Title: Standard Model Theory 2

Date: Jul 09, 2018 02:30 PM

URL: http://pirsa.org/18070020

Abstract:

Lie groups & algebras

a lot   SU(2)

* rotations space, spin

Lie groups & algebras

a lot   SU(2)

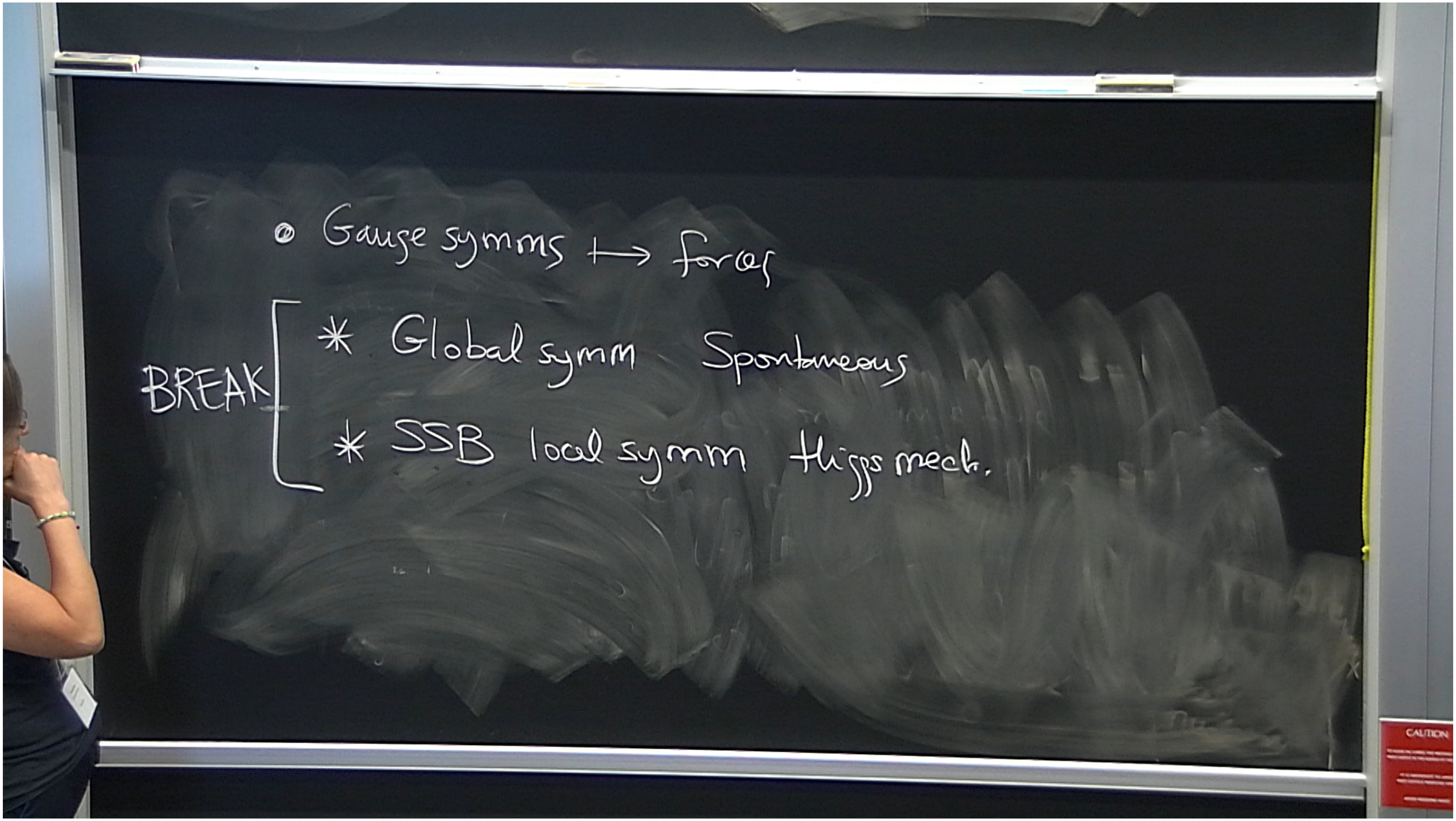
* rotations space, spin
* Lorentz symm S-t $\mapsto$ SU(2) $\times$ SU(2)
  $\hookrightarrow$ SR mechanics
* Isospin

Lie groups & algebras

a lot SU(2)

* rotations space, spin
* Lorentz symm S-t $\mapsto$ SU(2) × SU(2)
  $\hookrightarrow$ SR mechanics
* Isospin nuclear interactions $\leftrightarrow$ pions

- Gauge symms $\longmapsto$ forces


Gauge symms ⟼ forces
BREAK
* Global symm Spontaneous
* SSB local symm Higgs mech.
CAUTION

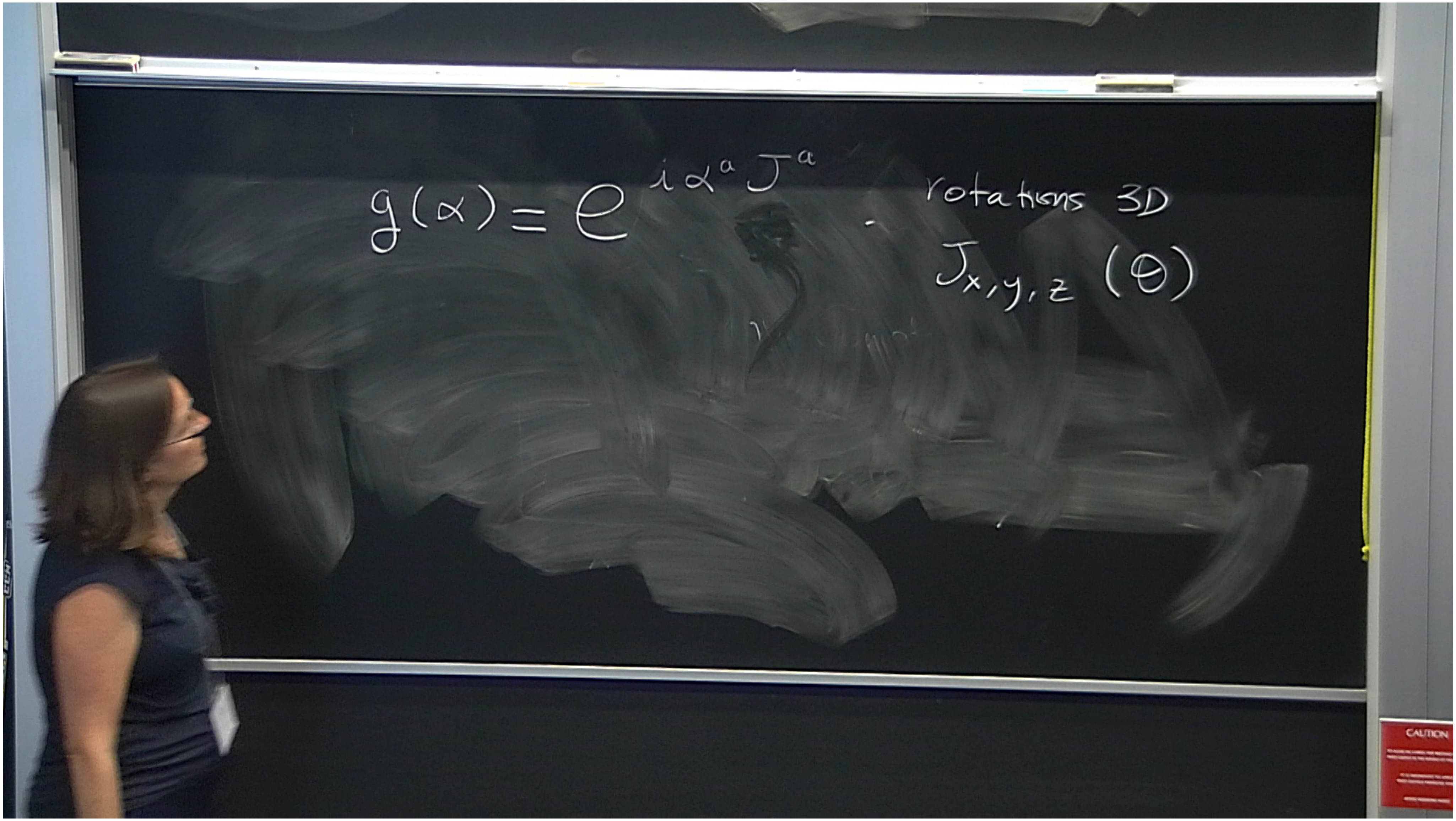
- Gauge symms $\longmapsto$ forces

BREAK
- * Global symm Spontaneous GB Goldstone bosons
- * SSB local symm Higgs mech.

$g(\alpha)$

rotations 3D

$J_{x,y,z}(\theta)$


$g(\alpha) = e^{i\alpha^a J^a}$
rotations 3D
$J_{x,y,z}(\theta)$

$$g(\alpha) = e^{i\alpha^a J^a}$$

rotations 3D

$J_{x,y,z}\ (\theta)$

$$\approx \mathbb{1} + i\,\alpha_a J^a \qquad a = 1,2,3$$

$$+\ \mathcal{O}(\alpha^2)$$

$$g(\alpha) = e^{i\alpha^a T^a}$$

rotations 3D

$J_{x,y,z}(\theta)$

$$\approx \mathbb{1} + i\alpha_a T^a + \mathcal{O}(\alpha^2)$$

$a = 1, 2, 3$

$$g(\alpha) = e^{i\alpha^a T^a}$$

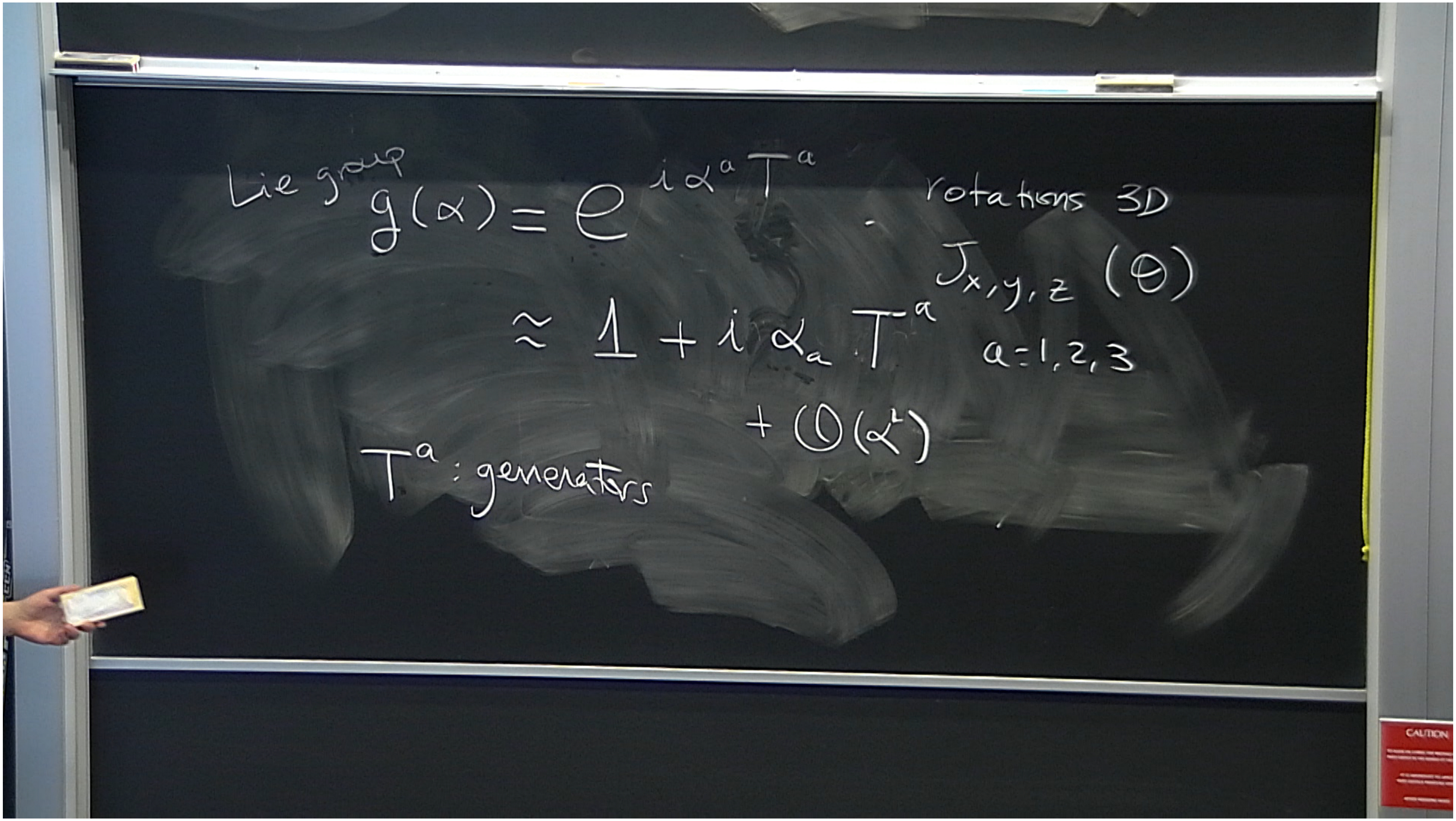
rotations 3D

$J_{x,y,z}(\theta)$

$$\approx \mathbb{1} + i\alpha_a T^a \qquad a = 1,2,3$$

$$+ \mathcal{O}(\alpha^2)$$

T^a : generators


Lie group
$g(\alpha) = e^{i\alpha^a T^a}$
$\approx \mathbb{1} + i\alpha_a T^a + \mathcal{O}(\alpha^2)$
rotations 3D
$J_{x,y,z}(\theta)$
$a=1,2,3$
T^a : generators

Lie group

$$g(\alpha) = e^{i\alpha^a T^a}$$

rotations 3D

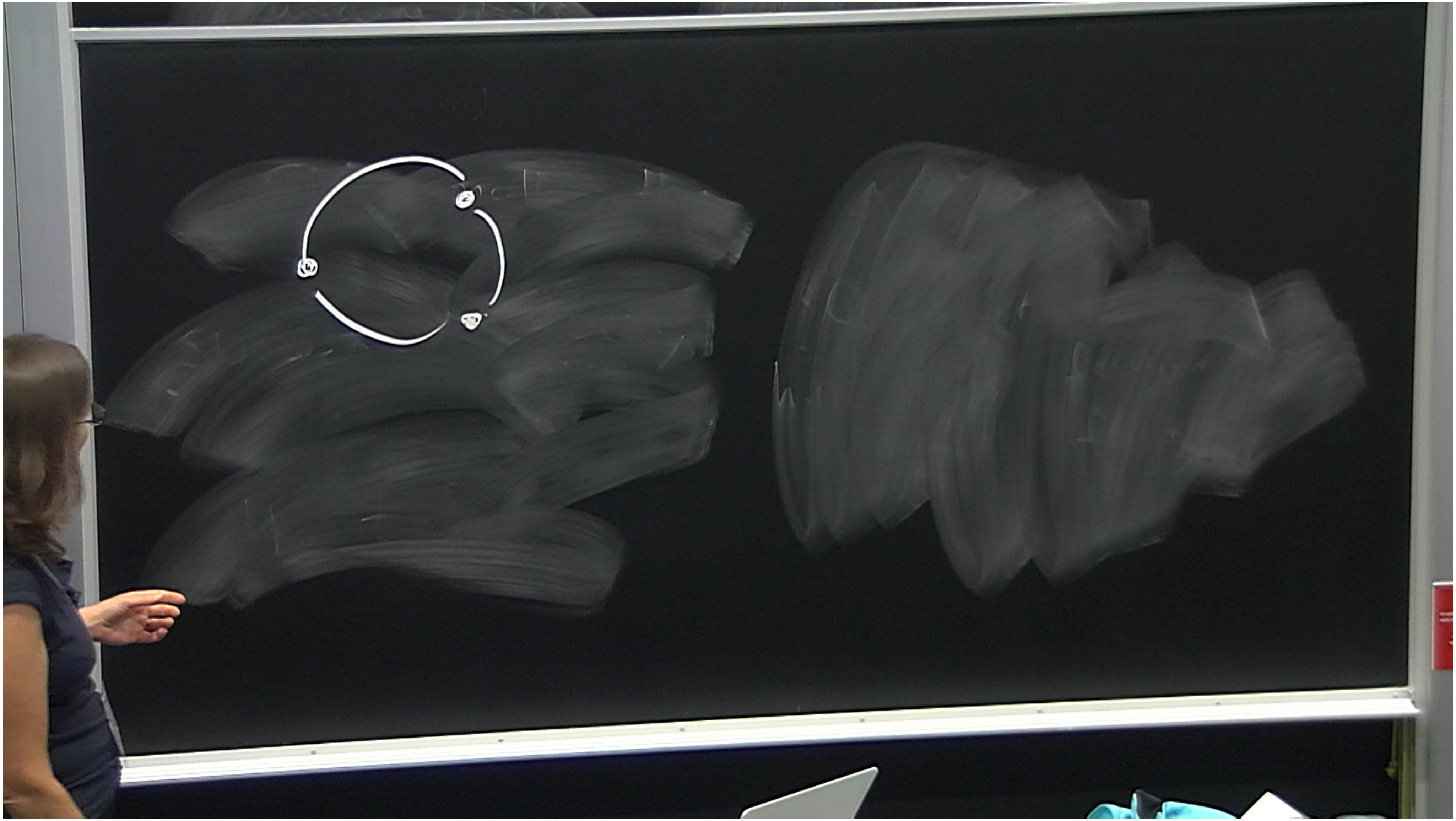
$J_{x,y,z}(\theta)$

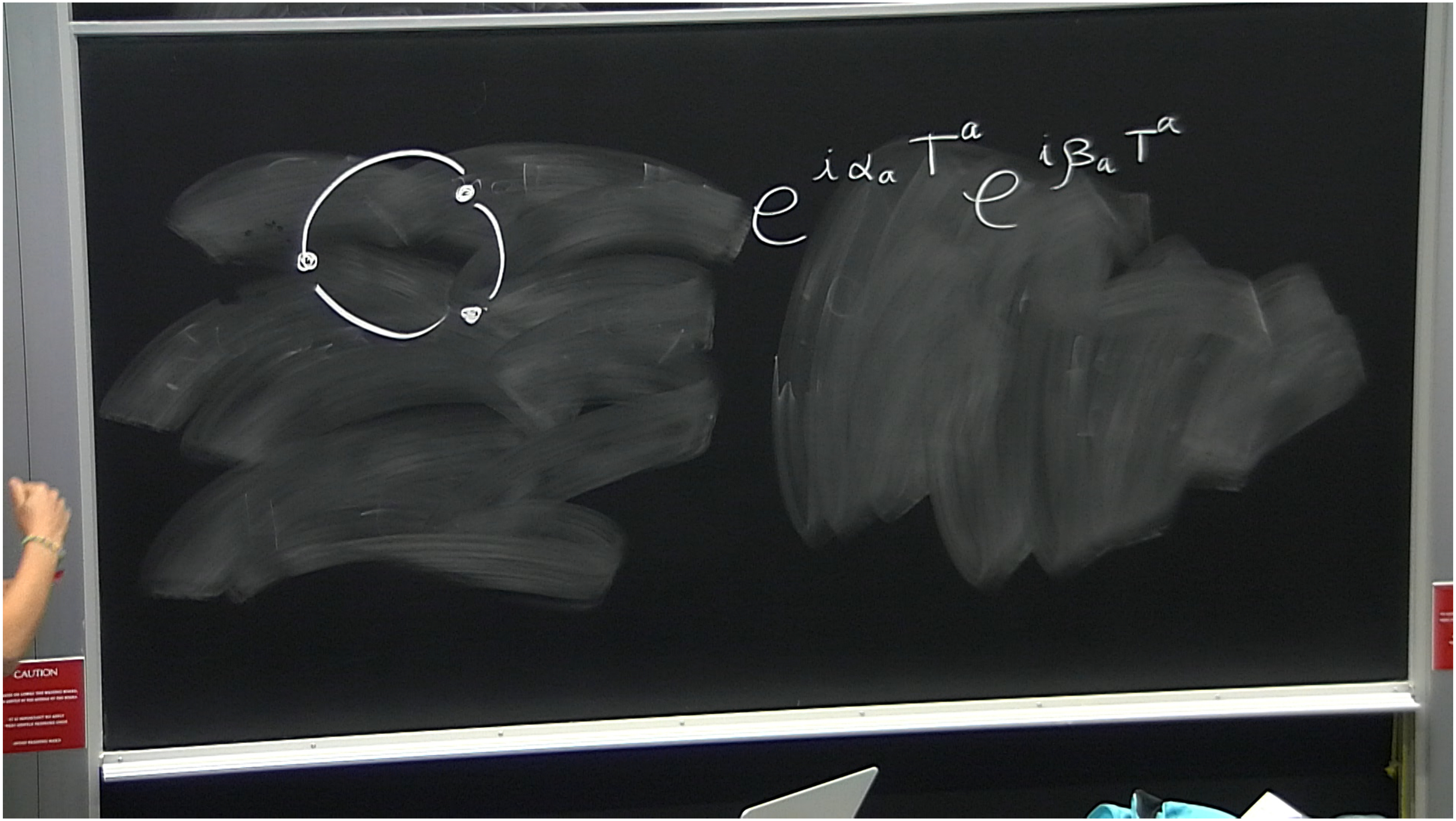
$$\approx \mathbb{1} + i\alpha_a T^a + \mathcal{O}(\alpha^2)$$

