Title: A device-independent approach to testing physical theories from finite data

Date: Jul 31, 2018 11:30 AM

URL: http://pirsa.org/18080030

Abstract: The device-independent approach to physics is one where conclusions are drawn directly and solely from the observed correlations between measurement outcomes. This operational approach to physics arose as a byproduct of Bell's seminal work to distinguish quantum correlations from the set of correlations allowed by locally-causal theories. In practice, since one can only perform a finite number of experimental trials, deciding whether an empirical observation is compatible with some class of physical theories will have to be carried out via the task of hypothesis testing. In this talk, I will review some recent progress on this task based on the prediction-based-ratio method and discuss how it may allow us to falsify, in principle, other classes of physical theories, such as those constrained only by the nonsignaling principle, and those that are constrained to produce the so-called "almost-quantum" set of correlations. As an application, I demonstrate how this method allows us to unveil the apparent violation of the nonsignaling conditions in certain experimental data collected in a Bell test. The lesson learned from this observation will be briefly discussed.

A device-independent approach to testing physical theories from finite data

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Foundations of Quantum Mechanics
30th Jul - 3rd Aug 2018

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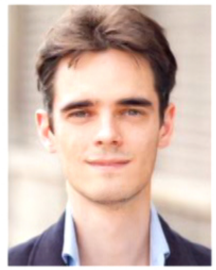

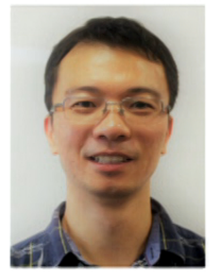

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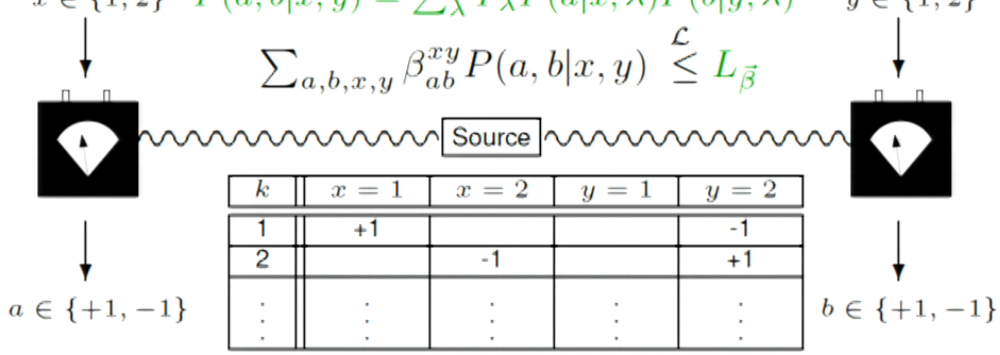
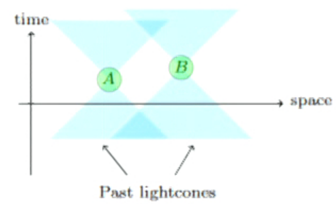
A device-independent approach to physics

$$P(a,b|x,y) \stackrel{\mathcal{L}}{=} \sum_\lambda P_\lambda P(a|x,\lambda)P(b|y,\lambda)$$

$$\sum_{a,b,x,y} \beta^{xy}_{ab} P(a,b|x,y) \stackrel{\mathcal{L}}{\leq} L_{\vec{\beta}}$$



k	$x=1$	$x=2$	$y=1$	$y=2$
1	+1			-1
2		-1		+1
⋮	⋮	⋮	⋮	⋮



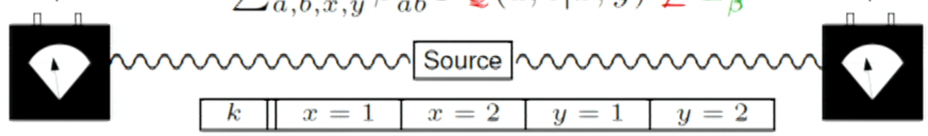
- Bell's theorem: No locally-causal theory can reproduce all quantum mechanical predictions.
  - Correlations between measurement outcomes allowed by *any* locally-causal theory must satisfy Bell inequalities.
  - Certain quantum prediction violates the derived inequality.
- An operational approach to physics.

Bell's theorem and quantum nonlocality

A device-independent approach to physics

$x \in \{1,2\}$ $\quad P(a,b|x,y) \overset{\mathcal{L}}{=} \sum_\lambda P_\lambda P(a|x,\lambda)P(b|y,\lambda)$ $\quad y \in \{1,2\}$

$$\sum_{a,b,x,y} \beta_{ab}^{xy} P_{\mathcal{Q}}(a,b|x,y) \not\leq L_{\vec{\beta}}$$



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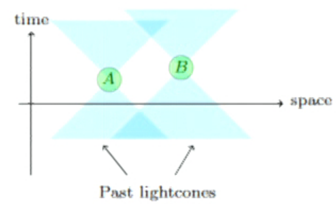
$a \in \{+1,-1\}$ $\quad b \in \{+1,-1\}$



- Bell's theorem: No locally-causal theory can reproduce all quantum mechanical predictions.
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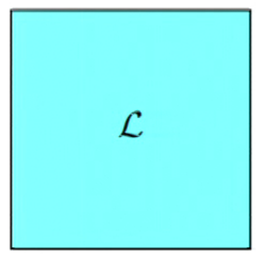
  - Certain quantum prediction violates the derived inequality.
- An operational, device-independent approach to physics.

