Title: Local quantum operations and causality

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Abstract: I give further details on a unification of the foundations of operational quantum theory with those of quantum field theory, coming out of a program that is also known as the positive formalism. I will discuss status and challenges of this program, focusing on the central new concept of local quantum operation. Among the conceptual challenges I want to highlight the question of causality. How do we know that future choices of measurement settings do not influence present measurement results? Should we enforce this, as in the standard formulation of quantum theory? Should this "emerge" from a fundamental theory? Does this question even make sense in a context without a fixed notion of time, such as quantum gravity? With a heavy dose of speculation (put also grounded in very concrete evidence) I find that fermionic theories might play an essential role.

Local quantum operations and causality

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Foundations of Quantum Theory
Perimeter Institute
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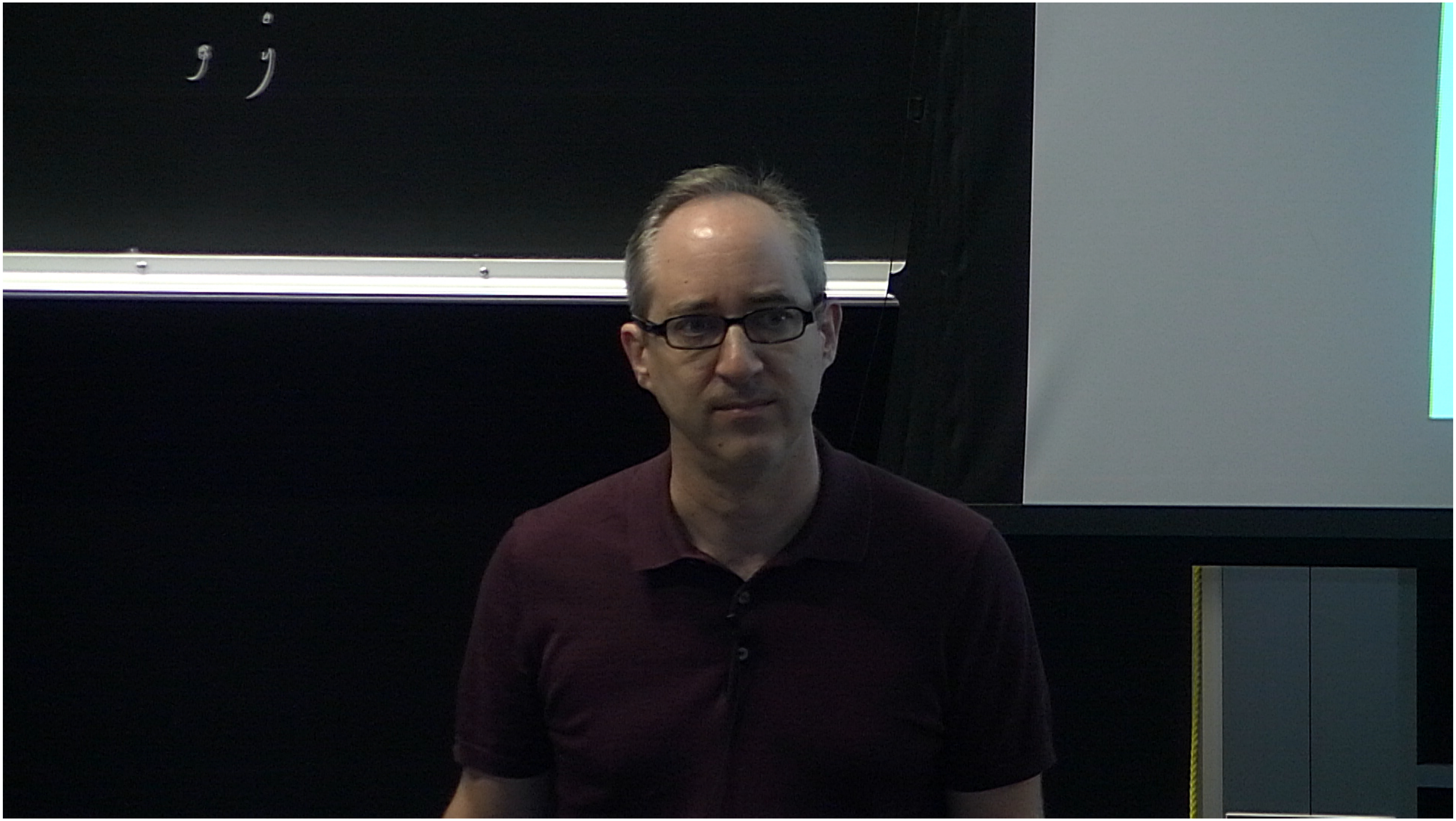
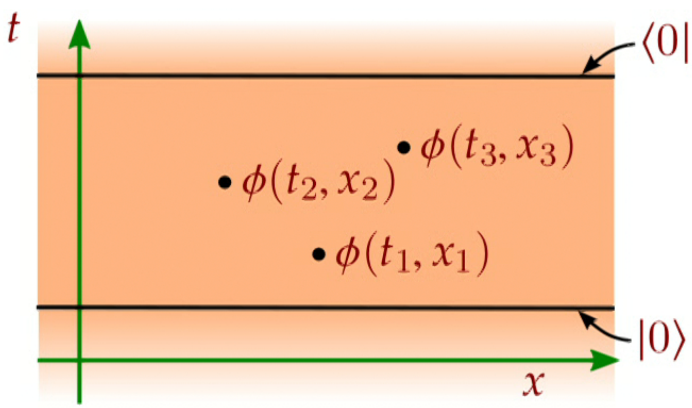


