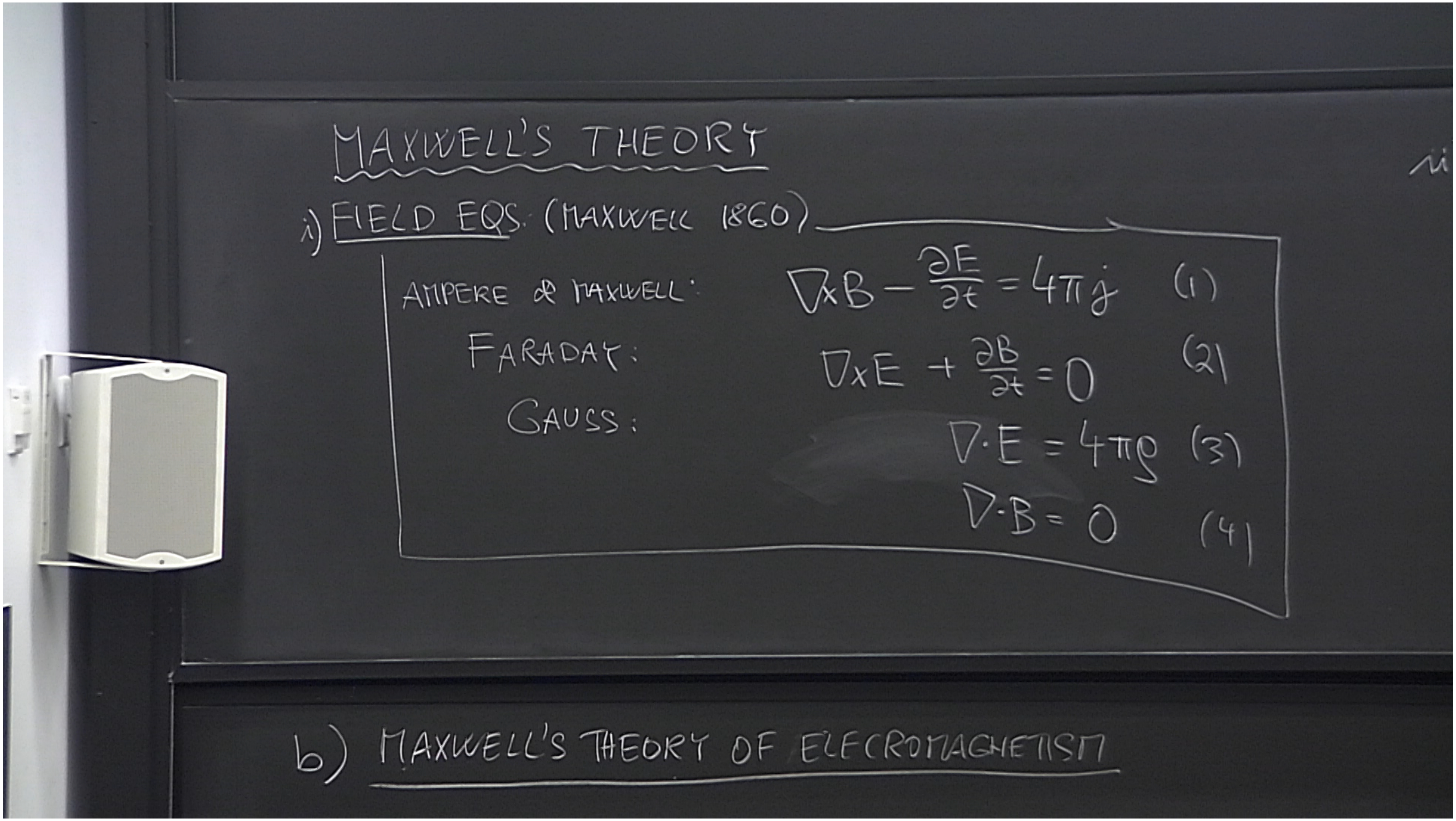
Title: PSI 2018/2019 - Classical Physics - Lecture 6

Date: Aug 21, 2018 09:00 AM

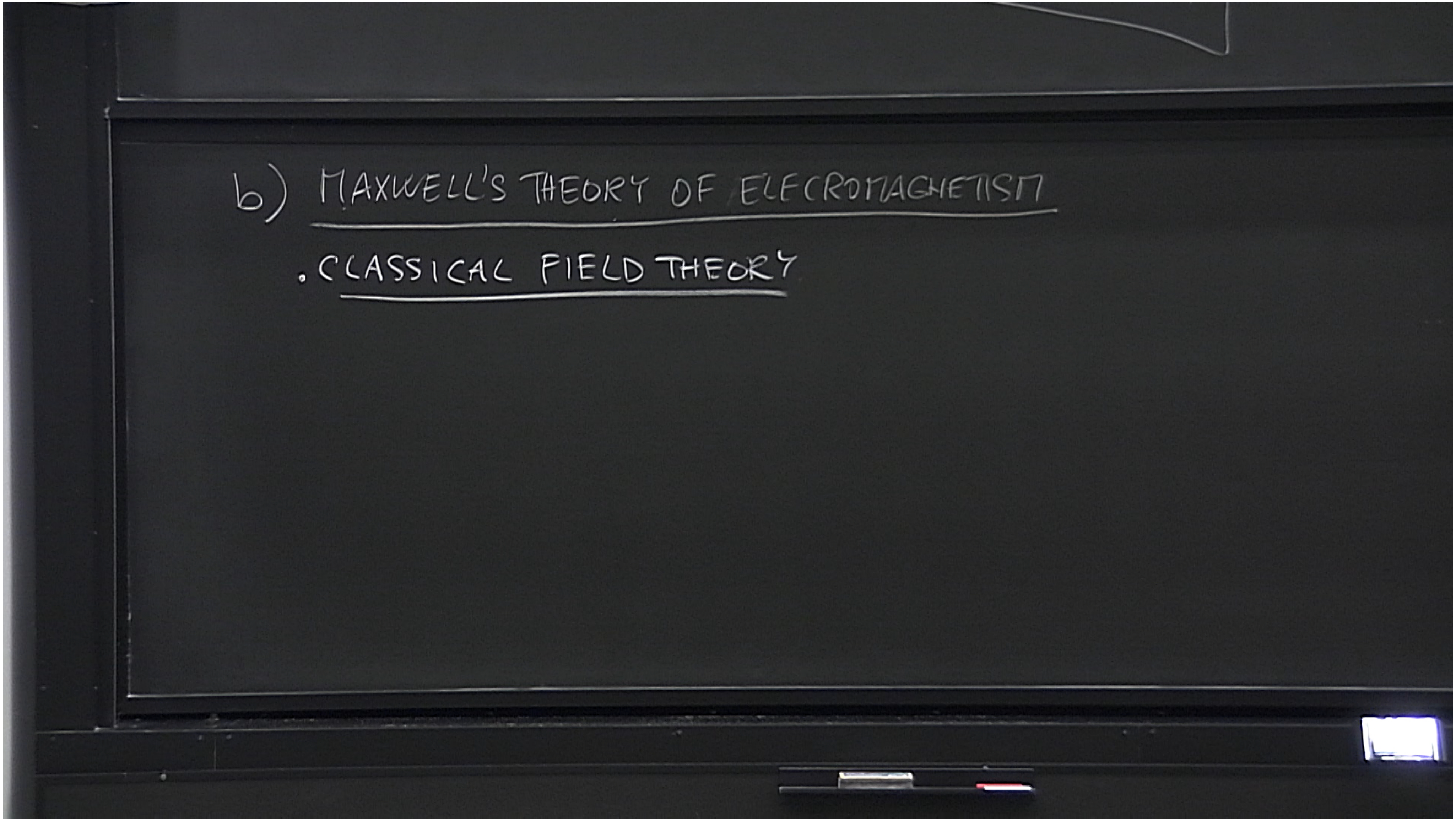
URL: http://pirsa.org/18080014

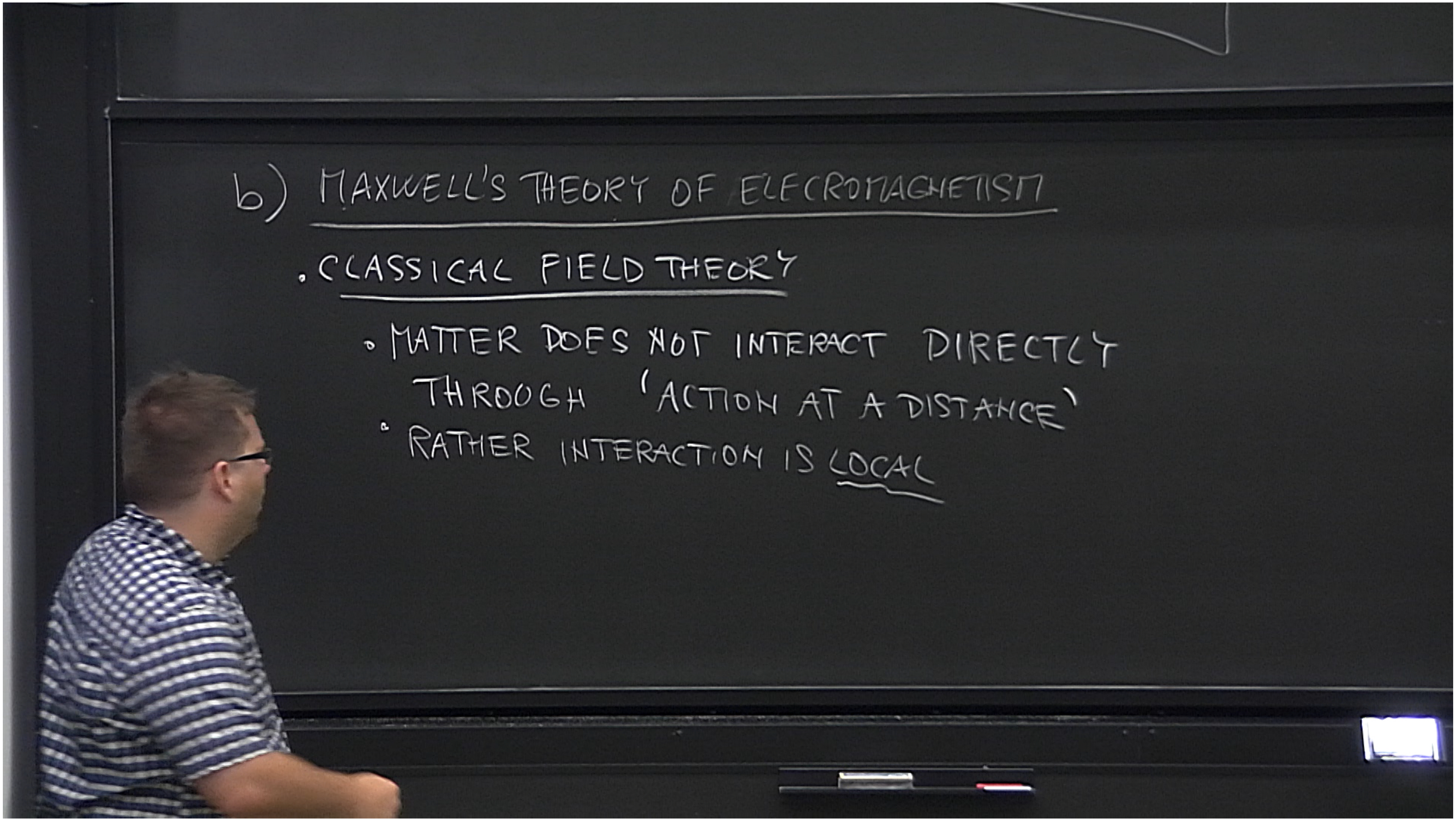
Abstract:

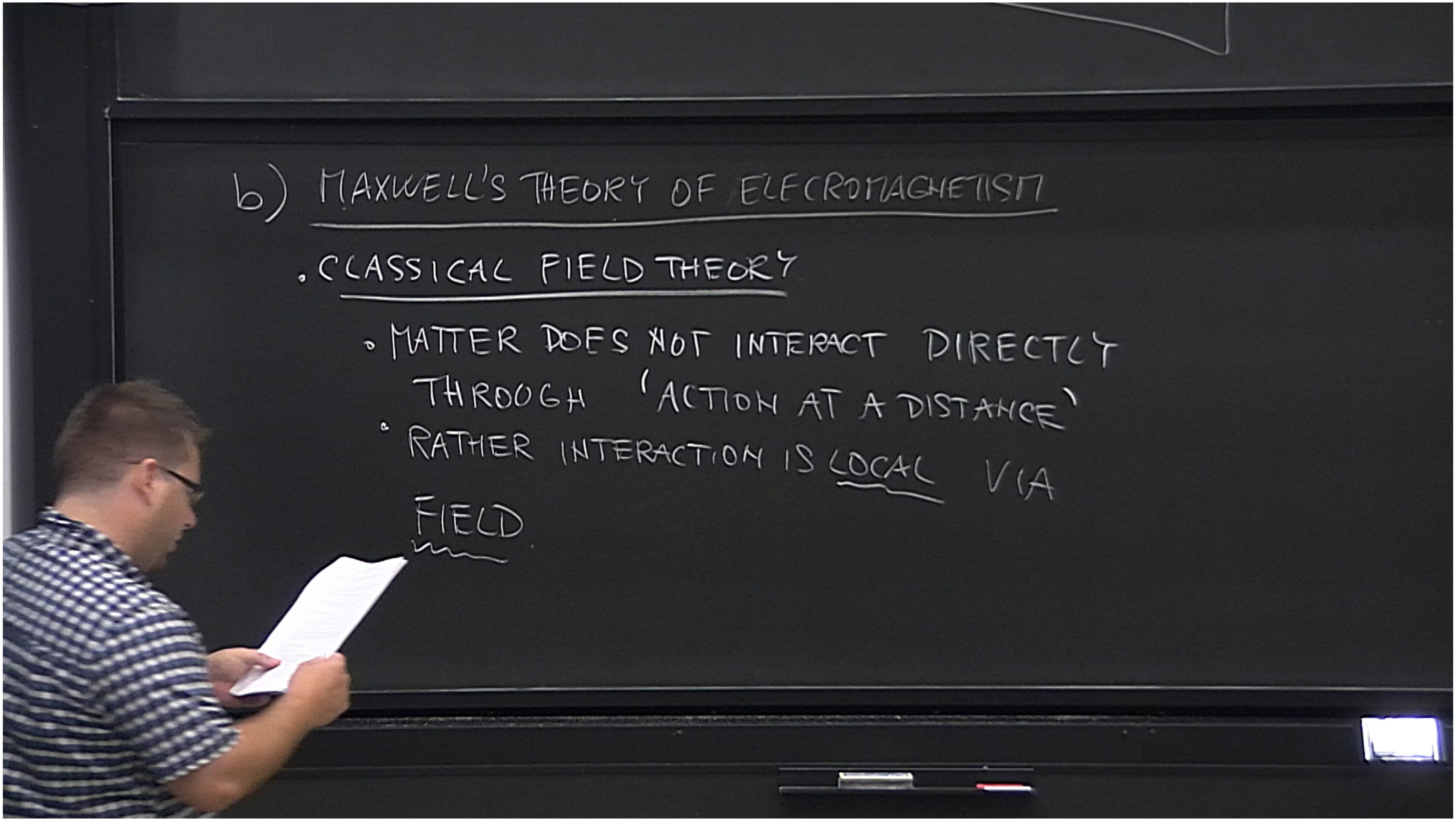

MAXWELL'S THEORY
i) FIELD EQS. (MAXWELL 1860)
AMPERE & MAXWELL: $\nabla \times B - \frac{\partial E}{\partial t} = 4\pi j$ (1)
FARADAY: $\nabla \times E + \frac{\partial B}{\partial t} = 0$ (2)
GAUSS: $\nabla \cdot E = 4\pi \rho$ (3)
$\nabla \cdot B = 0$ (4)
ii
b) MAXWELL'S THEORY OF ELECROMAGNETISM

ii) EQUATIONS OF MOTION FOR MATTER : LORENTZ FORCE LAW

$$\frac{dp}{dt} = F_L = e\left(E + v \times B\right)$$


b) MAXWELL'S THEORY OF ELECROMAGNETISM
. CLASSICAL FIELD THEORY


b) MAXWELL'S THEORY OF ELECROMAGNETISM
• CLASSICAL FIELD THEORY
• MATTER DOES NOT INTERACT DIRECTLY THROUGH 'ACTION AT A DISTANCE'
• RATHER INTERACTION IS LOCAL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• MATTER DOES NOT INTERACT DIRECTLY THROUGH 'ACTION AT A DISTANCE'
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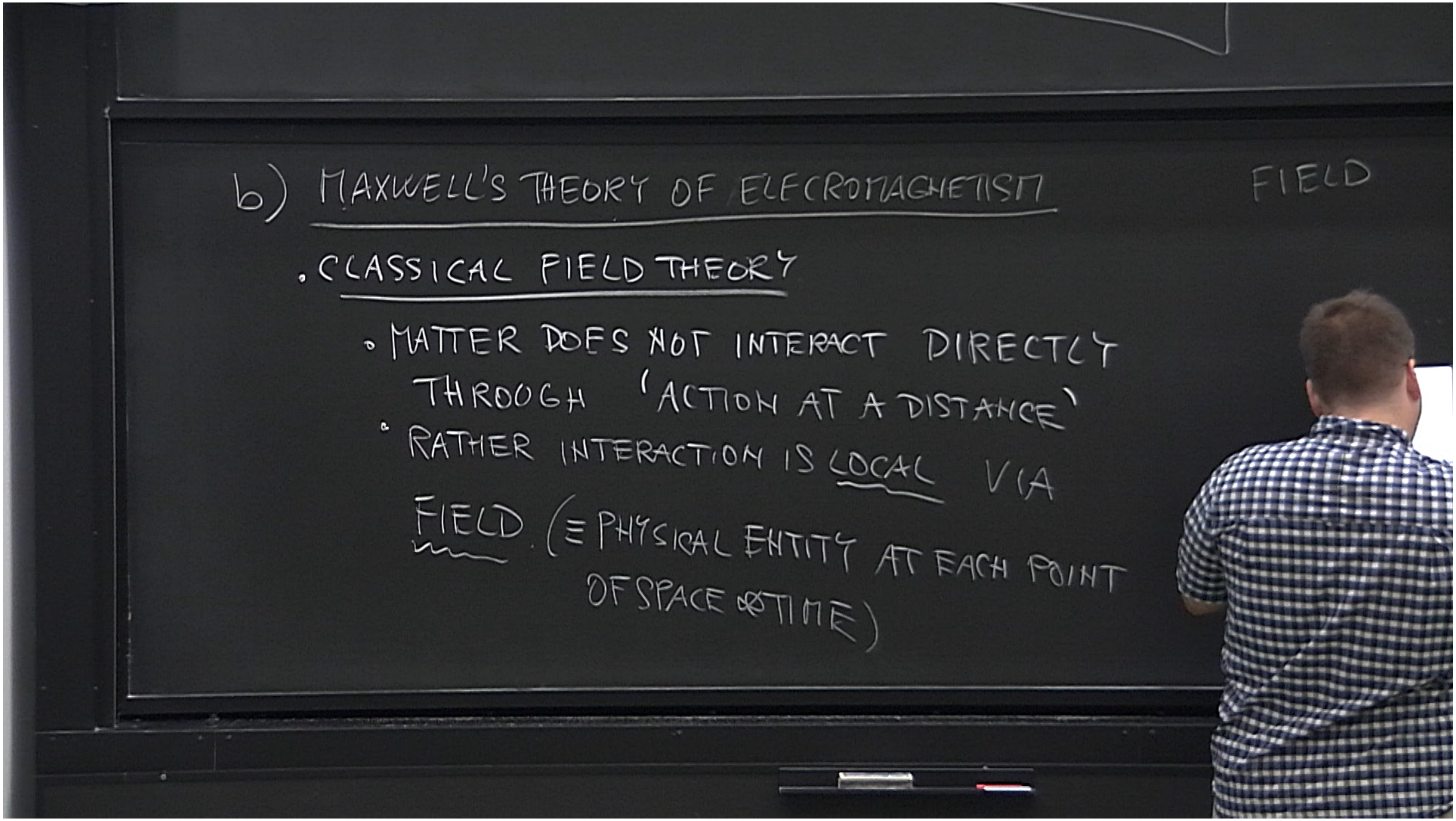
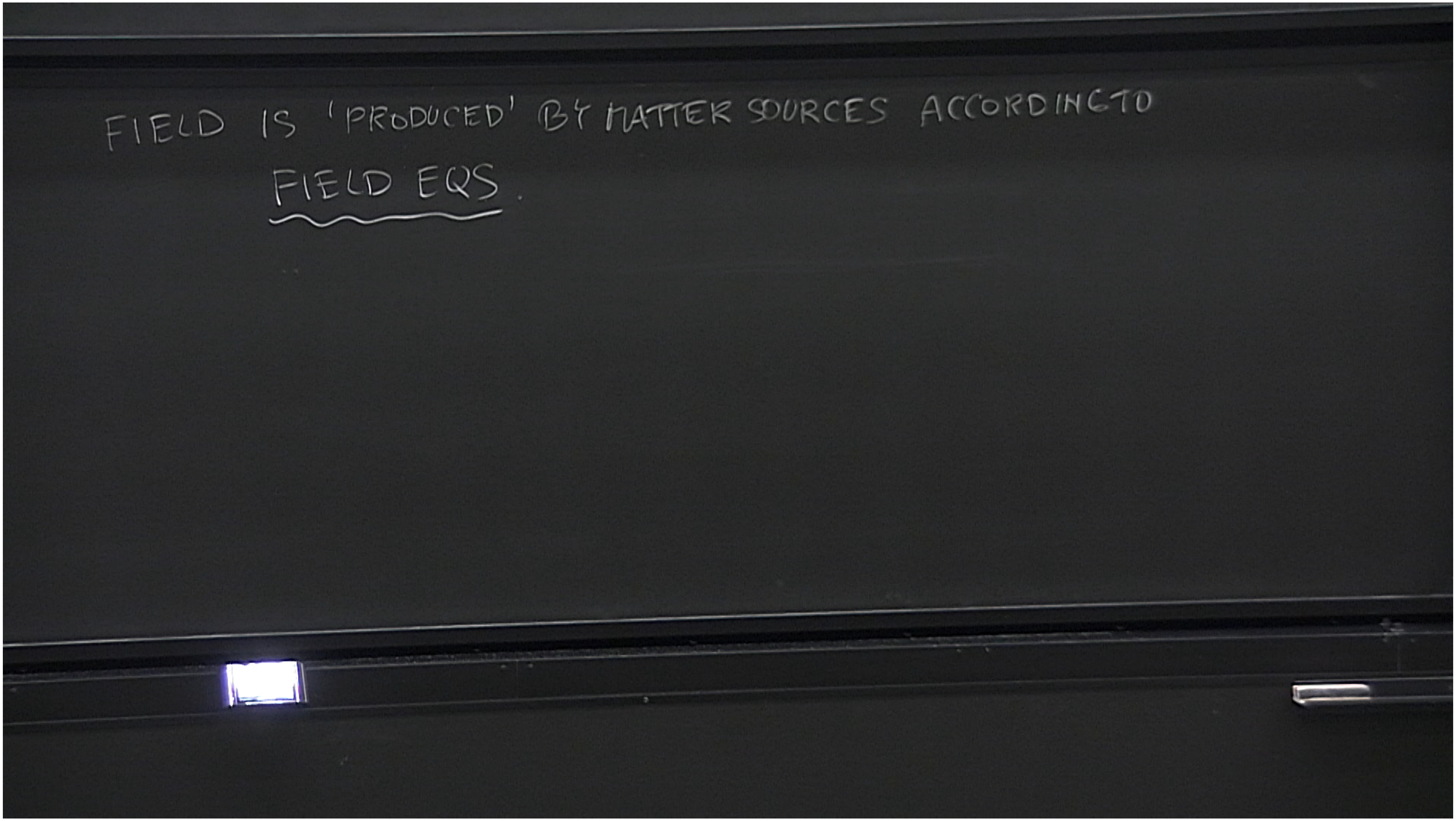
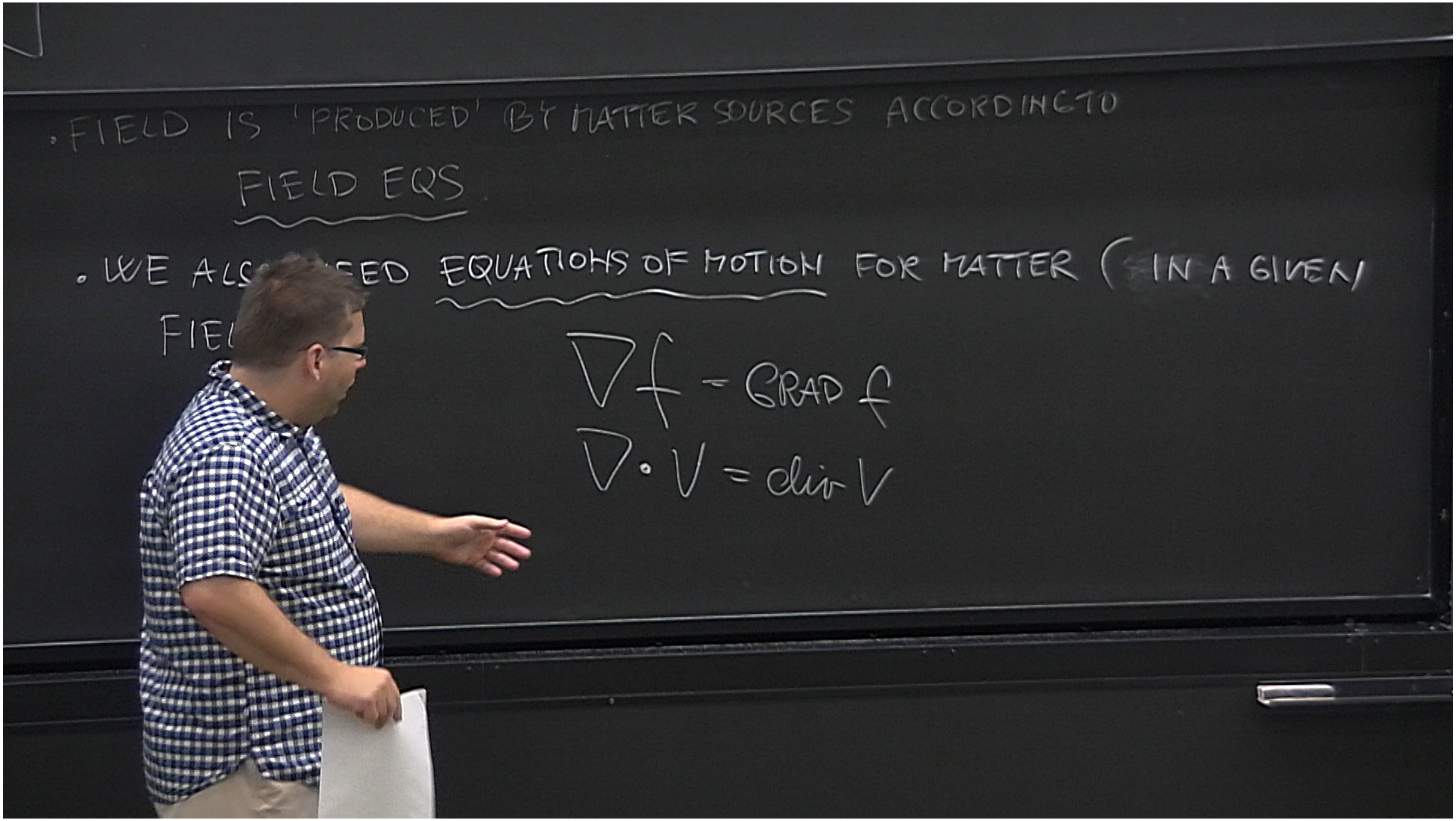

b) MAXWELL'S THEORY OF ELECROMAGNETISM
FIELD
• CLASSICAL FIELD THEORY
• MATTER DOES NOT INTERACT DIRECTLY THROUGH 'ACTION AT A DISTANCE'
• RATHER INTERACTION IS LOCAL VIA FIELD (≡ PHYSICAL ENTITY AT EACH POINT OF SPACE & TIME)


FIELD IS 'PRODUCED' BY MATTER SOURCES ACCORDING TO
FIELD EQS.

- FIELD IS 'PRODUCED' BY MATTER SOURCES ACCORDING TO FIELD EQS.
- WE ALSO NEED EQUATIONS OF MOTION FOR MATTER (IN A GIVEN FIELD)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∇f = GRAD f
∇ · V = div V

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• WE ALSO NEED EQUATIONS OF MOTION FOR MATTER (IN A GIVEN FIELD)

$$\nabla f = \text{GRAD } f$$

$$\nabla \cdot V = \text{div } V$$

$$\nabla \times V = \text{curl } V$$

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- WE ALSO NEED EQUATIONS OF MOTION FOR MATTER (IN A GIVEN FIELD)

$$\nabla f = \mathrm{GRAD}\, f \qquad \Delta = \nabla \cdot \nabla = \nabla^2$$

$$\nabla \cdot V = \mathrm{div}\, V$$

$$\nabla \times V = \mathrm{curl}\, V$$

• FIELD IS 'PRODUCED' BY MATTER SOURCES ACCORDING TO

FIELD EQS

• WE ALSO NEED EQUATIONS OF MOTION FOR MATTER (IN A GIVEN FIELD)

$$\nabla\cdot\nabla\times V = 0$$

$$\nabla f = \text{GRAD}\, f$$

$$\nabla\cdot V = \text{div}\, V$$

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$$\nabla \cdot \nabla \times V = 0$$

$$\nabla \times \nabla f = 0$$

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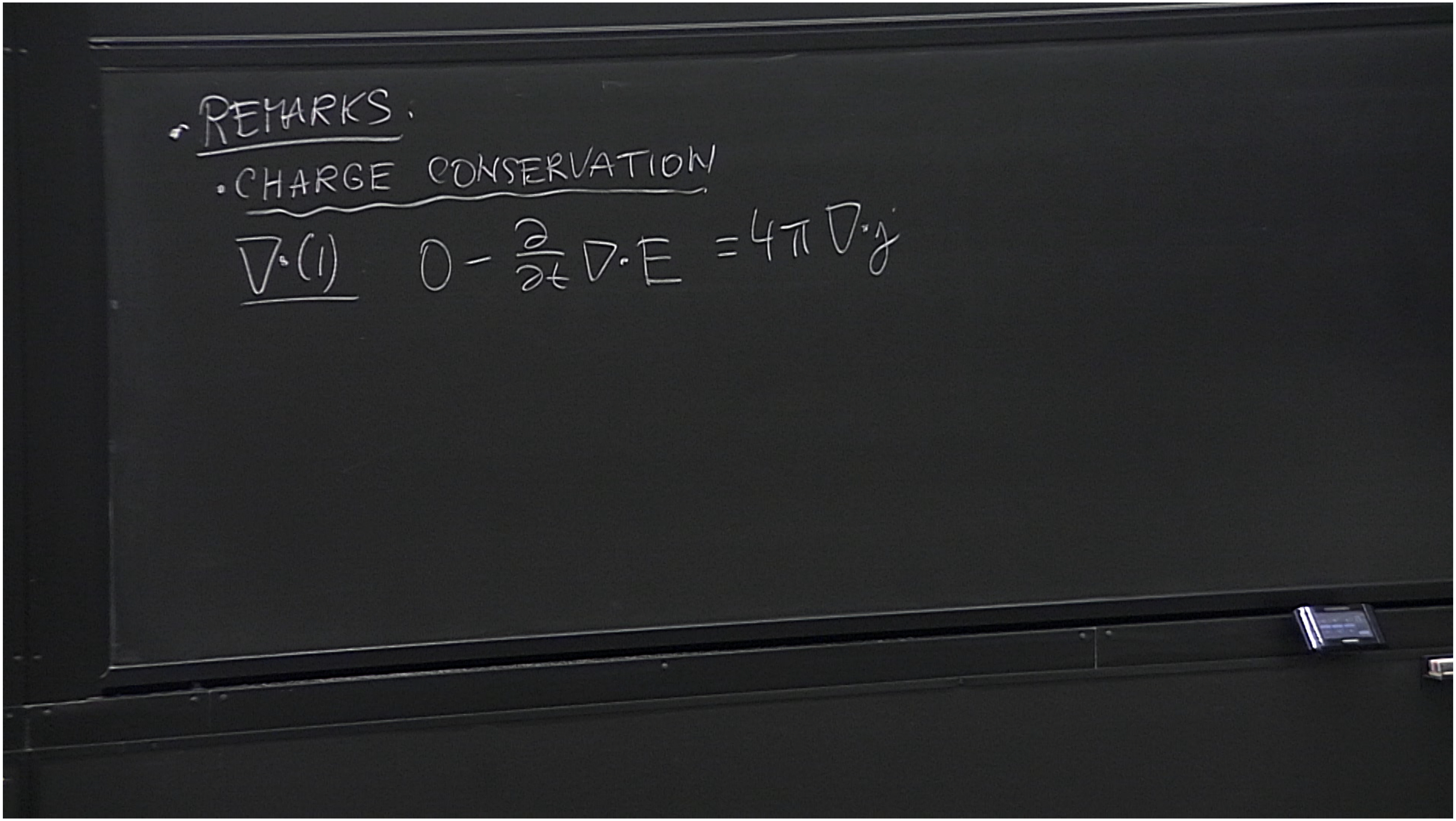
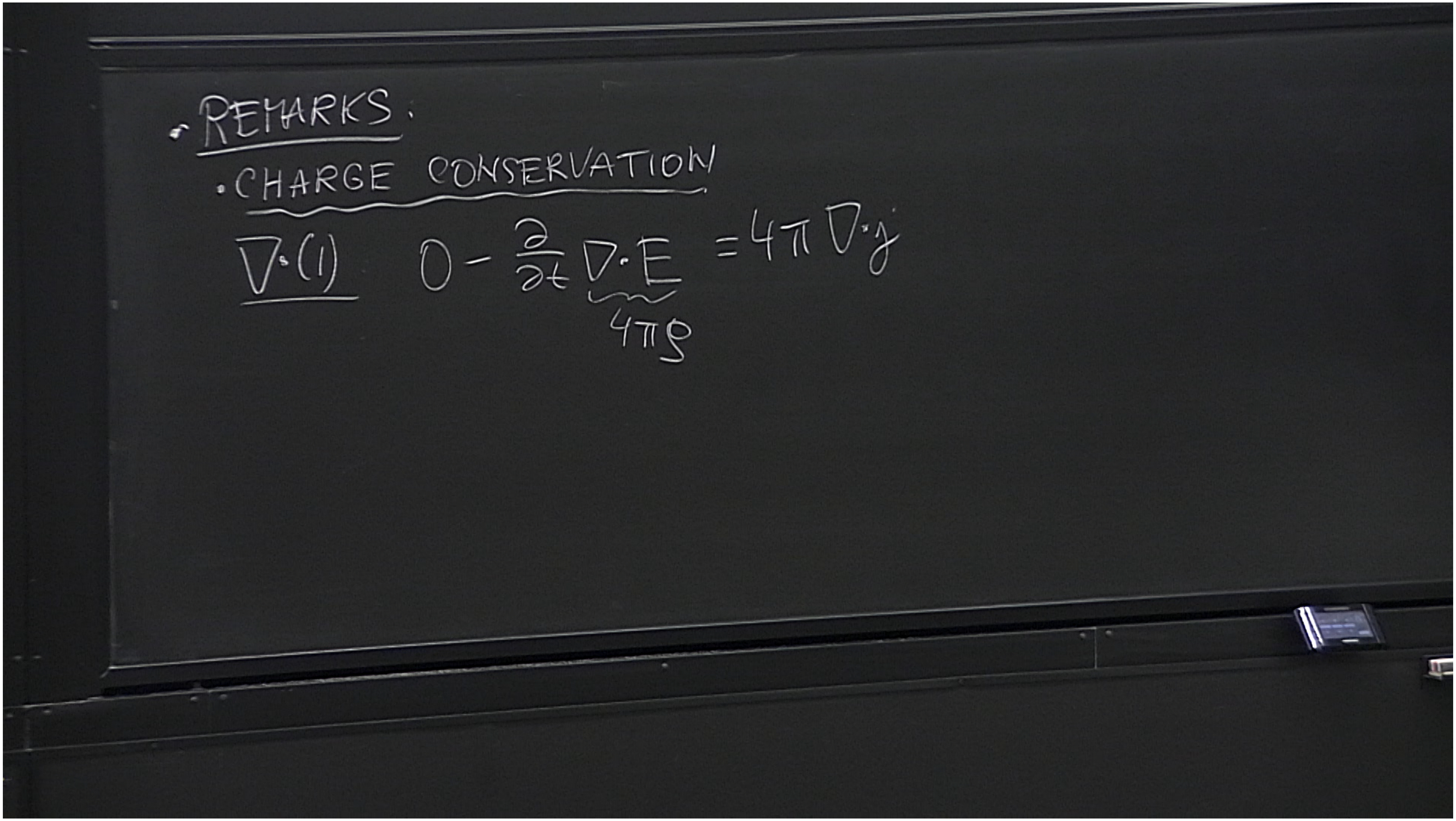

REMARKS:
CHARGE CONSERVATION
∇·(1) 0 − ∂/∂t ∇·E = 4π ∇·j



REMARKS:
CHARGE CONSERVATION
∇·(1) 0 − ∂/∂t ∇·E = 4π ∇·j
4πρ

REMARKS:

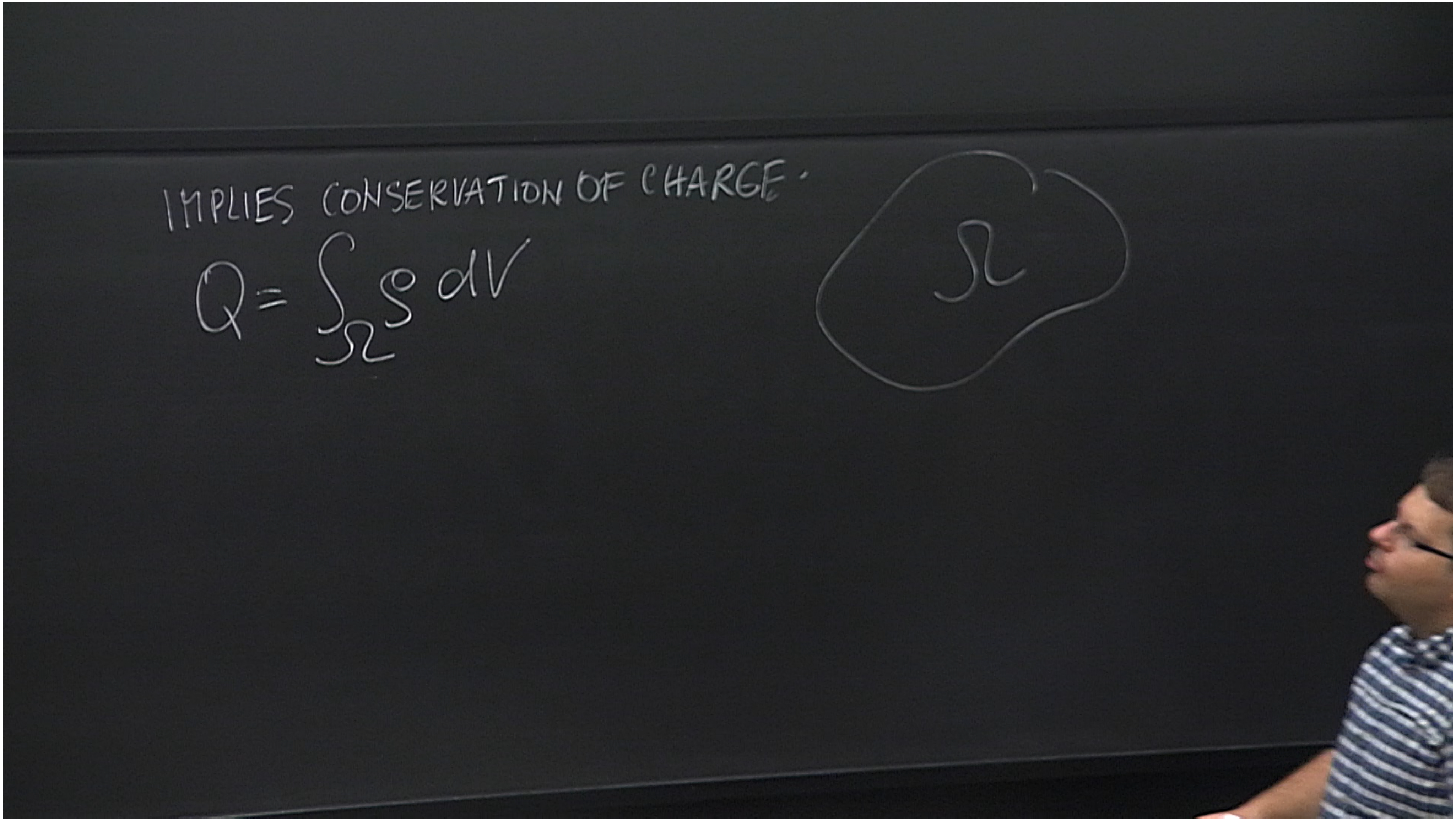
• CHARGE CONSERVATION

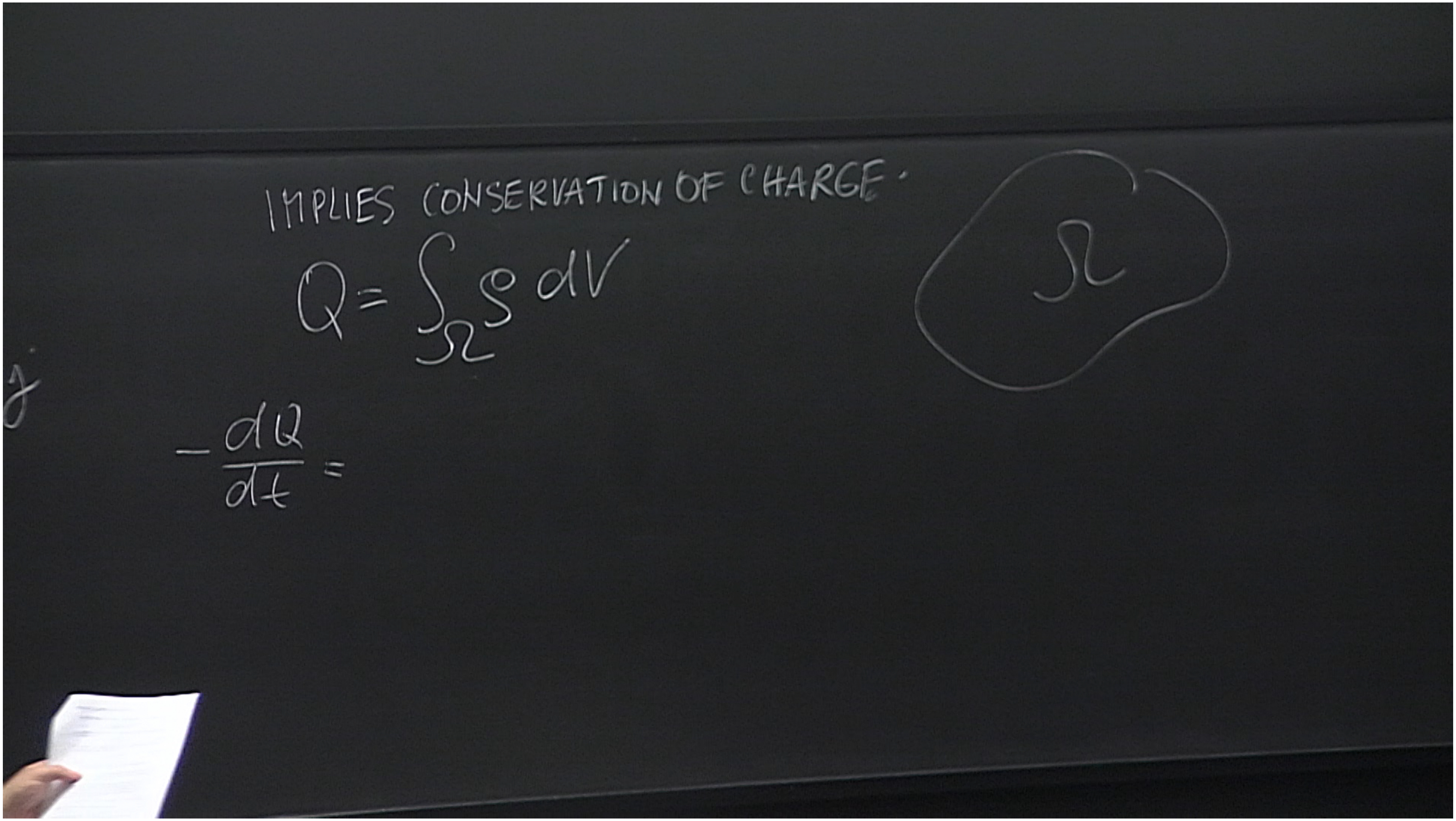
$\nabla\cdot(1)$ $0 - \frac{\partial}{\partial t}\nabla\cdot E = 4\pi \nabla\cdot j$

