Title: Infinite composite systems and cellular automata in operational probabilistic theories

Date: Jul 30, 2018 11:30 AM

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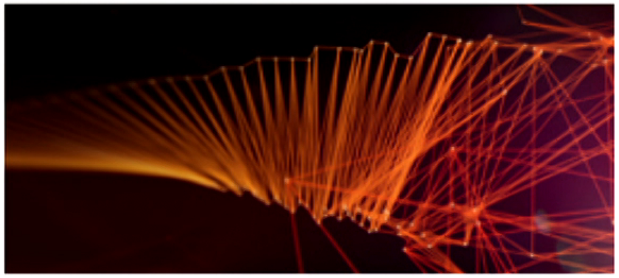
Abstract: <p>Cellular automata are a central notion for the formulation of physical laws in an abstract information-theoretical scenario, and lead in recent years to the reconstruction of free relativistic quantum field theory. In this talk we extend the notion of a Quantum Cellular Automaton to general Operational Probabilistic Theories. For this purpose, we construct infinite composite systems, illustrating the main features of their states, effects and transformations. We discuss the generalization of the concepts of homogeneity and locality, in an framework where space-time is not a primitive object. We show that homogeneity leads to a Cayley graph structure of the memory array, thus proving the universality of the connection between homogeneity and discrete groups. We conclude illustrating the special case of Fermionic cellular automata, discussing three relevant examples: Weyl and Dirac quantum walks, the Thirring automaton and the simplest families of automata on finite graphs.</p>



FOUNDATIONS OF QUANTUM MECHANICS

Perimeter Institute, July 30 2018

Infinite composite systems and cellular automata in operational probabilistic theories

Paolo Perinotti
Dipartimento di Fisica,
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JOHN TEMPLETON
FOUNDATION

QUit
quantum information
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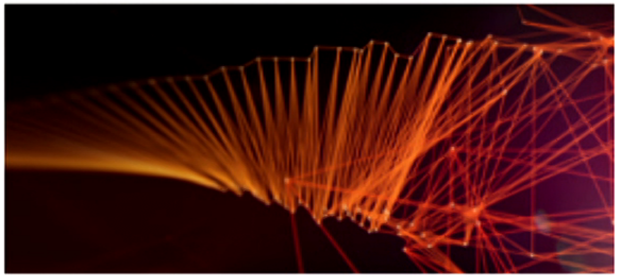

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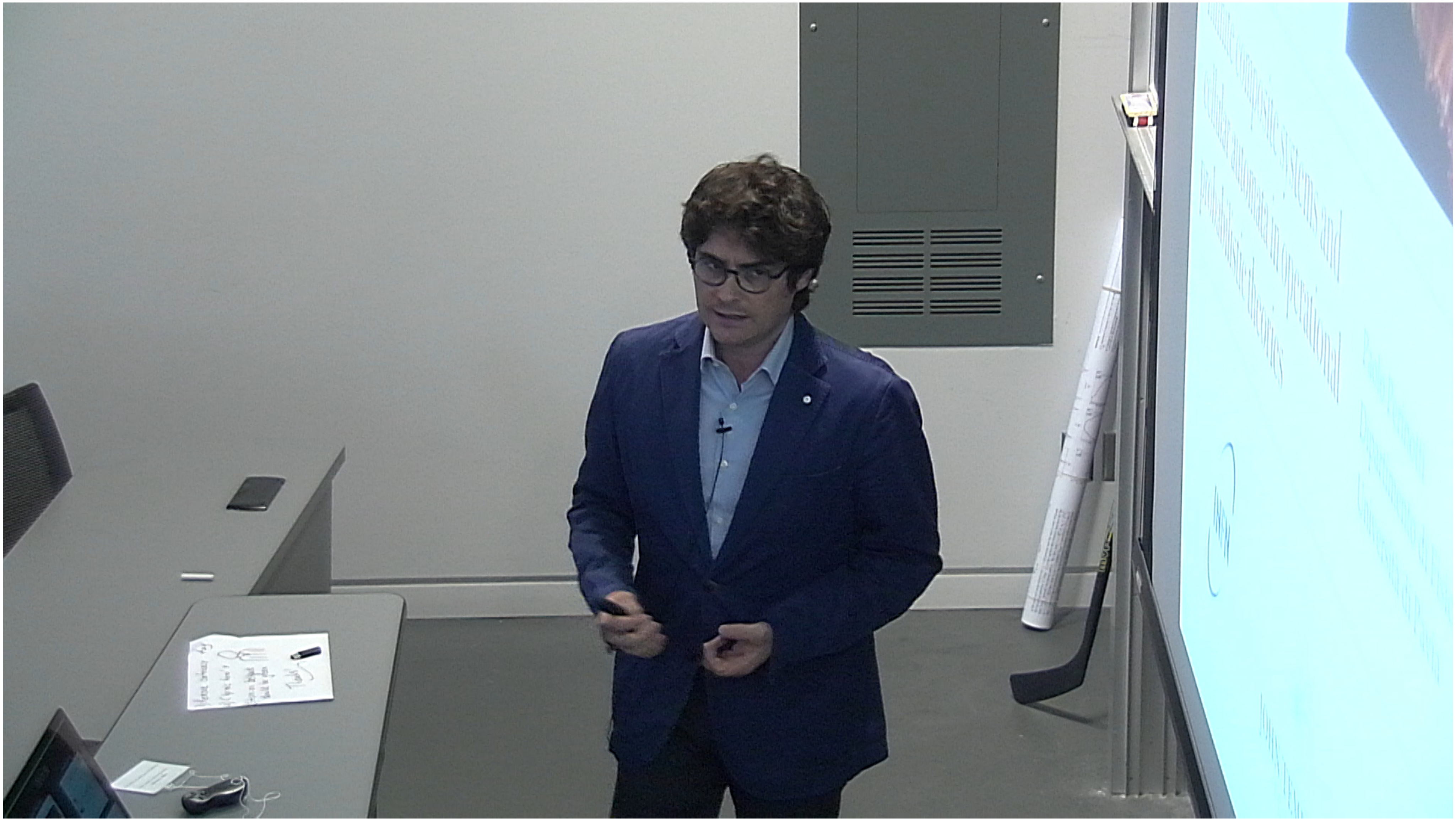
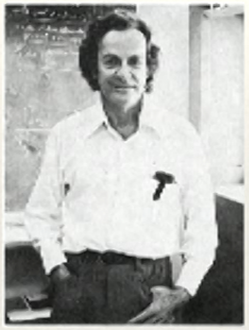


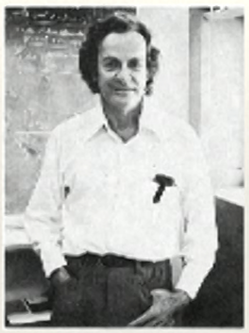
Digital Universe



"I want to talk about the possibility that there is to be an exact simulation, that the computer will do exactly the same as nature."

R. Feynman, Int. J. Theo. Ph. **21**, 467 (1982)

Digital Universe



"I want to talk about the possibility that there is to be an exact simulation, that the computer will do exactly the same as nature."

Physical law ↔ Algorithm

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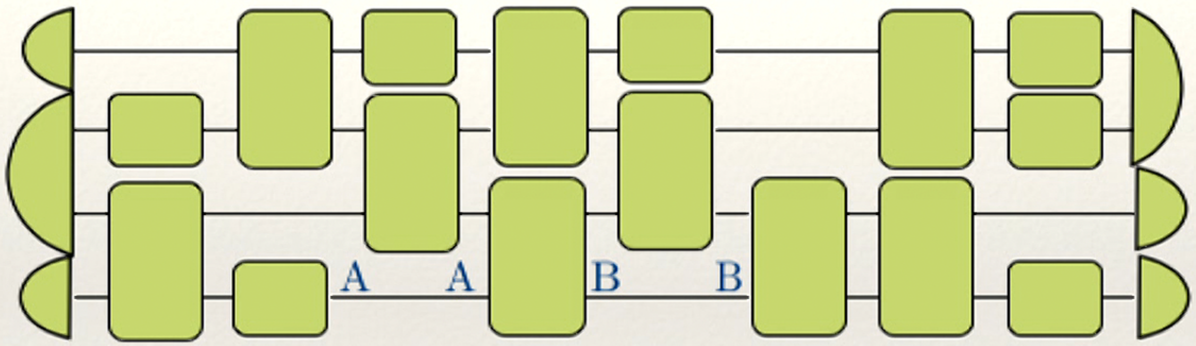
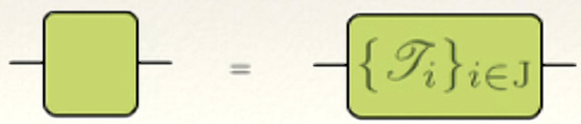
Operational Language

- Operational theory: tests with composition rules





G. Chiribella, G. M. D'Ariano, and PP, Phys. Rev. A **81**, 062348 (2010)

Main property

- Most important rule:

$$(\mathscr{A} \otimes \mathscr{B})(\mathscr{C} \otimes \mathscr{D}) = (\mathscr{A}\mathscr{C}) \otimes (\mathscr{B}\mathscr{D})$$

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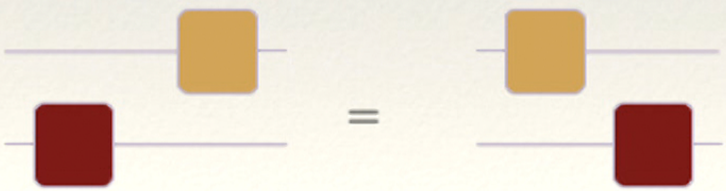
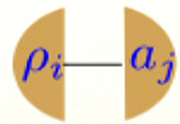
- Consequence:



Probabilistic theories

Every test of type I→I is a probability distribution 

$= \Pr(a_j, \rho_i)$

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Probabilistic theories

Every test of type I→I is a probability distribution ρ_i — a_j $= \Pr(a_j, \rho_i)$

States are functionals on effects and vice-versa

Real vector spaces $\mathsf{St}_{\mathbb{R}}(\mathrm{A}), \mathsf{Eff}_{\mathbb{R}}(\mathrm{A})$

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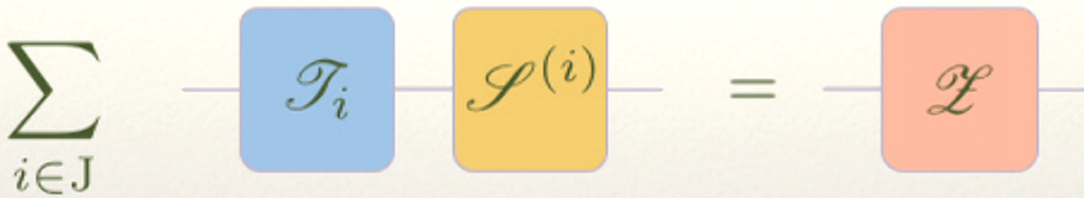
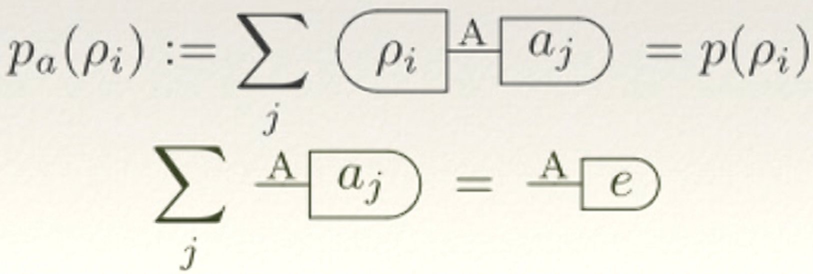
Causal theories

- Possibility of arbitrary conditional tests



- Causality implies no backward signalling



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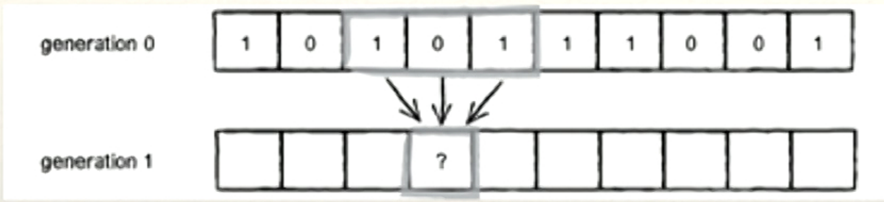
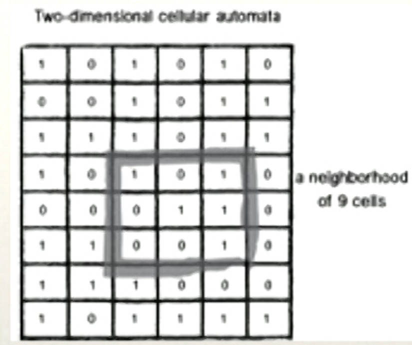
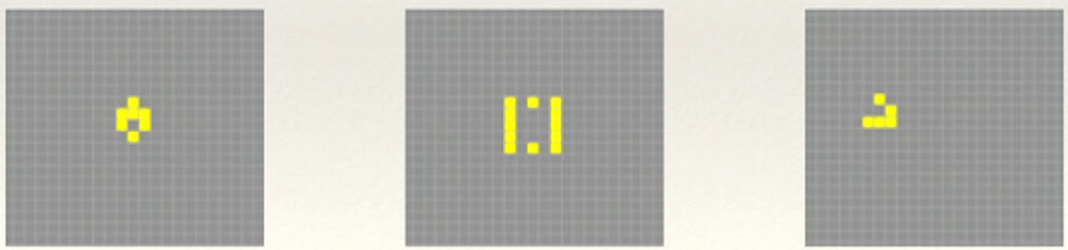
Cellular Automata

J. Von Neumann and A. W. Burks, "Theory of self-reproducing automata" 1966

Conway's *game of life*







http://web.stanford.edu/~cdebs/GameOfLife/

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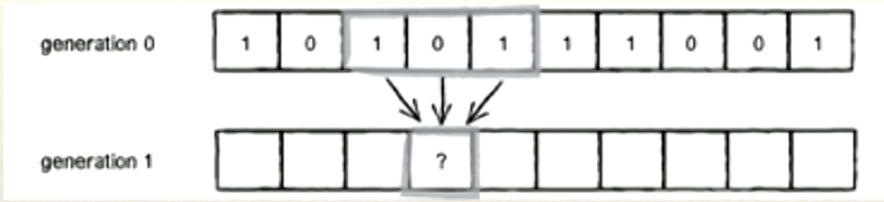

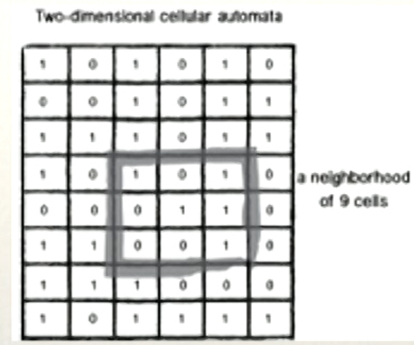

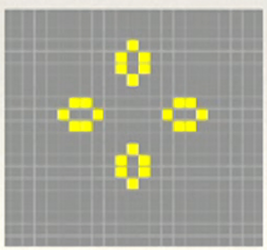

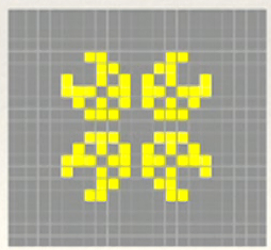

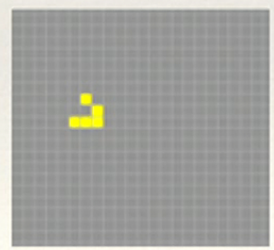

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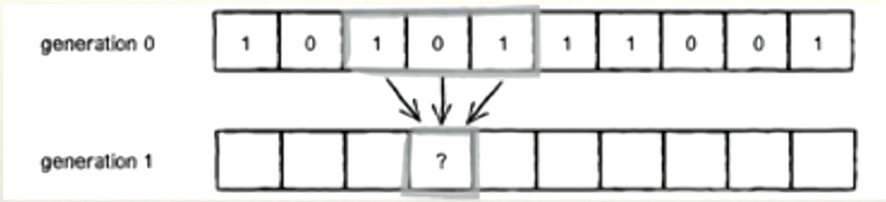

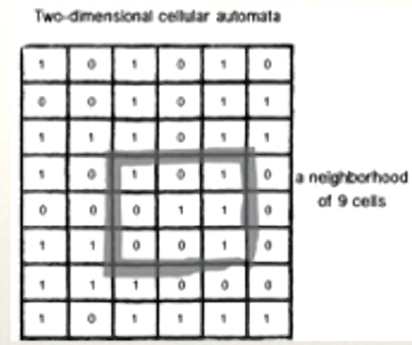

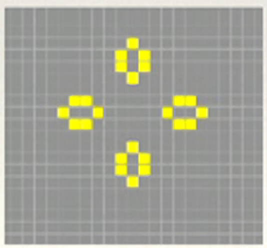

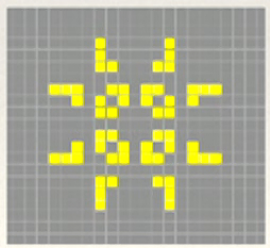

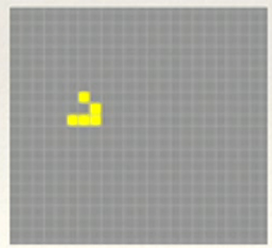

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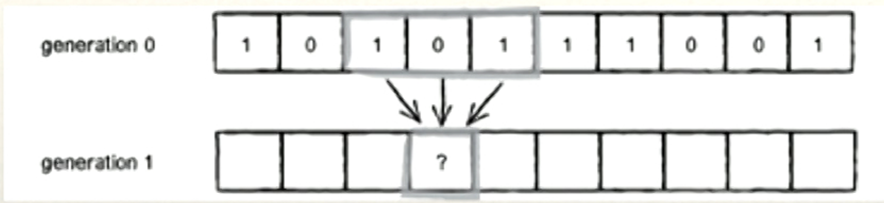

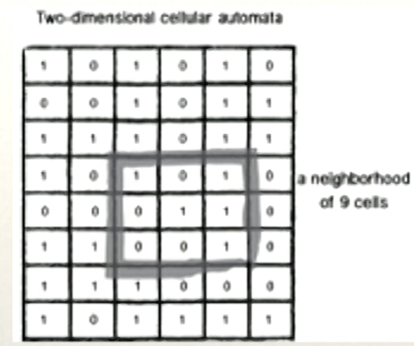

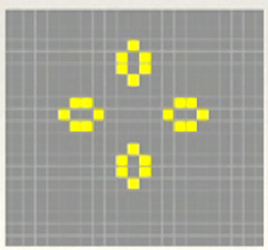

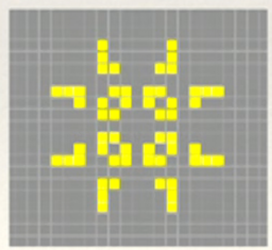

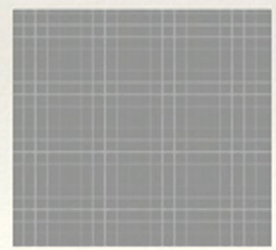

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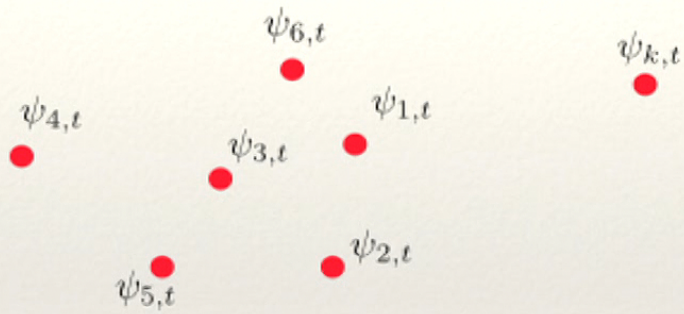
Linear Fermionic Cellular Automata

$$\{\varphi_i^\dagger, \varphi_j\} = \delta_{ij} I, \quad \{\varphi_i, \varphi_j\} = 0$$



$$\psi_{i,t+1} = \sum_{j \in N_i} A_{i,j} \psi_{j,t} \qquad A_{ij} \in M_{s_i \times s_j}$$

