Title: Gravitational Waves Theory 1

Date: Jul 18, 2018 11:00 AM

URL: http://pirsa.org/18070017

Abstract:

LIGO

GW "Theory"

Chad Hanna

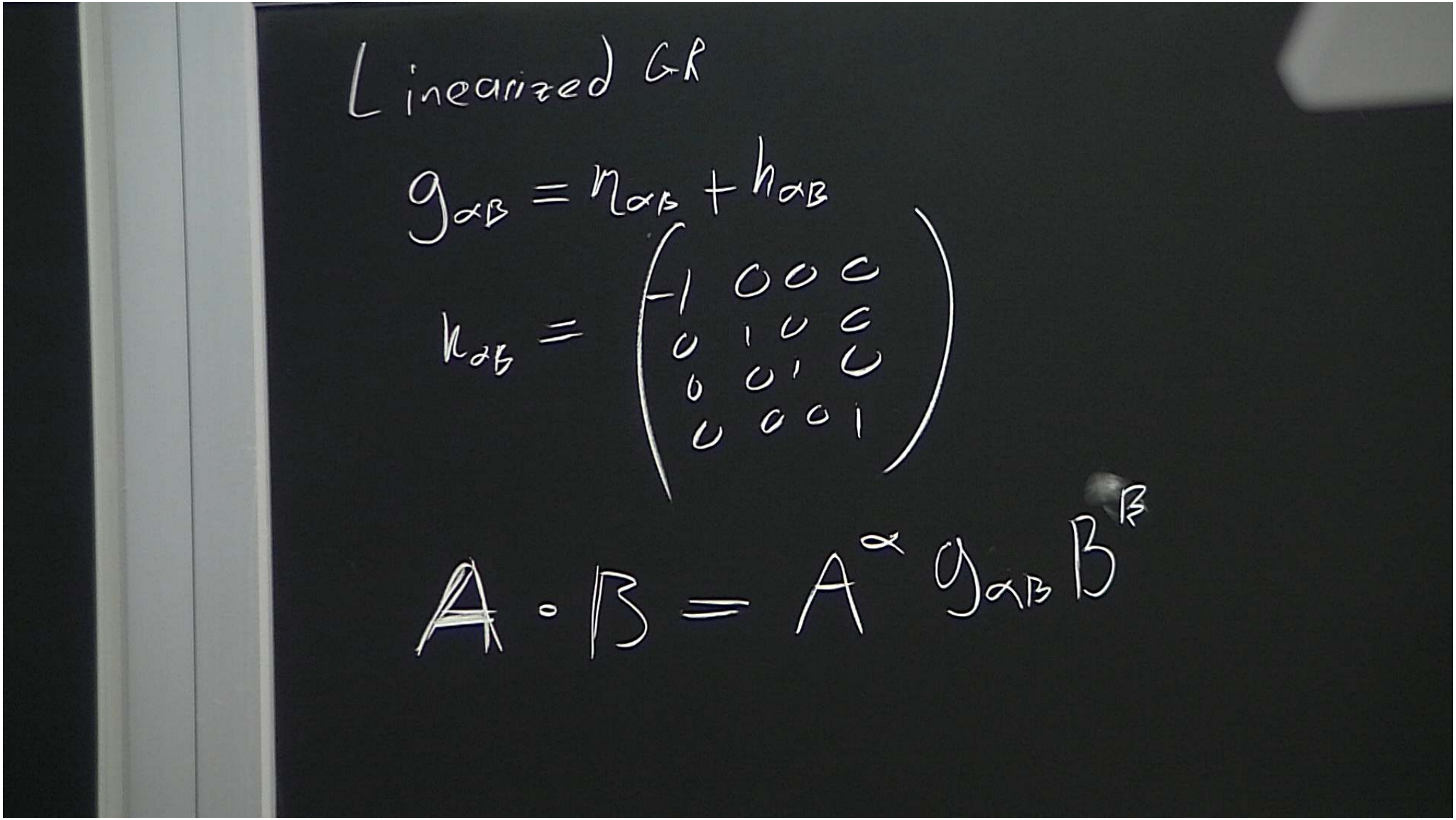
Outline:

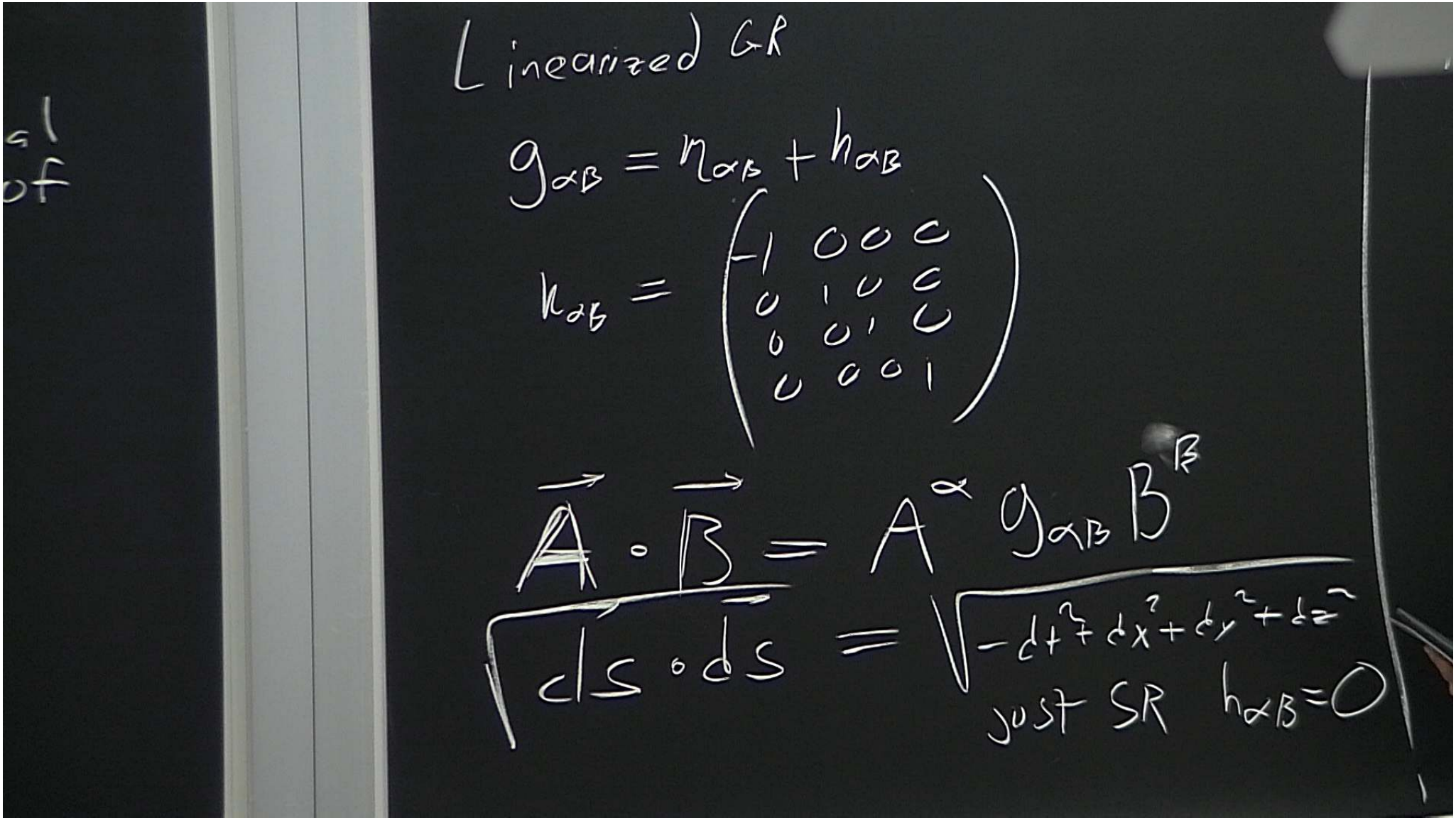
1) What are GWs
   a) Linearized gravity
   b) idealized sources
   c) predictions of amplitude
2) Compact binaries
   a) binary inspiral
   b) PSR B1913+16
   c) How we predict rate

