

Title: BSM Theory 3

Date: Jul 13, 2018 11:00 AM

URL: <http://pirsa.org/18070013>

Abstract:

The Strong-CP Problem



The Strong-CP Problem

$$\mathcal{L} = \dots + \frac{g^2}{32\pi^2} \theta \underbrace{\epsilon_{\mu\nu\alpha\beta} G^{\alpha\mu} G^{\nu\beta}}_{G\tilde{G}} + \dots$$

=

+

The Strong-CP Problem

$$\mathcal{L} = \dots + \frac{g^2}{32\pi^2} \theta \underbrace{\epsilon_{\mu\nu\alpha\beta} G^{\mu\nu} G^{\alpha\beta}}_{G\tilde{G}} + \dots$$

- Classical Symmetry \neq Quantum Symmetry
- Total Derivatives can matter.

Quantum Symmetries

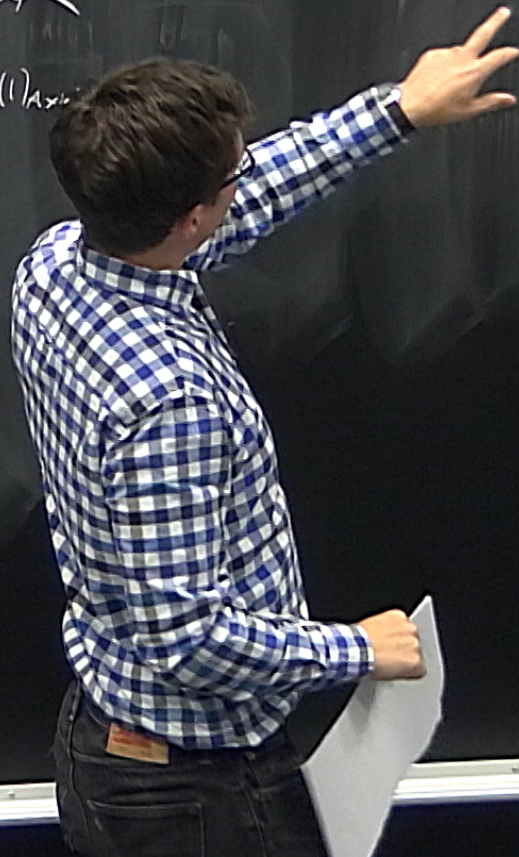
$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + i\bar{\Psi} \gamma^{\mu} D_{\mu} \Psi + \cancel{m\bar{\Psi}\Psi}$$

Quantum Symmetries

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + i\bar{\Psi} \gamma^{\mu} D_{\mu} \Psi + \cancel{m\bar{\Psi}\Psi}$$

Symmetry $\Psi \rightarrow e^{i\alpha\gamma_5} \Psi$: $U(1)_{\text{Axial}}$

$$\bar{\Psi} \gamma_5 \Psi + \Psi^{\dagger} \gamma_5 \Psi$$



CAUTION

Quantum Symmetries

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + i\bar{\Psi} \gamma^n D_n \Psi + \cancel{m\bar{\Psi}\Psi}$$

Symmetry $\Psi \rightarrow e^{i\alpha\gamma_5} \Psi$: $U(1)_{\text{Axial}}$

$$Z = \int \mathcal{D}\Psi \mathcal{D}\bar{\Psi} e^{i\int d^4x \mathcal{L}}$$

$$\bar{\Psi} \gamma_5 \Psi + \Psi^\dagger \gamma_5 \Psi$$

Quantum Symmetries

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + i\bar{\Psi} \gamma^n D_n \Psi + \cancel{m\bar{\Psi}\Psi}$$

Symmetry $\Psi \rightarrow e^{i\alpha\gamma_5} \Psi$: U(1)_{Axial}

$$Z = \int \underbrace{\mathcal{D}\Psi \mathcal{D}\bar{\Psi}}_{\text{Path Integral Measure}} e^{i\int d^4x \mathcal{L}}$$

$$\bar{\Psi} \gamma_5 \Psi + \Psi \gamma_5 \Psi$$

Quantum Symmetries

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + i\bar{\psi} \gamma^\mu D_\mu \psi + \cancel{m\bar{\psi}\psi}$$

Symmetry $\psi \rightarrow e^{i\alpha\gamma_3} \psi$: U(1)_{Axial}

$$\mathcal{Z} = \int \underbrace{\mathcal{D}\psi \mathcal{D}\bar{\psi}}_{\text{Path Integral Measure}} e^{i\int d^4x \mathcal{L}}$$

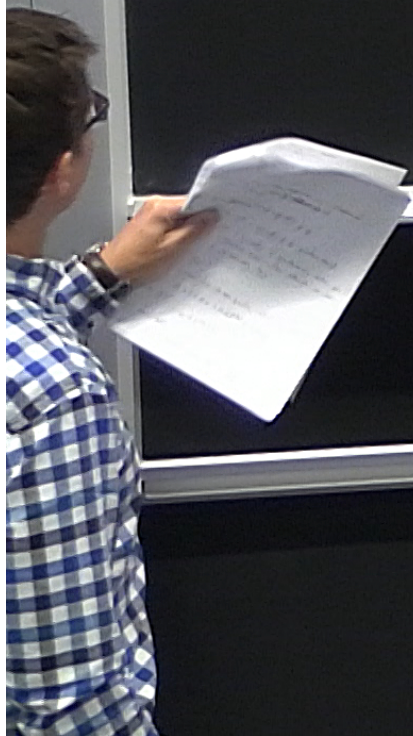
$$\Rightarrow \mathcal{D}\psi \mathcal{D}\bar{\psi} \rightarrow |\mathcal{J}|^{-2} \mathcal{D}\psi \mathcal{D}\bar{\psi}$$

$$\chi' \chi \chi + \psi^+ \psi \psi$$

CAUTION

$$Z = \int \underbrace{\delta\psi\delta\bar{\psi}}_{\text{Path Integral Measure}} e^{i\int d^4x \mathcal{L}} \Rightarrow \dots$$

$$|J|^{-2} = e^{i\int d^4x \frac{g^2\alpha}{32\pi} G\tilde{G}}$$



Quantum Symmetries

$$P_\mu + igA_\mu^a$$

↓

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + i\bar{\psi} \gamma^\mu D_\mu \psi + \cancel{m\bar{\psi}\psi}$$

$$x' \delta x + \psi' \delta \psi$$

Symmetry $\psi \rightarrow e^{i\alpha \gamma_5} \psi : U(1)_{\text{axial}}$

$$\mathcal{Z} = \int \underbrace{\mathcal{D}\psi \mathcal{D}\bar{\psi}}_{\text{Path Integral Measure}} e^{i\int d^4x \mathcal{L}}$$

$$\Rightarrow \mathcal{D}\psi \mathcal{D}\bar{\psi} \rightarrow |\mathcal{J}|^{-2} \mathcal{D}\psi \mathcal{D}\bar{\psi}$$

$$|\mathcal{J}|^{-2} = e^{i\int d^4x \frac{g^2 \alpha}{32\pi} \tilde{G}\tilde{G}}$$

$$Z = \int \underbrace{\Delta\psi\Delta\bar{\psi}}_{\text{Path Integral Measure}} e^{i\int d^4x \mathcal{L}}$$

$$\Rightarrow \Delta\psi\Delta\bar{\psi} \rightarrow |J|^{-2} \Delta\psi\Delta\bar{\psi}$$

$$|J|^{-2} = e^{i\int d^4x \frac{g^2\alpha}{32\pi^2} G\tilde{G}}$$

$$Z' = \int \Delta\psi\Delta\bar{\psi} e^{i\int d^4x \left(\mathcal{L} + \frac{\alpha g^2}{32\pi^2} G\tilde{G} \right)}$$

