Title: Gravitational Waves Theory 1

Date: Jul 18, 2018 11:00 AM

URL: http://pirsa.org/18070017

Abstract:

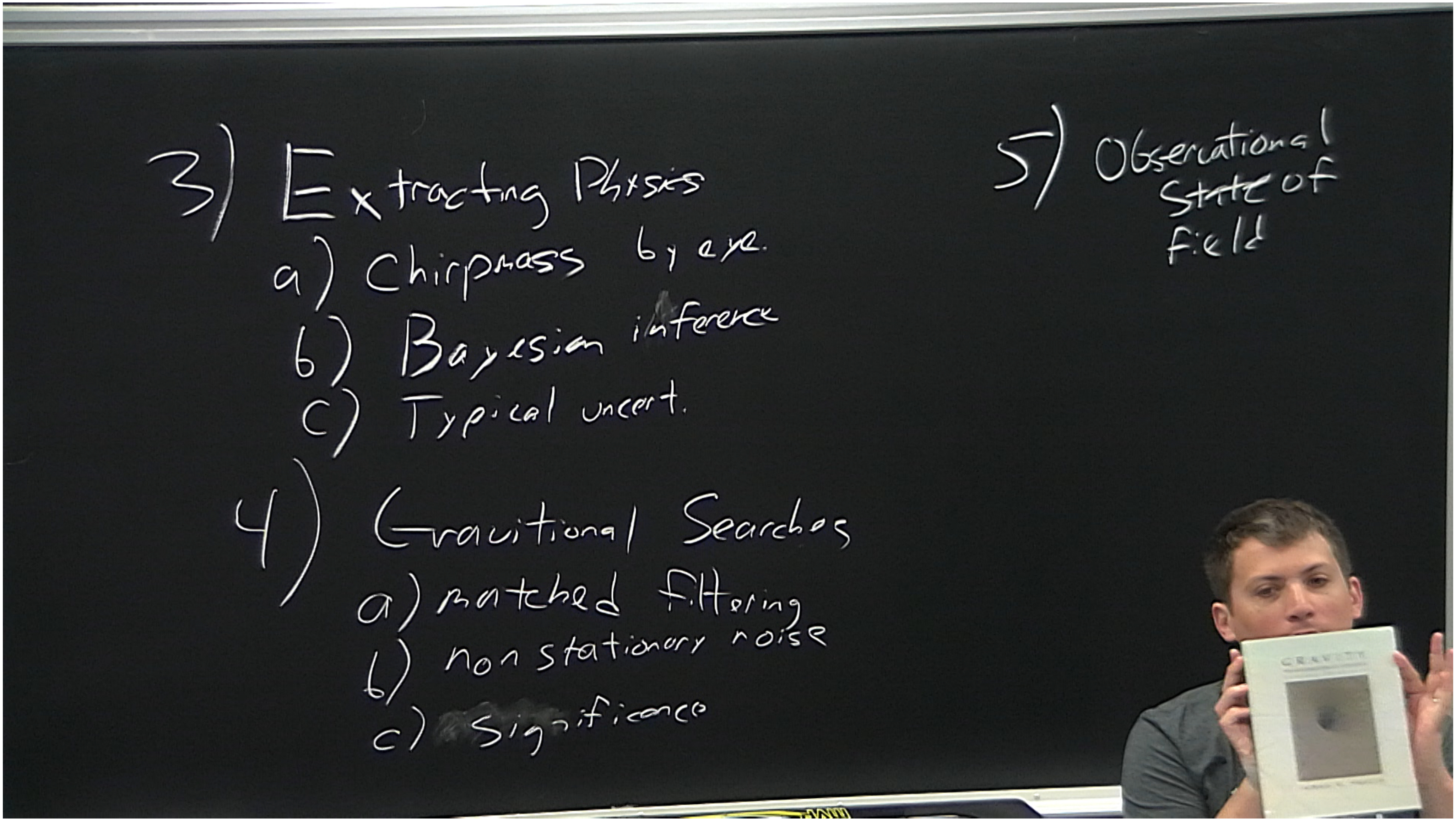

LIGO

GW "Theory"

Chad Hanna

Outline:

1) what are GWs
   a) linearized gravity
   b) idealized sources
   c) predictions of amplitude
2) Compact binaries
   a) binary inspiral
   b) PSR B1913+16
   c) How we predict rate


3) Extracting Physics
a) Chirpmass by eye.
b) Bayesian inference
c) Typical uncert.
4) Gravitional Searches
a) matched filtering
b) nonstationary noise
c) significance
5) Observational State of Field

