Title: Gravitational Waves Experiments 1

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Abstract:

Gravitational Waves: Experimental Techniques

1

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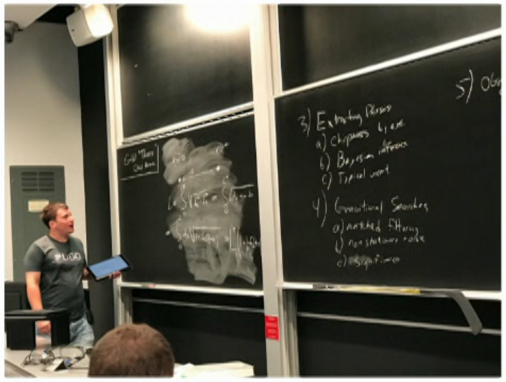
TRISEP 2018 - Perimeter Institute

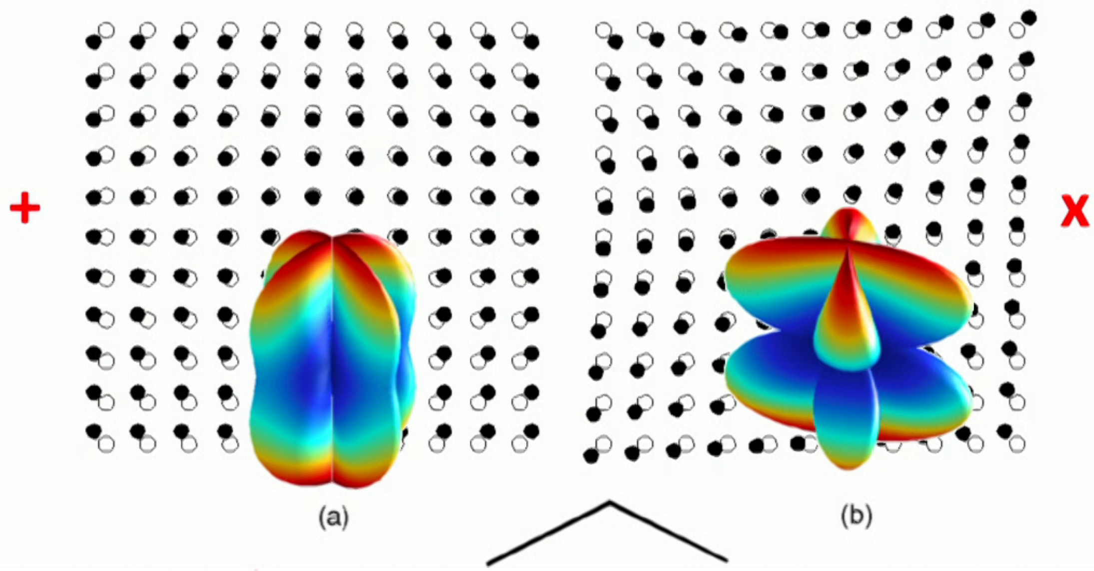
Outline

- **Lecture 1**: the basics of interferometric detection of gravitational waves
- **Lecture 2**: fundamental noise sources (seismic, thermal and quantum noises)
- **Lecture 3**: the dirty reality of ”technical noises” (scattered light, control noise, etc…) and prospects for the future

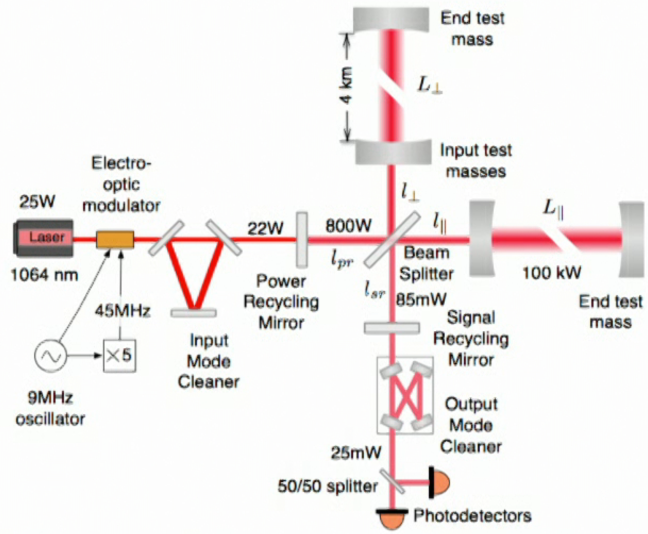
Gravitational waves



- All we need to know for this lecture:
  - Change in the distance between mirrors
  - For km scale distances, and audio frequencies, flat response in frequency
  - Differential displacement along orthogonal arms



Interferometric detectors



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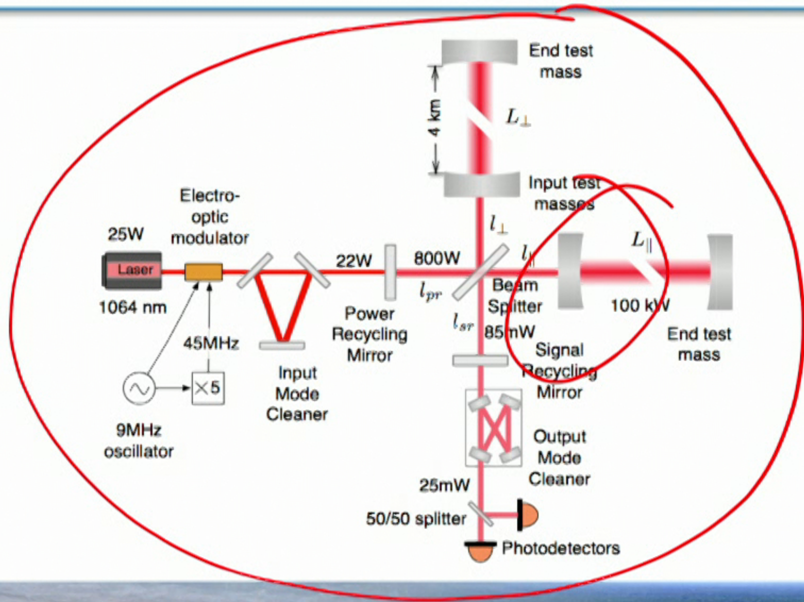

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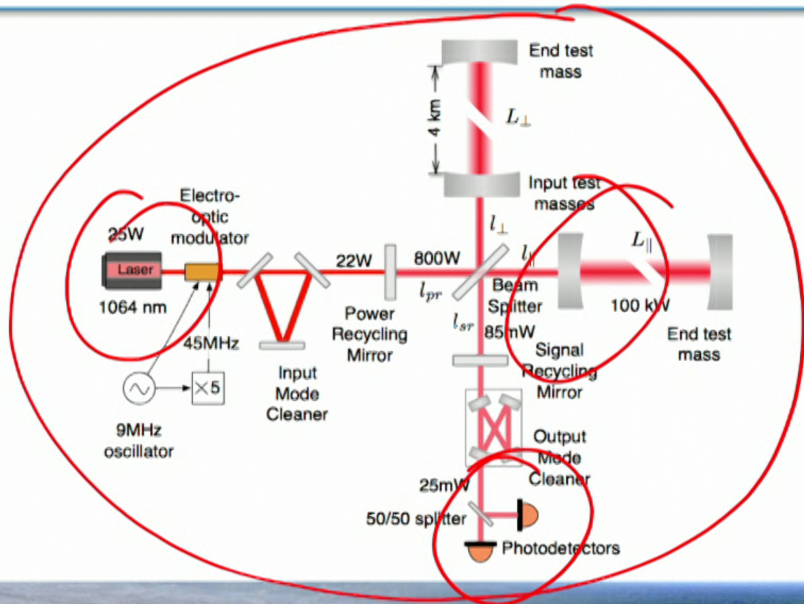

LIGO

Lasers

The Advanced LIGO Pre-Stabilized Laser

- Monochromatic light source
- Diffraction limited, but we can use a plane-wave approximation for most of the discussion

$$\psi(\vec{z},t) = \psi_0 e^{i\vec{k}\cdot\vec{z} - i\omega t}$$

$$|\vec{k}| = \frac{2\pi}{\lambda} \qquad \omega = 2\pi f$$

$$\lambda = 1064\,\text{nm}$$

$$P = 70\,\text{W}$$

LIGO

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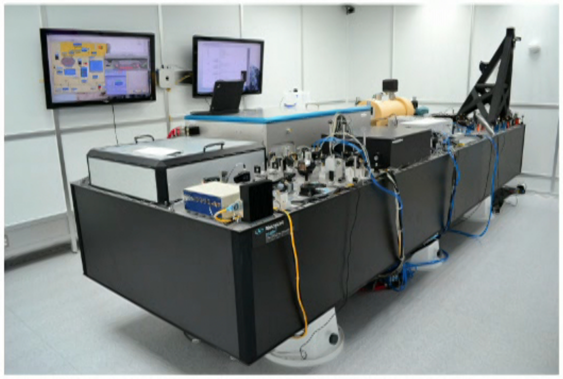


- Monochromatic light source
- Diffraction limited, but we can use a plane-wave approximation for most of the discussion

$$E(\mathbf{x},t) = Ee^{i\mathbf{k}\cdot\mathbf{x}-i\omega t}$$

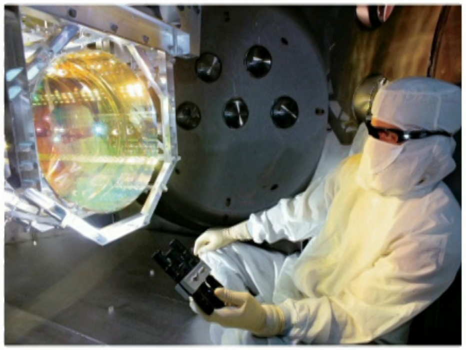
$$\omega = 2\pi f$$

$$\mathbf{k} = \frac{2\pi}{\lambda}\mathbf{n}$$

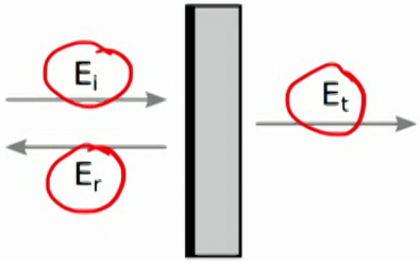
LIGO laser power 70 W
Wavelength 1064 nm

Mirrors

An Advanced LIGO mirror
suspended in the vacuum chamber



- Dielectric coatings
- Very low absorption
- High optical quality of both substrate and surface



$$E_r = rE_i$$
$$E_t = tE_i$$

$$|r|^2 + |t|^2 + \mathcal{L} = 1$$

Mirrors

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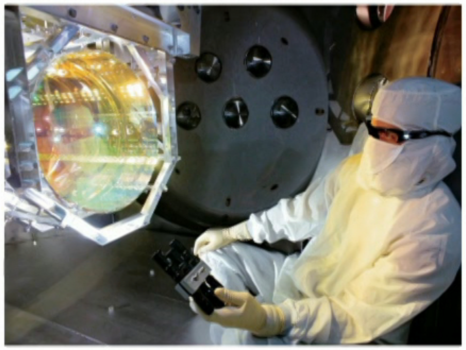

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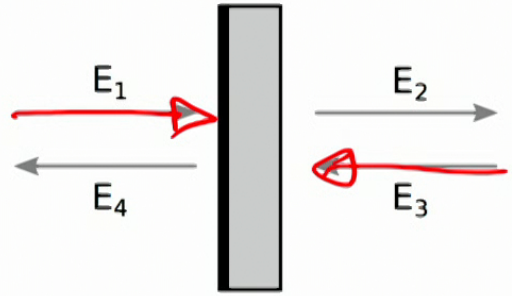

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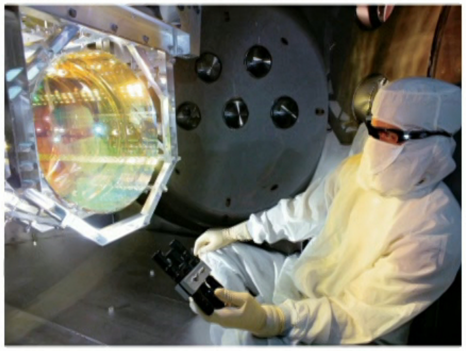

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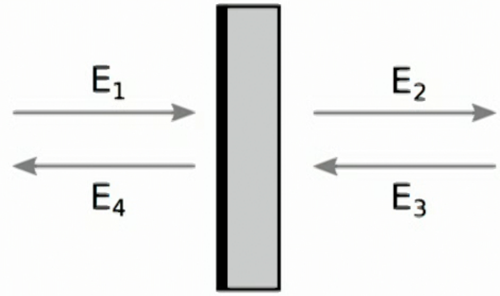

$$|E_2|^2 + |E_4|^2 = (1 - \mathcal{L})\left(|E_1|^2 + |E_3|^2\right)$$

$$\begin{aligned} E_r &= \pm r E_i \\ E_t &= t E_i \end{aligned}$$

$$E_4 = r E_1 + t E_3$$
$$E_2 = t E_1 - r E_3$$

Mirrors

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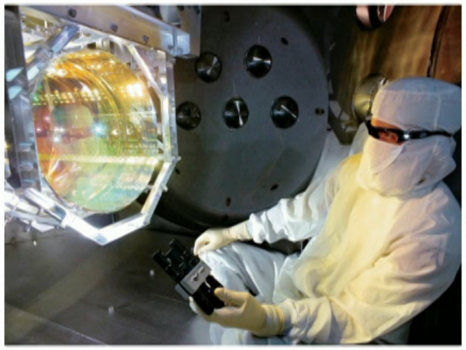

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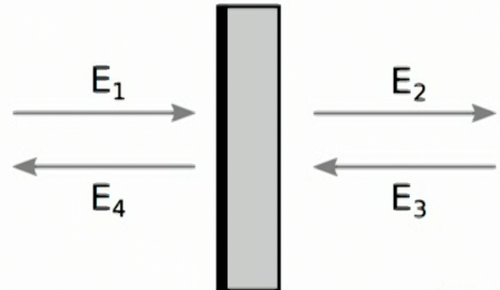

OUT IN

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$\in \mathbb{R}$

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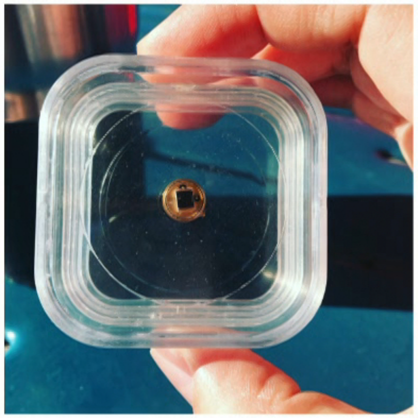
Photodetectors

The Advanced LIGO photodiode that detected the first GW



- They can detect only the power
- Output a current proportional to the impinging laser power
- Instantaneous absolute phase of field not accessible

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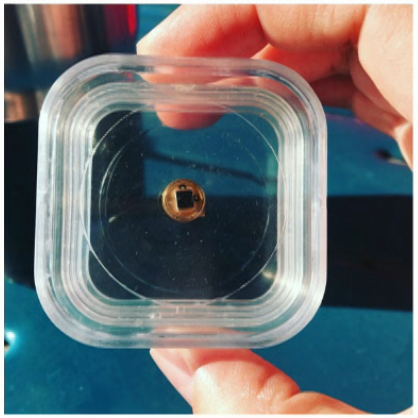

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$$P(\mathbf{x}) = |E(\mathbf{x},t)|^2$$

$$I = \eta q_e \frac{P}{\hbar\omega}$$

$$\begin{aligned} P &= |E_1 + E_2|^2 = |E_1|^2 + |E_2|^2 + 2\Re\left(E_1^* E_2\right) \\ &= |E_1|^2 + |E_2|^2 + |E_1| \cdot |E_2| \cos\phi_{12} \end{aligned}$$

LIGO

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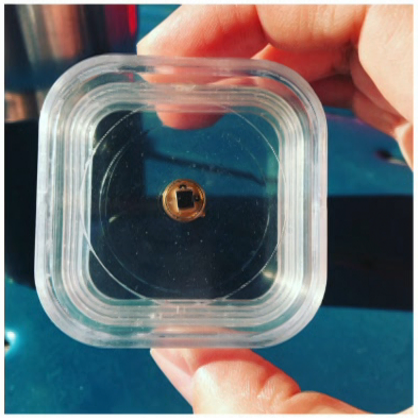

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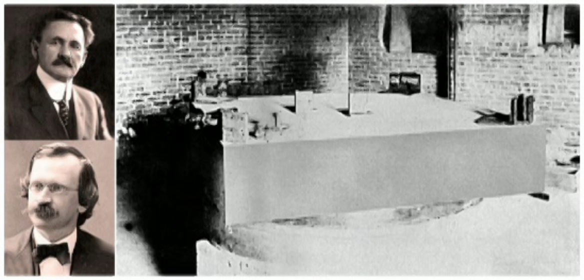
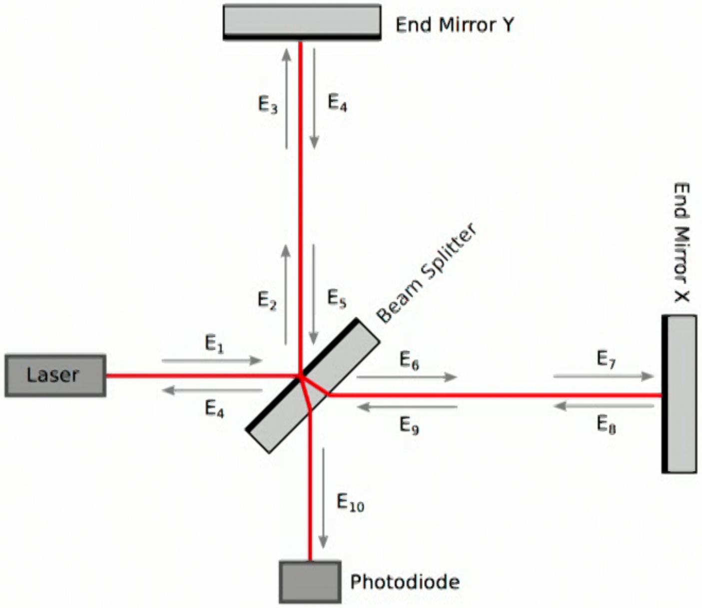
Michelson interferometer

The original Michelson and Morley experiment



- Arguably the most sensitive instrument to detect differential strain
- Based on interference of two laser beams propagating in orthogonal directions



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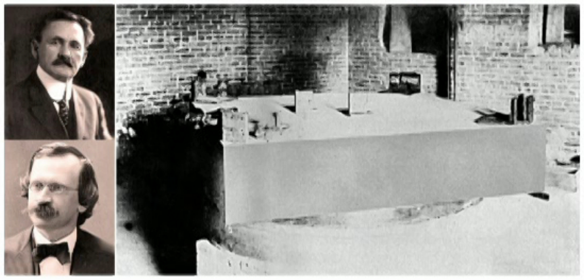