Symmetry $\psi \rightarrow \bar{\psi}$

$$Z = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{i \int d^4x \mathcal{L}}$$

Path Integral
Measure

$$\Rightarrow \mathcal{D}\psi \mathcal{D}\bar{\psi} \rightarrow |\mathcal{J}|^{-2} \mathcal{D}\psi \mathcal{D}\bar{\psi}$$

$$|\mathcal{J}|^{-2} = e^{i \int d^4x \frac{g^2 \alpha}{32\pi^2} G\tilde{G}}$$

$$Z' = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{i \int d^4x \left(\mathcal{L} + \frac{\alpha g^2}{32\pi^2} G\tilde{G} \right)}$$

Symmetry $\psi \rightarrow e^{i\alpha\sigma_3} \psi$: $U(1)_{\text{Axial}}$

$$Z = \int \underbrace{\Delta\psi\Delta\bar{\psi}}_{\text{Path Integral Measure}} e^{i\int d^4x \mathcal{L}}$$

$$\Rightarrow \Delta\psi\Delta\bar{\psi} \rightarrow |J|^{-2} \Delta\psi\Delta\bar{\psi}$$

$$|J|^{-2} = e^{i\int d^4x \frac{g^2\alpha}{32\pi^2} G\tilde{G}}$$

$$Z = \int \Delta\psi\Delta\bar{\psi} e^{i\int d^4x (\mathcal{L} + \frac{\alpha g^2}{32\pi^2} G\tilde{G})}$$

$U(1)_A$ is not a symmetry of quantum theory!

$$\mathcal{L} = \mathcal{L}_{\text{Gauge}} + i\bar{\Psi} \gamma^m D_m \Psi + \cancel{m\bar{\Psi}\Psi}$$

Symmetry $\Psi \rightarrow e^{i\alpha\gamma_5} \Psi$: $U(1)_{\text{Axial}}$

$$Z = \int \underbrace{\mathcal{D}\Psi \mathcal{D}\bar{\Psi}}_{\text{Path Integral Measure}} e^{i\int d^4x \mathcal{L}}$$

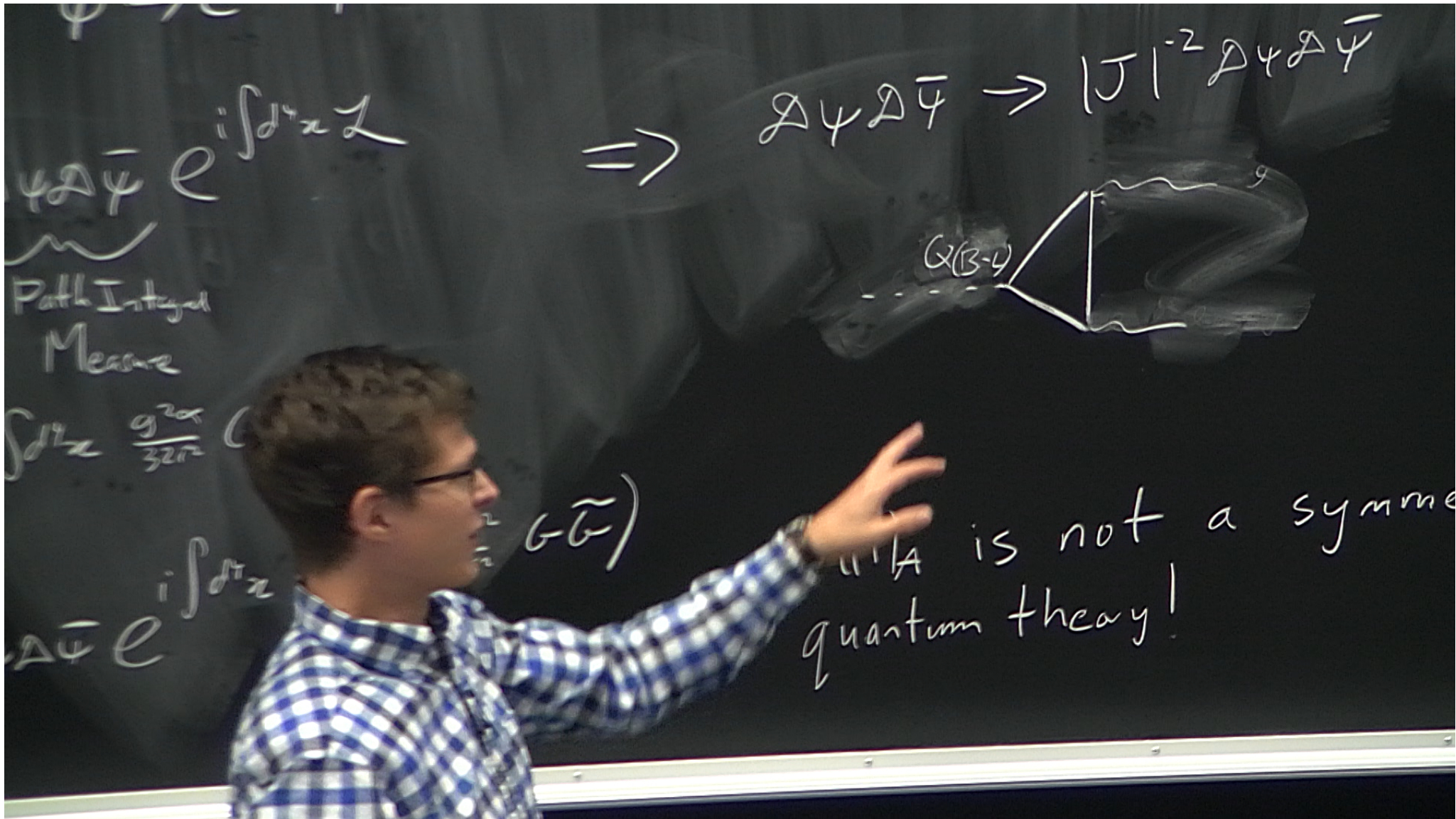
$$\Rightarrow \mathcal{D}\Psi \mathcal{D}\bar{\Psi} \rightarrow |\mathcal{J}|^{-2} \mathcal{D}\Psi \mathcal{D}\bar{\Psi}$$



$$|\mathcal{J}|^{-2} = e^{i\int d^4x \frac{g^2 \alpha}{32\pi} \dots}$$

$$Z' = \int \mathcal{D}\Psi \mathcal{D}\bar{\Psi} e^{i\int d^4x \mathcal{L}(\tilde{G})}$$

$U(1)_A$ is not a symmetry of quantum theory!

