Title: PSI 2018/2019 - Classical Physics - Lecture 1

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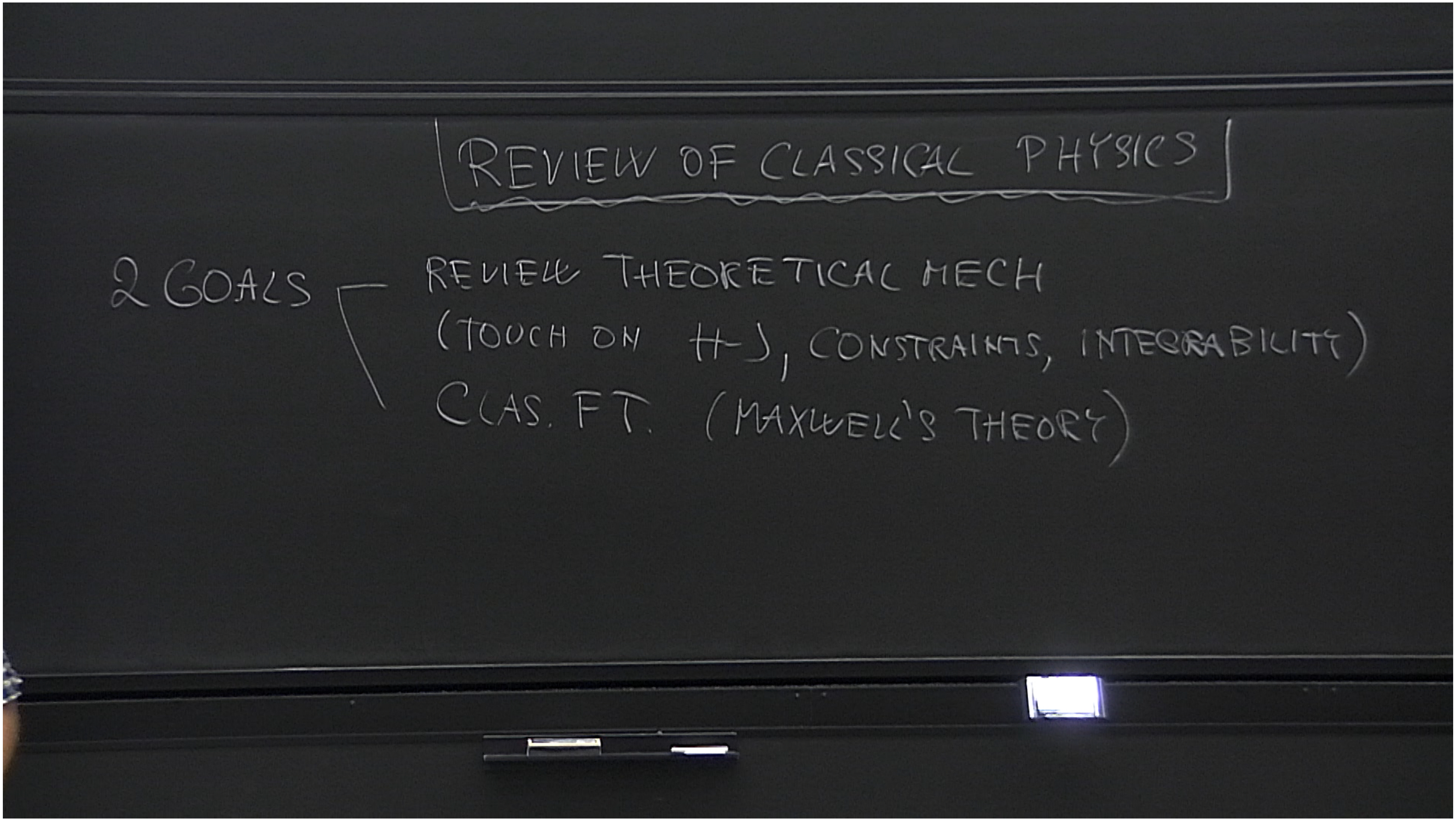
Abstract:

REVIEW OF CLASSICAL PHYSICS

2 GOALS — REVIEW THEORETICAL MECH

(TOUCH ON H-J, CONSTRAINTS, INTEGRABILITY)