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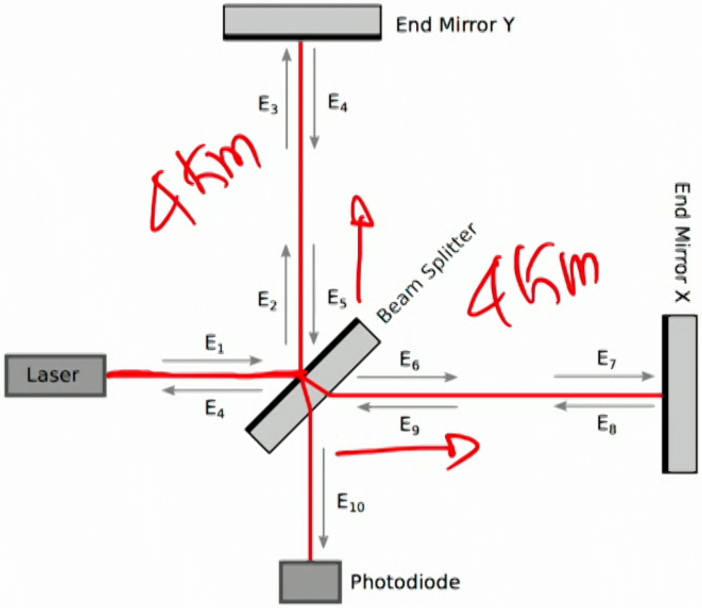
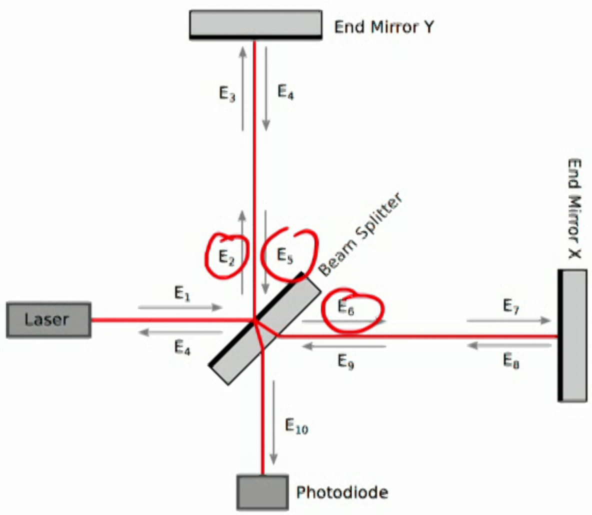
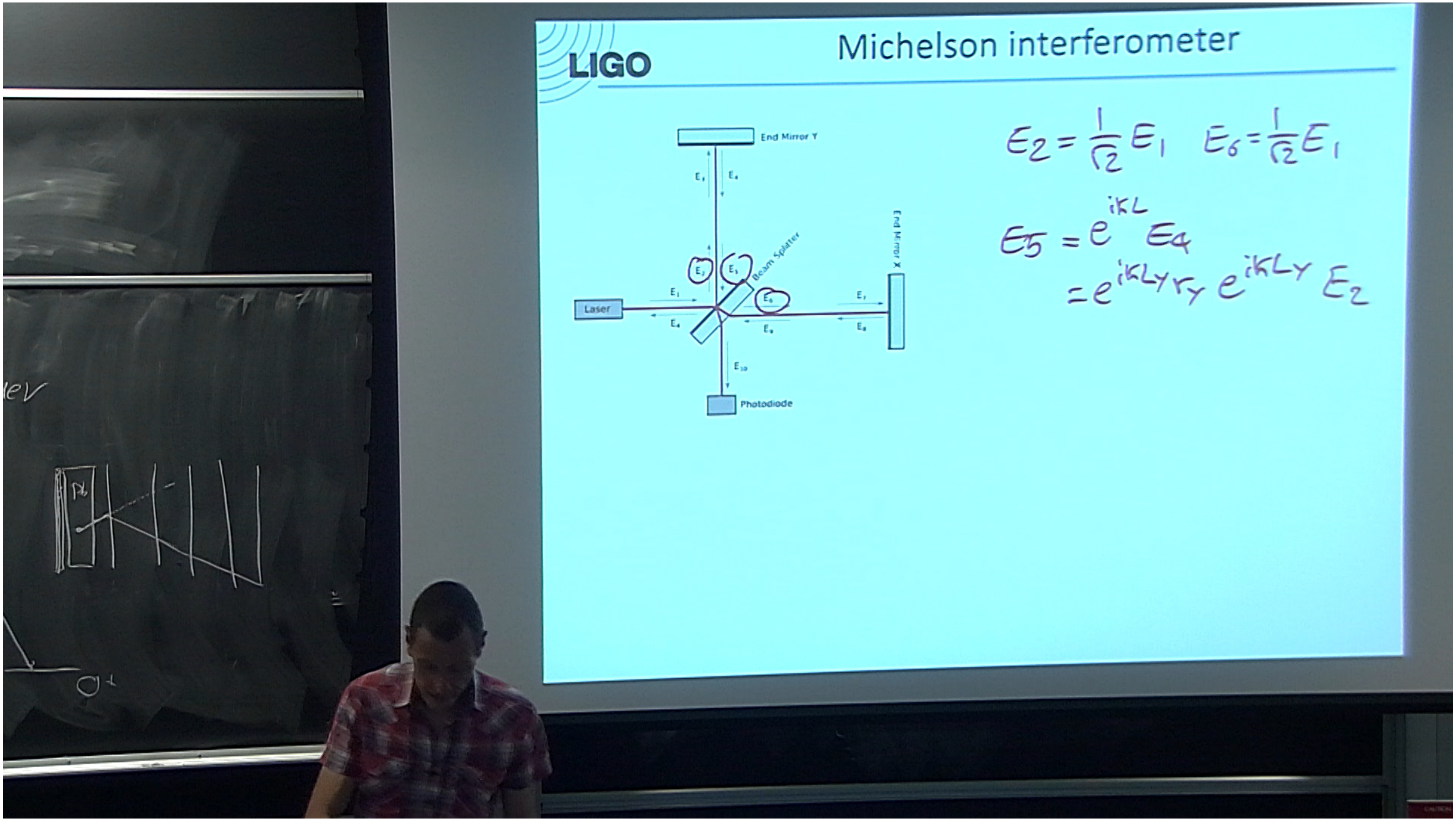


Michelson interferometer



$$E_2 = \frac{1}{\sqrt{2}} E_1 \quad E_6 = \frac{1}{\sqrt{2}} E_1$$

$$E_5 = e^{ikL} E_4$$
$$= e^{ikL} r_Y$$


LIGO
Michelson interferometer
End Mirror Y
Laser
End Mirror X
Photodiode
$E_2 = \frac{1}{\sqrt{2}} E_1 \quad E_6 = \frac{1}{\sqrt{2}} E_1$
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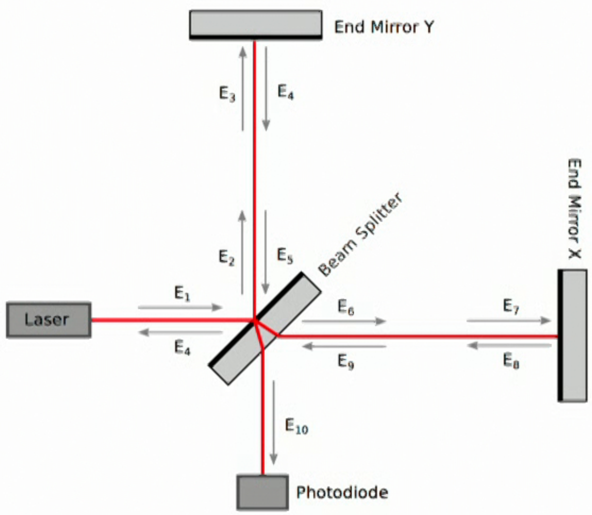
Michelson interferometer



$$E_2 = \frac{i}{\sqrt{2}} E_1 \qquad E_3 = e^{ikL_Y} E_2$$

$$E_6 = \frac{1}{\sqrt{2}} E_1 \qquad E_7 = e^{ikL_Y} E_6$$

$$E_4 = r_Y E_3 \qquad E_5 = e^{ikL_Y} E_4$$

$$E_8 = r_X E_7 \qquad E_9 = e^{ikL_X} E_8$$

$$E_{10} = -\frac{1}{\sqrt{2}} E_9 + \frac{1}{\sqrt{2}} E_5$$

$$E_A = E_{10} = -\frac{1}{2}\left[e^{2ikL_X} r_X - e^{2ikL_Y} r_Y\right] E_1$$

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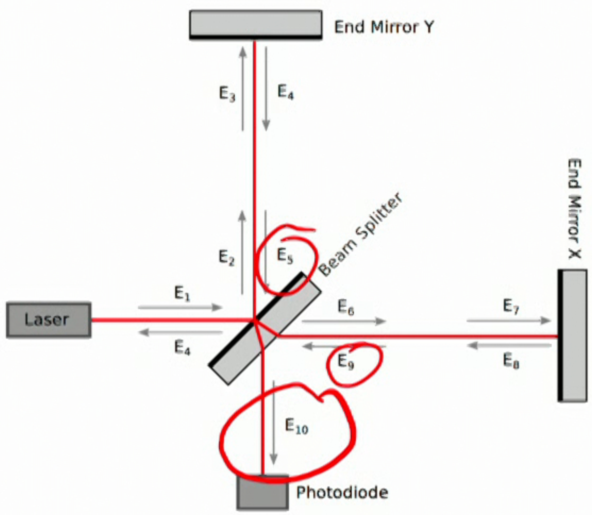
Michelson interferometer



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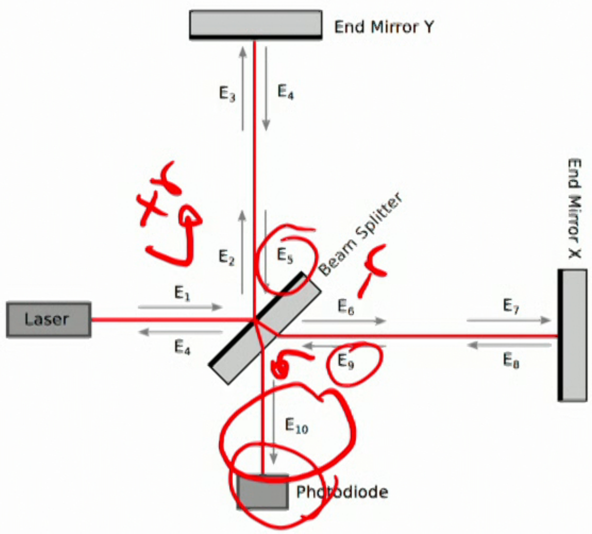

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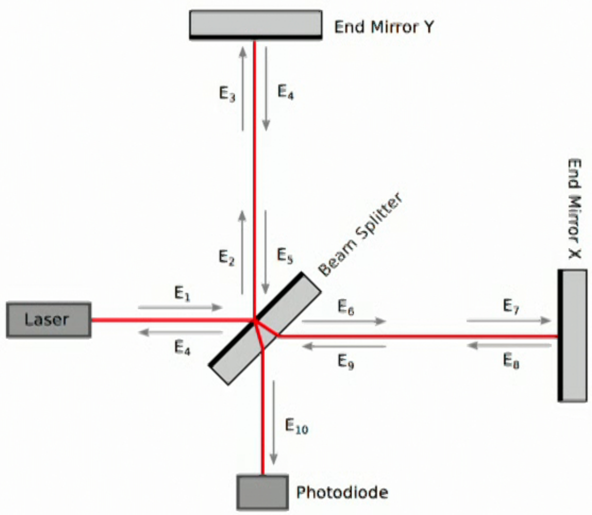

$$L = \frac{L_X + L_Y}{2} \qquad \Delta L = L_X - L_Y$$

$$E_A = -\frac{1}{2}e^{2ikL}\left[e^{ik\Delta L}r_X - e^{-ik\Delta L}r_Y\right]E_1$$

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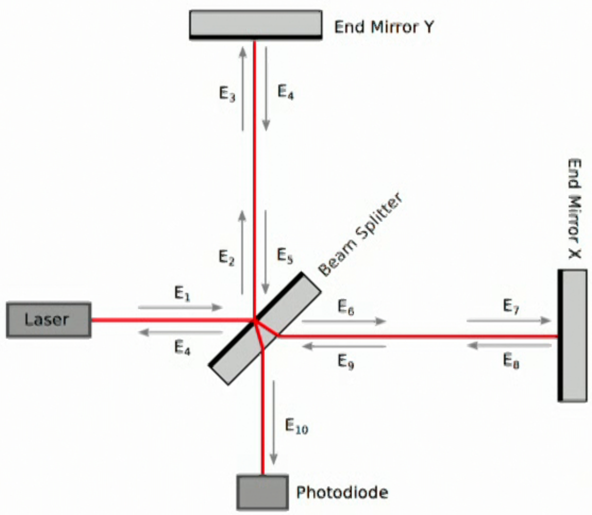

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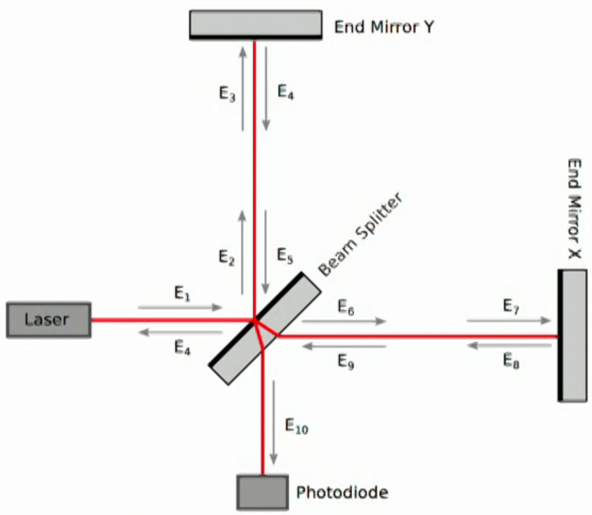
LIGO

Michelson interferometer



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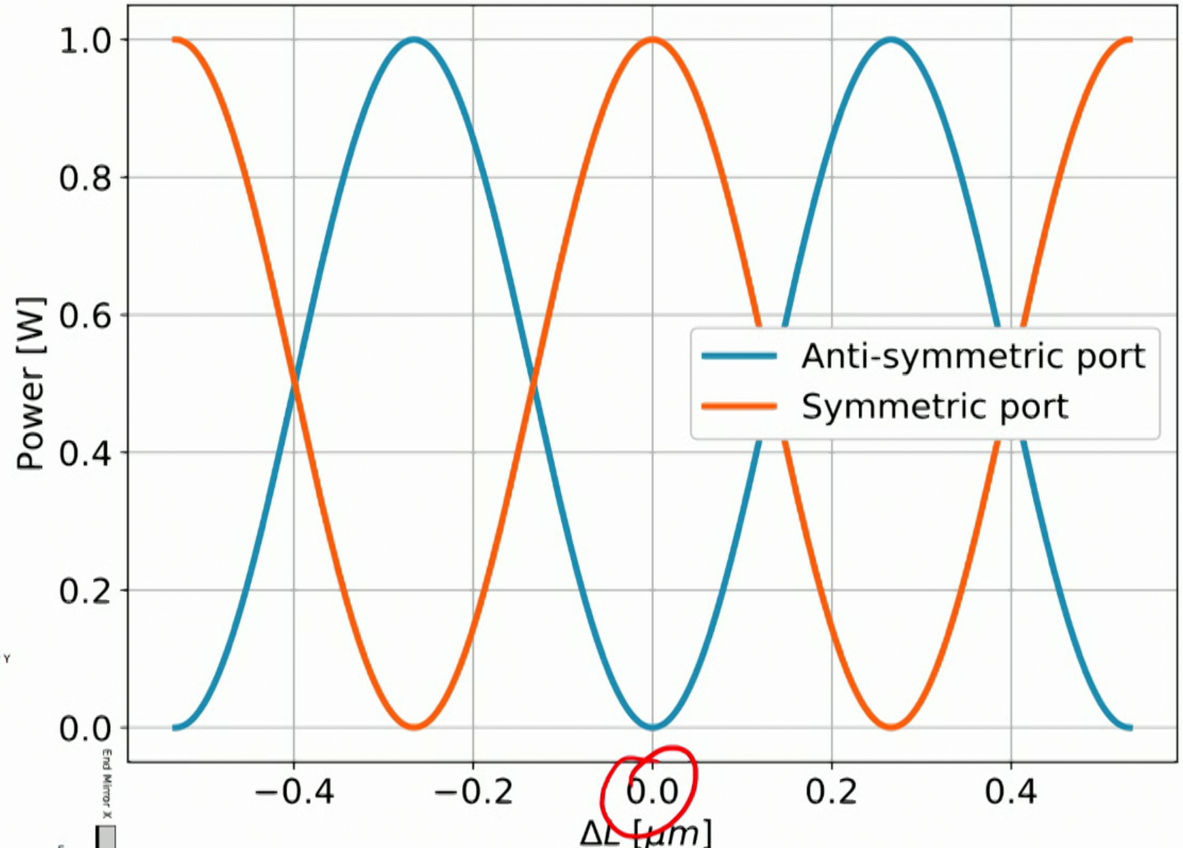
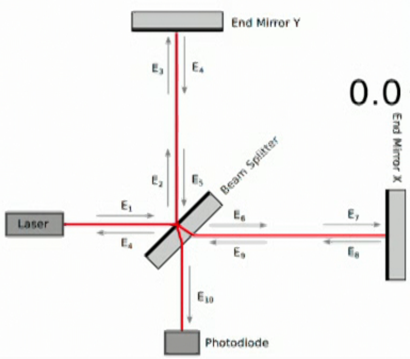
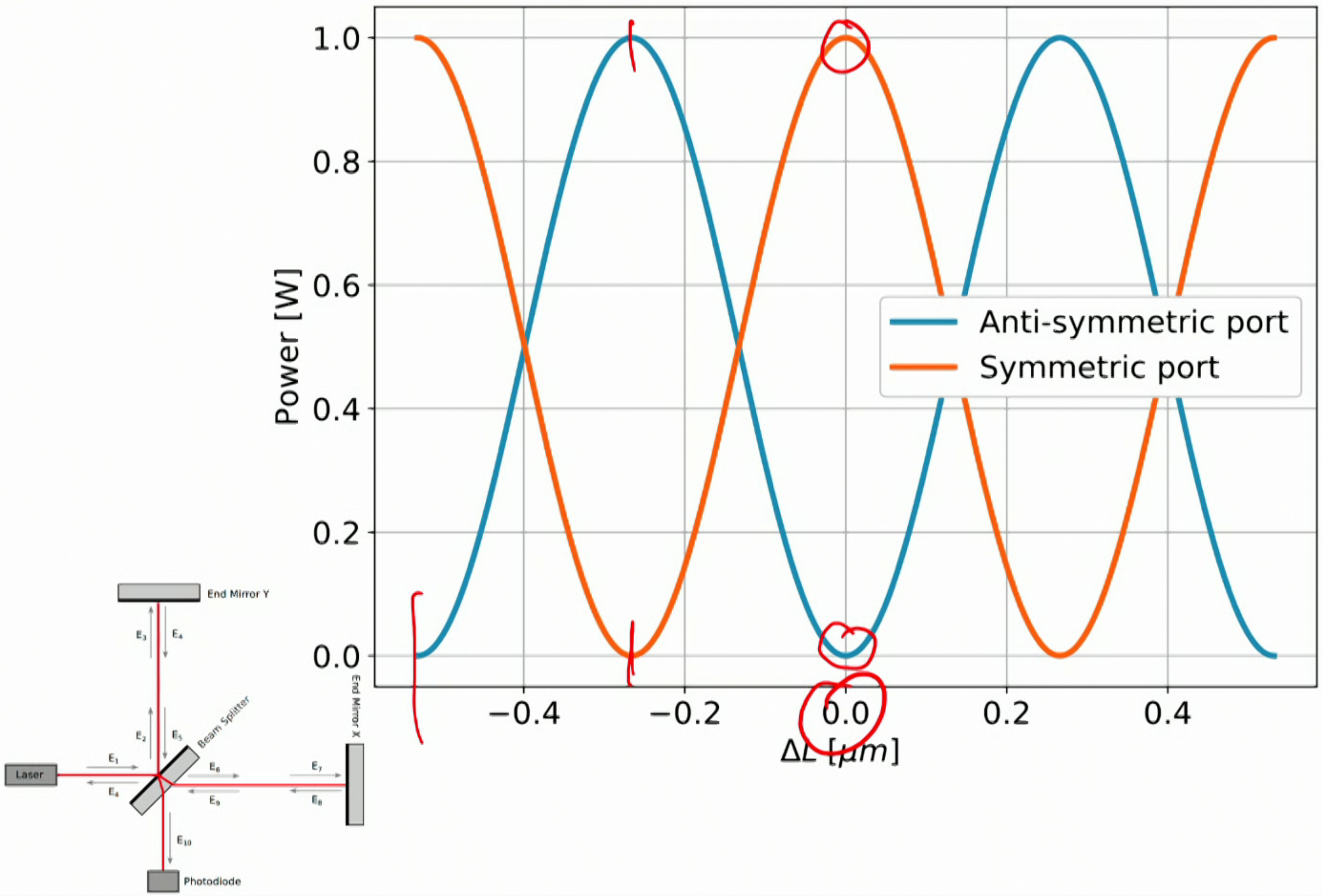
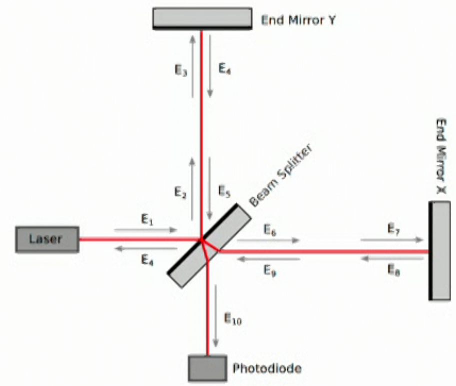