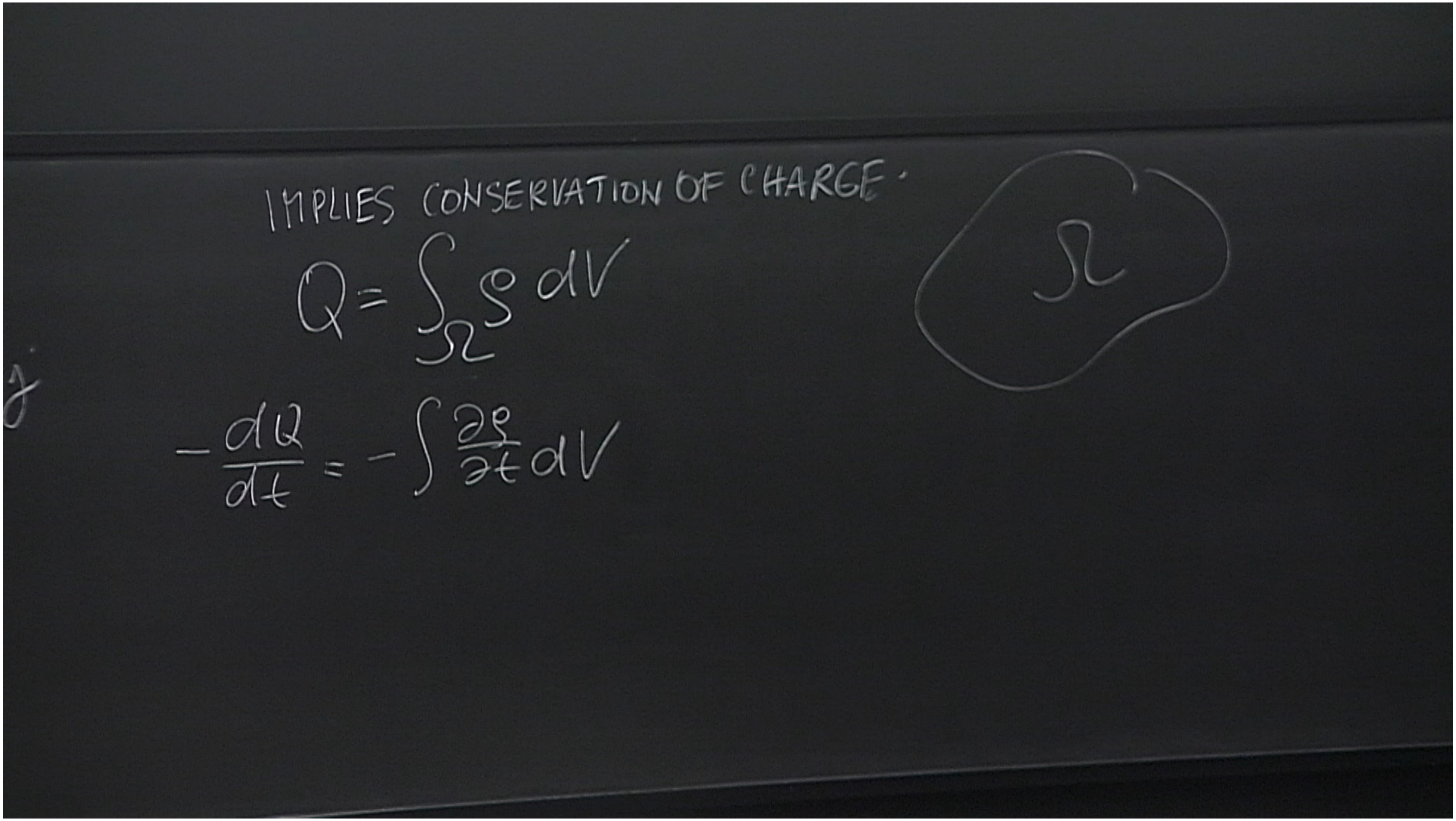
($\nabla\cdot E = 4\pi\rho$)

$$\frac{\partial \rho}{\partial t} + \nabla\cdot j = 0$$

CONTINUITY EQ.


IMPLIES CONSERVATION OF CHARGE.
$Q = \int_\Omega \rho \, dV$
Ω


IMPLIES CONSERVATION OF CHARGE.
$Q = \int_\Omega \rho \, dV$
$-\frac{dQ}{dt} =$
Ω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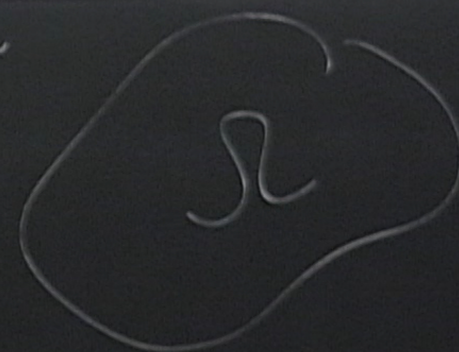
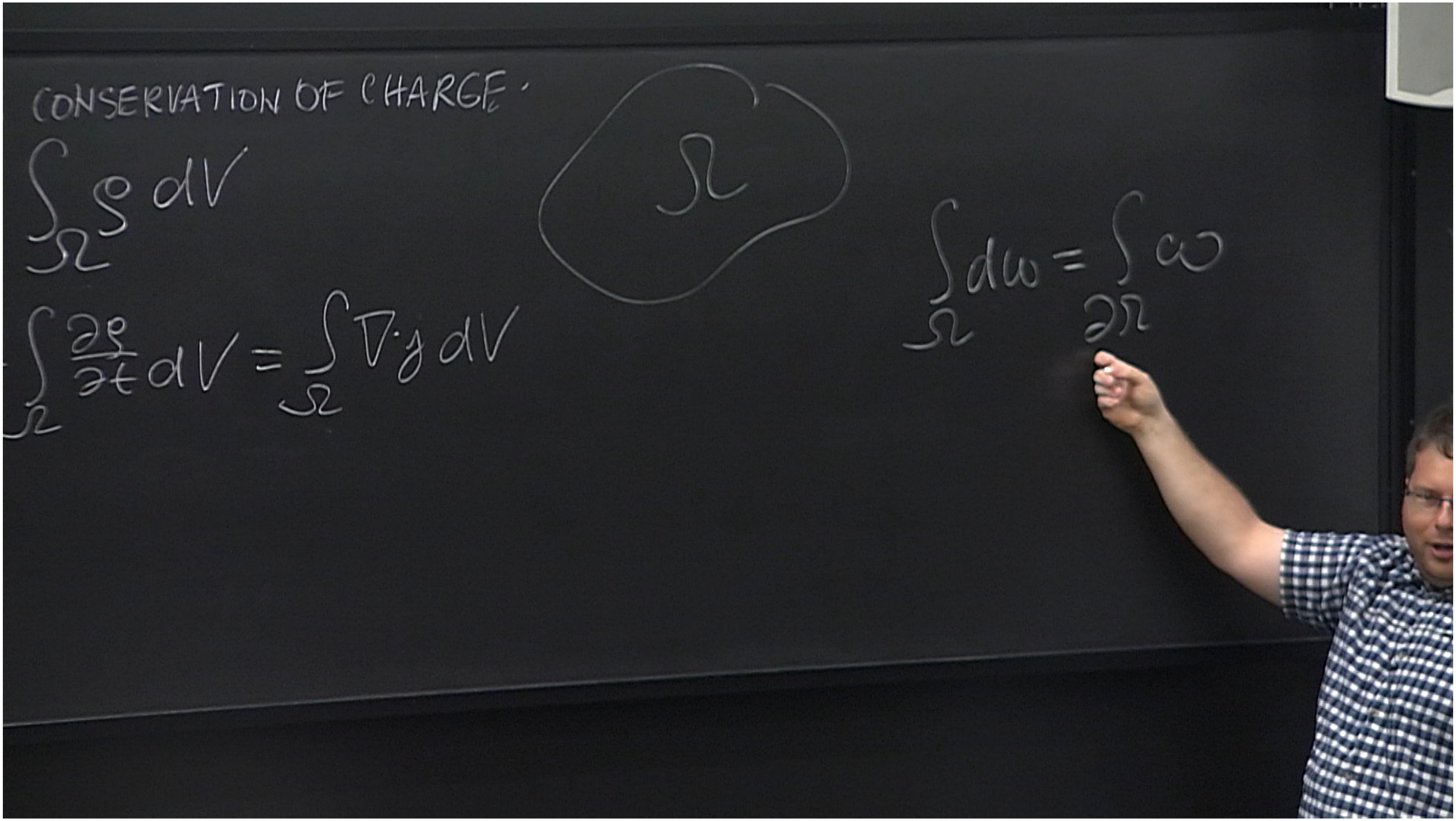
$Q = \int_{\Omega} \rho \, dV$
$-\frac{dQ}{dt} = -\int \frac{\partial \rho}{\partial t} dV$
Ω

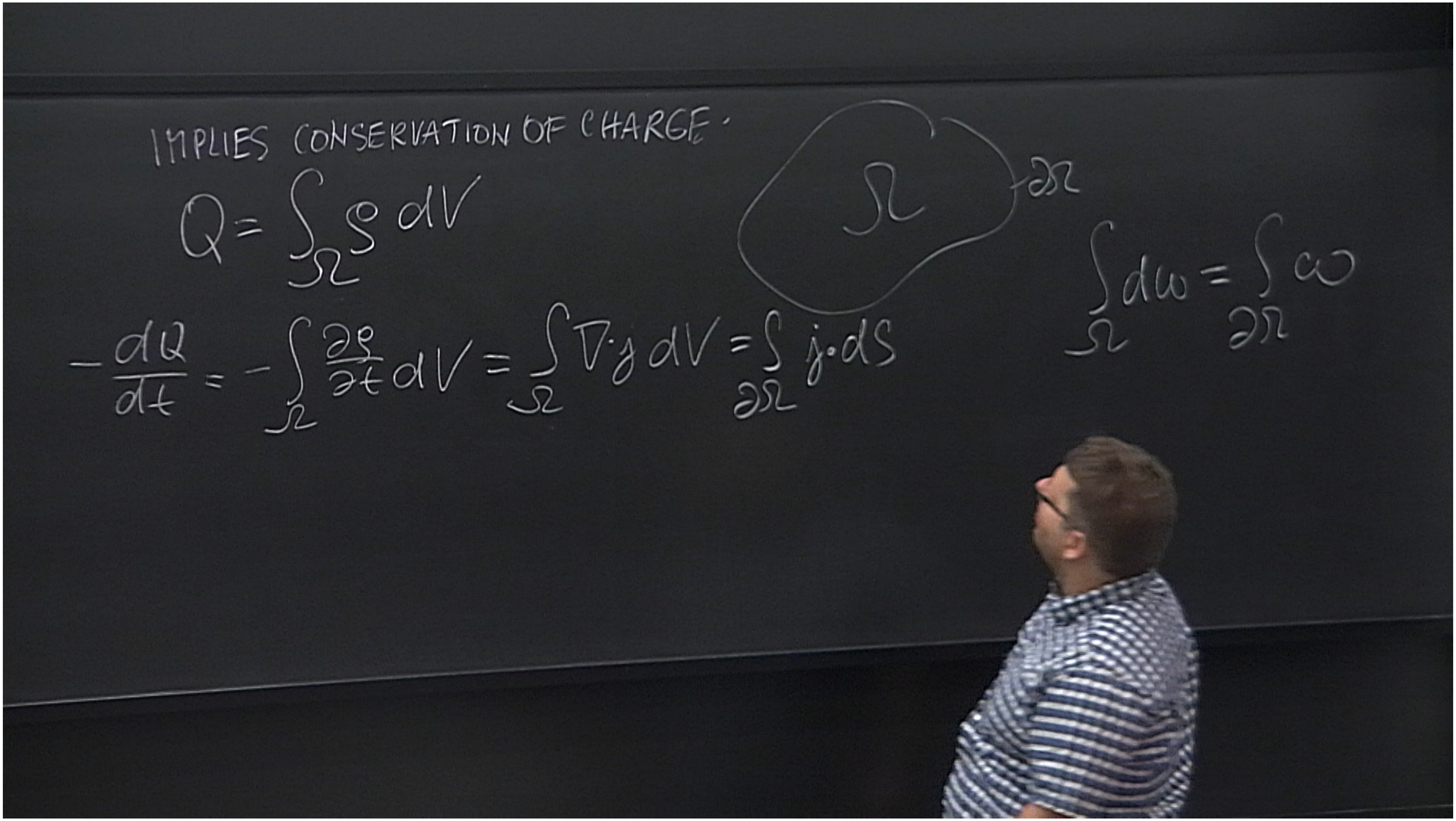
IMPLIES CONSERVATION OF CHARGE.

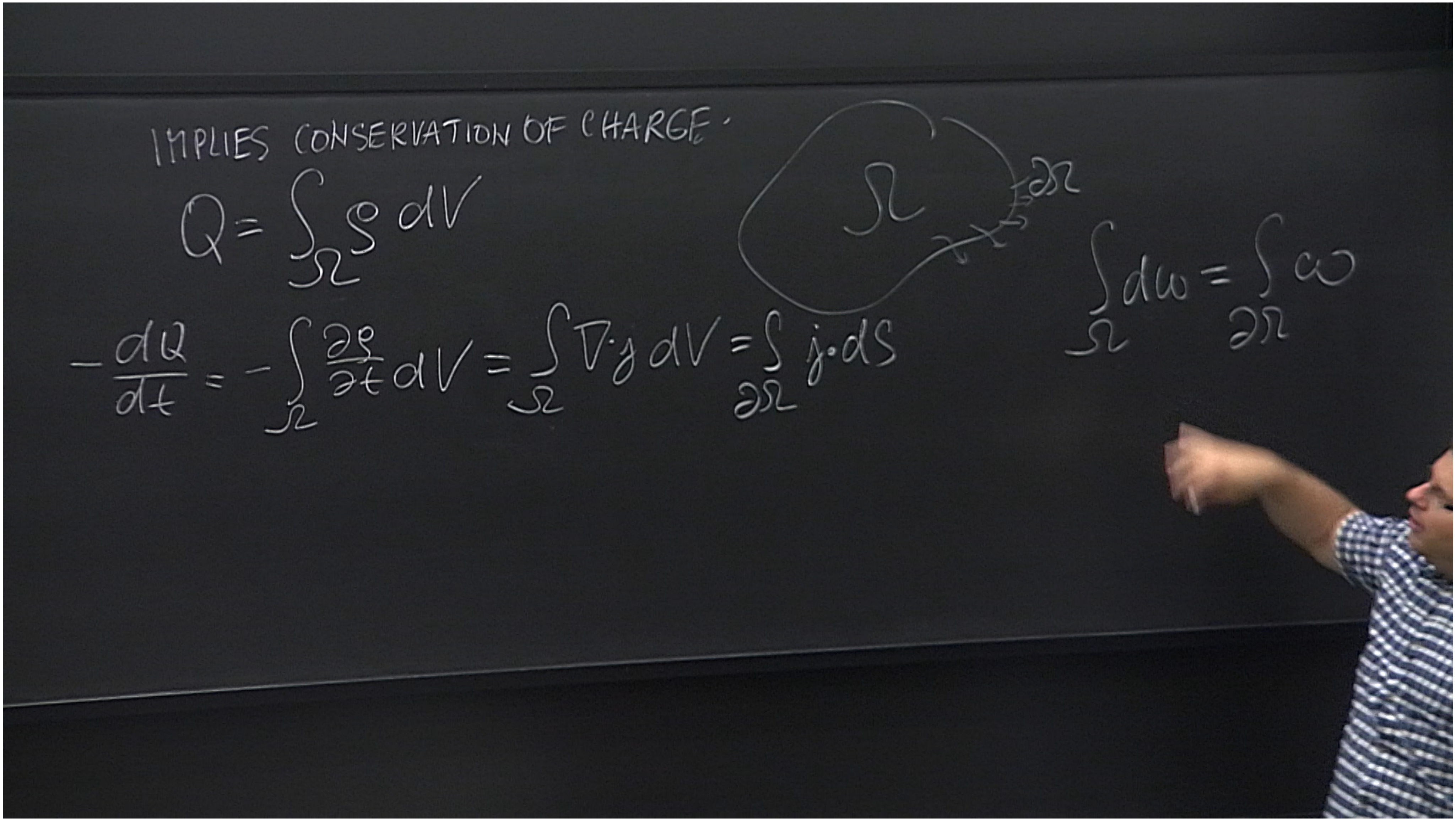
$$Q = \int_\Omega \rho \, dV$$



$$-\frac{dQ}{dt} = -\int_\Omega \frac{\partial \rho}{\partial t} dV = \int_\Omega \nabla \cdot j \, dV$$


CONSERVATION OF CHARGE.
$\int_\Omega \rho \, dV$
$\int_\Omega \frac{\partial \rho}{\partial t} dV = \int_\Omega \nabla \cdot j \, dV$
Ω
$\int_\Omega d\omega = \int_{\partial\Omega} \omega$


IMPLIES CONSERVATION OF CHARGE.
$Q = \int_\Omega \rho \, dV$
$-\frac{dQ}{dt} = -\int_\Omega \frac{\partial \rho}{\partial t} dV = \int_\Omega \nabla \cdot j \, dV = \int_{\partial\Omega} j \cdot dS$
Ω
$\partial\Omega$
$\int_\Omega d\omega = \int_{\partial\Omega} \omega$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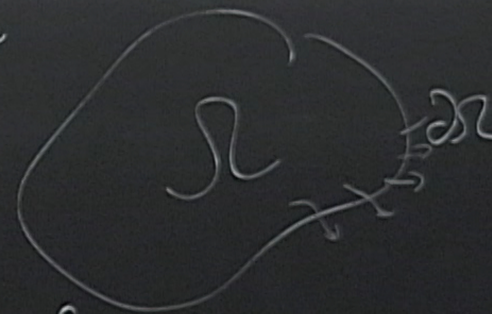


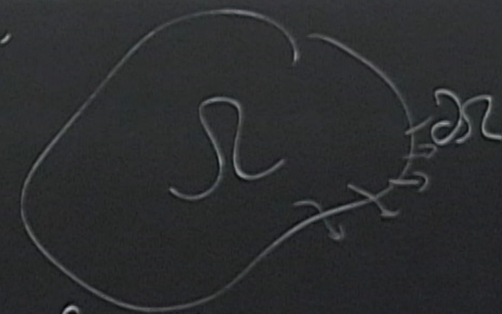
$$\int_{\Omega} d\omega = \int_{\partial\Omega} \omega$$

$$-\frac{dQ}{dt} = -\int_{\Omega} \frac{\partial \rho}{\partial t} dV = \int_{\Omega} \nabla \cdot j \, dV = \int_{\partial\Omega} j \cdot dS$$

= CURRENT FLUX THROUGH $\partial\Omega$.

IMPLIES CONSERVATION OF CHARGE.

$$Q = \int_\Omega \rho \, dV$$



$$\int_\Omega d\omega = \int_{\partial\Omega} \omega$$

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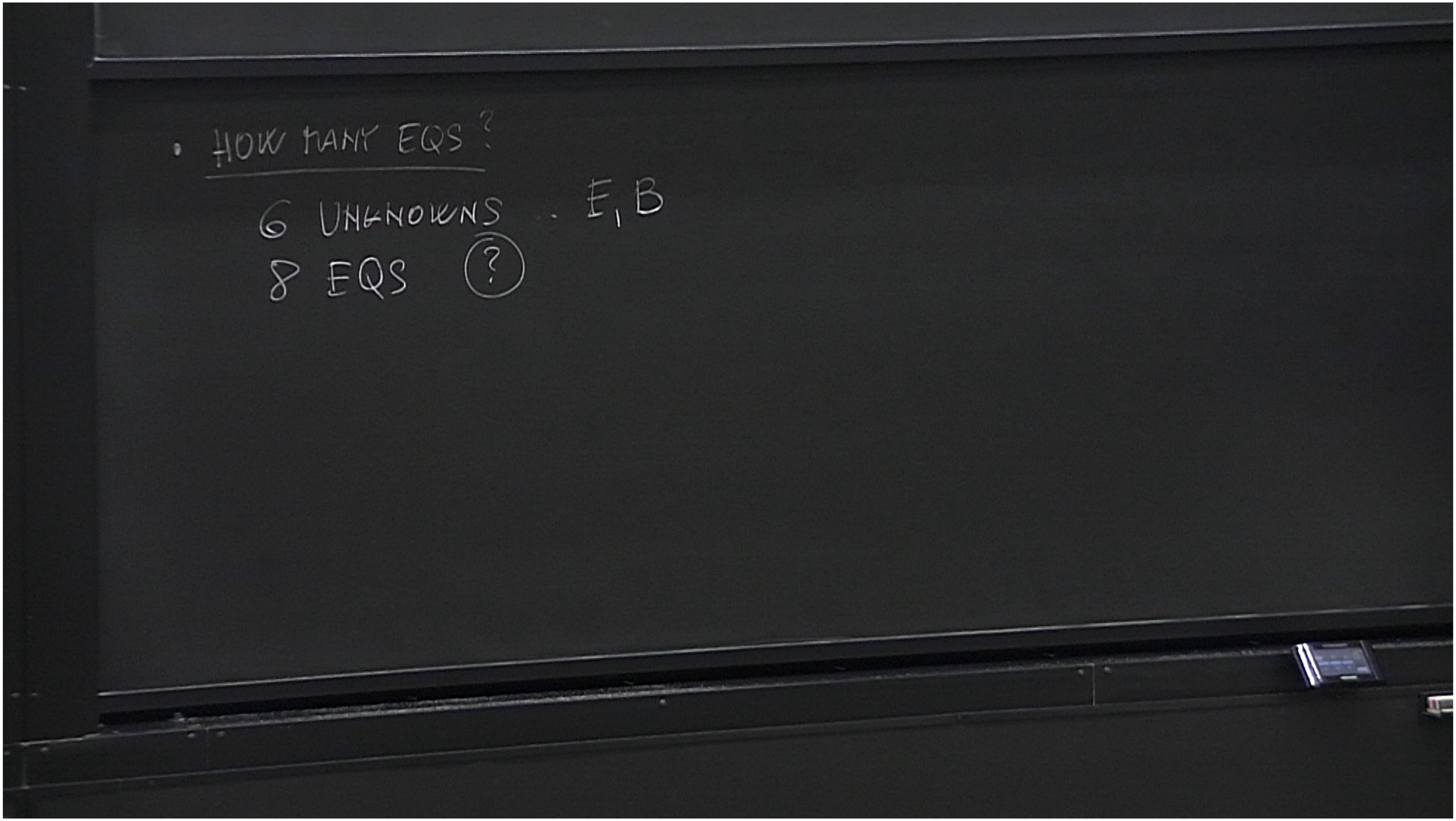
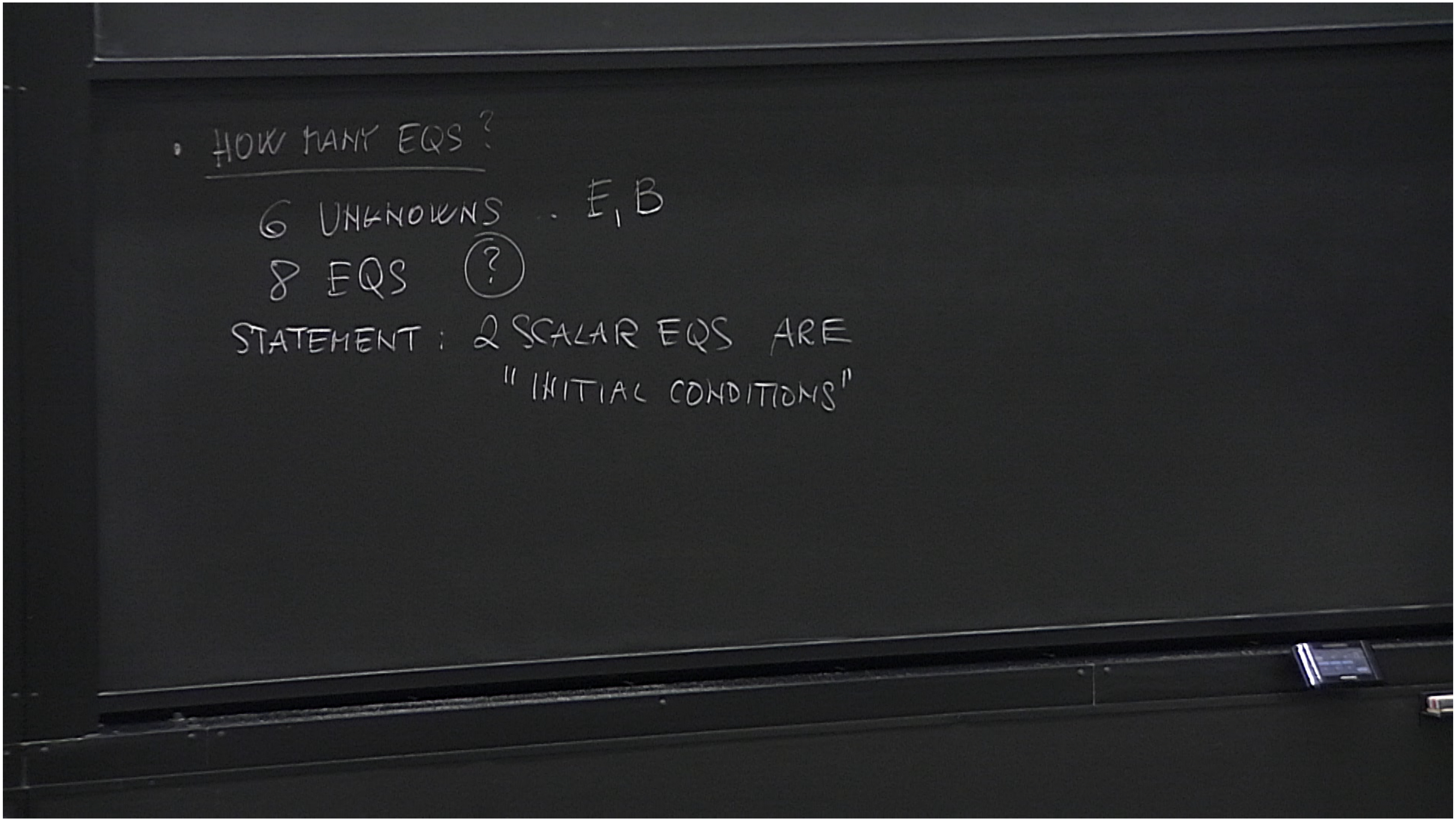
= CURRENT FLUX THROUGH $\partial\Omega$.

IF NO FLUX => Q IS CONSERVED


• HOW MANY EQS ?
6 UNKNOWNS .. E, B
8 EQS ?


• HOW MANY EQS ?
6 UNKNOWNS .. E, B
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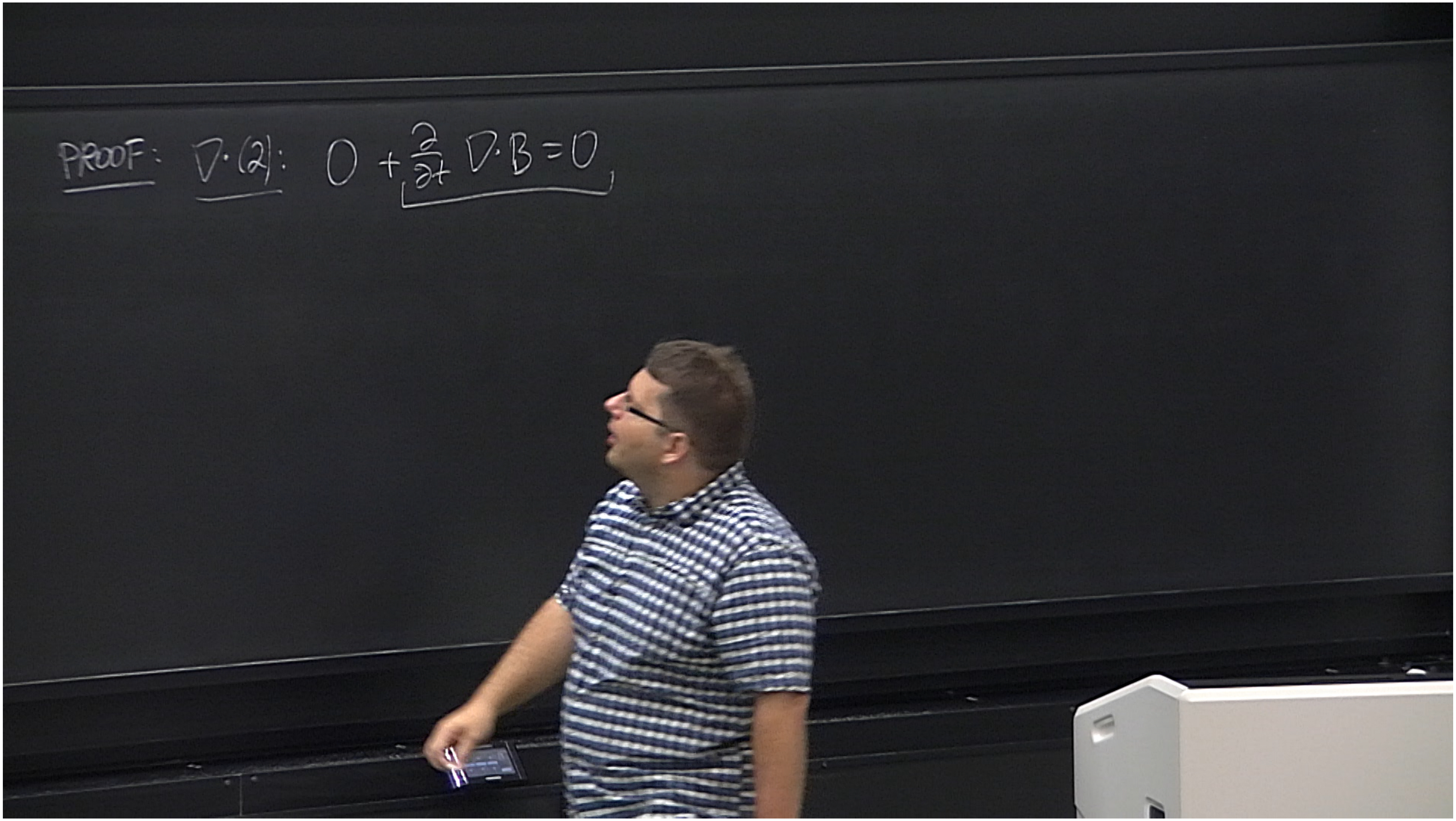
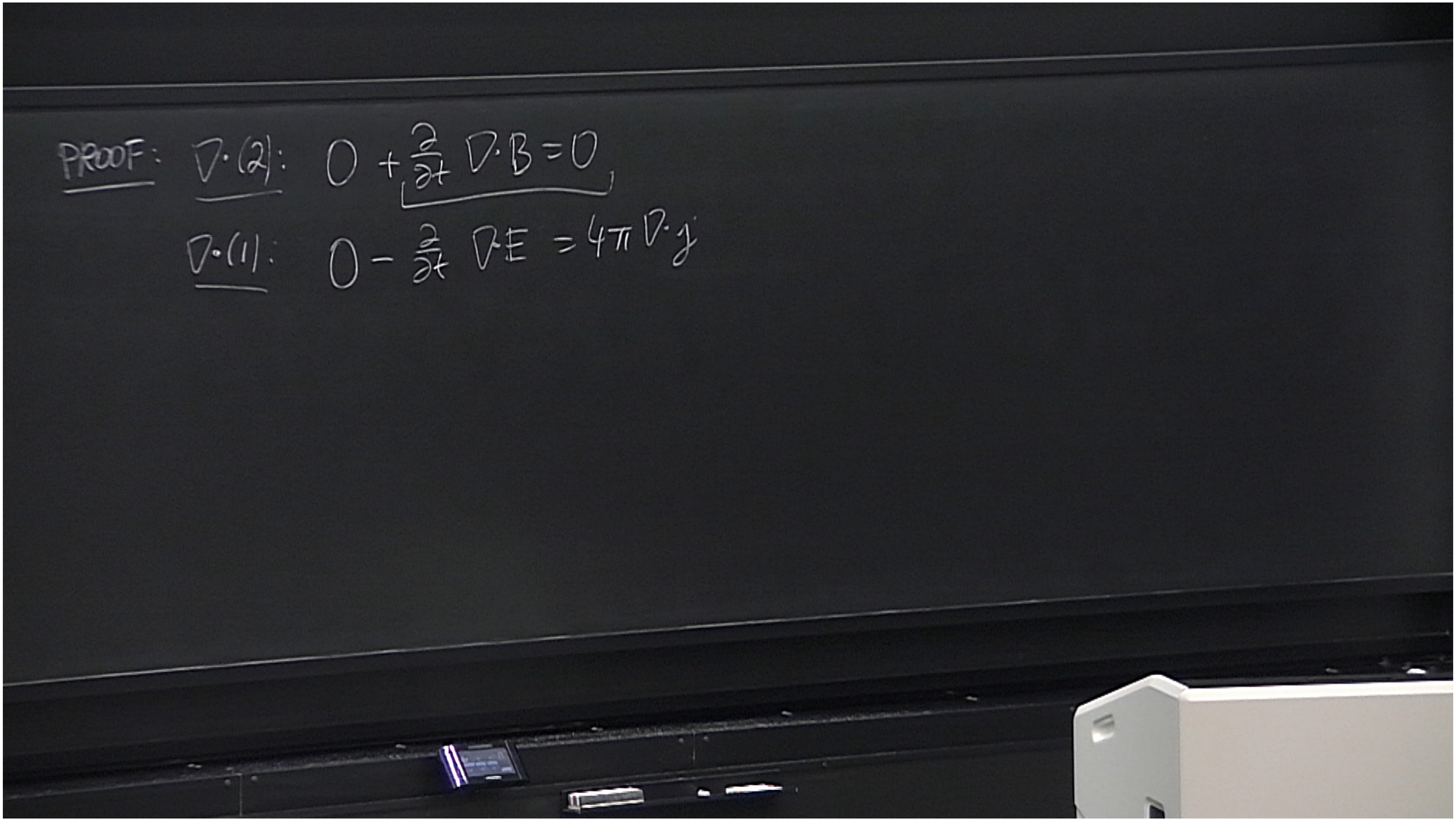
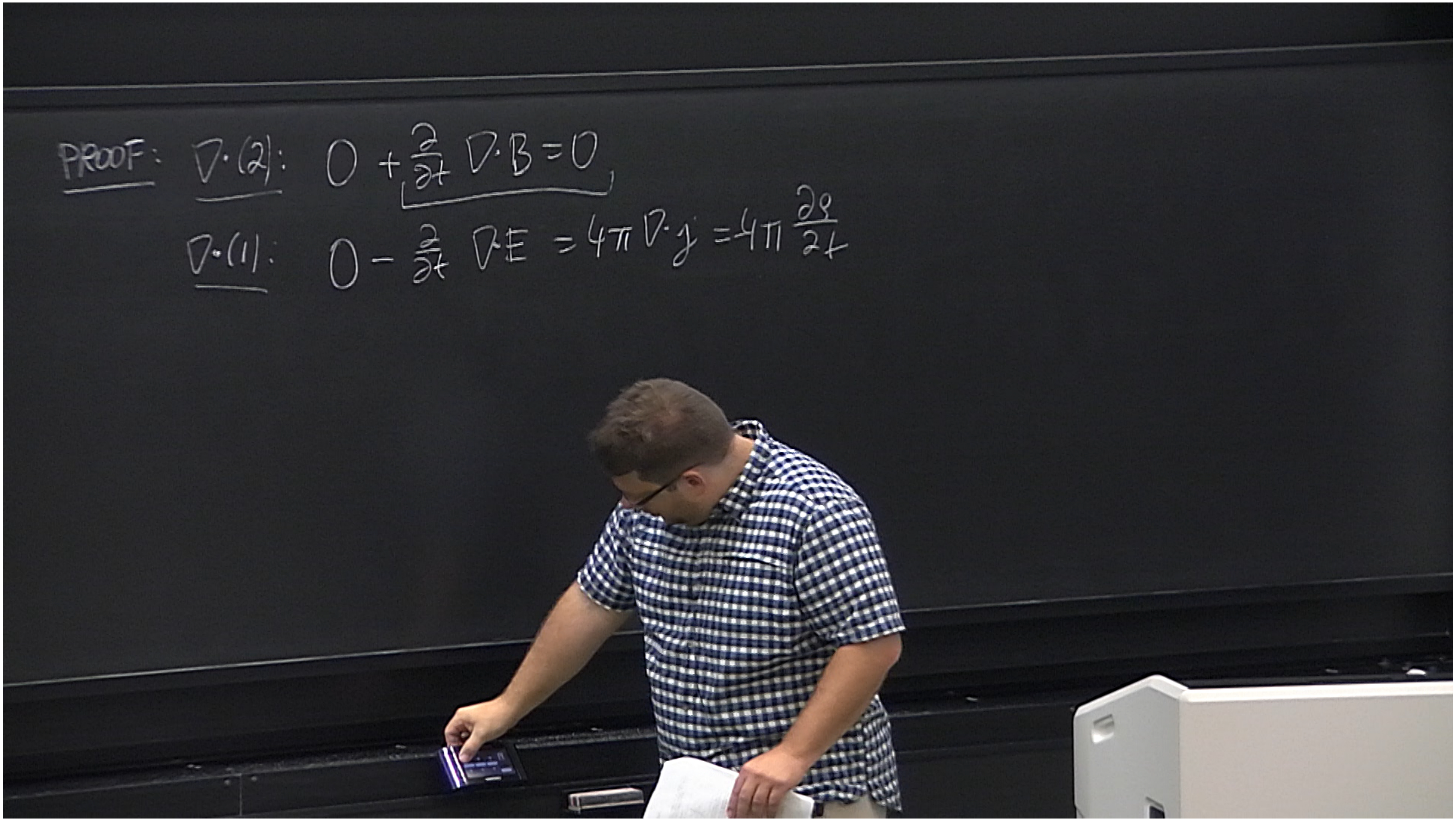
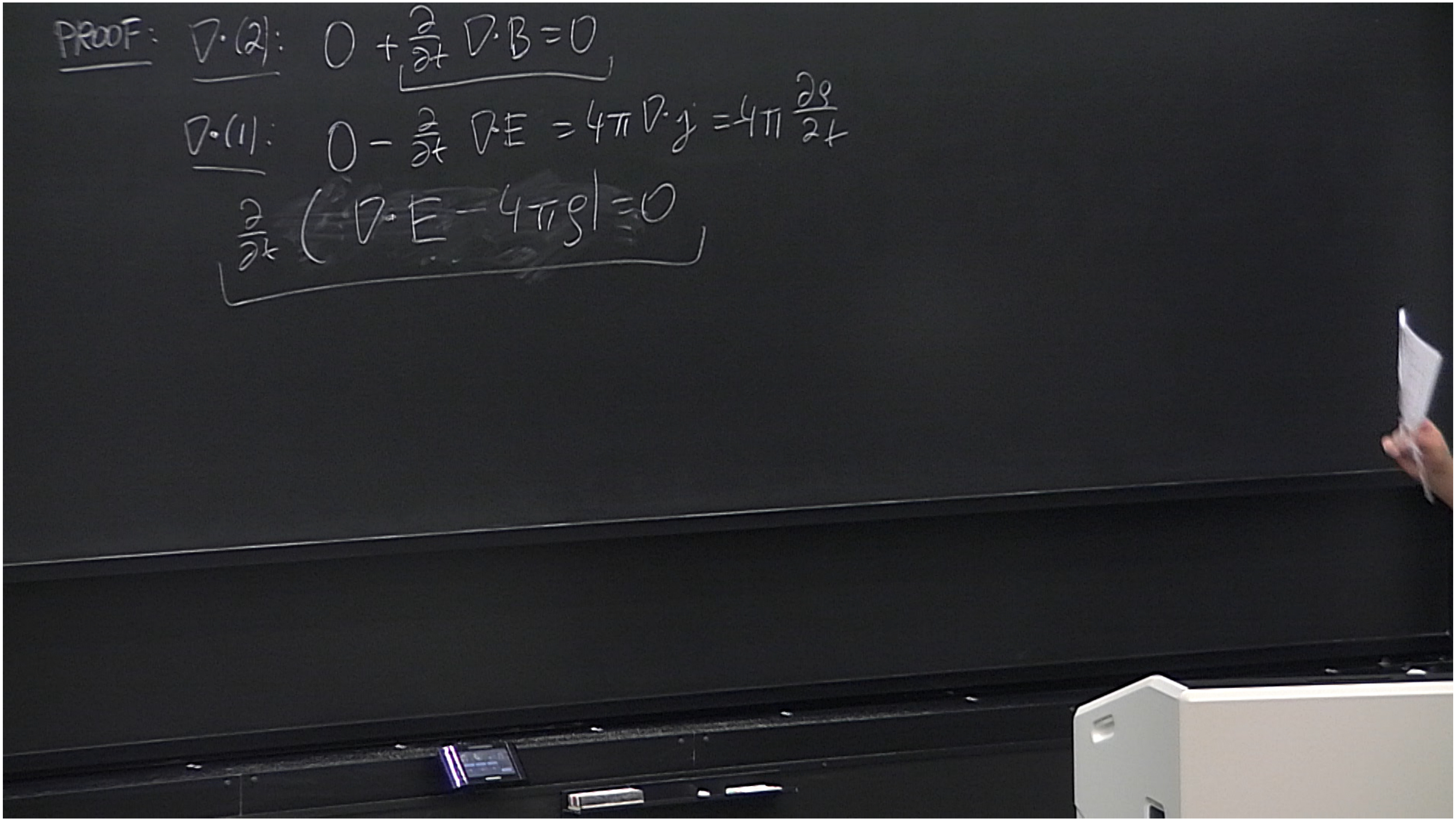
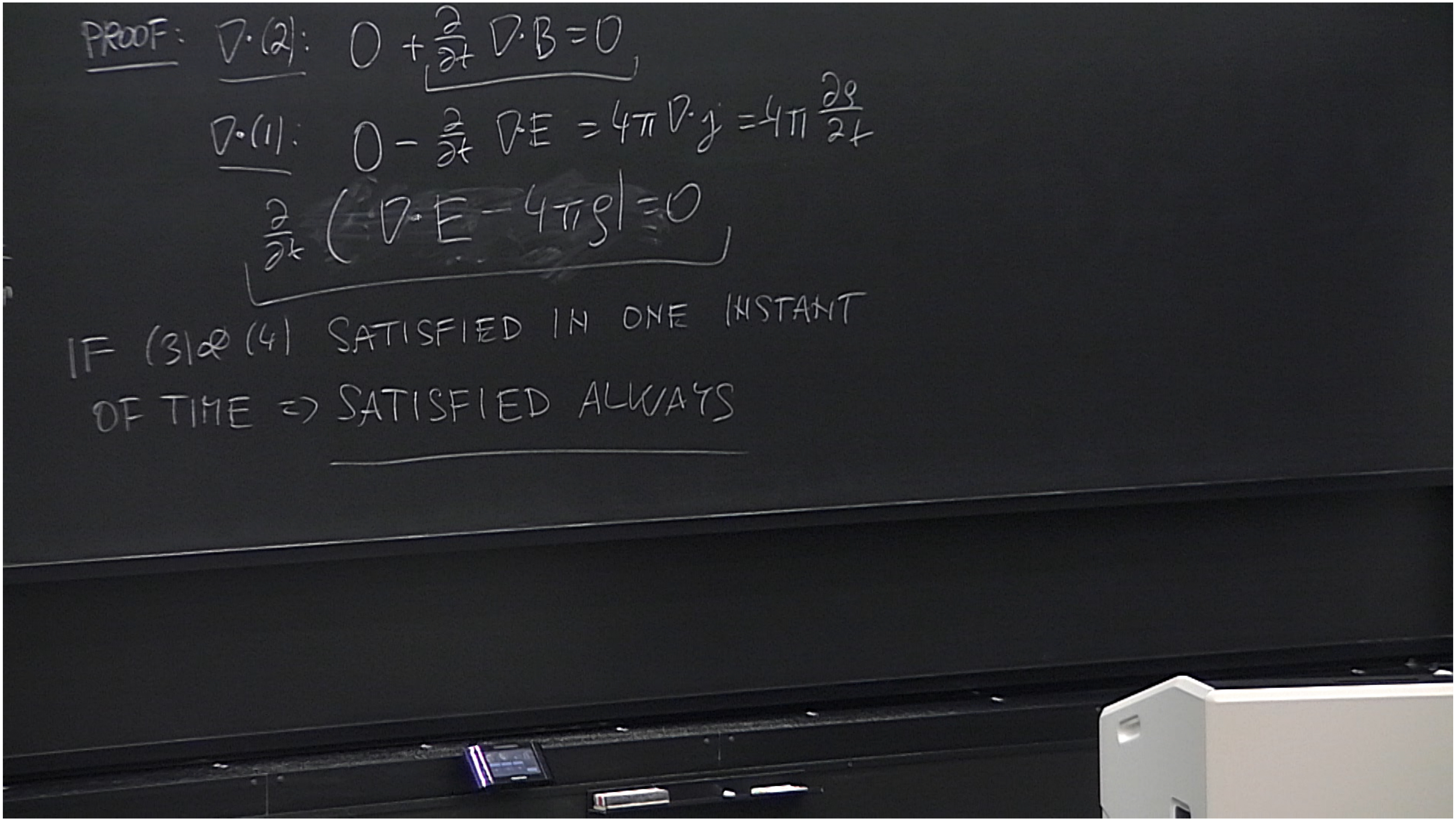
STATEMENT: 2 SCALAR EQS ARE
"INITIAL CONDITIONS"