A geometrical approach to various natural sets of correlations

- In the space of probability vectors $\vec{P} = \{P(a,b|x,y)\}_{a,b,x,y}$:
  - Locally-causal correlations form a convex polytope $\mathcal{L}$.
  - The set of quantum correlations $\mathcal{Q}$ is a strict superset of $\mathcal{L}$.
  - Correlations only constrained by nonsignaling $\mathcal{NS}$



$P(a,b|x,y) \stackrel{\mathcal{L}}{=} \sum_\lambda P_\lambda P(a|x,\lambda)P(b|y,\lambda)$

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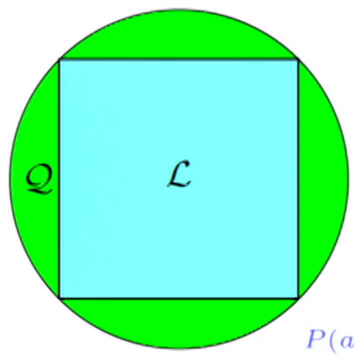

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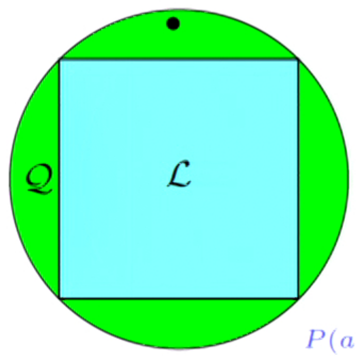

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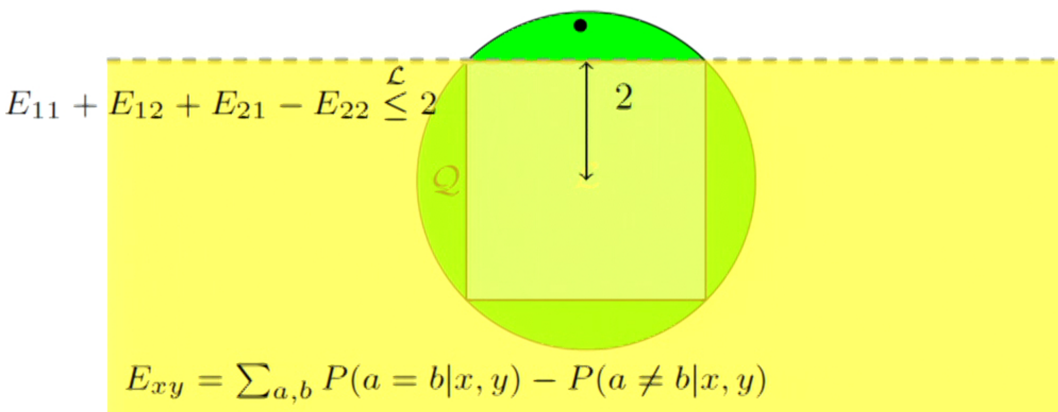

Generalized theory of nonlocality

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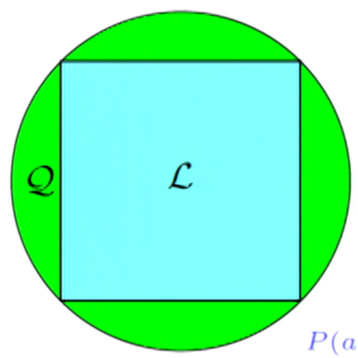

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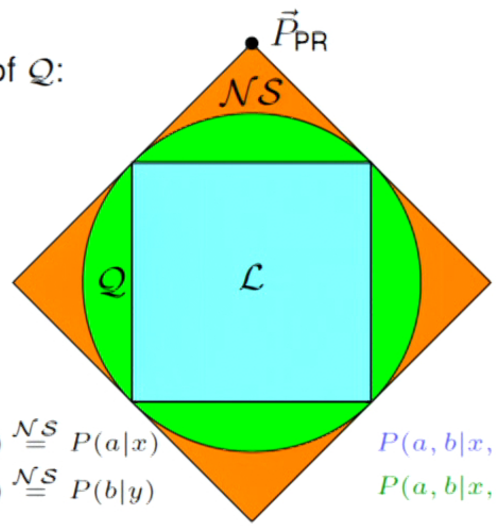
Outer approximations of $\mathcal{Q}$:

$\tilde{\mathcal{Q}} \supsetneq \tilde{\mathcal{Q}}_2 \supsetneq \ldots \supsetneq \mathcal{Q}$



$\downarrow$

impossibility to perform superluminal communication via correlations alone

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$$P(a,b|x,y) \overset{\mathcal{L}}{=} \sum_\lambda P_\lambda P(a|x,\lambda) P(b|y,\lambda)$$
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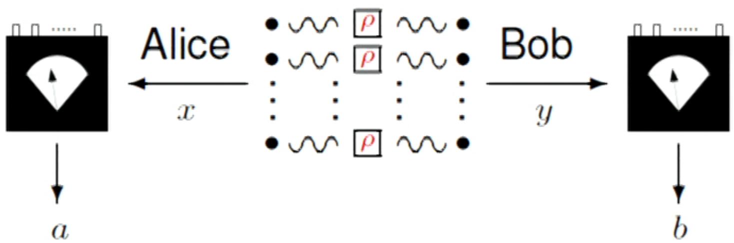
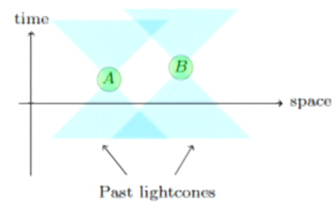
[1] Popescu & Rohrlich, FP (1994).

[2] Goh, Kaniewski, Wolfe, Vértesi, Wu, Cai, YCL, Scarani, PRA (2018).

A simplified account of what happens in practice . . .