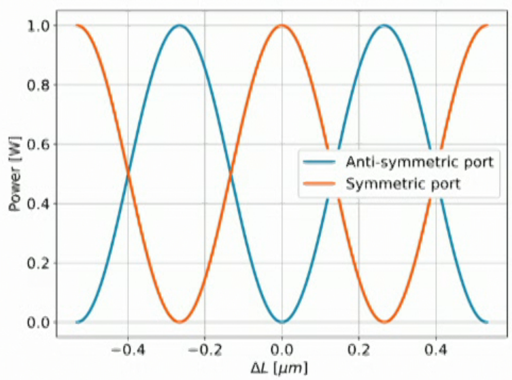


Michelson Interferometer



Response to GW signals

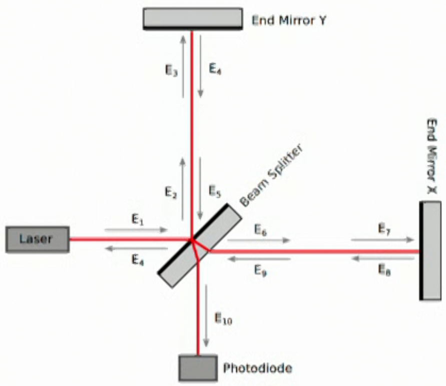


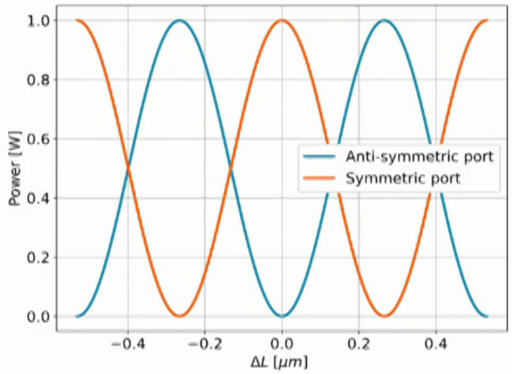


- We detect the power at the anti-symmetric port

$$P_A = \frac{P_i}{4}\left[r_X^2 + r_Y^2 - 2r_X r_Y \cos 2k\Delta L\right]$$

Response to GW signals





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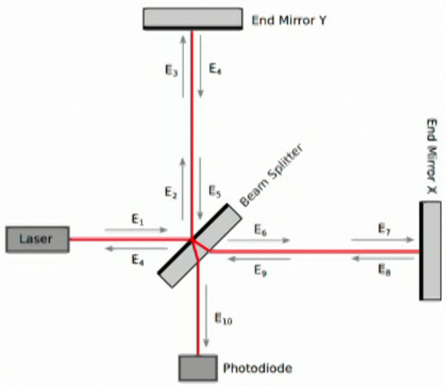
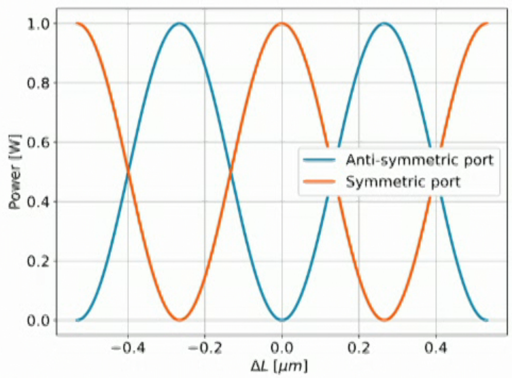
Response to GW signals





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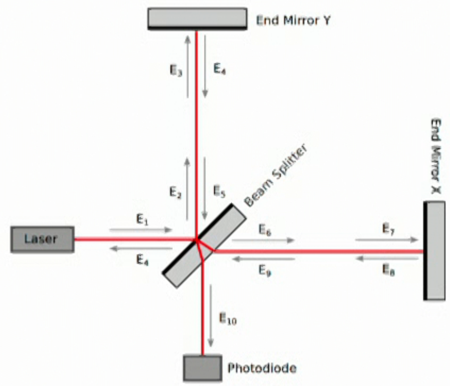

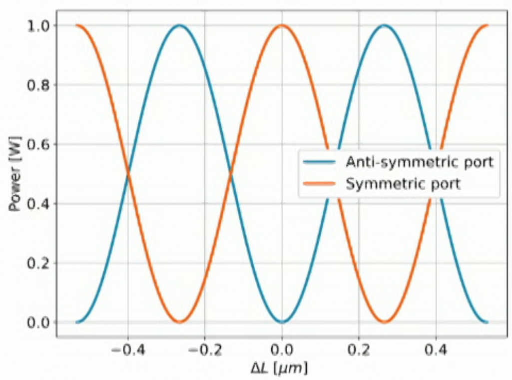

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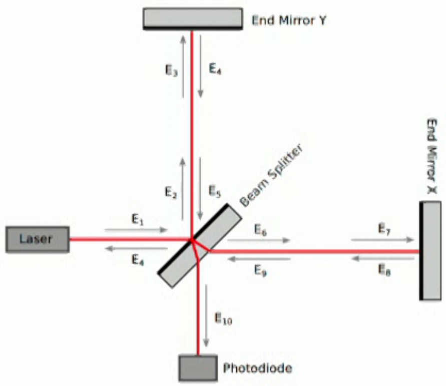
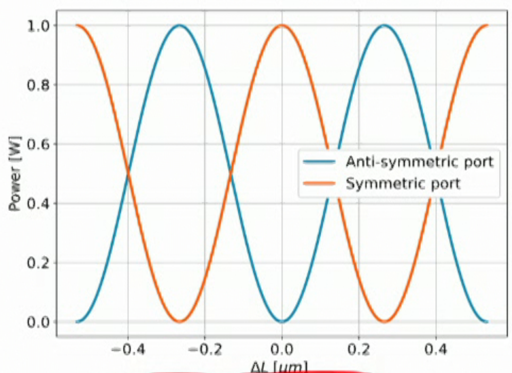




- We detect the power at the anti-symmetric port

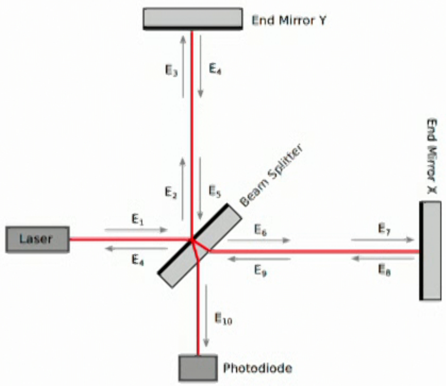
$$P_A = \frac{P_i}{4}\left[r_X^2 + r_Y^2 - 2r_X r_Y \cos 2k\Delta L\right]$$

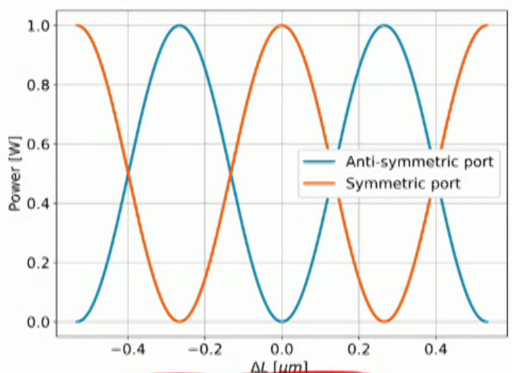
$$K = \frac{2\pi}{\lambda} = \frac{\omega_0}{c} \qquad \delta L = Lh$$

$$2k\Delta L = 2\frac{\omega_0 + \delta\omega}{c}(\Delta L_0 + \delta L)$$

$$2k\Delta L = 2\frac{\omega_0}{c}\Delta L_0 + 2\frac{\omega_0}{c}\delta L + 2\frac{\Delta L_0}{c}\delta\omega + 2\frac{\delta\omega\delta L}{c}$$

Response to GW signals





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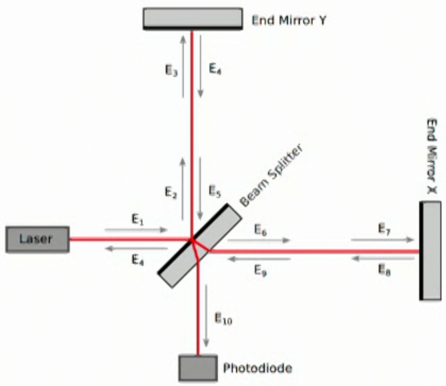

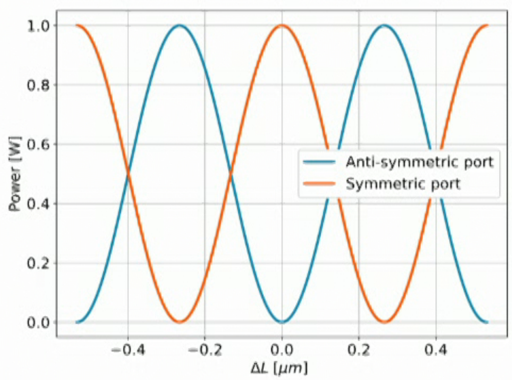

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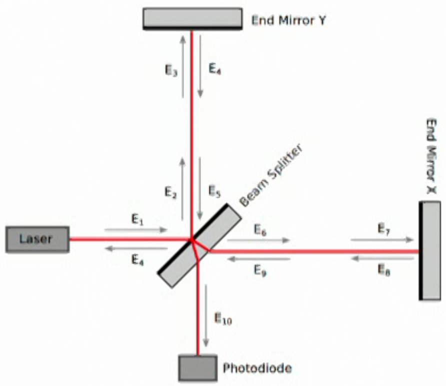

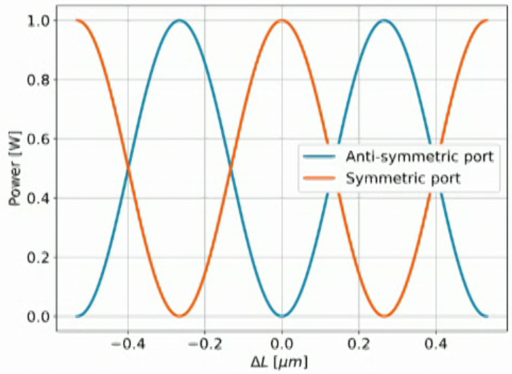

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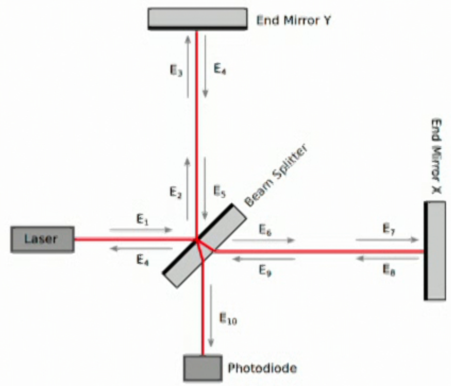
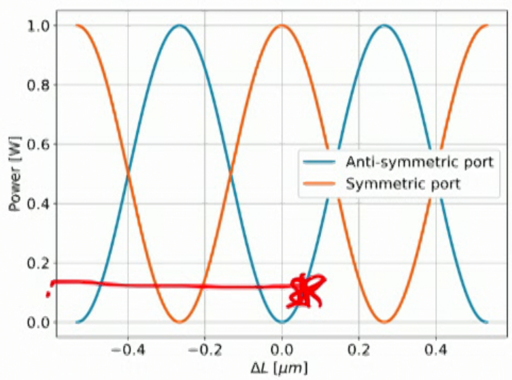
LIGO

Response to GW signals





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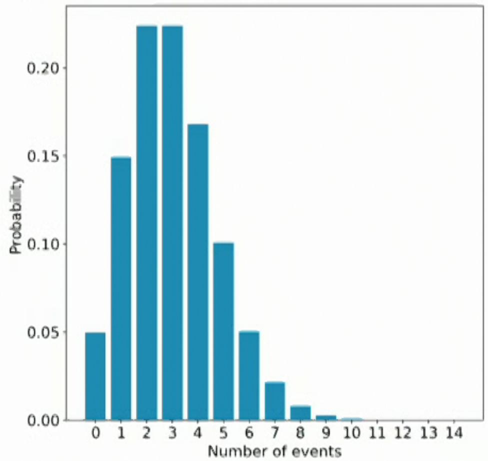
Shot noise

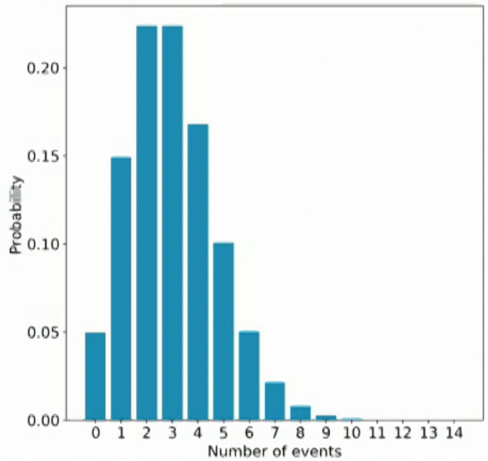
The Poisson distribution for an average number of events equal to 3



- Light is quantized
- Power is proportional to the number of photons
- Uncorrelated events: Poisson's distribution

Shot noise

The Poisson distribution for an average number of events equal to 3



- Light is quantized
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$$m = \mathcal{E}[n] = \frac{P\Delta t}{\hbar\omega}$$

$$\mathcal{E}[(P-\bar{P})^2\Delta t^2] = (\hbar\omega)^2\mathcal{E}[(n-\bar{n})^2]$$

$$= (\hbar\omega)^2\mathcal{E}[n] = \hbar\omega\bar{P}\Delta t$$

$$\mathcal{E}[(P(t_1)-\bar{P})(P(t_2)-\bar{P})\Delta t^2] =$$

$$= \mathcal{E}[(P-\bar{P})^2\Delta t^2]\delta(t_1-t_2)$$

$$= \hbar\omega\bar{P}\Delta t\delta(t_1-t_2)$$

$$S_P^{1/2}(\Omega) = \sqrt{2\hbar\omega\bar{P}}$$

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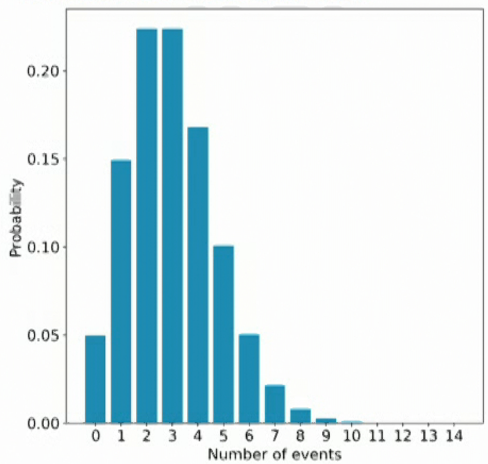

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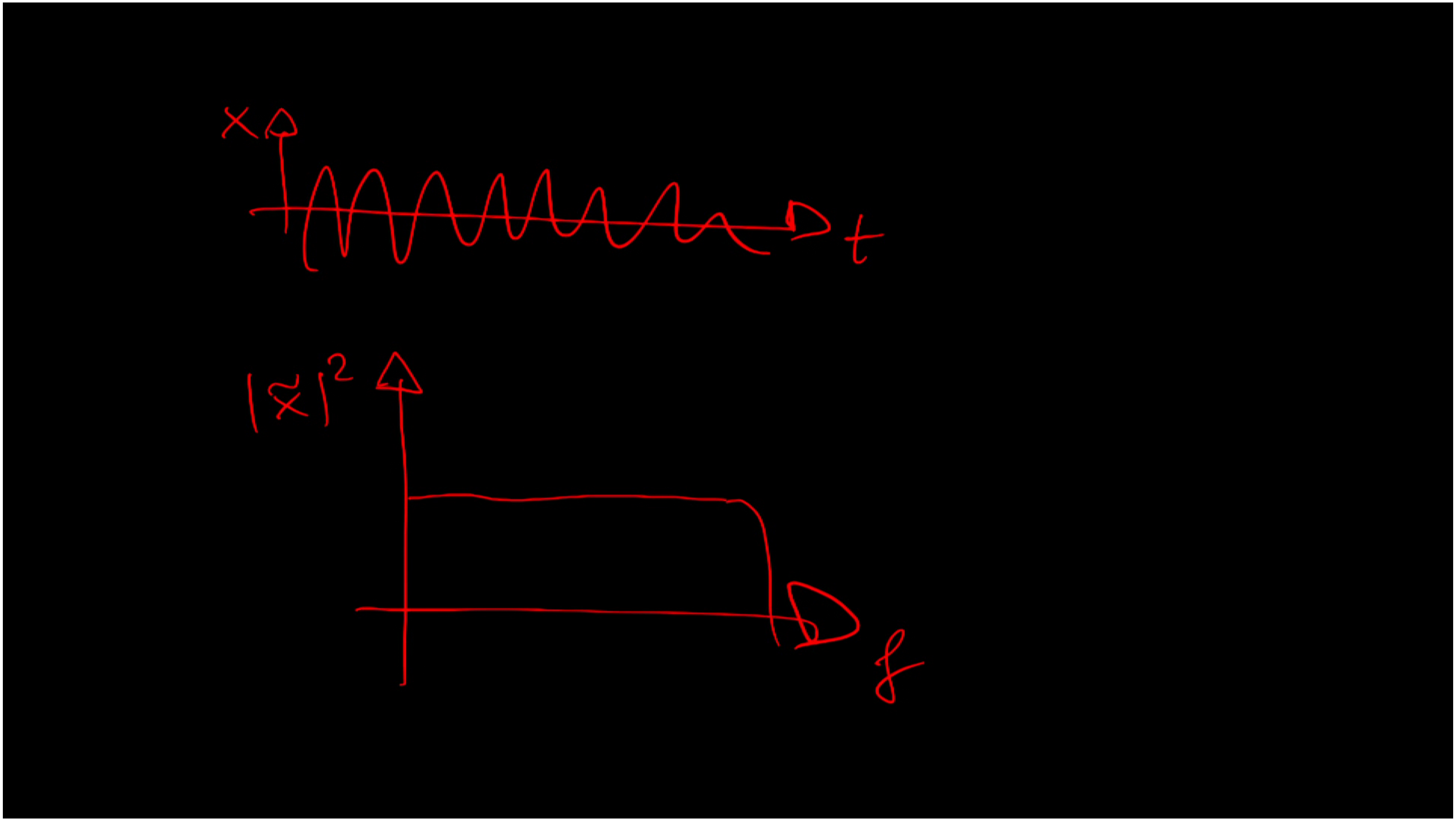
x
t
$|\tilde{x}|^2$
f

Signal to noise ratio

- Power at AS port: $$P_{A,0}(\delta L_0) = \frac{P_i}{4}\left[r_X^2 + r_Y^2 - 2r_x r_Y \cos 2k\delta L_0\right]$$
- Optical gain: $$G(\delta L_0) = \frac{4\pi L}{\lambda} r_X r_Y P_i \sin 2k\delta L_0$$

$$S_P^{1/2}(\Omega) = \left[\frac{4\pi L}{\lambda} r_X r_Y P_i \left|\sin 2k\delta L_0\right|\right] S_h^{1/2}(\Omega)$$

$$S_N^{1/2}(\Omega) = \sqrt{2\hbar\omega P_A} = \sqrt{2\hbar\omega \frac{P_i}{4}\left(r_X^2 + r_Y^2 - 2r_X r_Y \cos 2k\delta L_0\right)}$$

$$SNR = \frac{S_P^{1/2}}{S_N^{1/2}} = \sqrt{\frac{2\omega}{\hbar c^2}} \cdot \frac{L}{\sqrt{P_i}} \cdot \frac{r_X r_Y \sin 2k\delta L_0}{\sqrt{r_X^2 + r_Y^2 - 2r_X r_Y \cos 2k\delta L_0}} S_h^{1/2}$$

