Title: PSI 2018/2019 - Lie Groups and Lie Algebras - Lecture 1

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Abstract:

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Lie Groups and Lie Algebras

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Submitted by tali (175)
5 August, 2018 - 23:10

Groups: PSI 2019

Lecturer: Tibra Ali

Referenes:

1. HF Jones. Groups, Represnetations and Physics (for tensor methods and Young's tableau)
2. A Zee. Group Theory in a Nutshell for Physicists (classification of Lie algebras)
3. SD Joglekar. Mathematical Physics, volume II (discrete groups and introductory material)
4. M Nakahara. Geometry, Topology and Physics (applications to geometry)
5. MB Robinson, T Ali, GB Cleaver. A Simple Introduction to Particle Physics, Part II. (online book on arXiv.org) (Heuristic derivation of Lie algebras from Lie groups)

Course Outline:

- Motivations for studying Lie groups and Lie algebras
- Fundamental defintion and important concept of group theory
- Lie groups and how Lie algebras arise from them
- Examples of matrix Lie groups and Lie algebras
- Application of Lie groups to geometry and physics
- SU(2) and its representation theory
- Tensor methods and Young's tableau for SU(2), SU(3), ... , SU(N)
- Classification of classical Lie algebras
- Spinor representations of the Lorentz group [If time permits.]

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TIBRA ALI

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Motivations Symmetries of physical systems.

1905 – Einstein's Special Relativity. Maxwell's eqn. — Mi

Standard Model of particle physics $SU(3) \times SU(2) \times U(1)$ —

of physical systems.

s Special Relativity. Maxwell's eqn. — Minkowski geometry

el of particle physics $SU(3) \times SU(2) \times U(1)$ — Internal 'gauge' symmetry

Classical field theory — 100 years ago Emmy Noether. Sy

→ Breakthroughs in QFT

Wigner's theorem: Hilbert space of a quantum system furnish a un

Classical field theory — 100 years ago Emmy Noether

→ Breakthroughs in QFT

Wigner's theorem: Hilbert space of a quantum system furnish

→ Weinberg Vol I.

... $SU(3)\times SU(2)\times U(1)$ — Internal 'gauge' symmetry

...go Emmy Noether : Symmetries $\Longrightarrow$ Conserved quantities

...ntum system furnish a unitary & linear rep of the symmetry group of the system.

Fundamental Concepts:

Defⁿ Group: A set G equipped with a binary operation $\circ : G \times G \to$ under which the set remains closed. And satisfies the following.

1. Closure: If $a, b \in G$ then $a \circ b \in G$
2. Associative $(a \circ b) \circ c = a \circ (b \circ c)$
3. $\exists$ a unique element $e \to$ identity $a \circ e = a \; \forall a \in G$

...ch $\circ : G \times G \longrightarrow G$, known as group multiplication.

...following:

$= a \ \forall a \in G$

4 Inverses. For each a in G $\exists$ a unique inverse a^{-1}

st $a \circ a^{-1} = e$

$a^{-1} \to$ right inverse

Show: $a^{-1} \to$ left inverse

$$a^{-1} \circ a = e$$

$$\begin{aligned} a^{-1} a &= a^{-1} e a \\ &= a^{-1} (a a^{-1}) a \\ &= a^{-1} a a^{-1} a \\ &= (a^{-1} a)^2 \end{aligned}$$

$$\to e = (a^{-1} a)^{2-1} = a^{-1} a .$$

ght inverse

$^{-1} \to$ left inverse

$= e$

$$a^{-1}a = a^{-1}ea$$
$$= a^{-1}(aa^{-1})a$$
$$= a^{-1}aa^{-1}a$$
$$= (a^{-1}a)^2$$
$$\to e = (a^{-1}a)^{2-1} = a^{-1}a.$$

$$a = a \quad = a\cdot(a^{-1}\cdot a)$$
$$= \quad \cdot(a^{-1})^{-1}$$

$$a^{-1}a = a^{-1}a\,e$$
$$= a^{-1}a\,a^{-1}(a^{-1})^{-1}$$
$$= a^{-1}(aa^{-1})(a^{-1})^{-1}$$
$$= a^{-1}e(a^{-1})^{-1}$$
$$= a^{-1}(a^{-1})^{-1} = e$$

$ea = (aa^{-1})a = ae = a$

$G \rightarrow$ finite or infinite (countably or <u>uncountably</u>)

