Title: Almost quantum correlations violate the no-restriction hypothesis

Date: Jul 31, 2018 10:00 AM

URL: http://pirsa.org/18080029

Abstract: To identify which principles characterise quantum correlations, it is essential to understand in which sense this set of correlations differs from that of almost quantum correlations. We solve this problem by invoking the so-called no-restriction hypothesis, an explicit and natural axiom in many reconstructions of quantum theory stating that the set of possible measurements is the dual of the set of states. We prove that, contrary to quantum correlations, no generalised probabilistic theory satisfying the no-restriction hypothesis is able to reproduce the set of almost quantum correlations. Therefore, any theory whose correlations are exactly, or very close to, the almost quantum correlations necessarily requires a rule limiting the possible measurements. Our results suggest that the no-restriction hypothesis may play a fundamental role in singling out the set of quantum correlations among other non-signalling ones.

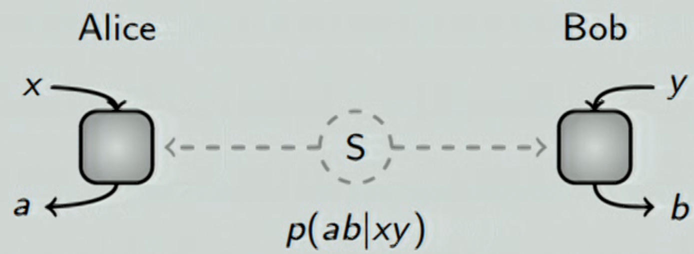
Almost quantum correlations violate the no-restriction hypothesis

Ana Belén Sainz

Yelena Guryanova, Antonio Acín, Miguel Navascués

PRL 120, 200402 (2018)

Bell scenarios



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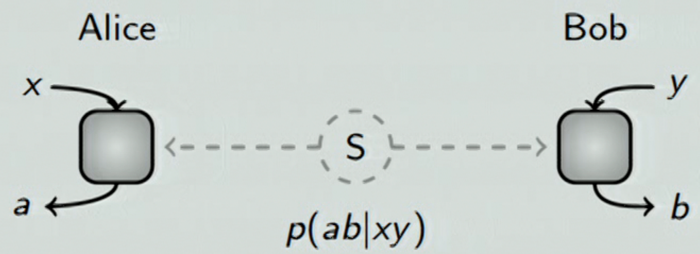

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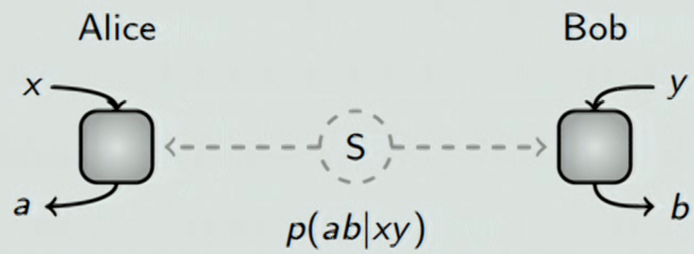

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- Classical (Local)

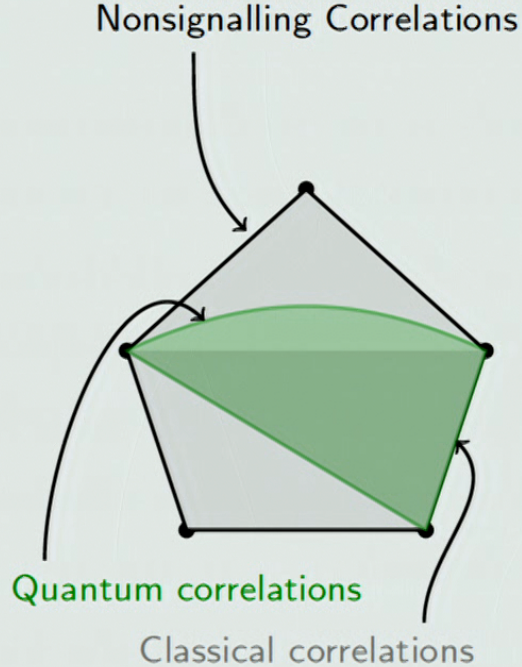
  "Classical common cause" – "local operations and shared randomness"