Linearized GR

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$$

$$\eta_{\alpha\beta} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A \cdot B = A^{\alpha} g_{\alpha\beta} B^{\beta}$$

al
of

Linearized GR

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$$

$$h_{\alpha\beta} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

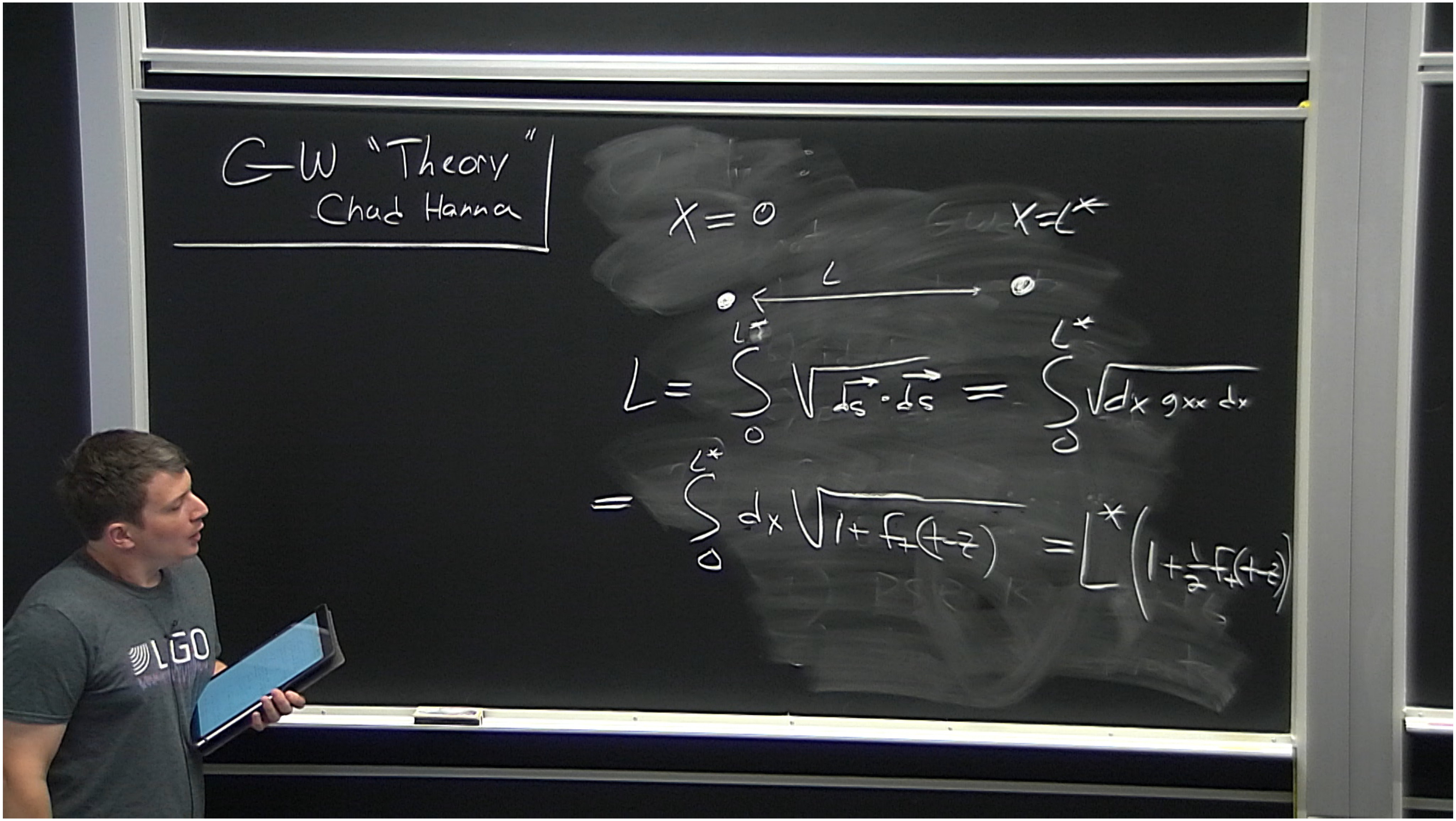
$$\vec{A} \cdot \vec{B} = A^{\alpha} g_{\alpha\beta} B^{\beta}$$

$$\sqrt{\vec{ds} \cdot \vec{ds}} = \sqrt{-dt^2 + dx^2 + dy^2 + dz^2}$$

just SR $h_{\alpha\beta} = 0$

B^{B}

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & f_+(t-z) & f_\times(t-z) & 0 \\ 0 & f_\times(t-z) & -f_+(t-z) & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

