

Title: PSI 2018/2019 - Classical Physics - Lecture 8

Date: Aug 23, 2018 09:00 AM

URL: <http://pirsa.org/18080016>

Abstract:

YESTERDAY. MAXWELL'S THEORY IN RELATIVISTIC NOTATION

- $\boxed{F_{[\mu\nu,\lambda]} = 0 = F_{[\mu\nu,\lambda]}c}$

- $\boxed{\partial_\mu F^{\mu\nu} = -4\pi J^\nu}$

HERE.

$$F^{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ & 0 & B_z & -B_y \\ & & 0 & B_x \\ & & & 0 \end{pmatrix}$$

$$F_{0i} = E_i, \quad F_{ij} = \epsilon^{ijk} B_k$$

$$J^\nu = (\rho, \vec{j})$$

YESTERDAY. MAXWELL'S THEORY IN RELATIVISTIC NOTATION

- $\boxed{F_{[\mu\nu,\lambda]} = 0 = F_{[\mu\lambda,\nu]} = 0}$ 4 EQS

HERE.

- $\boxed{\partial_\mu F^{\mu\nu} = -4\pi J^\nu}$ 4 EQS

$F_{\mu\nu} \rightarrow F$

$\boxed{dF=0}, \boxed{F=dA}$

$$F^{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ & 0 & B_z & -B_y \\ & & 0 & B_x \\ & & & 0 \end{pmatrix}$$

$F^{0i} = E_i, F^{ij} = \epsilon^{ijk} B_k$

$J^\nu = (\rho, \vec{j})$

RY IN RELATIVISTIC NOTATION

HERE.

4 EQS

$$F^{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ & 0 & B_z & -B_y \\ & & 0 & B_x \\ & & & 0 \end{pmatrix}$$

$$F^{0i} = E_i, \quad F^{ij} = \epsilon^{ijk} B_k$$

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LEVI-CIVITA TENSOR

EQS.

$$F = dA$$

$\partial_\mu A_\nu$

RY IN RELATIVISTIC NOTATION

HERE.

4 EQS

$$F^{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ & 0 & B_z & -B_y \\ & & 0 & B_x \\ & & & 0 \end{pmatrix}$$

$$F^{0i} = E_i, \quad F^{ij} = \epsilon^{ijk} B_k$$

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LEVI-CIVITA TENSOR

$$\epsilon^{\mu\nu\alpha\beta} = \begin{cases} +1 & (\mu\nu\alpha\beta) = (0,1,2,3) \\ -1 & \\ 0 & \end{cases}$$

RT IN RELATIVISTIC NOTATION

HERE.

4 EQS

$$F^{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ & 0 & B_z & -B_y \\ & & 0 & B_x \\ & & & 0 \end{pmatrix}$$

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LEVI-CIVITA TENSOR

$$\epsilon^{\mu\nu\alpha\beta} = \begin{cases} +1 & (\mu\nu\alpha\beta) = (0,1,2,3) \\ -1 & \\ 0 & \end{cases}$$

$$(*F)^{\mu\nu} = \frac{1}{2!} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$$

HODGE DUAL OF $F_{\alpha\beta}$

$$\frac{1}{2} \epsilon^{\mu\nu\alpha\lambda} \partial_\nu F_{\alpha\lambda} = 0$$

$$\partial_\nu (*F)^{\mu\nu} = 0$$

$$\partial_\mu (*F)^{\mu\nu} = 0$$

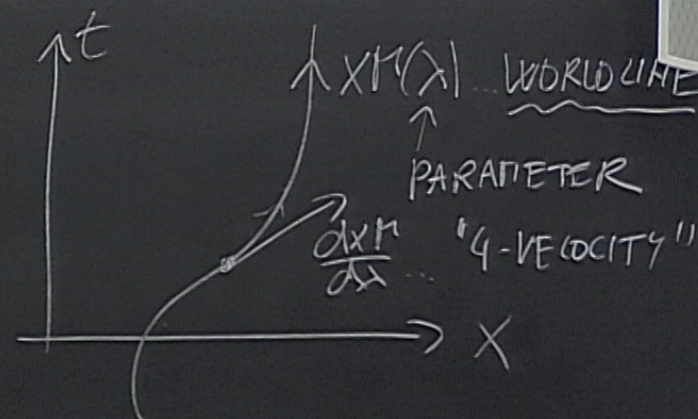
NO MAGNETIC
CURRENTS.