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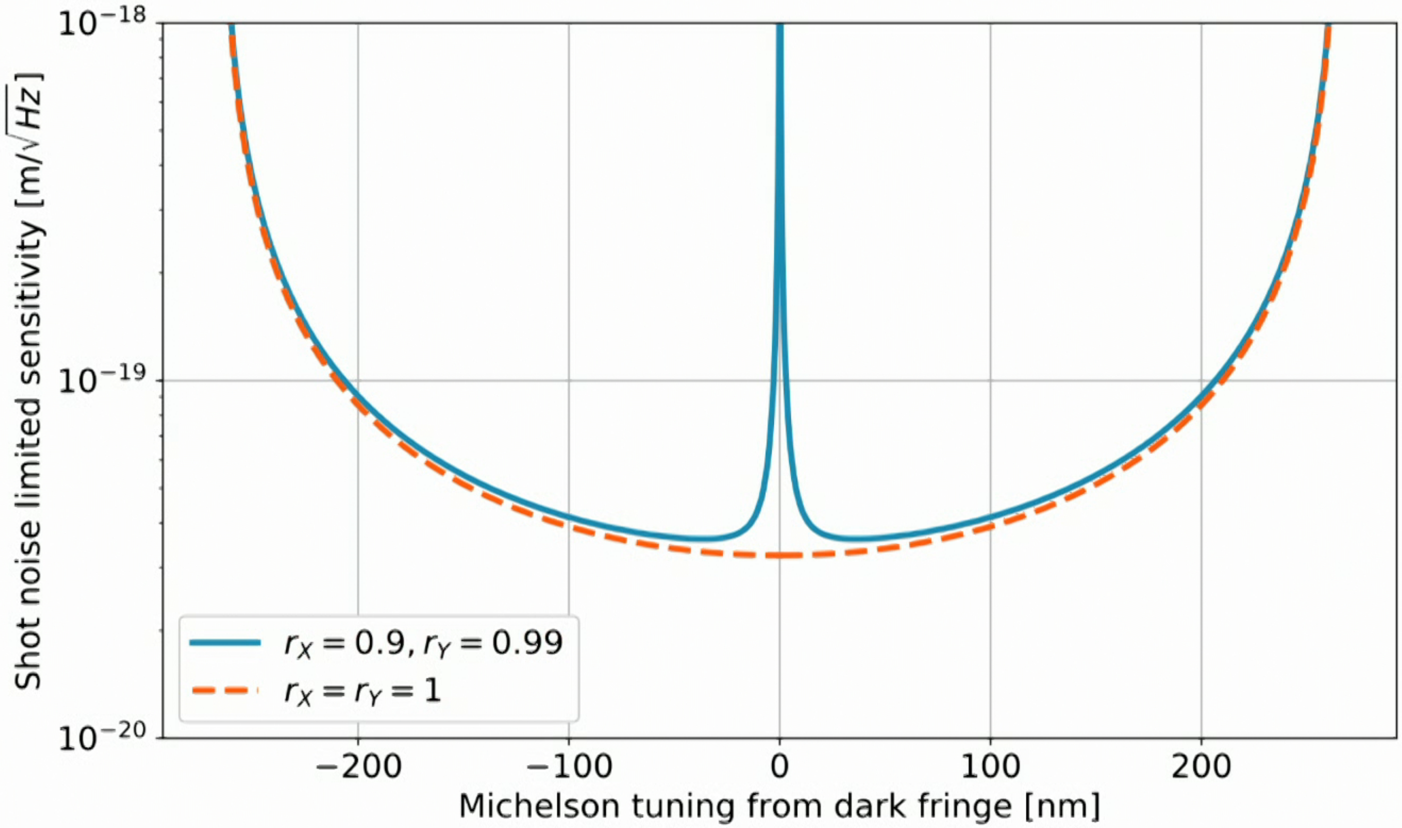

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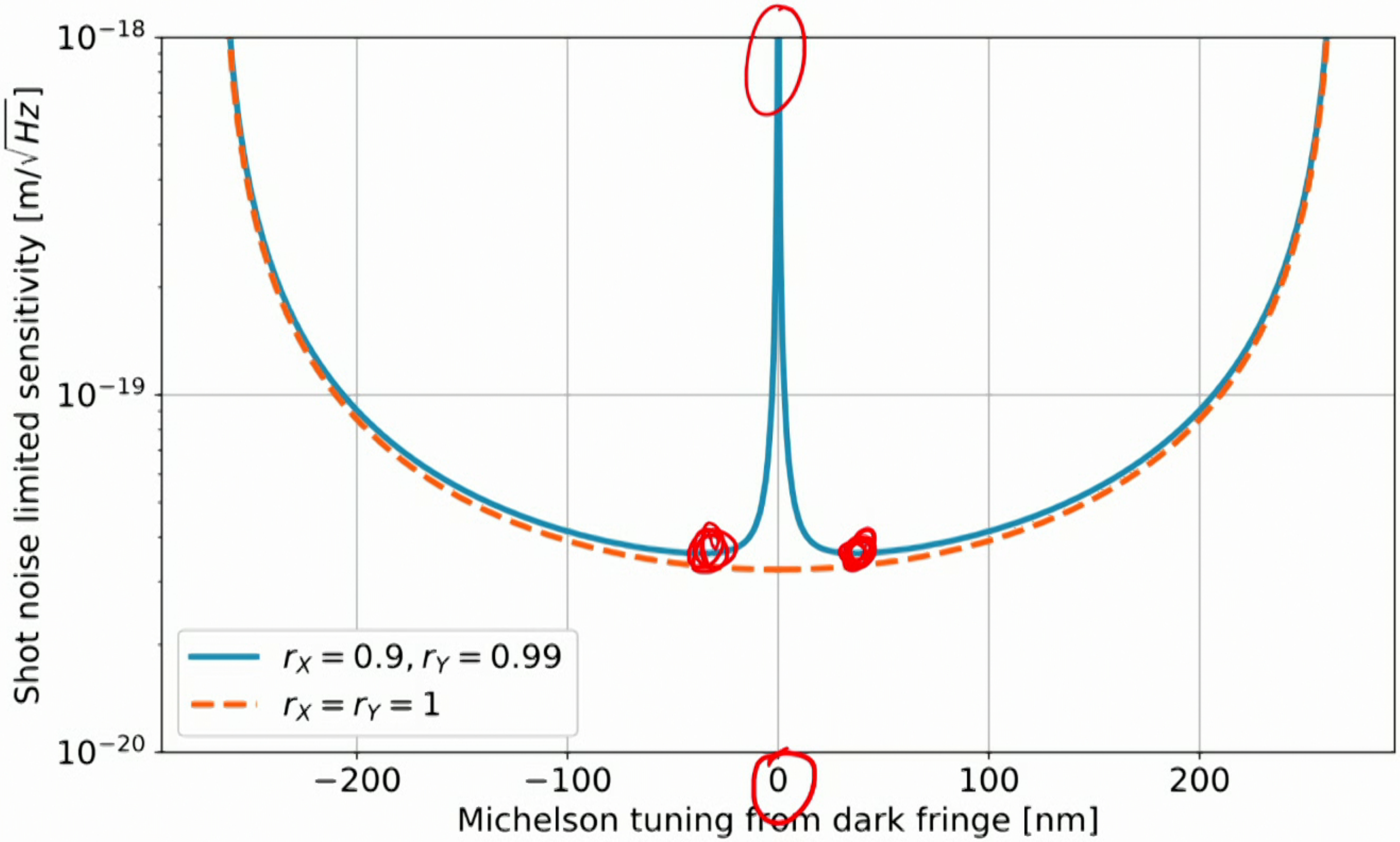
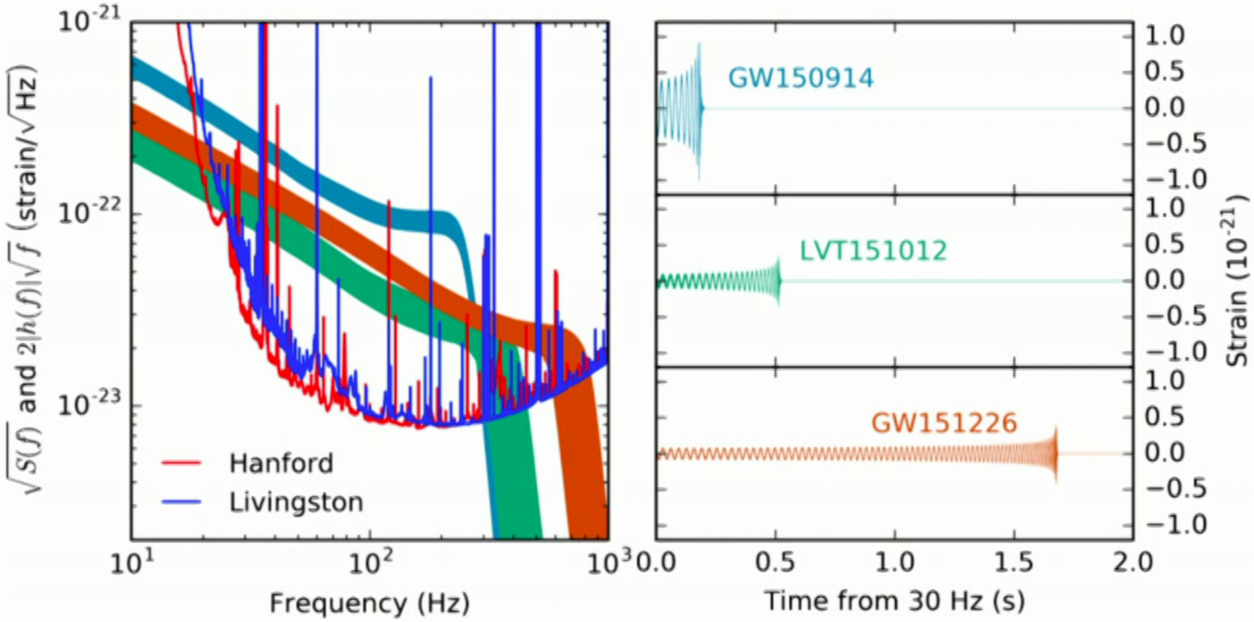


How to improve the sensitivity?

$$S_h^{1/2} > \sqrt{\frac{\hbar c^2}{\omega}} \frac{1}{L\sqrt{P_i}} \simeq 1.3 \times 10^{-20} \mathrm{Hz}^{-1/2}$$



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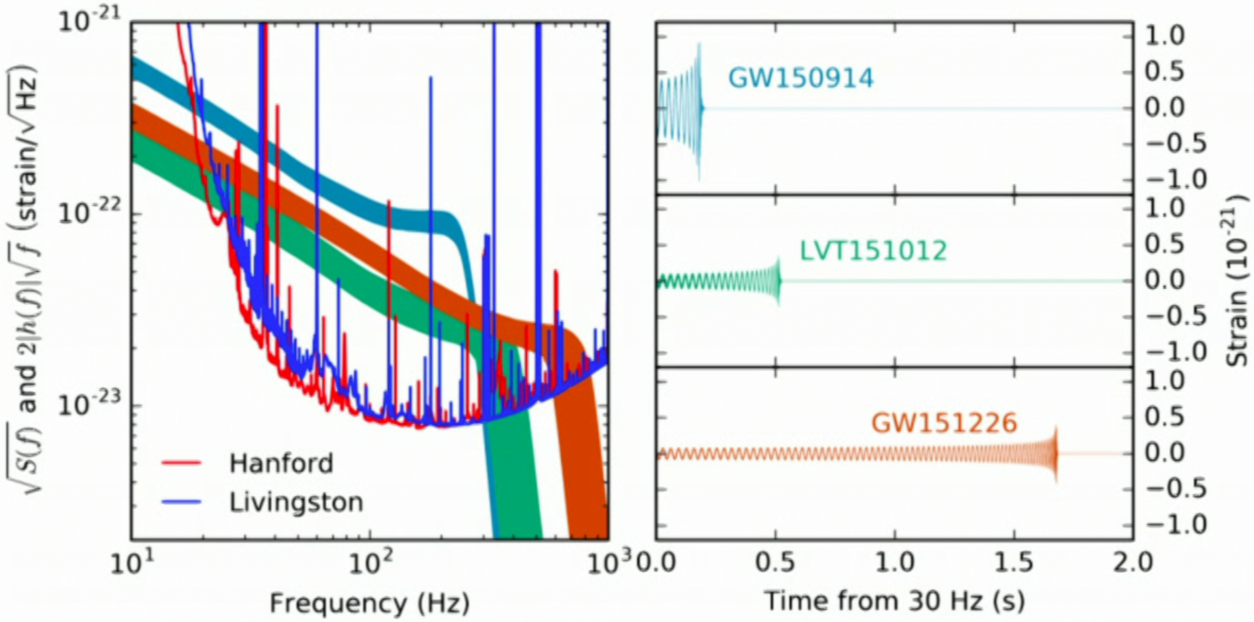
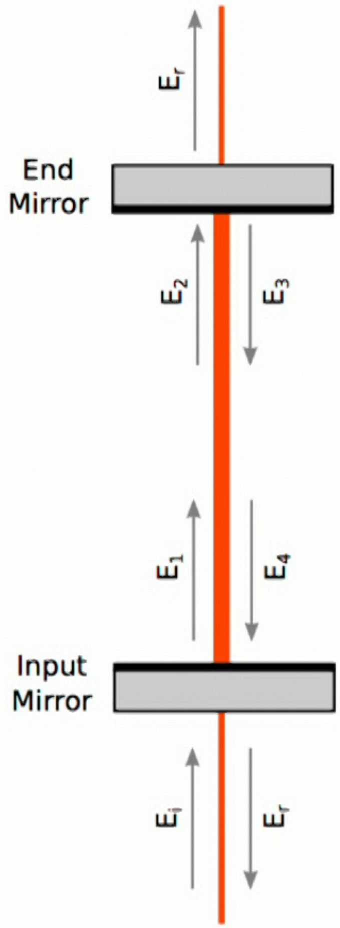
4 km

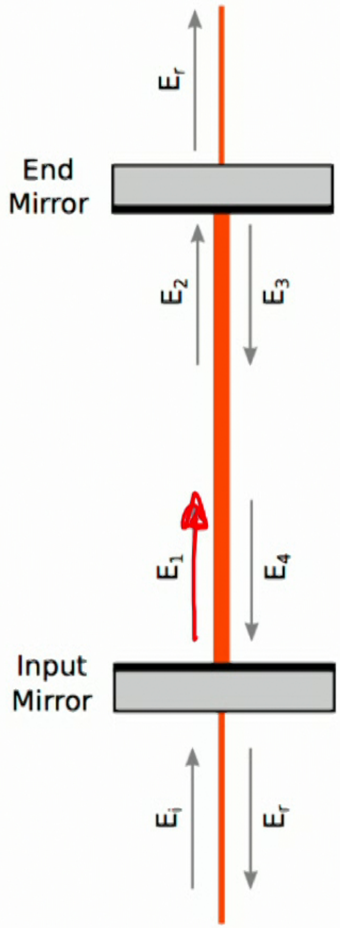
1 W



Fabry-Perot resonant cavities



Fabry-Perot resonant cavities



$$
\begin{aligned}
E_1 &= t_i E_i + r_i E_4 \\
E_4 &= e^{2ikL} r_e E_1 \\
E_r &= t_i E_4 - r_i E_i \\
E_t &= e^{ikL} t_e E_1
\end{aligned}
$$

$$
\begin{aligned}
E_1 &= \frac{t_i}{1 - r_i r_e e^{2ikL}} E_i \\
E_t &= \frac{t_i t_e e^{ikL}}{1 - r_i r_e e^{2ikL}} E_i \\
E_r &= -\frac{r_i - r_e (r_i^2 + t_i^2) e^{2ikL}}{1 - r_i r_e e^{2ikL}} E_i
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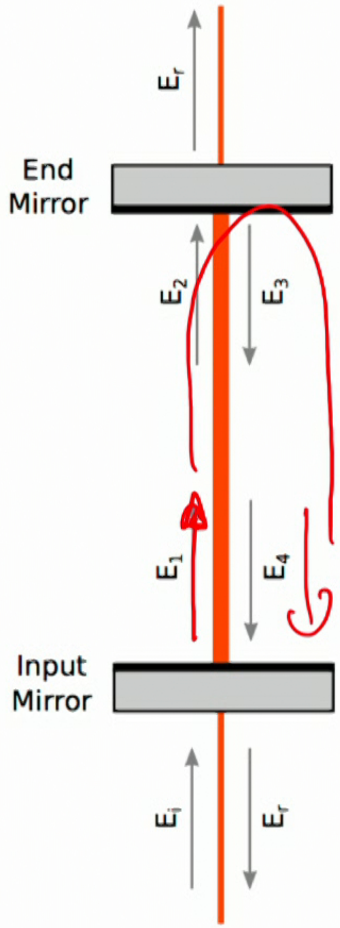
$$

Fabry-Perot resonant cavities



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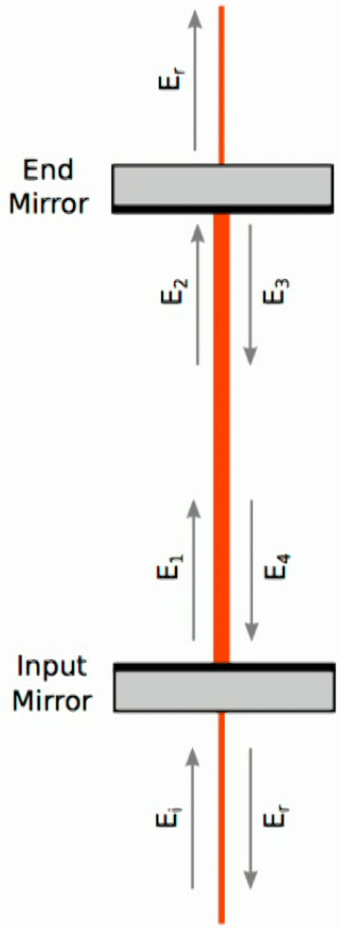
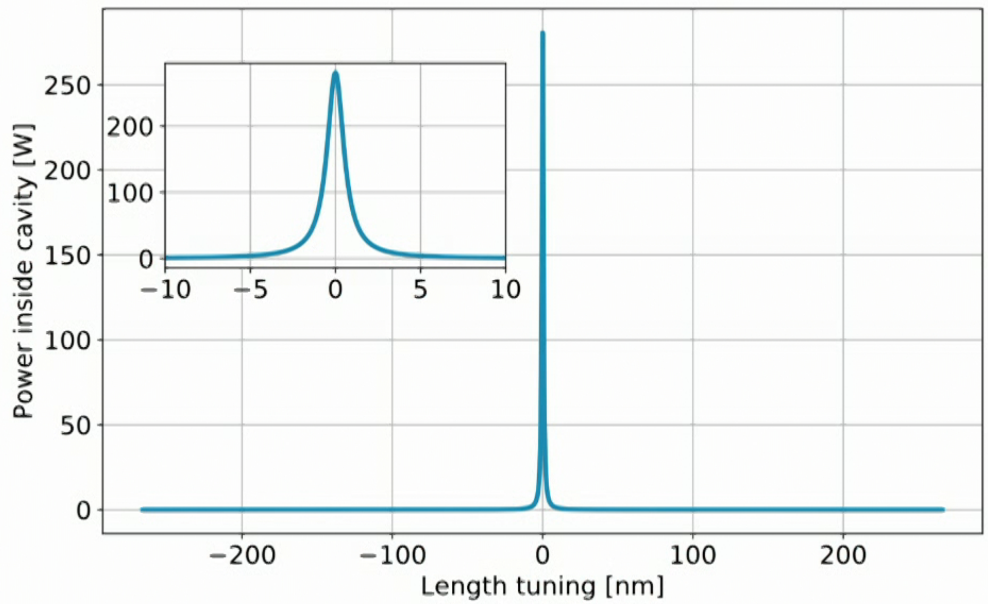
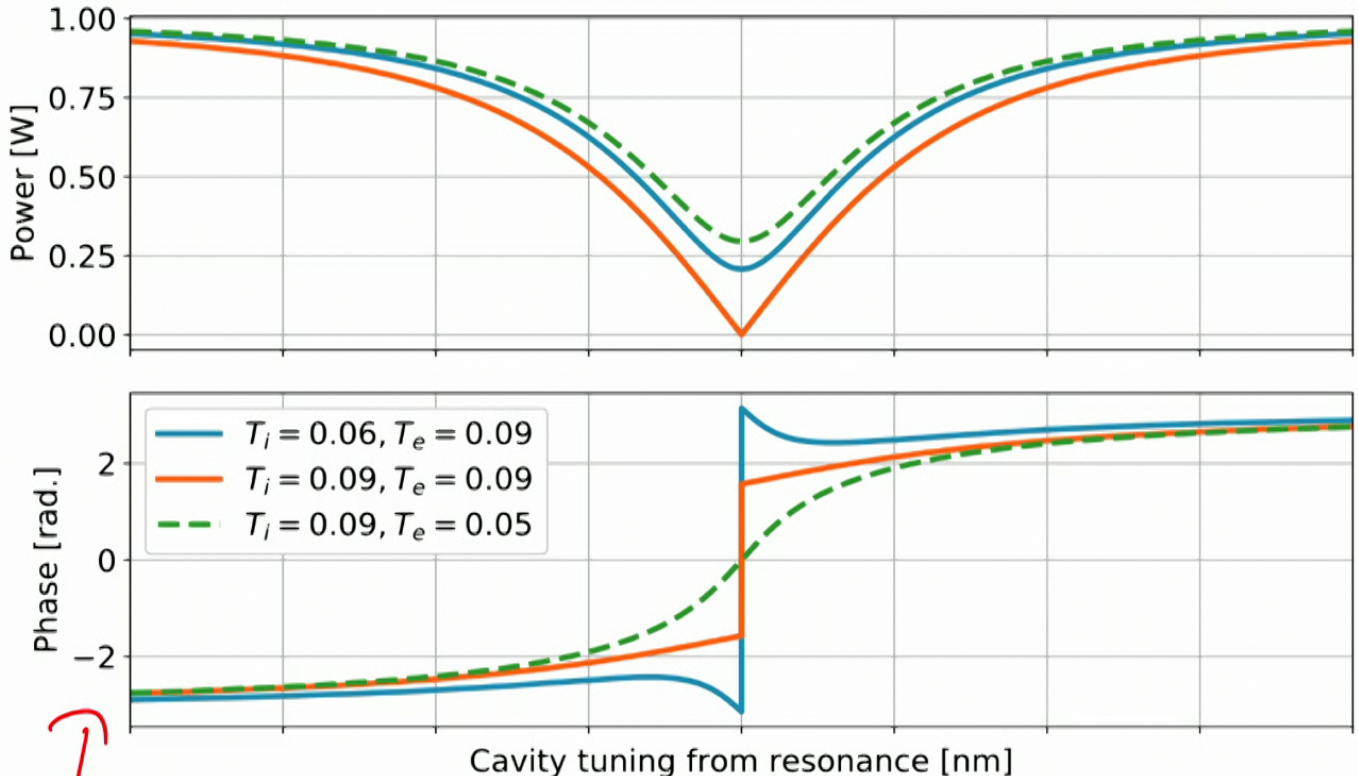


$$P_1 = \frac{t_i^2}{(1 - r_i r_e)^2} \frac{1}{1 + \frac{4 r_i r_e}{(1 - r_i r_e)^2} \sin^2 k\delta L} P_i$$

