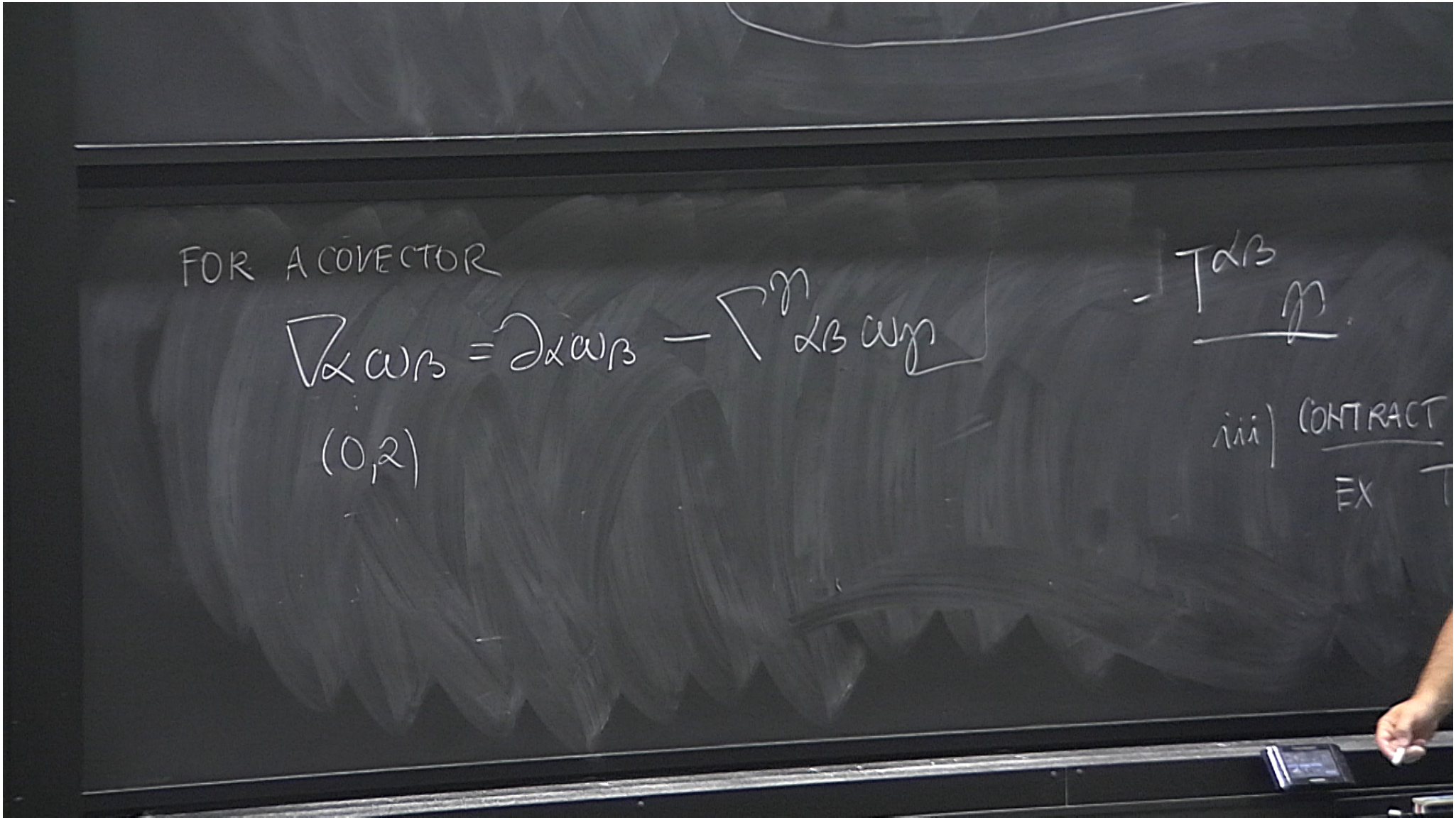
COVARIANT DERIVATIVE

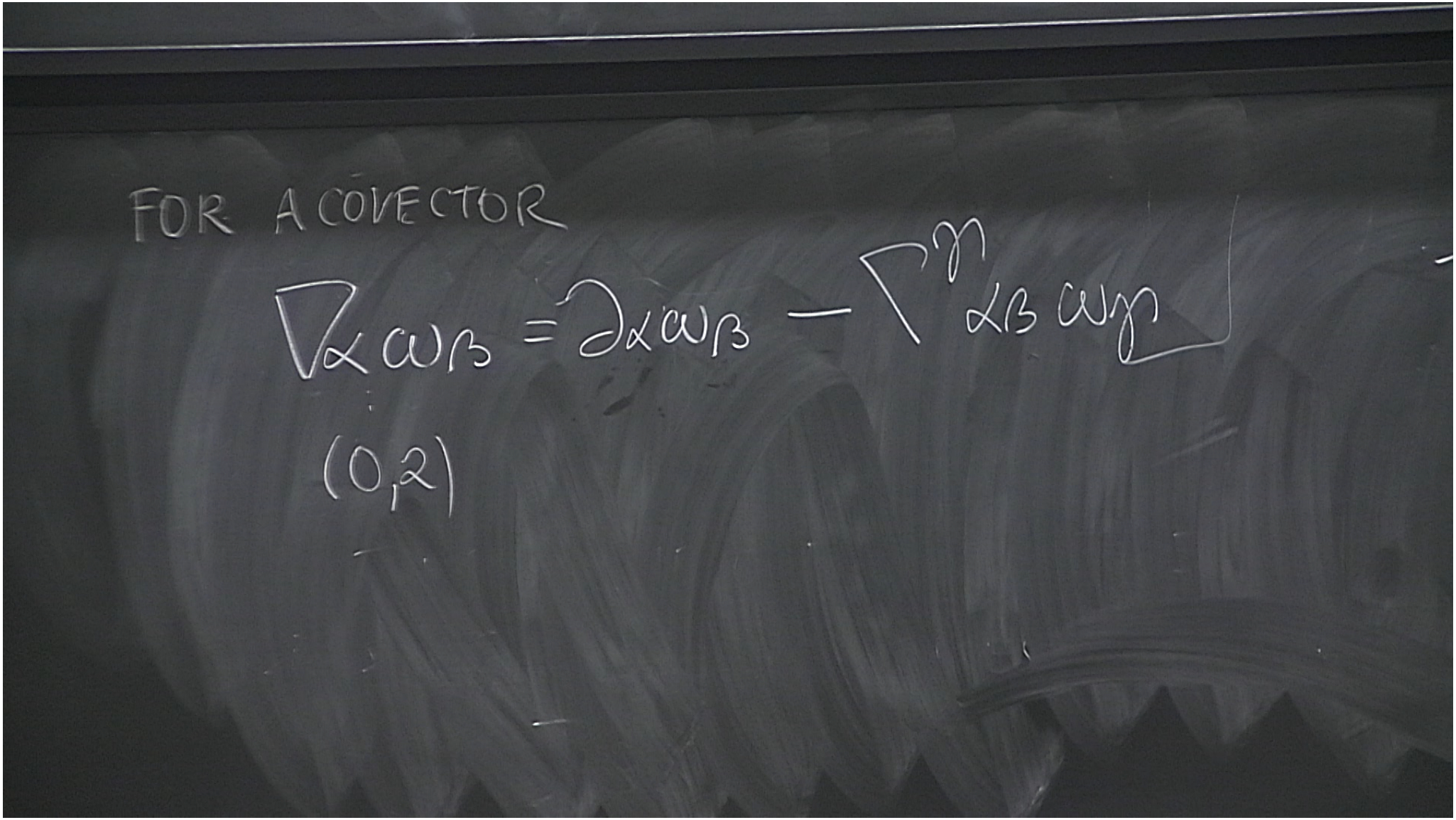
$$\nabla_{\mu} V^{\nu} = \partial_{\mu} V^{\nu} + \Gamma^{\nu}_{\mu\alpha} V^{\alpha} \quad (1,1)$$