$$\partial_\mu *F^{\mu\nu} = -4\pi J^\nu$$

$$F^{\mu\nu} = -4\pi J^{\nu}_\mu$$

• LORENTZ FORCE LAW
RELATIVISTIC PARTICLE

$$\text{"VELOCITY"}^2 = \eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}$$

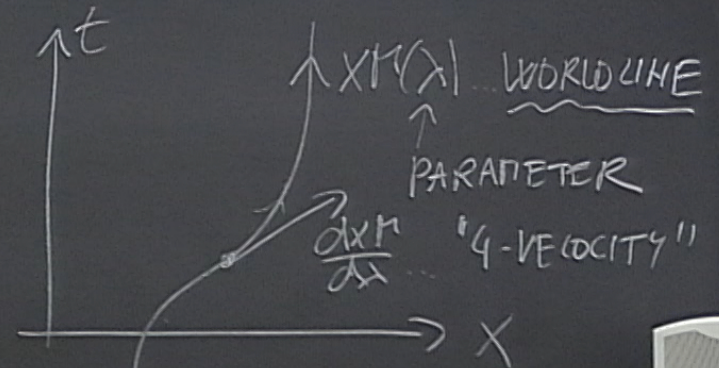


$$\mu^{\nu} = -4\pi J^{\nu}_M$$

• LORENTZ FORCE LAW
RELATIVISTIC PARTICLE

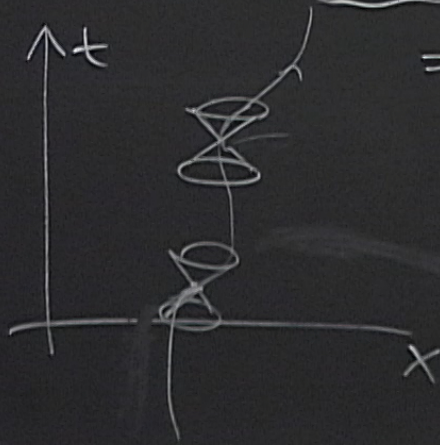
$$\text{"VELOCITY"}^2 = \eta_{\mu\nu} \frac{dx^{\mu}}{d\lambda} \frac{dx^{\nu}}{d\lambda}$$

$< 0 \dots$ TIMELIKE
 $> 0 \dots$ SPACELIKE
 $= 0 \dots$ NULL



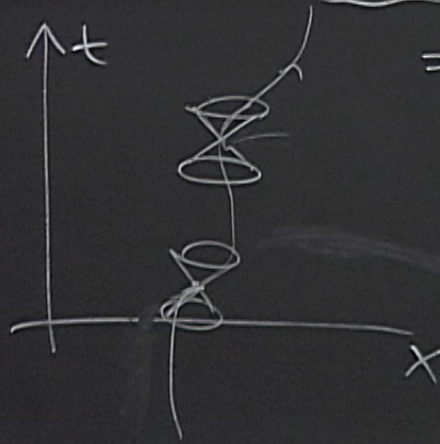
CURRENTS.