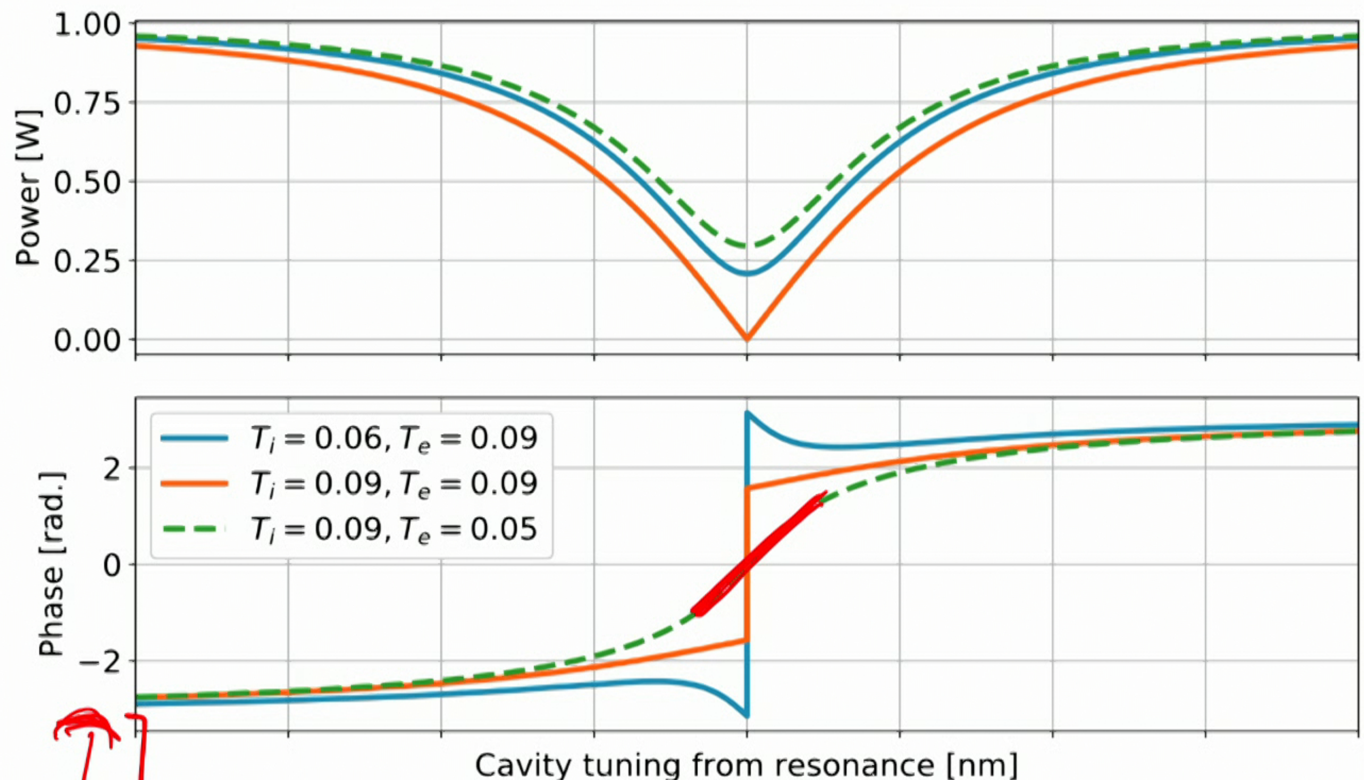


Fabry-Perot resonant cavities



$$\phi_{FP}(\delta L) = 2k\frac{2}{1 - r_i}\delta L = 2k\frac{2\mathcal{F}}{\pi}\delta L$$

Fabry-Perot resonant cavities



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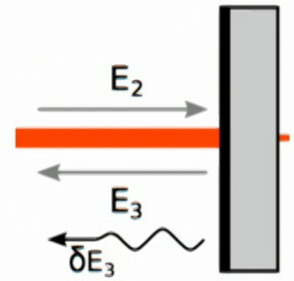
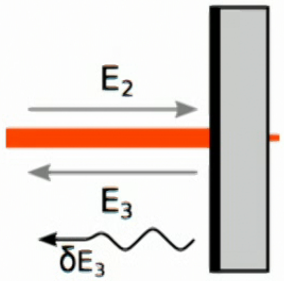
Frequency response



Frequency response

$$E_L = E_0 e^{iKL - i\omega t + ikz_0 \cos \Omega t}$$
$$= E_0 \left[e^{-i\omega t} + \frac{ikz_0}{2} \left(e^{-i(\omega+\Omega)t} + e^{-i(\omega-\Omega)t} \right) \right]$$



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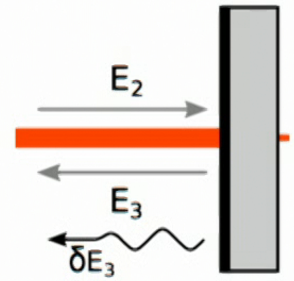
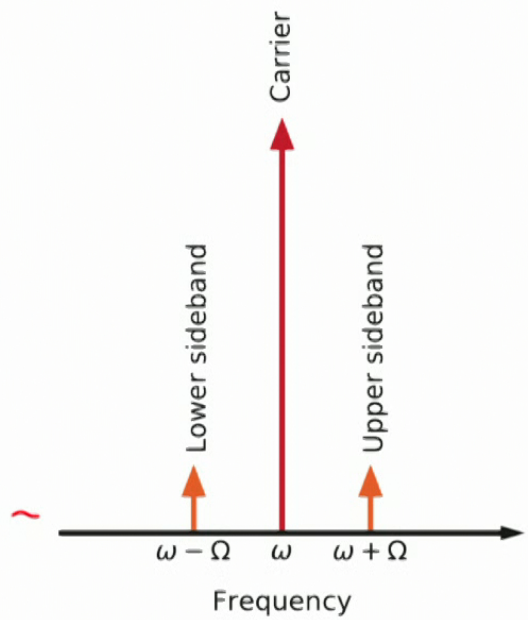
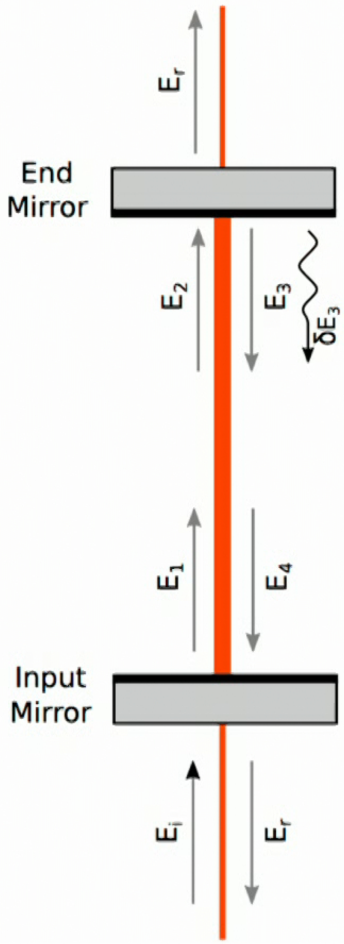
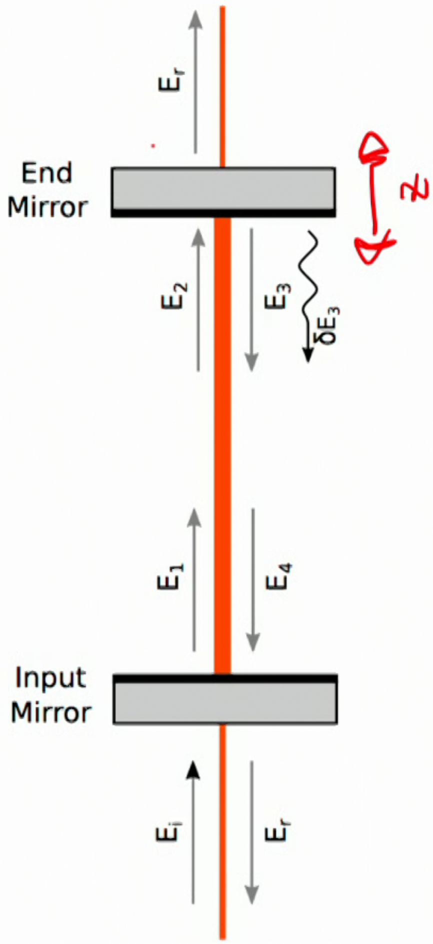




Frequency response



Frequency response



$$
\begin{aligned}
E_3 &= r_e E_2 e^{2ikz_0 \cos\Omega t} \simeq r_e E_2 \left[1 + ikz_0 \left(e^{i\Omega t} + e^{-i\Omega t}\right)\right] \\
&= E_3 + \delta E_3(\Omega) + \delta E_3(-\Omega)
\end{aligned}
$$

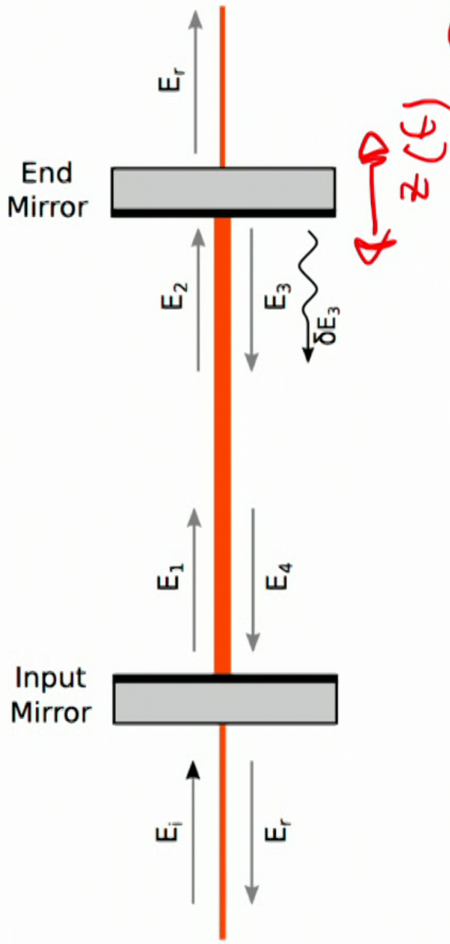
$$E_3(\Omega) = \frac{\delta E_3(\Omega)}{1 - r_i r_e e^{2ikL - 2i\frac{\Omega}{c}L}}$$

$$E_r(\Omega) = \frac{t_i E_i}{1 - r_i r_e} \frac{r_e t_i e^{-i\frac{\Omega}{c}L}}{1 - r_i r_e e^{-2i\frac{\Omega}{c}L}} ikz_0$$

$$E_r(\Omega) = ikz_0 r_e E_i \frac{2\mathcal{F}}{\pi} \frac{1}{1 - i\frac{\Omega}{2\pi\frac{c}{4\mathcal{F}L}}}$$

LIGO

Frequency response



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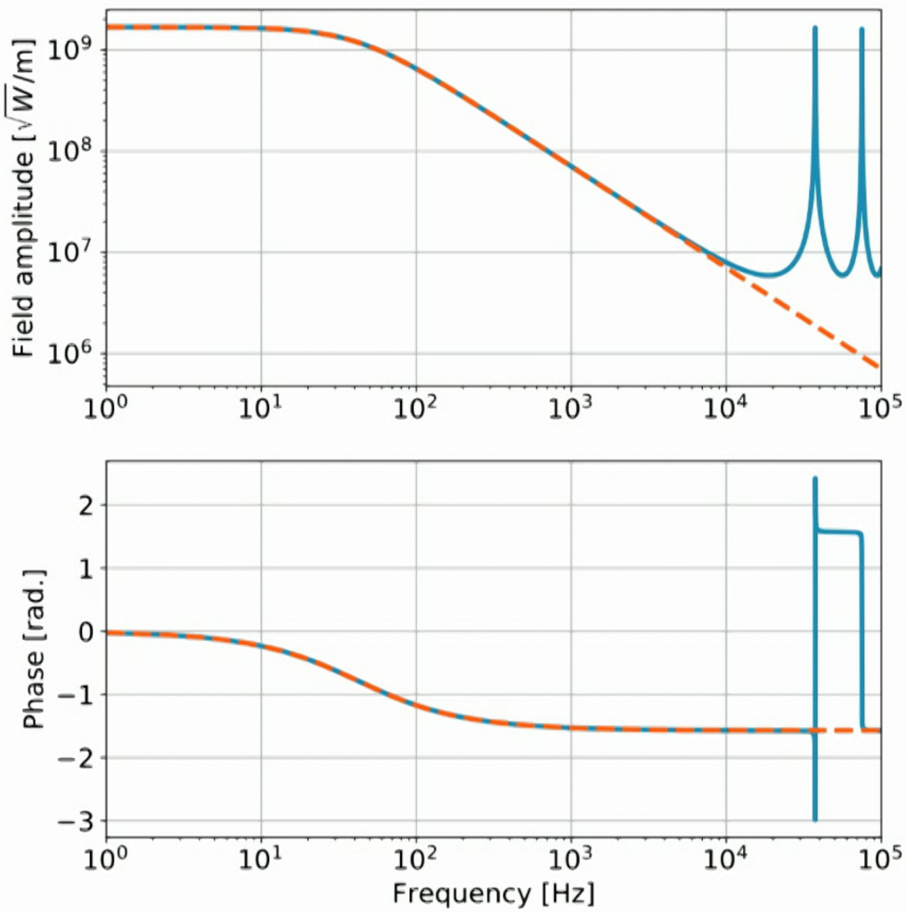
$$= E_3 + \delta E_3(\Omega) + \delta E_3(-\Omega)$$

$$E_3(\Omega) = \frac{\delta E_3(\Omega)}{1 - r_i r_e e^{2ikL - 2i\frac{\Omega}{c}L}}$$

$$E_r(\Omega) = \frac{t_i E_i}{1 - r_i r_e} \frac{r_e t_i e^{-i\frac{\Omega}{c}L}}{1 - r_i r_e e^{-2i\frac{\Omega}{c}L}} ikz_0$$

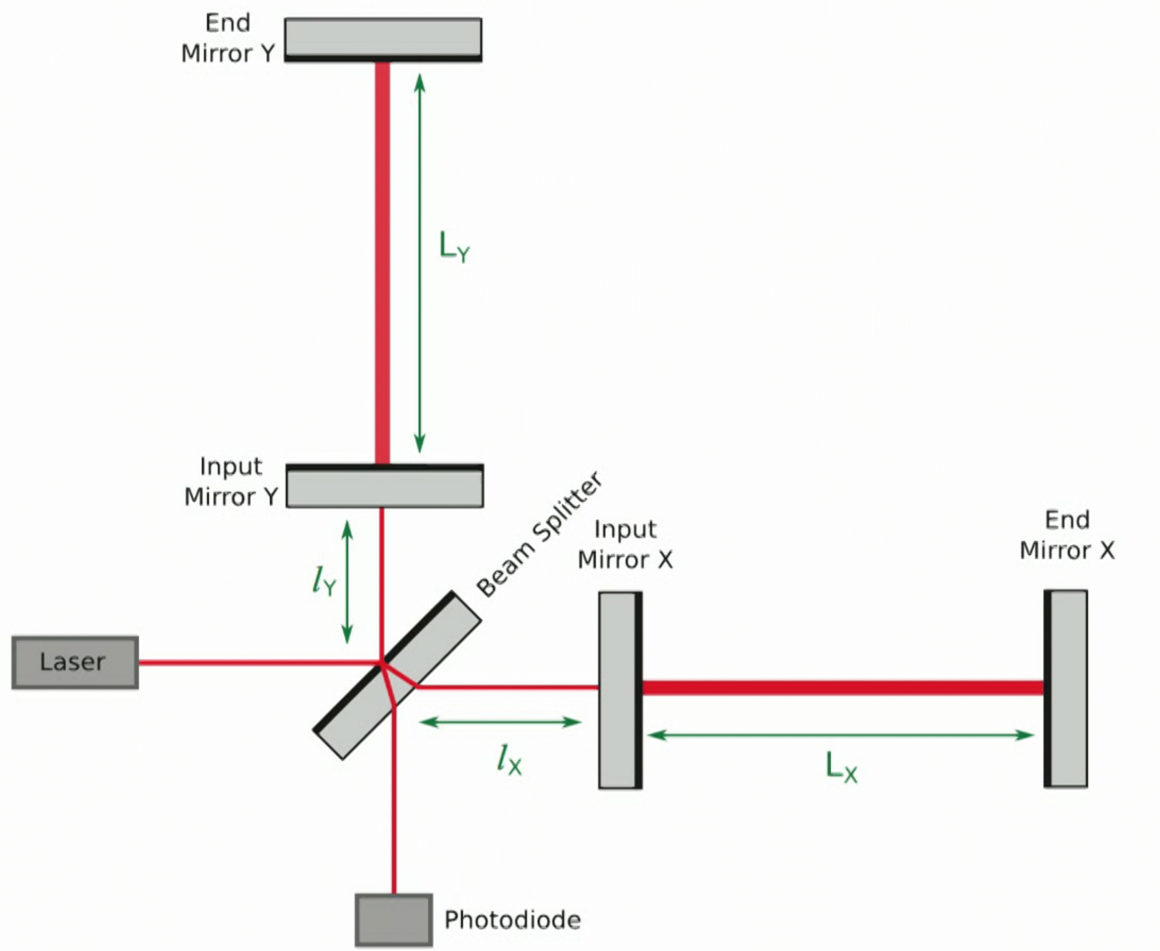
$$E_r(\Omega) = ikz_0 r_e E_i \frac{2\mathcal{F}}{\pi} \frac{1}{1 - i\frac{\Omega}{2\pi \frac{c}{4\mathcal{F}L}}}$$

Cavity pole



$$E_r(\Omega) = ikz_0 r_e E_i \frac{2\mathcal{F}}{\pi} \frac{1}{1 - i\frac{\Omega}{2\pi\frac{c}{4\mathcal{F}L}}}$$