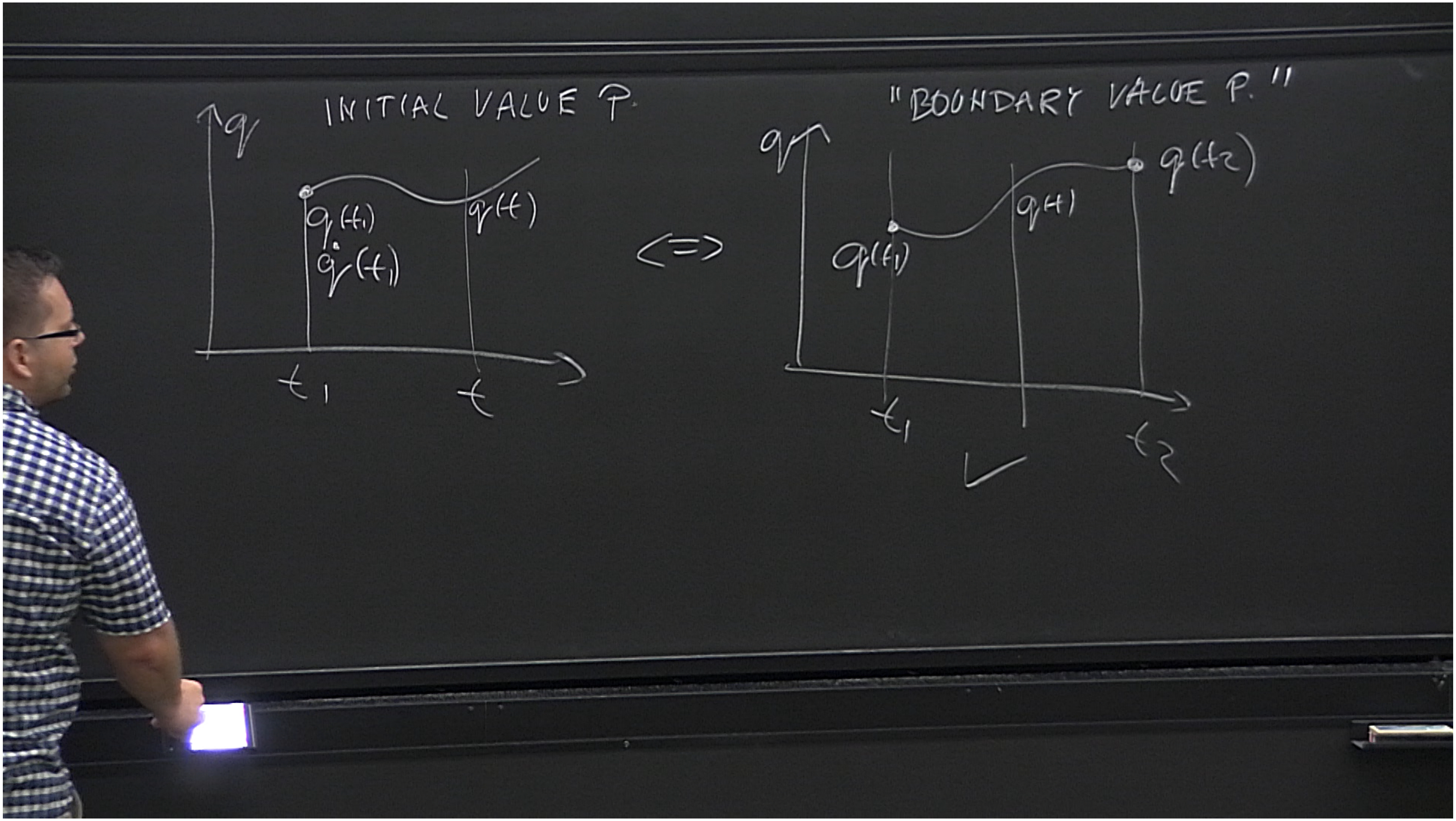

REVIEW OF CLASSICAL PHYSICS
2 GOALS
REVIEW THEORETICAL MECH
(TOUCH ON H-J, CONSTRAINTS, INTEGRABILITY)
CLAS. F.T. (MAXWELL'S THEORY)