G. M. D'Ariano and PP, Phys. Rev. A **90**, 062106 (2014).

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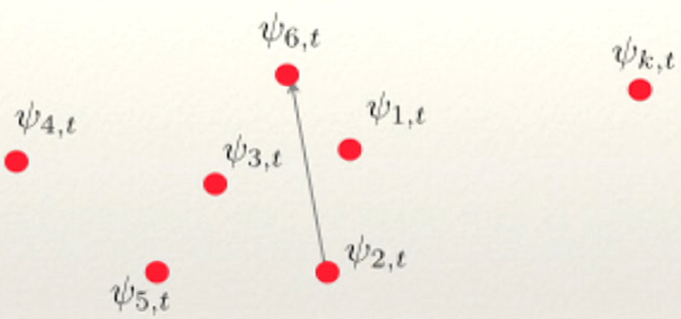

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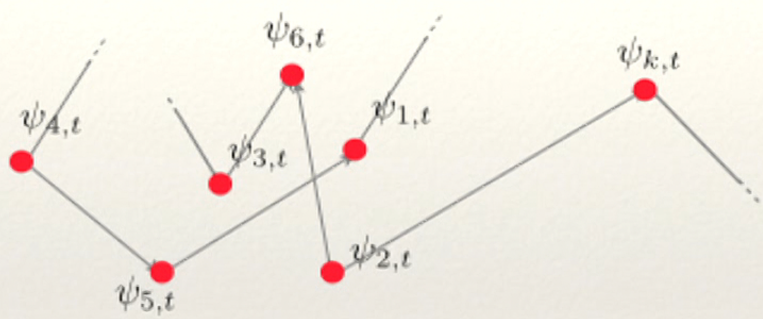

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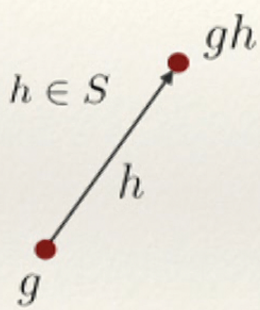
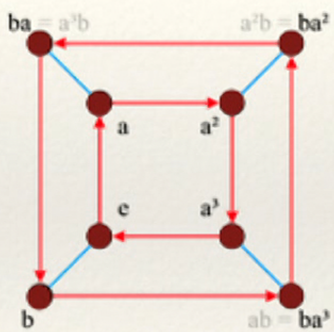
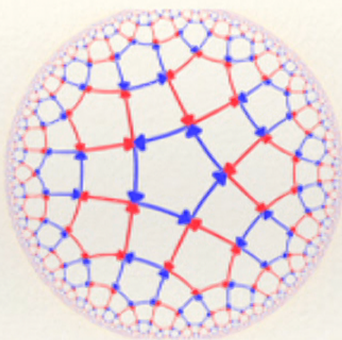
Homogeneity and Cayley Graphs

- Homogeneity: the memory array is structured as a Cayley graph







G. M. D'Ariano and PP, Phys. Rev. A **90**, 062106 (2014)
G. M. D'Ariano and PP, Front. Phys. **12**(1), 120301 (2017).

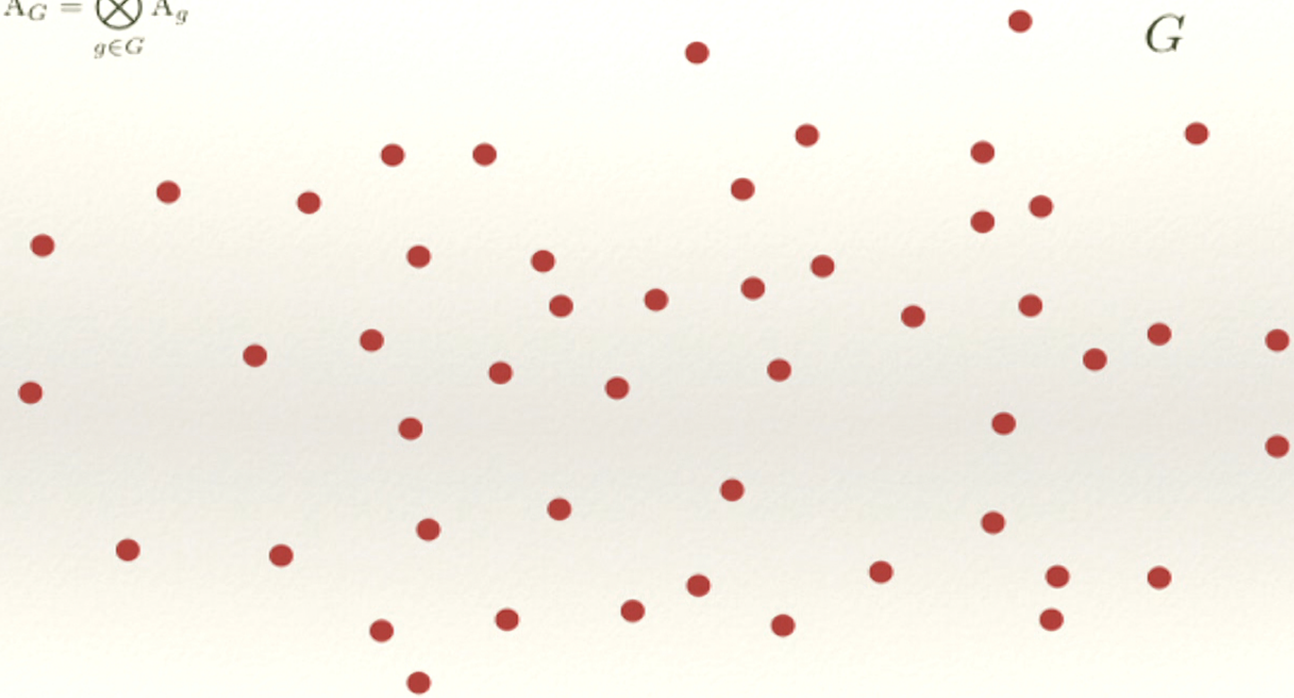
In general OPTs

- We do not have Hilbert spaces - only state/effect spaces
- No field operators - only transformations
  - There is no notion of a linear automaton

- Can we generalise the above construction?
- Are our results generic or specific of FQT?

(Quasi-)Local effects

$$\mathrm{A}_G = \bigotimes_{g \in G} \mathrm{A}_g$$



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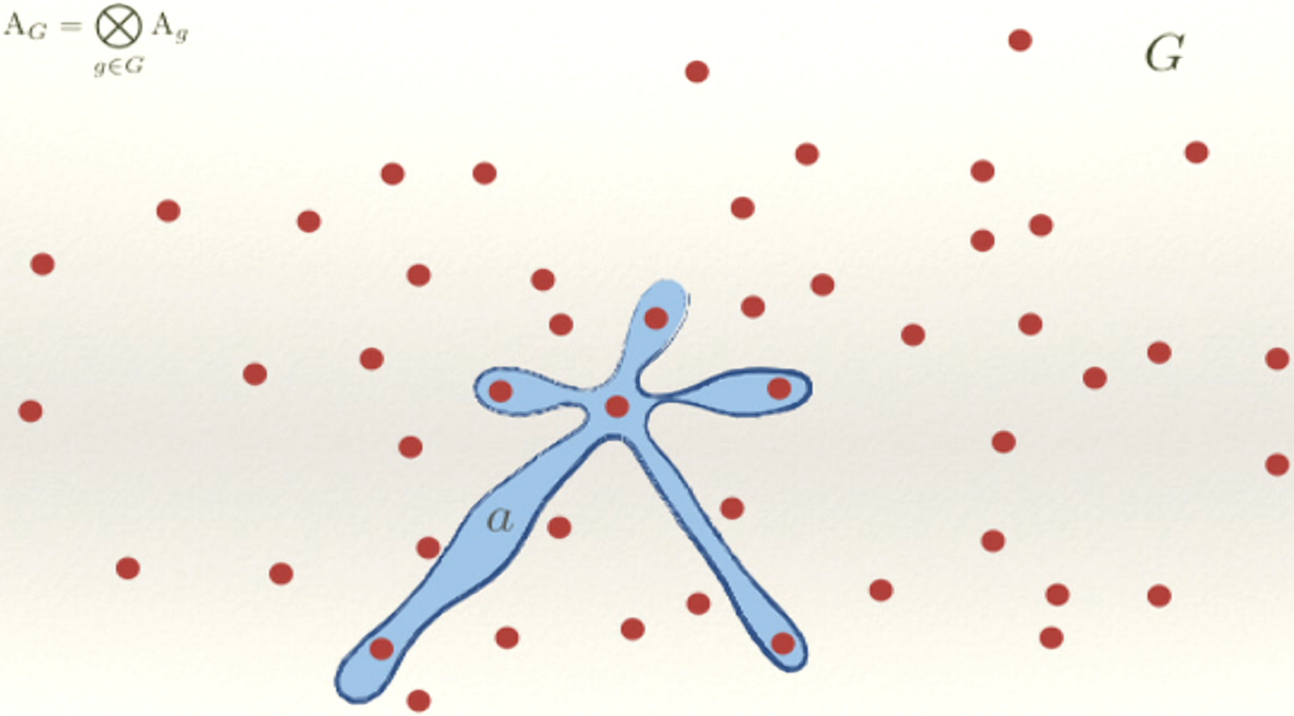