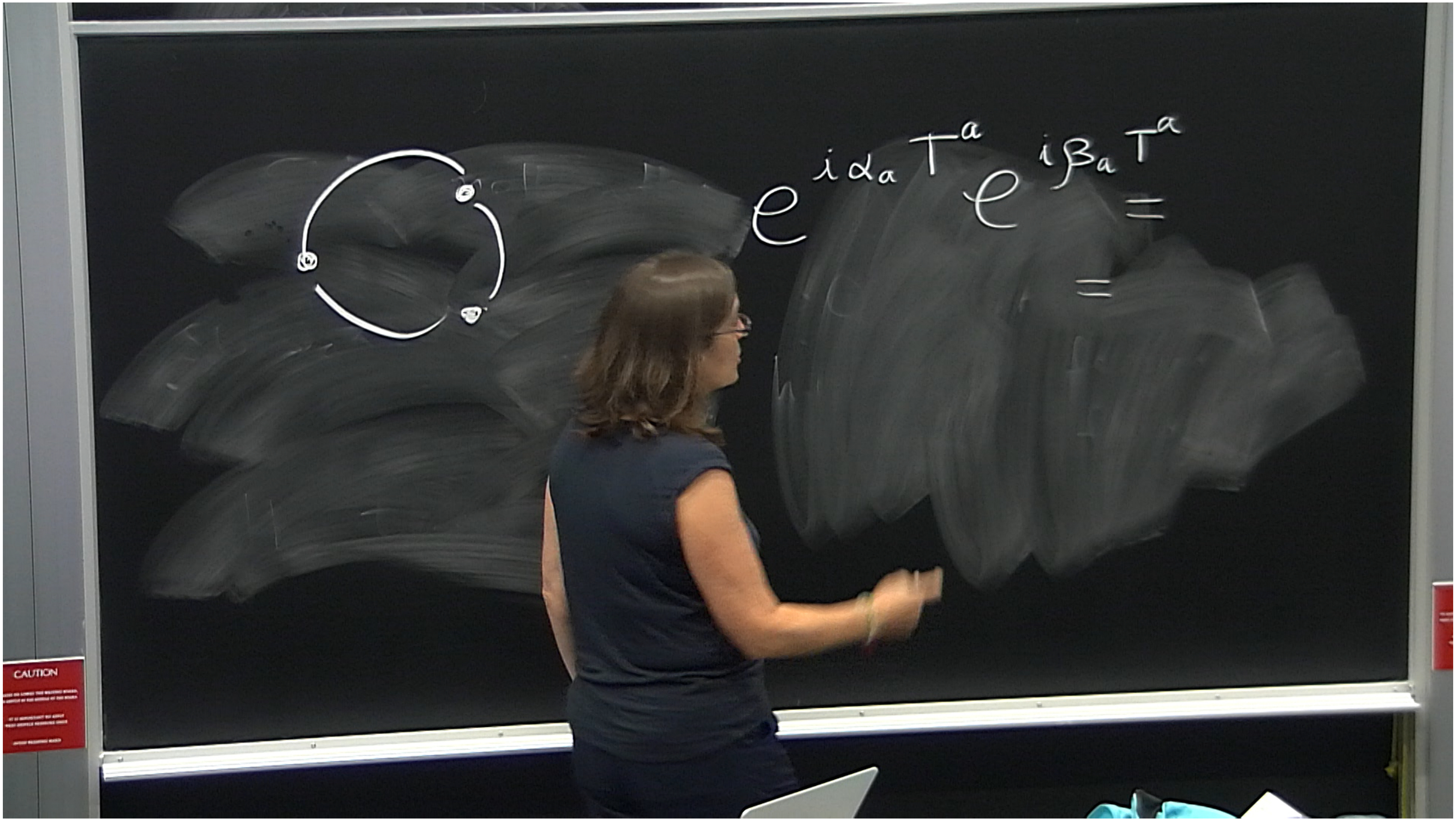
$a = 1, 2, 3$

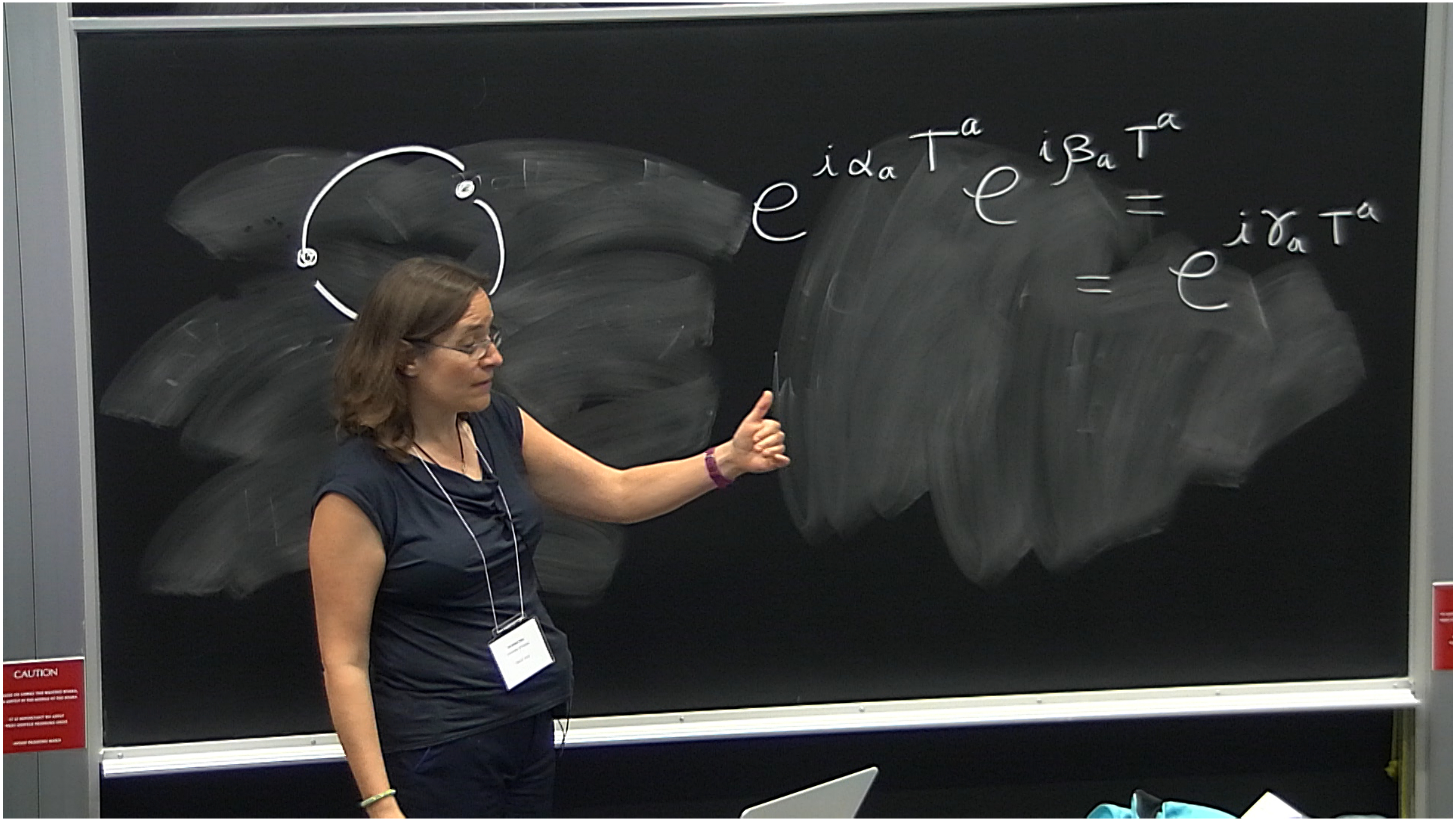
T^a: generators

Lie algebra

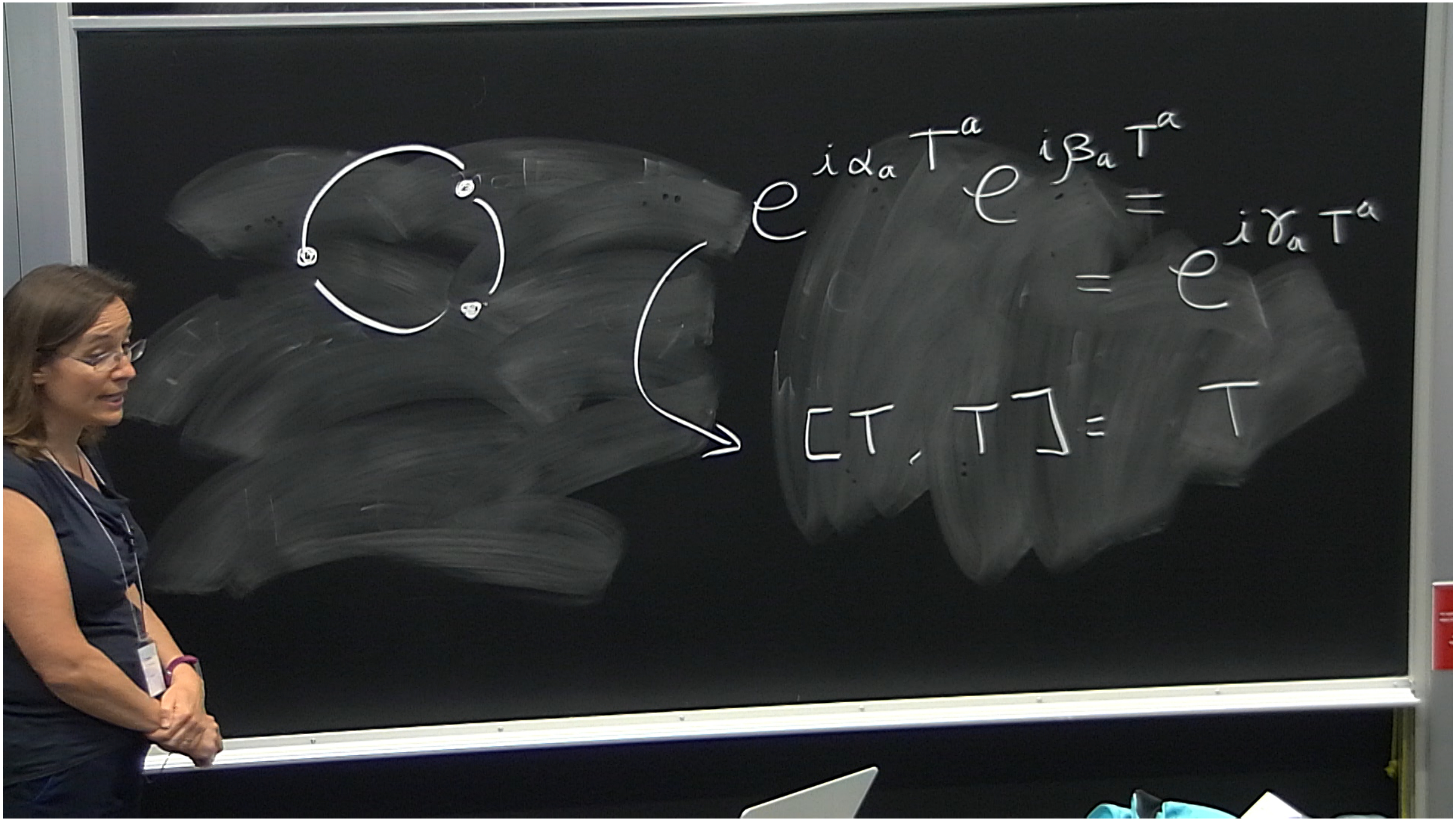


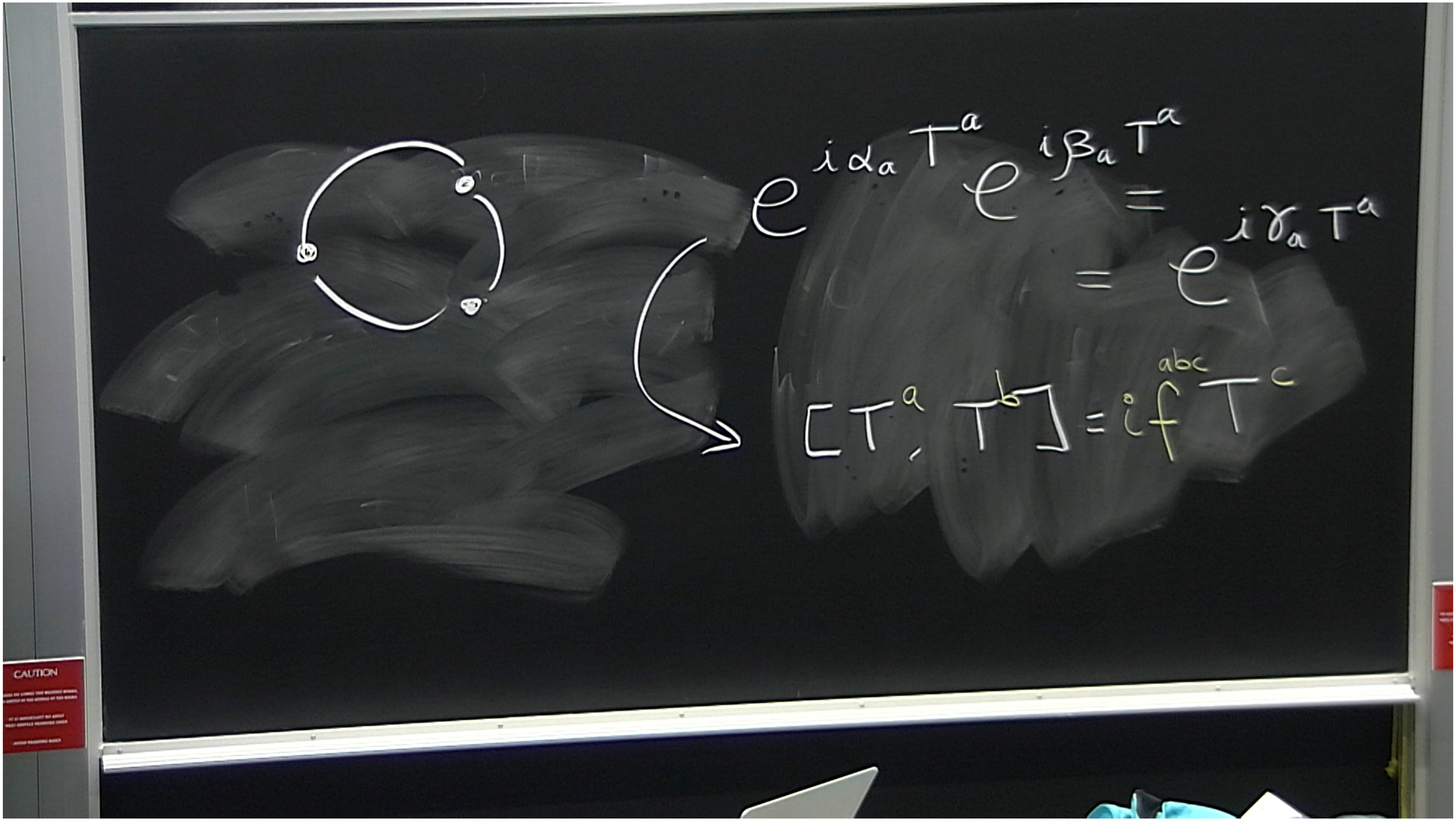

$e^{i\alpha_a T^a} e^{i\beta_a T^a}$

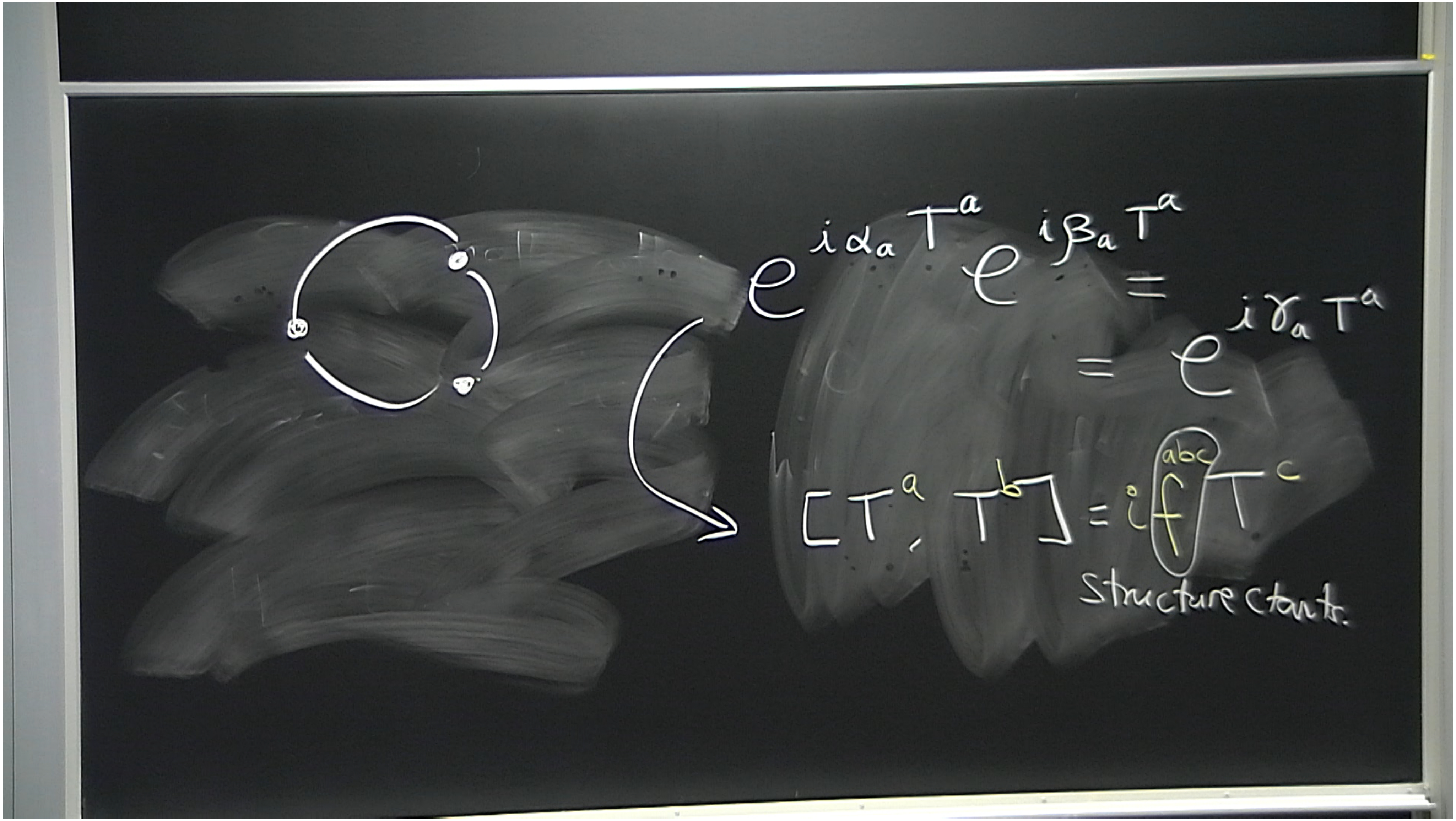

$e^{i\alpha_a T^a} e^{i\beta_a T^a} =$
=

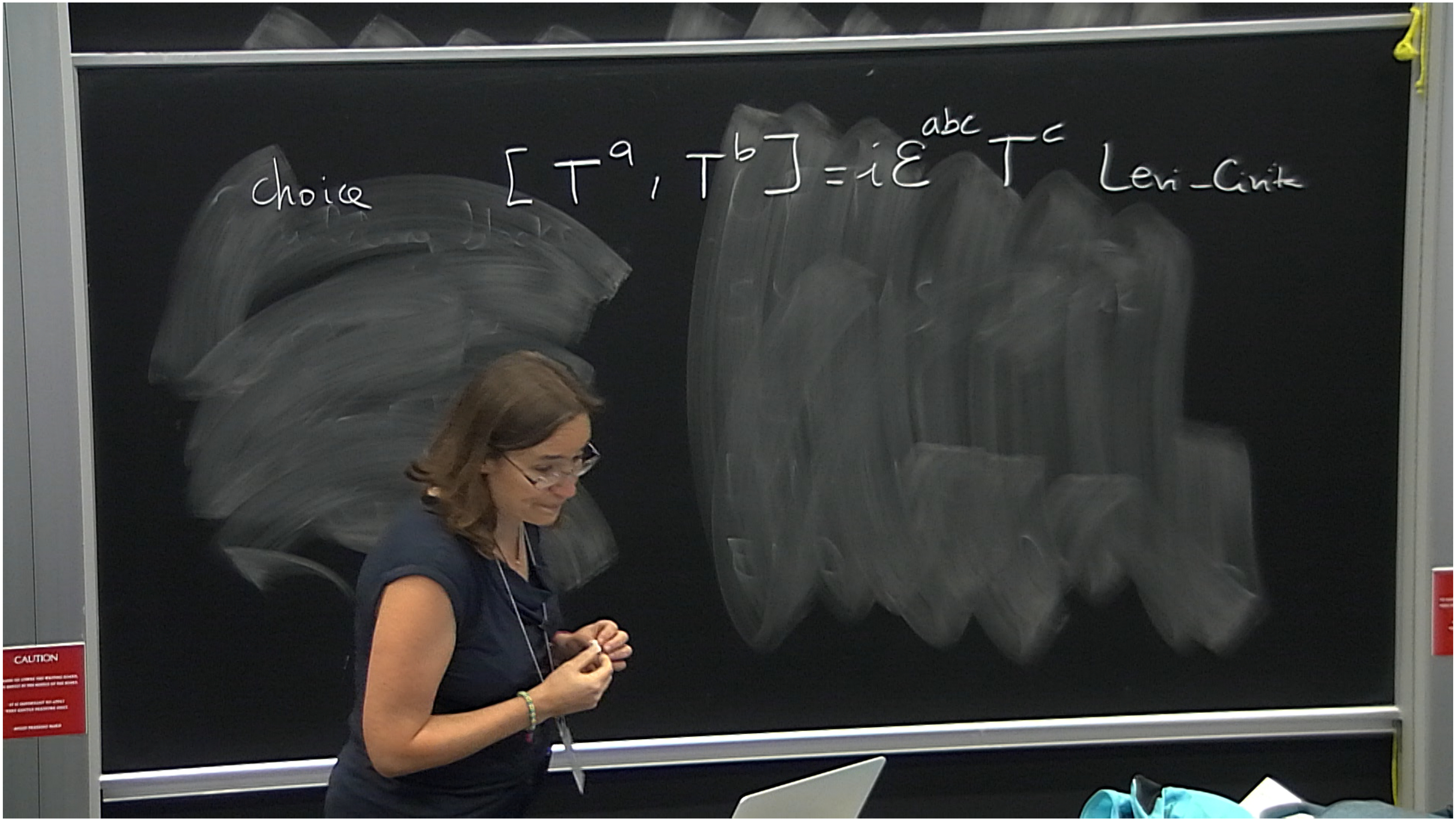


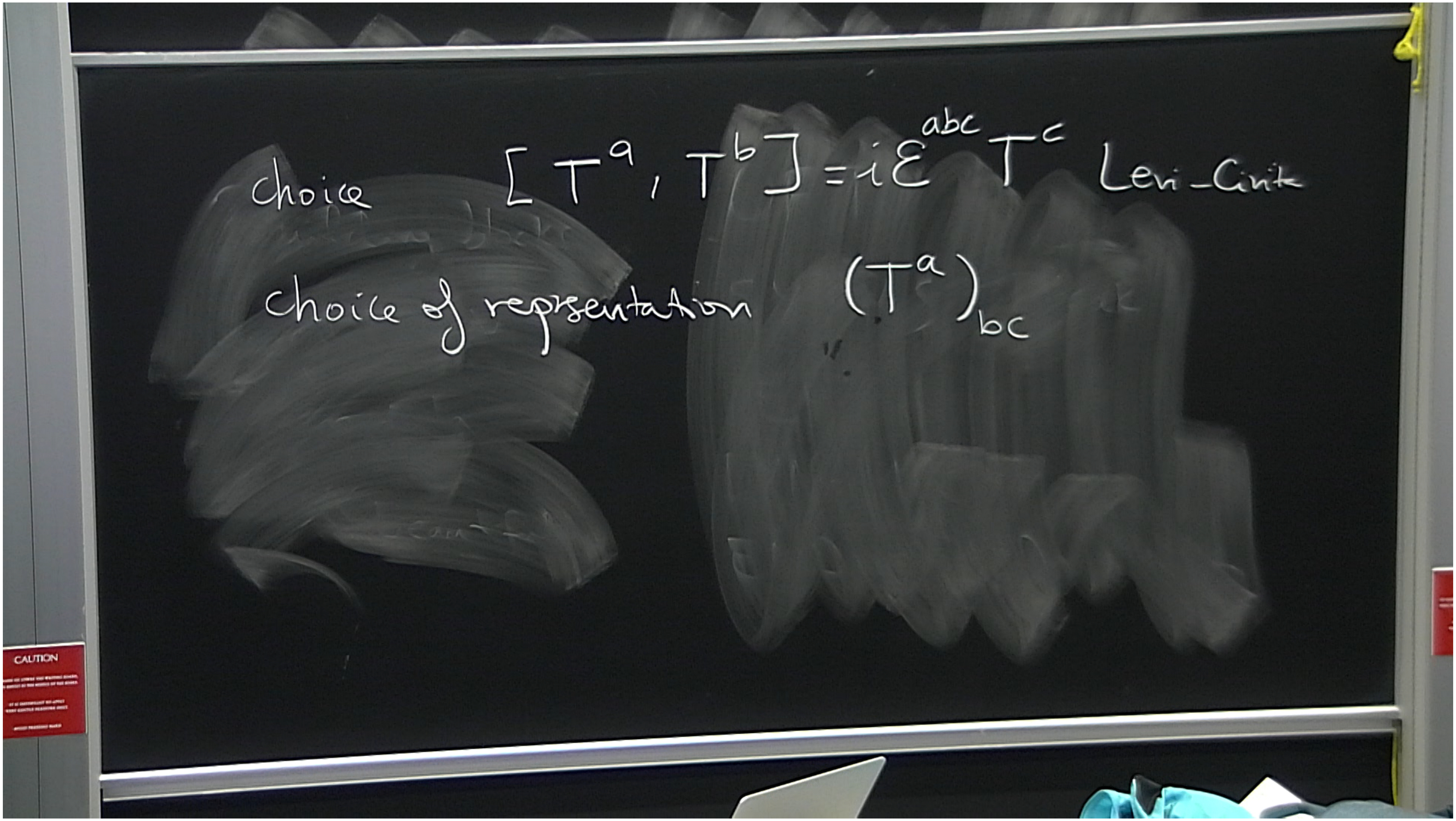
$e^{i\alpha_a T^a} e^{i\beta_a T^a} = e^{i\gamma_a T^a}$

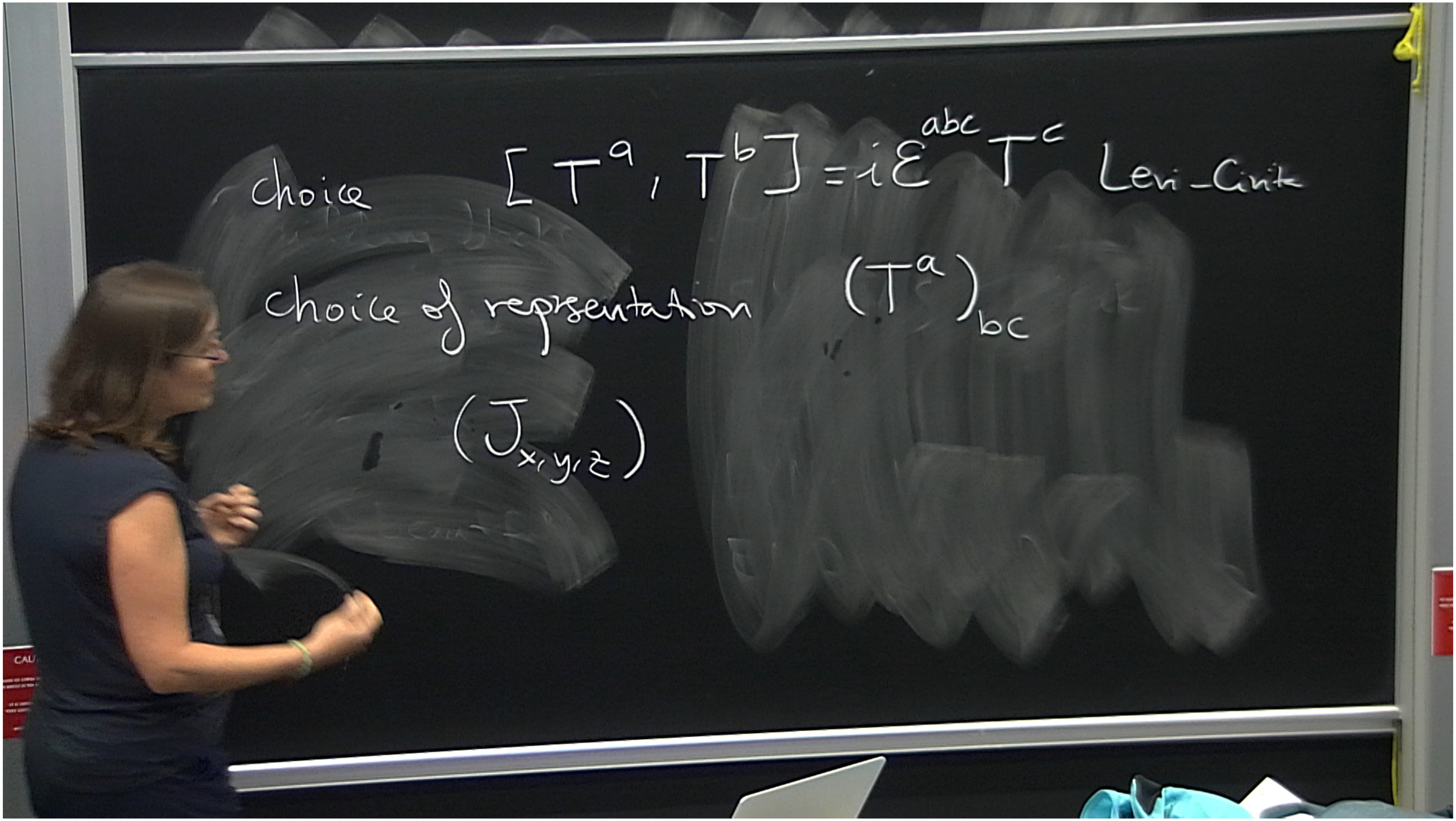

$e^{i\alpha_a T^a} e^{i\beta_a T^a} =$
$= e^{i\gamma_a T^a}$
$[T, T] = T$

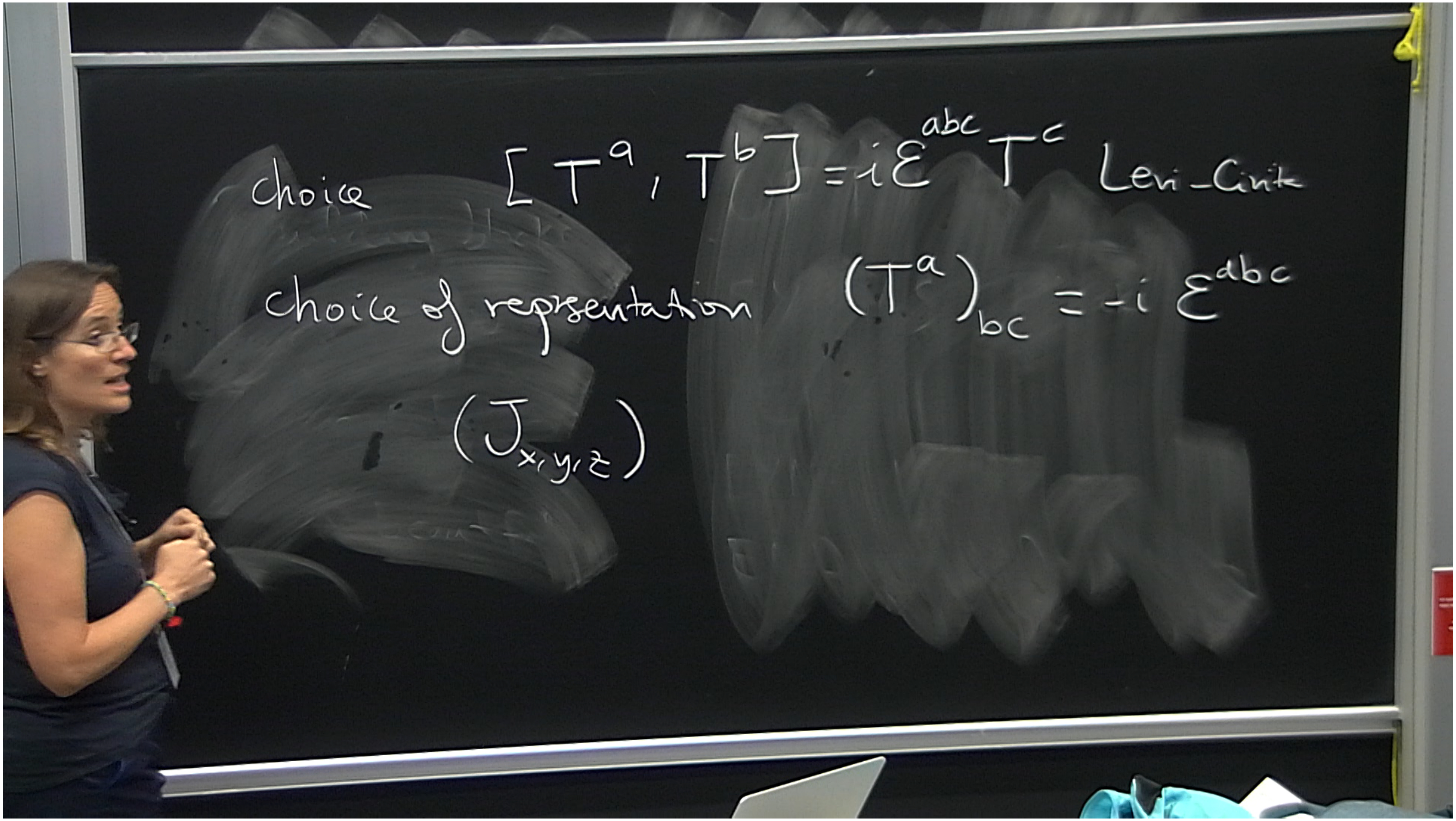

$e^{i\alpha_a T^a} e^{i\beta_a T^a} = e^{i\gamma_a T^a}$
$[T^a, T^b] = i f^{abc} T^c$
CAUTION

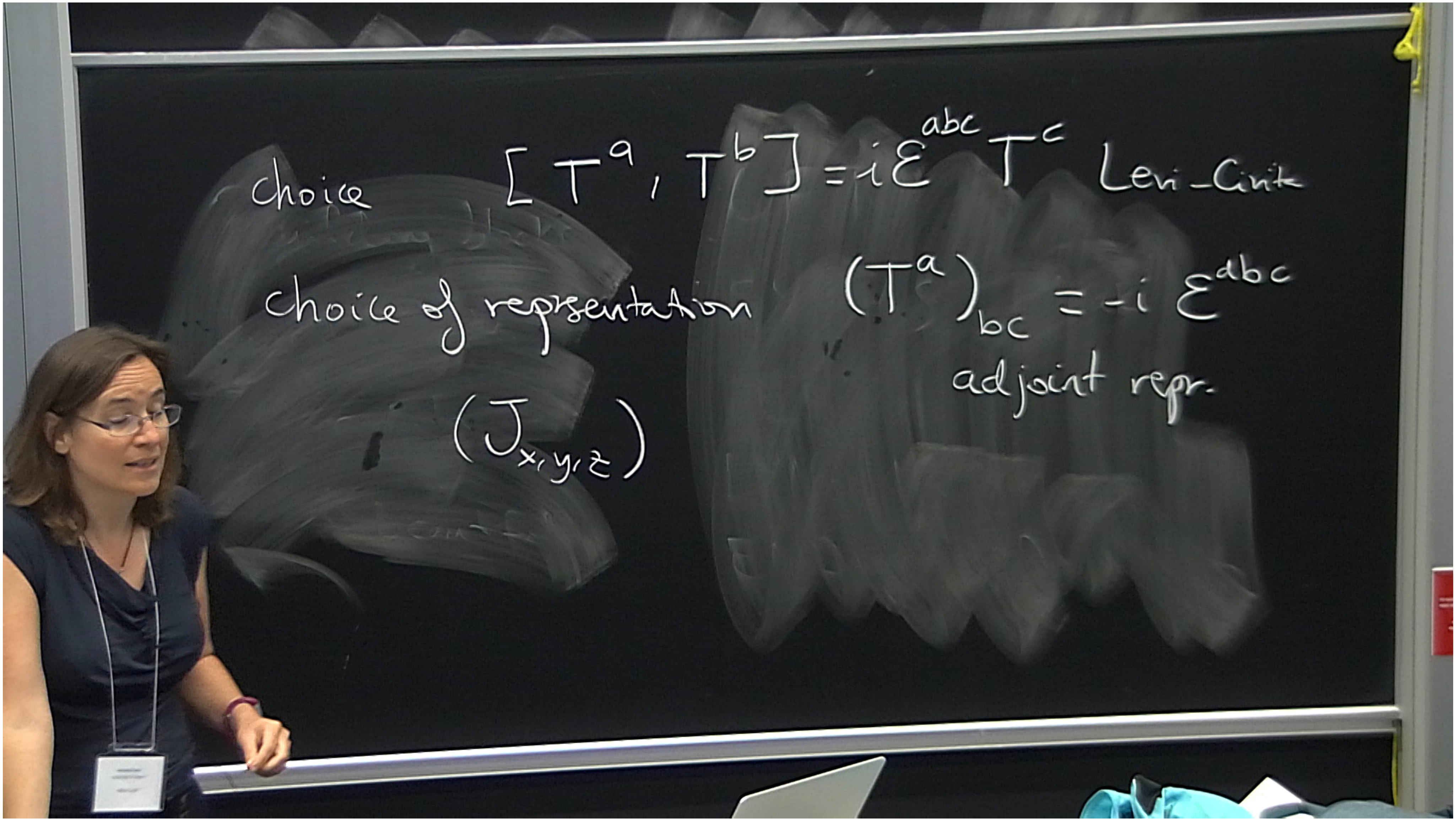

$e^{i\alpha_a T^a} e^{i\beta_a T^a} = e^{i\gamma_a T^a}$
$= e$
$[T^a, T^b] = i f^{abc} T^c$
Structure ctants.


