Title: Quantum mechanics and the covariance of physical laws in quantum reference frames

Date: Jul 17, 2018 03:30 PM

URL: http://pirsa.org/18070044

Abstract: <p>In physics, every observation is made with respect to a frame of reference. Although reference frames are usually not considered as degrees of freedom, in all practical situations it is a physical system which constitutes a reference frame. Can a quantum system be considered as a reference frame and, if so, which description would it give of the world? The relational approach to physics suggests that all the features of a system â€”such as entanglement and superpositionâ€” are observer-dependent: what appears classical from our usual laboratory description might appear to be in a superposition, or entangled, from the point of view of such a quantum reference frame. In this work, we develop an operational framework for quantum theory to be applied within quantum reference frames. We find that, when reference frames are treated as quantum degrees of freedom, a more general transformation between reference frames has to be introduced. With this transformation we describe states, measurement, and dynamical evolution in different quantum reference frames, without appealing to an external, absolute reference frame. The transformation also leads to a generalisation of the notion of covariance of dynamical physical laws, which we explore in the case of â€˜superposition of Galilean translationsâ€™ and â€˜superposition of Galilean boostsâ€™. In addition, we consider the situation when the reference frame moves in a â€˜superposition of accelerationsâ€™, which leads us to extend the validity of the weak equivalence principle to quantum reference frames.</p>

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Quantum mechanics and the covariance of physical laws in quantum reference frames

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Preprint at arXiv:1712.07207

CoQuS ComplexQuantumSystems

Perimeter Institute
Waterloo, 17 July 2018

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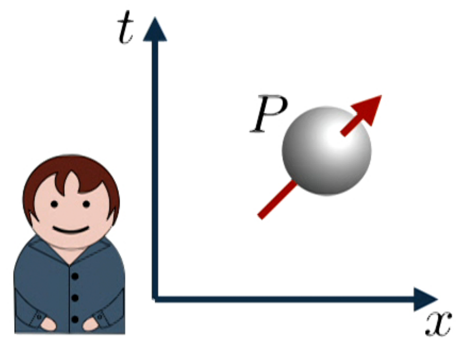
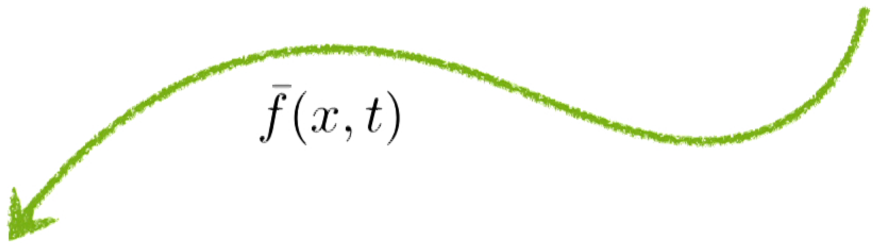
INTRODUCTION

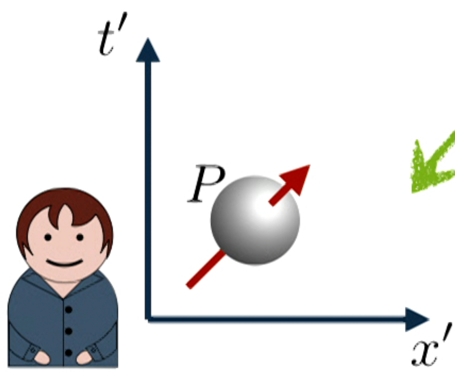
What is a reference frame?

Reference frames are abstract entities, used to fix the point of view from which observations are carried out.



The laws of physics are the same regardless of the choice of the reference frame. (**Principle of covariance**).



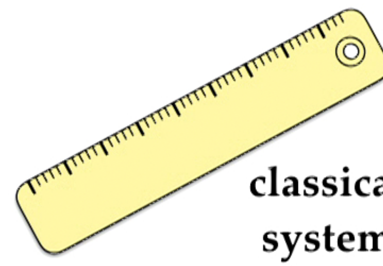


The reference frame enters the transformation as a **parameter**.

What is a reference frame?

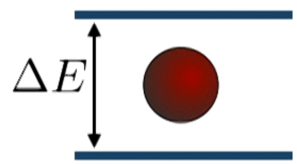
Assumptions for idealised reference frames:

- It is always possible to know their position and momentum with arbitrary precision;
- They don't have **dynamical degrees of freedom**;



classical system

Every observation is carried out by means of a physical system...