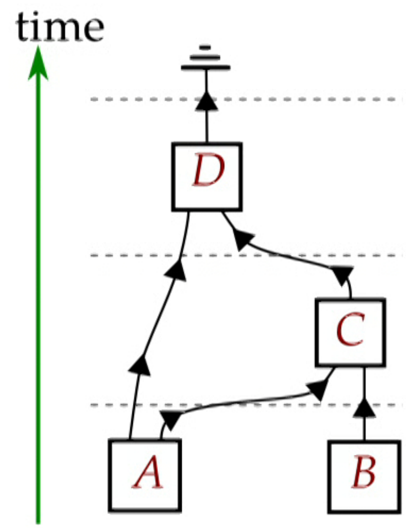
QFT vs Foundations of QT

We have two languages for fundamental physics (not counting GR!)

Quantum Field Theory
(Textbook Formulation)



Foundations of QT
(Standard Formulation)



We should find a unified language for both!

Standard Formulation

Ingredients

Convex operational framework / Standard Formulation of QT

$\mathcal{B}$ real **partially ordered vector space** with inner product $(\!|\cdot,\cdot|\!)$ and **order unit** $\mathbf{e} \in \mathcal{B}$.

In quantum theory:

- $\mathcal{B}$ space of self-adjoint operators on the Hilbert space $\mathcal{H}$
- $(\!|A, B|\!) = \mathrm{tr}(AB)$
- $\mathbf{e} = \mathrm{id}$ the identity operator on $\mathcal{H}$

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Diagrams:

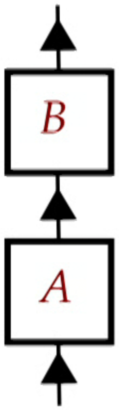
$A : \mathcal{B} \to \mathcal{B}$ operation

$B : \mathbb{R} \to \mathcal{B}$ preparation

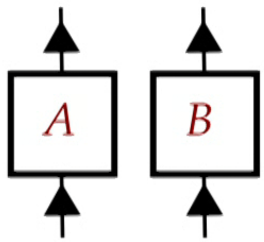
$C : \mathcal{B} \to \mathbb{R}$ effect

$\sigma \mapsto (\!|\mathbf{e}, \sigma|\!)$ discard

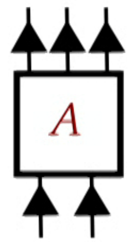
Composition



$B \circ A : \mathcal{B} \to \mathcal{B}$

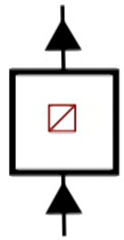
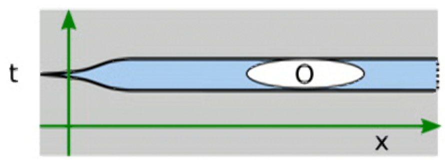
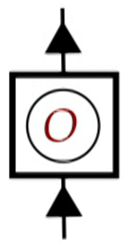


$A \otimes B :$
$\mathcal{B}_1 \otimes \mathcal{B}_2 \to \mathcal{B}_1 \otimes \mathcal{B}_2$



$A : \mathcal{B}_1 \otimes \mathcal{B}_2$
$\to \mathcal{B}_3 \otimes \mathcal{B}_4 \otimes \mathcal{B}_5$

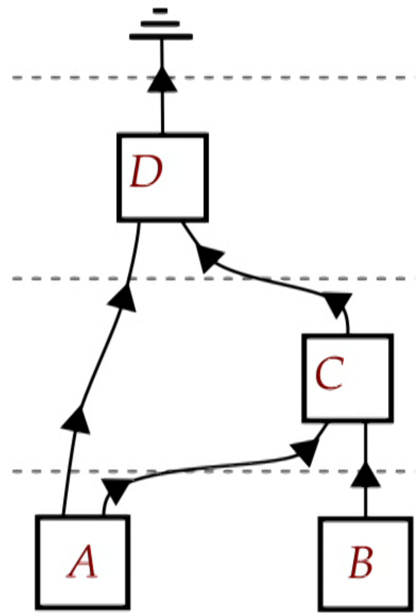
Quantum operations

spacetime	diag.	Hilbert space	(stat.) state space
		$U : \mathcal{H} \to \mathcal{H}$ unitary	$\hat{U} : \mathcal{B} \to \mathcal{B}$ $\hat{U}(\sigma) = U \sigma U^\dagger$
		$O : \mathcal{H} \to \mathcal{H}$ hermitian $O = \sum_j o_j P_j$	$\hat{O} : \mathcal{B} \to \mathcal{B}$ $\hat{O}[i](\sigma) = P_i \sigma P_i^\dagger$ $\hat{O}[*](\sigma) = \sum_j \hat{O}[j]$ $\hat{O}(\sigma) = \sum_j o_j \hat{O}[j]$
			$A : \mathcal{B} \to \mathcal{B}$ completely positive $A(\sigma) = \sum_j K_j \sigma K_j^\dagger$

Evaluation and probability

Example:

- preparations A, B
- measurements C, D
- a discard effect $\mathbf{e}$



Evaluation and probability

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- preparations A, B
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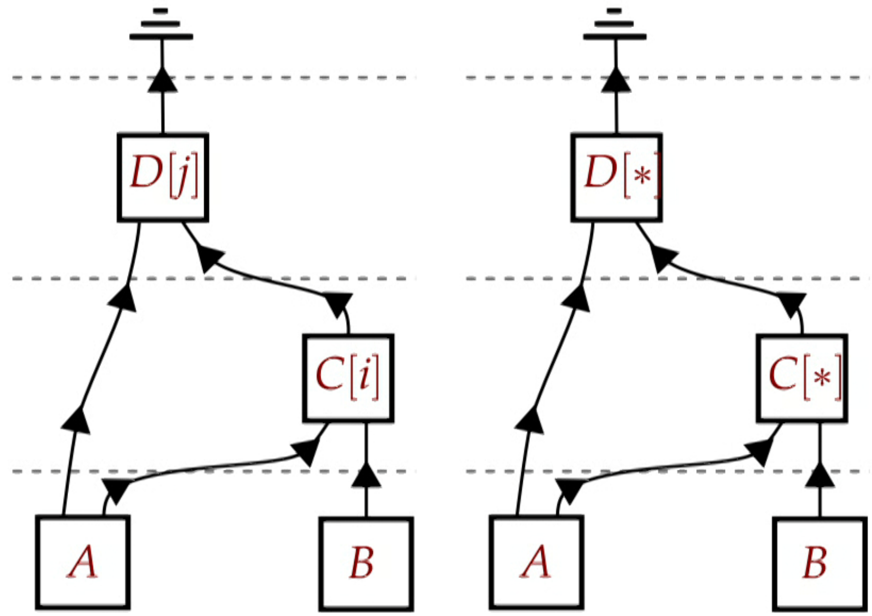
What is the probability for outcome (i, j) in measurements (C, D)?



Evaluation and probability

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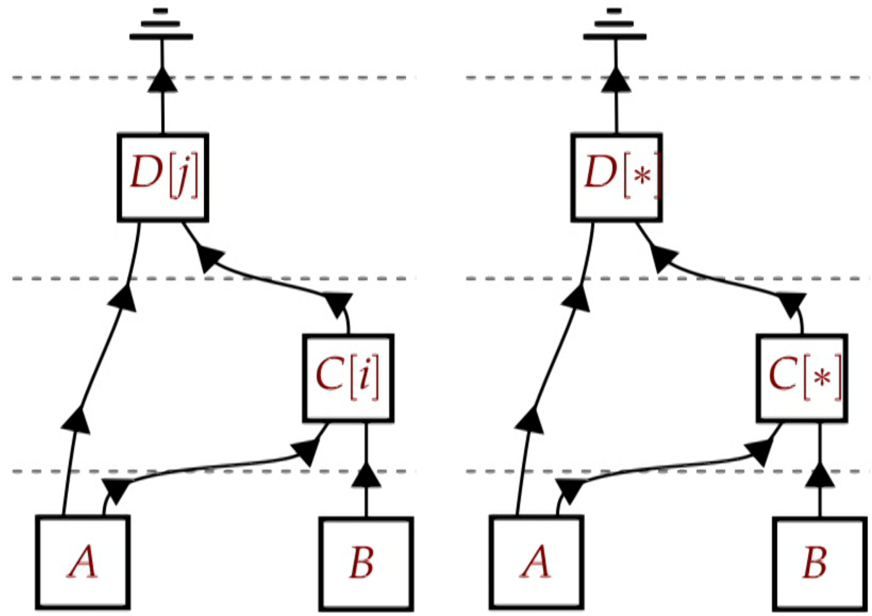

$A \diamond B \diamond C[i] \diamond D[j] \diamond \mathbf{e}$ $\quad$ $A \diamond B \diamond C[*] \diamond D[*] \diamond \mathbf{e}$

Probability: $$P(i, j) = \frac{A \diamond B \diamond C[i] \diamond D[j] \diamond \mathbf{e}}{A \diamond B \diamond C[*] \diamond D[*] \diamond \mathbf{e}}$$

Causality condition in standard formulation

Causality is implemented through a normalization condition on operations. This is **time-asymmetric**, requires directionality.

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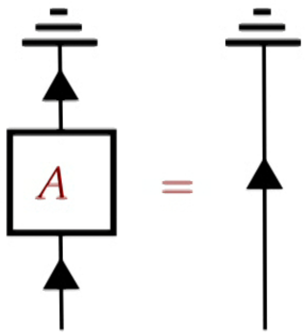
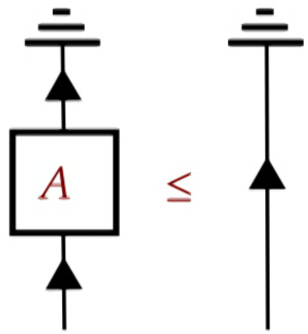
$A : \mathcal{B} \to \mathcal{B}$ **non-selective**