• TO PRESERVE CAUSALITY: MASSIVE PARTICLES HAVE
ALWAYS TIMELIKE TRAJECTORIES



=> ALWAYS STAY IN
THE "LIGHT CONE"

• TO PRESERVE CAUSALITY: MASSIVE PARTICLES HAVE
ALWAYS TIMELIKE TRAJECTORIES



=> ALWAYS STAY IN
THE "LIGHTCONE"

METRIC ELEMENT

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu$$

$\gamma > 0 \dots$ SPACELIKE
 $\gamma = 0 \dots$ NULL

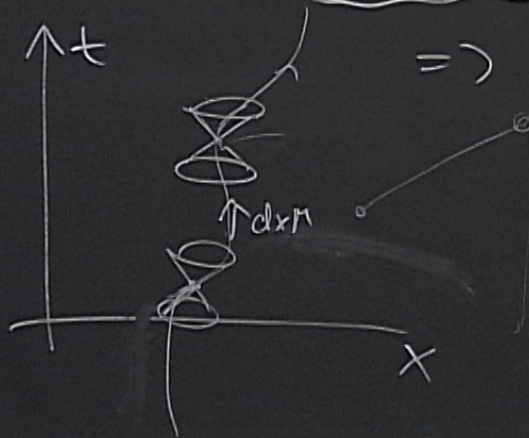
PARTICLES HAVE $\boxed{ds^2 < 0}$

PROPER TIME

$$\boxed{d\tau = \sqrt{-\eta_{\mu\nu} dx^\mu dx^\nu}}$$

"TIME OF OBSERVER SITTING ON
THE PARTICLE"

- TO PRESERVE CAUSALITY: MASSIVE PARTICLES HAVE ALWAYS TIMELIKE TRAJECTORIES



=> ALWAYS STAY IN THE "LIGHTCONE"

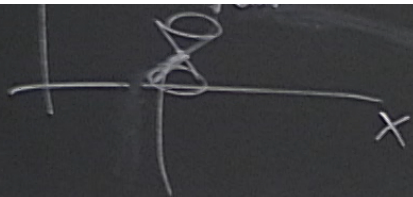
METRIC ELEMENT

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

PARTICLES

PROPER

TI



$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu$$

THE PARTICLE "

4-VELOCITY. $u^\mu = \frac{dx^\mu}{d\tau} = \left(\frac{dt}{d\tau}, \frac{dx^i}{d\tau} \right) = \left(\frac{dt}{d\tau}, \underbrace{\frac{dx^i}{dt} \frac{dt}{d\tau}}_{v^i} \right) = \frac{dt}{d\tau} (1, v^i)$

4-VELOCITY. $\left[u^\mu = \frac{dx^\mu}{d\tau} \right] = \left(\frac{dt}{d\tau}, \frac{dx^i}{d\tau} \right) = \left(\frac{dt}{d\tau}, \underbrace{\frac{dx^i}{dt}}_{v^i} \frac{dt}{d\tau} \right) = \frac{dt}{d\tau} (1, v^i)$

• $u^\mu u_\mu = \eta_{\mu\nu} u^\mu u^\nu = \eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}$

4-VELOCITY. $u^\mu = \frac{dx^\mu}{d\tau} = \left(\frac{dt}{d\tau}, \frac{dx^i}{d\tau} \right) = \left(\frac{dt}{d\tau}, \underbrace{\frac{dx^i}{dt}}_{v^i} \frac{dt}{d\tau} \right) = \frac{dt}{d\tau} (1, v^i)$

• $u^\mu u_\mu = \eta_{\mu\nu} u^\mu u^\nu = \eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -1$

4-VELOCITY. $\boxed{u^\mu = \frac{dx^\mu}{d\tau}} = \left(\frac{dt}{d\tau}, \frac{dx^i}{d\tau} \right) = \left(\frac{dt}{d\tau}, \underbrace{\frac{dx^i}{dt} \frac{dt}{d\tau}}_{v^i} \right) = \frac{dt}{d\tau} (1, v^i)$

• $u^\mu u_\mu = \eta_{\mu\nu} u^\mu u^\nu = \eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -1$
 $= \left(\frac{dt}{d\tau} \right)^2 (v^2 - 1)$