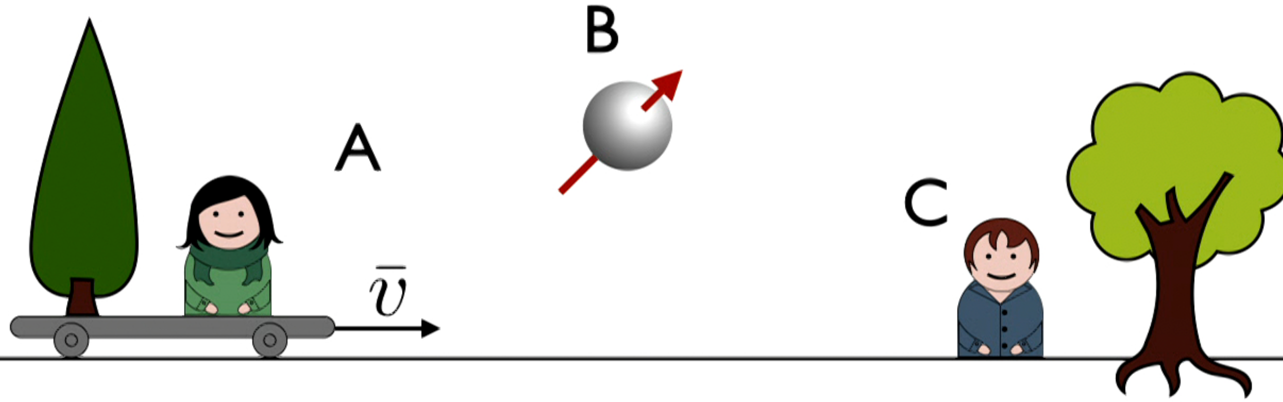


quantum system

... and taking as reference a physical object.

Is there any framework which does not rely on these assumptions?

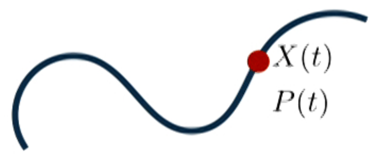
What is a reference frame?



A reference frame is a **physical system** and obeys the laws of physics.

We need to consider the **dynamical degrees of freedom** of the reference frame.

$$\begin{cases} X(t) \\ P(t) \end{cases}$$



The system can behave according to either **classical** or **quantum** mechanics.

What is a reference frame?

Relaxation of the two main assumptions

Classicality assumption	Absence of dynamical evolution
⇩	⇩
Quantum reference frames	**Dynamical reference frames**
Kinematics: Structure of the Hilbert space with superposition and entanglement properties	**Dynamics:** Temporal evolution, laws of motion

These two aspects are both necessary and complementary for a model of reference frames as physical systems.

Outline

- Kinematics: transformation to the quantum reference frame, notion of relative state, examples
- Dynamics: general approach, Schrödinger equation
- Superposition of spatial translations
- Superposition of Galilean boosts
- Weak equivalence principle in quantum reference frames

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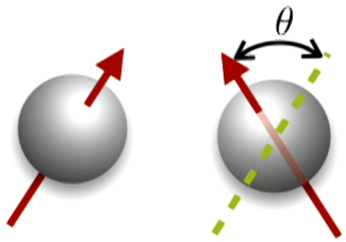
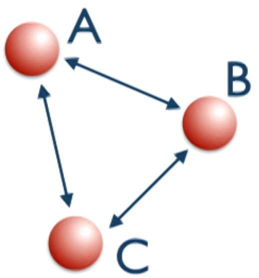
HILBERT SPACE STRUCTURE: KINEMATICS

Quantum reference frames



The description of the quantum state is given in terms of **relative** quantities.



Different observers assign a different quantum state to the system.

The usual definition of relative coordinates and momenta is:

$$x_i^r = x_i - x_0$$
$$p_i^r = \mu_{i0}\left(\frac{p_i}{m_i} - \frac{p_0}{m_0}\right) \qquad i = 1, ..., N$$

$\mu_{i0} = \frac{m_i m_0}{m_i + m_0}$ reduced mass

Not a canonical transformation!

$$\{x_i^r, p_j^r\} \neq 0 \qquad i \neq j$$

We are not guaranteed to have a hamiltonian formalism after the transformation.

We choose the relative quantity we are interested in and then complete the transformation canonically.