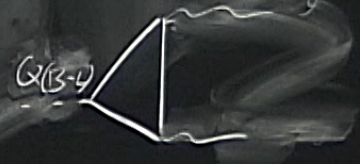


$$\mathcal{L} = \mathcal{L}_{\text{Dirac}} + i\bar{\psi} \gamma^\mu D_\mu \psi + \cancel{m\bar{\psi}\psi}$$

Symmetry $\psi \rightarrow e^{i\alpha\gamma_5} \psi : U(1)_{\text{Axial}}$

$$Z = \int \underbrace{\Delta\psi\Delta\bar{\psi}}_{\text{Path Integral Measure}} e^{i\int d^4x \mathcal{L}}$$

$$\Rightarrow \Delta\psi\Delta\bar{\psi} \rightarrow |\mathcal{J}|^{-2} \Delta\psi\Delta\bar{\psi}$$



$$\psi = \left(\begin{array}{c} \psi \\ \psi \end{array} \right)$$

$$|\mathcal{J}|^{-2} = e^{i\int d^4x \frac{g^2 \alpha^2}{32\pi^2} G\tilde{G}}$$

$$Z' = \int \Delta\psi\Delta\bar{\psi} e^{i\int d^4x \left(\mathcal{L} + \frac{\alpha g^2}{32\pi^2} G\tilde{G} \right)}$$

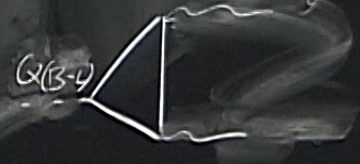
$U(1)_A$ is not a symmetry of quantum theory!

$$\mathcal{L} = \mathcal{L}_{\text{Gauge}} + i\bar{\Psi} \gamma^\mu D_\mu \Psi + \cancel{m\bar{\Psi}\Psi}$$

Symmetry $\Psi \rightarrow e^{i\alpha\sigma_3} \Psi$: U(1)_{Axial}

$$Z = \int \underbrace{\Delta\psi\Delta\bar{\psi}}_{\text{Path Integral Measure}} e^{i\int d^4x \mathcal{L}}$$

$$\Rightarrow \Delta\psi\Delta\bar{\psi} \rightarrow |J|^{-2} \Delta\psi\Delta\bar{\psi}$$



$$\Psi = \begin{pmatrix} e^{-i\alpha} \\ e^{i\alpha} \end{pmatrix}$$

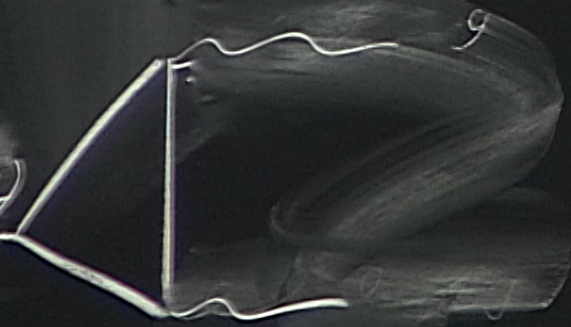
$$|J|^{-2} = e^{i\int d^4x \frac{g^2\alpha^2}{32\pi^2} G\tilde{G}}$$

$$Z' = \int \Delta\psi\Delta\bar{\psi} e^{i\int d^4x (\mathcal{L} + \frac{\alpha g^2}{32\pi^2} G\tilde{G})}$$

U(1)_A is not a symmetry of quantum theory!

$$\psi \Delta \bar{\psi} \rightarrow |\psi|^{-2} \Delta \psi \Delta \bar{\psi}$$

$G(B-4)$



$$\psi = \begin{pmatrix} e^{i\alpha} \chi \\ e^{i\alpha} \phi^+ \end{pmatrix}$$

Adding Mass

$$\mathcal{L} = i\bar{\Psi}\not{\partial}\Psi + m\bar{\Psi}e^{-2i\theta_2}\Psi$$

Adding Mass

$$\mathcal{L} = i\bar{\Psi}\not{\partial}\Psi + m\bar{\Psi}e^{-2i\theta_a\gamma_5}\Psi$$

$$\Psi \rightarrow e^{i\theta_a\gamma_5}\Psi$$

Adding Mass

$$\mathcal{L} = i\bar{\Psi}\not{\partial}\Psi + m\bar{\Psi}e^{-2i\theta_a\gamma_5}\Psi + \mathcal{L}_{\text{Gauge}}$$

$$\Psi \rightarrow e^{i\theta_a\gamma_5}\Psi$$

Adding Mass

$$\mathcal{L} = i\bar{\Psi}\not{D}\Psi + m\bar{\Psi}e^{-2i\alpha_2\gamma_5}\Psi + \mathcal{L}_{\text{Gauge}}$$

$$\Psi \rightarrow e^{i\alpha_2\gamma_5}\Psi$$

$$\mathcal{L}' = i\bar{\Psi}\not{D}\Psi + m\bar{\Psi}\Psi + \mathcal{L}_{\text{Gauge}} + \frac{\alpha_2 g^2}{32\pi^2} \tilde{G}\tilde{G}$$

Adding Mass

$$\mathcal{L} = i\bar{\Psi}\not{D}\Psi + m\bar{\Psi}e^{-2i\theta_2\gamma_5}\Psi + \mathcal{L}_{\text{Gauge}} + \frac{Qg^2}{32\pi^2}G\tilde{G}$$

$$\Psi \rightarrow e^{i\theta_2\gamma_5}\Psi$$

$$\mathcal{L}' = i\bar{\Psi}\not{D}\Psi + m\bar{\Psi}\Psi + \mathcal{L}_{\text{Gauge}} + \frac{(0+0_2)g^2}{32\pi^2}G\tilde{G}$$

Adding Mass

$$\mathcal{L} = i\bar{\Psi}\not{\partial}\Psi + m\bar{\Psi}e^{-2i\theta_n\gamma_5}\Psi + \mathcal{L}_{\text{Gauge}} + \frac{\theta g^2}{32\pi^2} G\tilde{G}$$

$$\Psi \rightarrow e^{i\theta_n\gamma_5}\Psi$$

$$\mathcal{L}' = i\bar{\Psi}\not{\partial}\Psi + m\bar{\Psi}\Psi + \mathcal{L}_{\text{Gauge}} + \underbrace{\frac{(\theta+\theta_n)g^2}{32\pi^2} G\tilde{G}}$$

Totally Derivative

$$(\theta+\theta_n)G\tilde{G} = (\theta+\theta_n)\partial^\mu J_\mu$$

$$J_\mu = \sum_{\mu\nu\alpha\beta} \left(A_\nu^a G_{\alpha\beta}^a - \frac{2}{3} f^{abc} A_\nu^a A_\alpha^b A_\beta^c \right)$$

Adding Mass

$$\mathcal{L} = i\bar{\Psi}\not{\partial}\Psi + m\bar{\Psi}e^{-2i\theta_a\gamma_5}\Psi + \mathcal{L}_{\text{Gauge}} + \frac{\theta g^2}{32\pi^2} G\tilde{G}$$

$$\Psi \rightarrow e^{i\theta_a\gamma_5}\Psi$$

$$\mathcal{L}' = i\bar{\Psi}\not{\partial}\Psi + m\bar{\Psi}\Psi + \mathcal{L}_{\text{Gauge}} + \underbrace{\frac{(\theta+\theta_a)g^2}{32\pi^2} G\tilde{G}}$$