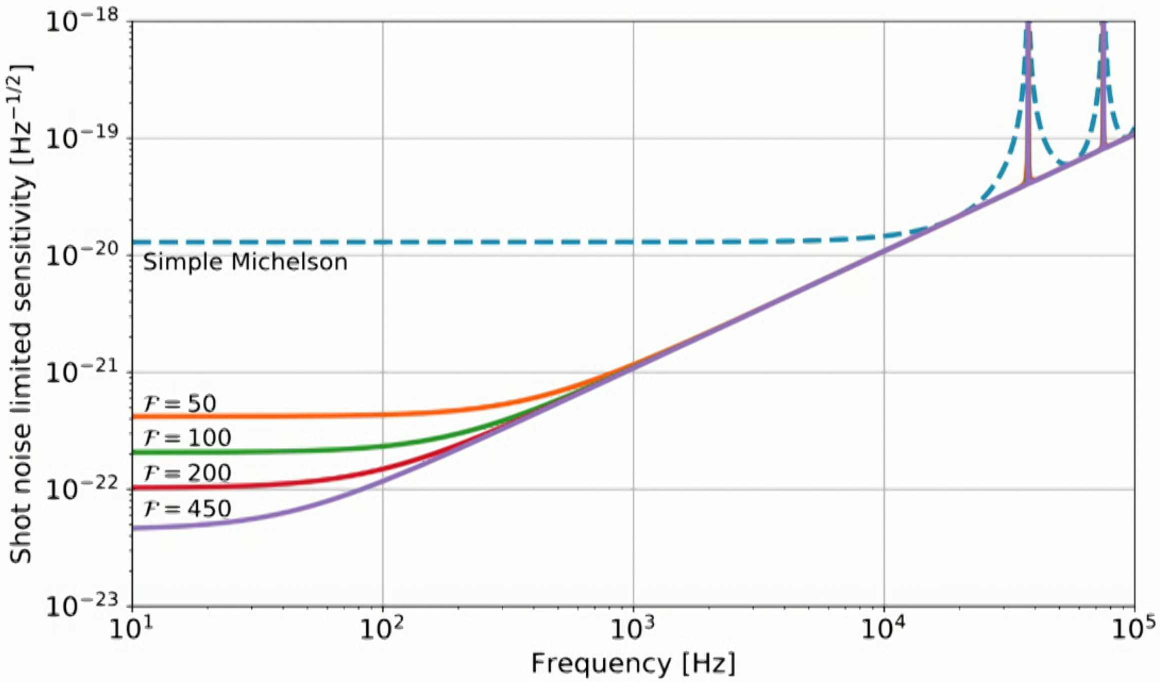
Fabry-Perot Michelson



Fabry-Perot Michelson

$$\phi_{space} = kLh \qquad \phi_{FP} = kLhr_e \frac{2\mathcal{F}}{\pi} \frac{1}{1 - i\frac{\Omega}{2\pi \frac{c}{4\mathcal{F}L}}}$$

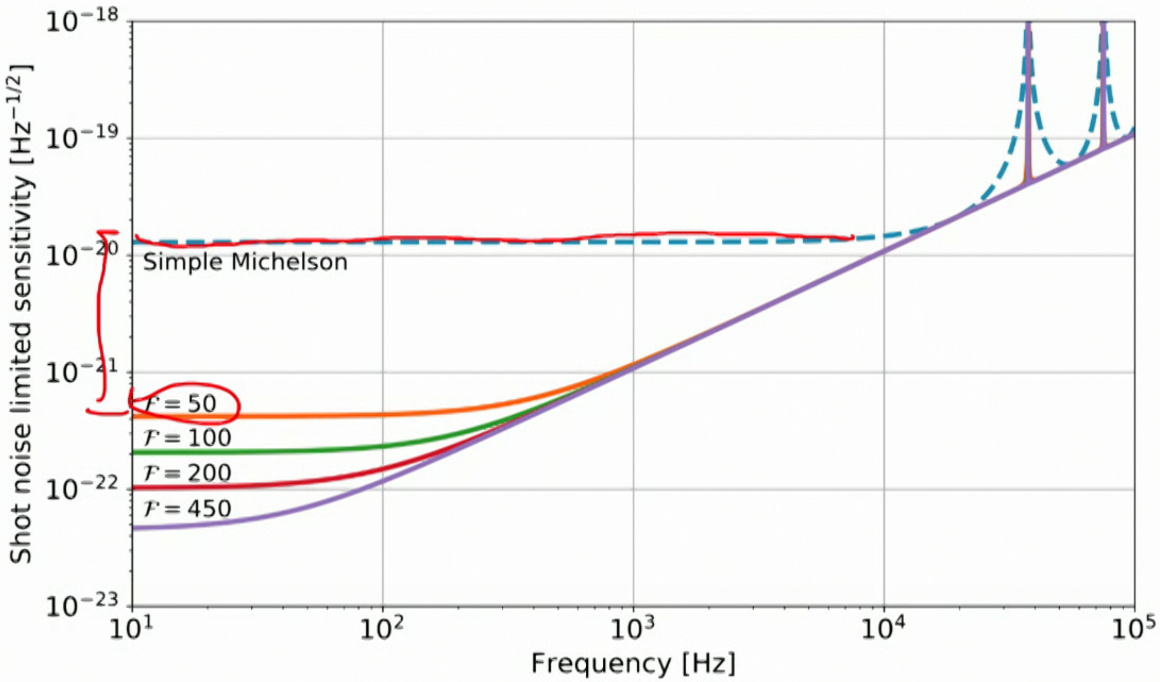
$$G(\delta L_0) = \frac{8\pi \mathcal{F} L}{\lambda} \frac{1}{1 - i\frac{\Omega}{2\pi \frac{c}{4\mathcal{F}L}}} r_X r_Y P_i \sin 2k\delta l$$



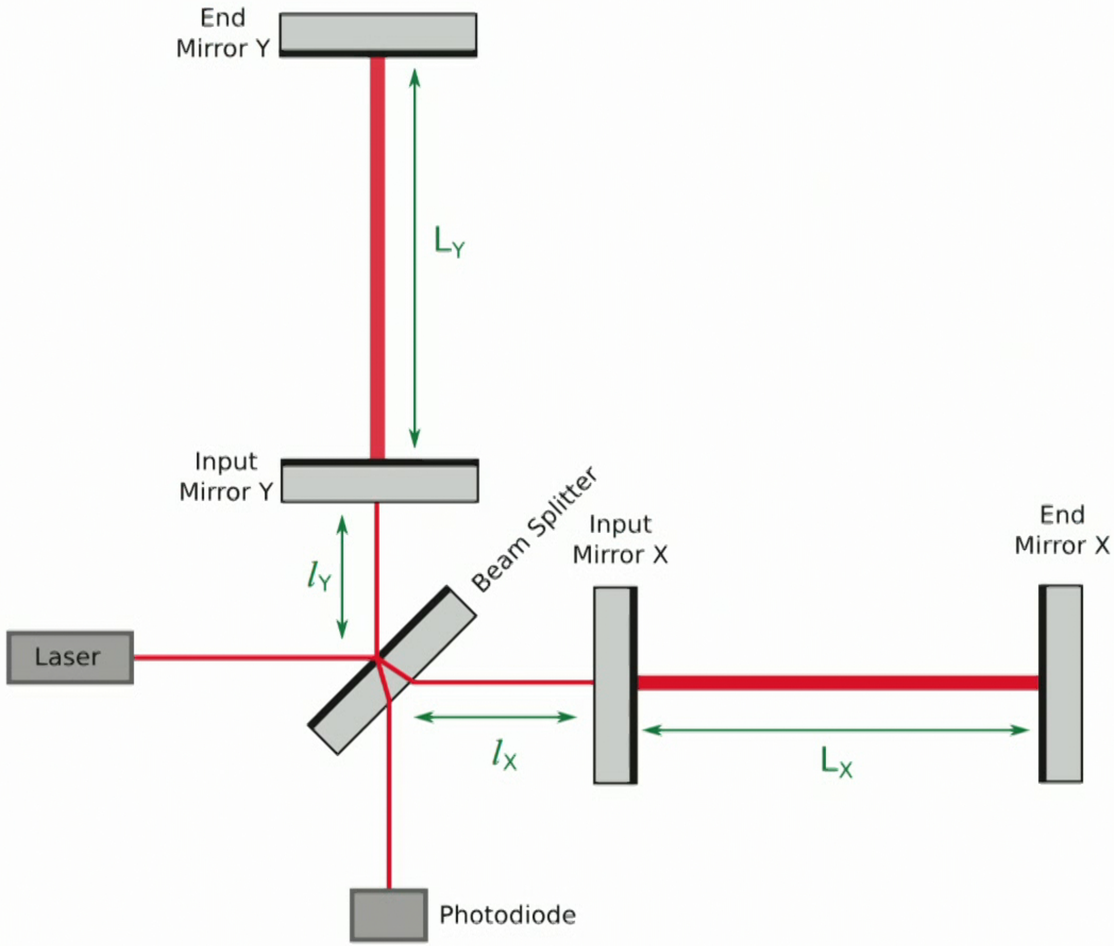
Fabry-Perot Michelson

$$\phi_{space} = kLh \qquad \phi_{FP} = kLhr_e \frac{2\mathcal{F}}{\pi} \frac{1}{1 - i\frac{\Omega}{2\pi \frac{c}{4\mathcal{F}L}}}$$

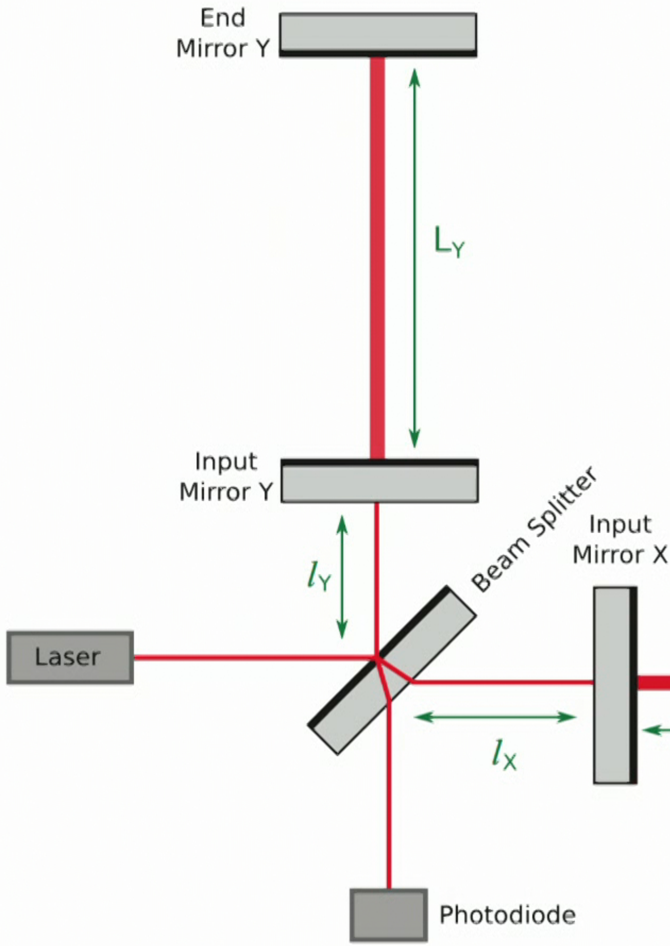
$$G(\delta L_0) = \frac{8\pi\mathcal{F}L}{\lambda} \frac{1}{1 - i\frac{\Omega}{2\pi \frac{c}{4\mathcal{F}L}}} r_X r_Y P_i \sin 2k\delta l$$



Power recycling



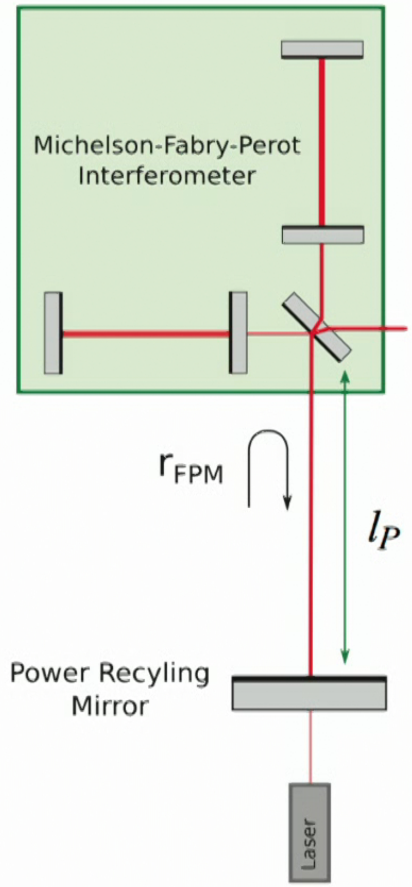
Power recycling



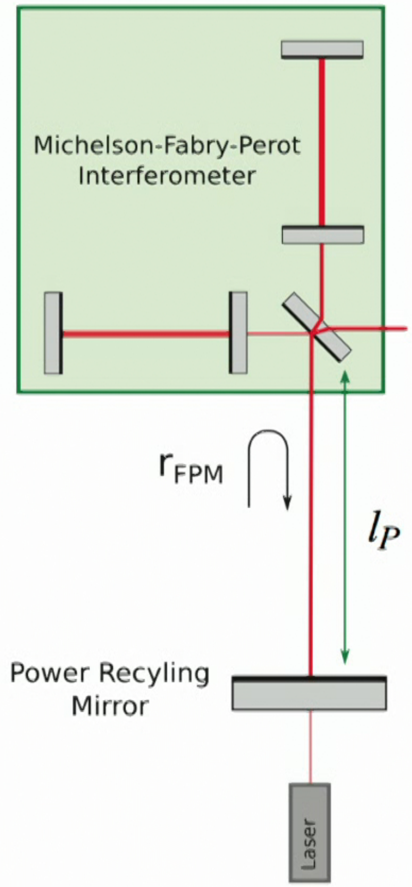
$$r_{FP} = -1 + \frac{\mathcal{F}}{\pi}(L_i + L_e)$$

$$r_{FPM} = -e^{2ikl}\left(1 - \frac{\mathcal{F}}{\pi}L_{RT}\right)$$

Power recycling



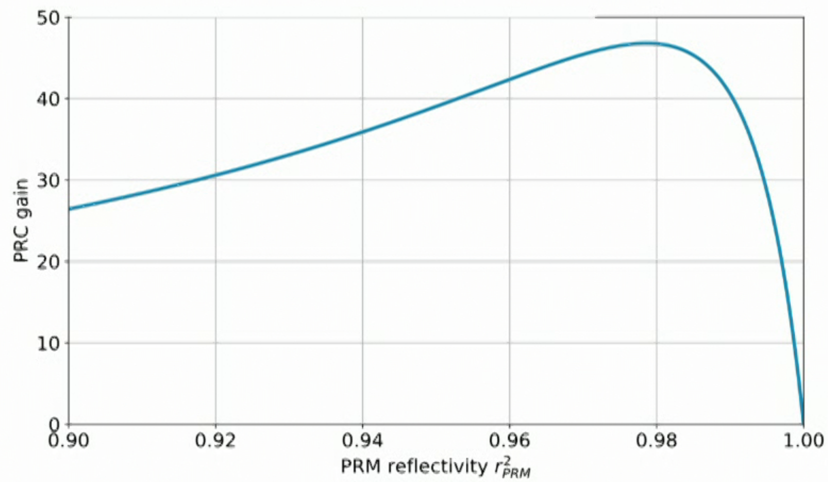
Power recycling



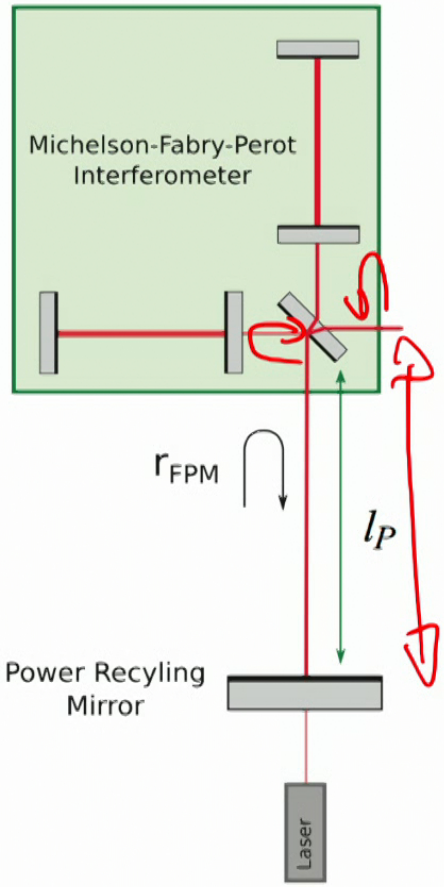
$$E_{PRC} = \frac{t_p}{1 - r_p r_{FPM} e^{2ikl_P + 2ikl}} E_{in}$$

$$l_{PRC} = l_P + \frac{l_X + l_Y}{2}$$

$$P_{PRC} = \frac{1 - r_p^2}{\left[1 - r_p \left(1 - \frac{\mathcal{F}}{\pi} L_{RT}\right)\right]^2} P_{in}$$



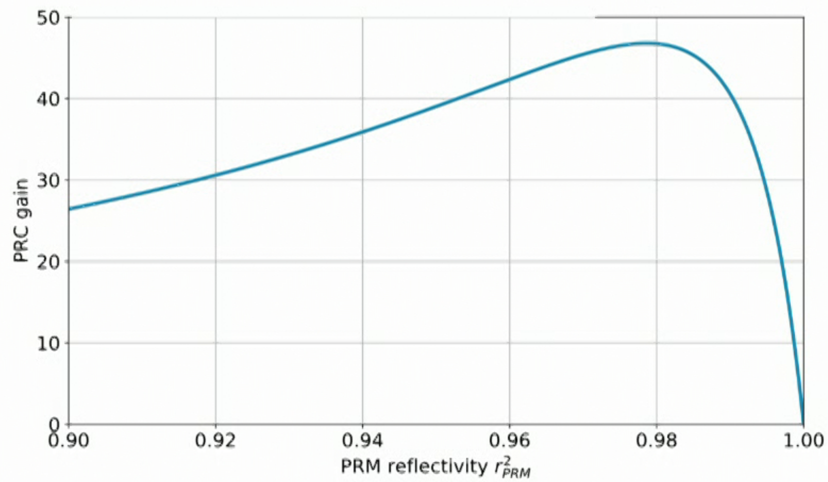
Power recycling



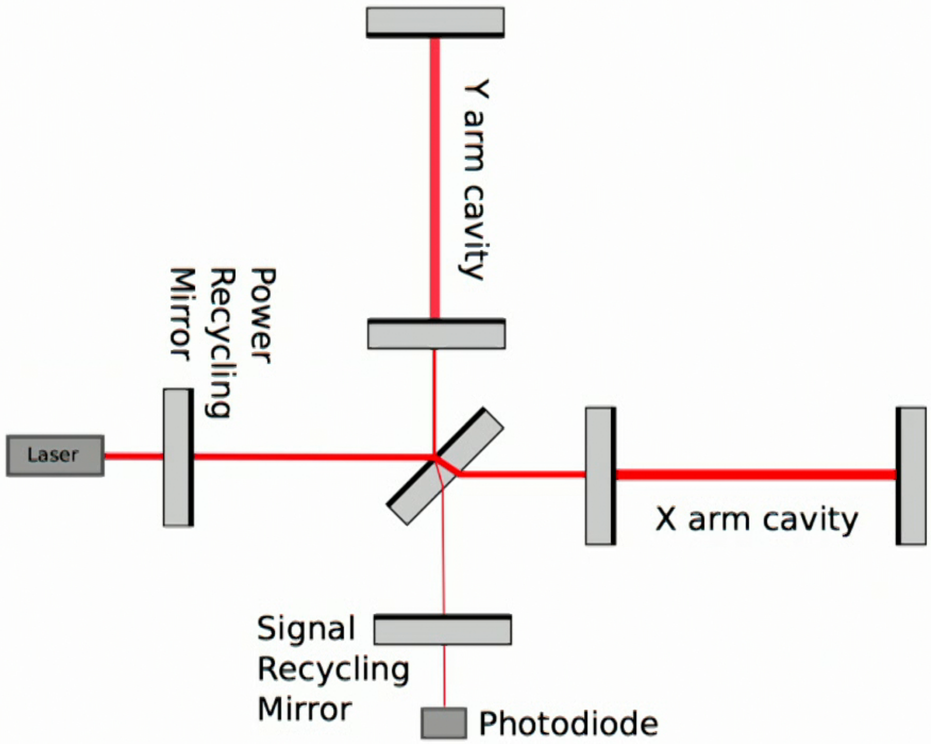
$$E_{PRC} = \frac{t_p}{1 - r_p r_{FPM} e^{2ikl_P + 2ikl}} E_{in}$$

$$l_{PRC} = l_P + \frac{l_X + l_Y}{2}$$

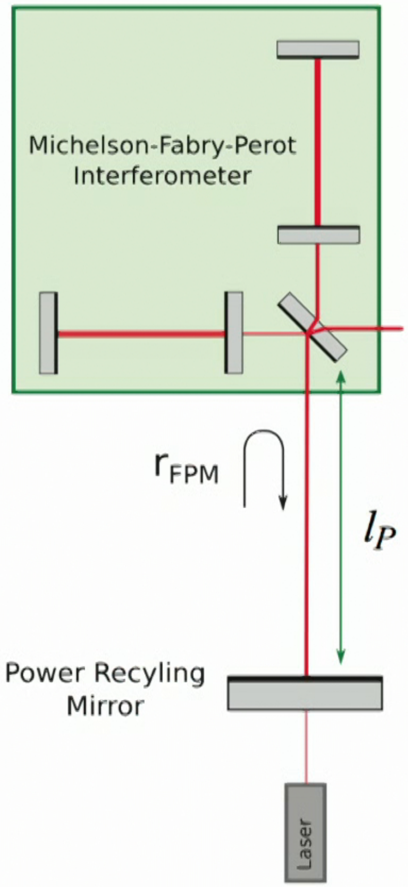
$$P_{PRC} = \frac{1 - r_p^2}{\left[1 - r_p\left(1 - \frac{\mathcal{F}}{\pi} L_{RT}\right)\right]^2} P_{in}$$