3) Extracting Physics
a) Chirpmass by eye.
b) Bayesian inference
c) Typical uncert.

4) Gravitational Searches
a) matched filtering
b) non stationary noise
c) significance

5) Observational State of field


Linearized GR
$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$
$\eta_{\alpha\beta} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
$A \cdot B = A^{\alpha} g_{\alpha\beta} B^{\beta}$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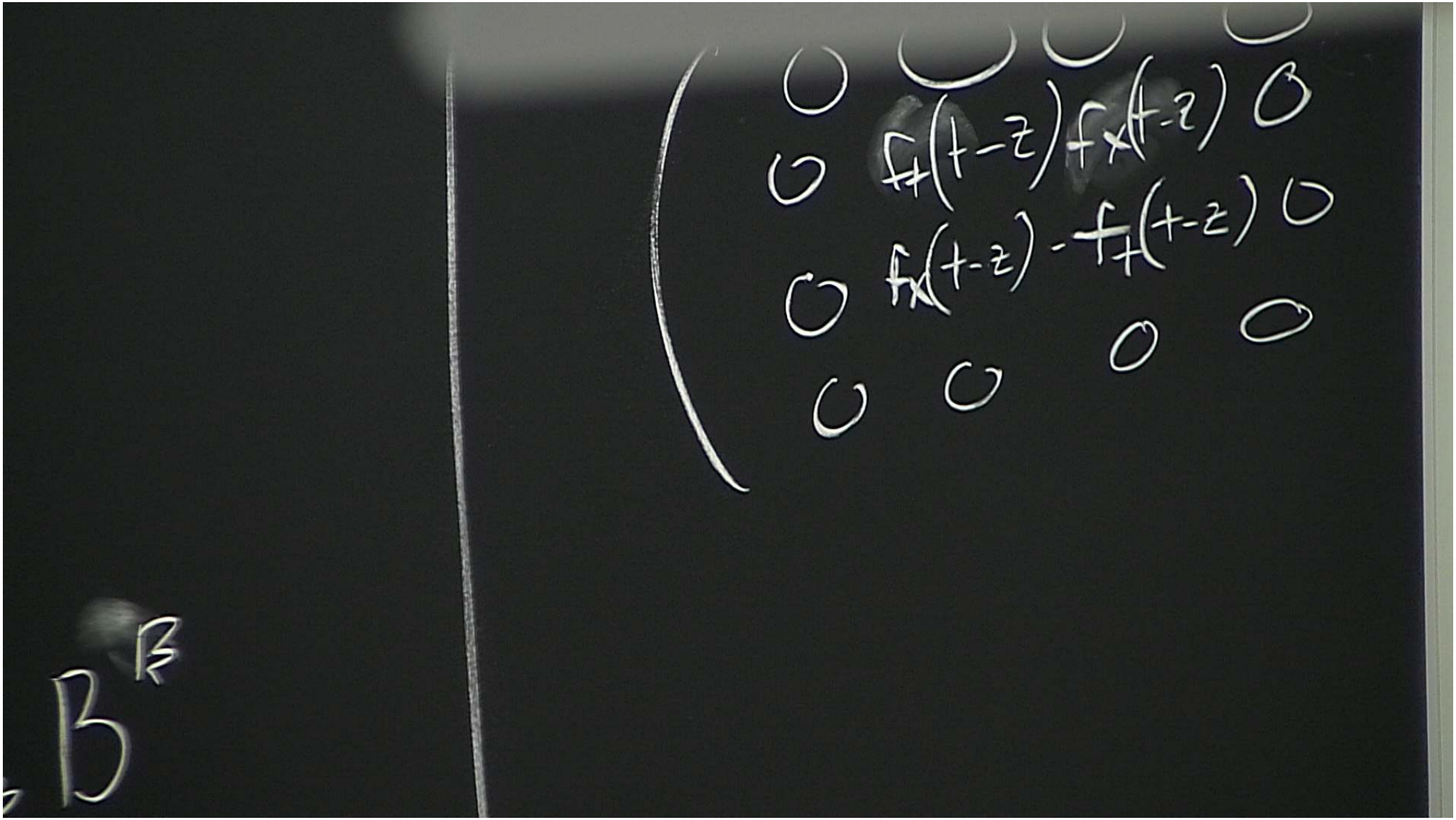
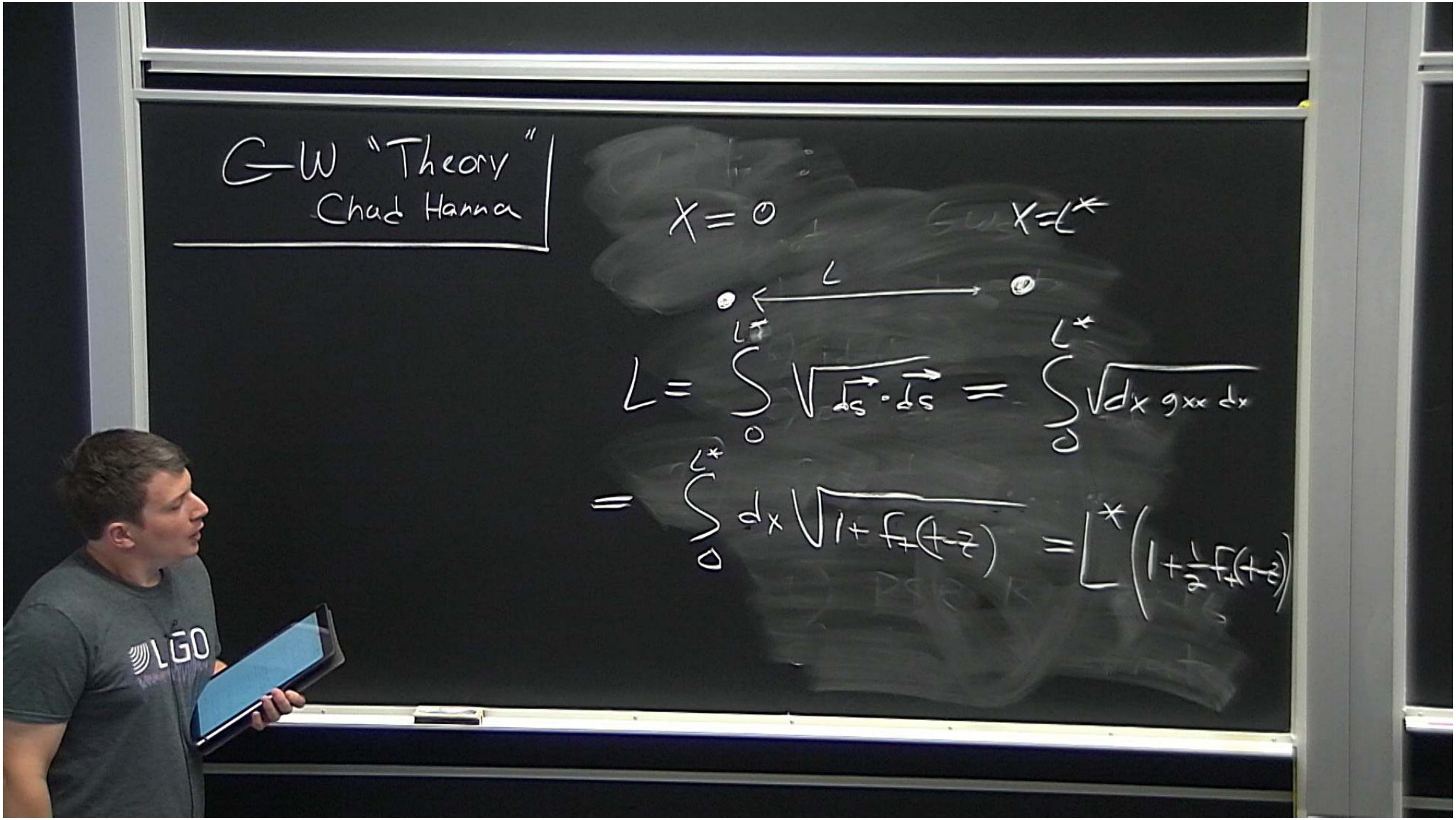
$\vec{A} \cdot \vec{B} = A^{\alpha} g_{\alpha\beta} B^{\beta}$
$\sqrt{ds \cdot ds} = \sqrt{-dt^2 + dx^2 + dy^2 + dz^2}$
just SR $h_{\alpha\beta} = 0$