1) LAGRANGIAN MECHANICS

a) HAMILTON'S PRINCIPLE OF LEAST ACTION

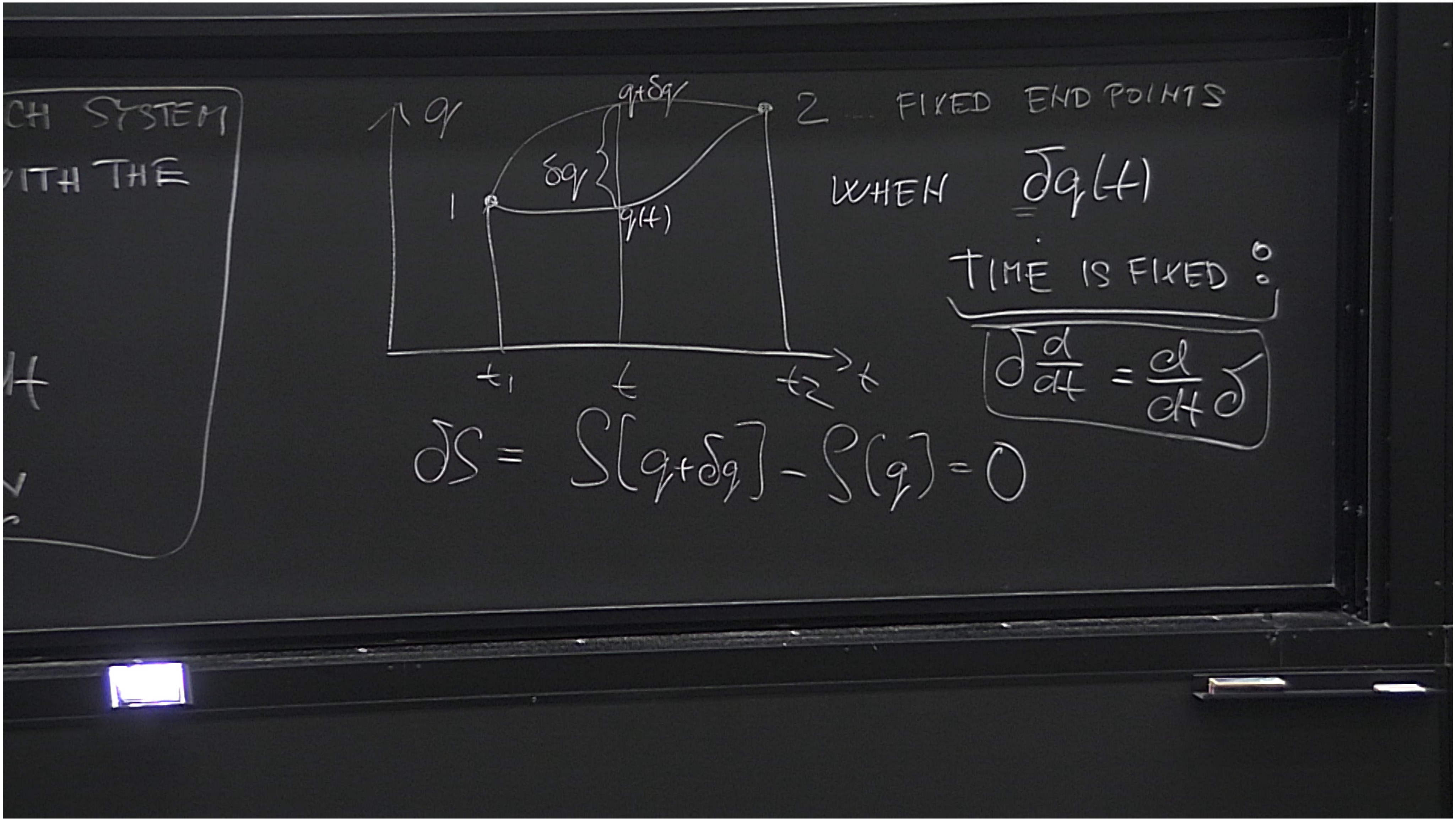
MOTION: $q^I(t)$

GENERALIZED COORDS
(AS MANY AS # DOF)


INITIAL VALUE P.
"BOUNDARY VALUE P."
q
q(t1)
q̇(t1)
q(t)
t1
t
<=>
q(t1)
q(t)
q(t2)
t1
t2

P. OF LEAST ACTION (HAMILTON): MOTION OF THE MECH SYSTEM IN TIME INTERVAL $t \in (t_1, t_2)$ COINCIDES WITH THE EXTREMAL OF THE ACTION FUNCTIONAL

$$S = S[q(t)] = \int_{t_1}^{t_2} \underbrace{L(q, \dot{q}, t)}_{\text{LAGRANGIAN}} \, dt$$