A closer look at what actually happens





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2		+1			-1	
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⋮	⋮	⋮	⋮	⋮	⋮	⋮

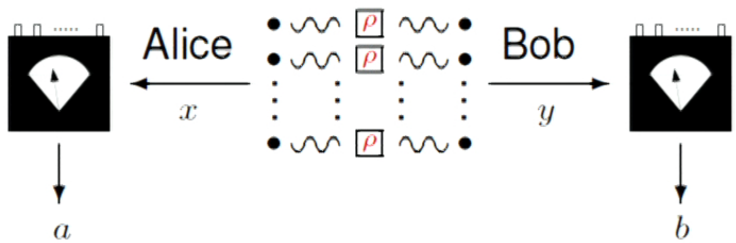
(a,b,x,y)	$N(a,b,x,y)$
(+1,+1,1,1)	1007
(+1,+1,1,2)	2533
⋮	⋮

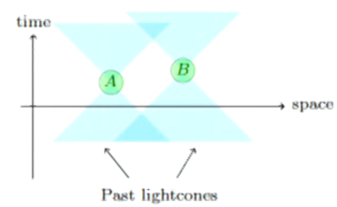
$$N(x,y) = \sum_{a,b} N(a,b,x,y)$$

$$N_{\text{total}} = \sum_{x,y} N(x,y)$$

Relative frequencies $\vec{f}$: $f(a,b|x,y) \equiv \frac{N(a,b,x,y)}{N(x,y)} \approx P_{\mathcal{Q}}(a,b|x,y)$

A closer look at what actually happens





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⋮	⋮	⋮	⋮	⋮	⋮	⋮

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A simplified account of what happens in practice . . .

Bell violation is real . . .

- "Loophole-free" Bell tests confirmed with high confidence that our world is not locally causal.[3]
- Estimate of the underlying distribution (via relative frequencies):

$$f(a,b|x,y) \equiv \frac{N(a,b,x,y)}{N(x,y)} \neq \text{tr}(\rho\, M^{A}_{a|x} \otimes M^{B}_{b|y})$$

is generically not quantum, and violates the nonsignaling conditions:

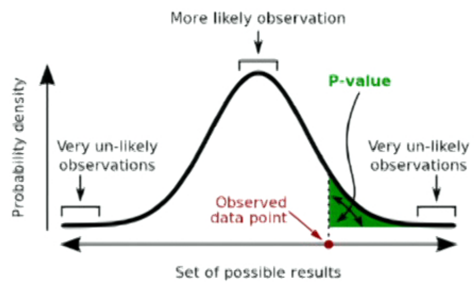
$$f(a|x,y) \equiv \sum_b f(a,b|x,y) \neq \sum_b f(a,b|x,y') \quad \text{if } y \neq y'$$

$$f(b|x,y) \equiv \sum_a f(a,b|x,y) \neq \sum_a f(a,b|x',y) \quad \text{if } x \neq x'$$

[3] Hensen *et al.*, Nature (2015); Giustina *et al.*, PRL (2015); Shalm *et al.*, PRL (2015); Rosenfeld *et al.*, PRL (2017).

Hypothesis testing in Bell test I

- Determine from the actual experimental data whether a certain (null) hypothesis is likely to be true:
  - $\mathcal{L}$: Local causality.
  - $\mathcal{NS}$: The nonsignaling conditions.
  - $\mathcal{Q}$: Quantum (Born's rule); $\tilde{\mathcal{Q}}$: Almost-quantum[4]; $\tilde{\mathcal{Q}}_k$[5]
- P-value (probability value): the probability, under the assumption of a hypothesis **H**, of obtaining a result equal to or more extreme than what was actually observed.



https://en.wikipedia.org/wiki/P-value

[4] Navascués, Guryanova, Hoban, Acín, Nat. Commun. (2015).
[5] Vallins, Sainz, YCL, PRA (2017).

Hypothesis testing in Bell test II

- Pioneering work from Gill for the null hypothesis that the observed correlation is compatible with local causality.[6]
- E.g.: A particular way of writing the CHSH inequality — in terms of the actual number of counts — is $Z \overset{\mathcal{L}}{\leq} 0$, then

$$\text{Prob}(Z \geq k\sqrt{N_{\text{total}}}|\mathcal{L}) \leq \exp(-\tfrac{1}{2}k^2)$$

- Probability that Weihs' *et al.*[7] (with $N_{\text{total}} = 14700$) experimental results ($\mathcal{S}_{\text{CHSH}} \approx 2.73$) are compatible with the assumption of $\mathcal{L}$ (modulo some loopholes) $\leq 2.64 \times 10^{-27}$.

[6] Gill, arXiv: 0301059 (2003).
[7] Weihs, Jennewein, Simon, Weinfurter, Zeilinger, PRL (1998).

Some useful tools

The Kullback-Leibler (KL) divergence and Bell inequalities

- The amount of evidence should be measured by the Kullback-Leibler (KL) divergence.[8]
- Given $\vec{P}_Q$ and some fixed input distribution P_{xy}, the KL divergence from $\vec{P}_Q$ to $\mathcal{L}$:

$$D_{\text{KL}}\left(\vec{P}_Q||\mathcal{L}\right) := \min_{\vec{P}\in\mathcal{L}} \sum_{a,b,x,y} P_{xy} P_Q(a,b|x,y) \log_2\left[\frac{\cancel{P_{xy}} P_Q(a,b|x,y)}{\cancel{P_{xy}} P(a,b|x,y)}\right]$$

- The unique[9] minimizer $\vec{P}_{\text{KL}}^{\mathcal{L},*}(\vec{P}_Q)$ can be used to construct an optimized Bell inequality:[10]

$$R(a,b,x,y) P_{xy} P(a,b|x,y) \overset{\mathcal{L}}{\leq} 1, \quad R(a,b,x,y) \equiv \frac{P_Q(a,b|x,y)}{P_{\text{KL}}^{\mathcal{L},*}(a,b|x,y)}$$

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[8] van Dam, Gill, and Grünwald, IEEE Trans. Inf. Theo. (2005).
[9] Lin, Rosset, Zhang, Bancal, YCL, PRA (2018).
[10] Acín, Gill, and Gisin, PRL (2005).

The Prediction-based-ratio method [Zhang *et al.*, PRA (2011)]

- Given $\vec{f}$ and some input distribution P_{xy}, the KL divergence from $\vec{f}$ to $\mathcal{L}$:[11]

$$D_{\mathsf{KL}}\left(\vec{f}||\mathcal{L}\right) := \min_{\vec{P}\in\mathcal{L}} \sum_{a,b,x,y} P_{xy} f(a,b|x,y) \log_2 \left[\frac{f(a,b|x,y)}{P(a,b|x,y)}\right]$$

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- This Bell inequality can then be used to compute a p-value bound for $\mathcal{L}$ even in the non-i.i.d scenario.