$\frac{dt}{d\tau} = \frac{1}{\sqrt{1-v^2}} = \gamma$

$$U^\mu = \left(\frac{dt}{d\tau}, \underbrace{\frac{dx^i}{dt} \frac{dt}{d\tau}}_{v^i} \right) = \frac{dt}{d\tau} (1, v^i)$$

$$\frac{dt}{d\tau} \frac{dx^\mu}{d\tau} = -1$$

4-MOMENTUM : $p^\mu = m_0 U^\mu$

- MASS OF PARTICLE .. DEPENDS ON VELOCITY.

$$m(v) = \gamma m_0$$

↑
REST MASS
(SCALAR)

$$\left(\frac{dx^i}{d\tau} \right) = \left(\frac{dt}{d\tau}, \underbrace{\frac{dx^i}{dt} \frac{dt}{d\tau}}_{v^i} \right) = \frac{dt}{d\tau} (1, v^i)$$

$$\frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -1$$

4-MOMENTUM

$$p^\mu = m_0 u^\mu$$

$\Rightarrow$ NEWTON'S LAW

$$\frac{dp^\mu}{d\tau} = F^\mu$$

• MASS OF PARTICLE .. DEPENDS ON VELOCITY.

$$m(v) = \gamma m_0$$

↑
REST MASS
(SCALAR)

4-FORCE

$$F = q(E + v \times B)$$

... LINEAR

IN

q
 E, B
 v

$$F_L^M = q F^{\mu\nu} u_\nu = \frac{dp^\mu}{d\tau}$$

$$F = e(E + v \times B)$$

... LINEAR

$$IH \leftarrow \begin{matrix} \mathcal{Q} \\ E, B \\ \mathcal{V} \end{matrix}$$

$$\begin{pmatrix} E_y & E_z \\ B_z & -B_y \\ 0 & B_x \\ & 0 \end{pmatrix}$$

$$\mathcal{E}^{ijk} B_{le}$$

$$F_L^M = -e F^{M\nu} u_\nu = \frac{dp^M}{d\tau}$$

$$-e F^{\mu\nu} u_\mu$$

$$\underline{\text{PROBLEM 1.}} \quad |u^\mu = \frac{dx^\mu}{d\tau}| = \left(\frac{dx^0}{d\tau}, \frac{dx^i}{d\tau} \right) = \left(\frac{dx^0}{d\tau}, \frac{dt}{d\tau} \frac{dx^i}{dt} \right)$$

a) RELATIVISTIC FIELD THEORIES

FIELD $\varphi(t, \vec{x}) = \varphi(x^\mu)$

$$S[\varphi] = \int \underbrace{d^4x}_{dt d^3x} \mathcal{L}(\varphi, \partial_\mu \varphi, x)$$

$$\begin{aligned}
 \delta S = 0 &= \int_{\Omega} d^4x \delta \mathcal{L} = \int_{\Omega} d^4x \left(\frac{\partial \mathcal{L}}{\partial \psi} \delta \psi + \frac{\partial \mathcal{L}}{\partial (\partial_{\mu} \psi)} \underbrace{\delta (\partial_{\mu} \psi)}_{\partial_{\mu} \delta \psi} \right) \\
 \text{BY PARTS} &= \int_{\Omega} \underbrace{\left(\frac{\partial \mathcal{L}}{\partial \psi} - \partial_{\mu} \left(\frac{\partial \mathcal{L}}{\partial (\partial_{\mu} \psi)} \right) \right)}_{=0} d^4x \\
 &\quad - \int_{\partial \Omega} d^3x n_{\mu} \frac{\partial \mathcal{L}}{\partial (\partial_{\mu} \psi)} \delta \psi
 \end{aligned}$$

A hand is visible on the left side of the chalkboard, pointing towards the first term of the second equation. To the right of the equations, there is a hand-drawn diagram of a region Ω in the shape of an irregular blob. The boundary of this region is labeled $\partial \Omega$.