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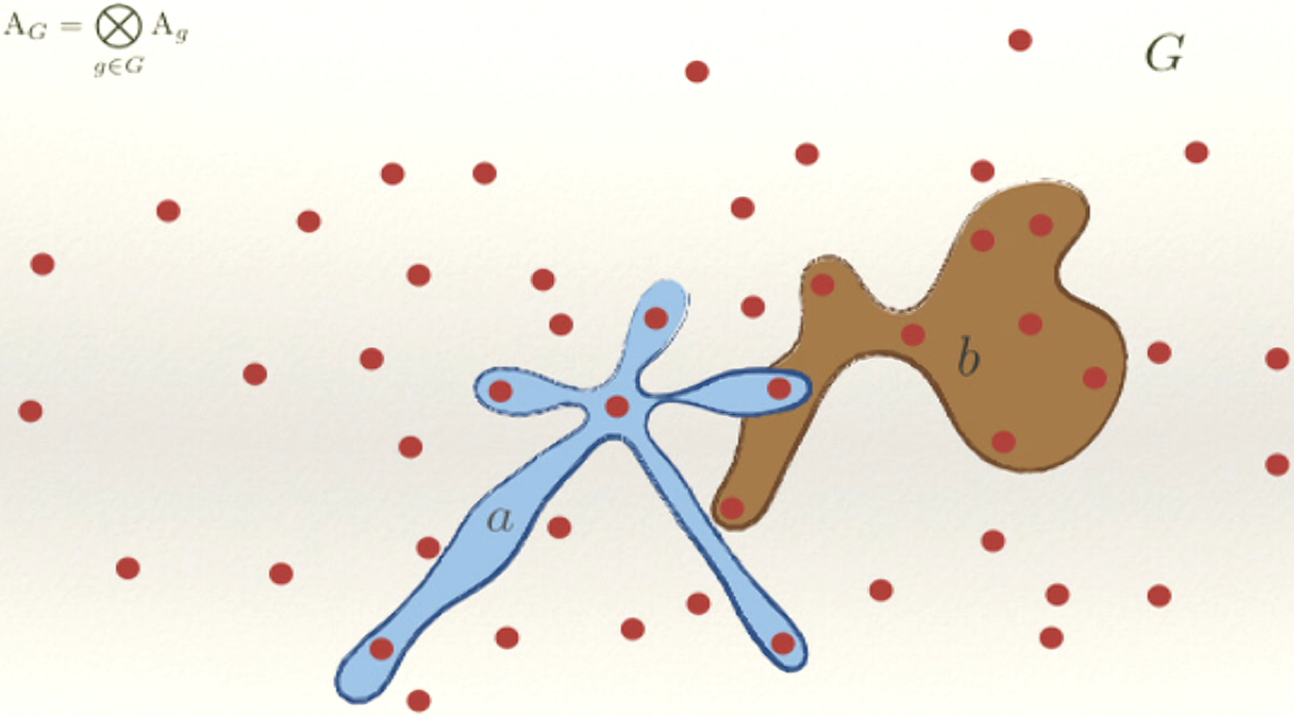

(Quasi-)Local effects

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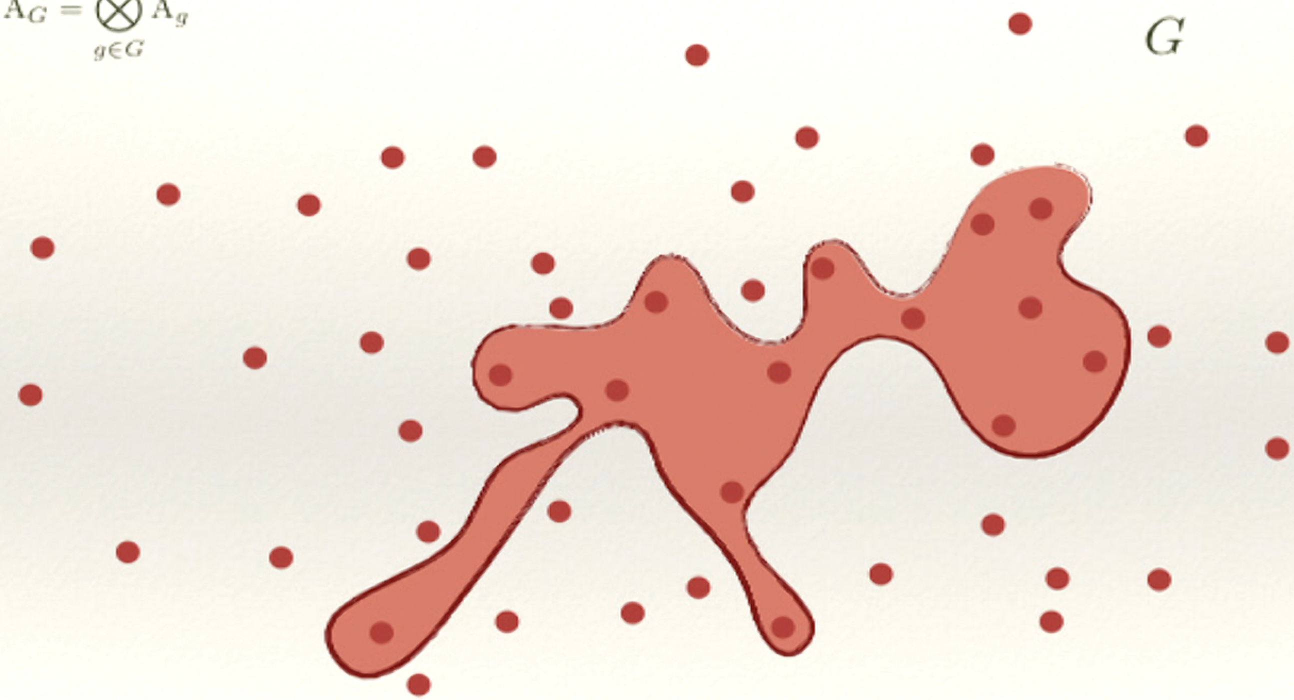
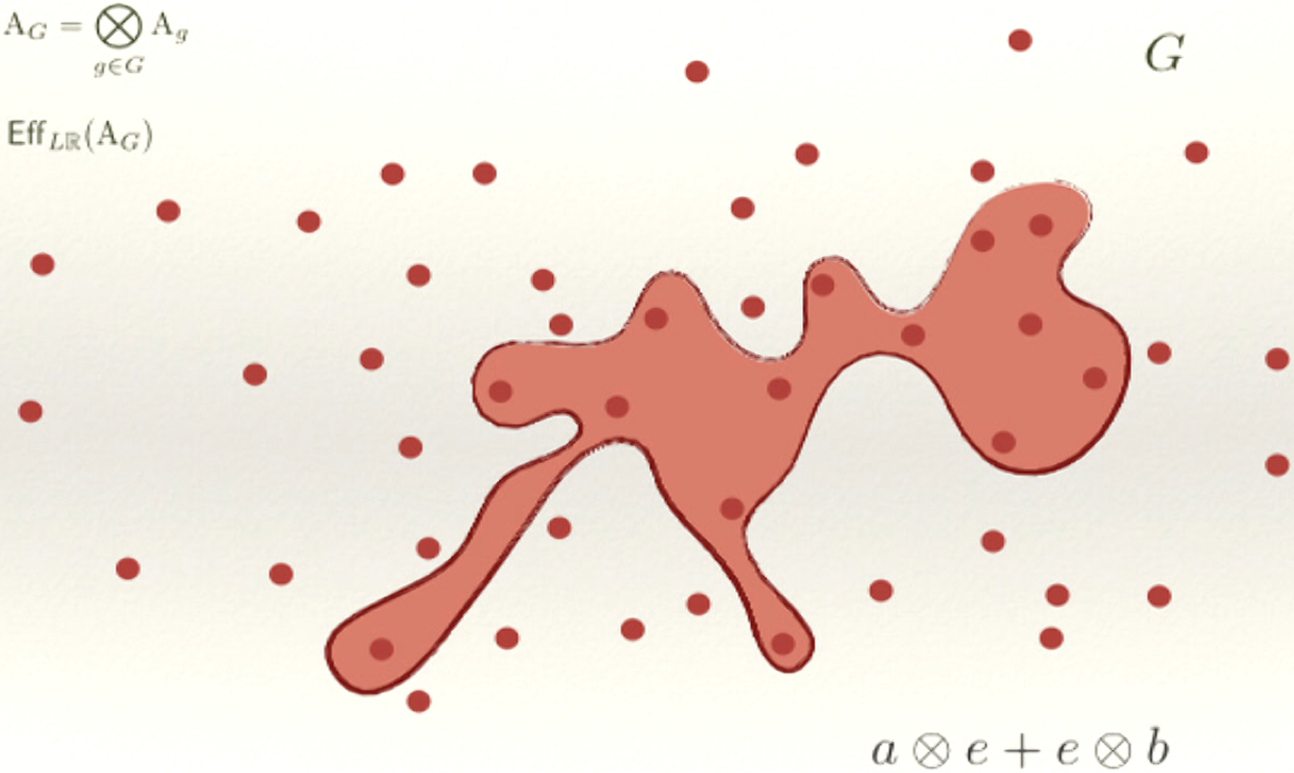