[11] Zhang, Glancy, Knill, PRA (2011).

Applying the "PBR" method

Initial data collection → Relative freq. $\vec{f}$ —minimize $D_{\mathsf{KL}}(f\|\mathcal{L})$→ Bell ineq. → Other data to be tested → Upper bound on the p-value for $\mathcal{L}$

- For the data to be tested, define the prediction-based-ratio in terms of the i-th trial's results $r_i = (x_i, y_i, a_i, b_i)$ as:

$$R(r_i) = R(x_i, y_i, a_i, b_i) := \frac{f(a_i, b_i|x_i, y_i)}{P_{\mathsf{KL}}^{\mathcal{L},*}(a_i, b_i|x_i, y_i)} \implies \langle R(r_i)\rangle \overset{\mathcal{L}}{\leq} 1 \quad \forall\, i$$

- Given a sequence of results $\vec{r}_n = (r_1, r_2, \ldots, r_n)$, the test statistics $T_j(\vec{r}_j) := \prod_{i=1}^{j} R(r_i)$ satisfies, for all $j \leq n$,

$$T_j(\vec{r}_j) \geq 0, \quad \langle T_j(\vec{r}_j)\rangle \overset{\mathcal{L}}{\leq} 1, \quad \langle T_j(\vec{r}_j)|\vec{r}_{j-1}\rangle \overset{\mathcal{L}}{\leq} T_{j-1}(\vec{r}_{j-1})$$

- By Markov's inequality, one gets a p-value upper bound:

$$\text{Prob}(\vec{r}_n|\mathcal{L}) \leq \min\left\{\frac{1}{\prod_{j=1}^{n} T_j(r_j)}, 1\right\}$$

Some useful tools

Generalization to other convex sets of correlations

- For the data to be tested, define the prediction-based-ratio in terms of the i-th trial's results $r_i = (x_i, y_i, a_i, b_i)$ as:

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- By Markov's inequality, one gets a p-value upper bound:

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Generalization to other convex sets of correlations

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Various natural sets of correlations

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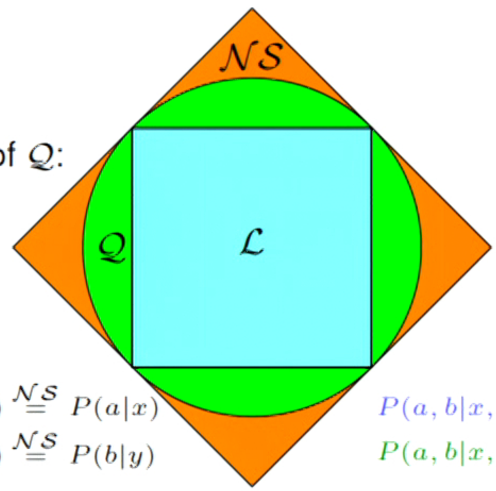
↓

impossibility to perform superluminal communication via correlations alone

Outer approximations of $\mathcal{Q}$:

$\tilde{\mathcal{Q}} \supsetneq \tilde{\mathcal{Q}}_2 \supsetneq \dots \supsetneq \mathcal{Q}$



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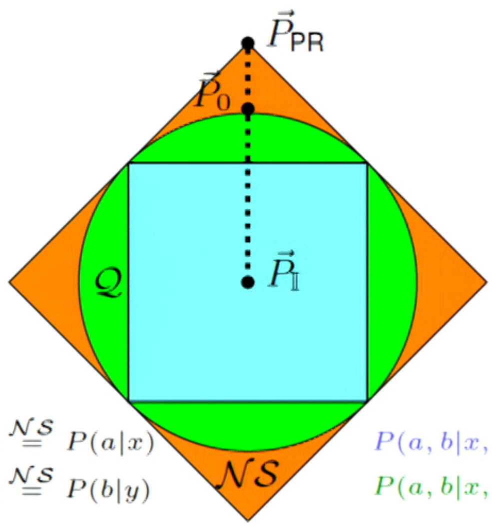
[1] Popescu & Rohrlich, FP, 1994.

Simulating Bell tests with an i.i.d. nonlocal source

- Consider $\vec{P}_0(v) = v\vec{P}_{\text{PR}} + (1-v)\vec{P}_{\mathbb{I}}$ on the isotropic line.
- Perturb $\vec{P}_0(v)$ to obtain $\vec{P} = (1-\epsilon)\vec{P}_0(v) + \epsilon\vec{P}_{\text{R}}, \quad \epsilon = 10^{-2}$.
- Simulate 500 Bell experiments with $N_{\text{total}} = 10^6$ trials each.
- Use first 10% of data to obtain opt. Bell-like ineq.
- Compute a p-value bound using the remaining data.