$(\!|\mathbf{e}, A\sigma|\!) = (\!|\mathbf{e}, \sigma|\!)$

$A : \mathcal{B} \to \mathcal{B}$ **selective**



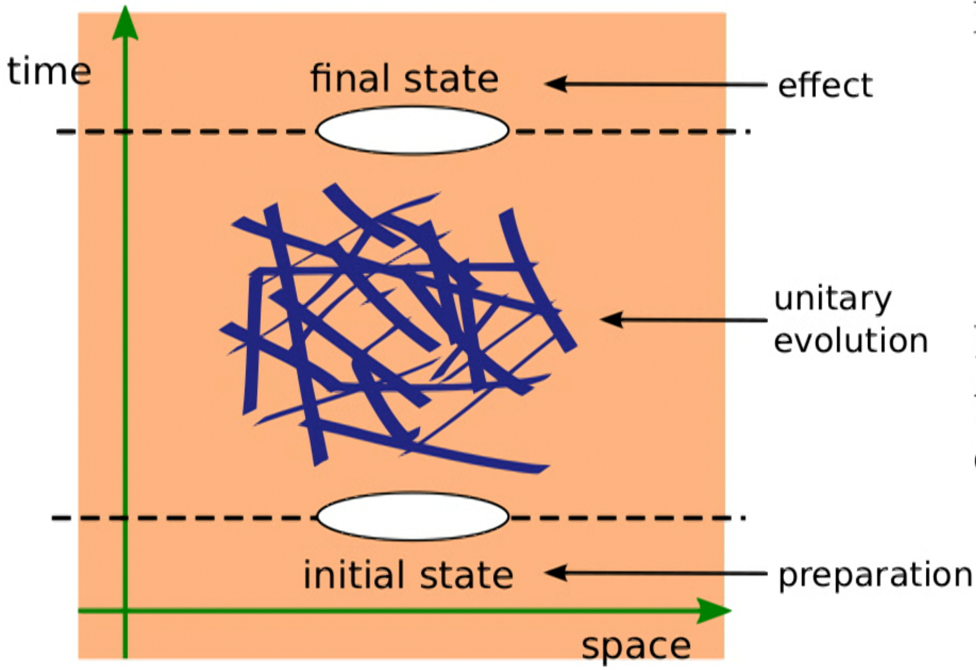
$(\!|\mathbf{e}, A\sigma|\!) \leq (\!|\mathbf{e}, \sigma|\!)$

Further consequence: exclusive **composites** of **non-selective operations** (including preparations and discard) evaluate to 1.
Example: $A \diamond B \diamond C[*] \diamond D[*] \diamond \mathbf{e} = 1$.

Quantum field theory

Measurement in QFT: S-matrix

In **quantum field theory** measurement is idealized to take place at **asymptotic infinite time**.



Measurement is

- a preparation at infinitely early time
- an effect at infinitely late time

Described by **S-matrix**, the asymptotic **time evolution operator**

$$\langle \mathrm{out}|S|\mathrm{in}\rangle$$

Requires no composition of measurements. Pure states are sufficient.

n-point functions

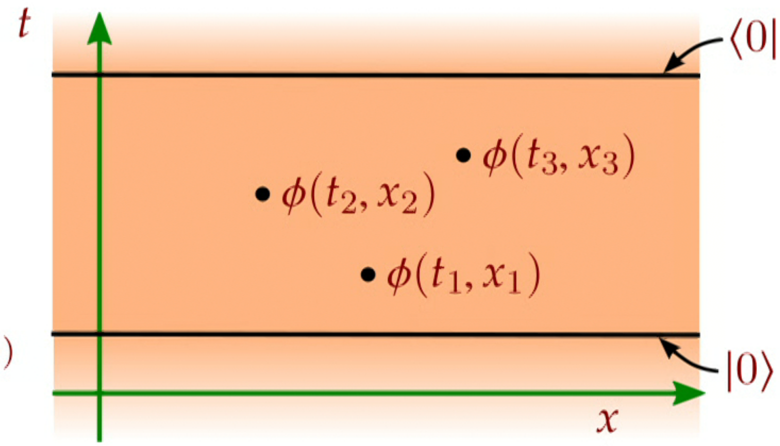
Key observables are **field operators** $\phi(t,x)$. The relevant product is the **time-ordered product**.

Typical object: **n-point function**

$$\langle 0|T\phi(t_3,x_3)\phi(t_2,x_2)\phi(t_1,x_1)|0\rangle$$

is given by the **path integral**

$$\int \mathcal{D}\phi\,\phi(t_1,x_1)\phi(t_2,x_2)\phi(t_3,x_3)e^{\mathrm{i}S(\phi)}$$



These structures exhibit **relativistic covariance**, allow for the implementation of **gauge symmetries** and are instrumental in the description of the **Standard Model of Elementary Particle Physics**.

Microcausality in QFT

In QFT there is a notion of **microcausality**. Observables must commute if they are relatively **spacelike** localized.

$$[\phi(t,x),\phi'(t',x')] = 0 \qquad \text{if } (t,x) \text{ and } (t',x') \text{ relatively spacelike}$$

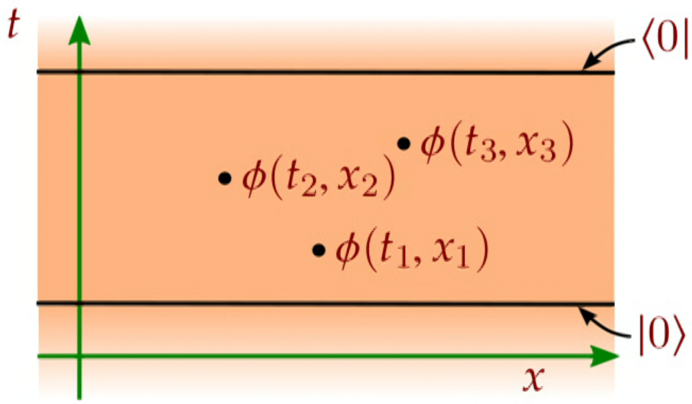
This is necessary for the consistency of the **time-ordered product**.

This can also be interpreted to imply that spacelike separated measurements cannot influence each other.

Note that this condition is **time-reversal symmetric**. It does not imply temporal causality.