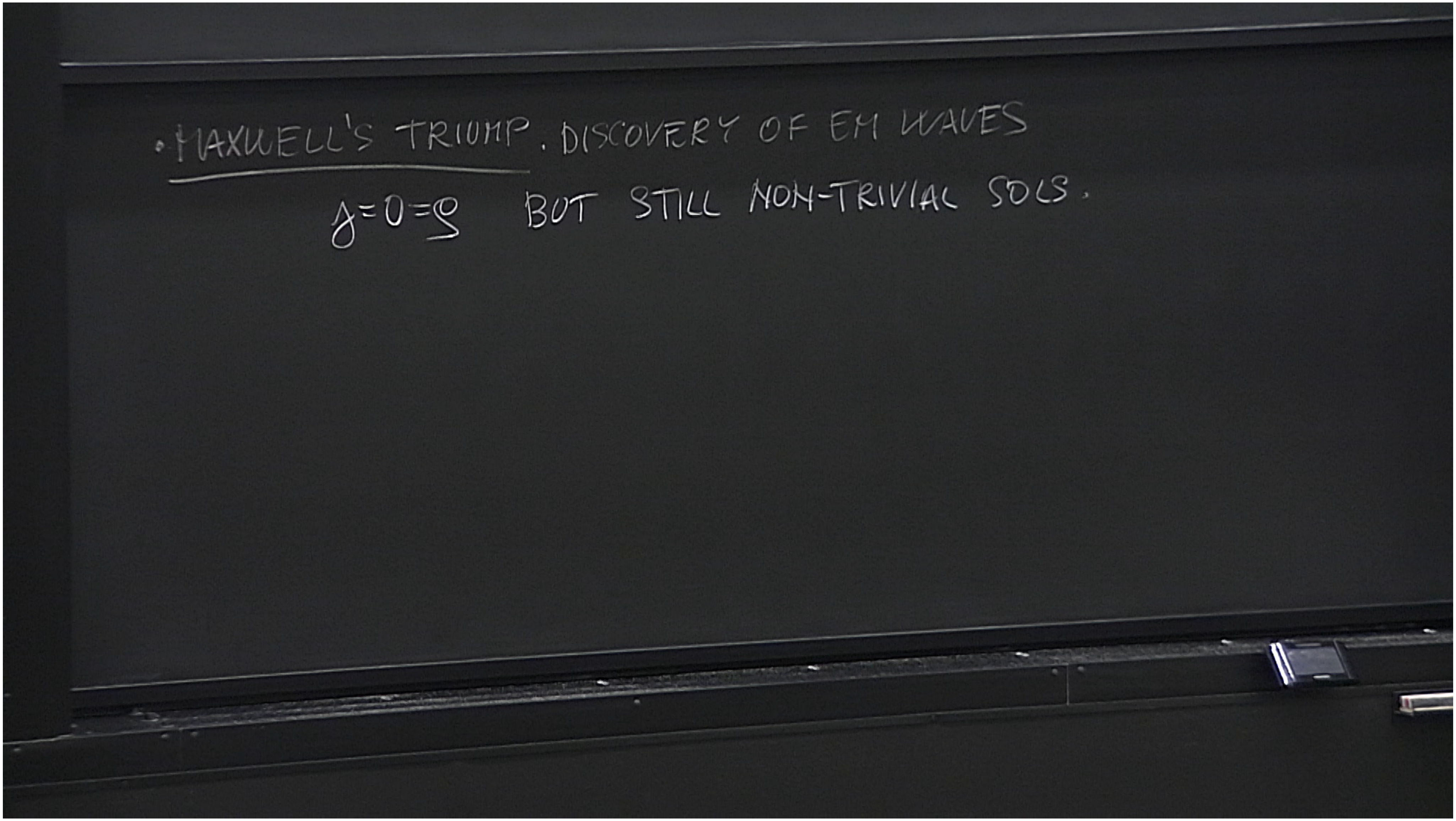

PROOF: ∇·(2): 0 + ∂/∂t ∇·B = 0

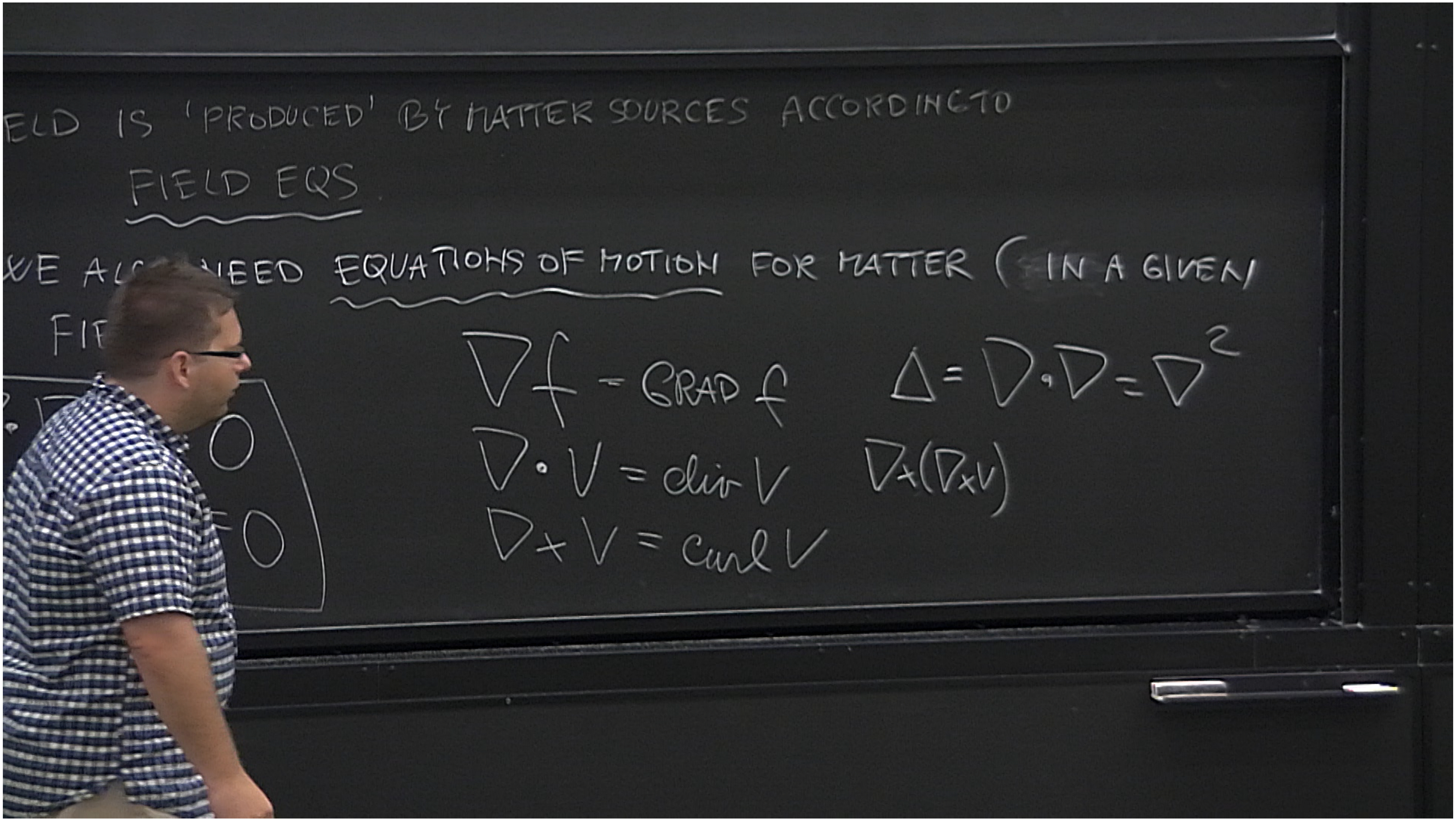

PROOF:
∇·(2): 0 + ∂/∂t ∇·B = 0
∇·(1): 0 − ∂/∂t ∇·E = 4π ∇·j


PROOF: ∇·(2): 0 + ∂/∂t ∇·B = 0
∇·(1): 0 − ∂/∂t ∇·E = 4π ∇·j = 4π ∂ρ/∂t


PROOF: $\nabla\cdot(2)$: $0 + \frac{\partial}{\partial t}\nabla\cdot B = 0$
$\nabla\cdot(1)$: $0 - \frac{\partial}{\partial t}\nabla\cdot E = 4\pi\nabla\cdot j = -4\pi\frac{\partial\rho}{\partial t}$
$\frac{\partial}{\partial t}(\nabla\cdot E - 4\pi\rho) = 0$


PROOF: ∇·(2): 0 + ∂/∂t ∇·B = 0
∇·(1): 0 − ∂/∂t ∇·E = 4π ∇·j = −4π ∂ρ/∂t
∂/∂t (∇·E − 4πρ) = 0
IF (3) & (4) SATISFIED IN ONE INSTANT
OF TIME ⇒ SATISFIED ALWAYS


• MAXWELL'S TRIUMP. DISCOVERY OF EM WAVES
j = 0 = ρ BUT STILL NON-TRIVIAL SOLS.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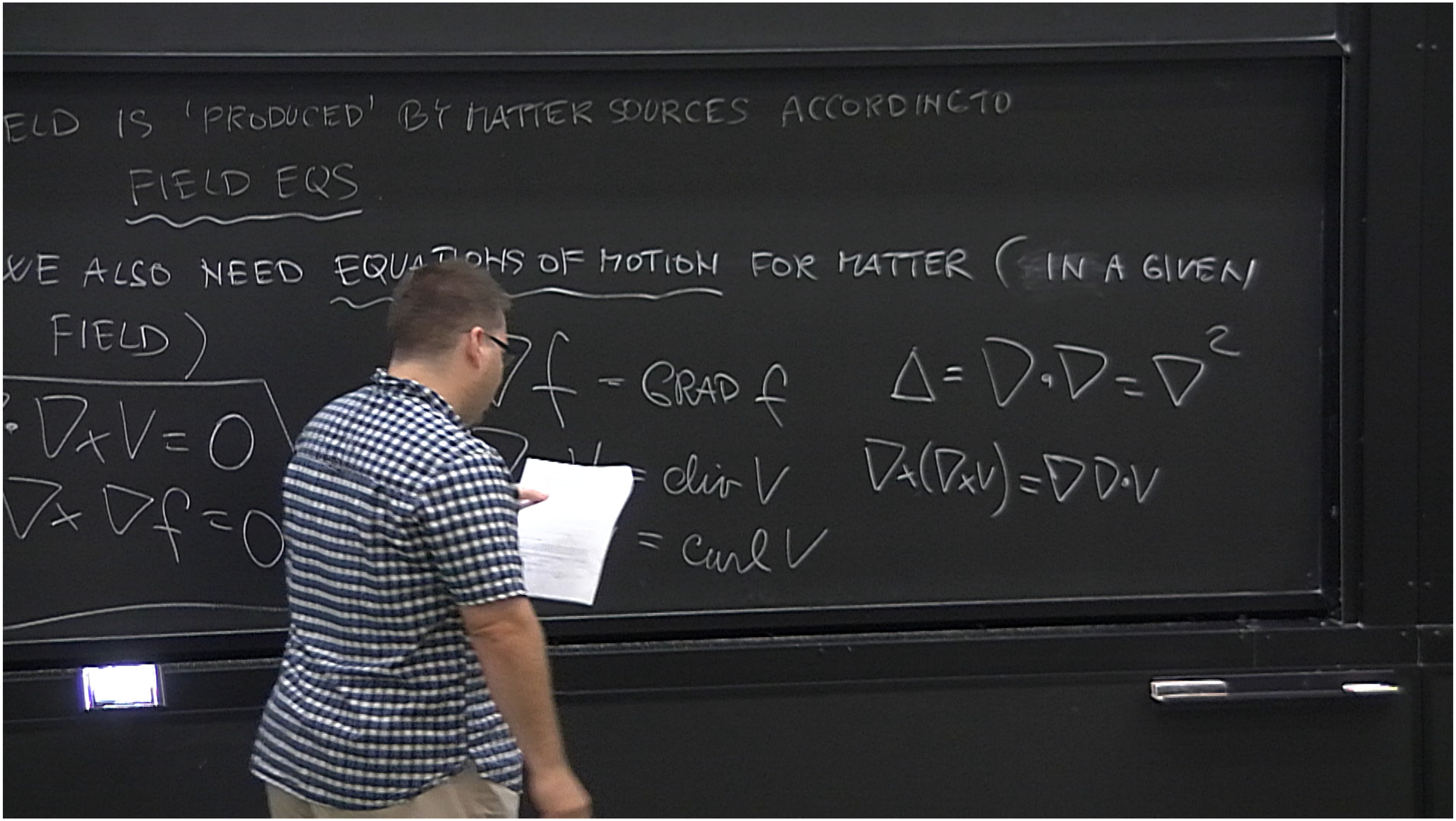
∆ = ∇·∇ = ∇²
∇·V = div V
∇×(∇×V)
∇×V = curl V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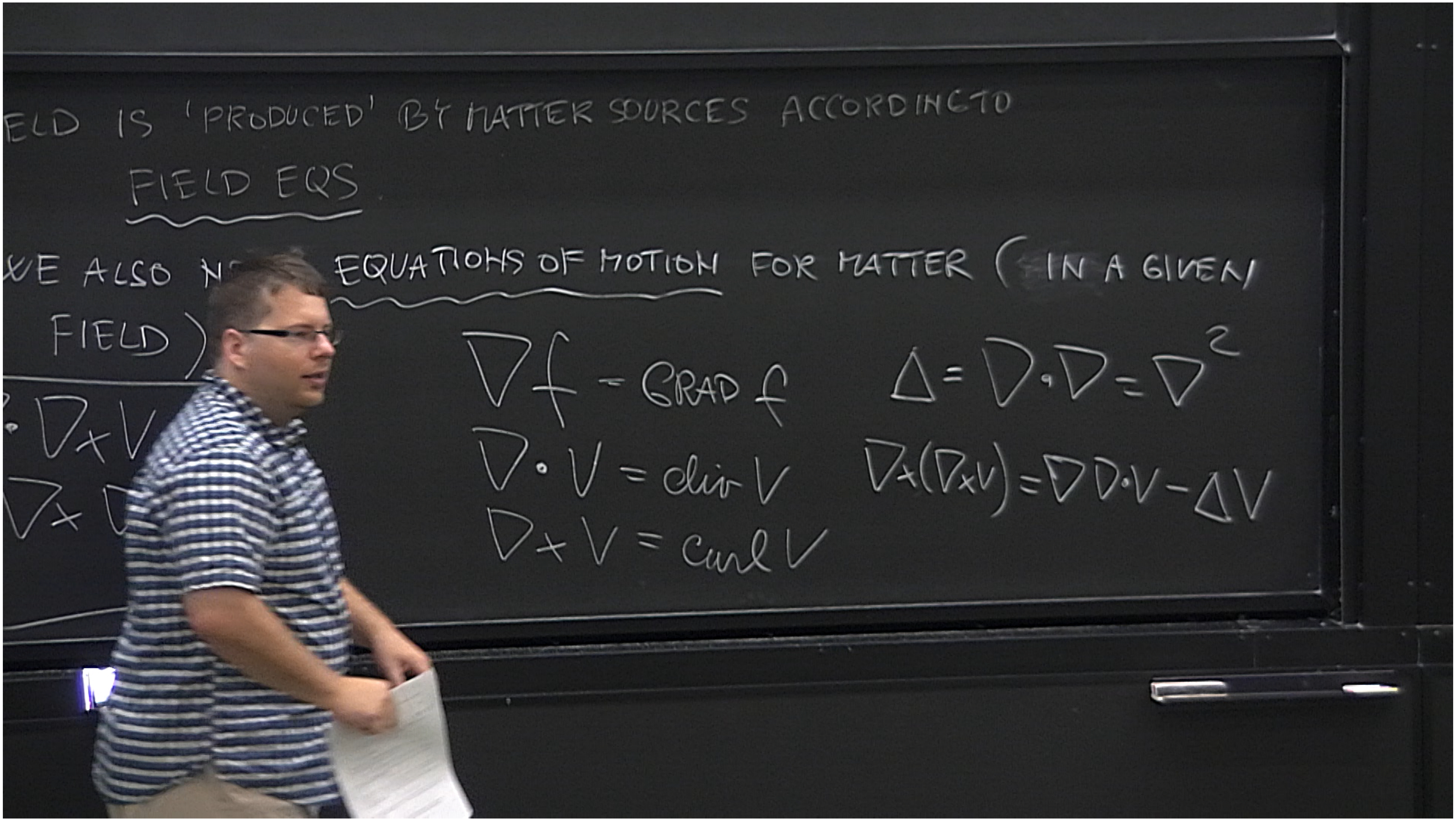
WE ALSO NEED EQUATIONS OF MOTION FOR MATTER (IN A GIVEN
FIELD)
f = GRAD f
Δ = ∇·∇ = ∇²
∇×V = 0
∇×∇f = 0
= div V
∇×(∇×V) = ∇∇·V
= curl V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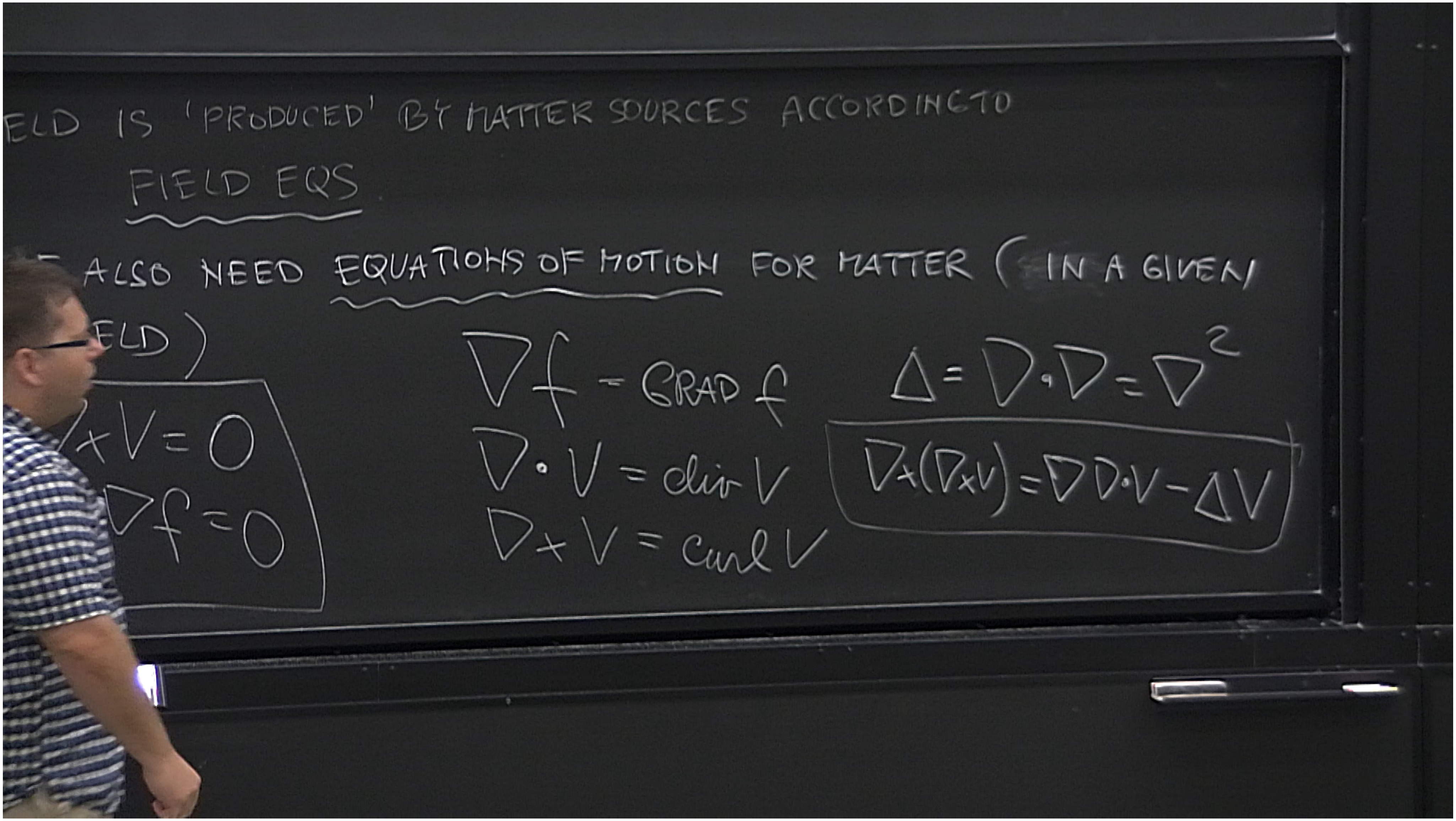
FIELD)
∇f = GRAD f
∇·V = div V
∇×V = curl V
Δ = ∇·∇ = ∇²
∇×(∇×V) = ∇∇·V − ΔV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Δ = ∇·∇ = ∇²
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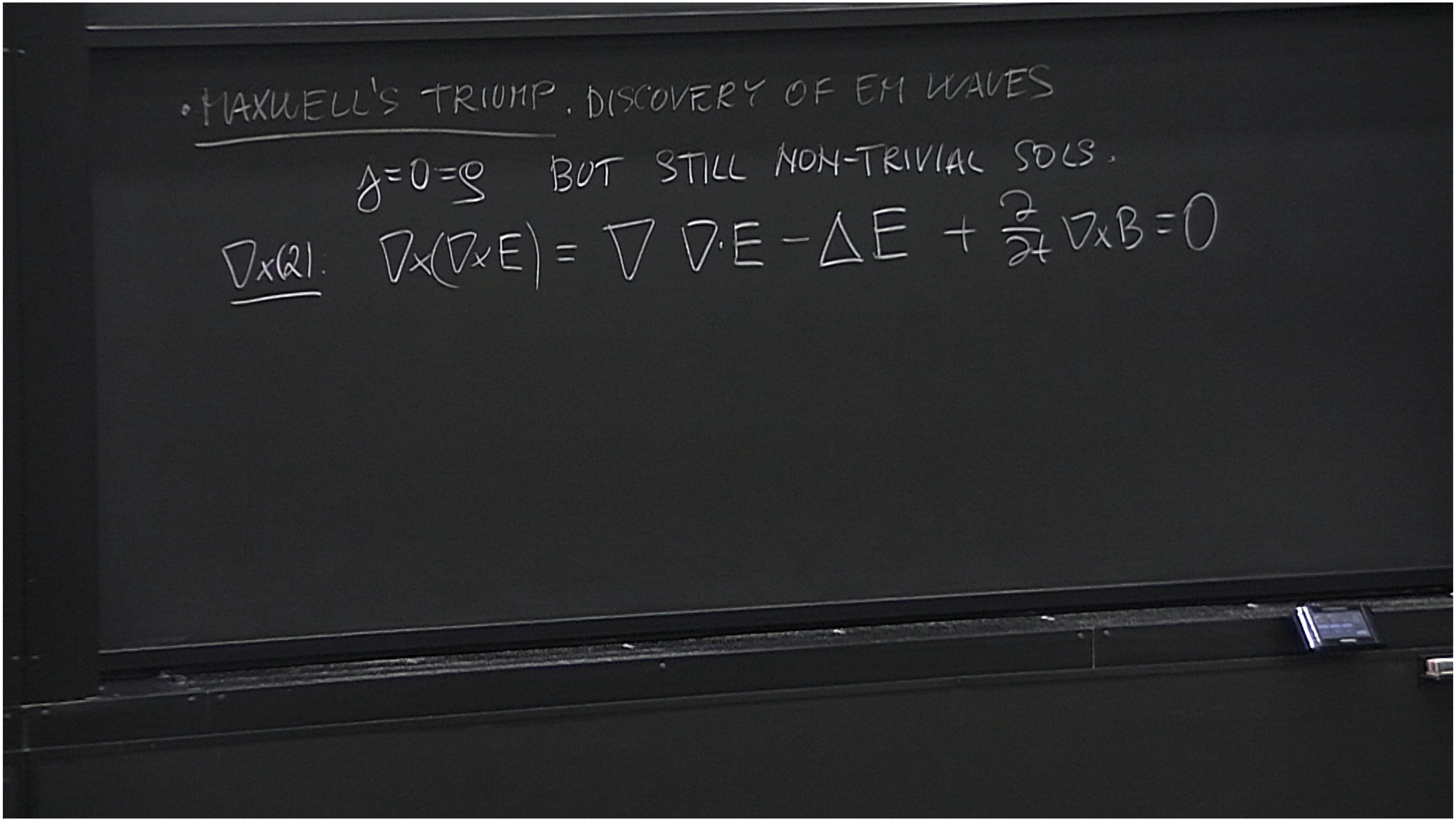
∇·V = div V
∇×(∇×V) = ∇∇·V − ΔV
∇f = 0
∇×V = curl V


• MAXWELL'S TRIUMP. DISCOVERY OF EM WAVES
j=0=ρ BUT STILL NON-TRIVIAL SOLS.
∇×(2): ∇×(∇×E) = ∇ ∇·E − ΔE + ∂/∂t ∇×B = 0

• MAXWELL'S TRIUMP. DISCOVERY OF EM WAVES

$j = 0 = \rho$ BUT STILL NON-TRIVIAL SOLS.

$\nabla \times (2)$: $\nabla \times (\nabla \times E) = \nabla \underbrace{\nabla \cdot E}_{0} - \Delta E + \frac{\partial}{\partial t} \nabla \times B = 0$