- Quantum

  $p(ab|xy) = \mathrm{tr}\left\{\Pi_{a|x} \otimes \Pi_{b|y}\, \rho\right\}$

Bounds on correlations

Possibilities and limitations of quantum resources



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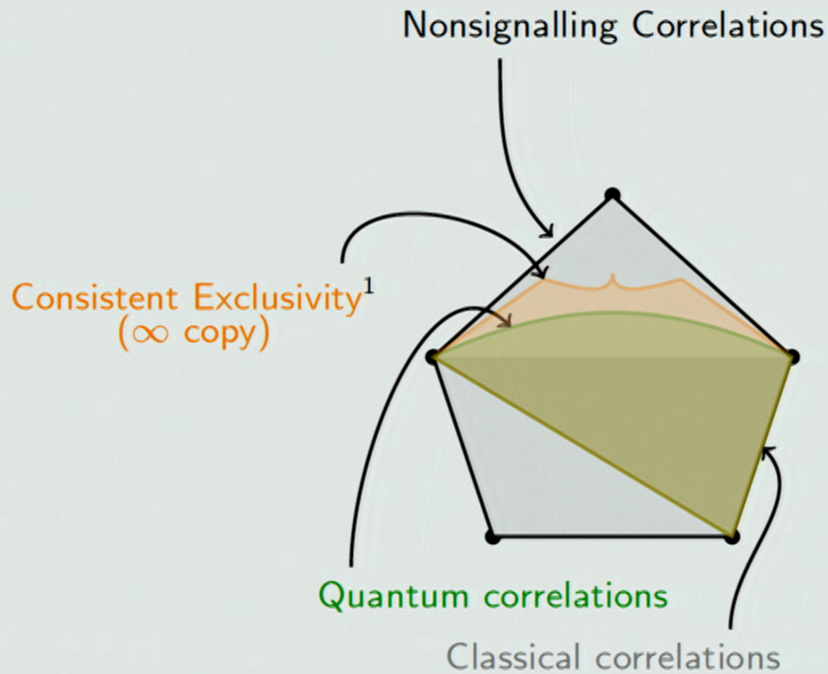
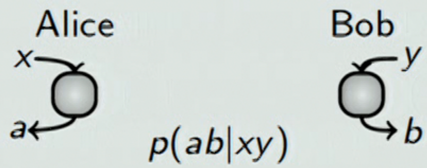
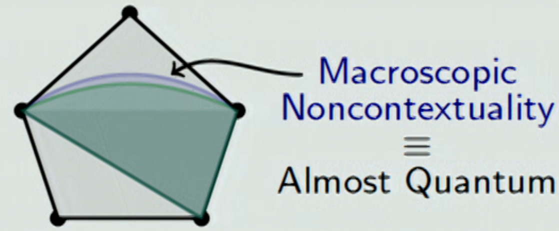


[1] CMP 334, 533-628 (2015)

Almost quantum correlations





Almost quantum correlations: $\mathcal{Q}^{1+\mathrm{AB}}$ [3] , $\mathcal{Q}_1$ [4] , $\tilde{\mathcal{Q}}$ [5]

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[5] Nat Comm 6, 6288 (2015)

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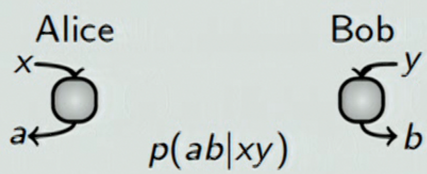

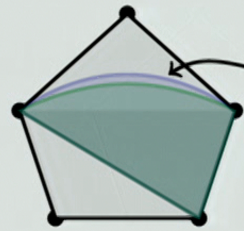

Almost quantum correlations: $\mathcal{Q}^{1+\mathrm{AB}}$[3] , $\mathcal{Q}_1$[4] , $\tilde{\mathcal{Q}}$[5]

$p(ab|xy) \in \tilde{\mathcal{Q}}$ iff

$\exists \mathcal{H}$, $|\Psi\rangle$, a PVM $\{\Pi_{a|x}\}_a$ for each x, a PVM $\{\Pi_{b|y}\}_b$ for each y,