ELD THEORIES

$$|\vec{x}\rangle = \varphi(x^\mu)$$

$$\int d^4x \mathcal{L}(\varphi, \partial_\mu \varphi, x)$$

(E-L)

$$\boxed{\frac{\partial \mathcal{L}}{\partial \varphi} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial (\partial_\mu \varphi)} \right) = 0}$$

$$\delta S = 0 = \int_\Omega d^4x \delta \mathcal{L} = \int_\Omega d^4x \left(\frac{\partial \mathcal{L}}{\partial \varphi} \delta \varphi + \frac{\partial \mathcal{L}}{\partial (\partial_\mu \varphi)} \underbrace{\delta (\partial_\mu \varphi)}_{\partial_\mu \delta \varphi} \right)$$

$$\stackrel{\text{BY PARTS}}{=} \int_\Omega \underbrace{\left(\frac{\partial \mathcal{L}}{\partial \varphi} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial (\partial_\mu \varphi)} \right) \right)}_{\theta} d^4x$$

$$- \int d^3x n_\mu \frac{\partial \mathcal{L}}{\partial (\partial_\mu \varphi)}$$

$$\delta S = 0 = \int_{\Omega} d^4x \delta \mathcal{L} = \int_{\Omega} d^4x \left(\frac{\partial \mathcal{L}}{\partial \psi} \delta \psi + \frac{\partial \mathcal{L}}{\partial (\partial_{\mu} \psi)} \underbrace{\delta \partial_{\mu} \psi}_{\partial_{\mu} \delta \psi} \right)$$

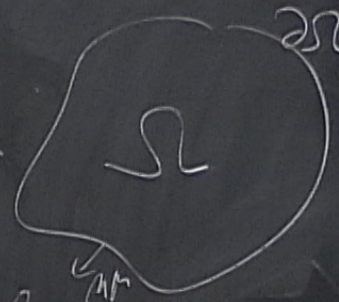
BY PARTS

$$= \int_{\Omega} \underbrace{\left(\frac{\partial \mathcal{L}}{\partial \psi} - \partial_{\mu} \left(\frac{\partial \mathcal{L}}{\partial (\partial_{\mu} \psi)} \right) \right)}_{\emptyset} d^4x$$

$$\boxed{\left(\frac{\partial \mathcal{L}}{\partial (\partial_{\mu} \psi)} \right) = 0}$$

$$- \int_{\partial \Omega} d^3x n_{\mu} \frac{\partial \mathcal{L}}{\partial (\partial_{\mu} \psi)} \delta \psi$$

$$\text{IF } \boxed{\delta \psi / \partial \Omega = 0}$$



RELATIVISTIC FIELD THEORIES

FIELD $\varphi(t, \vec{x}) = \varphi(x^\mu)$

$$S[\varphi] = \int \underbrace{d^4x}_{dt d^3x} \mathcal{L}(\varphi, \partial_\mu \varphi)$$