Totally Derivative

$$(\theta+\theta_a)G\tilde{G} = (\theta+\theta_a)\partial^\mu J_\mu$$

$$: J_\mu = \sum_{\mu\nu\alpha\beta} \left(A_\nu^a G_{\alpha\beta}^a - \frac{2}{3} f^{abc} A_\nu^a A_\alpha^b A_\beta^c \right)$$

$$\mathcal{L} = i\bar{\psi}\not{\partial}\psi$$

$$\psi \rightarrow e^{i\theta_a \gamma_5} \psi$$

$$\mathcal{L}' = i\bar{\psi}\not{\partial}\psi + m\bar{\psi}\psi + \underbrace{\mathcal{L}_{\text{gauge}} + \frac{(g+\theta_a)^2}{32\pi^2} \tilde{G}\tilde{G}}$$

Totally Derivative

$$(g+\theta_a)\tilde{G}\tilde{G} = (g+\theta_a)\partial^\mu J_\mu$$

$$: J_\mu = \sum_{\mu\nu\alpha\beta} \left(A_\nu^a G_{\alpha\beta}^a - \frac{2}{3} f^{abc} A_\nu^a A_\alpha^b A_\beta^c \right)$$

Aharonov - Bohm Effect:

QCD Instantons

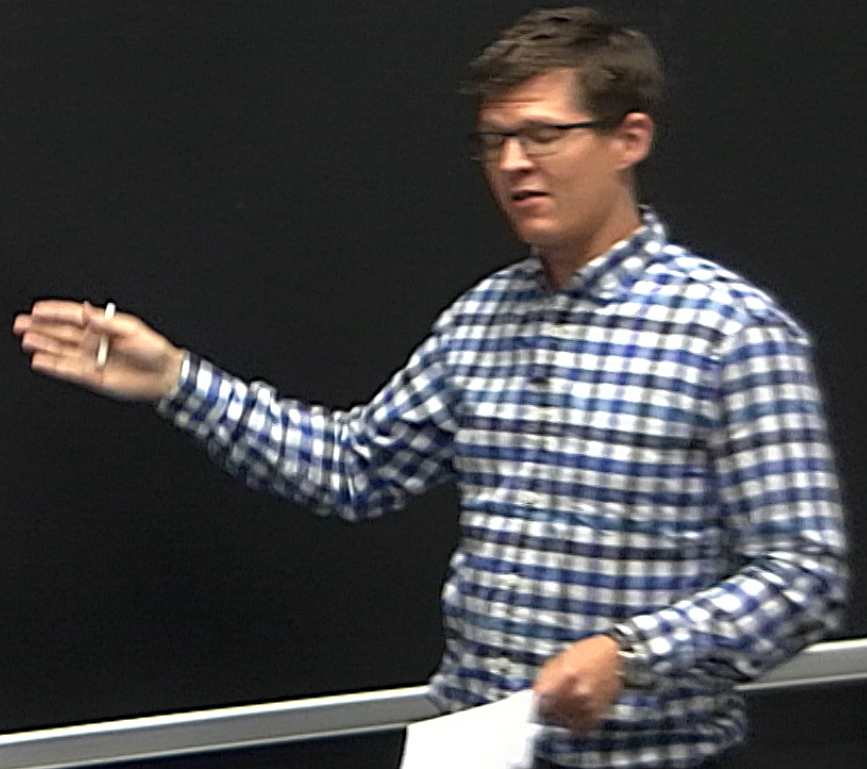
$A_{\text{IVE}} \rightarrow 0$

$$\int d^4x \mathcal{L}(\text{Instanton}) = \text{finite}$$

QCD Instantons

$A_{\text{IWE}} \rightarrow 0$

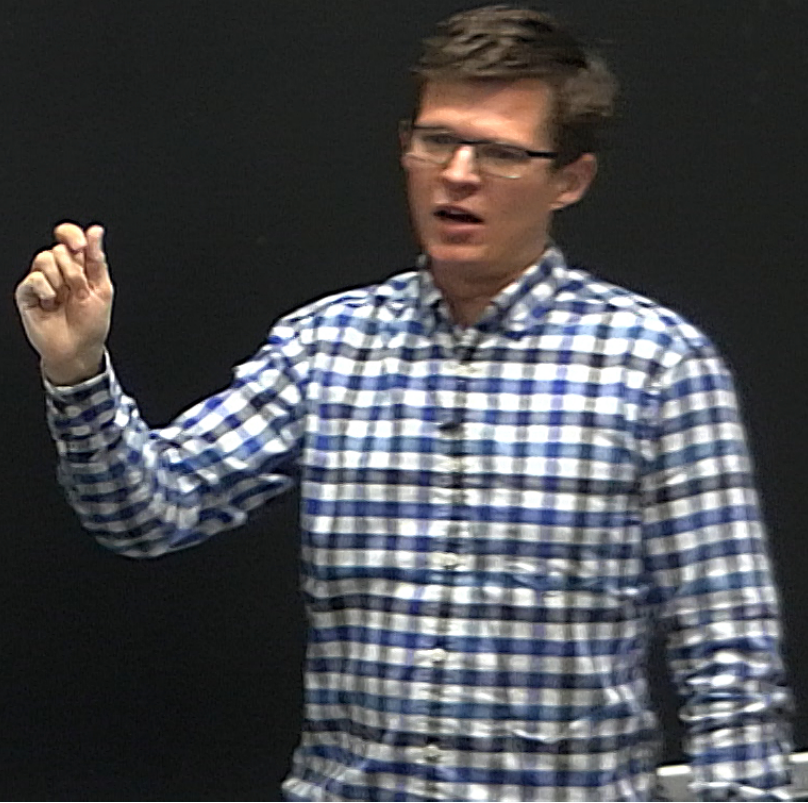
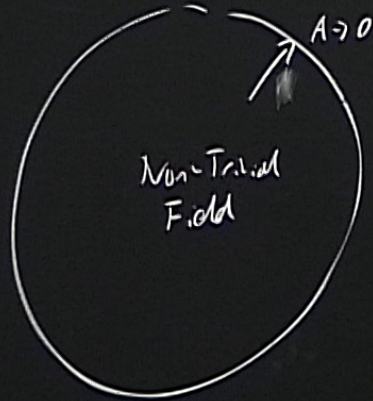
$$\int d^4x \mathcal{L}(\text{Instanton}) = \text{finite}$$



QCD Instantons

$A_{\text{INF}} \rightarrow 0$

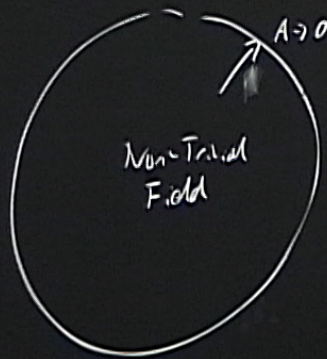
$$\int d^4x \mathcal{L}(\text{Instanton}) = \text{finite}$$