st

$$\Pi_{a|x}\Pi_{b|y}|\Psi\rangle = \Pi_{b|y}\Pi_{a|x}|\Psi\rangle$$

$$p(ab|xy) = \langle\Psi|\Pi_{a|x}\Pi_{b|y}|\Psi\rangle$$

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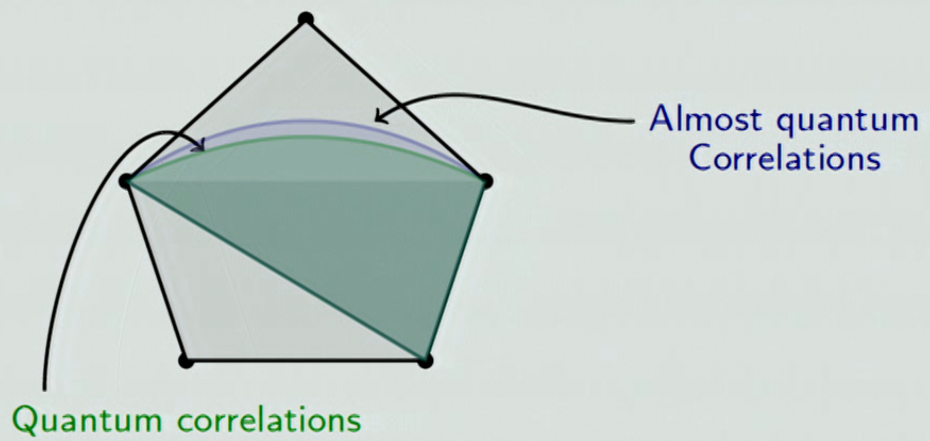
Quantum vs Almost Quantum



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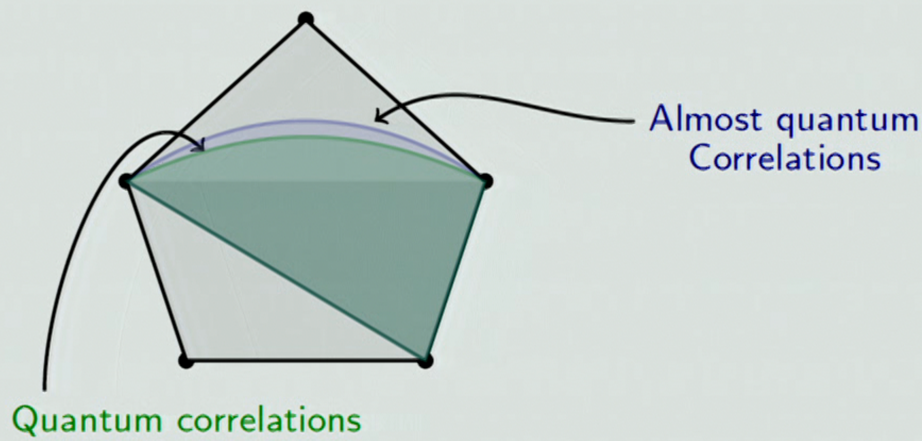

New idea: Combine with 'device dependent' principles

↓

Principles about the theory that predicts the correlations

Generalised Probabilistic Theories

- System types, composition rules $\mathcal{H}_d, \otimes$
- States: $\Psi \in S \quad \rightarrow \quad$ vector of outcome probabilities ρ
- Effects: $e \in \mathcal{E}$, $e : S \rightarrow [0,1]$, linear $0 \leq M \leq \mathbb{I}$

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No Restriction Hypothesis: $\mathcal{E} = S^*$

Normalised Bell Functionals

Bell inequality:

$$-2\sqrt{2} \underset{\tilde{Q}}{\leq} -2 \underset{C}{\leq} \sum_{a,b,x,y} (-1)^{(a\oplus b=xy)} p(ab|xy) \underset{C}{\leq} 2 \underset{\tilde{Q}}{\leq} 2\sqrt{2}$$

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Normalised Bell Functional: $W(a,b,x,y)$

Family of joint effects

$$p(ab|xy) = \mathbf{e}_{a|x} \otimes \mathbf{f}_{b|y}[\Psi_{AB}], \qquad \Psi_{AB} \in S_{AB}, \quad \mathbf{e}_{a|x} \in \mathcal{E}_A, \quad \mathbf{f}_{b|y} \in \mathcal{E}_B$$

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$$\overset{\text{No Restriction Hypothesis}}{\Rightarrow} \quad \mathbf{W} \in \mathcal{E}_{AB} \quad \forall\, \mathbf{e}_{a|x} \in \mathcal{E}_A\,, \quad \mathbf{f}_{b|y} \in \mathcal{E}_B$$