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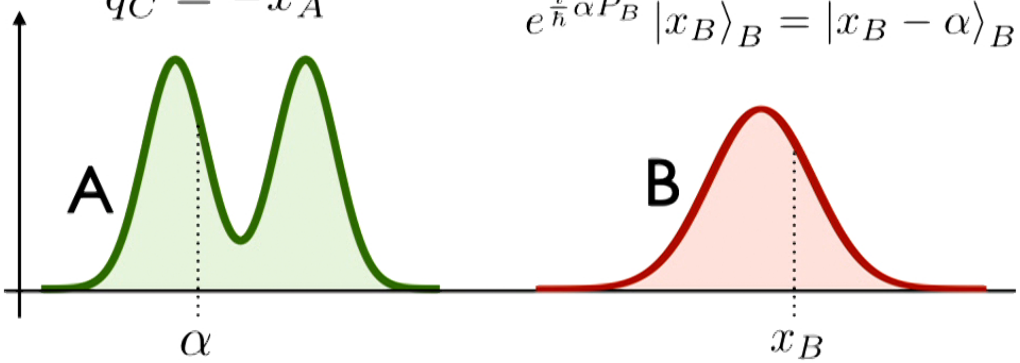
Transformation of the state

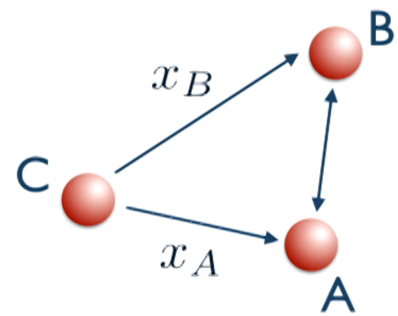
Simplest case: transformation to relative coordinates

$$q_B = x_B - x_A$$

$$q_C = -x_A$$

$$e^{\frac{i}{\hbar}\alpha \hat{P}_B} |x_B\rangle_B = |x_B - \alpha\rangle_B$$





For the whole state:

$$e^{\frac{i}{\hbar}\hat{X}_A \hat{P}_B} |\psi\rangle_A |\phi\rangle_B$$

usually a parameter of the group!

- Wavepackets instead of sharp position / velocities
- Quantum superposition, entanglement

$$\boxed{\hat{S}_x = \hat{\mathcal{P}}_{AC} e^{\frac{i}{\hbar}\hat{x}_A \hat{p}_B}}$$

$$\rho_{BC}^{(A)} = \hat{S}_x \rho_{AB}^{(C)} \hat{S}_x^\dagger$$

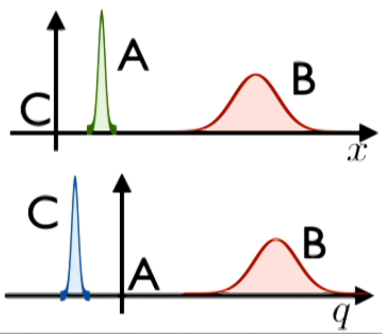
$\hat{\mathcal{P}}_{AC}$: parity operator + swap between A and C.

Example: Relative states

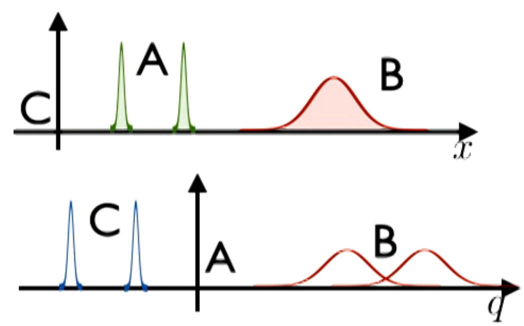
$$\hat{S}_x = \hat{\mathcal{P}}_{AC} e^{\frac{i}{\hbar}\hat{x}_A \hat{p}_B}$$