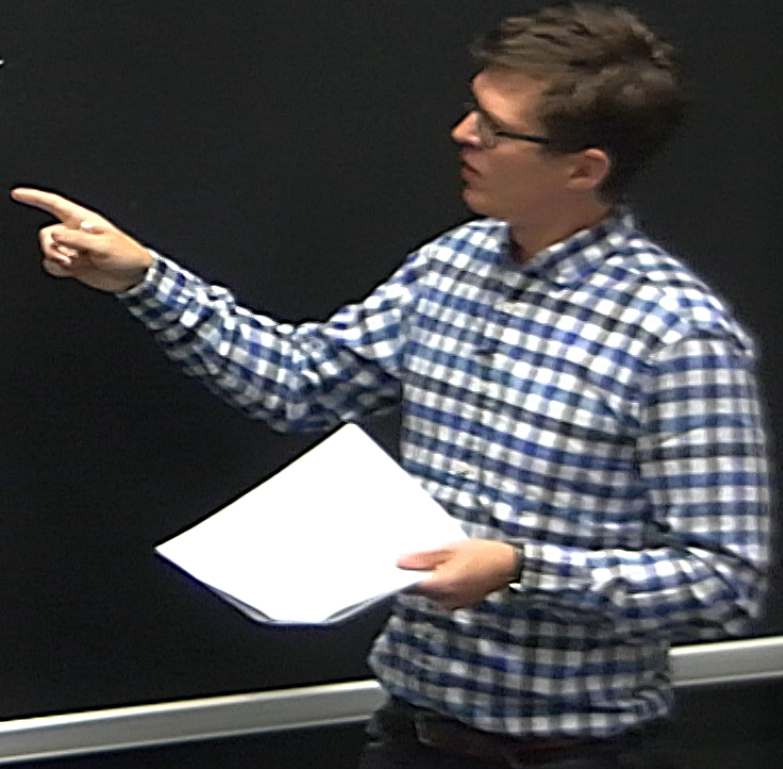
QCD Instantons

$$A_{\text{IUV}} \rightarrow 0$$

$$\int d^4x \mathcal{L}(\text{Instanton}) = \text{finite}$$



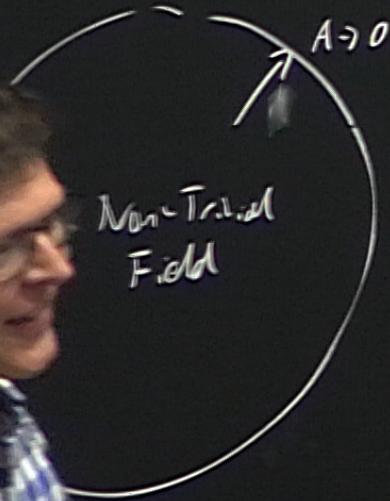
$$e^{-\frac{8\pi^2}{\alpha_s}}$$



QCD Instantons

$$A_{\text{IIVE}} \rightarrow 0$$

$$\int d^4x \mathcal{L}(\text{Instanton}) = \text{finite}$$

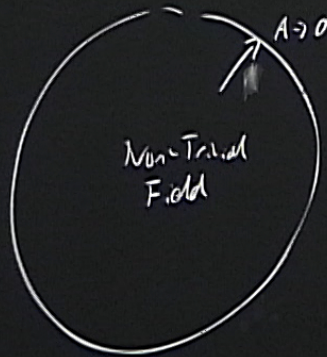


$$e^{-\frac{8\pi^2}{g\alpha(\mu)}}$$

QCD Instantons

$$A_{\text{IUV}} \rightarrow 0$$

$$\int d^4x \mathcal{L}(\text{Instanton}) = \text{finite}$$



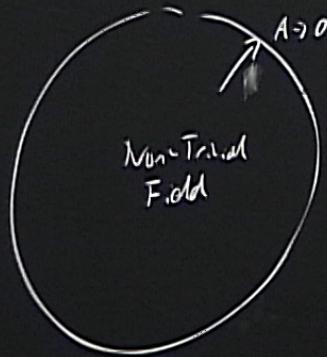
$$e^{-\frac{8\pi^2}{\alpha(\mu)}}$$

$$\alpha(\mu) \ll 1$$

QCD Instantons

$$A_{\text{IUV}} \rightarrow 0$$

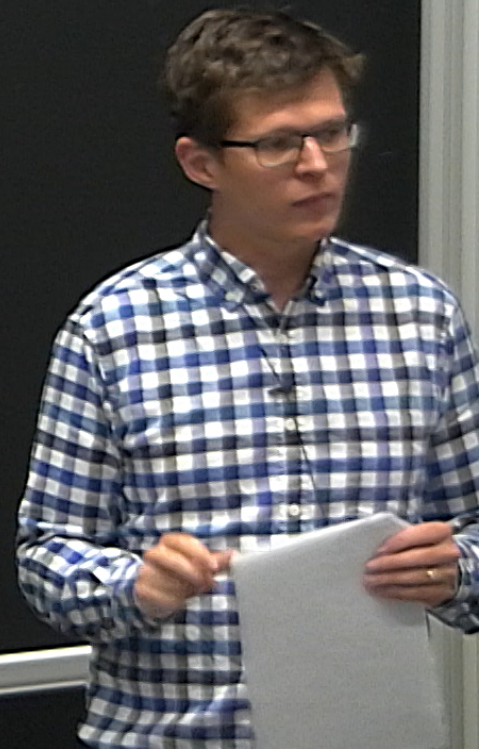
$$\int d^4x \mathcal{L}(\text{Instanton}) = \text{finite}$$



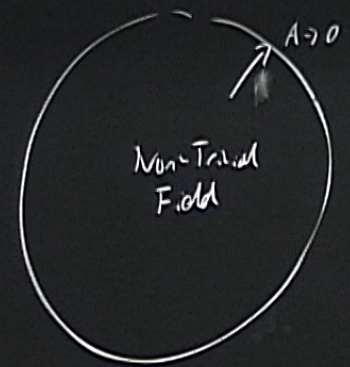
$$e^{-\frac{8\pi^2}{\alpha(u)}}$$

$$\alpha(u) \ll 1$$

$$\alpha(u) \sim 1$$



Five 70



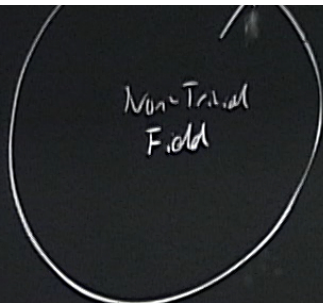
$$e^{-\frac{8\pi^2}{\alpha(\mu)}}$$

$$\alpha(\mu) \ll 1$$

$$\alpha(\mu) \sim 1$$

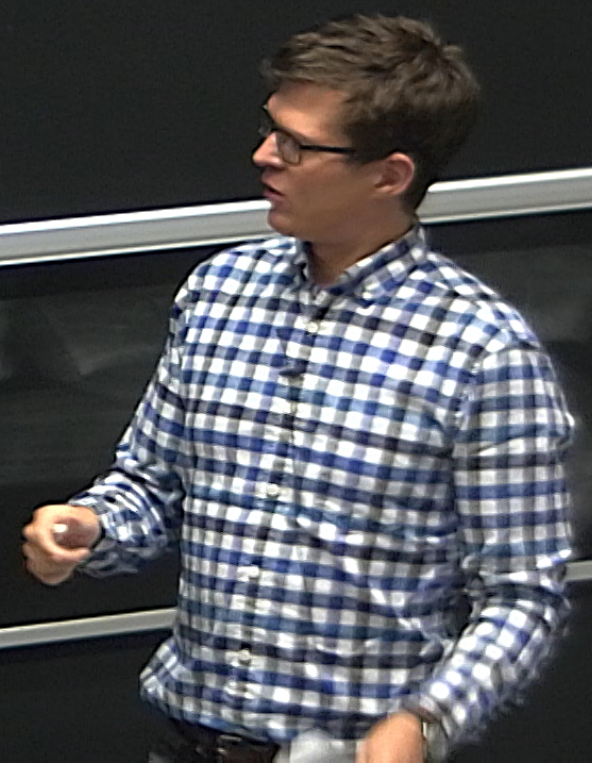
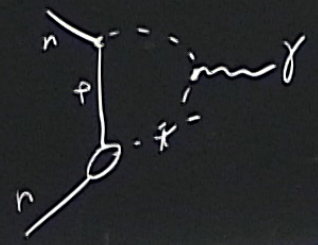
Physical Effects