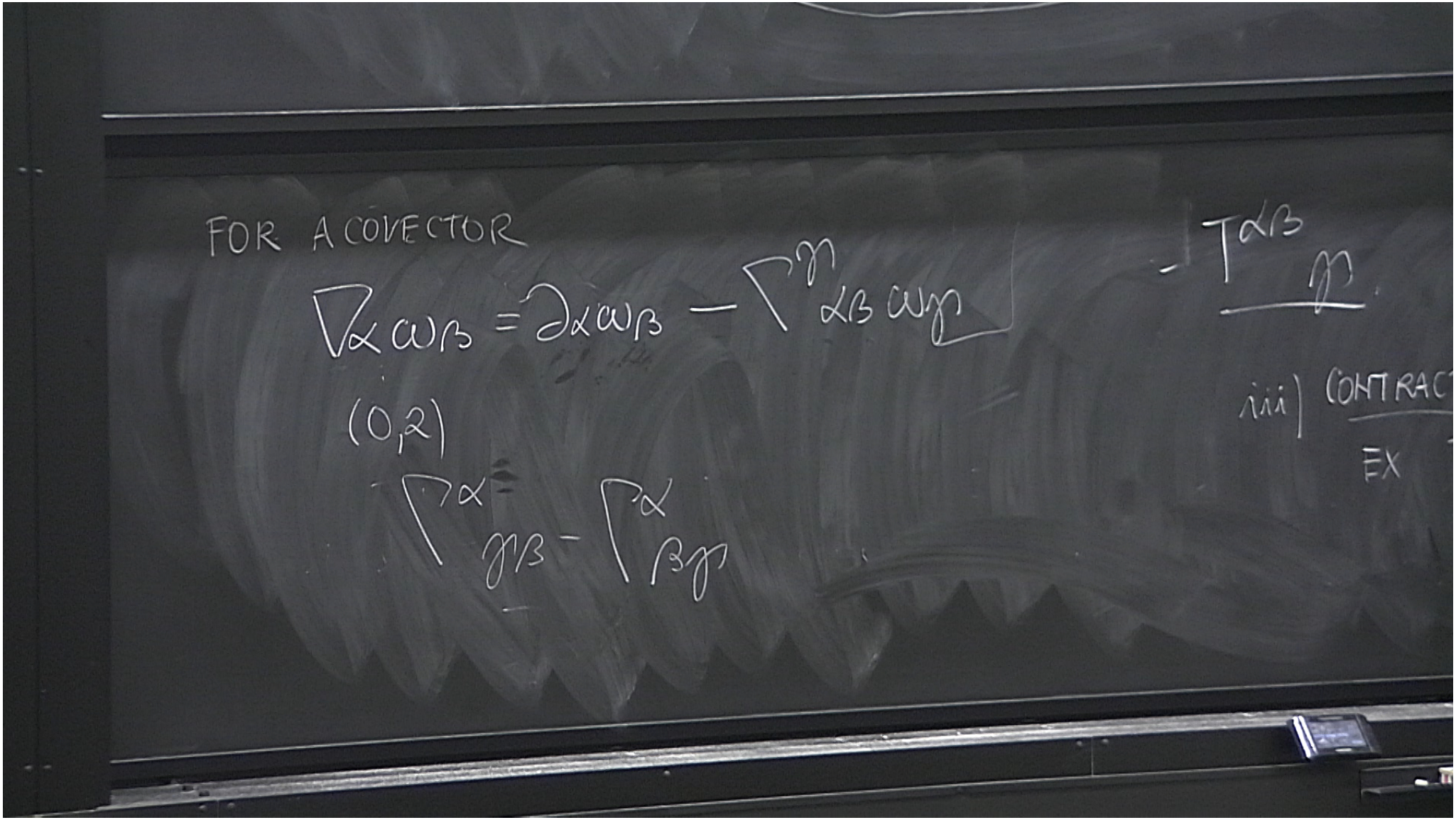

PROBLEM !
CONNECTION
(1830)