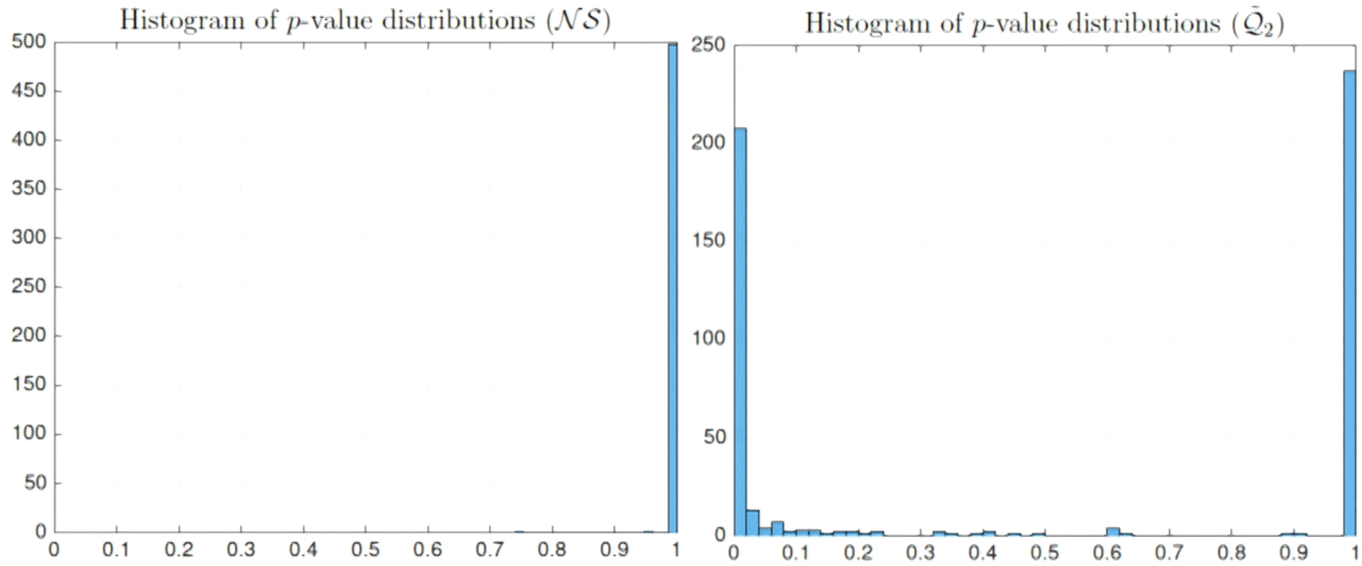
$$P(a|x,y) = \sum_b P(a,b|x,y) \overset{\mathcal{NS}}{=} P(a|x)$$
$$P(b|x,y) = \sum_a P(a,b|x,y) \overset{\mathcal{NS}}{=} P(b|y)$$

$$P(a,b|x,y) \overset{\mathcal{L}}{=} \sum_\lambda P_\lambda P(a|x,\lambda)P(b|y,\lambda)$$
$$P(a,b|x,y) \overset{\mathcal{Q}}{=} \text{tr}(\rho\, M^A_{a|x} \otimes M^B_{b|y})$$

Hypothesis testing for a source perturbed around the Tsirelson point

Example 1: an i.i.d. source $\vec{P} \in \mathcal{NS}$ but with $\vec{P} \notin \tilde{\mathcal{Q}}_2$

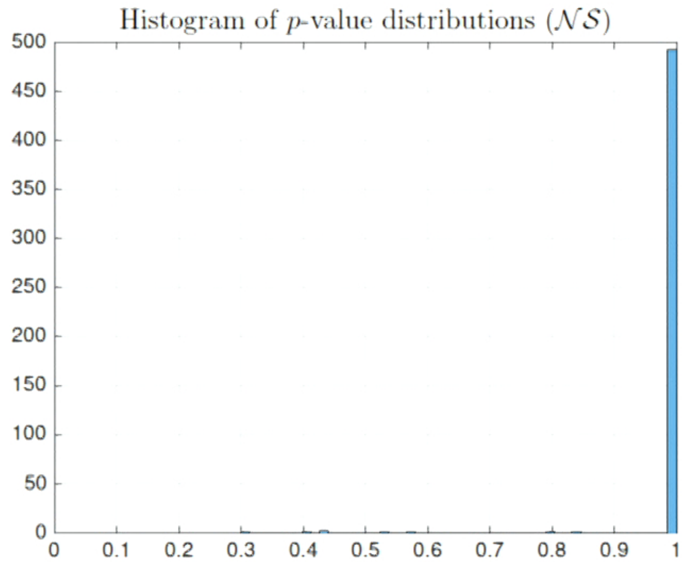
- Consider $\vec{P}_0(v) = v\vec{P}_{\text{PR}} + (1-v)\vec{P}_{\mathbb{I}}$ on the isotropic line.
- Perturb $\vec{P}_0(v)$ to obtain $\vec{P} = (1-\epsilon)\vec{P}_0(v) + \epsilon\vec{P}_{\text{R}}, \quad \epsilon = 10^{-2}$.
- Set $v = 0.72$ (cf., $v \leq 1/\sqrt{2} \approx 0.7071$ for $\vec{P}_0 \in \tilde{\mathcal{Q}}_2$).

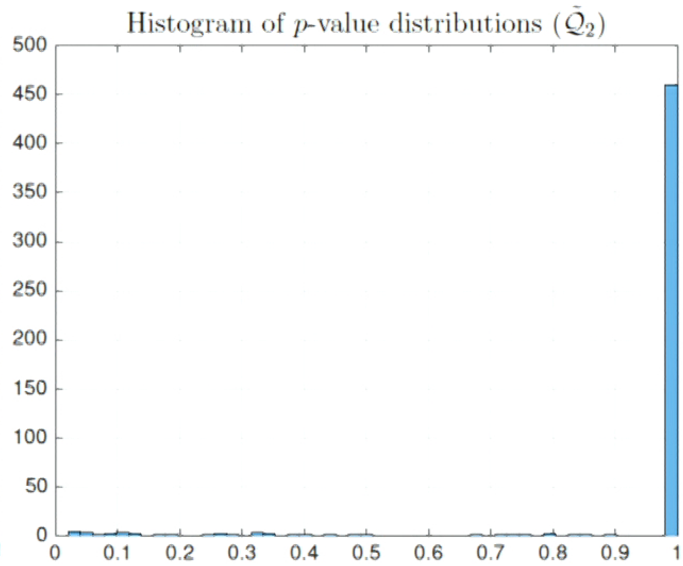


Hypothesis testing for a source perturbed around the Tsirelson point

Example 2: an i.i.d. source $\vec{P} \in \tilde{\mathcal{Q}}_2 \subset \mathcal{NS}$