• MAXWELL'S TRIUMP. DISCOVERY OF EM WAVES

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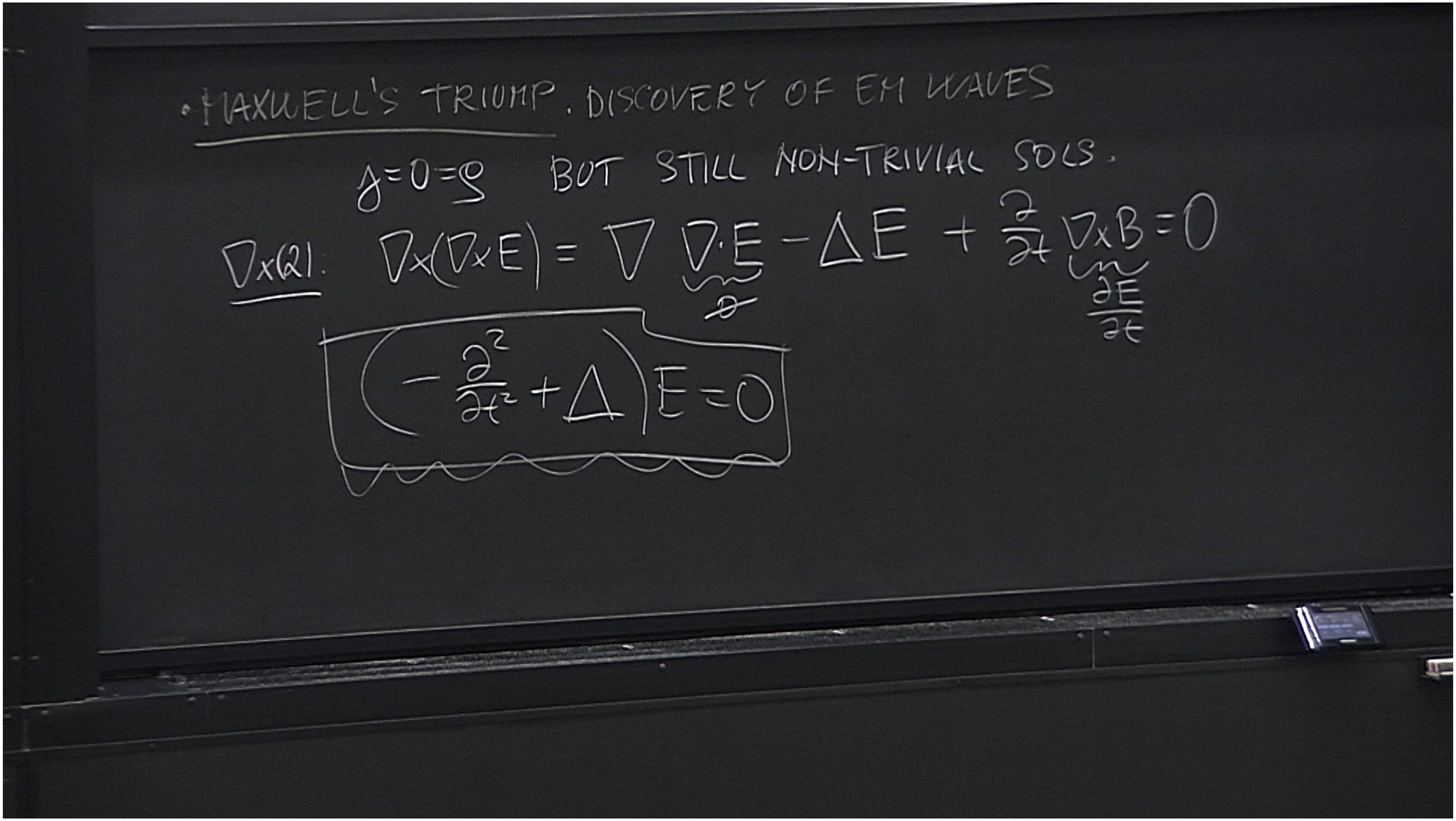
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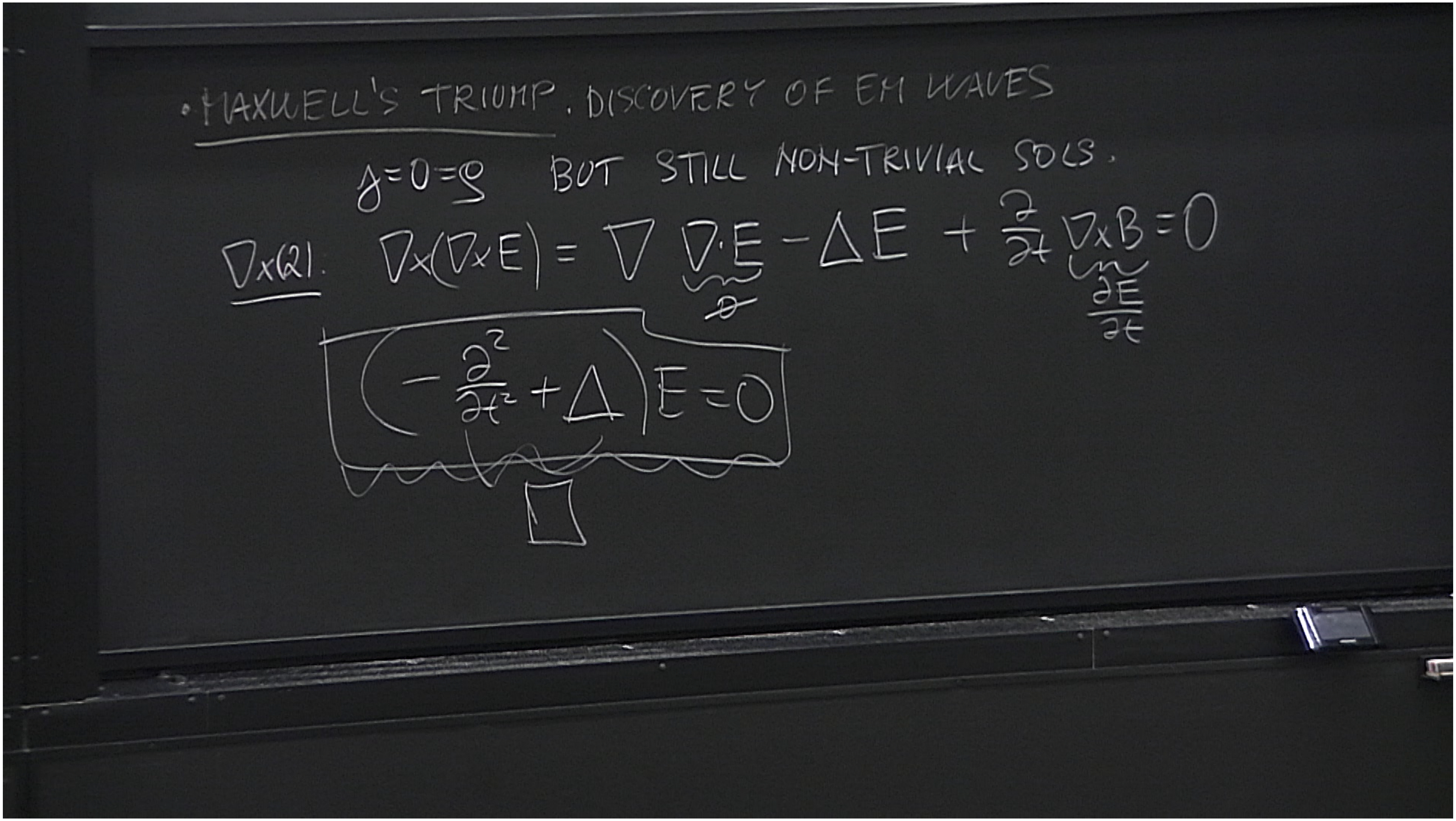
∇×(2): ∇×(∇×E) = ∇ ∇·E − ΔE + ∂/∂t ∇×B = 0
∂E/∂t
(−∂²/∂t² + Δ) E = 0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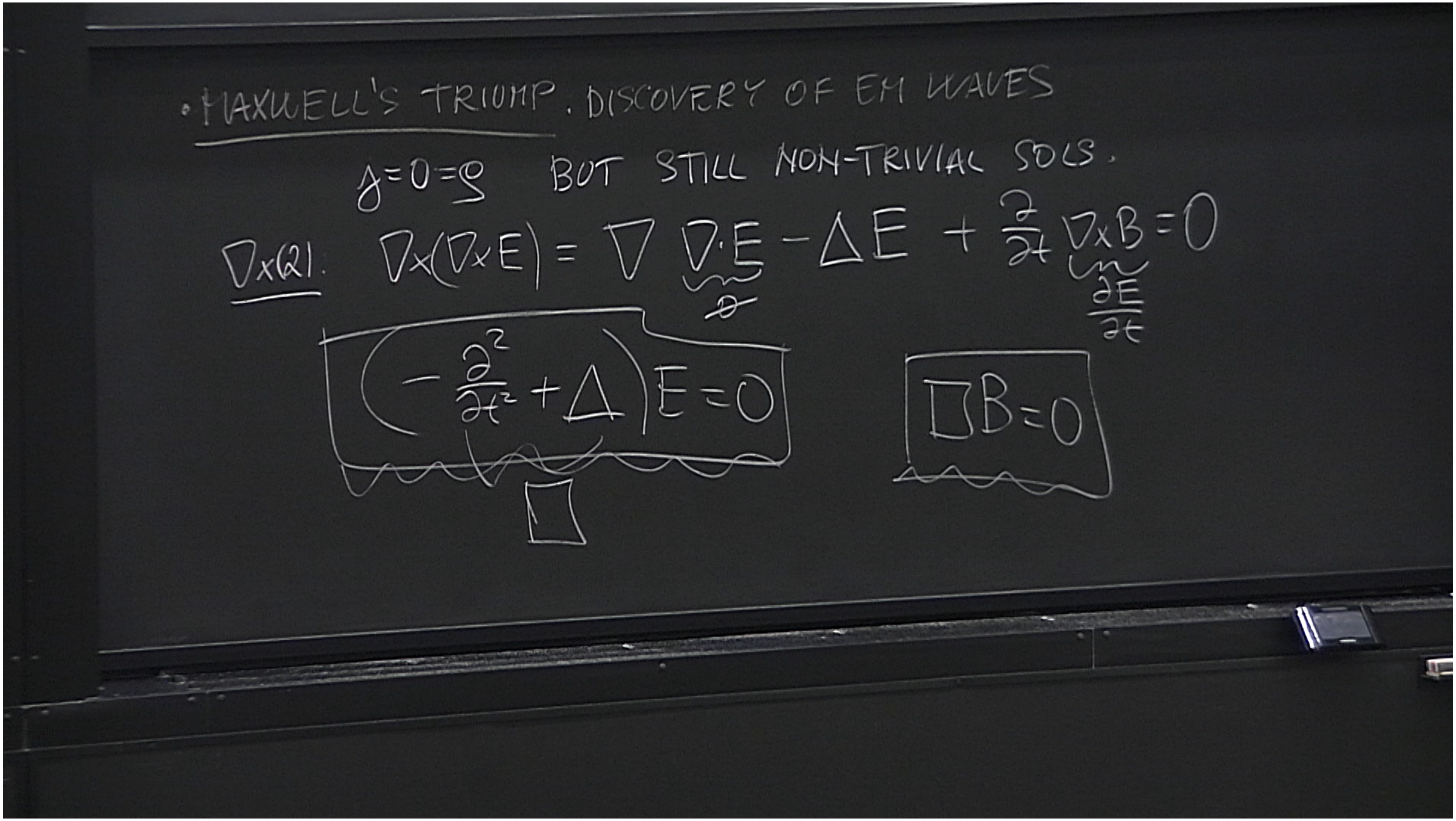
0
∂E/∂t
(−∂²/∂t² + Δ)E = 0
□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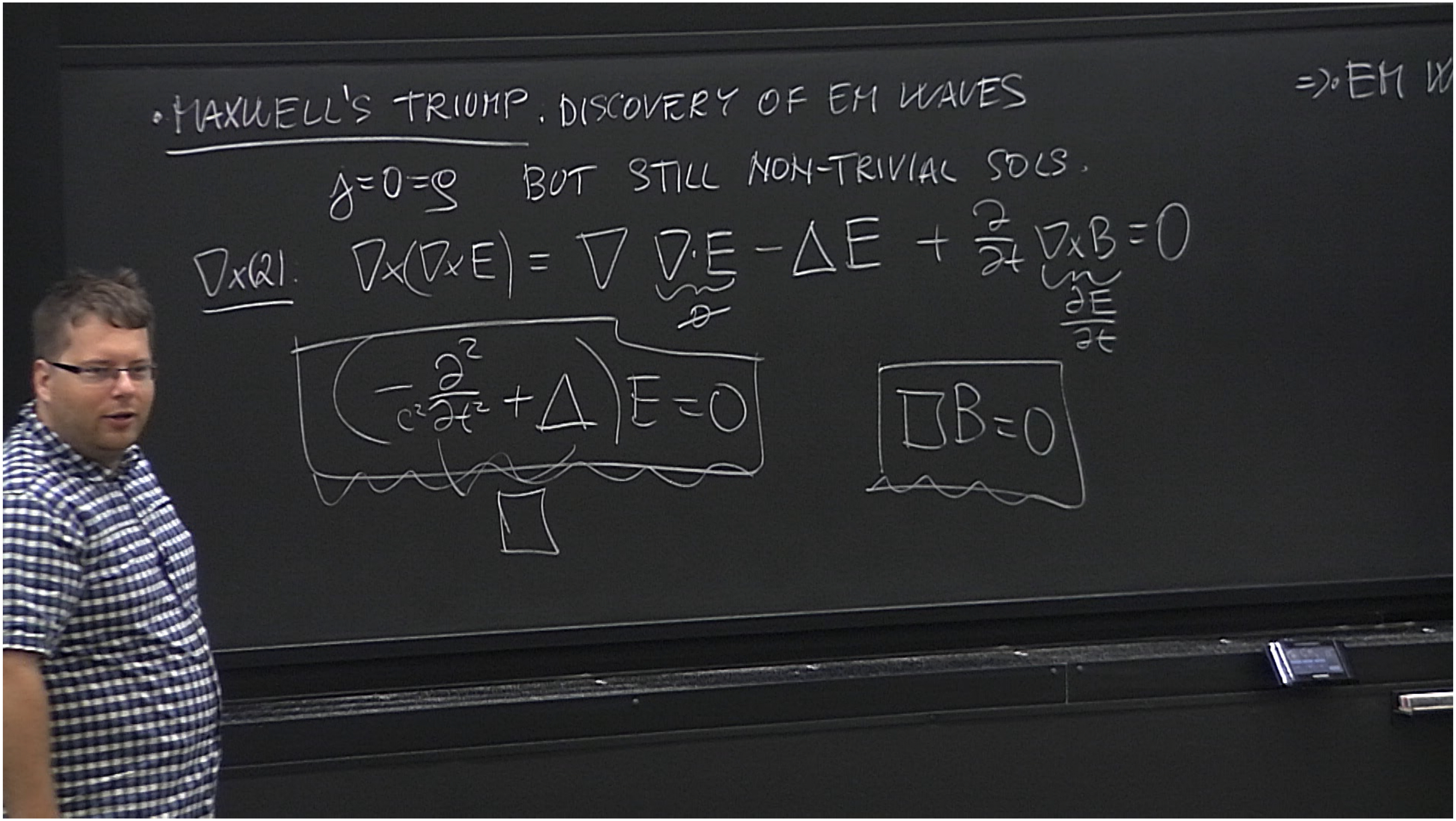
∂E/∂t
(−∂²/∂t² + Δ) E = 0
□B = 0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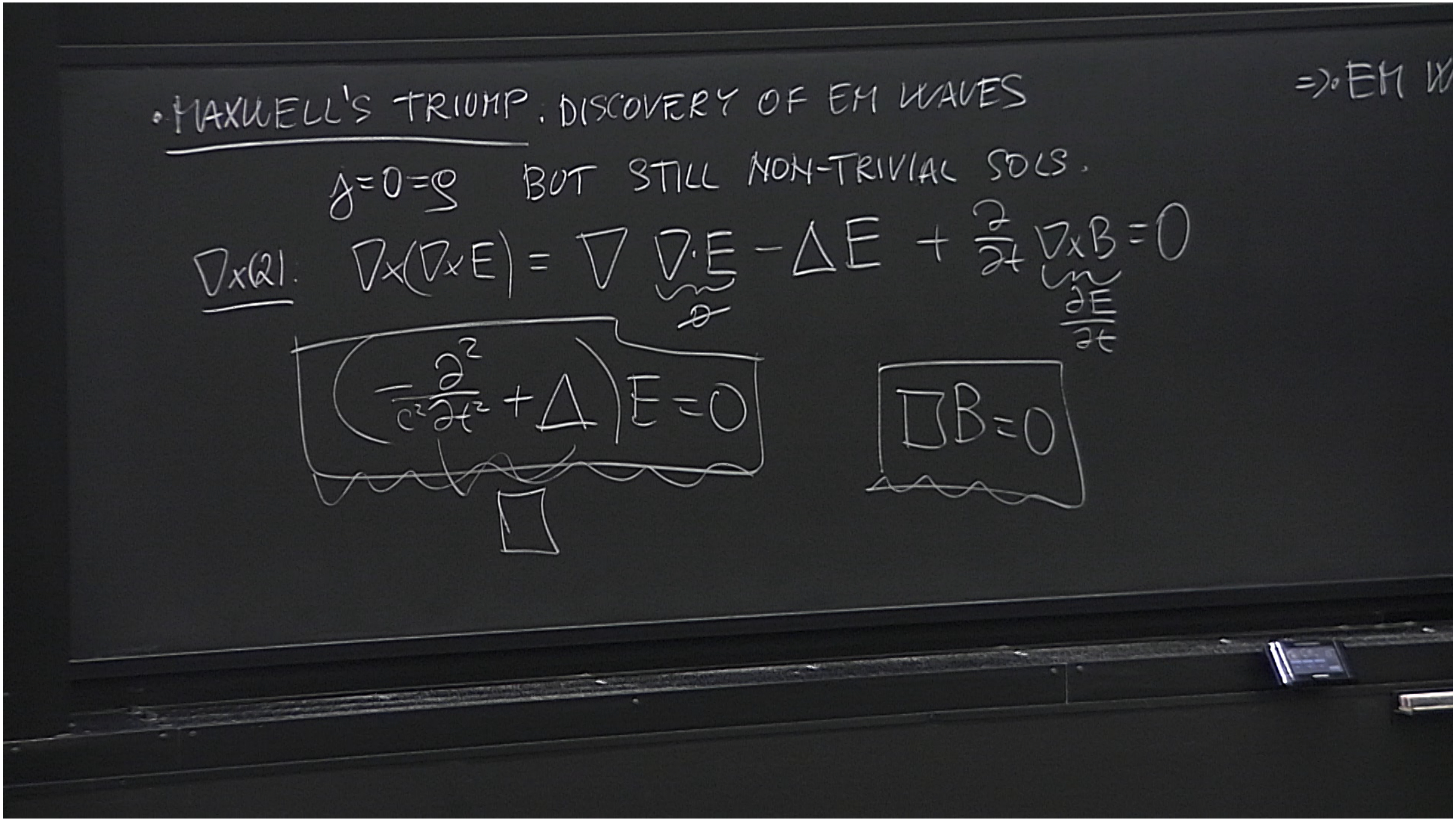
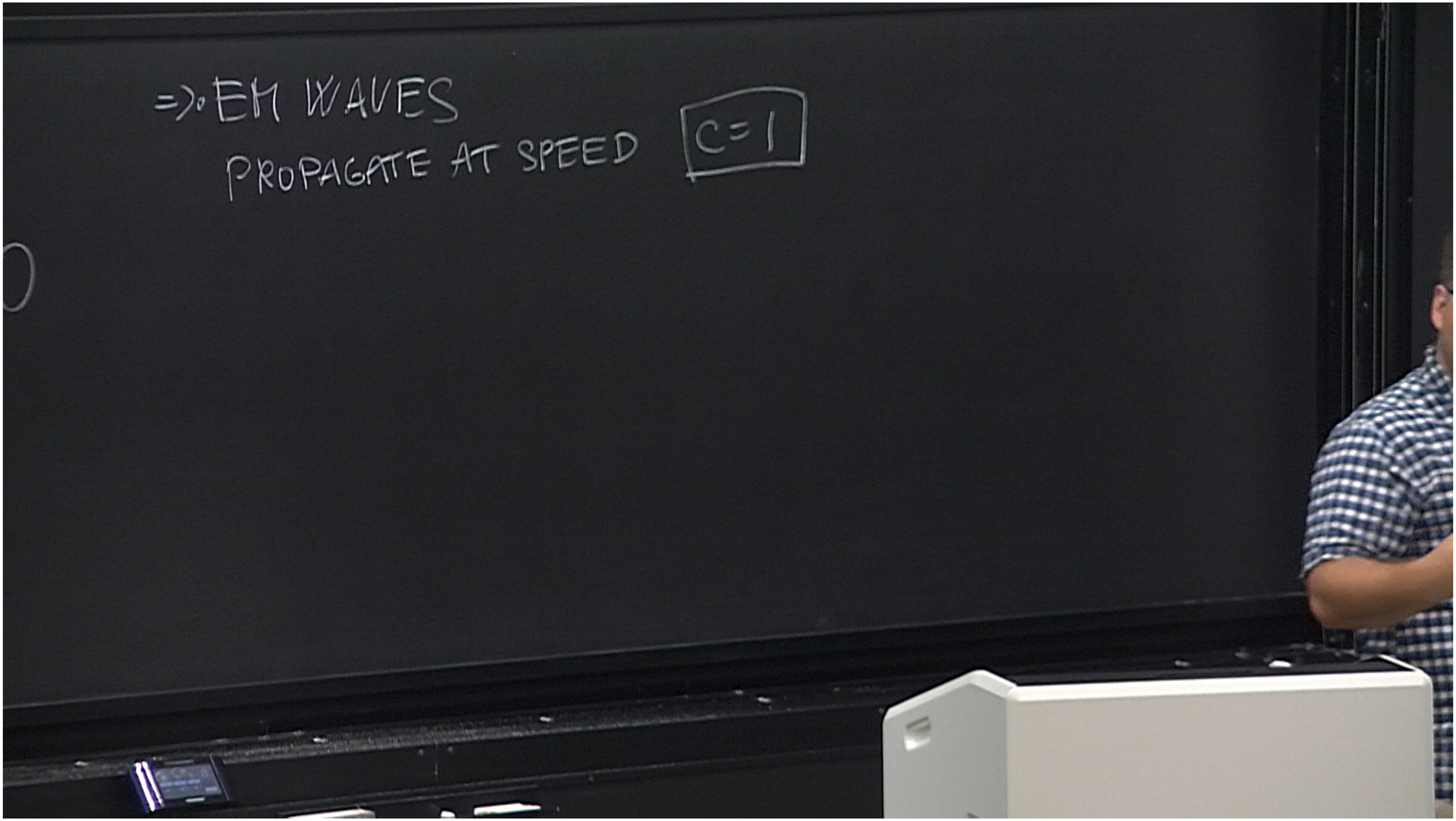
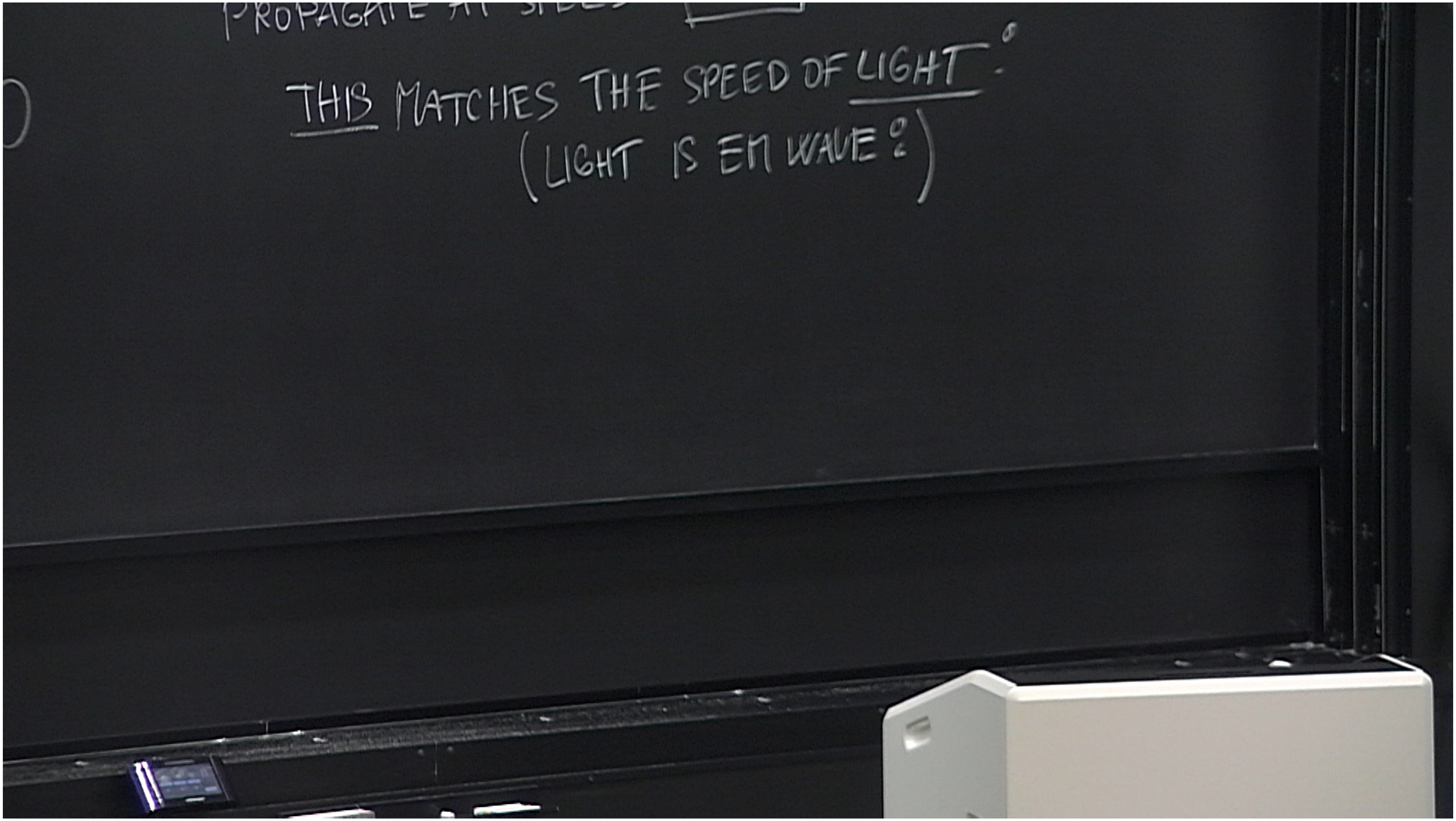
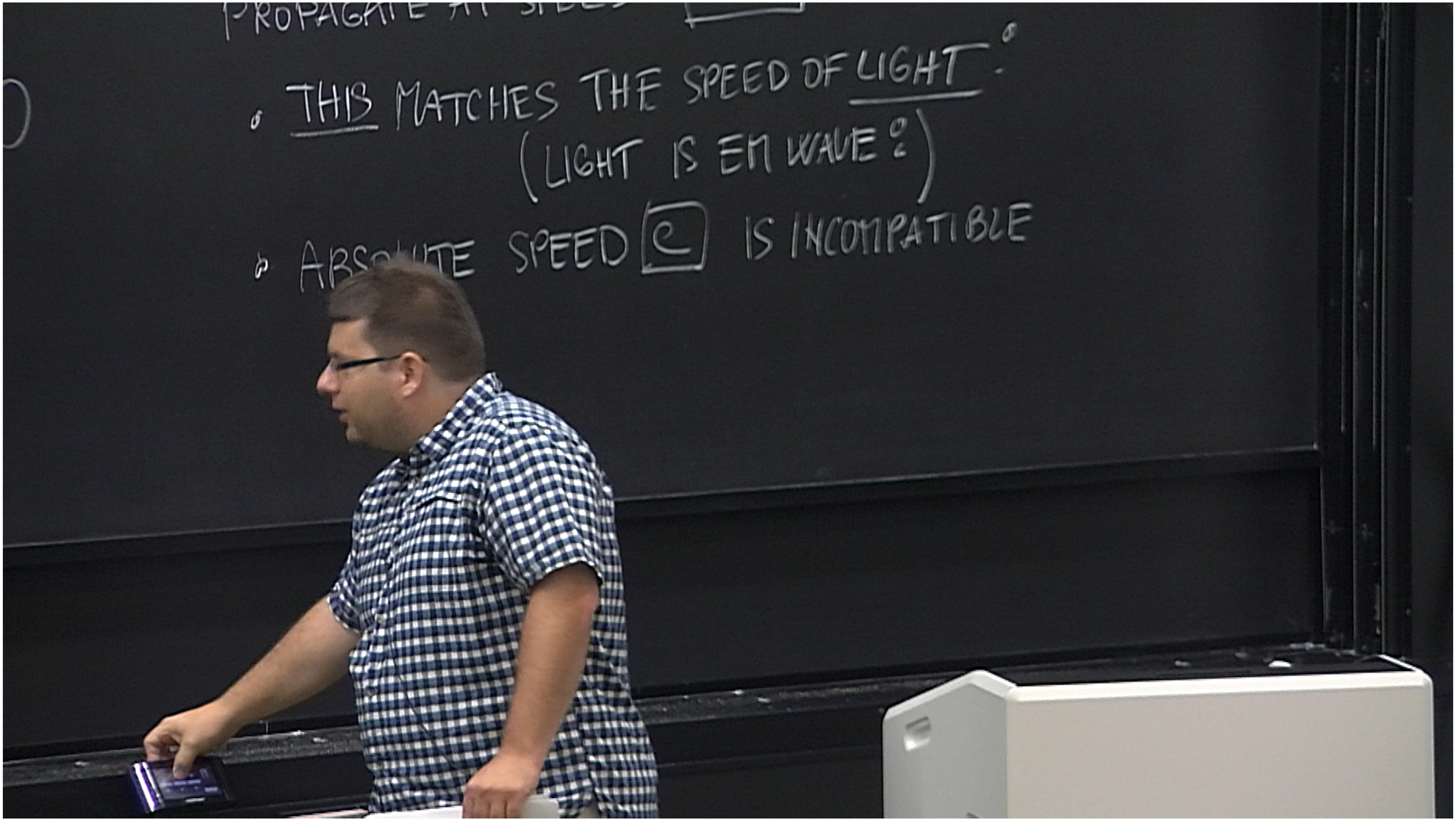
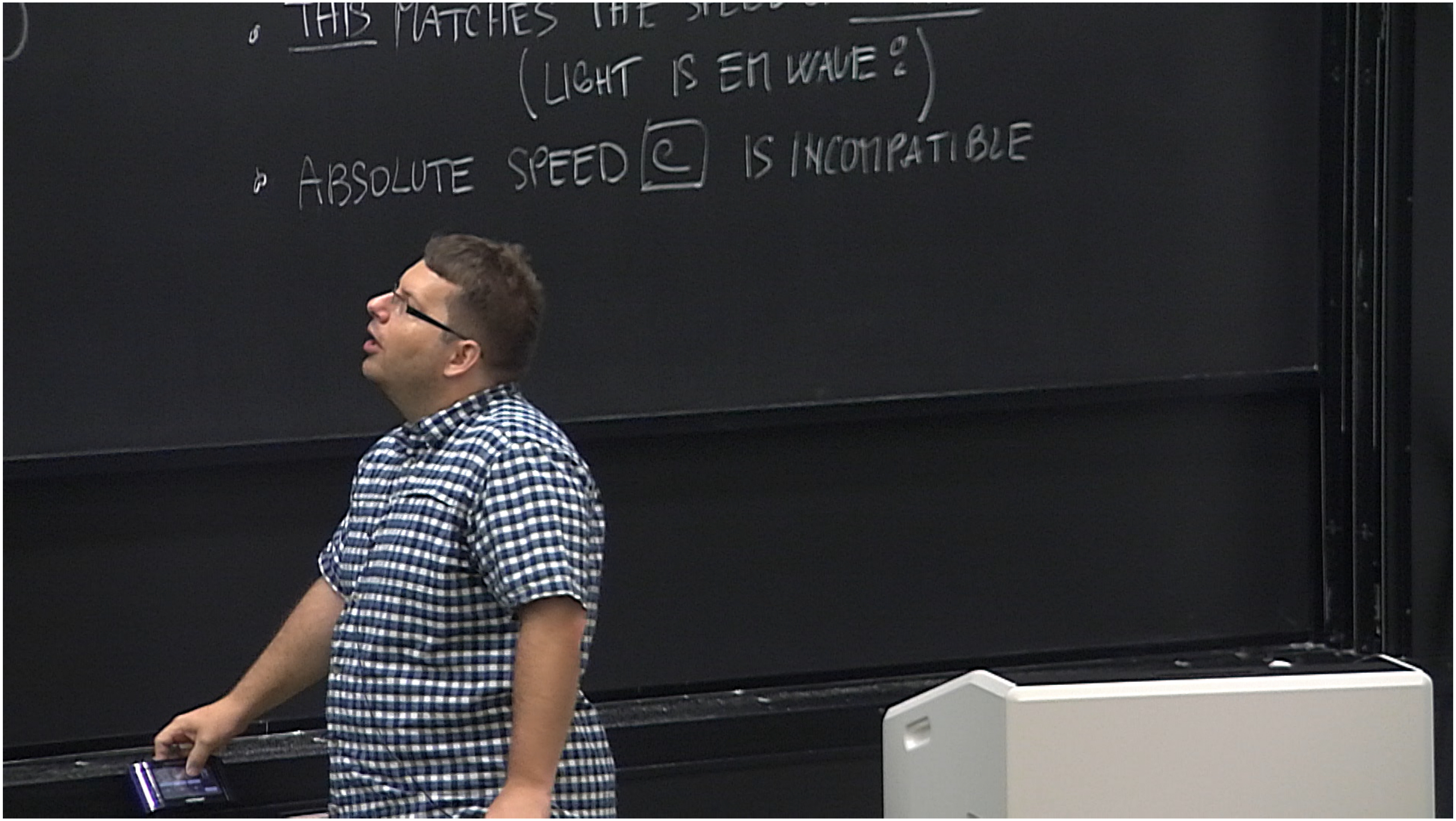
∇×(2). ∇×(∇×E) = ∇ ∇·E − ΔE + ∂/∂t ∇×B = 0
(−1/c² ∂²/∂t² + Δ)E = 0
□B = 0

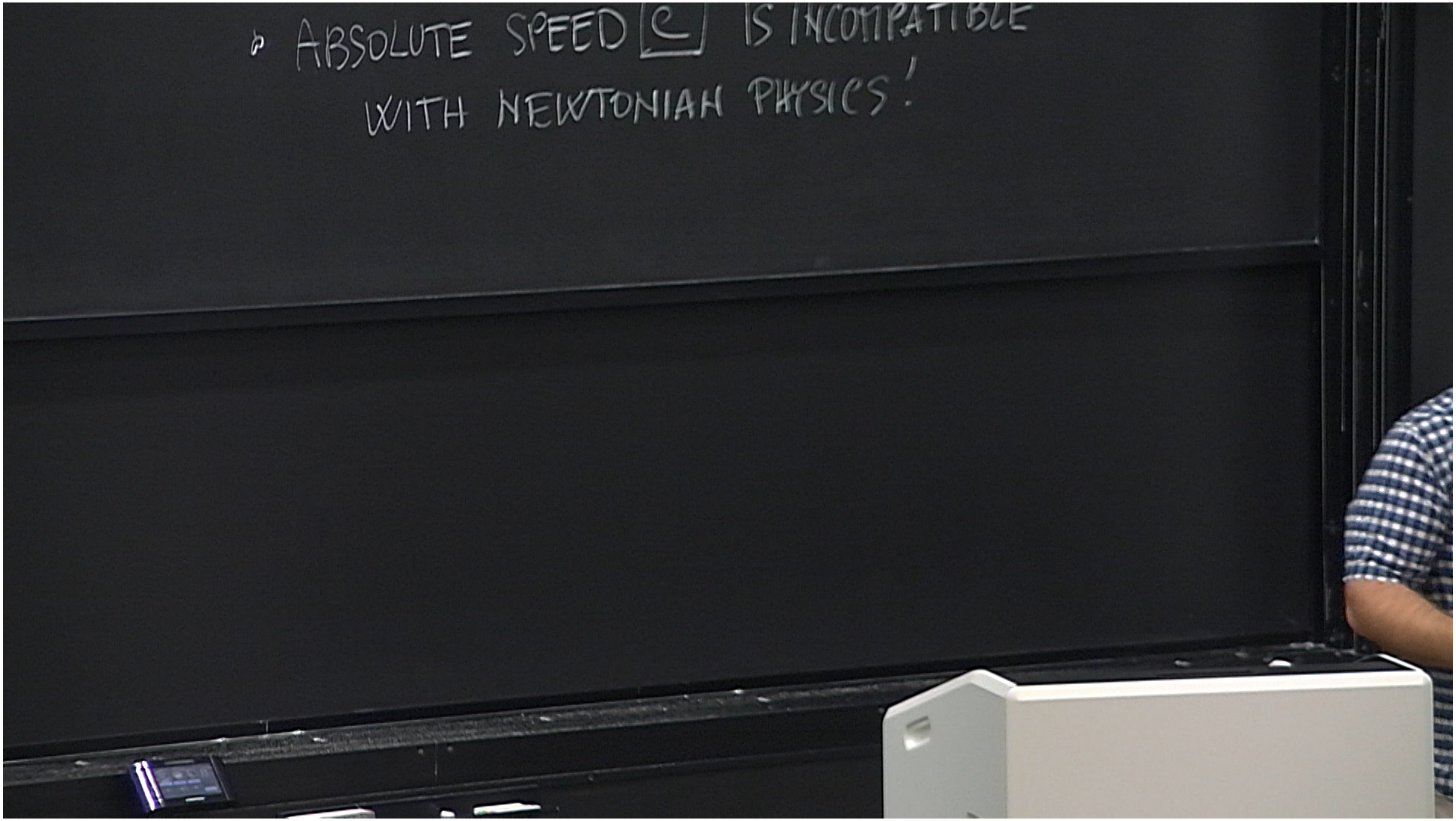

• MAXWELL'S TRIUMP. DISCOVERY OF EM WAVES
⇒ • EM W
j = 0 = ρ BUT STILL NON-TRIVIAL SOLS.
∇×Q1. ∇×(∇×E) = ∇ ∇·E − ΔE + ∂/∂t ∇×B = 0
(− 1/c² ∂²/∂t² + Δ) E = 0
□B = 0

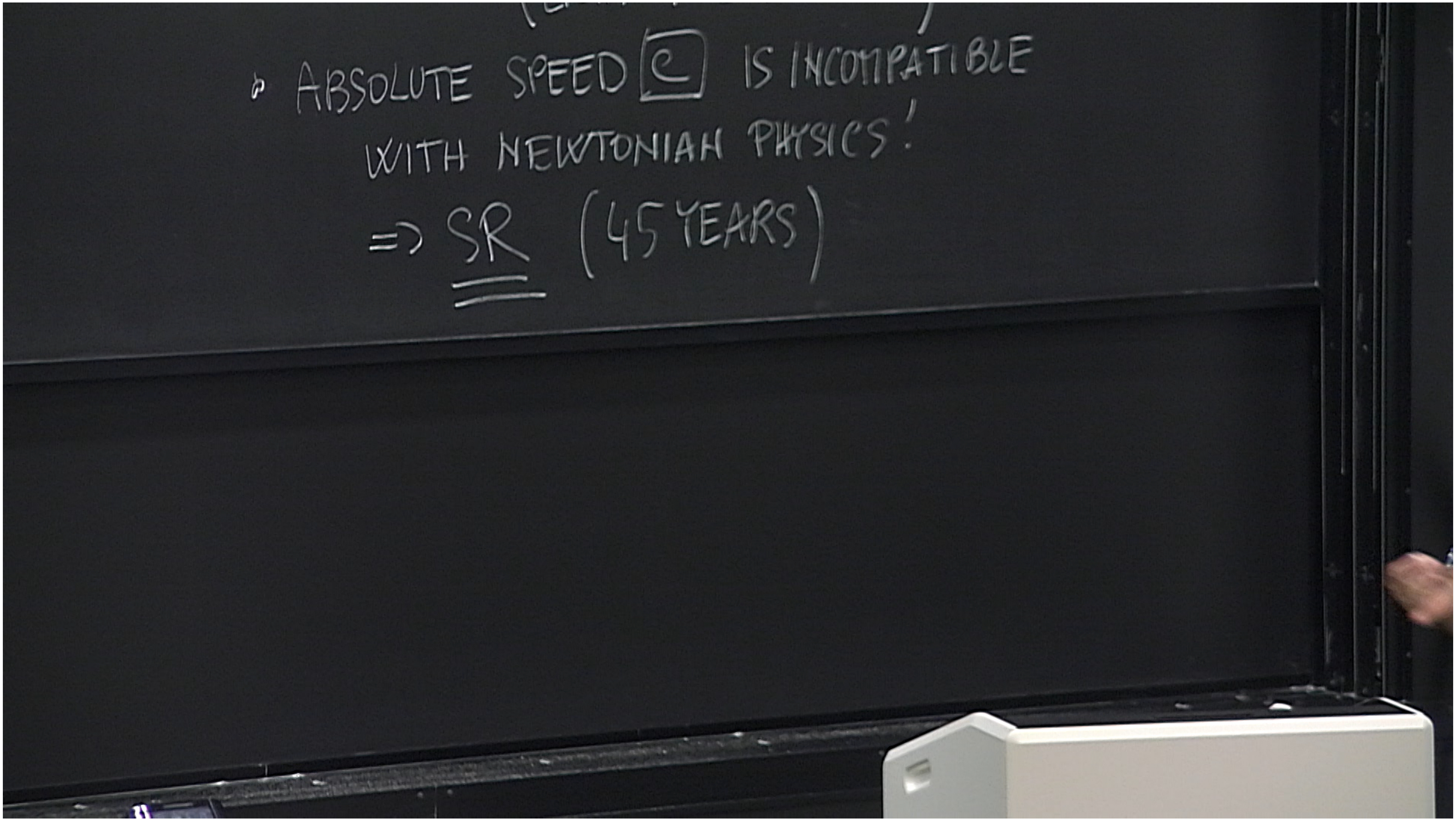

⇒ EM WAVES
PROPAGATE AT SPEED
c = 1


THIS MATCHES THE SPEED OF LIGHT!
(LIGHT IS EM WAVE ?)


PROPAGATE AT SPEED
THIS MATCHES THE SPEED OF LIGHT !
(LIGHT IS EM WAVE !)
ABSOLUTE SPEED c IS INCOMPATIBLE

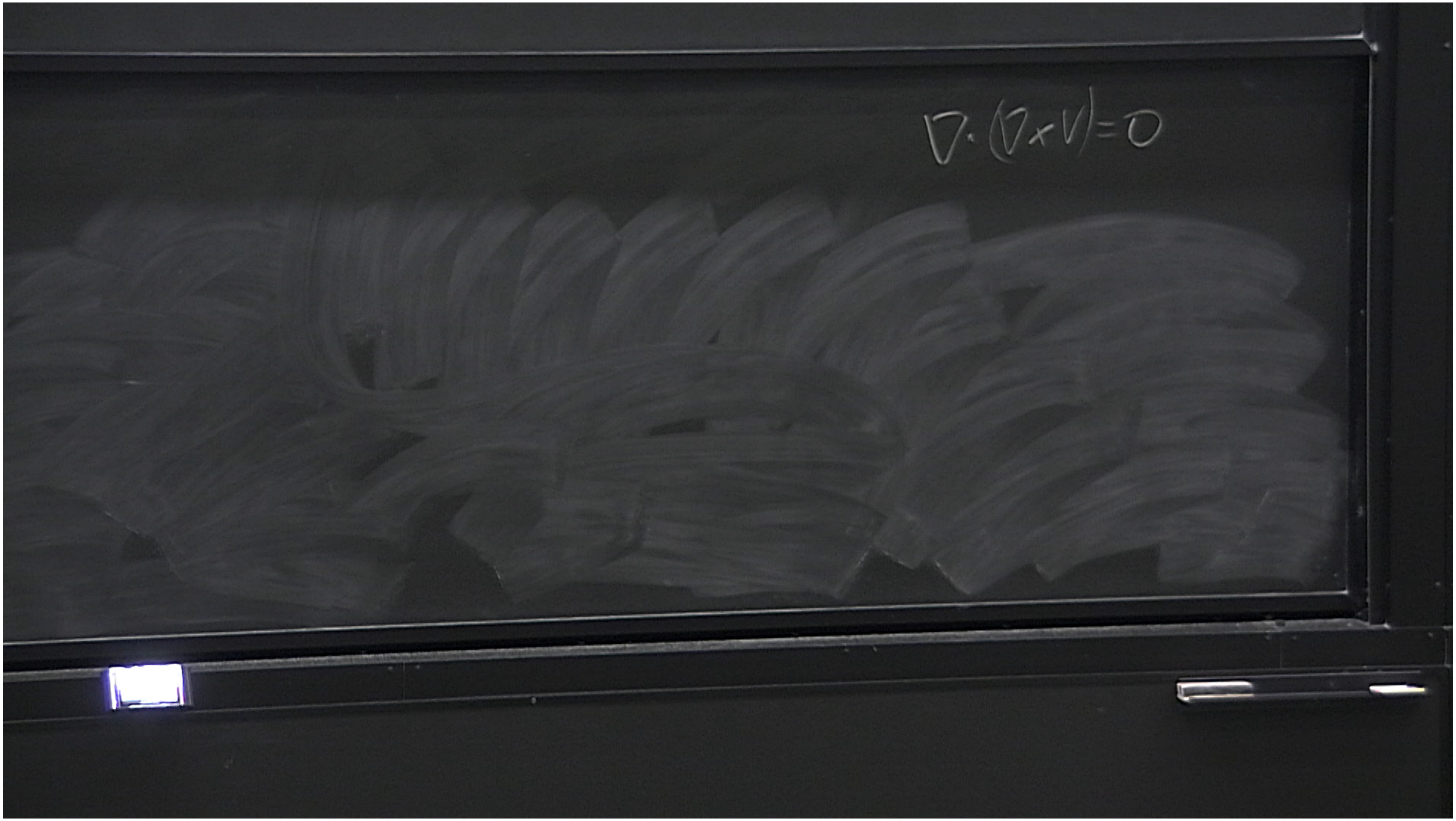

THIS MATCHES THE
(LIGHT IS EM WAVE !)
ABSOLUTE SPEED c IS INCOMPATIBLE

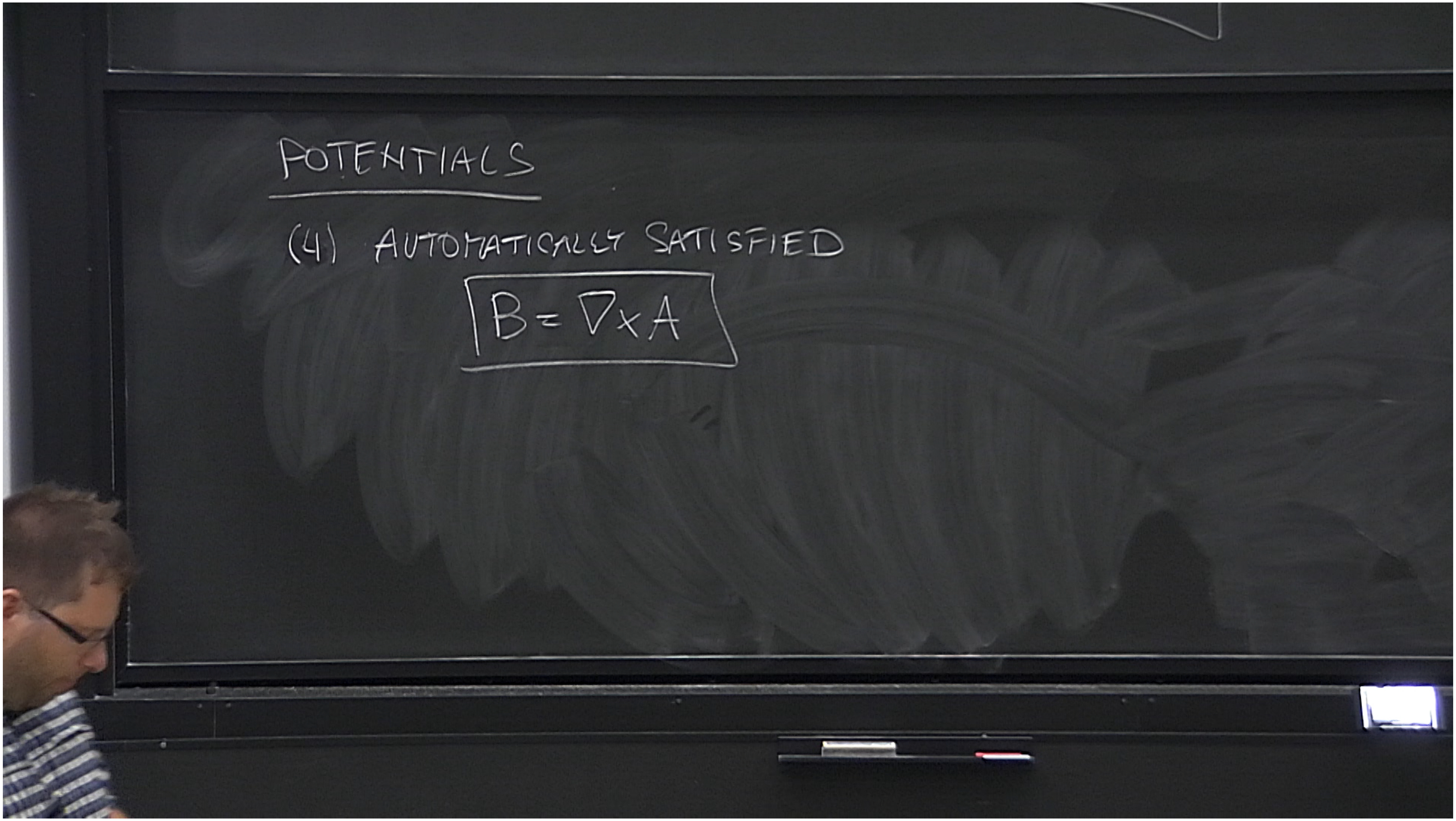

ABSOLUTE SPEED c IS INCOMPATIBLE
WITH NEWTONIAN PHYSICS!