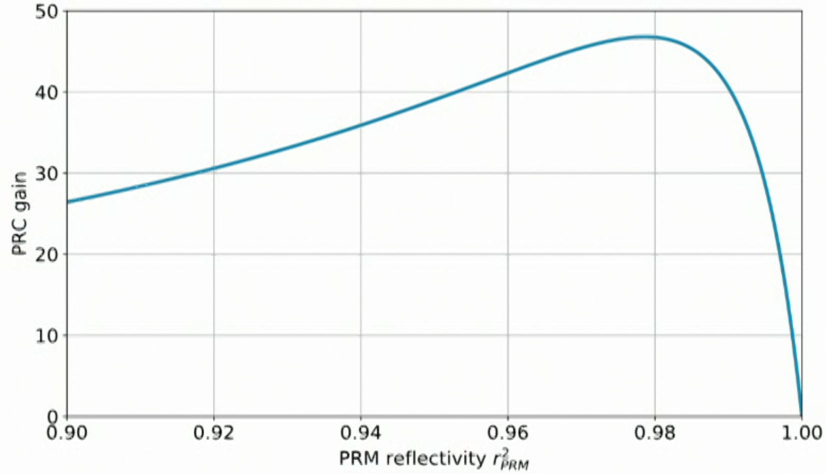


Signal recycling



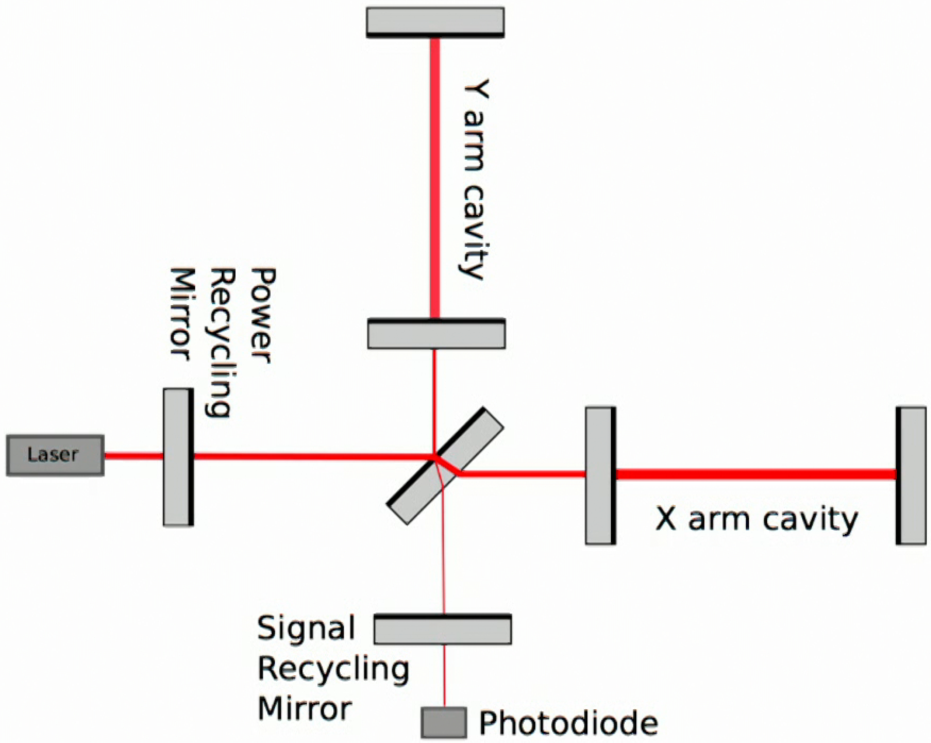
Power Recycling



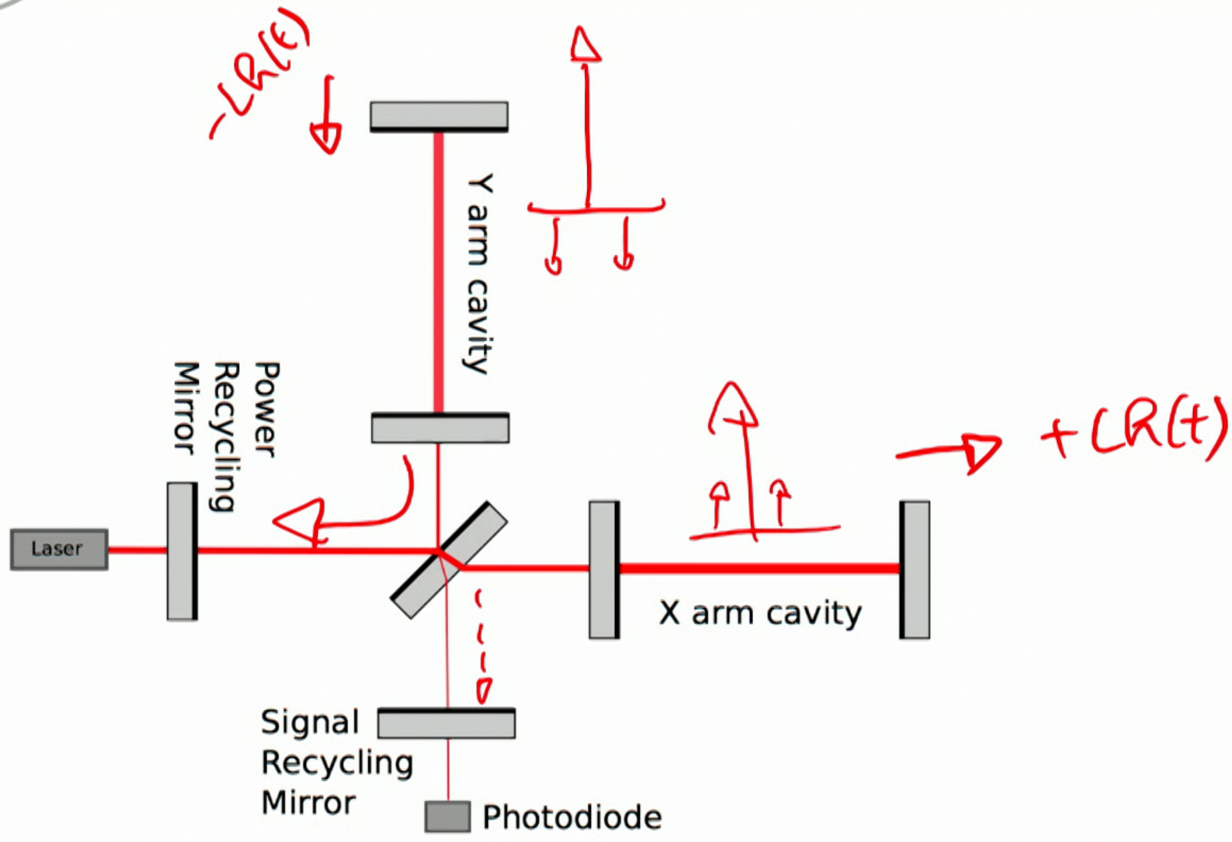


$$r_p^{(optimal)} = 1 - \frac{\mathcal{F}}{\pi} L_{RT} \qquad G_{PRC}^{(optimal)} = \frac{\pi}{2\mathcal{F} L_{RT}}$$

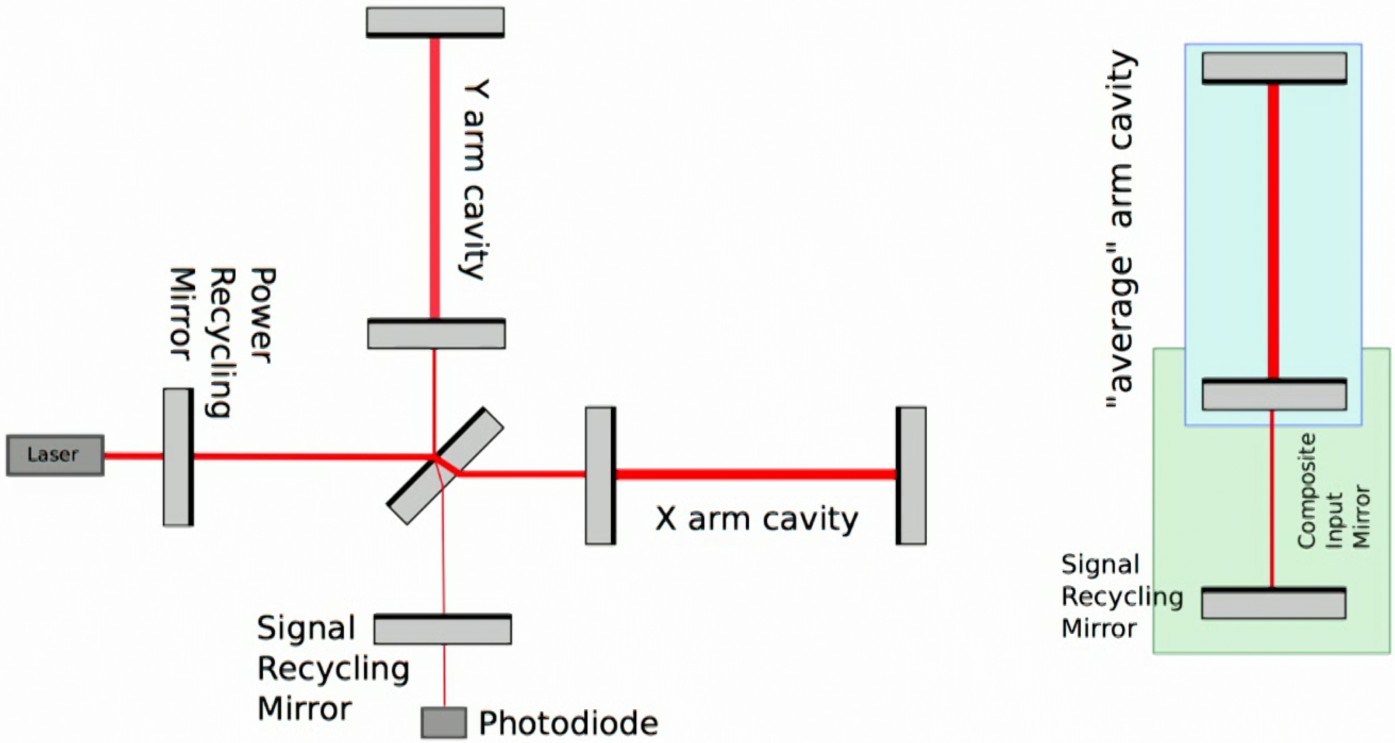
Signal recycling



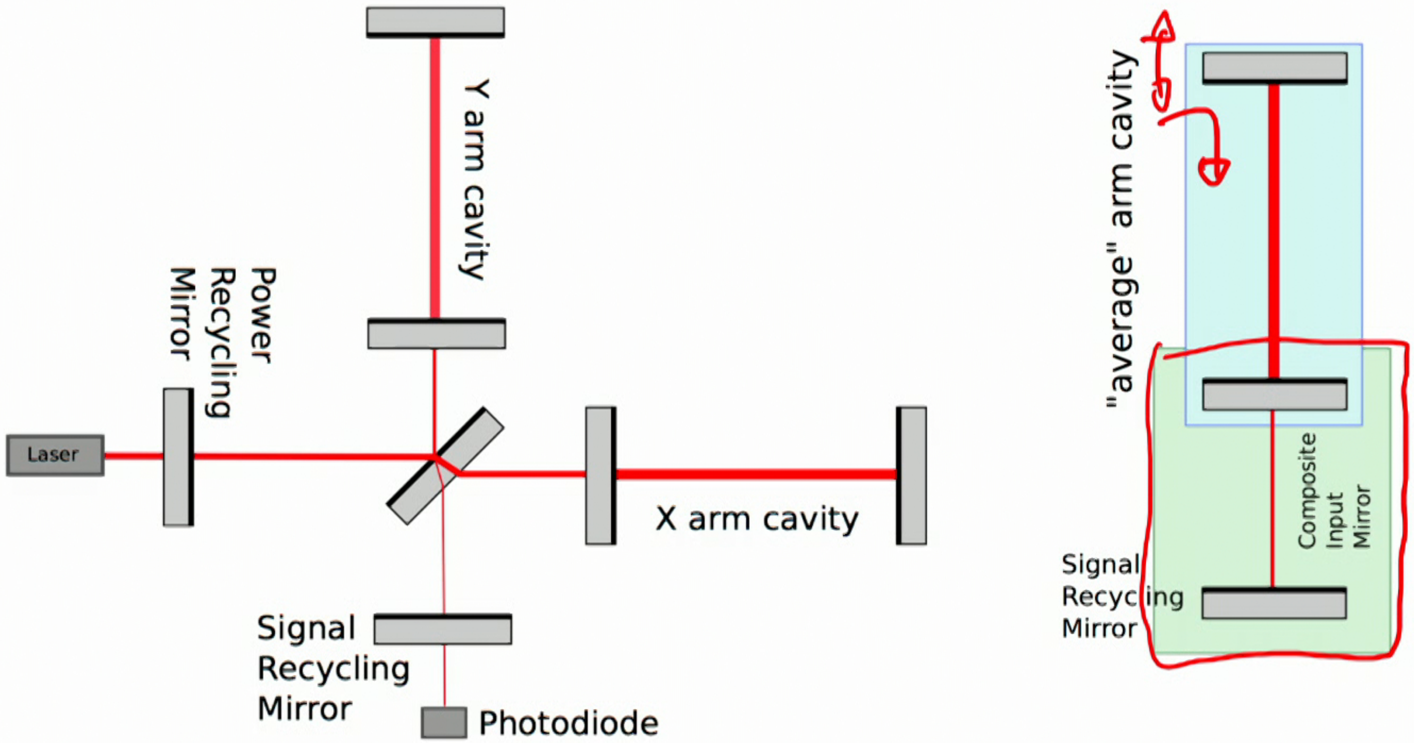
Signal recycling



Signal recycling

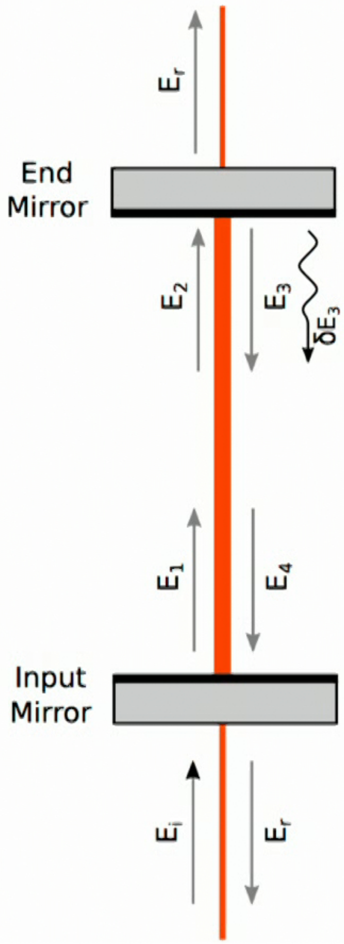


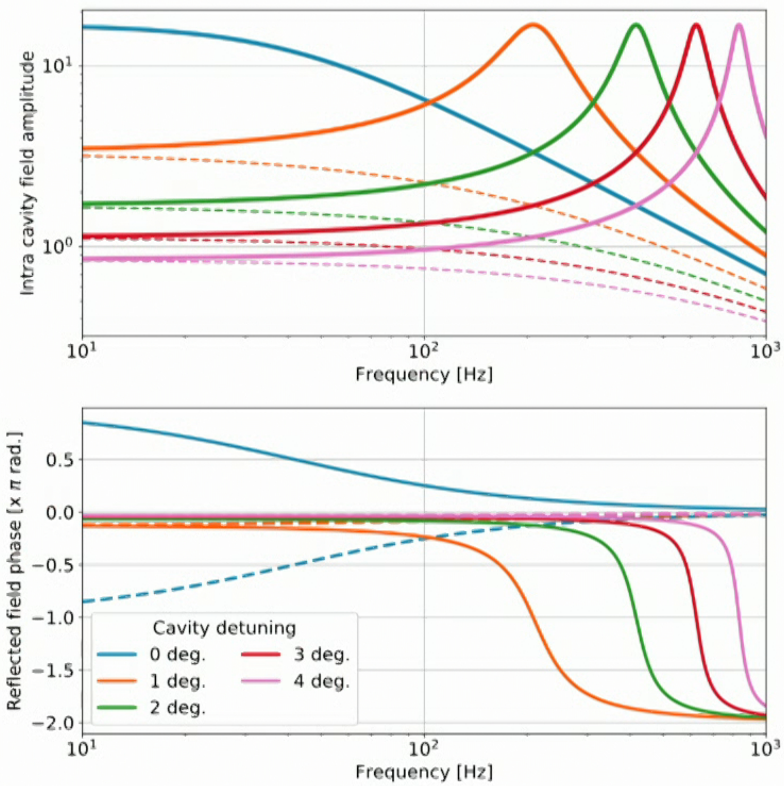
Signal recycling



Detuned cavity

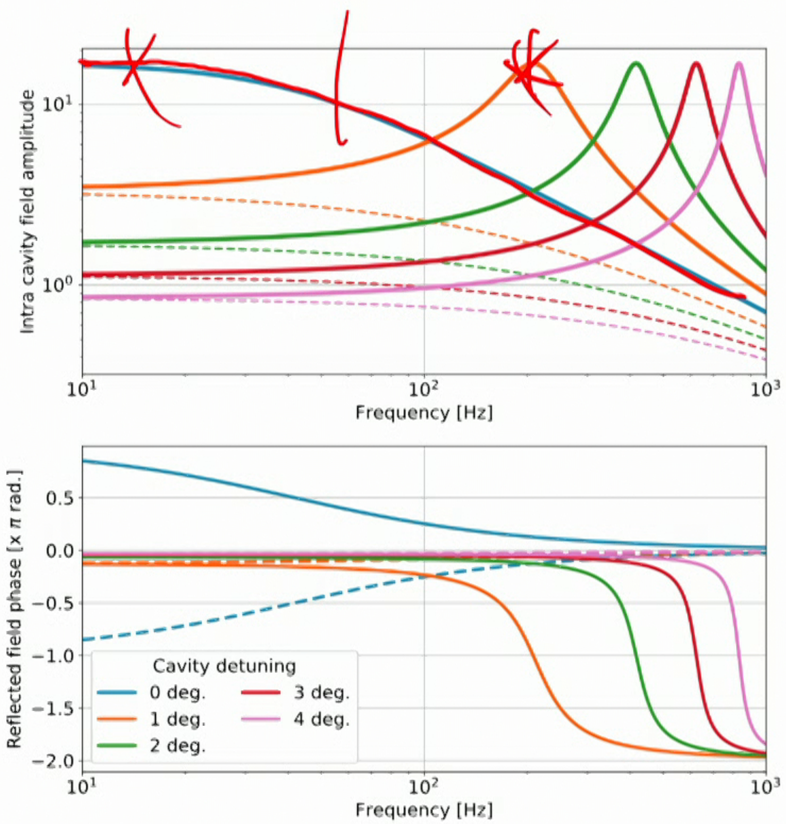
$$E_{intra}(\Omega) = \frac{t_i}{1 - r_i r_e e^{2i\phi} e^{2i\frac{\Omega}{c}L}} E_{in}(\Omega)$$



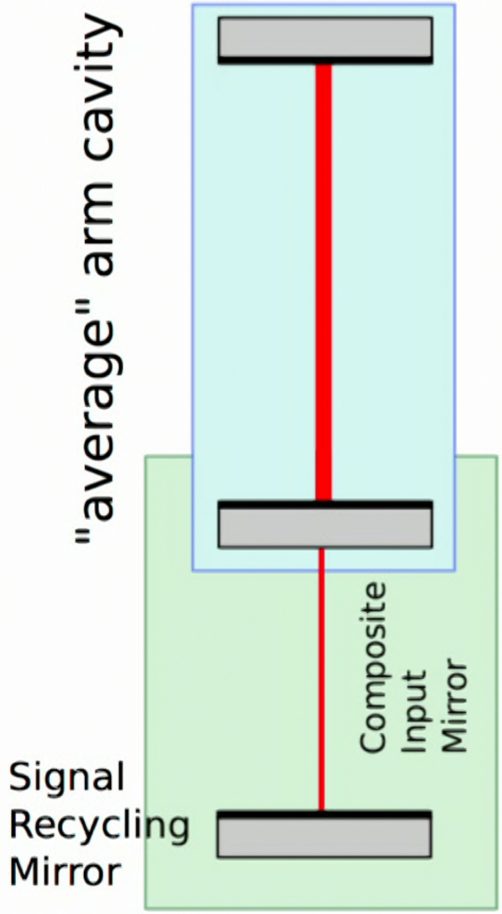


Detuned cavity

$$E_{intra}(\Omega) = \frac{t_i}{1 - r_i r_e e^{2i\phi} e^{2i\frac{\Omega}{c}L}} E_{in}(\Omega)$$