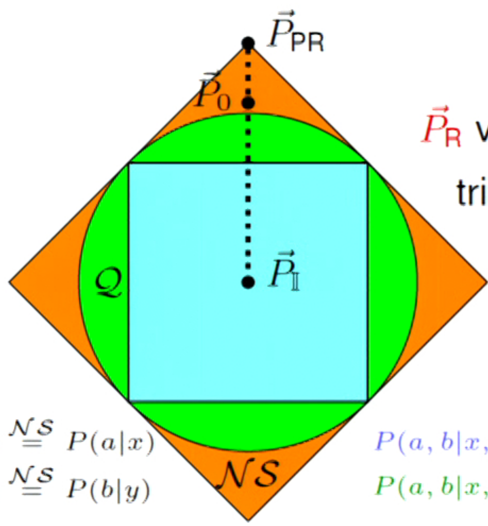
- Consider $\vec{P}_0(v) = v\vec{P}_{\text{PR}} + (1-v)\vec{P}_{\mathbb{I}}$ on the isotropic line.
- Perturb $\vec{P}_0(v)$ to obtain $\vec{P} = (1-\epsilon)\vec{P}_0(v) + \epsilon\vec{P}_{\text{R}}, \quad \epsilon = 10^{-2}$.
- Set $v = 1/\sqrt{2}$ (i.e., $\vec{P}_0 \in \tilde{\mathcal{Q}}_2$).





Simulating Bell tests with a *non*-i.i.d. nonlocal source

- Consider $\vec{P}_0(v) = v\vec{P}_{\text{PR}} + (1-v)\vec{P}_{\mathbb{I}}$ on the isotropic line.
- Perturb $\vec{P}_0(v)$ to obtain $\vec{P} = (1-\epsilon)\vec{P}_0(v) + \epsilon\vec{P}_{\text{R}}, \quad \epsilon = 10^{-2}$.
- Simulate 500 Bell experiments with $N_{\text{total}} = 10^6$ trials each.
- Use first 10% of data to obtain opt. Bell-like ineq.
- Compute a p-value bound using the remaining data.



$\vec{P}_{\text{R}}$ varies from one trial to another

$P(a|x,y) = \sum_b P(a,b|x,y) \overset{\mathcal{NS}}{=} P(a|x)$

$P(b|x,y) = \sum_a P(a,b|x,y) \overset{\mathcal{NS}}{=} P(b|y)$

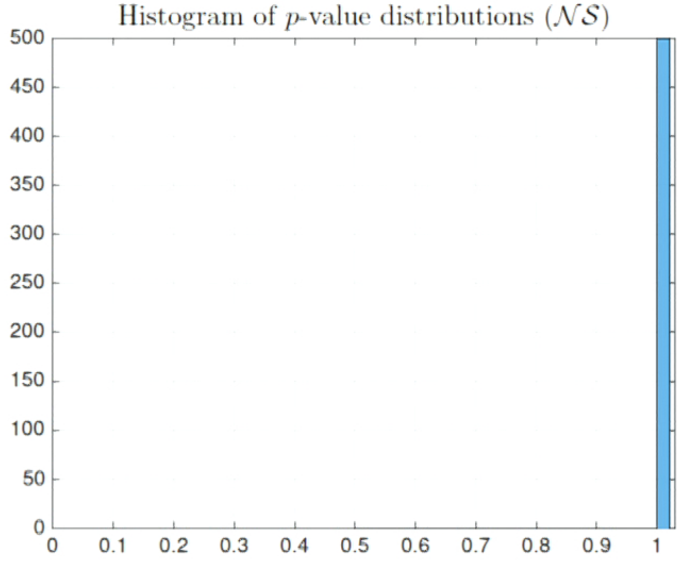
$P(a,b|x,y) \overset{\mathcal{L}}{=} \sum_\lambda P_\lambda P(a|x,\lambda)P(b|y,\lambda)$

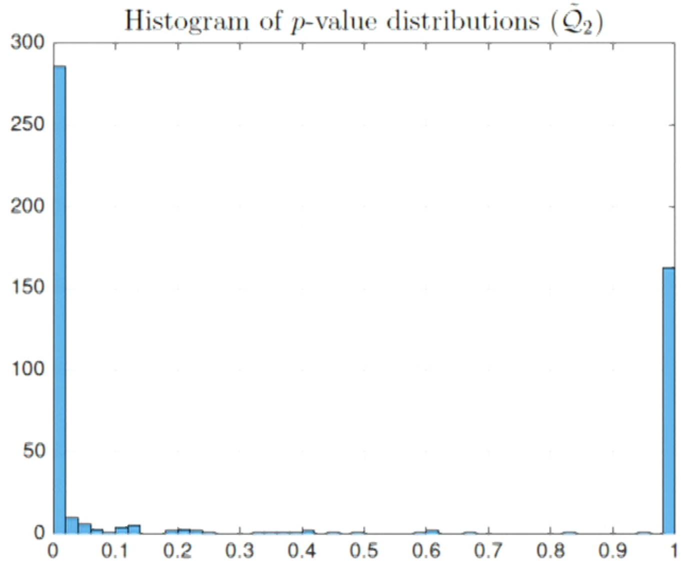
$P(a,b|x,y) \overset{\mathcal{Q}}{=} \text{tr}(\rho\, M^A_{a|x} \otimes M^B_{b|y})$

Hypothesis testing for a source perturbed around the Tsirelson point

Example 3: a non-i.i.d. source $\vec{P} \in \mathcal{NS}$ with $\vec{P}_0 \notin \tilde{\mathcal{Q}}_2$

- Consider $\vec{P}_0(v) = v\vec{P}_{\text{PR}} + (1-v)\vec{P}_{\mathbb{I}}$ on the isotropic line.
- Perturb $\vec{P}_0(v)$ to obtain $\vec{P} = (1-\epsilon)\vec{P}_0(v) + \epsilon\vec{P}_{\text{R}}, \quad \epsilon = 10^{-2}$.
- Set $v = 0.72$ (cf., $v \leq 1/\sqrt{2} \approx 0.7071$ for $\vec{P}_0 \in \tilde{\mathcal{Q}}_2$).