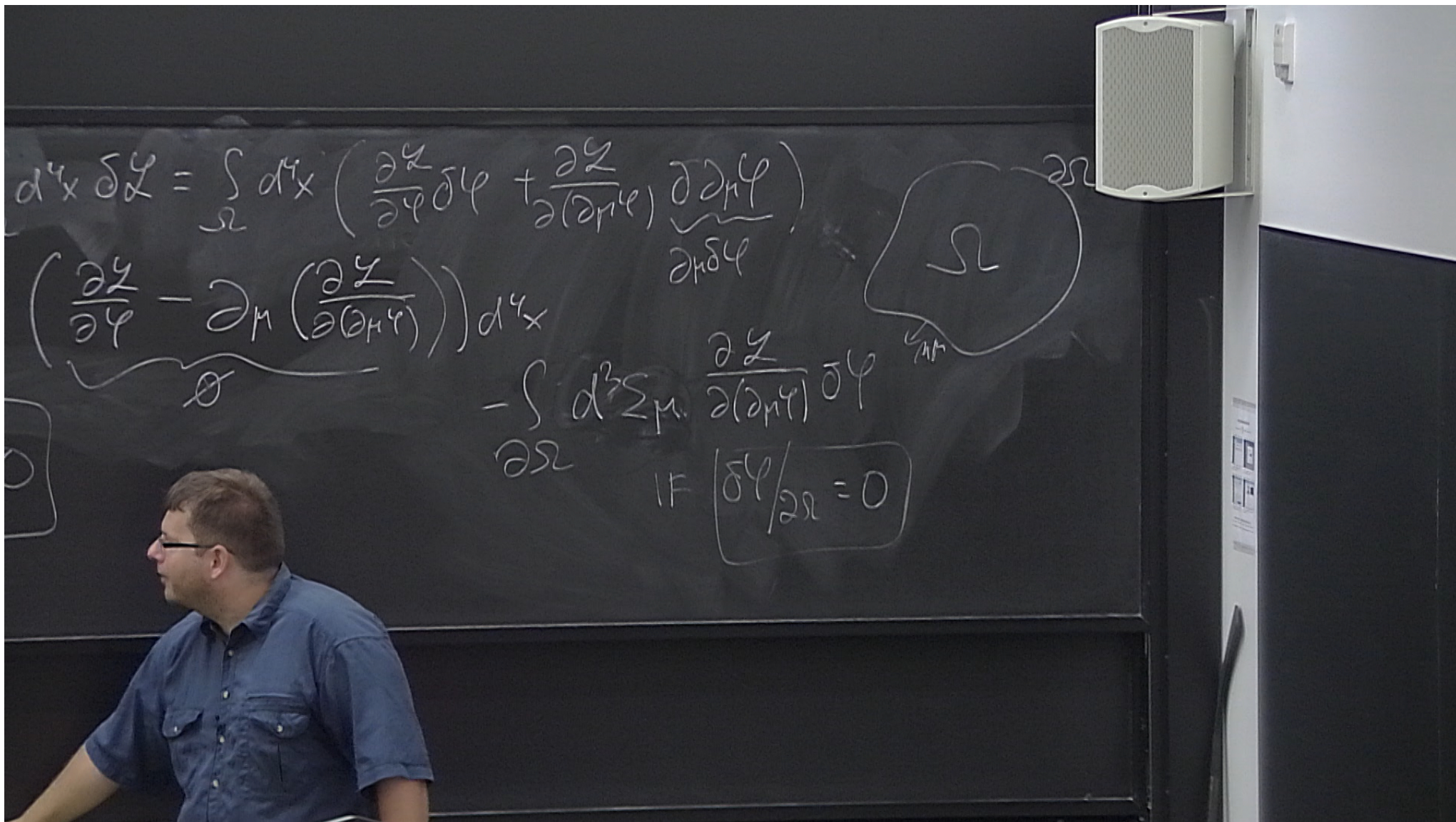
(E-L)

$$\boxed{\frac{\partial \mathcal{L}}{\partial \varphi} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial (\partial_\mu \varphi)} \right) = 0}$$

$$\frac{\partial L}{\partial q} - \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}} \right) = 0$$

$$\delta S = 0 = \int_\Omega d^4x \delta \mathcal{L} = \int_\Omega d^4x \left(\frac{\partial \mathcal{L}}{\partial \varphi} \delta \varphi + \frac{\partial \mathcal{L}}{\partial (\partial_\mu \varphi)} \delta (\partial_\mu \varphi) \right)$$

$$\stackrel{\text{BY PARTS}}{=} \int_\Omega \underbrace{\left(\frac{\partial \mathcal{L}}{\partial \varphi} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial (\partial_\mu \varphi)} \right) \right)}_{=0} \delta \varphi$$



NOETHER'S THEOREM (FIELD VERSION)

LET $\tilde{\delta}x^\mu, \tilde{\delta}\varphi$ GLOBAL CONT SYMMETRY,
THAT IS OFF-SHELL $\tilde{\delta}S=0 \Rightarrow$

$$\boxed{J^\mu = \underbrace{T^\mu_\nu}_{\text{CANONICAL ENERGY-MOM. TENSOR}} \tilde{\delta}x^\nu + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \tilde{\delta}\varphi}$$