$$a \otimes e + e \otimes b$$

(Quasi-)Local effects

$$A_G = \bigotimes_{g \in G} A_g$$

$\mathsf{Eff}_{L\mathbb{R}}(A_G)$

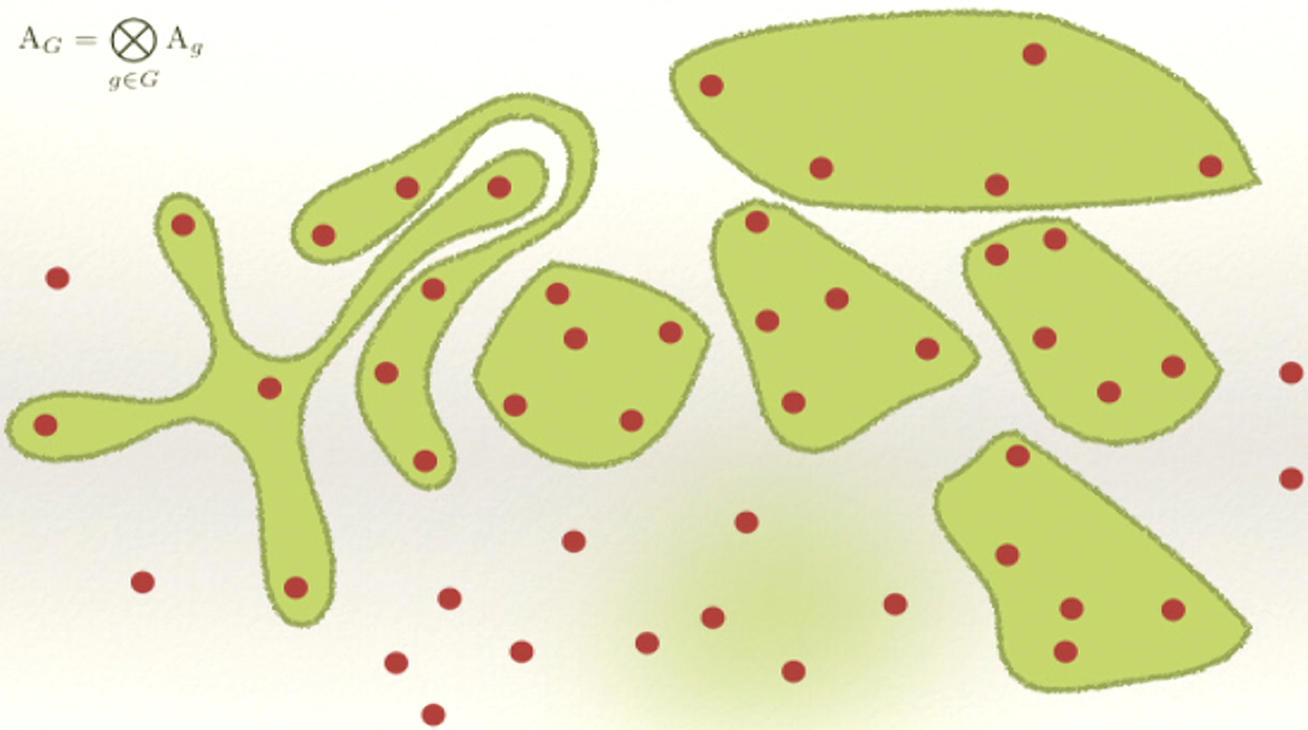


Remarks

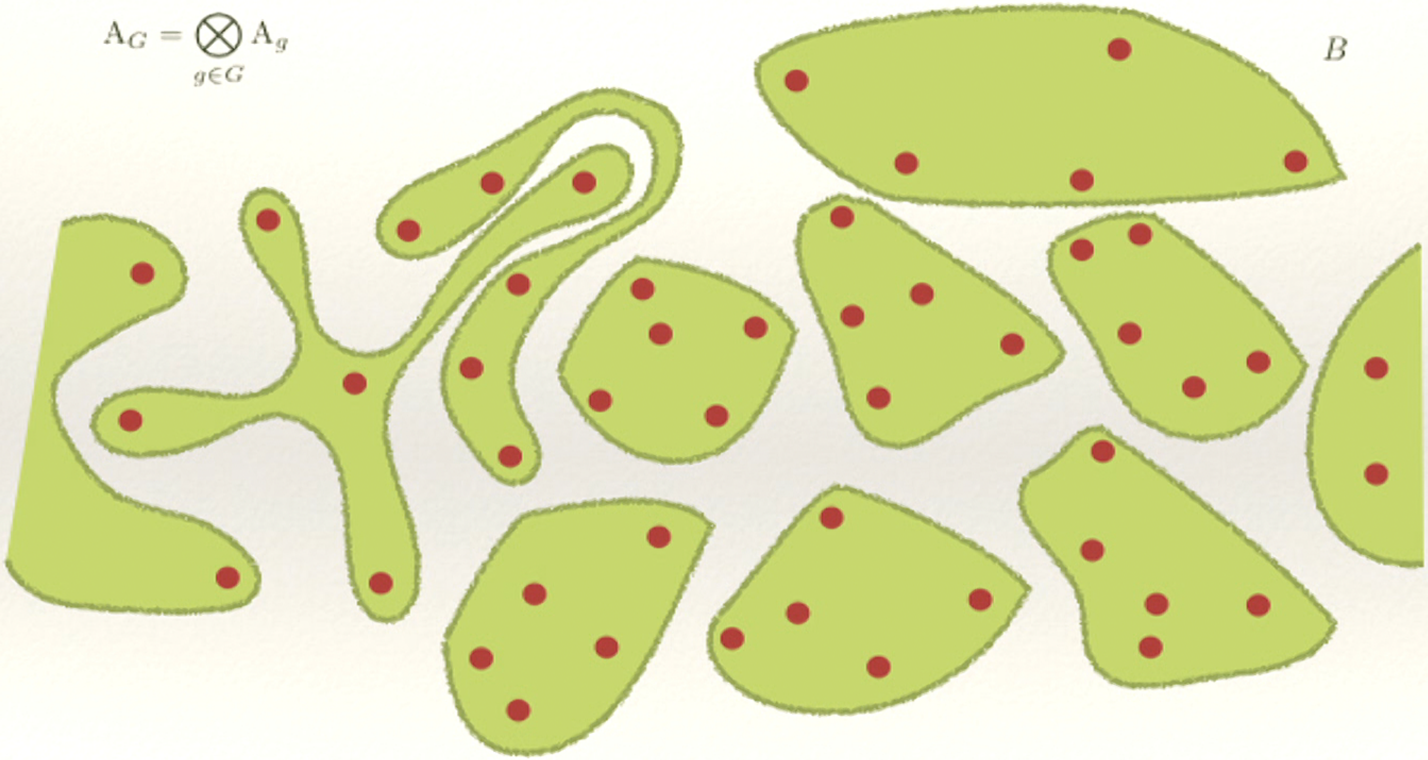
- Why starting from effects?
  - Causality: unique deterministic effect
  - State space: functionals on the space of effects
  - Much "larger" than the space of quasi-local states
- States: functionals that locally behave as states