B^B


GW "Theory"
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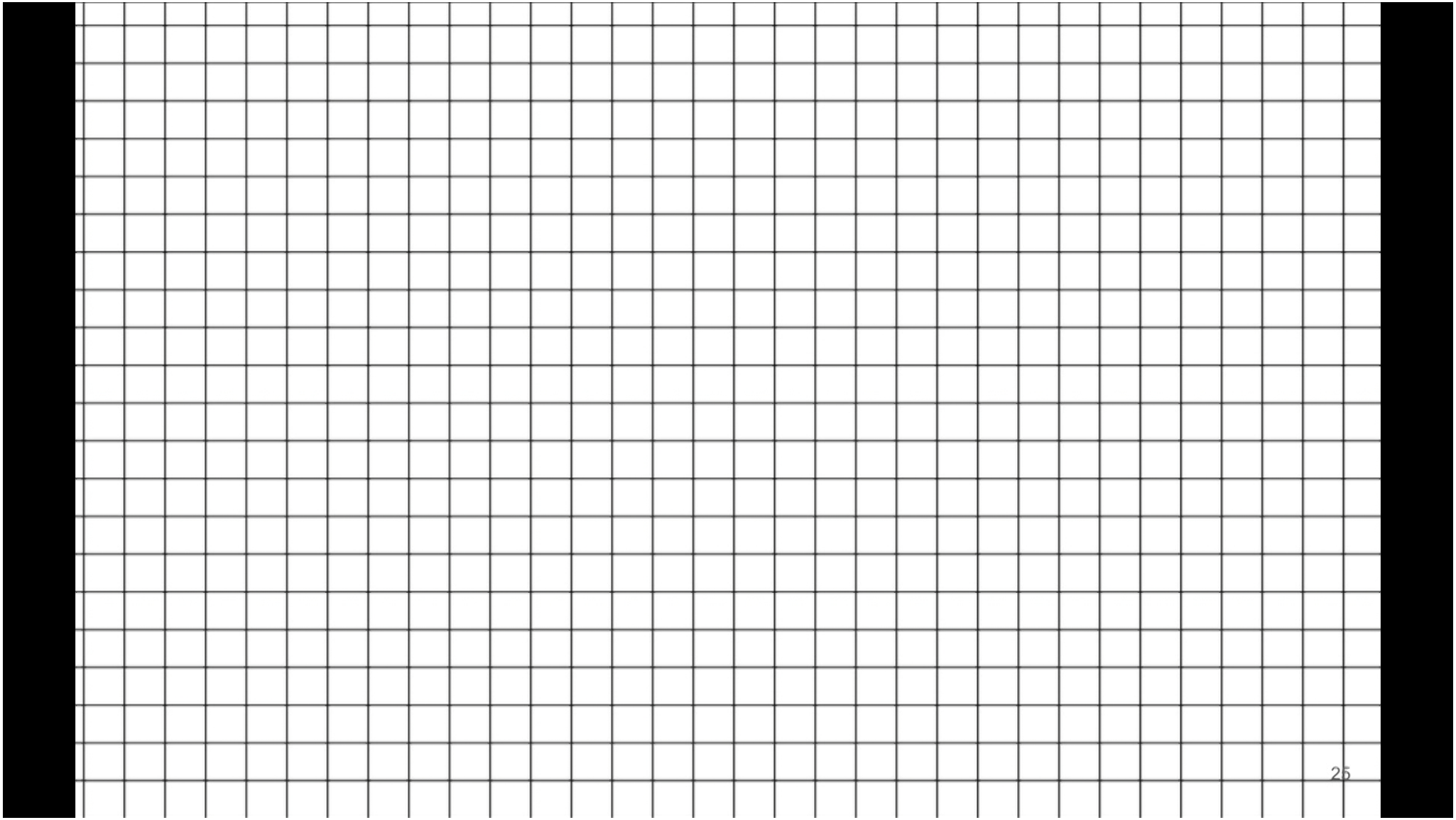
$x = 0$
$x = L^*$
L
$L = \int_0^{L^*} \sqrt{\vec{ds} \cdot \vec{ds}} = \int_0^{L^*} \sqrt{dx\, g_{xx}\, dx}$
$= \int_0^{L^*} dx \sqrt{1 + f_+(t-z)} = L^*\left(1 + \frac{1}{2} f_+(t-z)\right)$
LGO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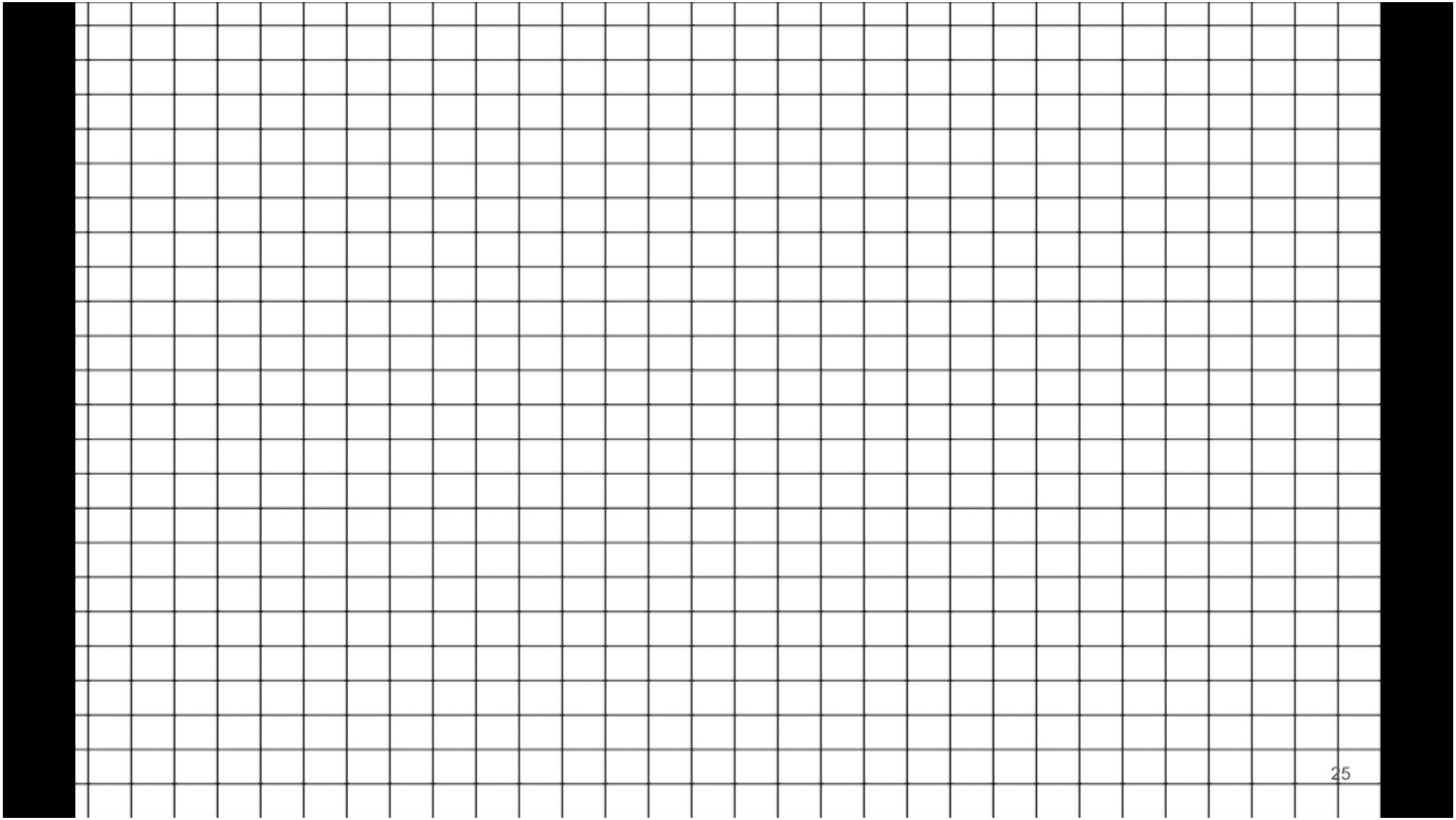
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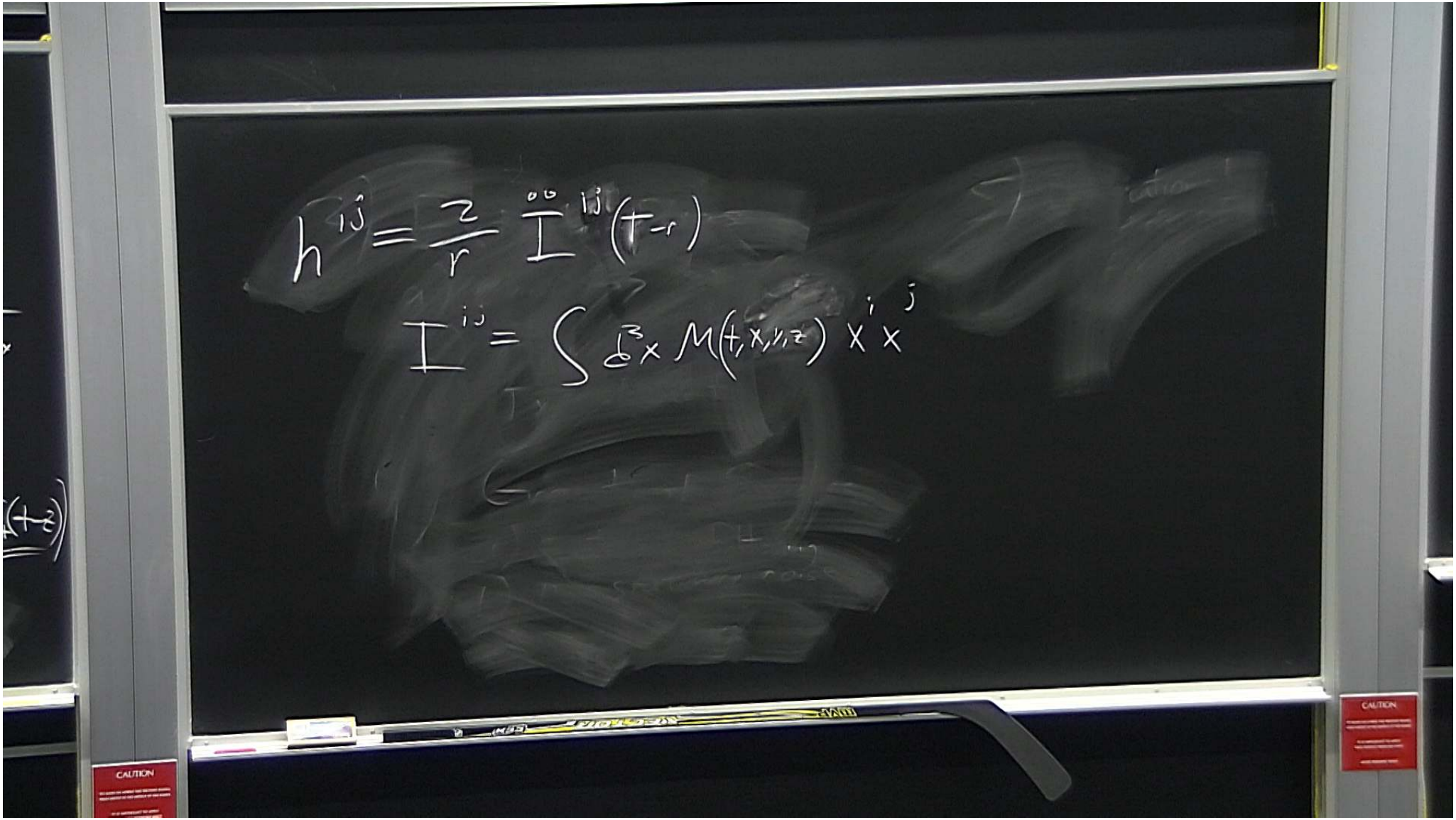