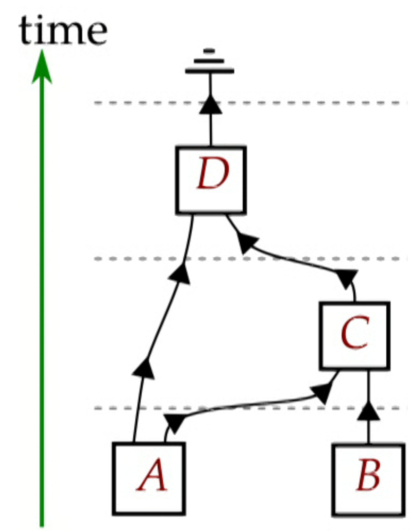
QFT vs Foundations of QT – comparison

Quantum Field Theory
(Textbook Formulation)



− restricted measurements
- (fixed) **spacetime** notion
+ special **relativistic**
+ exhibits **locality**
- time-reversal symmetric*

Foundations of QT
(Standard Formulation)

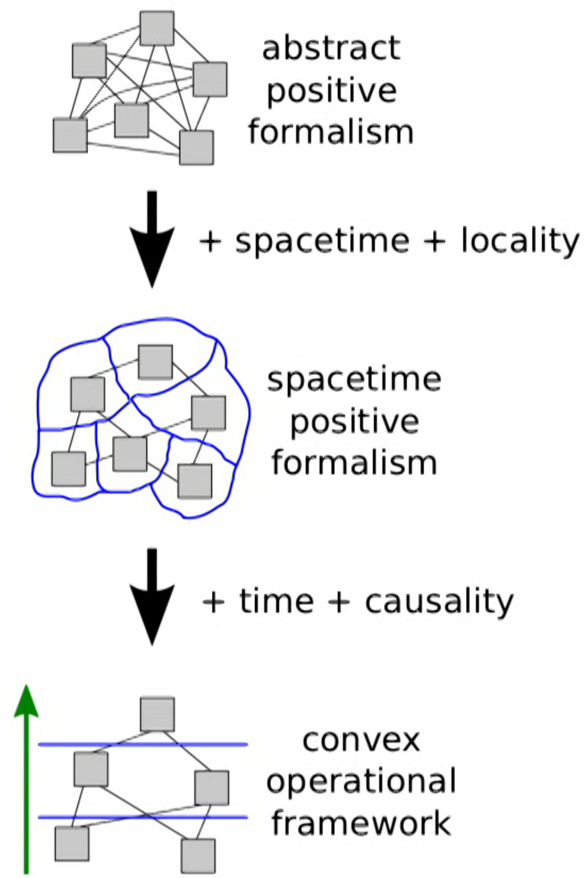


+ general measurements
− (fixed) **time** notion
− no space/locality
+ implements **causality**

Unification: The (local) positive formalism

Central notion:
Local quantum operation

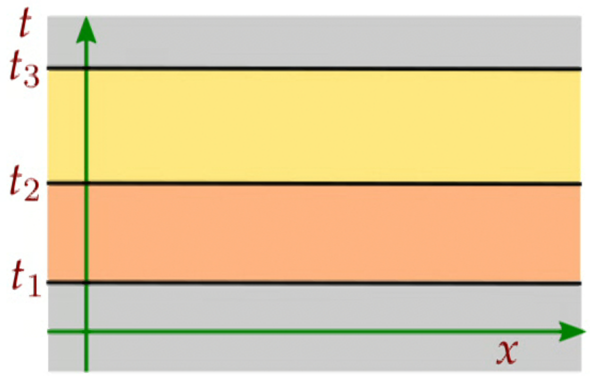
- general measurements
- (flexible) **spacetime** notion
- compatible with **general relativity**
- exhibits **locality**
- what about **causality**?


abstract
positive
formalism
+ spacetime + locality
spacetime
positive
formalism
+ time + causality
convex
operational
framework

QFT: Composition in time

Composition of time-evolutions

- in operator form: $U_{[t_1,t_3]} = U_{[t_2,t_3]} \circ U_{[t_1,t_2]}$
- in terms of matrix elements:
  $\langle\psi_3, U_{[t_1,t_3]}\psi_1\rangle = \sum_{i\in N}\langle\psi_3, U_{[t_2,t_3]}\zeta_i\rangle\langle\zeta_i, U_{[t_1,t_2]}\psi_1\rangle$



Write **transition amplitude** as **amplitude map** $\rho : \mathcal{H} \otimes \mathcal{H}^* \to \mathbb{C}$,

$$\rho(\psi \otimes \psi'^*) = \langle\psi', U\psi\rangle$$

Then composition takes form,

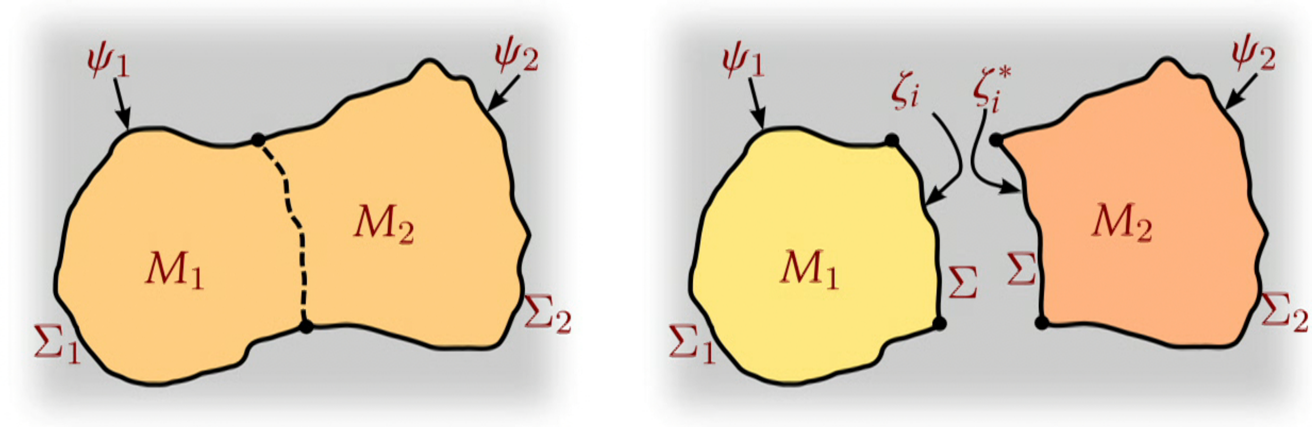
$$\rho_{[t_1,t_3]}(\psi_1 \otimes \psi_3^*) = \sum_{i\in N} \rho_{[t_1,t_2]}(\psi_1 \otimes \zeta_i^*)\rho_{[t_2,t_3]}(\zeta_i \otimes \psi_3^*)$$