Family of joint effects

A Normalised Bell Functional W induces a map from

a set of measurements $\{\{\mathbf{e}_{a|x}\}_a\}_x$ for Alice and

a set of measurements $\{\{\mathbf{f}_{b|y}\}_b\}_y$ for Bob

into

a joint linear functional $\mathbf{W} \in S^*_{AB}$

which the No Restriction Hypothesis renders as valid effect in $\mathcal{E}_{AB}$

The setup

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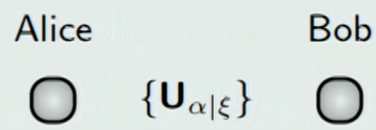

For each ξ, $\{\mathbf{U}_{\alpha|\xi}\}_\alpha$ is a measurement

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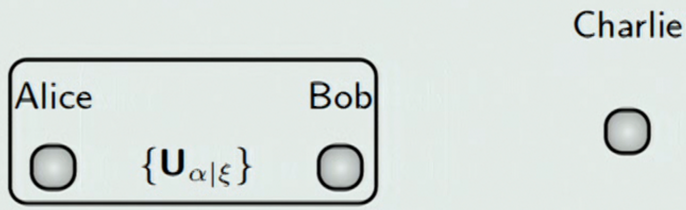

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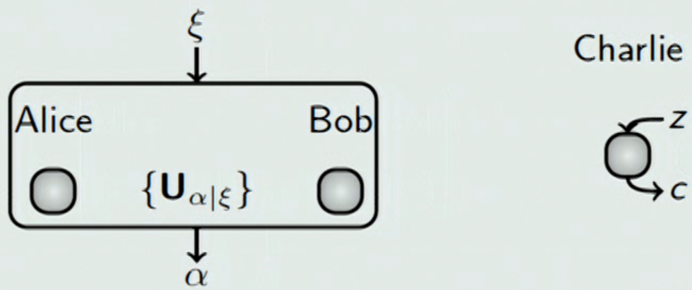

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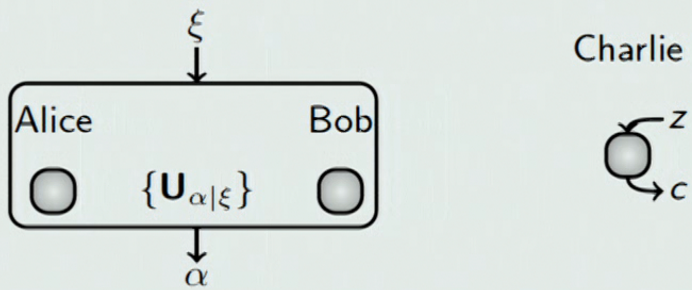

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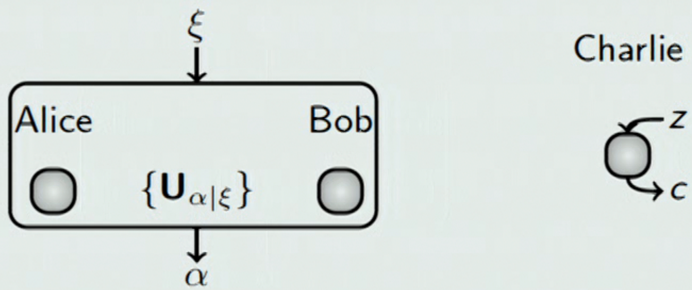

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$$p(\alpha c|\xi z) = \mathbf{U}_{\alpha|\xi} \otimes \mathbf{g}_{c|z}\,[\Psi_{ABC}] = \sum_{a,b,x,y} U_{\alpha,\xi}(a,b,x,y)\mathbf{e}_{a|x} \otimes \mathbf{f}_{b|y} \otimes \mathbf{g}_{c|z}\,[\Psi_{ABC}]$$

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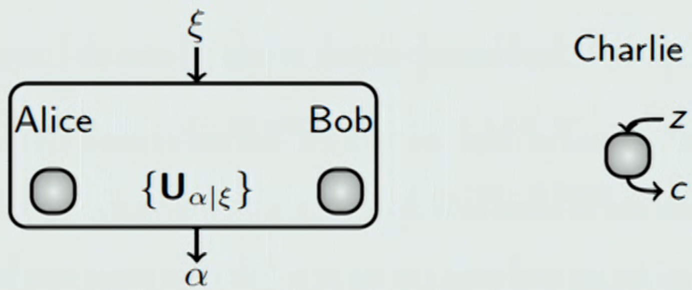