CONSERVED ON-SHELL

$$\boxed{\partial_\mu J^\mu = 0} \quad \hookrightarrow \quad \frac{\partial \mathcal{L}}{\partial t} + \nabla \cdot \mathbf{j} = 0$$

$$\boxed{T^\mu_\nu = \gamma^\mu_\nu \mathcal{L} - \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \partial_\nu \varphi}$$

EX: INVARIANCE W.R.T SPACETIME TRANSLATIONS:

$$\delta x^\mu = \epsilon^\mu, \quad \delta \varphi = 0$$

$$J^\mu_{(v)} = T^\mu_{\nu} v^\nu$$

4-CURRENTS

$v=0 \dots$ ENERGY
 $v=i \dots$ MOMENTUM,

ERVED ON-SHELL

$$J^\mu = 0$$

EX: INVARIANCE W.R.T SPACETIME TRANSLATIONS:

$$\delta x^\mu = \epsilon^\mu, \quad \delta \varphi = 0$$

$$J^\mu(v) = T^\mu_{\nu} v^\nu$$

4-CURRENTS

$v=0 \dots$ ENERGY
 $v=i \dots$ MOMENTUM,

$$\partial_\mu T^\mu_{\nu} v^\nu = 0$$

CONSERVATION
OF ENERGY-MOMENTUM,

ERVED ON-SHELL

$$J^\mu = 0$$

NEWTON'S LAW

$$\frac{dp^\mu}{\tau} = F^\mu$$

4-FORCE

CANONICAL ENERGY-MOM. TENSOR

$$\partial_\mu J^\mu = 0$$

4-CURRENT

$$T_{\mu\nu} = \eta_{\mu\nu} \mathcal{L} - \frac{\partial \mathcal{L}}{\partial(\partial^\mu \varphi)} \partial_\nu \varphi$$

ACTION FOR MAXWELL

$$S[A_\mu, y^\nu] = -\frac{1}{16\pi} \int d^4x$$

POTENTIAL ELECTRON'S TRAJECTORY

$$\begin{pmatrix} 0 & E_x & E_y & E_z \\ & 0 & B_z & -B_y \\ & & 0 & B_x \\ & & & 0 \end{pmatrix}$$

$$E_i, F^{ij} = \epsilon^{ijk} B_k$$

$$= (\vec{0}, \vec{B})$$

$$F = e(E + \vec{v} \times \vec{B})$$

... LINEAR

$$IH \leftarrow \begin{matrix} \mathcal{Q} \\ E, B \\ \vec{v} \end{matrix}$$

$$\boxed{F_L^M = e F^{M\nu} u_\nu = \frac{dp^M}{d\tau}}$$

$$\cancel{e F^{\nu\mu} u_\mu}$$

$$E^2 - B^2 \sim F_{\mu\nu} F^{\mu\nu}$$

CANONICAL ENERGY-MOM. TENSOR

$$\partial_\mu J^\mu = 0$$

$$T^\mu_{\nu} = \eta^\mu_{\nu} \mathcal{L} - \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \partial_\nu \varphi$$

ACTION FOR MAXWELL

$$S[A_\mu, y^\nu] = -\frac{1}{16\pi} \int d^4x F_{\mu\nu} F^{\mu\nu}$$

POTENTIAL ELECTRON'S TRAJECTORY



$$J^M = T_c^M \underset{\substack{\uparrow \\ \text{CANONICAL ENERGY-MOM TENSOR}}} v + \frac{\partial \mathcal{L}}{\partial (\partial_\mu \psi)} \underset{\substack{\uparrow \\ \text{CONSERVED ON-SHELL}}} \tilde{\partial} \psi$$