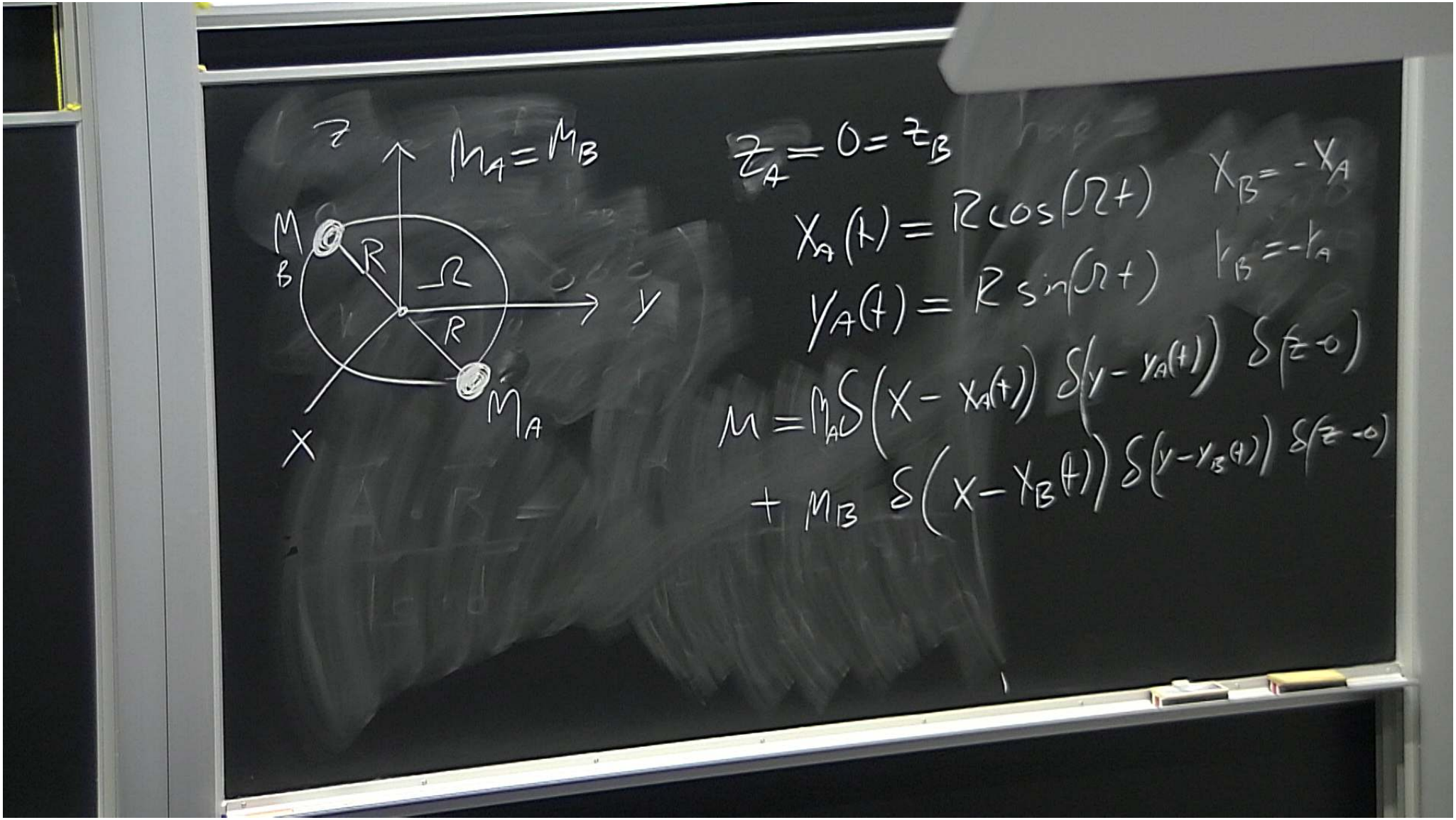
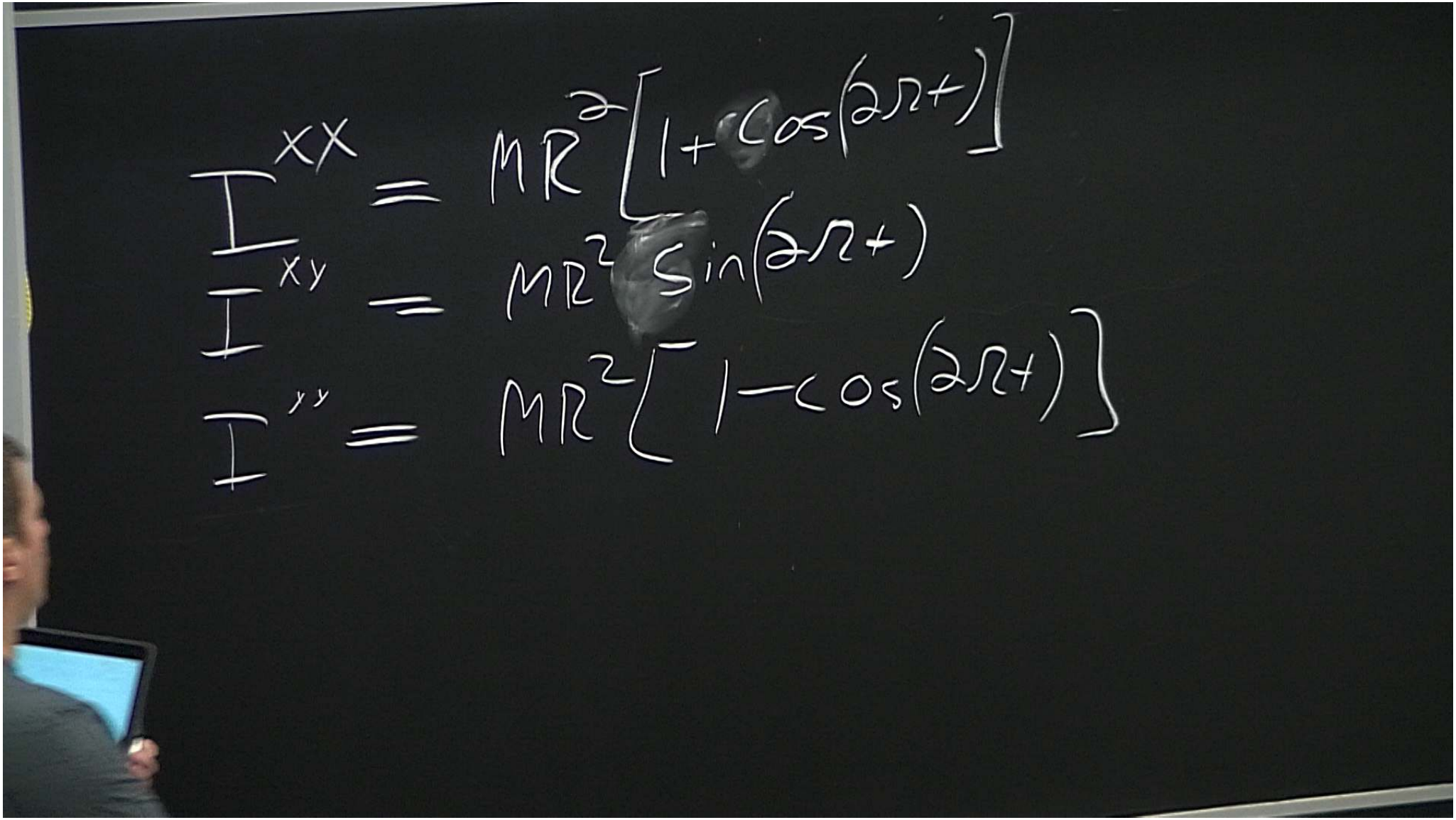
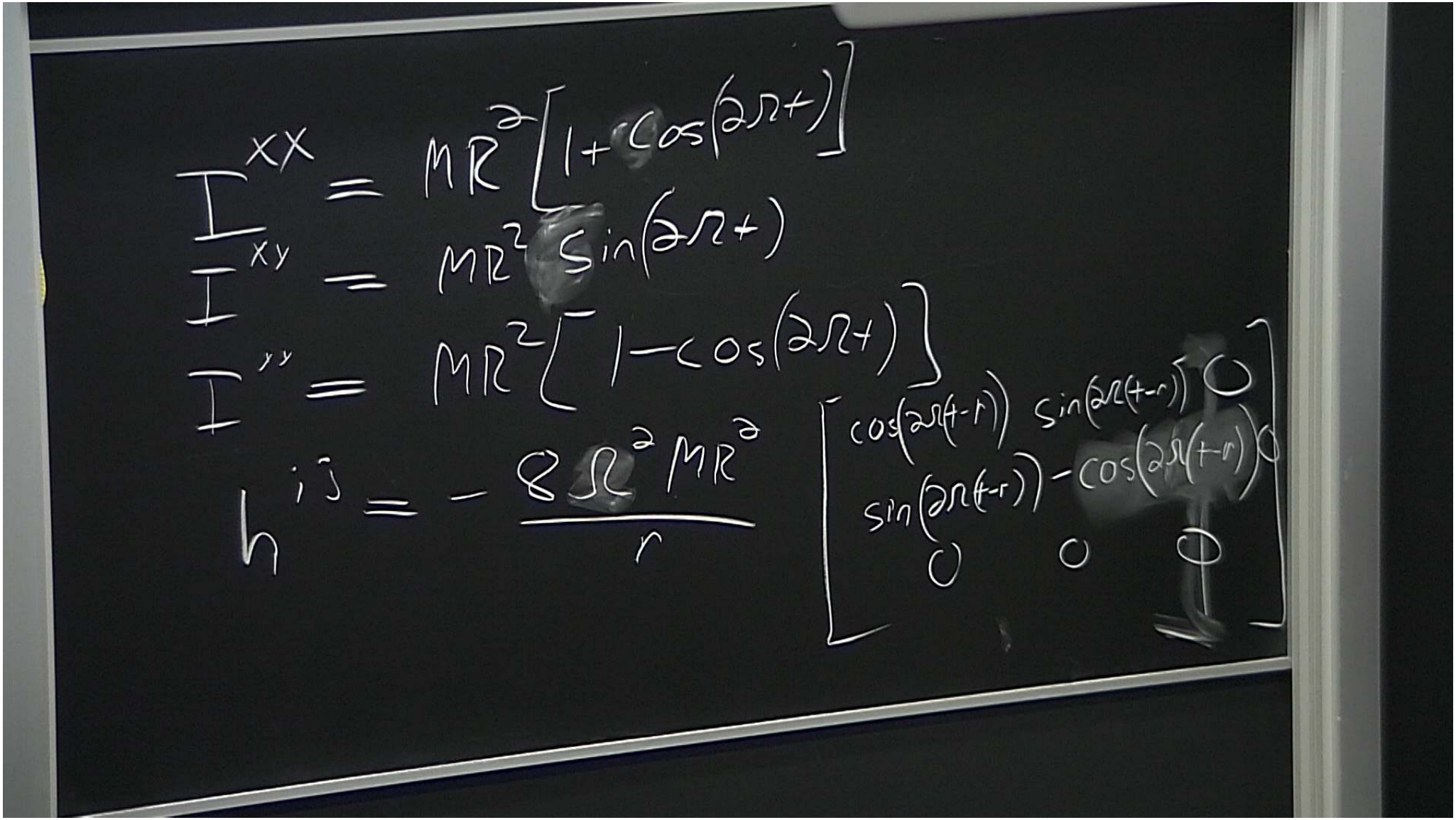
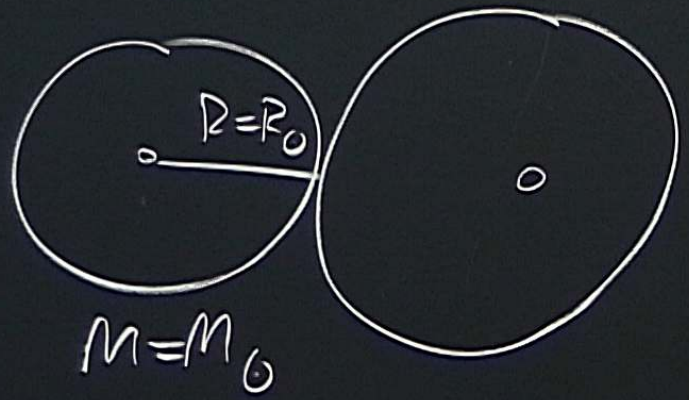