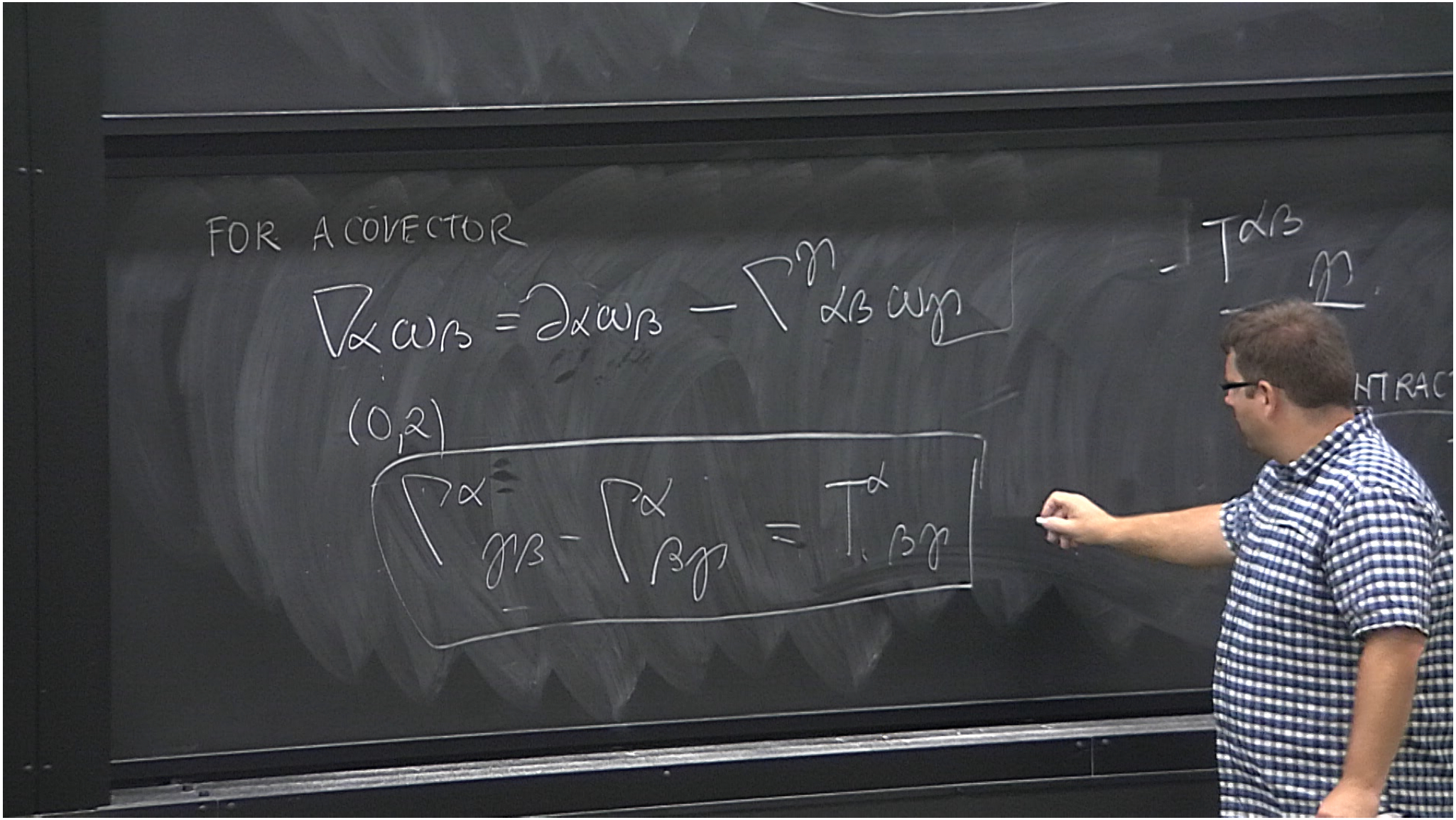

HAS TO TRANSFORM AS
$\Gamma'^{\alpha}_{\beta\eta} = \frac{\partial x'^{\alpha}}{\partial x^{\sigma}} \frac{\partial x^{\sigma}}{\partial x'^{\beta}} \frac{\partial x^{\varrho}}{\partial x'^{\eta}} \Gamma^{\sigma}_{\delta\varrho}$
NNECTION