Choice
$[T^a, T^b] = i\varepsilon^{abc} T^c$
Levi_Civit
CAUTION


Choice
$[T^a, T^b] = i\varepsilon^{abc} T^c$ Levi-Civita
Choice of representation $(T^a)_{bc}$
CAUTION


choice
$[T^a, T^b] = i\varepsilon^{abc} T^c$ Levi_Civita
choice of representation
$(T^a)_{bc}$
$(J_{x,y,z})$


Choice $[T^a, T^b] = i\varepsilon^{abc} T^c$ Levi_Civit
choice of representation $(T^a)_{bc} = -i\,\varepsilon^{abc}$
$(J_{x,y,z})$


Choice
$[T^a, T^b] = i\varepsilon^{abc} T^c$ Levi-Civita
choice of representation
$(T^a)_{bc} = -i\,\varepsilon^{abc}$
adjoint repr.
$(J_{x,y,z})$

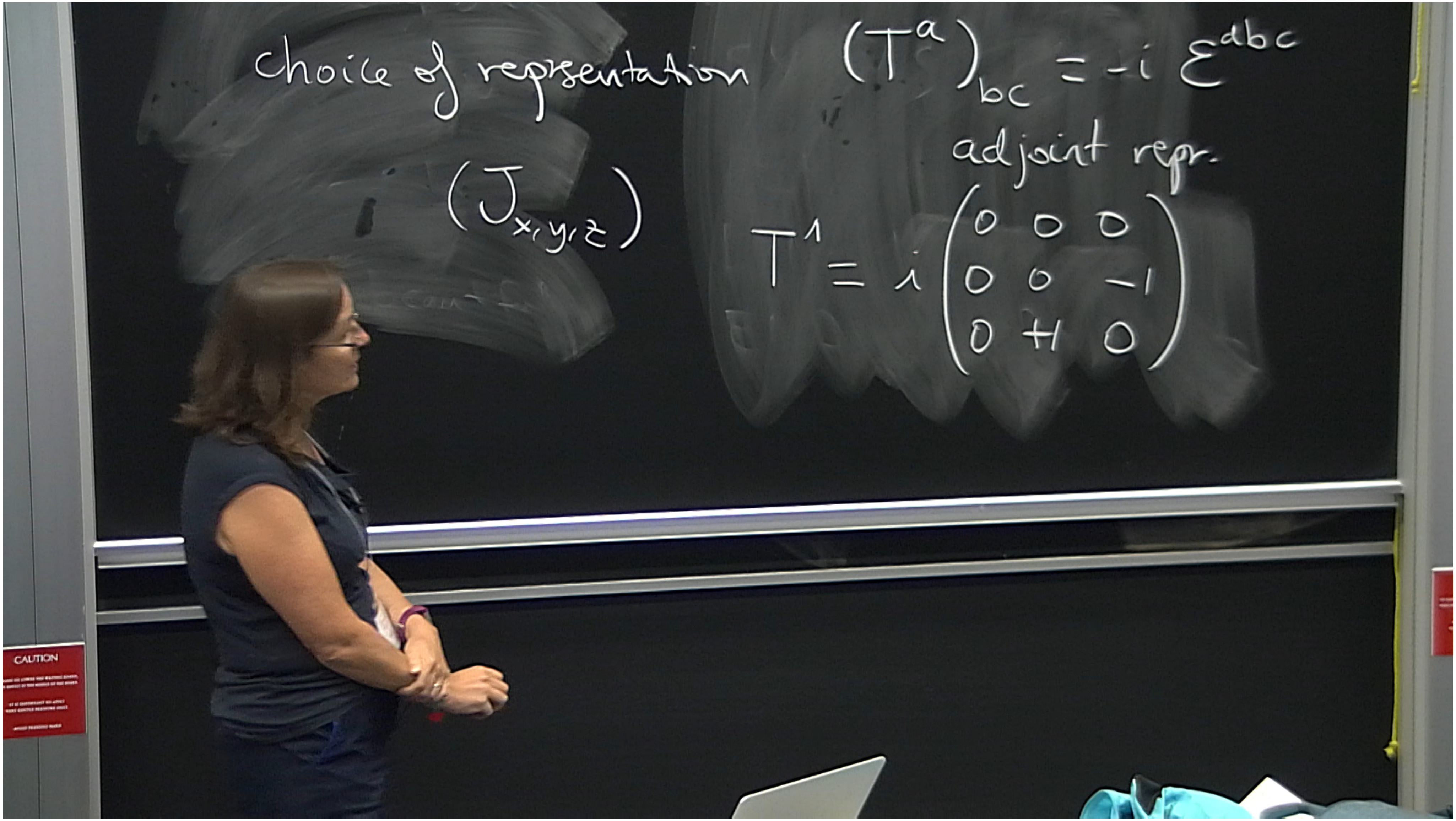
SU(2)

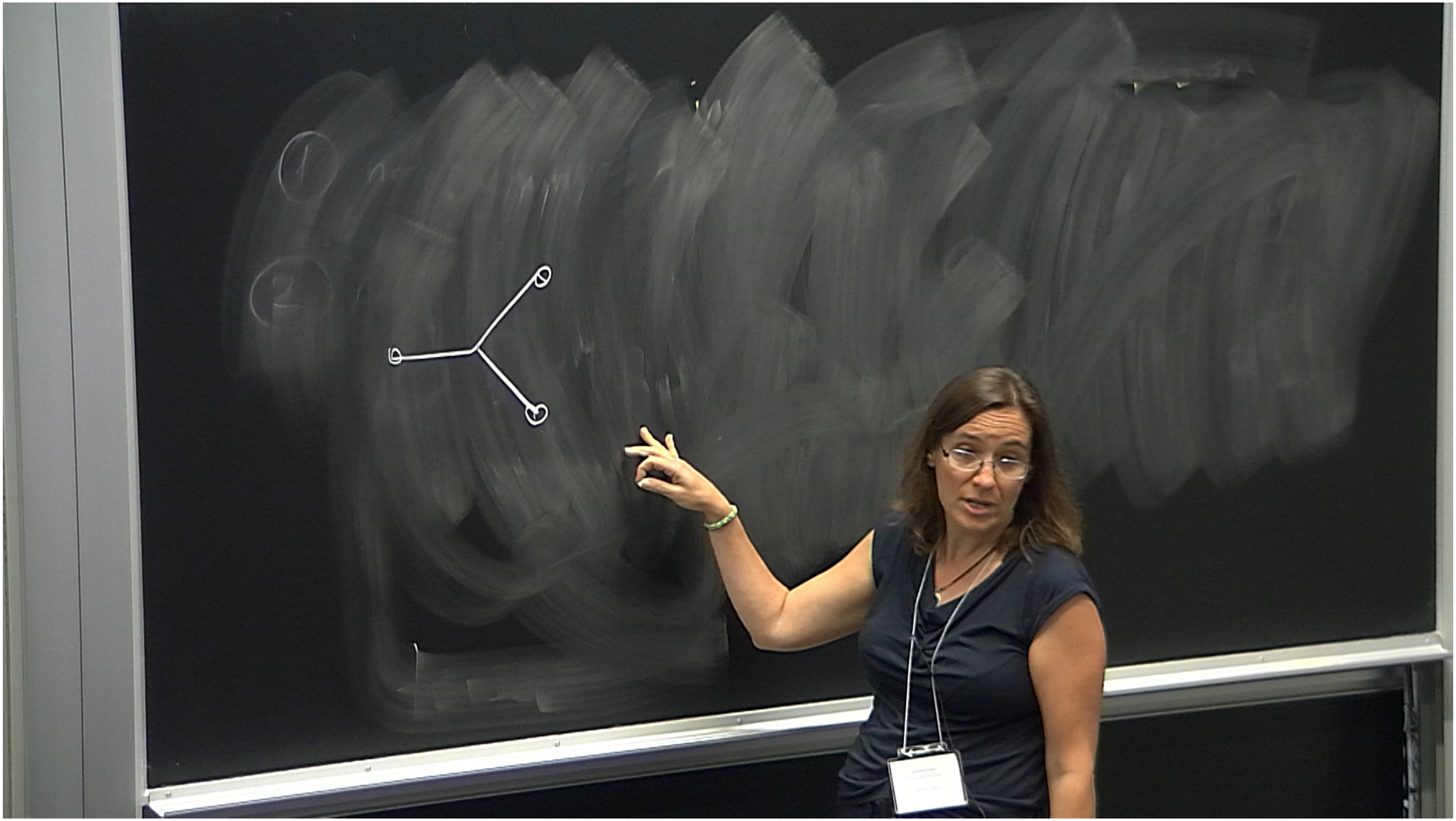
choice $[T^a, T^b] = i\varepsilon^{abc} T^c$ Levi-Civita

choice of representation $(T^a)_{bc} = -i\,\varepsilon^{abc}$

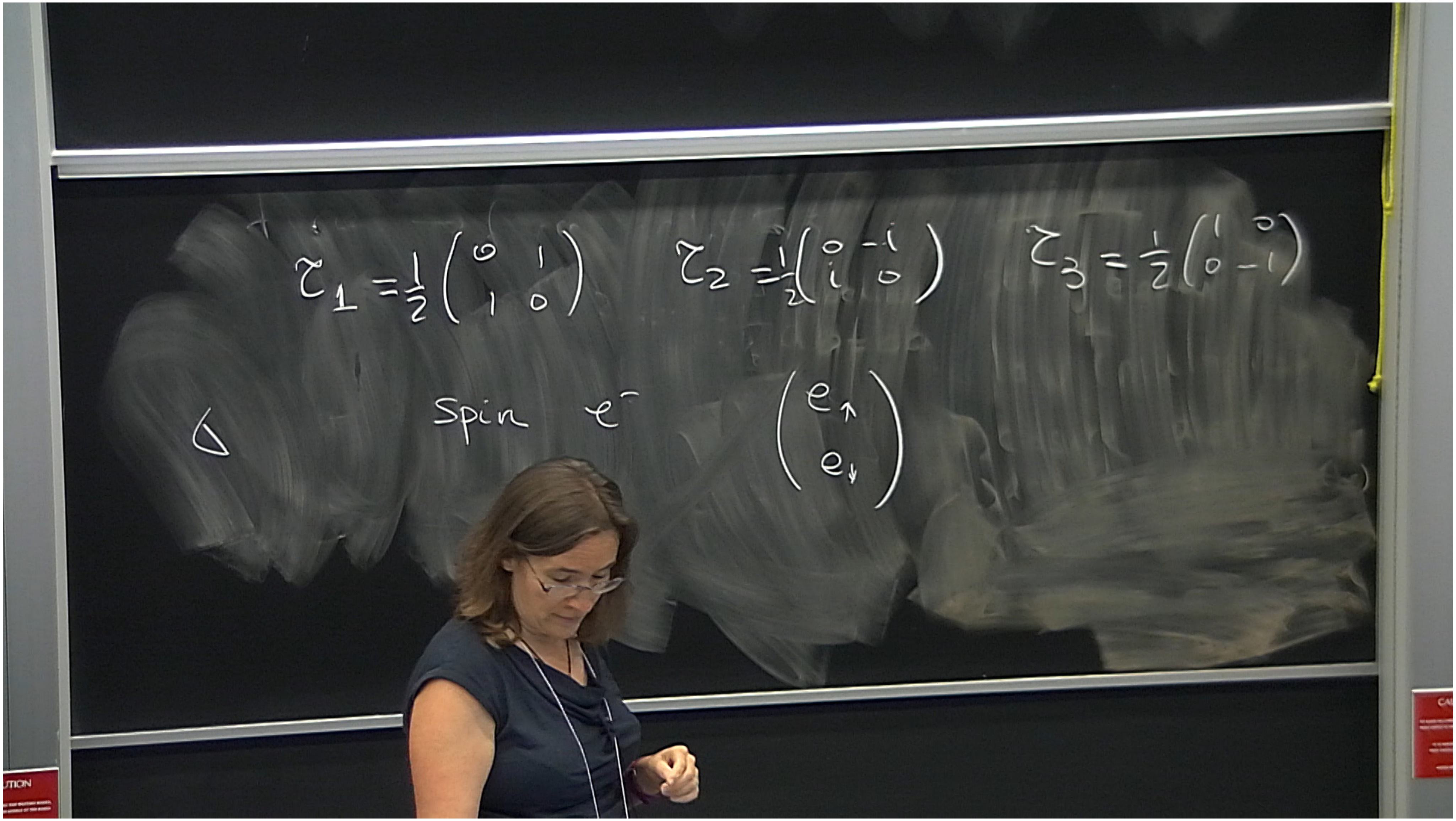
adjoint repr.

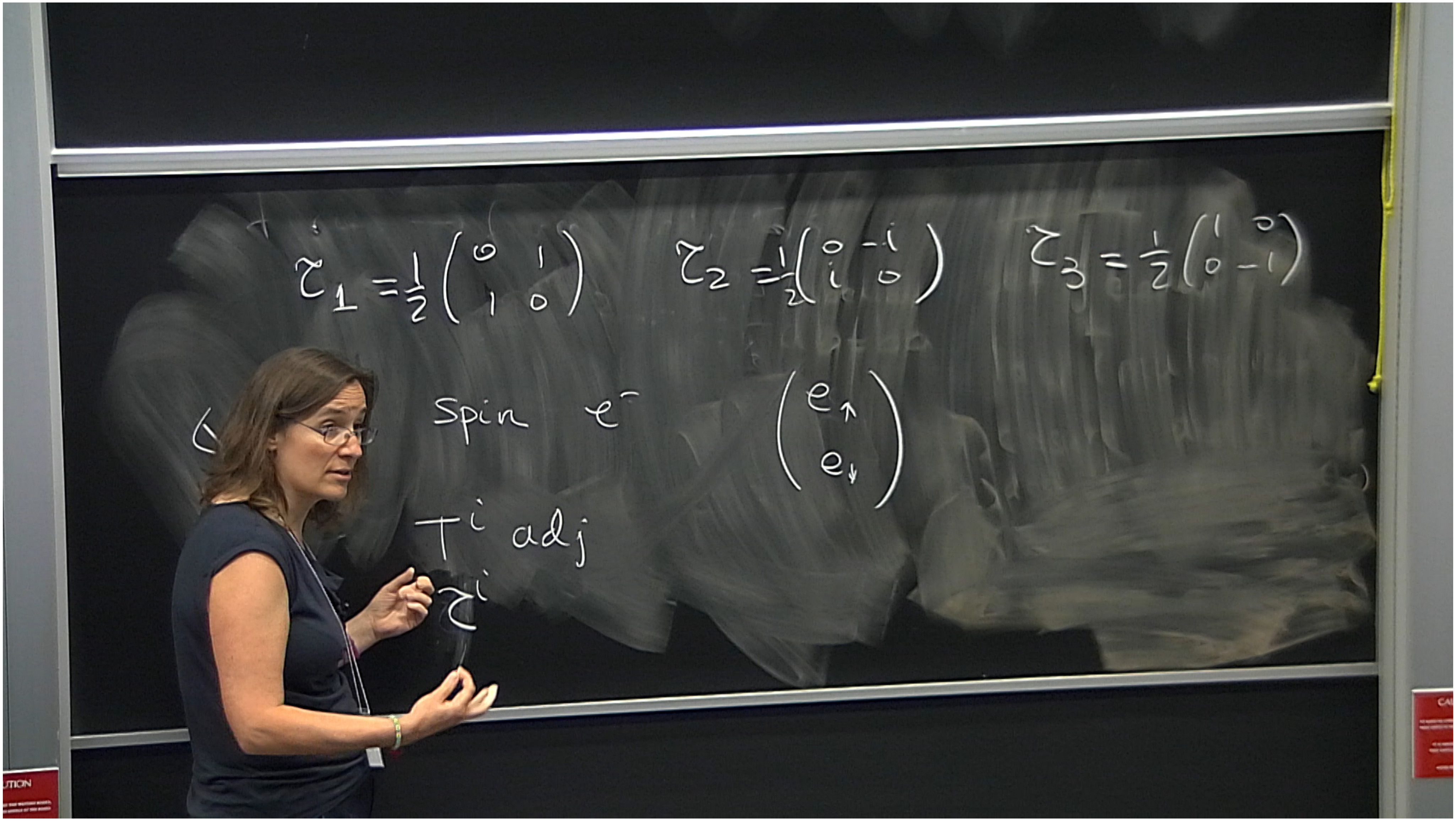
$(J_{x,y,z})$


choice of representation
$(J_{x,y,z})$
$(T^a)_{bc} = -i\,\varepsilon^{abc}$
adjoint repr.
$T^1 = i\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & +1 & 0 \end{pmatrix}$



$$\tau_1 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \tau_2 = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \tau_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$


$\tau_1 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$\tau_2 = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$
$\tau_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
Spin e^-
$\begin{pmatrix} e_\uparrow \\ e_\downarrow \end{pmatrix}$


$\tau_1 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$\tau_2 = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$
$\tau_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
spin e^-
$\begin{pmatrix} e_\uparrow \\ e_\downarrow \end{pmatrix}$
T^i adj
τ^i

$$\tau_1 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \tau_2 = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \tau_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Spin e^- $\begin{pmatrix} e_\uparrow \\ e_\downarrow \end{pmatrix}$