Size of G, order of G

$ab \neq ba$. If all elements of G commute we call gp Abelian. If $\exists$ a

$G \rightarrow$ non-Abelian

$y)$

we call gp Abelian. If $\exists$ at least 2 elements $a, b \in G$ st $ab \neq ba$ then

Examples: 1) $C_4 = \{\alpha, \alpha^2, \alpha^3, \alpha^4 \equiv e\}$

2)

$$\alpha^n = \underbrace{\alpha \cdots \alpha}_{n\text{-times}}$$

$e = \alpha^4$

$a = \alpha$

$b = \alpha^2$

$c = \alpha^3$

$\circ$	e	a	b	c
e	e	a	b	c
a	a	b	c	e
b	b	c	e	a
c	c	e	a	b

2) $\mathbb{Z}/4\mathbb{Z} = \{0, 1, 2, 3\}$

+	0	1	2	3
$e \leftarrow 0$	0	1	2	3
$a \leftarrow 1$	1	2	3	0
$b \leftarrow 2$	2	3	0	1
$c \leftarrow 3$	3	0	1	2

Group Isomorphism

$$C_4 \cong \mathbb{Z}/4\mathbb{Z}$$

Example of a transformation group

Non-Abelian $(23)\,(12) = (321)$

$(12) : (abc) \to (bac)$

$(23)\,(bac) \to (bca)$

$$(12)(23) = (123)$$

$$(23): (abc) \rightarrow (acb)$$

$$(12): (acb) \rightarrow (cab)$$

Cayley's theorem: Every finite group is a proper subgroup of a permutation group

Fundamental Concepts:

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3. $\exists$ a unique element $e \rightarrow$ identity $a \circ e = a$

operation $\circ : G \times G \longrightarrow G$, known as group multiplication.

fies the following:

$a \circ e = a \;\forall a \in G$

4 Inverses. For each a in G $\exists$ a unique inverse a^{-1}

st $a \circ a^{-1} = e$

4. Rotation in 2D

$$\underline{r} = \hat{\underline{i}}\,x + \hat{\underline{j}}\,y$$

Rotate $\underline{r}$ by some angle θ

$$\underline{r} \longrightarrow R(\theta)\,\underline{r}$$

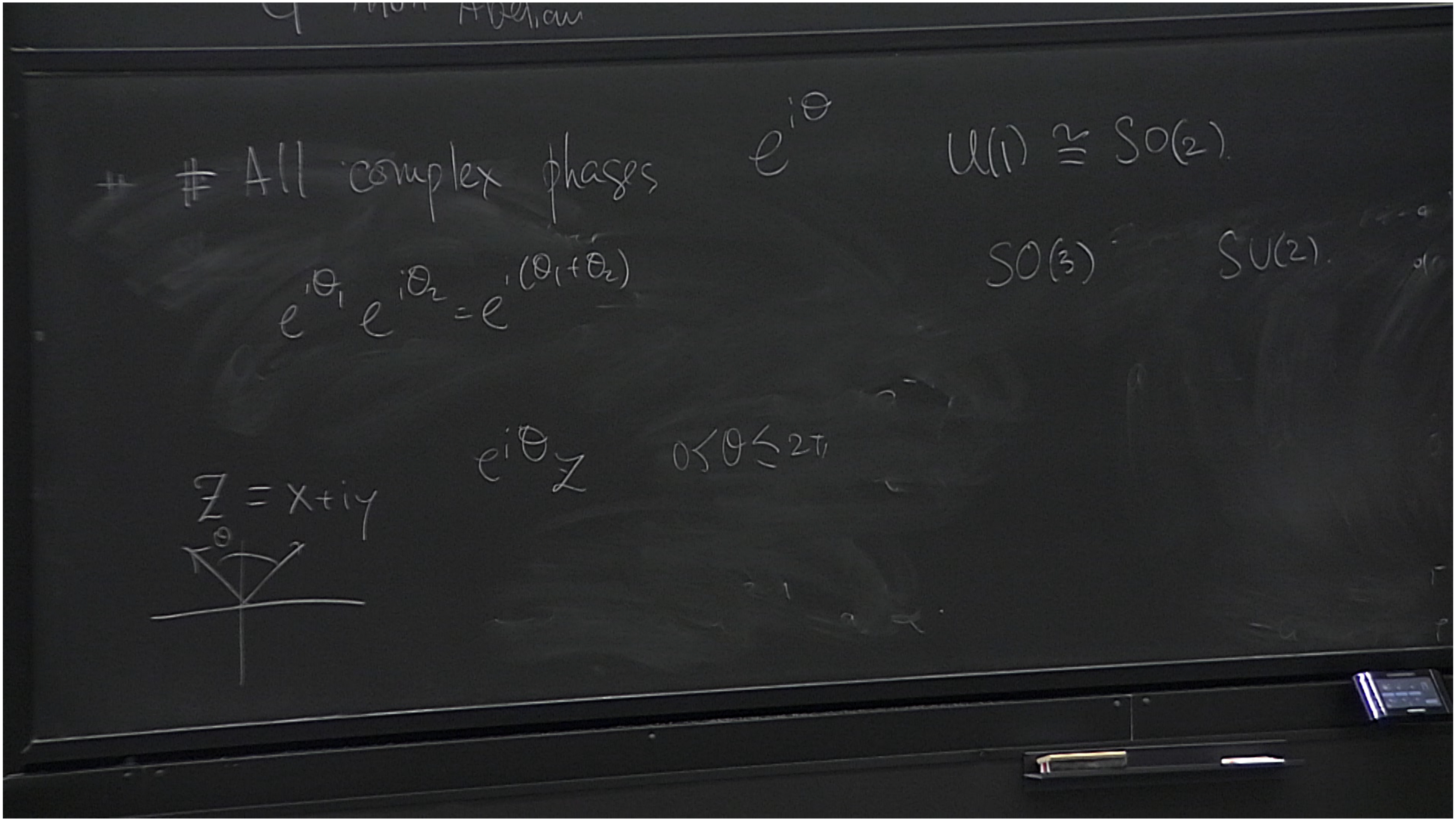
$$\Rightarrow \sqrt{\underline{r}\cdot\underline{r}} \rightarrow \text{unchanged.}$$

