

Title: Gravitational Waves Theory 1

Date: Jul 18, 2018 11:00 AM

URL: <http://pirsa.org/18070017>

Abstract:



GW "Theory"
Chad Hanna

Outline:

1) what are GWs

a) linearized gravity

b) idealized sources

c) predictions of amplitude

2) Compact binaries

a) binary inspiral

b) PSR B1913+16

c) How we predict rate

3) Extracting Physics

- a) Chirp mass by eye.
- b) Bayesian inference
- c) Typical uncert.

4) Gravitational Searches

- a) matched filtering
- b) non stationary noise
- c) significance

5) Observational State of field



Linearized GR

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$$

$$\eta_{\alpha\beta} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A \cdot B = A^\alpha g_{\alpha\beta} B^\beta$$

Linearized GR

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$$\vec{A} \cdot \vec{B} = A^\alpha g_{\alpha\beta} B^\beta$$

$$\sqrt{c^2 ds^2} = \sqrt{-dt^2 + dx^2 + dy^2 + dz^2}$$

just SR $h_{\alpha\beta} = 0$


$$\begin{pmatrix}
 0 & 0 & 0 & 0 \\
 0 & f_+(t-z) & f_+(t-z) & 0 \\
 0 & f_+(t-z) & -f_+(t-z) & 0 \\
 0 & 0 & 0 & 0
 \end{pmatrix}$$

$B^{\mathbb{R}}$

G-W "Theory"
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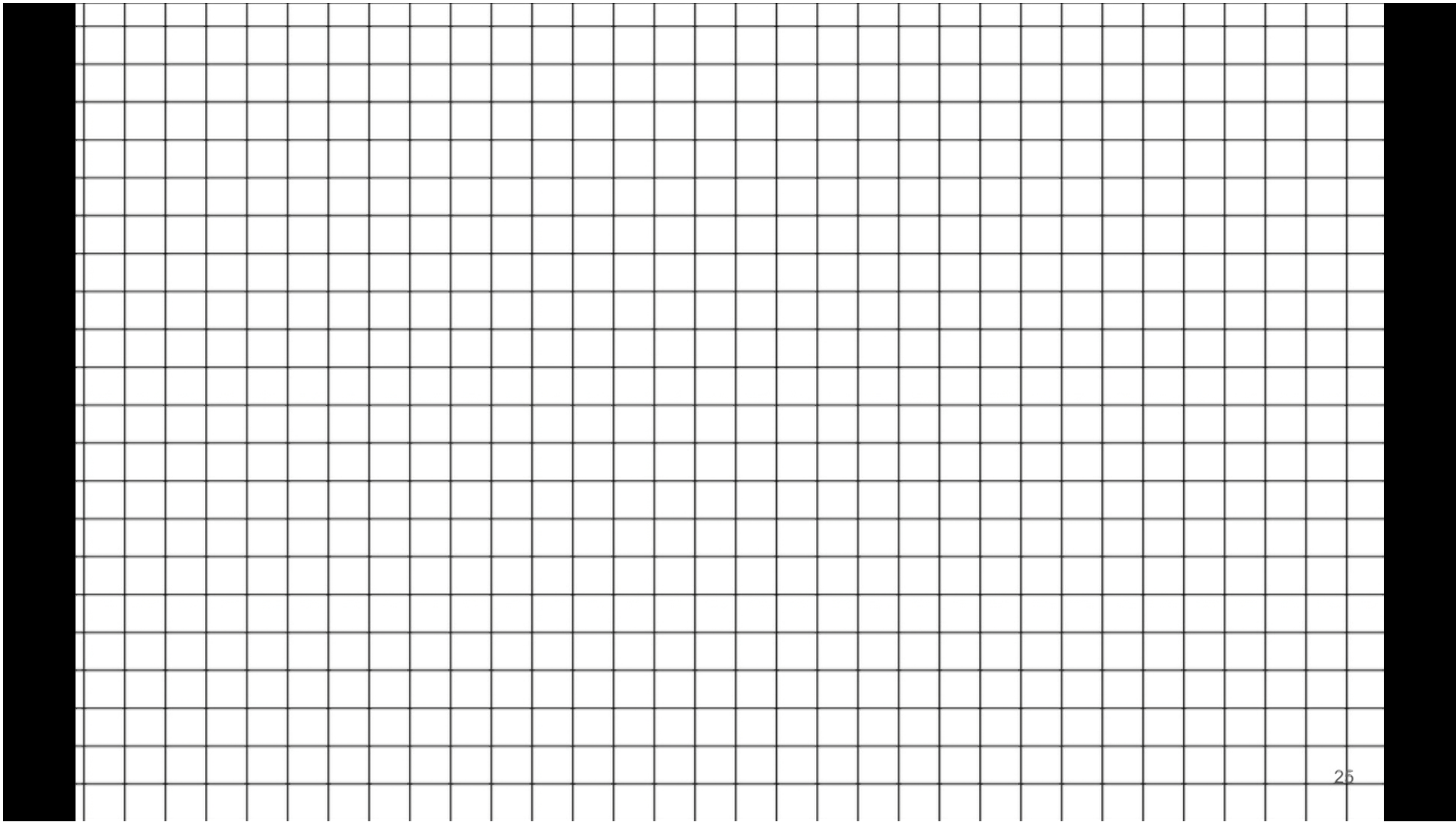
$x=0$