$SU(2)$

T^i adj $\rightarrow$ finite $\longrightarrow$ $SO(3)$

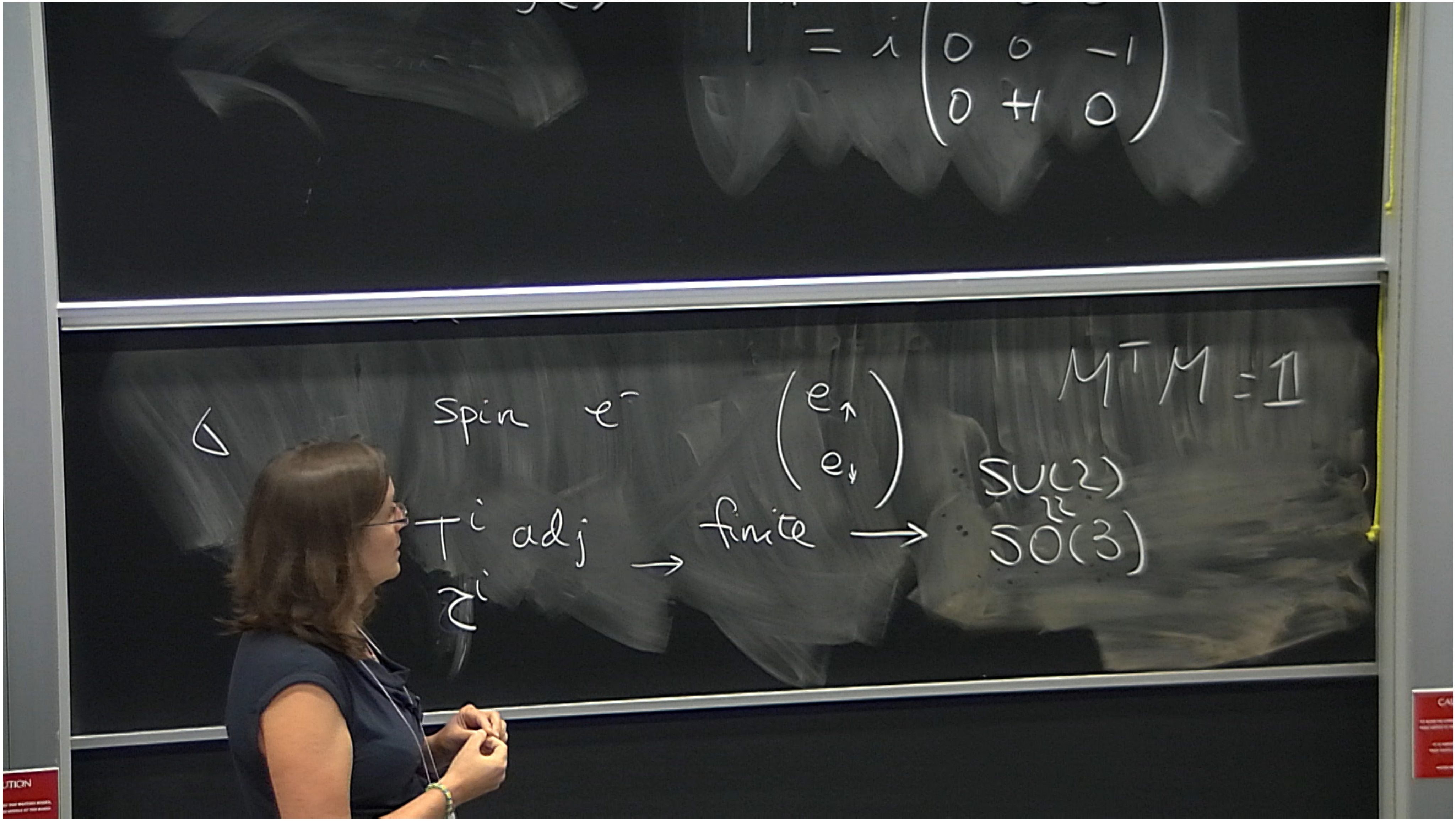
τ^i

$$= i\begin{pmatrix} 0 & 0 & -1 \\ 0 & +1 & 0 \end{pmatrix}$$

Spin e^- $\begin{pmatrix} e_\uparrow \\ e_\downarrow \end{pmatrix}$

T^i adj $\longrightarrow$ finite $\longrightarrow$ $SU(2) \simeq SO(3)$

τ^i

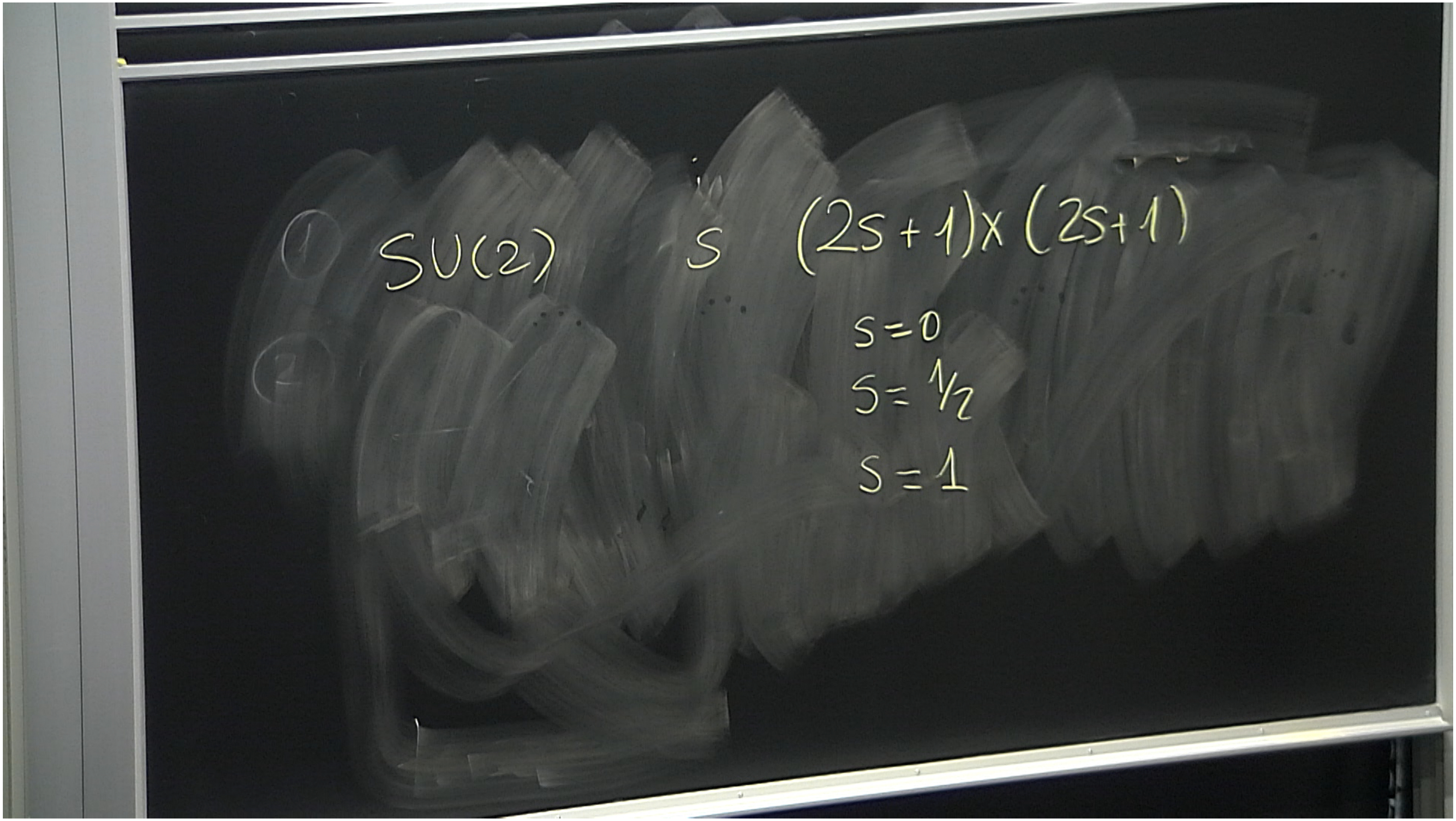

= i 0 0 -1
0 +1 0
Spin e⁻
e↑
e↓
MᵀM = 1
SU(2)
SO(3)
Tⁱ adj
finite

1 $SU(2)$ $\quad s \quad (2s+1)\times(2s+1)$

2

$s = 1/2$

$s = 1$



1 SU(2) S (2S+1) x (2S+1)
S=0
S=1/2
S=1
2

1 $SU(2)$ $\quad S \quad (2S+1)\times(2S+1)$

2

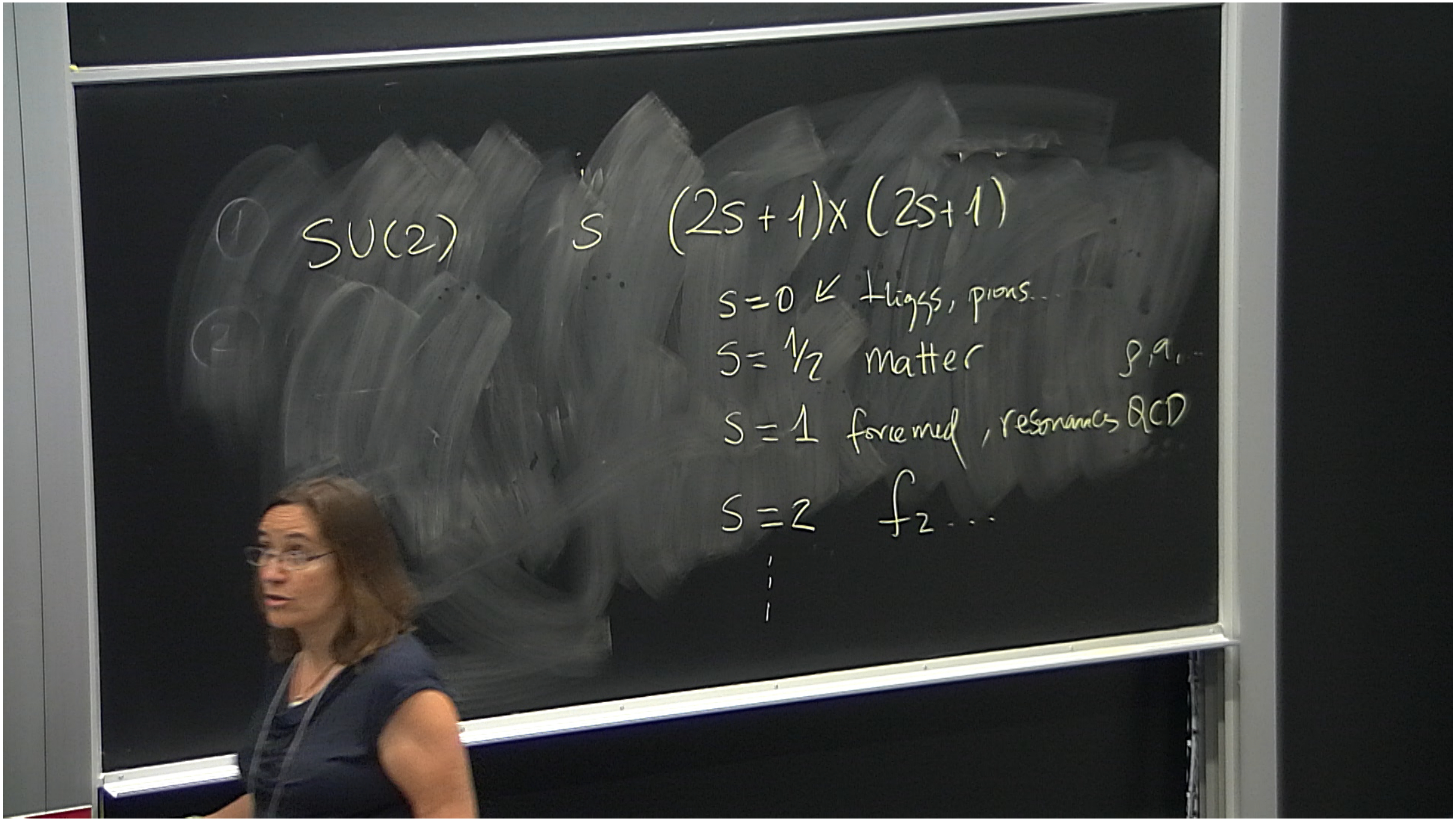
$S=0$ ↙ Higgs, pions...

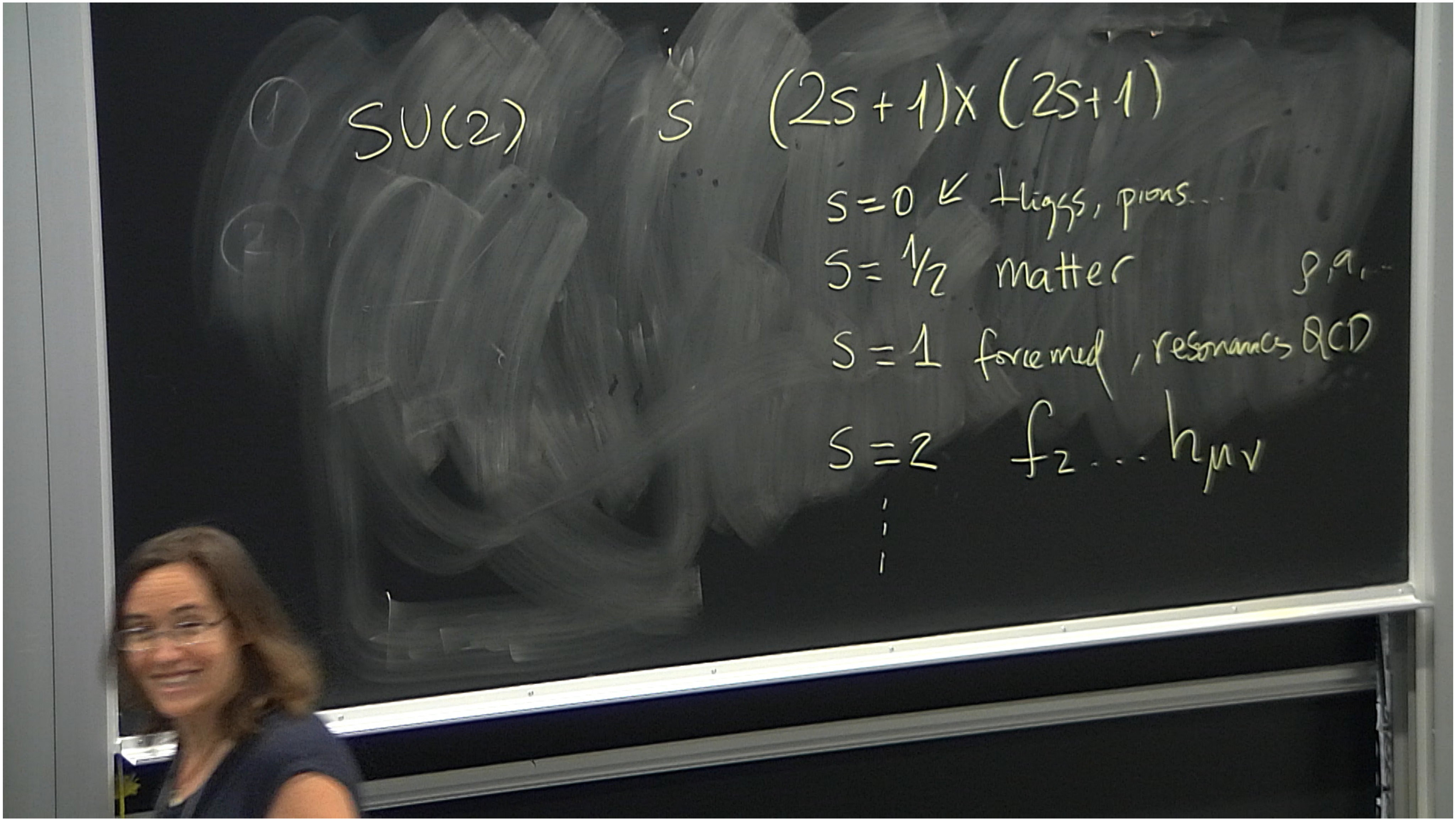
$S=1/2$ matter $\quad p, q, \ldots$

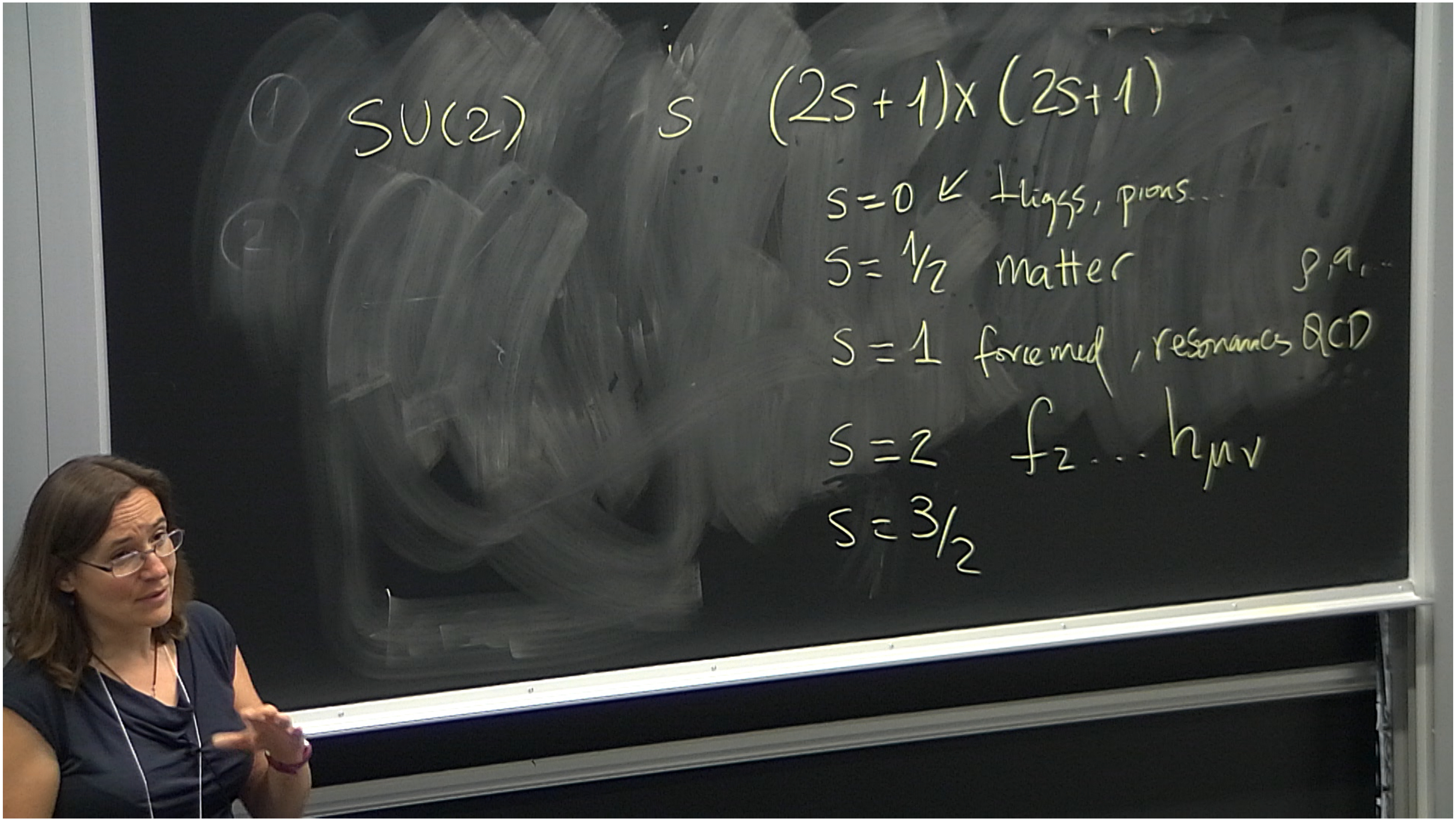
$S=1$ force med, resonances QCD

$S=2$

$\vdots$


SU(2)
S
(2S+1)x(2S+1)
S=0 Higgs, pions...
S=1/2 matter
S=1 force med, resonances QCD
S=2 f2 ...


1 SU(2) S (2S+1)x(2S+1)
S=0 Higgs, pions...
S=1/2 matter ρ, a, ...
S=1 force med, resonances QCD
S=2 f2 ... hμν


SU(2)
S
(2S+1)x(2S+1)
S=0 Higgs, pions...
S=1/2 matter
S=1 force med, resonances QCD
S=2 f2 ... hμν
S=3/2

1 $SU(2)$ $\quad s \quad (2s+1)\times(2s+1)$

2

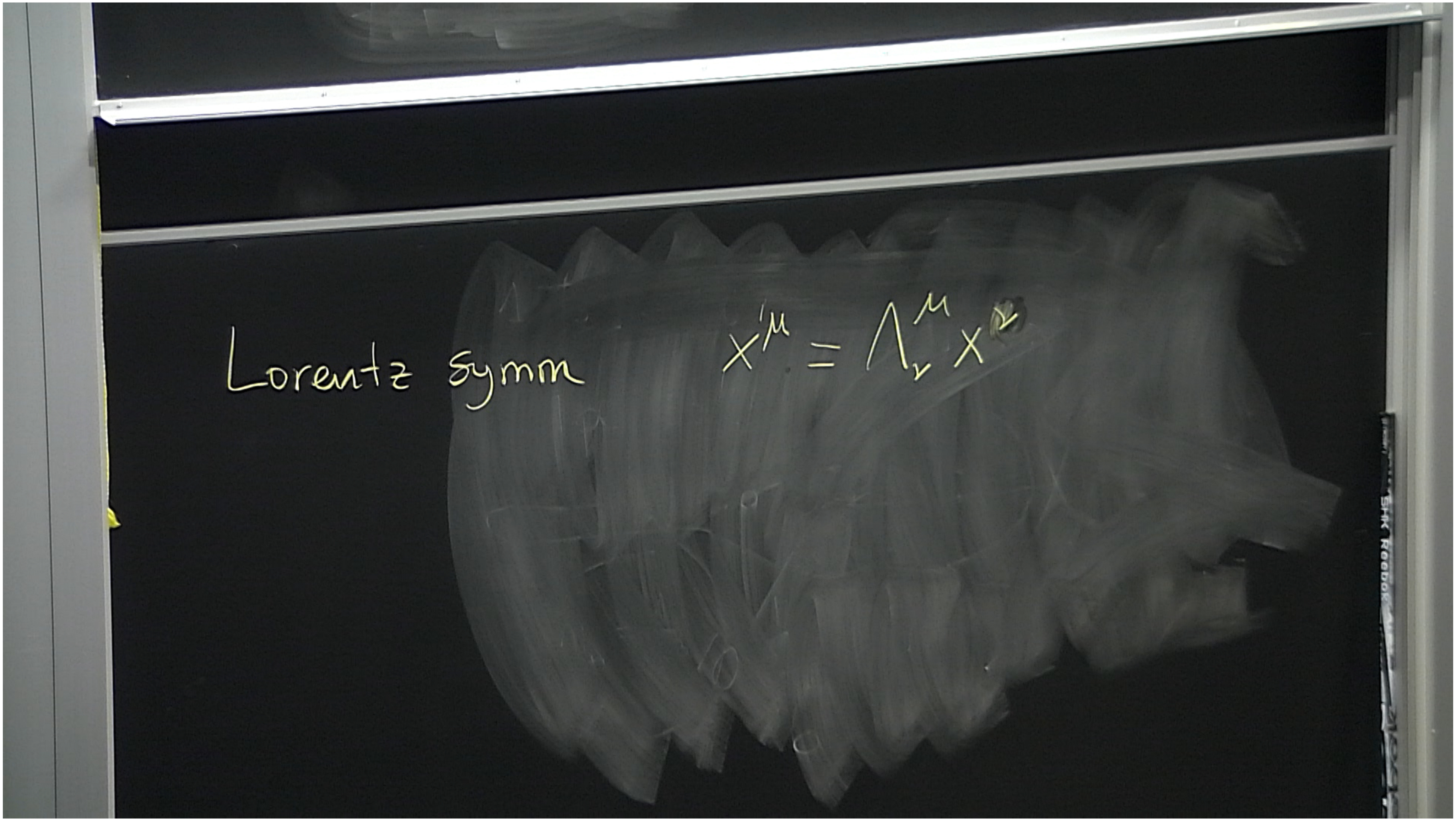
$s=0$ ↙ Higgs, pions...

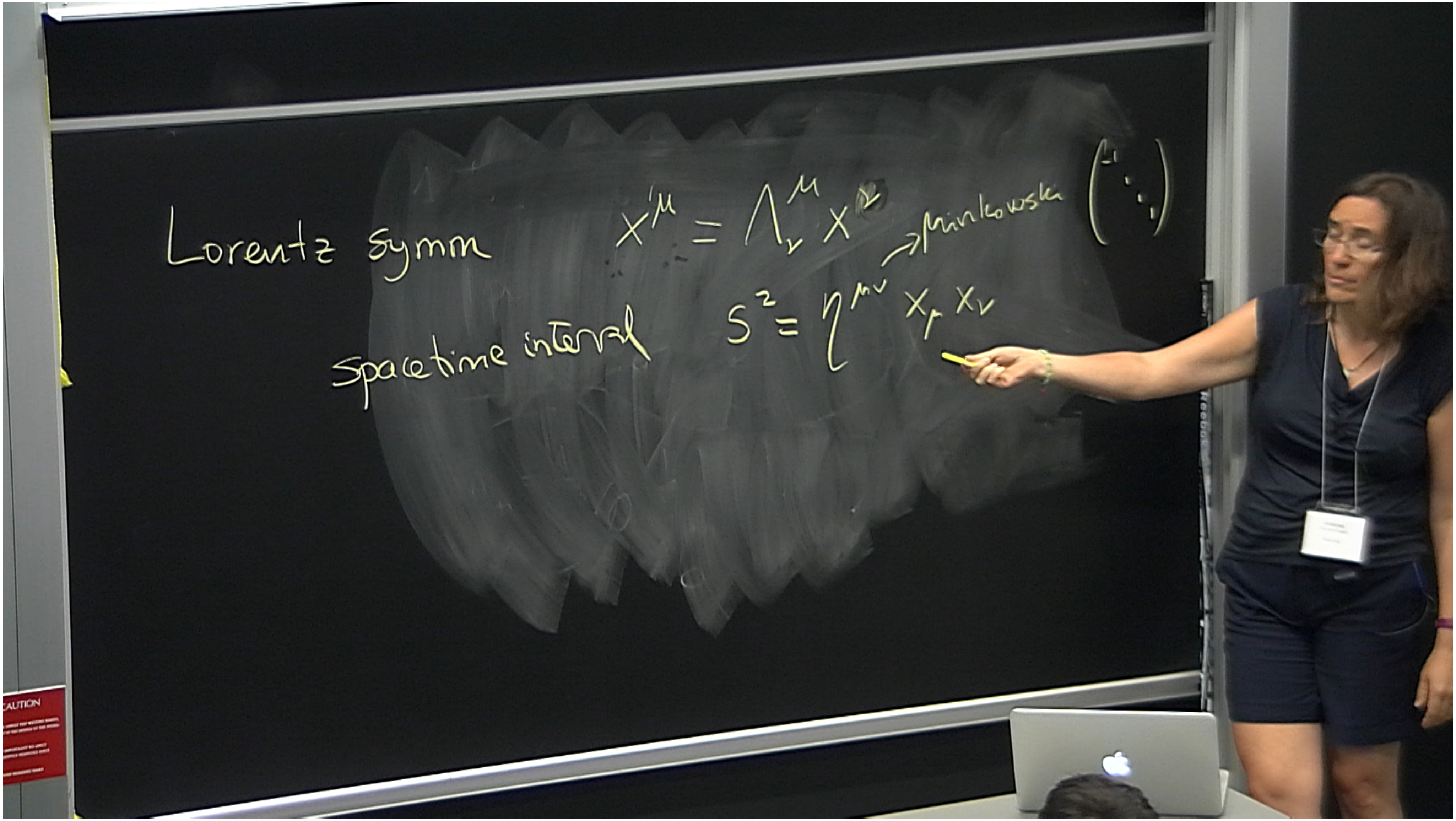
$s=1/2$ matter $\quad p, q, \ldots$

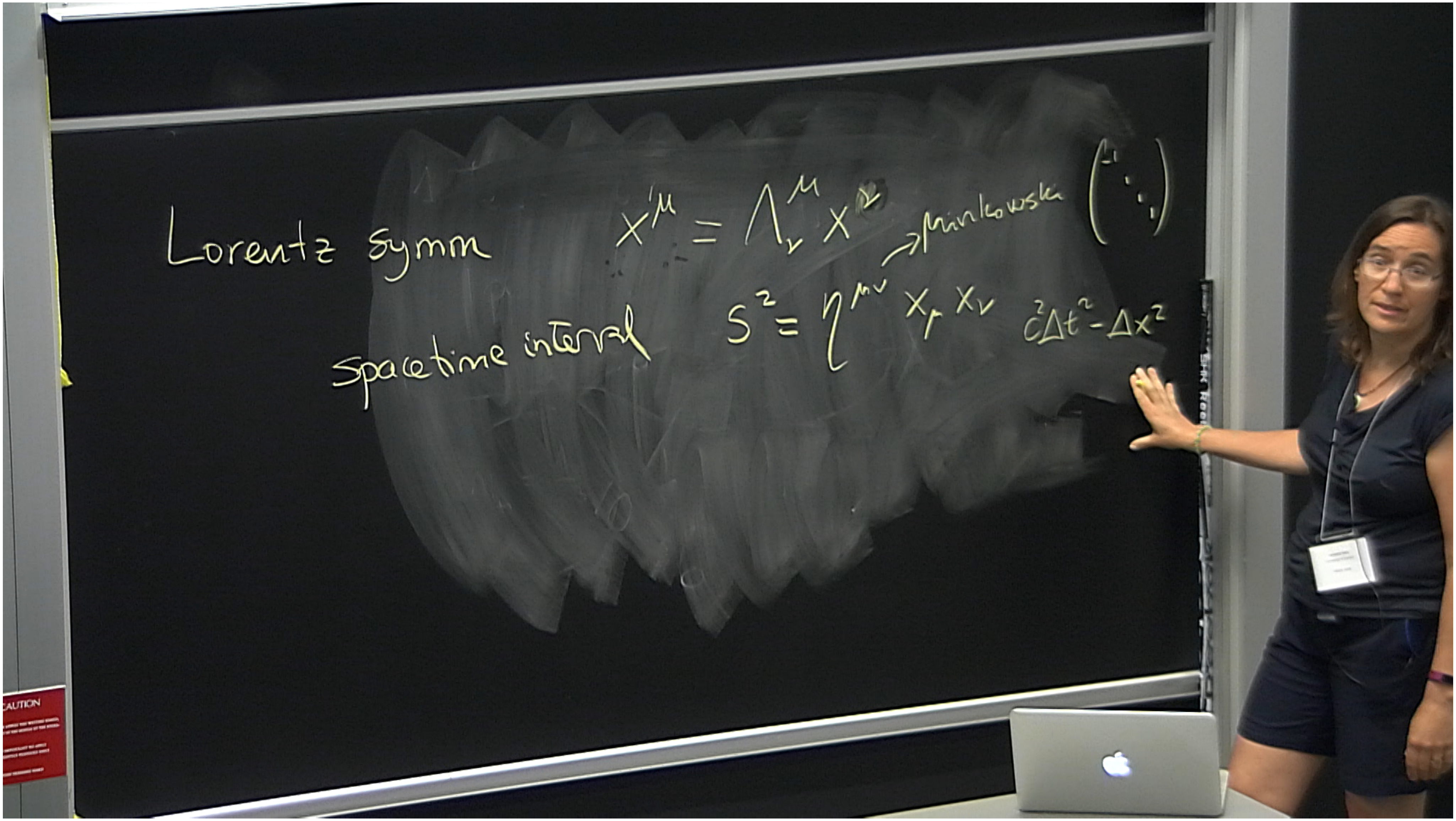
$s=1$ force med, resonances QCD

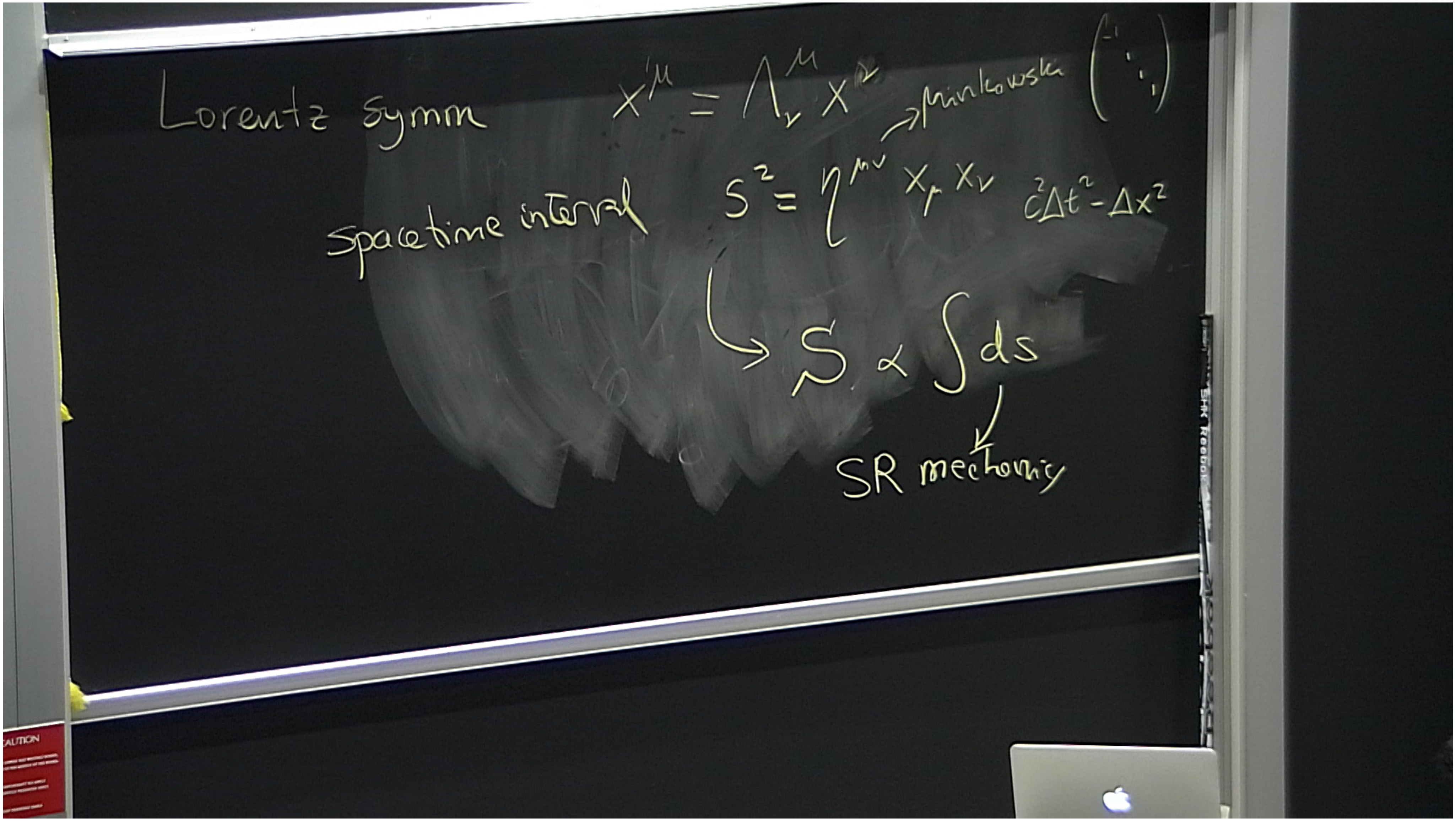
$s=2$ $\quad f_2 \ldots h_{\mu\nu}$

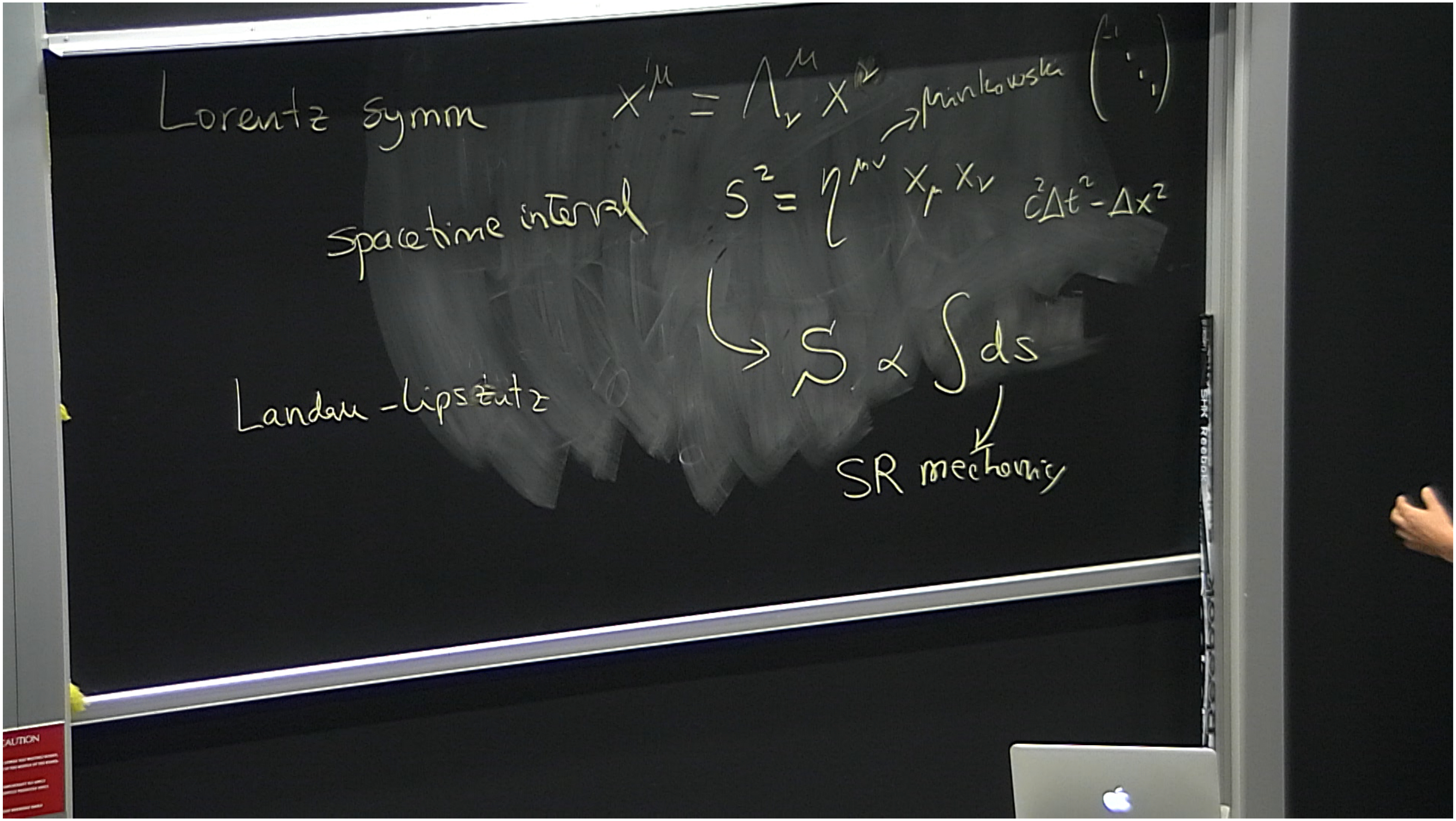
$s=3/2$ gravitino


Lorentz symm
$x'^{\mu} = \Lambda^{\mu}_{\ \nu} x^{\nu}$


Lorentz symm
$x'^{\mu} = \Lambda^{\mu}_{\nu} x^{\nu}$
→ Minkowski
spacetime interval
$s^2 = \eta^{\mu\nu} x_\mu x_\nu$
CAUTION