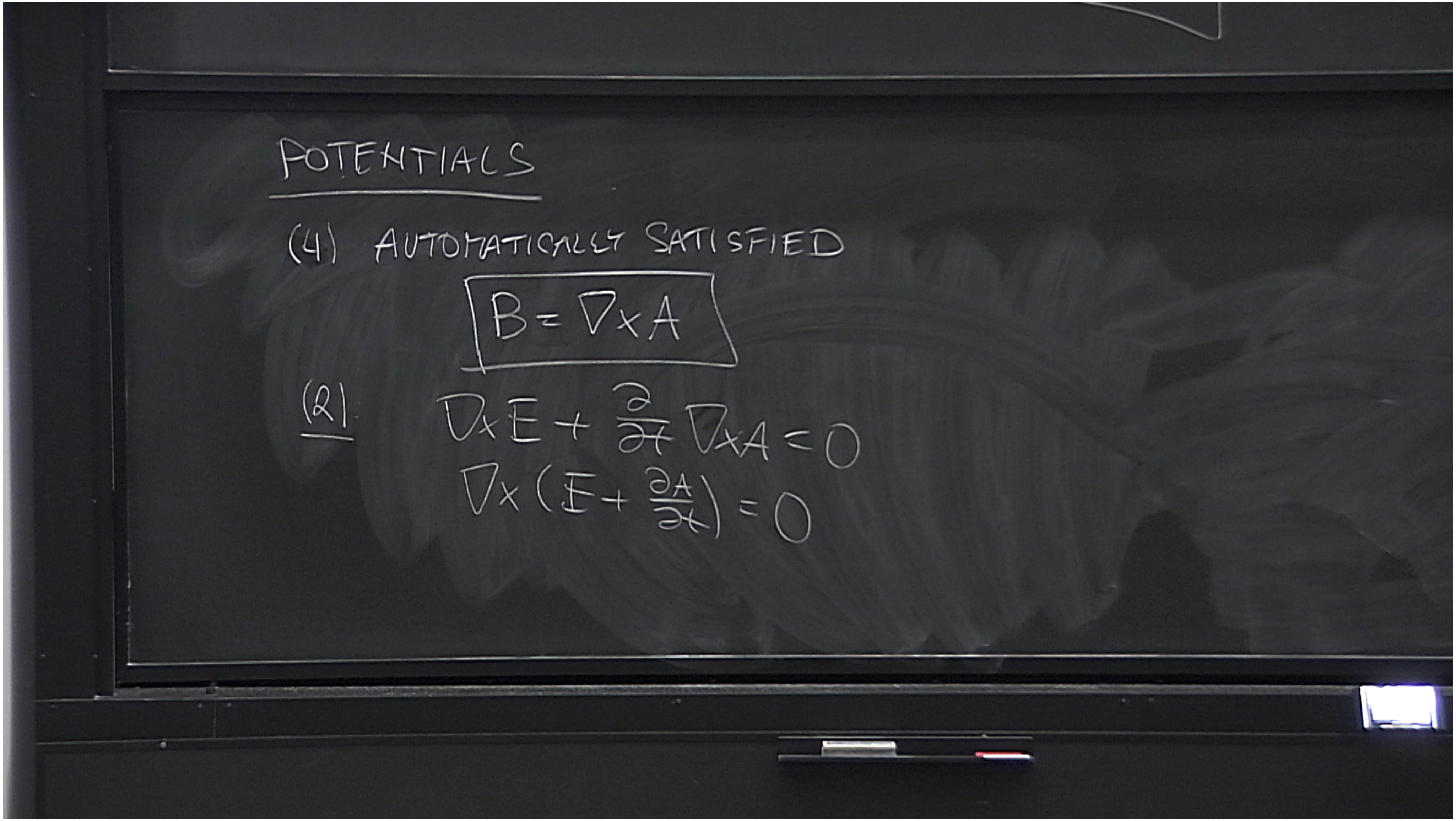

ABSOLUTE SPEED c IS INCOMPATIBLE
WITH NEWTONIAN PHYSICS!
=> SR (45 YEARS)

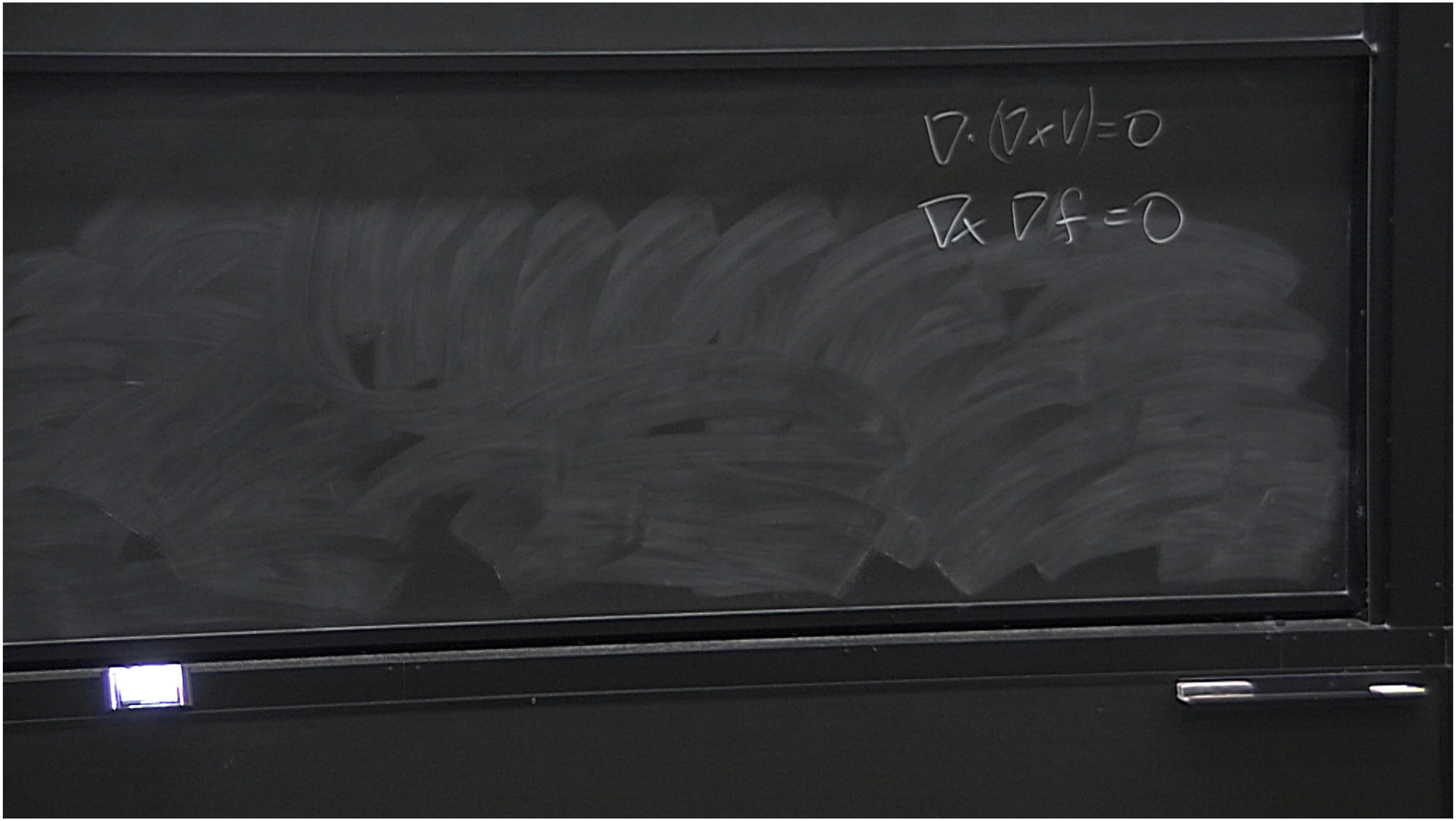
POTENTIALS

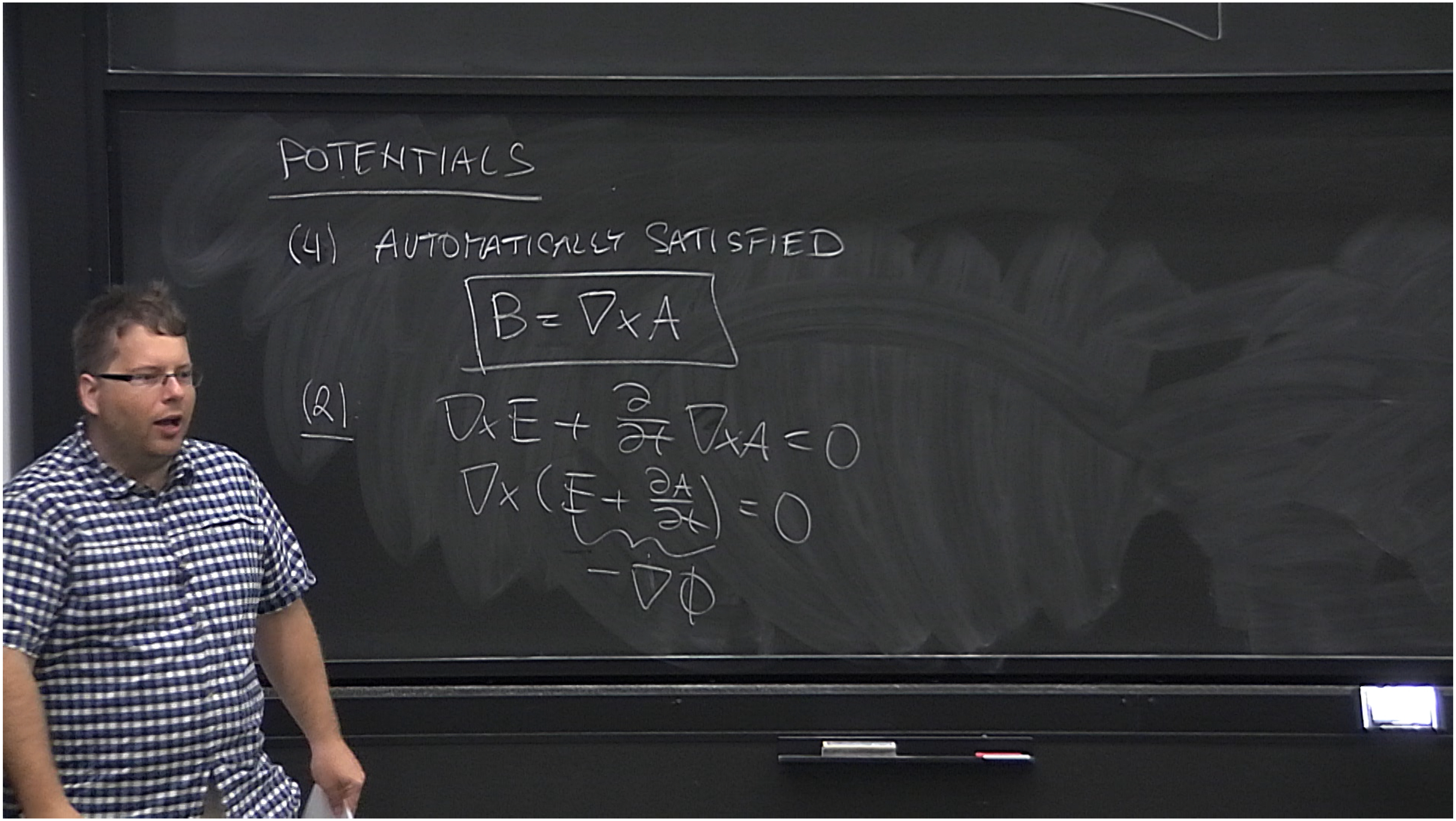
(4) AUTOMATICALLY SATISFIED


∇·(∇×V)=0


POTENTIALS
(4) AUTOMATICALLY SATISFIED
B = ∇ × A


POTENTIALS
(4) AUTOMATICALLY SATISFIED
B = ∇×A
(2) ∇×E + ∂/∂t ∇×A = 0
∇×(E + ∂A/∂t) = 0


$\nabla\cdot(\nabla\times V)=0$
$\nabla\times\nabla f=0$


POTENTIALS
(4) AUTOMATICALLY SATISFIED
B = ∇×A
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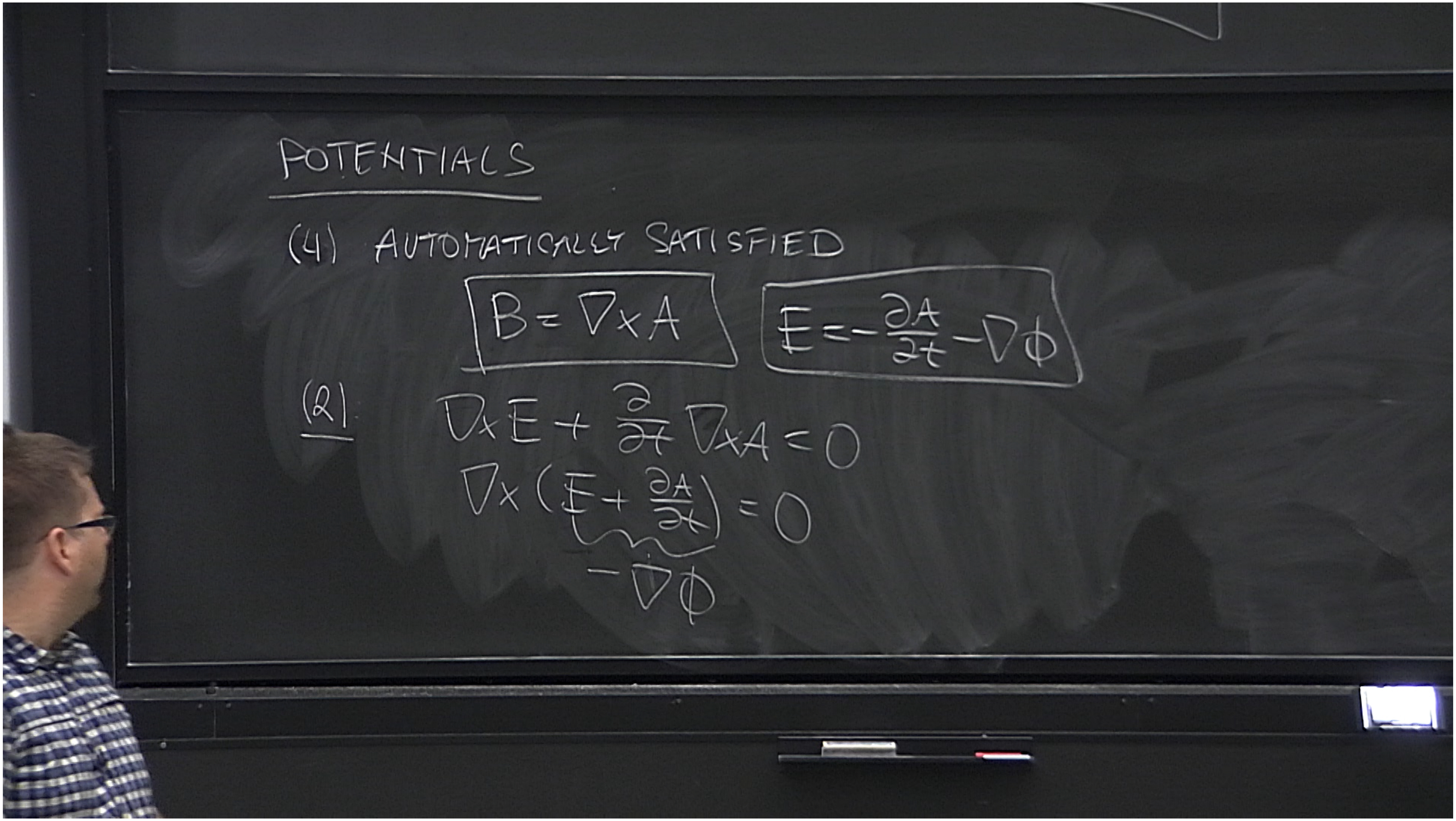
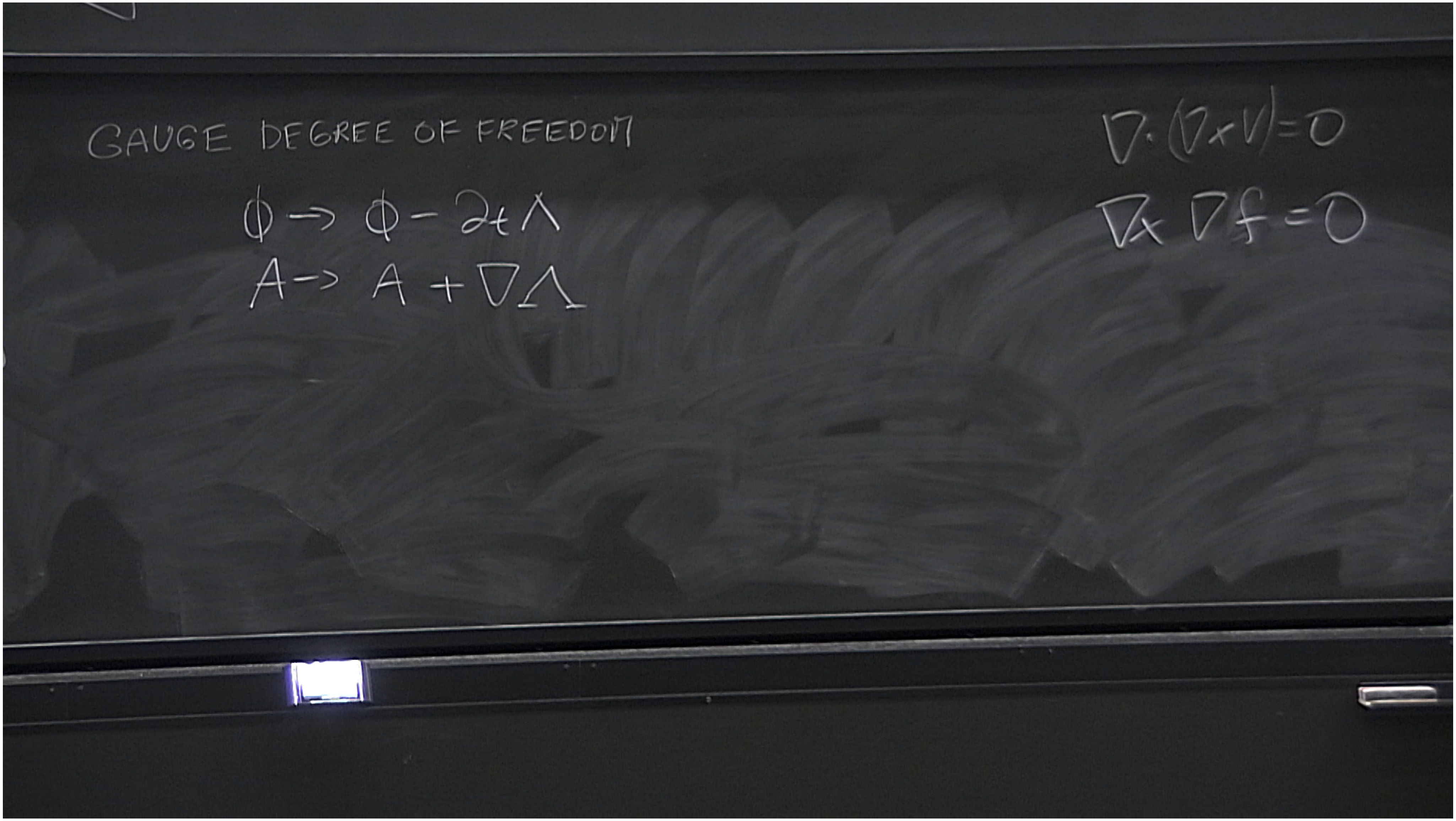
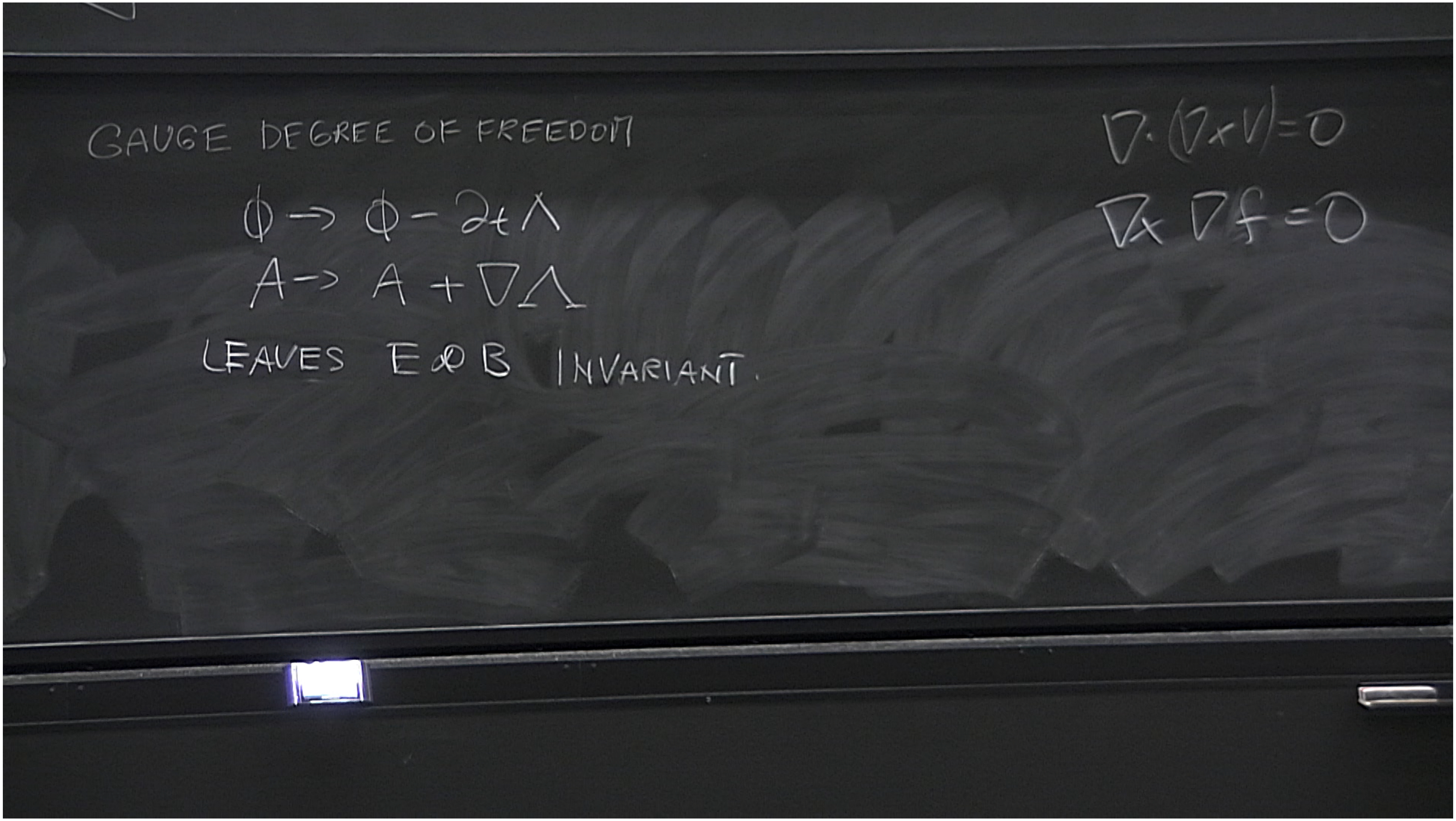
−∇ϕ


POTENTIALS
(4) AUTOMATICALLY SATISFIED
$B = \nabla \times A$
$E = -\frac{\partial A}{\partial t} - \nabla \phi$
(2)
$\nabla \times E + \frac{\partial}{\partial t} \nabla \times A = 0$
$\nabla \times \left(E + \frac{\partial A}{\partial t}\right) = 0$
$-\nabla \phi$


GAUGE DEGREE OF FREEDOM
$\phi \to \phi - \partial_t \Lambda$
$A \to A + \nabla \Lambda$
$\nabla \cdot (\nabla \times V) = 0$
$\nabla \times \nabla f = 0$


GAUGE DEGREE OF FREEDOM
$\phi \to \phi - \partial_t \Lambda$
$A \to A + \nabla \Lambda$
LEAVES E & B INVARIANT.
$\nabla \cdot (\nabla \times V) = 0$
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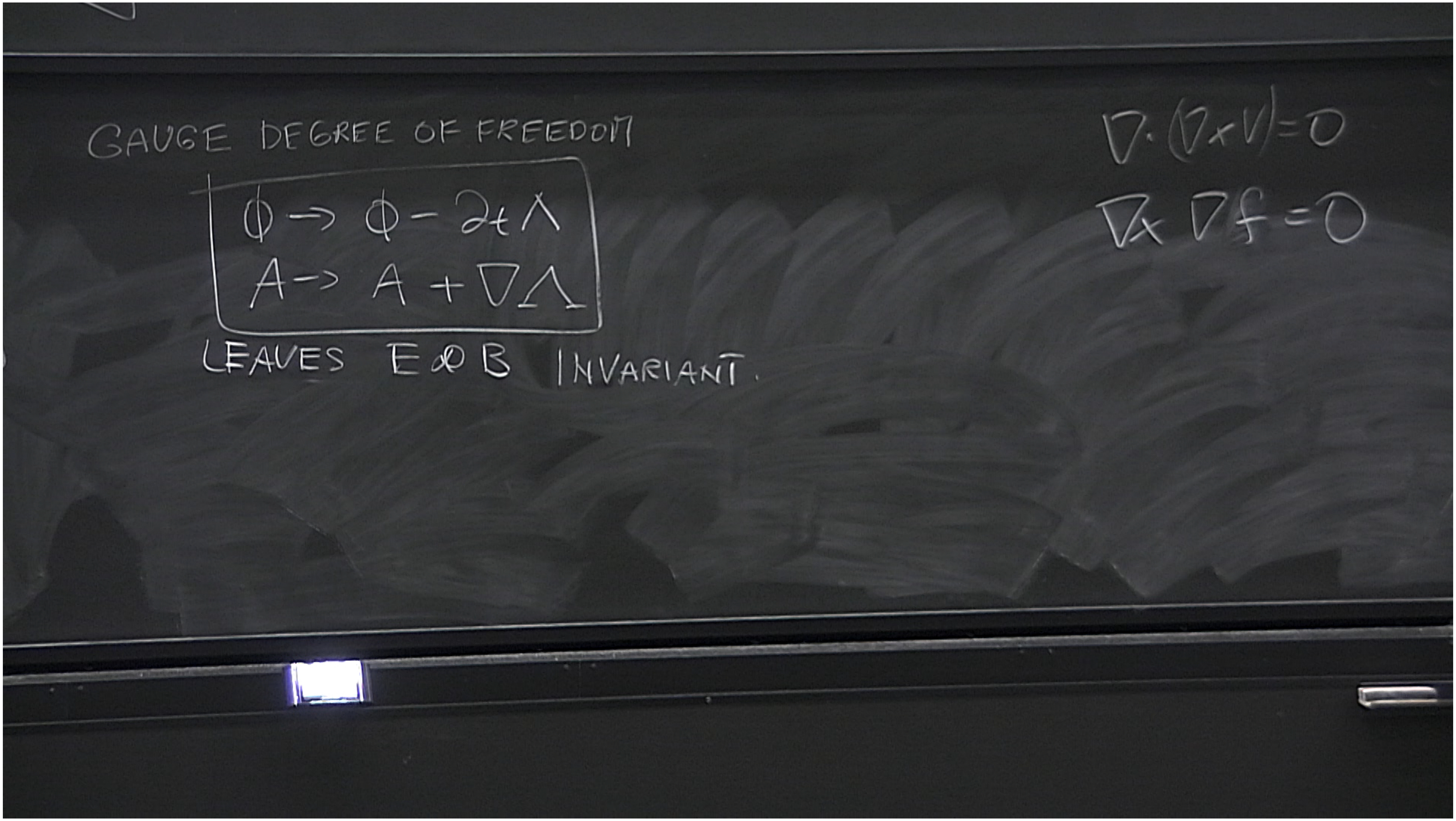
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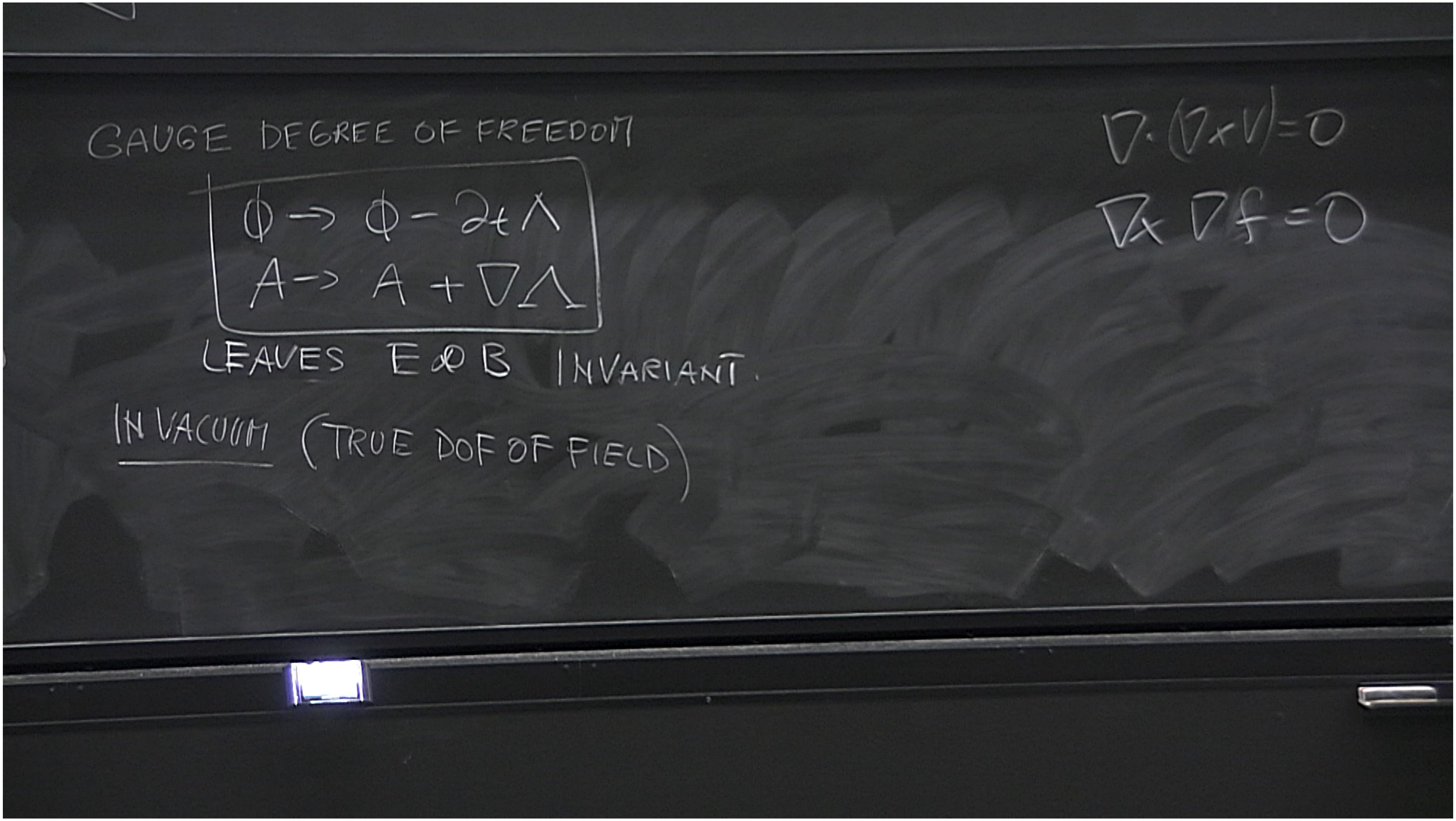

GAUGE DEGREE OF FREEDOM
$\phi \to \phi - \partial_t \Lambda$
$A \to A + \nabla \Lambda$
LEAVES E & B INVARIANT.
IN VACUUM (TRUE DOF OF FIELD)
$\nabla \cdot (\nabla \times V) = 0$
$\nabla \times \nabla f = 0$

GAUGE DEGREE OF FREEDOM

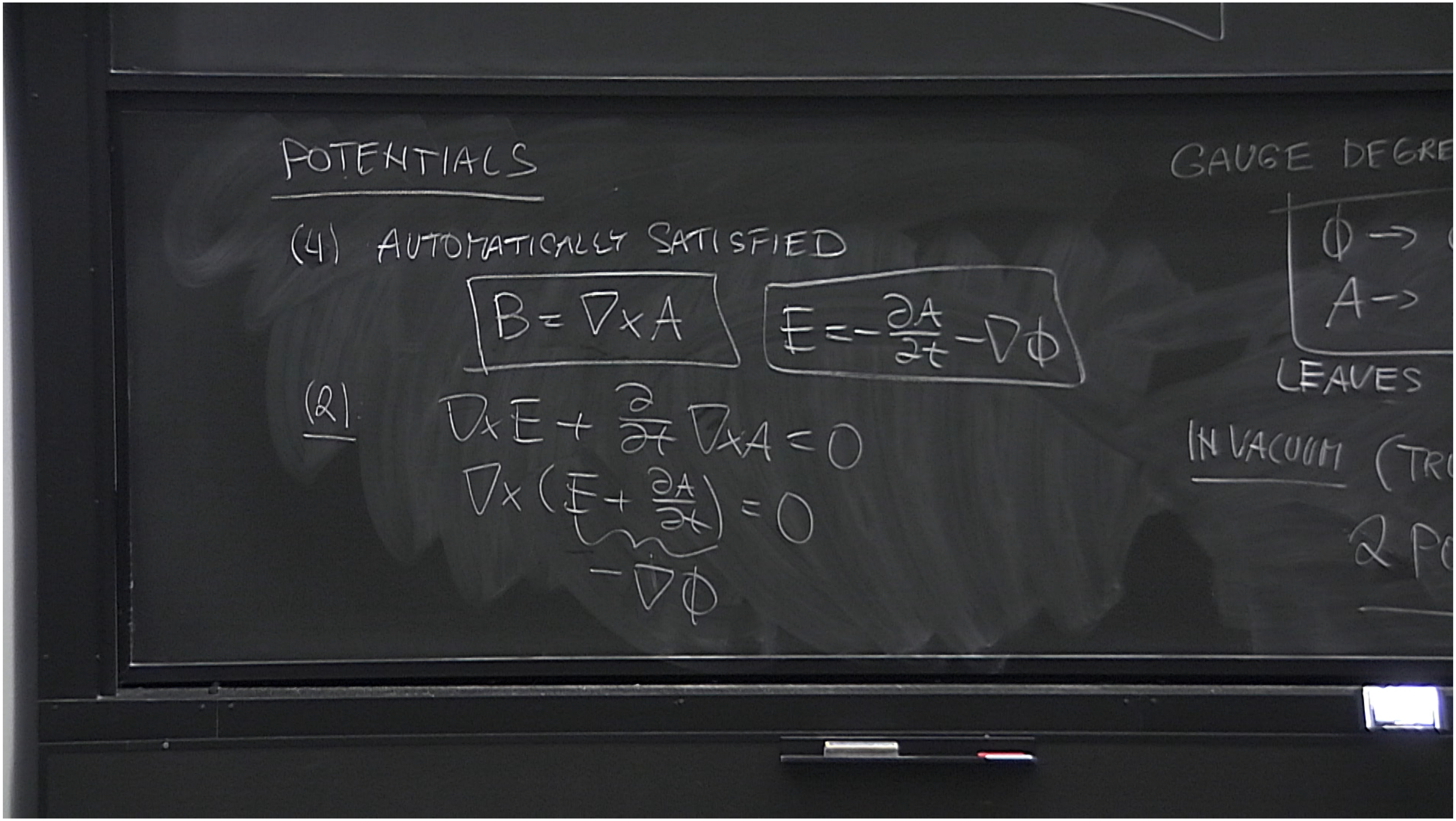
$$\phi \rightarrow \phi - \partial_t \Lambda$$
$$A \rightarrow A + \nabla \Lambda$$

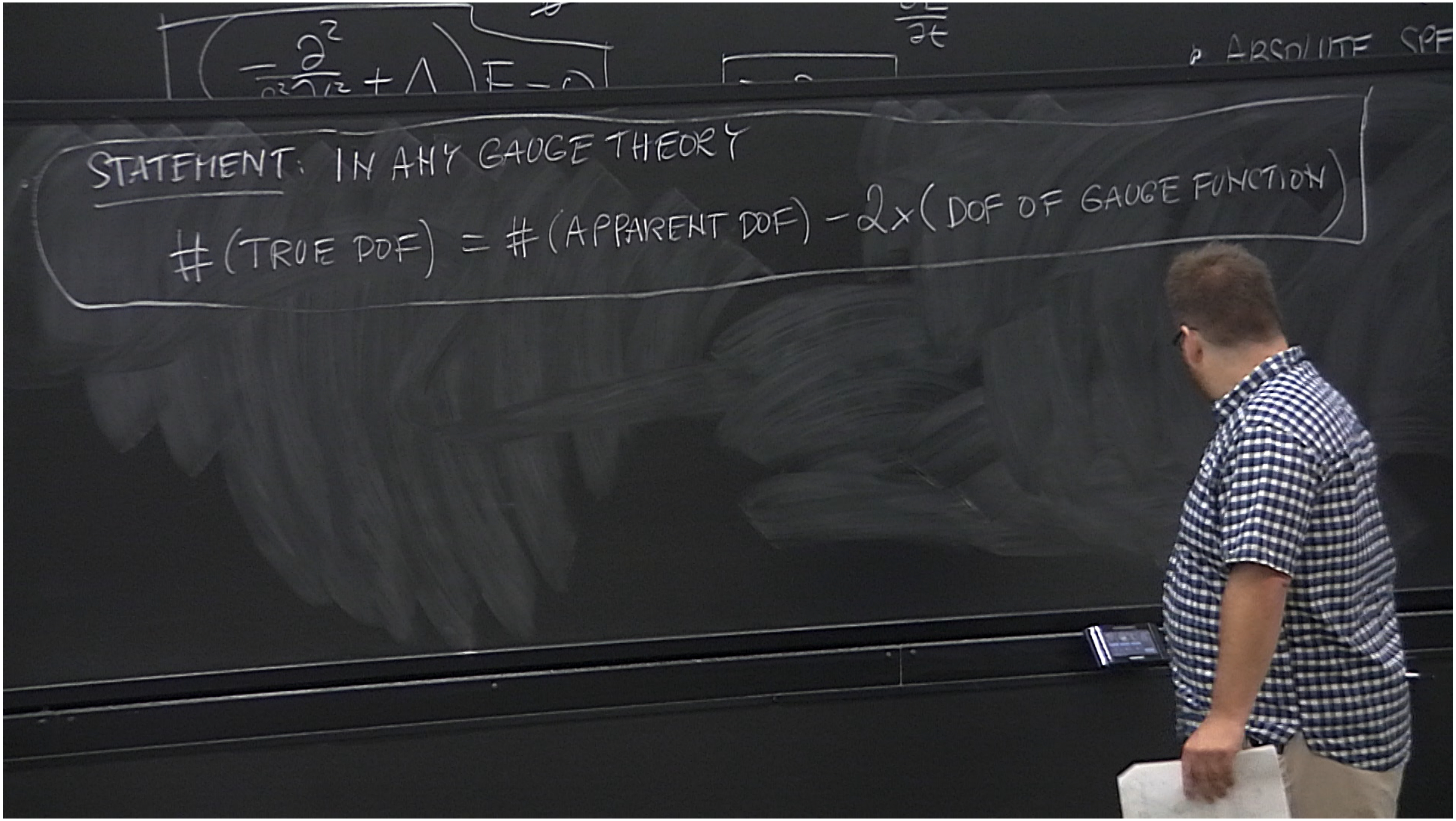
LEAVES E & B INVARIANT.