Physical Effects



$$e^{-\alpha(u)}$$

$$\alpha(u) \sim 1$$

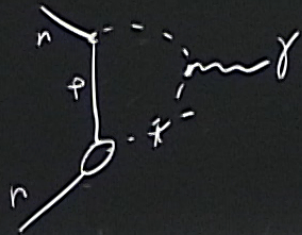


Non-Trivial Field

$$e^{-\alpha(u)}$$

$$\alpha(u) \sim 1$$

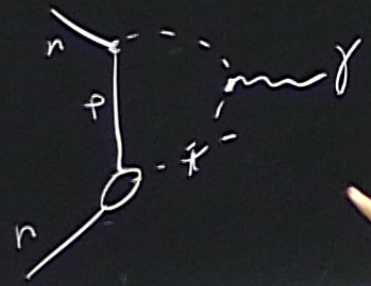
Physical Effects



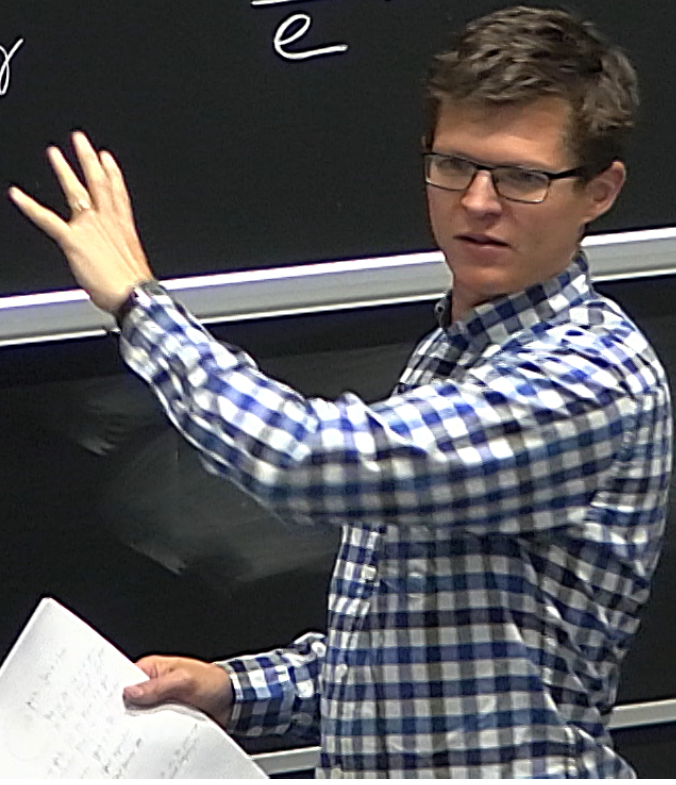
$$\frac{dn}{e} \propto -(\sigma + \sigma_a) \frac{m_a m_d}{(m_a + m_d)(2m_c - m_a - m_d)}$$

Non-Linear
Field

ical Effects



$$\frac{dn}{e} \propto - (\sigma + \sigma_a) \frac{m_n m_d}{(m_n + m_d)(2m_c - m_n - m_d)}$$

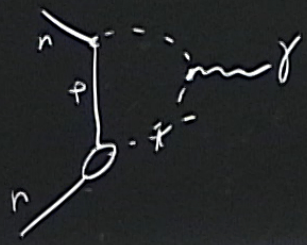


Non-Trivial Field

$$e^{-\alpha(x)}$$

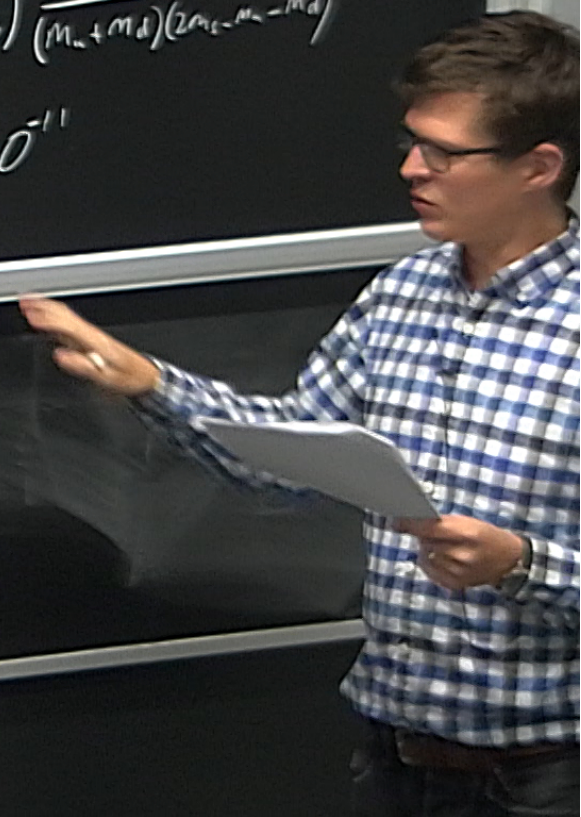
$$\alpha(x) \sim 1$$

Physical Effects



$$\frac{dn}{e} \propto -(\theta + \theta_0) \frac{m_a m_d}{(m_u + m_d)(2m_c - m_u - m_d)}$$

$$(\theta + \theta_0) \lesssim 10^{-11}$$



The Axion

Recipe:

$U(1)_{PQ}$:

$$\phi \rightarrow \frac{f + ip}{\sqrt{2}} e^{i\frac{\theta}{f}}$$

Heavy
↓

The Axion

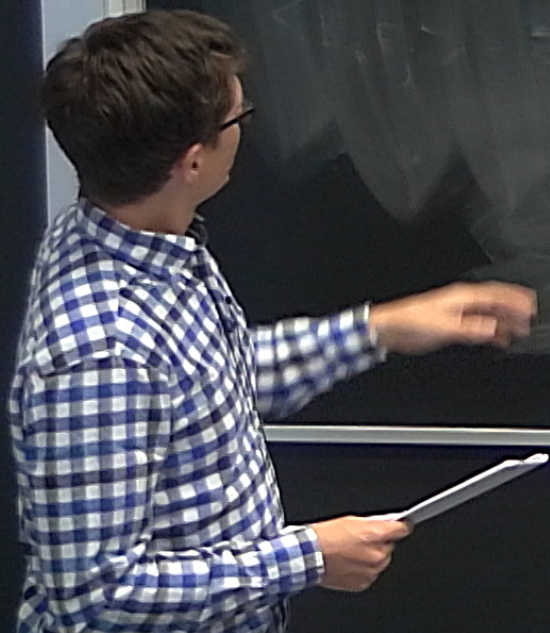
Recipe: $U(1)_{PQ}$: $\phi \rightarrow \frac{f+p}{\sqrt{2}} e^{i\frac{\theta}{f}}$ ^{heavy}

$$\mathcal{L} = \dots + \lambda \frac{\phi}{\Lambda} H \bar{\Psi} \Psi + h.c. + \dots + \frac{O g^2}{32 \pi^2} G \tilde{G}$$

$$= \dots + \tilde{\lambda} H \bar{\Psi} e^{i\frac{\theta}{f}(\theta_1 + \frac{q}{f})} \Psi + \dots$$

$$\mathcal{I}' = \dots + \tilde{\lambda} H \bar{\Psi} e^{i\frac{\theta}{f}(\theta_1 + \theta + \frac{q}{f})} \Psi + \dots$$

$$M\bar{\psi}\psi \rightarrow f_{\pi}^2 m_{\pi}^2$$



CAUTION
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$$M\bar{\psi}\psi \rightarrow f_a^2 m_a^2$$

$$\mathcal{L} = \dots + f_a^2 m_a^2 e^{i(\theta_a + \theta + \frac{a}{f_a})} + \text{h.c.}$$

$$= \dots + f_a^2 m_a^2 \cos\left(\theta_a + \theta + \frac{a}{f_a}\right) + \dots$$

$$V = -f_a^2 m_a^2 \cos\left(\theta_a + \theta + \frac{a}{f_a}\right)$$

$$M\bar{\psi}\psi \rightarrow f_a^2 m_a^2$$

$$\mathcal{L} = \dots + f_a^2 m_a^2 e^{i(\theta_a + \theta + \frac{a}{f})} + \text{h.c.}$$

$$= \dots + f_a^2 m_a^2 \cos\left(\theta_a + \theta + \frac{a}{f}\right) + \dots$$

$$V = -f_a^2 m_a^2 \cos\left(\theta_a + \theta + \frac{a}{f}\right)$$

$$\frac{\partial V}{\partial a} = 0$$

$$\frac{\langle a \rangle}{f} = -(\theta_a + \theta)$$

$$\langle MK\bar{\psi}\psi \rangle \rightarrow f_a^2 m_a^2$$

$$\mathcal{L} = \dots + f_a^2 m_a^2 e^{i(\theta_2 + \theta + \frac{a}{f})} + \text{h.c.}$$

$$= \dots + f_a^2 m_a^2 \cos(\theta_2 + \theta + \frac{a}{f})$$

$$V = -f_a^2 m_a^2 \cos(\theta_2 + \theta + \frac{a}{f})$$

$$\frac{\partial V}{\partial a} = 0$$

$$\frac{\langle a \rangle}{f} = -(\theta_2 + \theta)$$



$$\langle \bar{\psi} \psi \rangle \rightarrow f_a^2 m_a^2$$

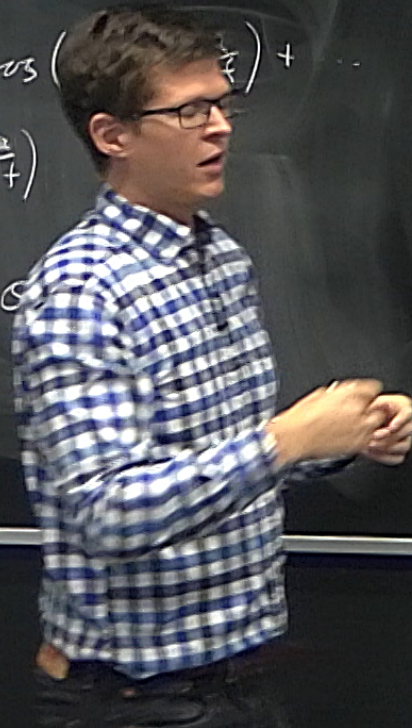
$$\mathcal{L} = \dots + f_a^2 m_a^2 e^{i(\theta_a + \frac{a}{f})} + \text{h.c.}$$

$$= \dots + f_a^2 m_a^2 \cos\left(\theta_a + \frac{a}{f}\right) + \dots$$

$$V = -f_a^2 m_a^2 \cos\left(\theta_a + \frac{a}{f}\right)$$

$$\frac{\partial V}{\partial a} = 0$$

$$\frac{\langle a \rangle}{f} = -(\theta_a + \theta)$$



The Axion

Recipe: $U(1)_{PQ}$: $\phi \rightarrow \frac{f+p}{\sqrt{2}} e^{i\frac{\theta}{f}}$ ^{heavy}

$$\mathcal{L} = \dots + \lambda \frac{\phi}{\Lambda} H \bar{\Psi} \Psi + h.c. + \dots + \frac{O_9}{32\pi^2} G\bar{G}$$

$$= \dots + \tilde{\lambda} H \bar{\Psi} e^{i\frac{(\theta_2 + \frac{q}{f})}{f}} \Psi + \dots$$

$$\mathcal{I}' = \dots + \tilde{\lambda} H \bar{\Psi} e^{i\frac{(\theta_2 + \theta + \frac{q}{f})}{f}} \Psi + \dots$$

$\tilde{\lambda} \nu = m_\nu$

$$M\langle\bar{\psi}\psi\rangle \rightarrow f_a^2 m_a^2$$

$$\mathcal{L} = \dots + f_a^2 m_a^2 e^{i(\theta_a + \theta + \frac{a}{f})} + \text{h.c.}$$

$$= \dots + f_a^2 m_a^2 \cos\left(\theta_a + \theta + \frac{a}{f}\right) + \dots$$

$$V = -f_a^2 m_a^2 \cos\left(\theta_a + \theta + \frac{a}{f}\right)$$

$$\frac{\partial V}{\partial a} = 0$$

$$\frac{\langle a \rangle}{f} = -(\theta_a + \theta)$$

$$\tilde{a} = a + (\theta_a + \theta)f$$



$$\langle \text{MK} \bar{\Psi} \Psi \rangle \rightarrow f_a^2 m_a^2$$

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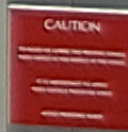
$$V = -f_a^2 m_a^2 \cos\left(\theta_a + \theta + \frac{a}{f}\right)$$

$$\frac{\partial V}{\partial a} = 0$$

$$\frac{\langle a \rangle}{f} = -(\theta_a + \theta)$$

$$\tilde{a} = a + (\theta_a + \theta) f$$

$$V = -f_a^2 m_a^2 \cos\left(\frac{\tilde{a}}{f}\right)$$



$$M\langle\bar{\psi}\psi\rangle \rightarrow f_{\pi}^2 m_{\pi}^2$$

$$\mathcal{L} = \dots + f_{\pi}^2 m_{\pi}^2 e^{i(\theta_2 + \theta + \frac{a}{f})} + \text{h.c.}$$