Lorentz symm
$x'^{\mu} = \Lambda^{\mu}_{\nu} x^{\nu}$
Minkowski
spacetime interval
$s^2 = \eta^{\mu\nu} x_\mu x_\nu$
$c^2\Delta t^2 - \Delta x^2$


Lorentz symm
$x'^{\mu} = \Lambda^{\mu}_{\nu} x^{\nu}$
Minkowski
spacetime interval
$S^2 = \eta^{\mu\nu} x_\mu x_\nu$
$c^2\Delta t^2 - \Delta x^2$
$S \propto \int ds$
SR mechanics


Lorentz symm
$x'^{\mu} = \Lambda^{\mu}_{\nu} x^{\nu}$
Minkowski
spacetime interval
$s^2 = \eta^{\mu\nu} x_\mu x_\nu$
$c^2\Delta t^2 - \Delta x^2$
$S \propto \int ds$
SR mechanics
Landau-Lipschutz
CAUTION

$$\eta_{\mu\nu} \Lambda^{\mu}{}_{\alpha} \Lambda^{\nu}{}_{\beta} = \eta_{\alpha\beta}$$

$$\eta_{\mu\nu} \Lambda^{\mu}{}_{\alpha} \Lambda^{\nu}{}_{\beta} = \eta_{\alpha\beta}$$

$$O(4) \mapsto O(3,1)$$

$$\eta_{\mu\nu} \Lambda^{\mu}{}_{\alpha} \Lambda^{\nu}{}_{\beta} = \eta_{\alpha\beta}$$

$$O(4) \mapsto O(3,1)$$

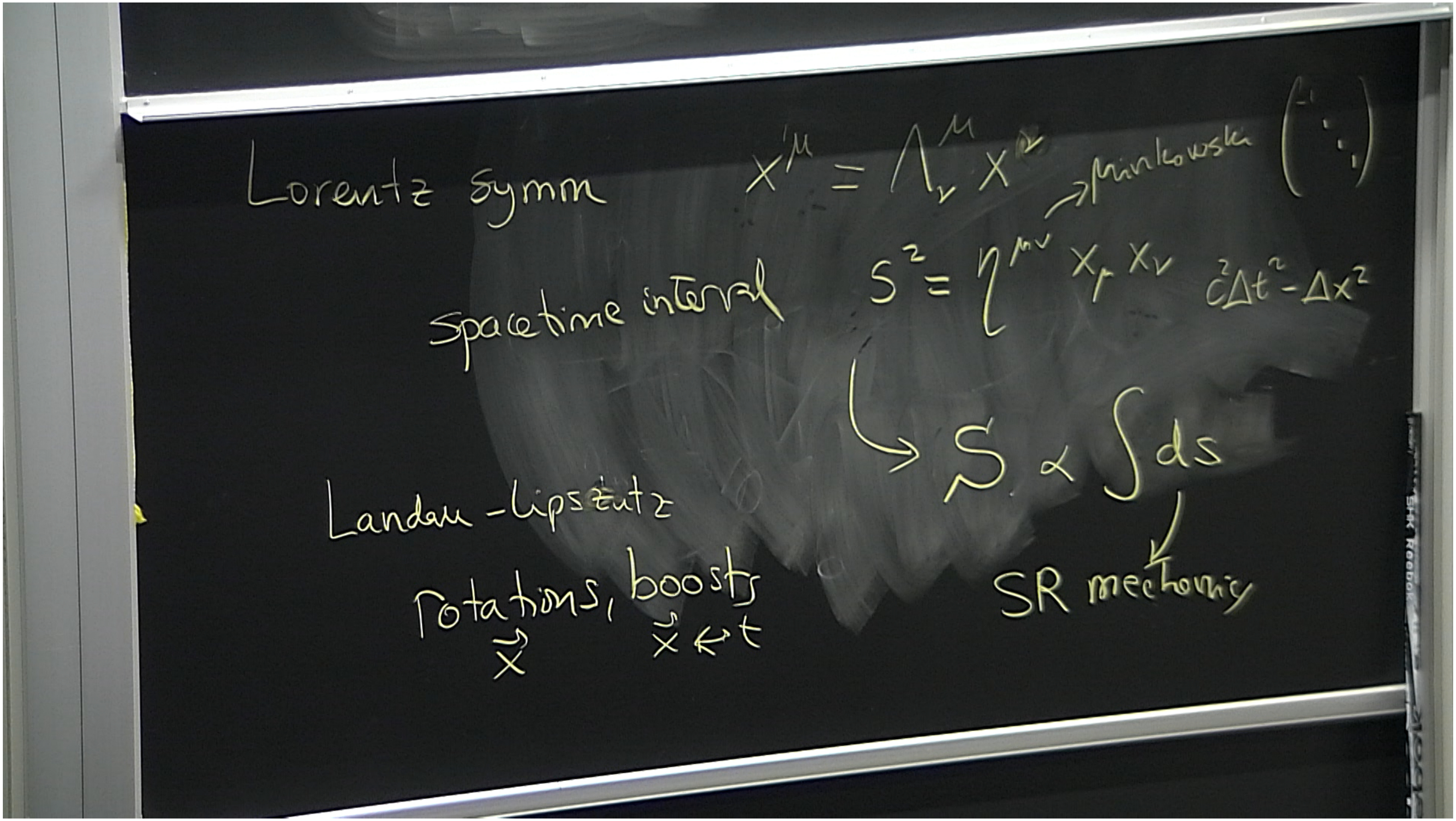
$$\Lambda^T \eta \Lambda = \eta$$

$$\eta_{\mu\nu} \Lambda^{\mu}{}_{\alpha} \Lambda^{\nu}{}_{\beta} = \eta_{\alpha\beta}$$

$$O(4) \mapsto O(3,1)$$

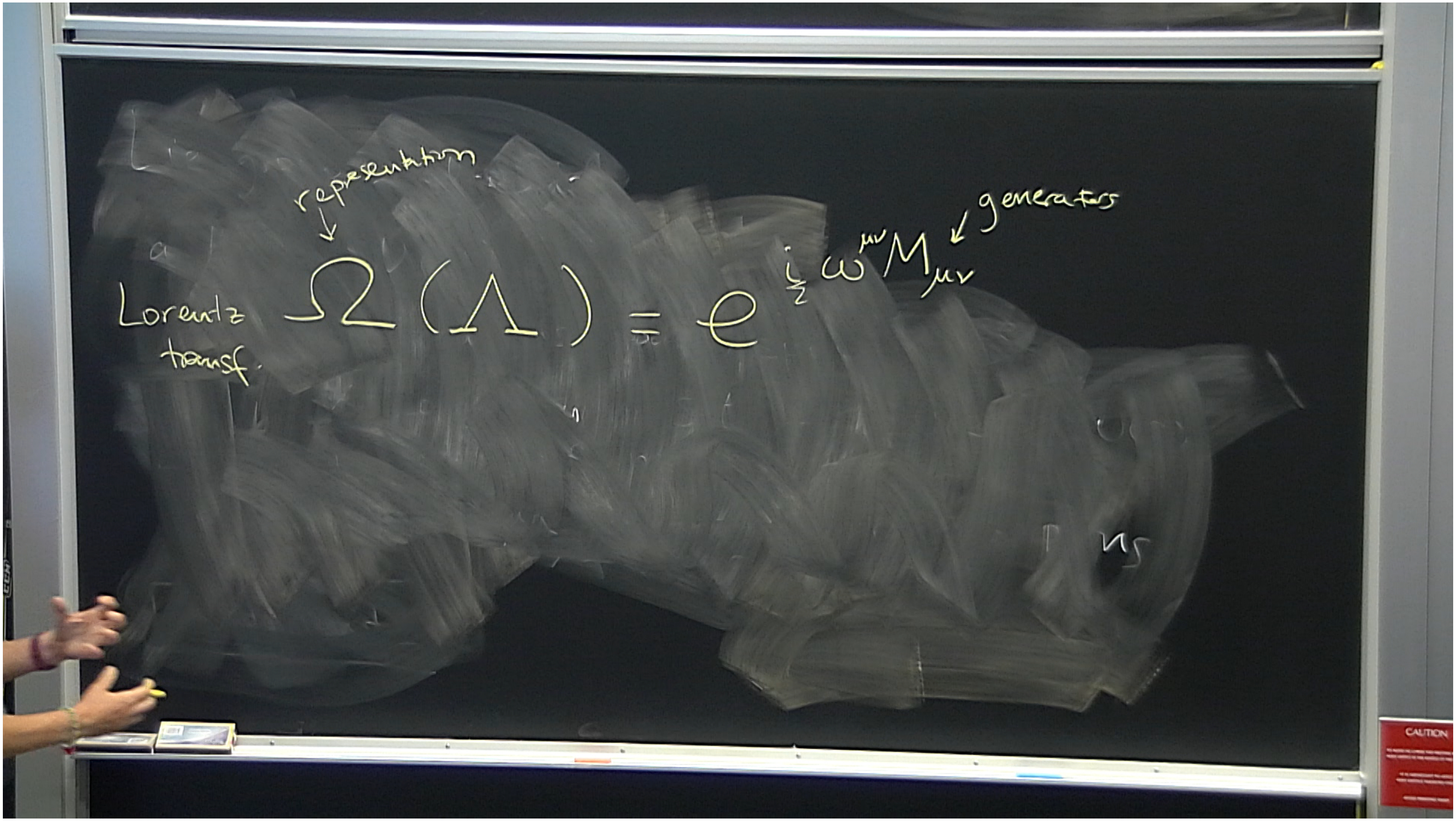
$$\Lambda$$

$$\Lambda^{T} \eta \Lambda = \eta$$


Lorentz symm
$x'^{\mu} = \Lambda^{\mu}_{\nu} x^{\nu}$
→ Minkowski
spacetime interval
$s^2 = \eta^{\mu\nu} x_\mu x_\nu$
$c^2\Delta t^2 - \Delta x^2$
$S \propto \int ds$
SR mechanics
Landau-Lipschutz
rotations, boosts
$\vec{x}$
$\vec{x} \leftrightarrow t$

representation

$$\Omega(\Lambda) = e^{\frac{i}{2}\omega^{\mu\nu}M_{\mu\nu}}$$

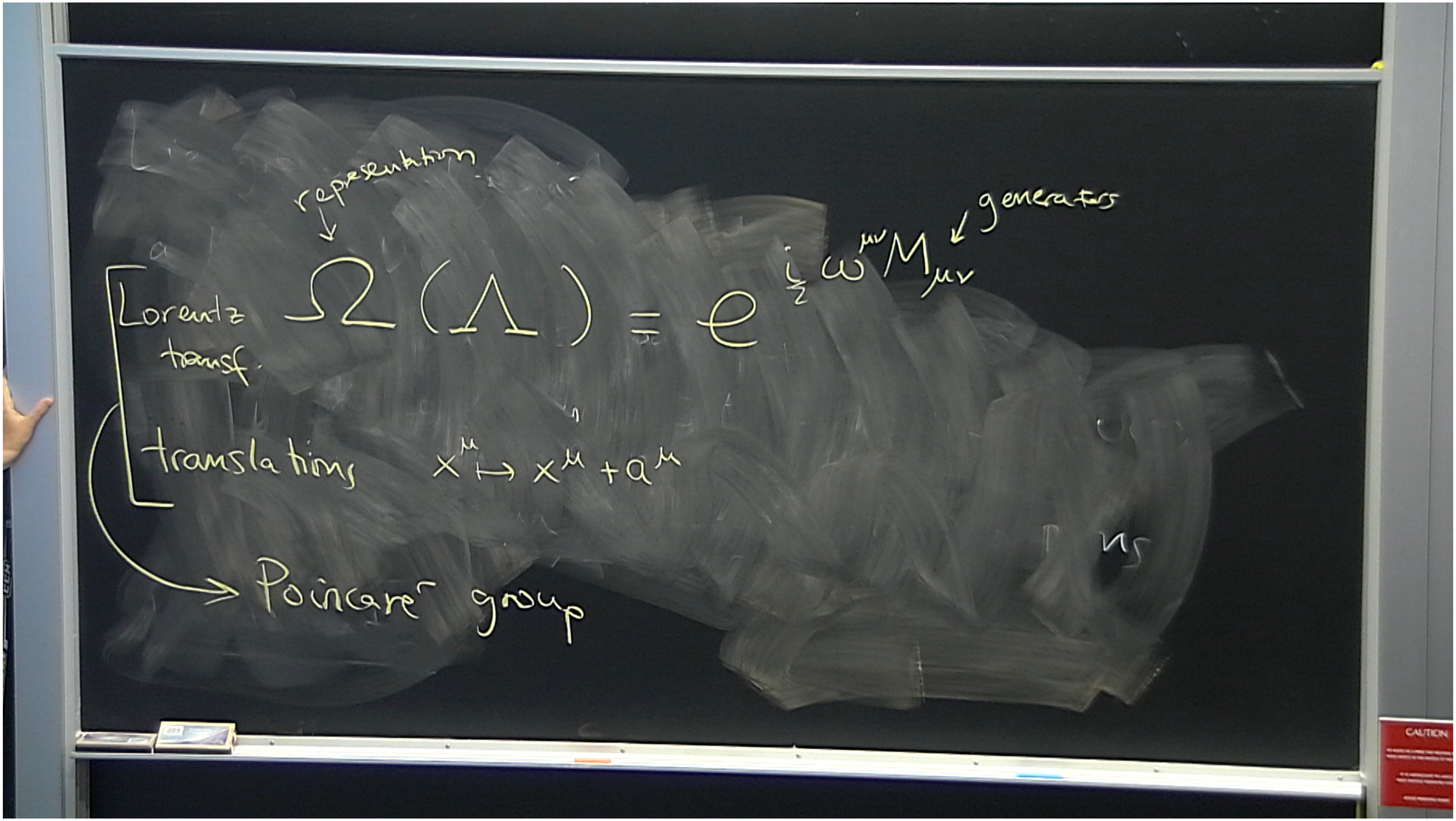

representation
generators
Lorentz transf.
$\Omega(\Lambda) \equiv e^{\frac{i}{2}\omega^{\mu\nu}M_{\mu\nu}}$

representation

generators

Lorentz transf. $\Omega(\Lambda) = e^{\frac{i}{2}\omega^{\mu\nu}M_{\mu\nu}}$

translations $x^\mu \mapsto x^\mu + a^\mu$


representation
generators
Lorentz transf.
$\Omega(\Lambda) = e^{\frac{i}{2}\omega^{\mu\nu}M_{\mu\nu}}$
translations $x^\mu \mapsto x^\mu + a^\mu$
Poincaré group

representation

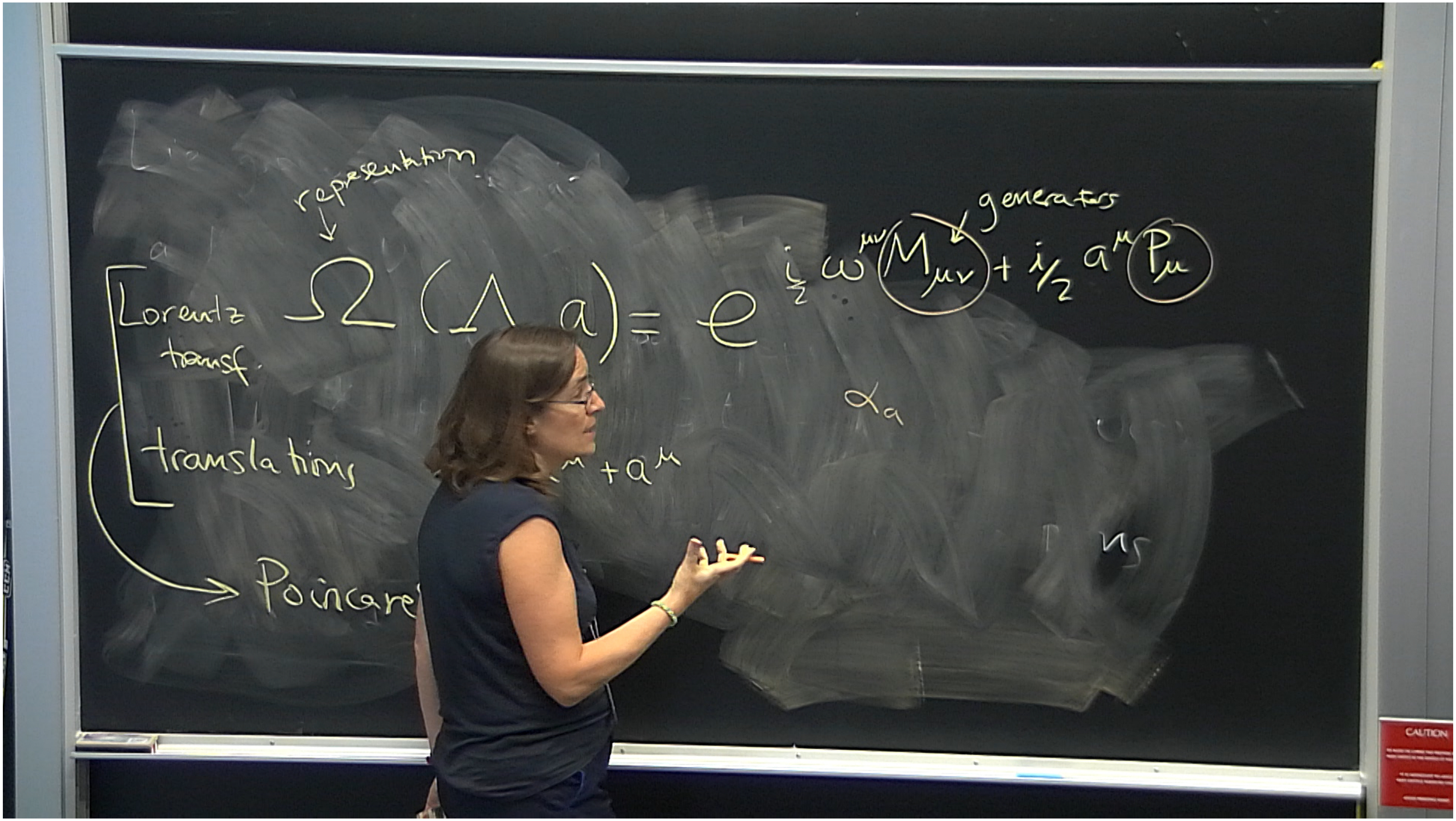
$$\Omega(\Lambda, a) = e^{\frac{i}{2}\omega^{\mu\nu}M_{\mu\nu} + \frac{i}{2}a^{\mu}P_{\mu}}$$

generators

Lorentz transf.

translations $x^{\mu} \mapsto x^{\mu} + a^{\mu}$

Poincaré group


representation
generators
Lorentz transf.
$\Omega(\Lambda\ a) \equiv e^{\frac{i}{2}\omega^{\mu\nu} M_{\mu\nu} + i/2\, a^\mu P_\mu}$
translations
$+a^\mu$
α_a
Poincaré

representation

generators

Lorentz transf.

$$\Omega(\Lambda, a) = e^{\frac{i}{2}\omega^{\mu\nu} M_{\mu\nu} + \frac{i}{2} a^{\mu} P_{\mu}}$$

α_a

translations $\quad x^{\mu} \mapsto x^{\mu} + a^{\mu}$

$\boxed{\text{Poincaré group}}$

representation

generators

$$\Omega(\Lambda, a) \equiv e^{\frac{i}{2}\omega^{\mu\nu} M_{\mu\nu} + \frac{i}{2} a^{\mu} P_{\mu}}$$

Lorentz transf.

translations $x^{\mu} \mapsto x^{\mu} + a^{\mu}$

$\boxed{\text{Poincaré group}}$

α_a

$[M, M]$

$[P, P]$

$[M, P]$

Lorentz transf.

$$\Omega(\Lambda, a) = e^{\frac{i}{2}\omega^{\mu\nu} M_{\mu\nu} + \frac{i}{2} a^{\mu} P_{\mu}}$$

translations $x^{\mu} \mapsto x^{\mu} + a^{\mu}$

$\boxed{\text{Poincaré group}}$

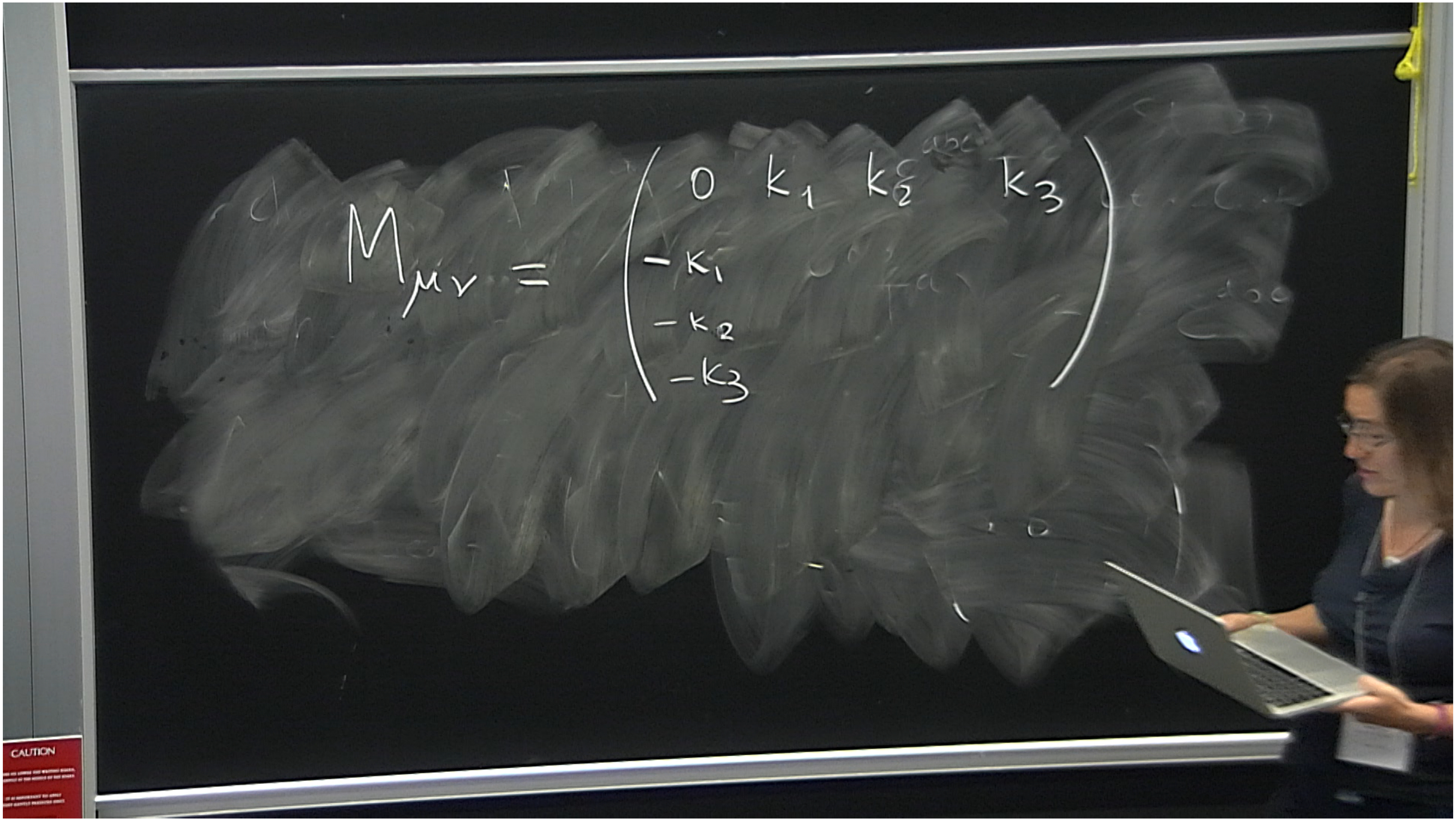
α_a

$[M, M]$

$[P, P]$

$[M, P]$

Lie algebra


$M_{\mu\nu} = \begin{pmatrix} 0 & k_1 & k_2 & k_3 \\ -k_1 & & & \\ -k_2 & & & \\ -k_3 & & & \end{pmatrix}$
CAUTION

$$M_{\mu\nu} = \begin{pmatrix} 0 & K_1 & K_2 & K_3 \\ -K_1 & 0 & -J_3 & J_2 \\ -K_2 & & 0 & -J_1 \\ -K_3 & & & 0 \end{pmatrix}$$