Signal recycling



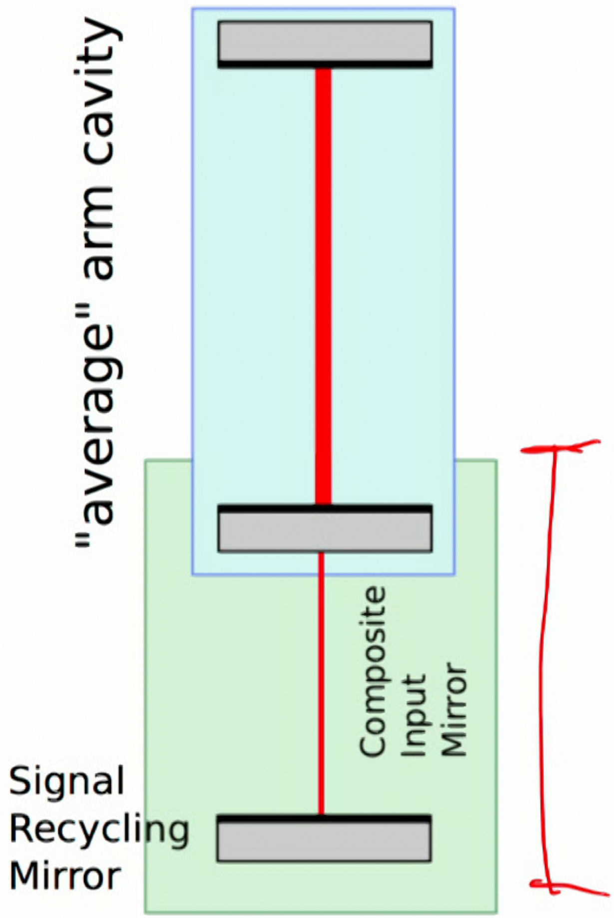
$$r(\Omega) = \frac{r_i - r_s e^{2i\phi_{SRC}} e^{2i\frac{\Omega}{c} l_{SRC}}}{1 - r_i r_s e^{2i\phi_{SRC}} e^{2i\frac{\Omega}{c} l_{SRC}}}$$

$$t(\Omega) = \frac{t_i t_s e^{i\phi_{SRC}} e^{i\frac{\Omega}{c} l_{SRC}}}{1 - r_i r_s e^{2i\phi_{SRC}} e^{2i\frac{\Omega}{c} l_{SRC}}}$$

$$r(\Omega) = \frac{r_i - r_s e^{2i\phi_{SRC}}}{1 - r_i r_s e^{2i\phi_{SRC}}}$$

$$t(\Omega) = \frac{t_i t_s e^{i\phi_{SRC}}}{1 - r_i r_s e^{2i\phi_{SRC}}}$$

Signal recycling



$$r(\Omega) = \frac{r_i - r_s e^{2i\phi_{SRC}} e^{2i\frac{\Omega}{c} l_{SRC}}}{1 - r_i r_s e^{2i\phi_{SRC}} e^{2i\frac{\Omega}{c} l_{SRC}}}$$

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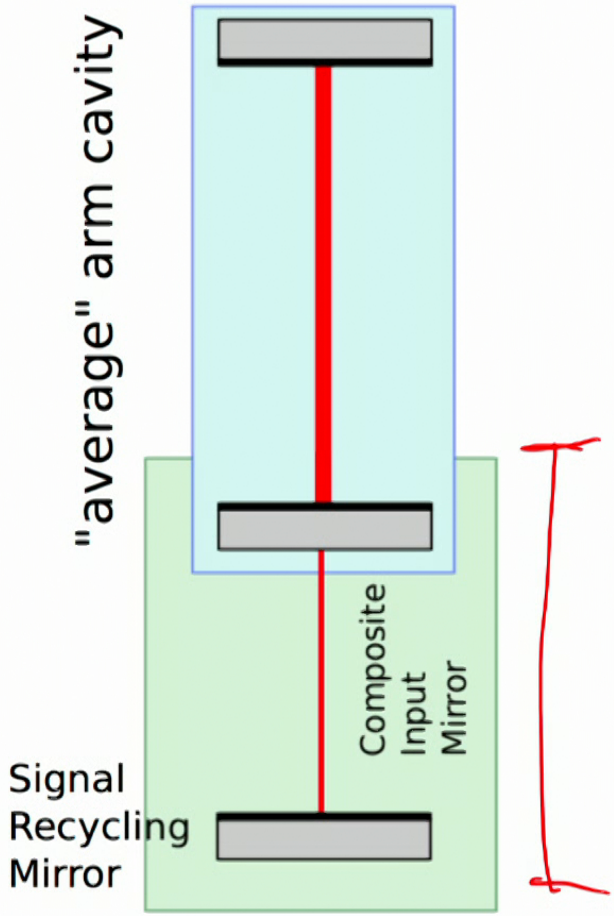
Signal recycling



$$r(\Omega) = \frac{r_i - r_s e^{2i\phi_{SRC}} e^{2i\frac{\Omega}{c} l_{SRC}}}{1 - r_i r_s e^{2i\phi_{SRC}} e^{2i\frac{\Omega}{c} l_{SRC}}}$$

$$t(\Omega) = \frac{t_i t_s e^{i\phi_{SRC}} e^{i\frac{\Omega}{c} l_{SRC}}}{1 - r_i r_s e^{2i\phi_{SRC}} e^{2i\frac{\Omega}{c} l_{SRC}}}$$

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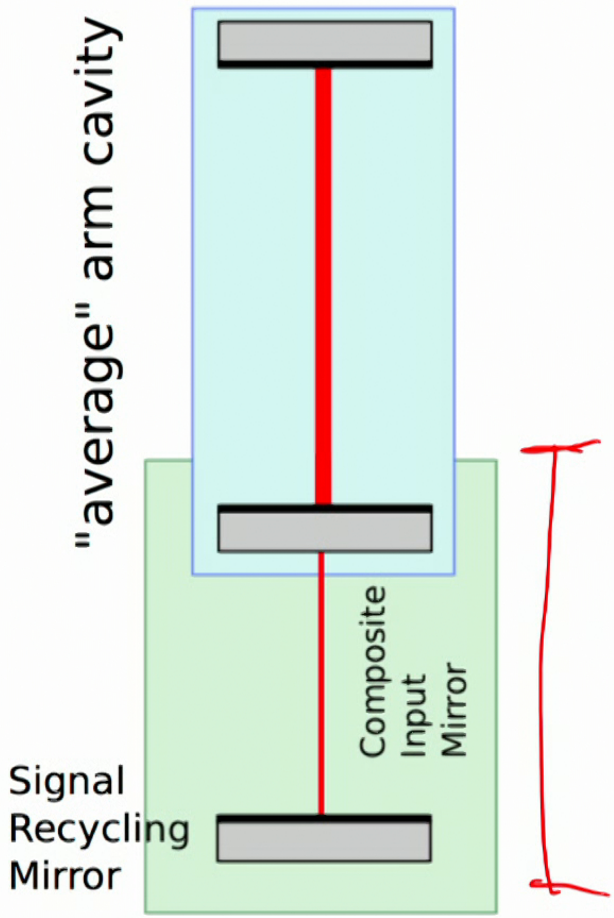
Signal recycling



$$r(\Omega) = \frac{r_i - r_s e^{2i\phi_{SRC}} e^{2i\frac{\Omega}{c} l_{SRC}}}{1 - r_i r_s e^{2i\phi_{SRC}} e^{2i\frac{\Omega}{c} l_{SRC}}}$$

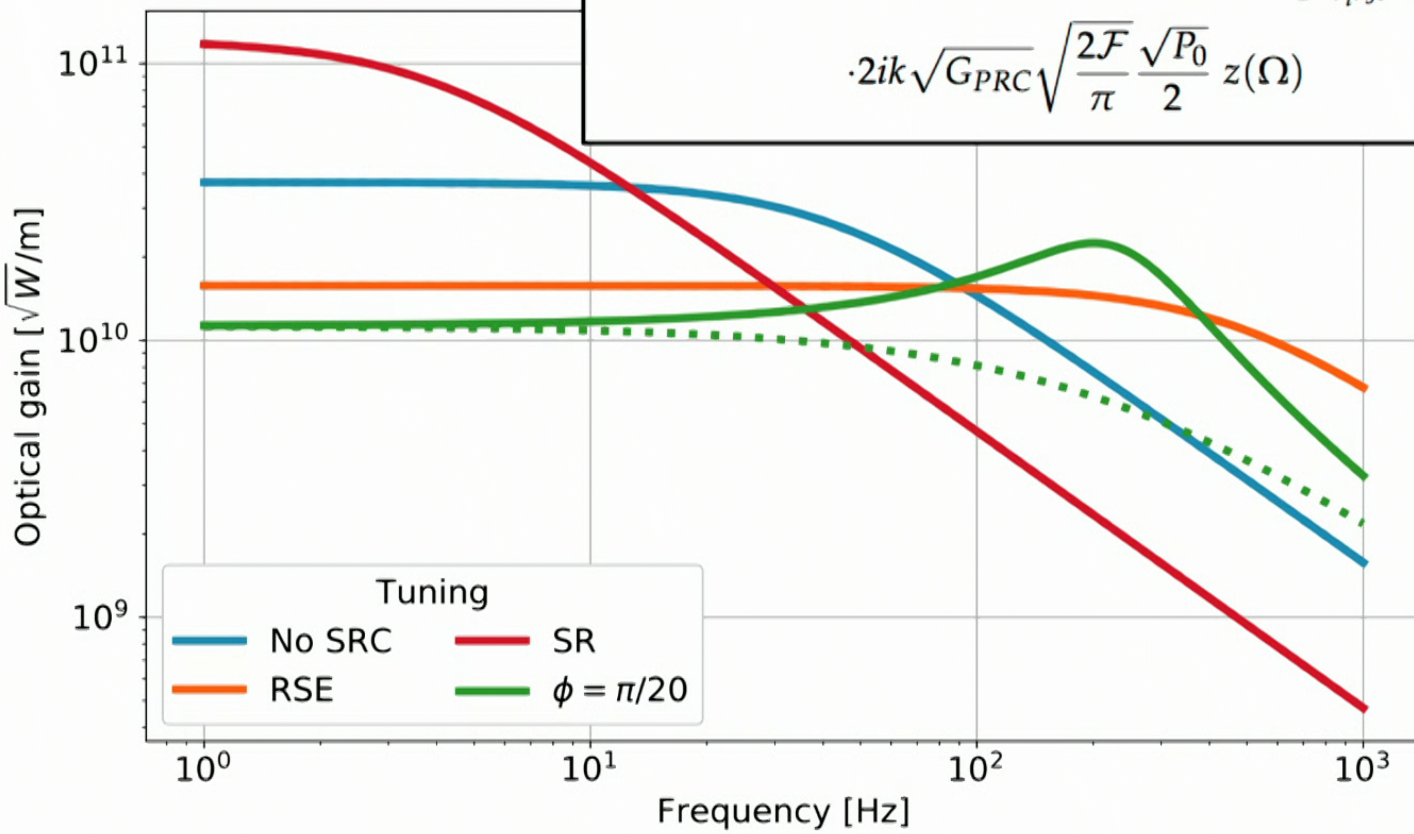
$$t(\Omega) = \frac{t_i t_s e^{i\phi_{SRC}} e^{i\frac{\Omega}{c} l_{SRC}}}{1 - r_i r_s e^{2i\phi_{SRC}} e^{2i\frac{\Omega}{c} l_{SRC}}}$$

$$r(\Omega) = \frac{r_i - r_s e^{2i\phi_{SRC}}}{1 - r_i r_s e^{2i\phi_{SRC}}}$$

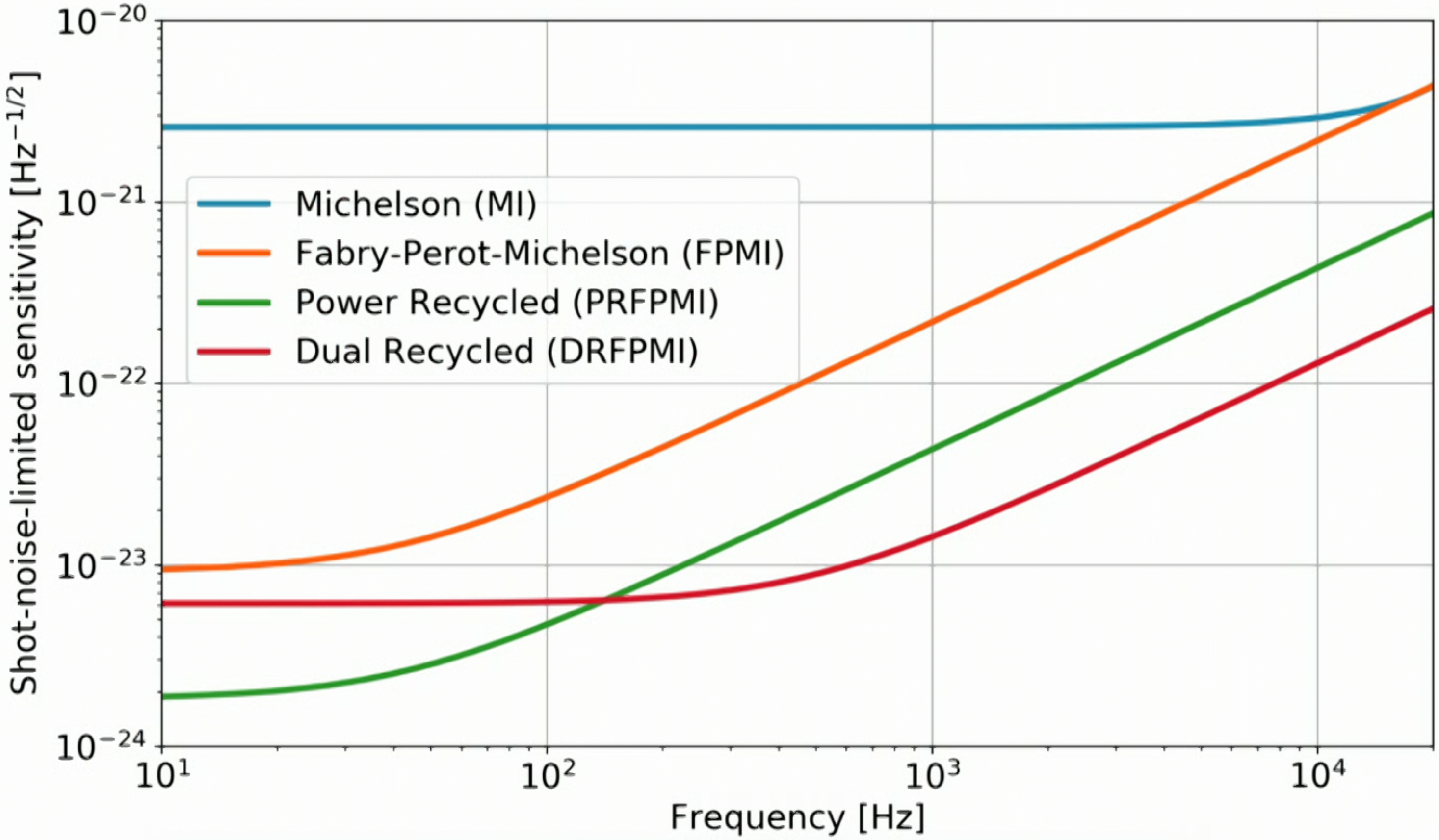
$$t(\Omega) = \frac{t_i t_s e^{i\phi_{SRC}}}{1 - r_i r_s e^{2i\phi_{SRC}}}$$

Signal recycling

$$E_A(\Omega) = \frac{t_i t_s e^{i\phi_{SRC}}}{1 - r_i r_s e^{2i\phi_{SRC}}} \cdot \frac{1}{1 - r_e e^{2i\frac{\Omega}{c}L} \frac{r_i - r_s e^{2i\phi}}{1 - r_i r_s e^{2i\phi}}} \cdot 2ik\sqrt{G_{PRC}}\sqrt{\frac{2\mathcal{F}}{\pi}} \frac{\sqrt{P_0}}{2} z(\Omega)$$

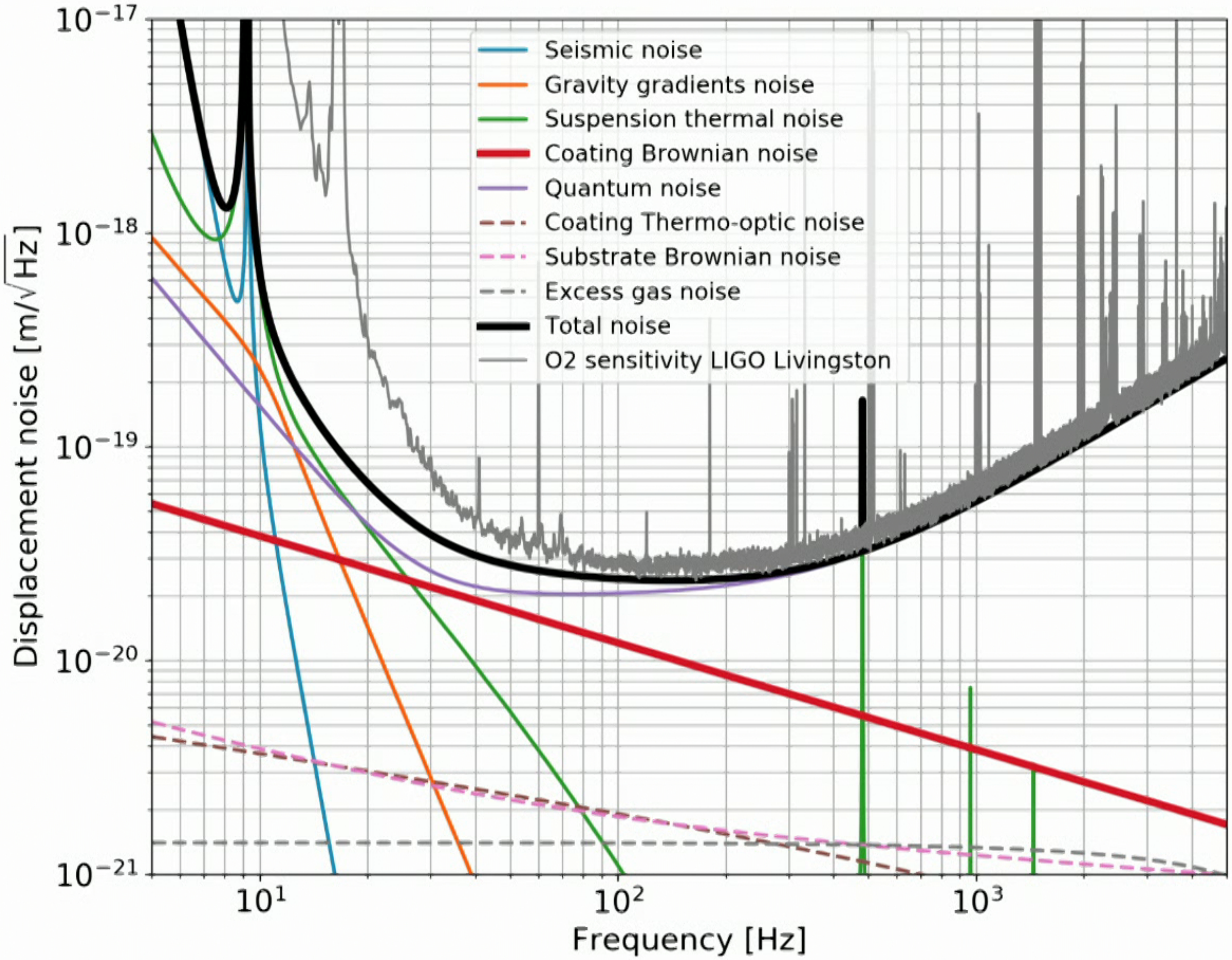


Summary



Michelson (MI)
Fabry-Perot-Michelson (FPMI)
Power Recycled (PRFPMI)
Dual Recycled (DRFPMI)
Shot-noise-limited sensitivity [Hz$^{-1/2}$]
10^{-20}
10^{-21}
10^{-22}
10^{-23}
10^{-24}
10^1
10^2
10^3
10^4
Frequency [Hz]

Advanced LIGO design



Advanced LIGO design