(Quasi-)Local states

$$\mathrm{A}_G = \bigotimes_{g \in G} \mathrm{A}_g$$



(Quasi-)Local states



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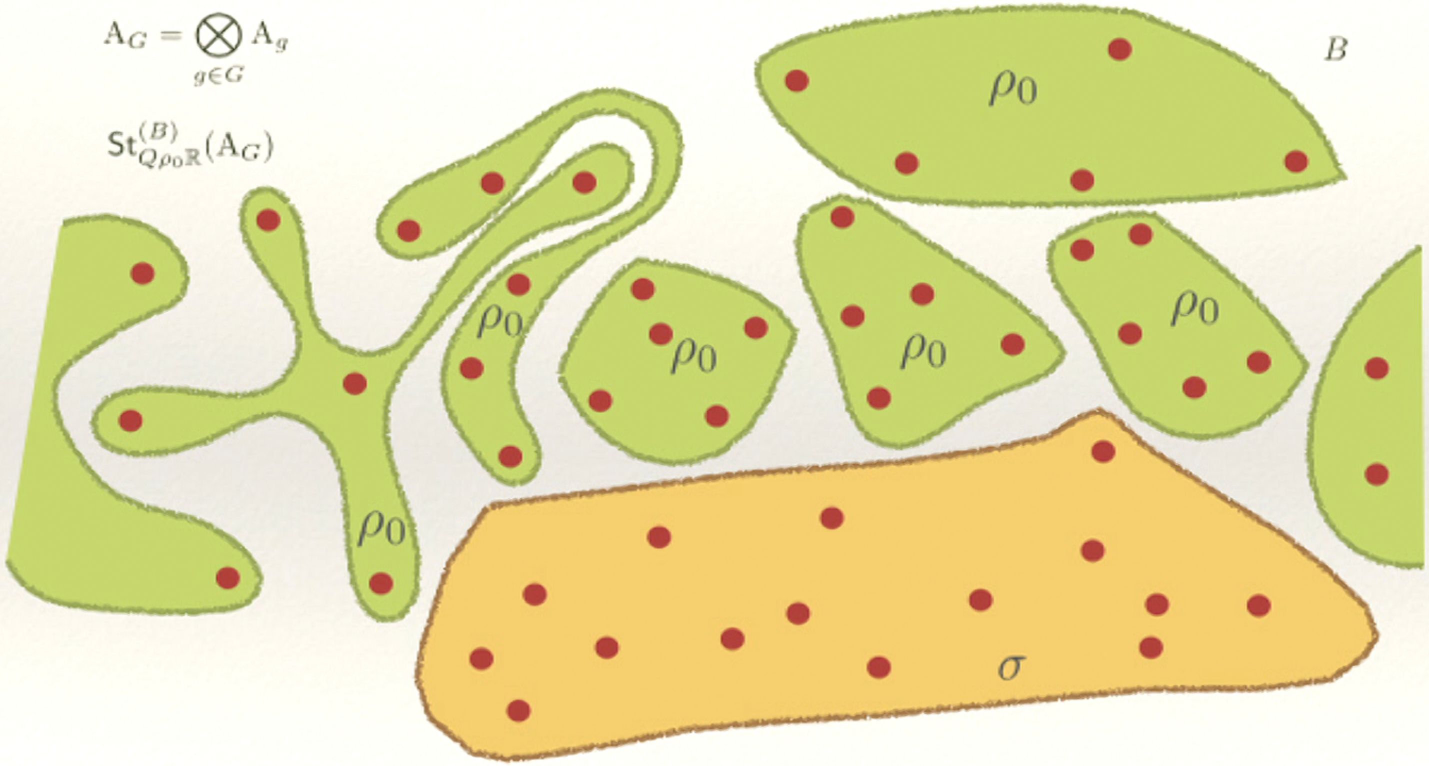
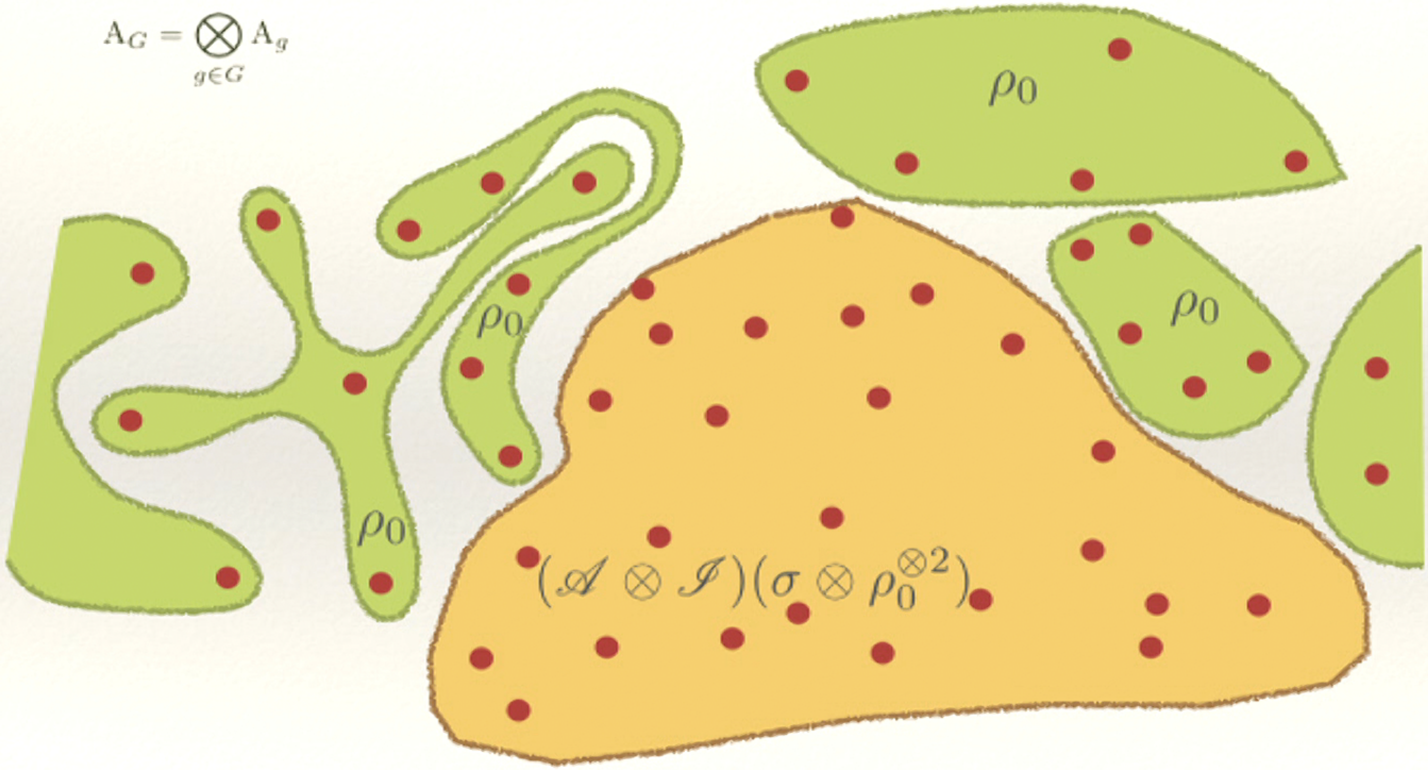


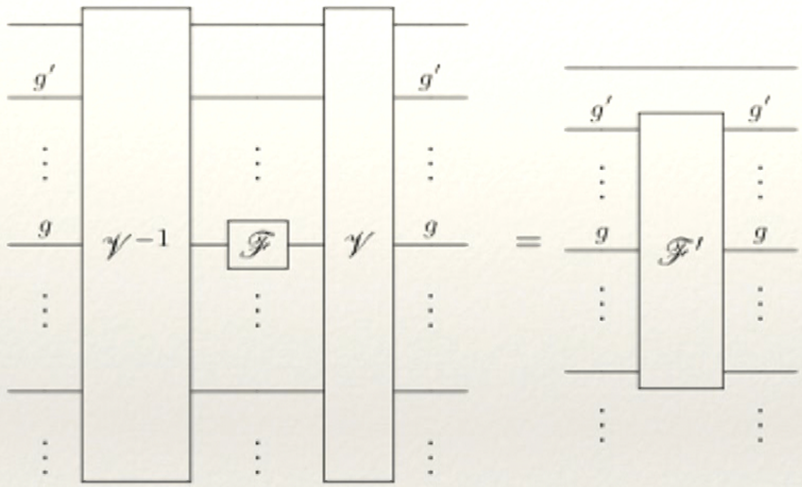
(Quasi-)Local transformations



Cellular Automata

- Maps $\mathscr{V} : \mathsf{Eff}_{Q\mathbb{R}}(\mathrm{A}_G) \to \mathsf{Eff}_{Q\mathbb{R}}(\mathrm{A}_G)$
  - Admissible $(\mathscr{V} \otimes \mathscr{I}_{\mathrm{C}})\mathsf{Eff}_Q(\mathrm{A}_G\mathrm{C})^* \subseteq \mathsf{Eff}_Q(\mathrm{A}_G\mathrm{C})^*$
  - Reversible $\mathscr{V}\mathscr{V}^{-1} = \mathscr{V}^{-1}\mathscr{V} = \mathscr{I}$
    with $\mathscr{V}^{-1}$ admissible
  - The map $\mathscr{V} \cdot \mathscr{V}^{-1}$ preserves local transformations

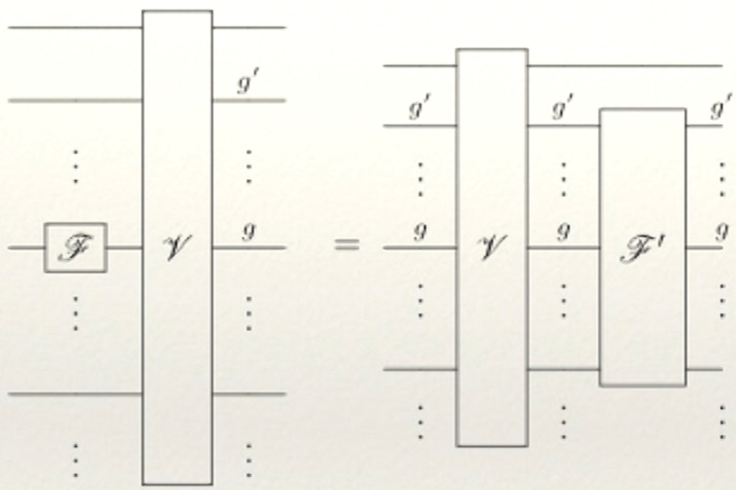
Causal influence



Then $g \rightarrowtail_g g'$

g directly influences g' through $\mathscr{V}$

Causal influence

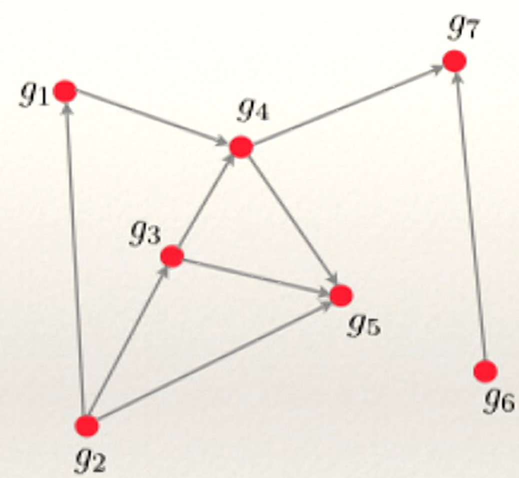


$$\mathscr{V}\mathscr{F}\mathscr{V}^{-1} = \mathscr{F}'$$
$$\mathscr{V}\mathscr{F} = \mathscr{F}'\mathscr{V}$$