$$R(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

Group under matrix multiplication: $R(\theta_1)R(\theta_2) = R(\theta_1+\theta_2)$

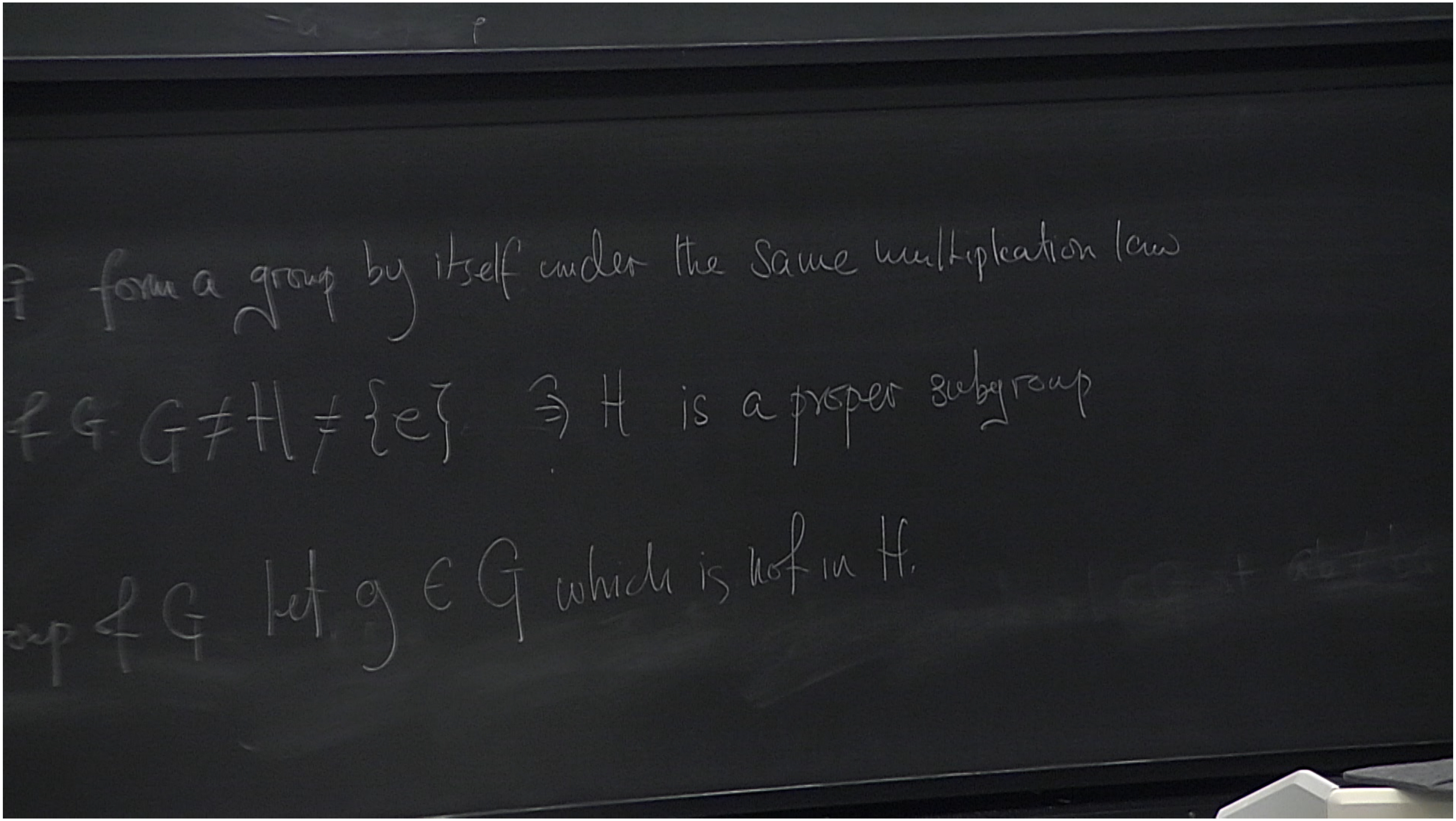
$\det R = 1$, $R^T R = R R^T = \mathbb{I}$