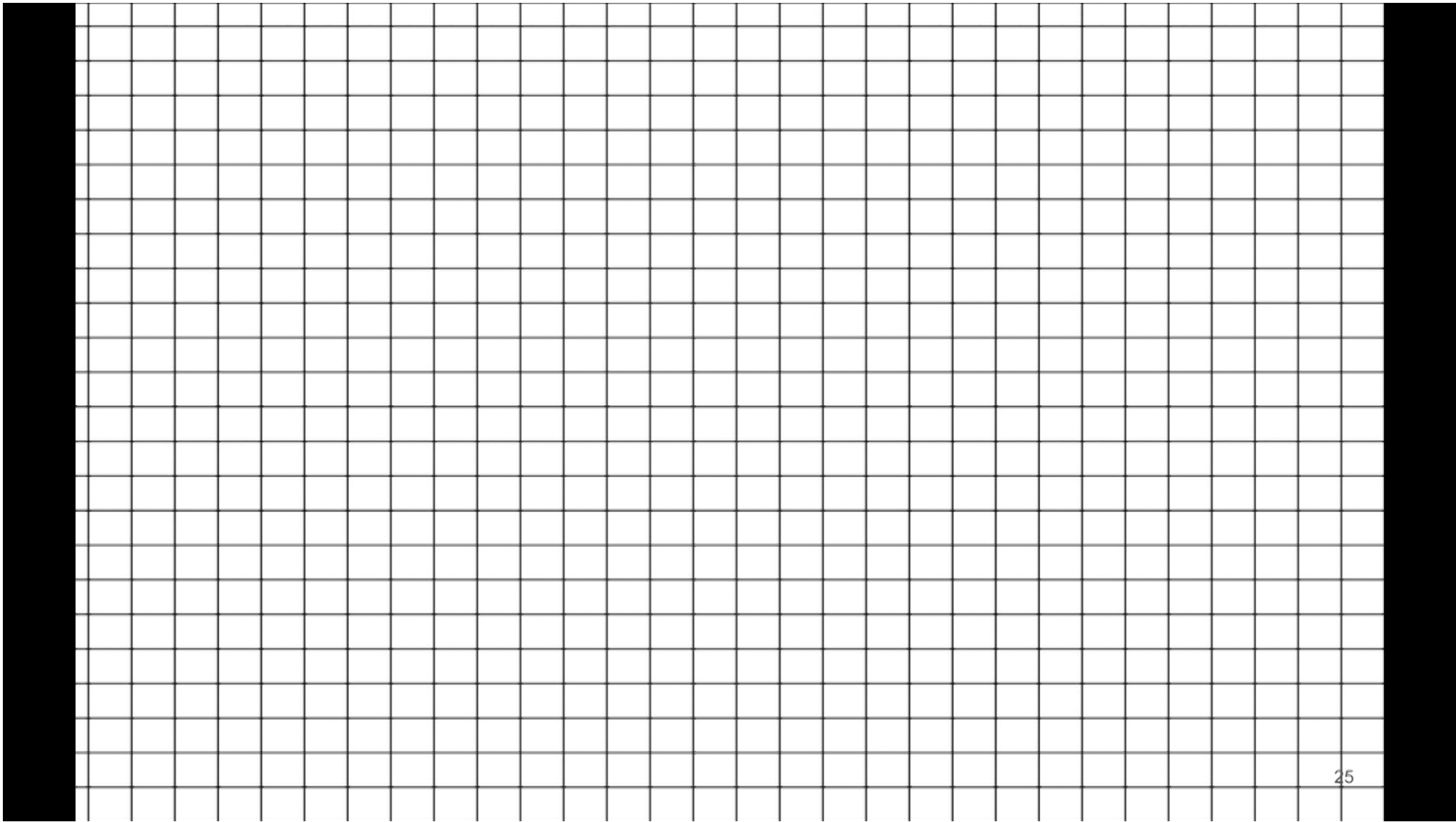
$x=L^*$



A diagram showing a horizontal line segment representing a rod. The left end is marked with a dot and labeled $x=0$. The right end is marked with a dot and labeled $x=L^*$. A double-headed arrow above the line indicates the length L . Below the line, the coordinate x is written at the left end, and L^* is written at the right end.