Here, $\{\zeta_i\}_{i\in N}$ is an ON-basis of $\mathcal{H}$.

GBQFT: Composition in spacetime

Associate a **Hilbert space** $\mathcal{H}_\Sigma$ to each **hypersurface** Σ.
Connected components correspond to **tensor factors**.

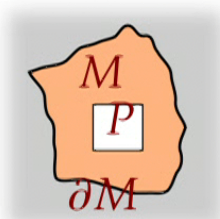
Replace **transition amplitudes** with **amplitude maps** $\rho_M : \mathcal{H}_{\partial M} \to \mathbb{C}$.



$$\rho_{M_1 \cup M_2}(\psi_1 \otimes \psi_2) = \rho_{M_1} \diamond \rho_{M_2}(\psi_1 \otimes \psi_2) := \sum_{i \in N} \rho_{M_1}(\psi_1 \otimes \zeta_i)\rho_{M_2}(\zeta_i^* \otimes \psi_2)$$

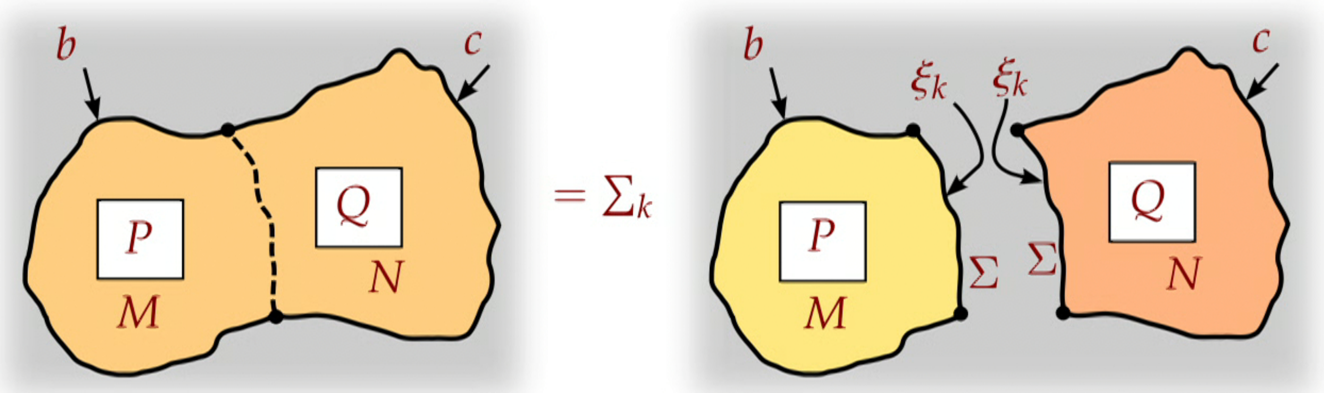
Here, $\psi_1 \in \mathcal{H}_{\Sigma_1}$, $\psi_2 \in \mathcal{H}_{\Sigma_2}$ and $\{\zeta_i\}_{i \in N}$ is an ON-basis of $\mathcal{H}_\Sigma$.

General boundary formulation

spacetime object	**amplitude formalism**	$\longrightarrow$ functor $\longrightarrow$	**positive formalism**
	Hilbert space $\mathcal{H}_\Sigma$	self-adjoint operators	ordered vector space $\mathcal{B}_\Sigma$
	amplitude map $\rho_M : \mathcal{H}_{\partial M} \to \mathbb{C}$	$\boxslash(\sigma) = \sum_i \overline{\rho(\zeta_i)}\rho(\sigma\zeta_i)$	null probe $\boxslash : \mathcal{B}_{\partial M} \to \mathbb{R}$ (positive!)
			probe $P : \mathcal{B}_{\partial M} \to \mathbb{R}$ (positive!)