$i = 1, 2, 3$

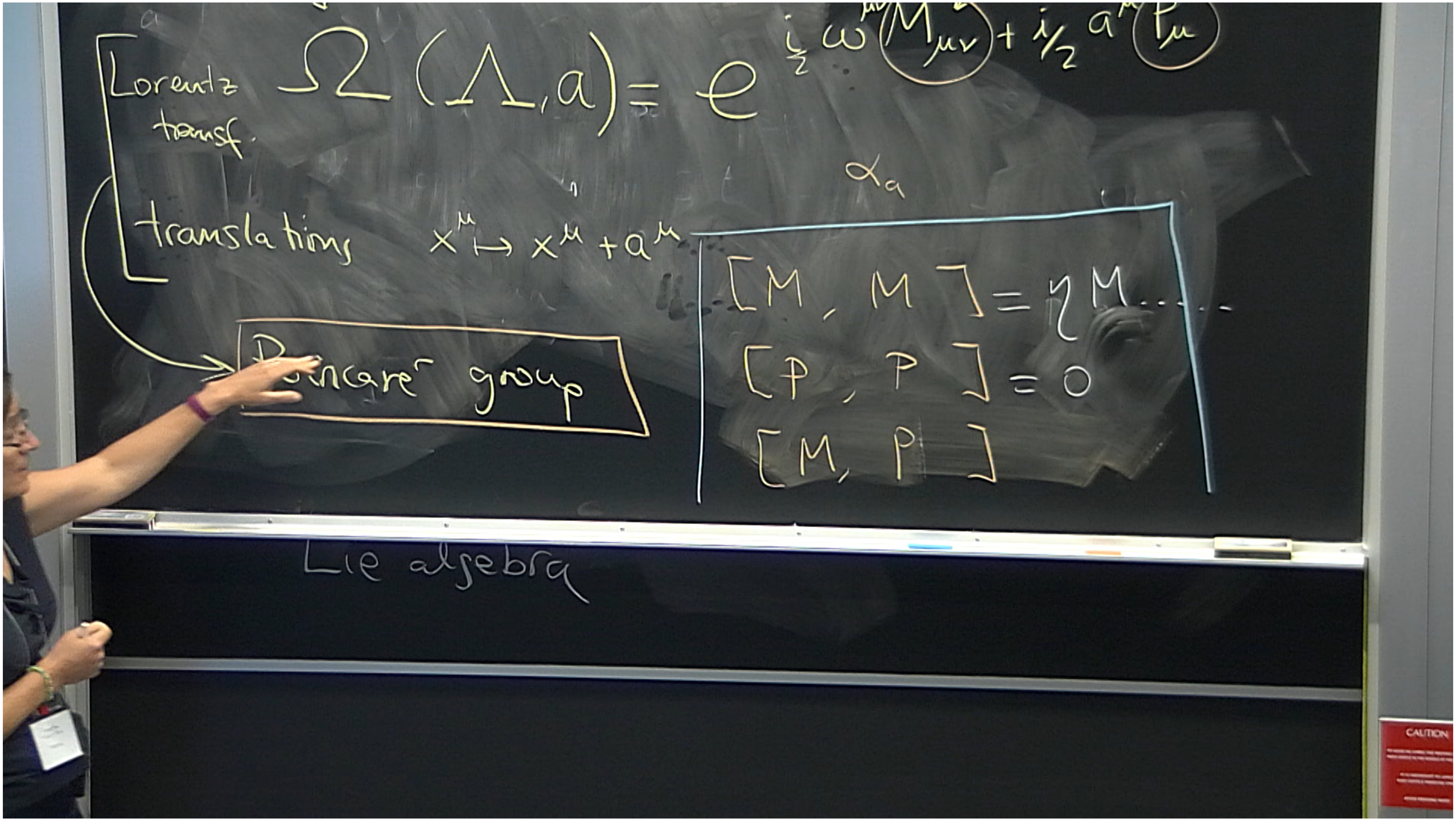
M_{0i}

$$M_{\mu\nu} = \begin{pmatrix} 0 & K_1 & K_2 & K_3 \\ -K_1 & 0 & -J_3 & J_2 \\ -K_2 & & 0 & -J_1 \\ -K_3 & & & 0 \end{pmatrix}$$

$i = 1, 2, 3$

$$M_{0i} = K_i$$

$$M_{ij} = -i\,\varepsilon_{ijk} J_k$$


Lorentz transf.
$\Omega(\Lambda, a) = e^{\frac{i}{2}\omega^{\mu\nu} M_{\mu\nu} + \frac{i}{2} a^{\mu} P_{\mu}}$
translations $x^{\mu} \mapsto x^{\mu} + a^{\mu}$
α_a
$[M, M] = \eta M \ldots$
$[P, P] = 0$
$[M, P]$
Poincaré group
Lie algebra
CAUTION

Lorentz transf.

translations $x^\mu \mapsto x^\mu + a^\mu$

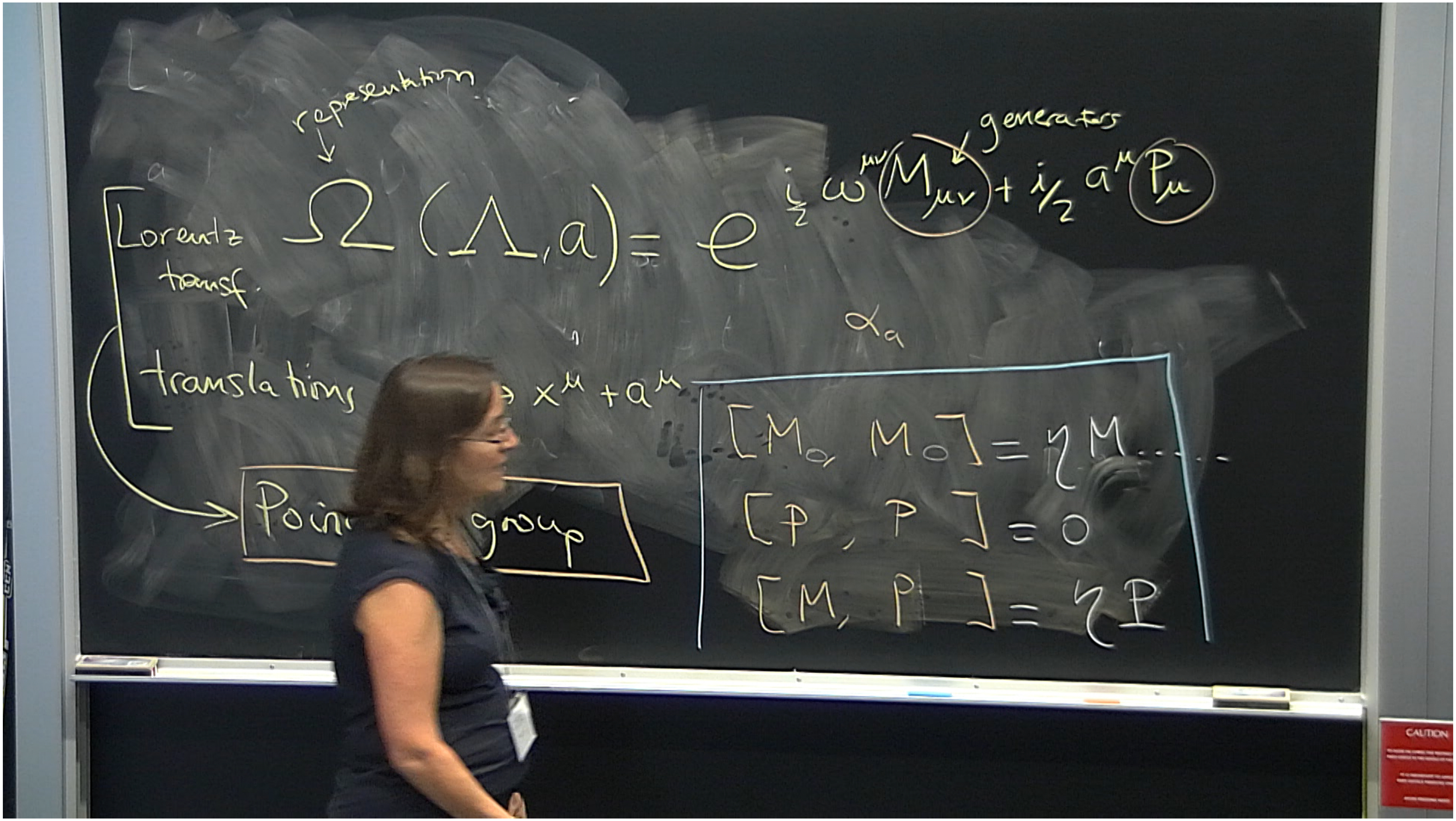
$$\Omega(\Lambda, a) = e^{\frac{i}{2}\omega^{\mu\nu} M_{\mu\nu} + i/2\, a^\mu P_\mu}$$

$\boxed{\text{Poincaré group}}$

α_a

$$[M, M] = \eta M \ldots$$
$$[P, P] = 0$$
$$[M, P] = \eta P$$

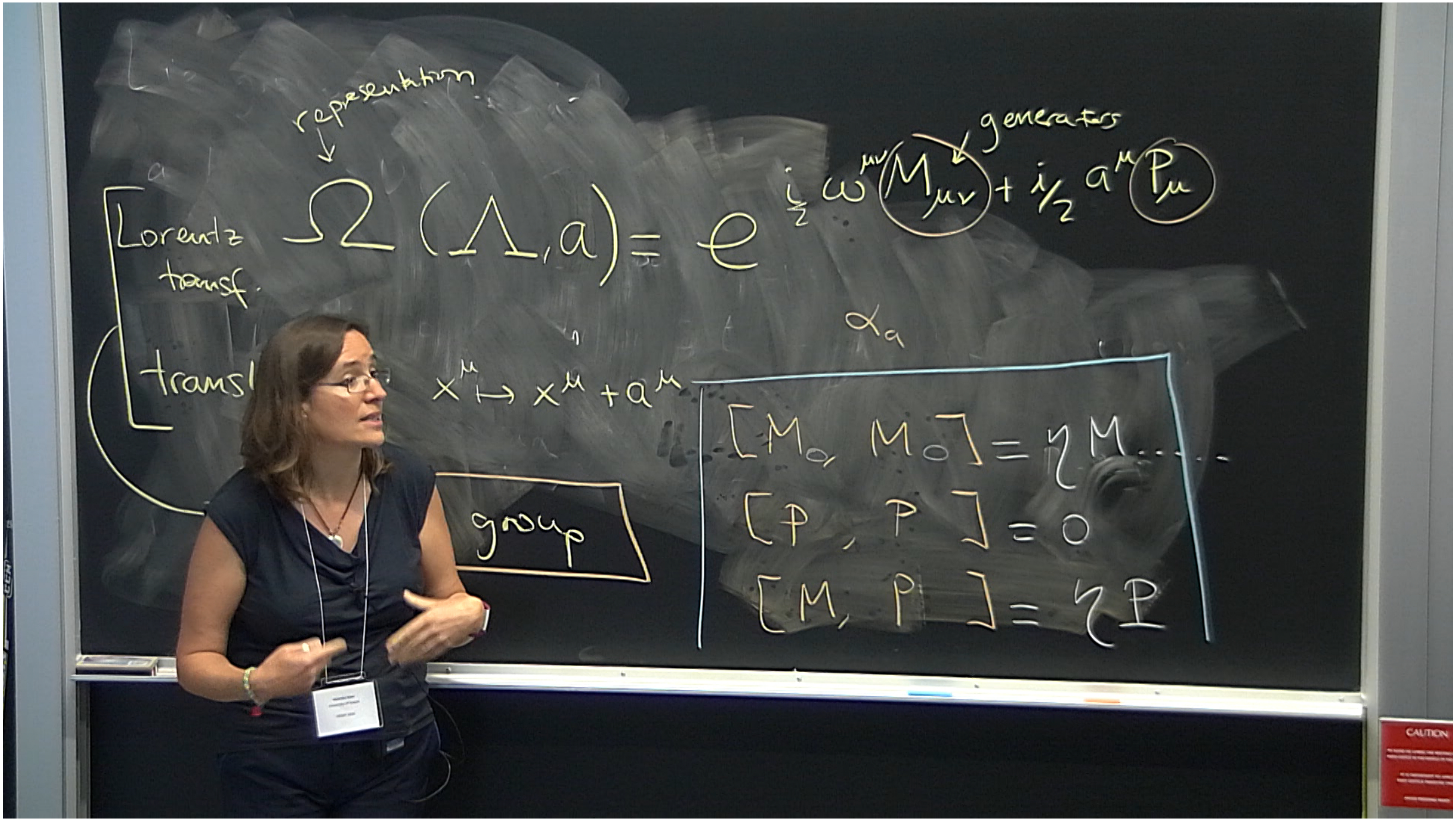
Lie algebra



representation
generators
Lorentz transf.
$\Omega(\Lambda, a) = e^{\frac{i}{2}\omega^{\mu\nu} M_{\mu\nu} + \frac{i}{2} a^\mu P_\mu}$
α_a
translations
$x^\mu + a^\mu$
group
$[M, M] = \eta M \ldots$
$[P, P] = 0$
$[M, P] = \eta P$

$$M_{\mu\nu} = \begin{pmatrix} 0 & k_1 & k_2 & k_3 \\ -k_1 & 0 & -J_3 & J_2 \\ -k_2 & & 0 & -J_1 \\ -k_3 & & & 0 \end{pmatrix}$$

$$[P_\mu, M_{\alpha\beta}] = i\left(\eta_{\alpha\mu}P_\beta - \eta_{\beta\mu}P_\alpha\right)$$


representation
generators
Lorentz transf.
transl.
$\Omega(\Lambda, a) = e^{\frac{i}{2}\omega^{\mu\nu} M_{\mu\nu} + i/2\, a^\mu P_\mu}$
α_a
$x^\mu \mapsto x^\mu + a^\mu$
group
$[M, M] = \eta M \ldots$
$[P, P] = 0$
$[M, P] = \eta P$

$$[J, J] = \ldots J$$

$$[K, K] = \ldots J$$

$$[K, J] = \ldots K$$

representation

generators

$$\Omega(\Lambda, a) = e^{\frac{i}{2}\omega^{\mu\nu} M_{\mu\nu} + \frac{i}{2} a^{\mu} P_{\mu}}$$

Lorentz transf.

translations $x^\mu \mapsto x^\mu + a^\mu$

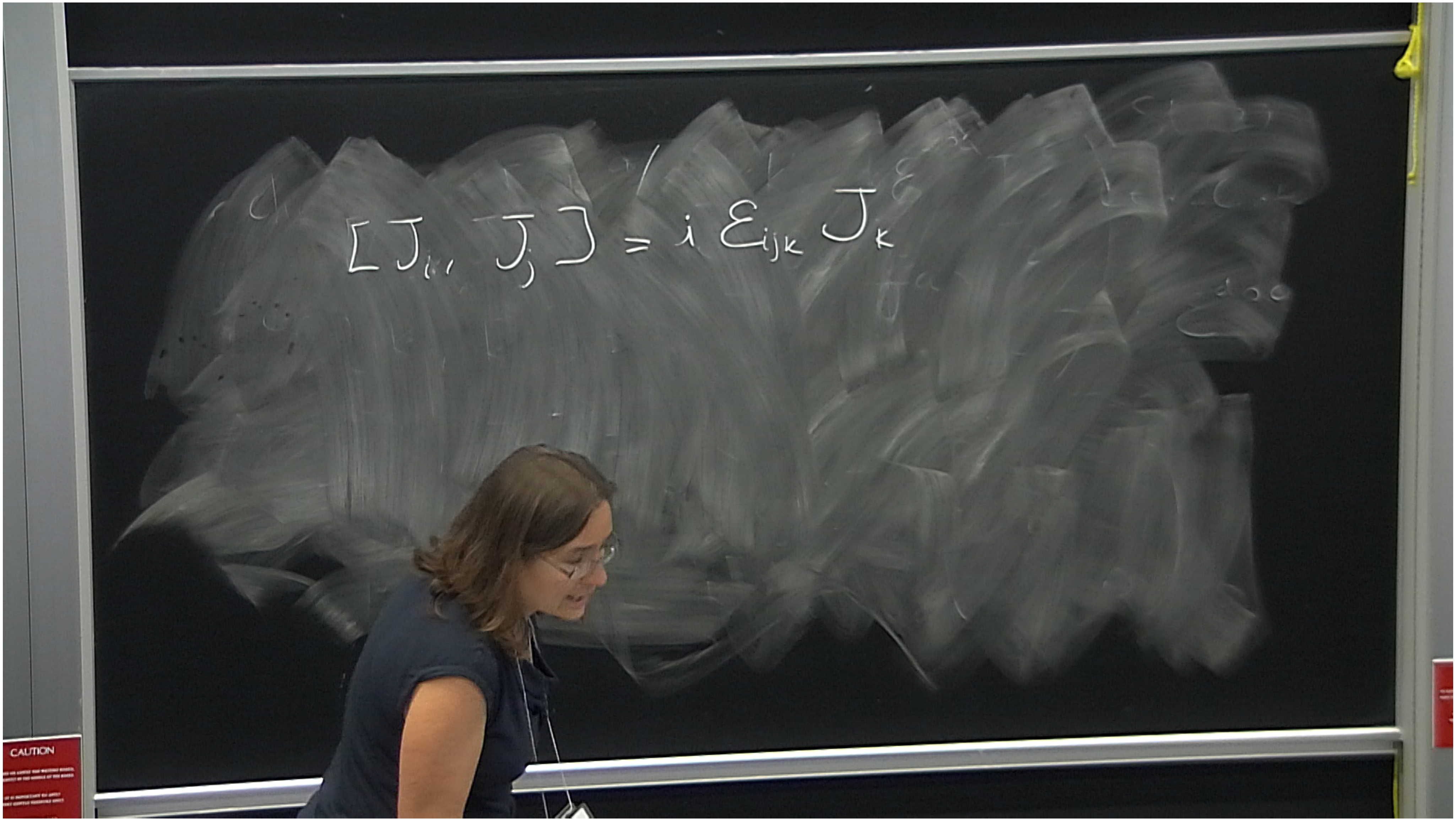
$\boxed{\text{Poincaré group}}$

α_a $SO(3,1)$

$$[M\ ,\ M\ ] = \eta M \ldots$$

$$[P\ ,\ P\ ] = 0$$

$$[M\ ,\ P\ ] = \eta P$$


$[J_i, J_j] = i\,\varepsilon_{ijk} J_k$
CAUTION

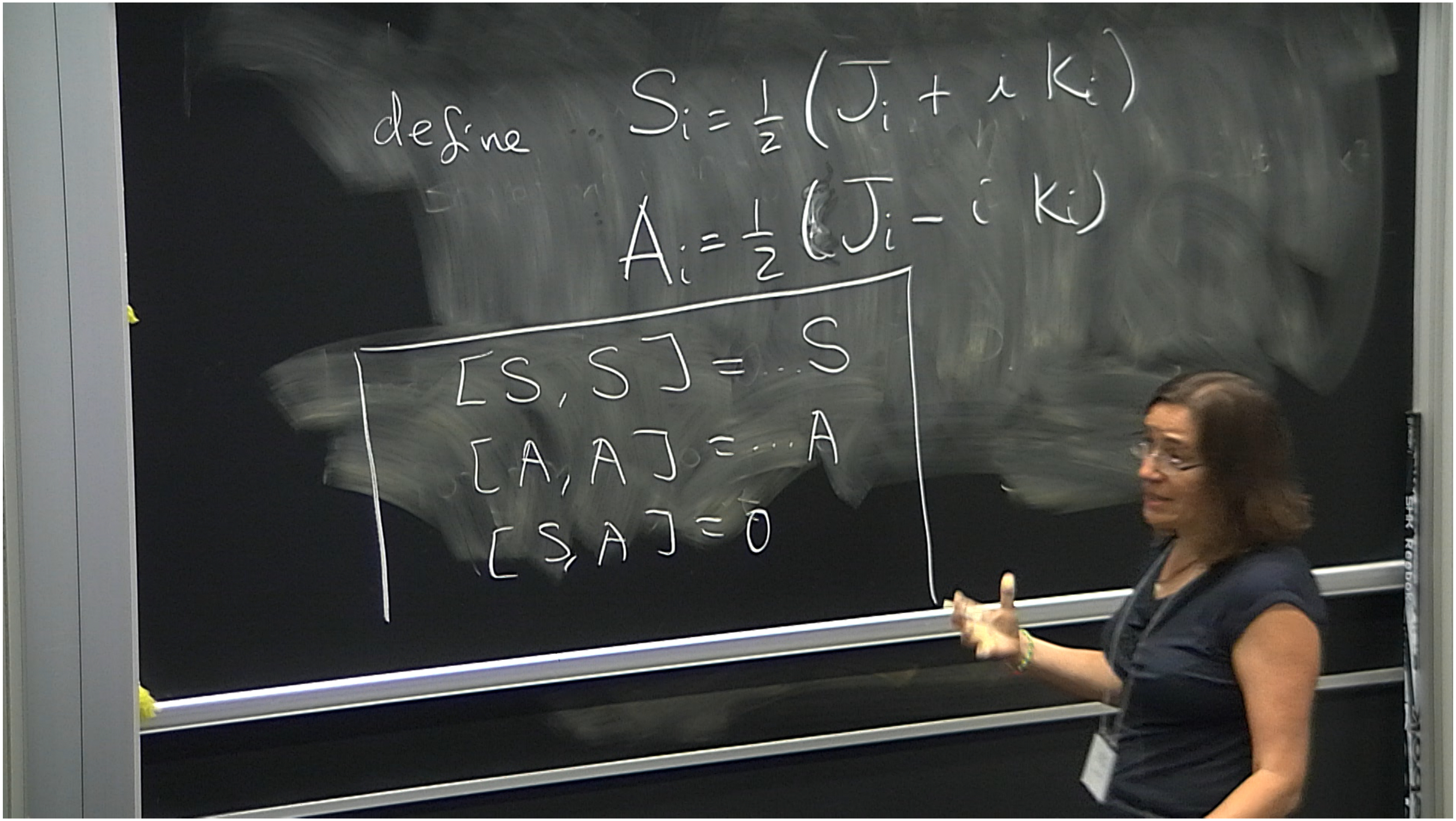
$$[J_i, J_j] = i\,\varepsilon_{ijk} J_k$$

$$[K_i, K_j] = i\,\varepsilon_{ijk} J_k$$

$$[K_i, J_j] = i\,\varepsilon_{ijk} K_k$$

define $S \quad J + iK$

$A \quad J - iK$


define $S_i = \frac{1}{2}(J_i + i K_i)$
$A_i = \frac{1}{2}(J_i - i K_i)$
$[S, S] = \ldots S$
$[A, A] = \ldots A$
$[S, A] = 0$

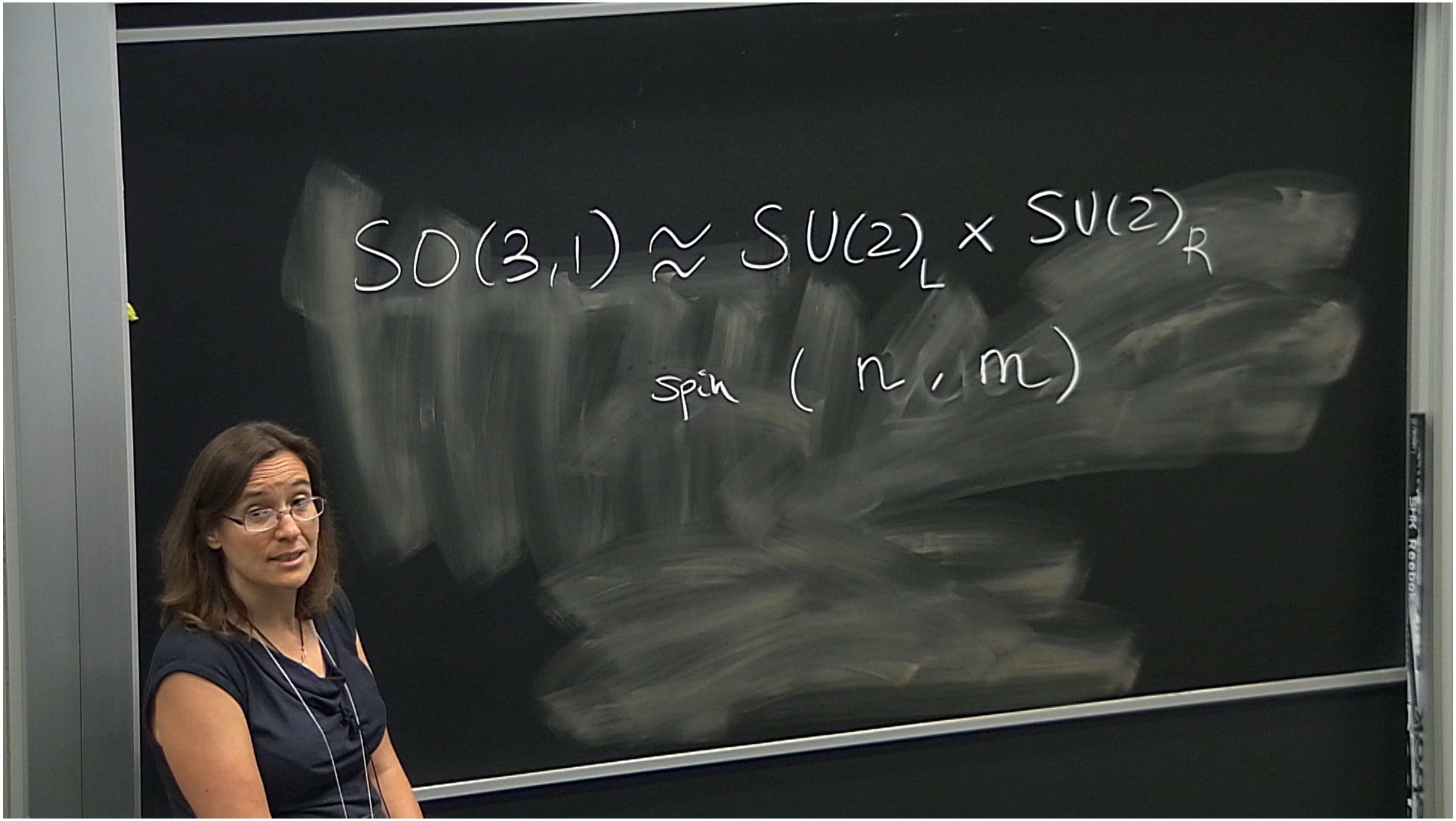
define $S_i = \frac{1}{2}(J_i + i K_i)$

$$A_i = \frac{1}{2}(J_i - i K_i)$$

$$[S, S] = \ldots S \longrightarrow SU(2)$$

$$[A, A] = \ldots A \longrightarrow SU(2)$$

$$[S, A] = 0 \longrightarrow \text{decoupled}$$


SO(3,1) ≈ SU(2)_L × SU(2)_R
spin (n , m)

$$SO(3,1) \approx SU(2)_L \times SU(2)_R$$

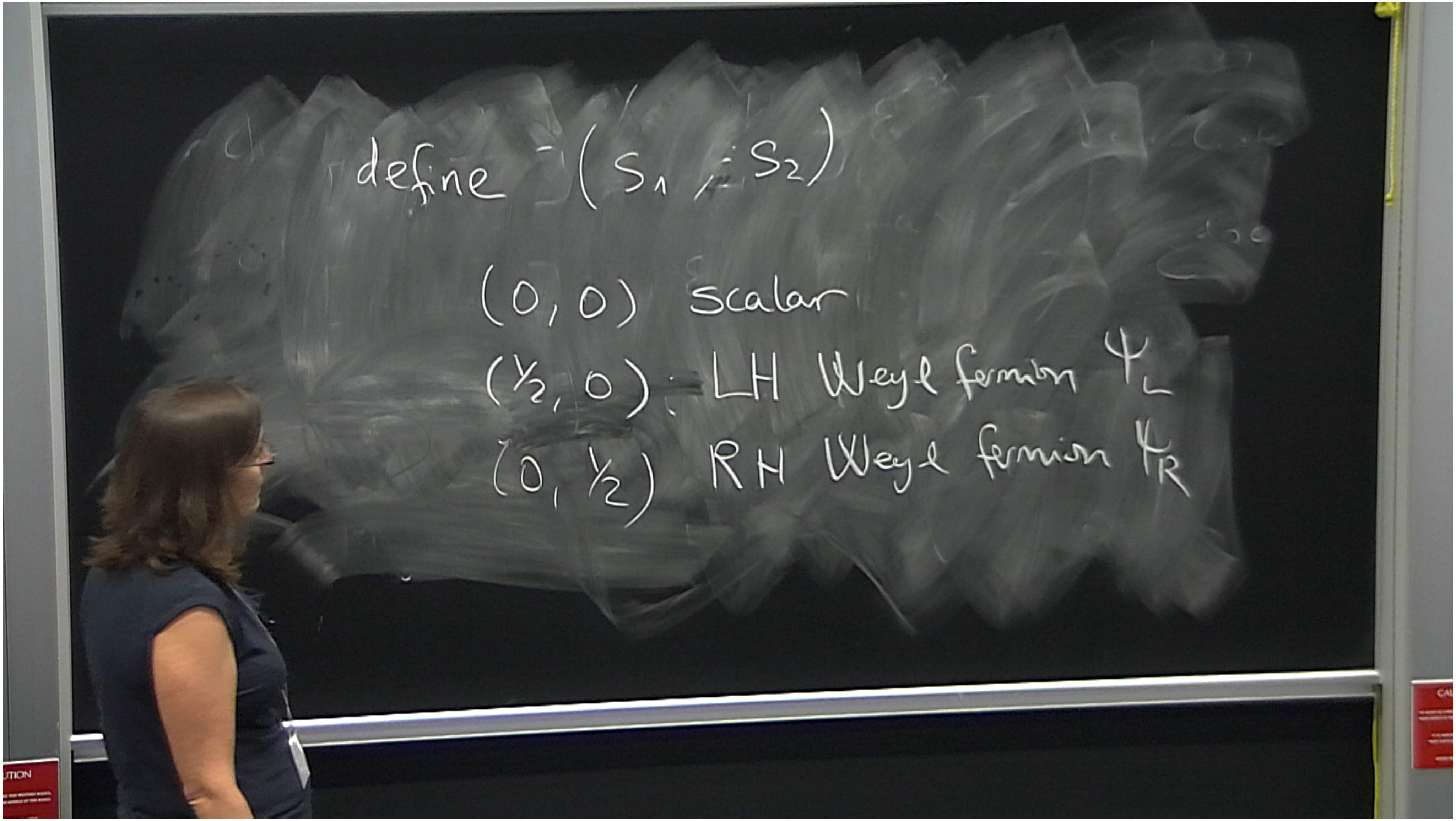
spin (n, m)

parity $K \to -K, \ J \to J$

$$S \leftrightarrow A$$

define (s_1 , s_2)