$$\rho^{(A)}_{BC} = \hat{S}_x \rho^{(C)}_{AB} \hat{S}_x^\dagger$$

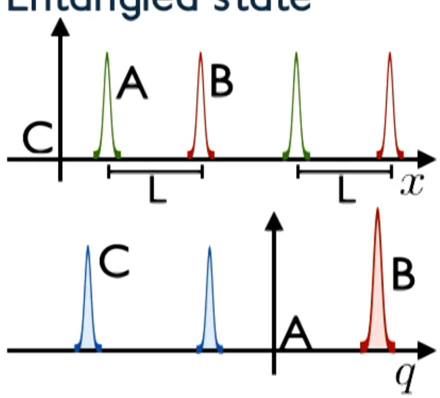
Localised state of A



Product state and spatial superposition

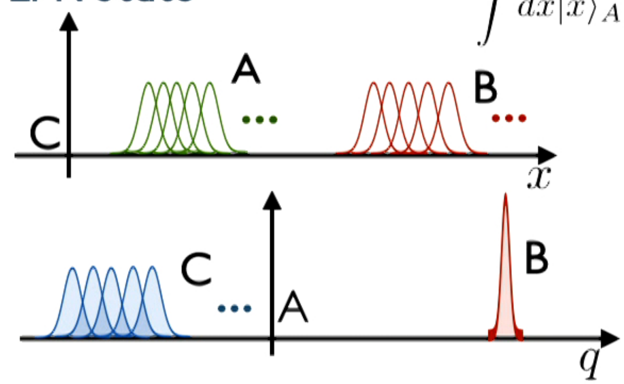


Entangled state



EPR state

$$\int dx |x\rangle_A |x + X\rangle_B$$



A: new reference frame; B: quantum system; C: old reference frame

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TEMPORAL EVOLUTION: DYNAMICS

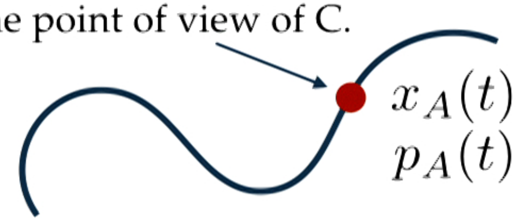
Dynamical reference frames

Reference frames are not anymore parameters in a transformation, but are associated to a hamiltonian.

C describes A and B: $H_{AB}^{(C)}$

A describes B and C: $H_{BC}^{(A)}$

A obeys Hamilton's equations of motion from the point of view of C.



The hamiltonian changes according to a canonical transformation, which is the same in the classical and quantum case:

$$H'(q_i, \pi_i) = H(x_i(q_i, \pi_i), p_i(q_i, \pi_i)) + \frac{\partial F}{\partial t}$$

new hamiltonian

old hamiltonian in new variables

generator of the canonical transformation

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The Schrödinger equation

Schrödinger equation in C's reference frame

$$i\hbar\frac{d\rho_{AB}^{(C)}}{dt} = \left[H_{AB}^{(C)}, \rho_{AB}^{(C)}(t)\right]$$

A: new reference frame
B: quantum system
C: old reference frame

To change to the frame of A we apply the transformation $\hat{S}$

$$i\hbar\frac{d\rho_{BC}^{(A)}}{dt} = \left[H_{BC}^{(A)}, \rho_{BC}^{(A)}(t)\right]$$

$$\hat{H}_{BC}^{(A)} = \hat{S}\hat{H}_{AB}^{(C)}\hat{S}^\dagger + i\hbar\frac{d\hat{S}}{dt}\hat{S}^\dagger$$

$$\hat{\rho}_{BC}^{(A)} = \hat{S}\hat{\rho}_{AB}^{(C)}\hat{S}^\dagger$$

The evolution in the new reference frame is unitary.

We define a symmetry transformation as:

$$\hat{S}\hat{H}\left(\{m_i, \hat{x}_i, \hat{p}_i\}_{i=A,B}\right)\hat{S}^\dagger + i\hbar\frac{d\hat{S}}{dt}\hat{S}^\dagger = \hat{H}\left(\{m_i, \hat{x}_i, \hat{p}_i\}_{i=B,C}\right)$$

General transformation

Hamiltonian of the new reference frame A as seen from C

$$\hat{S} = e^{-\frac{i}{\hbar}\hat{H}_C t}\hat{\mathcal{P}}^{(i)}_{AC}\Pi_n e^{\frac{i}{\hbar}\hat{f}^n_A(t)\hat{O}^n_B} e^{\frac{i}{\hbar}\hat{H}_A t}$$

generalised parity operator (switches the equations of motion of A and C)

generalisation of the standard change of reference frame

$$\hat{U}_i = \Pi_n e^{\frac{i}{\hbar}f^n(t)\hat{O}^n_B}$$

function describing the relationship between old and new reference frame

Translations in QRF

We move to the reference frame described by system A at time τ.

$$\hat{H}_{AB}^{(C)} = \frac{\hat{p}_A^2}{2m_A} + \frac{\hat{p}_B^2}{2m_B}$$

standard translation $X(t) = X_0 \Rightarrow \hat{X}_A(t) = e^{\frac{i}{\hbar}\frac{\hat{p}_A^2}{2m_A}\tau}\hat{x}_A e^{-\frac{i}{\hbar}\frac{\hat{p}_A^2}{2m_A}\tau}$ position of A at time τ

$$\hat{S}_T = \exp\left(-\frac{i}{\hbar}\frac{\hat{\pi}_C^2}{2m_C}(t-\tau)\right)\underbrace{\hat{\mathcal{P}}_{AC}^{(x)}\exp\left(\frac{i}{\hbar}\hat{x}_A\hat{p}_B\right)}_{\hat{S}_x}\exp\left(\frac{i}{\hbar}\frac{\hat{p}_A^2}{2m_A}(t-\tau)\right)$$