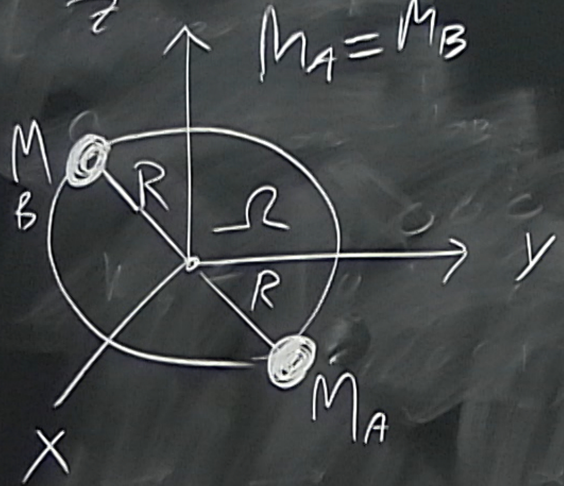

GW "Theory"
Chad Hanna
$X = 0$
$X = L^*$
L
$L = \int_0^{L^*} \sqrt{\vec{ds} \cdot \vec{ds}} = \int_0^{L^*} \sqrt{dx \, g_{xx} \, dx}$
$= \int_0^{L^*} dx \sqrt{1 + f_+(t-z)} = L^*\left(1 + \frac{1}{2} f_+(t-z)\right)$

25

$$h^{ij} = \frac{2}{r} \ddot{I}^{ij}(t-r)$$

$$I^{ij} = \int d^3x \, M(t,x,y,z) \, x^i x^j$$

$M_A = M_B$



$$z_A = 0 = z_B$$

$$x_A(t) = R\cos(\Omega t) \qquad x_B = -x_A$$

$$y_A(t) = R\sin(\Omega t) \qquad y_B = -y_A$$

$$M = M_A\,\delta\big(x - x_A(t)\big)\,\delta\big(y - y_A(t)\big)\,\delta(z-0)$$
$$+\, M_B\,\delta\big(x - x_B(t)\big)\,\delta\big(y - y_B(t)\big)\,\delta(z-0)$$

$$I^{xx} = MR^2\left[1+\cos(2\Omega t)\right]$$

$$I^{xy} = MR^2 \sin(2\Omega t)$$

$$I^{yy} = MR^2\left[1-\cos(2\Omega t)\right]$$

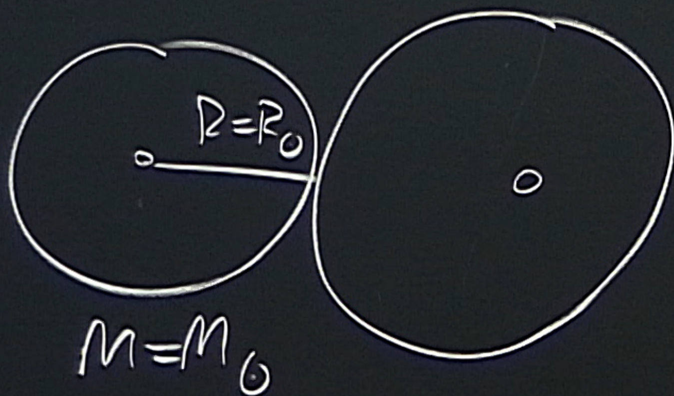
$$I^{xx} = MR^2\left[1+\cos(2\Omega t)\right]$$

$$I^{xy} = MR^2 \sin(2\Omega t)$$

$$I^{yy} = MR^2\left[1-\cos(2\Omega t)\right]$$

$$h^{ij} = -\frac{8\Omega^2 MR^2}{r}\begin{bmatrix} \cos(2\Omega(t-r)) & \sin(2\Omega(t-r)) & 0 \\ \sin(2\Omega(t-r)) & -\cos(2\Omega(t-r)) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