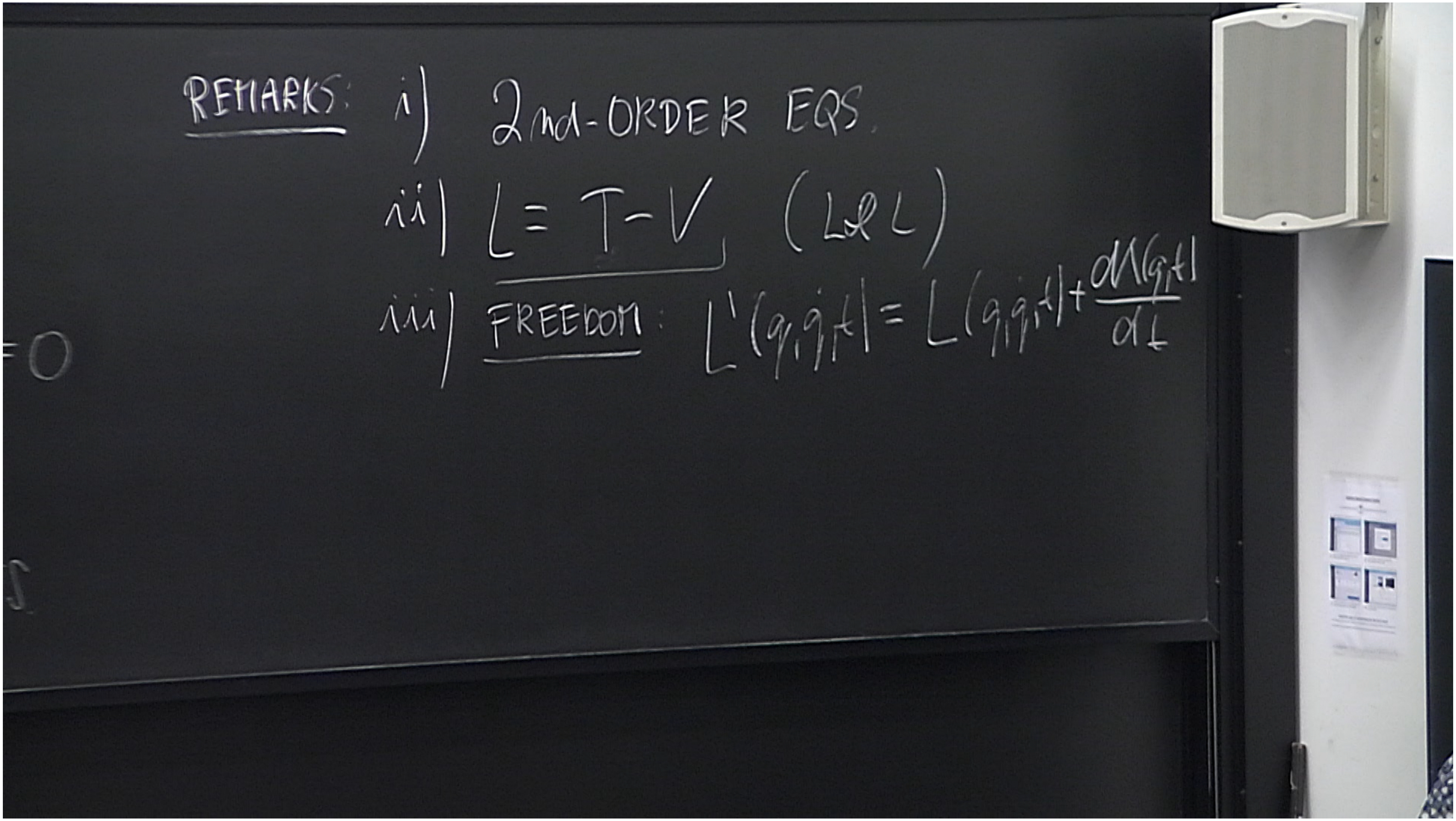

CH SYSTEM
ITH THE
q
q+δq
2 ... FIXED END POINTS
δq
1
q(t)
WHEN δq(t)
TIME IS FIXED:
δ d/dt = d/dt δ
t1
t
t2
t
δS = S[q+δq] - S[q] = 0

$$\delta S = \int_{t_1}^{t_2} \delta L \, dt = \int_{t_1}^{t_2} \left(\frac{\partial L}{\partial q}\delta q + \frac{\partial L}{\partial \dot q}\delta \dot q \right) dt$$

$$\delta \dot q = \delta \frac{d}{dt} q = \frac{d}{dt}\delta q$$

$$= \int_{t_1}^{t_2}\left(\frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{(\partial \dot q)}\right)\delta q\,dt + \underbrace{\left[\frac{\partial L}{\partial \dot q}\delta q\right]_{t_1}^{t_2}}_{0 \ldots \text{ FIXED END POINTS}} = 0$$