$\hat{S}_x$ translation to a reference frame which is frozen in time.

$$\hat{H}_{BC}^{(A)} = \frac{\hat{\pi}_B^2}{2m_B} + \frac{\hat{\pi}_C^2}{2m_C}$$

The free hamiltonian is symmetric under generalised translations.

$$H_{AB}^{(C)} = \frac{\hat{P}_A^2}{2m_A} + \frac{\hat{P}_B^2}{2m_B} + \frac{(\hat{p}_A + \hat{p}_B)^2}{2m_C}$$

$$p_A + p_B = -p_C -$$

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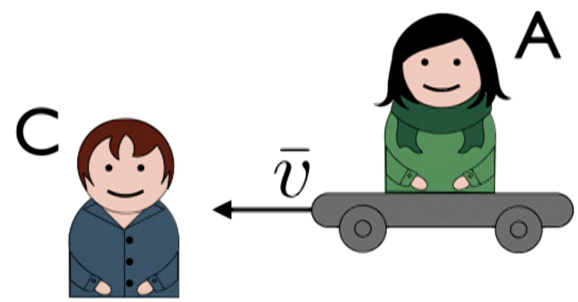
SUPERPOSITION OF GALILEAN BOOSTS

The Galilean boost

The boost describes the change to a reference frame moving with constant and uniform velocity with respect to the initial one (inertial).

$$U_v = e^{\frac{i}{\hbar} v \hat{G}}$$

$$\hat{G} = \hat{P}t - m\hat{X}$$



In standard reference frames the equations of motion are invariant under boost transformation

$$H' = U_v H U_v^+ + i\hbar \frac{dU_v}{dt} U_v^+ = H$$

when H is of the form

$$H = \frac{\hat{P}^2}{2m} + V(\hat{X} - x_0)$$

→ origin of the reference frame

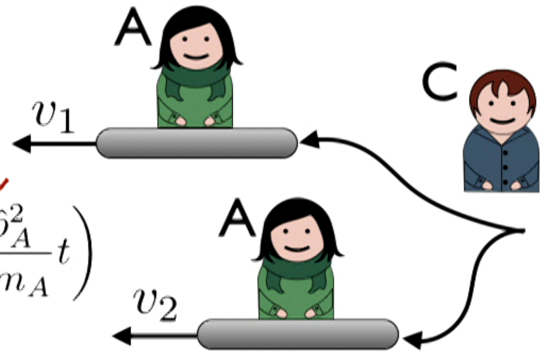
The galilean boost is a symmetry of the hamiltonian.

The boost in QRF

We want to consider a system (A) in a superposition of velocities from the point of view of the initial reference frame (C).

$$\hat{H}^{(C)}_{AB} = \frac{\hat{p}_A^2}{2m_A} + \frac{\hat{p}_B^2}{2m_B}$$

Free-particle evolution



$$\hat{S}_b = \exp\left(-\frac{i}{\hbar}\frac{\hat{\pi}_C^2}{2m_C}t\right)\hat{\mathcal{P}}^{(v)}_{AC}\exp\left(\frac{i}{\hbar}\frac{\hat{p}_A}{m_A}\hat{G}_B\right)\exp\left(\frac{i}{\hbar}\frac{\hat{p}_A^2}{2m_A}t\right)$$

- parity and swap
- sets velocity of C to the opposite of velocity of A

Galilean boost by the velocity of A

More general than the standard approach to reference frames.