$h^{ij} = \frac{2}{r} \ddot{I}^{ij}(t-r)$
$I^{ij} = \int d^3x \, M(t, x, y, z) \, x^i x^j$


z
$M_A = M_B$
M
B
R
Ω
R
y
x
M_A
$z_A = 0 = z_B$
$X_A(t) = R\cos(\Omega t)$
$X_B = -X_A$
$Y_A(t) = R\sin(\Omega t)$
$Y_B = -Y_A$
$\mu = M_A \delta(x - X_A(t))\,\delta(y - Y_A(t))\,\delta(z-0)$
$+ M_B\,\delta(x - X_B(t))\,\delta(y - Y_B(t))\,\delta(z-0)$


$I^{xx} = MR^2[1+\cos(2\Omega t)]$
$I^{xy} = MR^2 \sin(2\Omega t)$
$I^{yy} = MR^2[1-\cos(2\Omega t)]$


$I^{xx} = MR^2[1+\cos(2\Omega t)]$
$I^{xy} = MR^2 \sin(2\Omega t)$
$I^{yy} = MR^2[1-\cos(2\Omega t)]$
$h^{ij} = -\frac{8\Omega^2 MR^2}{r}$
$\begin{bmatrix} \cos(2\Omega(t-r)) & \sin(2\Omega(t-r)) & 0 \\ \sin(2\Omega(t-r)) & -\cos(2\Omega(t-r)) & 0 \\ 0 & 0 & 0 \end{bmatrix}$

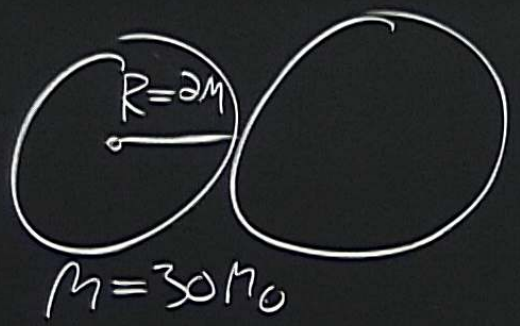
$$R = \left(\frac{M}{4\Omega^2}\right)^{1/3}; \quad |h| = \frac{2M^2}{rR}$$