probe = **local quantum operation**

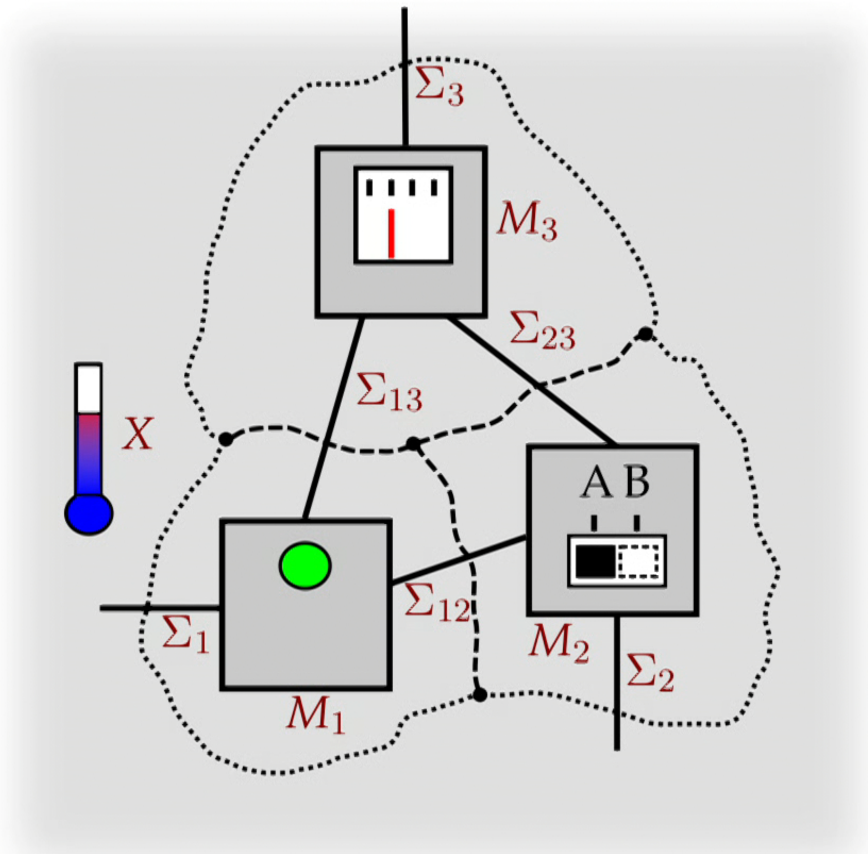
Composition of local quantum operations



$$P \diamond Q(b \otimes c) = \sum_{k \in N} P(b \otimes \xi_k) Q(\xi_k \otimes c)$$

Here, $\{\xi_k\}_{k \in N}$ is an orthonormal basis of $\mathcal{B}_\Sigma$.
Special case, $\boxslash_M \diamond \boxslash_N = \boxslash_{M \cup N}$.

Diagrammatics and evaluation



Regions carrying **local quantum operations** form a **cell complex**.

The **dual 1-complex** forms a **graph**.

Closed graphs evaluate to **non-negative numbers**. Graphs can be closed by considering states as operations.

Standard formulation: Undirected setting

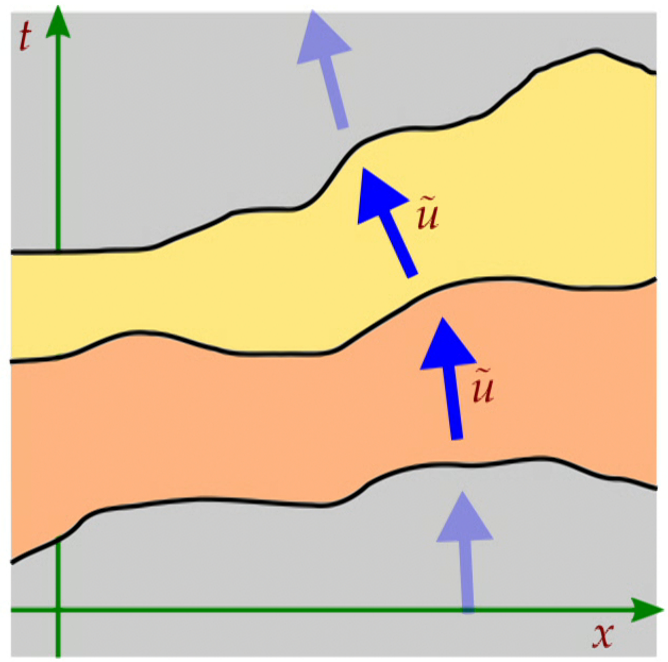
If we omit the **causality condition**, the **standard formulation** (respectively the **convex operational framework**)

1. is **time-symmetric**, i.e., invariant under reversal of all arrows
2. can even be formulated **undirected**, i.e., without arrows

If we add a notion of **spacetime** by associating operations to regions we obtain the very same **local positive formalism**.

Causality in the local positive formalism?

In order to implement the **causality condition** we would need **global directionality**, i.e., consistent arrows on the graph edges. This can be achieved with a consistent **time-orientation** on each hypersurface. That is, spacetime must have a **global time orientation** and each hypersurface must be completely **spacelike**.



$$\rho(\psi_1 \otimes \psi_2^*) = \langle \psi_2, \tilde{U}\psi_1 \rangle \quad \psi_i \in \mathcal{H}_{t_i}$$

Other geometric decompositions

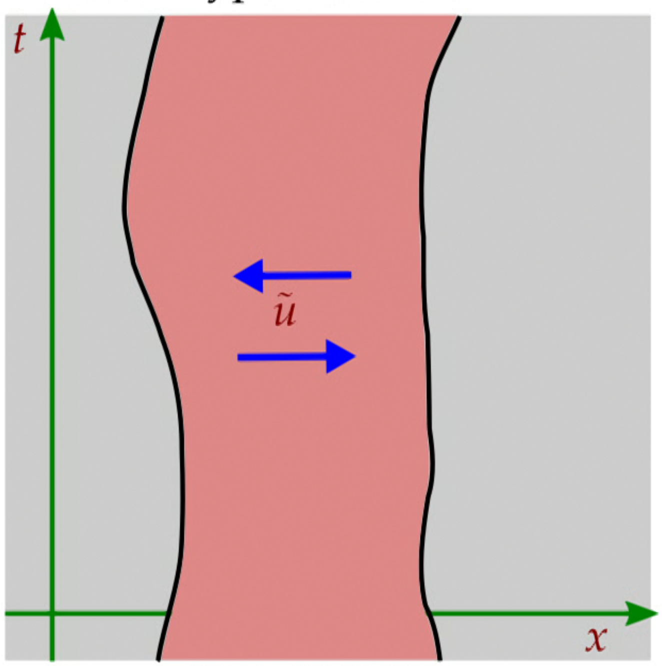
An evolution picture arises also in certain other types of regions. For example, for a region bounded by timelike hypersurfaces we may have