Then $\quad g \rightarrowtail_g g'$

g directly influences g' through $\mathscr{V}$

Graph of influence relations



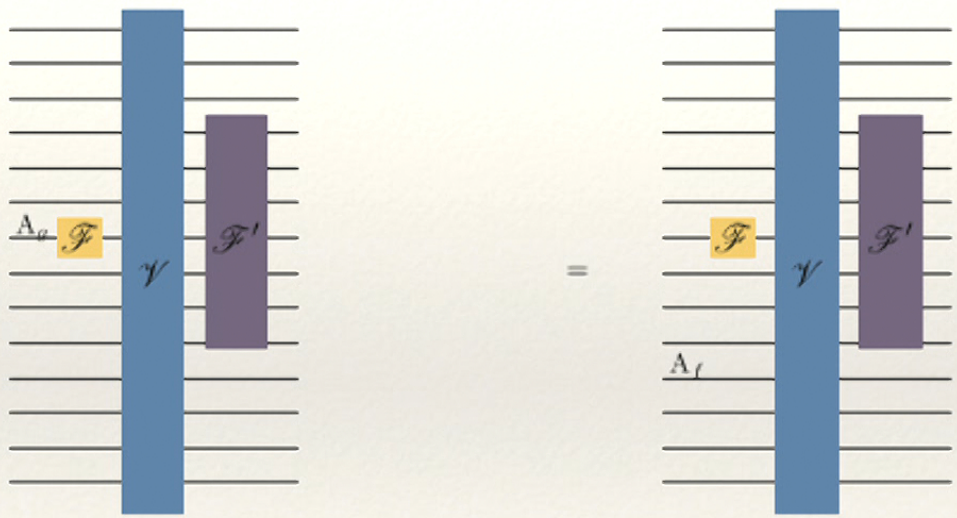
$g_1 \rightarrowtail g_4$ $\quad$ $g_2 \rightarrowtail g_1$ $\quad$ $g_3 \rightarrowtail g_4$ $\quad$ $g_4 \rightarrowtail g_5$ $\quad$ $g_6 \rightarrowtail g_7$

$g_2 \rightarrowtail g_3$ $\quad$ $g_3 \rightarrowtail g_5$ $\quad$ $g_4 \rightarrowtail g_7$

$g_2 \rightarrowtail g_5$

Homologous regions

$$f \sim_{\mathscr{V}} g$$



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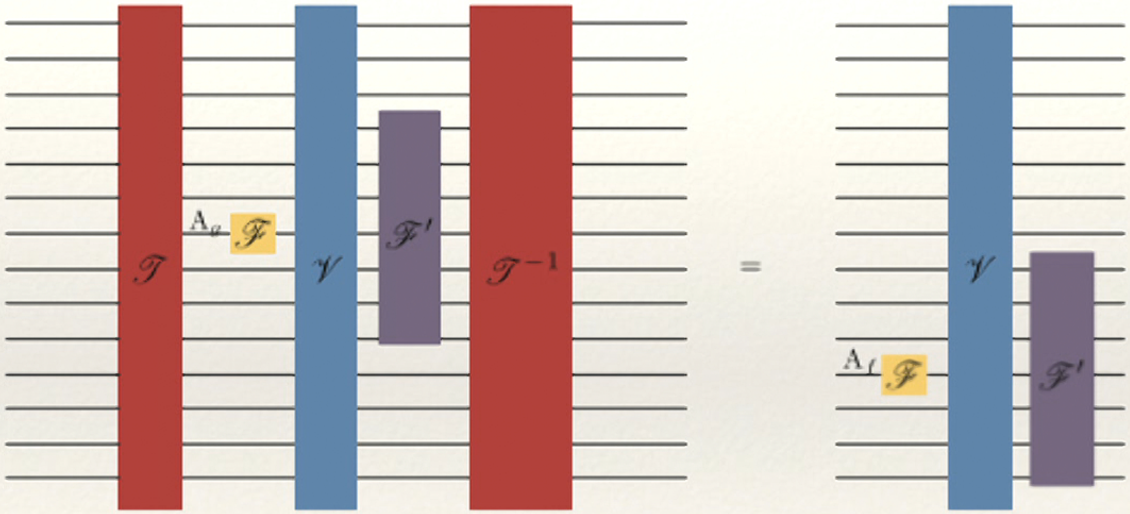


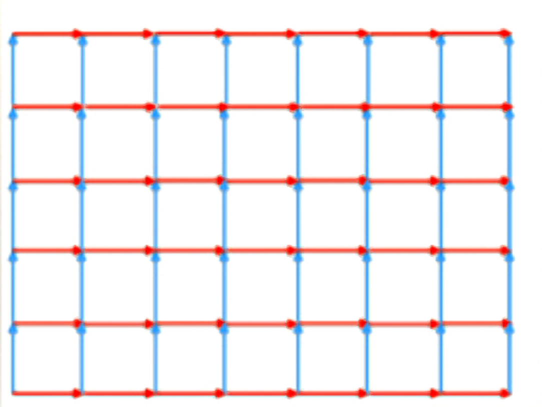
Homogeneity

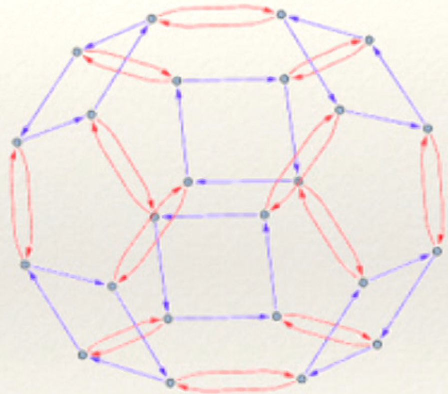
"The evolution of every two different systems can be distinguished
only with reference to a third system"

- For every f,g, f and g are homologous

- Two pairs (e,f) and (e,g) are never homologous

Homogeneity

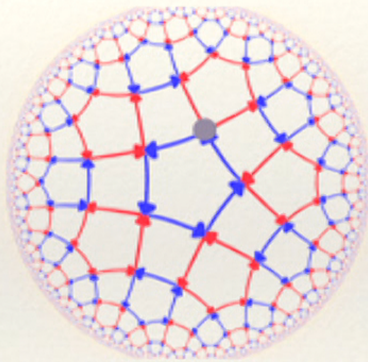




The influence graph is a Cayley graph: it corresponds to a group

Locality

"The global evolution rule can be reduced to the evolution of bounded regions"



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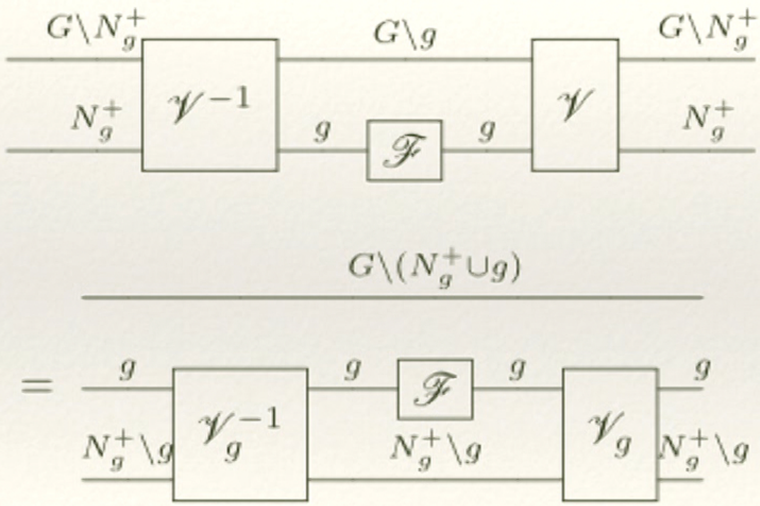
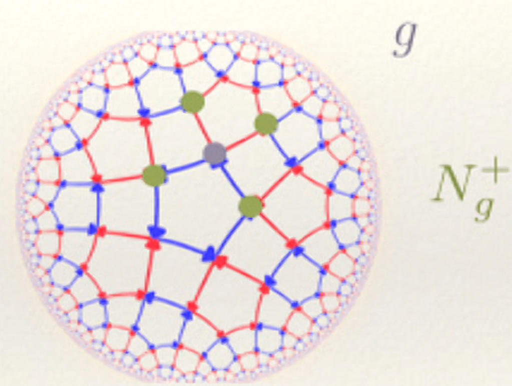
"The global evolution rule can be reduced to the evolution of bounded regions"



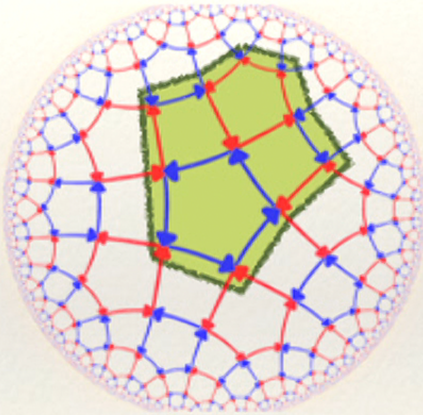


Locality

"The evolution rule can be reduced to the evolution of bounded regions"

i. Every vertex has finitely many arrows

ii.The elementary closed paths are bounded



The signalling graph is the Cayley graph of a finitely presented group

Emergent space

Homogeneity and locality of evolution

Essentially unique geometry

$$a + bd(\mathsf{E}(g), \mathsf{E}(g'))_R \leq d(g, g')_G \leq a + \frac{1}{b} d(\mathsf{E}(g), \mathsf{E}(g'))_R$$

Example: Fermionic QCA

- OPT: Fermionic theory
- Automata: can be expressed as $\mathscr{T}(\varphi_i) = \varphi'_i$

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- Automata: can be expressed as $\mathscr{T}(\varphi_i) = \varphi_i'$
- Wrapping lemma: the local rule for $\mathscr{T}$ defines automata also on finite groups

B. Schumacher and R. F. Werner, arXiv:quant-ph/0405174 (2004)
PP and L. Poggiali, arXiv:1807.08695

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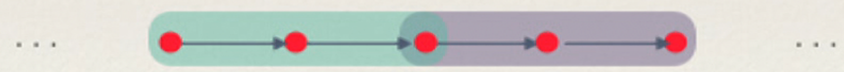

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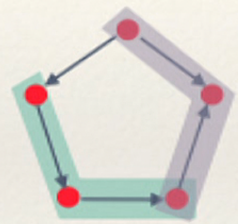

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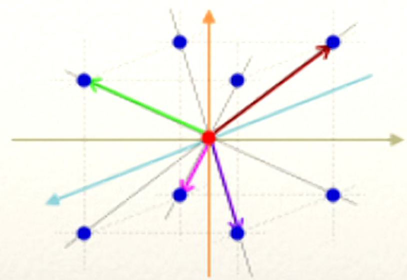
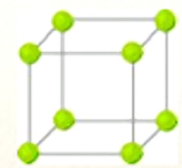
Linear Fermionic QCA