EULER-LAGRANGE: $\boxed{\dfrac{\partial L}{\partial q^I} - \dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot q^I}\right) = 0}$


REMARKS: i) 2nd-ORDER EQS.
ii) L = T - V (L&L)
iii) FREEDOM: L'(q, q̇, t) = L(q, q̇, t) + dΛ(q,t)/dt

REMARKS: i) 2nd-ORDER EQS.

ii) $\underline{L = T - V}$ (L&L)

iii) **FREEDOM:** $L'(q, \dot{q}, t) = L(q, \dot{q}, t) + \dfrac{d\Lambda(q,t)}{dt}$

$\Rightarrow$ E-L DO NOT CHANGE

$$S' = \int L' \, dt = S + \int \frac{d\Lambda}{dt} \, dt = S + \left[\Lambda\right]_{t_1}^{t_2}$$

$$\partial S' = \delta S + \underbrace{\left[\frac{\partial \Lambda}{\partial q}\delta q\right]_{t_1}^{t_2}}_{0 \,..\, \text{FIXED ENDPOINTS.}}$$

b) INTEGRALS OF MOTION

SOLVING EOM
- ANALYTICALLY .. INTEGRABLE SYSTM (VERY RARE BUT IMPORTANT)
- PERTURBATIONS
- NUMERICS

• **DEF:** INTEGRAL (CONSTANT) OF MOTION IS A FUNCTION

$$I = I(q, \dot{q}, t) \quad \text{S.T} \quad \frac{dI}{dt} = 0 \quad \text{FOR } q(t)$$

SOLVING EOM.

E. NOETHER $\boxed{\text{VI}}$ FOR EVERY GLOBAL CONTINUOS SYMMETRY OF THE SYSTEM, THERE IS A CORRESPONDING INTEGRAL OF MOTION

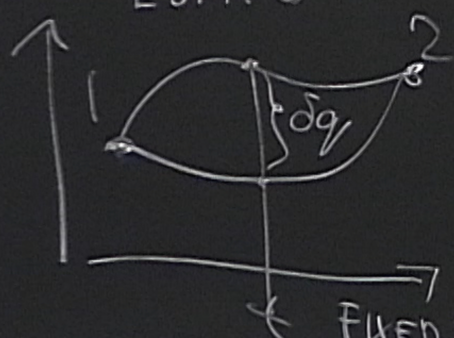
. GEOMETRIZES PHYSICS.

- TYPES OF SYMMETRIES

$$t \to t' = t + \tilde{\delta} t$$

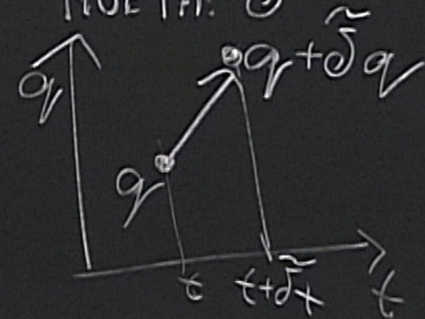
$$q \to q'(t') = q(t) + \tilde{\delta} q(t)$$

EOM: δ



FIXED ENDP.

NOETH: $\tilde{\delta}$



• $\tilde{\delta} t = \varepsilon \Delta t$ $\quad$ $\tilde{\delta} q = \varepsilon \Delta q$

PARAM.
(HOW BIG THE TRANSF IS)

"GENERATOR"
(WHAT TYPE OF TRANSF WE CONSIDER)

90° $\quad$ a

• ε .. FINITE GIVEN #

SYMMETRIES
- DISCRETE
- CONT. ... ε IS ARBITRARILY SMALL

- GLOBAL ... $\varepsilon = \text{CONST}$. NOETH S. CONSERV.
- LOCAL ... $\varepsilon = \varepsilon(t)$ .. GAUGE S. ... BIANCHI IDS.