$$\rho(\psi_1 \otimes \psi_2^*) = \langle \psi_2, \tilde{U}\psi_1 \rangle \quad \psi_i \in \mathcal{H}_{x_i}$$

This would be hard to make consistent in more than one spatial dimension.

Also, it would not implement the desired physical meaning.

timelike hypersurfaces

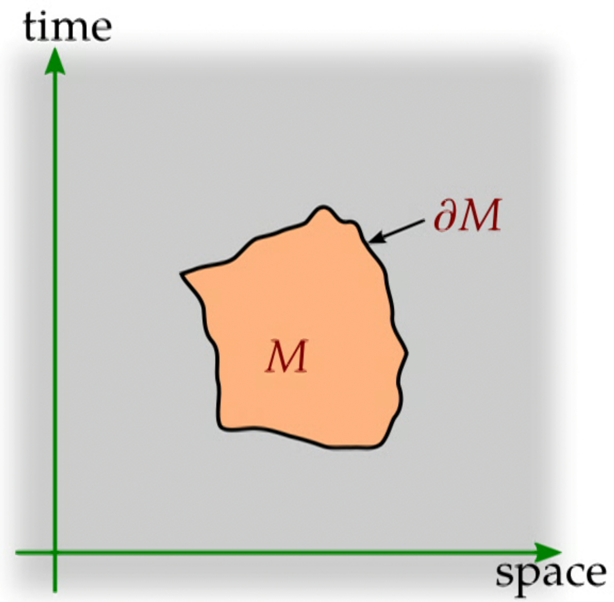


Algebraic decompositions

Generically, a spacetime region does not admit any natural **geometric decomposition** of its boundary Hilbert space.

But, there are many **algebraic decompositions**. That is, $\mathcal{H}_{\partial M} = \mathcal{H}_1 \otimes \mathcal{H}_2$ and $\tilde{U} : \mathcal{H}_1 \to \mathcal{H}_2$ such that

$$\rho(\psi_1 \otimes \psi_2^*) = \langle \psi_2, \tilde{U}\psi_1 \rangle \quad \psi_i \in \mathcal{H}_i$$



In general such decompositions will not be mutually consistent.

Also, their physical meaning is in general unclear.

Time emerging – in fermionic field theory

In **fermionic quantum field theory**, Hilbert spaces are replaced by **Krein spaces**. These are inner product spaces with a **positive definite** and a **negative definite** part.

For any region M something interesting happens:

- $\mathcal{H}_{\partial M}$ admits a canonical decomposition into a **tensor product** of a positive and a negative definite part, $\mathcal{H}_{\partial M} = \mathcal{H}_{\partial M}^{(+)} \otimes \mathcal{H}_{\partial M}^{(-)}$.
- There is a **graded unitary map** $\tilde{U}_M : \mathcal{H}_{\partial M}^{(+)} \to \mathcal{H}_{\partial M}^{(-)}$ such that

$$\rho_M(\psi_+ \otimes \psi_-^*) = \langle \psi_-, \tilde{U}_M \psi_+ \rangle \quad \psi_\pm \in \mathcal{H}_{\partial M}^{(\pm)}$$

- The unitary maps compose consistently.
- If M has spacelike boundary components $\partial M = \Sigma_{t_1} \sqcup \Sigma_{t_2}$, then $\mathcal{H}_{\partial M}^{(+)} = \mathcal{H}_{t_1}, \mathcal{H}_{\partial M}^{(-)} = \mathcal{H}_{t_2}$ and $\tilde{U}_M$ is the usual unitary evolution.

[Switch to science fiction mode]

This suggests the emergence of a notion of time from the dynamics of fermions.

Causality for fermions

The relevant structures are inherited in the positive formalism. That is, $\mathcal{B}_{\partial M}$ is a (real) Krein space that decomposes as $\mathcal{B}_{\partial M} = \mathcal{B}^{(+)}_{\partial M} \otimes \mathcal{B}^{(-)}_{\partial M}$. Moreover, there is a **graded unitary map** $\hat{U}_M : \mathcal{B}^{(+)}_{\partial M} \to \mathcal{B}^{(-)}_{\partial M}$ such that

$$\boxslash_M(\sigma_+ \otimes \sigma_-^*) = (\!|\sigma_-, \hat{U}_M \sigma_+|\!) \quad \sigma_\pm \in \mathcal{B}^{(\pm)}_{\partial M}.$$

[Switch to science fiction mode]

Causality condition for fermionic theories

Let $P : \mathcal{B}_{\partial M} \to \mathbb{R}$ be a **local quantum operation** in M. Require, (with $\mathbf{e} \in \mathcal{B}^{(-)}_{\partial M}$),

$$P(\sigma \otimes \mathbf{e}) = \boxslash_M(\sigma \otimes \mathbf{e}), \quad \forall \sigma \in \mathcal{B}^{(+)}_{\partial M}$$