$(0,0)$


define (s_1, s_2)
$(0,0)$ scalar
$(1/2, 0)$ LH Weyl fermion ψ_L
$(0, 1/2)$ RH Weyl fermion ψ_R

define (s_1, s_2)

$(0,0)$ scalar

$(\frac{1}{2}, 0)$ LH Weyl fermion ψ_L

$(0, \frac{1}{2})$ RH Weyl fermion ψ_R

$(\frac{1}{2}, \frac{1}{2})$ Dirac fermion Ψ

define (s_1, s_2)

$(0,0)$ scalar def 1

$(\tfrac{1}{2},0)$ LH Weyl fermion ψ_L 2

$(0,\tfrac{1}{2})$ RH Weyl fermion ψ_R 2

$(\tfrac{1}{2},\tfrac{1}{2})$ Dirac fermion Ψ 4

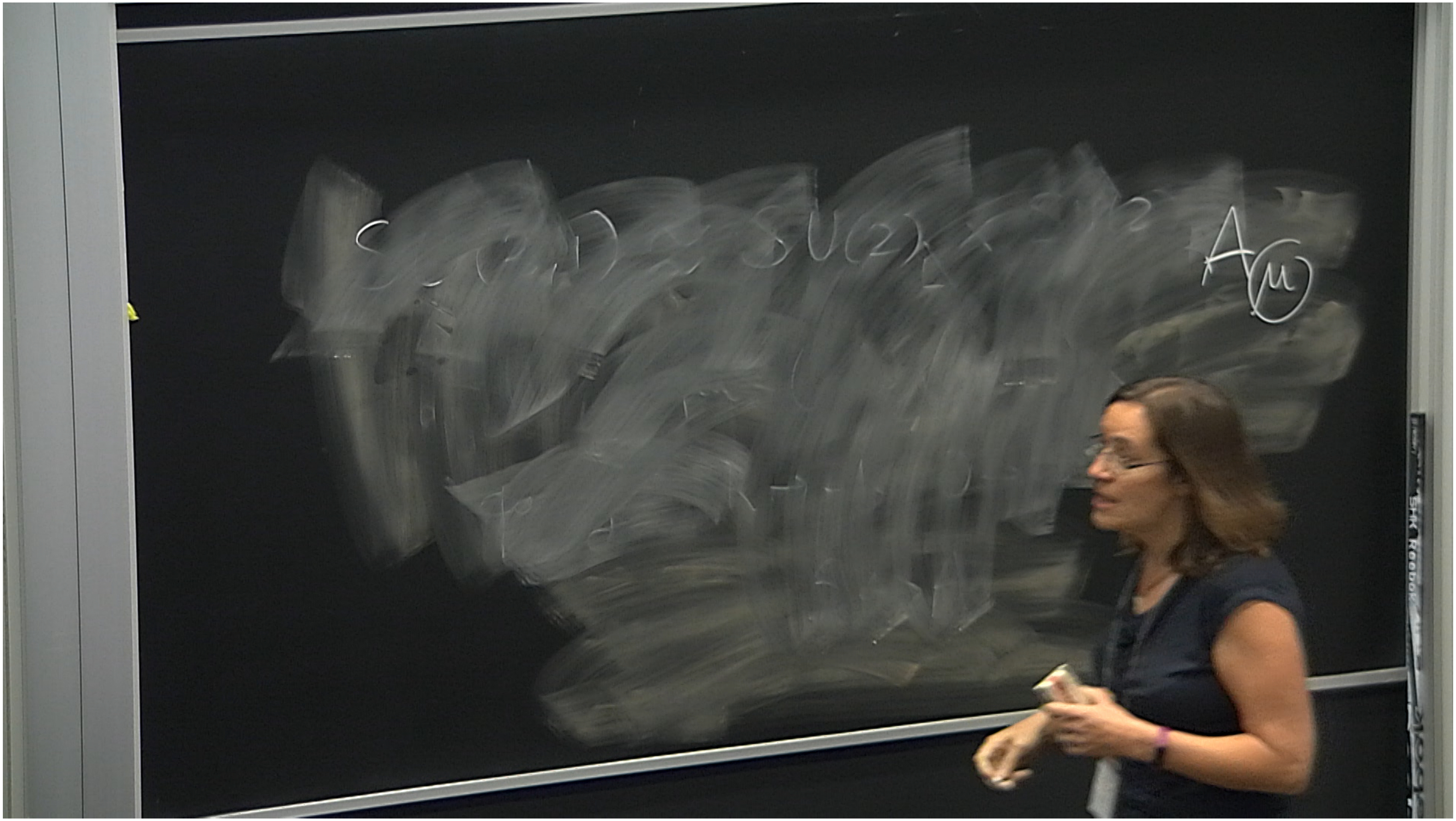
define (s_1, s_2)

$(0,0)$ scalar def 1

$(\tfrac{1}{2}, 0)$ LH Weyl fermion ψ_L 2

$(0, \tfrac{1}{2})$ RH Weyl fermion ψ_R 2

$\Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$ $(\tfrac{1}{2}, \tfrac{1}{2})$ Dirac fermion Ψ 4

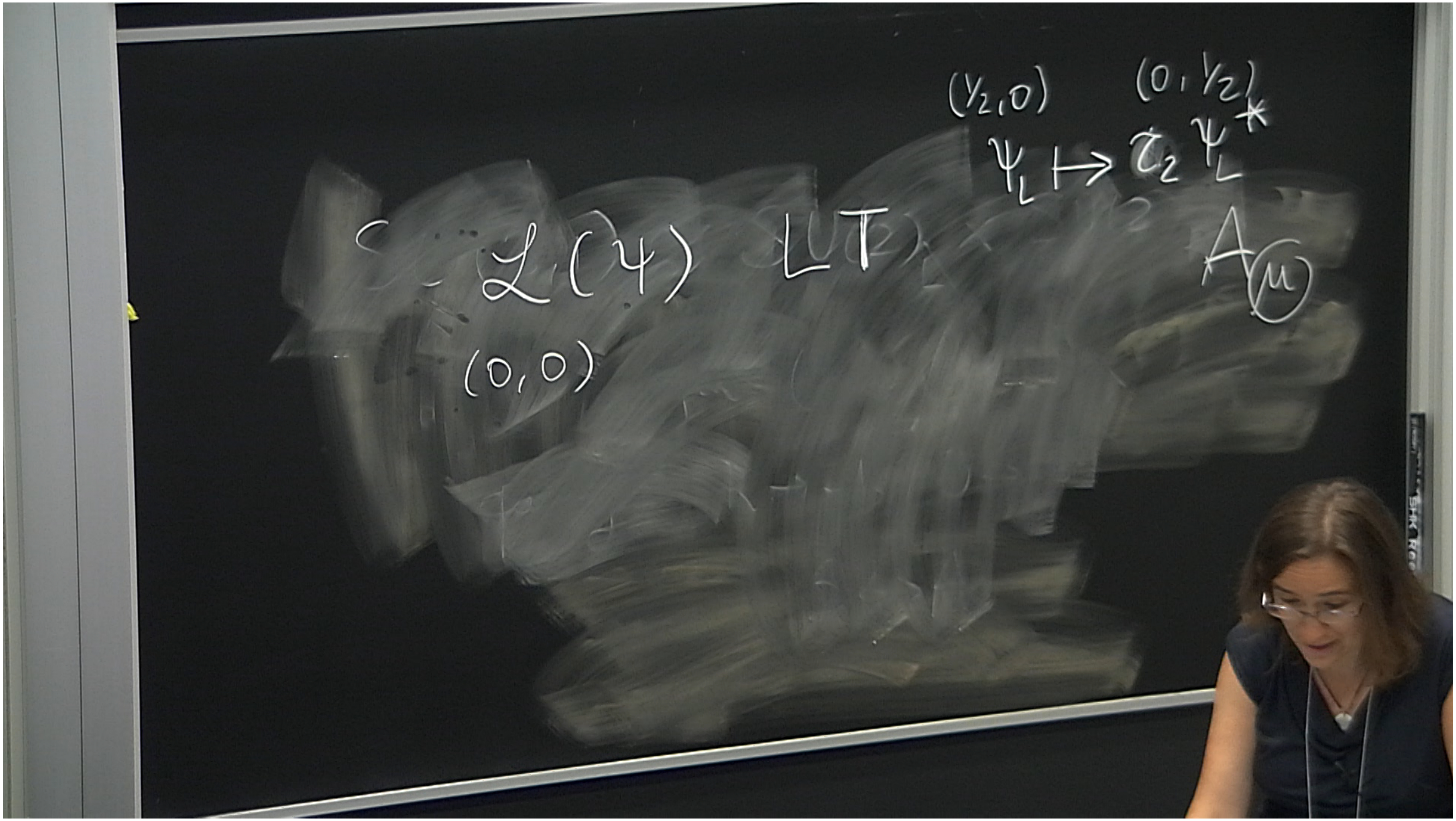

SU(2)
A μ

$(1/2, 0)$ $(0, 1/2)$

$\psi_L \mapsto \sigma_2 \psi_L^*$

$\mathcal{L}(\psi)$ LT

A_μ

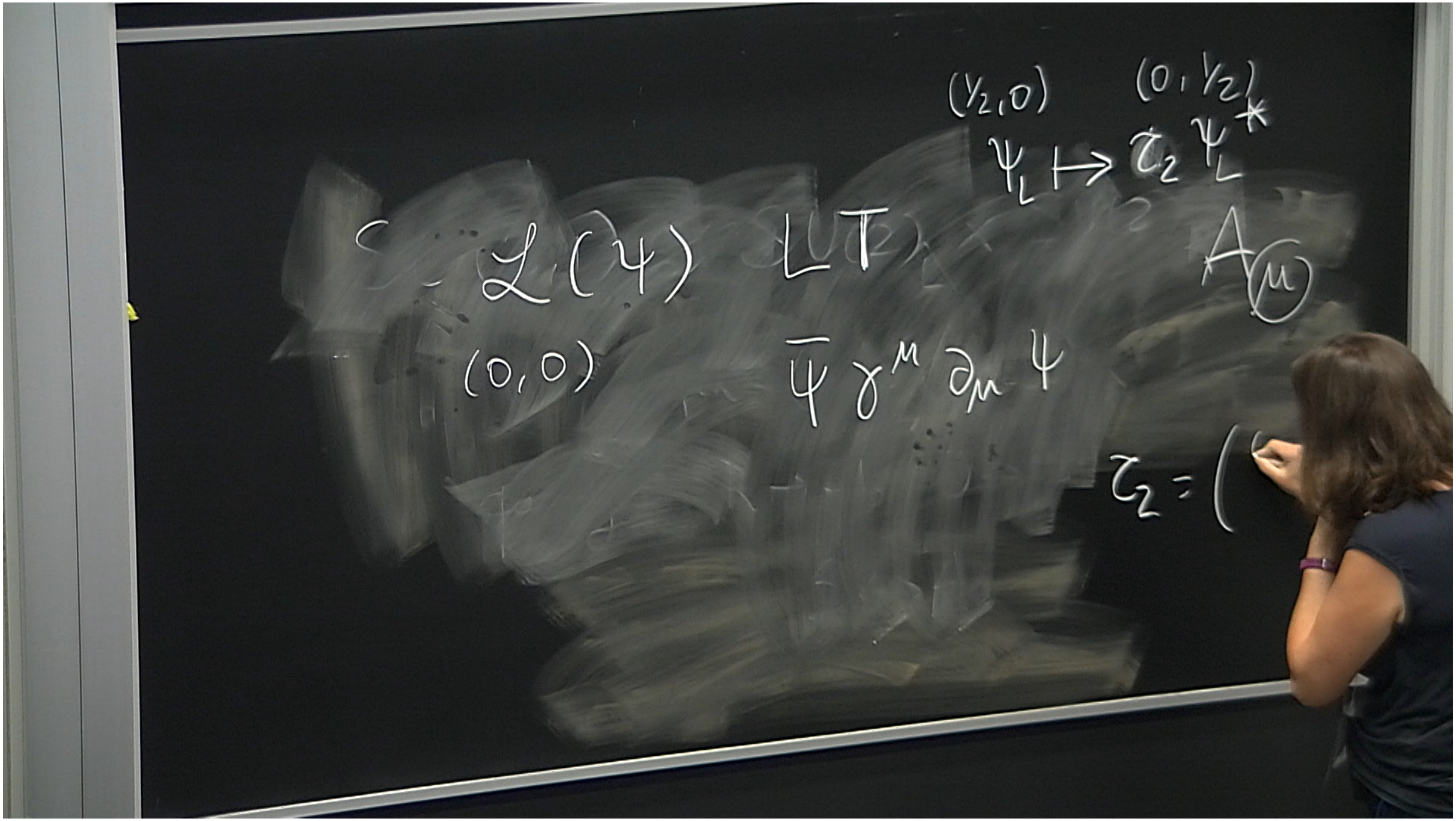

(1/2, 0)
(0, 1/2)
ψ_L ↦ τ_2 ψ_L^*
L(ψ)
LT
A_μ
(0, 0)

$(1/2, 0)$ $\quad$ $(0, 1/2)$

$$\psi_L \mapsto \tau_2 \psi_L^*$$

$\mathcal{L}(\psi)$ $\quad$ LT $\quad$ A_{μ}

$(0,0)$ $\quad$ $\bar{\psi} \gamma^{\mu} \partial_{\mu} \psi$


$(1/2, 0)$ $(0, 1/2)$
$\psi_L \mapsto \tau_2 \psi_L^*$
$\mathcal{L}(\psi)$ LT
A_μ
$(0,0)$
$\bar{\psi} \gamma^\mu \partial_\mu \psi$
$\tau_2 = ($

$(1/2, 0) \quad (0, 1/2)$

$$\psi_L \mapsto \tau_2 \psi_L^*$$

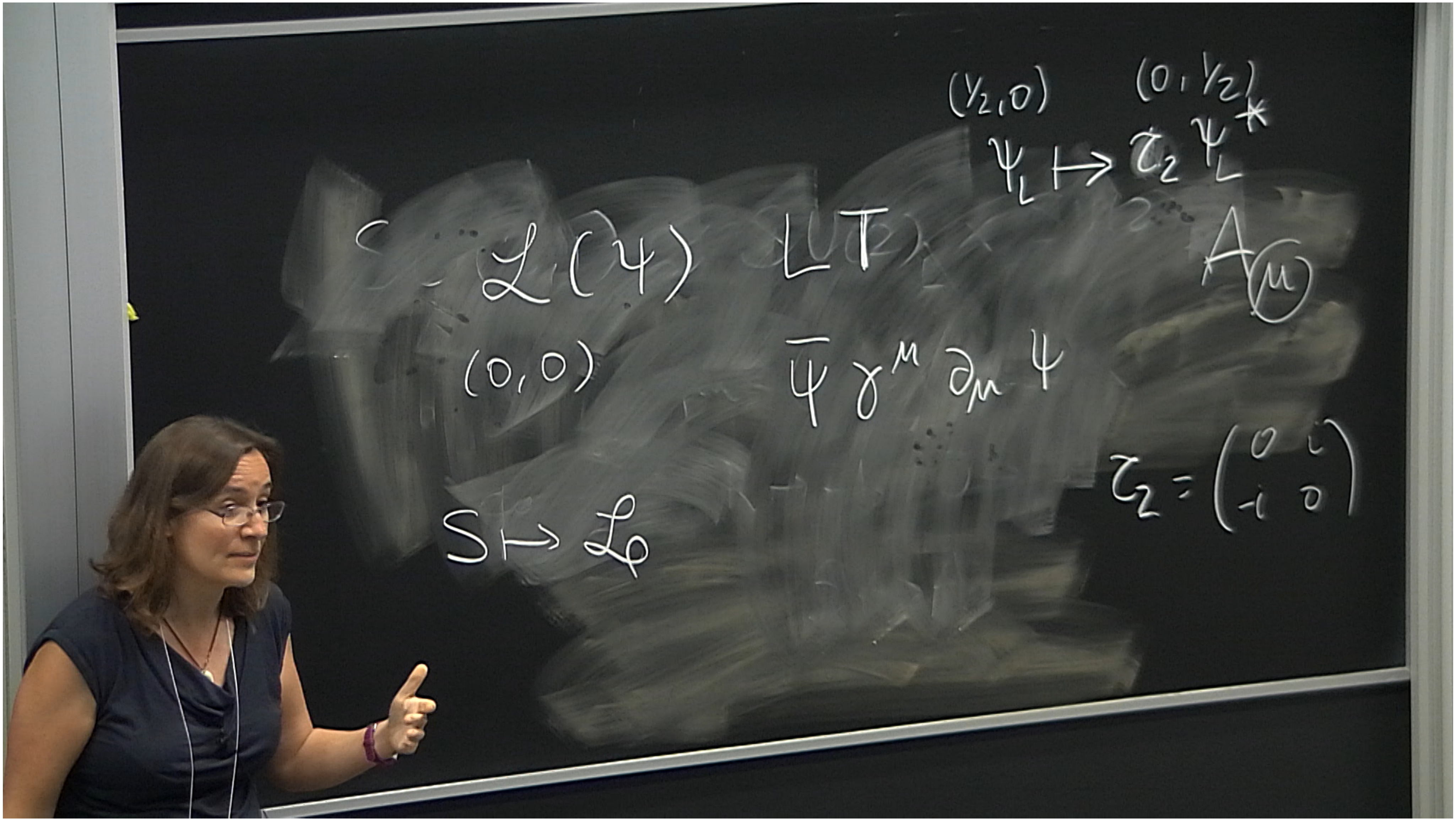
$\mathcal{L}(\psi)$ LT

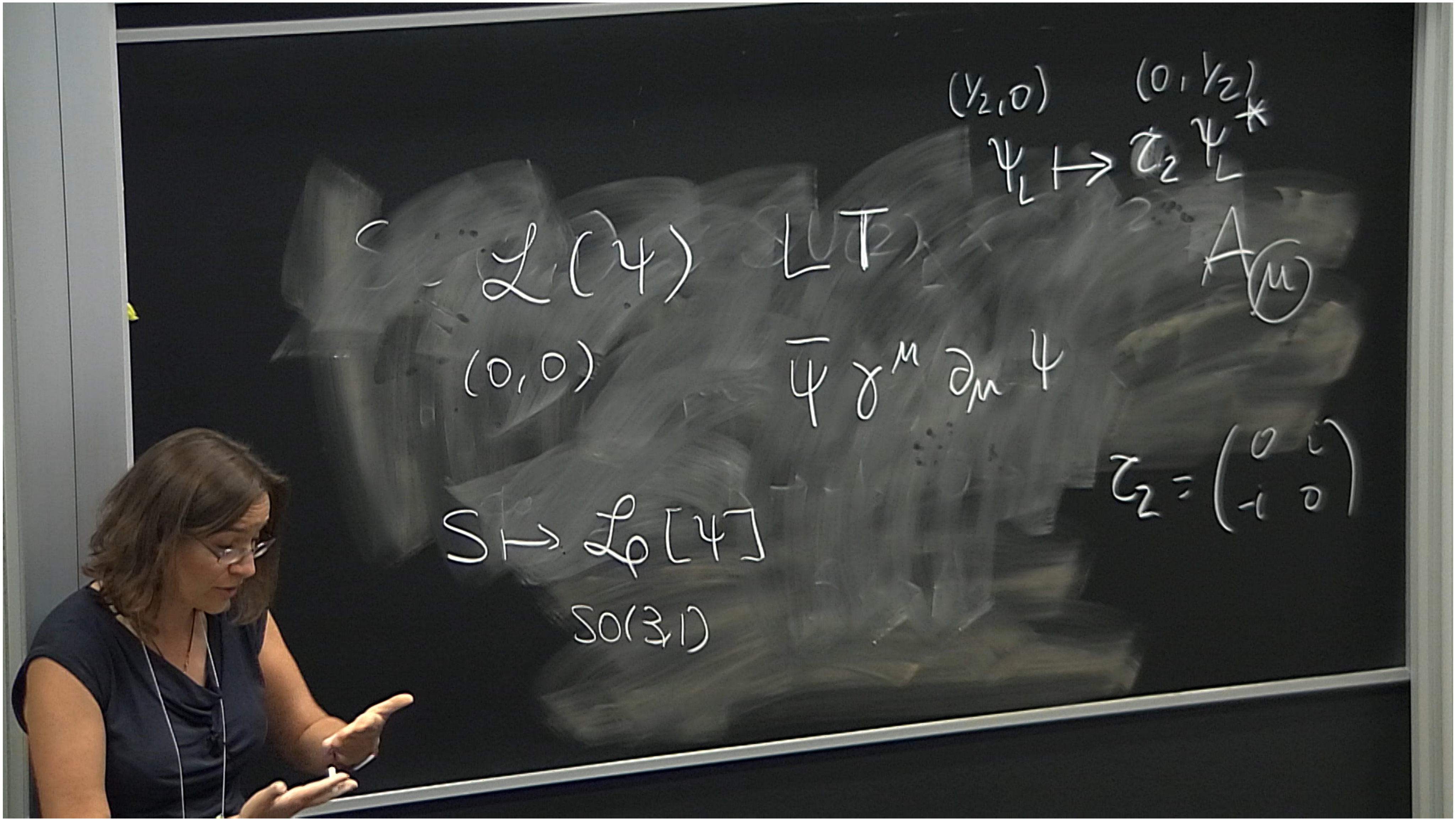
A_μ

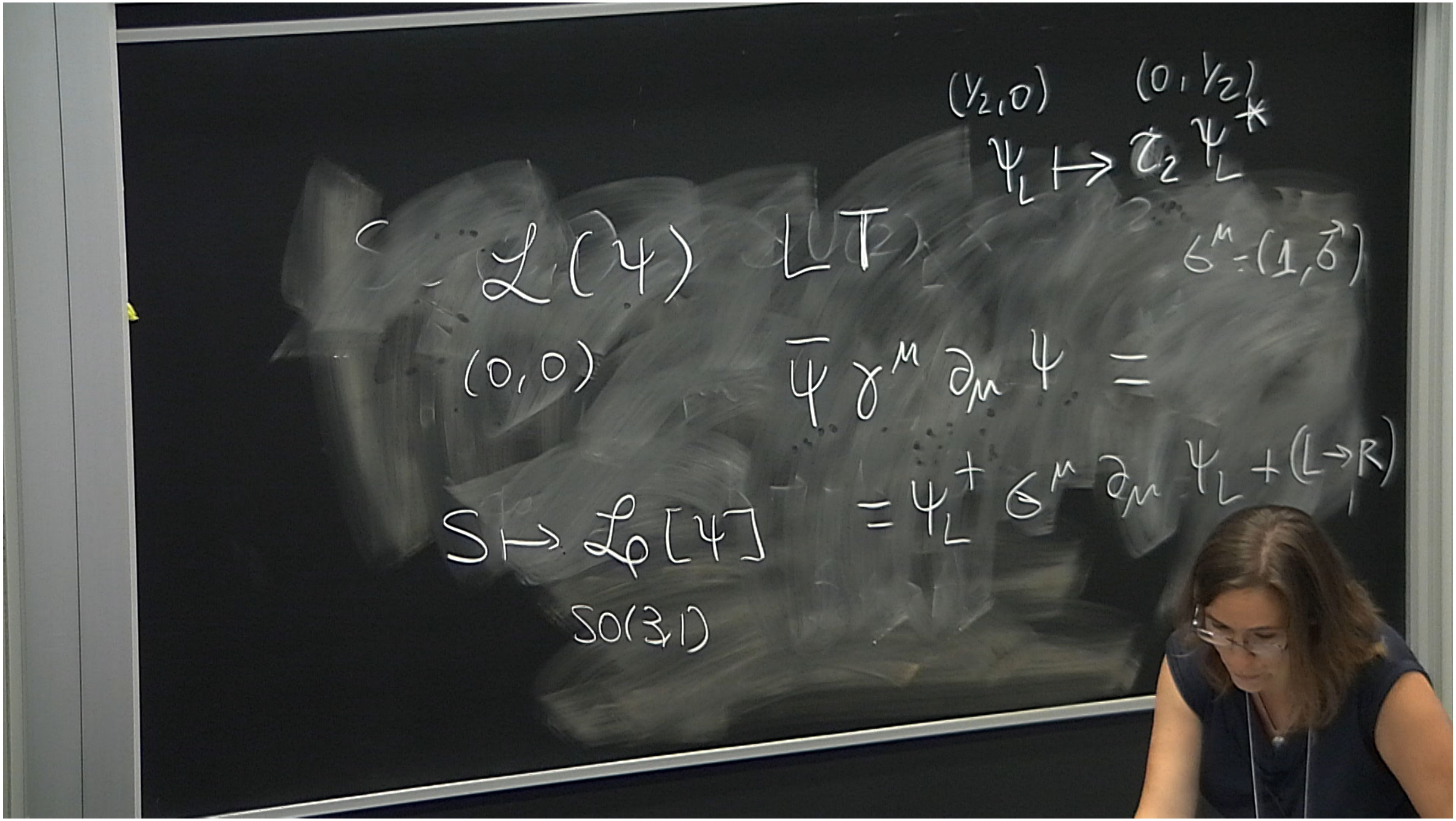
$(0,0)$

$$\bar\psi \gamma^\mu \partial_\mu \psi$$

$$\tau_2 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$$


$(1/2, 0)$
$(0, 1/2)$
$\psi_L \mapsto \tau_2 \psi_L^*$
$\mathcal{L}(\psi)$
LT
A_μ
$(0,0)$
$\bar{\psi} \gamma^\mu \partial_\mu \psi$
$\tau_2 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$
$S \mapsto \mathcal{L}_0$