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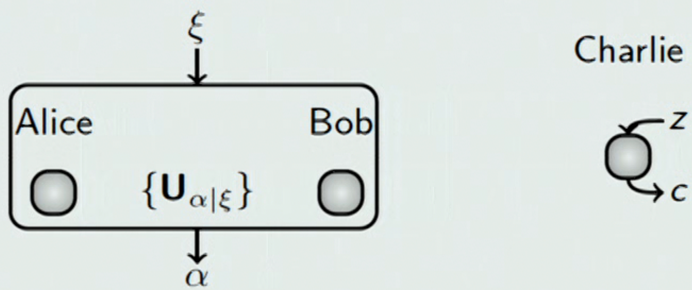
$$\Rightarrow \quad p(\alpha c|\xi z) = \sum_{a,b,x,y} U_{\alpha,\xi}(a,b,x,y)\, p(abc|xyz)$$

The contradiction



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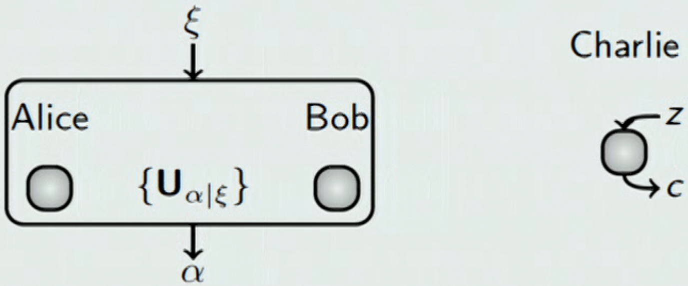

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$p(\alpha c|\xi z) \notin \tilde{\mathcal{Q}}$

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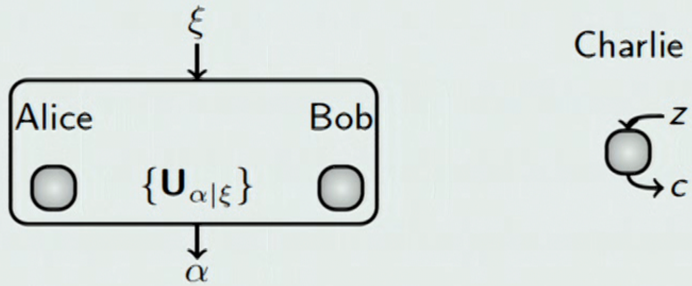

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Conclusions and open problems

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- First fundamental difference between quantum and almost quantum
- "Device-dependent" principles to characterise correlations

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  $\longrightarrow$ single out quantum predictions

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Thanks!

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Q-turn:
changing paradigms
in quantum science
26th - 30th November 2018 | Florianópolis, Brazil
qturnworkshop.wixsite.com/2018
IQC Institute for Quantum Computing
FQXi
IQI Vienna
Freie Universität Berlin
Microsoft
ETH zürich

Public lecture

Prof Anne Broadbent, University of Ottawa, Canada

Invited speakers (scientific program)

Prof Barbara Amaral, Federal University of São João del-Rei, Minas Gerais, Brazil (resource theories)

Dr Miriam Backens, University of Oxford, UK (complexity theory)

Prof Dominic Horsman, Université Grenoble Alpes, France (quantum causality)

Dr Gláucia Murta, QuTech, Delft, Netherlands (quantum cryptography)

Dr Nelly Ng, Freie Universität Berlin, Germany (quantum thermodynamics)

Invited speakers and panel discussion (awareness program)

Prof Anne Broadbent, University of Ottawa, Canada (diversity, women in science)

Prof Ariel Bendersky, University of Buenos Aires, Argentina (workers' rights)

Prof Havi Carel, University of Bristol, UK (unconscious bias, prejudice, solutions)

Prof Nara Guisoni, Universidad de La Plata, Argentina (diversity, harassment, solutions)

Prof Dominic Horsman, Université Grenoble Alpes, France (representation and inclusion)

Prof Renato Pedrosa, Unicamp, Campinas, Brazil (diversity and social inclusion)

Steering committee

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Juani Bermejo-Vega (Freie Universität Berlin, Berlin, Germany)

Yelena Guryanova (IQOQI, Vienna, Austria)

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Local organising committee

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Susane Calegari (Universidade Federal de Santa Catarina, Florianópolis, Brazil)

Deadlines

Talk/poster submission: **31st August**
Application for travel support: **8th September**

Super early-bird registration: **31st August**
Early-bird registration: **30th September**
Registration: **31st October**

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