NOETHER T. (V2 - EXPLICIT ONE)

LET $\tilde{\delta}$ BE A GLOBAL CONT. SYM., THAT IS OFF-SHELL WE FIND $\tilde{\delta} t, \tilde{\delta} q$ S.T.

$$\tilde{\delta} S = 0 \Rightarrow$$

$$I = \frac{\partial L}{\partial \dot{q}} \tilde{\delta} q + \left(L - \dot{q} \frac{\partial L}{\partial \dot{q}} \right) \tilde{\delta} q$$

ON-SHELL... EOM ARE REQUIRED!

OFF-SHELL... VALID ALL THE TIME.

EX: TIME TRANSLATION SYMMETRY. $L \neq L(t)$.

$$\tilde{\delta} t = \varepsilon, \; \tilde{\delta} q = 0 \; = \tilde{\delta} S = 0$$

$$\boxed{I = L - \dot{q} \frac{\partial L}{\partial \dot{q}}} \; \ldots$$ GENERALIZED ENERGY.

PROOF IT IS CONSERVED:

EX2.

$$\frac{dI}{dt} = \frac{d}{dt} L - \frac{d}{dt}\left(\dot{q}\,\frac{\partial L}{\partial \dot{q}}\right)$$

$$= \frac{\partial L}{\partial q}\dot{q} + \cancel{\frac{\partial L}{\partial \dot{q}}\ddot{q}} - \cancel{\ddot{q}\frac{\partial L}{\partial \dot{q}}} - \dot{q}\,\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = 0$$

EX2: SPACE TRANSLATION SYMMETRY.

$L \neq L(q_k)$ CYCLIC COORDINATE

$$\tilde{\delta} t = 0, \quad \tilde{\delta} q = \varepsilon \Rightarrow \tilde{\delta} S = 0$$

$$\left(\frac{\partial L}{\partial \dot{q}}\right) = 0$$

$I = \dfrac{\partial L}{\partial \dot{q}}$ ... GENERALIZED MOMENTUM

- TYPES OF SYMMETRIES

$t \to t' = t + \tilde{\delta} t$

- $\tilde{\delta} t = \varepsilon \Delta t$

PARAM

PROOF OF TV2:

- RELATIONSHIP BETWEEN δ & $\tilde{\delta}$:
  - $\tilde{\delta}\, dt = dt' - dt = d\tilde{\delta} t = \frac{d\tilde{\delta} t}{dt} dt$
  - $\tilde{\delta} q(t) = q'(t') - q(t) = q'(t) + \tilde{\delta} t \frac{dq'(t)}{dt} + \dots$

$t \to t' = t + \delta t$

PROOF OF TV2:

- RELATIONSHIP BETWEEN δ & $\tilde{\delta}$:
  - $\tilde{\delta}\, dt = dt' - dt = d\tilde{\delta} t = \frac{d\tilde{\delta} t}{dt}\, dt$
  - $\tilde{\delta} q(t) = q'(t') - q(t) = \underline{q'(t)} + \underline{\tilde{\delta} t \, \frac{dq'(t)}{dt}} + \dots \underline{- q(t)}$

$$= \delta q(t) + \tilde{\delta} t \, \frac{dq(t)}{dt}$$

$$\tilde{\delta} S \overset{\text{AS}}{\doteq} 0 = \int \left(\tilde{\delta} L \, dt + L \, \tilde{\delta} dt \right)$$

$$= \int \left(\delta L \, dt + \underbrace{\tilde{\delta} t \frac{dL}{dt} dt + L \, \tilde{\delta} dt}_{\frac{d}{dt}(L \tilde{\delta} t) \, dt} \right) = \delta S + \int \frac{d}{dt} \left(L \tilde{\delta} t \right) dt$$

$$= \int \left(\frac{\partial L}{\partial q} \delta q + \frac{\partial L}{\partial \dot{q}} \delta \dot{q} \right) dt + \frac{d}{dt} \left(L \tilde{\delta} t \right) dt$$

$$= \int \left(\underbrace{\frac{\partial L}{\partial q} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}}}_{0} \right) \delta q \, dt + \int \frac{d}{dt} \left(L \tilde{\delta} t + \frac{\partial L}{\partial \dot{q}} \delta q \right) dt$$