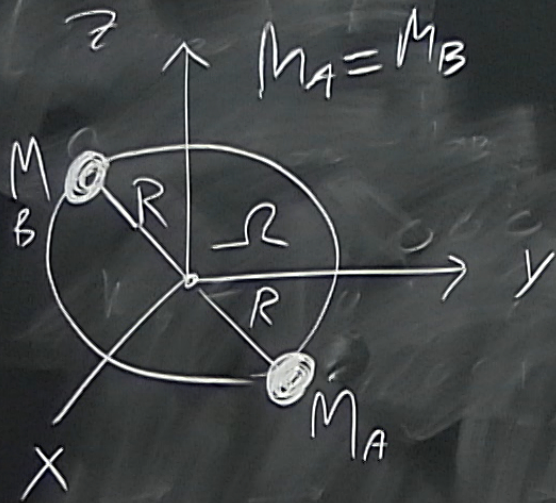
$$L = \int_0^{L^*} \sqrt{d\vec{s} \cdot d\vec{s}} = \int_0^{L^*} \sqrt{dx g_{xx} dx}$$
$$= \int_0^{L^*} dx \sqrt{1 + f_x(t, z)} = L^* \left(1 + \frac{1}{2} f_x(t, z) \right)$$





$$h^{ij} = \frac{2}{r} I^{ij}(t-r)$$

$$I^{ij} = \int d^3x M(t, x, i, j) x^i x^j$$



$$z_A = 0 = z_B$$

$$x_A(t) = R \cos(\Omega t) \quad x_B = -x_A$$

$$y_A(t) = R \sin(\Omega t) \quad y_B = -y_A$$

$$M = M_A \int \delta(x - x_A(t)) \delta(y - y_A(t)) \delta(z - 0) \\ + M_B \int \delta(x - x_B(t)) \delta(y - y_B(t)) \delta(z - 0)$$

$$\begin{aligned} I^{xx} &= MR^2 [1 + \cos(2\Omega t)] \\ I^{xy} &= MR^2 \sin(2\Omega t) \\ I^{yy} &= MR^2 [1 - \cos(2\Omega t)] \end{aligned}$$

$$I_{xx} = MR^2 [1 + \cos(2\Omega t)]$$

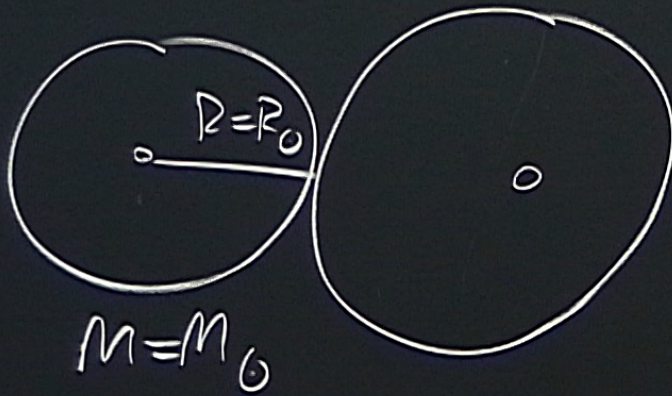
$$I_{xy} = MR^2 \sin(2\Omega t)$$

$$I_{yy} = MR^2 [1 - \cos(2\Omega t)]$$

$$h_{ij} = -\frac{8\Omega^2 MR^2}{r}$$

$$\begin{bmatrix} \cos(2\Omega(t-r)) & \sin(2\Omega(t-r)) & 0 \\ \sin(2\Omega(t-r)) & -\cos(2\Omega(t-r)) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

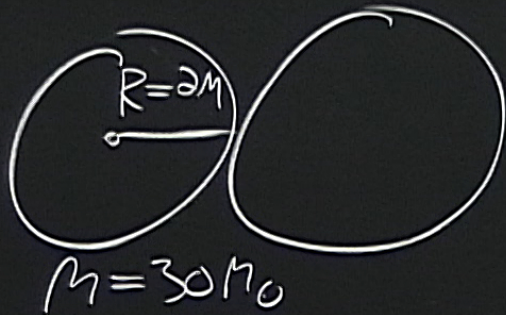
$$R = \left(\frac{M}{4\Omega^2} \right)^{1/3}, \quad |h| = \frac{2M^2}{rR}$$



$$|h| = \left(\frac{1.5}{7 \times 10^5} \right) \frac{2(1.5)}{10^{13}}$$

$$= 6 \times 10^{-19}$$

2 $30 M_{\odot}$ BHs (a) $r = 10^9 \text{ ly} = 10^{22} \text{ km}$



$$|h| = \left(\frac{30}{60} \right) \frac{(30)(2)(1.5)}{10^{22}}$$
$$= \sim 10^{-21}$$

$$L_{EW} = \frac{1}{5} \left\langle \overset{\circ\circ\circ}{I}_{ij} \overset{\circ\circ\circ}{I}_{ij} \right\rangle$$

$$\overset{\circ\circ\circ}{I}_{ij} = I_{ij} - \frac{1}{3} \delta^{ij} I_k^k$$

CAUTION
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$$\Omega = 2\pi/P$$

$$R_{\text{Kep.}} = \left(\frac{M P^2}{16\pi^2} \right)^{1/3}$$

$$\mathcal{L}_{\text{GW}} = \frac{128}{5} M^2 R^4 \Omega^6$$

$$\mathcal{L}_{\text{GW}} = \frac{128}{5} 4^{1/3} \left(\frac{\pi M}{P} \right)^{10/3}$$