What does the relative state look like?

$$|\Psi_t\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|m_A v_1\rangle_A + |m_A v_2\rangle_A\right)|\psi_t\rangle_B \quad \text{(from C)}$$

$$|\Psi'_t\rangle_{BC} = \frac{1}{\sqrt{2}}\left(e^{\frac{i}{\hbar}v_1\hat{G}_B}|\psi_t\rangle_B|-m_C v_1\rangle_C + e^{\frac{i}{\hbar}v_2\hat{G}_B}|\psi_t\rangle_B|-m_C v_2\rangle_C\right) \quad \text{(from A)}$$

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WEAK EQUIVALENCE PRINCIPLE IN QRFs

Weak equivalence principle

The physical effects as seen from a reference frame moving with constant and uniform acceleration are indistinguishable from those as seen in a uniform gravitational field.

$$m_I \vec{a} = m_g \vec{g} \quad \rightsquigarrow \quad \vec{a} = \vec{g}$$

Weak equivalence principle

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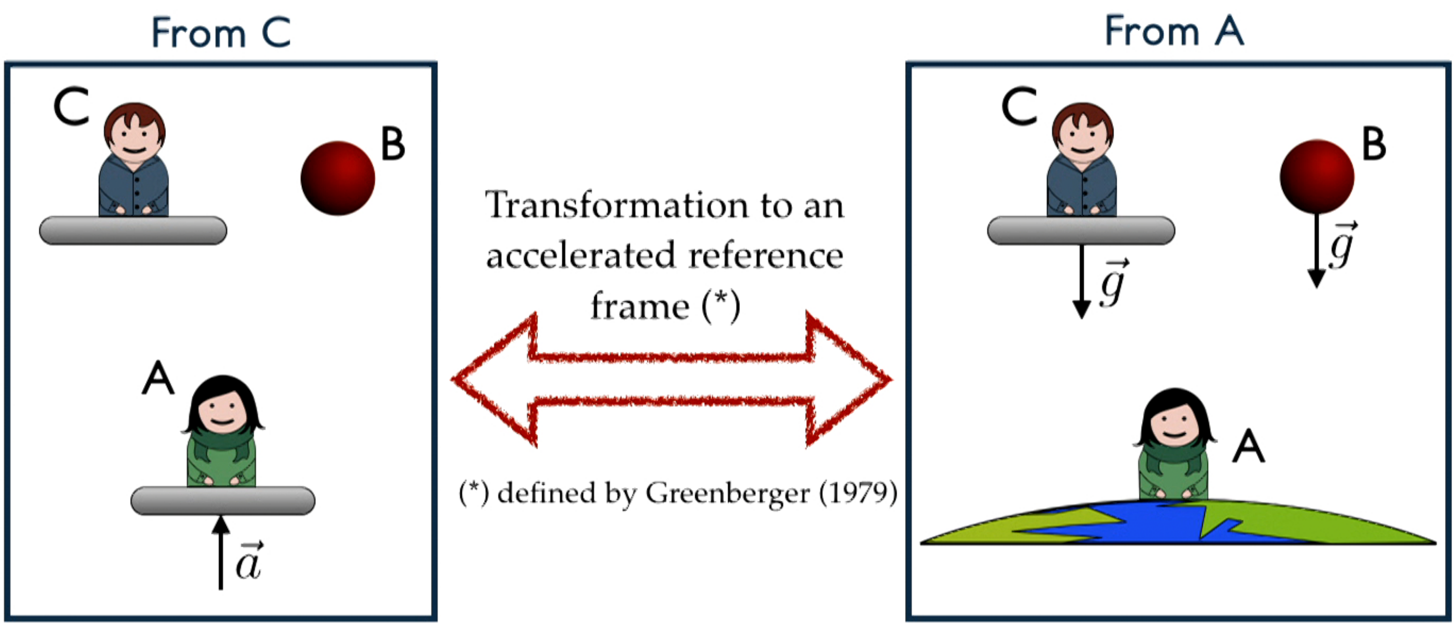