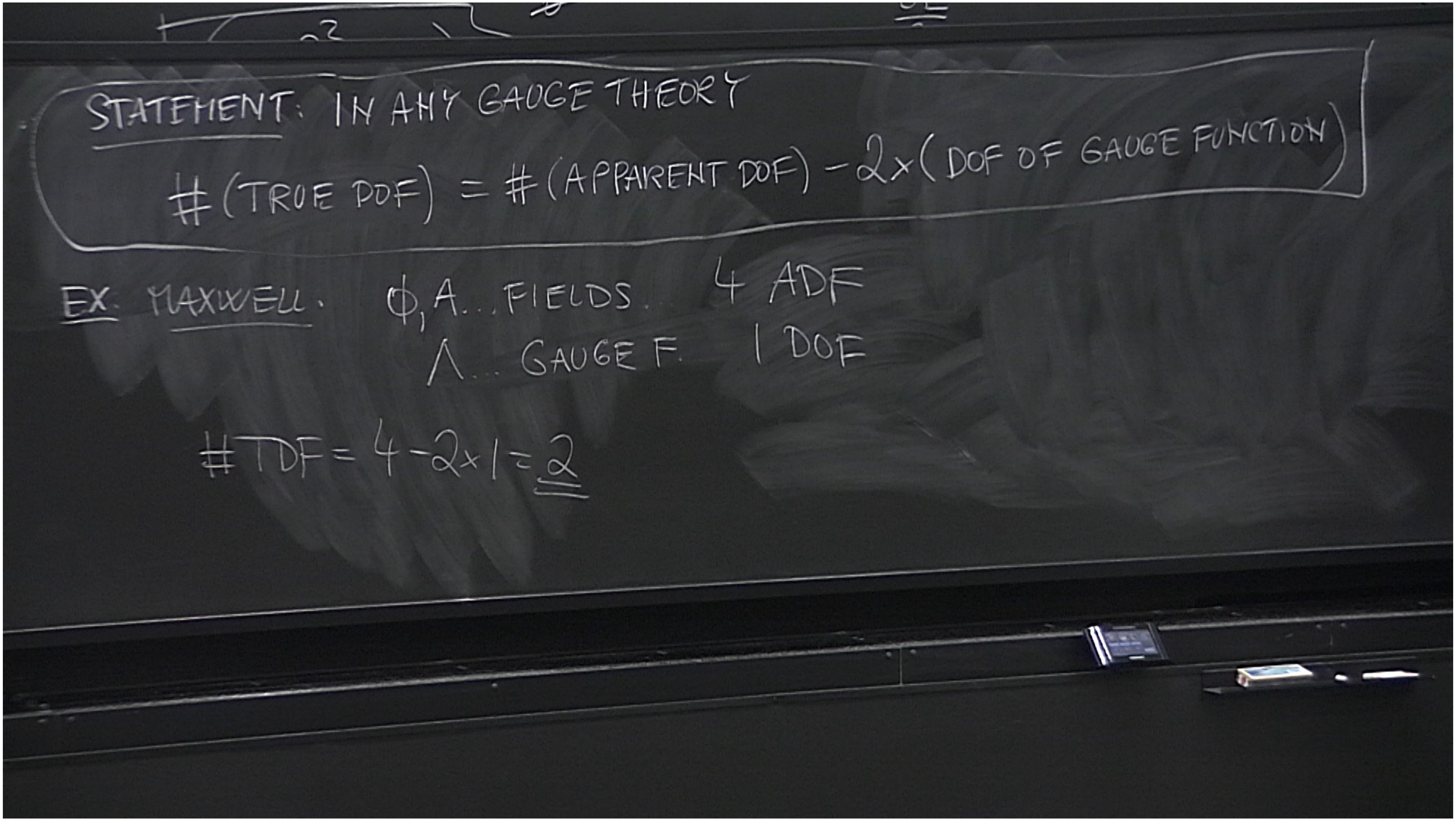
$$\nabla \cdot (\nabla \times V) = 0$$
$$\nabla \times \nabla f = 0$$

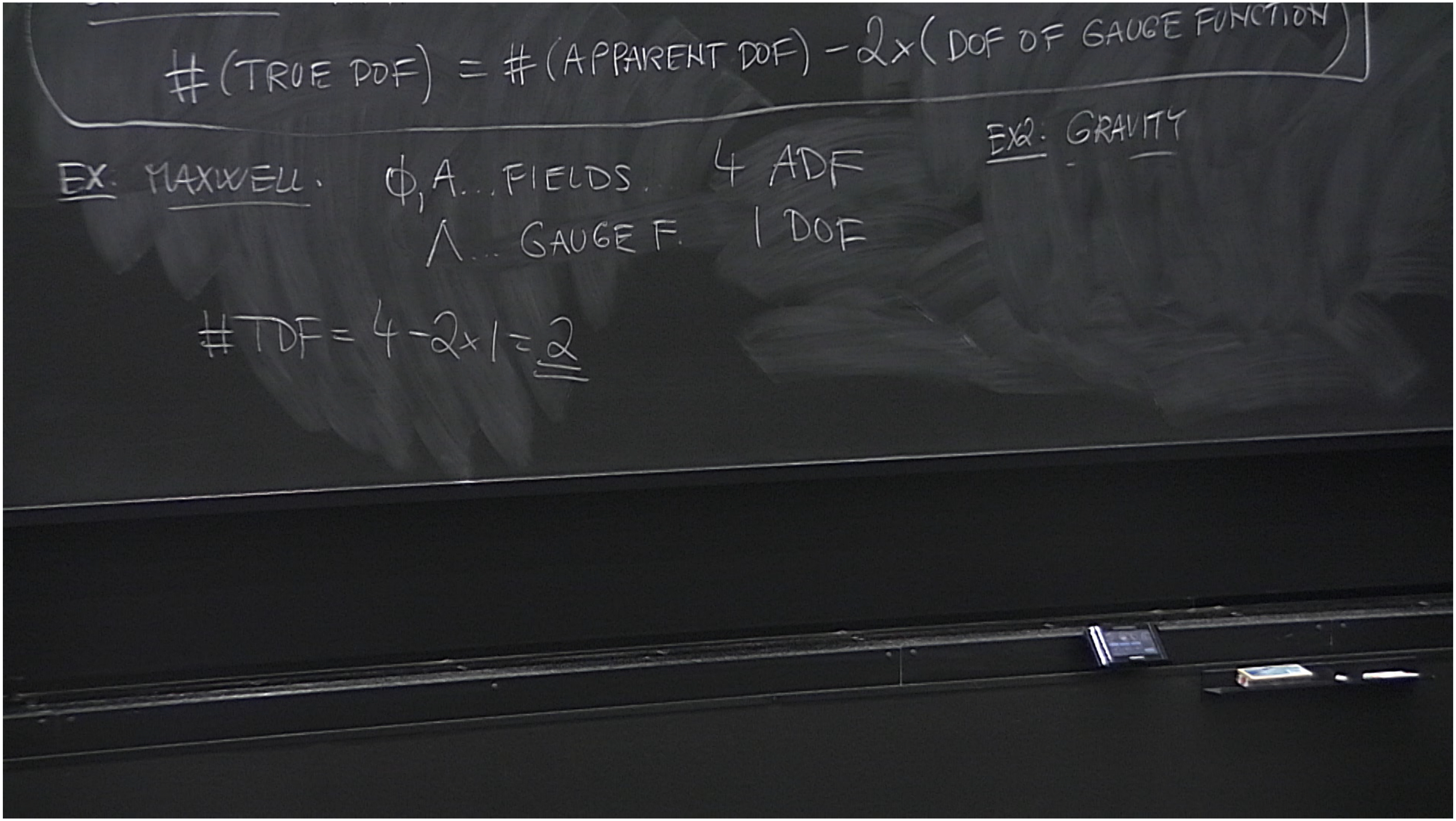
IN VACUUM (TRUE DOF OF FIELD)

2 POLARIZATIONS


POTENTIALS
(4) AUTOMATICALLY SATISFIED
$B = \nabla \times A$
$E = -\frac{\partial A}{\partial t} - \nabla \phi$
(2) $\nabla \times E + \frac{\partial}{\partial t} \nabla \times A = 0$
$\nabla \times \left(E + \frac{\partial A}{\partial t} \right) = 0$
$-\nabla \phi$
GAUGE
$\phi \to$
$A \to$
LEAVES
IN VACUUM


STATEMENT: IN ANY GAUGE THEORY
#(TRUE DOF) = #(APPARENT DOF) − 2×(DOF OF GAUGE FUNCTION)


STATEMENT: IN ANY GAUGE THEORY
#(TRUE DOF) = #(APPARENT DOF) − 2×(DOF OF GAUGE FUNCTION)
EX. MAXWELL. φ, A... FIELDS... 4 ADF
Λ... GAUGE F. 1 DOF
#TDF = 4 − 2×1 = 2


#(TRUE DOF) = #(APPARENT DOF) - 2x(DOF OF GAUGE FUNCTION)
EX: MAXWELL.
ϕ, A... FIELDS... 4 ADF
Λ... GAUGE F. 1 DOF
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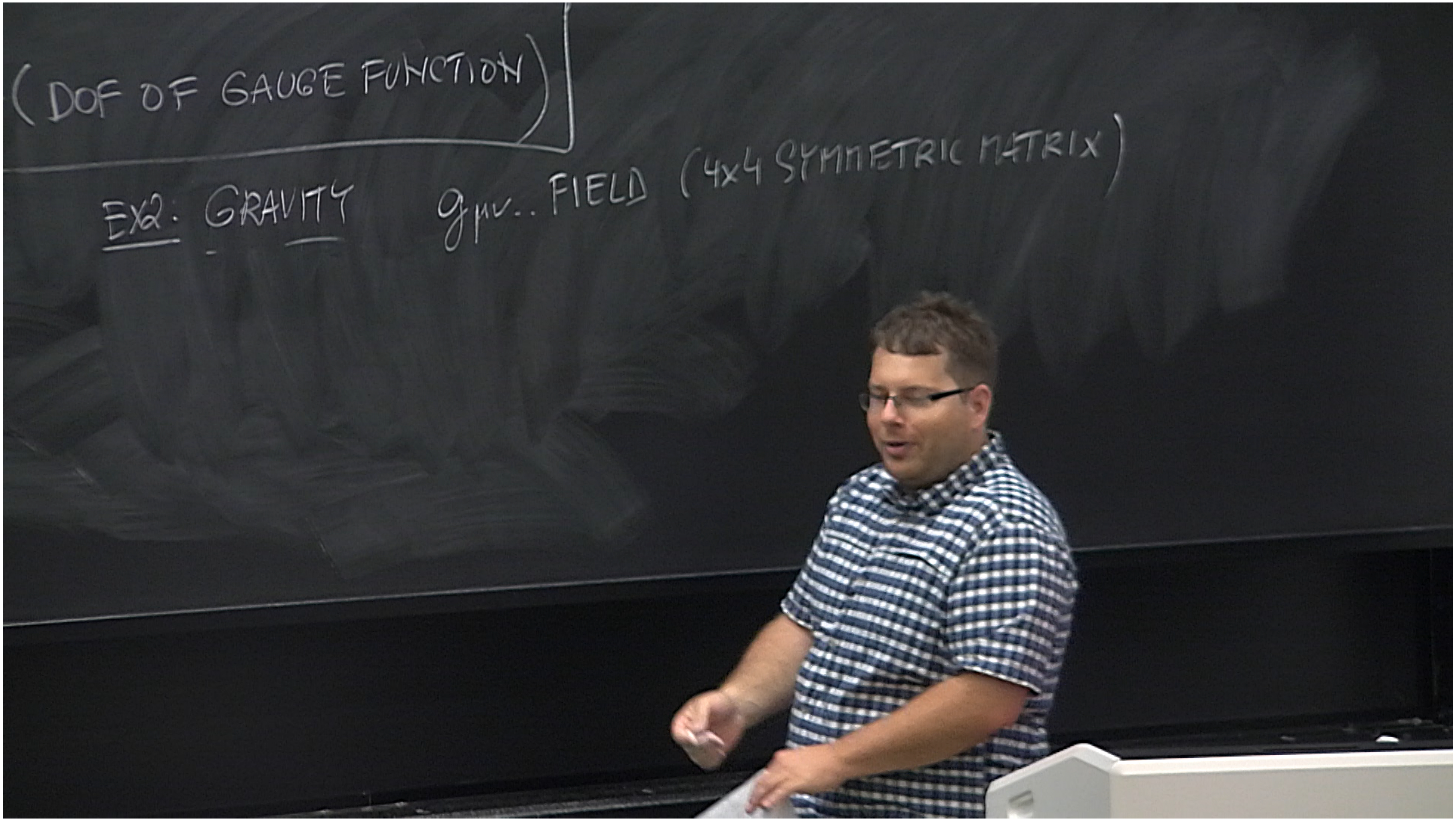
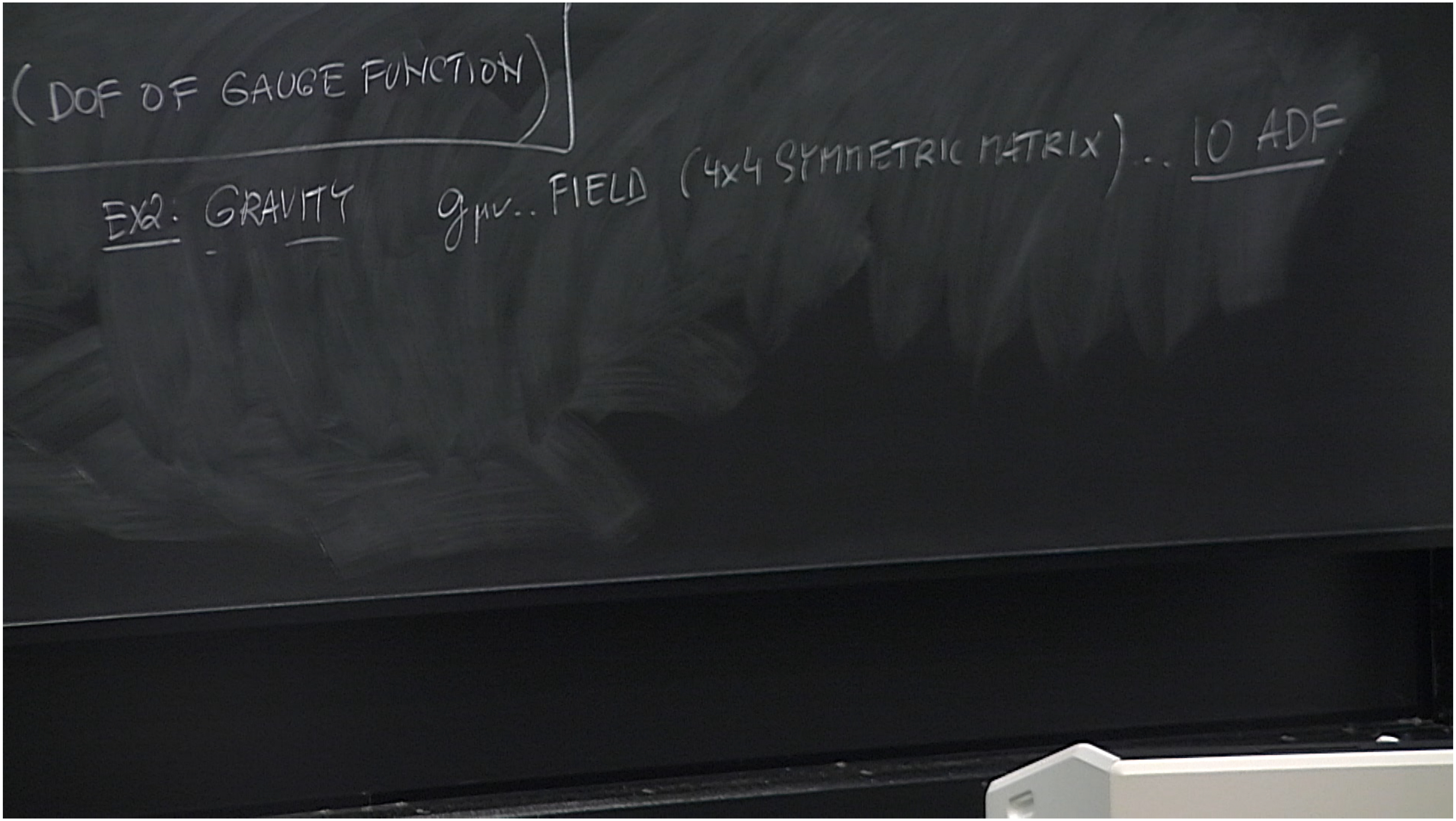
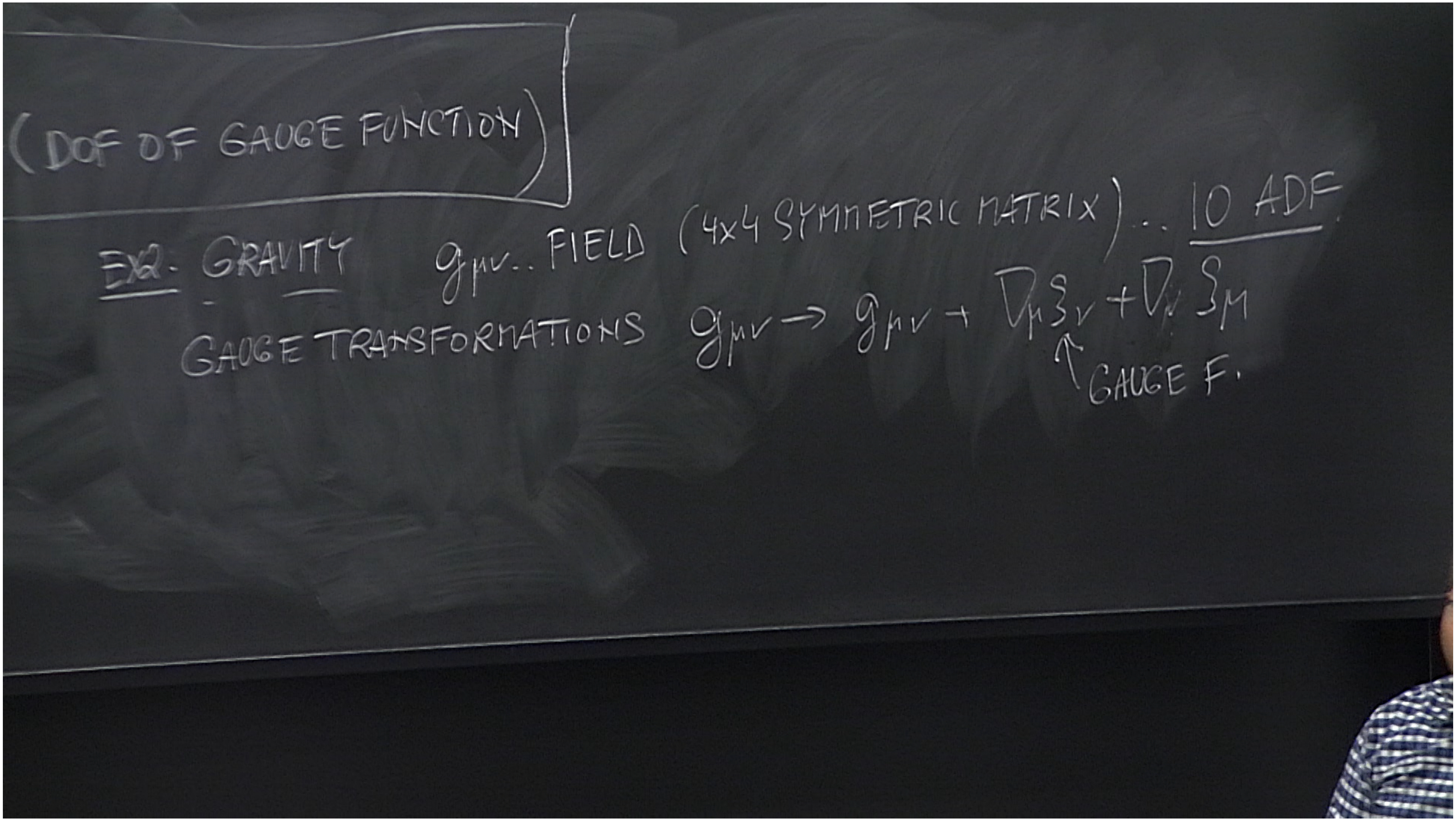
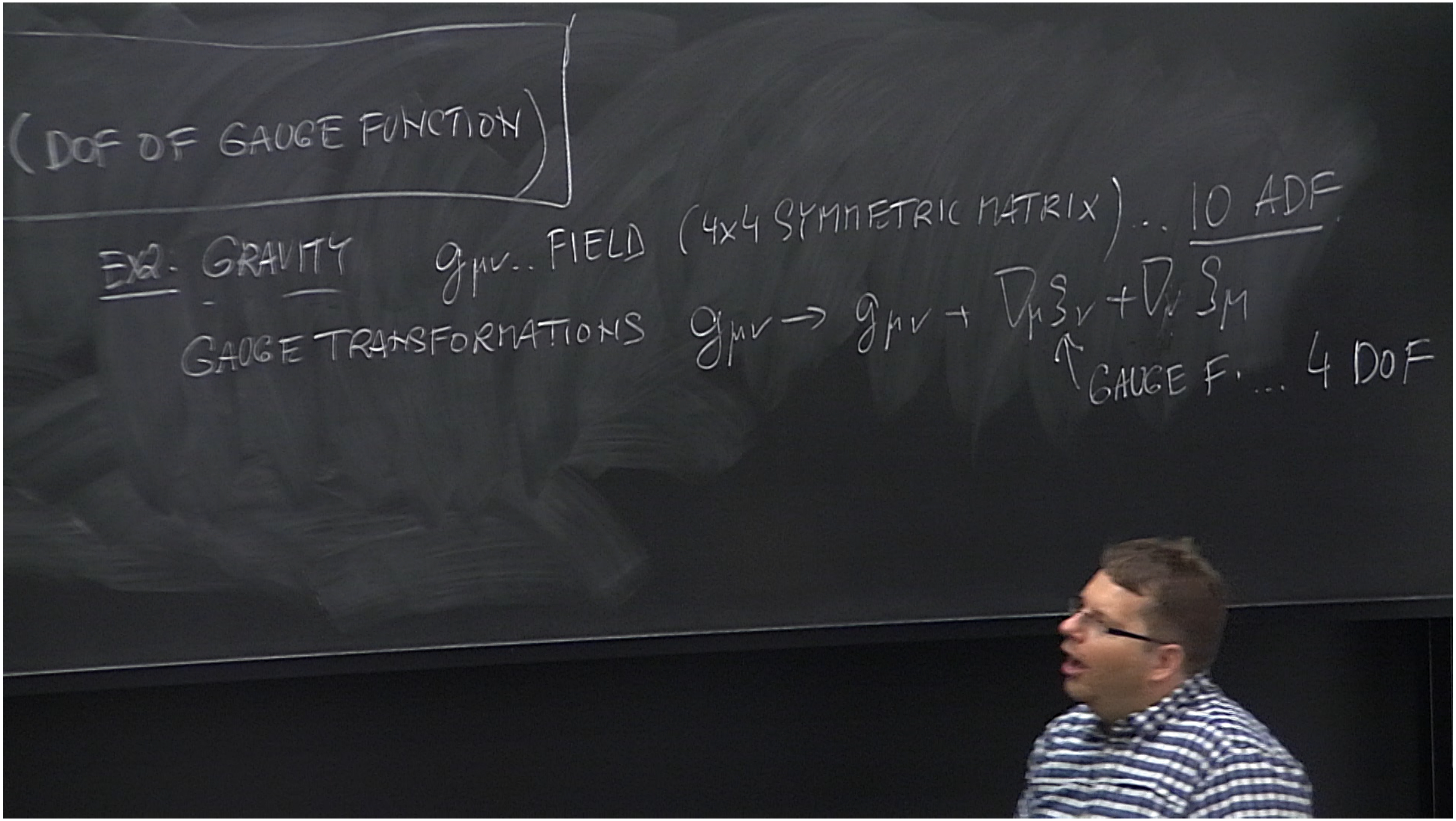
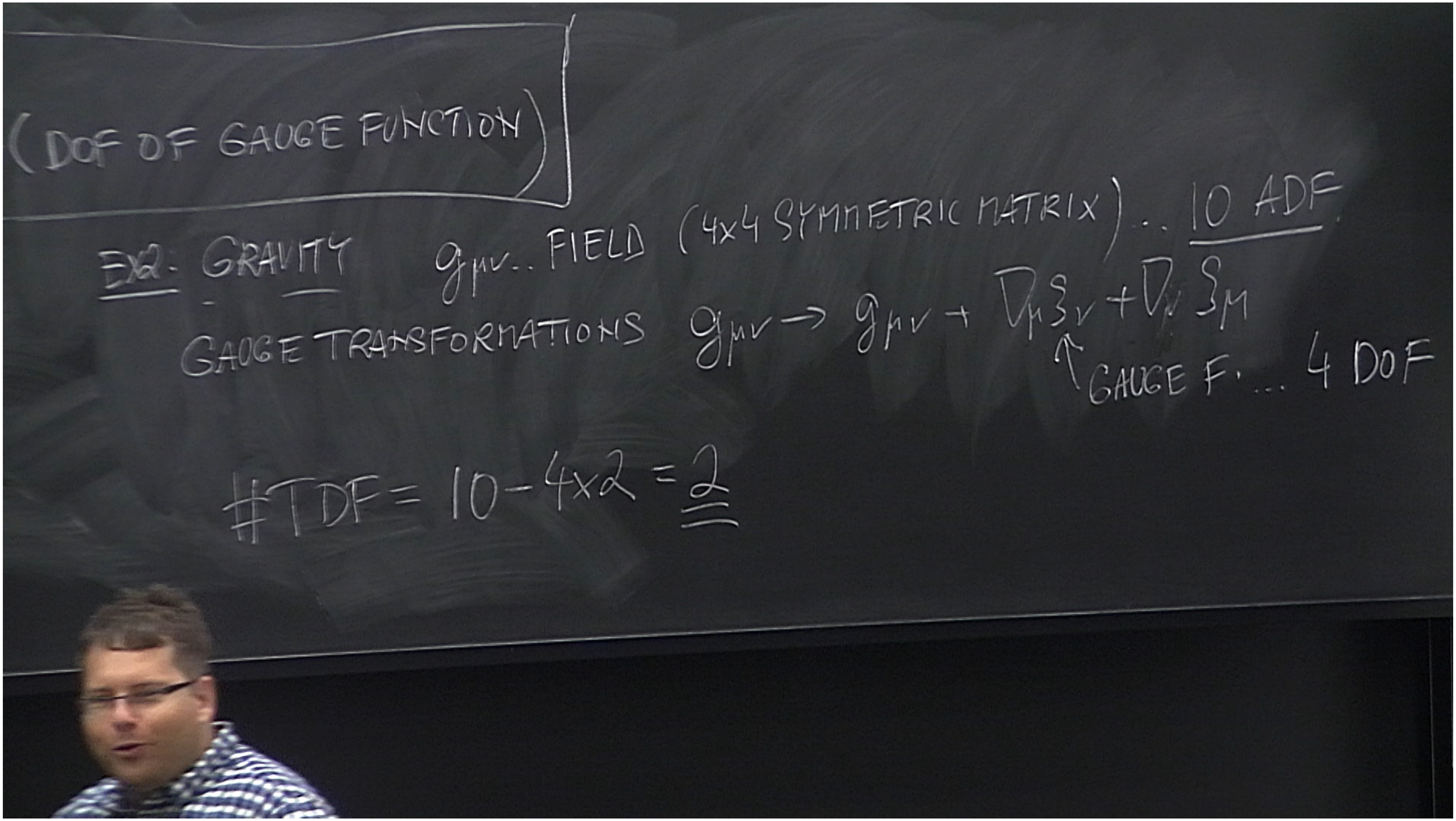
EX2: GRAVITY

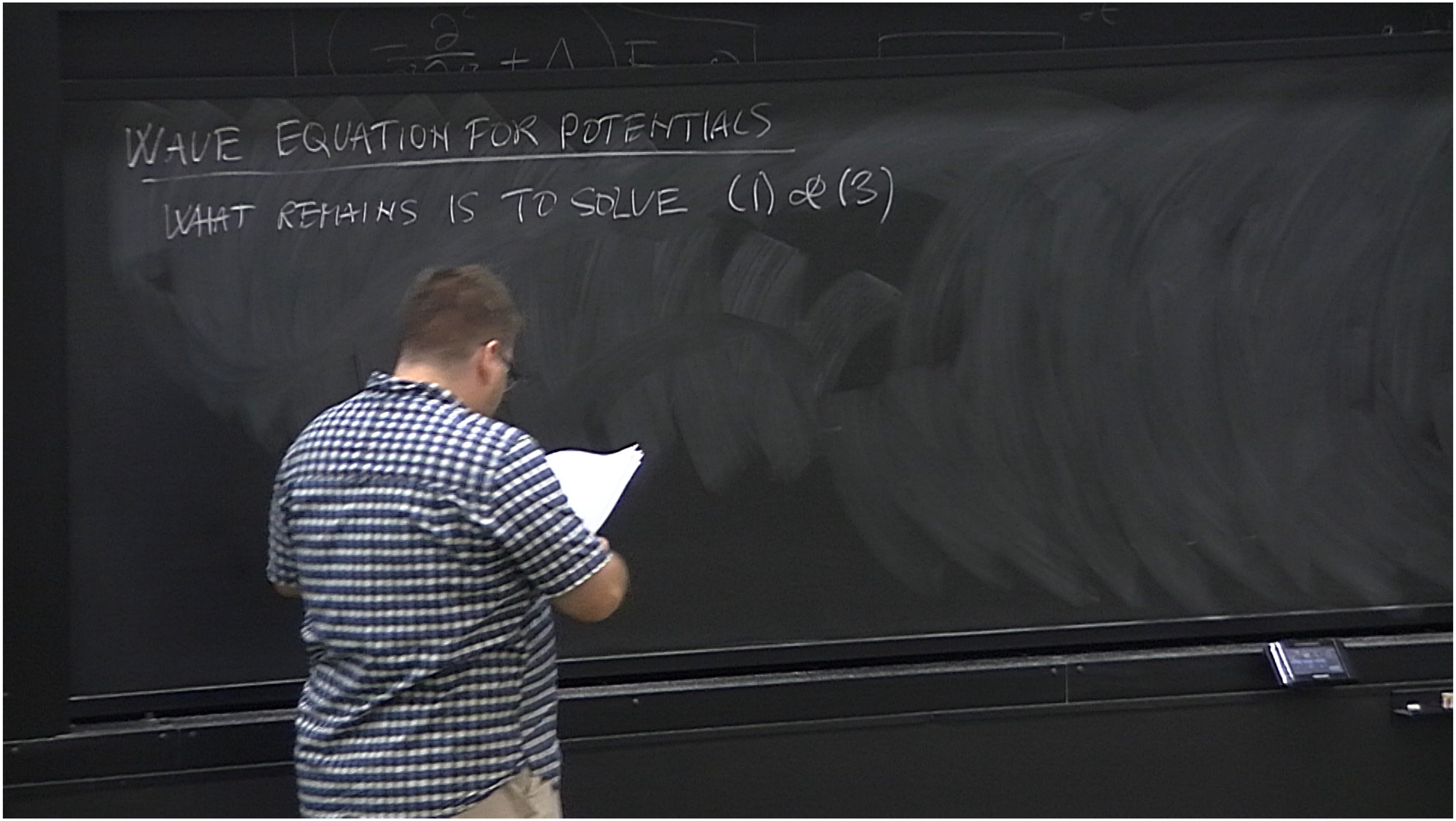

(DOF OF GAUGE FUNCTION)
EX2: GRAVITY
$g_{\mu\nu}$.. FIELD (4x4 SYMMETRIC MATRIX)


(DOF OF GAUGE FUNCTION)
EX2: GRAVITY
$g_{\mu\nu}$ .. FIELD (4x4 SYMMETRIC MATRIX) .. 10 ADF


(DOF OF GAUGE FUNCTION)
EX2. GRAVITY $g_{\mu\nu}$.. FIELD (4x4 SYMMETRIC MATRIX)... 10 ADF
GAUGE TRANSFORMATIONS $g_{\mu\nu} \rightarrow g_{\mu\nu} + D_\mu \xi_\nu + D_\nu \xi_\mu$
GAUGE F.


(DOF OF GAUGE FUNCTION)
EX2: GRAVITY
gμν .. FIELD (4x4 SYMMETRIC MATRIX) ... 10 ADF
GAUGE TRANSFORMATIONS gμν → gμν + Dμ ξν + Dν ξμ
GAUGE F. ... 4 DOF

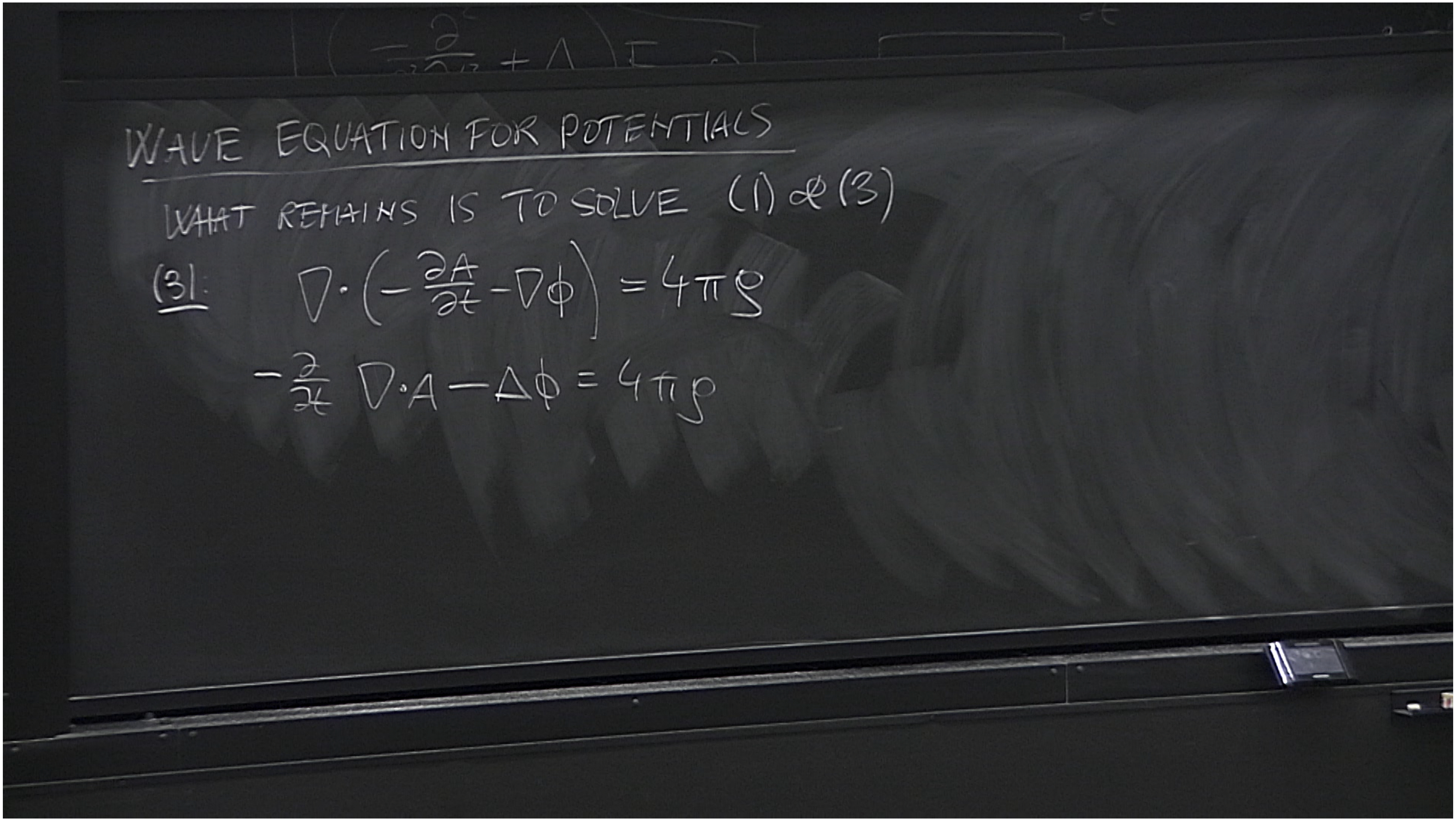

(DOF OF GAUGE FUNCTION)
EX2: GRAVITY
$g_{\mu\nu}$ .. FIELD (4x4 SYMMETRIC MATRIX) ... 10 ADF
GAUGE TRANSFORMATIONS $g_{\mu\nu} \to g_{\mu\nu} + D_\mu \xi_\nu + D_\nu \xi_\mu$
GAUGE F. ... 4 DOF
#TDF = 10 − 4x2 = 2

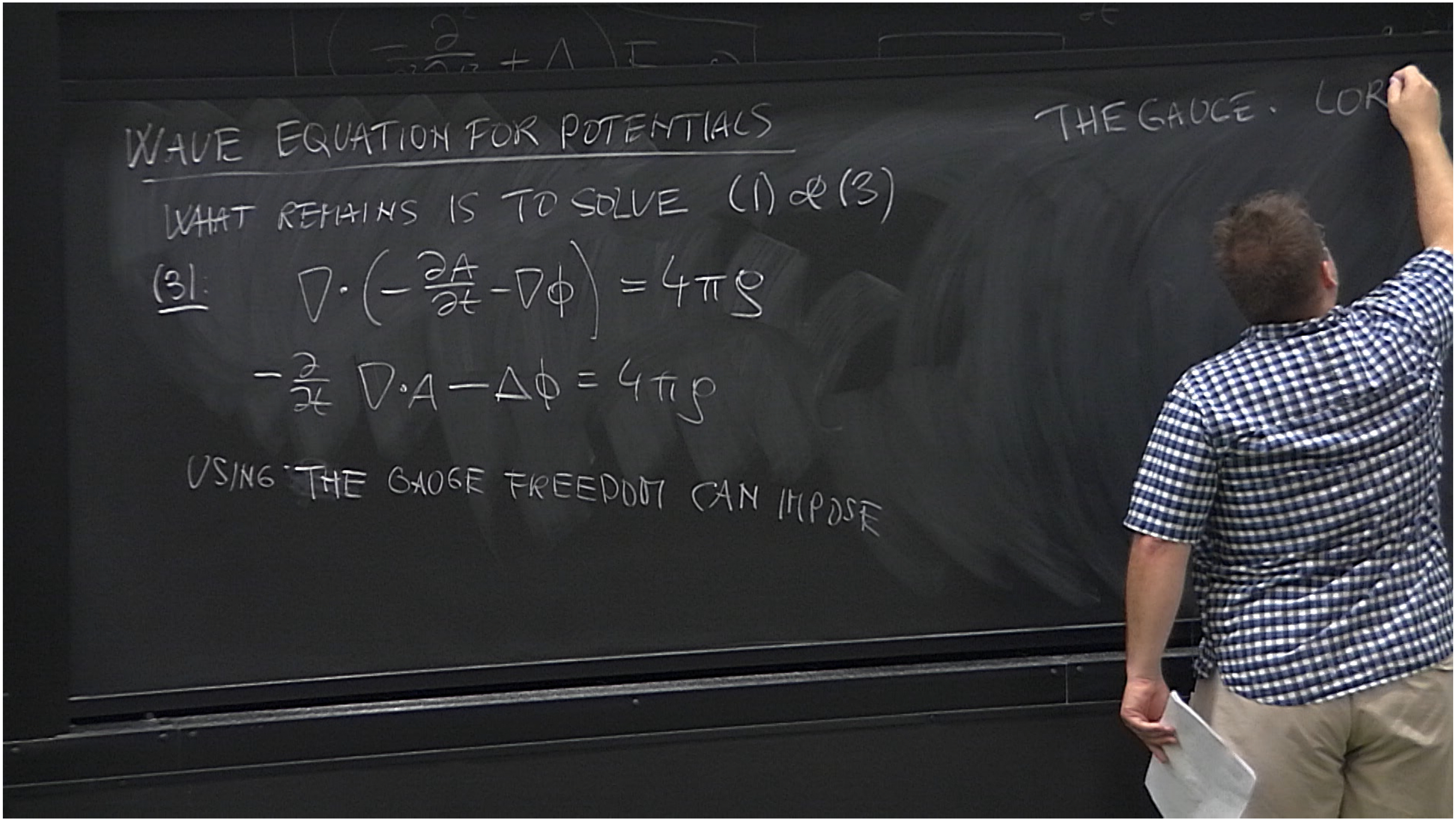

WAVE EQUATION FOR POTENTIALS
WHAT REMAINS IS TO SOLVE (1) & (3)