$$= \dots + f_{\pi}^2 m_{\pi}^2 \cos\left(\theta_2 + \theta + \frac{a}{f}\right) + \dots$$

$$V = -f_{\pi}^2 m_{\pi}^2 \cos\left(\theta_2 + \theta + \frac{a}{f}\right)$$

$$\frac{\partial V}{\partial a} = 0$$

$$\frac{\langle a \rangle}{f} = -(\theta_2 + \theta)$$

$$\tilde{a} = a + (\theta_2 + \theta)f$$

$$V = -f_{\pi}^2 m_{\pi}^2 \cos\left(\frac{\tilde{a}}{f}\right) \quad \left. \frac{\partial^2 V}{\partial \tilde{a}^2} \right|_{\tilde{a}=0} = m_{\pi}^2 = \frac{f_{\pi}^2 m_{\pi}^4}{f^2}$$

$$M\bar{\psi}\psi \rightarrow f_{\pi}^2 m_{\pi}^2$$

$$\mathcal{L} = \dots + f_{\pi}^2 m_{\pi}^2 e^{i(\theta_2 + \theta + \frac{a}{f})} + \text{h.c.}$$

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$$\frac{\partial V}{\partial a} = 0$$

$$\frac{\langle a \rangle}{f} = -(\theta_2 + \theta)$$

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$$M_a \approx 57 \times 10^{-6} \text{ eV} \times \left(\frac{10^{12} \text{ GeV}}{f} \right)$$

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The Ax

$$M_a \approx 57 \times 10^{-6} \text{ eV} \times \left(\frac{10^{12} \text{ GeV}}{f} \right)$$

Re...

KSVZ

DFFSV

$$\text{Mass} = m_a \approx 5.7 \times 10^{-6} \text{ eV} \times \left(\frac{10^{12} \text{ GeV}}{f} \right)$$

Relevant
 KSVZ DFSV

Couplings

$$\frac{c_j}{f} (\partial_\mu a) O_j^\mu$$

$$O_j^\mu \Rightarrow \bar{q} \gamma^\mu \gamma^5 q, \text{Tr} L, \dots$$

$$\begin{matrix} J_{EM}^\mu \\ \uparrow \\ \frac{a}{f} F \tilde{F} \end{matrix}$$



$$\text{Mass} = M_a \approx 57 \times 10^{-6} \text{ eV} \times \left(\frac{10^{12} \text{ GeV}}{f} \right)$$

Relevant

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DFSV

Couplings

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$$A \propto \left(\frac{E}{f} \right)^n$$

QCD Indentors

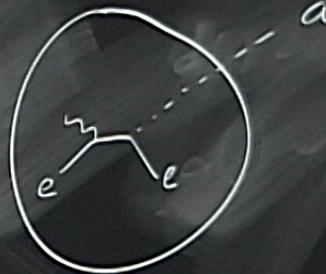
Axions and Stars

$$\gamma + Z_e \rightarrow a + Z_e$$

QCD Introduction

Axions and Stars

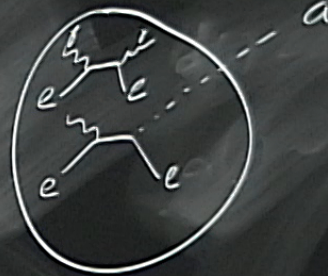
$$\gamma + Ze \rightarrow a + Ze$$



QCD Indirect

Axions and Stars

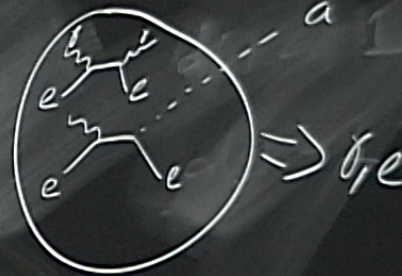
$$\gamma + Ze \rightarrow a + Ze$$



QCD Indenture

Axions and Stars

$$\gamma + Ze \rightarrow a + Ze$$



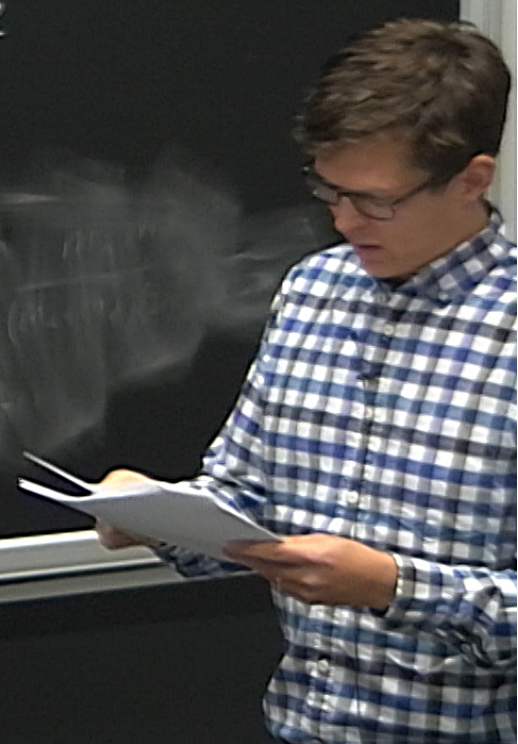
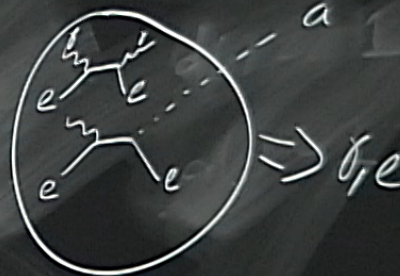
QCD Indentors

Axions and Stars

$$\gamma + Ze \rightarrow a + Ze$$

$$\Gamma_a \propto \left(\frac{E_a}{f}\right)^2 \times N$$

$$\Gamma_\gamma \propto (4\pi\alpha)^2 \times N^{2/3}$$



QCD Instantons

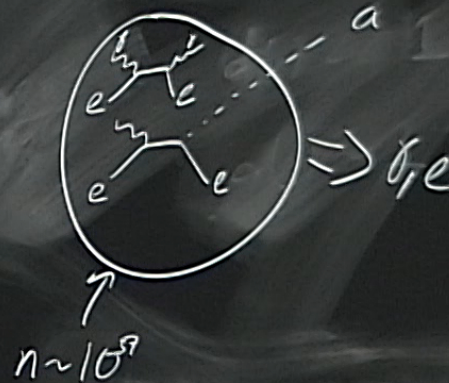
Axions and Stars

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When will $\Gamma_a \sim \Gamma_\gamma$?



QCD Instantons

Axions and Stars

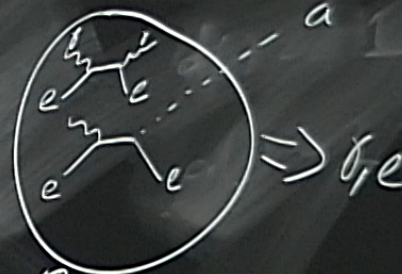
$$\gamma + Ze \rightarrow a + Ze$$

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When will $\Gamma_a \sim \Gamma_\gamma$?

$$f_a \sim \underline{\underline{10^5 \text{ GeV}}}$$



$$n \sim 10^9$$