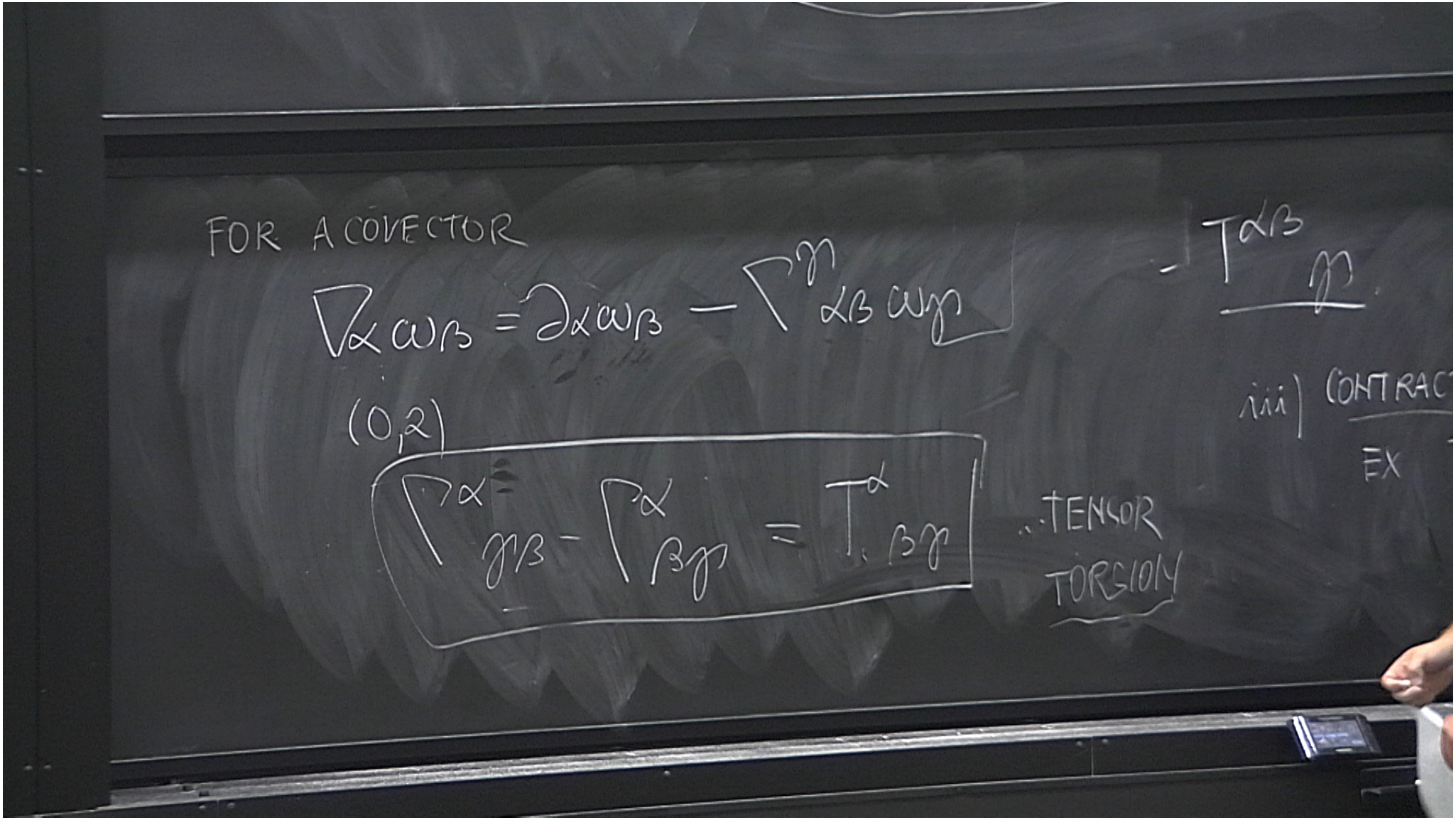

NECTION
(1,1)


FOR A COVECTOR
$\nabla_\alpha \omega_\beta = \partial_\alpha \omega_\beta - \Gamma^\eta_{\alpha\beta} \omega_\eta$
(0,2)
$- T^{\alpha\beta}{}_{\eta}$
iii) CONTRACT
EX


FOR A COVECTOR
$\nabla_\alpha \omega_\beta = \partial_\alpha \omega_\beta - \Gamma^\eta_{\alpha\beta} \omega_\eta$
(0,2)


FOR A COVECTOR
$\nabla_\alpha \omega_\beta = \partial_\alpha \omega_\beta - \Gamma^\gamma_{\alpha\beta}\omega_\gamma$
(0,2)
$\Gamma^\alpha_{\gamma\beta} - \Gamma^\alpha_{\beta\gamma}$
$T^{\alpha\beta}{}_{\gamma}$
iii) CONTRAC
EX


FOR A COVECTOR
$\nabla_\alpha \omega_\beta = \partial_\alpha \omega_\beta - \Gamma^\gamma_{\alpha\beta}\omega_\gamma$
$-T^{\alpha\beta}{}_{\gamma}$
(0,2)
$\Gamma^\alpha_{\gamma\beta} - \Gamma^\alpha_{\beta\gamma} = T^\alpha_{\cdot\,\beta\gamma}$


FOR A COVECTOR
$\nabla_\alpha \omega_\beta = \partial_\alpha \omega_\beta - \Gamma^\gamma_{\alpha\beta}\omega_\gamma$
$T^{\alpha\beta}{}_{\gamma}$
iii) CONTRAC
EX
(0,2)
$\Gamma^\alpha_{\gamma\beta} - \Gamma^\alpha_{\beta\gamma} = T^\alpha_{\cdot\,\beta\gamma}$
..TENSOR
TORSION