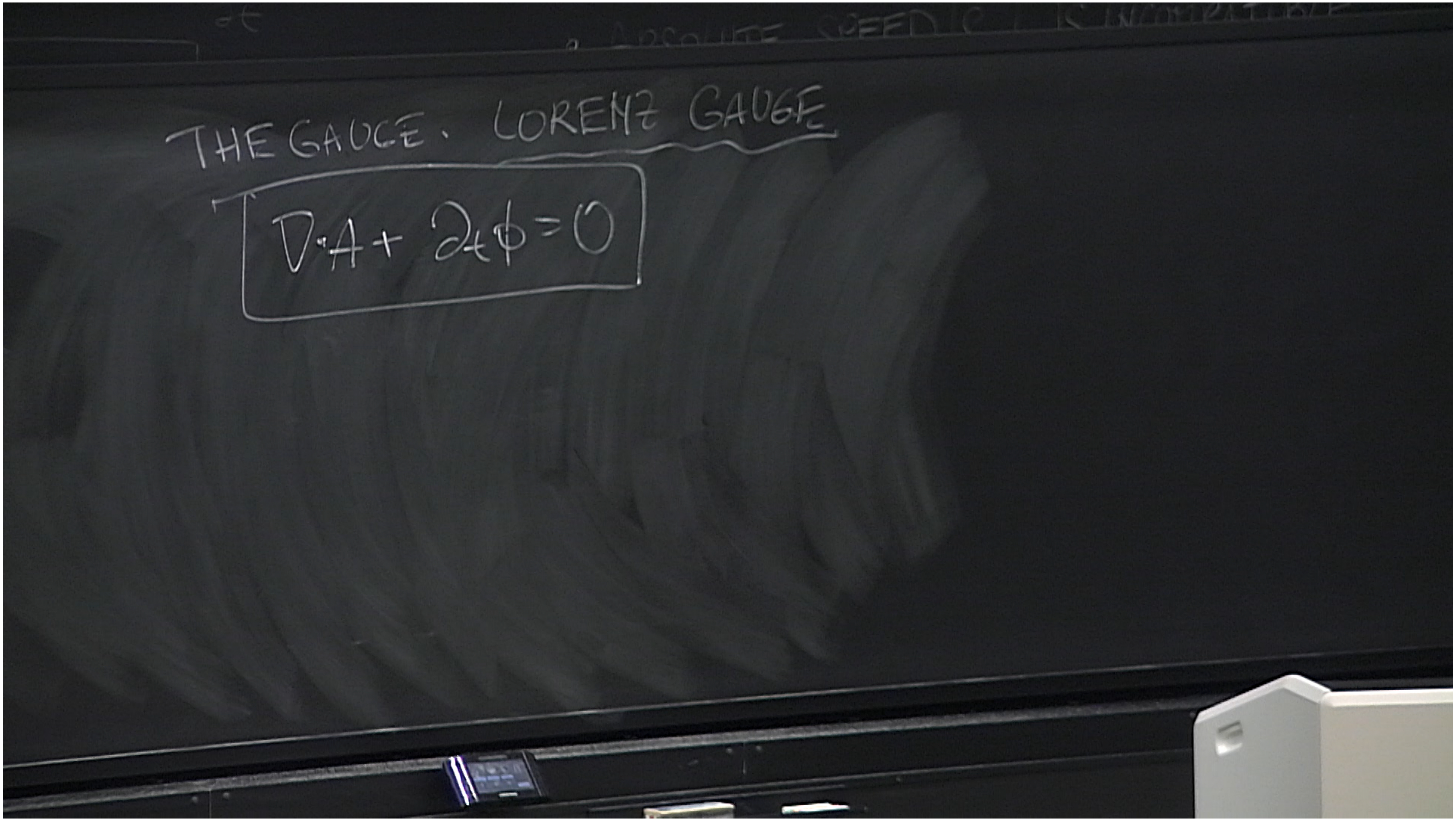
WAVE EQUATION FOR POTENTIALS

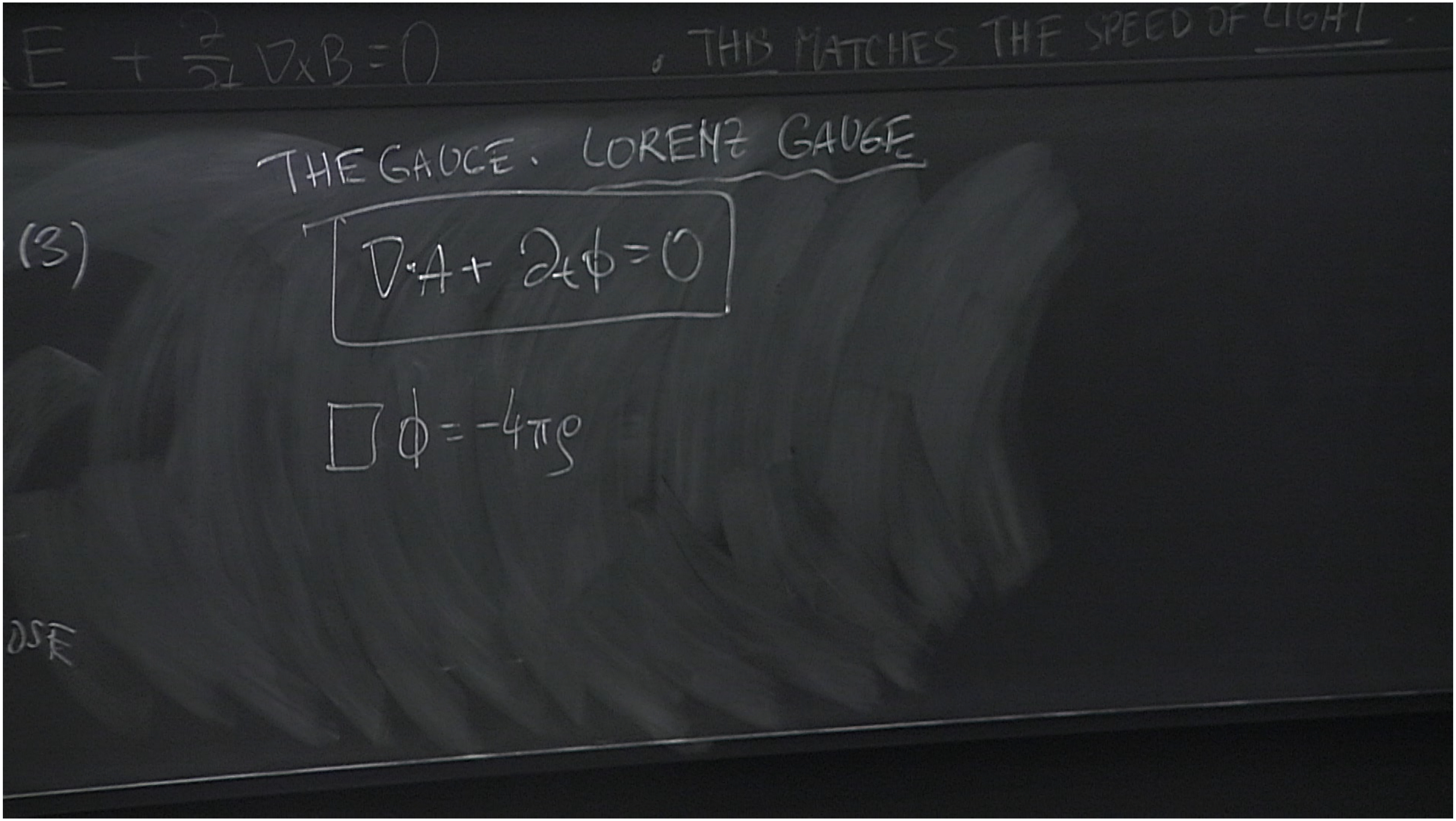
WHAT REMAINS IS TO SOLVE (1) & (3)

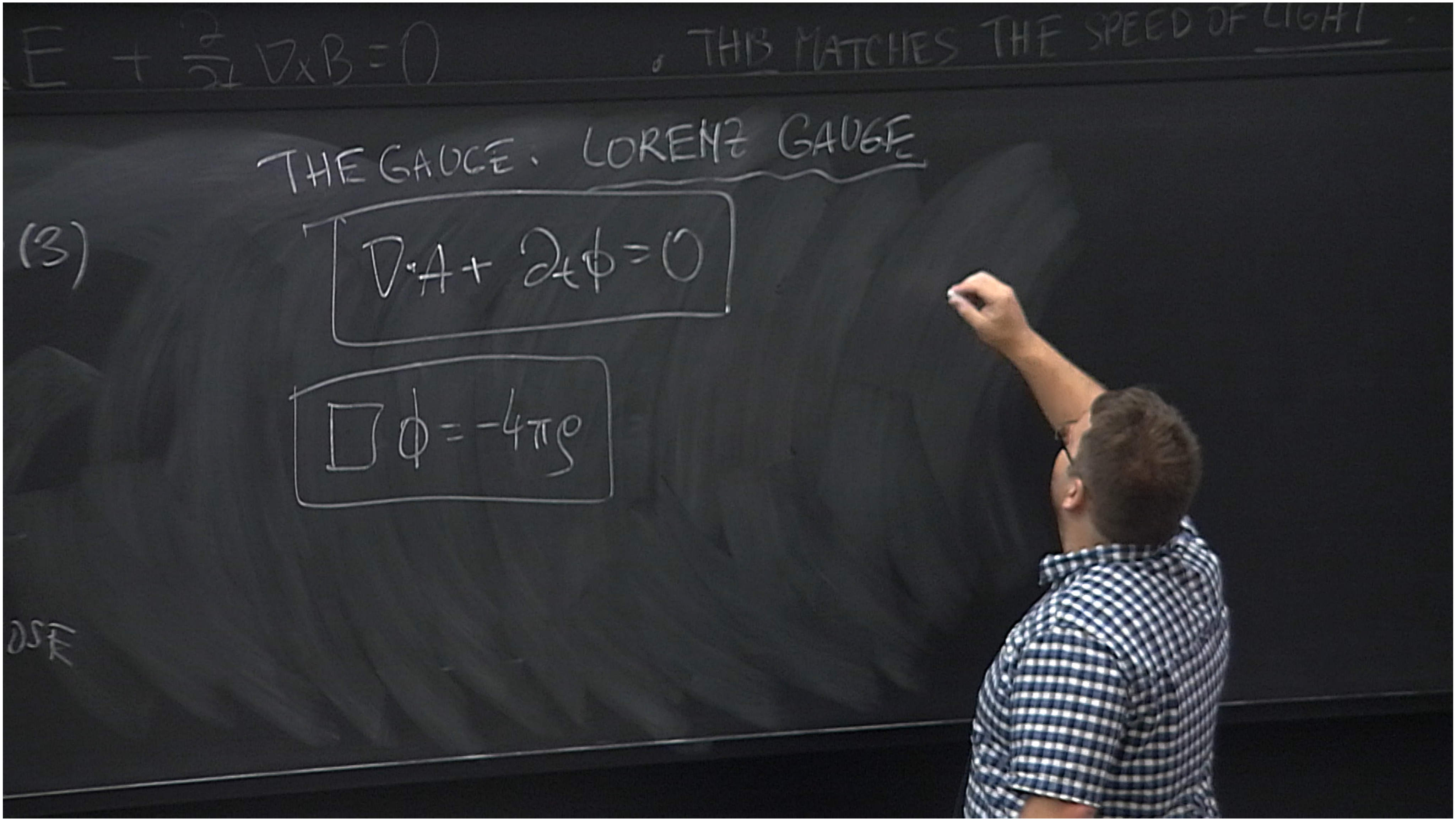
(3):
$$\nabla\cdot\left(-\frac{\partial A}{\partial t}-\nabla\phi\right)=4\pi\rho$$


WAVE EQUATION FOR POTENTIALS
WHAT REMAINS IS TO SOLVE (1) & (3)
(3): $\nabla\cdot\left(-\frac{\partial A}{\partial t}-\nabla\phi\right)=4\pi\rho$
$-\frac{\partial}{\partial t}\nabla\cdot A-\Delta\phi=4\pi\rho$


WAVE EQUATION FOR POTENTIALS
THE GAUGE. LOR
WHAT REMAINS IS TO SOLVE (1) & (3)
(3): $\nabla\cdot\left(-\frac{\partial A}{\partial t}-\nabla\phi\right)=4\pi\rho$
$-\frac{\partial}{\partial t}\nabla\cdot A-\Delta\phi=4\pi\rho$
USING THE GAUGE FREEDOM CAN IMPOSE


THE GAUGE. LORENZ GAUGE
$\nabla\cdot A + \partial_t \phi = 0$


E + ∂/∂t ∇×B = 0
THIS MATCHES THE SPEED OF LIGHT
THE GAUGE: LORENZ GAUGE
(3)
∇·A + ∂t φ = 0
□φ = −4πρ
OSE


THIS MATCHES THE SPEED OF LIGHT
THE GAUGE. LORENZ GAUGE
(3)
$\nabla\cdot A + \partial_t \phi = 0$
$\Box \phi = -4\pi\rho$

$\nabla \times B = 0$. THIS MATCHES THE SPEED OF LIGHT .

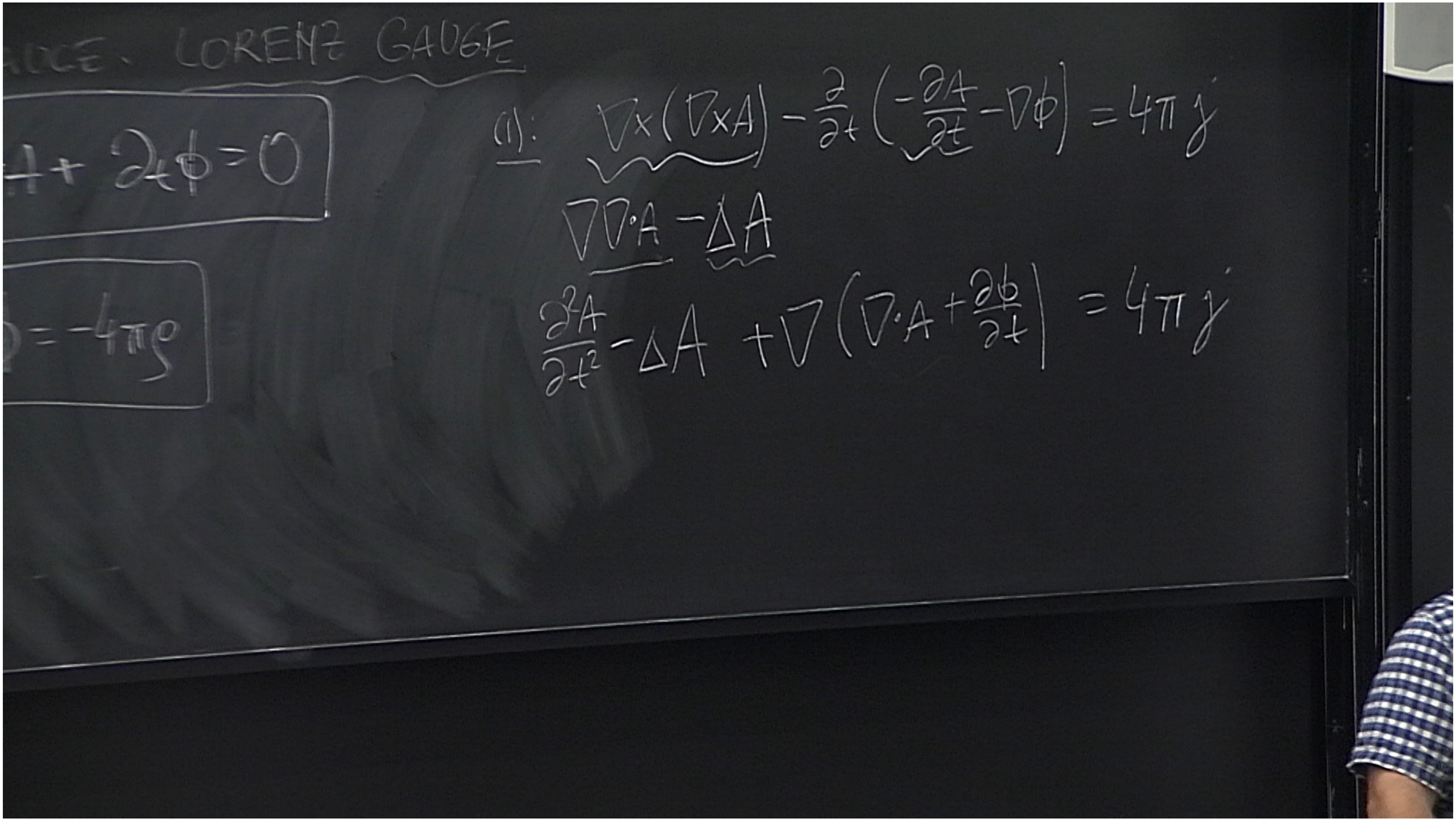
THE GAUGE. LORENZ GAUGE

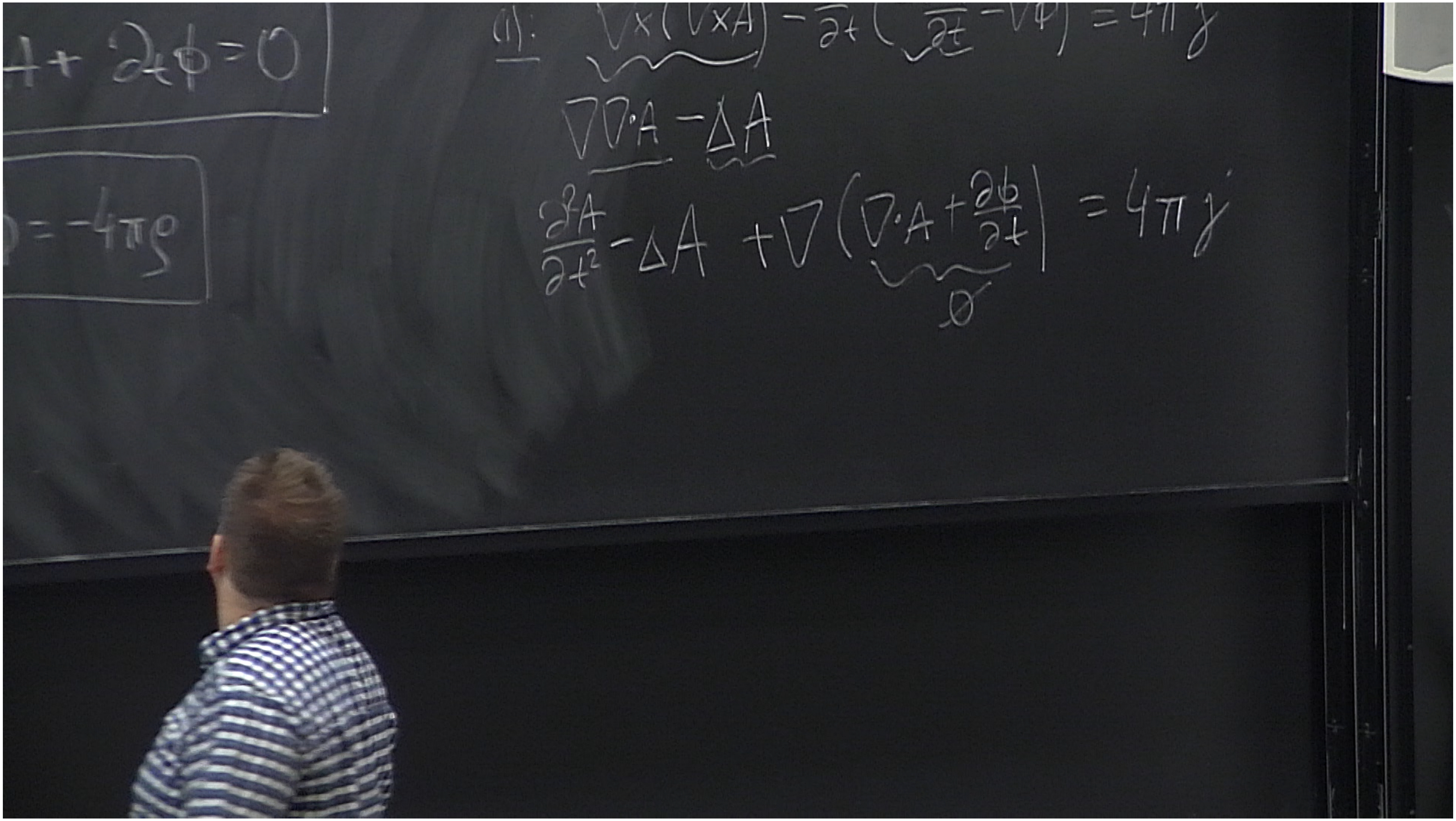
$$\boxed{\nabla \cdot A + \partial_t \phi = 0}$$

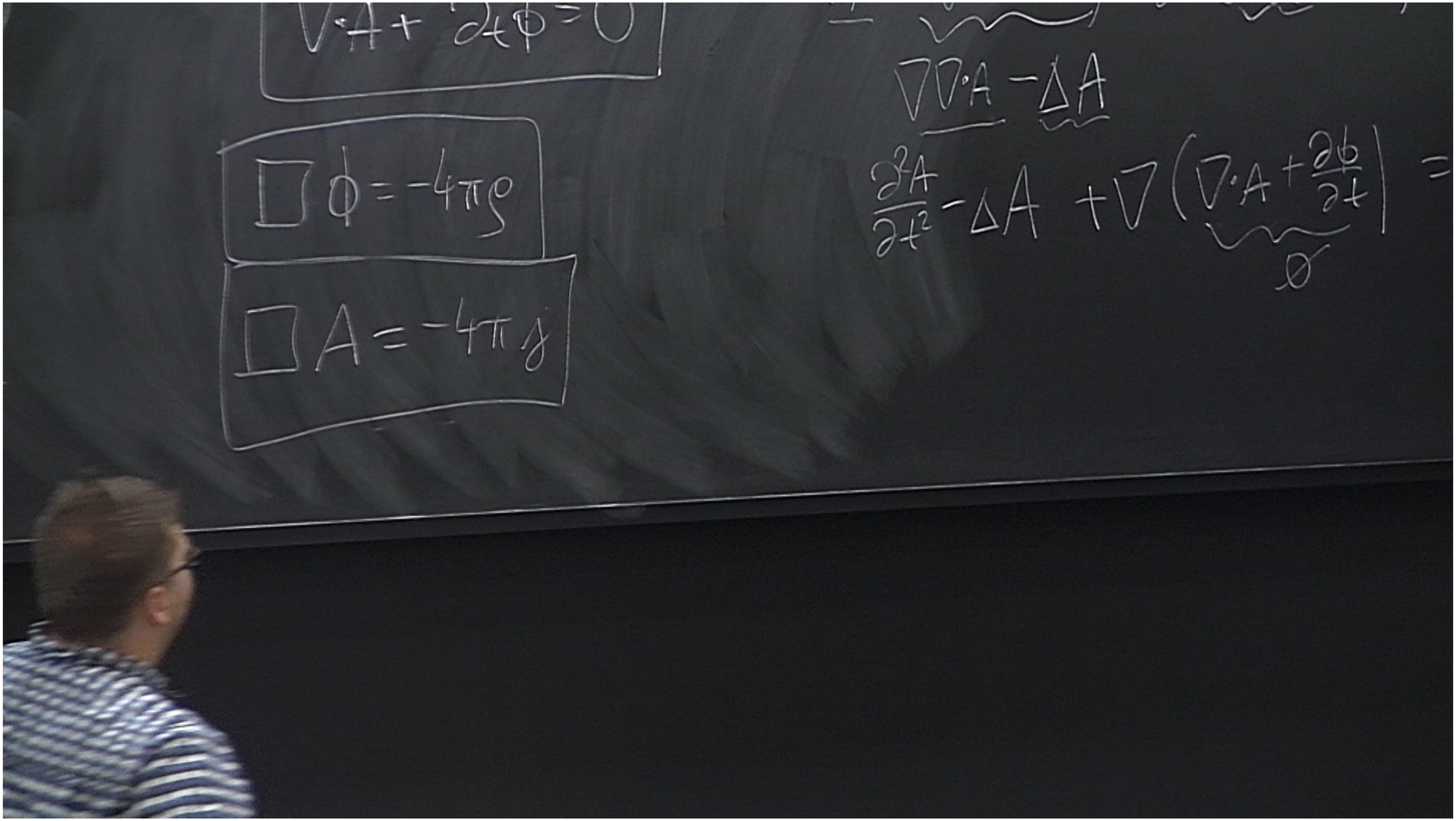
$$\boxed{\Box \phi = -4\pi \rho}$$

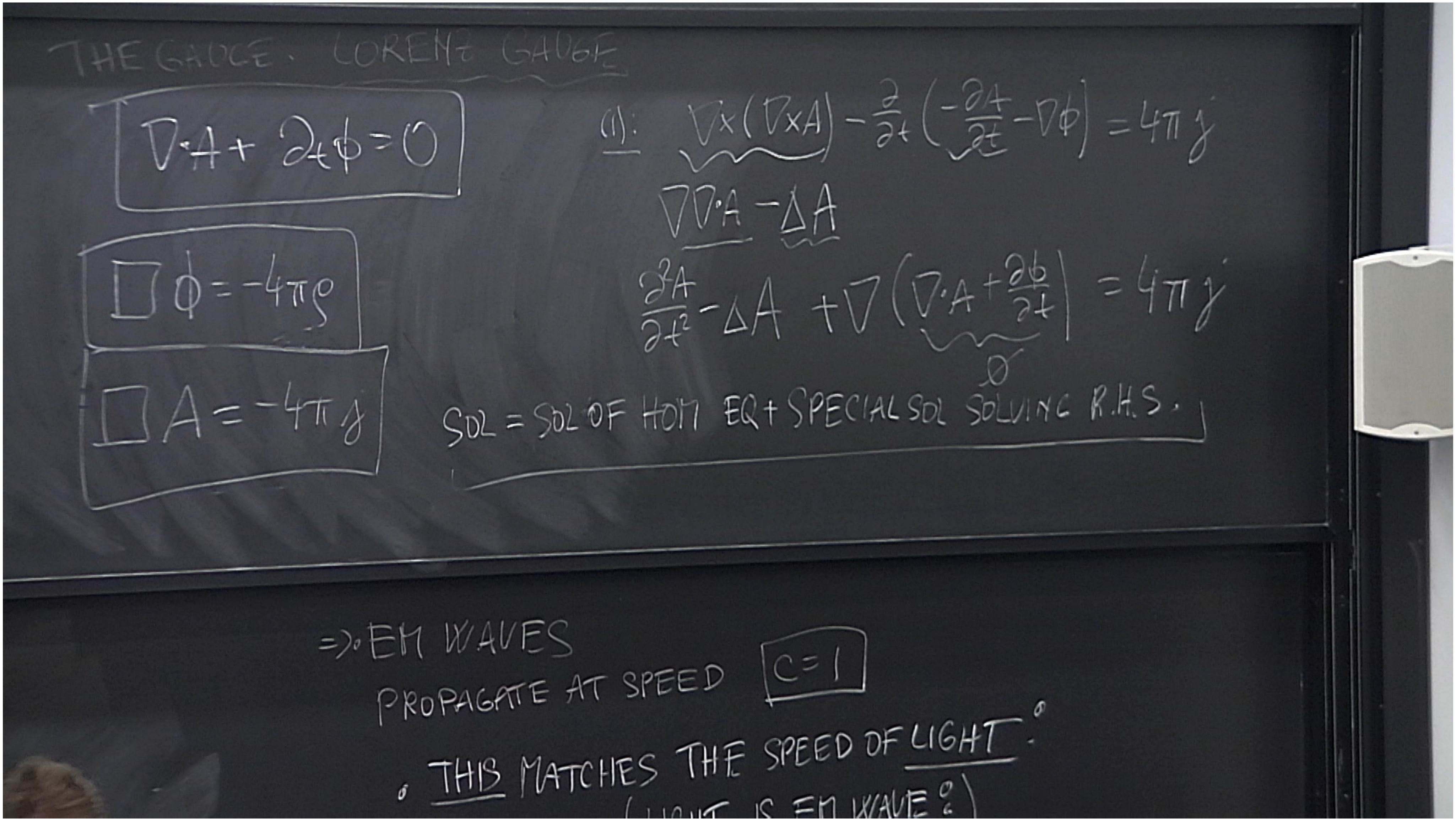
(1): $\nabla \times (\nabla \times A) - \frac{\partial}{\partial t}\left(-\frac{\partial A}{\partial t} - \nabla \phi\right) = 4\pi j$

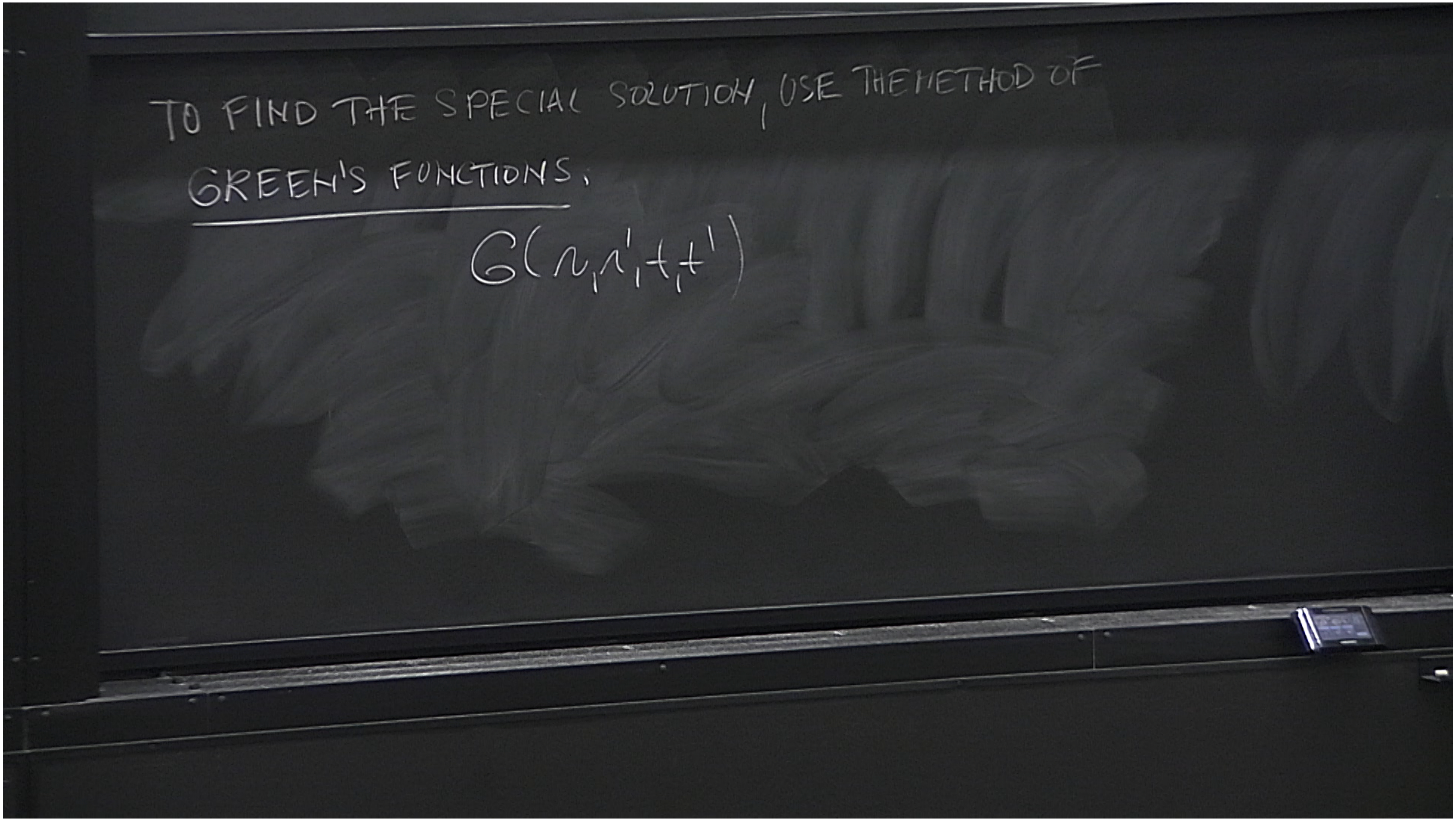
$\nabla \nabla \cdot A - \Delta A$


LORENZ GAUGE
$\partial_t \phi = 0$
$= -4\pi\rho$
(1): $\nabla \times (\nabla \times A) - \frac{\partial}{\partial t}\left(-\frac{\partial A}{\partial t} - \nabla\phi\right) = 4\pi j$
$\nabla \nabla \cdot A - \Delta A$
$\frac{\partial^2 A}{\partial t^2} - \Delta A + \nabla\left(\nabla \cdot A + \frac{\partial \phi}{\partial t}\right) = 4\pi j$


A + ∂tφ = 0
φ = −4πρ
(1): ∇×(∇×A) − ∂t(∂t − ∇φ) = 4πj
∇∇·A − ΔA
∂²A/∂t² − ΔA + ∇(∇·A + ∂φ/∂t) = 4πj
0


∇·A + ∂tφ = 0
□φ = −4πρ
□A = −4πj
∇∇·A − ΔA
∂²A/∂t² − ΔA + ∇(∇·A + ∂φ/∂t) =
0


THE GAUGE. LORENZ GAUGE
∇·A + ∂tφ = 0
□φ = −4πρ
□A = −4πj
(1): ∇×(∇×A) − ∂/∂t (−∂A/∂t − ∇φ) = 4πj
∇∇·A − ΔA
∂²A/∂t² − ΔA + ∇(∇·A + ∂φ/∂t) = 4πj
0
SOL = SOL OF HOM EQ + SPECIAL SOL SOLVING R.H.S.
=> EM WAVES
PROPAGATE AT SPEED
c = 1
THIS MATCHES THE SPEED OF LIGHT!

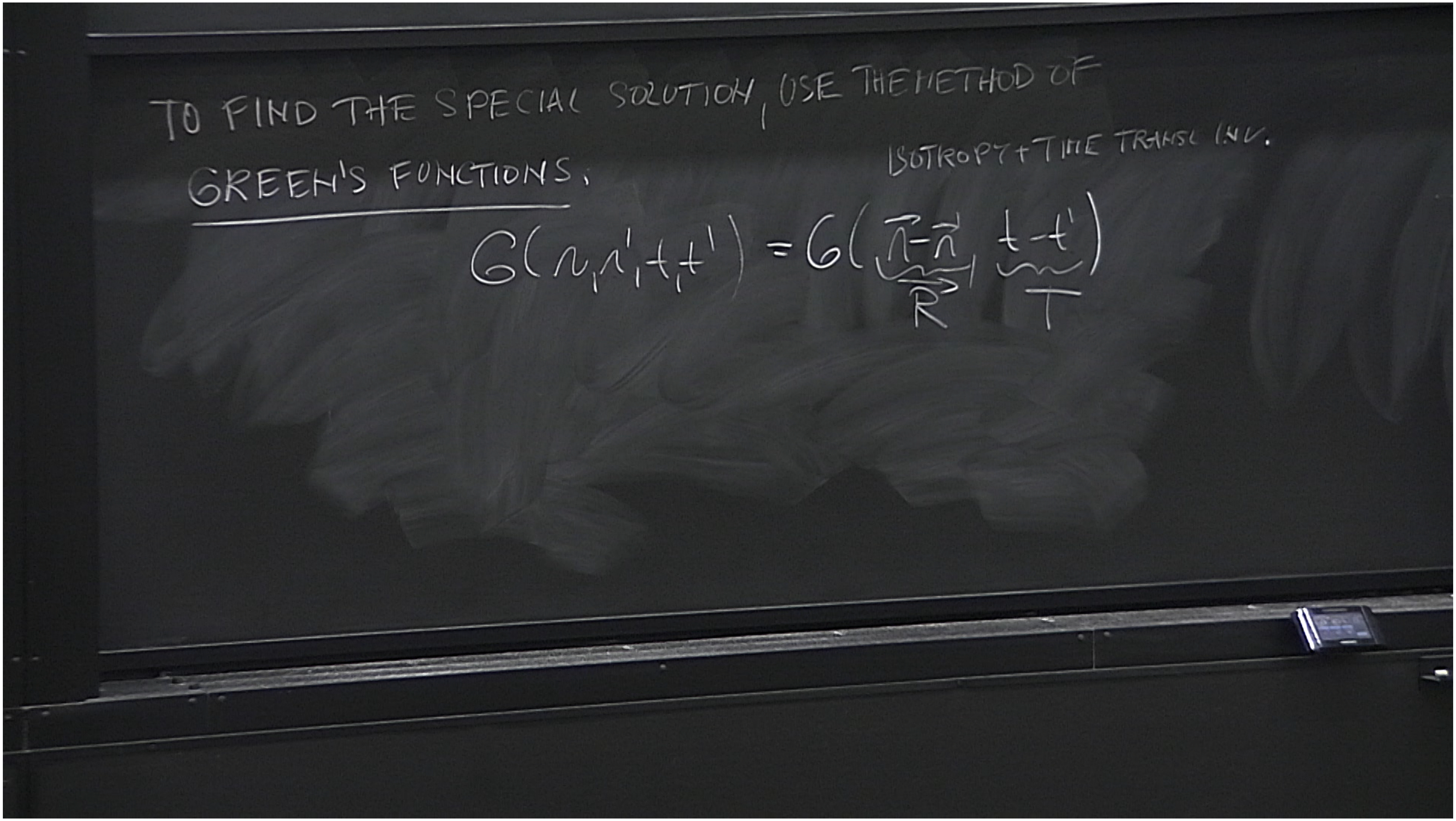

TO FIND THE SPECIAL SOLUTION, USE THE METHOD OF
GREEN'S FUNCTIONS:
G(r, r', t, t')

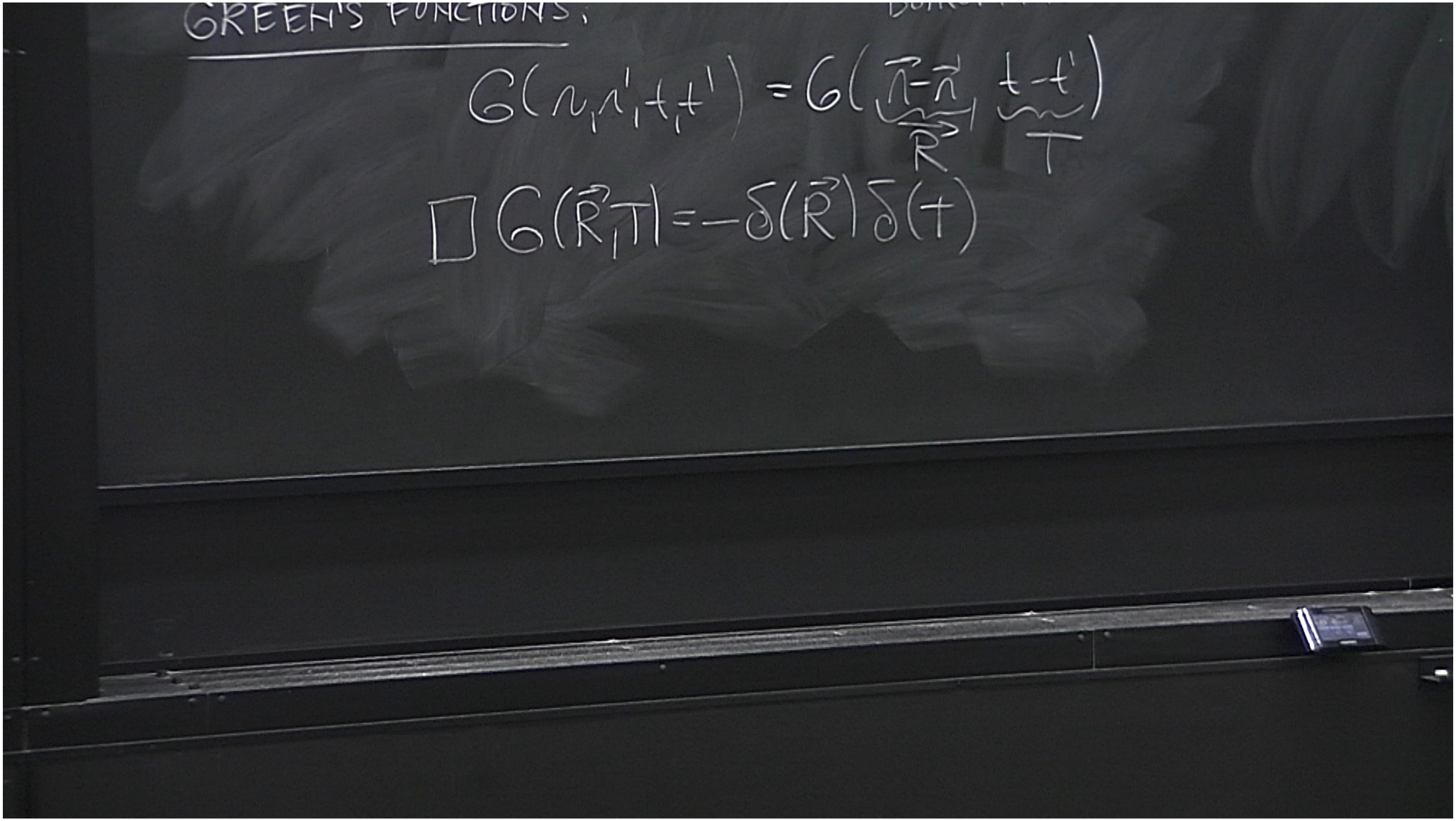
TO FIND THE SPECIAL SOLUTION, USE THE METHOD OF

GREEN'S FUNCTIONS.