Weak equivalence principle

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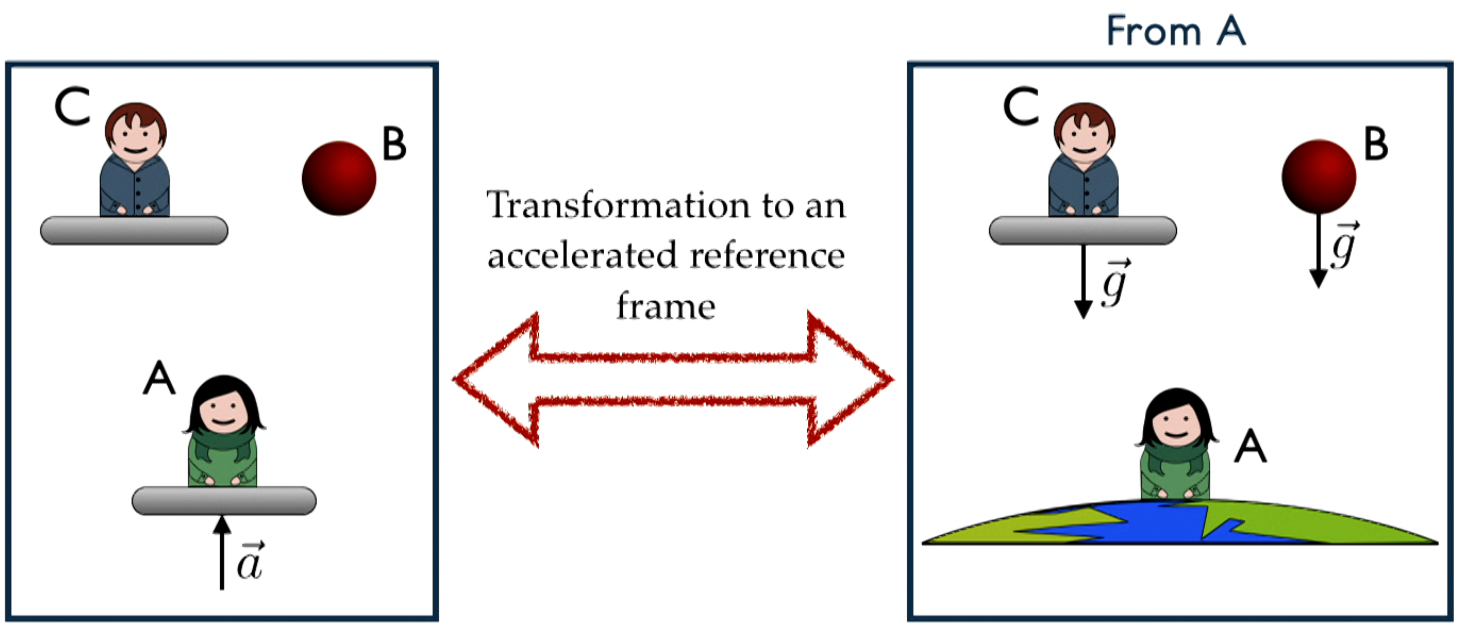

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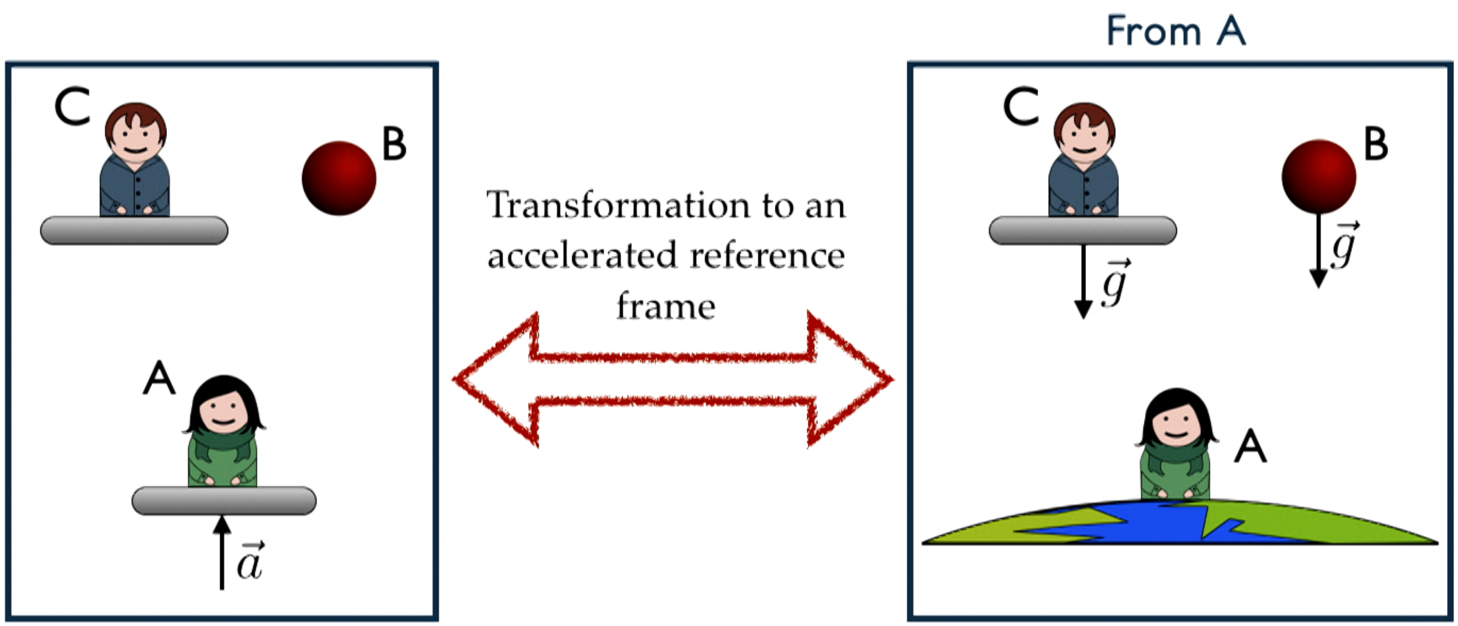
$$m_I \overset{\hat{a}}{\cancel{\vec{a}}} = m_g \overset{\hat{g}}{\cancel{\vec{g}}} \quad \rightsquigarrow \quad \cancel{\vec{a} = \vec{g}} \quad \hat{a} = \hat{g}$$