$(\frac{1}{2},0)$ $(0,\frac{1}{2})$
$\psi_L \mapsto \tau_2 \psi_L^*$
$\mathcal{L}(\psi)$
LT
A_μ
$(0,0)$
$\bar{\psi}\gamma^\mu \partial_\mu \psi$
$\tau_2 = \begin{pmatrix} 0 & i \\ -1 & 0 \end{pmatrix}$
$S \mapsto \mathcal{L}_0[\psi]$
SO(3,1)


$(1/2, 0)$ $(0, 1/2)$
$\psi_L \mapsto \tau_2 \psi_L^*$
$\mathcal{L}(\psi)$ LT
$\sigma^\mu = (1, \vec{\sigma})$
$(0,0)$
$\bar{\psi} \gamma^\mu \partial_\mu \psi =$
$= \psi_L^\dagger \sigma^\mu \partial_\mu \psi_L + (L \to R)$
$S \mapsto \mathcal{L}_\phi[\psi]$
$SO(3,1)$

$(1/2, 0) \qquad (0, 1/2)$

$$\psi_L \mapsto \sigma_2 \psi_L^*$$

$\mathcal{L}(\psi)$ LT

$\sigma^\mu = (1, \vec{\sigma})$

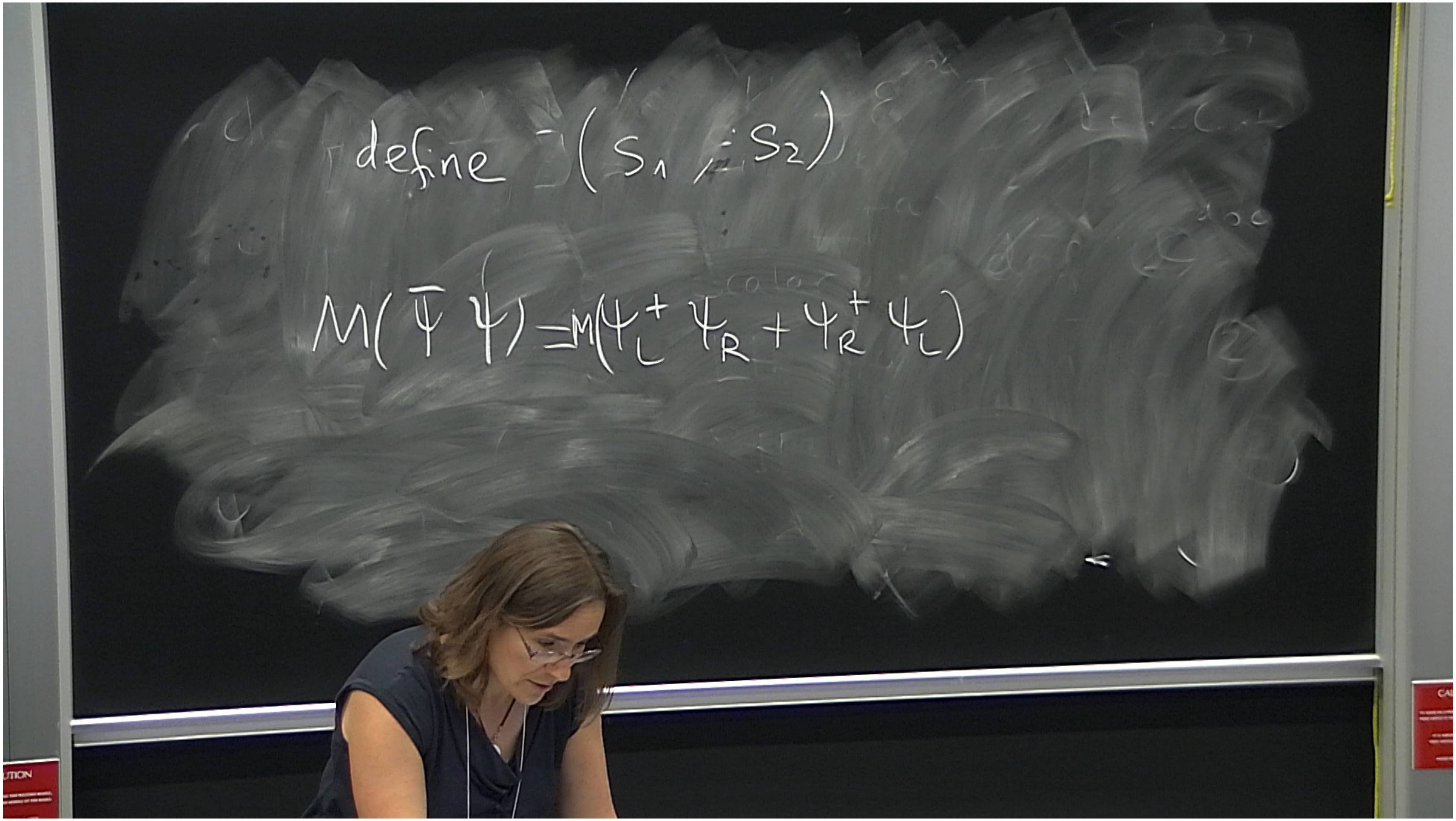
$(0,0)$

$$\bar{\psi} \gamma^\mu \partial_\mu \psi =$$

$$= \psi_L^\dagger \sigma^\mu \partial_\mu \psi_L + (L \to R)$$

$$S \mapsto \mathcal{L}_0[\psi]$$

$SO(3,1)$


define $(S_1 ; S_2)$
$M(\bar{\Psi}\Psi) = M(\Psi_L^+ \Psi_R + \Psi_R^+ \Psi_L)$

define $(S_1 ; S_2)$

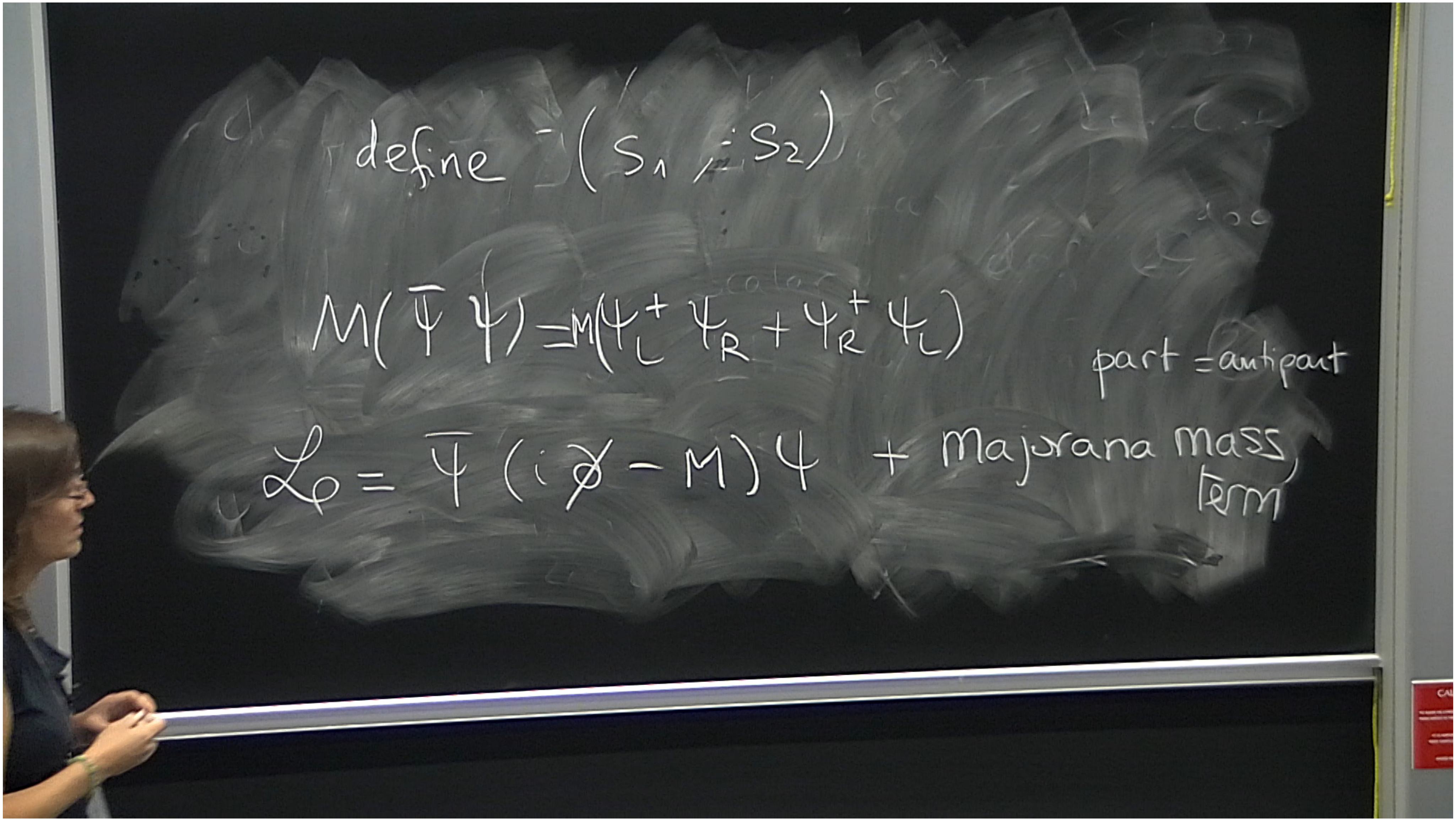
$$M(\bar{\Psi}\Psi) = M(\psi_L^{\dagger}\psi_R + \psi_R^{\dagger}\psi_L)$$

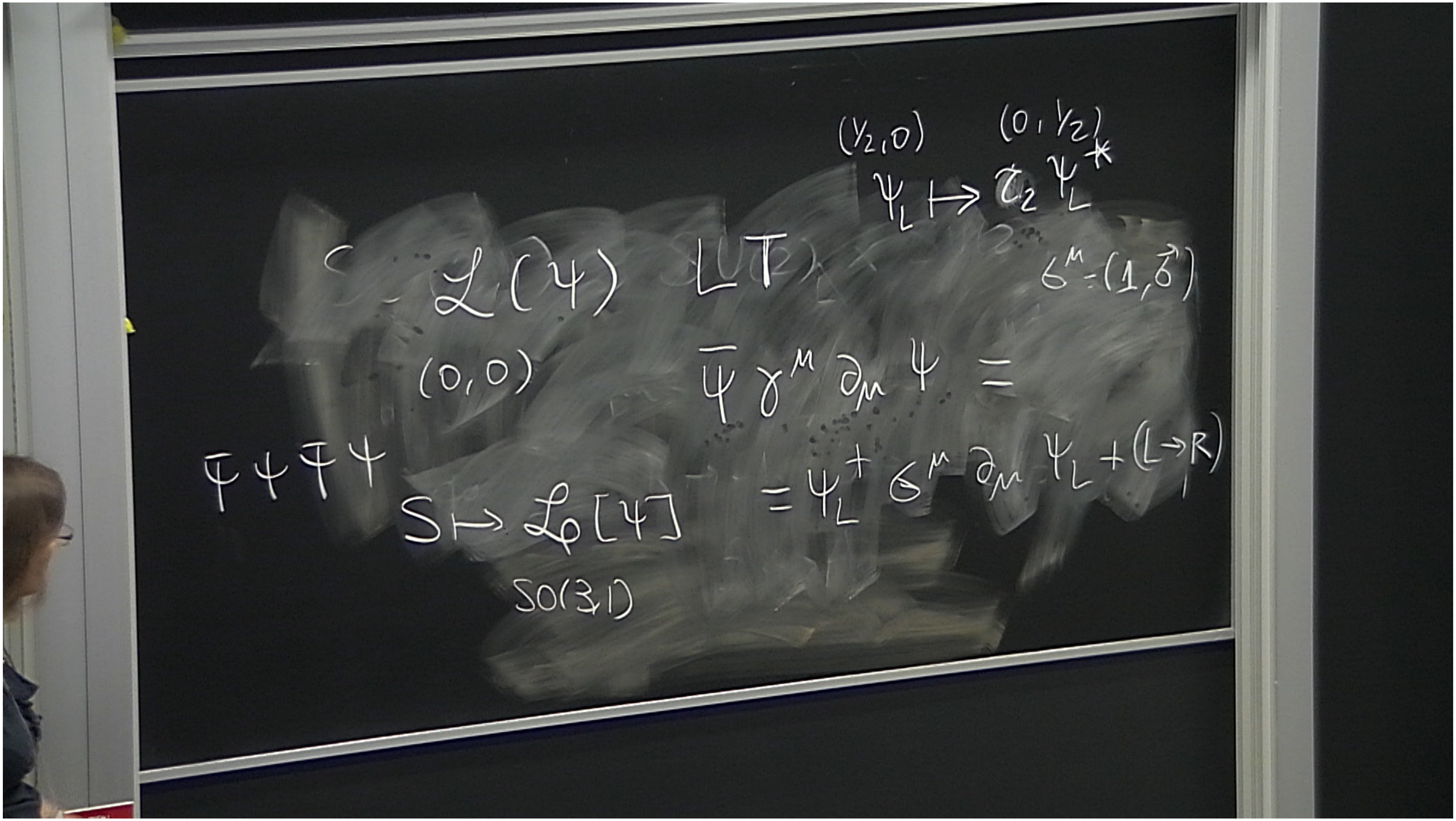
$$\mathcal{L}_0 = \bar{\Psi}(i\not{\partial} - M)\Psi$$

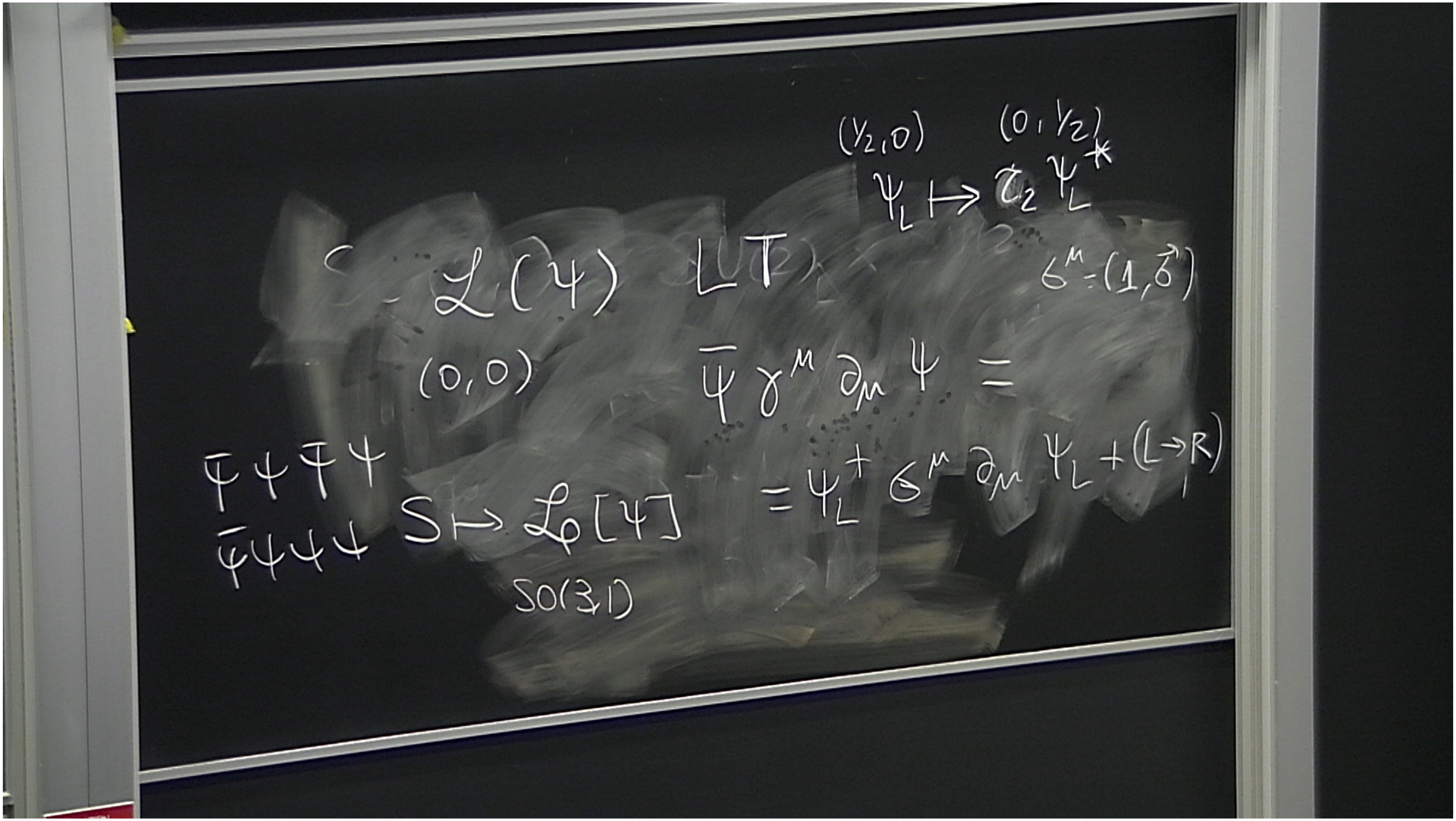
define (S_1, S_2)

$$M(\bar{\Psi}\Psi) = M(\Psi_L^{+}\Psi_R + \Psi_R^{+}\Psi_L)$$

$$\mathcal{L}_0 = \bar{\Psi}(i\not{\partial} - M)\Psi$$ + Majorana mass term


define (S_1, S_2)
$M(\bar{\Psi}\Psi) = M(\Psi_L^+ \Psi_R + \Psi_R^+ \Psi_L)$
part = antipart
$\mathcal{L}_0 = \bar{\Psi}(i\not{\partial} - M)\Psi$ + Majorana mass term


$(1/2, 0)$ $(0, 1/2)$
$\psi_L \mapsto \sigma_2 \psi_L^*$
$\mathcal{L}(\psi)$ LT
$\sigma^\mu = (1, \vec{\sigma})$
$(0,0)$
$\bar{\psi} \gamma^\mu \partial_\mu \psi =$
$\bar{\psi}\psi\bar{\psi}\psi$
$= \psi_L^\dagger \sigma^\mu \partial_\mu \psi_L + (L \to R)$
$S \mapsto \mathcal{L}_0[\psi]$
$SO(3,1)$


$(1/2, 0)$ $(0, 1/2)$
$\psi_L \mapsto \sigma_2 \psi_L^*$
S $\mathcal{L}(\psi)$ LT
$\sigma^\mu = (1, \vec{\sigma})$
$(0,0)$
$\bar{\psi} \gamma^\mu \partial_\mu \psi =$
$\bar{\psi}\psi\bar{\psi}\psi$
$= \psi_L^\dagger \sigma^\mu \partial_\mu \psi_L + (L \to R)$
$\bar{\psi}\psi\psi\psi$
$S \mapsto \mathcal{L}_0[\psi]$
$SO(3,1)$

define (s_1, s_2)

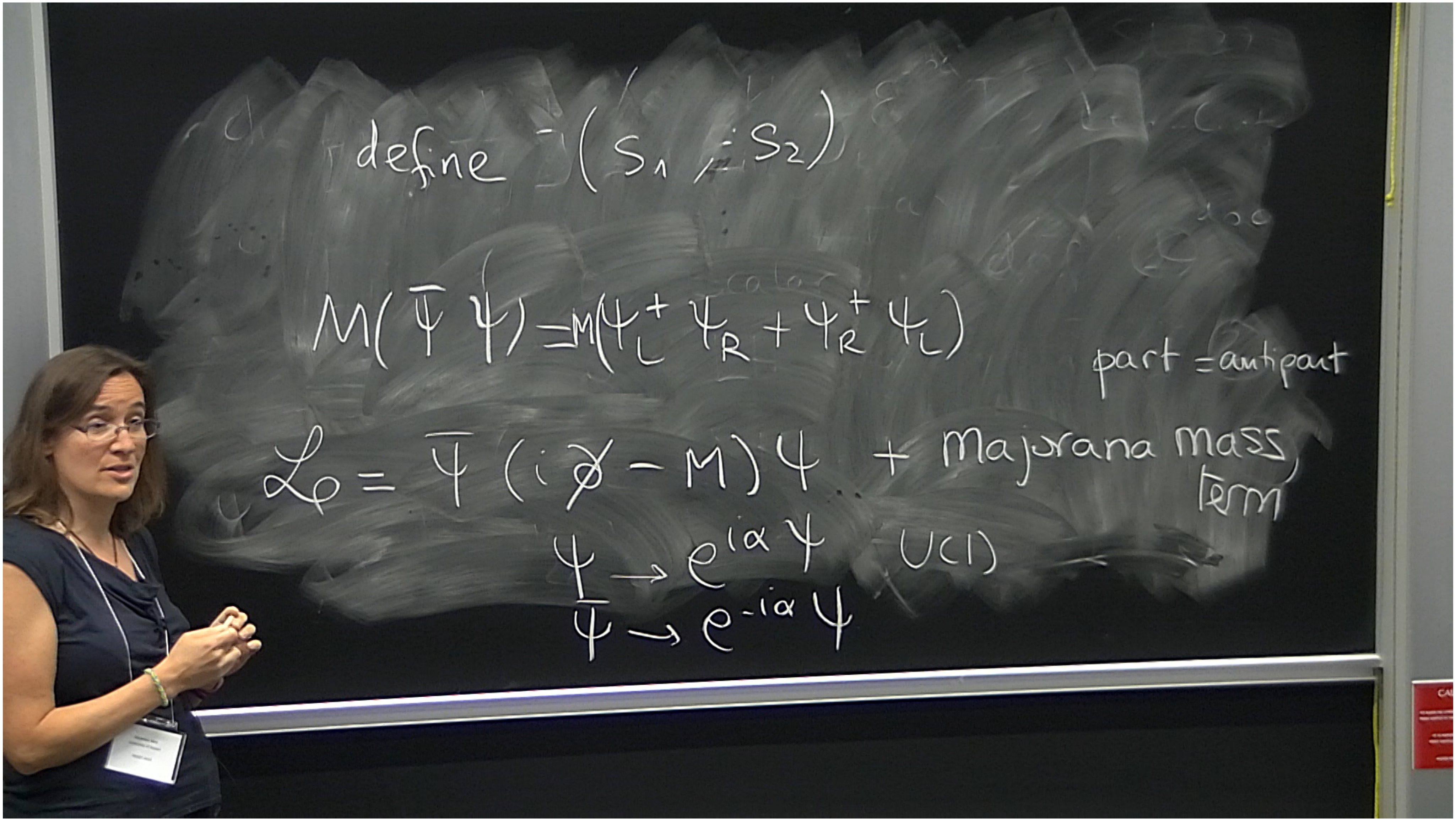
$$M(\bar{\psi}\psi) = M(\psi_L^{\dagger}\psi_R + \psi_R^{\dagger}\psi_L)$$

part = antipart

$$\mathcal{L}_{\psi} = \bar{\psi}(i\cancel{\partial} - M)\psi$$ + Majorana mass term

$$\psi \to e^{i\alpha}\psi$$

$$\bar{\psi} \to e^{-i\alpha}\psi$$


define (S1, S2)
M(Ψ̄Ψ) = M(ΨL+ ΨR + ΨR+ ΨL)
part = antipart
L0 = Ψ̄ (i∂̸ − M) Ψ + Majorana mass term
Ψ → e^iα Ψ U(1)
Ψ̄ → e^−iα Ψ

define (S_1, S_2)

$$M(\bar{\Psi}\Psi) = M(\psi_L^{+}\psi_R + \psi_R^{+}\psi_L)$$

part = antipart

$$\mathcal{L}_0 = \bar{\Psi}(i\not{\partial} - M)\Psi$$ + Majorana mass term

$\psi \to e^{i\alpha}\psi$ U(1) fermion number

$\bar{\psi} \to e^{-i\alpha}\psi$

define (S_1, S_2)

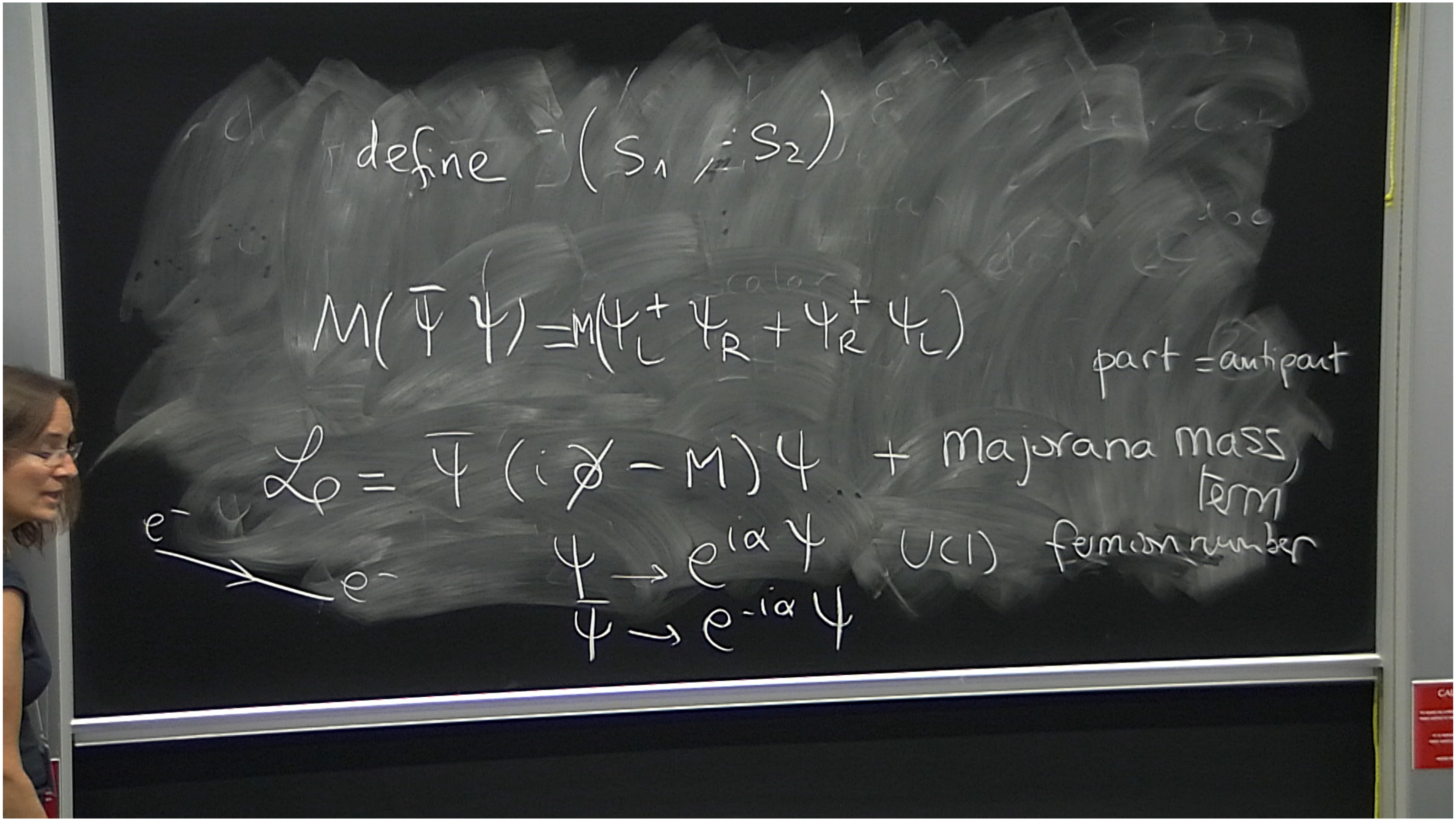
$$M(\bar{\Psi}\Psi) = M(\psi_L^{+}\psi_R + \psi_R^{+}\psi_L)$$

part = antipart

$$\mathcal{L}_{\varphi} = \bar{\Psi}(i\not{\partial} - M)\Psi$$ + Majorana mass term

$\psi \to e^{i\alpha}\psi$ U(1) fermion number

$\bar{\psi} \to e^{-i\alpha}\psi$


define (s_1, s_2)
$M(\bar{\psi}\psi) = M(\psi_L^+ \psi_R + \psi_R^+ \psi_L)$
part = antipart
$\mathcal{L}_0 = \bar{\psi}(i\not{\partial} - M)\psi$ + Majorana mass term
e^-
e^-
$\psi \to e^{i\alpha}\psi$
$\bar{\psi} \to e^{-i\alpha}\psi$
U(1) fermion number