ISOTROPY + TIME TRANSL INV.

$$G(r, r', t, t') = G(r - r', t - t')$$


TO FIND THE SPECIAL SOLUTION, USE THE METHOD OF
GREEN'S FUNCTIONS:
ISOTROPY + TIME TRANSL INV.
$G(r, r', t, t') = G(\vec{r} - \vec{r}', t - t')$
$\vec{R}$
T

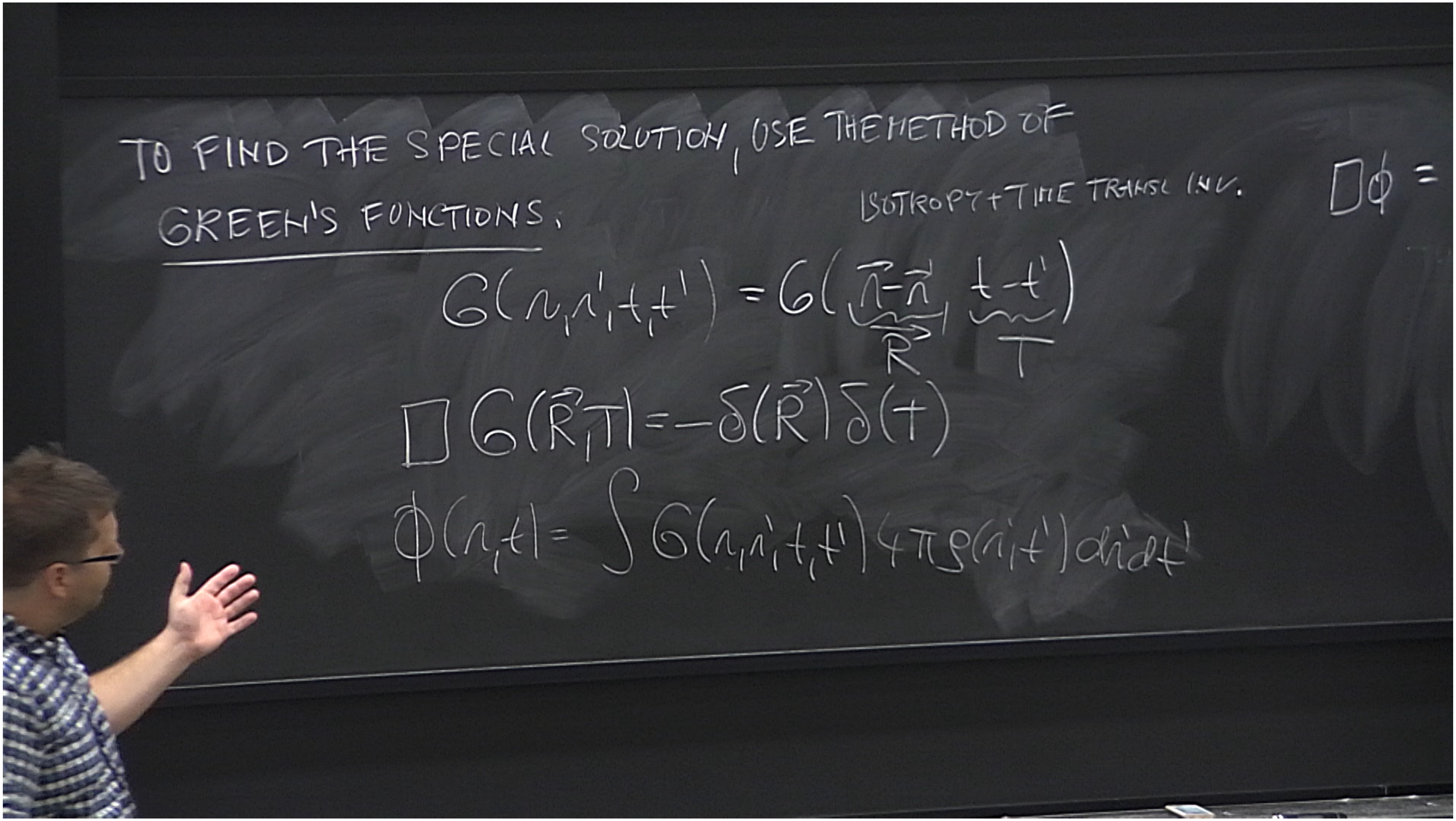

GREEN'S FUNCTIONS:
G(r, r', t, t') = G(r⃗ − r⃗', t − t')
R⃗   T
□ G(R⃗, T) = −δ(R⃗) δ(T)

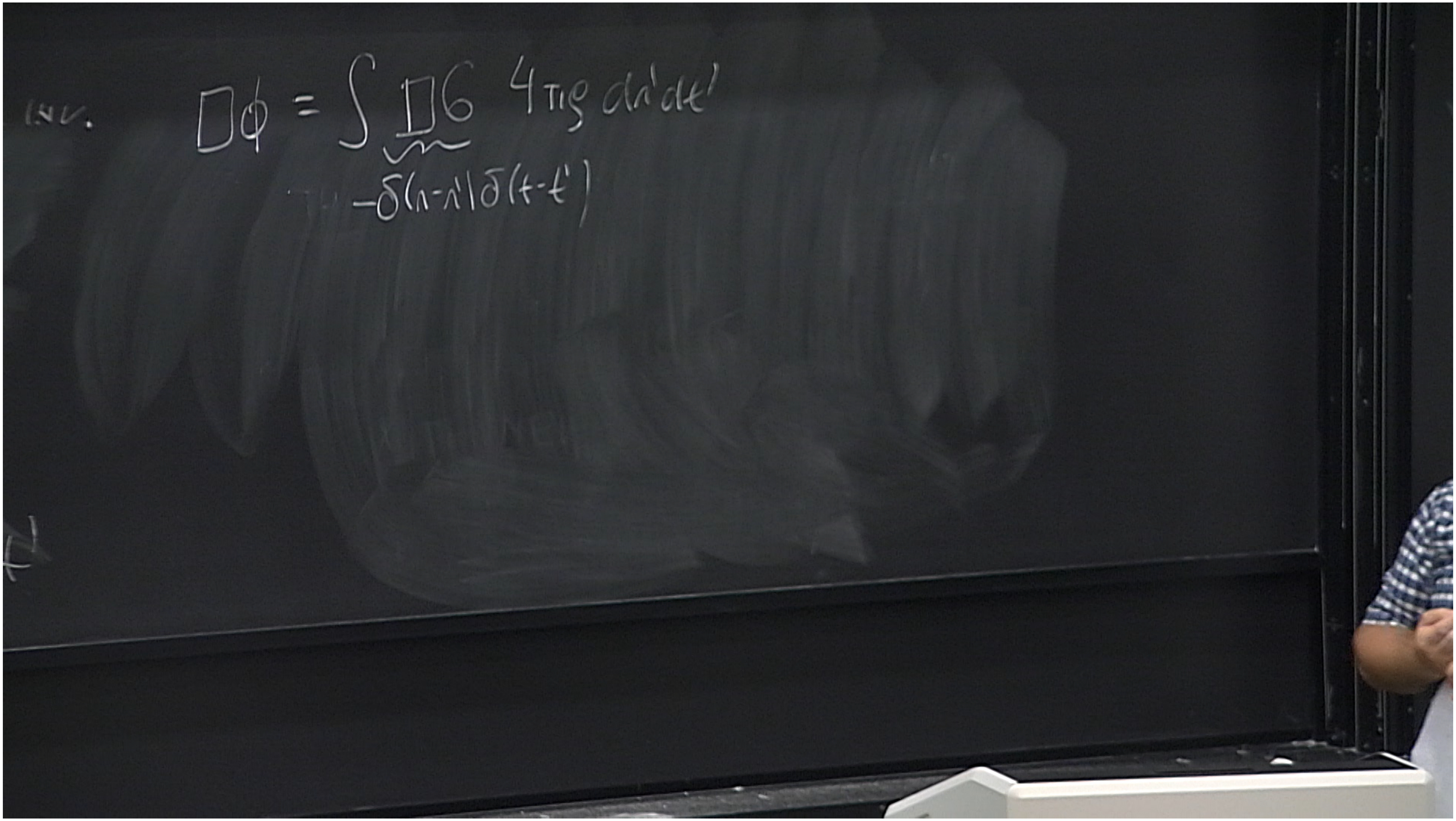
GREEN'S FUNCTIONS:

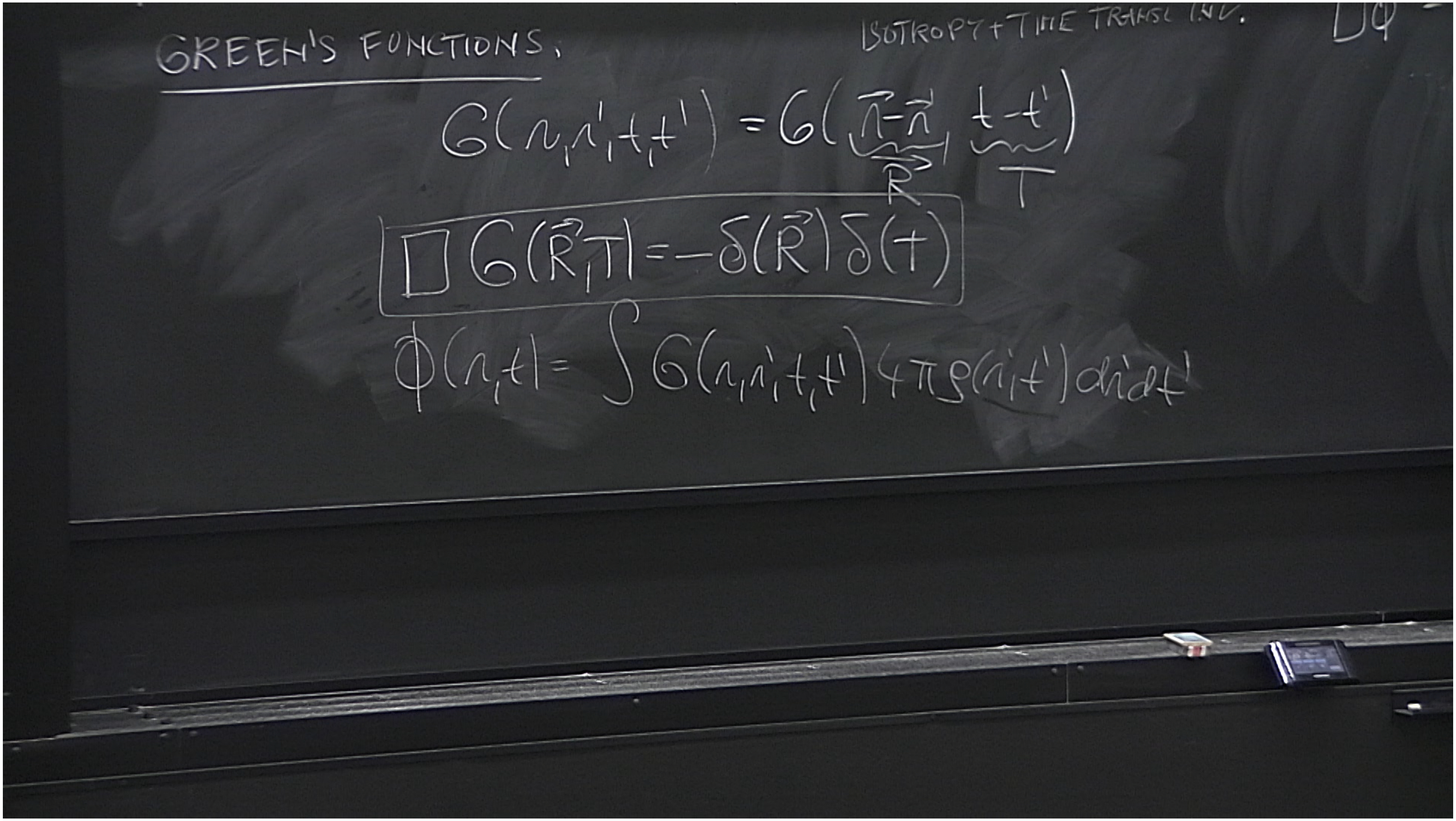
$$G(r,r',t,t') = G(\underbrace{\vec{r}-\vec{r}'}_{\vec{R}}, \underbrace{t-t'}_{T})$$

$$\Box\, G(\vec{R},T) = -\delta(\vec{R})\,\delta(t)$$

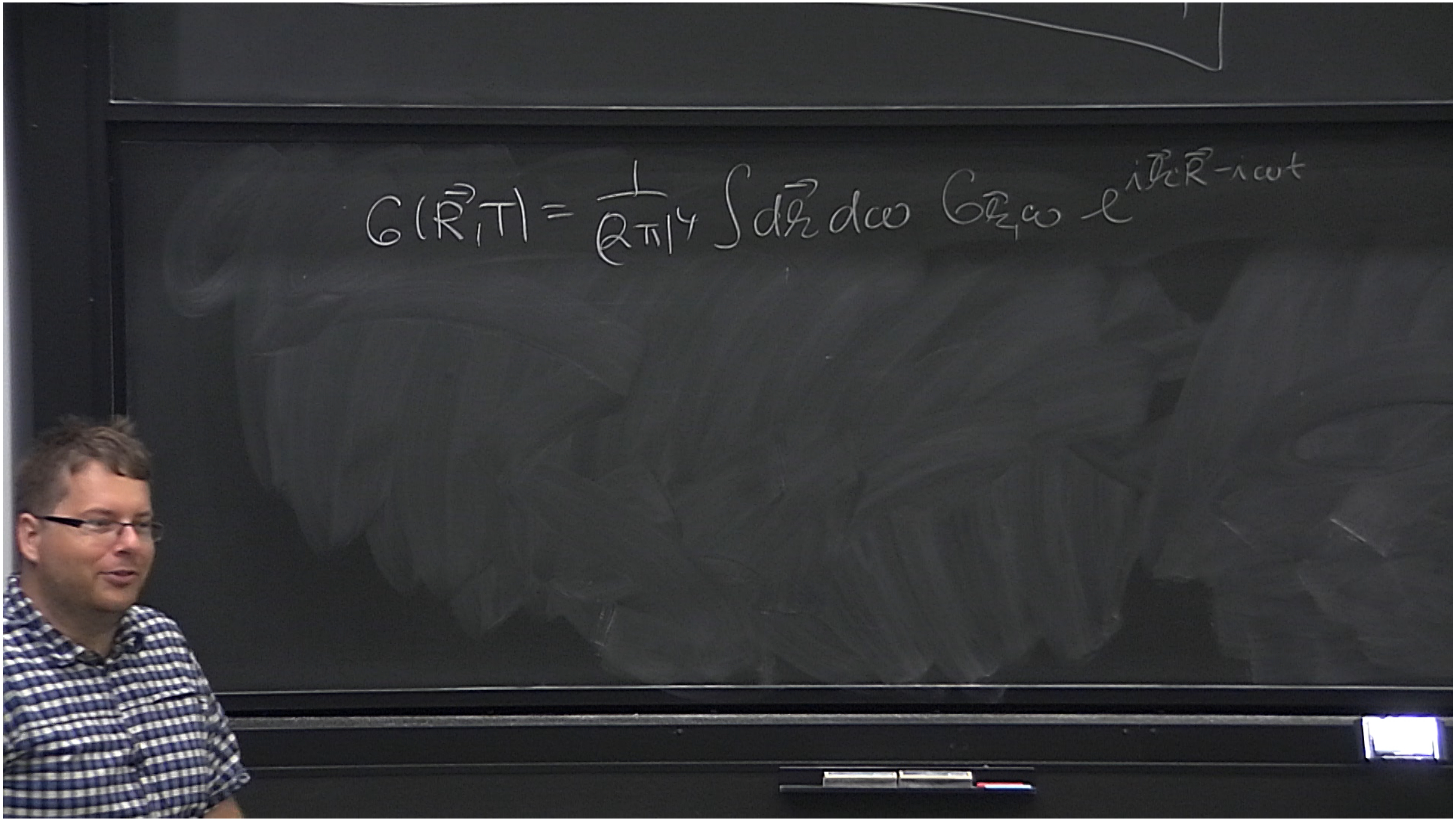
$$\phi(r,t) = \int G(r,r',t,t')\, 4\pi\rho\, dr'\, dt'$$

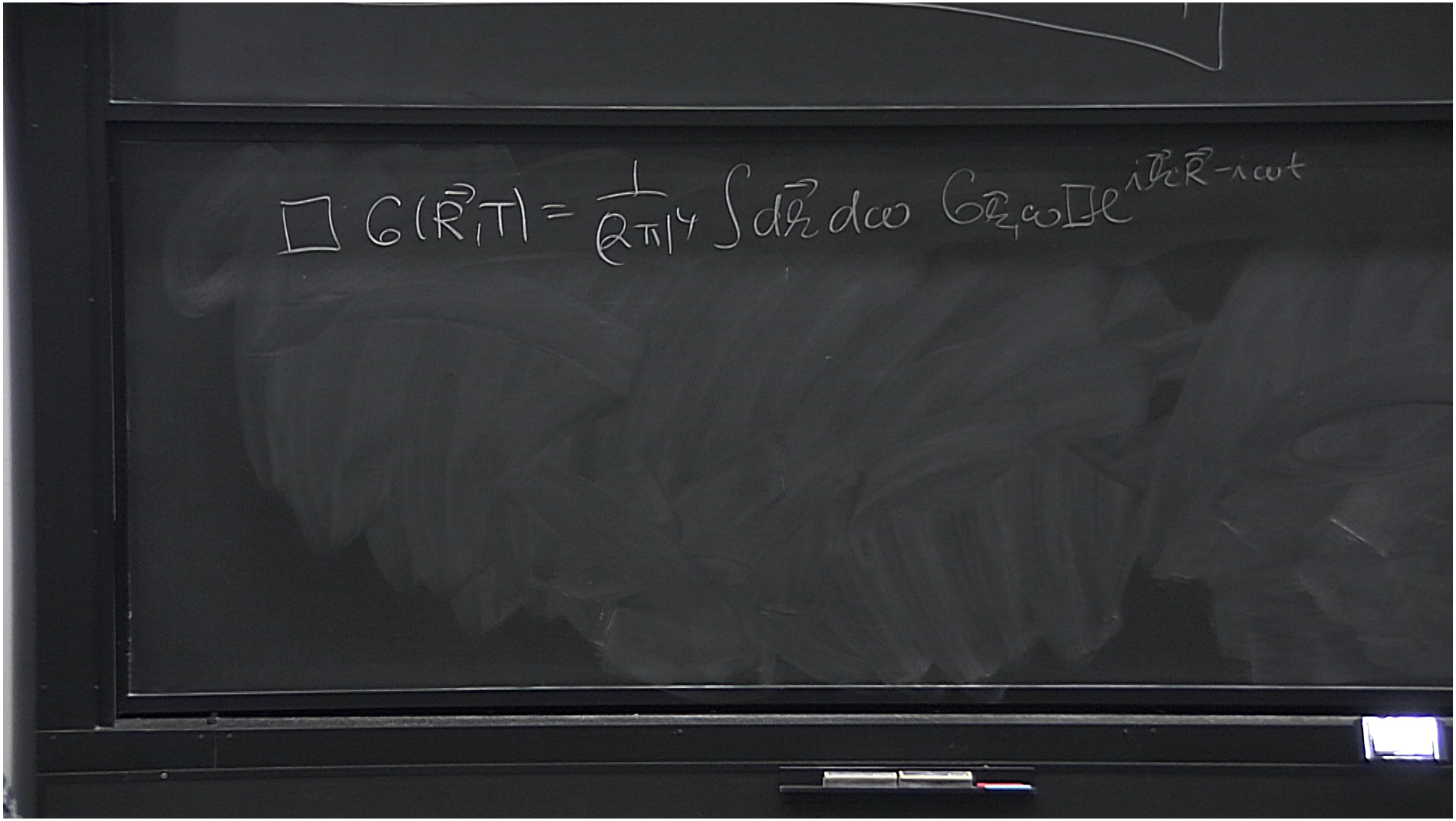

TO FIND THE SPECIAL SOLUTION, USE THE METHOD OF
GREEN'S FUNCTIONS.
ISOTROPY + TIME TRANSL INV.
□φ =
G(r, r', t, t') = G(r⃗ − r⃗', t − t')
R⃗   T
□ G(R⃗, T) = −δ(R⃗) δ(T)
φ(r, t) = ∫ G(r, r', t, t') 4π ρ(r', t') dr' dt'

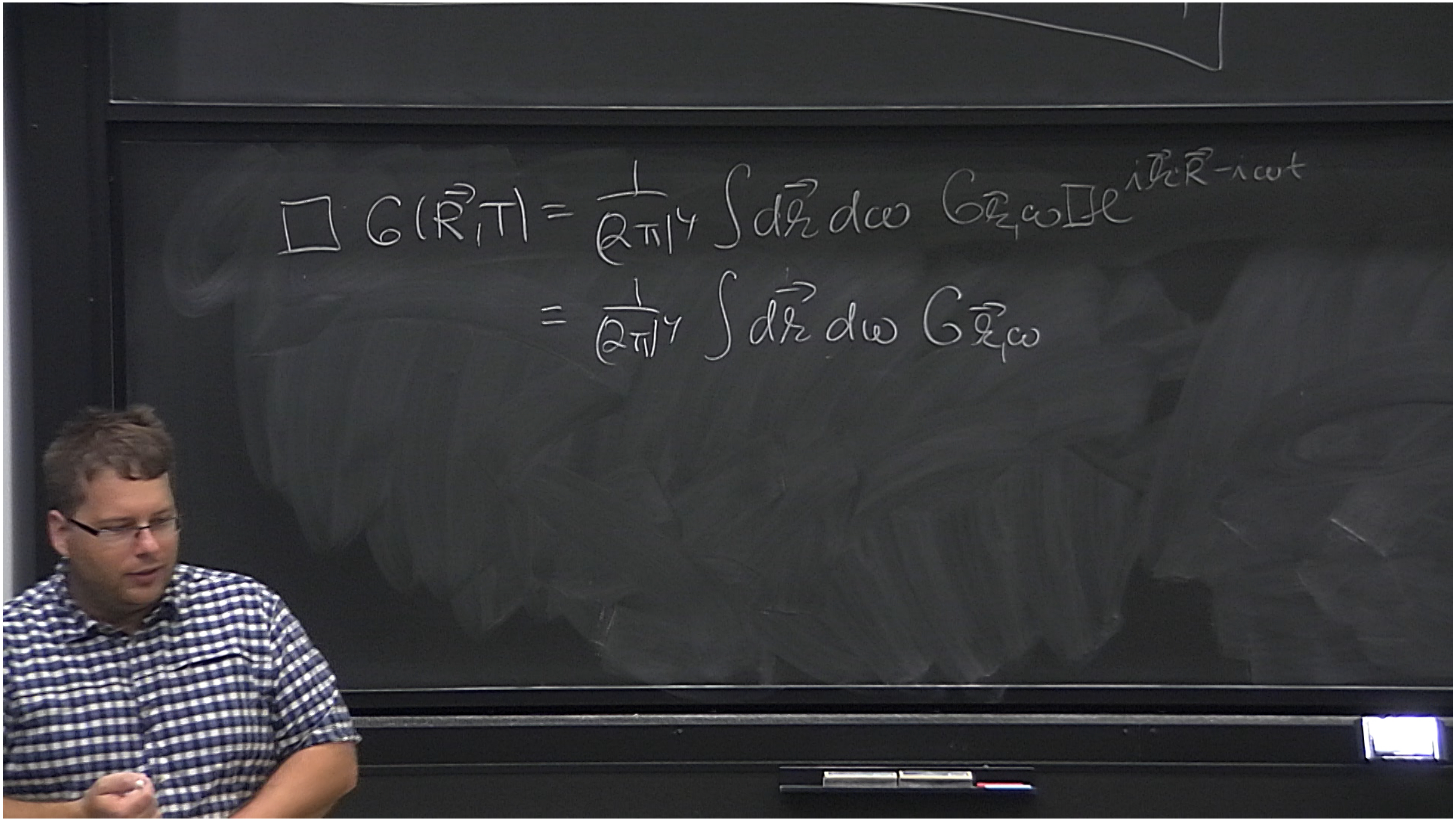

INV.
□ϕ = ∫ □G 4πρ dr'dt'
−δ(r−r')δ(t−t')

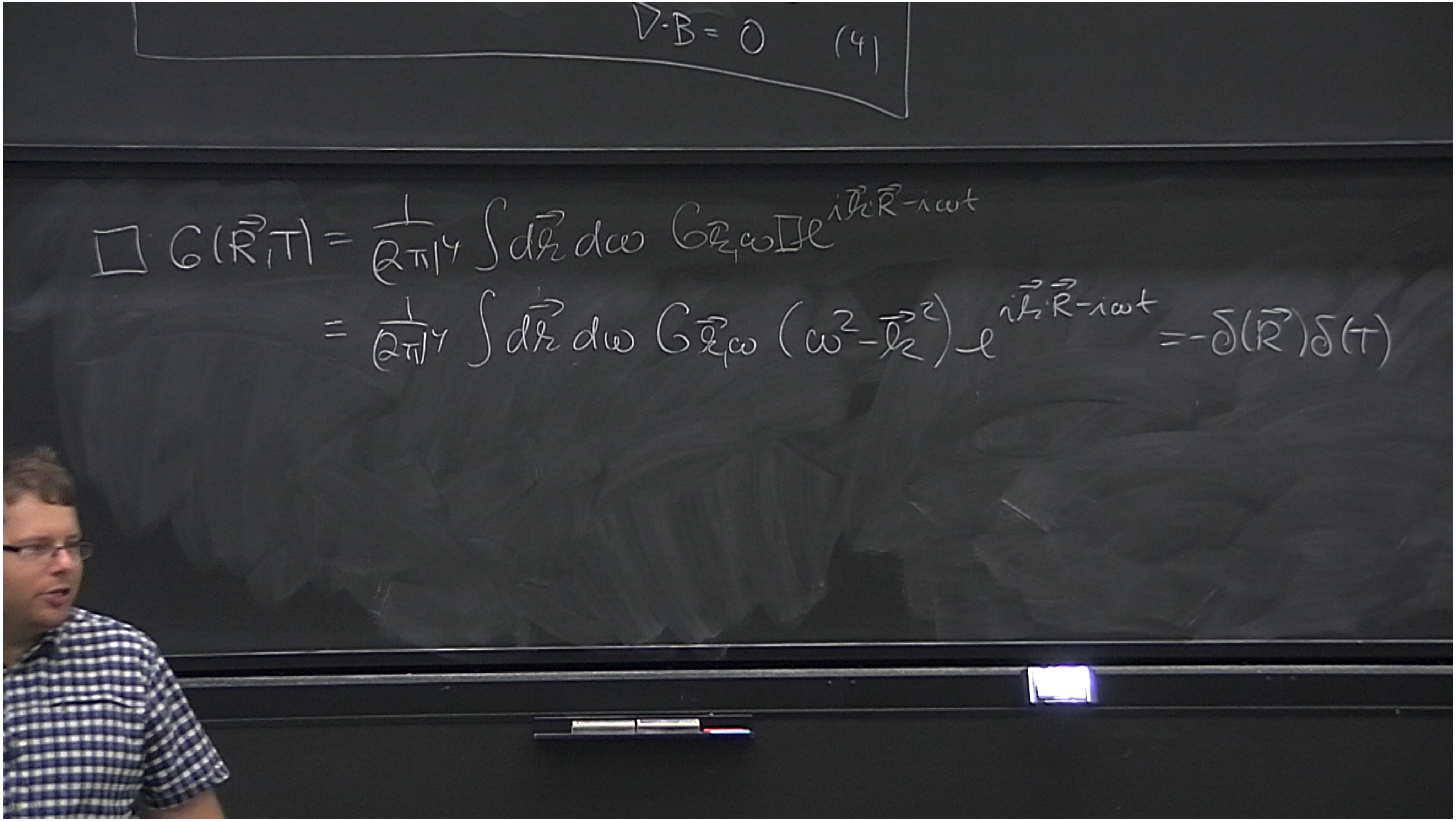

GREEN'S FUNCTIONS:
ISOTROPY + TIME TRANSL INV.
$G(r,r',t,t') = G(\vec{r}-\vec{r}', t-t')$
$\vec{R}$ T
$\Box G(\vec{R},T) = -\delta(\vec{R})\delta(T)$
$\phi(r,t) = \int G(r,r',t,t') 4\pi\rho(r',t') dr' dt'$

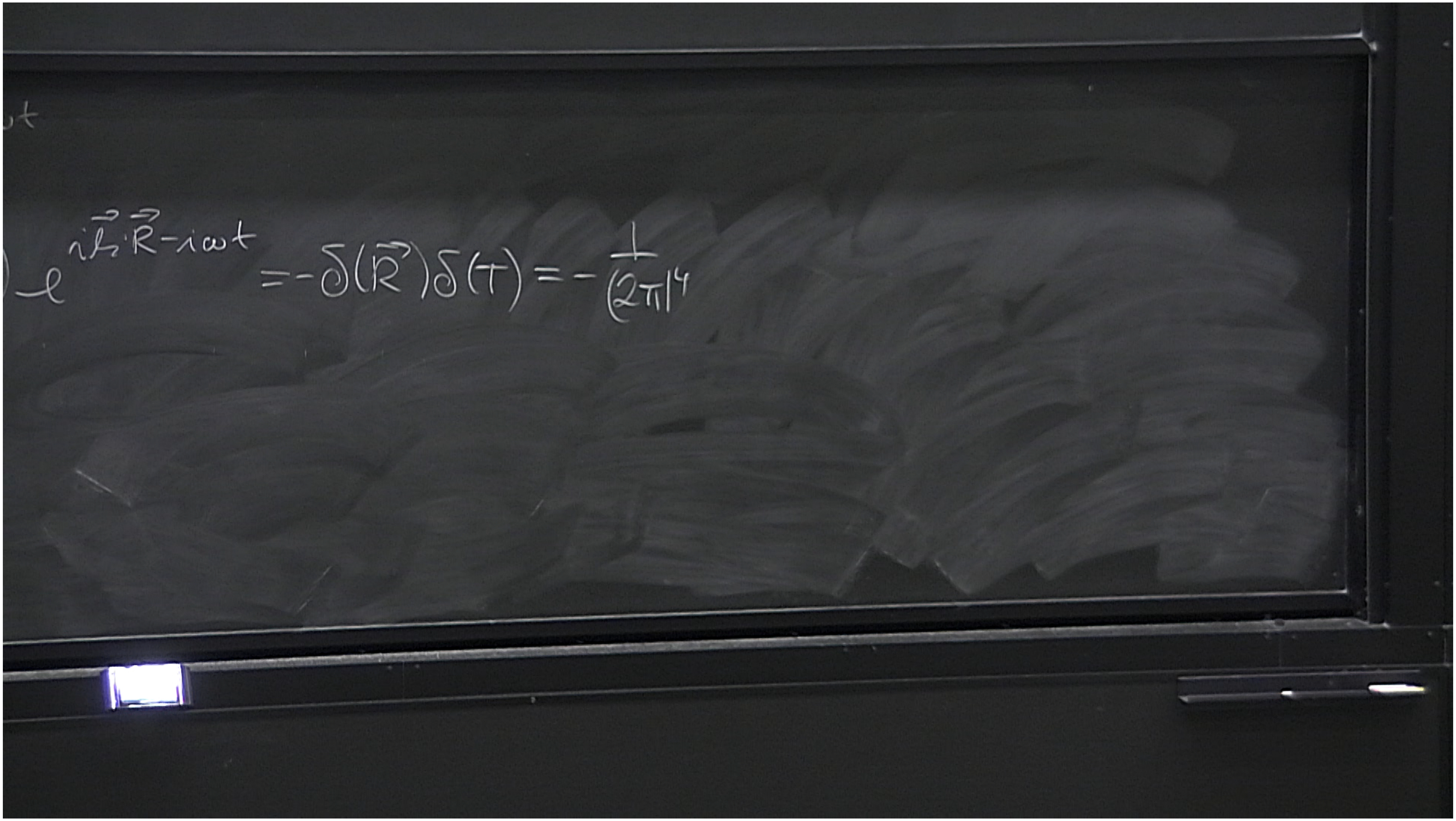
$$\Box\phi = \int \underbrace{\Box G}_{\to\, -\delta(r-r')\delta(t-t')} 4\pi\rho \, dr'\, dt' = -4\pi\rho(r,t)$$

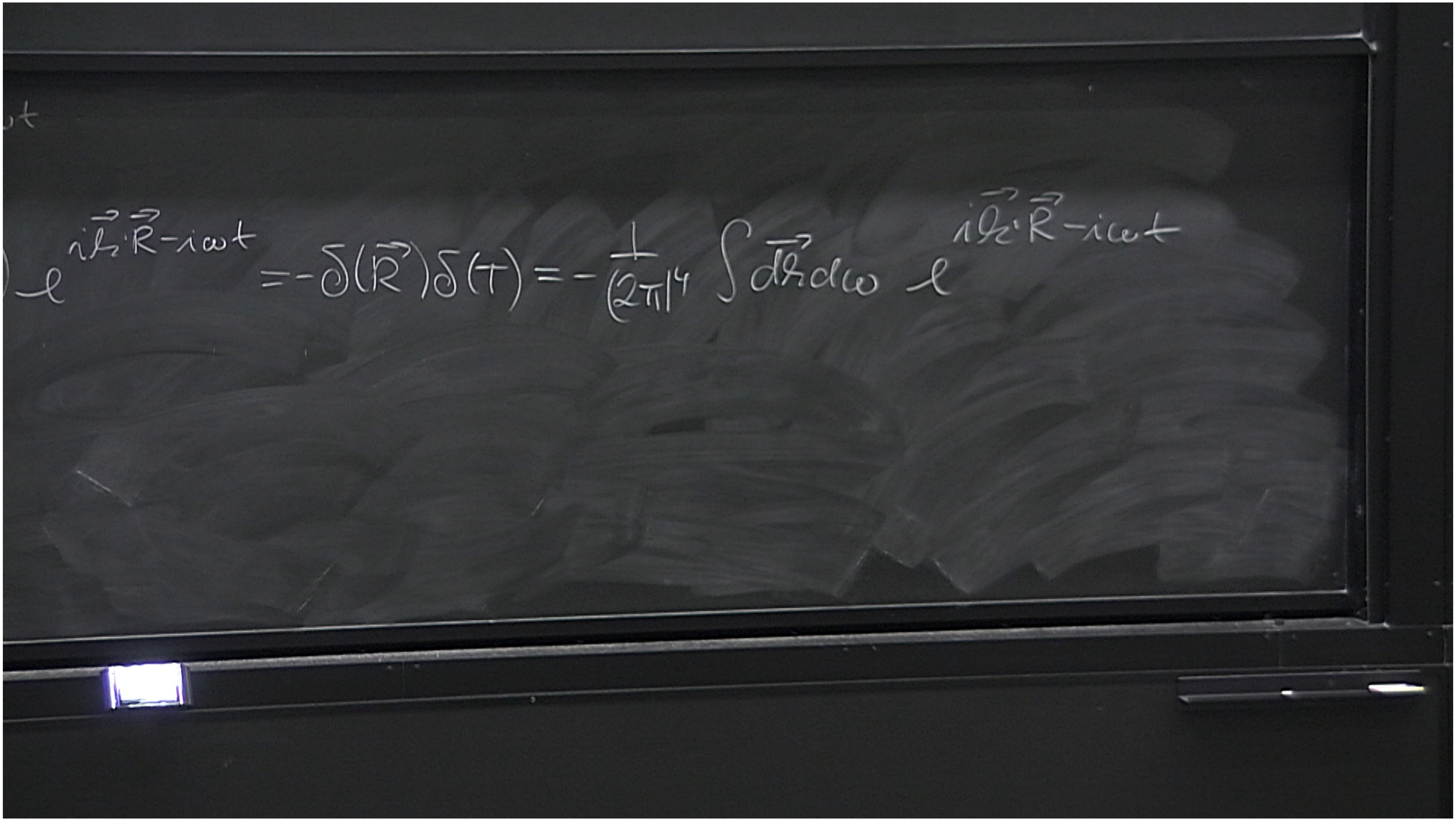

$G(\vec{R},T) = \frac{1}{(2\pi)^4} \int d\vec{k}\, d\omega \; G_{\vec{k},\omega} \, e^{i\vec{k}\cdot\vec{R} - i\omega t}$

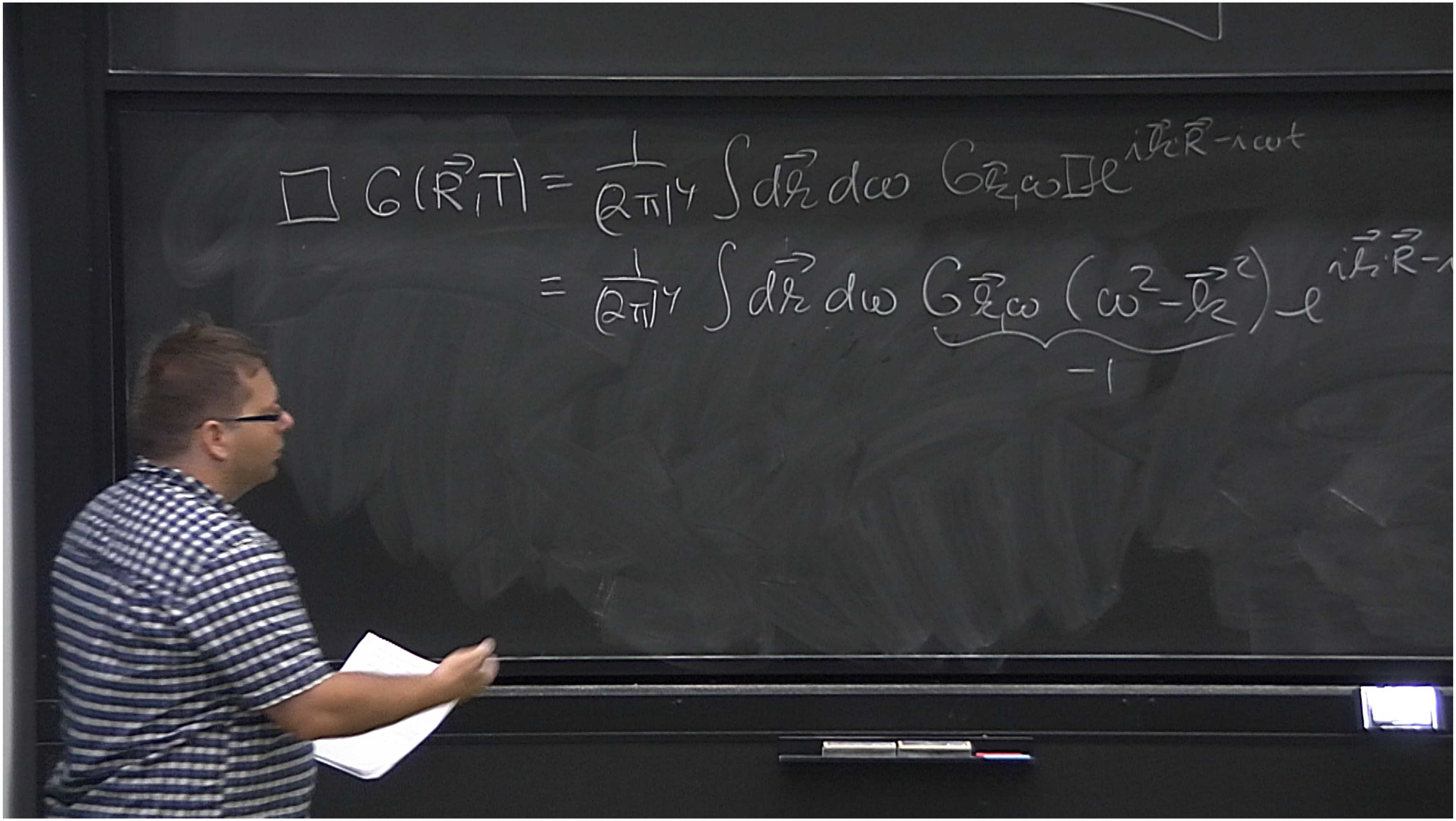

$\square \; G(\vec{R},T) = \frac{1}{(2\pi)^4} \int d\vec{k}\, d\omega \; G_{\vec{k},\omega}\, e^{i\vec{k}\cdot\vec{R} - i\omega t}$

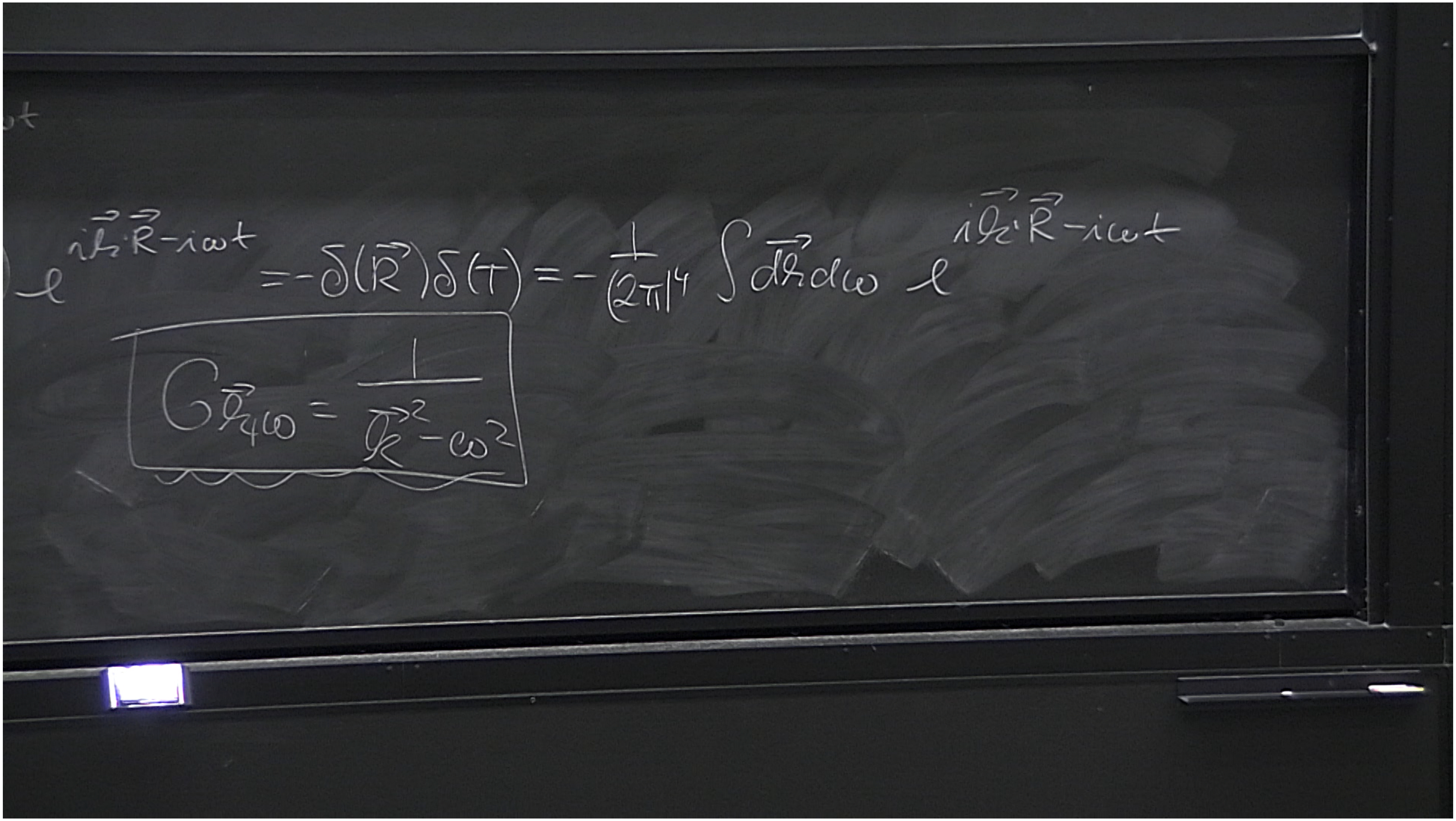

$\square\, G(\vec{R},T) = \frac{1}{(2\pi)^4}\int d\vec{k}\, d\omega\, G_{\vec{k},\omega}\,\square\, e^{i\vec{k}\cdot\vec{R}-i\omega t}$
$= \frac{1}{(2\pi)^4}\int d\vec{k}\, d\omega\, G_{\vec{k},\omega}$


$\nabla \cdot B = 0$ (4)
$\Box\, G(\vec{R},T) = \frac{1}{(2\pi)^4} \int d\vec{k}\, d\omega\, G_{\vec{k},\omega}\, \Box\, e^{i\vec{k}\cdot\vec{R} - i\omega t}$
$= \frac{1}{(2\pi)^4} \int d\vec{k}\, d\omega\, G_{\vec{k},\omega} (\omega^2 - \vec{k}^2)\, e^{i\vec{k}\cdot\vec{R} - i\omega t} = -\delta(\vec{R})\delta(T)$


ωt
e^{i k⃗·R⃗ − iωt} = −δ(R⃗)δ(t) = − 1/(2π)^4


e^{ik·R−iωt} = −δ(R)δ(t) = −1/(2π)^4 ∫ dk dω e^{ik·R−iωt}


□ G(R,T) = 1/(2π)^4 ∫ dk dω G_{k,ω} □ e^{ik·R − iωt}
= 1/(2π)^4 ∫ dk dω G_{k,ω} (ω^2 − k^2) e^{ik·R −
−1



$e^{i\vec{k}\cdot\vec{R}-i\omega t} = -\delta(\vec{R})\delta(t) = -\frac{1}{(2\pi)^4}\int d\vec{k}\,d\omega\, e^{i\vec{k}\cdot\vec{R}-i\omega t}$
$G_{\vec{k}\omega} = \frac{1}{\vec{k}^2-\omega^2}$


G(R,T) = 1/(2π)^4 ∫ dk dω G_{k,ω} e^{ik·R−iωt}