CONSERVED ON-SHELL

$$\partial_\mu J^M = 0$$

$$J_c^M(v) = T_c^M v$$

4-CURRENTS

$$\partial_\mu T_c^M v = 0$$

ACTION FOR MAXWELL

$$S[A_\mu, y^\nu] = \underbrace{-\frac{1}{16\pi} \int d^4x F_{\mu\nu} F^{\mu\nu}}_{\substack{\text{MAXWELL FIELD} \\ \text{ON ITS OWN}}} - \underbrace{m_0 \int d\tau \sqrt{-\eta_{\mu\nu} \frac{dy^\mu}{d\tau} \frac{dy^\nu}{d\tau}}}_{\substack{\text{MAXIMIZE PROPER}}} \\ \substack{\uparrow \\ \text{POTENTIAL}} \quad \substack{\uparrow \\ \text{ELECTRON'S} \\ \text{TRAJECTORY}}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$d\tau = \sqrt{1-v^2} d\lambda$$

$\Rightarrow$ NEWTON'S LAW

$$\frac{d\mathbf{p}}{d\tau} = \mathbf{F}$$

4-FORCE

$M + M$ $V=0$ ENERGY

$$-m_0 \int d\lambda \sqrt{-\eta_{\mu\nu} \frac{dy^\mu}{d\lambda} \frac{dy^\nu}{d\lambda}}$$

$$+ e \int \frac{dy^\mu}{d\lambda} A_\mu d\lambda$$

$$-m_0 \int d\tau$$

MAXIMIZE PROPER

$$e \int A^\nu J_\nu$$

INTERACTION TERM

4-MOMENTUM

$$p^\mu = m_0 u^\mu$$

REST MASS
(SCALAR)

FERMION'S LAW

$$\frac{dp^\mu}{d\lambda} = F^\mu$$

4-FORCE

$$-m_0 \int d\lambda \sqrt{-\eta_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}}$$

$$-m_0 \int d\tau$$

MAXIMIZE PROPER

$\delta y^\mu \Rightarrow$ LORENTZ FORCE LAW

$\delta A_\mu \Rightarrow$ MAXWELL EQS.

$$+ e \int \frac{dy^\mu}{d\lambda} A_\mu d\lambda$$

$$e \int A^\nu J_\nu$$

INTERACTION TERM

EX: INVARIANCE W.R.T SPACETIME TRANSLATIONS:

$$T_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

NOETHER'S THEOREM (FIELD VERSION)

LET $\tilde{\delta}x^\mu, \tilde{\delta}\varphi$ GLOBAL CONT SYMMET
 THAT IS OFF-SHELL $\tilde{\delta}S=0 \Rightarrow$

$$\boxed{J^\mu = \underbrace{T_c^\mu}_{\text{CANONICAL ENERGY-MOM TENSOR}}_\nu \tilde{\delta}x^\nu + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \tilde{\delta}\varphi}$$

$$\boxed{T_c^{\mu\nu} = \gamma^{\mu\nu} \mathcal{L} - \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \partial^\nu \varphi}$$

OH

$$= \begin{pmatrix} 0 & E_x & E_y & E_z \\ 0 & B_z & -B_y & 0 \\ 0 & B_x & 0 & 0 \end{pmatrix}$$

$$= \epsilon^{ijk} B_{jk} e_i$$

CANONICAL TENSOR IS NOT SYMMETRIC
& NOT GAUGE INVARIANT.

UPGRADED TENSOR

$$T^{\mu\nu} = \frac{1}{4\pi} \left(F^{\mu\alpha} F^{\nu}_{\alpha} - \frac{1}{4} \eta^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \right)$$

OH

$$= \begin{pmatrix} 0 & E_x & E_y & E_z \\ & 0 & B_z & -B_y \\ & & 0 & B_x \\ & & & 0 \end{pmatrix}$$

$$= E_i, F^{ij} = \epsilon^{ijk} B_k$$

$$v = (\phi, \vec{A})$$

CANONICAL TENSOR IS NOT SYMMETRIC
& NOT GAUGE INVARIANT.

UPGRADED TENSOR

$$T^{\mu\nu} = \frac{1}{4\pi} \left(F^{\mu\alpha} F^{\nu}_{\alpha} - \frac{1}{4} \eta^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \right)$$

$$(\partial^2 - m^2) \phi = 0$$