$G = \mathbb{Z}^3$

$s = 2$





G. M. D'Ariano and PP, Phys. Rev. A **90**, 062106 (2014)

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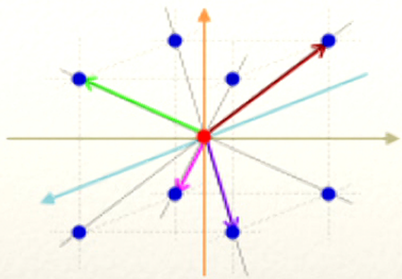


$$\psi_{\mathbf{k},t+1} = W_{\mathbf{k}}^{\pm} \psi_{\mathbf{k},t}$$

G. M. D'Ariano and PP, Phys. Rev. A **90**, 062106 (2014)

Free Quantum Field theory

- Composing Weyl automata
  - Dirac's equation

$$Z_{\mathbf{k}}^{\pm} = \begin{pmatrix} nW_{\mathbf{k}}^{\pm} & imI \\ imI & nW_{\mathbf{k}}^{\pm\dagger} \end{pmatrix}$$

$$i\partial_t \psi_{\mathbf{k},t} = (s\boldsymbol{\alpha} \cdot \mathbf{k} + c\beta)\psi_{\mathbf{k},t}$$

- Special class of entangled states
  - Maxwell's equations

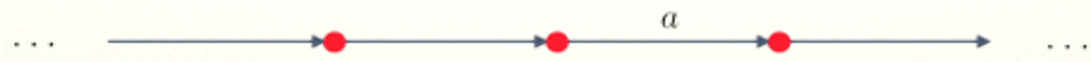
$$M_{\mathbf{k}}^{\pm} = W_{\mathbf{k}}^{\pm} \otimes W^{\pm}{}_{\mathbf{k}}^{*}$$

$$\partial_t \,\mathrm{Re}\,\mathbf{H}(\mathbf{x}, t) = \nabla \times \mathrm{Im}\,\mathbf{H}(\mathbf{x}, t),$$
$$\partial_t \,\mathrm{Im}\,\mathbf{H}(\mathbf{x}, t) = -\nabla \times \mathrm{Re}\,\mathbf{H}(\mathbf{x}, t),$$

G. M. D'Ariano and PP, Phys. Rev. A **90**, 062106 (2014),
A. Bisio, G. M. D'Ariano and PP, Ann. Phys. **368**, 177 (2016).

Non-linear FQCA

- $G = \mathbb{Z}$



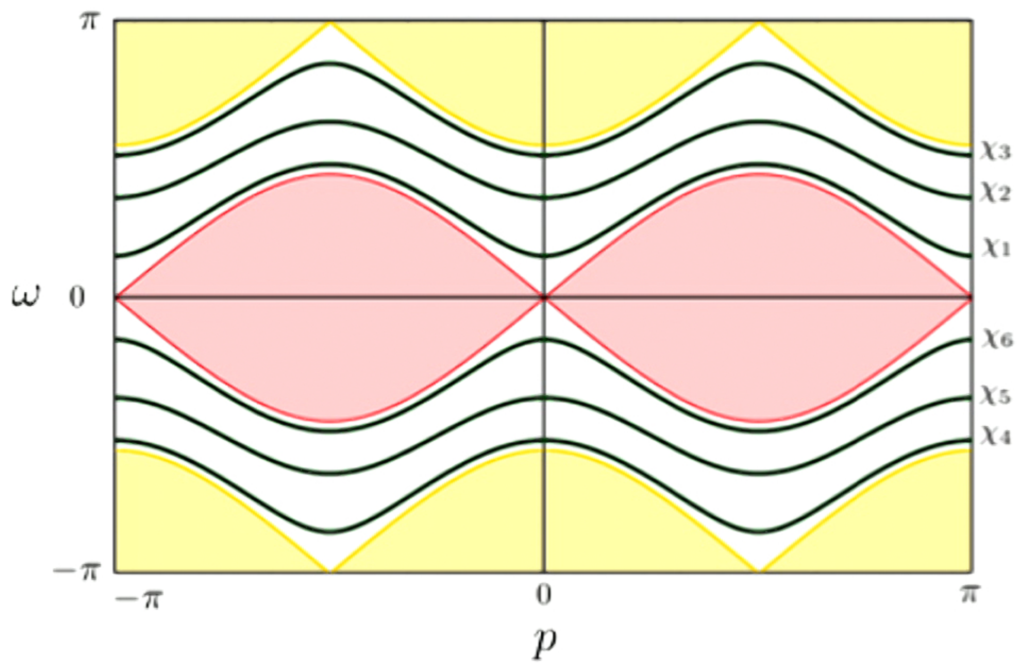
- $A = A_D V(\chi)$

A_D Dirac automaton: $A_D \psi_j(x) A_D^\dagger = W_{D\,jk}(x,y)\psi_k(y)$

$$V(\chi) := \exp[i\chi \psi_r^\dagger(x)\psi_r(x)\psi_l^\dagger(x)\psi_l(x)]$$

A. Bisio, G. M. D'Ariano, PP, and A. Tosini, Phys. Rev. A, **97,** 032132, (2018)

Non-linear FQCA

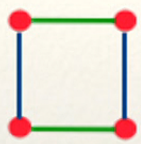


A. Bisio, G. M. D'Ariano, PP, and A. Tosini, Phys. Rev. A, **97,** 032132, (2018)

Non-linear FQCA: Case studies

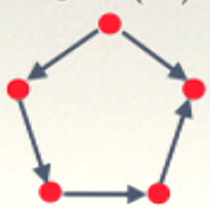
$$\mathscr{T}(\varphi_i) = \varphi_i'$$

$\mathbb{Z}_2 \times \mathbb{Z}_2$



$\alpha_{\mathbf{x}} \neq 0$ $\forall$ **x = a, b, e**	$\alpha_{\mathbf{x}} \neq 0$ **with x = a, b, e**	$\alpha_{\mathbf{x}} \neq 0$ **and** $\alpha_{\mathbf{y}} \neq 0$ **with x = a, b, e**
	$\alpha_x, \beta_{ix}, \beta_{jx}, \gamma_{ijx} \neq 0$	$\alpha_x, \alpha_y, \beta_{yx}, \beta_{xy} \neq 0$
	$\alpha_x\alpha_x^* = 1;\ \gamma_{ijx} = \beta_{ix}\beta_{jx}$	$\alpha_x\alpha_x^* + \alpha_y\alpha_y^* = 1$
No admitted solutions	$-2 \leq \mathrm{Re}(\beta_{ix}) \leq 0$	
	$\mathrm{Im}(\beta_{ix}) = -\sqrt{-\mathrm{Re}(\beta_{ix})(\mathrm{Re}(\beta_{ix})+2)}$	$2\lvert\alpha_i\rvert\cos(\theta_i - \theta_{ji}) = -\lvert\beta_{ji}\rvert$
	$-2 \leq \mathrm{Re}(\beta_{jx}) \leq 0$	$\theta_x - \theta_{yx} = \theta_y - \theta_{xy}$
	$\mathrm{Im}(\beta_{jx}) = -\sqrt{-\mathrm{Re}(\beta_{jx})(\mathrm{Re}(\beta_{jx})+2)}$	

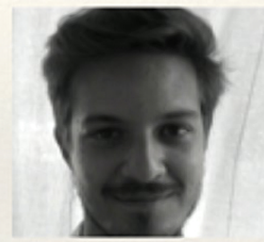
$\mathbb{Z}_5 \quad (\mathbb{Z})$



$\mu_{410} = \nu_{410} = 0$	$\mu_{410} = 0; \nu_{410} \neq 0$	$\nu_{410} = 0; \mu_{410} \neq 0$
	$\gamma_0, \eta_{10}, \eta_{40} \neq 0$	$\alpha_0, \beta_{10}, \beta_{40} \neq 0$
	$\eta_{10} = \eta_{40}$	$\beta_{10} = \beta_{40}$
Linear case	$\gamma_0\gamma_0^* = 1;\ \nu_{410}\gamma_0 = \eta^2$	$\alpha_0\alpha_0^* = 1;\ \mu_{410}\alpha_0 = \beta^2$
	$2\cos(\theta_{\eta_{ij}} - \theta_{\alpha_0}) = -\lvert\eta_{ji}\rvert$	$2\cos(\theta_{\beta_{ij}} - \theta_{\alpha_0}) = -\lvert\beta_{ji}\rvert$

PP and L. Poggiali, arXiv:1807.08695

L. Poggiali



Summary

- Paradigm of a physical law as an algorithm
- Emergence of space manifolds from Cayley graphs
  - Universal feature of automata in OPTs
- Fermionic cellular automata
  - Linear case
  - Finite groups and Z

Linear Fermionic QCA

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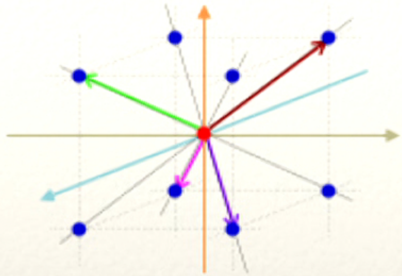

$$\psi_{\mathbf{k},t+1} = W^{\pm}_{\mathbf{k}} \psi_{\mathbf{k},t}$$

$$\downarrow$$

$$i\partial_t \psi_{\mathbf{k},t} = \mathbf{k} \cdot \boldsymbol{\sigma}^{\pm} \psi_{\mathbf{k},t}$$

Weyl's equations

G. M. D'Ariano and PP, Phys. Rev. A **90**, 062106 (2014)