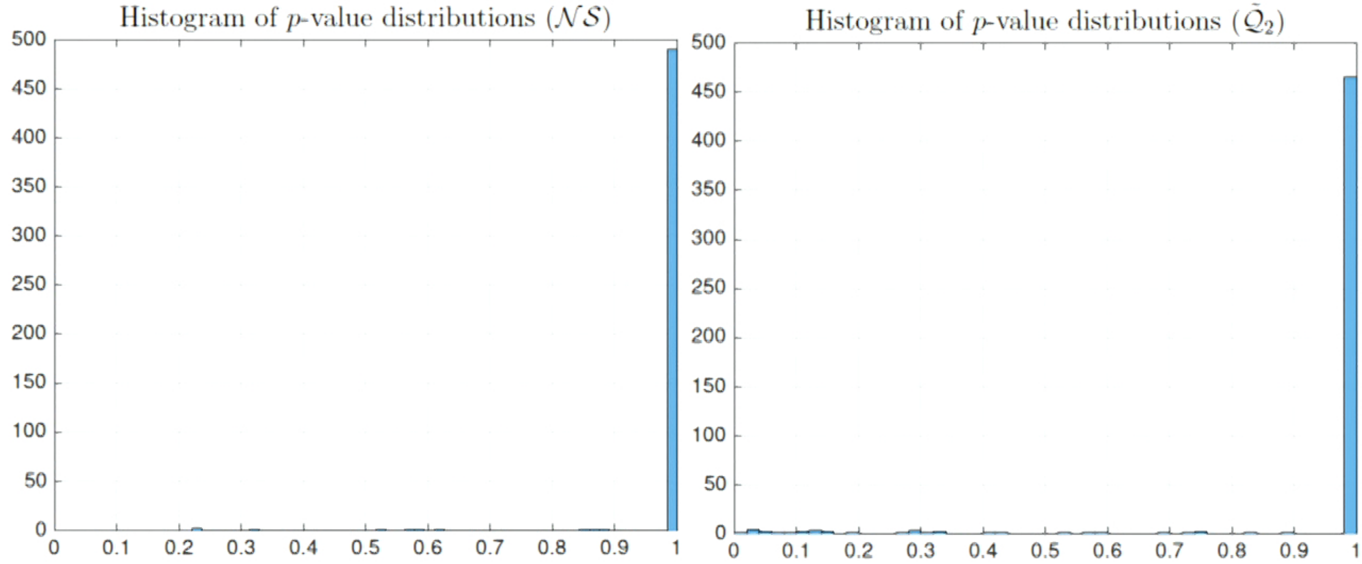




Hypothesis testing for a source perturbed around the Tsirelson point

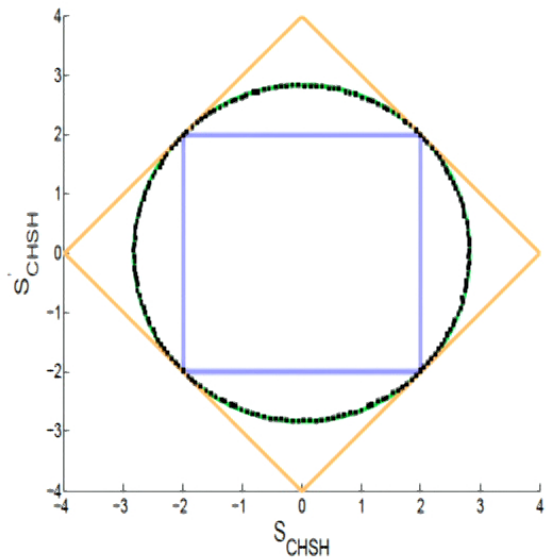
Example 4: a non-i.i.d. source $\vec{P} \in \mathcal{NS}$ with $\vec{P}_0 \in \tilde{\mathcal{Q}}_2$

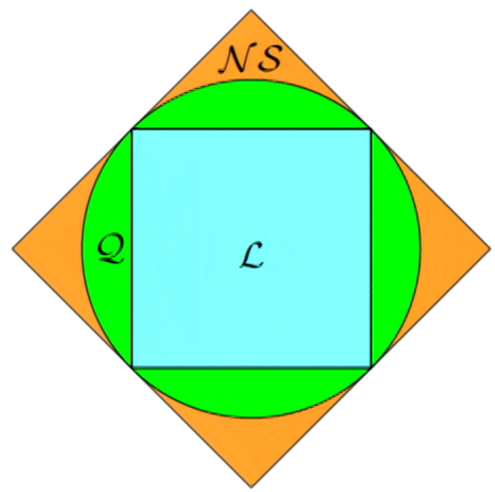
- Consider $\vec{P}_0(v) = v\vec{P}_{\text{PR}} + (1-v)\vec{P}_{\mathbb{I}}$ on the isotropic line.
- Perturb $\vec{P}_0(v)$ to obtain $\vec{P} = (1-\epsilon)\vec{P}_0(v) + \epsilon\vec{P}_{\text{R}}, \quad \epsilon = 10^{-2}$.
- Set $v = 1/\sqrt{2}$ (i.e., $\vec{P}_0 \in \tilde{\mathcal{Q}}_2$).



What's all these about?