$\Rightarrow R \in SO(2)$, $0 < \theta \leq 2\pi$


Abelian
# All complex phases e^{iθ}
U(1) ≅ SO(2)
SO(3)
SU(2)
e^{iθ₁} e^{iθ₂} = e^{i(θ₁+θ₂)}
e^{iθ} z
0 ≤ θ ≤ 2π
z = x+iy
θ

<u>Defn</u> : <u>Subgroup</u>: If a subset $H \subset G$ form a group by itself then H is subgroup of G. $G \neq H \neq \{e\}$

Right <u>Coset</u>: Let $H = \{h_1, h_2, \ldots\}$ be prop subgroup of G let $g \in G$


form a group by itself under the same multiplication law
of G. $G \neq H \neq \{e\} \Rightarrow H$ is a proper subgroup
of G let $g \in G$ which is not in H.

Defn: Subgroup: If a subset $H \subset G$ form a grou

then H is subgroup of G. $G \neq H$

Right Coset: Let $H = \{h_1, h_2, \ldots\}$ be prop subgroup of G Let

the set $\{gh : \forall h \in H\} \rightarrow$ Right Coset gH

$-\sin\theta$

$\cos\theta$

x multiplication: $R(\theta_1)R(\theta_2) = R(\theta_1+\theta_2)$

$R^T R = R R^T = \mathbb{I}$

(2) , $0 < \theta \leq 2\pi$

$h_1 h_1^{-1} = e$

$h_1(h_1^{-1} h) = h$

$\#$ gH in $G-H$

$\#$ $G = H + g_1 H + g_2 H + \cdots$

Coset decomposition

$\#$ Coset space $G \backslash H = \{H, g_1 H, g_2 H, \ldots\}$

SO(3)

SO(2)

$SO(3)/SO(2) = S^2$

Ex: Consider $P_2 \subset P_3$. $P_2 = \{(\ ), (12)\}$

Work out all the cosets.

Defn : Subgroup: If a subset $H \subset G$ form a group by itself un

then H is subgroup of G. $G \neq H \neq \{e\}$ $\Rightarrow$

Left Coset: let $H = \{h_1, h_2, \ldots\}$ be prop subgroup of G let $g \in G$ w

the set $\{gh : \forall h \in H\}$ $\rightarrow$ Right Coset gH

\# gH in $G-H$

$SO(3)$

Normal or Invariant Subgroups

If $\forall g \in G$ we have $gH = Hg$

then H is a normal or invariant subgroup

Def 4: Simple group: e & G are the only normal subgrou

$\underline{\text{Def}^n}$: Centre: A subset Z of G is called the centre if

$\forall z \in Z$ and $\forall g \in G$ we have $zg = gz$

groups of G then we say G is simple.