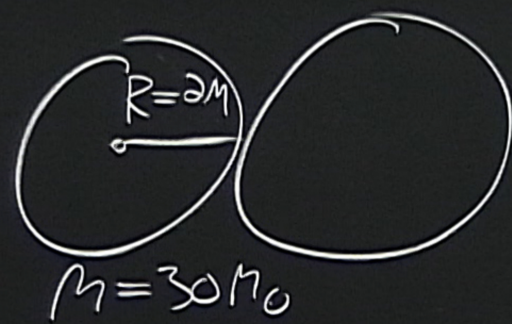
$$R = \left(\frac{M}{4\Omega^2}\right)^{1/3} ; \quad |h| = \frac{2M^2}{rR}$$



$$|h| = \left(\frac{1.5}{7\times10^5}\right)\frac{2(1.5)}{10^{13}}$$

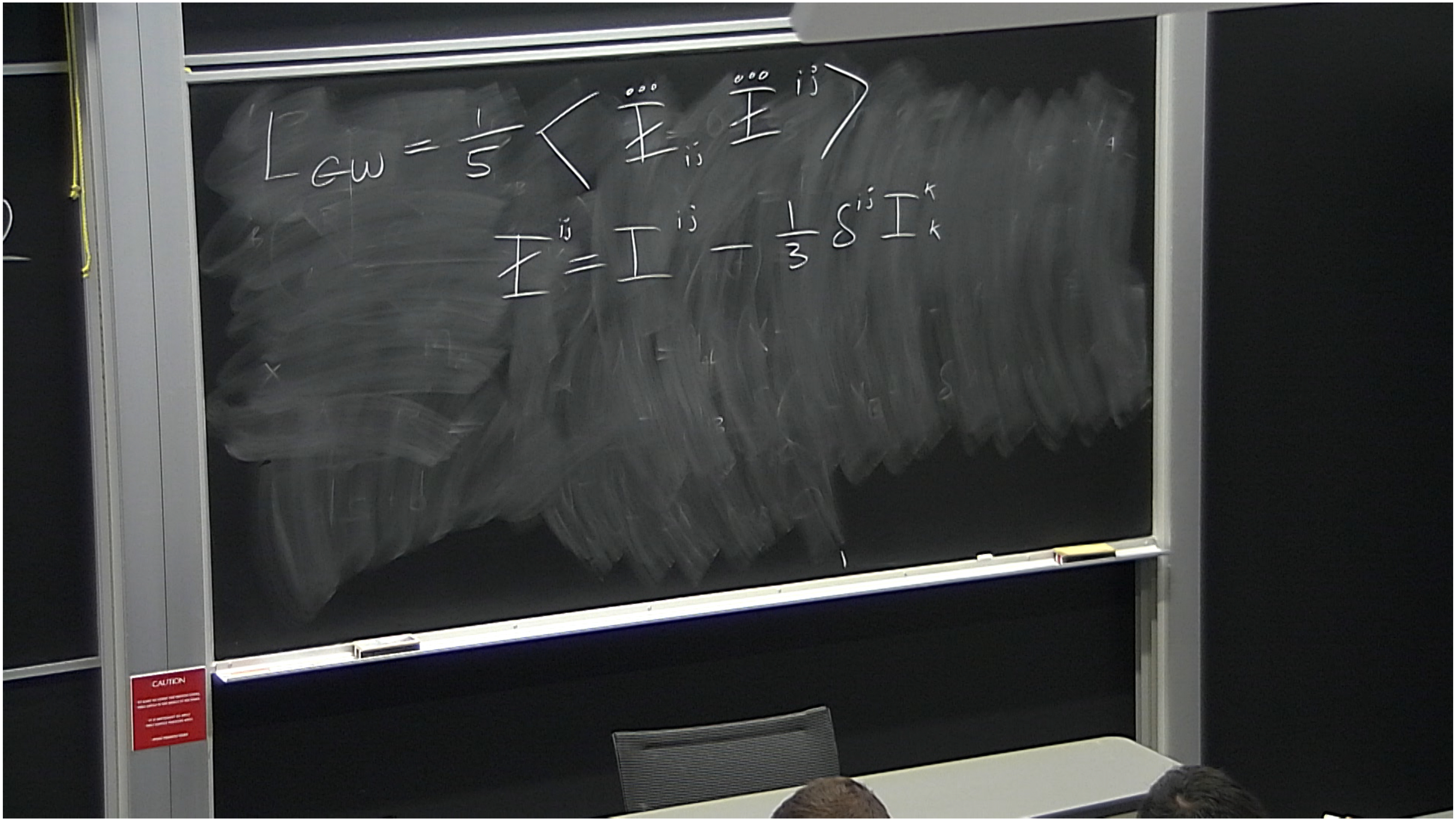
$$= 6\times10^{-19}$$

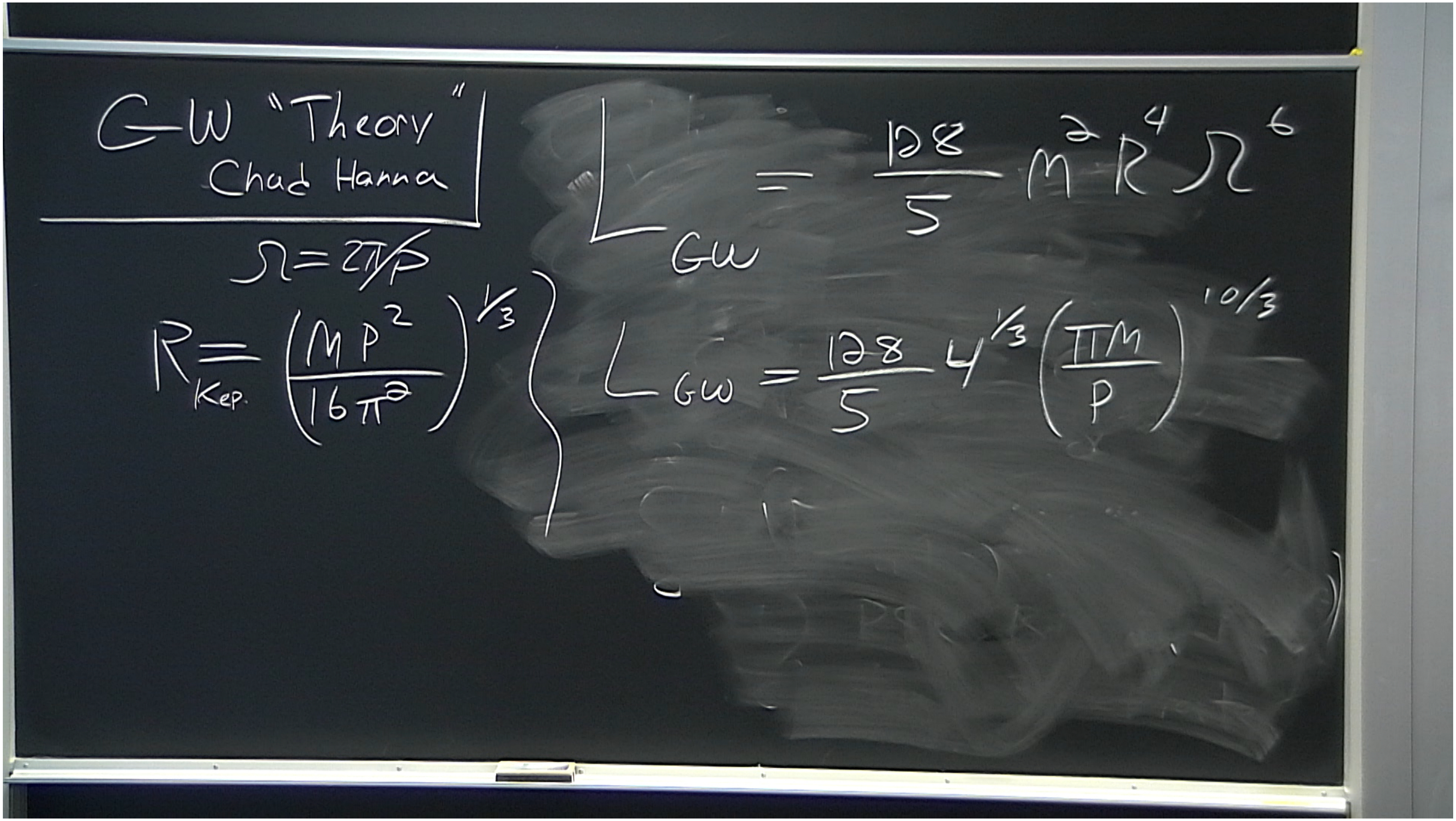
2 $30 M_\odot$ BHs ⓐ $r = 10^9 ly = 10^{22} km$



$$|h| = \left(\frac{30}{60}\right) \frac{(30)(2)(1.5)}{10^{27}}$$

$$= \sim 10^{-21}$$


$L_{GW} = \frac{1}{5} \langle \dddot{I}_{ij} \dddot{I}^{ij} \rangle$
$I^{ij} = I^{ij} - \frac{1}{3} \delta^{ij} I^k_k$


GW "Theory"
Chad Hanna
$\Omega = 2\pi/P$
$R_{Kep.} = \left(\frac{M P^2}{16\pi^2}\right)^{1/3}$
$L_{GW} = \frac{128}{5} M^2 R^4 \Omega^6$
$L_{GW} = \frac{128}{5} 4^{1/3} \left(\frac{\pi M}{P}\right)^{10/3}$