Scanning the boundary of the quantum set



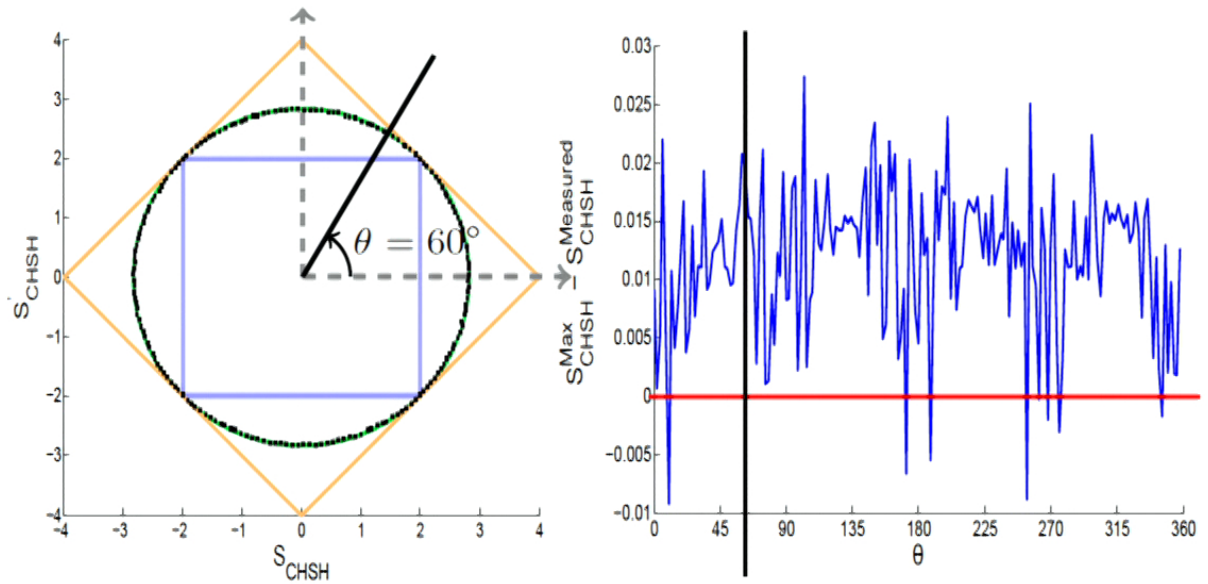


- 180 measured data points.[11]
- Agree exceptionally well with quantum predictions

[11] Christensen, YCL, Brunner, Gisin, Kwiat, PRX (2015).

What's all these about?

Scanning the boundary of the quantum set - an anomaly?



- 180 measured data points (labeled by $\theta \in \{\alpha_i\}_i$).[11]
- Agree exceptionally well with quantum predictions?

[11] Christensen, YCL, Brunner, Gisin, Kwiat, PRX (2015).

Testing the hypothesis of $\mathcal{NS}$ and $\tilde{\mathcal{Q}}_2$ – Part I

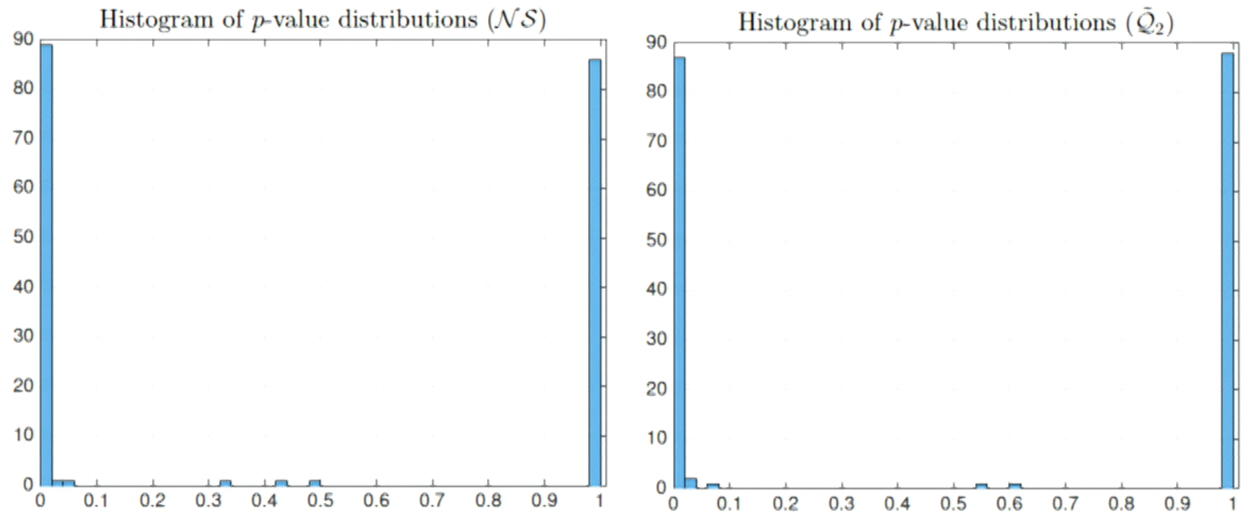
Results from $\theta = \alpha_{i-1}$ → Relative freq. $\vec{f}$ → Optimized Bell-like ineq. → Results from $\theta = \alpha_i$ → Upper bound on the p-value for $\mathcal{NS}$ / $\tilde{\mathcal{Q}}_k$

- Among the 8 θ, 3 of them give non-trivial p-value bound, the smallest of which are,
  - Prob(Results for $\theta|\mathcal{NS}) \leq 1.68 \times 10^{-6}$
  - Prob(Results for $\theta|\tilde{\mathcal{Q}}_2) \leq 1.22 \times 10^{-8}$
- Among all the 180 θ, 94 (92) give non-trivial p-value bound for the hypothesis of $\mathcal{NS}$ ($\mathcal{Q}$), the smallest of which are,
  - Prob(Results for $\theta|\mathcal{NS}) \leq 2.73 \times 10^{-55}$
  - Prob(Results for $\theta|\tilde{\mathcal{Q}}_2) \leq 3.75 \times 10^{-55}$

What's all these about?

Testing the hypothesis of $\mathcal{NS}$ and $\tilde{\mathcal{Q}}_2$ – Part 2

- This histograms do not like promising either ...



- ...
- The non-randomization of measurement settings invalidates the entire analysis based on the PBR method...

Summary

Possible take home messages ...

- There exists a general framework for (device-independent) hypothesis testings based on convex sets of correlations.
- Minimization of the KL divergence over convex sets admitting a LP/ SDP characterization is possible.[a]

[a] Lin, Rosset, Zhang, Bancal, YCL, PRA (2018).

On a more skeptical side:

- "Unexpected" features of experimental results can be easily overlooked without proper statistical analysis.
- Apparent violation of the nonsignaling constraints may be useful for identifying systematic errors in the setup! [12]

[12] See also Smania *et al.*, arXiv:1801.05739.