$$|h| = \left(\frac{1.5}{7\times10^{5}}\right)\frac{2(1.5)}{10^{13}}$$

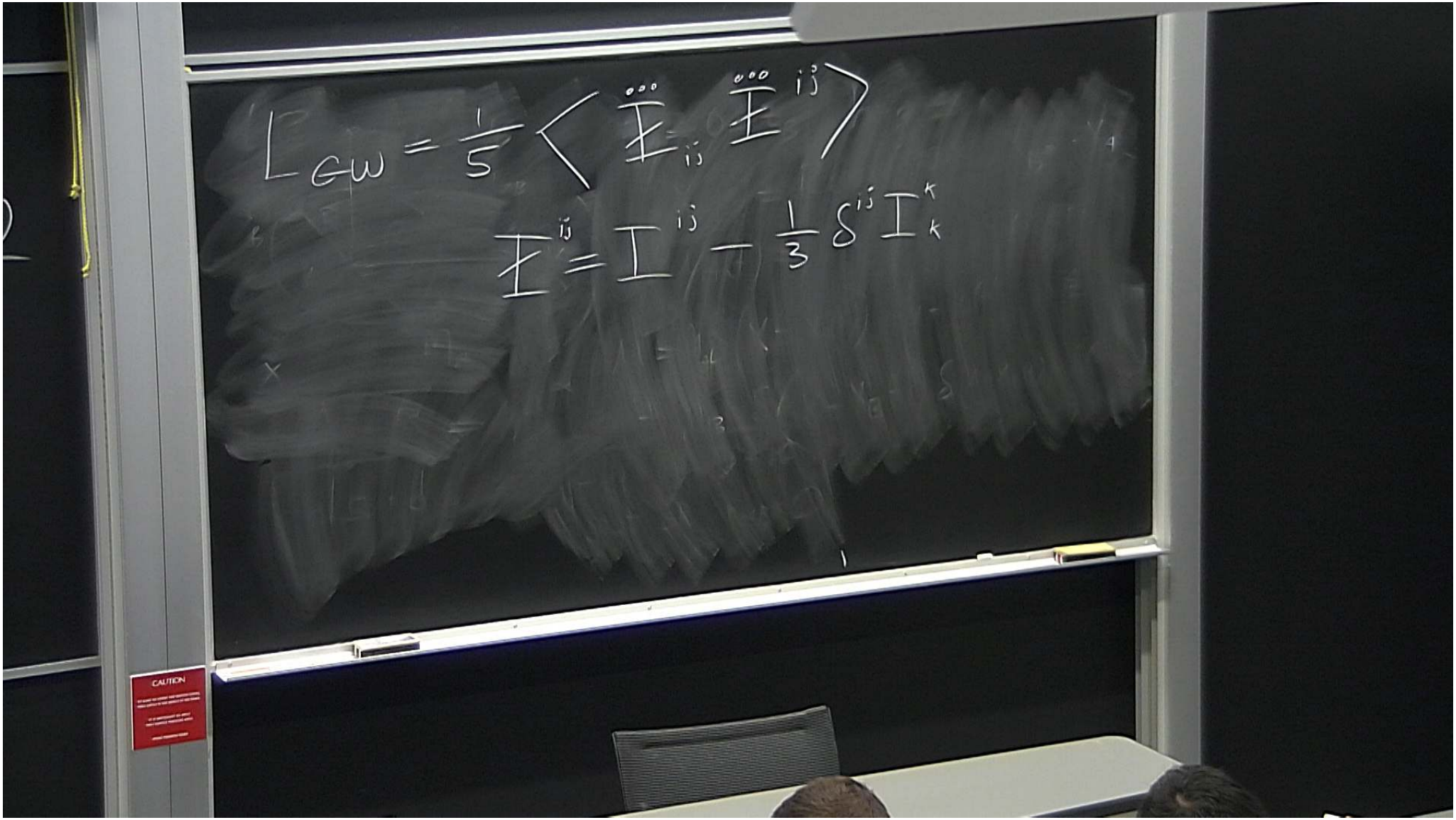
$$= 6\times10^{-19}$$

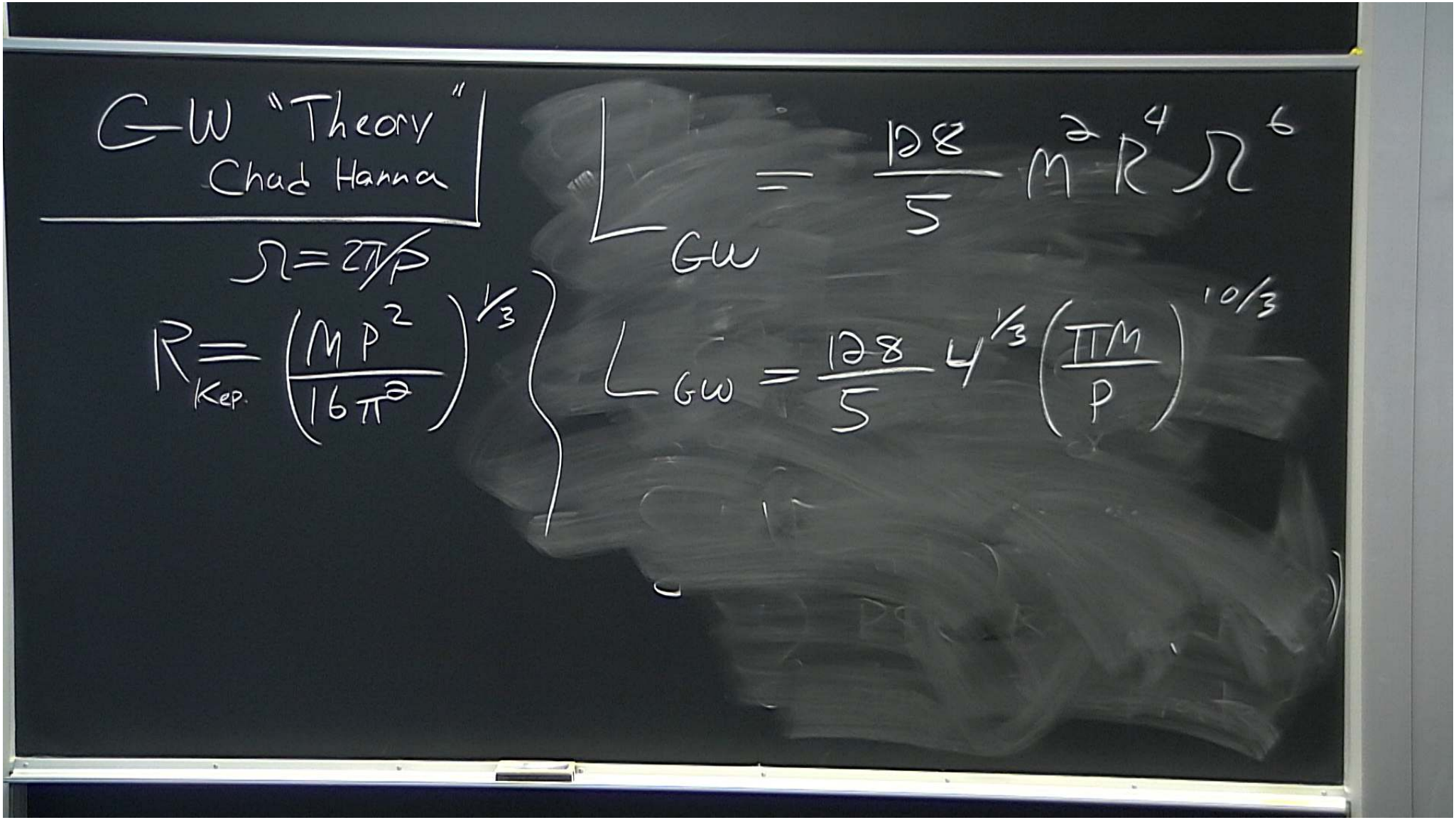
2 $30 M_\odot$ BHs @ $r = 10^9 ly = 10^{22}$ km



$$|h| = \left(\frac{30}{60}\right)\frac{(30)(1)(1.5)}{10^{22}}$$

$$= \sim 10^{-21}$$


$L_{GW} = \frac{1}{5} \langle \dddot{\mathcal{I}}_{ij} \dddot{\mathcal{I}}^{ij} \rangle$
$\mathcal{I}^{ij} = I^{ij} - \frac{1}{3}\delta^{ij} I^{k}_{k}$
CAUTION


GW "Theory"
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$L_{GW} = \frac{128}{5} m^2 R^4 \Omega^6$
$\Omega = 2\pi/P$
$R = R_{Kep.} = \left(\frac{MP^2}{16\pi^2}\right)^{1/3}$
$L_{GW} = \frac{128}{5} 4^{1/3} \left(\frac{\pi M}{P}\right)^{10/3}$