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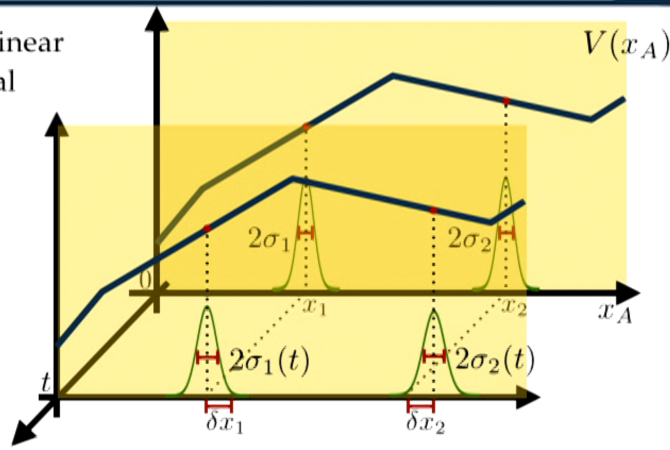
WEP for QRFs

$$\hat{H}_{AB}^{(C)} = \frac{\hat{p}_A^2}{2m_A} + \frac{\hat{p}_B^2}{2m_B} + V(\hat{x}_A)$$

Piecewise linear potential



State of A at time t:

$$|\psi_0(t)\rangle_A = \frac{1}{\sqrt{2}}\left(|\psi_1(t)\rangle_A + |\psi_2(t)\rangle_A\right)$$

$$a_1 = -\frac{1}{m_A}\frac{dV(\hat{x}_A)}{d\hat{x}_A}\Big|_{x_1(t)} \qquad a_2 = -\frac{1}{m_A}\frac{dV(\hat{x}_A)}{d\hat{x}_A}\Big|_{x_2(t)}$$

$$\hat{S}_{EP} = e^{-\frac{i}{\hbar}\left(\frac{\hat{\pi}_C^2}{2m_C} + \frac{m_C}{m_A}V(-\hat{q}_C)\right)t}\,\hat{\mathcal{P}}_{AC}^{(v)}\hat{Q}_t\, e^{\frac{i}{\hbar}\left(\frac{\hat{p}_A^2}{2m_A} + V(\hat{x}_A)\right)t}$$

Generalisation of the Greenberger operator

In the reference frame of A

$$\hat{H}_{BC}^{(A)} = \frac{\hat{\pi}_B^2}{2m_B} + \frac{\hat{\pi}_C^2}{2m_C} + \frac{m_C}{m_A}V(-\hat{q}_C) - \frac{m_B}{m_A}\frac{dV}{dx_A}\Big|_{-\hat{q}_C}\hat{q}_B$$

B moves as if it were in a superposition of gravitational fields!

Standard Greenberger operators for the accelerations 1 and 2

$$|\psi'(t)\rangle_{BC} = \frac{1}{\sqrt{2}}\left(|\alpha_1'(t)\rangle_C Q_t^1|\phi(t)\rangle_B + |\alpha_2'(t)\rangle_C Q_t^2|\phi(t)\rangle_B\right)$$

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Summary

Operational approach: reference frames as physical systems.

Problem split into:

Kinematics: transformations of quantum states

Dynamics: transformation of the hamiltonian and Schrödinger equation.

Question

How can we describe the world from the point of view of a non-idealised reference frame, i.e. associated to a quantum state and to a dynamical equation of motion?

Need to find a more general law to change the reference frame.

This leads to a **generalisation of the notion of covariance**, which has been explored in the two cases of the **superposition of spatial translations** and the **superposition of Galilean boosts**.

The **weak equivalence principle** can also be extended to quantum reference frames.